GLUON-CONDENSATION OF QUARK–GLUON PLASMA IN MEAN FIELD APPROXIMATION

István LOVAS [1], Walter GREINER
Institut für Theoretische Physik, Johann Wolfgang Goethe Universitat, D-5000 Frankfurt am Main, Fed Rep Germany

Péter HRASKÓ, Erika LOVAS
Central Research Institute for Physics, H-1525 Budapest, Hungary

and

Kornél SAILER
Institute for Experimental Physics, Kossuth Lajos University, H-4001 Debrecen, Hungary

Received 10 January 1985

Starting from the field equations of QCD and applying the mean field approximation, a self-consistent set of equations is derived for the description of the gluon-condensed state of quark–gluon plasma A solution of these equations has been found, which corresponds to a static, chromo-magnetic field, periodic in space. This field induces self-consistently a periodic oscillation of the spin-color density of the quarks

If the local gauge invariance of QCD is broken then the gluon fields may have non-zero expectation values. In this paper we investigate the properties of such a gluon-condensed state [1–4] of the quark–gluon plasma. We restrict our considerations to the case of SU(2).

The field equations of QCD in conventional notation are given by

$$\partial_\mu F_a^{\mu\nu} + g f_{abc} A_{\mu b} F_c^{\mu\nu} = j_a^\nu , \tag{1}$$

$$(\mathrm{i}\partial\!\!\!/ + g A\!\!\!/_a T_a - m)\psi = 0 , \tag{2}$$

where the field strength $F_a^{\mu\nu}(x)$ and the vector current $j_a^\nu(x)$ are defined as follows:

$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g f_{abc} A_b^\mu A_c^\nu , \tag{3}$$

$$j_a^\nu = g \bar{\psi} \gamma^\nu T_a \psi . \tag{4}$$

The generators and the structure constants of SU(2) are denoted by T_a and f_{abc}, respectively.

We assume that the field operators may have non-vanishing expectation values:

$$\langle A_a^\mu(x)\rangle = \bar{A}_a^\mu(x) \neq 0 . \tag{5}$$

In the decomposition

$$A_a^\mu(x) = \bar{A}_a^\mu(x) + \alpha_a^\mu(x) , \tag{6}$$

$A_a^\mu(x)$ and $\alpha_a^\mu(x)$ are field operators, while $\bar{A}_a^\mu(x)$ is a c-number and the expectation value of $\alpha_a^\mu(x)$ vanishes by definition:

$$\langle \alpha_a^\mu(x)\rangle = 0 . \tag{7}$$

Substituting the decomposition (6) into the field equation (1) and taking the expectation value on both sides, the following simple equation is obtained for the mean field $\bar{A}_a^\mu(x)$:

$$\partial_\mu \bar{F}_a^{\mu\nu} + g f_{abc} \bar{A}_{\mu b} \bar{F}_c^{\mu\nu} + M^2(\nu) \bar{A}_a^\nu = \langle j_a^\nu \rangle . \tag{8}$$

[1] Permanent address: Central Research Institute for Physics, Budapest, Hungary.

Deriving this equation it is assumed that in addition to $\alpha_a^\mu(x)$ all of its derivatives has vanishing expectation value and all the expectation values of products of independently fluctuating quantities are also zero. In this way all terms containing $\alpha_a^\mu(x)$ or its derivatives drop out except for some third-order terms having the following form:

$$g^2 f_{abc} f_{cde} \langle \alpha_{\mu b} \bar{A}_d^\mu \alpha_e^\nu + \alpha_{\mu b} \alpha_d^\mu \bar{A}_e^\nu \rangle . \quad (9)$$

In this expression products of non-independently fluctuating quantities occur, which may have a non-vanishing expectation value:

$$M^2(\nu) = -g^2 \sum_{\substack{\mu \neq \nu \\ c}} \langle \alpha_{\mu c} \alpha^{\mu c} \rangle . \quad (10)$$

The parameter $M^2(\nu)$ formally plays the role of the mass squared, associated with the excitation of the mean gluon field. For the sake of consistency the gluon operators $A_a^\mu(x)$ in the field equation of the quarks are substituted also by their expectation values $\bar{A}_a^\mu(x)$. In the spirit of this mean field approximation the coupling constant g must be substituted also by a renormalised one.

In the following we shall investigate a class of possible solutions [5,6] corresponding to a gluon-condensed state of the quark–gluon plasma, defined by

$$\bar{A}_c^\mu(x) = (1/g) a^\mu \theta_c(kx) , \quad (11)$$

$$\theta_1 = \sin kx , \quad \theta_2 = \cos kx , \quad \theta_3 = 0 , \quad (12)$$

$$kx = k_0 x_0 - \boldsymbol{k} \cdot \boldsymbol{x} . \quad (13)$$

By virtue of this ansatz the Dirac equation can be solved exactly. First of all one can get rid of the x-dependence performing a rotation around the third axis in color space:

$$\psi' = \exp(-\mathrm{i} kx T_3) \psi . \quad (14)$$

The solutions of the transformed Dirac equation are plane waves:

$$\psi' = \exp(-\mathrm{i} px) \mu . \quad (15)$$

The energy eigenvalue p_0 and the eight-component spinor U can be obtained by solving the following equation:

$$(\not{p} + \not{a} T_2 - \not{k} T_3 - m) U = 0 . \quad (16)$$

Multiplying by $(\not{p} + \not{a} T_2 - \not{k} T_3 + m)$, we arrive to the equation

$$[p_\mu p^\mu + \tfrac{1}{4} a_\mu a^\mu + \tfrac{1}{4} k_\mu k^\mu - m^2 + 2 p_\mu a^\mu T_2 - 2 p_\mu k^\mu T^3 - \tfrac{1}{2} \sigma_{\mu\nu} (a^\mu k^\nu - a^\nu k^\mu) T_1] U = 0 . \quad (17)$$

If the $\sigma_{\mu\nu}$ matrices are represented by the Pauli matrices in the conventional manner:

$$\sigma_{ij} = \begin{pmatrix} \sigma_k & 0 \\ 0 & \sigma_k \end{pmatrix} , \quad \sigma_{0k} = \begin{pmatrix} 0 & \mathrm{i}\sigma_k \\ \mathrm{i}\sigma_k & 0 \end{pmatrix} , \quad (18)$$

eq. (17) can be written in the following form:

$$\begin{pmatrix} B & C \\ C & B \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \end{pmatrix} = 0 , \quad (19)$$

where

$$B = p_\mu p^\mu + \tfrac{1}{4} a_\mu a^\mu + \tfrac{1}{4} k_\mu k^\mu - m^2 + 2 p_\mu a^\mu T_2 - 2 p_\mu k^\mu T_3 - \tfrac{1}{2} \epsilon_{trs} \sigma_t (a_r k_s - a_s k_r) T_1 , \quad (20)$$

and

$$C = \mathrm{i} \sigma_r (a_r k_0 - a_0 k_r) T_1 . \quad (21)$$

Because of the symmetry of eq. (19) two solutions exist:

$$U_2 = \pm U_1 , \quad (22)$$

and U_1 must satisfy the equation given by

$$(B \pm C) U_1 = 0 . \quad (23)$$

The secular equation is obtained in the form of an algebraic equation of fourth order:

$$p_0^4 + b p_0^2 + c p_0 + d = 0 , \quad (24)$$

where

$$b = -2 [\boldsymbol{p}^2 + m^2 + \tfrac{1}{4} (\boldsymbol{a}^2 + a_0^2 + \boldsymbol{k}^2 + k_0^2)] , \quad (25)$$

$$c = 2 [a_0 (\boldsymbol{a} \cdot \boldsymbol{p}) + k_0 (\boldsymbol{k} \cdot \boldsymbol{p})] , \quad (26)$$

$$d = [\boldsymbol{p}^2 + m^2 + \tfrac{1}{4} (\boldsymbol{a}^2 - a_0^2 + \boldsymbol{k}^2 - k_0^2)]^2 + \tfrac{1}{4} [a_0^2 \boldsymbol{k}^2 + k_0^2 \boldsymbol{a}^2 - \boldsymbol{a}^2 \boldsymbol{k}^2 + (\boldsymbol{a} \cdot \boldsymbol{k})^2 - 2 a_0 k_0 (\boldsymbol{a} \cdot \boldsymbol{k})] - (\boldsymbol{a} \cdot \boldsymbol{p})^2 - (\boldsymbol{k} \cdot \boldsymbol{p})^2 . \quad (27)$$

The energy eigenvalues have a twofold degeneracy and they are symmetric against the interchange $a_\mu \rightleftarrows k_\mu$.

Substituting the ansatz (11) into the left-hand side of eq. (8) the following self-consistent set of equations can be derived:

$$\{(k_0a_0 - \boldsymbol{k}\cdot\boldsymbol{a})k^\nu - [k_0^2 - \boldsymbol{k}^2 - M^2(\nu)]a^\nu\} = g\langle j_2'^\nu\rangle, \tag{28}$$

$$-[(k_0a_0 - \boldsymbol{k}\cdot\boldsymbol{a})a^\nu - (a_0^2 - \boldsymbol{a}^2)k^\nu] = g\langle j_3'^\nu\rangle . \tag{29}$$

The solution of this self-consistent set of equations provides us with the amplitude a^ν and the wave vector k^ν.

We assume that the quark–gluon plasma under consideration is found in a volume V and it is in thermodynamical equilibrium characterized by the temperature T, and by the baryonic chemical potential μ. Thus the thermodynamical properties of the system can be described by the following density matrix.

$$\rho = Z^{-1}\exp[-(H-\mu N)/T] , \tag{30}$$

where the partition function Z is given by

$$Z = \mathrm{Tr}\exp[-(H-\mu N)/T] . \tag{31}$$

The hamiltonian and the number of baryons is denoted by H and N, respectively. We introduce also the thermodynamical potential defined by

$$\Omega = -T\log Z . \tag{32}$$

In the mean field approximation the quarks behave as independent quasiparticles. In this approximation we managed to solve the Dirac equation without any further restriction, so we are able to calculate explicitly the thermodynamical potential and we obtain the following result:

$$\omega = \Omega/V = \epsilon - T\sum_\lambda \int \frac{d^3p}{(2\pi)^3} \times (\log\{1+\exp[-(E_\lambda - \mu)/T]\} + \log\{1+\exp[-(\bar{E}_\lambda + \mu)/T]\}) , \tag{33}$$

where

$$\epsilon = \frac{1}{2g^2}\left(\sum_{i=1}^{3} a_i^2[M^2(i) + \boldsymbol{k}^2 - k_0^2] + a_0^2[\boldsymbol{k}^2 - M^2(0)] - (\boldsymbol{a}\cdot\boldsymbol{k})^2 - 2a_0k_0(\boldsymbol{a}\cdot\boldsymbol{k})\right) . \tag{34}$$

The energy eigenvalues of the quarks and antiquarks are denoted by E_λ and $\bar{E}_\lambda$, respectively. The two independent spin and two independent color states are labelled by the index λ. The thermodynamical potential density is the function of the physical parameters of the system

$$\omega = \omega(T, \mu, a^\nu, k^\nu) . \tag{35}$$

The thermodynamical equilibrium is characterised by the minimum of the thermodynamical potential. This means that at prescribed values of μ and T the conditions

$$\partial\omega/\partial a^\nu = 0 , \quad \partial\omega/\partial k^\nu = 0 , \tag{36}$$

must be satisfied.

It is not difficult to prove that these equations, which are the necessary conditions of the thermodynamical equilibrium, are equivalent with the self-consistent set of equations (28), (29), derived previously. In addition to the self-consistent equations an auxiliary condition must be imposed in order to fix the color of the system. The difference of the color densities of the quarks is given by

$$\langle\bar\psi\gamma^0(\tfrac{1}{2}+T_3)\psi\rangle - \langle\bar\psi\gamma^0(\tfrac{1}{2}-T_3)\psi\rangle = 2\langle\bar\psi\gamma^0T_3\psi\rangle . \tag{37}$$

For a colorless system (with constant color densities) this quantity must vanish, consequently the left-hand side of the self-consistent equation (29) must vanish for $\nu = 0$. Assuming transversal waves ($\boldsymbol{k}\cdot\boldsymbol{a} = 0$) this condition is fulfilled if $k_0 = 0$.

Searching for transversal color waves, characterised by the parameters

$$k_3 = 1.0 , \quad k_1 = k_2 = 0, \quad k_0 = 0 ,$$

$$a_1 \leqslant 1.0, \quad a_2 = a_3 = 0 , \quad a_0 = 0 ,$$

a non-trivial solution has been found. This type of solution corresponds to a static, chromo-magnetic field, periodic in space and orthogonal both to $\boldsymbol{a}$ and $\boldsymbol{k}$ [7,8]

The length scale is defined by fixing the quark mass: $m = 1$. The temperature and the chemical potential were varied in the range $0.05 \leqslant T \leqslant 10.0$ and $0.1 \leqslant \mu \leqslant 1.2$, respectively. In these regions, at the given values of T and μ, the thermodynamical potential ω and the energy per baryon charge – that is the ratio of the energy density and the baryon density – is appreciably lower in the gluon-condensed state ($a_1 \neq 0$), than in the case of a non-interacting Fermi gas ($a_1 = 0$).

This difference is becoming less and less important as the temperature or/and the baryon density becomes higher and higher.

Since no a priori knowledge is available for the value of the effective coupling constant and the "effective gluon mass" $M^2(\nu)$ the following procedure was adopted. At fixed values of the parameters a^ν, k^ν, T and μ, the self-consistent equations were solved for $M^2(\nu)$ and g.

The value of the "effective gluon mass" $M^2(\nu)$ and the effective coupling constant g increase with decreasing temperature and/or baryon density. It must be pointed out that the value of the coupling constant was found to be very large ($g \geqslant 8.0$) in the investigated range of temperature and chemical potential.

Valuable discussions with V.N. Gribov, M. Huszár, B. Müller and J. Zimányi are gratefully acknowledged. This work was supported by a contract between the Deutsche Forschungsgemeinschaft and the Hungarian Academy of Sciences.

References

[1] G.K. Savidy, Phys. Lett. 71B (1977) 173.
[2] N.K. Nielsen and P. Olesen, Nucl. Phys. B144 (1978) 376.
[3] H.B. Nielsen and M. Ninomiye, Nucl. Phys. B156 (1979) 1.
[4] J. Balog and A. Patkós, Phys. Lett. 98B (1981) 205.
[5] B. Banerjee, N.K. Glendenning and M. Gyulassy, Nucl. Phys. A461 (1981) 326.
[6] I. Lovas, J. Németh and K. Sailer, Phys. Lett. 135B (1984) 258.
[7] A.W. Overhauser, Phys. Rev. 128 (1962) 1438.
[8] M. Noga, Nucl. Phys. B220 (1983) 185.