GRADE 9 MATHEMATICS LEARNERS' STRATEGIES IN SOLVING NUMBER PATTERN PROBLEMS

by

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DEDICATION

This research study is dedicated to my father, Joseph Shimane Aphane and my mother, Luccy Ramadimetsa Aphane, for bringing me up and providing me with an education; and for their support and encouragement, and also to my spouse, Thabo Serogole Mphahlele, and my son, Lethabo, for all their love, support and kindness. May the almighty God bless you all.

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ABSTRACT

Problem solving provides an opportunity for learners to explore ideas and to extend their creativity, specifically if they are exposed to strategies that translate text into mathematical expressions. Number pattern problems allow learners to make predictions and justify their reasoning when solving problems. However, the solving of number pattern problems is often regarded as difficult for learners to do. Many learners use irrelevant strategies to solve number pattern problems and cannot easily identify number patterns embedded in problems. They also lack an understanding of the mathematical concepts of number patterns, which results in them not being able to solve algebraic problems or translate algebraic problems into mathematical equations. A reason for these difficulties could be that teachers often do not expose learners to various strategies for solving number pattern problems. Therefore, the purpose of this study is to investigate grade 9 mathematics learners' strategies in solving number pattern problems. Knowledge about learners' strategies will assist teachers in the teaching of problem solving and guide them to introduce various strategies, which can assist in the solving of number-related problems.

Mathematics is defined as a human activity that involves observing, representing and investigating patterns and qualitative relationships in physical and social phenomena and between mathematical objects themselves. The beliefs about the nature of mathematics provide a foundation for teacher's methods of teaching and how learners learn mathematics. It could also affect how mathematics concepts are explained, demonstrated and taught to learners. The in-depth investigation of patterns in mathematics could add more value to the ability to problem solve. Patterns are a way for learners to recognise order and organise their world and are important in all aspects of mathematics. The study was guided by the problem solving (PS) conceptual framework of Singer and Voica (2013), which highlights progression in problem solving according to four phases, namely decoding, representing, processing and implementing. This PS conceptual framework reflects learners' levels of natural dispositions towards strategies for solving problems and offers insight into more the effective learning of mathematical problem solving.

The study adopted an exploratory qualitative case study research strategy. Ninety grade 9 learners were purposively selected from three rural schools (30 participants from each school) to participate in the study. Qualitative data were collected through a written activity and semi-structured one-on-one interviews. Data analysis was done by means of content analysis following a deductive approach. The levels of engagement in the four phases of the PS conceptual framework of Singer and Voica (2013) were used to analyse and interpreted participants' strategies.

The main finding revealed that learners utilise five main strategies in number pattern problems, namely (1) direct counting; (2) direct proportion; (3) recursive strategy; (4) mental image representation; and (5) mental model representation. Both direct counting and recursive strategies were evident during the decoding phase (DP). The strategy of mental image representation was evident during the representing phase (RP). The strategy of direct proportion, even inappropriately employed, was evident in both the processing phase (PP) and the implementing phase (IP). Mental modelling was evident in the (RP). The different strategies learners used to solve number pattern problems could sensitise teachers about other strategies to use when introducing number pattern problems to learners. This study also makes teachers aware of learners' interpretations and implicit thinking processes about the strategies they use in solving number patterns during the different problem-solving process phases, which may eventually influence learners' learning of mathematics.

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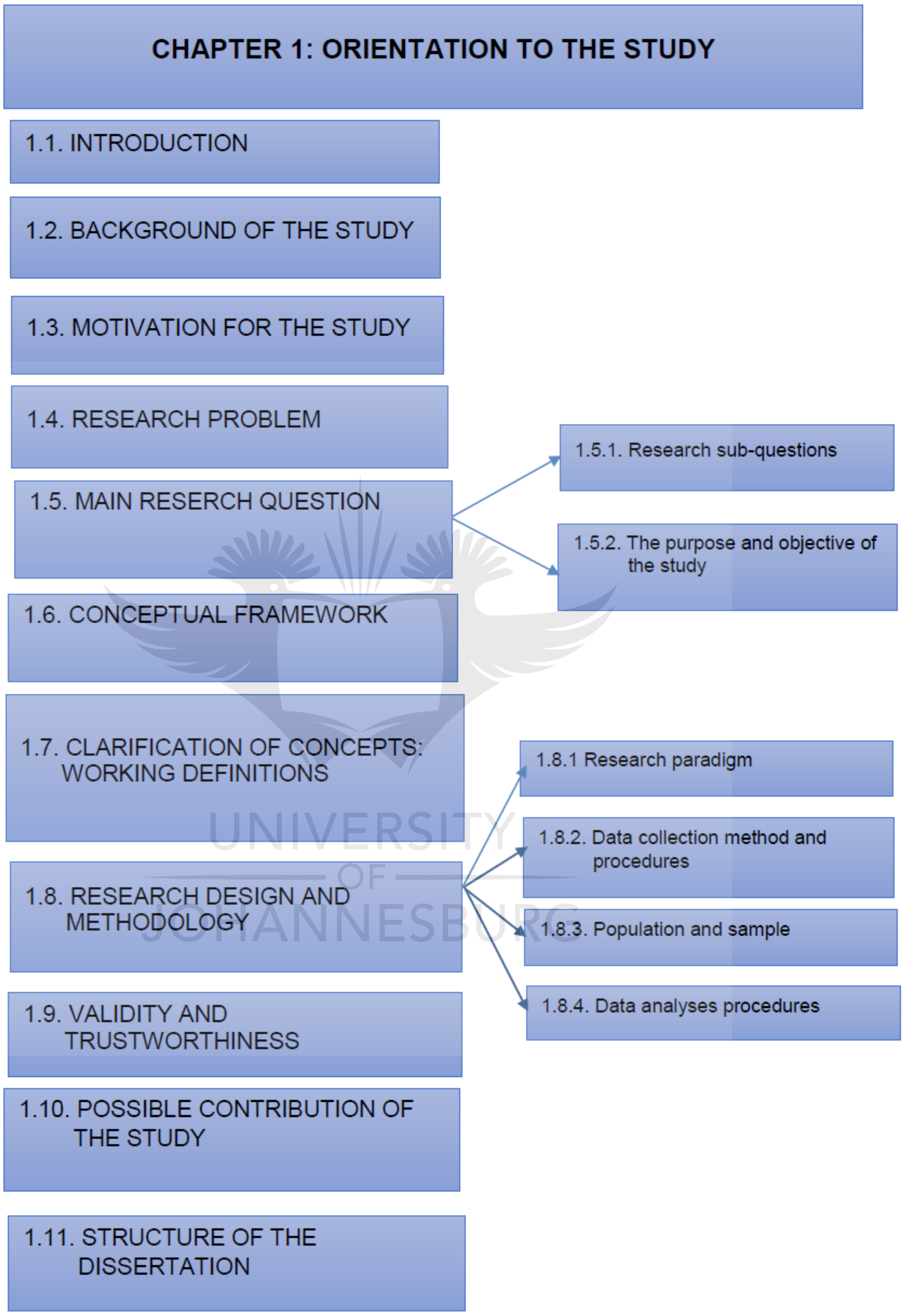
ANA	Annual National Assessment
CAPS	Curriculum and Assessment Policy Statement
DBE	Department of Basic Education
DP	Decoding phase
FET	Further Education and Training
FP	Foundation Phase
IP	Intermediate Phase
IP	Implementing phase
PP	Processing phase
PS	Problem solving
RP	Representing phase
SP	Senior Phase

CONTENTS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CHAPTER 1: INTRODUCTION AND CONTEXTUALISATION

1.1 INTRODUCTION

In South Africa, most learners across the primary grades have poor mathematics skills (Graven & Heyd-Metzuyanim, 2014), with the average performance steadily declining by approximately 10% each year (Department of Basic Education (DBE), 2014). The results of the Annual National Assessment (ANA) for mathematics in South Africa, is also of national concern because of the poor performance of grade 9 mathematics learners (Govender, 2013). The ANA tests are meant to serve as a diagnostic tool for identifying areas of strengths and weakness in teaching and learning, thus providing information for school-focused interventions. The tests also provide teachers with benchmark information and baseline data that can be used to improve “classroom assessment practices and inform the teaching and learning of literacy and numeracy” (DBE, 2012:4). Although the 2013 ANA national average increased slightly from 13% to 14% for grade 9 mathematics, the results are still poor. In addition, the overall performance in mathematics in the ANA of 2014 showed an increase in performance by a maximum of 8% for all grades, except for grade 9. Unfortunately, in 2015 the writing of the ANA was postponed (DBE, 2015).

The current forms of assessment available in grade 9 mathematics are tests and internal examinations, investigations, assignments and projects (DBE, 2013). Van Staden and Motsamai (2017) argue that South African learners continue to perform poorly in mathematics throughout all grades compared to their counterparts internationally, locally and regionally. The Head of the National Education Evaluation and Development Unit (NEEDU) Department of Basic Education, argues that poor learner performance is largely due to poor teacher subject knowledge, especially in mathematics, in most schools (Zingiswa, 2019).

Govender (2012) indicates that the DBE and other stakeholders have paid significant attention over the past years to the Further Education and Training (FET) phase (grades 10-12), especially grade 12, at the expense of the other phases. Govender (2012) argues that it is possible that this neglect has contributed to the poor state of mathematics teaching and learning in the lower grades. In particular, the majority of grade 9 mathematics learners in South Africa face difficulties in solving number pattern problems. Maluleka (2013) notes that the greatest difficulty in working with mathematical problems is to translate real-life issues into mathematical problems. A person first need to have knowledge of the issue before trying to attempt it.

Mathematical problem solving is complicated because learners need to read and comprehend written content which is stated in numerical relations (Tolar, Fuchs, Cirino, Fuchs, Hamlett, & Fletcher, 2012). Phonapichat, Wongwanich, and Sujiva (2014) identify some key difficulties that learners experience in mathematical problem solving, namely: understanding keywords appearing in problems; interpreting keywords in mathematical sentences; figuring out what information to assume, and what information is necessary to solve the problem. Wang, Fuchs, and Fuchs (2016) notice that many learners approach mathematical problems without giving in-depth consideration about how unrelated details could derail them from associating a new problem with and existing known problem. According to Tambychik and Meerah (2010:150):

> Learners' difficulties in mathematics problem solving are due to incompetency in acquiring many mathematics skills and lacking in cognitive abilities of learning. This lacking, results in uncertainty, confusion and inaccuracy in the decision and connection making among information, and therefore leads to errors in mathematics problem solving.

1.2 BACKGROUND TO THE STUDY

Since the 1970's much research has focused on problem solving. As explained by Allevato and Onuchic (2008:61):

> Concomitantly, at the beginning of the 1970's, systematic investigation of problem solving and its implications for curricula was initiated. Thus, the importance attributed to problem solving is relatively recent, and only in this decade did mathematics educators come to accept the idea that the development of problem-solving abilities deserved more attention. At the end of the 1970's, problem solving emerged, gaining greater acceptance around the world.

In 1980 the National Council of Teachers of Mathematics (NCTM) published a document recommending problem solving to be the main focus of school mathematics (NCTM, 2010) (see section 2.5). Kilpatrick, Swafford, and Findell (2001:421) state that:

> Problem solving must be in the centre of the Mathematics Curriculum. Problem solving should be the site in which all of the strands of mathematics proficiency converge. It should provide opportunities for learners to weave together the strands of proficiency and for teachers to assess learners' performance on all of the strands.

Lambdin (2009) expresses the belief that "the primary goals of mathematics learning are understanding and problem solving, and that these goals are inextricably related because learning mathematics with understanding is best supported by engaging in problem solving" (p. 6). Otten (2010) adds that problem solving develops creativity, flexibility and metacognitive skills that address professional and post-secondary demands. In other words, the study of problem solving in mathematics prepares learners for many aspects of their lives after school, for example, trades, professional careers and knowledgeable citizenship. In addition, Matlala (2015) states that the problem-solving approach in mathematics teaching may be a way to improve the quality of, and results in, school mathematics.

The importance of problem solving is also evident in the numerous studies on problem solving in mathematics that have been conducted in the South African context (Maluleka, 2013; Mochesela, 2007; Sepeng, 2010; Sepeng & Madzorera, 2014; Sepeng & Sigola, 2013; Sepeng & Webb, 2012). Maluleka (2013) discovered that grade 9 learners are trying to fix problems without comprehension; and that methods of interaction, reasoning and recording seem to be crucial to helping learners. Mochesela (2007:iii) found that "exposing learners to a variety of problem-solving strategies improves their problem solving performance and attitudes towards mathematics". Therefore, creating an environment where learners attempt to find variety of strategies for solving number-related problems empowers them to explore alternatives and develops confidence in mathematics problem solving. Sepeng (2010) found that the discussion and argumentation techniques in mathematical problems have a positive effect on learners' ability to consider reality during problem solving. Similarly, Sepeng and Webb (2012) found that debates in mathematics classrooms, as a learning approach, could enhance learners' efficiency in solving mathematical problems, as well as their capacity to make sense of real-life problems. Sepeng and Madzorera (2014) have shown that learners are struggling to define algebraic terms that are used both in problem statements and in educational vocabulary. Sepeng and Sigola (2013) explored sources of mistakes made by grade 9 learners when they solve mathematical problems. They found that learners' mistakes in problem solving appear to be due to a lack of knowledge of mathematical vocabulary used in a problem declaration.

Despite all the above-mentioned research studies on problem solving in South Africa, grade 9 learners still struggle to solve number patterns-related problems.

1.3 MOTIVATION FOR THE STUDY

Mathematical problems allow learners to think creatively and develop new mathematical reasoning skills (Depaepe, De Corte, & Verschaffel, 2010). One benefit of problem solving is that it is a learner-centred approach in which learners investigate and explore mathematical ideas on their own (Depaepe et al., 2010). In addition, problem-solving motivates learners to perform academically as it enhances creativity and mental behaviour of learners to develop their knowledge (Căprioară, 2015). The Curriculum and Assessment Policy Statements (CAPS) document for Senior Phase (SP) Mathematics in South Africa also claims that mathematical problem solving "enables learners to understand the world (physical, social and economic) around them, and, most of all, encourages teachers to teach learners to think creatively" (DBE, 2011:8).

"Number pattern problems allow learners to make predictions and justify their reasoning when solving problems" (DBE, 2011:9). For example, in number pattern problems, learners are given a sequence of numbers and they have to identify a pattern or relationship between consecutive terms in order to extend the pattern. Examples of number patterns are: -2, -5, -8, -11… or 3, 7, 11, 15, 19… where the second term depends on what happens to the first term, and the third term depends on the second term. Number pattern activities can lead to the development of problem-solving capabilities by highlighting the evaluation of specific instances, systematically organising information, and inferring and generalising information (Barbosa, Vale, & Palhares, 2012). Number pattern problems have an important place in mathematics, and pattern seeking is a valuable problem-solving strategy (Kurbal, 2015). According to Mahlobo and Ntombela (2014), problems involving number patterns provide "an opportunity to generalise and to give general algebraic descriptions of the relationship between terms and their position in a sequence" (p. 186).

The learning of patterns and sequences starts in the Foundation Phase (FP) where CAPS states that: "In the SP phase, learners work with both number patterns (e.g. skip counting) and geometric patterns (e.g. pictures)" (DBE, 2011:9). This topic continues through the Intermediate Phase (IP) and SP. Thus, in many nations, including South Africa, the use of symbols and variables to represent number patterns and generalisations is a significant element of the mathematics curriculum (Fray, Fish, & Taylor, 2015) (see section 2.3).

1.4 CONCEPTUAL FRAMEWORK

The problem-solving (PS) conceptual framework of Singer and Voica (2013) is adopted for this study and is briefly illustrated in Figure 1.1. This framework reflects the learners' natural disposition towards strategies for solving problems, and offers "insight for more effective learning of mathematical problem solving and can be used in problem posing and problem analysis in order to devise questions more relevant for deep learning" (Singer & Voica, 2013:11). It is designed to help learners to read the problem with understanding by, firstly, identifying the key words that could help them solve the problem. The conceptual framework of Singer and Voica (2013) is particularly relevant to this study as it reflects the new, focused research publication devoted to mathematics problem solving. The framework describes the strategies that could help learners to make sense of the information on the given text, and ultimately to arrive at the correct solution. The framework describes phases to reach that could help learners to make sense of the information for effective problem solving (see section 2.8).

Singer and Voica (2013) as cited by Irvine (2017) point out that correlations exist between problem solving, problem posing, and creativity. Multiple descriptions of creativity include problem finding, problem solving, and problem posing, and problem posing is frequently used in assessing creativity. Salazar Solórzano (2014) indicate that the PS conceptual framework of Singer and Voica (2013), which involves four operational categories, namely decoding, representing, processing and implementing, can be helpful when analysing the original problem, modifying it or posing a new problem. They further conclude that despite teachers being naturally predisposed to problem solving, they also need to be properly trained in this skill as part of their university studies in order to acquire an effective technique.

Munroe (2016) argues that Singer and Voica (2013)' s PS conceptual framework consider problem solving as a generative activity from which information on the levels of mathematical thinking, skills, and areas of weakness of the learners could be drawn. This framework allows leaners to interact with multiple problems, methods, and solutions simultaneously and this increases the possibility of developing creativity in leaners (Munroe, 2016).

The PS conceptual framework in Figure 1.1 highlights four phases, namely: decoding, representing, processing, and implementing.

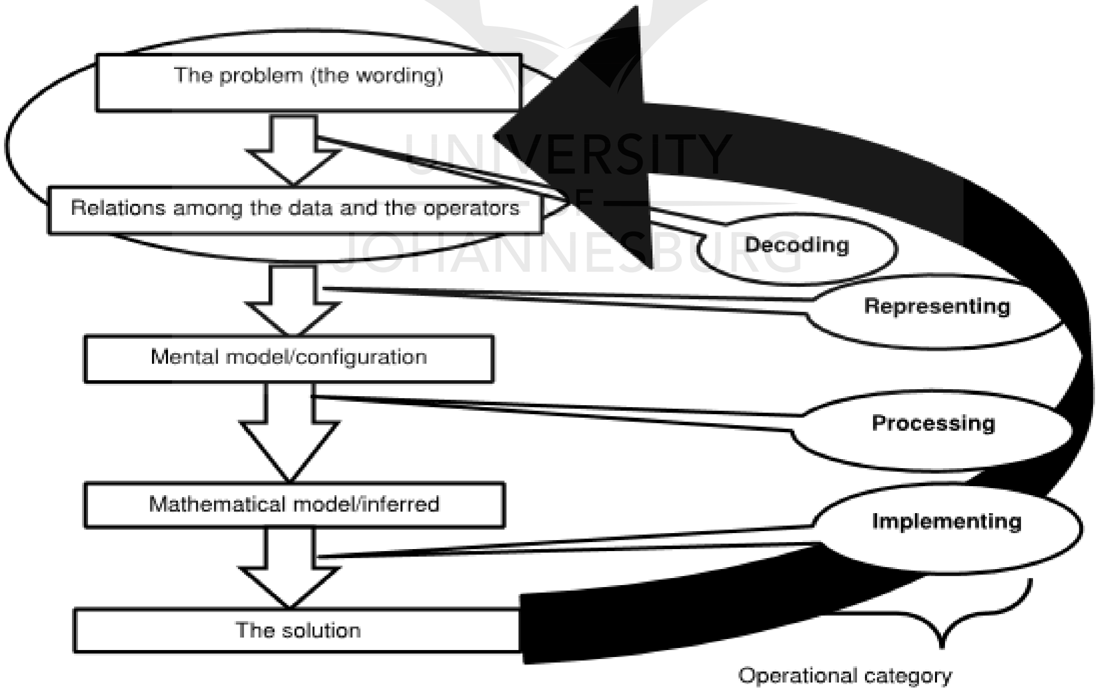

Figure 1.1: The phases of the PS conceptual framework of Singer and Voica (2013:13)

Phase 1: Decoding

According to Singer and Voica (2013), when a problem is given (the wording) the problem solver makes relations among the data and operators, which requires the solver to convert the text of the problem into understandable language. Lorenzo (2005) states that "breaking the problem into parts allows learners to focus on a few ideas at a time, when they have to work on a new unknown, which means that the likelihood of making mistakes would decrease" (p. 54) (See sub-section 2.8.1).

Phase 2: Representing

In this phase, the problem solver needs to represent the problem in the text by using appropriate mental representation (Singer & Voica, 2013). The representing process in mathematical problem solving permits learners to visually relate various types of information given in the problem statement to help them determine which mathematical expressions are useful in solving the problem (Jan & Rodrigues, 2012b) (see sub-section 2.8.2).

Phase 3: Processing

Singer and Voica (2013) state that in the processing phase (PP) the problem solver "uses the mental model suggested by the problem and personal mathematical competence to identify a mathematical model that can be associated with the problem" (p. 11). The learner creates a mental image to solve problems, which may produce mathematical models such as formulas or equations (see sub-section 2.8.3).

Phase 4: Implementing

During the implementing phase (IP), the problem solver implements what is already known about the problem in order to test a model or an equation. According to Singer and Voica (2013), this phase focusses on the "application of techniques that are specific to the found mathematical model and adaptable to the given particular situation, with the purpose to obtain final results for the problem" (p. 13) (see sub-section 2.8.4).

Singer and Voica (2013) argue that "when the process of solving is successful, from the text of the problem (the wording) to its solution, a solver need to work from the solution to the initial problem" (p. 13). Thus, a solver needs to interpret the solution in relation to the given problem (the wording). The IP involves the understanding of the solution from the context within which the problem is given, meaning that the solver should be able to connect the formal solution of the problem with the initial data. The bold arrow in Figure 1.1 illustrates the closing of the solving cycle (Singer & Voica, 2013).

1.5 RESEARCH PROBLEM

According to Boonen, Van der Schoot, Van Wesel, De Vries and Jolles (2013), solving number pattern problems presents difficulties if learners cannot identify the relationship between the known and the unknown variable. This is particularly true when learners face challenges in understanding the problem-description text provided. Jupri and Drijvers (2016) confirm that the major difficulties encountered by learners when dealing with number pattern problems are "to understand the words, to formulate a mathematical model from the problem, to solve the problem expressed in the model, and to interpret the solution in terms of the original problem" (p. 2499).

Working with number patterns is particularly difficult for learners as the patterns are often embedded in problem solving. Problem solving becomes difficult for learners when the problem is presented linguistically, since they require learners to "read and interpret the problem, represent the semantic structure of the problem, and choose a solution strategy" (Schumacher & Fuchs, 2012:608). Learners cannot easily identify number patterns embedded in problems. Barbosa et al. (2012) state that students tend to use numeric rather than visual methods and experience several difficulties when solving pattern exploration issues, particularly when generalising remote values. Usually, learners experience difficulties in problem solving when trying to translate algebraic problems into mathematical equations (Ahmad, Tarmizi, & Nawawi, 2010).

The SP Mathematics CAPS document states that "problem solving and cognitive development should be central to all mathematics teaching. Learning procedures and proofs without a good understanding of why they are important will leave learners ill equipped to use their knowledge in later life" (DBE, 2011). However, the analysis of the 2014 ANA results reveals that the solving of problems is challenging for learners (DBE, 2014). Problem solving is a complex process in which learners need to be coached (Klingler, 2012) and by which an unfamiliar situation is resolved (Killen, 2007). Notwithstanding the emphasis on the importance of problem solving, many researchers recommend further research in this regard (Boonen et al., 2013; Duru, Peker, Bozkurt, Akgün, & Bayrakdar, 2011; Jan & Rodrigues, 2012b; Mimbs, 2005; Peters, 2011; Sepeng & Sigola, 2013; Tambychik & Meerah, 2010).

Boonen et al. (2013) suggest that “follow-up studies should examine the effects of interventions in which elementary and secondary school learners are taught to systematically build visual-schematic (mental) representations during math problem solving” (p. 277). Duru et al. (2011) claim that teachers need to be informed about various strategies to solve problems in mathematics; and they need to understand learner difficulties in order to implement these various strategies. Jan and Rodrigues (2012a) suggest studies to determine which variables affect the capacity of students to understand mathematical problems in order to discover suitable teaching approaches to solve problems meaningfully.

Mimbs (2005) suggests that current and future teachers should be supported professionally with regard to strategies needed for problem solving. Peters (2011) recommends that future research “should be conducted in order to explore new or alternative strategies that can be added to the already known standard list of problem-solving strategies as suggested by Polya (1945)” (p. 93). In addition, Sepeng and Sigola (2013) recommend that strategies that can be used to solve problems should be made accessible to learners, especially “the use of models, images, tables, diagrams and other learning aids” (p. 332). According to Tambychik and Meerah (2010), further research based on learners’ ability to perform mathematical skills is necessary for a better understanding of problem solving. The identification of the mathematics skills needed for tackling problem-solving activities is essential for achieving better performance in these activities.

Barbosa et al. (2012) suggest that for learners to understand the meaning of numbers and variables, it is important that teachers provide tasks that encourage learners to use and understand the potential of visual strategies and to link numerical contexts with visual contexts. Number patterns problems can help learners to develop the ability to generalise, giving learners the opportunity to come up with a pattern rule or formula to find any term in a sequence. In addition, Sepeng and Sigola (2013) noticed that many learners experience difficulties in reading and making sense of mathematical problems and it appears they are struggling to understand problem solving. Therefore, it is essential to assist learners in developing skills and strategies to effectively solve problems related to number patterns (Sepeng & Sigola, 2013). The problem of number patterns can lead to increased problem-solving abilities by highlighting the assessment of particular cases, systematically organising data, conjecturing and generalising information. Therefore, learners need to be exposed to strategies to solve number pattern problems in their classroom activities

Regardless of all the mentioned inquiries on problem solving, grade 9 learners in South Africa are still struggling to solve number patterns problems. As a mathematics teacher, I have observed that many grade 9 learners have difficulty in solving number pattern problems in mathematics. In particular, they cannot solve algebraic number pattern problems or translate algebraic number pattern problems into mathematical equations due to a lack of understanding of mathematical concepts. Many of them do not have sufficient knowledge or exposure to strategies for solving number pattern problems. Yet, problem solving provides an opportunity for learners to explore ideas and gives them the chance to extend their creativity.

1.6 MAIN RESEARCH QUESTION

In the light of the importance of number pattern problems in mathematics, as well as the complexity of the topic for many SP learners in South Africa, the following research question is interrogated in this study:

What strategies do grade 9 mathematics learners use in solving number pattern problems?

1.6.1 Research sub-questions

To address the main question, the following sub-questions were posed:

- What are the strategies grade 9 mathematics learners engage in when solving number pattern problems?
- What are the views of grade 9 mathematics learners regarding the areas of difficulty (if any) they experience as they complete number pattern problems?

- What levels of engagement in the four phases of the PS conceptual framework are evident in grade 9 mathematics learners' strategies to a number pattern problem activity?

1.6.2 The purpose and objectives of the study

The purpose of this study is to investigate grade 9 mathematics learners' strategies in solving number pattern problems. Knowledge about learners' strategies will assist teachers in the teaching of number pattern problems and guide them to employ various strategies, which can assist in the solving of number pattern problems.

The objectives of this study were to:

- Determine the strategies grade 9 mathematics learners engage in when solving number pattern problems;
- Establish grade 9 mathematics learners' views regarding the areas of difficulty (if any) they experience as they complete number pattern problems; and
- Ascertain the levels of engagement in the four phases of the PS conceptual framework in grade 9 mathematics learners' strategies to a number pattern problems activity.

1.7 CLARIFICATION OF CONCEPTS: WORKING DEFINITIONS

The following section clarifies the concepts and definitions used in this study to ensure a clear understanding.

Mathematics:

"A diverse discipline that deals with data, measurements, and observations from science; with inference, deduction, and proof; and with mathematical models of natural phenomena, of human behaviour, and of social systems" (NRC, 1989).

Number patterns:

Number patterns are numbers, which cannot be transferred to "non-numeric patterns without loss of some crucial property of the pattern" (Liljedahl, 2004:4).

Problem solving:

A mathematical task that has the potential to provide intellectual challenges for enhancing learners' mathematical understanding and development (NCTM, 2010).

Strategies in problem solving:

Groupings of mental or physical actions designed to solve a problem (Biddlecomb & Carr, 2011). “Giving problems to learners that would explore all aspects of principles and concepts depending on subject matter” (Killen, 2007:260).

Decoding:

The ability to interpret the given statement and to identify the key words in order to solve the problem (Clement, 2008).

Representing:

“Understanding the problem through a generated mental model” (Singer & Voica, 2013:11).

Processing:

Using problem-suggested mental settings and private mathematical expertise to define a mathematical model that can be connected with the problem (Singer & Voica, 2013).

Implementing:

Application of “techniques that are specific to the found mathematical model and adaptable to the given particular situation, with the purpose” of obtaining final results for the problem (Singer & Voica, 2013:11).

1.8 RESEARCH DESIGN AND METHODOLOGY

The following section describes the research design used in this study and how the data collection process was followed (see section 3.2).

1.8.1 Research paradigm

An interpretivist paradigm with an “epistemological position that prioritises people’s subjective interpretations and their own actions” (Matthews & Ross, 2010:28) was used. This paradigm provided an in-depth insight into the inquiry (see sub-section 3.2.1). A qualitative research approach in the form of an exploratory case study was used “to explore those situations in which the intervention being evaluated has no clear, single set of outcomes” (Yin, 2003:102) (see section 3.2).

The qualitative research approach focused on meaning and understanding and provided a rich description of the phenomenon under investigation (Merriam, 2009). Working with qualitative data in this study involved interpreting the words, stories, accounts, and explanations of the participants about the strategies they use to solve problems on number patterns (see sub-section 3.2.2).

1.8.2 The data collection methods and procedures

In stage one, a written activity on number patterns prescribed by the SP mathematics CAPS, was used to collect qualitative data from learners (see section 3.3). In stage two, the researcher conducted one-on-one semi-structured interviews (after school hours) with selected participants to establish their views regarding the areas of difficulty they experience as they complete number pattern problems. Open-ended interviews were administered to probe for deeper understanding. Data collected from the interviews was transcribed (see section 3.3). Lastly, the researcher ascertained the levels of engagement in the four phases of the PS conceptual framework in the participants' strategies derived from the analysis of the written activity and the interviews in order to determine the implications for solving number pattern problems (see section 3.3).

1.8.3 Population and sample

The population of the study was grade 9 mathematics learners in Quintile 1 schools in the Capricorn district of the Limpopo Province. The population comprised 90 learners from three rural schools (A, B and C). The researcher purposively selected 30 grade 9 mathematics learners from each of these three sampled schools; thus a total of 90 learners consisting of both males and females of ages ranging from 14–16 (see section 3.3).

1.8.4 Data analysis procedures

In stage one and two, inductive content analysis was used to analyse the written activity and interview transcripts. A constant comparative analysis method was used to ascertain the levels of engagement in the four phases of the PS conceptual framework in the participants' strategies derived from the analysis of the written activity and the interviews (see section 3.3).

1.9 TRUSTWORTHINESS

According to Guion, Diehl, and McDonald (2002:1), "validity in qualitative research, refers to whether the findings of a study are true and certain", thus trustworthiness.

Trustworthiness was ensured by using triangulation, which involved the analyses and comparison of results from multiple qualitative data collection instruments, namely documents and interviews. Trustworthiness was also established by considering credibility, transferability, dependability, and confirmability (Lincoln & Guba, 1985).

To ensure credibility, audio-taped interviews were transcribed and the inconsistent data was carefully checked and examined. Member checking was used by returning all the analysed interview transcripts to the participants to check whether or not they agreed with emerging findings (Birt, Scott, Cavers, Campbell, & Walter, 2016). However, it was difficult to validate the emerging findings, as participants could have reached a particular phase of the PS conceptual framework in their minds, but chose not to write down the solution they arrived at, for not trusting that the solution is accurate. Therefore, it was done just to check the accuracy of the transcriptions. To ensure transferability, a dense description of the methodology, a literature control, and verbatim quotes were provided. To ascertain dependability, the researcher continued interviewing participants until data-saturation was reached. Confirmability was established by external audits. Expert teachers/researchers were consulted to assist the researcher with advice regarding clarity of the interview questions, and the design of the activity sheet and assessment grid (see section 3.4).

1.10 POSSIBLE CONTRIBUTION OF THE STUDY

The study contributes to practice by providing information about learners' strategies to solve problems, which could assist teachers in their future teaching of number patterns. Teachers could benefit from the interpretation of learners' strategies pertaining to the implicit thinking processes of learners when they solve problems, which may influence their teaching of mathematics. Curriculum developers may benefit from an increased awareness of the difficulties learners experience with problem solving. Subject advisors could use learners' strategies, in solving number pattern problems, in a meaningful and effective way to support teachers in their professional development of problem-solving proficiency. Policy developers were provided with guidelines on learners' strategies to solve problems involving number patterns, which may influence policy. The importance of this study in terms of academic value is that it expands on the existing PS conceptual framework of Singer and Voica (2013), which may be used for further research pertaining to the development and maintenance of effective strategies to solve problems involving number patterns.

1.11 STRUCTURE OF THE DISSERTATION

The dissertation consists of five chapters:

Chapter 1 provides an introduction to the background and rationale of the study to orientate the reader. It briefly introduces the study's research questions, purpose, objectives, and theoretical framework. The research paradigm, design, and methodology are also outlined. Important concepts are defined and possible contributions of the study are foregrounded.

Chapter 2 consists of an in-depth review of relevant literature, and explains the conceptual framework of the problem-solving process on which this study is based, namely: decoding, representing, processing, and implementing (Singer & Voica, 2013). The topics that are addressed in the literature review include: the nature of mathematics, problem solving in mathematics, problems in mathematics, number patterns, and strategies for problem solving.

Chapter 3 describes and justifies the research methodology and design used in this study. The data collection instruments, selection of the participants, data collection processes and data analysis procedures are discussed, including the validity and trustworthiness of the study.

Chapter 4 presents the results of the document analysis, and the analysis of the one-on-one semi-structured interviews; as well as the interpretation of the findings in light of the literature reviewed and the conceptual framework.

Chapter 5 summarises the study according to the research questions; makes recommendations for further research; discusses the implications of the research, and the limitations of the study. Lastly, this chapter concludes with a personal reflection on the study.


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CHAPTER 2: LITERATURE OVERVIEW AND CONCEPTUAL FRAMEWORK

2.1 INTRODUCTION

Chapter 2 provides an overview of literature pertaining to the nature of mathematics with reference to problem solving. Firstly, definitions of mathematics are provided followed by a discussion on beliefs about the nature of mathematics pertaining to teaching and learning practices. The nature of mathematics is viewed from the perspectives of mathematics as a discipline, knowledge of the teacher regarding mathematics, and how mathematics is learned and taught in a classroom (Siswono, Kohar, & Hartono, 2017). Then, problem solving focuses on problems in mathematics will be discussed. The discussion will be extended by focusing on learners' experiences of problem solving and strategies to solve problems. The majority of grade 9 mathematics learners in South Africa face difficulties in solving problems. Problems belong to one of the difficult and complex topics in mathematics that learners encounter during their elementary level of mathematical development.

A discussion will follow focusing on number patterns and the design of a number pattern problem-solving activity. Working with number patterns in particular can be difficult for learners as it is often embedded in problems. Learners cannot easily identify number patterns embedded in problems. This chapter will conclude by describing the conceptual framework of Singer and Voica (2013) for problem solving in mathematics. This conceptual framework will highlight and elaborate on four phases, namely decoding, representing, processing and implementing.

2.2 THE NATURE OF MATHEMATICS

2.2.1 Defining mathematics

According to the DBE (2011), mathematics is a discipline consisting of many topics such as algebra, geometry, trigonometry, statistics and probability. In particular, mathematics "is a human activity that involves observing, representing and investigating patterns and qualitative relationships in physical and social phenomena and between mathematical objects themselves" (DBE, 2011:8).

Various scholars (Graven, 2015; Mwakapenda, 2008; Peters, 2011; Schoenfeld, 2013) have also attempted to define mathematics. Peters (2011) agrees that mathematics is an “ordered field of knowledge with many branches such as arithmetic, algebra, geometry, trigonometry, statistics and analysis that are related to and dependent on each other” (p. 7). Peters (2011) further claims that mathematics is a common language that uses closely defined terms and symbols that allow people to think about, record, and transmit their thoughts about the features and relationship of quantity. Schoenfeld (2013) states that in doing mathematics we investigate and discuss details and make conjectures. Through problem-solving techniques those conjectures can be proven and verified. In addition, Graven (2015) defines mathematics as a subject where learners must be able to build new knowledge from previously learnt knowledge as they progress towards more abstract levels, which implies having strategies of solve the problem, and the process of reasoning. In contrast, Mwakapenda (2008) defines mathematics as the ability to notice significant details of patterns with accurate logical thinking, which leads to theories of abstract relations.

From the given definitions, it can be concluded that mathematics is way of trying to understand, make sense of, or describe how one’s world works. Mathematics allows learners to have knowledge that they can use outside of school. The definition of mathematics as a human activity requires teachers to change their approach to mathematics teaching and learning – from traditional methods to allowing learners to think creatively, investigate and explore mathematical ideas on their own.

2.2.2 Beliefs about the nature of mathematics

Ernest (1991) argues that having a good understanding of the nature of mathematics, and its philosophical underpinnings, is important both for the teaching and learning of mathematics.

Teachers' views about the essence of mathematics should take into account the understanding of the content and the way of teaching and learning to enrich the learners’ enjoyment of it (Spangenberg & Myburgh, 2017). Similarly, Beswick (2012:145) suggested that:

> There is a need to focus on the beliefs about the nature of mathematics that the teachers have constructed during their teaching experience as mathematics teachers in primary and secondary schools, as well as universities; and from years of involvement in the profession.

Many researchers (Ernest, 1991; Givvin, Stipek, Salmon, & MacGyvers, 2001; Lamichhane, 2017; Siswono et al., 2017; Űnlü & Aktaş, 2013) have investigated the beliefs that teachers hold about the nature of mathematics, and how these beliefs affect their practice. Ernest (1991) classified beliefs about the nature of mathematics into three main philosophical conceptions of mathematics, namely instrumentalist, Platonist and problem-solver. An instrumentalist view proposes that mathematics consists of certain operations, rules and skills. The role of a teacher is as an instructor, meaning that the teacher tells learners which rule and procedure to follow when solving a mathematical problem. The learner does not know if the answer is correct until the teacher tells them. The learner who is taught instrumentally can perform calculations and apply procedures, but will not necessarily understand the mathematics behind the rule or procedure (Skemp, 1976).

A Platonist view suggests that mathematics is a static, but unified body of certain rules (Ernest, 1991). Learning of mathematics is seen as a passive reception of knowledge, while a teacher is the possessor and explainer. In the problem-solver view, mathematics as a continually expanding field of human activity, creativity and discovery, in which patterns are generated and then distilled into knowledge (Ernest, 1991). A teacher facilitates while learners investigate and explore mathematical ideas on their own. The Platonist, instrumentalist and problem-solver views on mathematics teaching and learning can be summarised as follows (Beswick, 2012:30):

> A Platonist believes mathematics teaching is content focused with an emphasis on understanding, while mathematics learning is an active construction of understanding. An instrumentalist views mathematics teaching as content focused with an emphasis on performance, while mathematics learning is about skill mastery, thus a passive reception of knowledge. While problem solving is a learner-centred approach in which learners investigate and explore mathematical ideas on their own.

Lamichhane's (2017) research on the relationship between teachers' beliefs about mathematics and their instructional practices showed that teachers with an absolutist belief tended to "blend with technical interest of curriculum development that adopts an instructional pedagogical approach and thus their classroom practices become more transmissions, autocratic and disempowering" (p. 19). The learner depends on the knowledge of the teacher. Learners might consider mathematics as a subject composed of meaningless symbols that have to be remembered and manipulated correctly to get the answer.

Siswono et al. (2017) investigated three Indonesian lower secondary mathematics teachers' beliefs about the nature of mathematics, and how these beliefs related to their knowledge. The different philosophical views of teaching and learning mathematics were used, i.e., instrumentalist, Platonist and problem-solver. The study showed that "the instrumental teacher's belief was consistent with his insufficient knowledge about problem solving, while both Platonist and problem-solving teachers' beliefs were consistent with their sufficient knowledge of either content or pedagogical problem solving" (p. 6).

Űnlü and Aktaş (2013) investigated the beliefs of 104 pre-service elementary mathematics teachers about the nature of mathematics. They found that most pre-service teachers' hold a problem-solver view; meaning that they did not believe that mathematical problems could only be solved in ways shown in the book, but that different strategies could be used. They thought that mathematics was concerned with intelligence, mental thinking and creativity.

In conclusion, beliefs about the nature of mathematics provide the foundation for a teacher's method of teaching; and their views on how learners learn mathematics. Thus teachers' beliefs about the nature of mathematics can affect how mathematics concepts are explained, demonstrated, and taught to learners. These different philosophies of mathematics provide an important lens for the study of learners' errors and misconceptions in mathematics, and the relationship of these misconceptions to the different philosophies of mathematics learners might hold.

2.3 NUMBER PATTERNS IN PROBLEM SOLVING

2.3.1 Defining pattern

Peters (2011) defines mathematics as the science of finding patterns. Patterns are powerful tools in mathematics and can suggest several approaches, namely: knowledge of counting numbers, recognition of numbers, and relating numbers to the identification of pattern. Number-pattern tasks are problems that involve a number sequence. Recognising number patterns, as related to problem solving in this study, involves a process of looking out for numbers in a given sequence and forming a pattern, which allows the problem-solver to generalise a solution that can be applied in every given situation. The learning of patterns and sequences starts in the FP where CAPS states that: "In this phase, learners work with both number patterns (e.g. skip counting) and geometric patterns (e.g. pictures)" (DBE, 2011:9). The topic continues through to the IP and SP levels. "This understanding of patterns allows learners to make predictions and justify their reasoning when solving problems" (DBE, 2011:9). According to CAPS, number patterns in mathematics is about recognising, describing, and working with numerical and non-numerical patterns (DBE, 2011). In the mathematics curricula of many countries, including South Africa, the use of symbols and variables that represent patterns and generalisation are foregrounded (Fray et al., 2015). According to the NCTM (2000:91):

> Patterns are a way for learners to recognise, order and to organise their world and are important in all aspects of mathematics. Learners recognise patterns in their environment and through experiences in school, and should become more skilled in noticing patterns in arrangements of objects, shapes, and numbers, and in using patterns to predict what comes next in an arrangement.

2.3.2 Example of number pattern problems

According to SP CAPS, there are two kinds of patterns in mathematics, namely, number patterns and geometric patterns (DBE, 2011). Ilany and Margolin (2010) classify number patterns according to two criteria: number patterns that deal with mathematical relationships between numbers or object sizes; and mathematical number patterns that deal with real-life situations. Szabo and Andrews (2017) emphasise that number pattern problems must unfold the mathematical competences necessary for solving the problem, rather than the recall of previously solved problems to derive the answer.

With number patterns, learners are given a sequence of numbers and they have to identify a pattern or relationship between consecutive terms in order to extend the pattern. Examples of number patterns are: -2, -5, -8, -11… or 3, 7, 11, 15, 19… where the second term depends on what happens to the first term, and the third term depends on the second term. Liljedahl (2004:4) argues that number patterns contain numbers, which cannot be transferred to

> non-numeric patterns without loss of some crucial property of the pattern … For example, the pattern 1, 2, 3, 4, 3, 2, 1 is transferable to a b c d c b a; and 1, 2, 1, 1, 2, 1, 1, 1, 2… can be transferred to a b a a b a a a b… without loss of the nature of the pattern. Thus, these two patterns cannot be defined as number patterns.

While geometric patterns are number patterns represented diagrammatically, the diagrammatic representation reveals the structure of the number patterns (DBE, 2011), e.g., flower patterns or matchstick patterns. Such patterns usually require some form of generalisation of patterns, usually in terms of algebraic symbols. Moreover, there are a variety of different number patterns in mathematics, including: “linear or arithmetic sequences and quadratic sequences” (DBE, 2011:22).

A number pattern or geometric pattern is linear sequence if each number is obtained by adding a constant increment to the previous number; while a quadratic sequence is a sequence of numbers in which the second difference between each consecutive term differs by the same amount, e.g., -3; 8; 23; 42; 65. To confirm that the sequence is quadratic, the second difference must be found. According to the CAPS, patterns in mathematics are about describing and working with numerical and non-numerical patterns (DBE, 2011). Patterns may be represented in concrete, visual words or symbolic forms using shapes, pictures and colours.

2.3.3 Benefits of number pattern problems

Number pattern activities enable learners to be creative and improve their algebraic thinking. This is achieved through open-ended questioning to develop a deep understanding of most topics in mathematics in order to prepare learners for further learning and to develop problem-solving skills. Number patterns can help learners to develop the ability to generalise, giving learners the opportunity to come up with a pattern rule or formula to find any term in a sequence. According to Mahlobo and Ntombela (2014), solving number-pattern problems “is an opportunity to generalise and to give general algebraic descriptions of the relationship between terms and their position in a sequence” (p. 186). In addition, number pattern activities can lead to the development of problem solving skills by foregrounding the evaluation of specific instances, systematically organising data, and inferring and generalising information (Barbosa et al., 2012).

Barbosa et al. (2012) believe that the ways learners use strategies to solve problems on patterns may significantly contribute to teaching decisions that increase mathematical knowledge in learners and, more especially, in algebraic thinking. Barbosa et al. (2012) indicate that the in-depth investigation of pattern in mathematics can add more value to the ability to problem solving. “Number patterns, geometric and pictorial patterns is more beneficial in building a positive and meaningful image of mathematics and add more value to the development of several skills related to problem solving and algebraic thinking” (Barbosa et al., 2012:274).

2.4 PROBLEM SOLVING IN MATHEMATICS

2.4.1 Previous inquiries on problem solving in mathematics

Since the 1970’s much research has focused on problem solving. According to Allevato and Onuchic (2008:61-62):

> At the beginning of the 1970’s, an investigation of problem solving and its implications for curricula was initiated. In 1976, at the 3rd International Congress on Mathematical Education, in Karlsruhe, Germany, problem solving was one of the themes addressed.

In 1980 the National Council of Teachers of Mathematics (NCTM) published a document recommending that problem solving be the main focus of school mathematics (NCTM, 2010) (see section 2.5). Kilpatrick et al. (2001:421) state that:

> Problem solving must be in the centre of the Mathematics Curriculum. Problem solving should be the site in which all of the strands of mathematics proficiency converge. It should provide opportunities for learners to weave together the strands of proficiency and for teachers to assess learners’ performance on all of the strands.

Thinking processes and problem-solving teaching topics received increased attention from psychological researches in the 1980's (Dwiyogo, 2016). In South Africa, CAPS states that "problem solving and cognitive development should be central to all mathematics teaching and learning. Learning procedures and proofs without a good understanding of why they are important leaves learners ill-equipped to use their knowledge in later life" (DBE, 2011:8). Moreover, problem solving is also part of every content area in the South African IP, SP and FET Mathematics curricula. Anderson (2009) recognises problem solving as skill that contributes to the processes that involve thinking and reasoning, interpretation of the give statement, analysing, predicting, evaluating and finding strategies to solve the problem.

2.4.2 Defining problem solving in mathematics

Problem solving is a process requiring learners to understand the situation, be able to identify key words to help to solve the problem, create strategies to solve the problem, and apply them to arrive at the solution. In other words, by studying problem solving in mathematics, learners become better prepared for many aspects of their lives after school, for example, trades, professional careers, and knowledgeable citizenship. In addition, Matlala (2015) states that a problem-solving approach to mathematics teaching may be a way to improve the quality and results of school mathematics. According to Lester and Kehle (2003), "problem solving involves coordinating previous experiences, knowledge, familiar representations and patterns of inference, and intuition in an effort to generate new representations and related patterns of inference that resolve some tension or ambiguity (i.e., lack of meaningful representations and supporting inferential moves) that prompted the original problem-solving activity" (p. 510).

Vula and Kurshumlia (2015) emphasise that problem solving should be part of mathematics teaching and learning since it affects the development and application of learners' knowledge and abilities in mathematics. Problems in mathematics could involve real-life questions where information is provided to perform computations to solve them (Depaepe et al., 2010). Peters (2011) defines a problem in mathematics as a "verbal description of a problem situation wherein one or more questions are posed of which the answers can be obtained through the application of mathematical operations to information available in the text" (p. 7). According to Boonen and Jolles

(2015), the term ‘problem’ is used to refer to any mathematics activity were the information is presented as a story problem rather than in the form of notation. Schumacher and Fuchs (2012) points out that mathematical problem-solving tasks are presented linguistically and do not require learners to do straight forward calculations, learners have to read with understanding and be able to interpret the problem, represent the model or equation from the written statement, and choose a solution strategy.

Problems are used in mathematics to relate mathematical knowledge and learning that occur in the classrooms to the kind of mathematics that might be encountered in real-world contexts (Hill, Blunk, Charalambous, Lewis, Phelps, Sleep, & Ball, 2008). The use of problem solving involving number patterns in this study was to help learners make sense of a given problem and to devise their own strategy to solve the problem. Problem solving, is considered to be the basis of applying and integrating mathematics in the real world. This provides opportunities to practice how to solve real-life problem situations when they are encountered.

2.4.3 Benefits and difficulties of problem solving for learners

Problem solving in mathematics contains several benefits for learners. Problem solving in mathematics is a learner-centred approach allowing learners to investigate and to explore mathematical ideas on their own (Depaepe et al., 2010). In addition, Căprioară (2015) claims that problem solving activities motivate learners to develop intellectually as it enhances creativity of learners and inform their mental behaviour to a better understanding of mathematics. The CAPS document for SP mathematics in South Africa also indicates that mathematical problem solving “enables learners to understand the world (physical, social and economic) around them, and, most of all, encourages teachers to teach learners to think creatively” (DBE, 2011:8). The NCTM (2010:52) claims that problem solving is a mathematical activity that could provide intellectual challenges to learners to improve their knowledge of and development in mathematics:

> By learning problem solving in mathematics, learners should acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve well outside the mathematics classroom. Being a good problem solver in a workplace or in everyday life, in general, is advantageous.

According to Lambdin (2009), the primary goals of mathematics learning are understanding and problem solving, and they are inextricably related because learning with understainding is best supported by engaging in problem solving. Otten (2010) states that a problem-solving skill provides the creativity and flexibility control of thought that address professional and post-secondary demands. In other words, by studying problem solving in mathematics, learners become better prepared for many aspects of their lives after school, for example, trades, professional careers, and knowledgeable citizenship.

The importance of problem-solving skills is also evident in the numerous studies on problem solving in mathematics that have been conducted in the South African context and also in other countries (Maluleka, 2013; Mochesela, 2007; Sepeng, 2010; Sepeng & Madzorera, 2014; Sepeng & Sigola, 2013; Sepeng & Webb, 2012). Maluleka (2013) found that grade 9 learners are attempting to solve mathematical problems without understanding, whereby communication, reasoning and recording processes appear to be key in assisting them. Mochesela (2007:iii) discovered that "exposing learners to a variety of problem-solving strategies improves their problem-solving performance and attitudes towards mathematics". Sepeng (2010:iv) found that the "discussion and argumentation techniques in the learning of mathematics problems have a positive effect on learners' ability to consider reality during problem solving". Similarly, Sepeng and Webb (2012) found that discussion as a teaching strategy has positive results in improving learners' problem-solving skills in mathematics classrooms, and their ability to make sense of real world problems. Sepeng and Madzorera (2014) revealed that "learners struggle with defining algebraic terms used in the problem statements as well as in instructional vocabulary" (p. 217).

2.5 LEARNERS' EXPERIENCES OF PROBLEM SOLVING

Bohlmann and Pretorius (2008) claim that the "conceptual complexity and problem-solving nature of mathematics make extensive demands on the reasoning, interpretive and strategic skills of learners, especially when these activities are done in a language that is not their primary language" (p. 43). Sepeng and Sigola (2013) also note that many learners encounter difficulties in reading and making sense of the mathematical problem solving and it appears that the learners struggle to comprehend given mathematics problems.

Kilpatrick et al. (2001) recommends that mathematics problem solving should be the site in which all of the learning exploration of mathematics proficiency converge. It should provide opportunities for learners to weave together the experiences of proficiency and for teachers to assess learners' performance. Mathematics problems challenge learners to "read and interpret the problem, represent the semantic structure of the problem and choose a solution strategy" (Schumacher & Fuchs, 2012:608). Learners cannot easily identify number patterns embedded in problems. Barbosa et al. (2012) state that learners "tend to use numeric instead of visual approaches and experience several difficulties when solving problems involving pattern exploration, especially when they have to generalise for distant values" (p. 291). Learners normally face difficulties in problem solving initially from translating algebraic problems into mathematical equations (Ahmad et al., 2010). Therefore, working with number patterns, in particular, can be difficult for learners as it is often embedded in problems. Maluleka (2013) notes that the biggest challenge in working with mathematical problems is that there is no correlation between real-life practices and mathematical problems which require understanding of the problem first, before attempting to respond to it.

"Mathematical problem solving is complex because it requires learners to read and understand written material that expresses numerical relations" (Tolar et al., 2012:1). Phonapichat et al. (2014) identified some key difficulties that learners experience in mathematical problem solving, namely, understanding keywords appearing in problems; interpreting keywords in mathematical sentences; and figuring out what information to assume and what information from the problem are necessary to solve the problems. Wang et al. (2016) noted that "many learners approach problems without thinking deeply about how irrelevant information detracts them from recognising a novel problem as belonging to a known" (p. 4).

Sepeng and Sigola (2013) observed that many students are experiencing difficulties in reading and making sense of mathematical problems and it appears that they are struggling to understand problem solving. Sepeng and Madzorera (2014) revealed that “learners struggle with defining algebraic terms used in problem statements as well as in instructional vocabulary” (p. 217). Sepeng and Kunene (2015) noted that learners have difficulties in reading the problem statement as well as generating meaning about the situation that the problem-solving task holds. Sepeng (2010) explored language and mathematics issues when English second-language learners solve mathematical problems and discovered that computational mistakes made by learners, especially numerical abilities, seem to result from the failure to use language efficiently to fix problems in a realistic scenario. Maluleka (2013) showed that learners fail to break the issue into smaller, meaningful components for better comprehension, as well as experiencing difficulties in connecting real-life issues with mathematical material learned in class. According to Sepeng (2010), language in mathematics plays a really vital role in understating the concepts which must be dealt with when solving mathematical problems. Sepeng and Sigola (2013) “investigated sources of errors that grade 9 learners make when they solve mathematical problems in a classroom” (p. 325). The findings revealed that learners cannot read or interpret the given mathematics problems and they cannot make sense of the problems:

> It appeared that learners struggle to comprehend the given problems and the errors exhibited by learners in the solution of problems appeared to be as a result of lack of understanding of mathematical vocabulary that is used in a problem statement.

Sepeng and Madzorera (2014) explored grade 11 learners’ views on the what causes the difficulty in comprehending mathematical problems, the majority of learners struggled to formulate the correct equations that were needed to answer the problem tasks given. The study indicated learners’ lack of both vocabulary knowledge and conceptual knowledge (or understanding) to form linear equations during problem solving. “The challenges that learners face when solving algebraic problems are caused by their inability to read and understand the problem statement itself” (Ellion, 2016:268).

Raoano (2016) revealed several “challenges faced by learners in solving problems; including language challenges, lack of strategy knowledge, lack of arithmetic skills and lack of reflective skills” (p. 74). Barbosa et al. (2012) noticed that learners experience difficulties when solving problems related to number patterns; especially when they had to give the general formula for the n^{th} term. It was found that learners were able to get better results for far generalisation questions. According to Boonen et al. (2013), solving problems seems to be very difficult if learners cannot identify the relationship between the known and the unknown, especially when the learners face challenges in understanding the problem text given. In some cases, if a learner fails to identify some operations required to solve the problem, they might have difficulties arriving at the acceptable answers (Vula & Kurshumlia, 2015).

As a mathematics teacher, I often observe many grade 9 learners having difficulties in solving problems in mathematics. In particular, many cannot solve algebraic problems or translate algebraic problems into mathematical equations due to a lack of understanding of mathematical concepts. Also, I believe, many learners do not have sufficient knowledge or exposure to strategies for solving problems on number patterns. Learners are often overwhelmed by problems not because they cannot solve these, but because they do not comprehend the problem statement due to a language barrier. Jupri and Drijvers (2016) confirms this challenge by stating that the main difficulties encountered by learners who deal with problems are to understand the problem, to formulate a mathematical model from the problem, to solve the problem expressed in the model, and to explain the meaning of their solution in terms of the original problem given.

2.6 STRATEGIES FOR PROBLEM SOLVING

Biddlecomb and Carr (2011) define strategies as groupings of mental or physical actions designed to solve a problem. Killen (2007) identifies strategies in problem solving as “giving problems to learners that would explore all aspects of principles and concepts depending on subject matter” (p. 260). Various strategies can be utilised to solve problems related to number patterns in mathematics. Learners need to be exposed to those strategies naturally in their classroom activities. Ellion (2016) argues that it is important for teachers to expose their learners to various problem-solving strategies and to help them carry out those strategies to solve problem tasks. In agreement, Duru et al. (2011) argues that teachers need to be informed about various strategies to solve problems in mathematics and they need to understand learners’ difficulties in order to implement these various strategies.

Sepeng and Sigola (2013) suggest that "strategies that can be used for solving problems must be made available to the learners particularly the use of models, pictures, tables, diagrams and other learning aids" (p. 332). Creating an environment where learners attempt to find variety of strategies for solving problems empowers them to explore alternatives and develops confidence in mathematics problem solving. Taber (2013) suggested that strategies for solving mathematical problems must be taught to learners. Learners need to be exposed to these strategies so that when they translate the text into mathematical expressions they know how to tackle the problem. "Mathematical problems are not simply computational tasks, but also require appropriate selection of strategies and decisions that lead to logical solutions" (Ahmad et al., 2010:357). Boonen and Jolles (2015:5) recommended that:

> Research should focus on the development of effective problem-solving instruction. Adequate problem solving instructional programs that teach learners to solve problems are still limited, or they have not been implemented in the educational practice of elementary schools.

It is important for learners to be assisted in gaining the necessary skills and strategies in order to successfully solve problems (Sepeng & Sigola, 2013). There are various models describing problem-solving strategies. Therefore, to solve a problem, learners need to understand its context and develop a strategy to solve it. Tambychik and Meerah (2010) indicate that identification of mathematics strategies is important in responding to difficulties in mathematical problem solving. These strategies could help to motivate, manage, and assist in improving the learners' skills in mathematics problem solving.

An understanding of the strategies involved in solving number pattern is critical in assisting learners to solve the problems. Patterns are normally found in nature, art, music, movement, and also in numbers. Therefore, problems related to growing pattern can be taken from real-life situations where learners have to work on the known stages to be able to complete the unknown stage. García Cruz and Martinón (1997) analysed the generalisation procedures created by high school learners and have identified the following primary categories: counting strategy, recursive approach, and direct proportional strategy, while Ibrahim and Rebello (2013) investigated the mental representation categories with which learners operate during problem solving of distinct formats of representational tasks. The results indicate that leaners work at three levels of mental representation, namely propositional; mental images and mental model representation.

Barbosa et al. (2012) define a counting strategy as counting the elements of a particular number or figural term in a pattern. Counting is always a successful strategy, but is only useful in solving near generalisation questions, because It involves counting the amount of shape parts or numbers, in order to calculate the expected number or shape drawing of the next figure. Example of counting strategy: to add 4+3, learners start with the "4", and then count up, "5, 6, 7". Furthermore, when adding negative numbers to negative number (counting backwards). Example: -4+ -2. This would be read as negative four plus negative two. First, you have to disregard the plus sign and recognise that you are subtracting that amount by the second negative number. You would therefore believe of this issue as "negative four minus two." Start at -4, then count (subtract) 2 more backwards. Your response will then be -6.

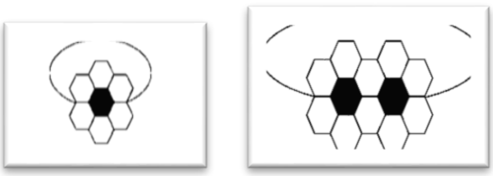
Becker and Rivera (2005) indicate that those learners who do not provide the overall formulae tend to begin with counting strategies; however, they lack the flexibility to attempt other methods and see possible links between distinct types of representation and overall strategies. Lannin et al. (2006) define a recursive approach as a connection between successive independent variable values in the scenario. Lannin et al. (2006) mention that learners may select a recursive strategy because they want to determine a general rule based on an understanding of a relationship that occurs in the situation. In recursive strategy leaners used the common difference between two consecutive terms of the sequence to solve some of the questions posed. Akkan's (2013) study on the learners' strategies and representations regarding generalisation patterns found that most of the learners who used the recursive strategy were able to find near and far terms accurately in the sequence.

An example of recursive strategy is as follows: A number pattern is a sequence or number list formed by a rule. Therefore, a number patterns can use any of the four operations (+, –, ×, ÷) or a combination of these. A recursive rule is to find the next number by doing something to the number. E.g. finding a general rule for the following pattern …. a leaner will 5; 13; 21; 29; 37…. first find the common difference which is

8. Therefore multiply by nth term, $n \times (8) = 8n$; n represents the number of term in the sequence and 8 is the common difference. Term 1 = $1 \times (8) = 8$, check whether it give us the first term of the sequence which is five, this does not give 5 which mean the learner need to subtract (-) 3 to give 5. This apply to the second term T2 =$2 \times (8) = 16$, this does not give me 13 which is the second, also subtract 3 to give us 13; therefore, the recursive rule will be have $Tn = 8n - 3$.

Barbosa et al. (2012) define direct proportion strategy as a situation, where the multiples of a specific term of a sequence is considered, and the problems presented in the test do not fit that model. Similarly, Lannin (2003) describe this strategy as using a portion as a unit to construct a larger unit using multiples of the units. For example, Joana needs 6 white beads and one black bead to make one flower. How many white and black beads will Joana need to make a necklace with 3 flowers? therefore, applying the direct proportion strategy a learner will say for 3 flowers will be (6 x 3 = 18 white beads and one three black beads). How many flowers will Joana be able to make if she uses 102 white beads? Joana will need 17 flowers with 102 white beads, the learners are counting how many white beads for the second flower, and looking for a rule that would work, therefore multiplied six which is the white beads by the number of flowers. Barbosa et al. (2012) indicate that learners who use the direct proportional strategy fail to make a final adjustment based on the context of the problem. While Ibrahim and Rebello (2013) state that learners with propositional mental representation tend to rotate and mechanically use definitions and manage mathematical formulations. There is no proof of the basic ideas being understood.

Ibrahim and Rebello (2013) define mental image representation as having the tendency to include a diagrammatic representation which may not be linked to the mathematical formulations used. For example; Joana likes to make necklaces using flowers. She uses white beads for the petals and black beads for the centre of each flower. The following diagrams shows a necklace with one flower and a necklace with two flowers, both made by her.



One flower two flowers three flowers

How many white and black beads will Joana need to make a necklace with 3 flowers? Draw a diagram of a necklace with 3 flowers. Although they are aware of the relationship among the white and the black beads to make a flower and may recognise the applicability of the qualitative method (taking the number of whites beads and the number of flower) to solve the problem, but they prefer manipulating equations. Meaning that this leaner will see this flowers as a disjoint unit which does not make a linear pattern. In the mental image representation Ibrahim and Rebello (2013) found that "students who construct a mental image handle the (generated) visual representations in isolation" (p. 14).

According to Ibrahim and Rebello (2013), representation of mental models provides a means to connect the syntactic (mathematical) and structural (visual) aspects of the task under consideration, it allows interpretation and understanding When leaners had to interpret the visual pattern, they will formulate a description referring to each flower as a joint unit to form a linear pattern. Each flower shares two white beads to have a complete flower, four white beads were added to the end of each prior flower to create a new flower, and then added two beads to give a complete flower, therefore a general rule will be $T_n=4n+2$ the variable n represent the number of flowers.

2.7 PROCESSES FOR PROBLEM SOLVING IN MATHEMATICS

Polya (1945) presents four phases of problem solving, focusing on supporting the teaching of problem-solving skills. The four phases are: understanding the problem, devising a plan to solve the problem, implementing the plan, and reflecting on the problem. Therefore, as learners are presented with a problem they have to interpret the problem in order to understand what the problem is about, devise a method to solve it to achieve the results, and then analyse the results to see if it is an acceptable solution to the problem presented. According to Singer and Voica (2013), Polya's strategy focuses on describing how teachers can help learners develop skills in problem solving, whereas this study intends to look for a strategy that describes different phases the learners utilise when solving problems. Krulik, Rudnick, and Milou (2003) regard problem solving as the means by which individuals take the skills and understandings previously developed and apply them to unfamiliar situations. The process begins with the initial confrontation of the problem and continues until an answer has been found and the learner has double checked the solution.

Johnston (1994) viewed the problem-solving process as comprising six critical steps, namely: representing the unknown by a variable, breaking down the problem into small parts, representing the pieces by an algebraic expression, arranging the algebraic expression in an equation, solving the equation and checking the solution. What challenges most learners in problem solving is that learners cannot solve algebraic problems or translate algebraic problems into mathematical equations due to the lack of understanding of mathematical concepts. Therefore, if learners cannot represent the unknown by a variable they cannot make sense of the problem.

Maluleka (2013) observed that the greatest difficulty in working with problems is to translate real-life issues into mathematical problems needing first knowledge of the issue before trying to react to it. Boonen et al. (2013) argue that solving problems using words seems to be very difficult if the learners cannot relate the known and unknown, especially when the learners face challenges in understanding the given problem text. This study, however, focuses on a framework to assist learners to move their attention from the wording of the problem to the relations among the data and the mathematical operators by means of decoding and identifying the key word that could assist them in making sense of the problem.

Ilany and Margolin (2010) argue that many learner difficulties in solving problems emerge from understanding the text literally and mathematically. Therefore, these authors developed a nine stages model for mathematical problems to address the gaps between natural language and mathematical language in problem solving, namely:

> Reading the problem, understanding the linguistic situation, understanding the mathematical situation, matching the mathematical situation to the linguistic situation, screening the ideas, building a mathematical model, finding the solution and control.

They claim that this “nine-stage instruction and learning model transforms into a complex thought process when fully understood and internalised” (Ilany & Margolin, 2010:142).

The difficulties faced by learners using this strategy are more noticeable during the first phase in problem solving. Learners are not able to transform the problem into mathematical sentences. Ahmad et al. (2010) point out that learners usually encounter difficulties in solving problems because problems are actually story problems and the learners should be given guidance on how to relate between the known and the unknown. The study is looking for the framework that will help to understand the problem via a generated mental model made up of images, configurations, drawings, schemes, and graphs (Singer & Voica, 2013).

2.8 CONCEPTUAL FRAMEWORK OF THE STUDY

Singer and Voica (2013) have identified a conceptual framework for the problem-solving process, which has been adopted for this study (see Figure 2.1). This framework reflects the learners' natural disposition towards strategies for solving problems, and offers "insight for more effective learning of mathematical problem solving and can be used in problem posing and problem analysis in order to devise questions more relevant for deep learning" (Singer & Voica, 2013:11). It is designed to help learners to read the problem with understanding by, firstly, identifying the key words that could help them solve the problem. The framework describes the strategies that could help learners to make sense of the information on the given text, and ultimately to arrive at the correct solution. The framework describes four phases that could help learners to make sense of the information for effective problem solving, namely, decoding, representing, processing, and implementing.

Singer and Voica (2013) argue that "when the process of solving is successful, from the text of the problem (the wording) to its solution, a solver need to work from the solution to the initial problem" (p. 11). Thus, a solver needs to interpret the solution in relation to the given problem (the wording). This conceptual framework for problem solving involves understanding the solution from the original statement within which the problem is given, meaning that the solver should be able to connect the answer of the problem with the problem statement. The bold arrow in this figure 2.1 illustrates the closing of the solving cycle.

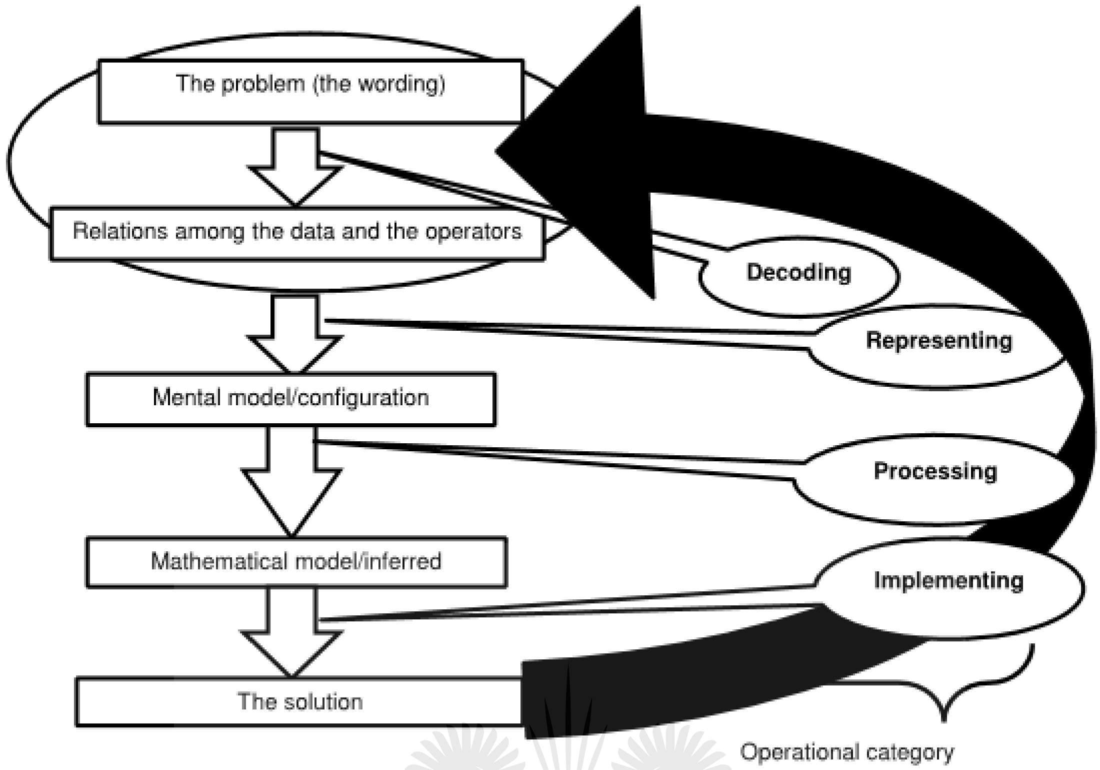


Figure 2.1: PS conceptual framework of Singer and Voica (2013:13)

2.8.1 Phase 1: Decoding

Clement (2008) defines decoding as the ability to interpret the given statement and to identify the key words in order to solve the problem. In order to be a successful problem solver one needs to know how to decode the problem. This decoding process, according to Singer and Voica (2013), involves understanding the wording given on the problem statement which will leads in describing clearly what the problem is about. In decoding, the problem text includes a background theme, (numerical) data, operators (or operating systems), information and operator limitations and limitations involving at least one unknown parameter value (Singer & Voica, 2013).

The background theme represents the problem in detail, the operating schemes are actions suggested by the text of the problem, which could be mathematical operations that will lead the problem solver to solve the problem, e.g., plot, draw, trace, intersect, cut, addition or subtraction (Singer & Voica, 2013). The data and operators are limitations that state the background theme's relationships with information and operators (operating schemes) (Singer & Voica, 2013).

According to Singer and Voica (2013), when a problem is given (the wording) the problem solver makes a connection between the data and the operators, which requires the solver to convert the given statement of the problem into more understandable language. Usually number patterns are actually numbers that require learners to make connections between the known and the unknown in order to solve the problem. According to Lorenzo (2005), by breaking the issue into sections enables students to concentrate on a few ideas at a moment when they need to work on a fresh unknown, thus reducing the probability of making errors.

2.8.2 Phase 2: Representing

For this phase, according to Singer and Voica (2013), the problem solver must represent the problem in the text by using a suitable mathematical image and provide a mathematical equation that can help solve the problem. Therefore, the learner creates a mental image to solve problems and this may produce mathematical models such as formulas, equations, pictures, and drawings. Appropriate depiction of the issue shows that the problem solver perceived the problem and serves as a guide to the solution plan for the learners. Apprentices with math issues will find it difficult to solve them (Sajadi, Amiripour, & Rostamy-Malkhalifeh, 2013).

The representing process in mathematical problem solving permits learners to visually relate various types of information given in the problem statement to help them determine which mathematical expressions are useful in solving the problem (Jan & Rodrigues, 2012a). According to Singer and Voica (2013), when a problem solver reads or hears the problem text with understanding, the problem statement should lead him/her to a mental model. “The mental model is a structured ensemble of mental representations induced by the wording, which is oriented by the purpose of solving the problem” (Singer & Voica, 2013:9). The mental model might also consist of translating the problem into a language that is more understandable to the solver.

The mental model in this study could take the form of images, drawings, schemes, constructions, or general formula. “Therefore, as a result of reading/hearing and understanding the wording, the solver builds a mental model that is expressed via images, movements, physical objects, schemes, or sentences in a more familiar (internal) language" (Singer & Voica, 2013:9). Tambychik and Meerah (2010) pointed out that mathematical language and representing are important in the process of understanding the problem. However, lack of these skills causes difficulties in bringing meaning to the information stated in the mathematical problems.

2.8.3 Phase 3: Processing

During the processing phase, the problem solver processes what is already known about the problem to test the model or the equation that is currently being chosen. Tambychik and Meerah (2010) state that, during this phase, the problem solver "uses a mental model suggested by the problem and personal mathematical competence to identify a mathematical model that can be associated with the problem" (p. 11). Ilany and Margolin (2010) state that processing in problem solving means that the problem solver need to be able to change the information into an algebraic equation. This can only be done by understanding the keywords in the problem, and the mathematical operations within the problem statement.

During the PP the problem solver need to use his/her mathematical knowledge and understanding to identify the mathematical model that is relevant to the problem. The PP is related to the ability to identify the mathematical model associated with the question. The question can be an equation, a system, the steps of a graphical representation and various computing algorithms (Singer & Voica, 2013). Therefore, in this study, processing is viewed as the exploration of possible situations through drawings, finding of the general formula of sequences, finding of the relevant equation of the n^{th} term of the sequence, using of the correct mathematical operation sign, or the transposition of the text into an equation.

2.8.4 Phase 4: Implementing

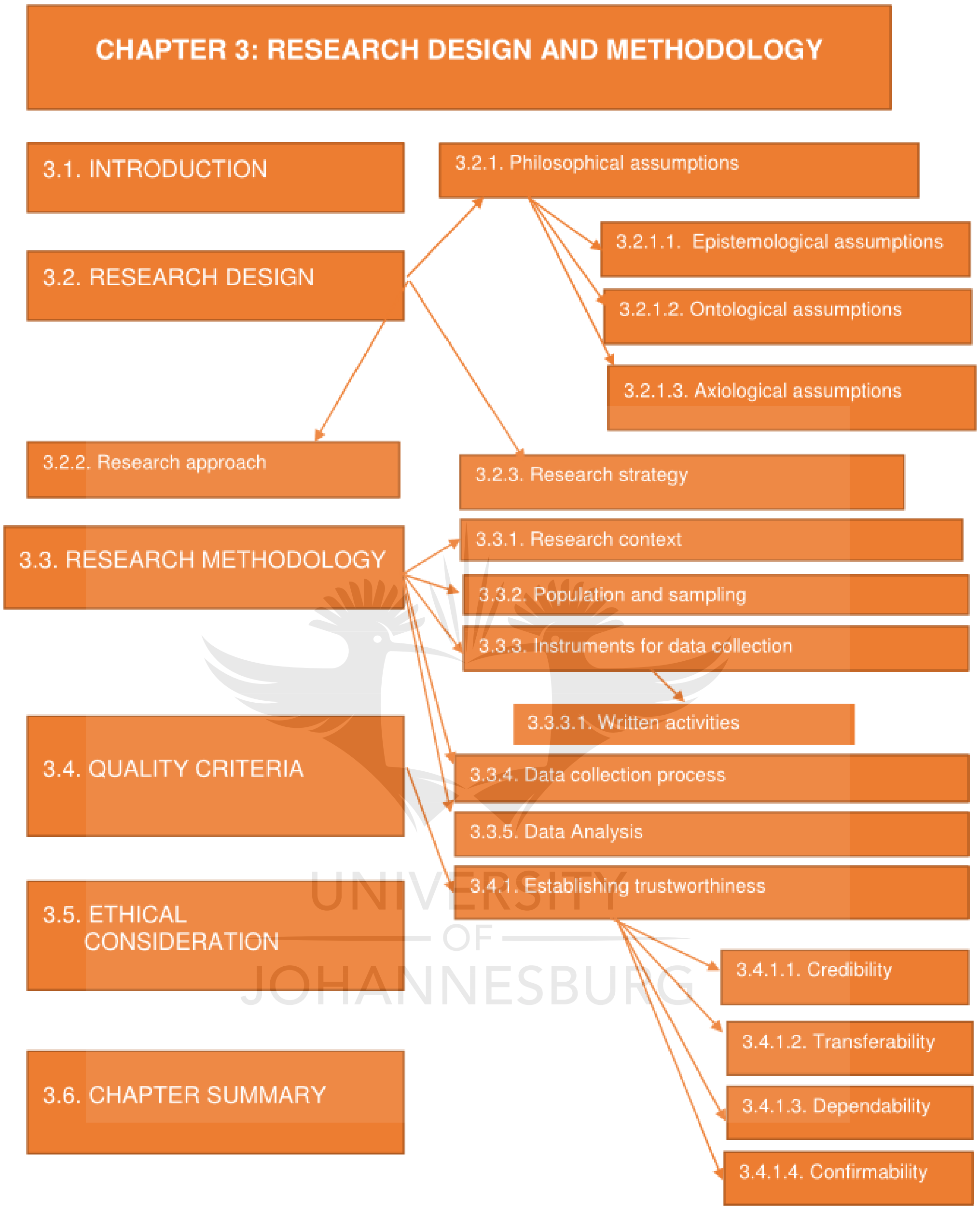
Implementation refers to the process of implementing the solution to the problem in problematic situations (D'Zurilla & Nezu, 2010). During this phase, the problem solver applies the solution strategy that has been used to arrive at the solution of the problem. According to Singer and Voica (2013), this phase focusses on the "application of techniques that are specific to the found mathematical model and adaptable to the given particular situation, with the purpose to obtain final results for the problem" (p. 11). Singer and Voica (2013) argue that "when the process of solving is successful, from the text of the problem (the wording) to its solution, a solver need to work from the solution to the initial problem" (p.11). Therefore, the problem solver needs to interpret the solution in relation to the given problem statement (the wording). This will involve understanding the solution from the context within which the problem is given. The problem solver should be able to connect the formal solution of the problem with the initial data.

Singer and Voica (2013) indicate that the implementation phase is "based on the application of already mastered techniques to a certain determined situation in certain known conditions" (p. 9). These techniques might involve, for example, recognising the different ways of proving the solution; minimising the values that do not satisfy the constraints of the problem; using helpful constructions, substitutions; or by using a known algorithm.

2.9 CHAPTER SUMMARY

This chapter reviewed literature concerning number pattern problems. A discussion on the nature of mathematics, including the definition of mathematics; problem solving; number patterns; the design of a number pattern problems; learners' experience of problem solving; strategies used in number patterns; and lastly the PS conceptual framework developed from Singer and Voica (2013), was provided. Mathematics is defined as a "human activity that involves observing, representing and investigating patterns and qualitative relationships in physical and social phenomena and between mathematical objects themselves" (DBE, 2011:8). Beliefs about the nature of mathematics provide a foundation for teacher's methods of teaching and how learners learn mathematics. Mathematical problems challenge learners to "read and interpret the problem, represent the semantic structure of the problem and choose a solution strategy" (Schumacher & Fuchs, 2012:608). Learners cannot easily identify number patterns embedded in problems.

Number patterns related to problem solving in this study is a process of looking out for numbers in the given sequence and forming a pattern, which will allow the problem solver to come up with a more general solution that can be applied in every given situation. Various strategies can be utilised to solve problems related to number patterns in mathematics. Learners need to be exposed to those strategies naturally in their classroom activities. The framework describes four phases that could help learners to make sense of the information for effective problem solving, namely, decoding, representing, processing, and implementing. According to the framework of Singer and Voica (2013), learners should be able to relate the initial wording of the problem with the implemented solution of the problem after they have solved the problem. Therefore, teachers should encourage learners to reflect on their solutions to check whether what they have answered is what the question required. Chapter 3 will discuss the research design and research methodology adopted for the study.


CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY
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3.2.1.1. Epistemological assumptions
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CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY

3.1 INTRODUCTION

This chapter discusses the research design and research methodology used for the study. It starts with a discussion of the research paradigm, which consists of the following philosophical assumption: epistemology, ontology, and axiology. Each philosophical assumption is explained, and the relationships between them explored. The research paradigm helped to shape an understanding of the interconnectivity of real-life elements in the study, and assisted in planning and carrying out the research. The research approach used in this study was qualitative in nature, since the study sought to interrogate grade 9 mathematics learners' strategies in solving number pattern problems, and to assist in setting guidelines to improve future learning of problems involving number patterns. A qualitative exploratory case study research design was used for the study, as it allowed participant strategies to be investigated for the solving of number pattern problems in three rural schools. The research methodology adopted for the study is discussed in more detail below, including an explanation of the research context, population, and sampling procedures. Thereafter, the data collection process and data collection instruments are discussed followed by an explanation of the data analyses processes. Finally, the chapter concludes with a discussion of the quality criteria used in the study, as well as the ethical measures considered.

3.2 RESEARCH DESIGN

Mouton (2001) describes "the research design as a plan or blueprint of how the researcher intends to conduct the research" (p. 55). In addition, Leedy and Ormrod (2010) describe a research design as a plan where you link research method and procedure used to achieve reliable and valid data for analysing and interpreting the findings of the study. This study's research design provided the researcher with the research paradigm, the epistemological, ontological, and axiological assumptions; as well as the most suitable research approach and research methods to use. The research design gives a clear description of the research methodology by indicating how the researcher will conduct the fieldwork; and it gives a clear indication of the data collection process, methods and procedures (Peters, 2011).

The design also sets the basis for interpretation and analysis of data. Therefore, the qualitative research design chosen for this study was an exploratory case study.

3.2.1 Philosophic assumptions

The epistemology, ontology, and axiology helped define the research paradigm for this study. The epistemological assumption is concerned with addressing the facts by asking what acceptable knowledge is. The ontological assumption deals with the nature of reality. Finally, the axiological assumption focuses on the value of the study and biases affecting information gathered from the field (Creswell, 2013). Each of these philosophical assumptions is discussed next as they were used in this study.

3.2.1.1 Epistemological assumptions

Matthews and Ross (2010) define epistemology as "the theory of knowledge and how we know things" (p. 18). Epistemology defines what acceptable knowledge is for a field of research and what information is known to be true – for this study, mathematical problems related to number patterns in mathematics. "The interpretive epistemology is one of subjectivism which is based on real world phenomena" (Scotland, 2012:11). This study seeks to explore learner strategies used in solving number pattern problems, and learner views and perspectives on these strategies. Thus, theories about learner strategies have an epistemological dimension since they describe how we acquire knowledge in problem solving, and what is important both to the theory and practice of mathematics education.

An interpretivist paradigm, with an "epistemological position that prioritises people's subjective interpretations and their own actions" (Matthews & Ross, 2010:28), provided an in-depth insight into this inquiry. "An interpretive approach sees people, and their interpretations, perceptions, meanings and understandings, as the primary data sources" (Mason, 2002:56). Goldkuhl (2012) states that "the aim of understanding the subjective meanings of persons in studied domains is essential in the interpretive paradigm" (p. 4). The interpretive paradigm, as used in this study, sought to explore the learners' strategies and their views and perspectives on these strategies. The learners were interviewed to provide them with an opportunity to describe their strategy and to allow researcher interpretation and understanding of the words, views and explanations of the strategies they used to solve number patterns-related problems.

3.2.1.2 Ontological assumptions

Ontology refers to the way in which the social world is seen and what can be assumed about the nature and reality of the social phenomena (Matthews & Ross, 2010). Similarly, Ponterotto (2005) explains ontology as the “nature of reality and being” (p. 130). Ponterotto (2005) indicates that reality is subjective and influenced by the context of the situation, which is the individual’s experience and perceptions. This study used the ontological position that learners’ knowledge, views, understandings, interpretations, experiences, and interactions with number pattern problems are important properties of their social reality.

3.2.1.3 Axiological assumptions

Axiology “concerns the role of researcher values in the scientific process” (Ponterotto, 2005:130). The axiology in this study is about the value of problem solving, such as good and bad, moral and immoral; thus, questions about what the meaning of problem solving is, and how we should value it. Ponterotto (2005) highlighted that axiological assumptions “maintain that the researcher’s values and lived experience cannot be divorced from the research process, the researcher should acknowledge and state his or her values, but not eliminate them” (p. 131). Therefore, the researcher values number pattern problems in mathematics as it allows creative thinking, and learners are able to develop new mathematical reasoning skills when solving the problem. A benefit of number pattern problems includes a learner-centred approach in which learners investigate and explore mathematical ideas on their own (Verschaffel, Greer, & De Corte, 2000). By studying problem solving in mathematics, learners become better prepared for many aspects of their lives after school, for example, trades, professional careers, and knowledgeable citizenship. Matlala (2015) states that by using problem solving in mathematics the quality and results of school mathematics could be improved in future. In addition, Căprioară (2015) indicates that problem solving stimulates motivation towards intellectual progress; it sharpens learners’ creativity and applies mental behaviour that aims to build a better structure of learners’ knowledge.

3.2.3 Research approach

A qualitative research approach was chosen because it provides a rich description of the phenomenon under investigation (Merriam, 2009), namely constructing an understanding of the strategies that learners use when solving number- pattern-related problems. In this study, the qualitative methods used examined learners' words, views, thinking, and their perceptions about number pattern problems in descriptive ways, and with the intention of representing the situation as experienced by the learners. Therefore, a qualitative research approach helped the researcher to understand the embedded actions of learners' strategies in more detail. "Qualitative research is concerned with the opinions, experiences and feelings of individuals producing subjective data. It describes social phenomena as they occur naturally and understanding of a situation is gained through a holistic perspective" (Kakulu, 2014:6).

3.2.4 Research strategy

The case study design allowed the researcher to investigate grade 9 mathematics learners' strategies in solving number pattern problems in three rural schools. Yin (2009) describes a case study as an empirical investigation that explores a modern phenomenon in depth and in its real-life context, particularly when the limits between phenomenon and context are not obvious.

Rule and John (2011) state that "a case study is a systematic and in-depth investigation of a particular instance in its context in order to generate knowledge" (p. 4). In addition, Rule and John (2011) show that a case study strategy enables you to examine a specific example in excellent depth, rather than superficially examining various cases.

Therefore, in the context of this study, the use of a case study refers to the process of conducting an investigation to understand learner strategies in three different schools. Such understanding assisted in the final written documents produced from the study, and the setting of guidelines to improve the future teaching and learning of problems regarding number patterns.

An exploratory qualitative case study inquiry was deemed most appropriate for this study. Rule and John (2011) indicate that "an exploratory case study often examines a phenomenon that has not been investigated before and can lay the basis for further studies" (p. 8). This exploratory case study assisted in developing insight into the strategies that learners use in solving number pattern problems in order to develop models or theories to improve the future teaching and learning of problems regarding number patterns (Kakulu, 2014).

The advantage of the exploratory case study used in this research was that open-ended questions required learners to elaborate on their strategies, and gave them the opportunity to respond in their own words and explain what they wrote, rather than forcing learners to choose from fixed responses such as 'yes' or 'no' questions. According to Neuman (2011), exploratory research is used when the subject is very new, or if we know little or nothing about it. As mentioned in Chapter 1, rural schools have been marginalised in South Africa; therefore, more research needs to be done, and reported on, in this context. The overall goal of this study fits well with the general intention of exploratory research, as it sought to provide a basis for formulating more precise questions about the strategies that rural high-school learners use when solving number pattern problems in mathematics, and which can be used to conduct further research.

A multiple-case study was used to find the strategies used by grade 9 mathematics learners to solve number pattern problems. According to Ary, Jacobs, and Sorensen (2010), "multiple case studies use several cases selected to further understand and investigate a phenomenon, population or general condition, while single case studies may not provide a detailed understanding of phenomenon being investigated" (p. 455). In this study the unit of analysis was learners in three public schools in the rural Capricorn district of Limpopo Province at Lepelle Nkumpi municipality. Using learners from three schools provided data on different types of strategies used and a better understanding of those strategies. Data were collected from two sources, namely, 1) a written activity on number pattern problems to establish the strategies used; and 2) semi-structured interviews with participants to obtain an in-depth understanding of the strategies used.

3.3 RESEARCH METHODOLOGY

Research methodology is “used to gather and analyse data related to the research question or hypothesis” (Crotty, 2003:3). This section will include an explanation of the research context, population, sampling procedures, and a discussion on the data collection instruments and data analysis methods.

3.3.1 Research context

Data were collected from three Quintile 1 schools in the Capricorn district of Limpopo Province. The DBE classifies schools according to quintiles. Quintile 1 schools are in deep rural areas and mostly inadequately resourced in terms of teaching and learning materials, and poor infrastructure. The Capricorn district, where this study was conducted, has a total land area of 21 704 km^2, and the three rural schools were 5 to 10 km apart in the Lepelle Nkumpi Municipality of the Lepelle circuit. Figure 3.1 gives a map of the Limpopo Province showing the Capricorn district in the Lepelle Nkumpi Municipality. The Capricorn district is one of the five districts of the Limpopo Province of South Africa. Prior to the study the researcher collected background information and demographic details of schools from the circuit office (see section 4.2). The map in Figure 3.1. shows the Capricorn district in the Lepelle Nkumpi Municipality were the Quintile 1 schools are situated.

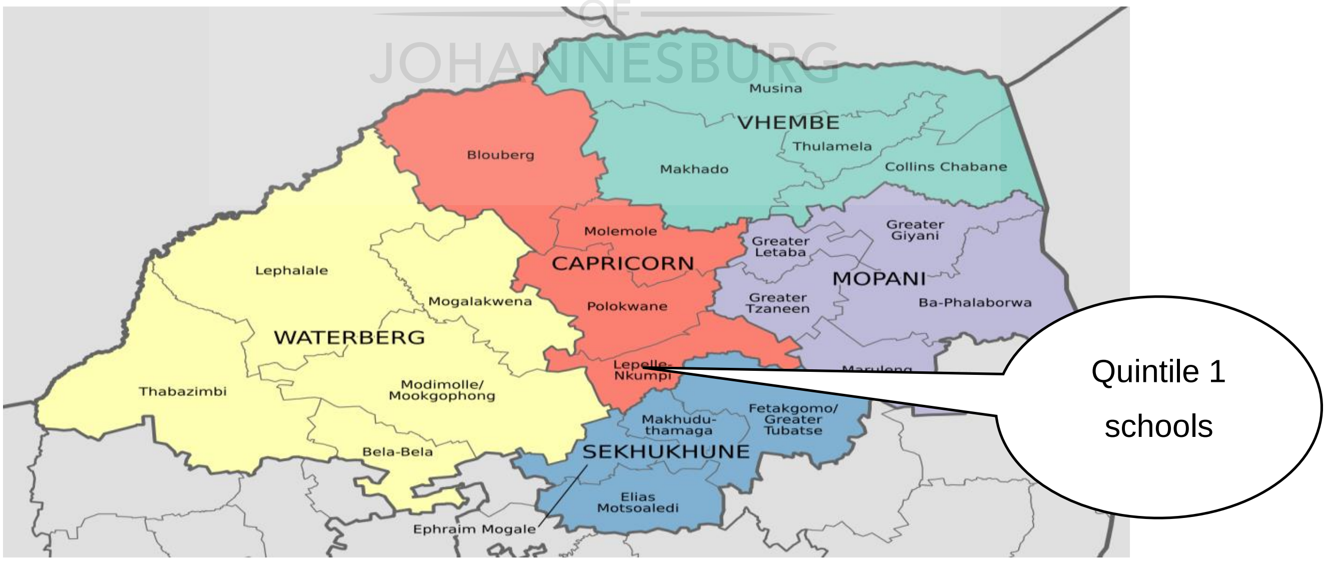


Figure 3.1: A map of Limpopo Province showing the Capricorn district in the Lepelle Nkumpi Municipality

3.3.1.1 School A

School A was established in 1971 with only two classroom blocks. The school was renovated in 2012 and now consists 15 blocks including a library and a laboratory. It was the first secondary school in the village to be used for this study. The school's enrolment at the time of the study was 320 learners, with 8 teachers including the principal. The school had 31 grade 9 learners with 20 females and 11 males, but only 20 females and 10 males participated, thus, in total 30 learners participated. The ages of the selected learners ranged from 15–17 years. The grade 12 pass rate in 2017 was 58%. This school was classified as an underperforming school in 2018. According to the DBE, this means that the school's grade 12 results were below the benchmark of 65%.

3.3.1.2 School B

School B was established in 1990, and was the second secondary school in the village to be used for this study. At the time of the study, the school had an enrolment of 118 learners, with 5 teachers including the principal. The school had one class of 33 grade 9 learners with 21 males and 12 females. The age of learners ranged from 15–16. Ten females and 20 males participated in this study, thus, in total 30 learners participated. The grade 12 passed rate in 2017 was 45% and it was declared a chronically underperforming school in 2018, meaning the school had been underperforming for three consecutive years (according to the DBE).

3.3.1.3 School C

School C is currently the largest in the village in terms of learner enrolment, but did not have sufficient equipment or classrooms to cater for all learners. This was the third secondary school used for this study. The school was established in 1994 and the grade 12 pass rate for the school has never been less than 80%. In 2017, the grade 12 passed rate was 90%.

At the time of the study, the school had an enrolment of 884 learners and 24 teachers (including the principal and the deputy principal). The school had 142 grade 9 learners evenly distributed in two classrooms; grade 9A with 63 learners (27 are males and 36 females) and grade 9B with 79 learners (38 are males and 41 females). The ages of learners ranged from 14–16. Fifteen females and 15 males participated in this study, thus, in total 30 learners participated. Table 4.2 gives a summary of the selected schools and demographic information about the participants.

3.3.2 Population and sampling procedures

The population for this study comprised grade 9 mathematics learners in Quintile 1 schools in the Capricorn district of the Limpopo Province. The sample comprised 90 learners from three rural schools (A, B and C). The researcher purposively selected 30 grade 9 mathematics learners from each of the three sampled schools in order to have equal distribution of participants among schools – a total of 90 learners. Kunene (2014) defines "sampling as a process of identifying relevant participants, subjects or people who are rich informants according to the researcher from which data can be collected" (p. 20). Similarly, Rule and John (2011) define sampling as where individuals are intentionally selected as research participants because of their suitability in advancing the purpose of the research. Therefore, participants were chosen because of their relevant knowledge, interest and experience in relation to the case.

The schools were selected using purposive sampling. Purposive sampling, as Merriam (1998) indicates, is mostly based on the assumption that the investigator wants to discover, understand and gain insight. The researcher, therefore, selects a sample from which most can be learned. According to Creswell (2013), purposive sampling involves selecting participants based on characteristics that make them holders of the information needed for the study. Kakulu (2014) also indicates that, in purposive sampling, the researcher selects the population according to the purpose or aims of the study using categories such as age, gender, status, role or function in an organisation. Moreover, in purposive sampling, researchers intentionally select individuals and sites that will help them to learn about, or understand, the central phenomenon (Creswell, 2008). In the context of this study, the researcher purposively selected participants that were doing mathematics in grade 9, as they were supposed to have knowledge of number pattern problems as prescribed in the CAPS document for the specific grade. The researcher wanted to investigate and understand grade 9 mathematics learners' strategies in solving number pattern problems to assist in setting guidelines to improve future learning of problems regarding number patterns. Table 3.1 indicates the criteria used for selecting the participants.

Table 3.1: Criteria used for selecting participants

Criteria
Public schools
Rural schools
Quantile-one schools
Voluntary participation
Grade 12 performance
Grade 9 mathematics learners

The selection of schools was based on the two criteria: 1) public rural schools; and 2) schools in rural areas that continue to experience challenges, including a lack of necessary resources. Learners from the selected schools were sharing textbooks, workbooks, and other mathematical instruments.

According to the principals of those schools, their learners were from various backgrounds, with the majority coming from poor socio-economic backgrounds, and with parents who were uneducated. The schools lack parental involvement because most parents are employed in faraway large cities, such as Johannesburg. Quantile-one schools, as selected for this study, are classified as no-fee schools according to the South African School Act (84 of 1996), and are situated in deep rural areas (Dass & Rinquest, 2017).

The selected schools enrol learners from grade 8 to grade 12. Two of the selected schools were declared as underperforming schools according to the DBE[1], while the other school's grade 12 performance results were at a higher level, as they had not obtained below 65% for the past number of years. Thus, the grade 9 mathematics learners who participated in this study came from selected public rural schools. They also participated voluntarily in the study.

The motivation for choosing Quintile 1 rural schools in the Capricorn district in Limpopo was that these schools were the most in need of assistance in mathematics. They are situated in deep rural areas with inadequate resources in terms of teaching and

[1] Underperforming schools are schools that obtain less than the DBE's benchmark of 65% in the grade 12 final results.

learning materials and infrastructure, and the majority of schools in the district were classified as underperforming schools according to their grade 12 results since 2016. The schools also comprised a convenience sample as the researcher lives and work in the Capricorn district. Moreover, the selected schools shared common features, such as poverty, absence of parental and community engagement, and a number of curriculum challenges. This study also focused on these rural schools due to the context, i.e., that the learners' needs differ from those in suburban schools; and mathematics teachers from these schools receive less professional training than those from suburban schools. Adedeji and Olaniyan (2011:21) state:

> Teaching is often of poor quality and is poorly supported in rural schools. Isolated conditions in rural areas fail to attract high-quality teachers. This situation is made worse by the fact that poor infrastructure obstructs support from advisory agencies

3.3.3 Instruments for data collection

3.3.3.1 Written activity

A written activity on number pattern problems was used to collect qualitative data for this study (see Appendix A). According to Matthews and Ross (2010), "documents are often readily available and frequently contain large amounts of information and they are socially constructed, they can tell the researcher more than just the information that they contain" (p. 277). The aim of the written activity was to promote higher levels of learner engagement in number pattern problems, creativity, and the utilisation of different strategies to solve the problems. The setting of the activity was informed by the SP Mathematics CAPS. Furthermore, the activity allowed participants to engage in the process of problem solving by reaching different phases as outlined in the PS conceptual framework of the study. It is important to note that by its nature, this activity requires learners to engage in all phases to arrive to a solution.

The first phase to reach was the DP. The DP indicated the ability to move focus from the wording of the problem (understanding the text) to the relations among the data and the operating schemes that can be deduced from the given constraints (Singer & Voica, 2013).

The second phase to reach was the RP. The RP concerned the ability to understand the problem via a generated mental model. The mental model was made of images, configurations, drawings, schemes and graphs. The mental model also required a rephrasing of the problem statement into a language more accessible to the solver (Singer & Voica, 2013).

The next phase was the PP. The PP related to the ability to identify the mathematical model associated with the question. The question can be an equation, a system, the steps of a graphical representation, or various computing algorithms (Singer & Voica, 2013).

The last phase to reach was the IP. The IP indicated the ability to apply techniques that are specific to the found mathematical model and adaptable to the given situation. The IP serves to obtain the final results for the problem (Singer & Voica, 2013).

The activity was learner centred and required participants to come up with their own strategies to solve the problem (see Appendix A). The written activity in this study intended to investigate the participants' strategies they engaged in when e solving of number pattern problems that deal with mathematical relationship between consecutive numbers and real-life situations. Thus, the data collected from the responses to the written activity were used to answer the first research question, namely: What are the strategies grade 9 mathematics learners engage in when solving number pattern problems?

This activity requires an understanding of numeric patterns and geometric patterns. This type of content requires learners to have a high level of understanding in order to arrive at a strategy to solve the problem. Barbosa et al. (2012) state that there are two main reasons for including linear pattern presented in numbers and pictures: "it allows application of a diversity of generalisation strategies, i.e. numeric, visual or mixed and the observation of the structure of the figure is enough to determine the general rule of the pattern" (p. 282). Szabo and Andrews (2017) also emphasise that problem-solving activities must utilise the mathematical competences necessary for solving the problem, rather than the recall of previously solved problems to obtain the answer to the next problem.

The written activity consisted of two questions adapted from Barbosa et al. (2012). These questions were aligned with the SP CAPS, which states that “investigating number patterns is an opportunity to generalise and to give general algebraic descriptions of the relationship between terms and their position in a sequence and to justify solutions” (DBE, 2011:126). Furthermore, the SP CAPS states that, in order to complete the sequence of numbers, learners have to identify the constant difference between consecutive numbers. The SP CAPS also indicates that learners are requested to prepare the rule that can describe the relationship between the numbers in this sequence; and they must be able to use that rule to find the n^{th} term in that sequence (DBE, 2011). Similarly, Singer and Voica (2013) state that “the problem solver needs to represent the problem in the text by using appropriate mathematical image and come up with mathematical equation that can help with solving the problem” (p. 11). Barbosa et al. (2012) argues that the kinds of activities that involve generalisation allow teachers to analyse the learners’ strategies used to solve the problem, as well as their level of understanding.

In question 1, as illustrated in Table 3.2, the researcher asked learners to continue the given sequences by indicating the next term of the sequence, and also to continue the sequence to n^{th} term. This question required that learners had knowledge and understanding of linear patterns and decreasing patterns. Thus, this question involved near and far generalisation and allowed the researcher to analyse how learners understood patterns in different contexts, as well as the strategies they used. Moreover, the researcher intended to analyse participants’ ability to interpret and continue sequences and to recognise the pattern.

Table 3.2: Question 1: Numeric patterns

1.1 Complete the table by indicating the next terms (f(x)) of the sequence. If the sequence is continued to the n^{th} term, please write the general formula for the n^{th} term.

x	1	2	3	4	5	6	7	8	N
$f(x)$	-2	-5	-8	-11	-14				

For question 1, participants were expected to give a decreasing pattern by noticing that the values of *f(x)* which shrank, by a constant difference each time to get the next term. Participants needed to explore numbers by using mathematical operators, drawing, words, and symbols. When creating decreasing patterns, learners first need to choose a starting point, and then decide on the amount by which the number decreases.

During the interview, participants had to describe their pattern in question 1 of the written activity by clearly explaining how the value of the terms changes from one term to the next. Participants were required to share their number patterns and the strategies they used to create their pattern. Therefore, the first question in this study involved number patterns, requiring participants to carefully move through the phases of the Singer and Voica's PS conceptual framework from decoding, representing, processing to implementing the sequence of numbers. According to Singer and Voica's PS conceptual framework:

> A solver starts from the wording of the problem (decoding); then searches for relationships among the data and the operators, which lead to mental representations (representing); deduces relationships that call for a known mathematical model adequate to the problem (processing); and finally uses techniques adequate to the identified mathematical model to get to the final result(s)/solution(s) of the problem (implementation) (Singer & Voica, 2013:11).

First, during the decoding phase (DP) in this question; participants were required to observe the sequence of numbers and to be able to use the correct mathematical operators in order to extend the pattern to the next number. Therefore, they had to understand and to identify mathematical operators, which would help them to realise that this problem involves a decreasing numbers pattern. The decoding of decreasing numbers facilitates an integrated understanding of numbers and operations in mathematics.

Secondly, moving up the representing phase (RP), participants were required to find the connection between the data and the unknown in the given problem statement in order to produce a mental model (with or without visual support) suggested by the problem (such as, drawing, words, symbols, graphs or table values).

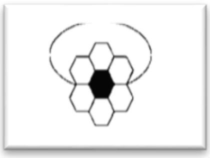
Thirdly, moving further up to the PP, the participants had to produce mathematical models, such as formulas, equations, pictures, and drawings relevant to the problem statement. Furthermore, when identifying an algebraic equation, the variables had to describe the number patterns in a general form. Therefore, if participants applied the correct mathematical model to a given problem, these would provide opportunities to understand how learners represent problems, and what strategies they use to solve the problem.

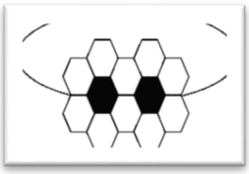
Lastly, to reach the implementation phase (IP), the participants were required to implement the mathematical model suggested by the problem to find any n^{th} term of the sequence.

Question 2, as illustrated in Table 3.3, represented an increasing linear pattern, presented in a visual context. This question required a description of figural growth patterns numerically, and translation between their figural and numerical pattern, as well as the use mathematical operations to find the relationship of the unknown data and known data. It also required learners to engage in near and far generalisation. Near generalisation problems are problems that can be solved by using a sketch or a counting strategy, such as discovering the second, third, fourth or fifth item of the series. On the other hand, far generalisation problems imply the finding of a general rule of the sequence (Barbosa et al., 2012).

Table 3.3: Question 2: Geometric patterns

2. Joana likes to make necklaces using flowers. She uses white beads for the petals and black beads for the centre of each flower. The figure below shows a necklace with one flower and a necklace with two flowers, both made by her.





One flower two flowers three flowers

2.1. How many white and black beads will Joana need to make a necklace with 3 flowers? Draw a diagram of a necklace with 3 flowers.

2.2. How many flowers will Joana be able to make if she uses 102 white beads?

In question 2, participants were expected to describe an increasing geometric pattern, and to relate the concepts of a linear function to the geometric diagram. An increasing pattern in this study is a growing pattern where the size of the flowers increases in a predictable way. Therefore, as the participants described increasing shape patterns in a drawing, they were expected to also recognise that each term had a numeric value. For example, participants had to describe a given increasing pattern by stating the pattern rule. A pattern rule tells us how to make the pattern, and can be used to extend an increasing pattern.

The participants were given the first two increasing patterns, which presented a real-life situation in the form of making necklaces using flowers. The goal was to work visually, thinking about how the number of beads grows and whether participants could see the extra white and black beads. Moreover, participants had to see the different ways to visualise, and to explain, how patterns change.

Barbosa et al. (2012) recommended that learners should be motivated to understand the problem based on the real-life situation, where they are encouraged to “use the potential of visual strategies and to relate number contexts with visual contexts in order to then understand the meaning of numbers and variables” (p. 219).

During the first phase, namely the DP, it was important to emphasise the essential relationship between the data and the operating schemes. Therefore, the second question represented an increasing linear pattern presented in a visual context, which required participants to have the ability to interpret the given statement and to understand what the problem is about. The numerical data of the problem referred to the number of beads needed to make a flower. Therefore, decoding required noticing a transition from smaller flower to growing flower. Incomplete decoding would show that the participants had a perception of the process described in the text, but could not see the need for reversibility (Singer & Voica, 2013).

During the second phase, namely the RP, participants had to progress by establishing a mental model of this problem by means of a drawing. Drawing involves creating a mental image of how many white and black beads will be needed to make a necklace with three flowers. Therefore, at this phase the participants had to explain how their extension followed the pattern. The question demands a change of the problem statement into a language that is more accessible to the solver (Singer & Voica, 2013).

During the next phase, namely the PP, participants had to advance from drawing to the awareness that the problem involved the use of the following mathematical tools: linear pattern, algebra equation, mathematical operators, and meaning of variables. The processing in this problem was the transposition of the text into an equation to produce a mathematical model (Singer & Voica, 2013). The participants needed to construct a mathematical model and then use the model to generalise a formula to show how the number of white beads, at the end, depend on the number of white beads. Participants were expected to formulate a pattern and to generalise a rule for the linear pattern (generalising). Participants had to extend the pattern by identifying the rule, and then use the rule to build and draw the next flowers.

Lastly during the IP, participants had to develop from recognising mathematical tools to applying techniques that were specific to the found mathematical model in order to obtain final results. They were also required to refer back to the previous pattern using their general rule to check if their solution worked best. The participants had to explore the possible situations through a formula to check whether it could be applied to any n^{th} term.

3.3.3.2 Semi-structured one-on-one interviews

In this study, semi-structured one-on-one interviews were used to understand participants' strategies, and their explanations as to why they chose certain strategies to solve number pattern problems, rather than simply interpreting strategies from the written activity. The interviews also helped the researcher to ensure correct interpretation of the participant explanations given in the written activity (in cases of uncertainty). Maluleka (2013) confirms that interviews aim to get a deeper understanding of the views and opinions from the participants by talking to them individually. Interviews are one of the data collection methods in qualitative research that are used to gain a deeper understanding of the research question (Petty, Thomson, & Stew, 2012). Similarly, Gill, Stewart, Treasure, and Chadwick (2008) state that interviews provide a deeper understanding to the problem, and also explore the views, experiences, and beliefs of an individual. Furthermore, interviews aim "to explore people's individual and collective understandings, reasoning processes and social norms" (Mason, 2002:56).

This study used one-on-one semi-structured interviews in order to answer the sub-research question 2, namely: What are the views of grade 9 mathematics learners regarding the areas of difficulty (if any) they experience as they complete number pattern problems? and to gain explanations and a deeper understanding of the strategies grade 9 mathematics learners engage in when solving these problems. "Semi-structured interviews consist of several key questions that help to define the areas to be explored, but also allows the interviewer or interviewee to diverge in order to pursue an idea or response in more detail" (Gill et al., 2008:91). Peters (2011) adds that "a one-on-one semi-structured interview aims to explore concerns relating to a topic by obtaining information from respondents" (p. 38). In the context of the one-on-one semi-structured interviews in this study, the participants were allowed to freely explain their strategies and their thinking while they were involved in solving the problems.

The interviews also allowed the researcher to understand what the participants wrote by asking for clarifications and further explanations on their strategies. Nine open-ended questions base on the four phases of the PS conceptual framework of Singer and Voica (2013) were administered to probe deeper in attempting to understand the strategies employed in the written activity (see Appendix B). Participants were asked to explain their strategy and how they implemented it to arrive at the solution. In addition, they were required to answer explanatory questions such as “How did you think?”; “How did you solve?”; What”; and “Why”.

The interview questions were as follows:

1. Let’s look at question 1, what type of pattern is that and why? This question demanded the participant to interpret the decreasing pattern in a table form. Participants had to describe their pattern by clearly explaining how it changes from one term to the next and whether the sequence is linear or quadratic. Therefore, the participant could reach the DP
2. How did you complete the sequence in question 1? Explain your strategy. This question demanded the participant to be able to describe the decreasing linear pattern. The participant was required to explain the strategy she/he used to complete the table and also explain the mathematical operator used to extend the pattern to the next number. Therefore, the participant could reach the DP
3. How did you get your general formula? This question required the participant to explain the connection between the data and the unknown in the given problem statement in order to produce a mental model (with or without visual support) suggested by the problem. Therefore, the participant could reach RP. For the participant to reach PP, the participants had to explain mathematical models, such as formulas, equations, pictures, and drawings relevant to the problem statement. Finally, for the participant to reach IP, the participants were required to implement the mathematical model suggested by the problem.
4. When working with question 2 about Joana making necklaces; how did you identify the key words to solve the problem? This question demanded the understanding of the wording given on the problem statement which will leads in describing clearly what the problem is about. Therefore, the participant could reach DP
5. How did you make a necklace with 3 flowers? The question required the participant to explain the connection between two known flowers and the unknown flowers in order to produce a mental model (with or without visual support) suggested by the problem. Therefore, the participant could reach RP

6. How many flowers will Joana be able to make if she uses 102 white beads? Explain your answer. This question, the participants had to explain mathematical models, such as formulas, equations, pictures, and drawings relevant to the problem statement. Furthermore, participants were required to implement the mathematical model suggested by the problem. Therefore, the participant could reach PP and IP
7. Can you come up with a general formula for this problem? The question required the participants to construct a mathematical model and then use the model to generalise a formula to show how the number of white beads, at the end, depend on the number of white beads. Participants were expected to formulate a pattern and to generalise a rule for the linear pattern (generalising). Therefore, the participant could reach PP
8. How do you determine if the formula used is correct? This question required the participants to apply techniques that were specific to the found mathematical model in order to obtain final results. They were also required to refer back to the previous pattern using their general rule to check if their solution worked best. Therefore, the participant could reach IP
9. Is there anything else you want to tell me with regard to better understanding of number pattern problems?

3.3.4 Data collection process

Data collection was done at three rural schools in the Capricorn district, Limpopo Province. The circuit manager, as well as the three principals of the schools, was contacted to discuss the study and request permission to conduct the study. Three letters of invitation were sent to each school, one addressed to the principal (see Appendix F), one to the parents of the learners (see Appendix G) and one to the grade 9 learners (see Appendix H).

During data collection, all arrangements were made directly with the principal, as well as grade 9 mathematics teachers. Data pertaining to grade 9 learners' strategies on solving number pattern problems were collected from two sources, namely the participants' written activity in their scripts, and one-on-one semi-structured interviews with three of the participants.

The data were collected at two stages. In stage one, participants were given a written activity (see Appendix A) to complete individually during school contact time in their classrooms. The participants were given 40 minutes to complete the activity. The written activity was marked by the researcher and analysed pertaining to the levels of engagement in the four phases of the PS conceptual framework of Singer and Voica (2013). It took the researcher a period of three days to complete stage one in all three schools. The schools that had more than 30 learners, for example school C with 147 learners, only 30 learners were randomly selected. Those non selected learners were also given the written activity, but their scripts were not considered for analysis.

During stage two, the researcher conducted individual interviews with three randomly selected learners after school hours based on the criteria given in Table 3.4. The three participants – A, B and C – were selected, one from each of the three schools, and had participated in the written activity during stage one. The researcher took a period of eight days to complete stage two. During the individual interviews, each participant was given their script which asked them to give a detailed explanation about the strategy they used to solve the problem. The interview questions were based on the conceptual framework of the study in order to view the learners' strategies with regard to the levels of engagement in the four phases of PS conceptual framework (see section 2.8). Some open-ended questions were used based on the activity to probe for deeper understanding, and to allow the participants to explain their strategies in detail and to further elaborate on the interview questions. Furthermore, the interview questions were clarified with learners in cases where they did not understand the question clearly.

The interview questions (see Appendix B) were designed based on the levels of engagement in the four phases of the PS conceptual framework. Table 3.4 gives the criteria for selecting participants for the interviews.

Table 3.4: Criteria for selecting participants for the interviews

• Undocumented number pattern strategy related to the problem – left a blank space (Participant A). • Irrelevant number pattern strategy related to the problem (Participant B). • Appropriate use of a specific strategy and/or comments with the potential for further discussion (Participant C).

The first criterion, undocumented number pattern strategy, as related to the problem, refers to a case where the researcher could not ascertain the participant's strategy (meaning that there was no response to the problem or the participant left a blank space). The interview was conducted with such a participant to explore his/her thinking processes.

The second criterion, the irrelevant number pattern strategy, as related to the problem, refers to a case where the participant used an inappropriate strategy to solve the problem. The participant did not fail to decode the problem correctly, but failed to progress in identifying the correct mathematical model to solve the problem. The participant was directly extracting numbers from the given problems, meaning that they used the number pattern strategy, but did not address the problem statement.

The interview was conducted with such a participant to probe his/her thinking processes and the explanation provided for using that strategy, or to obtain other similar responses. The last criterion, appropriate use of a specific strategy or comments with the potential for further discussion, relates to the case where the participant was able to indicate his/her levels of engagement in the four phases of the PS conceptual framework. The interview was conducted with such a participant to obtain an explanation for the chosen strategy, and also to obtain a deeper understanding of the strategy used.

An audio tape recorder was used to record the participants' responses during the interviews in order to transcribe the responses later. "The recording of the interview makes it easier for the researcher to focus on the interview content and the verbal prompts and thus enables the transcriptionist to generate verbatim transcript of the interview" (Jamshed, 2014:87). To ensure that there was no disruption during the interviews, the researcher arranged a suitable place for privacy in the school building (staff room). The length of each interview was approximately 30 minutes. The interview phase took place during the period from 18th to 22nd July 2018, and the researcher took 3 days to complete the interviews with the three participants. Table 3.5 illustrates the management plan and the profile of the participants who were involved in the interviews.

Table 3.5: Management plan for the interviews

Schools	A	B	C
Date	18 July 2018	20 July 2018	22 July 2018
Time	15H00–15H30	15H00–15H30	16H00–16H30
Venue	Staff room of school A	Staff room of school B	Staff room of school C

3.3.5 Data analysis

In this study each participant's written activity, and the three participants' one-on-one interview transcripts, were analysed and interpreted separately to form a picture of the learners from each school. The findings from the data-analyses were organised, discussed, and interpreted according to the levels of engagement in the four phases of the PS conceptual framework (Singer & Voica, 2013). Data were analysed in two stages.

In stage one, participant scripts, with their written activity, were collected by the researcher. Thirty (30) scripts per school were collected to make a total of 90 scripts. These scripts were marked by the researcher using a marking guideline (see Appendix D). Content analysis following a deductive approach was used to analyse the collected data from the written activity. Maree (2007) defines content analyses as a systematic qualitative data analysis approach that identifies and summaries the message. In the context of this study the learners' scripts were read thoroughly and repeatedly for 'sense making' and to identify the strategies that had been used, and how the participants had solved the question in order to create a theory. A deductive approach was used to analyse the data according to the levels of engagement in the four phases of the PS conceptual framework, which highlights the four phases for effective problem solving, namely the decoding phase (DP), the representing phase (RP), the processing phase (PP), and the implementing phase (IP) (see section 4.3). Participants who left blank space to all questions were coded as DP0, RP0, PP0, and IP0 and grouped together, the total was calculated. This section used both the protocol and the indicators to analyse the level of engagement in the written activity by learners per school (see section 4.3).

In stage two, one-on-one semi-structured interviews were transcribed into a written form that was further analysed using content analysis. The initial coding scheme for number pattern problem was again based on the levels of engagement in the four phases of the PS conceptual framework, as for the written activity. For example, indicators for decoding were coded as DP0, DP1, DP2 and DP3 (see section 4.3.3). In DP3 responses the participants showed an understanding of the level of engagement in the DP by accurately, appropriately and flexibly interpreting, recognising and expanding a number pattern, and relate numbers and operations with the problem statement. Participants with a response coded as DP2 showed an acceptable level of decoding with minor errors. This participant did not focus on finding relationships between the data and mathematical operators, and showed a lack of integration and interpretation of negative numbers. This means that the participant showed no knowledge of how to use the mathematical operation of subtraction and negative signs. The participant showed an incorrect understanding of negative numbers. Participant responses coded as DP1 indicated decoding with major mathematical errors. Lastly, DP0 indicated that participants left blank spaces, or did not respond to the question or showed no engagement in the DP

Lastly, a constant comparative analysis was used to ascertain the levels of engagement in the four phases of the PS conceptual framework in the participants' strategies derived from the analysis of the written activity and the interviews (see section 4.6). Matthews and Ross (2010) defined a "constant comparison method as comparing data from different sources and from different places and times to support the analysis, along with the search for negative cases" (p. 400). "Constant comparison serves to uncover and explain patterns and variations" (Bitsch, 2005:79).

3.4 QUALITY CRITERIA

3.4.1 Establishing trustworthiness

According to Guion et al. (2002), "validity in qualitative research, refers to whether the findings of a study are true and certain" (p. 1). Direct quotations from the participants, and a detailed description of the data, was provided to ensure validity. Trustworthiness was ensured by using triangulation. "Triangulation is a method used by qualitative researchers to check and establish validity in their studies"; it uses two or more methods of data collection (Guion et al., 2002:1). Creswell (2008) defines triangulation as "a validity procedure where researchers search for convergence among multiple and different sources of information to form themes or categories in a study" (p. 126). The different sources of data that were analysed and triangulated were participants' written activity and interviews. The findings were then compared to ascertain the levels of engagement in the four phases of the PS conceptual framework in the participants' strategies derived from the analysis of the written activity and the interviews. This approach increased the likelihood that the phenomenon of interest was being understood from various points of view (Maykut & Morehouse, 1994). Trustworthiness was also established by considering credibility, transferability, dependability, and confirmability (Lincoln & Guba, 1985; Peters, 2011).

3.4.1.1 Credibility

Credibility ensures that the study measures what is actually intends to measure and determines how congruent the findings are with reality (Shenton, 2004). Ary et al. (2010) define credibility in qualitative research as the "concerns of the truthfulness of the inquiry's findings" (p. 498). Therefore, participants were given a written activitiy to complete in the presence of a researcher and interviews were conducted based on what participants wrote in their activity. To further ensure credibility, a Mathematics Education lecturer from another university was consulted to assist with specialised advice regarding the clarity of the interview questions, the design of the activity sheet and assessment grid, and the establishment of the indicators for the four phases of the PS conceptual framework. In order to enhance the credibility of the findings, data were interpreted and compared with reference to the levels of engagement in the four phases of the PS conceptual framework of Singer and Voica (2013). During the interview, participants were asked structured questions and the researcher behaved impartially, without showing any personal interest, in order to avoid bias and enhance the degree of objectivity. Before drafting the final report, interview transcripts were presented to the participants to confirm the accuracy of the transcriptions. The raw data, along with the interpretation of the findings, were forwarded by the researcher to another university for peer reviewing.

3.4.1.2 Transferability

Transferability is the "provision of background data to establish context of study and detailed description of phenomenon in question to allow comparisons to be made" (Shenton, 2004:73). To ensure transferability, a dense description of the findings from the written activity and interviews were provided to make judgements about the similarities and differences between case studies with regard to strategies used to solve number pattern problems. To further ensure transferability, verbatim quotes from the interviews were provided to demonstrate how the findings and the researcher's interpretations arose from the data.

3.4.1.3 Dependability

Dependability refers to reliability in qualitative studies. A study is reliable if it is repeated in the same context using the same data collection methods and participants with the same characteristics and similar results are obtained (Shenton, 2004). To ensure dependability of the study, the researcher continued interviewing participants until data-saturation was reached. According to Fusch and Ness (2015), "data saturation is reached when there is sufficient information to replicate the study, when the ability to obtain additional new information has been attained, and when further coding is no longer feasible" (p. 1408). An audit trail was kept to enable readers to evaluate the context of this study (Ary et al., 2010:502). The audit trail contains all the raw data gathered in interviews and the written activity, and records of the researcher's decisions about whom to interview and why. These raw data (audio recording and learners' written activity) have been stored for verification.

3.4.1.4 Confirmability

Confirmability ensures that the "worker's findings are the result of the experiences and ideas of the participants, rather than the characteristics and the preferences of the researcher" (Shenton, 2004:72). Member checking was applied to validate the emerging findings from the data analysis. Bitsch (2005) suggested the following questions to ask pertaining to member checking: “have data and interpretations been re-checked with the participants? Did those who provided the data agree with findings and interpretations? Have they been heard and did they contribute to the final findings and conclusions?” (p. 84). Therefore, participants who were interviewed were asked to verify the accuracy of the verbal quotations that were recorded in the interview. Those participants acknowledged that transcripts were true reflection of what they said.

3.5 ETHICAL CONSIDERATIONS

The Faculty of Education Academic Ethics Committee of the University of Johannesburg (Appendix C) and the Limpopo Department of Education (Appendix D) granted the researcher ethical clearance to conduct this research study. Before the study commenced the researcher informed the participants, School Governing Bodies (SGB) and the school principals about the purpose and process of the study and requested permission (Appendix E). Written permission was obtained from the respective school principals and SGBs (Appendix F). Parents were informed about the purpose and process of the study and were asked to sign a letter of consent (Appendix G) to allow their children to participate. Participants were asked to sign letters of consent (Appendix H) to participate, which included a description of the research process and purpose. Ninety participants took part in the study voluntarily. No harm or discomfort was associated with participation in this study. Participants were at liberty to withdraw from the study at any time, without penalty or pressure to provide reasons to the researcher. Every effort was made to guarantee the participants' confidentiality and privacy.

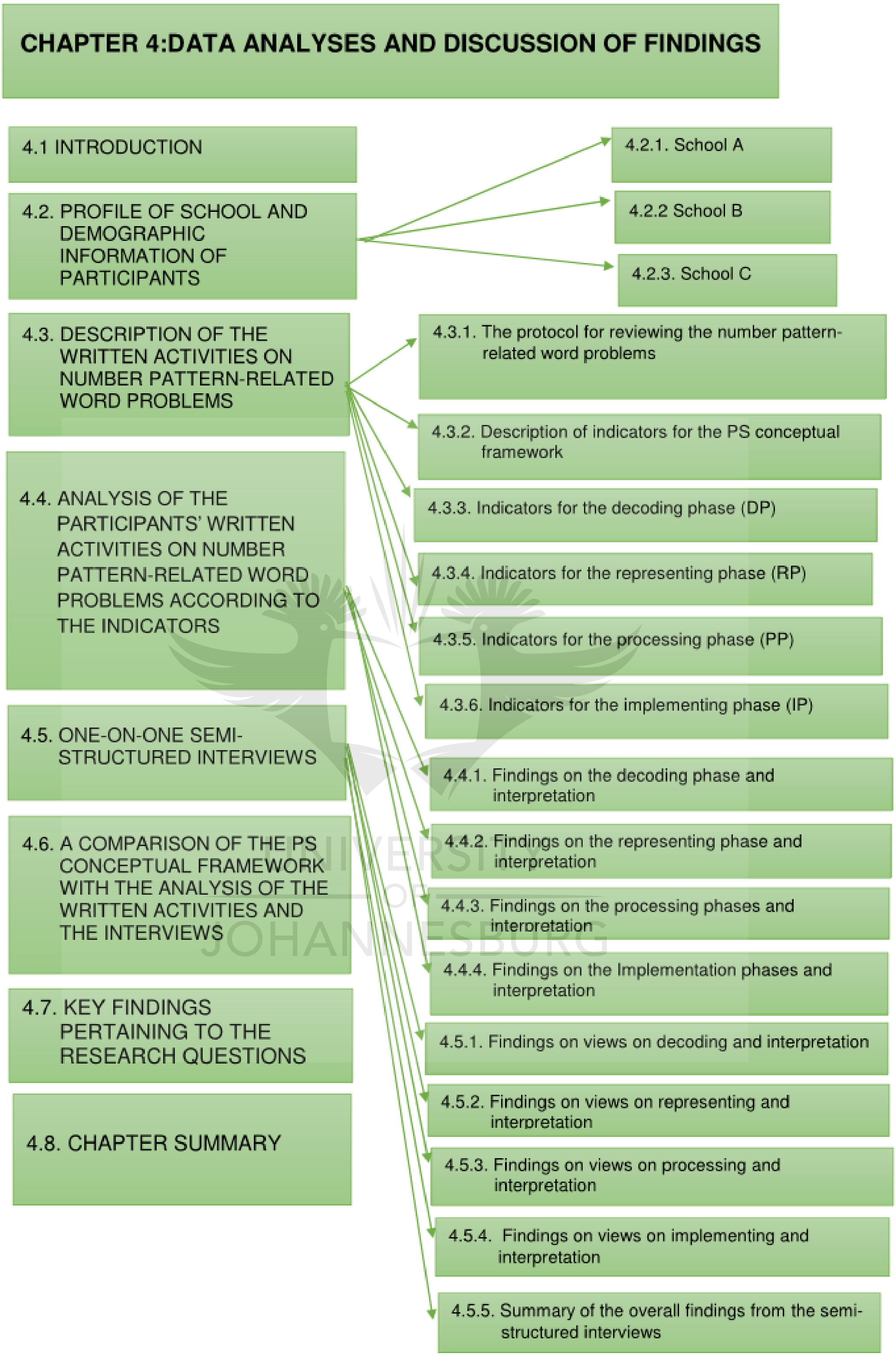
To ensure anonymity and secure the privacy of the participants, no names were requested throughout the data collection process. The researcher protected the identities of schools and those of participants. The following procedure was used to establish name codes for the participants in the three selected schools: letters of the alphabet (A, B and C) were assigned to each school to protect their identities. A implying the first school, B implies the second school and C implies the third school.
To distinguish one participant from another, for example in School A, participants were named A1, A2, A3, etc. For School B, participants were named B1, B2, B3, etc. The same applies for School C. Participants were named C1, C2, C3, etc... All data

collected were treated anonymous and kept under lock and key. Only the researcher had access to the data. Data were securely stored and will be kept for no longer than two years after completion of the study. Thereafter, all collected data will be destroyed. Although there could be risk of group or cohort identification in research publications, the participants' personal identity will always remain confidential.

3.6 CHAPTER SUMMARY

Chapter 3 deals with the research design and research methodology used in the study. In particular, the chapter describes the philosophical assumptions (the epistemological, ontological, and axiological assumptions) of the study. The qualitative research approach followed in this study assisted the researcher to develop a deep understanding of the learners' strategies used on number pattern problems. The researcher used an exploratory case study.

A clear indication of the sampling techniques, data collection processes, and data analysis procedures are also provided. Purposive sampling was used for this qualitative study. The researcher selected participants that were doing mathematics in grade 9. The data were collected through a written activity and semi-structured interviews. Data analysis was carried out by means of content analysis, following a deductive approach, to analyse participant scripts and interviews. The phases of the PS conceptual framework (Singer & Voica, 2013), namely, decoding, representing, processing and implementing, formed the categories for the analysis. Finally, the levels of engagement in the four phases of the PS conceptual framework in strategies from the analyses of the written activity and the interviews were compared. Trustworthiness of the study was discussed with reference to credibility, transferability, dependability, and confirmability. Lastly, ethical considerations taken into account were reported. The next chapter will focus on data analysis and the interpretation of findings.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CHAPTER 4: DATA ANALYSES AND DISCUSSION OF FINDINGS

4.1 INTRODUCTION

In Chapter 3 it was mentioned that qualitative research provides a rich description of the phenomenon under investigation (Merriam, 2009); and was therefore chosen for this study. Specifically, this study focussed on meaning and understanding of learners' strategies in solving number pattern problems. Therefore, this study, gathered data from participants' scripts containing the written activity; to analyse grade 9 mathematics learners' strategies used to solve number pattern problems. Also, data were collected from one-on-one interview with the participants to establish the strategies grade 9 mathematics learners use to solve number pattern problems. This chapter deals with the analysis of these collected data and discusses and interprets the findings regarding the strategies grade 9 mathematics learners use in solving number pattern problems from three rural schools. Specifically, five steps were followed to analyse data. These steps are outlined next.

The first step was to obtain the profile of schools and biographic information of participants. Step two provided a brief description on how the written activity was analysed, including the protocol for analysing the number pattern problems, and a description of the phases from the PS conceptual framework. Step three discussed the findings from the analyses by interpreting the levels of engagement in the written activity by the participants separately according to each of the three schools (school A, school B and school C) in terms of the levels of engagement in the four phases of the PS conceptual framework of Singer and Voica (2013) (see section 2.8). Step four involved data analyses from the one-on-one semi-structured interviews with participants to understand their views on the PS conceptual framework strategies and their explanation for using these strategies to solve number pattern problems, rather than simply interpreting strategies from the written activity. Finally, in step five, the levels of engagement in the four phases of the PS conceptual framework of Singer and Voica (2013) in strategies from the analysis of the written activity and the interviews were compared. Table 4.1 that follows provides a summary of the five steps followed to analyse data in this study.

Table 4.1: Summary of the steps followed in the data analysis

Step 1	**Profile of schools and biographic information of participants** • **School A** • **School B** • **School C**
Step 2	**Analysis of the written activity on number pattern problems** • Analysing of questions on number pattern problems • Description of phases of the PS conceptual framework
Step 3	**Findings from the analysis of the written activity on number pattern problems according to the phases** • Findings and discussion of participant responses on number pattern problem
Step 4	**One-on-one semi-structured interviews with three participants** • Findings on views of decoding and interpretation • Findings on views of representation and interpretation • Findings on views of processing and interpretation • Findings on views of implementing and interpretation
Step 5	The levels of engagement in the four phases of the PS conceptual framework of Singer and Voica (2013) in strategies from the analysis of the written activity and the interviews

4.2 PROFILE OF SCHOOLS AND DEMOGRAPHIC INFORMATION OF PARTICIPANTS

The following sections provide the background information and demographic details of participants, which were collected form the circuit office prior to the study.

4.2.1 School A

School A was established in 1971 with only two classroom blocks. The school was renovated in 2012 and now consists 15 blocks including a library and a laboratory. It was the first secondary school in the village to be used for this study. The school's enrolment at the time of the study was 320 learners, with 8 teachers including the principal. The school had 31 grade 9 learners with 20 females and 11 males, but 20 females and 10 males participated, thus, in total 30 learners participated. The ages of the selected learners ranged from 15–17 years. The grade 12 pass rate in 2017 was 58%. This school was classified as an underperforming school in 2018. According to the DBE, this means that the school's grade 12 results were below the benchmark of 65%.

4.2.2 School B

School B was established in 1990, and was the second secondary school in the village to be used for this study. At the time of the study, the school had an enrolment of 118 learners, with 5 teachers including the principal. The school had one class of 33 grade 9 learners with 21 males and 12 females. The age of learners ranged from 15–16. Ten females and 20 males participated in this study, thus, in total 30 learners participated. The grade 12 passed rate in 2017 was 45% and it was declared a chronically underperforming school in 2018, meaning the school had been underperforming for three consecutive years (according to the DBE).

4.2.3 School C

School C is currently the largest in the village in terms of learner enrolment, but did not have sufficient equipment or classrooms to cater for all learners. This was the third secondary school used for this study. The school was established in 1994 and the grade 12 pass rate for the school has never been less than 80%. In 2017, the grade 12 passed rate was 90%.

At the time of the study, the school had an enrolment of 884 learners and 24 teachers (including the principal and the deputy principal). The school had 142 grade 9 learners evenly distributed in two classrooms; grade 9A with 63 learners (27 are males and 36 females) and grade 9B with 79 learners (38 are males and 41 females). The ages of learners ranged from 14–16. Fifteen females and 15 males participated in this study, thus, in total 30 learners participated. Table 4.2 gives a summary of the selected schools and demographic information about the participants.

Table 4.2: Summary of selected schools and demographic information of participants

Schools	A	B	C
Year established	1971	1990	1994
2018 learner enrolment	320	118	884
Number of teachers	8	5	24
2017 grade 12 performance	58%	45%	90%
Number of grade 9 learners enrolled in the schools and who participated in the study	31 enrolled 30 participated	33 enrolled 30 participated	147 enrolled 30 participated

4.3 ANALYSIS OF THE WRITTEN ACTIVITY ON NUMBER PATTERN PROBLEMS

4.3.1 Protocol for analysing the number pattern problems

A protocol for analysing the written activity (Appendix J) was developed to identify the levels of engagement in the four phases of the PS conceptual framework. This protocol, as well as the marking guideline (Appendix I), was used to analyse learners' responses to the written activity. For example, if the protocol showed that participants reached a certain phase of the PS conceptual framework, e.g. the decoding phase (DP), representing phase (RP), processing phase (PP), or implementing phase (IP), then the indicators for DP, RP, PP, and IP were used to analyse participants' responses. In the analysis of the participants' scripts the focus was on identifying strategies used by learners when solving number pattern problems.

4.3.2 Description of phases of the PS conceptual framework

For the study to analyse the participants' responses from the written activity, and to identify the strategies used for number pattern problems, phases of the PS conceptual framework were used. Literature discussed in Chapter 2 helped to unpack the PS conceptual framework and the strategies that participants used for number pattern problems. Indicators from the four phases of the PS conceptual framework are discussed next. When learners' responses were analysed, these indicators determined the competency that learners displayed in their response to number pattern problems.

4.3.3 Indicators for the decoding phase (DP)

In Chapter 2 (section 2.8) the researcher discussed the PS conceptual framework of Singer and Voica (2013), who argue that decoding is the ability to move focus from the wording of the problem (understanding the text) to relations among the data and the operating schemes that are induced by the given constraints. Clement (2008) defines decoding as the ability to interpret the given statement and to identify the key words in order to solve the problem (see sub-section 2.8.1). Data concerning the DP were analysed using the key given in Table 4.3.

Table 4.3: Key for analysing the decoding phase (DP)

Codes	Actions of participants
DP3	Participants at level 3 could show an understanding of the level of engagement in the DP accurately, appropriately and flexibly by interpreting, recognising and expanding a number pattern, and related numbers and operations with the problem statement.
DP2	Participants at level 2 could show acceptable level of engagement in the DP with minor errors. However, they could not show knowledge of how to use the mathematical operation of subtraction.
DP1	Participants at level 1 could display some insight into decoding, but with major mathematical errors. The participants made major mistakes when interpreting the mathematical structure and a numerical pattern. The participants concentrated only on the relationship between a single pair of beads (white beads) and used it as a general rule.
DP0	Participants at level 0 left blank spaces, did not respond to the question, or showed no level of engagement in the DP.

A participant response to the written activity coded as DP3 showed an understanding of the level of engaging in the DP accurately, appropriately and flexibly. For question 1, the participants were required to observe the sequence of numbers and to be able to use the correct mathematical operators in order to extend the pattern to the next number. Therefore, they had to understand and to identify mathematical operators, which would help them to realise that this problem involves a decreasing numbers pattern.

The second question represented an increasing linear pattern in a visual context, which required participants to have the ability to interpret the given statement and to understand what the problem is about. Therefore, decoding required noticing a transition from smaller flower to growing flower. Therefore, those participants were expected to be able to move beyond the data, by generating and predicting the problem statement, thus, the participants had to understand the problem and the mathematical operations within the problem statement. Moreover, the participants had to be able to comprehend mathematical concepts on number patterns that would be helpful in solving the problem.

Participants with a response coded as DP2 had to show an acceptable level of engagement in the DP with minor errors. However, these participants could not focus on finding relationships between the data and mathematical operators, and showed a lack of integration and interpretation of negative numbers in question 1. This means that the participants showed no knowledge of how to use the mathematical operation of subtraction and negative signs. The participants showed an incorrect understanding of negative numbers. For example: $-4-(-2)$, this would be read as negative four plus two, but they said negative four minus 2 and ignored the negative two. For the second, the participants coded DP2 were not able to interpret the given statement and to understand what the problem is about, which resulted them not noticing a transition from smaller flower to growing flower.

Participants with a response coded as DP1 reached some insight of decoding but with major mathematical errors; taking information as it is from the data, which means reading the data with little understanding of, for example, locating or translating. For question 2, the participants made major mistakes when decoding the structure. The participants concentrated only on the relationship between a single pair of beads (white beads) (n; f (n)) and used it as a general rule. For example, the participants saw that there were six white beads in the first flower f (1) = 6 × 1 = 6, and then used the rule f (n) = 6n to find the number of beads with 3 flowers f (3) = 6× 3 = 18; hence the participants used a direct proportional strategy.

Participants who gave a DP0 response left blank spaces, did not respond to the question, or showed no levels of engagement in the DP.

4.3.4 Indicators for the representing phase (RP)

Singer and Voica (2013) explain representing as the ability to understand a problem by generating a mental model. In the context of this study, a mental model is an explanation of learners' thought processes about how number pattern problems work in the real world (see sub-section 2.8.2). Three types of mental representations were taken into account in the analysis of the data, namely: proportional representations, mental models, and mental images. Data concerning the RP were analysed using the key given in Table 4.4.

Table 4.4: Key for analysing the representing phase (RP)

Codes	Actions of participants
RP3	Participants were classified at level 3 if they could focus on the comprehension of the situation, as well as the mathematics concepts in the activity. Those participants who were classified as having a mental model at level 3 prioritised a qualitative approach, such as an explanation of how to get the next term of the sequence to complete the table value and to draw the correct diagram.
RP2	Participants were classified as having a mental image representation at level 2 if they could show an awareness of the relationship among the flowers and the beads, and could mention the previous term and the next term. They could also recognise the applicability of the qualitative method to solve the problem. Participants could provide reasons for the problem even if they failed to handle the mathematical part of the problem. For example, they could recognise the common difference between consecutive numbers, but incorrectly used it to determine the next term. Although they could reason about the number of beads and the flower, they used the incorrect diagram.
RP1	Participants at level 2 were classified with propositional mental representation if they tended to use definitions and handled mathematical formulations in a rote and mechanical manner. They showed no evidence of understanding of the underlying concepts of a number pattern. They manipulated equations by directly applying multiplication of the common difference when the pattern increased.
RP0	Participants at level 0 left blank spaces or did not respond to the question.

Participants whose responses were coded as RP3 could present a mental model showing "comprehension of the situation, as well as the mathematics concepts highlighted in the activity. Moreover, before dealing with equations they could include a diagrammatic representation in their problem solution" (Ibrahim & Rebello, 203:4). For question 1, the participants reaching RP3, were required to find the connection between the previous term and the next term in order to produce a mental model (with or without visual support) suggested by the problem (such as, drawing, words, symbols, graphs, table values or number line). The participants who were classified as having a mental model prioritised a qualitative approach, such explanation of how to get the next term of the sequence to complete the table value.

For question 2, participants had to progress by establishing a mental model of this problem by means of a correct drawing. The participants had to be aware that each flower must be a joint unit to form a linear pattern. Each flower shared two white beads to have a complete flower, four white beads were added to the end of each prior flower to create a new flower, and then added two beads to give me a complete flower. Drawing involves creating a mental image of how many white and black beads will be needed to make a necklace with three flowers. Therefore, at this phase the participants had to explain how their extension followed the pattern. Those participants who were classified as having a mental model also emphasised the identification and understanding of number pattern principles or concepts.

Participants whose responses were coded as RP2 partially created concrete representations of the pattern. Participants could provide reasons for the problem even if they failed to handle the mathematical part of the problem (Ibrahim & Rebello, 203). For question 1, the participants could provide reason on how to find the common difference but the sign that the common difference should take was also a challenge, they could not subtract -3 from each term to get the next term on the sequence. For question 2, the participants who were coded RP2, demonstrated a diagrammatic representation (flowers) which may not be linked to the mathematical formulations used. Those participants were aware of the relationship among the flowers and the beads and they could recognise the applicability of the qualitative method to solve the problem, but used the incorrect diagram.

Participants whose responses were coded as RP1 were unable to create concrete representations of the number pattern problems. The participants could only "focus on given or apparent information and only prioritised manipulation of equations with rote memorisation and pattern matching of information" (Ibrahim & Rebello, 203:4). For question 1, they applied a multiple of common difference without making a final adjustment to give the generalise formula. For question 2, the participants attempted the tasks structured with a symbolic presentation by using a quantitative approach to determine a value of white beads that Joana needed to make three flowers.

Finally, participants who did not engage in the written activity or left blank spaces were coded as RP0.

4.3.5 Indicators for the processing phase (PP)

The PP was the phase where a mathematical model associated with the problem could be identified, either in the form of an equation, a formula, a system, steps of a graphical representation, or various computing algorithms (Singer & Voica, 2013) (see sub-section 2.8.3). In the context of this study, the model focused on describing pattern and external representations (drawing, table, or symbols) to build an understanding of the system that is modelled. Data concerning the PP were analysed using the key given presented in Table 4.5.

Table 4.5: Key for analysing the processing phase (PP)

Codes	Actions of participants
PP3	Participants at level 3 could construct a mathematical model and then used it to show how the number of a previous term depended on the number of the next term.
PP2	Participants at level 2 could create a mathematical model, with minor errors in finding the term of the sequence. They made application errors by thinking that "n" is the term that follows the previous term and applied numerical expressions instead of using an algebraic expression.
PP1	Participants at level 1 created an incorrect formula to generalise the sequence. They applied the formula of the common difference that is T2-T1;T3-T2… as a general formula.
PP0	Participants at level 0 left blank spaces or did not respond to the question.

Participants whose responses were coded as PP3 could construct a mathematical model and then use it to generalise a mathematics formula to show how the number of the previous term depends on the number of the next term. The participants had to produce mathematical models, such as formulas, equations, pictures, and drawings relevant to the problem statement. Furthermore, when identifying an algebraic equation, the variables had to describe the number patterns in a general form.

For question 2, participants had to advance from drawing to the awareness that the problem involved the use of the following mathematical tools: linear pattern, algebra equation, mathematical operators, and meaning of variables. The participants needed to construct a mathematical model and then use the model to generalise a formula to show how the number of white beads, at the end, depend on the number of white beads. Participants were expected to formulate a pattern and to generalise a rule for the linear pattern (generalising). Participants had to extend the pattern by identifying the rule, and then use the rule to build and draw the next flowers.

Participants whose responses were coded as PP2 created a model (general formula) with minor errors in finding the terms of the sequence, for example, participants made an application error in determining the unknown term, such as the n^{th} term. For question 1, the participants correctly completed the table value, but performed an application error by thinking that “n” is the term that follows -23 and applied numerical expressions instead of using an algebraic expression. The participants were not able to find adequate rule (question 2), revealing difficulties in finding a functional relationship between the number of white and black beads to make flowers and making mistakes like the application of a direct proportion model when not adequate. The participants started by processing the situation with a drawing, at the end they were unable to discover the pattern due to the application of inadequate strategies: counting (using a confusing diagram), or considering a direct proportional model. Considering the rule to find the number of beads in three flowers was $T_n=6\times n$, n being the number of flowers.

Participants whose responses were coded as PP1 created an incorrect formula to generalise the sequence. For question 1, the participants were coded on PP1 if they created an incorrect formula to generalise the sequence to find the nth term. The participants had evidence on how to calculate the common differences, but mistakenly taking the calculation of the common difference to be the calculation of the general formula of the sequence. When the participants were asked about the rule of the pattern when the number of flower increases (question 2), they stated that they only perceived the difference between two terms as a rule. These participants were able to make local generalisations and rise until PP1 level.

Finally, participants coded as PP0 level left blank spaces or did not respond to the question.

4.3.6 Indicators for the implementing phase (IP)

The IP is concerned with applying techniques that are specific to the found mathematical model, using a multiple solution strategy and producing mathematical explanation and justification for the solution, and generally questioning the model (Singer & Voica, 2013) (see sub-section 2.3.4). These techniques might involve, for example, recognising the different ways of proving the solution; minimising the values that do not satisfy the constraints of the problem; using helpful constructions, substitutions or using a known algorithm. During the IP, the participants could also apply knowledge of what is already known about the problem to test the model or the equation that is chosen. Data concerning the IP were analysed using the key given in Table 4.6.

Table 4.6: Key for analysing the implementing phase (IP)

Codes	Actions of participants
IP3	Participants at level 3 could apply techniques that were specific to the found mathematical model. The participants could implement the correct general rule to obtain the correct solution.
IP2	Participants at level 2 could implement the solution in a manner that addressed the problem statement, but ignored relevant contextual factors. The participants could indicate that the rule to find the number of white beads was Tn = 6n.
IP1	Participants at level 1 implemented the solution in a manner that did not directly address the problem statement. The participants were aware of the common difference between consecutive numbers, but conducted the wrong mathematical model and used a direct proportional model to implement the solution. e.g. Tn=-3n.
IP0	Participants left blank spaces or did not respond to the question.

Participants whose responses were coded as IP3 could apply techniques specific to the found mathematical model; could produce mathematical explanations and justification of the solution; and could generally question the model. To reach the IP3 (question 1), the participants were required to implement the mathematical model suggested by the problem to find any nth term of the sequence. For question 2, participants coded IP3, had to develop from recognising mathematical tools to applying techniques that were specific to the found mathematical model in order to obtain final results. They were also required to refer back to the previous pattern using their general rule to check if their solution worked best. The participants had to explore the possible situations through a formula to check whether it could be applied to any nth term.

Participants, whose responses were coded as IP2 could implement the solution in a manner that addressed the problem statement, but ignored relevant contextual factors. These participants could not produce a mathematical model, and also failed to substitute the correct value to arrive at the correct solution both for question and two. The participants indicated that the rule to find the number of white beads was Tn = 6n (question 2), and six was incorrect common difference for this problem. This rule indicated that they did not properly analyse the structure of the flower, but thought each flower is a disjoint unit. Therefore, the participants used a direct proportional strategy which was not relevant to the given problem.

Participants whose responses were coded as IP1 implemented the solution in a manner that did not directly address the problem statement. Even though the participants were aware of the common difference in question 1, they conducted the wrong mathematical model, such as Tn=-3n, and used a direct proportional model to implement the solution in question 1. Participants used a direct proportional strategy without adjustment to the problem, meaning that participants were directly picking up numbers from the given problems and using multiplication.

Participants who left blank spaces or did not respond to the question were coded as IP0.

4.3.7 Coding strategy

Various strategies can be utilised to solve number pattern problems. García Cruz and Martinón (1997) analysed the generalisation procedures on number pattern problems created by high school learners and have identified the following primary categories: counting strategy, recursive approach, and direct proportional strategy. While Ibrahim and Rebello (2013) investigated the mental representation categories with which learners operate during problem solving of distinct formats of representational tasks. The results indicate that leaners work at the three levels of mental representation, namely: propositional; mental images and mental model representation.

Barbosa et al. (2012) define a counting strategy as counting the elements of a particular number or figural term in a pattern. Lannin et al. (2006) define a recursive approach as a connection between successive independent variable values in the scenario. Lannin et al. (2006) mention that learners may select a recursive strategy because they want to determine a general rule based on an understanding of a relationship that occurs in the situation. Both direct counting and recursive strategies were evident during the decoding phase (DP). Besides, the participants using recursive, counting and direct proportional strategies, the tendency to convert shape pattern problems into number sequence problems was very high. In the counting strategy used in this study, the participants knew the starting point of the sequence, and counted up or counted down to generate the unique sequence. They also started making mathematical predictions, such as finding the next term of the sequence. For the recursive strategy, the participants used the common difference to find the next term of the sequence and adjust it to find the general formula.

Barbosa et al. (2012) define direct proportion strategy as a situation, where the multiples of a specific term of a sequence is considered, and the problems presented in the test do not fit that model. Similarly, Lannin (2003) describe this strategy as "using a portion as a unit to construct a larger unit using multiples of the units. The strategy of direct proportion, even inappropriately employed, was evident in both the processing phase (PP) and the implementing phase (IP). Those participants who used direct proportional they indicated some techniques for manipulating expressions and equations, but without a basic underlying understanding of what the variables and numbers represent. Hence, they used the direct proportional strategy to manipulate the equation. They could only focus on given information and only prioritised manipulation of equations with rote memorisation and pattern matching of information.

For example, in question 2, Joana needs 6 white beads and one black bead to make one flower. How many white and black beads will Joana need to make a necklace with 3 flowers? therefore, for 3 flowers will be (6 x 3 = 18 white beads and one three black beads). Those participants tended to inadequately use a direct proportion model, in some way familiar to them. This may indicate that they did not properly analyse the structure of the sequence, thinking of each flower as a disjoint unit. Most of them considered that each flower had six white beads and one black, so a necklace with eight flowers would have forty-eight white and eight black beads and a necklace with twenty-five flowers would have hundred and fifty white and twenty-five black beads.

Ibrahim and Rebello (2013) define mental image representation as having the tendency the tendency to include a diagrammatic representation which may not be linked to the mathematical formulations used. The strategy of mental image representation was evident during the representing phase (RP). Both for questions 1 and 2, the participants started by portraying the situation with a drawing, but ultimately they were unable to discover the pattern due to inadequate strategies (using a confusing diagram).

According to Ibrahim and Rebello (2013), mental models provide a means to connect the syntactic (mathematical) and structural (visual) aspects of the task under consideration, which allow for interpretation and understanding. Mental model representation was evident in the RP. In question 1, one participant stated that "the fast way to get -3 is to multiply the nth by (-3), n × (-3) = -3n, therefore substitute the first position to see if it gives you a first term. If it does not give a first term, then I added 1 to give -2".

4.4 FINDINGS FROM THE ANALYSIS OF THE WRITTEN ACTIVITY ON NUMBER PATTERN PROBLEMS ACCORDING TO THE PHASES

The indicators for the phases of the PS conceptual framework were used as codes to categorise the actions that participants displayed in their responses to the written activity, evident in participants' scripts (a total of 90 scripts – 30 from each of the three selected schools). The written activity consisting of two questions can be found in Appendix A. The 90 participants' responses to the written activity were marked by the researcher using the making guideline (see Appendix I). The indicators developed for the four phases of the PS conceptual framework were used to code the participants' work as they move from DP to IP (see Appendices P, Q and R for examples of coded written activities from each school).

In some cases, it is found that the participants were not able to complete the cycle, that is moving to the highest level of the phases, namely DP3, RP3, PP3 and IP3. For example, nine participants were able to showed an understanding of decoding accurately and were coded at DP3, but only four of them focused on comprehension of the situation, as well as the mathematics concepts in the activity and they were coded at RP3. The other five participants from the total of nine who were unable to move to RP3, could, however, provide reasons for the problem even if they failed to handle the mathematical part of the problem and they were coded at RP2. Three participants from those four participants who were coded RP3 moved to PP3. They constructed a mathematical model and then used the model to show how the number of previous term depends on the number of the next term. Finally, only two participants applied techniques that were specific to the found mathematical model and were coded at IP3. The participants who were coded at DP1 reached some insight into decoding, but made major mathematical errors. These participants could not move to RP2, but were coded at RP1, meaning the participants focused on given or apparent information or prioritised the manipulation of equations with rote memorisation or pattern matching of information. These participants created an incorrect formula to generalise the sequence PP1.

The participants who were able to implement the solution in a manner that did not directly address the problem statement were coded IP1. Table 4.7 shows the analysis of the number of participants (per school), who engaged in the four phases of the PS conceptual framework per indicator with regard to their responses in the written activity. The grey-shaded row (DP0, RP0, PP0 and IP0) represents the number of participants who left blank spaces or did not respond to the questions.

Table 4.7: Number participants (per school) who responded according to the phases of the PS conceptual framework

School A					
Indicators	**Phases**				**Total number of participants (*N*)**
	Decoding (DP)	**Representing (RP)**	**Processing (PP)**	**Implementing (IP)**	
0	5	5	5	5	20
1	11	11	10	9	41
2	5	5	5	4	19
3	9	5	4	2	20
Total number for phases	30	26	24	20	100

School B					
Indicators	**Phases**				**Total number of participants (*N*)**
	Decoding (DP)	**Representing (RP)**	**Processing (PP)**	**Implementing (IP)**	
0	10	10	10	10	40
1	12	10	9	7	38
2	6	4	3	3	16
3	2	2	1	1	6
Total number for phases	30	26	23	21	100

School C					
Indicators	**Phases**				**Total number of participants (*N*)**
	Decoding (DP)	**Representing (RP)**	**Processing (PP)**	**Implementing (IP)**	
0	5	5	5	5	20
1	7	7	6	6	26
2	8	7	5	4	24
3	10	8	6	6	30
Total number for phases	30	28	26	25	100

Overall (school A, B and C)					
Grand total per Indicators	**Phases**				**Total number of participants (*N*)**
	Decoding (DP)	**Representing (RP)**	**Processing (PP)**	**Implementing (IP)**	
0	20	20	20	20	80
1	30	28	25	22	105
2	19	16	13	11	56
3	21	15	11	9	59
Grand total for phases	90	79	69	62	300

The analysis of the indicators for the decoding phase (DP), the representing phase (RP), the processing phase (PP), and the implementing phase (IP) were descriptive in nature and were qualitatively analysed. The discussion on each of the indicators per school follows.

4.4.1 Findings on the decoding phase (DP) and interpretation

4.4.1.1 School A

At school A, five participants (5 out 30) engaged in the activity on DP2, while 11 out of 30 participants were on DP1, and five out of 30 participants were on DP0. Only nine of the participants performed on DP3. The vignette in Figure 4.1 shows an example of question 1 of the written activity, from school A, of participant A1 response coded as DP2.

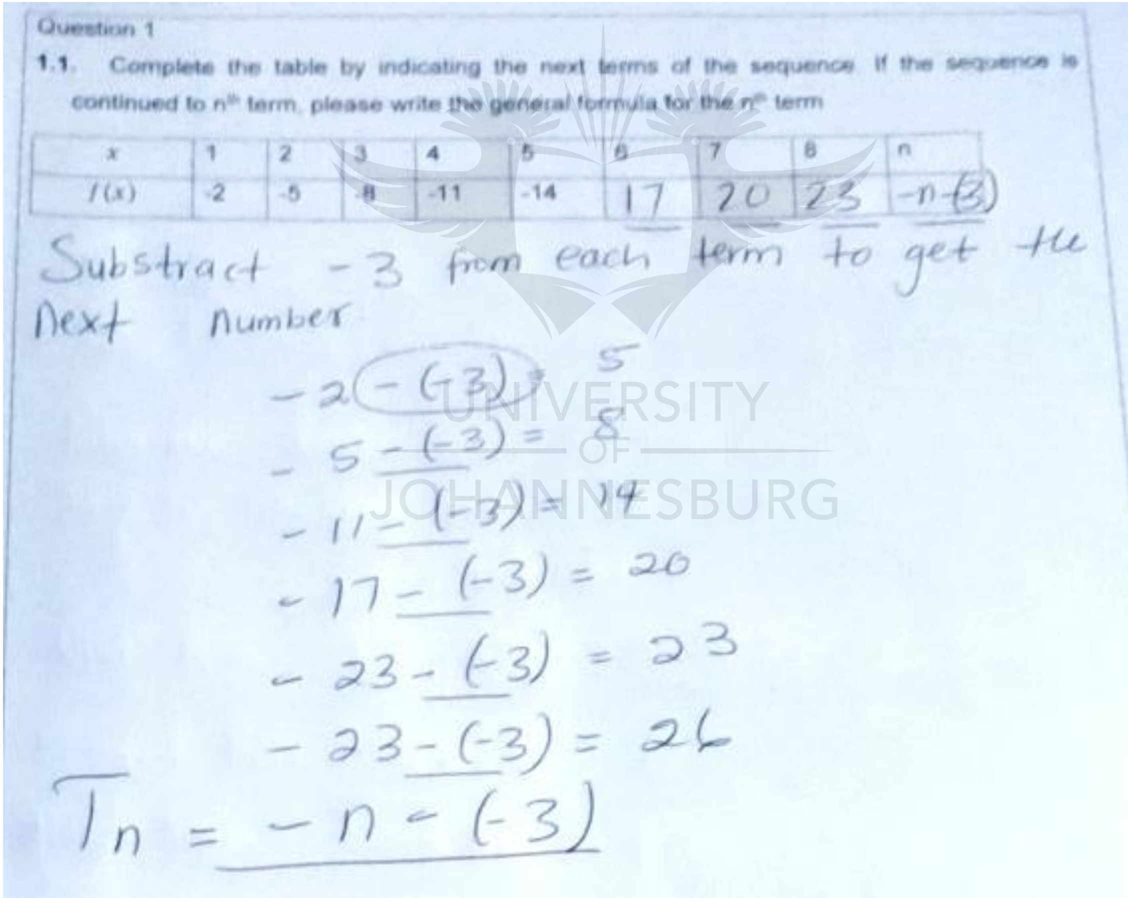


Figure 4.1: Participant A1's response from school A coded as DP2

Participant A1 was one of the 5 participants who showed acceptable levels of engagement in the DP with minor errors, and was coded as DP2. The participant incorrectly decoded the mathematical operations by indicating that "subtract -3 from each term to get the next term". The sign that the common difference should take was also a challenge, because no clear method seems to have been followed, so it could not be traced. So, in this case, there was a minor error of decoding that seems to have occurred. Thus, this participant did not focus on finding relationships between the data and mathematical operators, and showed a lack of integration and interpretation. This means that the participant showed no knowledge of how to use the mathematical operation of subtraction and negative signs. The participant showed an incorrect understanding of negative numbers. Makonye and Fakude (2016) also found that one of the errors learners make is poor interpretation of number lines when dealing with directed numbers. They found that learners could easily calculate numbers, which have positive and addition operation frames, but could not easily accommodate negative numbers or the subtraction operation involving negative integers. Similarly, Carvalho and Da Ponte (2017) revealed that the procedure of counting operations is one of the greatest problem areas for learners.

4.4.1.2 School B

At school B, less than half of the participants (12 out 30) could engage in the activity on DP1 level, while six of the 30 participants were on DP2, and five of the 30 participants were on DP0. Only two participants progressed to DP3. The vignette in Figure 4.2 shows an example of part of question 2 of the written activity of participant B2's response from school B coded as DP1.

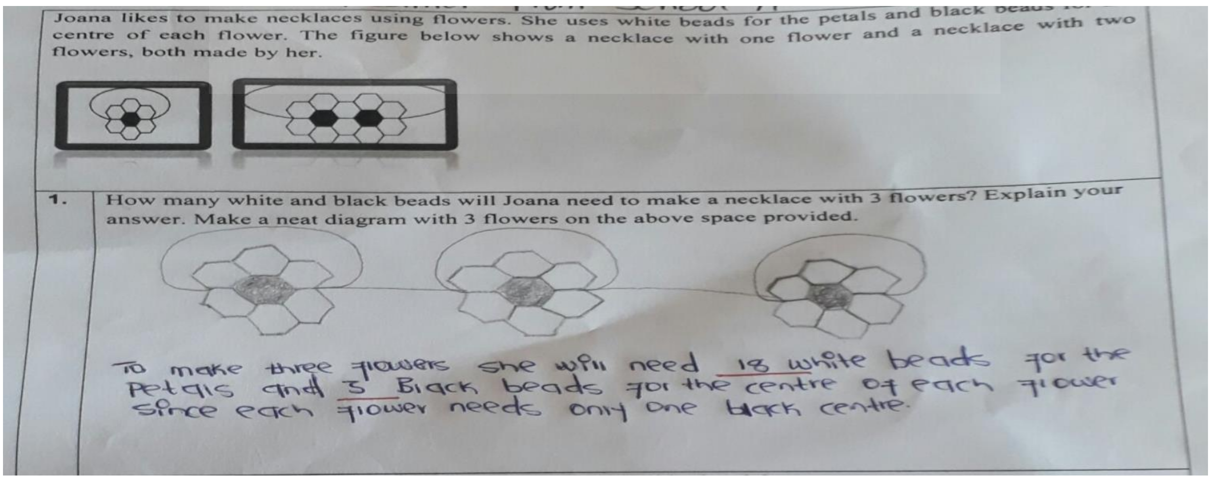
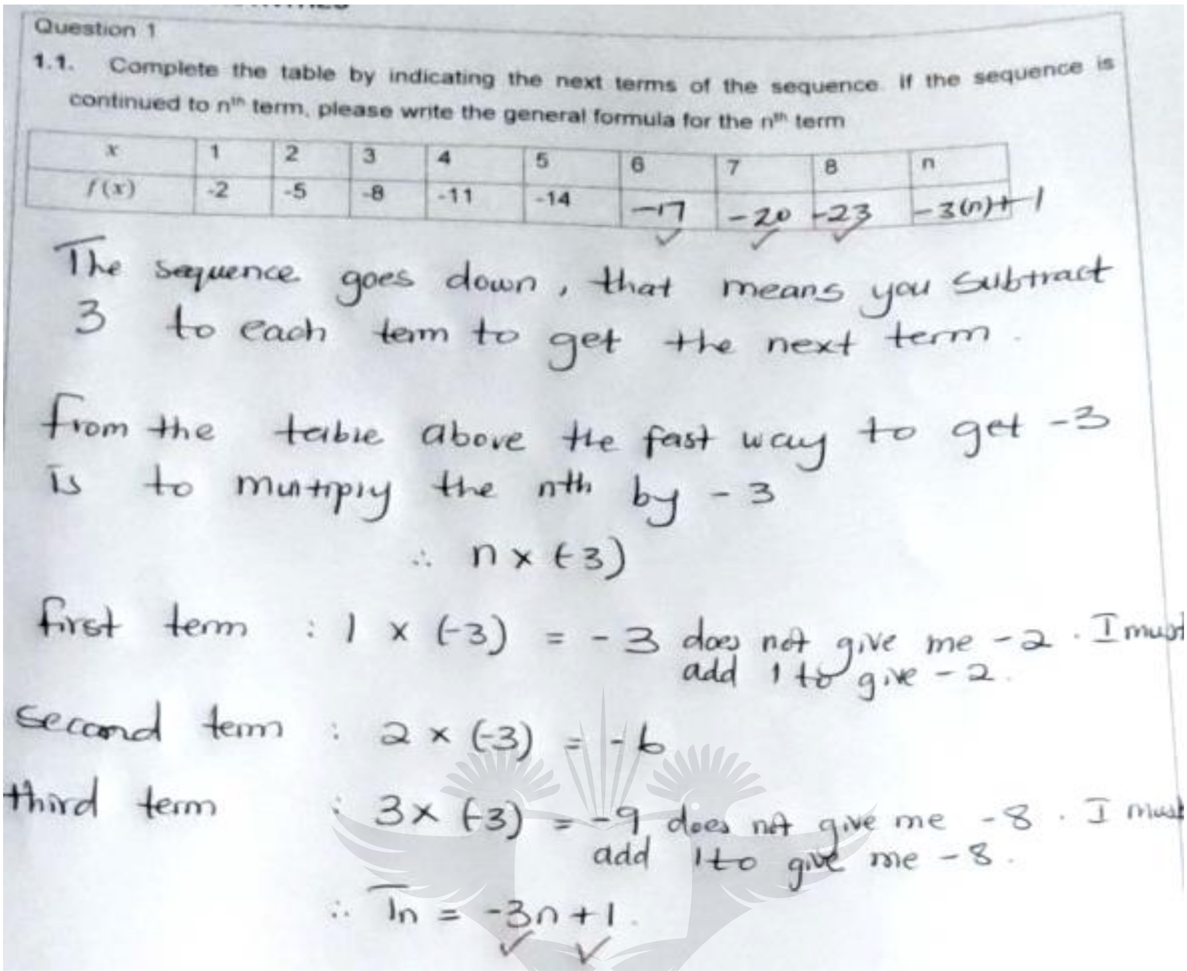


Figure 4.2: Participant B2's response from school B coded as DP1

Participant B2 was one of the 12 participants who reached some insight of decoding, but with major mathematical errors and was coded DP1. The participant focused on using information as it is from the data, which means reading the data with little understanding. The participant was only locating and translating the given information. Participant B2 indicated that “to make three flowers she will need 18 white beads for the petals and 3 black beads for the centre of each flower since each flower needs only one black centre”. The participant made major mistakes when decoding the structure. The participant concentrated only on the relationship between a single pair of beads (white beads) $(n; f(n))$ and used it as a general rule. For example, the participant saw that there are six white beads in the first flower $f(1) = 6 \times 1 = 6$, and then used the rule $f(n) = 6n$ to find the number of beads with 3 flowers $f(3) = 6 \times 3 = 18$; hence the participant used a direct proportional strategy. Lannin, Barker, and Townsend (2006) agree that learners who use a direct proportional strategy immediately attempt to calculate particular values in the given statement problem by means of multiplication, and fail to adjust for any over- or under-counting.

4.4.1.3 School C

At school C, only a third of the participants (10 out 30) could engage in the written activity on DP3 level, while eight of the 30 participants were on DP2, and seven of the 30 participants were on DP1. Five of the 30 participants were on DP0 level. The vignette in Figure 4.3 shows an example of question 1 of the written activity, from school C, of participant C6’s response coded as DP3.



Question 1

1.1. Complete the table by indicating the next terms of the sequence. If the sequence is continued to n^{th} term, please write the general formula for the n^{th} term

x	1	2	3	4	5	6	7	8	n
f (x)	-2	-5	-8	-11	-14	-17	-20	-23	-3(n)+1

The sequence goes down, that means you subtract 3 to each term to get the next term.

From the table above the fast way to get -3 is to multiply the nth by -3

∴ n × (-3)

first term : 1 × (-3) = -3 does not give me -2. I must add 1 to give -2.

second term : 2 × (-3) = -6

third term : 3 × (-3) = -9 does not give me -8. I must add 1 to give me -8.

∴ Tn = -3n + 1

Figure 4.3: Participant C6's response from school C coded as DP3

Participant C6 was one of 10 participants who were able to decode the numerical pattern that involved negative numbers and he/she also showed a good understating of a decreasing pattern. Participant C6 was able to identify the mathematical operator needed to solve the problem. Moreover, this participant was able to make connections between the data and the unknown in a given problem by indicating the next terms of the sequence.

As the sequence continued, the participant knew the starting point of the sequence, and the common difference between terms, then generated the unique sequence and started to make mathematical predictions, such as finding the value of the 9^{th} term. Participant C6 indicated that:

> From the table the fast way to get -3 is to multiply the n^{th} by (-3), n × (-3) = -3n, therefore substitute the first position to see if it gives you a first term. If it does not give a first term, then I added 1 to give -2.

Barbosa et al. (2012) concur that it is important to provide number pattern questions that encourage learners to use and understand the potential of interpreting patterns. Such questions assist learners to relate number contexts with operations in order to understand the meaning of numbers and variables.

4.4.1.4 Summary on findings on decoding

The first phase was categorised as the DP and required participants to interpret, recognise and expand a number pattern, and relate numbers and operations with the problem statement. Fewer participants from schools A and B than school C could identify the pattern related to numbers, and the majority of the participants failed to identify the pattern in a geometric figure. They could only engage in the written activity on DP1 and DP2 level and fewer progressed to DP3. At school C, participants could progress to DP3. Those participants had the ability to move their focus from the wording of the problem to the relationship among the data and the operating schemes. They successfully investigated and extended numeric and geometric patterns. Therefore, Barbosa et al. (2012) suggest that more attention should be paid to the understanding of mathematical concepts of pattern in mathematics teaching (represented as numeric and geometric patterns) by creating environments and opportunities to develop abilities related to this skill.

4.4.2 Findings on the representing phase (RP) and interpretation

4.4.2.1 School A

At school A, majority of participants (11 out of 30) could only engage in the written activity on RP1 level, while nine of the 30 participants were on RP2. Five participants did not engage or reach this phase and were coded as RP0. This finding shows difficulty with mental representing. Only five participants progressed to RP3 level. The vignette in Figure 4.4 shows an example of part of question 2 of the written activity of participant A4's response from school A coded as RP1

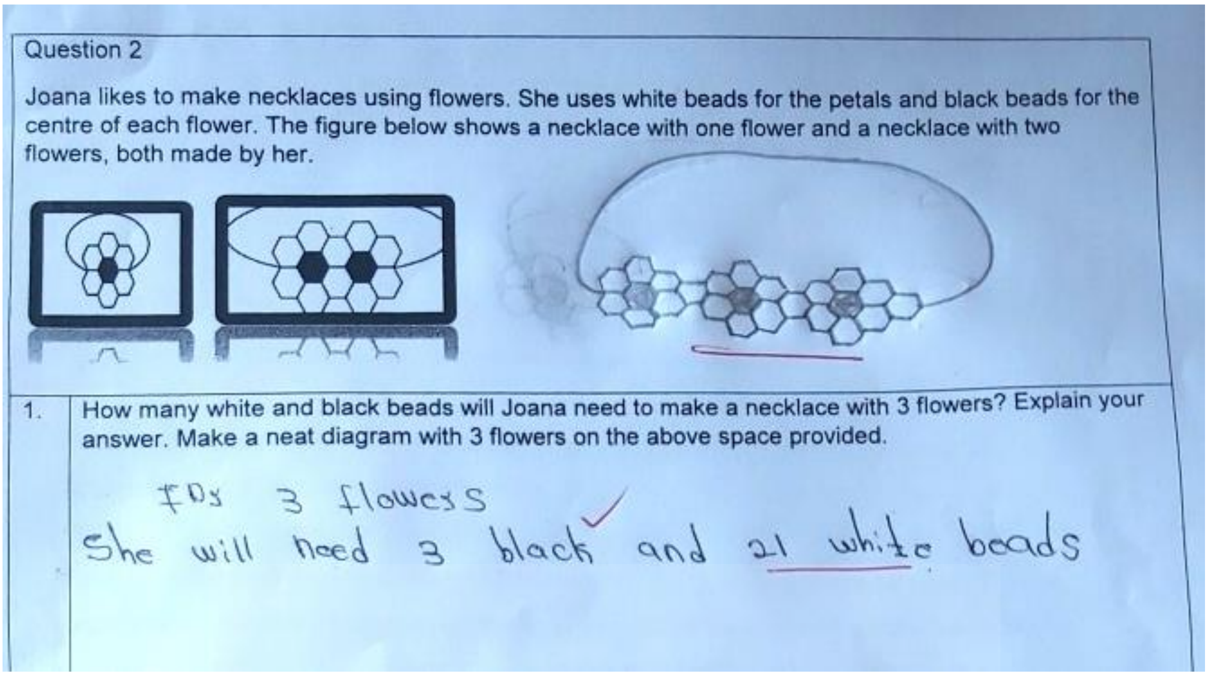
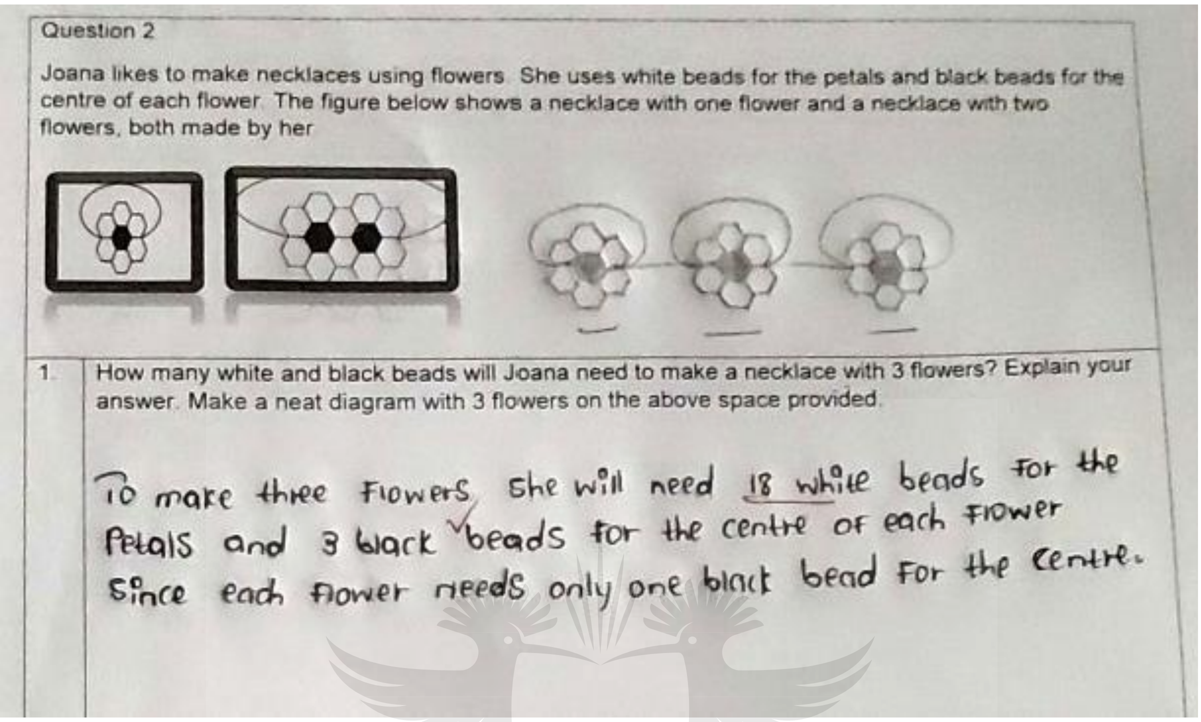


Figure 4.4: Participants A4's response from school A coded as RP1

Participant A4 was one of the 11 participants who were coded as RP1, which is categorised as proportional representing. The participant could only focus on given or apparent information, or prioritised the manipulation of equations with rote memorisation or pattern matching of information. The participant had an idea of joining each flower, but had no idea what the structure of the sequence with 3 flowers would look like. Therefore, the incorrect representation resulted in an over-counting of 21 white beads for three flowers. The participant's response indicated how the flowers were joined to make a pattern but he/she did not realise that there was an overlap of two white beads in each flower. Ibrahim and Rebello (2013) agree that students, who use proportional representation, focus on symbolic representations independently from other forms of visual representations, which shows poor conceptual understanding.

4.4.2.2 School B

At school B, ten participants (10 out of 30) could not represent their problems mentally. They left blank spaces or did not respond to the question and were, therefore, on RP0. 10 out of the 30 participants were on RP1, and four of the 30 participants were on RP2, only two progressed to RP3. The vignette in Figure 4.5 shows an example of part of question 2 of the written activity of participant B’s response from school B coded as RP2.


Question 2

Joana likes to make necklaces using flowers. She uses white beads for the petals and black beads for the centre of each flower. The figure below shows a necklace with one flower and a necklace with two flowers, both made by her

1. How many white and black beads will Joana need to make a necklace with 3 flowers? Explain your answer. Make a neat diagram with 3 flowers on the above space provided.

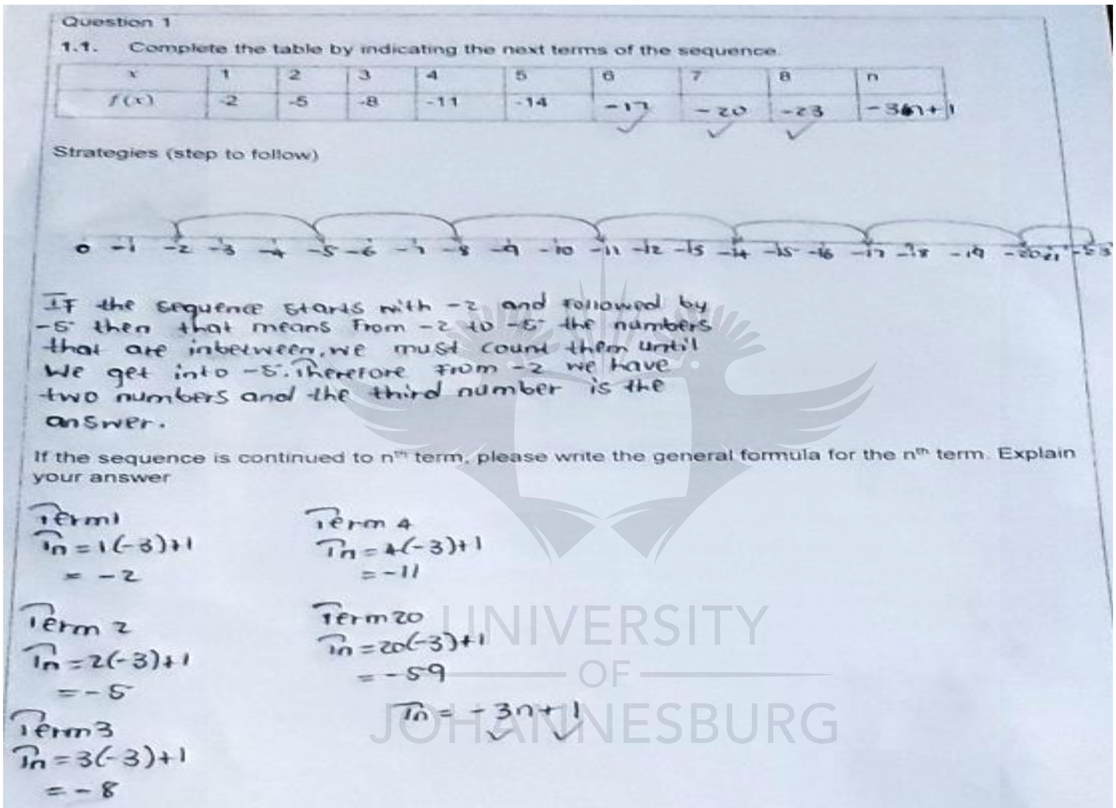
To make three Flowers she will need 18 white beads for the petals and 3 black beads for the centre of each flower since each flower needs only one black bead for the centre.

Figure 4.5: Participant B2's response from school B coded as RP2

Participant B2 was one of four participants who could provide reasons for the problem even if he/she failed to handle the mathematical part of the problem. The participant’s response was categorised as a mental image representation. The participant had an image of a model that represented the flower and responded to a vision of the model by joining the white beads to make a necklace with three flowers from perception or imagination. The participant could provide reasons for the problem by means of a drawing even if he/she failed to handle the mathematical part of the problem correctly. The participant indicated that “the diagram has a sum of 21 beads, 18 white and 3 black centre beads”. Ibrahim and Rebello (2013) found that students who use a mental image fail to translate information from the task presented with a symbolic format into a visual format for solving a problem. Additionally, although they do translate information from a linguistic to a visual format, they do not necessarily use the visual representation to generate a numerical solution.

4.4.2.3 School C

At school C, few participants (8 out of 30) could engage in the written activity on RP3. Seven out of the 30 participants were on RP2, also seven of the 30 participants were on RP1. The participants who did not engage or reach this phase had difficulty in understanding the questions, which were five out of 30 participants who were on RP0. The vignette in Figure 4.6 shows an example of participant C5's response on question 1 of the written activity from school C coded as RP3.


Question 1

1.1. Complete the table by indicating the next terms of the sequence

x	1	2	3	4	5	6	7	8	n
f(x)	-2	-5	-8	-11	-14	-17	-20	-23	-3n+1

Strategies (step to follow)

If the sequence starts with -2 and followed by -5 then that means from -2 to -5 the numbers that are inbetween, we must count them until we get into -5. Therefore from -2 we have two numbers and the third number is the answer.

If the sequence is continued to n^th term, please write the general formula for the n^th term. Explain your answer

Term 1: $T_n = 1(-3)+1 = -2$

Term 2: $T_n = 2(-3)+1 = -5$

Term 3: $T_n = 3(-3)+1 = -8$

Term 4: $T_n = 4(-3)+1 = -11$

Term 20: $T_n = 20(-3)+1 = -59$

$T_n = -3n+1$

Figure 4.6: Participant C5's response from school C coded as RP3

Participant C5 was one of nine participants who progressed to RP3. The participant was able to find the relations among the data and mathematical operators by creating a mental model using a number line. The participant successfully used a number line to find the next terms of the sequence. He/she identified the equal spacing on the number and started skipping counting by two. The participant successfully kept the spaces equal when drawing the marks on a number line, which indicated a good representation on the numerical linear pattern. Reading off values on a number line requires the participant to divide the space between two numbered points into parts. He/she noted: "from -2 to -5 the number that in between we must count them until we get into -5, so that means from -2 we have two numbers and the third number is the answer”. Makonye and Fakude (2016) recommend the use of number line as a model “to help learners understand the concept of directed numbers and how to add and subtract directed numbers” (p. 9).

4.4.2.4 Summary on findings on the representing phase (RP)

The second phase was categorised as the level of mental representation where participants had to build mental representation to create an equation of a linear pattern. Participants from school A partially created concrete representations of the pattern. Few participants could generate mental images to reason about the problem, and they failed to handle the mathematical part of the problem.

Participants from school B were unable to create concrete representations of number pattern problems. The participants could proportionally represent problems, meaning that they focused on given numbers in the problem and manipulated an equation from those numbers with rote memorisation from the prior knowledge.

Participants from school C were able to create concrete representations of patterns and perform alternative solution methods. Those participants were classified as having a mental model. They could focus on comprehending the situation and could highlight the mathematics concepts in the activity.

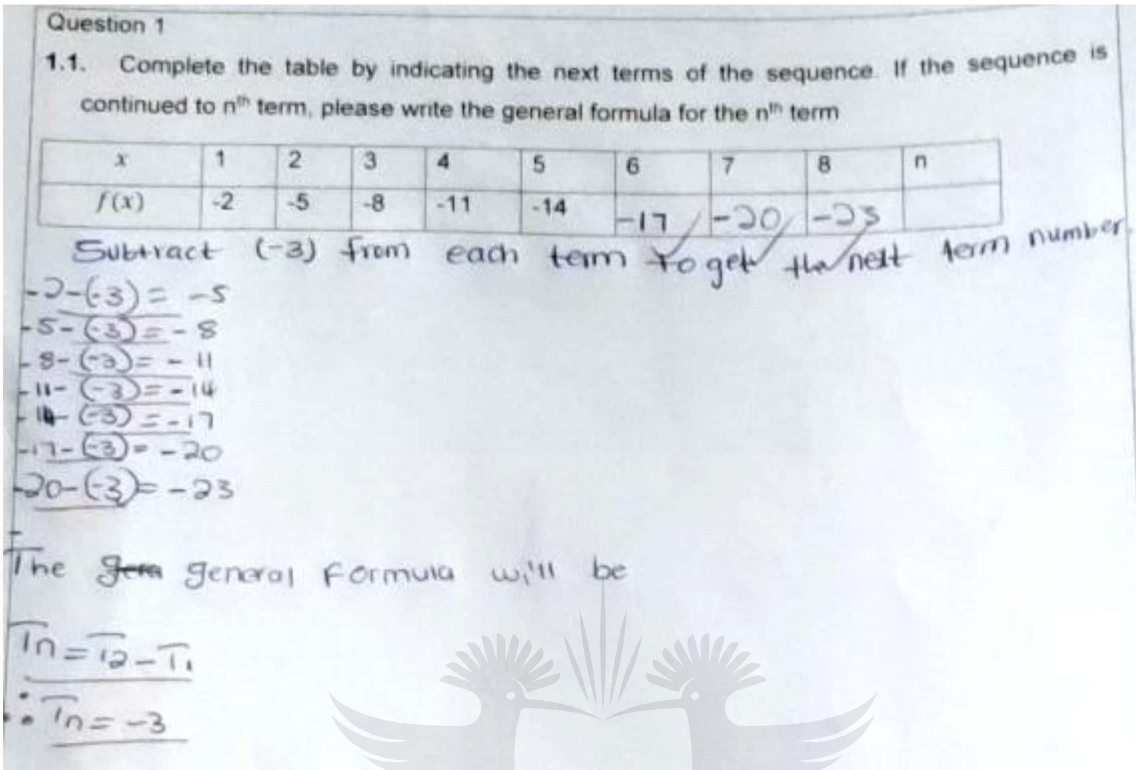
Liljedahl, Santos-Trigo, Malaspina, and Bruder (2016) acknowledge that when learners are given a number pattern problem activity, teachers are actually exposing learners to mathematical reasoning. Mathematical reasoning requires learners to represent their own thinking, and to identify and explore their understanding, which could support learners in the learning the number patterns.

4.4.3 Findings on the processing phase (PP) and interpretation

4.4.3.1 School A

At school A, less than half of the participants (5 out 30) could engage in the written activity on PP2, while 10 of the 30 participants were on PP1, and five of the 30 participants were on RP0. Only four participants progressed to PP3. The vignette in

Figure 4.6 shows an example of question 1 of the written activity of participant A1's response from school A coded as PP1.



Question 1

1.1. Complete the table by indicating the next terms of the sequence. If the sequence is continued to n[th] term, please write the general formula for the n[th] term

x	1	2	3	4	5	6	7	8	n
$f(x)$	-2	-5	-8	-11	-14	-17	-20	-23	

Subtract (−3) from each term to get the next term number

−2−(−3) = −5
−5−(−3) = −8
−8−(−3) = −11
−11−(−3) = −14
−14−(−3) = −17
−17−(−3) = −20
−20−(−3) = −23

The general formula will be

Tn = T2 − T1

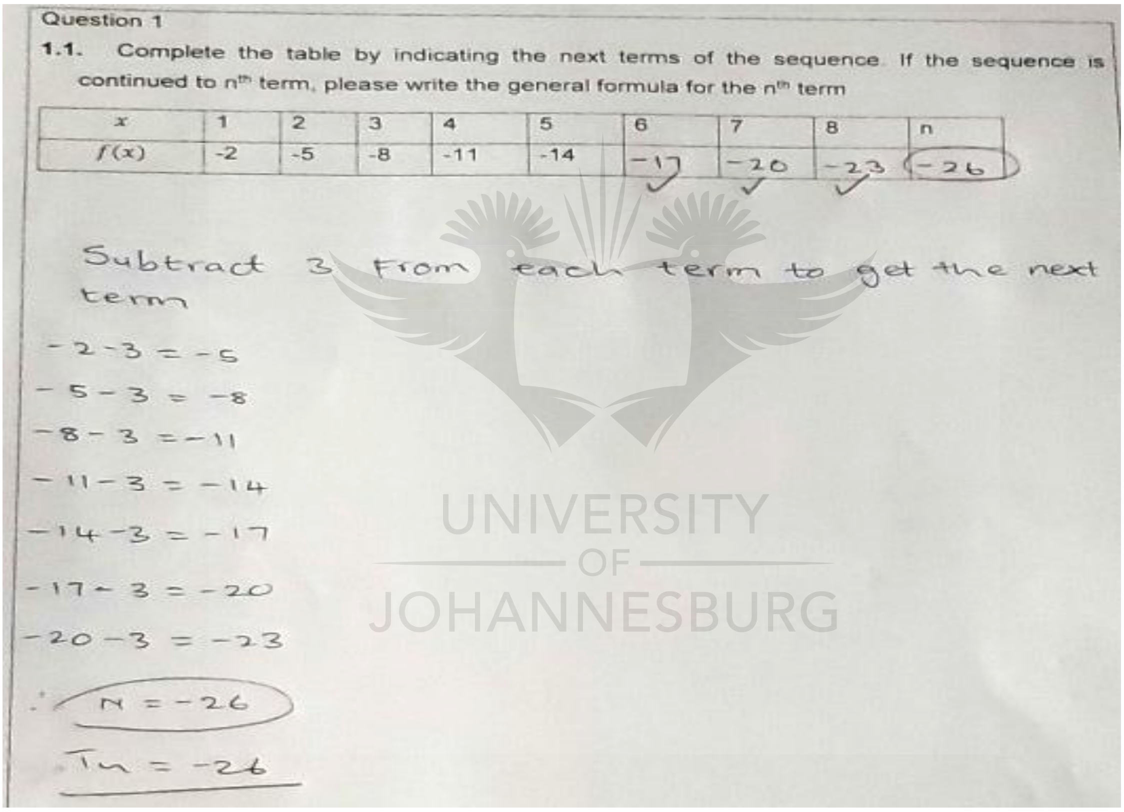
∴ Tn = −3

Figure 4.7: Participant A1's response from school A coded as PP1

Participant A1 was one of nine participants who created a wrong formula to generalise the sequence and was coded on PP1. The participant had knowledge of common differences between consecutives, but failed to describe the relationship between the number of terms and the position of numbers. During the processing to find the n[th] term, the participant could reason about the problem, but failed to generalise the sequence. The participant had evidence on how to calculate the common differences, but mistakenly taking the calculation of the common difference to be the calculation of the general formula of the sequence. The participant indicates that "the general formula will be $Tn = T2 - T1$, therefore $Tn = -3$". The assumption is that this formula for calculating the common difference was taught to learners without meaning and those learners lack understanding of linear function concepts. Foster (2007) agrees that if learners are taught abstract ideas without meaning, there will be no understanding. Learners need experiences with number patterns concepts to learn and develop meaning in solving the problem. Therefore, if a teacher wants learners to know the mathematics content in number patterns, learners must understand the mathematical concepts and not memorise the formula.

4.4.3.2 School B

At school B, three participants (3 out of 30) could engage in the written activity on PP2. Less than a half of the participants (9 out of 30) were on PP1, and ten of the 30 participants were on RP0. Only one participant progressed to PP3. The vignette in Figure 4.8 shows an example of question 1 of the written activity of participant B9's response from school B coded as PP2.



Question 1

1.1. Complete the table by indicating the next terms of the sequence. If the sequence is continued to n^{th} term, please write the general formula for the n^{th} term

x	1	2	3	4	5	6	7	8	n
$f(x)$	-2	-5	-8	-11	-14	-17	-20	-23	-26

Subtract 3 from each term to get the next term

-2-3 = -5
-5-3 = -8
-8-3 = -11
-11-3 = -14
-14-3 = -17
-17-3 = -20
-20-3 = -23
∴ n = -26
Tn = -26

Figure 4.8: Participant B9's response from school B coded as PP2

Participant B9 was one of three participants who could create a model (general formula) with a minor error in finding the term of the sequence and was coded on PP2. The participant correctly completed the table value, but performed an application error by thinking that "n" is the term that follows -23 and applied numerical expressions instead of using an algebraic expression. Another difficulty that emerged from the participant's response was the lack of experience in the use of algebraic symbolism to reason about and to express those pattern generalisations. Güner, Ersoy, and Temiz

(2013) mention that "patterns have an important role as a bridge between generalisation and algebra in primary level for providing constitution of algebraic thinking that is the base of formal algebra" (p. 39). There is a concern that the learners' level of reasoning for near generalisation questions is higher than the generalisation when solving number patterns problems (Jurdak & El Mouhayar, 2014).

4.4.3.2 School C

At school C, few participants (6 out of 30) could engage in the written activity on PP3. Six of the 30 participants were on PP2, and five of the 30 participants were on PP0. Six of the 30 participants were on PP1. The vignette in Figure 4.9 shows an example of part of question 2 of the written activity of participant C5's response from school C coded as PP3.

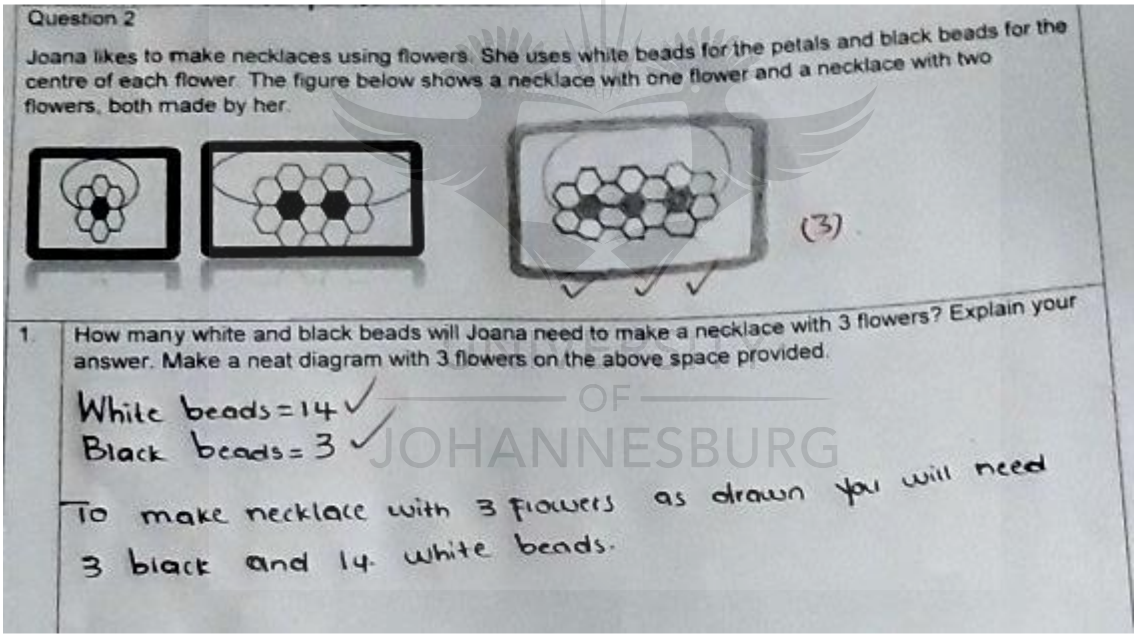


Figure 4.9: Participant C5's response from school C coded as PP3

Participant C5 was one of the nine participants who were able to join the flowers to make a pattern. The participant constructed a mathematical model and then used the model to generalise the pattern to show how the number of previous term depends on the number of the next term. The participant was able to use a mathematical model with a degree of success. The participant's drawing show how the flower was joined to make an increasing linear pattern. The participant used an appropriate mathematical model and came up with a mathematical equation that could help with solving the problem on generalisation. The participant indicated that “to make necklace with 3 flowers, as drawn, we will need 3 black and 14 white beads”.

Therefore, the participant was able to identify a correct mathematical model relevant to the given problem. The participant made no errors in identifying pattern rules and he/she was able to apply pattern rules to the find the correct solution. Mulligan and Mitchelmore (2009:45) indicate that learners “with high levels of awareness of mathematical pattern structure become knowledgeable about spatial structures”, and would have a tendency to look and to explore patterns. Mulligan and Mitchelmore (2009) also regard the finding and understanding of mathematical structure in patterns as pre-algebraic thinking**.**

4.4.3.4 Summary on findings on processing phase (PP)

The PP is characterised by creating correct mathematical models for finding the terms of sequences. The majority of participants from school A were unable to reach this phase and created incorrect formulas to generalise the sequence. Less than a third of the participants from school B created a model, in this case a general formula, with minor errors in finding the terms of the sequence. Few participants from school C could construct a mathematical model and used it to generalise a formula to show how the number of previous term depend on the number of the next term.

According to Greefrath and Vorhölter (2016), learning to work with a mathematical model in number patterns develops learners’ algebraic thinking, which is foundational for working with more abstract mathematics in higher grades. Therefore, processing is needed for exploring different mathematical models, which might be used to build real models in order to generalise solutions.

Mathematical models in the PP provided rich opportunities for the participants to integrate their mathematical knowledge with the usage of patterns and structures. According to Van de Walle and Lovin (2006), algebraic reasoning is directly related to mathematical modelling in patterns because this reasoning focuses on making generalisations based on mathematical experiences and recording these generalisations by using symbols or variables.

4.4.4 Findings on the implementing phase (IP) and interpretation

4.4.4.1 School A

At school A, few participants (4 out of 30) could engaged with the written activity and reached the level of IP2. Less than a half of 30 (9 out of 30) participants only reached the level of IP1, while five of the 30 participants did not engaged and were on IP0. Only two participants progressed to IP3. The vignette in Figure 4.10 shows an example of part of question 2 of the written activity of participant A7's response from school A coded as IP2.

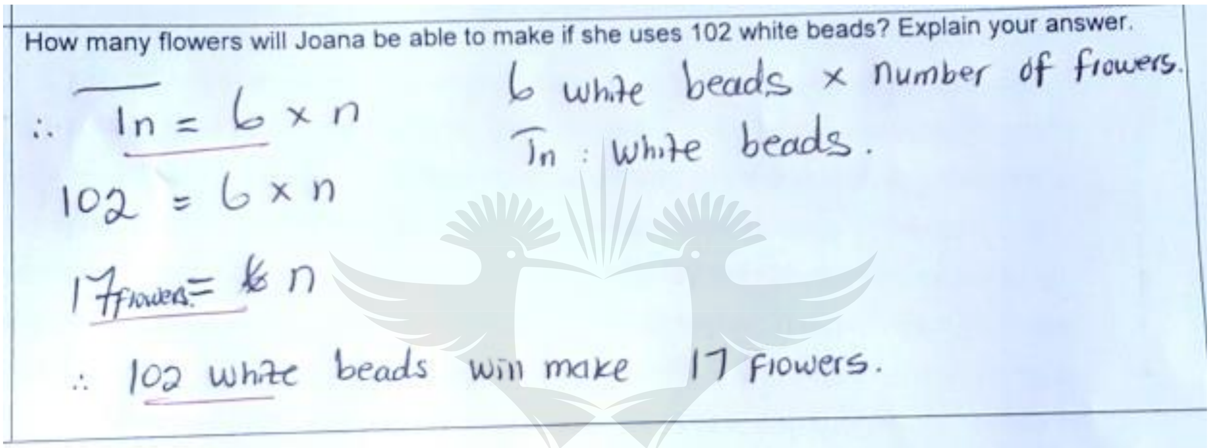
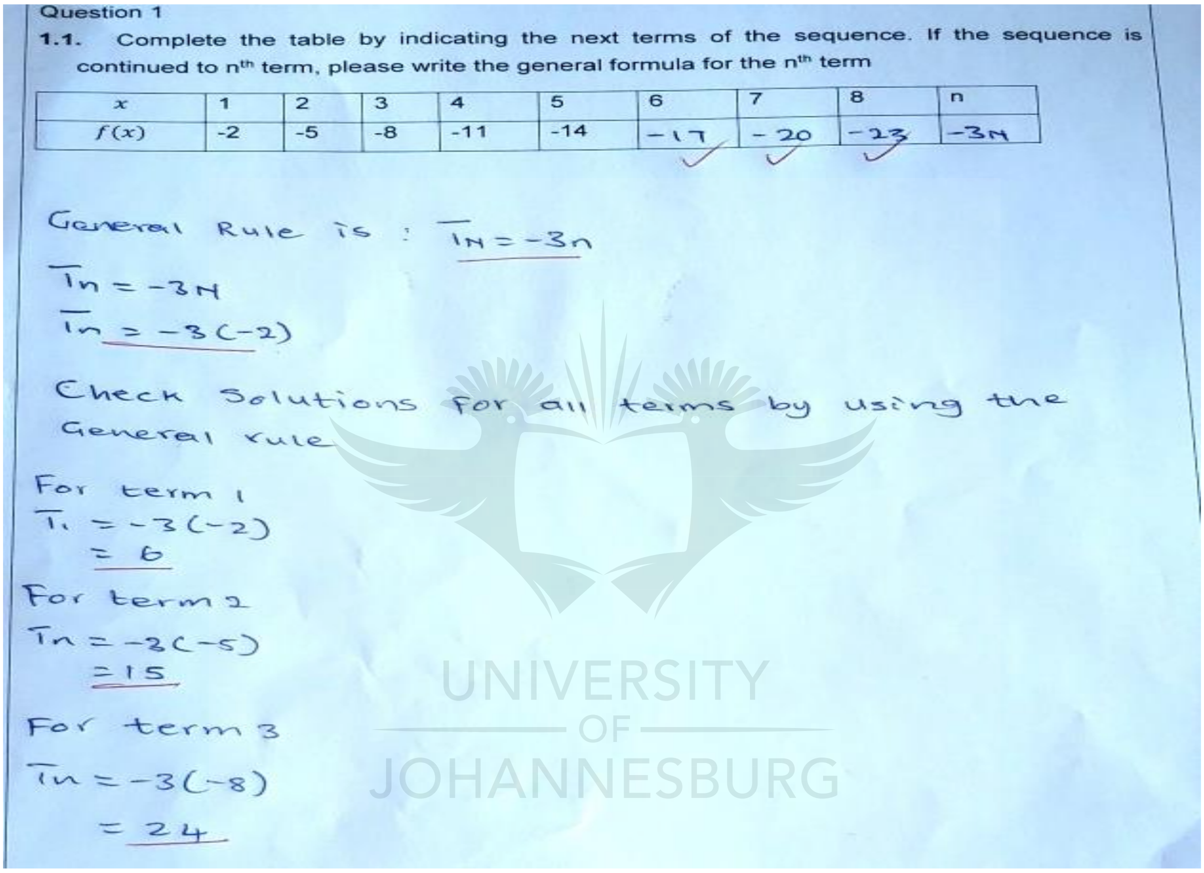


Figure 4.10: Participant A7's response from school C coded as IP2

Participant A7 was one of seven participants who implemented the solution in a manner that addressed the problem statement, but he/she ignored relevant contextual factors. The participant used a direct proportional strategy and did not make a final adjustment based on the context of the problem. Barbosa et al. (2012) indicate that "once it considers multiples of a specific term of a sequence, and the problems presented in the test do not fit that model it is called direct proportional strategy" (p. 283). The participant's tendency to manipulate numbers increased the difficulty to implement the general rule, which was noticed by the order of the flowers that got higher. The participant indicated that the rule to find the number of white beads was Tn = 6n. This rule indicated that he/she did not properly analyse the structure of the flower, but thought each flower is a disjoint unit. Therefore, the participant used a direct proportional strategy which was not relevant to the given problem.

4.4.4.2 School B

At school B, three participants (3 out of 30) could engage in the written activity on IP2. Less than a half of 30 participants (7out of 30) were on IP1, and ten of the 30 participants were on IP0. Only one participant progressed to IP3. The vignette in Figure 4.11 shows an example of question 1 of the written activity of participant B8's response from school B coded as IP1.



Question 1

1.1. Complete the table by indicating the next terms of the sequence. If the sequence is continued to n[th] term, please write the general formula for the n[th] term

x	1	2	3	4	5	6	7	8	n
$f(x)$	-2	-5	-8	-11	-14	-17	-20	-23	-3n

General Rule is : Tn = -3n

Tn = -3n

Tn = -3(-2)

Check Solutions for all terms by using the General rule

For term 1

T1 = -3(-2)

= 6

For term 2

Tn = -3(-5)

= 15

For term 3

Tn = -3(-8)

= 24

Figure 4.11: Participant B8's response from school B coded as IP1

Participant B8 was one of 16 participants who implemented the solution in a manner that did not directly address the problem statement and was coded on IP1. The participant conducted the wrong mathematical model and used a direct proportional model to implement the solution. Participant B8's response aligns with a concern raised by Jurdak and Mouhayar (2014) that learners' level of reasoning for near generalisation questions is higher than that for the far generalisation when solving number patterns. Smith and Thompson (2007) argue that incorrect implementation of mathematical models in solving number patterns results from elementary curricula that fail to develop learners' abilities to reason about complex additive and multiplicative relationships. Furthermore, according to Magiera, Van den Kieboom, and Moyer (2013), learners whose strategies are inappropriate to the problem statement, make more errors related to mathematical properties of number patterns, such as inappropriate organisation of information, identification of a pattern, description of the rule and justification of it.

4.4.4.3 School C

At school C, five participants (5 out of 30) left a blank space and, thus, did not reach the phase on IP0. Six of the 30 participants could only engage in the written activity on IP1. four of the 30 participants were on IP2, and six of the 30 participants were on IP3. The vignette in Figure 4.12 shows an example of part of question 2 of the written activity of participant C5's response from school C coded as IP3.

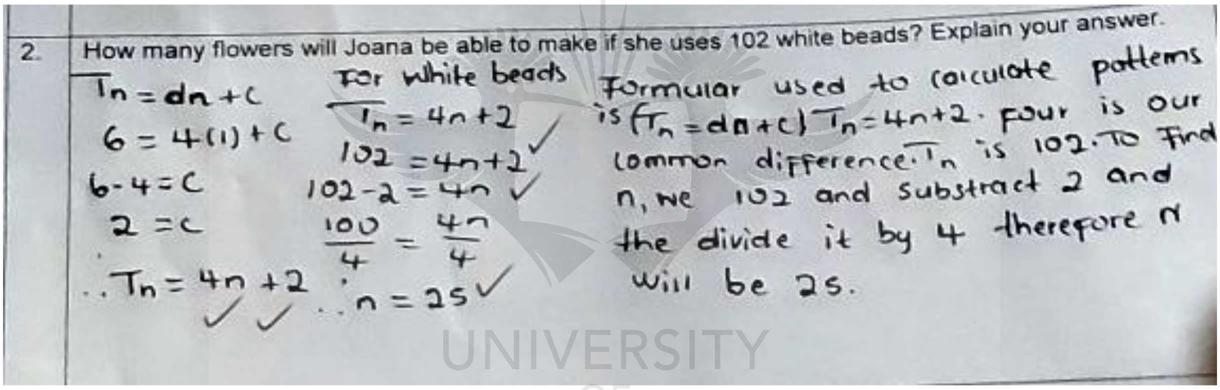


Figure 4.12: Participant C5's response from school C coded as IP3

Participant C5 was one of nine participants who applied techniques that were specific to the mathematical model and was coded on IP3. The participant implemented the correct general rule to obtain the correct solution. The participant used an explicit strategy to implement the solution. The participant applied techniques that were specific to the mathematical model and produced a mathematical explanation and justification of the solution. Güner et al. (2013) acknowledge that for the development of algebraic thinking in mathematics, it is important for learners have knowledge about how to construct patterns, to use cognitive processes and to think in these constructions.

4.4.4.4 Summary on findings on the implementing phase (IP)

The IP relates to the application of techniques, which are specific to the mathematical model, and also the production of mathematical explanation, the justification of the solution, and the general questioning of the model. Some participants from school A, B and C implemented the solution in a manner that did not directly address the problem statement, while several participants from school B implemented the solution in a manner that addressed the problem statement, but ignored relevant contextual factors.

Incorrect implementation shows that the participants had a perception of the process described in the text, but could not see the need for reversibility (Singer & Voica, 2013). Most participants who responded in the IP, were able to express the sequence for near generalisation, but were unable to express the sequence for far generalisation. Finally, some participants from school C applied techniques that were specific to the mathematical model and also produced correct explanations of their solutions. During the IP the participants had to check critically and to reflect on their solutions to reveal creativity. The participants, who were unable to implement the solutions to obtain the correct answers, did not appear to have an understanding of the mathematical model of the patterns as they found it difficult to check and reflect on their solutions. According to John, David, and Townsend (2006), formulating a mathematical model of number patterns recognises the need to make final adjustment based on the context of the problem to avoid over or undercounting of numbers.

4.4.5 Overall findings from the written activity

Most of the participants (30 out of 90), which include participants from all three schools in the study, could only engage in the written activity on the DP1. In contrast, 28of the 90 participants were able to engage in the activity on the RP1, 25 of the 90 participants were able to engage in the written activity on the PP1, and 22of the 90 participants could engage in the written activity on to the IP1. Those were the participants who implemented the solution in a manner that did not directly address the problem statement.

Less than a quarter of the participants (19 out of 90), could only engage in the written activity on the DP2. Only 16 of the 90 participants were able to engage in the written activity on the RP2, 13 of the 90 participants were able to engage in the written activity on the PP2, and 11 of the 90 participants could engage in the written activity on to the IP2. Those were participants who implemented the solution in a manner that addresses the problem statement, but they ignored relevant contextual factors. Furthermore 21 of the participants (21 out of 90), could only engage in the written activity on the DP3. 15 of the 90 participants were able to engage in the written activity on the RP3, 11 of the 90 participants were able to engage in the written activity on the PP3, and only 9 of the 90 participants could engage in the written activity on to the IP3. Those were the participants who applied techniques that were specific to the found mathematical model during implementation.

Magiera et al. (2013) concur that there is a need to understand the learners' thinking processes and also to determine what they already know or do not know when assessing them on number patterns. According to SP CAPS, grade 9 learners are exposed to number patterns generalisation through algebraic concepts such as variables, equations, algebraic expressions in the previous grades (DBE, 2011). Most participants whose strategies were fragmented during the RP and PP when solving the number pattern problem, recalled prior knowledge that was within the context of the problem. However, some elements of their solution strategies were incorrectly manipulated that were caused by incorrect representation and processing of the problem. When representing their strategies, most participants confused the roles of the independent and dependent variables. Thus, they had an incorrect understanding of positive and negative numbers when describing the rule and justifying it.

Less than a half of the participants (43 out of 90) whose strategies were inappropriate to the number pattern problem, failed to implement the mathematical model to obtain the correct answer. Although they had a mathematical model of the described situation, it was not entirely satisfactory for the problem text. Only 12 participants out of 90 whose implementation was appropriate to number pattern problems, could recall and manipulate number patterns strategies that were relevant to the context of the problem. Those participants were developing an own understanding through meaningful decoding, representation and processing that allowed them to deeply understand the concepts and processed what was already known about the problem. They could use the model as a building block to implement the solution and their understanding of the given problem was at the advanced level. They were able to move beyond the data as they were able to generate and predict the number patterns. Thus, their written activity showed that they had a deeper understanding of mathematical structures and mathematical concepts related to number patterns.

4.5 ONE-ON-ONE SEMI-STRUCTURED INTERVIEWS

Semi-structured interviews were conducted with one participant from each of the three schools. The three participants (Participant A, Participant B and Participant C) were selected to be interviewed based on the following criteria: undocumented strategy on number pattern problems; inappropriate strategy on number pattern problems; and appropriate used of a specific strategy on number pattern problems (see sub-section 3.5.3).

The transcripts of these three interviews were qualitatively analysed using the content analysis method. Ezzy (2002) indicates that content analysis starts with the unit of analysis (for example, words or sentences) and the categories to be used for analysis. The texts were reviewed in order to code them and to place them into categories. Then, the occurrences of words, codes and categories were counted and logged.

In the context of this study, the unit of analysis were the phases; therefore, categories were based on the indicators of the phases of the PS conceptual framework. These indicators were developed and discussed in sections 4.3 to 4.6 and were used to analyse the participants' responses from the interviews. The participants' responses from the interviews were coded according to the levels of engagement in the four phases of the PS conceptual framework as per indicator (see Appendix M, N and O). The number of responses per indicator was counted. Table 4.8 presents the number of participants' interview responses showing their levels of engagement in the four phases of the PS conceptual framework regarding their solutions to the written activity.

Table 4.8: Number of participants' interview responses showing the levels of engagement in the phases of the PS conceptual framework

Phases	Indicators	Number of responses			Total number of responses per phase
		Participant A	Participant B	Participant C	
Decoding	DP1	0	3	0	11
	DP2	3	0	0	
	DP3	1	1	3	
Representing	RP1	1	1	0	7
	RP2	2	1	0	
	RP3	0	0	2	
Processing	PP1	0	0	0	7
	PP2	2	2	0	
	PP3	0	0	3	
Implementing	IP1	1	1	0	6
	IP2	1	0	0	
	IP3	0	0	3	
Total number of responses per participant		11	9	11	31

4.5.1 Findings on views of decoding and interpretation

The grand total number of responses on the DP was 11 in total for participants A, B and C. Participant A15 responded three times on DP2 and once on DP3. As participant A1 was completing the sequence from the table, he/she mentioned that "I am counting the numbers from the sequence by subtracting 3 up until I arrive to the n^{th} term". The counting strategy in near generalisation was the preferred strategy for the participant; as the participant was unable to give the general rule for far generalisation.

Barbosa et al. (2012) define a counting strategy as counting the elements of a particular number or figural term in a pattern. Counting is always a successful strategy, but is only useful in solving near generalisation questions (see section 2.6). Samson and Schäfer (2007) acknowledge that a noticeable positive change in the counting strategy is evident when three consecutive purely pictorial terms are used instead of two non-consecutive pictorial terms.

Participant B2 responded three times on DP1 and once on DP3. Participant B2 stated that: “the sequence goes down and we called it a decreasing pattern and we represent the common difference between consecutive numbers by the variable d; I have calculated the common difference (d) between consecutive numbers which is -3. E.g. T2-T1=d; -5-(-2) = -3; T3-T2=d the common difference between two consecutive terms is -3, and then I subtracted the first term by -3 to get the next term”.

Participant B2 used the recursive strategy, but was unable to give the n^{th} term. He/she could not process the correct general formula. Lannin et al. (2006) define a recursive strategy as a connection occurring in the scenario between the independent variable's successive values. This finding does not correspond with Lannin et al. (2006) arguing that most learners used recursion when they appeared to have a strong visual picture of the situation and when they focused on decontextualized numerical relationships. This finding can be interpreted in the way participant B2 responded in his/her decoding of the decreasing numerical pattern. Although the participant had a strong visual picture of the situation by focusing on the relationship between the common difference and the consecutive number to complete the table, he/she obtained results in near generalisation rather than in far generalisation, hence he/she could not process to the general formula.

Participant C5 responded on three times DP3. Participant C5 showed an understating of decoding accurately, appropriately, and flexibly. Participant C5 could see that each flower shares two white beads to have a complete flower; four white beads were added to the end of each prior flower to create a new flower. This finding is consistent with Mulligan and Mitchelmore (2009) claiming that if learners are conscious of mathematical patterns and structure, they tend to look for patterns and discover similarities and differences between them and learn more easily fresh constructions. This finding can be interpreted in the way participant C5 responded in his/her decoding of the mathematical pattern. This finding concurs with Mulligan and Mitchelmore (2009) claiming that if students are aware of mathematical patterns and structure, they have a tendency to look for patterns and to explore similarities and differences between them and learn new structures more easily.

Participant C5 was conscious of mathematical pattern and structure, he/she correctly decoded the mathematical structure on the pictorial pattern when responding that: "the flowers should be a joint unit which to make this pattern a linear pattern".

During the DP, an understanding of position-to-term relationship was evident in the interviews with participant A15, B2 and C5. Participants A1 and B2 could show an understating of decoding accurately, appropriately, and flexibly by finding a pattern to discover terms in the near position using the counting strategy. However, only participant C5 was able to proceed to the far position using the general rule. All three participants used mathematical operators correctly and were able to find the relationships among the data and the operators.

Participants A15 and B2, who used the counting strategy, applied direct proportion to determine the term in the far position. This strategy was incorrect for the given problem. Both participants A and B lacked an understanding of linear pattern for far generalisation during decoding. They could not identify how each figure and number in the pattern differed from each previous figure or number as the pattern increased. In their description of the number patterns for the far position, both participants A and B did not recognise that each term had a numeric value. These participants were unable to demonstrate a way of establishing a general formula of the n^{th} term of the sequence number in a table form. This finding correspond with Barbosa et al. (2012) which disclosed that in solving problems involving pattern exploration, learners encountered several difficulties, particularly when they had to generalise for remote values. In problems of near-generalisation they accomplished better outcomes than in issues of far-reaching generalisation.

This finding can be interpreted in the way participant A15 and B2 responded in their decoding of the numerical pattern and geometric pattern, they began using the counting approach, although they moved to a direct proportion model for the far-reaching generalisation. They were focusing on calculating the number of components of a shape or configuring in order to calculate the expected drawing a shape

4.5.2 Findings on views of representing and interpretation

Participant A15 responded twice on RP2 and once on RP1. Participant A15 focused on given or apparent information or prioritised the manipulation of equations with rote memorisation or pattern matching of information. Participant A15 mentioned that "I am weak in mathematics and poor in visualisation to see a pattern, but now as I am speaking to you I see light". Participant A15 could, therefore, provide reasons for the problem even if he/she failed to handle the mathematical part of the problem. Participant A15 mentioned that "6 white beads and one black bead make one flower therefore for 3 flowers will be (6x3=18 beads)". The mathematical structure of the sequence was not properly represented. Participant A15 interpreted each flower as a disjoint unit and therefore used the direct proportional strategy.

This finding does not correspond with Barbosa et al., (2012) suggesting that it is important to provide learners with questions that encourage them to use and understand the potential of visual patterns. Such visual patterns will enable learners to relate number contexts with visual contexts in order to enhance their understanding of the meaning of numbers and variables. This finding can be interpreted in the way participant A15 responded in his/her representing of the visual pattern. Although the participant had a visual patterns of the situation but was unable to relate number contexts with visual contexts in order to enhance his/her understanding of the meaning of numbers and variables, hence he/she did not make a final adjustment based on the context of the problem.

Participant C5 responded twice on RP3. When participant C5 was asked what type of pattern is presented in the sequence, he/she answered that: "the pattern is an increasing linear pattern they sometimes call it arithmetic sequence", and indicated:

> Each flower must be a joint unit to form a linear pattern. Each flower shares two white beads to have a complete flower, four white beads were added to the end of each prior flower to create a new flower, and then added two beads to give me a complete flower, therefore my general rule will be Tn=4n+2 the variable n represent the number of flowers.

Participant C5's response was categorised as having a mental model, referring to the participant's ability to conduct high-level reasoning. Participant C5 focussed on comprehension of the situation and highlighted the mathematics concepts in the activity. Moreover, before dealing with equations he/she was able to include a diagrammatic representation in his/her solution of the problem. This finding correspond with Ibrahim and Rebello (2013) who acknowledge that the construction of the mental model provides a means of linking the syntactic (mathematical) and structural (visual) aspects of the task under consideration, enabling interpretation and understanding to be taken into account. This finding can be interpreted in the way participant C5 responded in his/her mental model representation. The participant linked the mathematical concepts and structural aspects of number pattern to construct the number of beads to make flowers with understanding.

Participant B2 responded once on RP2 and once on RP1. Participant B2 mentioned that "the sequence increases by 6 white beads in ever flower, but I don't know the type of pattern". Participant B2 claimed that she/he understood how the subsequent terms were obtained. Yet, she/he was unable to establish the rule for the general term. Both participants A15 and B2 had a mental image of the problem. They demonstrated a diagrammatic representation (flowers) which may not be linked to the mathematical formulations used. This finding correspond with Becker and Rivera (2005), arguing that variables used simply as non-significant placeholders except as a generator for linear number sequences show a lack of representation. This finding can be interpreted in the way participant A15 and B2 responded in question 2, they could count number of beads from the sequence, but made no sense of what the n^{th} term in the linear pattern represented, nor what the variable in the linear pattern represented. Although they are aware of the relationship among the three flowers and the beads and they may recognise the applicability of the qualitative method to solve the problem, they prefer manipulating equations.

4.5.3 Findings on views of processing and interpretation

Participant A15 responded twice on PP2. When asked to complete number patterns on a table until the n^{th} term, participant A15 responded: "I counted the numbers from the sequence by subtracting 3 up until I arrive to the n^{th} term, I think n will be a number that follow the previous number in the sequence". Participant A15 used direct counting to solve near generalisation for the given number patterns. For far generalisation Participant A15 claimed that “the general formula will be $Tn = -3 \times n$”.

This finding correspond with Barbosa et al. (2012) arguing that learners who are “not able to find adequate explicit rules, reveal difficulties in finding a functional relationship and make many mistakes, such as the application of a direct proportion model when it is not applicable” (p. 291). This finding can be interpreted in the way participant A15 responded in his/her general formula. By using the counting strategy, participant A15 found it difficult to produce a mathematical model, specifically where far generalisation was involved, and hence used direct proportion.

Participant B2 responded three times on PP2 and mentioned:

> I found out that the pattern decreases by 3 less than before. I tried the different possibilities i.e. n^{-3}, -3n, 3×n and trying to add other digits that can give the first -2. I then substituted n by 1; 2; 3 so that it can give me the pattern of the sequence. Then I found the algebraic expression for this kind of pattern then I choice -3n therefore I have Tn=-3n+….

Participant B2 could write some general formulas, but did not focus on the relationship between the term and the value of the term, nor did he/she check the accuracy of formulas by comparing the number and value of the step. Thus, he/she was unable to find the general formula. Participant B2 failed to elaborate clearly on how he/she arrived at his/her rules used in the written activity. Becker and Rivera (2005) concur that learners who are unable to provide the general formulae tend to start with numerical strategies; however, they lack the flexibility to try other approaches and see possible connections between different forms of representation and processing strategies.

This finding can be interpreted in the way participant A15 and B2 responded in his/her representation and strategy for processing. Participant B2 only knew the letter n represented a number, whereas participant A15 took it to mean the previous term. Clearly, participants A15 and B2 were unaware of the meaning of n as used in the symbolic rule.

Participant C5 responded three times on PP3. Participant C5 mentioned:

$n \times (-3) = -3n$; n represents the number of term in the sequence and -3 is the common difference. Term 1 = $1 \times (-3) =$, this does not give -2 which mean I must add 1 Term 2 =$2 \times (-3) = -6$, this does not give me -5 as the second term I must add 1; therefore I will have $Tn = -3n + 1$.

Participant C5 had reasoning competence on producing a mathematical model as he/she could demonstrate a way of establishing a general formula for the n^{th} term of the sequence number by drawing a logical conclusion. Thus, participant C5 showed an understanding of the mathematical model during the PP.

Participant A15 preferred to use a direct proportional strategy far generalisation, while participant B2 tried to use guess and check for far generalisation. This guessing and checking strategy took the form of scribbling down the algebraic expressions by participant B2. Participant B2 tested and adjusted the number in order to fit the expression to the pattern.

4.5.4 Findings on views of implementing and interpretation

Participant A15 responded once on IP1 and IP2. He/she implemented the solution in a manner that addressed the problem statement, but ignored relevant contextual factors. When asked to use the mathematical model to calculate the number of flowers Joana will need if she uses 102 white beads, participant A15 mentioned that "6 white beads and one black bead make one flower therefore for 3 flowers will be (6 x 3 = 18 beads)". Participant A15 was asked to express the pattern rule in then algebraic expression Tn = 6n, and responded as follows:

> Joana will need 17 flowers with 102 white beads, I was counting how many white beads for the second flower, and I was looking for a rule that would work therefore I multiplied six which is the white beads by the number of flowers.

Participant A15's explanation for using direct proportion in this study indicates that her/his response was on IP2, thus focussing on taking information as is from the data by reading the data with little understanding. Barbosa et al. (2012) found that students who use direct proportional strategy were not able to find adequate general rules, revealing difficulties in finding a functional relationship between the data and the mathematical operations. This finding can be interpreted in the way participant A15 responded to the number of flower will Joana be able to make if she uses 102 white beads. The participants used direct proportional strategy and was not able to find adequate general rule, revealing difficulties in finding a functional relationship between the number of beads and the number of flowers.

Participant B2 responded once on IP1. Participant B2 implemented the solution in a manner that did not directly address the problem statement. Participant B2 used an incorrect strategy for implementing the decreasing pattern. Participant B2 could hardly apply a mathematical operation, and this kind of error also appeared in the analysis of participant B2's written activity. In the case of $-2-(-3)$, the participant confessed: "I added 2 and 3 to give me 5 but know that the common difference of the sequence was 3 so I ignored the subtraction sign and the negative sign added 2 and 3 to give me the next term." This finding is in agreement with Akkan (2013) claiming that most learners who use the recursive (additive) strategy to generalise patterns are able to find near terms accurately, but are unable to implement a model to find later terms in a sequence. This finding can be interpreted in the way participant B2 responded in his/her implementation. Although the participant focusing on the relationship between the common difference and the consecutive number but still applied an error on the mathematical operation and could not find near terms accurately and was not able to implement a model to find the next term in the sequence.

Participant C5 responded three times on IP3. Participant C could construct a mathematical model and use the constructed model to generalise a pattern. Specifically, he/she could show how the number of a previous term depends on the number of the next term. Participant C5 could thus establish the relationship between terms and could use this relationship to express the general formula of the pattern. Participant C5's response shows that he/she processed knowledge about the obtained formula and, therefore, used a recursive strategy to generalise the pattern. Barbosa et al. (2012) state that, when learners use a recursive strategy correctly, they should be able to find the pattern for near and far generalisations. Using the recursive strategy, this learner used the common difference between two consecutive terms of the sequence to solve the questions posed. This finding correspond with Barbosa et al. (2012) arguing that a learner can extend the sequence and generate the general formula using the common difference, but when using multiples of the common difference without final adjustment, it would result in inaccurate answers.

This finding can be interpreted in the way participant C5 responded the number of bead need to make three flowers. The participant indicated that each flower shares two white beads to have a complete flower, four white beads were added to the end of each prior flower to create a new flower which is the common difference, and then make final adjustment by adding two beads to give me a complete flower.

4.5.5 Summary of the overall findings from the semi-structured interviews

Eleven of 31 responses by participants indicated that they have reached the DP, 7 of 33 responses showed evidence of moving to the RP and PP, and 6 of 33 responses implied some level of engagement at the IP. The interviews shed light on a number of reasons influencing the strategies participants used during the solving of the number pattern problem. Apart from their personal opinions and prior learning experiences, other reasons ranged from a lack of understanding of mathematical operations, such as subtraction and addition, to a lack of visualisation of patterns, which resulted in overlapping shapes, and finally, to a lack of understating of the n^{th} term when completing a table of values.

According to the responses in the interviews, it can be inferred that, during the IP, participants who used the direct proportional strategy did not make a final adjustment based on the context of the problem to obtain a final solution. As a result, those participants may have lacked mental representing and processing of the mathematical model. Even though the participants were able execute the IP, their solutions were incorrect. They 'picked up' numbers directly from the given problems and applied multiplication to those numbers, hence their responses were coded on IP2. Participant B could successfully find the mathematical model to generalise the pattern, and was able to use the rule for far generalisation.

Participant C's visual images appeared to contribute to the success in generating correct pattern rules. Therefore, participant C was able to engage in all four phases of Singer and Voica's (2013) PS conceptual framework. The overall finding emerging from the participants' responses showed that the participants could successfully recognise patterns and generalise them as algebraic formulas.

4.6 THE LEVELS OF ENGAGEMENT IN THE FOUR PHASES OF THE PS CONCEPTUAL FRAMEWORK IN STRATEGIES FROM THE ANALYSIS OF THE WRITTEN ACTIVITY AND THE INTERVIEWS

This section ascertains the levels of engagement in the four phases of the PS conceptual framework in the participants' strategies derived from the analysis of the written activity and the interviews Table 4.9 illustrates these levels of engagement in the four phases of the PS conceptual framework in the solutions of the problems of the written activity of the participants, who participated in the written activity and the semi-structured one-on-one interviews.

Table 4.9: Levels of engagement in the four phases of the PS conceptual framework according to the responses in the written activity and interviews

Phases	No of participants (*N*=90)	Frequency of responses for three participants (*f*=31)	Comparison	Interpretation
	Written activity	Interviews		
Decoding	21 out of 90	11 out of 31 responses	Similarities between the written activity and the interviews: • Able to find a pattern to discover terms in near positions. • Able to relate number patterns and mathematical operation. • Used the counting strategy. Differences between the written activity and interviews: • Found a pattern to discover terms in far positions. • Used the recursive strategy or direct proportion.	The participants were similar in the sense that they could find a pattern to discover the term in a near position by using the counting strategy. They could all start by writing sequence number of -2, -5, -8, -11, -14, -17, -20, -23, -26, which showed their comprehension of the decreasing pattern. The finding indicates that the participants were able to understand the problem and could construct a sequence number. They were able to identify the relationship between numbers and operations.

Representing	16 out of 90	7 out of 31 responses	Similarity between the written activity and interviews: • Used proportional representations. Differences between the written activity and interviews: • Used mental models and the mental images.	The participants who used proportional representation during the written activity, changed to use mental images during the interviews.
Processing	14 out of 90	7 out of 31 responses	Similarity between the written activity and interviews: • Concluded a new structure with (6×n) as the strategy for the next stage.	Participants did not understand how to use symbolic notation to continue the pattern. They used numbers instead of algebraic expressions. The lacked in-depth observation of patterns. They found it difficult to find meaning into an abstract algebraic expression. Participants A and B found it challenging to find a pattern to determine a term in the far position. They used direct proportion incorrectly to find the value of n^{th} term. Participant C successfully found the n^{th} term using the recursive strategy.
Implementing	12 out of 90	6 out of 31 responses	Similarity between the written activity and the interviews: • Drew a conclusion to implement the direct proportional strategy.	Participants were unsuccessful to use substitution to find the unknowns patterns, since they relied on the direct proportional strategy. They found it challenging to check the mathematical model to implement the solution.

All three participants' number of indicators for decoding, representing, processing and implementing increased from the responses in their written activity to the responses in their interviews as indicated in table 4.9. This finding correspond with Wang et al. (2016) recognising that learners often have difficulties with the problem solving that vary from teaching problems when solving problems. This finding can be interpreted in the way participants responded during the interviews, where the researcher connected a problem statement with previous knowledge on number pattern problems so that those participants who had difficulties during written activity could recognise and recall their prior knowledge. Participant A15's response from the written activity posits that the participant left a blank space in all the phases. The researcher could not ascertain participant A's number patterns strategy; hence an interview was conducted to access his/her thinking process.

During the decoding, participant A15's written activity shows that the participant did not understand the information within the given problem statement in order to solve the problem. There was evidence of the counting strategy on decoding and proportional representation during his/her interview. This finding could indicate that there was a problem in understanding the language of instructions in the written activity. A similarity was found between the responses of participants' B and C. Both participants completed the number sequence by using the counting strategy, which indicates correct decoding as they could comprehend the decreasing pattern.

By reviewing the written activity on number sequence, participant A15, who did not response to the question, made a unique plan to complete the sequence during his/her response in the interview by stating that "this sequence number was difficult to calculate in order to find a general form of the n^{th} term when I was writing the activity that is why I left it blank".

During representing, participant B2, who used proportional representation during the written activity, changed his/her response to a mental image during the interview. He/she was able to visualise the pattern and could provide reasons for the problem, even while failing to handle the mathematical part of the problem. Both participants A and B did not understand how to use symbolic notation to continue the pattern during their responses in the interviews, but they were able to provide images of how the pattern grows. They used numbers instead of algebraic expressions to find the value of the n^{th} term. Participant C5, who responded to the written activity at RP2, changed his/her response to a mental model during representation. Participant C5 could focus on comprehension of the situation, and highlighted the mathematics concepts in the interview questions.

During processing, a similarity was found between participants A15 and B2 from the written activity to the interview. Both left a blank space in the written activity. During the interview, both of them indicated some techniques for manipulating expressions and equations, but without a basic underlying understanding of what the variables and numbers represent. Hence, both of them used the direct proportional strategy to manipulate the equation. They could only focus on given information and only prioritised manipulation of equations with rote memorisation and pattern matching of information.

Therefore, their mathematical model did not support their understanding of what was happening when they manipulated the algebraic expression. They found it difficult to find meaning into an abstract algebraic expression. During the written activity, participant A15 did not response to the question, while in the interview he/she could identify what the problem was asking in terms of calculation, and could then perform the calculation by translating the problem statement into numbers. Participant A15 could found an algebraic expression (formula) for the number of objects in each pattern in the sequence by using multiplication of numbers. Participant C5, before dealing with equations could include a diagrammatic representation in their problem solution and he/she was successfully in developing the correct mathematical model.

During the implementation, participants A15 and B2 were unsuccessful to use substitution to find the unknowns pattern. They all relied on the direct proportional strategy, namely multiplication of numbers. They found it challenging to checking their mathematical models to implement the solution during both the written activity and the interviews. Only participant C5 was able to apply techniques that were specific to the mathematical model, which was adaptable to the given particular situation, with the purpose to obtain final results for the problem.

Participant C5 revealed a high-level thinking capability and reasoning level in all the phases. He/she was the only one who could progress to DP3, RP3, PP3 and IP3 during the written activity and who was able to explain the strategies used in every phase of the PS conceptual framework during the interview. Participant C5 reached all the phases in the sense of being able to make an individual idea creatively by constructing new knowledge based on his/her pre-existed knowledge. Participant C5 demonstrated a way of establishing a general formula of n^{th} term of the sequence number through drawing a logical conclusion in both the written activity and the interview.

4.7 KEY FINDINGS PERTAINING TO THE RESEARCH QUESTIONS

The overarching findings for the main research question will be addressed in this section, namely: *What strategies do grade 9 mathematics learners use in solving number pattern problems?* Table 4.10 illustrates the data analysis collection instruments and the key findings related to the PS conceptual framework with regard to the sub-research questions.

Table 4.10: Findings related to the PS conceptual framework of the study in relation to the research questions

Sub-question 1: What are the strategies grade 9 mathematics learners engage in when solving number pattern problems?	Written activity (see section 4.4)	Decoding: • Most participants used the counting strategy to find a pattern to determine terms in near position. Fewer participants changed to the recursive strategy to find a pattern to determine the term in the far position Representing: • Few participants were able to use mental models and mental images; they used diagrams and pictures to find the n^{th} term. • Most participants used proportional representation and did not use pictures or diagrams as guidance in finding the n^{th} term Processing: • Most participants used an incorrect formula to find the general rule. They were incorrectly interpreting the unknown variables. Hence, participants used the direct proportional (multiplication of numbers) strategy without making adjustment to the problem, Implementation: • Almost half of the participants were able to identify adequate ways to describe or to prove the number patterns and to eliminate the values that did not satisfy the constraints of the problem. Just more than half of the participants had a tendency to manipulate numbers to let the order of the sequence got higher, which increased the difficulty of implementing the model to find a correct solution

Sub-question 2: What are the views of grade 9 mathematics learners regarding the areas of difficulty (if any) they experience as they complete number pattern problems?	Interviews (see section 4.5)	Decoding: Almost a third of the responses indicated that participants were able to engage or reach the DP using the counting strategy for a near position of the terms. The participants indicated that they had experienced difficulties to find a pattern to determine a term in the far position in comparison to the near position. Decoding of numbers and mathematical structures were a challenge to participants and they used direct proportion incorrectly when finding the value of the n^{th} term. Representing: A third of the responses showed that participants were able to represent the problem. The participants indicated that they had experienced difficulties in using symbolic notation to continue with the pattern, but they were able to provide an image of how the pattern grows. They used numbers instead of algebraic expressions to explain their strategies. Processing: Few responses indicated that participants reached the PP. The participants indicated that they had experienced difficulties in finding meaning into an abstract algebraic expression. The participants indicated some techniques for manipulating algebraic expressions and equations, but without a basic underlying understanding of what the variables and numbers represent. Implementation: Few responses indicated that participants reached the IP. The participants were unsuccessful to use substitution to find the unknowns pattern, since they relied on the direct proportional strategy (multiplication of numbers).

Sub-question 3: What levels of engagement in the four phases of the PS conceptual framework are evident in grade 9 mathematics learners' strategies to a number pattern problem activity?	Synthesis from written activity and interviews (see section 4.6)	During the written activity and interview, participant C was able to progress to DP3, RP3, PP3 and IP3. Participant C used recursive and strategy during DP3. He/she was able to move beyond the data by generating and predicting the problem statement. Participant C understood the problem and the mathematical operations within the problem statement and could comprehend mathematical concepts of number patterns that would be helpful in solving the problem. During RP3, it could be interpreted that participant C had high-level thinking capabilities reasoning levels, as he/she was able to use mental models. He/she focussed on the comprehension of the situation, as well as the mathematics concepts in the activity. During PP3, participant C was able to successful formulate a geometric diagram into a number patterns and could change a difficult problem into a simpler problem to produce the correct mathematical model. Participant C was able to overcome difficulties related to the phase without much intervention during the interview. Participant C could demonstrate a way of establishing a general formula of n^{th} term of the number sequence through drawing a logical conclusion. Therefore, participant C reached all the phases by being able to make an individual idea creative by constructing new knowledge based on pre-existed knowledge during the IP. Participants' A and B responses were coded on DP1, RP1, PP1 and IP1. These participants used direct proportional and direct counting strategies. They indicated some techniques for manipulating expressions and equations, but without a basic underlying understanding of what the variables and numbers

		represent. Hence, they used the direct proportional strategy to manipulate the equation. They could only focus on given information and only prioritised manipulation of equations with rote memorisation and pattern matching of information.

The strategies indicated in the written activity and the responses from the interviews on the number pattern problems revealed four strategies, namely (1) counting; (2) recursive strategy; (3) direct proportion; (4) mental image representation; and (5) mental modal representation (see section 2.6).

The *counting strategy* was evident in the DP. Specifically, during the DP, the counting strategy was the preferred strategy for participants in near generalisation while recursive was for far generalisation. The participants that used the counting strategy were unable to give a general rule for far generalisation. This strategy was applied by the majority of the participants in executing numerical linear patterns and this preference has increased as compared to the execution of pictorial patterns. The participants who used the counting strategy could not recognise a pattern for far generalisation than, those who used recursive strategy.

The participants who used counting strategies indicated that they were more successful in observing the sequence of numbers by completing the pattern indicating the next terms or elements, therefore this indicate correct decoding of numbers for near generalisation when given a number pattern. However, participants who used counting strategy were not able to find a pattern in far generalisation. Becker and Rivera (2005) indicate that those learners "who fail to provide the general formulae tend to start out with counting strategies; however, they lack the flexibility to try other approaches and see possible connections between different forms of representation and generalisation strategies" (p. 128).

The *recursive strategy* was also evident in the DP. The participant who use the recursive strategy, they were able to decode the problem statement, and also able to use the mental model of the problem during the RP. Moreover, they were able to proceed to PP by indicating the correct general rule of the problem. Lannin et al. (2006)

mention that learners may select a recursive strategy because they want to determine a general rule based on an understanding of a relationship that occurs in the situation. Akkan's (2013) study on the learners' strategies and representations regarding generalisation patterns found that most of the learners who used the recursive strategy were able to find near and far terms accurately in the sequence.

The *direct proportional strategy* was evident in the RP, PP and the IP. During the RP, the majority of participants used the proportional representation. These participants could only focus on given or apparent information and only prioritised manipulation of equations of common difference and multiplication (rote memorisation) when finding the n^{th} term.

During the PP, the participants who used the direct proportional strategy, did not have a strong visual image of the situation. As a result, they focussed only on the numbers given in the problem. Those participants who attempted to generalise the problem, began with an incorrect drawing and directly counted the numbers from a pattern. They used the direct proportional strategy (multiplication of numbers) to generalise the sequence.

During the IP, only a minority of participants used the recursive strategy, while the majority used direct proportional strategies. Participants' interpretations of the mathematical structures involving patterns contributed to their choice of strategies used.

The participants who used the direct proportional strategy without adjustment to the problem, were directly picking up numbers from the given problems and applied multiplication to those numbers during the IP, hence they failed to obtain the correct answer.

The participants who used the direct proportional strategy showed a lack of representing, processing and implementation. Those participants did not understand the information within a given problem statement in order to solve the problem. Barbosa et al. (2012) indicate that learners who use the direct proportional strategy fail to make a final adjustment based on the context of the problem Therefore, participants who used direct proportion in this study responded at an elementary level. They focused on taking information as it is from the data which means they were reading the data with little understanding. This finding is in agreement with Ibrahim and Rebello (2013) claiming that during the proportional representation “most of the students deal with symbolic representations independently of other forms of visual representations” (p. 14). Even though some participants were able to move to the IP, their solutions were incorrect because they tend to directly picking up numbers from the given problems and applying multiplication of those numbers.

The *mental image representation* was evident in the RP. While the minority of the participants using mental image representation could provide reasons for the problem, they failed to handle the mathematical operations of the problem statement. Ibrahim and Rebello (2013) found that “students who construct a mental image handle the (generated) visual representations in isolation” (p. 14). The competency in mental model representing was noted when the participants stated the problem in language they could understand and were able to represent the problem by means of mental model. The participants focus on the image or diagram to provide reasons for the problem even if they fail to handle the mathematical part of the problem.

Mental models were also evident in the RP. The few participants who used mental model representation, focussed on comprehension of the situation, as well as the mathematics concepts of the activity. They dealt with equations by including diagrammatic representations in their solutions of the problem. The competency of developing mental models was specifically noted when the participants focused on comprehension of the pattern, as well as the mathematics concepts highlighted in the problem. According to Ibrahim and Rebello (2013), “mental model provides a medium for making links between the syntactic (mathematical) and structural (visual) aspects of the task under consideration, thus allowing interpretation and comprehension to take place” (p. 14).

4.8 CHAPTER SUMMARY

The profile of schools and demographic information of participants were provided. Thereafter a description of the written activity, including the protocol for reviewing the number pattern problems, was given. The written activity on number pattern problems provided opportunities for the participants to decode, represent, process and implement the mathematical ideas on number patterns. A discussion of participants' responses on a number pattern problem for school A, school B, school C aligned with the phases of the PS conceptual framework was provided. The PS conceptual framework helped participants to model and interpret problem situations, to understand number patterns mathematical concepts, to clarify and to communicate their thinking, and to make connections between related mathematical ideas. Finally, the levels of engagement in the four phases of the PS conceptual framework of Singer and Voica (2013) in the strategies in the analysis of the written activity and the interviews were compared to align participants' strategies with the four phases of the PS conceptual framework.

The phases of the PS conceptual framework were used to analyse and interpreted the levels of engagement by the participants in their response to the written activity and interviews. The documents in the form of a written activity were analysed and discussed separately for each of the three schools in terms of four phases of the PS conceptual framework. During the analysis of the participants' responses, the phases, namely decoding, representing, processing and implementing determined the competencies that participants displayed in their responses to number pattern problems. The findings showed that participants who left blank spaces during the written activity were able to give their strategies during the interviews. Also, participants whose strategies were inappropriate to number pattern problems on the written activity, were able to practically describe their representing during the interviews. They were able to move from proportional representing to mental image representing even though their solution was not entirely satisfactory for the problem text during the IP.

Both direct counting and the recursive strategy were evident during the DP. Those who correctly decoded the number patterns used recursive strategy, saw the patterns or trends in the problem statement. While counting strategy was the preferred strategy for participants in near generalisation, the strategies of mental image and mental model representation were evident during the RP. The strategy of direct proportion, even inappropriately employed, was evident in both the PP and the IP. Most participants used direct proportion during the PP and the IP, which was noted when the participants constructed a rule that did not work for all the terms in their number pattern.

CHAPTER 5: CONCLUSIONS, RECOMMENDATIONS AND LIMITATIONS

5.1. INTRODUCTION

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5.10. FINAL REFLECTION

CHAPTER 5: CONCLUSIONS, RECOMMENDATIONS AND LIMITATIONS

5.1 INTRODUCTION

The solving of number pattern problems is often regarded as a difficult topic for learners. Learners often use inappropriate strategies to solve number pattern problems. Many of them cannot easily identify number patterns embedded in problems and lack the mathematical concepts to do so. This study, therefore, focused on the strategies used by participants to solve number pattern problems in line with the PS conceptual framework of Singer and Voica (2013). This chapter provides an overview of the study based on Chapter 1 to Chapter 4. A summary of the findings for each research objectives, as well as a summary of the overarching findings for the main research question, is provided. The implications of the findings of the research study are outlined. Thereafter, the contribution of the study will be given, followed by the limitations of the study and recommendations for future research. Lastly, the study is concluded and a final reflection on the study is offered.

5.2 OVERVIEW OF STUDY

Chapter 1 stated the purpose of the study, namely, to investigate grade 9 mathematics learners' strategies in solving number pattern problems. This purpose was envisaged to ultimately assist learners with guidelines to improve their learning of number patterns-related problems. Thus, the main research question was: What strategies do grade 9 mathematics learners use in solving number pattern problems? To address the main question, the following sub-questions were set:

- What are the strategies grade 9 mathematics learners engage in when solving number pattern problems?
- What are the views of grade 9 mathematics learners regarding the areas of difficulty (if any) they experience as they complete number pattern problems?
- What levels of engagement in the four phases of the PS conceptual framework are evident in grade 9 mathematics learners' strategies to a number pattern problem activity?

Chapter 2 focused on an overview of literature concerning number pattern problems. A discussion on the nature of mathematics, including the definition of mathematics; problem solving; number patterns; the design of number pattern problems; learners' experience of problem solving; strategies used in number patterns; and lastly the PS conceptual framework developed from Singer and Voica (2013), was provided.

Mathematics was defined as a human activity that involves observing, representing, and investigating patterns and qualitative relationships in physical and social phenomena and between mathematical objects themselves (DBE, 2011). Ernest (1991) argues that having a good understanding of the nature of mathematics and its philosophical underpinnings are important for both the learning and teaching mathematics. Beliefs about the nature of mathematics provide a foundation for teacher's methods of teaching, and how learners learn mathematics. It can also affect how mathematics concepts are explained, demonstrated, and taught to learners.

Barbosa et al. (2012) argues that an in-depth investigation of patterns in mathematics can add more value to the ability to problem solve. Patterns are a way for learners to recognise order and organise their world, and are important in all aspects of mathematics. Therefore, number patterns and geometric/pictorial patterns are beneficial in building positive and meaningful images of mathematics, and add value to the development of several skills related to problem solving and algebraic thinking. More specifically Schumacher and Fuchs (2012) indicate that mathematical problems challenge learners to read and interpret the problem, represent the semantic structure of the problem, and choose a solution strategy. The study was guided by the PS conceptual framework of Singer and Voica (2013). Singer and Voica's PS conceptual framework highlights four phases, namely, decoding, representing, processing, and implementing (see section 2.8)

Chapter 3 outlined the research design and research methodology used in this study. The chapter described the philosophical assumptions (epistemological, ontological, and axiological) of the study. The qualitative research approach followed in this study assisted the researcher to develop a deep understanding of the learners' strategies used in solving number pattern problems. The researcher used an exploratory qualitative case study research strategy. Purposive sampling was utilised to select 90 grade 9 mathematics learners from three rural schools (30 from each school). The data were collected through written a activity and semi-structured one-on-one interviews. Data analysis was carried out by means of content analysis following a deductive approach to analyse the participants' written activity and interviews. Trustworthiness of the study was discussed with reference to credibility, transferability, dependability, and confirmability. Lastly, ethical considerations taken into account were reported.

Chapter 4 focused on data analysis and the interpretation of findings. The profile of the schools, and demographic information on the participants, was provided. Thereafter, a description of the written activity, including the protocol for reviewing the number pattern problems, was given. The written activity on number pattern problems provided opportunities for the participants to decode, represent, process, and implement the mathematical ideas on number patterns. A discussion of how participant responses (to a number pattern problem for school A, B and C) aligned with the phases of the PS conceptual framework was provided. Finally, the levels of engagement of the four phases of the PS conceptual framework in strategies from the analysis of the written activity and the interviews were compared. A summary of the findings according to the research objectives are given next.

5.3 SUMMARY OF FINDINGS ACCORDING TO THE RESEARCH OBJECTIVES

The following section provides a summary of the findings according to the research objectives.

5.3.1 Research objective 1: Determine the strategies grade 9 mathematics learners engage in when solving number pattern problems

To achieve this objective, documents in a form of participant scripts were used. The participant scripts were marked by the researcher using a marking guideline (see Appendix I) set according to the indicators developed from the four phases of the PS conceptual framework (see section 4.5). During the DP, most participants used the *counting strategy* to find a pattern to determine terms in the near position. Fewer participants changed to the *recursive strategy* to find a pattern to determine terms in the far position.

During the RP few participants were able to use mental models and mental images; they used diagrams and pictures to find the n^{th} term. Most participants used *proportional representation* and did not use pictures or diagrams as guidance in finding the n^{th} term. The *mental image representation* was also evident in the RP. While the minority of the participants using mental image representation could provide reasons for the problem, they failed to handle the mathematical operations of the problem statement.

During the PP most participants used an incorrect formula to find the general rule. They were incorrectly interpreting the unknown variables. Hence, participants used the *direct proportional* (multiplication of numbers) strategy without making adjustments for the problem. Finally, during the IP, only a minority of participants used the *recursive strategy*, while the majority used direct *proportional strategies*. Almost half of the participants were able to identify adequate ways to describe or to prove the number patterns and to eliminate the values that did not satisfy the constraints of the problem. Just over half of the participants tended to manipulate numbers in order to allow the sequence to go higher, which increased the difficulty of implementing the model to find a correct solution.

5.3.2 Research objective 2: Establish grade 9 mathematics learners' views regarding the areas of difficulty (if any) they experience as they complete number pattern problems

To achieve this objective, the researcher conducted one-on-one semi-structured interviews with three participants in order to obtain explanations and experiences from participants for using specific strategies to solve number pattern problems.

Almost a third of the responses indicated that participants were able to engage or reach decoding using the counting strategy for a near position of the terms. The participants indicated that they had experienced difficulties in finding a pattern to determine a term in the far position, as compared to the near position. Decoding of numbers and mathematical structures were a challenge to participants and they used direct proportion incorrectly when finding the value of the n^{th} term.

A third of the responses showed that participants were able to represent the problem. The participants indicated that they had experienced difficulties in using symbolic notation to continue with the pattern, but they were able to provide an image of how the pattern grows. They used numbers instead of algebraic expressions to explain their strategies.

Few responses focused on processing. The participants indicated that they experienced difficulties in finding meaning in an abstract algebraic expression. However, they mentioned some techniques for manipulating algebraic expressions and equations, but without a basic underlying understanding of what the variables and numbers represented.

Few responses referred to implementation. The participants were unsuccessful in using substitution to find the pattern for the unknowns, since they relied on the direct proportional strategy (multiplication of numbers).

5.3.3 Research objective 3: Ascertain the levels of engagement in the four phases of the PS conceptual framework in grade 9 mathematics learners' strategies to a number pattern problem activity

To achieve this objective, the levels of engagement in the four phases of the PS conceptual framework of Singer and Voica (2013) in the strategies established from the results from the written activity with the interviews were compared. The findings showed that participants who left blank spaces during the written activity were able to give their strategies during the interviews. Also, participants whose strategies were inappropriate to number pattern problems in the written activity, were able to describe their representing practically during the interviews. They were able to move from proportional representation to mental-image representation even though their solution was not entirely satisfactory for the problem text during the IP.

5.4 SUMMARY OF OVERARCHING FINDINGS

Findings from the three research objectives were considered to present an answer to the main research question: *What strategies do grade 9 mathematics learners use in solving number pattern problems?*

The findings from the written activity and the responses from the interviews on the number pattern problems revealed four strategies, namely: (1) direct counting, (2) recursive strategy, (3) direct proportion, (4) mental-image representation (5) mental-modal representation.

Both direct counting and the recursive strategy were evident during the DP. The competency of decoding was demonstrated when the participants recognised and found a pattern in both the near and far position. The findings indicate that the participants who used direct counting during the DP, were able to discover a term in the near position, but failed in the far position. Those who correctly decoded the number patterns by means of the recursive strategy saw the patterns or trends in the problem statement.

The strategy of mental image representation was evident during the RP. The competency of representing was noted when the participants stated the problem in language they could understand and were able to represent the problem by means of mental model. The participants, who used mental model representations, could provide reasons for the problem even in cases where they failed to handle the mathematical part of the problem.

The strategy of direct proportion, even if it was inappropriately employed, was evident in both the PP and the IP. Processing occurred when participants could create a general formula for the sequence, i.e., a mathematical model. Most participants used direct proportion inappropriately during the PP and the IP, which was noted when the participants constructed a rule that did not work for all the terms in their number patterns. The participants, who were unable to use the correct mathematical model, struggled to implement their answers in the IP.

5.5 IMPLICATIONS OF THE STUDY

The findings of this study have several important implications. To develop the skill of decoding, learners should be provided with content knowledge of patterns. Teachers should use strategies that assist learners to use symbolic algebraic expressions and to generalise the sequence. Learners should be developed to use a recursive generalisation.

Teachers need to inform learners on how to find the value of a term by giving the value of the preceding term. It is important that learners are asked to explain their thinking. Having them describe their reasoning can also help them realise that often there is more than one way to look at a pattern.

Secondly, to acquire the skill of representing, learners should be guided on how to construct mental models, which will provide them with opportunities to use algebra to prove conjectures or rectify different solutions or formulae for number patterns. By drawing learners' attention to the relationships between the physical representation of a pattern and its symbolic expression, learners can be led to recognise that different symbolic expressions may represent the same physical situation.

Thirdly, to foster the skill of processing, learners should be assisted to develop accurate concepts on number pattern. The concept of algebraic expressions, to form equations to solve number patterns problems, is a powerful concept of elementary algebraic reasoning. Teachers should not only assist learners in using algebraic expressions, but also provide them with strategies to manipulate symbols in number patterns. Teachers should provide learners with content knowledge of number patterns, i.e., the strategies that can be used in both numbers and pictorial patterns.

Finally, to develop the skill of implementing, teachers should assist learners to focus on the relationship between the data in a problem and the mathematical operation. Learners should be guided to find and to discover a pattern, and to generalise and to implement a correct mathematical model. Thus, learners should be encouraged to reason about the strategies they use when solving number pattern problems.

5.6 LIMITATIONS OF THE STUDY

While a qualitative study is known to dig deep in terms of examining a phenomenon under study, the large sample size of 90 participants from three schools for the written activity could not achieve the depth-ness of the investigation. On the other hand, three participants for the interviews did not adequately address the issue of depth in terms of gaining knowledge that informs learners' problem-solving strategies in number pattern problem solving tasks. Therefore, the findings cannot be generalised to all schools in South Africa. Also, the context was restricted to rural schools and, therefore, the findings could be different for urban schools. Furthermore, the study was limited to grade 9 learners and did not consider learners from primary schools, where the basic foundations of the topic of number patterns are introduced.

Due to the limited scope of this study, only one mathematics topic was investigated, namely, linear number pattern problems. Patterns are also evident in linear and quadratic functions, which could reveal different findings. Only two data collection instruments were used due to time constraints, namely, interviews and a written activity. A shortcoming in this study is that no observations or think-aloud protocol were done while the participants engaged in the activity on number pattern problems. Therefore, participants' thinking processes could not be established during the various phases of problem solving. The written activity was completed during school hours, which could have led to learners rushing through the activity without putting cognitive effort into the mathematics problems due to insufficient time. The participants were also from three different schools and were taught by different mathematics teachers. It could also be possible that different teaching styles might have influenced the types of strategies used on number pattern problems. Conducting observations was not feasible as the researcher is also a teacher at another school and did not have permission to leave her school during contact time. The study was qualitative and focussed on participants' interpretations of their strategies to solve number pattern problems. A last limitation is that the study did not investigate the effect of the strategies on participant learning of, or performance in, number pattern problems.

5.7 RECOMMENDATIONS FOR FUTURE RESEARCH

This study aimed to lay a foundation for future studies regarding different strategies to employ in solving number pattern problems. Future studies may involve learners from the FP and IP at primary schools, in order to compare strategies on solving number pattern problems across different phases. Furthermore, a recommendation is to expand this research enquiry on learner strategies in solving number pattern problems to other contexts, both locally and internationally within the same grades. The focus of research could also shift from learners' strategies used in learning to teachers' strategies used in the teaching problems. Other frameworks, different from the PS conceptual framework of Singer and Voica (2013) could also be explored. A future study on number pattern problems, which includes a larger sample size, may contribute to the transferability of the study. Having a smaller sample size in this study did not provide sufficient information on the strategies that grade 9 mathematics learners use in solving number pattern problems.

In future, it is recommended that quantitative data is also collected on how learners' strategies in solving number pattern problems relate with those of the teachers' teaching practice in this topic. Lastly, it can be suggested that a longitudinal study on how learners could be developed to solve problems by using the phases of the PS conceptual framework of Singer and Voica (2013).

5.8 CONTRIBUTION OF STUDY

The study contributes to practice by establishing different strategies learners use to solve number pattern problems, which could sensitise teachers to introduce alternative strategies to learners when solving number pattern problems. This study also makes teachers aware of learners' interpretations and implicit thinking processes regarding the strategies they use when solving number patterns during the different process phases of problem solving, which may influence learners' learning of mathematics.

Curriculum developers may benefit from an increased awareness of the difficulties learners experience when solving number pattern problems, and may include examples of different strategies to solve problems in curriculum documents to guide teachers in this regard. Mathematics subject advisors at district level may introduce teachers to meaningful and effective strategies in solving number pattern problems to support them in their professional development of problem-solving proficiency. Guidelines on learners' strategies to solve number pattern problems may also be added to the mathematics CAPS.

The importance of this study in terms of academic value is that it expands on the uses of the existing PS conceptual framework of Singer and Voica (2013), which may be used for further research pertaining to the development and maintenance of effective strategies in the solving of number pattern problems. The study also adds to the limited research in South Africa on problem solving, specifically pertaining to the topic of number patterns, which has not been sufficiently addressed in previous studies.

5.9 CONCLUSION

This chapter summarises the findings of this study on grade 9 mathematics learners' strategies in solving number pattern problems. The findings of this study, together with its implications for practice and recommendations for future research, were discussed. The study investigated grade 9 mathematics learners' strategies in solving number pattern problems in order to assist in setting guidelines to improve future learning of problems regarding number patterns. The research question focused on the strategies used by grade 9 mathematics learners in solving number pattern problems. The qualitative research approach followed in this study assisted the researcher to develop a deep understanding of the strategies learners use for number pattern problems. The focus was on the levels of engagement in the four phases of the PS conceptual framework that participants displayed, namely: decoding, representation, processing, and implementation (Singer & Voica, 2013). The four phases of the PS conceptual framework were used to analyse participants' written activity and the interviews on the strategies used in solving number pattern problems.

The study concluded by revealing that the counting strategy was widely used across the various process phases of problem solving. Those participants who struggled to generalise the pattern, preferred to use the direct proportional strategy. Furthermore, many participants lacked an understanding of mathematical operations, such as subtraction and addition, and could not visualise the pattern, which resulted in overlapping shapes and misunderstanding of the n^{th} term when completing the table of values. Most participants used direct proportion inappropriately during processing and implementing.

From the findings, it can be concluded that there is no way a problem can be correctly solved if it is not properly decoded. A suggestion to teachers is that they should encourage learners to decode a problem before they attempt to find a solution. Also, when number pattern problems are solved, learners should be advised to check the results by means of substituting the calculated values in the initial statement, and also to control the results against the context within which the problem is given.

Definitely, the PS conceptual framework of Singer and Voica (2013) can be adopted and relevant strategies can be used to help solve number pattern problems effectively. In future, teachers could also adjust the PS conceptual framework and use it as a tool to analyse the strategies they use to teach number pattern problems.

5.10 FINAL REFLECTION

Completing this dissertation has been quite a challenge and it has taken much time and effort to complete it to the best of my ability. The biggest mistake that I made when writing this dissertation, was to write about something before I completely understood it. As a result, it impacted on my time management in completing the dissertation as I had to spend additional time on refocusing, including additional information and rephrasing of information. I now realise how my own thinking processes have improved since the beginning of my proposal to end of my dissertation. I have grown personally as a mathematics teacher, and academically as a researcher. As a mathematics teacher, it has been good to develop my knowledge and understanding of learners' strategies used to solve number pattern problems by using the PS conceptual framework. This new knowledge can help me in my future teaching of the topic, and also to share my expertise with my colleagues. As a researcher, this study has taught me how to report my findings and to review literature. I have learned how to analyse, to write up a report, and to think through what I can say and what I need to do, thus to conceptualise a research inquiry. Developing academic writing skills is a process, which I still need to improve as a whole. These skills will help me with other types of writing that I have to do, including writing used on a day-to-day basis throughout my teaching career.

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APPENDICES

APPENDIX A: WRITTEN ACTIVITY

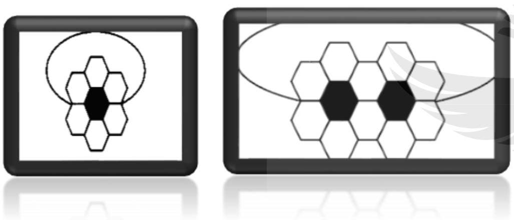
Question 1

1.1. Complete the table by indicating the next terms (f(x)) of the sequence. If the sequence is continued to nth term, please write the general formula for the nth term. Explain your answer

x	1	2	3	4	5	6	7	8	n
$f(x)$	-2	-5	-8	-11	-14				

Question 2

Joana likes to make necklaces using flowers. She uses white beads for the petals and black beads for the centre of each flower. The figure below shows a necklace with one flower and a necklace with two flowers, both made by her.



2.1	How many white and black beads will Joana need to make a necklace with 3 flowers? Explain your answer. Make a neat diagram with 3 flowers on the above space provided.
2.2	How many flowers will Joana be able to make if she uses 102 white beads? Explain your answer.

APPENDIX B: ONE-ON-ONE SEMI-STRUCTURED INTERVIEW

TITLE: GRADE 9 MATHEMATICS LEARNERS' STRATEGIES IN SOLVING NUMBER PATTERN PROBLEMS

Research Question:

The main research question:

What strategies do grade 9 mathematics learners use in solving number pattern problems?

Participant: School:

Date:

Interview Questions

1. Let's look at question 1, what type of pattern is that and why?

____

____

____

2. How did you complete the sequence in question 1? Explain your strategy

____

____

____

3. How did you get your general formula?

____

____

4. When working with question 2 about Joana making necklaces; how did you identify the key words to solve the problem?

____

____

5. How did you make a necklace with 3 flowers?

____

____

____

6. How many flowers will Joana be able to make if she uses need if she uses 102 white beads? Explain your answer

____

____

____

7. Can you come up with a general formula for this problem?

____

____

____

8. How do you determine if the formula used is correct?

____

____

____

9. How do you relate your final answer with the original statement to check whether the formula is correct?

____

____

____

APPENDIX C: UNIVERSITY OF JOHANNESBURG ETHICAL CLEARANCE LETTER

NHREC Registration Number REC-110613-036

UNIVERSITY OF JOHANNESBURG

ETHICS CLEARANCE

Dear KP Aphane

Ethical Clearance Number: 2017-029

Grade 9 mathematics learners' strategies in solving number pattern-related word problems.

Ethical clearance for this study is granted subject to the following conditions:

- If there are major revisions to the research proposal based on recommendations from the Faculty Higher Degrees Committee, a new application for ethical clearance must be submitted.
- If the research question changes significantly so as to alter the nature of the study, it remains the duty of the student to submit a new application.
- It remains the student's responsibility to ensure that all ethical forms and documents related to the research are kept in a safe and secure facility and are available on demand.
- Please quote the reference number above in all future communications and documents.

The Faculty of Education Research Ethics Committee has decided to

☒ Grant ethical clearance for the proposed research.
☐ Provisionally grant ethical clearance for the proposed research
☐ Recommend revision and resubmission of the ethical clearance documents

Sincerely,

Prof Geoffrey Lautenbach
Chair: FACULTY OF EDUCATION RESEARCH ETHICS COMMITTEE
19 April 2017

APPENDIX D: LIMPOPO DEPARTMENT OF EDUCATION

LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

Ref: 2/2/2 Enq: MC Makola PhD Tel No: 015 290 9448 E-mail:MakolaMC@edu.limpopo.gov.za

Aphane KP
P O Box 1066
Marble hall
0450

RE: REQUEST FOR PERMISSION TO CONDUCT RESEARCH

1. The above bears reference.

The Department wishes to inform you that your request to conduct research has been approved. Topic of the research proposal: **"GRADE 9 MATHEMATICS LEARNERS STRATEGIES IN SOLVING NUMBER PERTENS-RELATED WORD PROBLEMS."**

2. The following conditions should be considered:

3.1 The research should not have any financial implications for Limpopo Department of Education.

3.2 Arrangements should be made with the Circuit Office and the schools concerned.

3.3 The conduct of research should not anyhow disrupt the academic programs at the schools.

3.4 The research should not be conducted during the time of Examinations especially the fourth term.

3.5 During the study, applicable research ethics should be adhered to; in particular the principle of voluntary participation (the people involved should be respected).

3.6 Upon completion of research study, the researcher shall share the final product of the research with the Department.

REQUEST FOR PERMISSION TO CONDUCT RESEARCH: APHANE K.P

CONFIDENTIAL

Cnr. 113 Biccard & 24 Excelsior Street, POLOKWANE, 0700, Private Bag X9489, POLOKWANE, 0700
Tel: 015 290 7600, Fax: 015 297 6920/4220/4494

The heartland of southern Africa - development is about people!

4 Furthermore, you are expected to produce this letter at Schools/ Offices where you intend conducting your research as an evidence that you are permitted to conduct the research.

5 The department appreciates the contribution that you wish to make and wishes you success in your investigation.

Best wishes.

Ms NB Mutheiwana
Head of Department

07/04/2017
Date

APPENDIX E: LETTER TO THE PRINCIPALS

P.O.BOX 1060
MARBLE HALL
0450
18 May 2017

Dear Principal

REQUEST FOR PERMISSION TO CONDUCT RESEARCH

Ethical clearance number 2017-029 has reference.

The Faculty of Education Academic Ethics Committee of the University of Johannesburg and the Limpopo Department of Education granted me an ethical clearance to conduct a research study in schools. See attachments.

I hereby request permission to conduct research from Grade 9 mathematics learners. I am an M Ed student in Mathematics Education at the University of Johannesburg. My research topic is Grade 9 mathematics learners' strategies in solving number pattern problems.

Most learners across the primary grades in South Africa have poor mathematics skills, with the average performance steadily declining by approximately 10% each year. The results for the ANA for mathematics in South Africa also raise a national concern regarding the poor performance of grade 9 mathematics learners. Even though the 2013 ANA results slightly increased, from 13% to 14% in the national average, for grade 9 mathematics, the results are still poor. Therefore, the purpose of my study is to investigate grade 9 mathematics learners' strategies in solving number pattern problems in order to provide guidelines to assist future learning of number patterns.

The data collection process will be as follows: During this term (second term 2017), educators will be asked to provide documentary source such as learners' record of performance in mathematics. I will give learners an activity sheet consisting of problems involving number patterns to complete. I will observe those learners as they complete activity sheets during school contact time for about two lessons on different days in one week and it will be done at a time convenient to the educator and should not disrupt the educator's timetable. The observations will be video recorded; this will allow for a clear and accurate record of the learners strategies in solving number pattern problems. I will also conduct one-on-one semi-structured interviews with learners after school hours for about two weeks in order to verify strategies used in the activity sheets and to ensure correct interpretation in cases of uncertainty. I would like to assure you in advance that my study in the selected schools will in no way interrupt the normal teaching, learning and assessment activities. Please note that it is unlikely that there will be any potential risks to teachers or learners participating in the study. Also note that learners are at liberty to withdraw from this study at any time, without penalty or pressure to provide reasons to me, as the researcher. In this regard, I will undertake to ensure that participating in this study does not disadvantage the participants.

You are hereby assured that the information the learners give will be treated with utmost confidentiality and that their identity as well as that of your school will be kept private. Data collected from this study will be kept safe for until the study is completed and destroyed afterwards. The published results of this study will, however, be made available to you, the circuit manager and the Limpopo Department of Education.

No direct or indirect financial benefits shall derive from carrying out this study, nor shall your learners' participation herein incur any costs.

For any information, please, contact me at 079 84 66 265 or email me at phithyk@gmail.com

Yours in Education

____

Aphane K.P. (Mathematics Educator)
Cell: 0798466265

APPENDIX F: PERMISSION TO CONDUCT RESEARCH FROM SCHOOLS

Emis Number: 924640897
Enquiries: Manamela M.I
Cell No: 0787384341

05 JUNE 2017

TO: APHANE K.P

Subject: Permission to conduct research

The above matter bears reference that:

1. We acknowledge receipts of your request for conducting research at this school
2. The school informs you that your request was accepted

We hope that you will enjoy conducting your study at this school

Yours Faithfully

Manamela M.I

(Principal)

PHALAKGORO-MOTHOA HIGH SCHOOL

Enq: KHOMO B.I
Tel: 073 259 9758

EFFORT DETERMINES PRIZE

P.O. Box 223
Gompies
0633
2017/05/17

TO: APHANE KP

PERMISSION TO CONDUCT RESEARCH

The above matter bears reference

We acknowledge receipts of your request for conducting research at this school

Yours faithfully

KGOMO B.I

(PRINCIPAL)

DEPARTMENT OF EDUCATION
PHALAKGORO-MOTHOA HIGH SCHOOL
P.O. BOX 223 GOMPIES 0631
CELL: [illegible] (B.M. KHOMO)
LIMPOPO PROVINCE

ENQIRIES: MAHLAELA M.W.

CELL: 060 668 2992

PERMISION TO CONDUCT RESEARCH

1. The above matter bears reference
2. We acknowledge receipts of your request for conducting research at your school
3. The school informs you that your request was accepted

We hope that you will enjoy conducting your research at this school

Yours Faithfully

MAHLAELA M.W

2017/04/20

(PRINCIPAL)

APPENDIX G: LETTER TO THE PARENTS

P.O. Box 1066
Marble Hall
0450
18 May 2017

Dear Parents/Guardians of Grade 9 Mathematics learners

The purpose of this letter is to request your permission to involve your Grade 9 Mathematics child in my research study. Approval from the Limpopo Department of Education and the circuit manager to conduct research in Schools has been given.

Most learners across the primary grades in South Africa have poor mathematics skills, with the average performance steadily declining by approximately 10% each year. The results for the ANA for mathematics in South Africa also raise a national concern regarding the poor performance of grade 9 mathematics learners. Even though the 2013 ANA results slightly increased, from 13% to 14% in the national average, for grade 9 mathematics, the results are still poor. The overall performance in mathematics in the ANA of 2014 showed an upward trend in performance with the average percentage scores increasing by a maximum of 8% in mathematics in all grades, except in grade 9.

As part of my M Ed Degree at the University of Johannesburg, I am conducting research on Grade 9 mathematics learners' strategies in solving number pattern problems. Research in this area will investigate grade 9 mathematics learners' strategies in solving number pattern problems in order to provide guidelines to assist future learning of number patterns. I will conduct classroom observation and an in-depth interview with the learners to identify learners' strategies to solve problems involving number patterns.

Please note that learners are at liberty to withdraw from this study at any time, without penalty or pressure to provide reasons to me, as the researcher. In this regard, I will undertake to ensure that participating in this study does not disadvantage the participants.

All the information supplied will be treated with confidentiality and outcomes of the research will be made available on request. Data will be kept under lock and key and will be destroyed after completion of the research study. Should you have any queries or comments regarding this research, you are welcome to contact me via the School. Your cooperation is highly appreciated.

Yours in Education

__________________ ____________________

Aphane K.P
(Mathematics Educator)
Researcher

Dr E D Spangenberg
(Supervisor)

✂ --

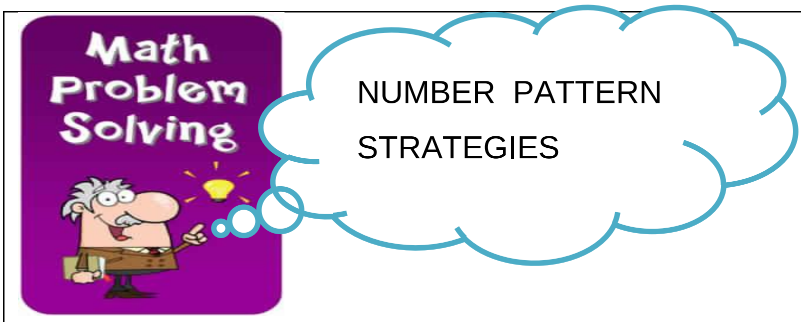
I, ____, the parent/guardian of ____ give my consent that he/she may participate in the study and that the information may be used for research purposes. Furthermore, I give consent that the school may provide his/her marks to the researcher on condition that all the information will be treated as confidential at all times.

____

____

Signature of parent Date

APPENDIX H: LETTER TO LEARNERS



Dear learner

You are kindly requested to complete the activity sheets which will be handed to you the next few days and to conduct one-on-one interview with you. The aims of the activity sheets and the interview are:

- To identify learners' strategies to solve problems involving number patterns,
- To assess learners' strategies when solving problems involving number pattern;

The purpose of my study is to investigate grade 9 mathematics learners' strategies in solving number pattern problems in order to provide guidelines to assist future learning of number patterns. It will take approximately 45 minutes to complete the activity and 30 minute for an interview. The activity will be completed during class under supervised examination conditions and the interview will be conducted after school hours.

I assure you that your identity and your responses to the activity and the interview will be treated as CONFIDENTIAL at all times and that it will NOT be made available to any unauthorised user.

Please note that you are at liberty to withdraw from this study at any time, without penalty or pressure from me, as the researcher, to provide reasons. In this regard, I will undertake to ensure that participating in this study does not disadvantage you.

Should you have any queries or comments regarding this research, you are welcome to contact me via your educator.

Your cooperation is highly appreciated.

Yours in Education

APHANE K.P.

CONSENT

I, ____ have read and understand the aims of this research study. On condition that the information provided by me is treated as confidential at all times, I hereby give consent that it may be used for research purposes. Furthermore, I give consent that the school may provide my marks to the researcher on condition that these will also be treated as confidential.

______________________________ ______________________

Signature of participant Date

APPENDIX I: MARKING GUIDELINE

Question 1

1.1 Complete the table by indicating the next terms (f(x)) of the sequence. If the sequence is continued to n^{th} term, please write the general formula for the n^{th} term

x	1	2	3	4	5	6	7	8	n
$f(x)$	-2	-5	-8	-11	-14	-17 √	-20 √	-23 √	-3n+1 √√

Continue the sequences indicating the next three elements (find a pattern, to discover terms in near) (3 marks)

Recognise a pattern and expand it, and be able to relate numbers and operations

e.g. $T2 - T1 = d$ $(common\ difference$

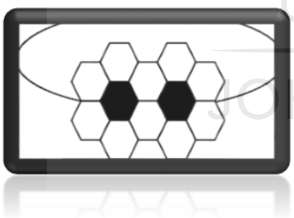
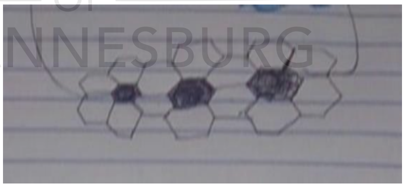
Correct rule given i.e. find a pattern, to discover terms in far position

$Tn = -3n + 1$ (2 marks)

√ √

Question 2

Joana likes to make necklaces using flowers. She uses white beads for the petals and black beads for the centre of each flower. The figure below shows a necklace with one flower and a necklace with two flowers, both made by her.

2.1 How many white and black beads will Joana need to make a necklace with 3 flowers? Explain your answer

Able to explain a pattern and expand it (2 marks)

14 white beads √

3 black beads √

Build concrete representation which can be expressed in a form of drawing. Make a neat diagram with 3 flowers on the above space provided (3 marks for the diagram)

2.2	How many flowers will Joana be able to make if she uses 102 white beads? Explain your answer Provide the correct rule of Tn=4n+2 (2marks) √ √ Correct substitution in the formula and calculation (3 marks) $102 = 4(n) + 2$ √ $102 - 2 = 4n$ √ $\frac{100}{4} = n$ $n = 25$ √ (2)

APPENDIX J: PROTOCOL TO REVIEW THE WRITTEN ACTIVITY

1.1 Complete the table by indicating the next terms (f(x)) of the sequence. If the sequence is continued to nth term, please write the general formula for the nth term. Explain your answer

x	1	2	3	4	5	6	7	8	n
$f(x)$	-2	-5	-8	-11	-14				

- DP; continue the sequences, recognise a pattern and expand it, and be able to relate numbers and operations. The question demanded knowledge of linear pattern and the mathematical operation of subtraction.
- RP; find a pattern, to discover terms in near and far positions. The question demanded the use mental model to create concrete representations of patterns using, tables, word and symbols.
- PP; correct rule
- IP; implementing the rule

If the sequence number is continued to n^{th} term, please write the general formulation of the n^{th} term.

- DP: continue the sequences, recognise a pattern and expand it, and be able to relate numbers and operations
- RP: build mental representation to create an equation of linear pattern. This question demanded knowledge of recognition of variables and construct relations between variables, be able to identify the unknown quantity by generalising a rule for a linear pattern
- PP; create a correct formula for finding the element of the sequence. The question demanded the use formula of arithmetic sequence to find the n^{th} term
- IP; substituting the correct values, simplifying, and using known algorithm. The question demanded the applying techniques that are specific to the found mathematical model, using multiple solution strategy and producing mathematical explanation and justification of the solution and generally questioning the model (Singer and Voica, 2013).

Joana likes to make necklaces using flowers. She uses white beads for the petals and black beads for the centre of each flower.

How many white and black beads will Joana need to make a necklace with 3flowers?

- DP; recognise a pattern and expand it, and be able to relate numbers and operations**.**
- RP; build concrete representation which can be expressed in a form of drawing. The question might also demand changing of problem statement into a language that is more accessible to the solver (Singer & Voica, 2013).
- PP; create a correct formula to generalise linear patterns. The question demanded the application of linear formula to find the nth term of the sequence and knowledge of mathematical expression to represent a pattern.
- IP; substituting the correct values, simplifying, and using known algorithm.

2.2. How many flowers will Joana be able to make if she uses 102 white beads?

- DP; recognise a pattern and expand it, and be able to relate numbers and operations**.**
- RP; build concrete representation which can be expressed in a form of drawing.
- PP; create a correct formula to generalise linear patterns.
- IP: Substituting the correct values and solving linear equation.

APPENDIX K: BIBLIOGRAPHIC CONTROL

271 Editing

Tel: 082 879 5799
E-mail: 271editing@gmail.com

To whom it may concern

This letter is to confirm that Ms Aphane Koko Pithmajor submitted her Master's dissertation to me for bibliographic control according to the prerequisites of Harvard style as used by the University of Johannesburg.

Regards

Kirchner van Deventer

10 June 2019

APPENDIX L: LANGUAGE AND STYLE EDITING

Word for Word
Academic Editing and Writing Support

Xenia Kyriacou (PhD Science Education)
+27614252802
xeniaxsk7@gmail.com

Confirmation of editing

This letter serves to confirm that the article below has been language and style edited:

GRADE 9 MATHEMATICS LEARNERS' STRATEGIES IN SOLVING NUMBER PATTERN PROBLEMS

Kind Regards
Xenia Kyriacou

11 June 2019

APPENDIX M: ONE-ON-ONE SEMI-STRUCTURED INTERVIEW WITH PARTICIPANT A15

TITLE: GRADE 9 MATHEMATICS LEARNERS' STRATEGIES IN SOLVING NUMBER PATTERN PROBLEMS

The sub-research question:

What are the views of grade 9 mathematics learners regarding the areas of difficulty (if any) they experience as they complete number pattern problems?

Participant A: undocumented strategy on number pattern problems

Interview Questions

1. **Let's look at question 1, what type of pattern is that and why?**

   Participant A: I was thinking that the pattern is linear because we have the common difference of a constant number of white beads in each flower. (Coded DP2)

2. **How did you complete the sequence in question 1? Explain your strategy**

   Participant A: I am counting the numbers from the sequence by subtracting 3 up until I arrive to the nth term (Coded DP3) showed an understanding of decoding accurately (in the table) for the first three elements. I think n will be multiplied by the common difference (coded DP2) or n^{th} term can be the number that follow the previous number in the sequence which is -26.

3. **How did you get your general formula?**

   Participant A: the general formula will be $Tn = -3 \times n$. I counted the numbers from the sequence by subtracting 3 up until I arrive to the n^{th} term. (Coded PP2, RP1, IP1)

4. **When working with question 2 about Joana making necklaces; how did you identify the key words to solve the problem?**

   Participant A: I cannot not figure it out, but I was thinking that the pattern is linear because we have the common difference of 6 white beads in each flower. I was counting the number of beads in each flower and figuring out what the third flower would look like. (Coded DP2)

5. **How did you make a necklace with 3 flowers?**

   Participant A: Since we are having six white beads in the first flowers therefore for three flowers will be Tn=6×3(flower)=18 white beads and 3 black beads. Coded (RP2)

6. **How many flowers will Joana be able to make if she uses 102 white beads? Explain your answer**

   Participant A: from my general formula; Tn = 6n, therefore Joana will need 17 flowers with 102 white beads, I was counting how many white beads for the second flower, and I was looking for a rule that would work therefore I multiplied six which is the white beads by the number of flowers. (Coded RP2)

7. **Can you come up with a general formula for this problem?**

   Participant A: the general formula will be $Tn = 6 \times n$. Six is the number of white beads and n is the number of flower. (Coded PP2)

8. **How do you determine if the formula used is correct?**

   Participant A: I think by substituting the correct values, that is Tn=6×1 is 6 therefore for flower 1 total of seven beads. (Coded IP2)

9. **Is there anything else you want to tell me with regard to better understanding of number pattern problems?**
   Participant A: I am weak in mathematics and poor in visualisation to see a pattern, but now as I am speaking to you I see light. 6 white beads and one black bead make one flower therefore for 3 flowers will be (6x3=18 beads)

The participant A15's responses						
3 DP2	1DP3	2PP2	1IP1	1RP1	2RP2	1IP2

APPENDIX N: ONE-ON-ONE SEMI-STRUCTURED INTERVIEW WITH PARTICIPANT B2

TITLE: GRADE 9 MATHEMATICS LEARNERS' STRATEGIES IN SOLVING NUMBER PATTERN PROBLEMS

The sub-research question:

What are the views of grade 9 mathematics learners regarding the areas of difficulty (if any) they experience as they complete number pattern problems?

Participant B2: irrelevant strategy on number pattern problems

Interview Questions

1. **Let's look at question 1, what type of pattern is that and why?**

   Participant B: I am going to subtract negative 3 in each term to give me the next term and negative multiple by negative is positive, but I don't know the type of pattern (Coded DP1)

2. **How did you complete the sequence in question 1? Explain your strategy**

   Participant B: the sequence goes down and we called it a decreasing pattern and we represent the common difference between consecutive numbers by the variable d; I have calculated the common difference (d) between consecutive numbers which is -3. E.g. T2-T1=d; -5-(-2) = -3; T3-T2=d the common difference between two consecutive terms is -3, and then I subtracted the first term by -3 to get the next term (Coded DP3)

3. **How did you get your general formula?**

   Participant B: I tried the different possibilities because I know the common difference -3 i.e. n^{-3}, -3n, 3×n and trying to add other digits that can give the first -2. I then substituted n by 1; 2; 3 so that it can give me the pattern of the sequence. Then I found the algebraic expression for this kind of pattern then I choice -3n therefore I have Tn=-3n+... (coded RP2, PP2)

4. **When working with question 2 about Joana making necklaces; how did you identify the key words to solve the problem?**

   Participant B: my key word was the Joana making necklace. (Coded DP1)

5. **How did you make a necklace with 3 flowers?**

   Participant B: for 3 flower she will need 18 white beads and 3 black beads (Coded RP1)

6. **How many flowers will Joana be able to make if she uses need if she uses 102 white beads? Explain your answer**

Participant B: Mmmmm… (*pause)* I think it will be 17 flowers because 18 white beads for 3 flowers. (Coded PP2)

7. **Can you come up with a general formula for this problem?**

   Participant B: Yes…I think it will be Tn=6n (Coded PP2)

8. **How do you determine if the formula used is correct?**

   Participant B: for question 2; the pattern grows by 6 white beads each time. Therefore, Tn=6×17(flower)=102 white beads. For question 1; I added 2 and 3 to give me 5 but know that the common difference of the sequence was 3 so I ignored the subtraction sign and the negative sign added 2 and 3 to give me the next term (Coded IP1)

9. **Is there anything else you want to tell me with regard to better understanding of number pattern problems?**
   Participant B: I understand how the consecutive terms are obtained in a pattern, but I'm still not sure if I understand how many white beads and black beads you will need for 3 flowers

The participant B2's responses					
3DP1	1DP3	1RP2	3PP2	1IP1	1RP1

APPENDIX O: ONE-ON-ONE SEMI-STRUCTURED INTERVIEW WITH PARTICIPANT C5

TITLE: GRADE 9 MATHEMATICS LEARNERS' STRATEGIES IN SOLVING NUMBER PATTERN PROBLEMS

The sub-research question:

What are the views of grade 9 mathematics learners regarding the areas of difficulty (if any) they experience as they complete number pattern problems?

Participant C5: appropriate used of a specific strategy on number pattern problems

Interview Questions

1. **Let's look at question 1, what type of pattern is that and why?**

   Participant C5: it is a decreasing linear pattern. They sometimes call it arithmetic sequence. An arithmetic sequence is simply a sequence that is expanded by the same number of integers for each new term. I this case we have the constant difference of -3. (Coded DP3)

2. **How did you complete the sequence in question 1? Explain your strategy**

   Participant C5: The pattern has a common difference of -3; therefore, I used the common difference to get the next term which will be -2-3=-5, then continue like that until I arrive to the n^{th} term. For the nth term I used the general formula (Coded DP3)

3. **How did you get your general formula?**

   Participant C5: $n \times (-3) = -3n$; n represents the number of term in the sequence and -3 is the common difference. Term 1 = $1 \times (-3) =$, this does not give -2 which mean I must add 1 Term 2 =$2 \times (-3) = -6$, this does not give me -5 as the second term I must add 1; therefore I will have $Tn = -3n + 1$. (coded RP3, PP3, IP3)

4. **When working with question 2 about Joana making necklaces; how did you identify the key words to solve the problem?**

   Participant C5: I have looked at the number of white beads in flower 1 and flower 2 to observe the difference between the number of white beads (coded DP3)

5. **How did you make a necklace with 3 flowers?**

Participant C5: Each flower must be a joint unit to form a linear pattern. Each flower shares two white beads to have a complete flower, four white beads were added to the end of each prior flower to create a new flower, and then added two beads to give me a complete flower (Coded RP3).

6. **How many flowers will Joana be able to make if she uses 102 white beads? Explain your answer**

Participant C5: the flowers should be a joint unit which to make this pattern a linear pattern. I will multiply 3(flowers) by 4(common difference), which is 12 white beads plus 3 black beads. Therefore 2 flowers have 6+4=10 white and 1+1=2 black beads, 3 flowers will need 10+4=14 whit and 2+1=3black beads. Therefore, for 102 white beads she will make 25 flowers. (Coded PP3, IP3)

7. **Can you come up with a general formula for this problem?**

Participant C5: my general rule will be Tn=4n+2 the variable n represents the number of flowers. (Coded PP3)

8. **How do you determine if the formula used is correct?**

Participant C5: I will substitute 102 white beads into my formula. (coded IP3)

9. **Is there anything else you want to tell me with regard to better understanding of number pattern problems?**

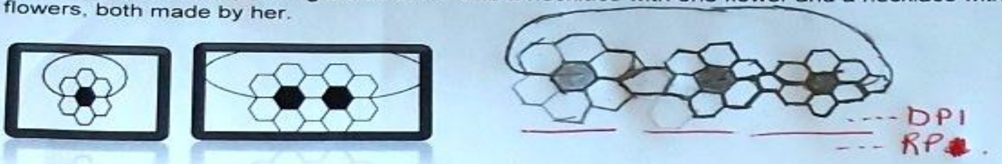
Participant C5: I think I do understand number pattern

The participant C5's responses			
3DP3	2RP3	3PP3	3IP3

APPENDIX P: WRITTEN ACTIVITY OF PARTICIPANT A4

Question 2

Joana likes to make necklaces using flowers. She uses white beads for the petals and black beads for the centre of each flower. The figure below shows a necklace with one flower and a necklace with two flowers, both made by her.



1. How many white and black beads will Joana need to make a necklace with 3 flowers? Explain your answer. Make a neat diagram with 3 flowers on the above space provided.

DP1

For 3 flowers

She will need 3 black and 21 white beads ------ PP1

2. How many flowers will Joana be able to make if she uses 102 white beads? Explain your answer.

For 102 white beads

She will need 17 flowers. IP1

Question 1

1.1. Complete the table by indicating the next terms of the sequence. If the sequence is continued to n^{th} term, please write the general formula for the n^{th} term

x	1	2	3	4	5	6	7	8	n
$f(x)$	-2	-5	-8	-11	-14	17	20	23	-n-(3)

DP2

Substract -3 from each term to get the next number. ---------- DP2

$-2-(-3) = 5$

$-5-(-3) = 8$

$-11-(-3) = 14$

$-17-(-3) = 20$

$-23-(-3) = 23$

$-23-(-3) = 26$ ------ IP1

$T_n = -n-(-3)$ -------- PP2.

APPENDIX Q: WRITTEN ACTIVITY OF PARTICIPANT B2

Question 1

1.1. Complete the table by indicating the next terms of the sequence. If the sequence is continued to n^{th} term, please write the general formula for the n^{th} term

x	1	2	3	4	5	6	7	8	n
$f(x)$	-2	-5	-8	-11	-14	-17	-20	-23	

DP3

Subtract (-3) from each term to get the next term number. ------DP1

-2-(-3) = (-5)
-5-(-3) = -8
-8-(-3) = -11
-11-(-3) = -14
-14-(-3) = -17
-17-(-3) = -20
-20-(-3) = -23

The ~~gen~~ general formula will be

$T_n = T_2 - T_1$ .....PP1.

$\therefore T_n = -3$ IP1

Question 2

Joana likes to make necklaces using flowers. She uses white beads for the petals and black beads for the centre of each flower. The figure below shows a necklace with one flower and a necklace with two flowers, both made by her.

--RP2 ; DP2

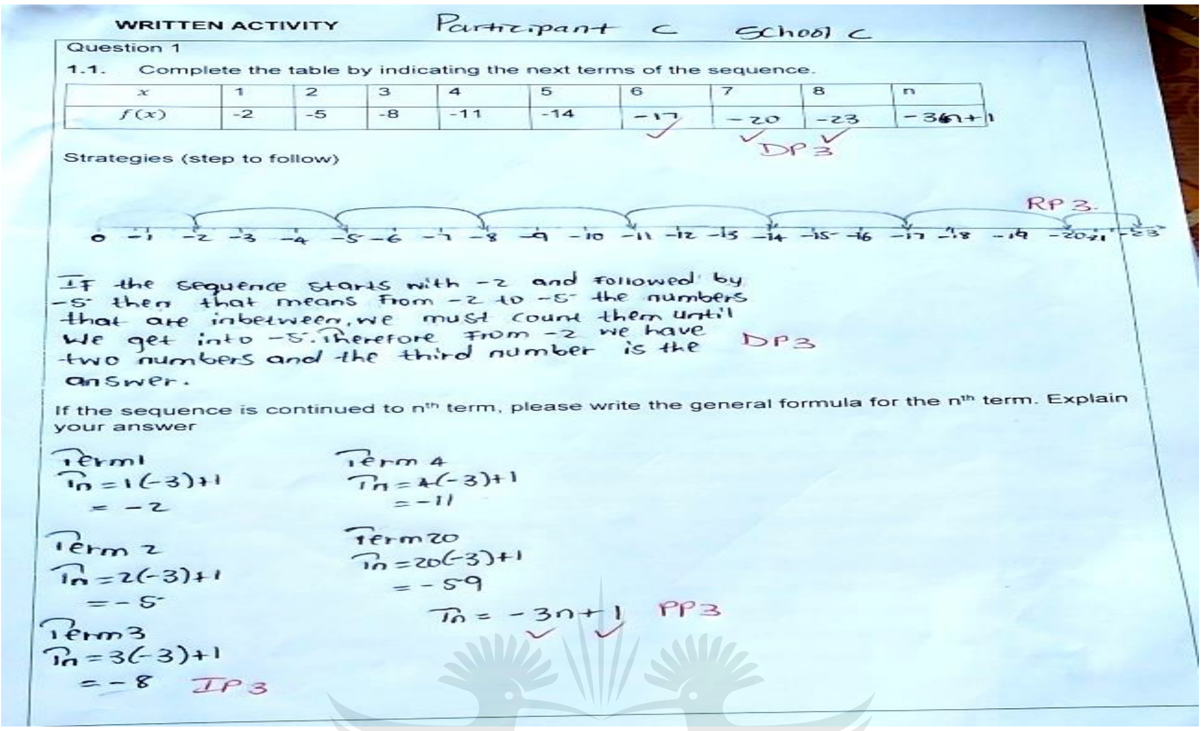
1. How many white and black beads will Joana need to make a necklace with 3 flowers? Explain your answer. Make a neat diagram with 3 flowers on the above space provided.

To make three flowers she will need 18 white beads for the petals and 3 black beads for the centre of each flower since each flower needs only one black bead for the centre.

2. How many flowers will Joana be able to make if she uses 102 white beads? Explain your answer.

$T_n = 6 \times n$ PP2
$102 = 6 \times n$
17 flowers $= n$ ....IP2.
$\therefore$ 102 white beads will make 17 flowers.

APPENDIX R: WRITTEN ACTIVITY OF PARTICIPANT C5



WRITTEN ACTIVITY Participant C School C

Question 1

1.1. Complete the table by indicating the next terms of the sequence.

x	1	2	3	4	5	6	7	8	n
$f(x)$	-2	-5	-8	-11	-14	-17	-20	-23	-3n+1

DP3

Strategies (step to follow)

RP3

0 -1 -2 -3 -4 -5 -6 -7 -8 -9 -10 -11 -12 -13 -14 -15 -16 -17 -18 -19 -20 -21 -23

If the sequence starts with -2 and followed by -5 then that means from -2 to -5 the numbers that are inbetween, we must count them until we get into -5. Therefore from -2 we have two numbers and the third number is the answer. DP3

If the sequence is continued to n[th] term, please write the general formula for the n[th] term. Explain your answer

Term 1
$T_n = 1(-3)+1$
$= -2$

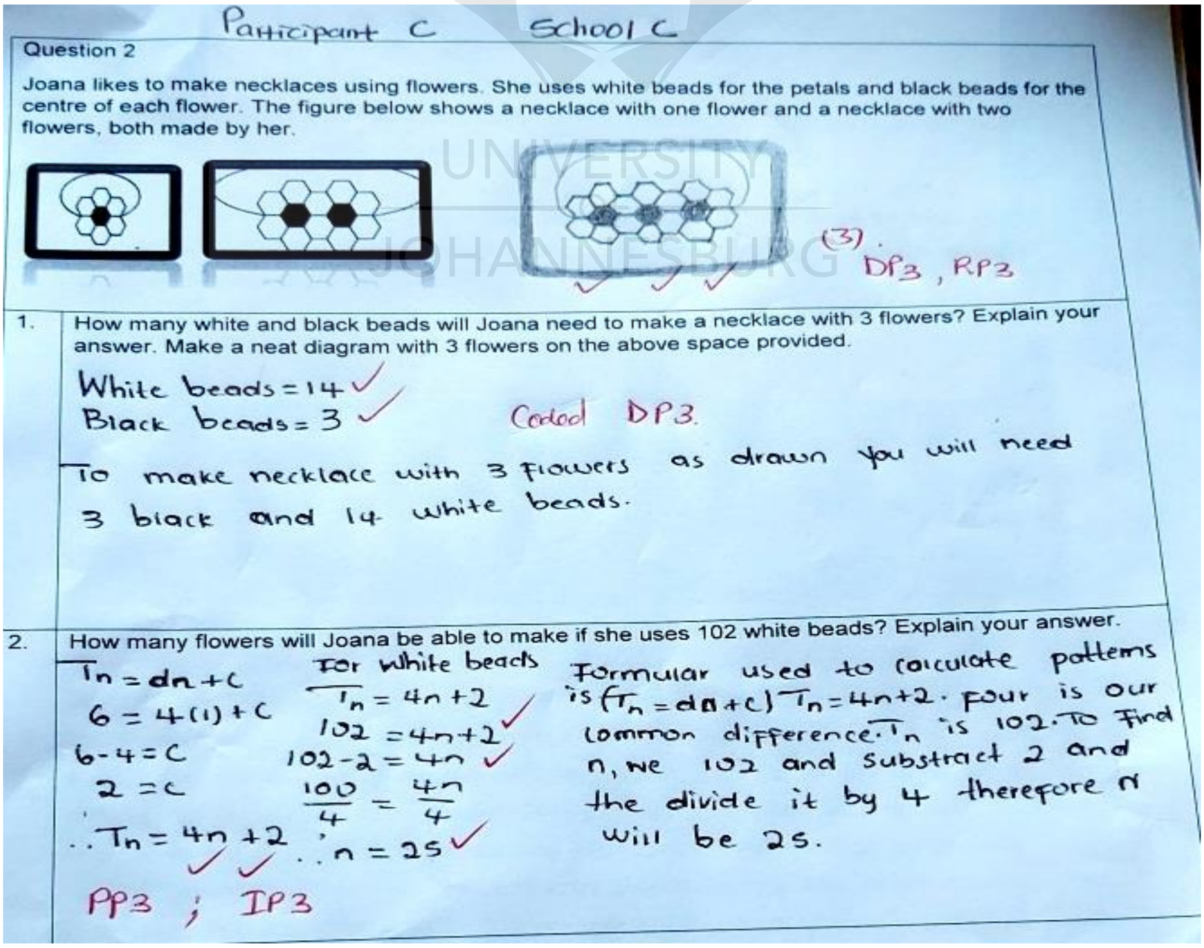
Term 2
$T_n = 2(-3)+1$
$= -5$

Term 3
$T_n = 3(-3)+1$
$= -8$ IP3

Term 4
$T_n = 4(-3)+1$
$= -11$

Term 20
$T_n = 20(-3)+1$
$= -59$

$T_n = -3n+1$ PP3



Participant C School C

Question 2

Joana likes to make necklaces using flowers. She uses white beads for the petals and black beads for the centre of each flower. The figure below shows a necklace with one flower and a necklace with two flowers, both made by her.

(3). DP3, RP3

1. How many white and black beads will Joana need to make a necklace with 3 flowers? Explain your answer. Make a neat diagram with 3 flowers on the above space provided.

White beads = 14
Black beads = 3 Coded DP3.

To make necklace with 3 flowers as drawn you will need 3 black and 14 white beads.

2. How many flowers will Joana be able to make if she uses 102 white beads? Explain your answer.

$T_n = dn + c$
$6 = 4(1) + c$
$6 - 4 = c$
$2 = c$
$\therefore T_n = 4n + 2$

For white beads
$T_n = 4n + 2$
$102 = 4n + 2$
$102 - 2 = 4n$
$\frac{100}{4} = \frac{4n}{4}$
$\therefore n = 25$

Formular used to calculate patterns is ($T_n = dn + c$) $T_n = 4n+2$. Four is our common difference. T_n is 102. To find n, we 102 and substract 2 and the divide it by 4 therefore n will be 25.

PP3 ; IP3