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Baking process design based on modelling and simulation: Towards optimization of bread baking

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Abstract

This paper presents a theoretical approach for optimal design of the baking process. Conventional baking of bread was taken as subject of study, and simulation of previously validated models was used to investigate the process. The proposed approach is based on the definition of two different times for the baking process: a critical time, i.e. a minimum baking time assessed by the complete starch gelatinization in the product, and a quality time, i.e. the time necessary to achieve a target value for a given quality attribute. In this work, browning determined the quality time due to its relevance with regard to sensory and nutritional aspects. As a result, feasible solutions are obtained involving a minimum baking (acceptable products) and a minimum thermal input for a given value of browning, which helps to reduce the formation of acrylamide. Optimum solutions can be then obtained by defining specific objectives; weight loss can be minimized by lowering the value of heat transfer coefficient. Furthermore, obtained results can be helpful to build more efficient ovens.

Keywords: Heat and mass transfer; Multi-objective optimization; Energy demand; Process control; Cooking; Drying.

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Nomenclature

a_w	water activity
C_p	specific heat (J kg^{-1} K^{-1})
D	water (liquid or vapour) diffusion coefficient of product (m^{2} s^{-1})
D_{va}	water vapour diffusion coefficient in air (m^{2} s^{-1})
E_a	activation energy of starch gelatinization (J mol^{-1})
h	heat transfer coefficient (W m^{-2} K^{-1})
K	rate constant of starch gelatinization (s^{-1})
K_0	pre-exponential factor in Eq. (19) (s^{-1})
k	thermal conductivity (W m^{-1} K^{-1})
k_b	rate constant of browning (min^{-1})
k_g	corrected mass transfer coefficient (kg Pa^{-1} m^{-2} s^{-1})
k_g^*	mass transfer coefficient from Eq. (16) (kg Pa^{-1} m^{-2} s^{-1})
L^*	lightness
M	molecular mass (g mol^{-1})
P	water vapour pressure (Pa)
Pr	Prandlt number
Q	heat uptake in starch gelatinization (J)
R, r	radius (m)
R_g	universal gas constant (8.314 J K^{-1} mol^{-1})
RH	relative humidity (%)
Sc	Schmidt number
T	temperature (K)
t	time (s)

W	water (liquid or vapour) content (kg kg^{-1})

Greek symbols

α	degree of starch gelatinization
δ	Delta-type function
ΔT	temperature range of phase change (K)
ε	emissivity
λ_v	latent heat of evaporation (J kg^{-1})
ρ	density (kg m^{-3})
σ	Stefan-Boltzmann constant (5.67×10^{-8} W m^{-2} K^{-4})

Subscripts

∞	ambient
air	air
atm	atmospheric
f	phase change
s	solid or surface
sat	saturated
w	water

1. Introduction

Baking is the final and most important step in bread production, and can be defined as the process which transforms dough, basically made of flour, water and leavening agents, in a food with unique sensory features by application of heat inside an oven. In particular, white or French bread is the most popular type of bread, and is distinguished for having a crunchy and golden-yellow (or brown) crust, a sponge and light crumb with soft texture and intermediate moisture, and a typical flavour. All these quality aspects are the result of a series of physical and chemical changes produced by simultaneous heat and mass transfer occurring within the product during baking (Mondal & Datta, 2008; Purlis, 2010; Sablani, Marcotte, Baik, & Castaigne, 1998; Scanlon & Zghal, 2001; Vanin, Lucas, & Trystram, 2009).

Optimization of the bread baking process is a subject of great importance for food industry. On the one hand, bread is a staple food and thus its production is relevant from a commercial point of view, besides its cultural relevance. On the other hand, baking is an energy-intensive process due to water evaporation occurring in the product (e.g. latent heat of water vaporization is 2.257 MJ/kg at 100 °C). The energy demand for a conventional baking process is around 3.7 MJ/kg, though it can be higher (up to 7 MJ/kg) depending on specific products and operating conditions. In this sense, baking is similar to (conventional) drying, both demanding a high amount of energy in comparison with chilling, freezing, and canning, which need less than 1 MJ/kg (Le Bail et al., 2010). In addition, ovens are often operated in an empirical way by trial-and-error, since information about manipulating the oven settings for an optimum production is still lacking and poorly understood (Broyart & Trystram, 2002). As a result, inconsistency in the quality of bakery products is common in most industrial processes, besides an inefficient use of energy, leading to economical losses (Wong, Zhou, & Hua, 2007).

The end point of the bread baking process is generally established by assessing sensory attributes, e.g. surface colour, texture and flavour of bread, which play a key role in the acceptance of the product by consumers (Purlis & Salvadori, 2007). In particular, surface browning is a practical indicator of baking advance, since can be easily monitored during the process by means of in-line sensors, and therefore can be used as a control parameter (McFarlane, 1990). Furthermore, the development of browning caused by the Maillard reaction is associated with nutritional issues, such as acrylamide formation and decrease of nutritional value of proteins (Purlis, 2010). This way of assessment of the bread baking process, i.e. by subjective (sensory) parameters which also depend on type of consumers, culture and even regulations, makes difficult the task of developing a general (and objective) methodology to design, optimize, and control this process.

On the other hand, since quality changes depend on transport phenomena, it is essential to perform a comprehensive analysis involving both aspects. In this way, two approaches have been used to optimize or design baking. The first approach includes semi-empirical studies where quality attributes are experimentally determined as a function of operating conditions, with a subsequent application of surface response methodology (Demirekler, Sumnu, & Sahin, 2004; Sevimli, Sumnu, & Sahin, 2005) or nonlinear programming techniques (Dingstad, Egelandsdal, Mevik, & Færgestad, 2004; Therdthai, Zhou, & Adamczak, 2002). The second methodology consists in considering transport models coupled with quality kinetic models as a starting point with the aim of describing all changes occurring during the process. Afterwards, process design and optimization can be performed by applying optimization algorithms (Hadiyanto, Boom, van Straten, van Boxtel, & Esveld, 2009; Hadiyanto, Esveld, Boom, van Straten, & van Boxtel, 2008). Besides the advantages, drawbacks, and restrictions of each specific procedure, it is clear the need of adopting a comprehensive point of view with the aim of developing baking strategies considering practical applications. In particular, baking is a special case of food preservation processes and operations, since no microbiological risk has to be considered (as long as good manufacturing practices are carried out), so all objectives to be optimized with regard to the product are quality objectives. Thus, experimental data related to sensory attributes is always necessary to define an objective function, no matter which optimization procedure will be applied.

In this context, the objective of this paper was to propose a theoretical approach to design heating strategies with focus on optimization and control of the baking process. For this aim, mathematical modelling and process simulation were implemented to investigate the bread baking process. This work seeks to contribute to a more comprehensive understanding of the baking process in order to design and control the process in a more efficient way. In addition, this investigation can also help to oven designers and manufacturers to build more efficient equipment.

2. Theory

From the transport phenomena point of view, bread baking is considered as a simultaneous heat and mass transfer (SHMT) process occurring in a porous medium, where phase change (i.e. water vaporization) takes place in a moving front (details are given later in the description of the mathematical model). Amongst all physical and chemical changes that are generated during baking, which actually determine the quality attributes of final product, starch gelatinization and browning development are taken as

reference reactions in this work. The complete starch gelatinization ensures the sensory acceptability of the product because determines the transformation of dough into crumb, i.e. a minimum baking (Zanoni, Peri, & Bruno, 1995a). Surface colour is one of the main (and generally the first) quality features considering preference of consumers, and therefore is often used to judge the completion of baking (Ahrné, Andersson, Floberg, Rosén, & Lingnert, 2007). In bakery products, surface colour is an important sensory attribute associated with aroma, taste, appearance, and with the overall quality of food, and certainly has an important effect on the consumer judgment: colour influences the anticipated oral and olfactory sensations because of the memory of previous eating experiences (Abdullah, 2008). Other product quality descriptors such as specific volume, porosity, and mechanical properties are also important in baking design since they are associated with other sensory attributes (e.g. texture). However, these parameters are also affected by product formulation, i.e. type of flour, fat components, and specific additives or improvers, or by a change in baking technology, e.g. introduction of microwave heating (Demirekler et al., 2004; Sevimli et al., 2005).
Recently, a technological study of bread baking was presented analyzing simultaneously quality and process aspects (Purlis, 2011). It was found that when surface colour is used to determine the end point of the process, which is a common practice actually, it is possible to not achieve a complete baking due to an incomplete starch gelatinization. In particular, such situation occurs when slightly browned products are sought and intense heating is applied: because of high internal resistance to heat transfer due to low thermal conductivity of bread, surface browning is developed at higher rate than starch gelatinization at product centre. In addition, this is favoured with an increase in bread radius via the diminution of thermal gradient. A control parameter should be established to overcome this problem: a minimum value of 96 ºC at the product centre (or coldest

point) has been proposed as a practical solution (Purlis, 2011). Therefore, as browning and starch gelatinization have different reaction rates, partly because they are assessed at different locations undergoing different heat and mass transfer processes, operating conditions should be controlled in order to balance such reactions and generate correctly baked products presenting the desired quality attributes.

Based on previous hypotheses and results, two different times are identified in the baking process: a *critical time* (CT) and a *quality time* (QT). The CT is the minimum baking time, defined as the time necessary to achieve a complete transition of dough into crumb given by a complete starch gelatinization. The CT has to be assessed at the coldest point of bread, where temperature has to reach 96 ºC at least. The QT is defined as the time required to achieve the target value of a given quality attribute, relevant with regard to sensory acceptability of the product. For example, a target value of surface lightness representing the desired surface colour of bread, which can be established by sensory data obtained from preference of consumers. So, the proposed approach establishes that an optimum baking process will present the same value for CT and QT, i.e. at the same time, bread is completely baked and the requirements about sensory attributes are satisfied. In addition, nutritional quality should not be impaired.

Obviously, CT and QT can be unequal depending on heat and mass transfer fluxes established by operating conditions and product properties. For a given situation, if CT is greater than QT, the product will present the desired quality attribute (e.g. surface colour) but will remain unbaked since a complete starch gelatinization is not achieved. Alternatively, if the process time is prolonged to overcome this issue, over-baking will generate different values of the chosen quality attribute associated with QT, and even can lead to poor quality products due to excessive thermal input. Prolonged baking times can produce high temperature values at bread surface, leading to nutritional losses (including the formation of toxic compounds) and more weight loss (this is related to mechanical properties of crust and thus texture attributes). On the other hand, if QT is greater than CT, extra time will be needed to accomplish the target value of the chosen sensory attribute, while the product is already baked in terms of dough/crumb transformation. The described situations generated by non-optimum baking processes produce economical losses since unacceptable products are obtained and additional energy is consumed. Therefore, the ultimate objective is to design an optimum baking process based on the proposed approach.

3. Methodology

The subject of study is conventional baking of French bread (without mould or tin) in a static or batch, indirect (e.g. electric) oven. This is a typical case of traditional bread baking at small and medium scale production, which generally present a low level of process automation and technology, in contrast with continuous baking in large installations equipped with tunnel ovens, which is almost restricted to large scale production of tin bread, as well as biscuits, cakes and similar batter products (Maroulis & Saravacos, 2003). Batch ovens usually have forced convection provided by a fan that recirculates hot air within the baking chamber, which helps to increase the heat and mass flux (and thus transfer coefficients) from air to product. In general, fan velocity is fixed (on/off system) so air velocity and then heat (and mass) transfer coefficient cannot be modified for a given oven and product.

To study such process and apply the hypotheses previously proposed, we use the concept of modern food process design (Maroulis & Saravacos, 2003). This concept is based on engineering principles, mathematical modelling, and process simulation; the objective is to economically produce food products, with emphasis on product quality in addition to the conventional engineering considerations of energy, process cost, and environmental impact. In this way, process simulation is performed using a mathematical model for SHMT in bread during baking, which was previously developed and validated using experimental data of the process; discussion about validation and sensitivity analysis regarding the parameters of the model can be found in Purlis and Salvadori (2009a, 2009b, 2010). Kinetic models for starch gelatinization (Zanoni et al., 1995a; Zanoni, Schiraldi, & Simonetta, 1995b) and browning development (Purlis & Salvadori, 2009c) are coupled to the transport model to describe product quality changes as a function of state variables.

3.1. Heat and mass transfer model

The SHMT model includes the main distinguishing features of bread baking, i.e. the rapid heating of bread core and the development of a dry outer crust. The former has been explained by the evaporation-condensation mechanism (de Vries, Sluimer, & Bloksma, 1989; Sluimer & Krist-Spit, 1987; Wagner, Lucas, Le Ray, & Trystram, 2007), while the later is due to the formation and advancing of an evaporation front towards the bread core (Zanoni, Peri, & Pierucci, 1993; Zanoni, Pierucci, & Peri, 1994). So, bread baking is considered as a moving boundary problem (MBP) where SHMT with phase change occurs in a porous medium. Bread is modelled as a system containing three different regions: (1) *crumb*: wet inner zone, where temperature does not exceed 100 ºC and dehydration does not occur; (2) *crust*: dry outer zone, where temperature exceeds 100 ºC and dehydration occurs; (3) *evaporation front*: between the crumb and crust, where temperature is ca. 100 ºC and water evaporates (liquid-vapour transition).

Mathematically, the MBP is formulated using a physical approach, where the enthalpy jump corresponding to phase change is incorporated in the model by defining equivalent thermophysical properties (Bonacina, Comini, Fasano, & Primicerio, 1973). Such definition states that evaporation occurs within a temperature range rather than at a fixed temperature. Other major assumptions of the model are the following: (1) bread is homogeneous and continuous; the concept of porous medium is included through effective or apparent thermophysical properties; (2) heat is transported by conduction inside bread according to Fourier's law, but an effective thermal conductivity is used to incorporate the evaporation-condensation mechanism in heat transfer; (3) only liquid diffusion in the crumb and only vapour diffusion in the crust are assumed to occur (Luikov, 1975); (4) volume change is neglected. For a detailed description of the model, including thermophysical properties, the reader is referred to Purlis and Salvadori (2009a, 2009b, 2010).

3.1.1. Governing equations

Bread (French type) is considered as an infinite cylinder of radius *R*, so the problem is reduced to a single dimension via the axial symmetry assumption. For initial conditions, uniform temperature and water content are assumed.

Heat balance equation:

$$\rho C_p \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rk \frac{\partial T}{\partial r} \right) \qquad (1)$$

Mass balance equation:

$$\frac{\partial W}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r D \frac{\partial W}{\partial r} \right) \quad (2)$$

3.1.2. Boundary conditions

Heat arrives to the bread surface by convection and radiation, and is balanced by conduction inside the bread:

$$-k\frac{\partial T}{\partial r} = h(T_s - T_\infty) + \varepsilon\sigma(T_s^4 - T_\infty^4) \quad (3)$$

Water migrating towards the bread surface is balanced by convective flux:

$$-D\rho_s\frac{\partial W}{\partial r} = k_g\,(P_s(T_s) - P_\infty(T_\infty)) \quad (4)$$

where $P_s = a_w\, P_{sat}(T_s)$ and $P_\infty = (RH/100)\, P_{sat}(T_\infty)$.

At the centre, i.e. $r = 0$:

$$\frac{\partial T}{\partial r} = 0 \quad (5)$$

$$\frac{\partial W}{\partial r} = 0 \quad (6)$$

3.1.3. Thermophysical properties

According to the MBP formulation, equivalent thermophysical properties are defined by including the phase transition occurring during the process, thus an equivalent property is valid for dough/crumb and crust. A smoothed Heaviside function with continuous derivative is used to incorporate the phase transition into thermophysical properties, with parameters $T_f = 100$ ºC and $\Delta T = 0.5$ ºC. In addition, the delta-type function $\delta(T -$

T_f, ΔT) that simulates the enthalpy jump (Eq. (7)) is defined by the sum of two smoothed Heaviside functions with different sign.

Specific heat:

$$C_p(T,W) = C_p^*(T,W) + \lambda_v W \delta(T - T_f, \Delta T) \tag{7}$$

$$C_p^*(T,W) = C_{p,s}(T) + W C_{p,w}(T) \tag{8}$$

$$C_{p,s} = 5T + 25 \tag{9}$$

$$C_{p,w} = 5207 - 7.317T + 1.35\times10^{-2} T^2 \tag{10}$$

Thermal conductivity:

$$k(T) = \begin{cases} 0.9/\left[1+\exp(-0.1(T-353.16))\right] + 0.2 & if \quad T \le T_f - \Delta T \\ 0.2 & if \quad T > T_f + \Delta T \end{cases} \tag{11}$$

Density:

$$\rho(T) = \begin{cases} 180.61 & if \quad T \le T_f - \Delta T \\ 321.31 & if \quad T > T_f + \Delta T \end{cases} \tag{12}$$

Density for solid (ρ_s) that appears in Eq. (4) is equal to 241.76 kg m^{-3}.

Mass diffusivity:

$$D(T) = \begin{cases} 1\times10^{-10} & if \quad T \le T_f - \Delta T \\ 1.32\times10^{-3} D_{va}(T) & if \quad T > T_f + \Delta T \end{cases} \tag{13}$$

$$D_{va}(T) = 2.302\times10^{-5} \frac{p_0}{p}\left(\frac{T}{T_0}\right)^{1.81} \tag{14}$$

where $p_0 = 0.98\times10^5$ Pa and $T_0 = 256$ K (Eckert & Drake, 1959); $p = P_{atm} = 101325$ Pa.

Water activity:

$$a_w(T,W)=\left[\left(\frac{100\,W}{\exp(-0.0056T+5.5)}\right)^{-1/0.38}+1\right]^{-1} \qquad (15)$$

The heat transfer coefficient (*h*) is a model input for process simulation (see Section 3.4), and the mass transfer coefficient (k_g) is determined by using the Chilton-Colburn (or heat-mass) analogy and a correction factor (Purlis & Salvadori, 2009b):

$$\frac{h}{k_g^*}=\frac{M_{air}}{M_w}P_{atm}C_{p,air}\left(\frac{Sc}{Pr}\right)^{2/3} \qquad (16)$$

$$k_g=7.83\times10^{-2}\,k_g^* \qquad (17)$$

With regard to heat transfer by radiation, the emissivity of bread surface is considered equal to 0.9 (Hamdami, Monteau, & Le Bail, 2004).

3.2. Kinetic model for starch gelatinization extent

Zanoni et al. (1995a, 1995b) developed and validated a kinetic model of starch gelatinization for bread, which is temperature dependent. The extent of starch gelatinization follows first-order kinetics and the reaction rate constant is temperature dependent according to the Arrhenius equation:

$$\frac{d(1-\alpha)}{dt}=-K(1-\alpha) \qquad (18)$$

$$K=K_0\exp\left(\frac{-E_a}{R_gT}\right) \qquad (19)$$

where $K_0 = 2.8\times10^{18}$ s^{-1} and E_a = 139 kJ mol^{-1}. The gelatinization degree (α) is defined as:

$$\alpha(t) = 1 - \frac{Q(t)}{Q_{max}} \quad (20)$$

where $Q(t)$ and Q_{max} are the heat uptakes for partially baked and raw dough, respectively. At initial condition, $\alpha = 0$, i.e. $Q = Q_{max}$ (raw dough).

It can be assumed a complete starch gelatinization when the coldest point of the product achieves a value of $\alpha \geq 0.98$ (Therdthai et al., 2002; Zanoni et al., 1995a, 1995b). This parameter is used to verify the assessment of the minimum baking time (CT) by using the core temperature (≥ 96 ºC) as a technological solution. It is worth mentioning that this model is applied to crumb but not to crust, where the starch gelatinization process is more complex due to variation in water content (Primo-Martín, van Nieuwenhuijzen, Hamer, & van Vliet, 2007; Vanin, Michon, Trystram, & Lucas, 2010).

3.3. Kinetic model for browning development

The formation of colour, i.e. browning is the result of non-enzymatic chemical reactions (Maillard reaction and caramelization of sugars) that produce coloured compounds, which are accumulated in the product during baking. This phenomenon is a dynamic process depending on local temperature and water activity, so it should not be decoupled from transport phenomena (Purlis, 2010). Purlis and Salvadori (2009c) developed and validated a kinetic model for browning development based on a non-isothermal kinetic approach and assuming a general mechanism of browning, which can be described by the variation of lightness (L^* parameter of the CIE $L^*a^*b^*$ colour space). Browning advance is described by first-order kinetics, and the rate constant is a function of temperature and water activity of bread:

$$\frac{dL^*}{dt} = -k_b L^* \quad (21)$$

$$k_b = \left(7.9233\times10^6 + 2.7397\times10^6/a_w\right)\exp\left(-\frac{8.7015\times10^3 + 49.4738/a_w}{T}\right) \quad (22)$$

Browning is initiated when temperature exceeds 120 ºC; raw dough has an initial value of L^* = 85 (standard recipe for French bread: 100% wheat flour, 54.1% water, 1.6% salt, 1.6% sugar, 1.6% margarine, 1.2% dry yeast).

3.4. Simulations

The bread baking process was simulated for several operating conditions. Input parameters to the SHMT model were oven temperature (180, 190, 200, 210, 220, 230, and 240 ºC), heat transfer coefficient (5, 10, 15, 20, and 25 W m^{-2} K^{-1}), and product radius (0.025, 0.03, and 0.035 m). These values were selected according to reported data for conventional baking ovens and common industrial practice (Baik, Grabowski, Trigui, Marcotte, & Castaigne, 1999; Baik, Marcotte, & Castaigne, 2000; Carson, Willix, & North, 2006; Li & Walker, 1996; Sakin, Kaymak-Ertekin, & Ilicali, 2009; Therdthai et al., 2002; Zareifard, Boissonneault, & Marcotte, 2009). Initial temperature and water content were assumed to be uniform and equal to 25 ºC and 0.65 kg kg^{-1} (dry basis), respectively. Relative humidity (or water vapour pressure) in oven ambient was assumed to be negligible (i.e. conventional baking without steam injection).

The system of nonlinear partial differential equations describing the stated MBP was solved using the finite element method. The numerical procedure was implemented in COMSOL Multiphysics 3.2 (COMSOL AB, Sweden) and MATLAB 7.0 (The MathWorks Inc, USA). The method of lines is used in COMSOL Multiphysics for discretization of the partial differential equations, so a differential algebraic equation system is obtained. This new system is solved using an implicit time-stepping scheme (backward differentiation), i.e. a Newton's method together with a COMSOL Multiphysics linear system solver (UMFPACK). The time step taken by the algorithm is variable (COMSOL AB, 2005), but it was ensured to be small enough (< 5 s) to do not miss the latent heat peak corresponding to phase transition. The finite element mesh consisted in 240 elements in all cases. Finally, a medium order Runge-Kutta routine (function *ode45* from MATLAB) was used to solve (numerically) the quality kinetic models from temperature and moisture content profiles obtained through transport model simulation, using the same criterion for time step as before.

Baking time used for process simulation was long enough (90 min) to ensure covering a wide range of practical situations. Afterwards, CT was calculated by interpolating the time-temperature curve of product centre for a temperature value of 96 ºC. For this time, other variables were determined: surface temperature, water content and water activity, weight loss, surface lightness, and starch gelatinization extent at product centre. Also, the time-temperature curve of product surface was used to assess the *thermal input* (TI), i.e. the combination of temperature and time to which the product is subjected during the process (FoodDrinkEurope, 2011):

$$TI = \int_0^{CT} T_s \, dt \qquad (23)$$

A recursive adaptive Simpson quadrature routine (function *quad* from MATLAB) was used to evaluate numerically the integral in Eq. (23), using the same criterion for time step as before.

4. Results and discussion

To investigate the proposed approach, bread baking was simulated for 105 operating conditions according to input parameters established in Section 3.4. For each baking condition, the minimum baking time (CT) was determined, and afterwards other variables were calculated. Therefore, all results shown are feasible solutions considering the proposed theory of an optimum baking process. If the target value for the desired attribute is achieved at this time, i.e. QT = CT, then feasible conditions become optimum conditions. In other words, if the value of surface lightness reached at CT is the designed target value, then the heating strategy used is optimum. Otherwise, more time will be necessary while the product is already baked, thus consuming extra energy. This non-optimum condition can appear when the end point of baking is established by colour formation, as described before. To analyze this situation with regard to the proposed approach, we will refer to data reported in Purlis (2011). It is worth to note that temperature and moisture content profiles (and other microscopic data) will not be discussed here. The intention is not to avoid a discussion on transport phenomena but concentrate on the engineering aspects of design, optimization, and control of bread baking. A microscopic perspective of the process can be found in the cited literature. Results obtained from simulations are included in Table 1; Figure 1 is introduced to have a visual reference guide of browning development in bread when analyzing the results. Firstly, it is confirmed that a minimum value of 96 ºC at the coldest point of the product is an effective control parameter to assess the minimum baking time, which corresponds to a complete starch gelatinization. Nevertheless, not all operating conditions produce a marked development of browning. In some cases, browning is not even initiated since surface temperature does not exceed 120 ºC. Although this is an advantageous situation with regard to nutritional quality because toxic compounds associated with browning reactions can not be generated, the products are valueless from a commercial point of view since (French) bread is characterized by a yellow/golden-brown crust. Also, limited dehydration (i.e. low values of weight loss) occurring under these conditions affects sensory attributes associated with texture due to a limited formation of crust. This situation is mainly produced by natural convection heating mode, represented by values of heat transfer coefficient not greater than 10 W m^{-2} K^{-1}, approximately (Purlis & Salvadori, 2009b). In addition, a small radius (characteristic length) favours such situation since CT is reduced and there is less time for the development of browning. On the other hand, as h increases and thus forced convection becomes the heating mode, and oven temperatures above 200 ºC are used, the development of browning is noticeable.

This observation (which can be interpreted as a practical recommendation) seems to be in disagreement with (technological) considerations arisen in Purlis (2011): intense heating (e.g., h greater than 15 W m^{-2} K^{-1} and oven temperature above 220 ºC) as a baking strategy was not recommended because unbaked foods could be produced and high values of surface temperature are achieved, thus generating harmful compounds. In fact, rather than a contradiction there is a conceptual difference that lies in the criterion used in both cases to establish the end point of the baking process. In the previous study, a target value of surface lightness determined the end of baking, with the aim of reproducing a common industrial practice. So, such recommendation was funded on the risk of obtaining unbaked foods while surface colour is acceptable. The approach proposed in this work eliminates this possible problem, and the search is now oriented towards optimum conditions of baking. Nevertheless, the nutritional quality issue is still relevant. In this regard, the Confederation of the Food and Drink Industries of the European Union suggests avoiding excessive browning in the crust to reduce acrylamide formation during baking (FoodDrinkEurope, 2011). In addition, it has been found that the thermal input (combination of temperature and heating time) is a key factor in this subject. For instance, a lower temperature combined with a prolonged baking time does not result in lower acrylamide contents if the same browning of the product is to be achieved (Amrein, Schönbächler, Escher, & Amadò, 2004).

By applying the proposed theory, it is observed an increasing trend of the thermal input (TI) with browning development, for a given product dimension (note that assessing TI via the evolution of surface temperature instead of oven temperature allows comparing the results obtained by using different values of heat transfer coefficient) (Figure 2). As expected, an increase in radius produces an increase in TI since longer times are needed to achieve 96 ºC at bread centre. Therefore, the recommendation of avoiding excessive browning during baking to diminish acrylamide formation is still applicable. The scope of this paper is limited to develop optimum heating strategies and derive some practical recommendations. In this sense, the ultimate decision about the reduction of acrylamide generation via reduction of browning development requires a fundamental change with respect to the production and consumption of baked products, which will be not discussed here (although it is an urgent debate). Nevertheless, an additional consideration is necessary. Thermal input was also calculated for data reported in Purlis (2011), where the end point of the process was determined for three different values of surface lightness, e.g. L^* = 80, 75, and 70 (results not shown). When comparing these supplementary results with the ones presented in this work, it was found that no further reduction in TI can be done as by applying the proposed approach, for given values of L^* and radius. A further diminution of TI implies that CT is greater than QT, and thus unbaked products are obtained. Although this observation can be derived from previous considerations elaborated in Section 2, now is inferred from numerical results.

Different combinations of oven temperature and heat transfer coefficient can produce the same (minimum) thermal input, for fixed values of final L^* and radius. For example, let analyze the case of $L^* = 80$ (approximately), for $R = 0.03$ m and $R = 0.035$ m (results are extracted from Table 1 and summarized in Table 2 for readability). Firstly, the minimum TI value is balanced by opposite variations in oven temperature and heat transfer coefficient, as can be expected from transport phenomena concepts if the driving force has to be balanced to produce the same TI. Secondly, CT shows a diminishing tendency with the increase of h and the balanced diminution of oven temperature, while weight loss presents an opposite trend; final values of surface temperature do not show a marked behaviour in this regard. This observation reveals a higher influence of the heat transfer coefficient than oven temperature to establish more rapidly the evaporation front at the beginning of baking (in the tested range of operating conditions). This would also explain the higher weight loss produced by increasing the heat transfer coefficient, and thus by the earlier formation of the evaporation front in the product. Weight loss by dehydration of the outer zone of the product is the consequence of the advance of the evaporation front towards the core, which also increases the thickness of the crust (Purlis & Salvadori, 2009a; Zanoni et al., 1993, 1994).

In summary, the proposed approach of baking optimization could lead to multiple *optimum* solutions or baking strategies to apply, so a new problem is established to decide which baking strategy should be finally applied. Therefore, such solutions are now feasible solutions for the *ultimate decision problem*. In this sense, the developed theory leads to a two-step optimization problem: the first step consists in finding feasible solutions (or multiple *optimum* solutions), and the second step involves the final decision about the baking strategy to be applied. This second step of the global problem requires a variety of considerations, including sensory (subjective) aspects. Indeed, such global problem represents the design of a baking process. In order to be as general as possible, we will limit further discussion to objective factors, focusing on engineering aspects of the baking process. The main factor to analyze is the heat transfer coefficient, i.e. the oven (flow) characteristics. If the value of h can not be modified (e.g. there is already an oven with a characteristic h value), then the problem is simplified from the beginning, and the only way of optimizing the process is by the proposed approach, i.e. equalling CT to QT. This situation can limit the extent of browning development within the space of feasible solutions with minimum thermal input. If possible, an increase in the characteristic length of the product can lead to a wider range of browning since the CT is increased, so more time is available to colour formation (see Table 1).

On the other hand, we have the case of a variable h (not fixed a priori), which represents an entire design problem. In this case, other factors become important to make the final decision since multiple solutions can appear, as in previous examples. Two engineering parameters are the weight loss of the product and the energy demand during the baking process. It has been reported that about 20% of total energy related to the baking process is used for evaporation of water in the product (Le Bail et al., 2010). Based on this information, the optimum baking strategy should be the (feasible) one involving the lower value of heat transfer coefficient. Nevertheless, it should be noted that production costs and economy aspects of the process can not be assessed in a general way, and thus the optimum solution may change depending on each particular case. In any case, there is a compromise situation typical of multi-objective optimization problems, which are solved by assigning a relative weight factor to each objective using empirical data.

Finally, the results and discussion derived from the proposed theory could also be helpful to develop and improve baking equipment. In this sense, Zareifard et al. (2009) remarked the need of improving oven performance taking into account the quality and appearance of baked products. An interesting alternative to bread manufacturers would be specialized ovens that allow adjusting the heat transfer coefficient. Therefore, in addition to the improved efficiency sought by oven builders, versatility in terms of design, optimization, and control of the baking process would be delivered to baking industry.

5. Conclusions

Optimal design of a baking process is a complex and challenging problem that involves several aspects including both quality and operating variables, where multiple objectives have to be taken into account. In addition, baked products are mainly evaluated in a subjective or sensory manner, which makes difficult the task of developing a general approach to design, optimize, and control this traditional food process. To deal with these issues, a theoretical approach was developed and applied to the bread baking process.

The presented approach establishes a method to obtain firstly feasible heating strategies that ensure a minimum (critical) baking and minimize the thermal input provided to the product, which is essential for reducing the formation of acrylamide during the process. In this sense, is always recommended to avoid an excessive browning in the product. Afterwards, optimum baking strategies can be established according to different objectives. In general terms, minimization of weight loss should be desirable, and can be achieved by using a low heat transfer coefficient when possible. Finally, the investigation shows a balance between the heat transfer coefficient and baking temperature, which can be used to control the process towards optimum conditions, and also design more efficient ovens.

Other food processes could be studied under the developed theory by redefining the critical and quality times, as well as identifying the key operating parameters or factors affecting the process. In this sense, the methodology used in this work (modelling and simulation) or the case of study (bread baking) are not restrictive for the application of the presented approach.

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Figure captions

Figure 1. Image gallery of bread samples with the corresponding value of lightness L^* (Purlis & Salvadori, 2009c).

Figure 2. Thermal input (Eq. (23)) as a function of lightness for different values of bread radius (indicated in the figure) and heat transfer coefficient (symbols, in W m^{-2} K^{-1}).

Table 1. Results obtained from simulations of the bread baking process. For all conditions $\alpha \geq 0.98$. Units: h in W m^{-2} K^{-1}, T in ºC, CT in min, WL (weight loss) in %, TI in ºC min.

h	T_∞	R = 0.025 m					R = 0.03 m					R = 0.035 m				
		CT	WL	T_s	L^*	TI	CT	WL	T_s	L^*	TI	CT	WL	T_s	L^*	TI
5	180	9.45	1.27	105.46	85[a]	845.93	12.49	1.67	108.15	85[a]	1154.42	15.95	2.14	111.35	85[a]	1520.07
	190	8.93	1.52	107.80	85[a]	817.34	12.00	1.95	110.81	85[a]	1139.89	15.35	2.50	115.14	85[a]	1502.99
	200	8.56	1.80	110.71	85[a]	803.24	11.43	2.77	116.74	85[a]	1130.40	14.90	3.05	120.16	85[a]	1506.28
	210	8.21	2.03	113.45	85[a]	787.59	11.13	2.70	119.08	85[a]	1112.29	14.43	3.62	125.63	84.45	1506.46
	220	7.83	2.31	117.32	85[a]	769.25	10.77	3.30	124.81	84.64	1113.68	14.05	4.05	130.88	83.97	1504.41
	230	7.64	2.65	121.60	84.93	770.27	10.39	3.71	129.51	84.35	1104.47	13.61	4.91	138.30	83.10	1519.74
	240	7.44	3.16	126.94	84.66	774.57	10.09	4.89	138.54	83.49	1136.52	13.36	5.96	146.88	81.80	1562.90
10	180	8.54	2.23	109.62	85[a]	801.75	11.43	2.91	113.50	85[a]	1111.00	14.93	3.62	117.60	85[a]	1500.41
	190	8.08	2.60	112.60	85[a]	775.15	10.94	3.48	117.78	85[a]	1093.15	14.43	4.44	123.34	84.62	1501.17
	200	7.82	3.03	116.25	85[a]	770.32	10.71	3.91	122.10	84.83	1097.14	13.91	5.14	129.13	83.95	1492.58
	210	7.60	3.64	120.99	84.94	768.83	10.45	4.67	127.95	84.38	1106.45	13.66	5.87	135.52	83.19	1516.31
	220	7.38	4.22	125.92	84.67	769.26	10.25	5.29	133.44	83.96	1123.55	13.42	6.65	142.52	82.23	1545.41
	230	7.12	4.94	131.81	84.38	765.72	9.90	6.20	140.93	83.23	1124.05	13.11	7.59	150.53	80.59	1576.82
	240	7.08	5.20	135.93	84.20	778.25	9.73	7.12	148.83	82.11	1149.22	12.90	8.31	158.09	78.70	1607.71
15	180	7.91	3.08	113.50	85[a]	769.43	10.77	4.05	118.74	85[a]	1090.17	14.20	4.93	123.68	84.53	1492.29
	190	7.68	3.61	117.41	85[a]	767.18	10.44	4.70	123.74	84.65	1089.42	13.69	5.78	129.90	83.73	1492.45
	200	7.42	4.44	122.99	84.82	766.35	10.21	5.57	130.11	84.12	1105.14	13.36	6.60	136.60	82.79	1511.65
	210	7.24	5.04	128.06	84.52	769.19	9.96	6.40	136.71	83.45	1117.94	13.24	7.28	143.35	81.79	1547.97
	220	6.99	5.85	134.25	84.15	768.29	9.74	7.26	143.89	82.56	1134.85	12.89	8.39	151.98	79.75	1584.71
	230	6.90	6.43	140.13	83.69	779.96	9.59	7.96	150.92	81.47	1158.93	12.72	9.07	159.63	77.78	1620.52
	240	6.78	7.29	147.43	82.95	796.50	9.45	8.70	158.53	80.01	1182.53	12.63	9.49	166.62	75.57	1660.89
20	180	7.56	3.98	117.80	85[a]	760.47	10.41	4.94	123.35	84.70	1092.21	13.61	5.96	128.83	83.78	1485.40
	190	7.34	4.75	123.09	84.79	762.43	10.13	5.81	129.57	84.09	1102.63	13.33	6.87	135.86	82.78	1513.50
	200	7.17	5.39	128.35	84.48	766.78	9.92	6.59	136.03	83.47	1113.25	13.03	7.93	143.72	81.38	1548.64
	210	6.96	6.20	134.61	84.11	770.12	9.69	7.42	142.99	82.55	1132.64	12.81	8.58	150.74	79.92	1580.21
	220	6.83	6.90	140.84	83.55	781.47	9.48	8.45	151.23	81.21	1161.33	12.64	9.30	158.55	78.00	1623.51
	230	6.69	7.86	148.61	82.71	798.62	9.39	8.97	158.12	79.95	1187.14	12.42	10.27	167.36	74.87	1676.23
	240	6.60	8.49	155.45	81.91	815.33	9.33	9.60	165.53	78.04	1229.97	12.33	10.79	175.23	71.88	1718.23
25	180	7.23	4.92	122.24	84.83	752.07	10.05	5.93	128.14	84.16	1091.45	13.29	6.95	133.78	83.00	1504.56
	190	7.09	5.69	128.01	84.45	762.19	9.91	6.49	133.89	83.68	1106.90	13.09	7.76	140.92	81.91	1535.93
	200	6.92	6.51	134.28	84.04	770.88	9.64	7.74	142.26	82.59	1132.35	12.83	8.56	148.27	80.48	1568.50
	210	6.77	7.33	141.00	83.40	782.12	9.43	8.71	150.11	81.30	1159.14	12.55	9.68	157.16	78.27	1618.89
	220	6.63	8.21	148.32	82.65	797.47	9.26	9.52	157.92	79.85	1186.59	12.41	10.18	164.55	76.33	1662.00
	230	6.54	8.95	155.69	81.77	816.08	9.17	10.12	165.41	78.14	1220.61	12.22	11.05	173.34	73.42	1710.84
	240	6.46	9.51	162.73	80.68	835.66	9.06	10.77	173.43	75.65	1255.87	12.17	11.40	180.84	70.94	1772.20

[a] Browning has not been initiated because surface temperature does not exceed 120 ºC, and thus L^* corresponds to its initial value (85).

Table 2

Results corresponding to operating conditions that produce a final value of L^* = 80 (approximately). Units: R in m, h in W m^{-2} K^{-1}, T in ºC, CT in min, WL (weight loss) in %, TI in ºC min.

R	h	T_∞	CT	WL	T_s	L^*	TI
0.03	15	240	9.45	8.70	158.53	80.01	1182.53
	20	230	9.39	8.97	158.12	79.95	1187.14
	25	220	9.26	9.52	157.92	79.85	1186.59
0.035	10	230	13.11	7.59	150.53	80.59	1576.82
	15	220	12.89	8.39	151.98	79.75	1584.71
	20	210	12.81	8.58	150.74	79.92	1580.21
	25	200	12.83	8.56	148.27	80.48	1568.50

Figure 1 – Purlis

$L^* =$ 85.04 79.08 76.67 73.05 70.12

Figure 1. Image gallery of bread samples with the corresponding value of lightness L^* (Purlis & Salvadori, 2009c).

Figure 2 – Purlis

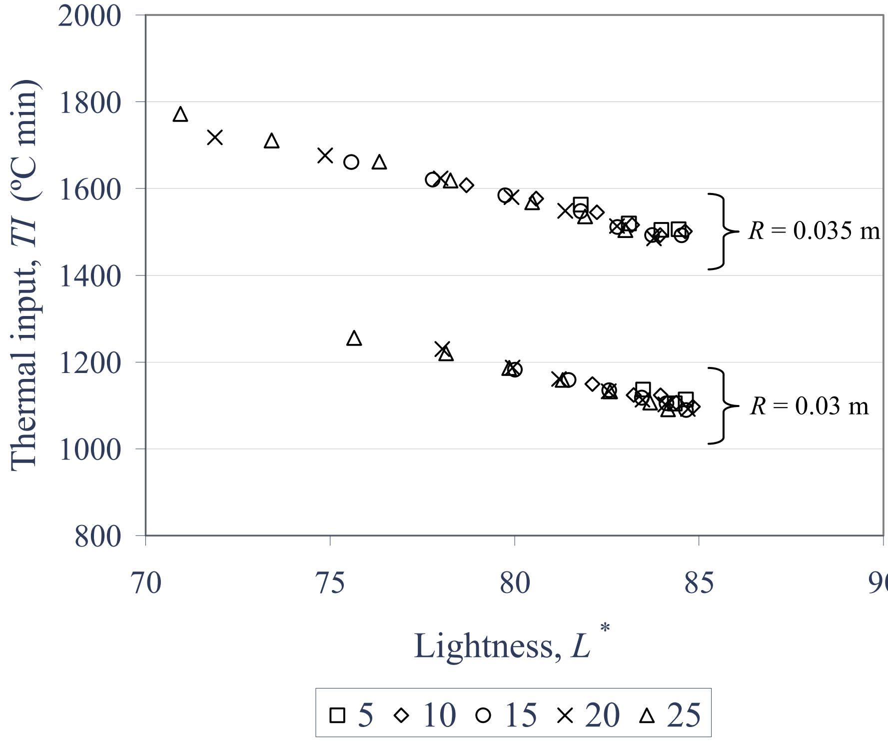

Figure 2. Thermal input (Eq. (23)) as a function of lightness for different values of bread radius (indicated in the figure) and heat transfer coefficient (symbols, in W m^{-2} K^{-1}).