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Old and new observables for α_s from e^+e^- to hadrons

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We present a computation of energy-energy correlation in e^+e^- annihilation at next-to-next-to-leading order accuracy in perturbative QCD matched with the next-to-next-to-leading logarithmic resummed calculation for the back-to-back limit. Using these predictions and state-of-the-art Monte Carlo tools to model hadronization corrections, we perform an extraction of the strong coupling from available data sets. We also show next-to-next-to-leading order results for soft-drop thrust, an observable specifically constructed to have reduced hadronization corrections. We study the impact of the soft drop on the convergence of the perturbative prediction and find that generally grooming improves perturbative stability. This improved stability, together with the reduced sensitivity to non-perturbative corrections makes soft-drop thrust a promising observable for precision measurements of the strong coupling at lepton colliders.

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Introduction

Accurate measurements of event shape distributions in e^+e^- annihilation continue to be one of the most precise tools for extracting the strong coupling α_s value from data [1, 2]. Such determinations are typically based on the comparison of differential distributions with perturbative predictions supplemented with hadronization corrections derived either from analytic models or Monte Carlo tools. As new data for e^+e^- annihilation are not foreseen in the near future, progress in such measurements relies solely on improved theoretical understanding of the $e^+e^- \to$ hadrons process.

When discussing the accuracy of theoretical predictions for event shape distributions, two very different sources of uncertainty present themselves. The first of these is simply the uncertainty coming from terms that are not evaluated exactly in perturbation theory. These can be higher-order terms in the coupling that are simply neglected in a fixed-order calculation, or subleading logarithmic terms that are not controlled in an all-order resummation. A second source of uncertainty is that associated with the description of the parton to hadron transition.

These two types of uncertainties have a rather different nature and so their reduction must be addressed in different ways. Clearly, uncertainties associated to the perturbative description of an observable may be reduced, at least in principle, by increasing the perturbative and/or logarithmic order at which the predictions are computed. These days, state-of-the-art computations include exact fixed-order corrections at next-to-next-to-leading order (NNLO) accuracy for three-jet event shapes [3, 4, 5], as well as next-to-next-to-leading logarithmic (NNLL) (see e.g., Ref. [6] and references therein) and even next-to-next-to-next-to-leading logarithmic (N^{3}LL) resummation [7, 8, 9] in the two-jet limit. However, it is less obvious how non-perturbative uncertainties could be similarly reduced. In this respect, one idea is to investigate observables that are less sensitive to hadronization corrections. In particular, borrowing ideas from jet grooming, new event shape observables can be defined for which hadronization corrections are much reduced as compared to traditional ones [10].

In this contribution we first present an extraction of the strong coupling α_s from the energy-energy correlation of particles in e^+e^- collisions, highlighting the role that higher-order perturbative corrections play in reducing the uncertainty of the measurement. Then, we investigate soft-drop thrust, an observable constructed to mitigate the impact of non-perturbative corrections. In particular, we point out that in addition to decreased hadronization corrections, this new observable also exhibits an increased perturbative stability, making it an appealing candidate for a precise determination of the strong coupling.

An old observable: energy-energy correlation

Energy-energy correlation (EEC) was one of the first infrared- and collinear-safe event shapes to be considered in the literature [11]. It is defined as the normalized energy-weighted distribution with respect to angles χ between the three-momenta of particles in an event,

$$\frac{1}{\sigma_t}\frac{d\Sigma(\chi)}{d\cos\chi} = \frac{1}{\sigma_t}\int\sum_{i,j}\frac{E_iE_j}{Q^2}d\sigma_{e^+e^-\to ij+X}\,\delta(\cos\chi - \cos\theta_{ij})\,, \tag{1}$$

where E_i and E_j are particle energies, Q is the total center-of-mass energy, θ_{ij} is the angle between the three-momenta of particles i and j[1] and σ_t is the total hadronic cross section.

The fixed-order prediction for EEC in perturbative QCD has been known numerically at NLO accuracy for some time (see e.g., Ref. [14] and references therein), while the NNLO correction has been computed more recently [15] using the CoLoRFulNNLO method [5, 16, 17]. At the renormalization scale μ the fixed-order result can be written as

$$\left[\frac{1}{\sigma_t}\frac{d\Sigma(\chi,\mu)}{d\cos\chi}\right]_{(\text{f.o.})} = \frac{\alpha_s(\mu)}{2\pi}\frac{d\bar{A}(\chi,\mu)}{d\cos\chi} + \left(\frac{\alpha_s(\mu)}{2\pi}\right)^2\frac{d\bar{B}(\chi,\mu)}{d\cos\chi} + \left(\frac{\alpha_s(\mu)}{2\pi}\right)^3\frac{d\bar{C}(\chi,\mu)}{d\cos\chi} + \mathscr{O}(\alpha_s^4)\,, \tag{2}$$

where the perturbative coefficients at LO, NLO and NNLO, $\bar{A}$, $\bar{B}$ and $\bar{C}$, have been normalized to the total hadronic cross section. On the left panel of Fig. 1 we show the physical predictions for EEC in fixed-order perturbation theory up to NNLO accuracy together with data measured by the OPAL collaboration [18]. The bands in the plot represent the effect of varying the renormalization scale by a factor of two around its central value of $\mu = Q$.

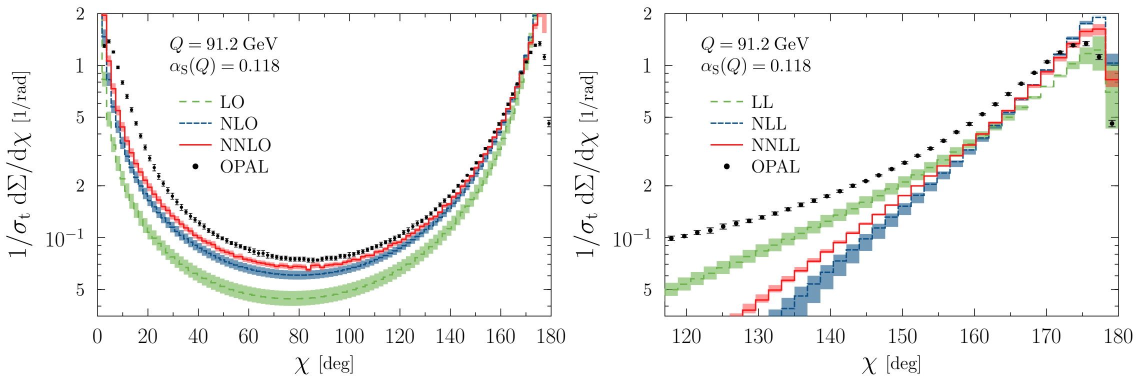

Figure 1: Fixed-order (left) and resummed (right) predictions for EEC.

Clearly the inclusion of higher-order corrections improves the agreement of the prediction and the data, although there are pronounced differences around the forward ($\chi \to 0°$) and back-to-back ($\chi \to 180°$) regions, where in fact the fixed-order predictions diverge. This is due to the presence of large logarithmic corrections of infrared origin in these phase space regions that must be resummed to all orders to obtain a physically valid description of EEC around the endpoints of the distribution. The resummation of large logarithms of

$$y = \cos^2\frac{\chi}{2} \tag{3}$$

in the back-to-back region around $\chi = 180°$ has been known for some time at NNLL accuracy [12]. The resummed prediction can be written as

$$\left[\frac{1}{\sigma_t}\frac{d\Sigma(\chi,\mu)}{d\cos\chi}\right]_{(\text{res.})} = \frac{Q^2}{8}H(\alpha_s(\mu))\int_0^\infty db\, J_0(bQ\sqrt{y})S(Q,b)\,, \tag{4}$$

[1] Refs. [12, 13] use the opposite $\chi = 180° - \theta_{ij}$ convention such that the back-to-back region corresponds to $\chi \to 0$.

where the logarithmically enhanced terms are collected in the Sudakov form factor

$$S(Q,b) = \exp\left\{-\int_{b_0^2/b^2}^{Q^2} \frac{\mathrm{d}q^2}{q^2}\left[A(\alpha_s(q^2))\ln\frac{Q^2}{q^2} + B(\alpha_s(q^2))\right]\right\}. \tag{5}$$

The functions A, B and H are free of logarithmic corrections and can be computed in perturbation theory. Their explicit expressions can be found in Refs. [12, 13]. On the right panel of Fig. 1 we present purely resummed predictions in the back-to-back limit for EEC up to NNLL accuracy together with OPAL data. The resummed calculation is finite and captures the trends of the data correctly for angles χ close to 180°, but does not do a good job of describing the measurement away from the back-to-back region.

From Fig. 1 it is evident that data is best described by fixed-order or resummed results over different angular ranges. In particular, fixed-order predictions are reliable for moderate to large y ($\alpha_s \ln^2 y \ll 1$), while the resummed calculation applies to small y ($y \ll 1$). Predictions that are valid over a wide kinematical range can be obtained by combining the fixed-order and resummed results. The matched predictions are obtained in the log-R scheme. The details of this procedure are presented in Ref. [13]. Note that the description of the EEC distribution over the full angular range would require resummation also in the forward limit.

In order to extract the strong coupling α_s from measurements of EEC, the theoretical prediction described above must be combined with hadronization corrections. We modeled these non-perturbative effects using the state-of-the-art particle-level Monte Carlo generators SHERPA [19] and Herwig 7 [20]. The exact Monte Carlo generation setups employed are discussed in Ref. [21] as well as by the contribution of A. Verbytskyi in these proceedings. Hadronization corrections were derived on a bin-by-bin basis as ratios of the EEC distribution at hadron and parton level in the simulated samples.

The perturbative results, corrected for hadronization effects as described above, were confronted with available data sets from the SLD, OPAL, L3, DELPHI, TOPAZ, TASSO, PLUTO, JADE, CELLO, MARKII, and MAC experiments. The details of data selection are described in Ref. [21]. The optimal value of α_s was determined by a chi-squared minimization procedure employing the MINUIT 2 program [22], see Ref. [21] and A. Verbytskyi's contribution in these proceedings for details. In Fig. 2 we show representative results of fits to data obtained with theoretical predictions at NNLO+NNLL as well as NLO+NNLL accuracy. Our best fit value for α_s at NNLO+NNLL accuracy is

$$\alpha_s(m_Z) = 0.11750 \pm 0.00287\,(\mathrm{comb.})\,, \tag{6}$$

in agreement with the world average as of 2017 [23]. The quoted combined error takes into account uncertainties associated with the variation of the renormalization and resummation scales in the perturbative calculation, the choice of hadronization model employed (Lund string fragmentation or cluster model), as well as fit uncertainty (obtained with the $\chi^2 + 1$ criterion as implemented in MINUIT 2). A detailed description of the estimation of the various uncertainties is given in Ref. [21] (see also A. Verbytskyi's contribution in these proceedings).

In order to highlight the impact of NNLO corrections on the determination, the fit was repeated with theoretical predictions computed at NLO+NNLL accuracy. The corresponding best fit value

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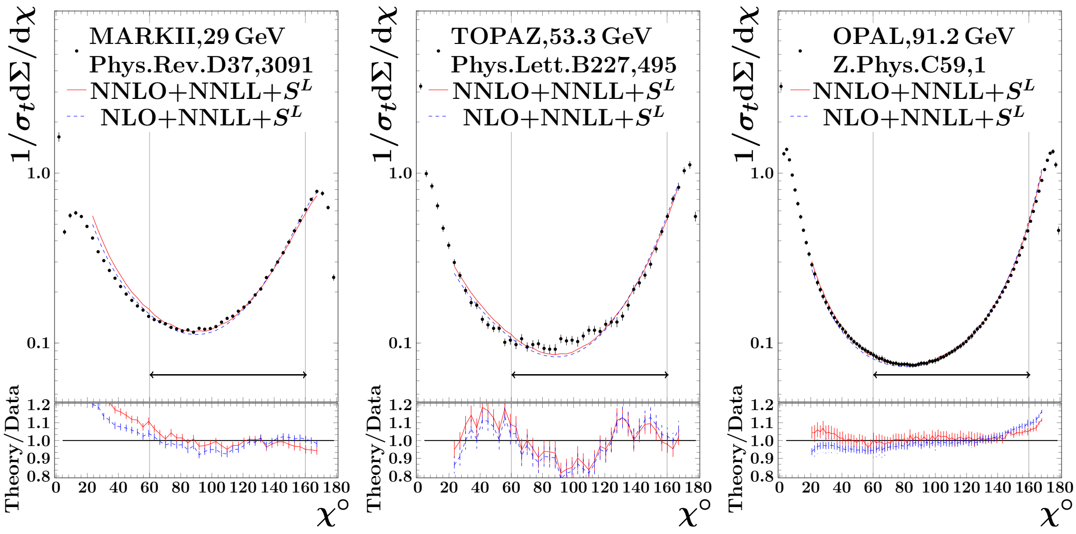

Figure 2: Selected results of fits to data at NNLO+NNLL and NLO+NNLL accuracy.

for α_s is $\alpha_s(m_Z) = 0.12200 \pm 0.00535\,(\text{comb.})$. We see that the inclusion of the NNLO correction has a moderate but non-negligible effect on the extracted value of α_s, while the uncertainty of the determination is reduced substantially, by a factor of two.

New observables: soft-drop event shapes

We now turn to the issue of how the uncertainty associated with the estimation of hadronization corrections might be mitigated in measurements of α_s. As mentioned in the introduction, one possible approach is to construct observables with reduced sensitivity to non-perturbative effects. The idea is simple: if the overall size of the hadronization correction is small, then even a sizable relative uncertainty on this contribution will correspond to a small overall uncertainty on α_s. Thus the limited precision of the hadronization correction becomes less of an issue.

Soft-drop event shapes constitute a generic class of observables that are constructed to have reduced hadronization uncertainties. Indeed, soft drop is a kind of grooming procedure, designed to remove soft and wide-angle radiation from jets that are defined in an event. For Cambridge–Aachen jets of radius R, soft-drop grooming is defined as follows [24]:

1. Undo the last step of clustering for jet J and split it into two subjets.

2. Check if the subjets pass the soft-drop condition, which for e^+e^- collisions reads

$$\frac{\min\{E_i,E_j\}}{E_i+E_j} > z_{\text{cut}} \left(\frac{1-\cos\theta_{ij}}{1-\cos R}\right)^{\beta/2} \quad \text{or} \quad \frac{\min\{E_i,E_j\}}{E_i+E_j} > z_{\text{cut}}\,(1-\cos\theta_{ij})^{\beta/2} \tag{7}$$

   for jets of radius R or hemisphere jets, respectively.

3. If the splitting fails this condition, the softer subjet is discarded and the groomer continues to the next step in the clustering.

4. If the splitting passes, the procedure ends and J is the soft-drop jet.

The grooming parameter $z_{\rm cut}$ sets an energy threshold for discarding soft radiation ($z_{\rm cut} \to 0$ corresponds to no grooming), while β controls how strongly wide-angle emissions are rejected ($\beta \to \infty$ corresponds to no grooming).

With the soft-drop procedure, one can define event shapes by first performing a special kind of grooming of the event, and then computing the value of the event shape from the groomed event. As an example, consider soft-drop thrust (more specifically $T'_{\rm SD}$), which was defined for e^+e^- collisions in Ref. [10]:

(a) Compute the thrust axis, $\vec{n}_T$, and divide the event into two hemispheres.

(b) Apply soft-drop grooming to each hemisphere.

(c') The set of particles left in the two hemispheres after the soft-drop constitute the soft-drop hemispheres $\mathscr{H}^L_{\rm SD}$ and $\mathscr{H}^R_{\rm SD}$, on which the soft-drop thrust $T'_{\rm SD}$ is defined as

$$T'_{\rm SD} = \frac{\sum_{i\in\mathscr{H}^L_{\rm SD}} |\vec{n}_L \cdot \vec{p}_i|}{\sum_{i\in\mathscr{E}_{\rm SD}} |\vec{p}_i|} + \frac{\sum_{i\in\mathscr{H}^R_{\rm SD}} |\vec{n}_R \cdot \vec{p}_i|}{\sum_{i\in\mathscr{E}_{\rm SD}} |\vec{p}_i|}\,, \tag{8}$$

where $\vec{n}_L$ and $\vec{n}_R$ are the jet axes of the original left and right hemispheres and $\mathscr{E}_{\rm SD}$ is the soft-drop event, $\mathscr{E}_{\rm SD} = \mathscr{H}^L_{\rm SD} \cup \mathscr{H}^R_{\rm SD}$.

Hadronization corrections to soft-drop thrust were studied in Ref. [10]. There it was demonstrated that non-perturbative corrections are indeed much reduced over a wide range of the event shape, with the precise magnitude of the reduction depending on the choice of grooming parameters $z_{\rm cut}$ and β. This property makes soft-drop event shapes attractive candidates for extractions of α_s, however, it should be noted that grooming also reduces the cross section, hence the soft-drop parameters must be chosen carefully to avoid the loss of too much data.

Furthermore, the precision of potential α_s measurements based on soft-drop event shapes is also influenced by the perturbative stability of the observables. Hence, it is important to investigate how grooming affects the convergence of perturbative predictions. In order to assess this, in Ref. [25] we computed the QCD corrections to soft-drop thrust ($T'_{\rm SD}$), hemisphere jet mass ($e_2^{(2)}$), and narrow jet mass (ρ). (The precise definitions of $e_2^{(2)}$ and ρ are given in Ref. [10].) We quantify the convergence of the perturbative results with K-factors, defined as the ratios of distributions at subsequent orders in perturbation theory,

$$K_{\rm NLO}(\mu) = \frac{{\rm d}\sigma_{\rm NLO}(\mu)}{{\rm d}O} \bigg/ \frac{{\rm d}\sigma_{\rm LO}(Q)}{{\rm d}O} \qquad \text{and} \qquad K_{\rm NNLO}(\mu) = \frac{{\rm d}\sigma_{\rm NNLO}(\mu)}{{\rm d}O} \bigg/ \frac{{\rm d}\sigma_{\rm NLO}(Q)}{{\rm d}O}\,. \tag{9}$$

Clearly the less the K-factors deviate from unity, the better the convergence of the perturbative prediction.

We present our results for soft-drop thrust in Fig. 3, where the left panel shows the distribution of $\tau'_{\rm SD} \equiv 1 - T'_{\rm SD}$ at LO, NLO, and NNLO accuracy for grooming parameters $z_{\rm cut} = 0.1$ and $\beta = 0$. The bands represent the effects of varying the renormalization scale by a factor of two around the central value $\mu = Q$. The dependence of the K-factors on grooming parameters is studied on the right panel of Fig. 3, where $K_{\rm NLO}(\mu)$ and $K_{\rm NNLO}(\mu)$ are plotted in dashed blue and solid red. We

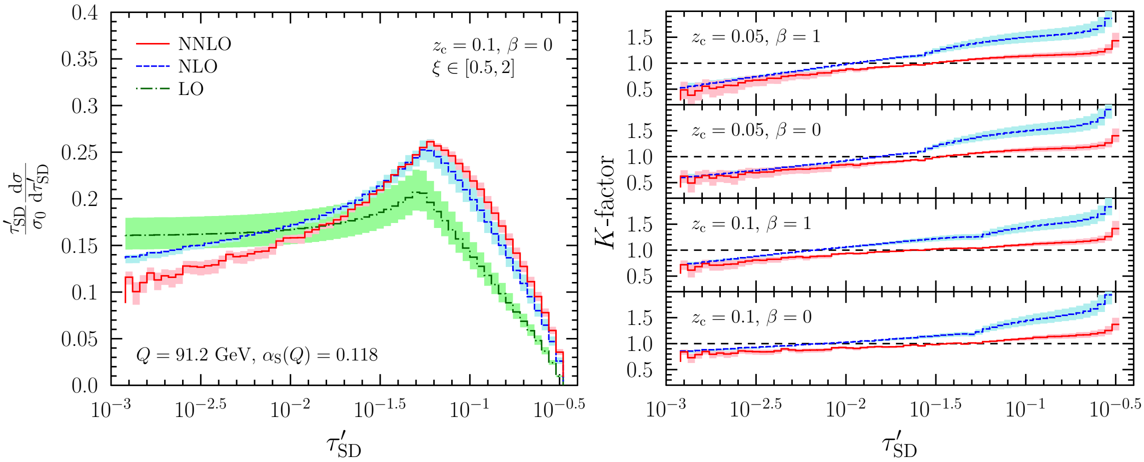

Figure 3: The soft-drop thrust distribution at LO, NLO and NNLO accuracy with $z_{\rm cut} = 0.1$ and $\beta = 0$ (left), and K-factors as defined in Eq. (9) for different choices of grooming parameters (right).

observe that stronger grooming (larger $z_{\rm cut}$ and smaller β) leads to K-factors closer to unity and hence a more stable perturbative prediction. This conclusion is of course not unexpected. For $\tau'_{\rm SD} \gtrsim 10^{-2}$, i.e., in the range where the bulk of the cross section is generated, we see that the perturbative result is the most stable for $z_{\rm cut} = 0.1$ and $\beta = 0$.

Summary

In this contribution we examined potential ways of increasing the precision of α_s measurements from e^+e^- annihilation into hadrons. On the one hand, we stressed the important role that higher-order perturbative corrections play in reducing the uncertainty of the determination. We highlighted this by presenting an extraction of α_s from measurements of energy-energy correlation, based on theoretical predictions with NNLO+NNLL accuracy and hadronization corrections derived using modern Monte Carlo tools. We find that the inclusion of NNLO corrections has a dramatic effect on the uncertainty of the measurement, which is reduced by a factor of two as compared to the result at only NLO+NNLL accuracy. Our analysis provides a determination of $\alpha_s(m_Z)$ with the highest numerical and theoretical precision obtained from this observable to date,

$$\alpha_s(m_Z) = 0.11750 \pm 0.00287\,(\text{comb.})\,.$$

On the other hand, we pointed out that a possible strategy for reducing uncertainties in measurements of α_s associated with the modeling of hadronization corrections is to employ observables for which these corrections are small. In particular, we examined soft-drop event shapes, observables specifically tailored so as to show less sensitivity to non-perturbative effects. Through the example of soft-drop thrust, we demonstrated that in addition to reducing hadronization corrections, soft-drop grooming also enhances the perturbative stability of the theoretical predictions. These features make soft-drop event shapes promising candidates for precision measurements of the strong coupling at lepton colliders.

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