

# Application of Rexroth Controlling for Inverted Pendulum

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*Abstract. This paper deals with the control of an inverted pendulum. Balancing techniques are used in great many controlling problems. The inverted pendulum problem is often used as a benchmark. The theoretical background is well-known and easy to treat. A commercially available Rexroth axis controller and a CKK compact module are used to control the input of the system, which is usually applied in industrial fields. A test bench has been designed and built. A PLC based program has been developed to swing up the pendulum from the rest location to inverted position and LQR controller is designed to balance the system.*

## Introduction

This research work deals with balancing of an inverted pendulum [1]. It is a well-known problem in control fields and dynamics. A great many applications are published, which use different balancing processes. One of the recent application is a precision vertical landing of rockets in space applications [2], which ensures the rockets reusable. Furthermore there are several robotic adaptations, e.g., humanoid robots utilizing spherical inverted pendulum model [3]. There are entertainment applications of the balancing problems, e.g., Segway [4], Hoverboard.

Main goals of the research work is to create a system, which can automatically swing up the pendulum then control it in order to keep in inverted position. A Rexroth IndraDrive HCS02 axis controller is used to control the inverted pendulum. The system is mounted on a Rexroth Compact Module CKK 15-110.

The rest of the paper is organized as follows: the theoretical formulation of the inverted pendulum is described in Section 1. The Lagrange's equation of the second kind is written in Section 1.1. Section 1.2. contains the linear quadratic regulator technique (LQR), the state space representation of the pendulum's second order differential equation and the determination of the gains. The readymade system and its PLC program are described in Section 2. The concluding remarks and the future plans are written in Section Conclusions.

## 1. Theoretical background of the inverted pendulum

Lagrange's equation of second kind can be used to derive the differential equation of the system. The mechanical model of the pendulum is shown in Figure 1, where  $L$  is the length of the rod,  $m$  denotes the mass,  $g$  is the gravitational acceleration,  $\varphi$  is the angle of the rod measured from the vertical position and  $u(t)$  is the prescribed position of the carriage.

The motion of the physical pendulum can be modelled with the mathematical counterpart. The coordinates of the mass can be written as

$$x = u + L \sin(\varphi), \quad y = L \cos(\varphi). \quad (1)$$

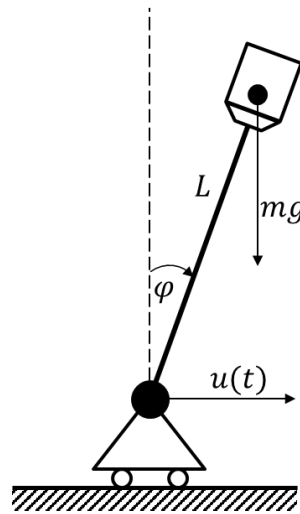


Figure 1. Mechanical model of the system

### 1.1. Lagrange's equation of the second kind

In aware of the kinetic  $E$  and potential energy  $V$  of the system, the Lagrange function is written as

$$\mathcal{L} = E - V = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - mgy. \quad (2)$$

Substituting (1) into (2) and performing straightforward manipulations  $\mathcal{L}$  is

$$\mathcal{L} = \frac{1}{2}m\dot{u}^2 + m\dot{u}L\dot{\varphi} \cos(\varphi) + \frac{1}{2}mL^2\dot{\varphi}^2 - mgL \cos(\varphi). \quad (3)$$

The generalized coordinate is the angle  $\varphi$ , therefore the second order differential equation is given as

$$\frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial \dot{\varphi}} \right) - \frac{\partial \mathcal{L}}{\partial \varphi} = 0. \quad (4)$$

After substituting the Lagrange function (3) and performing the partial and total time derivations, the second order differential equation is obtained as

$$\ddot{\varphi} + \frac{1}{L}\ddot{u} \cos(\varphi) - \frac{1}{L}g \sin(\varphi) = 0. \quad (5)$$

It is assumed that the angle of the rod is small during the balancing. Linearizing (5) the differential equation of the mathematical pendulum is given as:

$$\ddot{\varphi}(t) - \frac{g}{L}\varphi(t) = -\frac{1}{L}\ddot{u}(t). \quad (6)$$

## 1.2. LQR controlling technique

The differential equation (6) can be transformed also in state space representation:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}\ddot{u}, \quad (7)$$

where  $\mathbf{A}$  and  $\mathbf{b}$  are the state matrices,  $\mathbf{x}$  is the state space vector. The matrix form of the equation is the following

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ g/L & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -1/L \end{bmatrix} \ddot{u}, \quad (8)$$

where  $x_1 = \varphi$  and  $x_2 = \dot{\varphi}$ .

Based on the state space theory an optimal controller can be designed with the use of LQR technique [1]. A quadratic cost functional can be defined

$$J(\mathbf{x}, \ddot{u}) = \frac{1}{2} \int [\mathbf{x}^T \mathbf{Q} \mathbf{x} + r \ddot{u}^2] dt, \quad (9)$$

where  $\mathbf{Q} = \mathbf{Q}^T$ ,  $\mathbf{Q} \geq \mathbf{0}$  and  $r > 0$ , these parameters are the design or the penalty parameters. The first part of the functional  $\mathbf{x}^T \mathbf{Q} \mathbf{x}$  penalize the deviation from the demanded state. In our case the  $\mathbf{Q}$  is an identity matrix. The parameter  $r$  is associated with the limits of the servo motor, which drives the compact module. If the motor has limited velocity parameter then the value of  $r$  is relatively large ( $r \gg 1$ ).

The main goal of LQR technique is to determine gains of the state space, which provides the  $\ddot{u}(t)$ . The inverted pendulum is controlled by state feedback:

$$\ddot{u} = -\mathbf{k}^T \mathbf{x}, \quad (10)$$

where  $\mathbf{k}^T$  is the column vector of gains. The optimal gain vector can be written

$$\mathbf{k}^T = r^{-1} \mathbf{b}^T \mathbf{P}, \quad (11)$$

where  $\mathbf{P}(t)$  is symmetric and positive definite matrix, which is determined by the control algebraic Riccati equation (CARE):

$$\mathbf{A}^T \mathbf{P} + \mathbf{P} \mathbf{A} - \mathbf{P} \mathbf{b} r^{-1} \mathbf{b}^T \mathbf{P} + \mathbf{Q} = \mathbf{0}. \quad (12)$$

In PLC programming process the problem is handled with *MC\_MoveVelocity* function block. Its inputs are the acceleration and deceleration and also the velocity of the carriage:

$$\dot{u} = -\mathbf{k}^T \int \mathbf{x} dt. \quad (13)$$

## 2. Controlling the system

Inventor 2016 software is used to create the 3D model of the pendulum system. It consists of an aluminium hollow section 60x40, shaft  $\varnothing$  8 mm supported with two ball bearings and a rod diameter  $\varnothing$  8 mm, the length is  $L = 630 \text{ mm}$ , the mass is  $m = 0.35 \text{ kg}$ . Thereafter it was manufactured and mounted on the carriage of the Rexroth compact module (see in Fig. 2).



Figure 2. Inverted pendulum mounted on a Rexroth compact module

The angle of the pendulum is measured by PMI360D\_F130-IE8\_V15 inductive sensor [5]. The analogue input of the PLC is voltage based. The output of the sensor is 4-20 mA, which covers the angle of 0-360°. Therefore two 1 k $\Omega$  resistors in parallel connection were wired to produce 2-10 V voltage input for the PLC (see Fig. 3).

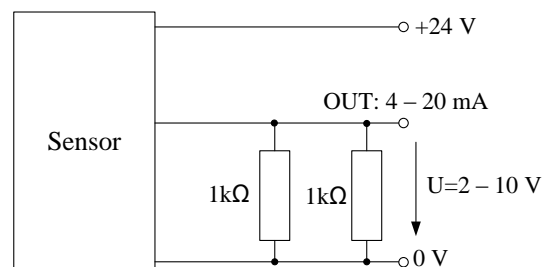


Figure 3. Connection between the PLC and the sensor

The carriage of the CKK module is driven by a MSK030C Bosch Rexroth servo motor and controlled by a Rexroth IndraDrive HCS02 axis controller. The maximum rotational speed of the motor is 9000 rpm [6], which can produce up to 1.5 m/s carriage speed.

The Rexroth system can be programmed under the standard PLC programming languages and also with a Continuous Function Chart (CFC). The flowchart of the inverted pendulum is shown in Figure 4.

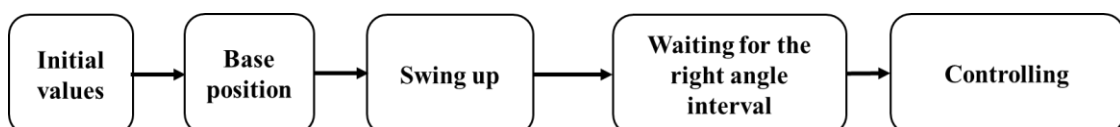


Figure 4. The flowchart of the pendulum controller

The main program, which contains the steps has been written in SFC (Sequential Function Chart). The initial values are given in ST (Structured Text) programming language. The rest of the blocks are written in CFC using the standard library functions, special purpose function blocks and user defined blocks.

The initial values are given in the block of *Initial values* (see Figure 4), then the carriage is moved to its base position (*Base position*) using *MC\_MoveAbsolute* function block. Thereafter the pendulum is swung up to the inverted position. In the next step the system is waiting for the rod to reach its inverted position, from which the balancing is started in the *Controlling* block. The PLC program of the balancing process is shown in Figure 5.

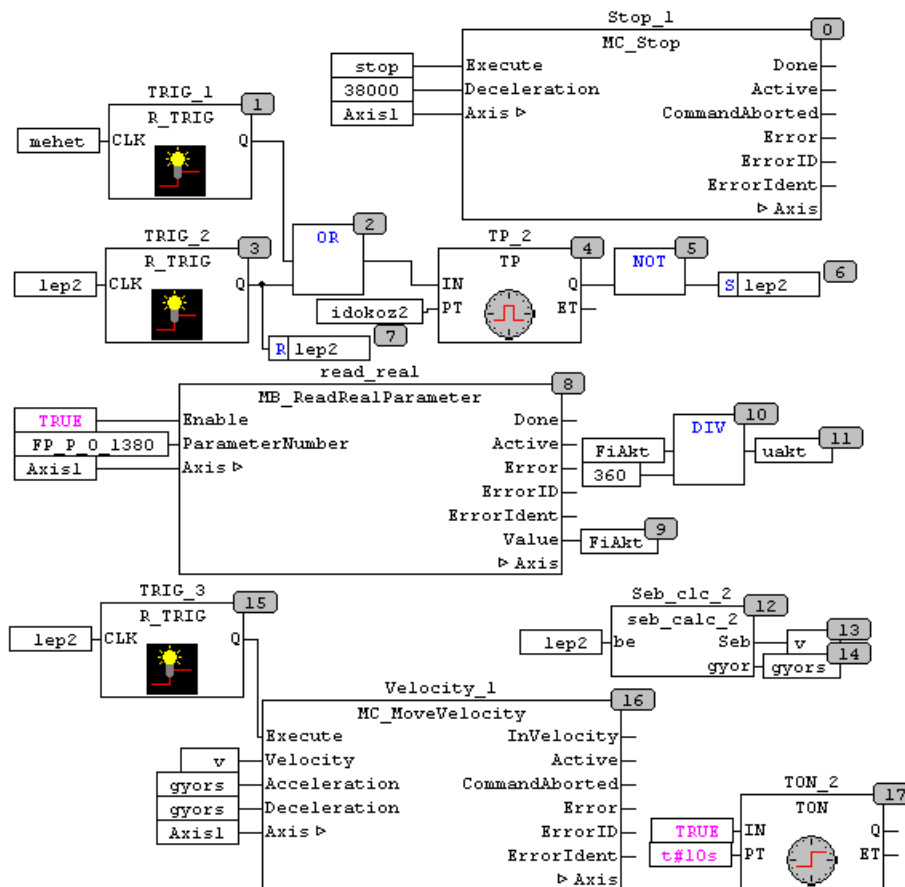


Figure 5. The CFC program of the controlling step

A time pulse (*TP*) function block was used to produce 50 ms time increments to calculate the new value of the velocity and the acceleration. The velocity and acceleration calculation block (*seb\_calc\_2*) is a user defined function block. These blocks were written in ST programming language. The integral of the angle of rotation in (13) is calculated numerically using trapezoid rule:

$$\int_0^{t_{i+1}} \varphi dt \cong \int_0^{t_i} \varphi dt + \Delta t \frac{\varphi_i + \varphi_{i+1}}{2}, \quad (14)$$

where  $\varphi_{i+1}$  is the actual value of the angle of rotation,  $\varphi_i$  is its previous value and  $\Delta t$  is the time increment.

Due to rising edge detection (*R\_TRIG*) one clock pulse can be generated, which is depending on the logic level of the triggering variable. A special function block (*MB\_ReadRealParameter*) is used to obtain the angle of the rod. The motion of the carriage is controlled by *MC\_MoveVelocity* special function block, which can produce relative movements during a  $\Delta t$  time increment.

A safety block (*MC\_Stop*) and also a timer on (TON) function block has been inserted. The *TON* block is used to stop the system after 10 s its working period.

## Conclusions

A well-known balancing problem has been investigated. A Rexroth type compact module and a HCS02 axis controller were used to control an inverted pendulum. The pendulum system was designed in Inventor 2016 software and was manufactured in order to begin the programming of the system. The main program was written in SFC language. The rest of the controlling steps were written in ST and CFC languages.

The test bench with our specific Rexroth type controller and compact module is working properly. Since the system can perform only 1.5 m/s carriage velocity therefore its application for such balancing is limited to a certain angle and velocity disturbances. The disturbance range of the angle is within  $\pm 10^\circ$ . The speed limit is below 0.5 m/s.

## Acknowledgement

The described article was carried out as part of the EFOP-3.6.1-16-2016-00011 “Younger and Renewing University – Innovative Knowledge City – institutional development of the University of Miskolc aiming at intelligent specialisation” project implemented in the framework of the Szechenyi 2020 program. The realization of this project is supported by the European Union, co-financed by the European Social Fund.

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