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Exploring Pedagogical Empathy of Mathematics Graduate Student Instructors

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EXPLORING PEDAGOGICAL EMPATHY OF MATHEMATICS GRADUATE STUDENT INSTRUCTORS

by

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Interpersonal relationships are central to the teaching and learning of mathematics. One way that teachers relate to their students is by empathizing with them. In this study, I examined the phenomenon of pedagogical empathy, which is defined as empathy that influences teaching practices. Specifically, I studied how mathematics graduate student instructors conceptualize pedagogical empathy and analyzed how pedagogical empathy might influence their teaching decisions. To address my research questions, I designed a qualitative phenomenological study in which I conducted observations and interviews with 11 mathematics graduate student instructors who were teaching precalculus courses at the University of Nebraska—Lincoln.

In the first part of my dissertation, I analyze how the participants in my study conceptualized pedagogical empathy and discuss their views on eight dimensions of empathy identified by researchers in the field of psychology. In the second part of my dissertation, I identify five factors that have the potential to influence instructors' pedagogical empathy: instructors' past experiences, students' mathematical experiences, students' experiences outside the classroom, the importance of communicating pedagogical empathy to students, and the course structure. Finally, in the third part of my dissertation, I examine how four of my participants reflected on their decision-making through a lens of empathy and argue that pedagogical empathy can be conceptualized as a

filter that connects pedagogical content knowledge and emotional knowledge to decision making. My findings suggest that instructors can develop a more empathic disposition towards students over time through reflection. In addition, the conclusions drawn from this research can help inform professional development targeted at novice postsecondary mathematics instructors.

DEDICATION

For my dad, I love you and I miss you.

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CHAPTER 1: INTRODUCTION

Background and Problem Context

Imagine the following scenario. You are observing a precalculus class where students are working in small groups on problems related to logarithmic functions. As the students work together on the problems with their classmates, their teacher, Jack, walks around the classroom facilitating group work and answering student questions. After a while, Jack stops at a table to help a group who is working on a difficult word problem. One of the students at the table, Connor, tries to explain what he did to come up with a solution to the problem. After Conner finishes his explanation, a group member turns to him and says, "I'm so confused by what you just said." She then asks Jack, "Can you just tell us how we should do it?" Conner looks away, seeming discouraged. What does Jack choose to do in this moment? What experiences or knowledge does he draw upon as he decides how to respond? How might he respond in a way that will acknowledge Conner's contributions and move the group forward?

Relationships between teachers and students are central to many conceptualizations of teaching. Adopting the stance that cognition occurs through participation in sociocultural contexts (Lave & Wenger, 1991; Op't Eynde, De Corte, & Verschaffel 2001; Vygotsky, 1978), I view learning and teaching as processes that take place through interactions in social settings. Since the goal of teaching is to support others in their learning, which involves both cognitive and affective processes, teachers

should attend not only to the content that is being learned, but also to the way that their students are experiencing that content. Understanding and responding to students' learning experiences involves how teachers empathize with their students. Thus, the phenomenon considered in this dissertation centers on the empathy that may or may not arise due to interactions between teachers and students in educational settings.

Empathy is a multifaceted construct. Researchers have described empathy in various ways and have developed different characterizations depending on the context in which empathy is referenced (Cuff, Brown, Taylor, & Howat, 2016). The history and definition of empathy are discussed in more detail in Chapter 2, but before going further, it is important to provide a working definition of empathy as it relates to the context of teaching. In this dissertation, I use the term *pedagogical empathy* to refer to the empathy that a teacher experiences as a result of interacting with a student or a group of students. Drawing on the work of Feshbach and Feshbach (2009), who have studied empathy in educational contexts, I define empathy as "a social interaction between any two individuals wherein one individual experiences the feelings of a second individual" (Feshbach & Feshbach, 2009, p. 85). Building on this definition, I define pedagogical empathy[1] as a social interaction between a teacher and a student wherein the teacher experiences the feelings of the student and uses that experience to *guide teaching decisions*. Thus, pedagogical empathy can influence teachers' decisions about what actions to take in the classroom. I adopt this definition because it positions empathy in a teaching context as a lens for examining teaching decisions. In addition, this definition is

[1] I propose the use of the term pedagogical empathy to emphasize the theoretical implications of this concept on teaching practices. To my knowledge, the term pedagogical empathy has not been used in any published literature.

broad enough to encompass other conceptualizations of empathy, which are discussed in more detail in the literature review.

Although it arises out of social interactions, pedagogical empathy may also influence broader instructional decisions, as well as decisions that teachers make during interactions with students. For example, some of the participants in my study discussed how empathy might influence the ways in which they prepared for class. My data collection and analysis, however, focused primarily on decisions that were related to context-specific interactions between my participants and their students. Therefore, this study examines how pedagogical empathy might influence instructional decisions that involve interactions between teachers and students. Instructional or teaching decisions are defined as an outcome of teacher noticing and are discussed further in Chapter 2.

In this dissertation, I use various terms to describe the phenomenon of pedagogical empathy. The phrase "pedagogical empathy" is used as a noun and refers to the experience of perceiving someone else's feelings in a teaching context as well as how that experience might influence teaching decisions. I use the phrase "empathizing with" as a verb, to denote the action of experiencing pedagogical empathy. Finally, I use the word "empathic" as an adjective to describe situations, experiences, responses, dispositions, and other nouns that can be modified by empathy.

Problem Statement, Literature Gap, and Significance

Mathematics education researchers have studied student affect and teacher affect independently within the context of learning and teaching mathematics (e.g., Gómez-Chacón, 2000, 2017; Hen & Sharabi-Nov, 2014; Hen, Walter, & Sharabi-Nov, 2014; Op't Eynde et al., 2001; Schukajlow, Rakoczy, & Pekrun, 2017). From my searches of

mathematics education literature, however, I found few studies that focused on the intersection of student affect and teacher affect where pedagogical empathy is situated. Moreover, much of the research that focuses on affect in mathematics education is at the K-12 level. Thus, studying pedagogical empathy within the context of postsecondary mathematics education adds to the research literature and can help connect student and teacher affect.

Further exploration is needed to understand how pedagogical empathy might influence the learning and teaching of mathematics. Research on pedagogical empathy has the potential to benefit several areas of research in postsecondary mathematics education. In a review of recent literature, Biza, Giraldo, Hochmuth, Khakbaz, and Rasmussen (2016) identified four important areas of research that fall under the broad category of postsecondary mathematics teaching: mathematics teaching practices; the influence of teachers' perspectives, background and research practices on their mathematics teaching; resources and preparation for teaching and teachers' professional development; and alternative approaches to teaching. Examining the construct of pedagogical empathy in the context of learning and teaching postsecondary mathematics has potential implications for all four of these research areas. I address each of these areas below and provide examples of how this research might benefit the field of postsecondary mathematics education.

First, because teaching involves social interactions, it is reasonable to expect that empathy might influence teaching decisions. Pedagogical empathy is situated at the intersection of student and teacher affect, and so it may factor into how teachers respond to verbal and nonverbal student cues in the context of teaching practices. As Speer,

Smith, and Horvath (2010) observed, “for each student contribution, teachers must listen and interpret, decide how to react, and then make their response” (p. 109). These responses shape, in turn, how students experience the material, as well as further teaching actions.

Teachers’ perspectives, backgrounds, and beliefs about learning and teaching mathematics may shape the way they experience and express pedagogical empathy. Past research in postsecondary mathematics education has suggested that teachers’ epistemological beliefs and research backgrounds influence their teaching practices (Biza et al., 2016). Pedagogical empathy has the potential to connect these two areas of research. For example, teachers’ prior experiences as a student may affect the way they empathize with students. Viewing teacher-student interactions through a lens of pedagogical empathy may help us better understand how or why a teacher’s beliefs about teaching and learning influence teaching decisions. Further research on pedagogical empathy needs to be conducted to help us theorize possible connections between mathematics teachers’ perspectives, background, and beliefs and their teaching practices.

Research on pedagogical empathy can also inform professional development efforts and the preparation of postsecondary mathematics teachers. Graduate student instructors (GSIs) play a significant role in teaching undergraduate mathematics courses in the United States (Ellis, 2014). Many mathematics departments with GSIs provide some kind of professional development or training to prepare GSIs for their teaching responsibilities (Deshler, Hauk, & Speer, 2015). In particular, leaders of professional development can use the findings from studies about pedagogical empathy to develop activities that promote the awareness of pedagogical empathy.

Alternate approaches to teaching may also provide different opportunities for experiencing pedagogical empathy. Pedagogical empathy depends on interactions between students and teachers (Feshbach & Feshbach, 2009). Classroom models that allow more time for personal interactions might afford teachers more opportunities to encounter pedagogical empathy. Flipped and inquiry-based pedagogies (see Bergmann & Sams, 2012 and Laursen & Rasmussen, 2019) are two examples of alternative teaching approaches where more time is spent on discussions and interactions between students and teachers. Past research suggests that students in these types of classrooms are more likely to feel that their teachers care about their learning and take their questions seriously (Cronhjort, Filipsson, & Weurlander, 2018). A closer examination of how pedagogical empathy manifests itself in these classrooms can add to our understanding of teachers' experiences and their impact on students' experiences.

Study Context

For this study, I chose to explore pedagogical empathy of mathematics graduate student instructors (GSIs). GSIs are defined as students who are pursuing a graduate degree in mathematics and are the instructors of record for undergraduate mathematics courses. GSIs are an interesting population to study when examining pedagogical empathy because of their unique position as both teachers and students. It is reasonable to assume that individuals pursuing a graduate degree in mathematics have had past success with learning mathematics and enjoy studying it. This proclivity for mathematics may create potential barriers to empathizing with students who may not have similar experiences or sentiments. At the same time, GSIs are also graduate students who are often taking challenging graduate level mathematics courses. Being students in these

courses might help GSIs reflect on their roles as instructors and how to empathize with their students. In addition, GSIs are critical to the present and future of undergraduate mathematics education (Speer, Gutmann, & Murphy, 2005), and few studies have focused on GSIs' growth as instructors (Miller et al., 2018). Thus, this research examines the ways in which mathematics GSIs might empathize with their students and adds to the growing body of knowledge about GSIs.

Purpose Statement and Research Questions

The purpose of this study was to explore how GSIs conceptualize pedagogical empathy and how that empathy might influence their teaching decisions. The research questions that guided this inquiry were

RQ1. How do GSIs conceptualize pedagogical empathy with respect to how psychological researchers conceptualize empathy?

RQ2. How might GSIs' definitions of pedagogical empathy change along different dimensions of empathy?

RQ3. What factors might influence the ways in which GSIs empathize with their students?

RQ4. How might GSIs' pedagogical empathy influence their decision making in the context of interactions with their students?

To answer these research questions, I conducted a qualitative study examining how GSIs view their teaching interactions with students through the lens of pedagogical empathy. I drew on phenomenological research methods in order to develop an in-depth understanding of this concept (Creswell, 2013). My data collection involved observing and video-recording classes taught by GSIs to capture instances of GSIs interacting

individually and in small group settings with their students. I then conducted stimulated video recall interviews where I used specific interactions between GSIs and their students to guide questions about pedagogical empathy and instructors' decision-making processes.

In the first three chapters of my dissertation, I give a brief introduction to my study, provide a broad overview of the research related to pedagogical empathy, and explain the methods that I used to study this phenomenon in further detail. The next three chapters—Chapters 4, 5, and 6—are intended to stand alone as individual papers for publication. In Chapter 4: Graduate Students Instructors' Conceptualizations of Pedagogical Empathy, I focus on RQ1 and RQ2. In this chapter, I examine how GSIs' define pedagogical empathy and how their conceptualizations of empathy compare to those found in psychology and social psychology literature. In Chapter 5: Factors That Influence Graduate Student Instructors' Pedagogical Empathy, I address RQ3. To answer this research question, I explore what factors can motivate pedagogical empathy and what factors might relate to the enactment of empathy in the classroom. I also discuss the connections between these factors and teaching practice. In Chapter 6: The Influence of Pedagogical Empathy on Teaching Decisions, I consider RQ4 and describe how four instructors' conceptualizations of pedagogical empathy might have influenced different interactions with their students. Finally, in Chapter 7, I discuss the major implications from the findings of this dissertation study and describe how my research can be used to inform graduate student instructor professional development programs.

CHAPTER 2: A REVIEW OF THE LITERATURE

Overview of Literature

Pedagogical empathy involves how teachers empathize with their students and the subsequent effects that this empathy might have on teaching decisions. Therefore, this study builds on several areas of research: empathy, affect in mathematics education, teacher noticing, in-the-moment decision making, and the teaching practice of providing students with feedback. The literature on empathy offers a broad view of how the concept of empathy has evolved over time, uncovers how empathy has been defined in different contexts, and provides an organizational model for describing empathy. The research on affect in mathematics education examines how student affect and teacher affect have been studied individually within the context of teaching and learning mathematics and highlights the need to bring these two areas of research together. The literature on teacher noticing describes how teachers notice and attend to events that happen in the classroom, while the literature on in-the-moment decision making provides a theory for modeling how and why teachers make certain choices in the classroom. This literature is also connected to the research on providing feedback, which explains how constructive feedback can have a powerful effect on student learning and achievement. Finally, this study builds on the growing area of research related to graduate student instructors and their development as teachers. In what follows, I highlight results from each of these research areas and connect them to my dissertation study.

Empathy

The History of Empathy

Although the concept of empathy has been around for centuries, the term, itself, is relatively new to the English language. The term empathy derives from the German word *Einfühlung*, which was used in the late 1800s to describe how aesthetic objects (like a piece of artwork) can evoke feelings and change the perceptions of a person viewing that object (Verducci, 2000). After the British psychologist Titchener translated the word into English in 1909, psychologists began to use the term empathy to explain the phenomenon that allowed therapists to understand the mind and feelings of their clients (Verducci, 2000). This transition of empathy from being used in aesthetics to being used in psychology shifted the focus from the affective or emotional components of empathy to a more cognitive perspective of the concept. Verducci (2000) theorized that one reason behind this shift is due to the fact that psychology was "a rising new science" (p. 71) and psychologists wanted it to be seen as a valid and objective discipline, like other hard sciences.

The Definition of Empathy

Due to its varied use in diverse fields, researchers conceptualize empathy in different ways (Cuff, Brown, Taylor, & Howat, 2016). As a result, multiple definitions of empathy exist in the literature (Decety & Jackson, 2004; de Vignemont & Singer, 2006; Cuff et al., 2016). In an effort to synthesize current definitions and conceptualizations of empathy, Cuff et al. (2016) examined 43 definitions of empathy and identified eight dimensions that contribute to a coherent understanding of how empathy has been

characterized in the research literature. These eight dimensions, along with short descriptions of each dimension, are shown below in Table 1.

Table 1

Eight Dimensions of Empathy (summarized from Cuff et al., 2016)

Dimension of Empathy	Short Description
Distinguishing Empathy from Other Concepts	Merging definitions of empathy and other associated concepts (e.g., sympathy or compassion) or clearly differentiating between them
Cognitive or Affective	Defining empathy in terms of only affective components, only cognitive components, or both
Congruent or Incongruent	Noting how accurately the observer, who is empathizing with another individual, understands the emotions and perspective of the observed individual
Subject to Other Stimuli	Making an explicit assumption about whether or not an "emotional other" needs to be present in order to elicit empathy
Self/Other Distinction or Merging	Considering whether the observer is aware that their emotional experience is coming from an external source and can distinguish between their thoughts and emotions and those of the observed individual
Has a Behavioral Outcome	Taking a stance on whether empathy includes a behavioral outcome or response
Trait or State Influences	Referring to empathy as a trait/ability or suggesting that empathy is context specific and dependent upon situational or "state" factors
Automatic or Controlled	Determining whether empathy is automatically elicited or subject to control

I provide detailed summaries of each of these dimensions of empathy in Chapter 4: Graduate Student Instructors' Conceptualizations of Pedagogical Empathy. In addition, it is important to note that my definition of pedagogical empathy includes a behavioral

outcome. That is, I take the explicit stance that pedagogical empathy has behavioral outcomes because this research focuses on how empathy influences teaching decisions. This stance was informed by the analysis and findings that I discuss in Chapter 4.

An Organizational Model of Empathy

The dimensions of empathy discussed above are all important to understanding and defining empathy. Davis (1996) cautioned against adopting an exclusory definition of empathy:

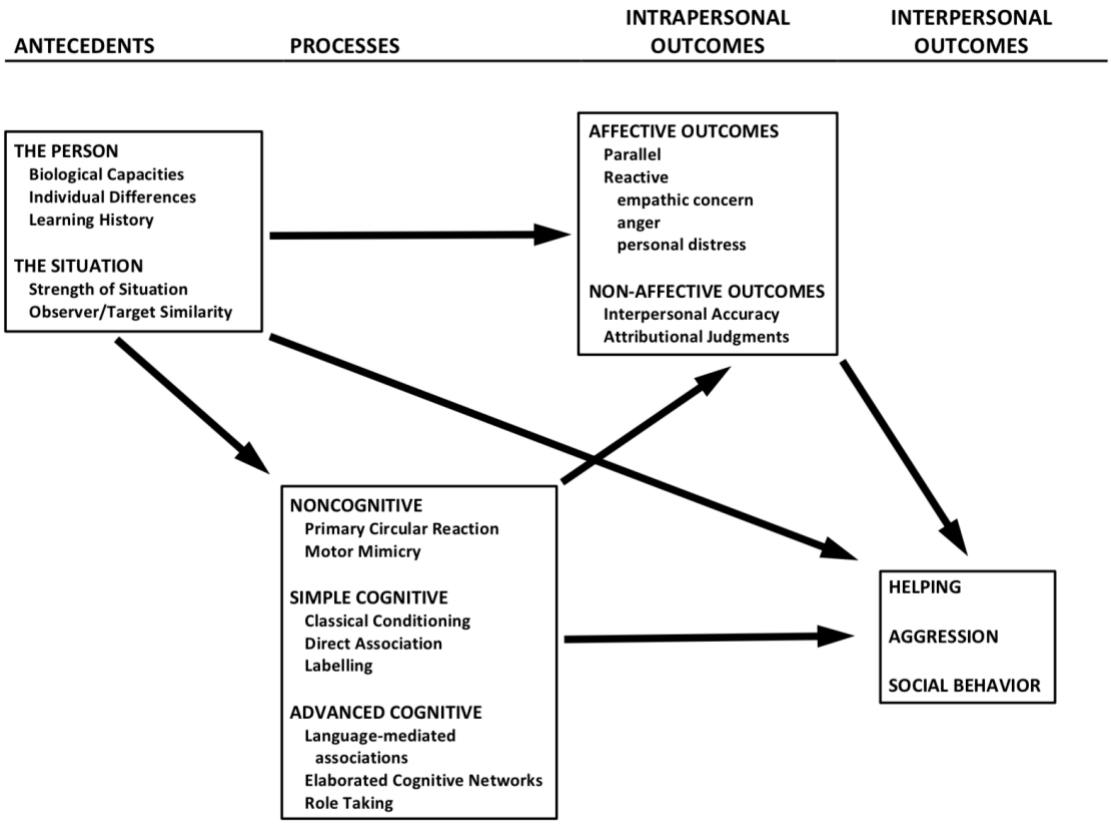
> One danger posed by the current multiplicity of empathy definitions is the possibility that when empathy is defined in a particular manner, any constructs excluded by the definition are in some sense seen as peripheral. Thus, if empathy is defined as an affective response, then cognitive role taking isn't empathy and becomes less important. If empathy is more specifically defined as experiencing similar affect, then dissimilar feelings fall outside the area of interest. The unintended result of such a series of exclusive definitions is to Balkanize the study of empathy (Davis, 1996, p. 12).

To avoid this pitfall, Davis proposed an organizational model based on an inclusive definition of empathy and suggested that the goal of this model is to emphasize the connectedness of these different constructs. In his organizational model, he defined empathy broadly as "a set of constructs having to do with the responses of one individual to the experiences of another" (p. 12) and noted that a typical empathic experience involves an observer being exposed in some way to a target, after which the observer experiences a cognitive, affective, and/or behavioral response (Davis, 1996). In his model, Davis identified four related constructs within this empathic experience:

antecedents, *processes*, *intrapersonal outcomes*, and *interpersonal outcomes*. Figure 1 shows a diagram of this organizational model connecting the four constructs.

Figure 1

An Organizational Model of Empathy (Davis, 1996, p. 13)



Antecedents. Antecedents refer to characteristics of the observer, target, or situation (Davis, 1996). This construct emphasizes that both trait and state factors may influence empathy. For example, trait characteristics of observers, including both their capacity for empathy and their previous learning histories, influence their likelihood of engaging in an empathy-related process (Davis, 1996). On the other hand, situational factors also influence how empathy is experienced. As Davis noted, "all responses to another person, whether cognitive or affective, emerge from some specific situational context" (p. 14). He also highlighted the importance of two situational factors: *strength of*

the situation and *degree of similarity*. Situations vary in terms of their power to evoke an intense empathic response in an individual (Davis, 1996). In particular, strong displays of negative emotion often lead to powerful empathic responses (Davis, 1996). The degree of similarity between the individuals involved in the empathic situation is another factor that may affect how empathy is experienced (Davis, 1996; Eklund, Andersson-Stråberg, & Hansen, 2009). In general, greater similarity between the observer and the target tends to "increase the likelihood and/or intensity of the observer's empathic response, whether affective or non-affective" (Davis, 1996, p. 15).

Processes. The second construct in the organizational model refers to the processes by which empathic outcomes are produced (Davis, 1996). These processes can require varying degrees of cognitive effort, ranging from what Davis termed as noncognitive processes to advanced cognitive process. Noncognitive processes can be thought of as processes that are automatic or unconscious in nature, which require very little cognitive effort (Davis, 1996). An example of this is the tendency of newborn infants to cry in response to hearing other infants cry, a phenomenon that occurs so early in life that it is unlikely to be a learned response (Davis, 1996). Simple cognitive processes require at least some cognitive effort on the part of the observer (Davis, 1996). These lower level cognitive processes may occur when an individual encounters a familiar situation and makes inferences about how someone else might react without taking into account actual cues that may be present (Davis, 1996). Finally, advanced cognitive processes, such as role taking or perspective taking, involve the attempt to imagine and understand another's perspective, requiring a more advanced level of cognitive sophistication (Davis, 1996). In addition to describing these cognitive

processes, Davis was careful to point out the distinction between the *processes* associated with understanding another's experience and the *outcomes* of these processes.

Intrapersonal Outcomes. Intrapersonal outcomes are the affective and cognitive responses that result from experiencing empathy (Davis, 1996). Affective responses are defined as the emotional reactions experienced by an observer and may be either parallel or reactive with what the target is feeling (Davis, 1996). In general, cognitive outcomes occur due to more advanced cognitive processes. Two such outcomes deal with interpersonal accuracy, or successfully understanding another's thoughts or feelings, and attributional judgments, which refers to inferring the reasons behind another's behavior (Davis, 1996).

Interpersonal Outcomes. The final stage of the organizational model, interpersonal outcomes, refers to the behavioral responses directed toward the target (Davis, 1996). Both cognitive and affective components of empathy are thought to contribute to the likelihood of an empathic helping response, and these intrapersonal outcomes are viewed as having the most direct impact on interpersonal outcomes (Davis, 1996). Furthermore, "the effect of empathy on behaviors that occur within *social relationships,* a topic which has only recently begun to attract consistent research interest, also falls into this category" (Davis, 1996, p. 19).

The Study of Affect in Mathematics Education

It is widely acknowledged that mathematical thinking is influenced by a variety of affective factors (Hannula, 2014). Like empathy, affect has been conceptualized in different ways. For example, in his review of the research on mathematics-related affect, McLeod (1992) used affect as a general term and provided a framework that organizes

affective research into three major areas: beliefs, attitudes, and emotions. For the purposes of this literature review, the term affect will be used as a broad concept that refers to beliefs, attitudes, emotions, and all other non-cognitive aspects of human thought (Hannula, 2012, 2014). In what follows, I provide a brief overview of the study of affect in mathematics education and show how researchers have attended to both student affect and teacher affect.

Research on affect in mathematics education started to grow in the 1960s and 1970s focusing mainly on students' mathematics anxiety and students' attitudes toward mathematics (Zan, Brown, Evans, & Hannula, 2006). Many studies on mathematics anxiety drew on social psychology research to examine the relationship between anxiety and test performance and used instruments such as the *Mathematics Anxiety Rating Scale* (MARS: Richardson & Suinn, 1972). Similarly, studies examining attitude toward mathematics were informed by the field of social psychology and were grounded in the beliefs that attitudes toward mathematics influences student learning and that attitudes are related to student achievement (Zan et al., 2006).

Later research investigated how affect is involved in mathematical problem solving. Importantly, researchers found that both expert and novice problem solvers experience a range of positive and negative emotions while solving problems and these emotions play a significant role in the mathematical problem-solving process (Hannula, 2012).

It is interesting to note that up until the early 1990s, affective research in mathematics education had largely focused on student affect. This observation is apparent in McLeod's (1992) review of the research on affect in mathematics education, where he

presented in-depth summaries of several studies on mathematics-related student affect but did not include information about teacher affect. Likewise, a later review conducted by Philipp (2007) reported that, “although McLeod addressed affect generally, he noted that teacher affect in mathematics education had been studied little, a trend that seems to have continued” (p. 262).

A more recent review on emotions and motivation in mathematics education research conducted by Schukajlow, Rakoczy, and Pekrun (2017) referenced several articles examining student emotions or affective measures (e.g., Goetz, Lüdtke, Nett, Keller, & Lipnevich, 2013; Gómez-Chacón, 2017; Hannula, 2015). However, Schukajlow and his colleagues (2017) noted that studies that have investigated relationships between teaching and affective variables in students have not focused specifically on emotions. As a result, Schukajlow et al. (2017) concluded that the findings from research on mathematics-related affect are still “too scarce to derive firm conclusions based on cumulative, consistent evidence across studies” (p. 318) and recommended that researchers “simultaneously consider parents’, teachers’, and students’ emotions and motivation” (p. 318).

Teacher Noticing

Mathematics education researchers conceptualize teacher noticing in different ways (Sherin, Russ, & Colestock, 2011). Definitions of teacher noticing can include a range of components. For example, Star and Strickland (2008) used a narrow definition of teacher noticing, focusing only on what teachers pay attention to and what they miss. Other definitions include what teachers attend to and how they interpret those experiences in their definition of teacher noticing (e.g., Colestock & Sherin, 2009; Sherin

et al., 2011; Sherin & van Es, 2009). Colestock and Sherin (2009) claimed that "as teachers view classroom practices, they attend to certain events and use what they perceive to reason and make sense of what is happening in the classroom" (p. 10). This definition of teacher noticing assumes that knowledge of students and mathematics influences the way teachers interpret experiences and that their interpretations are just as important as what they notice in the classroom (Sherin et al., 2011). Jacobs, Lamb, and Philipp (2010) provided an even broader conceptualization of noticing including not only what teachers attend to and how they interpret their experiences in the classroom, but also how teachers intend to respond.

In-The-Moment Decision Making

In-the-moment teaching decisions made in the classroom depend upon what teachers notice (Schoenfeld, 2011a). What teachers see or miss in a classroom influence what actions they decide to take. Thus, teacher noticing is inextricably connected to teachers' decision making. In his theory of goal-oriented decision making, Schoenfeld (2010) asserted that decisions are a function of an individual's resources, goals, and orientations. The decisions that teachers make can be viewed as choosing goals that are consistent with teachers' resources and orientations (Schoenfeld, 2011b). Thus, it is important to examine how these three components of decision making are interconnected, rather than selecting one to study in isolation (Schoenfeld, 2011b).

Resources

A teacher's resources include his or her knowledge as well as social and material resources (Schoenfeld, 2011b). Schoenfeld (2010) defined an individual's knowledge as "the information that he or she has potentially available to bring to bear in order to solve

problems, achieve goals, or perform other such tasks" (p. 25). Thus, a teacher's knowledge provides both affordances and constraints regarding what that teacher is able to do in the classroom (Schoenfeld, 2011b). There are various types of knowledge that teachers use while teaching (e.g., Ball, Thames, & Phelps, 2008).

In addition to knowledge, teachers also have social and material resources available to them in the classroom (Schoenfeld, 2011b). For example, teachers' personal and interpersonal skills are social resources, which shape the interactions teachers have with their students (Schoenfeld, 2011b). Material resources such as technology, curricula, time, and other physical resources also contribute to teachers' decision-making processes (Schoenfeld, 2011b). For example, a teacher with three minutes left in a class period may choose to respond differently to a student question than if there were 20 minutes left in the class period.

Goals

Goals are defined as the objectives that a teacher sets out to achieve (Schoenfeld, 2011b). Goals may be conscious or unconscious, immediate or long term, and relatively important or unimportant (Schoenfeld, 2010). They may work in combination or in tension with one another and can operate at multiple levels of varying grain sizes (Schoenfeld, 2010, 2011b). In his theory of decision making, Schoenfeld (2010) asserted that most human behavior is goal-oriented and can be modeled explicitly by a goal-oriented structure.

Orientations

A teacher's orientation refers broadly to his or her dispositions, beliefs, values, preferences, and tastes (Schoenfeld, 2010, 2011b). Over time, "teachers develop

understandings and perceptions about the nature of mathematics, about pedagogy, and about students on the basis of their experience" (Schoenfeld, 2011b, p. 460). These worldviews, attitudes, perceptions, and beliefs influence the way that teachers interpret and react to situations that occur in the classroom (Schoenfeld, 2010, 2011b).

Feedback

Feedback is a critical part of formative assessment and a core component of the learning process (Nicol, Thomson, & Breslin, 2014; Sadler, 1989, 2010). When used effectively, feedback can shape, improve, and accelerate learning behaviors thereby influencing learning and achieving outcomes (Sadler, 2010). In their review of feedback, Hattie and Timperley (2007) defined feedback as "information provided by an agent (e.g., teacher, peer, book, parent, self, experience) regarding aspects of one's performance or understanding" (p. 81). For this study, I adopt this definition and focus on verbal exchanges between teachers and students where teachers provide information to students about aspects of their performance or understanding.

Feedback and instruction are often interrelated. Hattie and Timperley (2007) argued that the relationship between feedback and instruction can be considered along a continuum. At one end of the continuum, there is a clear distinction between providing feedback and providing instruction, while at the other end, feedback and instruction become "intertwined" to take on a new instructional purpose (Hattie & Timperley, 2007, p. 82). To use feedback for this instructional purpose, teachers must provide specific information about the gap between what is understood and what needs to be understood (Hattie & Timperley, 2007; Sadler, 1989). Students can then use this information to reflect on the strengths and weaknesses of their performance so that successful aspects of

their performance can be reinforced, and unsatisfactory aspects can be improved (Sadler, 1989).

Sadler (2010) claimed that there are two main functions of feedback: to provide a statement of performance through a teacher's assessment of a student's response and to provide advice or suggestions as to how to improve that response. In order to support formative learning, feedback should be both specific to the task a student is working on and generalizable to other related principles (Sadler, 2010). Feedback should also be constructive and should include information about: the strength of student work, the nature of the deficiencies in student work and where they occurred, what would improve the student work, and what can be done differently next time the student is faced with a similar problem (Sadler, 2010).

Graduate Student Instructors

In recent years, researchers in undergraduate mathematics education have begun to study how mathematics GSIs develop as teachers. GSIs are responsible for teaching their own courses at many large universities across the United States (Belnap & Allred, 2009). In addition, many of these GSIs who graduate with their PhD in mathematics will become future faculty members (Deshler, Hauk, & Speer, 2015). Therefore, the quality of GSIs' teaching has the potential to impact the learning of many undergraduate students, both now and in the future. Much of the research on GSIs has focused on GSI professional development programs and their role in supporting GSIs' teaching practices. Deshler and her colleagues (2015) discussed three common types of GSI preparation programs, ranging from programs that provide their GSIs with little support to programs grounded in best practices and informed by research that provide their GSIs with

extensive support. While studying these professional development programs is useful, research focusing explicitly on GSIs and their teaching practices is lacking. As Miller et al. (2018) reported in their literature review, "graduate student instructors' growth as teachers is a largely unexamined practice" (p. 186). Thus, this study sought to gain a better understanding of how GSIs empathize with their students in order to add to the knowledge base of GSIs' teaching practices and to help inform leaders of professional development programs.

Implications for Dissertation

Pedagogical Empathy

With this literature in mind, I now connect these ideas to my dissertation study. As stated before, the term *pedagogical empathy* refers to the empathy that a teacher experiences as a result of interacting with a student or a group of students, which ultimately influences teaching decisions. This phenomenon provides a lens through which to view interactions between teachers and students and helped me unpack my participants' perceptions of their decision-making. In my analysis, I used the eight dimensions identified by Cuff et at. (2016) to help characterize how GSIs conceptualize empathy and address RQ1 and RQ2. I also related my findings that address RQ3 to the organizational model presented by Davis (1996).

Connections Between Teacher Noticing and Pedagogical Empathy

Empathic experiences must involve both an observer and a target (Davis, 1996). For this study, I am considering a teacher to be the observer and a student (or group of students) to be the target. Given my definition of pedagogical empathy, a teacher must be able to take the perspective of a student in order to experience this phenomenon. Taking

the perspective of a student requires a teacher to observe and notice something about that student, thus connecting pedagogical empathy with teacher noticing.

For this study, I adopt the conceptualization of teacher noticing provided by Jacobs et al. (2010) and define noticing as a threefold process where a teacher attends to an event, interprets that event, and then decides how to respond. Thus, teachers' decisions are an outcome of teacher noticing. Applying this definition to pedagogical empathy, a teacher must first pay attention to a student's actions in order to become aware of what that student might be experiencing. The teacher can then interpret the student's actions by reflecting on prior experiences and trying to understand the student's perspective. Lastly, the teacher must decide how to respond to that student, which is a function of the teacher's resources goals, and orientations. It is also important to note that responding may or may not involve an action by the teacher. A teacher may consciously choose not to take action after attending to and interpreting a student's perspective. This non-action can still be considered as a response.

In this study, I consider both actions and non-actions as responses to students. As an outside observer, it was difficult to tell when one of my participants decided not to take a certain action. However, during many post-observation interviews, my participants discussed both what they decided to do and what they chose not to do during the teaching interactions that we analyzed. Therefore, I was able to capture some of this data, which is included in my findings.

Connections Between Decision Making and Pedagogical Empathy

Decision making is an important part of teacher noticing. A teacher's interpretations about what he or she has noticed may influence how that teacher decides

to respond. In addition, teachers draw on their resources, goals, and orientations when making teaching decisions (Schoenfeld, 2010). These components are also likely to affect what teachers attend to originally and how they interpret what they observe. Thus, there is a dynamic relationship between teacher noticing and decision making.

Schoenfeld (2011b) argues that teaching decisions are motivated by goals and that these goals are usually consistent with a teacher's resources and orientations. Pedagogical empathy can be thought of both in terms of resources and in terms of orientations. Empathic experiences may depend on a teacher's knowledge, prior experiences, and his or her ability to take the perspective of a student. Thus, pedagogical empathy can be thought of as a type of resource that can be used and called upon while teachers are making decisions. Empathic experiences may also depend on a teacher's orientations, including what they value and believe about teaching and learning mathematics. For example, a teacher might believe that it is important to create a safe environment for students to ask questions because of prior experiences that they have had as a student. Therefore, pedagogical empathy is also connected to teachers' orientations.

For this study, I aim to better understand the connections between pedagogical empathy, teachers' resources, and teachers' orientations and how pedagogical empathy might manifest itself in the goals that motivate teachers' decision making. To this end, my fourth research question focuses on how GSIs perceive pedagogical empathy as influencing their decision making.

Connections Between Feedback and Pedagogical Empathy

Many interactions between teachers and students involve some kind of feedback, and teachers must make several decisions during these interactions. For example, when

providing feedback to a student, a teacher must decide when, why, and how best to respond to that student. Answering the "why" behind why a teacher provides feedback to a student relates back to the goals that a teacher has in mind when he or she responds to that student, and these goals may be influenced by pedagogical empathy. Pedagogical empathy may also influence when and how teachers decide to give feedback to their students. Teachers should aim to provide constructive and supportive feedback (Sadler, 2010). In addition, "teachers need to view feedback from the perspective of the individuals engaged in the learning" (Hattie & Timperley, 2007, p. 101). Taking on a student's perspective may help a teacher decide when to interact with a student and provide them with formative feedback during class. Likewise, empathy might influence how a teacher delivers feedback to a student if he or she is able to understand what the student might be feeling or thinking.

CHAPTER 3: METHODS

Research Design

Qualitative research is a type of research that focuses on uncovering "how people interpret their experiences, how they construct their worlds, and what meaning they attribute to their experiences" (Merriam & Tisdell, 2016, p. 6). The goal of qualitative research is to develop a complex, in-depth understanding of an issue, and it begins with assumptions and the use of theoretical frameworks, which inform the design of the study (Creswell, 2013). For this research study, I explored how GSIs experience pedagogical empathy, including how they interpreted and constructed their experiences. I used a phenomenological design (Creswell, 2013) to examine GSIs' conceptualizations of pedagogical empathy and how reflecting on pedagogical empathy might influence GSIs' decision making. Phenomenological designs are used to explore the common shared experiences of individuals with respect to a central concept or phenomenon (Creswell, 2013). Since I was interested in studying pedagogical empathy, phenomenological methods were an appropriate choice to address my research questions. Here, I view pedagogical empathy as a reoccurring phenomenon amongst GSIs that can be explored by examining their individual teaching experiences and interactions with students. In addition, "a phenomenological approach is well suited to studying affective, emotional, and often intense human experiences" (Merriam & Tisdell, 2016, p. 28). This description relates directly to the affective or emotional part of experiencing pedagogical empathy.

Researcher Positioning

As a mathematics GSI myself, I am aware of the unique perspective that GSIs have with respect to teaching and learning. GSIs are both students in graduate classes and instructors in undergraduate classes. In my opinion, these simultaneous responsibilities help me as a teacher to better relate to my students. One way I do this is by reflecting and drawing on my own experiences and emotions as a learner of mathematics and then using these reflections to empathize with my students and gain a better understanding of what kind of support they need in order to learn mathematics.

The ideas that sparked my interest about this topic of study grew out of my interactions with professors and GSIs who seemed to have various degrees of connection with their students. I began to think about the concept of pedagogical empathy and was curious if it is something that instructors can develop over time. This curiosity led to the development of this study in which I wanted to design a way to observe and characterize what pedagogical empathy might look like for GSIs.

In order to remain as objective as possible, it is recommended that researchers conducting phenomenological studies bracket their assumptions (Creswell, 2013). Since I was familiar with the participants in my study, I needed to set aside personal opinions of them in order to focus on their experiences and responses. One assumption that I had at the start of this study is that some GSIs will display more pedagogical empathy than others during their interviews and observations based on my prior interactions with them. However, in order to be an objective observer, I worked to intentionally shelve my preconceived notions of what qualities an individual might display and concentrated my attention on remaining unbiased towards my participants. One strategy I used was to

critically reflect on my own roles as a graduate student instructor and my potential biases during the formulation of my research design as well as during data collection and analysis. For example, my classroom observation protocol included questions to help me reflect on my relationship with the participant and my thoughts, feelings, and experiences prior to the observation (see Appendix A). I also regularly discussed my analysis of the data, which was deidentified, with an outside researcher in order to verify whether my interpretation of the data was valid.

Data Collection

Context

The participants in my study were all graduate student instructors in the Department of Mathematics at the University of Nebraska–Lincoln (UNL) who were fully supported by graduate teaching assistantships. At UNL, most mathematics GSIs are responsible for leading 75-minute Calculus I or Calculus II recitations during their first year of graduate school. In their second year, GSIs are usually given the opportunity to teach either Intermediate Algebra or College Algebra and take on the sole responsibility as the instructor of record for these courses. Intermediate Algebra and College Algebra are both 3-credit course that meet three times a week for 50 minutes and 75 minutes, respectively. After the first two years, there are a range of different courses that GSIs are generally assigned to teach including Precalculus (5 credits), Trigonometry (2 credits), Contemporary Mathematics (3 credits), and Mathematics Matters (3 credits)[2]. GSIs are

[2] Contemporary Mathematics is a terminal course for students who are not majoring in mathematics, i.e., a "liberal-arts" mathematics course. Mathematics Matters is a course for pre-service elementary school teachers that helps students develop a deep understanding of numbers and operations.

able to request what they want to teach each semester, and their preferences are taken into consideration when teaching assignments are selected.

Faculty in the mathematics department at UNL have set up several supports to help graduate students develop as teachers during the first few years of graduate school. For many GSIs, graduate school is the first time they will experience being instructors of a mathematics course, so these supports are essential to help them become more confident and comfortable in the classroom.

One form of support is a week-long orientation aimed at helping incoming graduate students prepare for their first semester of graduate school. During this orientation, new GSIs are introduced to active learning methods (e.g., facilitating group work, eliciting student thinking, leading class discussions) and the set of learning activities that are used in calculus recitations. They also have opportunities to ask questions and participate in a microteaching seminar. During this microteaching seminar, GSIs present a challenging mathematics problem to their peers and then facilitate group work on that problem to simulate the experience of leading a recitation section. In addition to the orientation for incoming graduate students, a separate orientation is held for returning second year GSIs who are about to begin teaching their own precalculus courses[3]. This orientation involves three days of workshops that are designed to help transition GSIs from being recitation leaders to instructors of record. At this orientation, GSIs participate in workshops about working with and motivating students, lesson planning and developing instructional materials, teaching practices such as leading productive discussions and managing group work, and equity. GSIs are required to attend

[3] In this mathematics department, "precalculus courses" refer to courses leading up to the Calculus sequence, i.e. Intermediate Algebra, College Algebra, Trigonometry, and Precalculus.

these orientations if they are first- or second-year graduate students, or if they are involved in teaching any of the precalculus courses.

In addition to orientation, second year GSIs also have the benefit of taking a pedagogy course during the school year to learn about theories of learning and teaching mathematics, another layer of support provided by the mathematics department. This course meets twice a week in the fall and once a week in the spring for a total of three credits, and the department requires all second year GSIs take this course. As an incentive, GSIs are given a three-credit course reduction in their teaching load during their second year. Assignments for this pedagogy course include reading and discussing mathematics education research articles, writing essays and reflecting on teaching practices, observing mathematics classes taught by other instructors in the department, and learning about various learning theories.

Another important type of support comes in the form of curriculum materials. Several course materials for precalculus classes and calculus recitations are provided for GSIs to use as part of their teaching responsibilities. These course materials include suggested lesson guides and course packets with group learning activities that students taking precalculus and calculus courses at UNL are required to buy. GSIs use these course packets during class to facilitate group work and help students apply and learn about mathematics. GSIs are encouraged to spend time working with small groups during class on problems in the course packets to help engage their students in the course material. Teaching in this kind of active learning environment provides the opportunity for a GSI to work individually with small groups of students and potentially form deeper interpersonal relationships with them.

Participants

For this study, I focused on precalculus courses to explore how pedagogical empathy might be expressed in the classroom. The participants in this study were 11 GSIs[4] who were instructors of record for one of four precalculus courses at UNL: Intermediate Algebra, College Algebra, Trigonometry, and Precalculus. They were selected by their willingness and availability to participate in this study. All of the GSIs had at least two semesters of teaching experience (either as a recitation instructor or as an instructor of record). The most experienced GSI in my study had a total of 3 full years teaching as the instructor of record. More in-depth descriptions of these GSIs are given in Chapters 4, 5, and 6. As a mathematics GSI myself, I had access to this population and recruited volunteers by sending out individual emails to graduate students teaching precalculus courses.

I chose to study GSIs because they are not only instructors of undergraduate mathematics courses, but also students in graduate mathematics courses. Having these simultaneous roles may make it more likely for GSIs to empathize with their students as they may have recently had similar or shared experiences as students themselves, an important factor that has been shown to be associated with empathy (Davis, 1996; Eklund, Andersson-Stråberg, & Hansen, 2009). In addition, GSIs are an important population to study, as many of them continue teaching at institutions of higher education after they graduate. The insights gained from this study can be used to inform GSI

[4] For the overall qualitative study, a total of 11 GSIs were recruited. One of the GSIs, Beth, participated during summer 2018, rather than fall 2018. Consequently, this participant had different final interview questions. Data from her interviews are not included in the analysis from Chapter 4, which draws on questions that were added to the final interview after Beth had participated in this study. However, data from Beth's interviews are included in both Chapters 5 and 6.

professional development efforts and design activities to help instructors recognize similarities between themselves and their students.

I chose to study GSIs who were teaching precalculus courses because these courses are structured so that GSIs have opportunities to interact with individuals and small groups during a typical class period. Thus, I was able to observe several instances of GSIs interacting with their students during my classroom observations. Without these interactions, I would have been unable to observe instances when pedagogical empathy might be influencing a GSI's decisions about how to respond to their students. Therefore, it was important for me to observe classrooms where there are interactions between GSIs and their students.

To reach saturation, I recruited 11 GSIs. Saturation occurs when no new insights would be gained from further participant sampling (Merriam & Tisdell, 2016). Morse (2000) provides advice for approximating the number of participants needed for saturation:

> Estimating the number of participants in a study required to reach saturation depends on a number of factors, including the quality of data, the scope of the study, the nature of the topic, the amount of useful information obtained from each participant, the number of interviews per participant, the use of shadowed data, and the qualitative method and study design used (Morse, 2000, p. 3).

For this phenomenology, I conducted multiple observations and interviews with each participant, focusing on pedagogical empathy, which is a very specific topic of interest. I also began analyzing this data while I was still collecting it (see Data Analysis section for further details). Thus, I was able to determine that I had reached a point of saturation with

the data from my 11 participants. This number is also consistent with recommendations made by Creswell (2013) and Morse (2000) for typical phenomenological studies.

Initial Interviews

I collected data in three stages. The first stage consisted of initial interviews with participants to begin exploring their conceptualizations of pedagogical empathy. The initial interview was a semi-structured, one-on-one interview that was audio-recorded for later analysis. During the first part of the interview, I asked participants to write down a definition of empathy and then had them illustrate that definition through a description of a teaching experience. Since empathy is a complicated phenomenon, the goal of this interview was to gain insights into how GSIs conceptualize pedagogical empathy. This interview also primed participants to start thinking about how empathy might be influencing their teaching decisions and raise their awareness of certain classroom interactions that may be linked to pedagogical empathy. Participants' initial definitions of empathy also provided a reference to look back on during subsequent classroom observations and interviews. A copy of my initial interview protocol can be found in Appendix A and a copy of the Definition of Empathy Activity that I had each participant fill out at the start of the initial interview is included in Appendix A.

Classroom Observations

The second stage of my data collection involved classroom observations to view in-the-moment interactions between participants and their students. The purpose of these observations was to capture instances of GSIs interacting with their students in a typical classroom setting. I observed each GSI three times over the course of the semester, with approximately three to four weeks in between each observation. During classroom

observations, I video-recorded the entire class period, focusing specifically on the GSIs and their interactions with their students. GSIs wore a microphone to pick up sound throughout the classroom. I also took detailed observation notes, specifically focusing on any teaching interactions or events that might have been notable. Immediately following each classroom observation, after all students had left the room, I would ask my participants if any interactions with their students stood out to them. A copy of the observation protocol that I used can be found in Appendix A.

Post-Observation Analysis

After each classroom observation, I would sit down and watch the video that I had recorded of the class. Using my observation notes, I then selected two or three important interactions that I observed which involved a meaningful exchange of ideas and feedback between the GSI and a student or group of students. I also made sure to select any interactions that GSIs had pointed out to me immediately after the classroom observation as being noteworthy. When GSIs did not point out specific interactions with students, I chose interactions with extended dialogue that consisted of more than one verbal response from the GSI and a back and forth exchange between the GSI and a student. These extended interactions gave GSIs more time to notice and attend to student thinking and were thus more likely to involve teaching decisions that were influenced by empathy.

After selecting these interactions from the observation video, I formulated specific questions to ask my participants in the post-observation interviews. In addition to these unique questions, I also asked my participants the same set of standard questions about every interaction we discussed. Thus, the interactions between the GSIs and their

students that I observed in the classroom informed the third and final stage of my data collection, the post-observation interviews.

Post-Observation Interviews

After each classroom observation, I conducted a post-observation interview. Typically, these interviews lasted between 45 minutes to an hour. During this interview, I asked participants about specific interactions between them and their students that I observed using stimulated-recall interview techniques. The post-observation interviews were typically scheduled in advance and were conducted as soon as possible after the observation occurred to help with memory and recall. I began each post-observation interview by returning to the participant's original written definition of empathy and asking if there were any modifications that they wanted to make to their definition. After reviewing their definition, we watched video clips of the two or three interactions that either I or the participant identified as important from their observation. I also gave my participants a transcript of the video-recorded interactions so that they could reference specific parts of the interaction more easily. After each video, I asked specific questions to help GSIs' reflecting on their goals, knowledge, and beliefs behind how they were responding to their student and how empathy may or may not have played a role in their teaching decisions. An example of a post-observation interview protocol can be found in Appendix A.

The final post-observation interview included additional questions to target GSIs' conceptualizations of empathy and how they might have changed over the course of the study. The first part of the final interview began in the same way as the other post-observation interviews: by returning to GSIs' initial definition of empathy and asking if

they wanted to make any changes. I then gave participants a list of approximately five other definitions of empathy constructed by other GSIs and asked them to reflect on various parts of these definitions. After looking through these definitions, I asked specific questions about the eight dimensions of empathy identified by Cuff et al. (2016) in order to address RQ1 and RQ2. These additional questions can be found in Appendix A.

Data Analysis

As recommended by Miles, Huberman, and Saldaña (2014), I began analyzing data at the same time as collecting data to help process the existing data and generate new strategies and methods for future data collection. During the first step of analysis, I prepared and organized data electronically by converting observation and interview notes into electronic word documents. In addition, I transcribed interviews using the qualitative research software, MAXQDA 2020 (VERBI Software, 2019), in order to prepare for future data analysis. I also watched and analyzed video recordings of classroom observations before post-observation interviews. While organizing the data, I recorded my initial impressions of the data and reflected on how responses might relate to my research questions. During this stage of data analysis, I also engaged in a two-cycle coding process, as described by Miles et al. (2014).

First-Cycle Coding

I began my data analysis by creating codes using a combination of elemental and affective coding methods. Three elemental methods serve as a foundation for first cycle coding: descriptive coding, in vivo coding, and process coding (Miles, Huberman, & Saldaña, 2014). Descriptive coding involves assigning words or short phrases to data to summarize the basic meaning of a passage of qualitative text (Miles et al., 2014). In vivo

coding is similar to descriptive coding but uses participants' words or phrases as codes to prioritize and capture their perspective (Miles et al, 2014). The third elemental coding method, process coding, uses gerunds as codes and is helpful to denote participant actions, interactions, and consequences that emerge from the data (Miles et al., 2014). During this part of the data analysis process, I used these elemental coding methods to create an initial set of conceptual codes from my data.

In addition to using elemental methods, Miles et al. (2014) recommended using affective coding methods, which I also used to analyze my data. The two affective coding methods that I focused on were emotion coding and values coding. Emotion coding labels the emotions recalled or experienced by the participant (Miles et al., 2014). Furthermore, "emotion coding is particularly appropriate for studies that explore intrapersonal and interpersonal participant experiences and actions" and can provide insight into a participant's perspective or worldview (p. 75). Value coding organizes codes in terms of a participant's values, attitudes, and beliefs, where values refer to the importance one attributes to a person, thing, or idea, attitudes are the way we think and feel about those things, and beliefs include personal knowledge, experiences, opinions, morals, and other interpretive perceptions of the social world (Miles et al., 2014). I used both emotion coding and values coding to highlight areas of the data where my participants expressed feelings related to empathy and/or described empathy as an important part of teaching.

Throughout the initial process of coding, I used inductive methods to develop codes that were grounded in the data. As I collected and analyzed more data, I revised and refined my existing codes to develop themes related to my research questions. This process of data analysis, in conjunction with data collection, allowed me to determine

when no new insights were emerging from my data and I had reached saturation. As I analyzed my data, I developed working definitions of codes as my coding progressed so that I was able to apply codes consistently across the data. After making revisions to codes, I revisited previously coded data in order to ensure that it was coded appropriately. In addition, I met with another researcher throughout this process to discuss my codes and confirm my findings. During these meetings, I would show this researcher my deidentified data and he would tag sections of it using my revised codes. We would also discuss questions such as "Does this code apply to other individuals in my study?" and "Is this interpretation true to the data" in order to evaluate the research (Pyett, 2003).

Second-Cycle Coding

The first cycle of coding helped me categorize the data into meaningful pieces, which I then further analyzed in a second cycle of coding. During the second cycle of coding, I used pattern coding to further analyze my data. This process involved grouping together first-cycle codes into smaller interrelated groups of codes and going back through the data to see whether any further patterns emerged. Thus, during this second cycle of coding, I grouped together my codes by different categories, themes, causes, and explanations that I saw repeated in the data across multiple cases. After grouping the data in this way, I composed a narrative description of each pattern code by drawing connections between first cycle codes and using direct supporting evidence from participants' interviews. Throughout this process, I organized my analysis into a coherent set of explanations by generating a set of assertions and propositions, which reflect the findings and conclusions of my dissertation.

Connections to Theoretical Framing

Throughout the coding process, I referred back to the literature on empathy, teacher noticing, and decision making to inform my first and second cycle of coding. To help gain an understanding of how GSIs might conceptualize empathy, I also used the eight dimensions described by Cuff et al. (2016) to code my participants' definitions of empathy and their initial and final interviews. In addition, I coded my post-observation interviews to unpack what my participants noticed about specific interactions with their students. In particular, I categorized what they attended to in these interactions, how they interpreted what they attended to, and how they decided to respond to their students. I also used Schoenfeld's (2010) decision making framework to identify resources, goals, and orientations that GSIs mentioned or discussed in our post-observation interviews.

CHAPTER 4: GRADUATE STUDENT INSTRUCTORS' CONCEPTUALIZATIONS OF PEDAGOGICAL EMPATHY

Empathy is a complicated construct to study. Although it has been examined in many fields and contexts, "empathy is not a well defined notion" (Cuff, Brown, Taylor, & Howat, 2016, p. 144), and numerous definitions of empathy exist in the literature (Decety & Jackson, 2004; de Vignemont & Singer, 2006). For example, Decety and Jackson (2004) define empathy as "a sense of similarity between the feelings one experiences and those expressed by others" (p. 71), while Hein and Singer (2008) define it as "an affective state, caused by sharing of the emotions or sensory states of another person (p. 7).

When it comes to empathy in a classroom setting, yet more definitions abound. In their analysis of the empathic dispositions of pre-service teachers, Tettegah and Anderson (2007) define "teacher empathy" as "the ability to express concern and take the perspective of a student" (p. 50). Cooper (2004) provides another definition of empathy as it relates to teacher student interactions:

> Empathy is a quality shown by individuals which enables them to accept others for who they are, to feel and perceive situations from their perspective and to take a constructive and long-term attitude towards the advancement of their situation by searching for solutions to meet their needs (p. 14).

While these definitions provide a foundation for understanding the phenomenon of empathy, they may or may not capture the perspective of individual teachers and their conceptualizations of empathy.

Having an understanding of how teachers view empathy, and its role in interacting with students, may be a fundamental component in the study of teachers' decision making. In the context of teaching mathematics, students' emotions and other affective processes are an integral part of their learning (Op't Eynde, De Corte, & Verschaffel, 2006). While a significant amount of general education research has focused on creating an optimal emotional learning environment (Hannula, 2014), few research studies have focused on how teachers empathize with students and the effect that empathy can have in the classroom. However, as Feshbach and Feshbach (2009) suggest, when teachers communicate empathy to their students, their students are more likely to experience greater understanding and acceptance, thus developing a more positive attitude towards themselves and schooling over time. Therefore, empathy can make a huge difference in a mathematics classroom, especially for students who might have a negative attitude towards mathematics.

This chapter is part of a larger study that seeks to understand how graduate student instructors (GSIs) experience empathy in the context of teaching lower division mathematics courses. It is important to study empathy in this context because low retention rates and negative attitudes towards mathematics are far too common in lower division mathematics courses (Gordon, 2008; PCAST 2012). Furthermore, many GSIs are future mathematics faculty members who will continue teaching at institutions of higher education after they graduate. Finally, GSIs are an important and unique

population to study because they are not only teachers of undergraduate mathematics courses, but also students themselves in graduate mathematics courses. Having these simultaneous roles may make it more likely for GSIs to empathize with their students as they may have recently had similar or shared experiences as students themselves, an important factor that has been shown to be associated with empathy (Eklund, Andersson-Stråberg, & Hansen, 2009).

For this study, I sought to understand my participants' notions of empathy as they relate to teaching and compare them to the dimensions in Cuff et al.'s (2016) framework that contributed to a coherent understanding of empathy. Throughout this chapter, I use the term *pedagogical empathy* to refer to the empathy that a teacher experiences through an interaction with a student or group of students. I argue that GSIs' conceptualizations of empathy can change over time and that what distinguishes pedagogical empathy from other types of empathy is the resulting influence of pedagogical empathy on behavioral outcomes, i.e., teaching decisions.

Conceptual Framework

Dimensions of Empathy

In an effort to synthesize definitions of empathy and reduce confusion in the field of psychology, Cuff et al. (2016) analyzed 43 definitions of empathy found in research literature. The authors used a snowballing procedure to identify formal and informal definitions of empathy in key articles that spanned a range of different viewpoints. They examined these definitions for similarities and differences and identified eight dimensions that they believed to be crucial to understanding empathy. In this way, they sought to formulate "a new conceptual summary of empathy that can be used by future

researchers/practitioners" (Cuff et al., 2016, p. 145). The eight dimensions of empathy identified by Cuff et al. (2016) are summarized below in Table 2, which is followed by a more in-depth description of each dimension.

Table 2

Eight Dimensions of Empathy (summarized from Cuff et al., 2016)

Dimension of Empathy	**Short Description**
Distinguishing Empathy from Other Concepts	Merging definitions of empathy and other associated concepts (e.g., sympathy or compassion) or clearly differentiating between them
Cognitive or Affective	Defining empathy in terms of only affective components, only cognitive components, or both
Congruent or Incongruent	Noting how accurately the observer, who is empathizing with another individual, understands the emotions and perspective of the observed individual
Subject to Other Stimuli	Making an explicit assumption about whether or not an "emotional other" needs to be present in order to elicit empathy
Self/Other Distinction or Merging	Considering whether the observer is aware that their emotional experience is coming from an external source and can distinguish between their thoughts and emotions and those of the observed individual
Has a Behavioral Outcome	Taking a stance on whether empathy includes a behavioral outcome or response
Trait or State Influences	Referring to empathy as a trait/ability or suggesting that empathy is context specific and dependent upon situational or "state" factors
Automatic or Controlled	Determining whether empathy is automatically elicited or subject to control

Distinguishing Empathy from Other Concepts

Empathy is often associated with other related concepts such as sympathy and compassion (Cuff et al., 2016). In their analysis, Cuff and his colleagues found that several definitions of empathy appear to merge empathy with related concepts, while other definitions make a clear distinction between these concepts. In order to avoid confusion, Cuff et al. argue that it is important to understand how these concepts are related and highlight the differences between them. Wispe (1986) defines sympathy as "the heightened awareness of the suffering of another person as something to be alleviated" (p. 318). She differentiates this from empathy by describing empathy as an attempt "to understand the subjective experiences of another self" (p. 314). Furthermore, Wispe (1986) emphasizes the difference between the types of emotions felt while experiencing sympathy and empathy, stating that, "sympathy is defined in terms of negative emotions" (p. 318) while empathy can refer to understanding both the positive and negative experiences of another person.

Compassion is also another concept that is often associated with both empathy and sympathy. In an attempt to characterize and define compassion, Goetz, Keltner, and Simon-Thomas (2010) identified and assessed three arguments about how to conceptualize compassion by conducting a review of empirical research. From their analysis, Goetz et al. (2010) found that compassion is a distinct emotion that arises in response to feelings of distress and is associated with feelings of concern. Empathy is involved in the elicitation and experience of compassion, but differs in that empathy, itself, is not an emotion (Goetz et al., 2010). Moreover, compassion shares commonalities with both positive emotions like love and negative emotions like sadness, distinguishing

it from sympathy, which is associated with only negative emotions such as sadness and fear (Goetz et al., 2010).

Cognitive or Affective

Many definitions of empathy include both cognitive and affective components (Cuff et al., 2016). The cognitive component of empathy involves understanding another person's experience while the affective component involves sharing the emotional experience of another person (Decety & Jackson, 2004; Baron-Cohen & Wheelwright, 2004). Studies focusing on personality and developmental disorders have shown that cognitive empathy and affective empathy are two different constructs (Cuff et al., 2016). For example, Baron-Cohen & Wheelwright found that individuals with high-functioning autism had difficulty judging, explaining, anticipating, and interpreting another's behavior, but were able to respond emotionally when someone else pointed out how the other person might have felt, thus expressing affective, but not cognitive empathy. In addition, neurological studies have shown that cognitive empathy and affective empathy are associated with different regions of brain functioning (e.g., Shamay-Tsoory, Aharon-Peretz, & Perry, 2009). Nevertheless, researchers maintain that it is essential to include both cognitive and affective components in the definition of empathy, because in many instances it is difficult to separate cognitive empathy from affective empathy (Baron-Cohen & Wheelwright, 2004).

Congruent or Incongruent

Congruency refers to accurately perceiving the emotions of an individual (Cuff et al., 2016). Some researchers explicitly state that empathy must involve "sharing" or "experiencing" another's emotions, while others suggest that congruency may occur but

is not a necessary component of empathy (Cuff et al., 2016). Much work has been done in the area of empathic accuracy defined as "the degree to which a perceiver is able to accurately infer the specific content of another person's successive thoughts and feelings" (Ickes, 2009, p. 57). Batson (2009) argues that accurately perceiving what another person is experiencing is important when trying to address their needs. However, the degree of congruency depends upon several factors such as past experiences, individual differences in expressing emotions, and situational interpretations, suggesting that empathic congruency may be difficult to achieve (Cuff et al., 2016).

Subject to Other Stimuli

Empathy involves an observer perceiving and interpreting the thoughts and emotions of another person. Many conceptualizations of empathy assume that the "emotional other" is present during this experience (Cuff et al., 2016, p. 148). However, some researchers contend that another person does not need to be present to elicit an empathic response (Cuff et al., 2016). In these situations, inferring or imagining what another might be experiencing may be enough to motivate an empathic response. For example, people may experience empathy when reading about an emotional experience in a book even though there is no other person physically present (Decety & Jackson, 2004). Following this line of thinking, Cuff et al. argue that there is little difference between empathizing with a real, fictional, or imagined other, since the main focus of empathy is on the experience of the observer rather than the target.

Self/Other Distinction or Merging

Some conceptualizations of empathy include a distinct self/other separation in that the observer maintains a clear awareness that empathy is being elicited by an external

source (Cuff et al., 2016). The reason why this distinction is important to clarify is that it separates empathy from related concepts such as emotional contagion (de Vignemont & Singer, 2006). With emotional contagion, an observer unconsciously shares the emotions of another person, and is not aware that the other person is the source of these feelings (de Vignemont & Singer, 2006). While recognizing the origin of an emotional response is important to the definition of empathy, some self/other merging may also be necessary to provide a connection between the self and the other (Cuff et al., 2016). Without self/other merging it would be difficult for an observer to understand another person's perspective and experience cognitive empathy (Cuff et al., 2016).

Trait or State Influences

Empathy can be viewed as both a trait and a state. Many researchers describe empathy as an "ability" or "capacity" which refers to empathy as a trait (Cuff et al., 2016). For example, Lamm, Batson, and Decety (2007) define empathy as "the capacity to understand and respond to the unique affective experiences of another person" (p. 42). This stance suggests that empathy is a trait, which is stable across time, and that some individuals are more empathic than others (Cuff et al., 2016). While many researchers acknowledge that empathy is an individual trait, some suggest that situational factors may also play a role in experiencing empathy (Cuff et al., 2016). This interpretation views empathy as a state that is activated by certain situations or contexts (Cuff et al, 2016).

Has a Behavioral Outcome

Another issue that arises when trying to define empathy is whether or not empathy has a behavioral outcome. While research has shown that empathy is often associated with a behavioral response, many researchers argue that the two should be

treated separately (Cuff et al., 2016). Others include behavioral responses in their definition of empathy and consider it to be an integral part of the empathic process (e.g., Barker, 2008; Davis, 1996). With respect to this study, I aim to explore how empathy influences teaching decisions and thus include behavioral outcomes and responses as part of my conceptualization of pedagogical empathy. However, because my research aims to understand how GSIs perceive the influence of empathy on feedback, behavioral outcomes are not the primary object of study.

Automatic or Controlled

A final point of discussion is whether empathy may be automatically elicited or subject to control. Hodges and Wegner (1997) argue that empathy can be separated into processes that are automatic and "simply happen to people" (p. 311) and processes that people can consciously and intentionally control. Studies in the field of neuroscience have shown that observing or imagining another person in a particular emotional state can automatically elicit empathic brain activity (e.g., Preston & de Waal, 2002; Singer et al., 2004). However, empathy is also highly dependent on individual differences and can be consciously regulated through reflection and cognition (Cuff et al, 2016).

Purpose

In this chapter, I explore how mathematics graduate student instructors (GSIs) conceptualize empathy and compare their conceptualizations to eight dimensions of empathy identified by Cuff, et al. (2016). I chose to focus on their conceptualizations of empathy, rather than present them with an existing definition of empathy from the literature, to better inform future work regarding my participants' interactions with their students in the classroom. My purpose in analyzing GSIs' conceptualizations of empathy

is twofold: first, I aim to connect my study on pedagogical empathy to broader research on empathy; and second, I aim to generate a nuanced understanding of individual GSIs' conceptualizations of empathy to better comprehend how they experience empathy in the classroom and how those experiences influence their teaching practices. Thus, this chapter focuses on the following two research questions:

1. *How do GSIs conceptualize pedagogical empathy with respect to how psychological researchers conceptualize empathy?*
2. *How might GSIs' definitions of pedagogical empathy change along different dimensions of empathy?*

Methods

To gather data on GSIs' conceptualizations on pedagogical empathy, I conducted four semi-structured interviews with 10 participants over the course of a semester.[5] The findings in this chapter are drawn primarily from the initial interview and final interview with the participants, which were separated by approximately three months. These two interviews yielded the most data relating to GSIs' conceptualizations of empathy since they involved asking participants questions that were directly related to their definitions of empathy and Cuff et al.'s (2016) eight dimensions of empathy.

Participants

The GSIs who participated in my study were all instructors of record for one of the following courses: Intermediate Algebra, College Algebra, Trigonometry, or Precalculus. A total of 10 GSIs with varying levels of teaching experience were recruited.

[5] For the larger qualitative study, a total of 11 GSIs were recruited. One of the GSIs, Beth, participated during summer 2018, rather than fall 2018. Consequently, this participant had different final interview questions. Data from her interviews are not included in this chapter.

All participants were native English speakers who had completed their bachelors' degrees at American universities, and participants selected their own pseudonyms, which are used throughout this chapter. Table 3 provides descriptions of each of the GSIs who participated, listed in alphabetical order by their chosen pseudonym.

Table 3

Descriptions of Participants

Instructor	Course	Year in Graduate Program
Aaron	College Algebra	2
Amanda	Intermediate Algebra	2
Bill	Precalculus	4
Chris	College Algebra	2
Henry	Trigonometry	3
Jack	Precalculus	3
Jane	College Algebra	2
Joe	Precalculus	4
Mark	Precalculus	5
Oliver	Trigonometry	4

Data

The goal of the initial interview was to gather my participants' thoughts on pedagogical empathy before classroom observations and post-observation interviews where we discussed specific student interactions through a lens of empathy. The final interview served as a capstone for how their views on empathy had evolved over the

course of the semester, after each participant had gone through the process of observation and reflection twice.

I began the initial interview by asking my participants to write down a definition of empathy as it relates to a teaching context and then provide an example to illustrate their definition (see the Definition of Empathy Activity in Appendix A). We then discussed their definition of empathy and illustrative example before moving on to other questions. Following the guidance of Hermanowicz (2013) for developing longitudinal qualitative interviews, at the beginning of each subsequent interview, we returned to the participant's original definition, and I asked if there was anything that the participant wanted to change about their definition. We then discussed any changes the participant wished to make and their reasoning for wanting to make that change to their definition.

I began the final interview by returning to the participant's definition of pedagogical empathy and asking if they wanted to modify anything about that definition. To further elicit their thoughts on the definition of empathy, I then presented them with a short list of definitions from other GSIs and asked them to point out any parts of these definitions with which they agreed or disagreed. Next, we went through a series of questions corresponding to the eight dimensions of empathy identified by Cuff et al. (2016). A complete list of questions can be found in the Additional Questions for the Final Interview in Appendix A.

Qualitative Coding

The eight dimensions of empathy identified by Cuff et al. (2016) were used as initial codes for qualitative analysis. After transcribing all of the interviews using MAXQDA 2020 (VERBI Software, 2019), I tagged the beginning of each interview,

where participants discussed and revisited their definition of empathy, using the eight dimensions of empathy as my codes. I also tagged each written definition with the dimension codes, using as little inference as possible. For example, Bill's definition of empathy was, "being able to put yourself in the position of a student to better relate to their fears, anxieties, or confusions." I coded this definition as a trait, since Bill used the phrase "being able to," and as affective since Bill discussed students' "fears, anxieties," and "confusions." Throughout this process, I also conferred with another researcher to verify my applications of the coding scheme and substantiate my findings. I met with this researcher weekly during this analysis process to discuss the data and my findings. This researcher also tagged deidentified interviews and served as a second coder to verify that I was applying the codes consistently across the data. During our meetings we would reconcile our coded interviews and discuss any discrepancies that arose.

Analysis

After coding the interviews using Cuff et al.'s eight dimensions of empathy, I extracted each excerpt into an excel file. I then went through all of the excerpts corresponding to each dimension and compared participants' responses across them. After analyzing the data across the eight dimensions, I also examined how GSIs' written and stated definitions of empathy changed over the four interviews. This process involved identifying the dimensions of empathy that were present in GSIs' initial written definitions and relating them to other dimensions that they brought up during the subsequent interviews. Finally, I drew comparisons between GSIs' written and stated definitions of empathy and their responses to the interview questions about the eight dimensions of empathy to see whether there were inconsistencies between them. During

this process, I also analyzed how GSIs' conceptualizations of empathy changed from the beginning of the semester to the end of the semester with respect to the eight dimensions of empathy. I labeled any additions that GSIs made to their definitions as "changes" even though the addition may have been part of their conception of empathy all along.

Results

GSIs' Conceptualizations of Pedagogical Empathy

Below I describe how GSIs' conceptualizations of empathy relate to the eight dimensions of empathy identified by Cuff et al. (2016). Participants' responses to the final interview questions related to these eight dimensions are summarized below in Table 4. Following the discussion of these eight dimensions, I address how GSIs' conceptualizations of pedagogical empathy changed over the course of the semester with respect to the dimensions of empathy. I also examine the differences between GSIs' professed definitions of pedagogical empathy and their responses to the questions corresponding to the eight dimensions of empathy.

In the results presented below, I use several quotes from my participants to illustrate their views on empathy. By using their words, I aim to support my interpretations of the data and give voice to my participants (Corden & Sainsbury, 2006). I chose to use these quotes because they are representative of my participants' conceptualizations of empathy based on the data that I gathered.

Table 4

GSIs' Stances on Dimensions of Empathy

Dimension	Able to Clearly Distinguish Empathy from Other Concepts	Categorized Empathy as Cognitive, Affective, or Both	Viewed Congruence as Important	Able to Experience Empathy Without Students Present	Viewed Self/Other Distinction as Important	Viewed Empathy as a Trait, State, or Both	Believes Empathy Has a Behavioral Outcome	Categorized Empathy as Automatic, Controlled, Or Both
Aaron	Yes	Both	Yes	Yes	Yes	Both	No	Both
Amanda	Yes	Affective	Yes	Yes	Yes	State	Yes	Both
Bill	No	Both	Yes	Yes	Yes	Both	Yes	Controlled
Chris	Yes	Both	Yes	Yes	Yes	State	Yes	Automatic
Henry	Yes	Both	Yes	Yes	Yes	Trait	Yes	Both
Jack	Yes	Affective	Yes	No	Yes	Trait	Yes	Automatic
Jane	Yes	Both	Yes	Yes	Yes	Trait	No	Automatic
Joe	No	Cognitive	Yes	Yes	Yes	Both	Yes	Both
Mark	No	Both	Yes	Yes	Yes	State	Yes	Both
Oliver	Yes	Both	No	Yes	No	Both	Yes	Automatic

Distinguishing Empathy from Other Concepts

When it came to distinguishing empathy from other related concepts like sympathy or compassion, GSIs fell into two categories: those who were able to clearly articulate differences they saw between empathy and other related concepts and those that struggled to do so.

Some of the GSIs were able to articulate distinct differences between empathy and sympathy. Jane discussed how empathy involves "experiencing the emotion or the feeling that your student or someone else is feeling," whereas with sympathy "you're acknowledging it and maybe feeling bad, for example, for the person, but you're not actually feeling how they're feeling." Amanda made a similar distinction and remarked how the definition of empathy involves relating to students through a shared experience and feeling how they feel in that moment. She went on to explain how "sympathy is more like when a student is in a situation that you can't relate to, but you understand that it's challenging for them."

To distinguish between empathy and sympathy, Aaron provided an example from his experience as a graduate student and explained how one of his math professors gave lectures that were "unempathetic" but not in an "unsympathetic" way:

> *His lectures seem to lack empathy, but not so much in the sense that he doesn't sympathize with his students.... It doesn't seem like he's uncaring or mean or something like that, but rather that he doesn't make the effort to think about how a student might be thinking when they see the material that's being presented…. There isn't an effort to understand the student's perspective and inform the lecture in any way.*

Several GSIs brought up reactions or responses when talking about the differences between empathy and related concepts. Oliver discussed how "empathy delves more into how to react" when comparing it to sympathy and that sympathy and compassion together can "create empathy." Similarly, Jack categorized compassion and sympathy as two of the responses of empathy, and Chris described compassion as acting on empathy.

The remaining GSIs were uncertain of how empathy differed from related concepts. When discussing a situation that had arisen with one of his students, Bill noted: "I have a hard time distinguishing between empathy and caring about people in general. I don't know. Maybe that is empathy." After being asked directly about how empathy differs from other concepts, Bill tried to clarify his thoughts by reverting back and elaborating on his definition of empathy, but ultimately, he was unable to state any clear differences between empathy, sympathy, and compassion. He ended his response by saying, "I don't know. I guess I think they are all similar." Similarly, Joe acknowledged that there was a difference between these concepts, but not one that he could vocalize. As a comparison, he brought up the mathematical difference between a group and a ring:

> *In my brain, a group and a ring are very clearly distinguished categories. Like this is a group and this is a ring. Where sympathy versus empathy, those have blurry boundaries and one bleeds into the other, and I don't know…. I'd maybe have some intuitions about when one thing is versus another versus compassion, but I can't draw sharp lines.*

Mark was another GSI who was unsure of the difference between empathy and other concepts. However, he implied that their differences might not matter in the context of teaching because the way you respond should essentially be the same:

> *Well sympathy, empathy, compassion are – I still don't have a perfect working definition for what distinguishes one from the other. I mean in the end I think they're all sort of getting at the same thing. Well the feelings may be different, but I think the implied actions, what you ought to do ends up maybe being sort of the same thing. You ought to try and make your students feel like they can succeed, feel welcome, feel like you're on their side, and also present things in such a way that they can succeed and whether that's the result of compassion or sympathy or empathy…. I think for all of these kinds of things, the general resultant actions are the same sorts of things.*

Cognitive or Affective

Consistent with the literature, most of the participants thought that experiencing empathy is both a cognitive and affective process. Chris explained that his initial response is mainly affective: "at first it seems affective, where someone's feeling bad and I just immediately emotionally respond to that and I start feeling bad. But sometimes, it can require more thought." He went on to describe how feelings can be difficult to perceive and how students who are struggling might be reluctant to ask for help, so empathy might require more cognition in certain situations. Jane also brought up the fact that one process might follow another and that thinking about a student's perspective might cause you to feel the way the student is feeling:

> *You have to think about the situation that they're in in order to understand how they're feeling, like the different factors of the situation and how they affect your student. But then you also, part of empathizing with others is feeling how they're feeling, so there definitely is an emotional aspect involved.*

Henry discussed how the cognitive or affective experience of empathy might depend on the situation and whether or not he has had a similar experience that can help him relate to the student. Without having had a similar experience, empathy is a much more cognitive process for Henry: "I'm not really feeling what they're feeling in this context, but I'm trying to simulate what they're feeling just cognitively or trying to simulate what they're thinking just cognitively."

Aaron distinguished between different teaching activities and explained how they might elicit a more cognitive or affective approach to empathy:

> *When one is lesson prepping, the process of thinking: what is the student going to think about this material? What are my students specifically going to think? How are they going to react? I think of that as being something like a cognitive exercise where you kind of decide consciously to cast your brain, it's not as much of a feeling as much as it is a conscious thing that I'm kind of thinking out. But when I'm in the moment and I'm confronted with a particular situation that I respond to, I feel like that's less of a cognitive thing.*

In response to the question of whether empathy was more of an affective or cognitive process, Mark stated, "I'm not convinced that there's a huge difference between affect and cognition." He went on to describe how there was not a "clear line" between thoughts and feelings and conjectured that "feelings are a specific type of thought."

Ultimately, Mark concluded that both "thought" and "physical feeling" are involved in the experience of empathy although "it's sort of the same."

Two of the GSIs considered empathy to be more of an affective process. When comparing definitions, Amanda pointed out the difference between her perspective and a definition that someone else had come up with:

> *For me, I feel like empathy is more about emotion and less about understanding, like material understanding. When I think about empathy, I think more, "this student is frustrated because they don't understand it," versus, "they don't understand it." So I think that kind of distances me from this definition.*

Amanda also discussed how she is more concerned about understanding what her students are feeling and why they are feeling that way, rather than assessing their understanding of the mathematical content in certain situations. For example, Amanda discussed a situation where a non-traditional student had come to office hours to talk with her about the pace of the course. Amanda's student told her that she was "overwhelmed" and that "she just wasn't able to comprehend everything as it was coming at her." Amanda's response was to tell her student that she had also felt like that and "it is normal to feel like that. It's normal to feel overwhelmed by a large amount of material that you haven't seen before coming at you, and so it doesn't mean that you won't understand it or anything." Thus, from Amanda's point of view, conversations like this are more about telling her students that "it's okay to feel like this," leading to a more "emotional" experience for Amanda that is "not really about the understanding of material."

Another GSI, Jack, also asserted that empathy is more of an affective process, but with "training" it could become more cognitive. Jack noted that "you have to be

careful…because some of your gut instincts are to feel some of these [negative] emotions" towards students. Jack went on to say that "you don't want to assume that they're [students are] just tired or lazy or whatever," so teachers should be "aware" of this potential for negative affect towards students and "remember what it's like to be a student."

Only one of the participants, Joe, described having a predominantly cognitive approach to empathy, although he acknowledged that it could happen on more of an "instinctual level" or "affective level" for others:

> *For me, I'd say it's primarily cognitive or I think about it in a cognitive way. I will often look at a student's face and say, "oh I am seeing x emotion on their face. I should respond in this way." And so for me, it feels cognitive. But I can imagine that another instructor does it, and I'm sure I do this too, but sort of on an instinctual level. Just being like, "oh, it looks like they're getting lost." And without this rising to the conscious level of what they think, but just, "oh, it looks like they're getting lost. I need to smile at them or make a joke to lighten the mood or whatever." And I'm sure it also works on an emotional level for some other people as well, at the affective level where they purely from their emotions are responding in the appropriate way.*

When asked why those differences exist, Joe explained that "people deal with the world in different ways, in either a more cognitive or instinctual or emotional way, and have more or less success with each of those ways and so people fall back on them depending on the circumstance."

Congruent or Incongruent

A majority of the participants said it was important to accurately perceive students' feelings and thoughts because it helps instructors respond to students in an appropriate way. Aaron used an analogy to explain his thoughts:

> *To empathize strongly, but then completely misunderstand their actual thoughts and feelings is equivalent to donating a large sum of money to a charity and feeling good about it, but then not having it actually do anything to help those in need…. Good intentions without proper actions is nice, but at the end of the day, we're instructors who are trying to teach effectively. And if you're not teaching effectively, then that's something to work on.*

Thus, from Aaron's perspective, having an accurate understanding of students' thoughts and feelings is important when experiencing empathy because that understanding can help instructors make appropriate decisions. Without that congruency, instructors might misunderstand what actions could best help their students.

Amanda was another GSI who thought it was important to understand both when students are feeling something and why they are feeling that way. She argued that without this understanding, a response could be "demoralizing" to students, even if you are trying to help them. Moreover, accurately perceiving what students are going through can help instructors better relate to their students: "If you're getting at it in the correct way, that's going to make you seem more accessible to them, I think. Like they're going to see you more as a person and less as the keeper of mathematics answers." Amanda's comment suggests that she might see a connection between congruency, or accurately

understanding students' thoughts and feelings, and students' agency in the classroom, a point that is expanded on in the discussion.

Mark reflected on a question he posed to himself, which was: "Can you experience empathy in error?" He concluded that you could, in fact, "get it wrong" but that "you're going to be more effective if you get it right." He repeatedly returned to this question in other parts of the interview and eventually defined accurately empathizing with students as "not reading their minds, but accurately coming to an understanding of what they're feeling, what they're going through and how you can help."

Oliver's views differed slightly from the other participants, as he was the only GSI to say that accuracy was "irrelevant." In his opinion, just attempting to empathize with students and then trying to help them was more important. He also questioned how to measure accuracy and gave an example of why he thought accuracy was unimportant:

> *I approach a student and it seems like they're having difficulty, then that's just my perspective. And I try to do my best at not making them feel bad and sort of helping them along the way, but perhaps I'm misunderstanding it. It may not be that they're misunderstanding it. Maybe they just don't care. Maybe they're unmotivated by the question or they don't care, for any particular reason that day, they don't want to be involved. So I think accuracy is kind of an irrelevant question. It's just a matter that — I mean by me trying to help them, it may help engage them.*

Subject to Other Stimuli

Several GSIs talked about empathizing with students as they reflected on prior experiences with them. They also acknowledged that this reflection might occur outside

of the classroom and that they could empathize with students even when they are not physically present. A few participants explicitly mentioned thinking about students while lesson planning. Oliver stated, “In lesson planning, very often I’m thinking about my students and their previous reactions to whatever I may have said in the past.” Similarly, Joe stated that he tries to “empathize with the average student” and predict what will help them learn when lesson planning. Amanda noted the importance of not only “responding to how your students are feeling, but also predicting how they’re going to feel about something” and “trying to address that before you teach, before you see them.”

A couple of the GSIs discussed how it was easier for them to empathize with students without them physically being present (e.g., while grading or lesson planning) towards the end of the semester after they had gotten to know them better. In particular, Bill said that “it was a matter of getting to know them well enough” and that it was easier for him to empathize with students who had been coming to class consistently or who had sought help outside of class. Amanda also talked about getting to know her students over the course of the semester and how she was able to empathize with them while grading:

> *At this point, knowing some of my students like when I’m going through their exams or reading through their quizzes or something, I can see kind of where they’re getting derailed, if they are. If they do well on a quiz, I’m excited for them.*

Other participants mentioned having a difficult time empathizing with students who were “disengaged” or “checked out” of class. For one GSI, Aaron, that was because he had a hard time understanding what the students were feeling: “The fact that they weren’t being really engaged, active students didn’t make me less empathetic; it made the signals dimmer.” Another GSI, Jack, had a difficult time relating to these students:

I think that's one of the things I struggle with. If someone wants to turn it off, I don't know how to boot it back up again. I guess maybe one of the things they're feeling is, 'this is boring,' and that is something I have a hard time empathizing with, so while I understand what you're feeling, I don't agree with that.

Self/Other Distinction or Merging

The participants responded in a variety of ways to the question of whether it is important to distinguish between your own thoughts and feelings and students' thoughts and feelings when empathizing with them. Some argued that the more you think about your own thoughts and feelings, the less able you are to empathize with your students. As Henry stated, "if you're mainly thinking about what you're feeling, then you're not experiencing empathy very much because you can't be thinking as much about what the other person is feeling." Aaron voiced similar concerns: "The more that you're thinking about your own thoughts and feelings about the situation, the more it clouds your judgment and occupies your mental energy." He went on to explain that "the more you allow your own thoughts and judgments, the more it occupies your mind, the less you are able to figure out where the student is, what they're thinking, and what they need."

Other GSIs thought it was important to distinguish between their thoughts and feelings and their students' thoughts and feelings because, as Amanda put it, "you don't want to be projecting your own insecurities onto your students." Chris also talked about projecting onto his students, saying how you don't want to make assumptions about where students are "stuck" because "maybe something seems unintuitive to myself, but the student's able to pick up on it right away, and I don't want to project my own lack of intuition onto the student."

Along the same lines, Jane mentioned how she did not want her thoughts and feelings to affect how she perceived her students' thoughts or feelings. From her perspective, making this distinction is especially important if a teacher has preconceived notions about a student:

> *In the end, you just want the student to learn and feel comfortable doing math and maybe just gain some critical thinking out of the class. And so you want to put your own opinions and feelings aside in order to make sure that you're giving all of your students an equal opportunity to learn.*

Three of the GSIs related this dimension back to whether or not it is important to accurately understand what students are experiencing. Aaron, Bill, and Mark all expressed that distinguishing between their thoughts and feelings and their students' thoughts and feelings was similar to accurately perceiving their students' experiences. As Bill remarked, "I think that would just be a result of how accurately I interpreted their emotions at the time." Similarly, Aaron discussed how making a distinction between his thoughts and feelings and those of his students "goes right back to the importance of clearly identifying their thoughts and feelings." He went on to say, "if it's all my thoughts and feelings that I'm ultimately thinking about, then I'm not correctly empathizing with them."

Trait or State Influences

GSIs were split on the question of whether empathy is an ability or a state of mind. Some participants described empathy as an ability because "you can learn to do it better." Other GSIs classified empathy as an ability because it is "something that you do." Jack discussed how you could try to bring empathy into your state of mind, but ultimately

"it's just something that you do and you probably don't think about very much." Jane expressed a similar sentiment:

> *My instinct would be that some people do experience more empathy than others, so maybe it is an ability in some way. I don't know if I would think that it's a state of mind. It's more like something that you do: you empathize.*

Other GSIs held contrasting views. For example, Amanda discussed how she thought empathy was more of a state of mind:

> *Everybody can experience empathy…I don't really think it's an ability. I think you have to really reflect on your own experiences and how that relates to what you want your students to be feeling in your class and how you can help set that up.*

Similarly, Oliver explained how everyone has the ability to empathize, but getting into that state of mind might depend on the situation:

> *I think everybody has the ability to do it, but it's a state of mind…. Some people who are brilliant may not think that this is very hard and perhaps can't empathize, but not because of their ability, just because they haven't really experienced a hardship like that and can't relate or adjust the curriculum or the way they interact to adjust to the students.*

While some GSIs said that empathy was either an ability or a state of mind, others argued that empathy could be both. Bill thought that experiencing empathy is an ability because "you can certainly get better at this," but cautioned that there was a difference between being able to do something and being willing to do something, and in that way, empathy was also a state of mind. Aaron also highlighted the importance of considering empathy as both an ability and a state of mind: "Both pieces are important and if you lack

one or the other, things are going to go awry." He claimed that empathy is a state of mind because you have to have "the right mindset" and an "empathetic intent" towards your students, but noted that you also have to act on that intent, which requires the ability to use empathy in order to help students.

Mark was initially uncertain as to whether or not empathy is an ability. At first, he argued that, "the state of experiencing empathy for students is a state of mind," but then questioned whether getting into that state of mind was an ability. Later on in the interview, Mark discussed empathy as a "skill" and asserted that it eventually becomes "second-nature" through practice, although he specifies that the skill is being able to get into this "state of empathy." Joe also talked about empathy in a similar way saying, "I have the ability to go into this state of mind."

Has a Behavioral Outcome

Most of the participants considered resulting actions or behavioral outcomes to be a part of empathizing with students. Actions and responses were commonly mentioned as concepts that set definitions apart from one another. For example, when comparing definitions, Jack pointed out that his included the phrase, "and respond accordingly" while another definition did not "mention a response," which made him think that definition was insufficient. Another GSI, Mark, said that after looking through other definitions he wanted to add the phrase "and acting according" to his definition because that was a "missing piece." He went on to explain:

> *When I gave that definition, I guess I was sort of assuming without really thinking about it that the 'and acting accordingly' was a natural consequence, but I guess to be maybe a little pedantic, it ought to be in there.*

Oliver was another GSI that brought up responding to students while talking about the definition of empathy. Like Jack and Mark, he thought that the definition of pedagogical empathy needed to include how you respond in the context of teaching. Oliver noted that trying to understand what students are experiencing is important, but that "it's your responsibility to assess the situation and adjust the way you approach something or the way you interact with students. It would be almost un-moralistically responsible to not act."

Other GSIs also discussed how part of what distinguishes empathy in a teaching context is taking appropriate actions or making adjustments. While Aaron did not think that empathy itself was an action, he did think that responding to students is "how we use empathy" and that it "clarifies empathy in the teaching context." He also acknowledged that appropriate actions are important for "teaching effectively" and that "actionable empathy" can help instructors make decisions about what to do. Joe also thought that "if you are empathizing with your students, it will change how you act and how you behave" and that "on the whole, in a classroom setting, you should probably be acting on your empathy if you perceive something."

Another idea that a few GSIs emphasized was responding to different students in different ways. Beth talked about thinking of students as individuals and acting on the differences between them:

> *Part of the empathy that teachers have or need to use is changing between student to student, like knowing how all of your students learn in a way, and that seems like a lot to have to know to teach to so many people. But actually understanding how to go about the situations when you're talking to different students in the*

> *class, like whether they're doing really well or they're not doing well, you have to approach that differently—so being able to actually act on those differences and treat a student the same.*

Henry expressed a similar sentiment to Beth and emphasized the importance of tailoring instruction to different students' needs:

> *If you're experiencing empathy for a student, then you're more likely to be asking them questions about what they think or what they're feeling, and you're more likely to be dealing with different individual students differently because different students are experiencing and feeling different things. As opposed to just treating everyone the same because you're only thinking about – you're not thinking of them as individuals.*

Only a few of the GSIs did not think empathy included some kind of response. While Aaron did think that empathy could be "actionable," he did not think that empathizing with a student included responding to them. Aaron explained his perspective on the relationship between empathy and resulting actions by comparing empathy to courage:

> *I think you can act on your empathy, but I don't think that it is itself an action – just like you can act on courage, but the decision to stab the dragon or whatever is not courage in and of itself.*

Jane also thought of the resulting responses as being "separate" from empathy because "it's how you choose to react to the empathy that you're feeling." She also talked about how there are other factors to consider when making decisions about how to respond to a student. Specifically, Jane mentioned being aware of time constraints and

treating students fairly: “If you’re sitting down with a student and they’re really struggling, you have to decide how much time you can give to this one specific student compared to other students.” She went on to talk about a student who might need accommodations due to a personal problem:

> *You have to decide: ‘Do I give this student an extension on their homework? How many times do I do this for them? How many absences do I excuse?’ I think since these are all decisions that you have to make, you want to be fair with how you treat your students.*

Automatic or Controlled

Several GSIs said that empathy is both an automatic and controlled process. Aaron talked about how “parts of it [empathy] are automatic and parts of it are controlled, but as one practices some elements cease being controlled and start becoming automatic.” He went on to discuss how when you first start teaching “you have to be thinking about all of the data and processing it because you haven’t formed a coherent model. But then after you’ve gotten experience, you start making more snap decisions, for better or for worse.” Mark expressed a similar sentiment saying that empathy is a controlled process, which can become more automatic with practice:

> *I would say it’s controlled, but with practice probably can become automatic. So, I mean it’s like any skill that becomes second nature, for a certain amount of time it’s very controlled, there’s a lot of thought to get there — you know, balancing on a bicycle or something, right? But eventually it becomes second nature. And I think the skill in this case would be the skill of sort of getting into this state of empathy with your students and accurately, as we were talking about before,*

> *accurately sort of not reading their minds, but accurately coming to an understanding of what they're feeling, what they're going through and how you can help.*

Other participants discussed how situational factors might affect whether empathy is an automatic or controlled process. Henry described how he sometimes needed to "consciously" think about empathizing with students and be more "intentional" about it, but at other times empathy was more automatic: "When I have a previous experience that is similar to what a student is experiencing, it tends to be more automatic. And if not, then I would need to be thinking much harder about it and be more controlled." Oliver also talked about how empathy might not be an automatic process for people who had never faced certain challenges:

> *Maybe for them, it would be different. Perhaps it's something that they're aware they didn't have a difficulty with, but maybe they've come to a level of maturation that they can assess that and say, "okay because I didn't have difficulty, doesn't mean that other people can't." And maybe it's not automatic for them, but it's sort of like a mental check that they need to do, and maybe they've realized this, and they do this. So perhaps it's different for different people.*

When discussing whether empathy was a controlled or an automatic process, Joe distinguished between "feeling empathy" and "action on that feeling." From his perspective, feeling empathy, which he defined as "an awareness of student thoughts," is usually more automatic, but could be improved by thinking about what the student is experiencing:

> *I think that it's good to distinguish the feeling of it and the acting on that feeling. And feeling empathy, like this awareness of student thoughts, that is usually automatic, but if you think really long and hard and be like, "wait a second. How are they feeling right now?" And you puzzle through it, then you can probably do that better. And on the other side, the acting on that feeling, I think that that is primarily active and chosen, like you choose to act on this. But I think also there are a few unconscious parts of that, like if you're smiling or your posture or whatever or like the tone of your voice, a lot of those things people don't automatically control and it comes from whatever they're thinking or feeling and that comes through regardless of whether or not you act on it or think about acting on it.*

Changes along Dimensions of Empathy

As the semester progressed, all of the GSIs, except for Jack, refined their written definitions of pedagogical empathy with additional clarifying phrases. Some of these modifications also corresponded to changes in participants' views on the eight dimensions of empathy. Appendix B provides GSIs' initial written definitions of empathy, along with any verbal changes that they made to their definitions during subsequent interviews. Table 5 provides an overview of how each GSIs' definition changed, with respect to the eight dimensions of empathy, from the initial interview to the final interview. Table 5 was constructed by analyzing both GSIs' initial written definitions of empathy and their revisions to these definitions. An "x" in the table means that GSIs addressed that dimension of empathy in either their initial interview and/or in the final interview.

Table 5

Changes in GSIs' Definitions of Empathy From the Initial to Final Interview

Dimension*	Subject to Other Stimuli		Self/Other Distinction or Merging		Trait or State Influences		Has a Behavioral Outcome	
	Initial Definition	Final Definition	Initial Definition	Final Definition	Initial Definition	Final Definition	Initial Definition	Final Definition
Aaron		x				x		x
Amanda		x			x	x		x
Bill				x	x	x		x
Chris					x	x		x
Henry		x			x	x	x	x
Jack					x	x	x	x
Jane		x			x	x		x
Joe	x	x			x	x	x	x
Mark					x	x		x
Oliver					x	x		x

*Three dimensions are not included in this table as they did not appear in any of the GSIs' definitions of empathy: Distinguishing Empathy from Other Concepts, Congruent vs. Incongruent, and Automatic vs. Controlled. The dimension of Cognitive vs. Affective is not included in the table as all the GSIs' initial definitions were categorized as attending to either cognitive or affective components.

As an example, Jane's original written definition of empathy was:

> *Being able to understand the class from the students' perspective and understand how students feel/why they feel the way they do, whether it's about the material, the class, environment, classmates, or teacher and how it affects their experience in the classroom.*

Thus, in her initial definition, Jane displayed having a trait perspective of empathy ("being *able* to understand…," emphasis mine). In a later interview, Jane discussed how it was important to predict "before you go into class how your students are going to feel about the material." Predicting how a student might feel about mathematical material involves imagining another's experience without them being physically present. Therefore, Jane's addition corresponds to the "Subject to Other Stimuli" dimension of empathy. Moreover, this prediction element can also be seen as influencing behavioral outcomes, since predicting how students might react to material may influence the way Jane interacts with her students in class. Thus, both Jane's initial and final definition of empathy reflected taking a "trait" stance on empathy, while the "Subject to Other Stimuli" and "Has a Behavioral Outcome" dimensions were reflected in her final definition.

Aaron's changes to his definition were similar to the changes that Jane made to her definition, especially with respect to the "Subject to Other Stimuli" and "Has a Behavioral Outcome" dimensions. Aaron's initial definition—"having an understanding of a student's unique perspective, history, and conditions within the context of a learning environment"—only reflected the cognitive dimension of empathy. However, Aaron made two changes to his definition over the course of the semester. During his final

interview, Aaron discussed "being able to predict and use that predictive power." Thus, similar to Jane, Aaron showed a change in the "Subject to Other Stimuli" and "Has a Behavioral Outcome" dimensions of empathy, as well as viewing empathy as a trait.

Besides Jack, who did not make any changes to his original definition of empathy, only one other GSI, Joe, made changes to his definition of empathy throughout the semester without changing his overall perspective on different dimensions. In his initial interview, Joe defined empathy as:

> *An awareness of student thoughts, especially their understanding of content and comfort with content and the ability to "relate to" those feelings. Being able to predict what students are thinking and understand what they're thinking and use that to help your teaching.*

In his initial definition, Joe described multiple dimensions of empathy. Joe referred to the process of predicting what students think, thus addressing the "Subject to Other Stimuli" dimension, and also took a trait stance on empathy, calling it an "ability." In addition, Joe explicitly discussed using empathy to "help" his teaching, thereby including behavioral outcomes as part of the experience of empathy. Throughout the semester, Joe added two phrases to his definition: "understanding students' comfort with you" and "empathy is something you should have and part of being a good teacher is having this empathy." Since these additions aligned with the dimensions of empathy present in Joe's initial definition, Joe did not show any change in these dimensions over the semester.

Discussion and Conclusion

The GSIs in my study held many differing views on the eight dimensions of empathy. The three dimensions with the largest variation of responses were: cognitive or

affective, trait or state influences, and automatic or controlled. This variation between GSIs' conceptualizations of empathy is unsurprising as these dimensions of empathy are also disagreed upon by many researchers. However, GSIs' views were mostly aligned on many of the other dimensions of empathy (i.e., congruent or incongruent, subject to other stimuli, self/other distinction or merging, and has a behavioral outcome). For these dimensions, only one or two GSIs took a different stance than others. Moreover, all statements by the GSIs fit into a dimension proposed by Cuff et al. (2016). Thus, the eight dimensions of empathy put forth by Cuff et al. (2016) is a useful framework for understanding how GSIs view empathy and provides a meaningful way to examine the concept of pedagogical empathy.

A striking finding is that there were no "profiles" of GSIs that emerged from the data. That is, GSIs did not respond in similar ways to similar questions. For example, Jack and Aaron had similar responses to the question relating to the dimension of congruence. Both thought that it was important to accurately understand what their students were feeling so that they could decide how to respond appropriately. However, Jack and Aaron held conflicting views on whether empathy was automatic or controlled. Jack thought empathy was primarily automatic but with more thought could become a controlled process, while Aaron discussed how empathy was controlled but could be become more automatic with practice. It is important to note that my sample size for creating profiles was somewhat limited with only ten participants in my study. However, because empathy is a complex phenomenon and each of my participants had a different understanding of what defines "empathy," I hypothesize that even with more participants, distinct profiles would still have been hard to discern from the data. A mixed-methods

research approach could be taken in future studies to investigate this hypothesis more carefully.

By the end of the semester, the GSIs in my study had expanded on their definitions of pedagogical empathy and refined their ideas relating to the phenomenon. In later interviews, several of the GSIs remarked how participating in this study had prompted them to think more carefully about empathizing with students. With the exception of a few definitions, GSIs' initial definitions of empathy primarily focused on two of the eight dimensions of empathy identified by Cuff et al. (2016). All of the GSIs' initial definitions of empathy reflected taking a cognitive and/or affective stance toward empathy; and several initial definitions reflecting having a trait or state perspective of empathy. After repeatedly returning to their definition of empathy, GSIs began addressing a few of the other dimensions of empathy. Thus, this analysis suggests that change in the way that GSIs' view pedagogical empathy is possible through repeated reflection.

The most evident change in GSIs' definitions was their stance on whether the experience of pedagogical empathy includes some sort of behavioral outcome or response. Only three of the GSIs addressed this dimension during their initial interview: Henry, Jack, and Joe. However, by the final interview all of the GSIs had included some form of responding to students as part of their definition of empathy. Most of the GSIs discussed this behavioral response in terms of in-the-moment decision making. However, two of the GSIs, Aaron and Jane, took a slightly different stance. When asked directly both Aaron and Jane responded that a behavioral response was not part of an empathic experience. Yet they both included elements of responding in their definitions of

empathy. While discussing their definitions of empathy, Aaron and Jane both brought up predicting how students will react to a certain topic and using that to make teaching decisions. Therefore, while they may not have discussed in-the-moment decision making as part of pedagogical empathy, they still felt that empathy could influence teaching decisions made during face-to-face interactions with students in some behavioral way.

Limitations

A primary limitation of this study is its generalizability to different populations and contexts. The graduate student instructors who participated in my study were relatively novice instructors in a mathematics department with a well-developed GSI professional development program. Their conceptualizations of pedagogical empathy may be very different from those of tenured faculty members or other more experienced undergraduate mathematics instructors due to differences in background and teaching preparation. In addition, all of the GSIs in my study expressed the belief that pedagogical empathy is an important aspect of teaching. Other instructors who may not hold this belief might have very different reactions to how the eight dimensions of empathy apply in a teaching context. Future research to examine these differences would be valuable.

Another limitation is that all of the GSIs in my study were, at the time, instructors of a Precalculus course. While some of the instructors had previously taught upper level mathematics courses (e.g., Linear Algebra), it is likely that the instructors in my study focused their responses more narrowly on empathizing with students in precalculus courses. This empathic experience might differ greatly from that of an instructor teaching a more advanced mathematics course with a different population of students. Instructors teaching upper level mathematics courses might remember their own experiences as

students learning that content more vividly or might see more similarities between themselves and their students. Moreover, all of the instructors in my study used some form of active learning and group work. It is likely that responses to questions about pedagogical empathy would vary with instructors using other teaching methods. Again, these differences provide avenues for future research, with the intent of drawing comparisons between different populations of both instructors and students.

Implications

My findings suggest that empathy has the potential to influence the ways in which GSIs interact with their students. The change in GSIs' perceptions of whether or not empathy includes a behavioral response is notable. This finding suggests that *responding accordingly* after empathizing with a student is a defining component of GSIs' conceptualizations of empathy in a teaching context. Therefore, we can view pedagogical empathy as a factor that might influence teaching decisions in the classroom, although further investigation is needed to better understand this relationship.

One question emerging from my findings is how different dimensions of empathy might be linked to students' experiences in the classroom. In her discussion of congruency, Amanda pointed out that accurately understanding what students are experiencing can help instructors seem more "accessible" to students rather than being seen as "the keeper of mathematics answers." This comment suggests that empathizing with a student might be one way to help increase that student's sense of agency in a mathematics class. Further research exploring students' perspectives on empathy is necessary to understand how different dimensions of empathy might impact students' experiences in the classroom.

A final takeaway from this chapter is that GSIs were able to expand on their definitions of empathy throughout the semester. Thus, incorporating reflective activities on pedagogical empathy into GSI professional development can help GSIs become more aware of this phenomenon. My findings suggest that GSIs conceptualize empathy in nuanced ways and the lack of emergence of typical "profiles" suggests that GSIs could learn about different dimensions of empathy by discussing these concepts with each other. For example, Jack and Aaron, who held similar views on congruence but differing views on whether empathy was automatic or controlled, could benefit from discussing their points of view with each other and seeing these similarities and differences. In addition, it is important to consider how GSIs think about these eight dimensions of empathy as different professional development activities might be more impactful for GSIs' holding some views over others. By focusing on this topic, professional development providers can help GSIs form a better understanding of pedagogical empathy and how it might influence specific interactions between instructors and students.

Conclusion

The purpose of this chapter was to compare GSIs' conceptualizations of empathy in a teaching context to dimensions of empathy found in research literature. While the instructors in my study shared similar views on multiple dimensions, there were no clear profiles that emerged from the data with respect to the eight dimensions of empathy. Similar to the research literature, GSIs' conceptualizations of empathy addressed different dimensions, and there was no consensus on any of the dimensions of empathy. However, by the final interview, all of the GSIs in my study had discussed the affect that

empathy can have on responding to students. This distinction separates empathy in a teaching context from other research contexts. Using this distinction, researchers can further explore the phenomenon of pedagogical empathy and how it might influence teaching decisions. Also, being aware of this phenomenon can help instructors understand their thought processes and make better informed teaching decisions.

CHAPTER 5: FACTORS THAT INFLUENCE GRADUATE STUDENT INSTRUCTORS' PEDAGOGICAL EMPATHY

Interpersonal relationships are central in the teaching and learning of mathematics. Since learning involves both cognitive and affective processes, teachers should attend not only to the content that is being learned, but also to the ways that their students are experiencing that content. One way that teachers can relate to their students is by empathizing with them. This experience is often situated at the intersection of teacher and student affect and has the potential to influence teaching practices; yet, little research has been conducted in this area (Zembylas, 2007). Exploring how empathy influences teaching can lead to a deeper understanding of how teaching practices are enacted and have far reaching implications for the broader education community.

This study aims to explore the factors that influence how mathematics graduate student instructors (GSIs) conceptualize empathy as it relates to the teaching and learning of mathematics. The conclusions drawn from this study will add to the emerging body of literature about GSIs' knowledge, beliefs, and teaching practices (e.g., Speer, Murphy, & Gutmann, 2009). In addition, this research can be used to inform professional development efforts for both novice and experienced instructors.

Theoretical Perspective and Framing

Learning mathematics involves both cognitive and affective processes (McLeod, 1992). To align research on affect with historical approaches of cognition and learning,

Op't Eynde, De Corte, and Verschaffel (2001, 2006) presented a socio-constructivist perspective of learning and discussed how this perspective is connected to students' affect and motivation. They used the term *socio-constructivist* to describe "a theoretical position that is grounded in a Vygotskian theory on cognition and learning, but accentuates more explicitly the mutual interaction between the individual and the context" (Op't Eynde et al., 2001, p. 151). Thus, this perspective is characterized by its focus on the situated context of learning and on the close interactions between cognitive and affective factors in students' learning in social settings (Op't Eynde et al., 2006).

Following the work of Op't Eynde and his colleagues (2001, 2006), I hold this socio-constructivist perspective on mathematics learning and problem solving. This perspective emphasizes that emotions and other affective processes are an integral part of learning mathematics and are influenced by the social contexts in which the learning is occurring. In this study, I use this perspective to examine how teachers conceptualize empathizing with their students.

Empathy is a multifaceted construct. Researchers have described empathy in various ways and have developed different characterizations depending on the context in which empathy is referenced (Cuff, Brown, Taylor, & Howat, 2016). It is therefore important to provide a working definition of empathy as it relates to the context of teaching. Drawing on the work of Feshbach and Feshbach (2009), I define empathy as "a social interaction between any two individuals wherein one individual experiences the feelings of a second individual" (p. 85). In this study, I use the term *pedagogical empathy* to refer to the empathy that a teacher experiences through an interaction with a student or group of students. I further propose that pedagogical empathy has the potential to

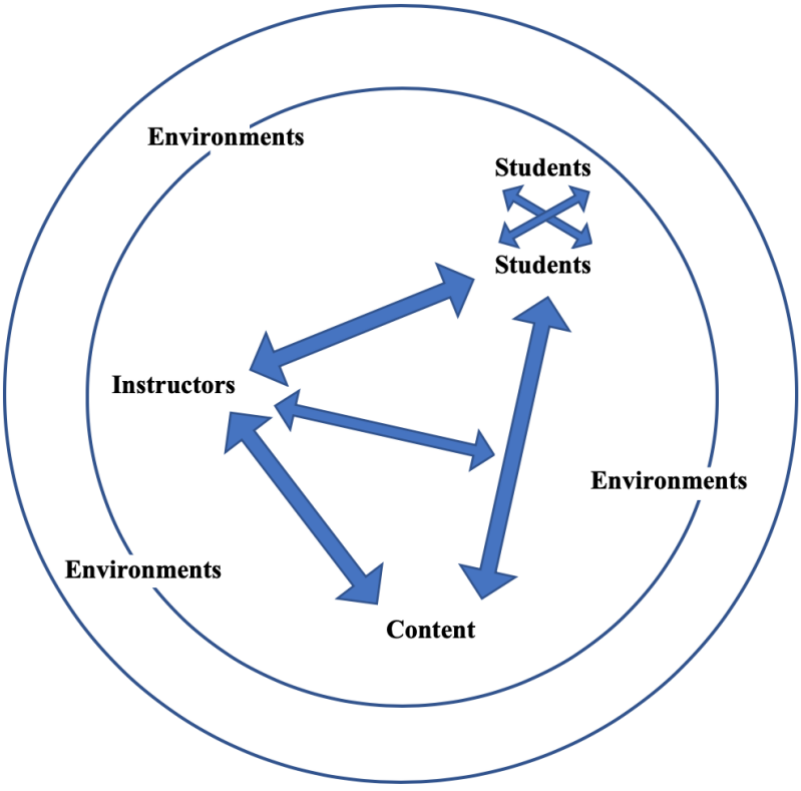
influence teaching decisions and involves how teachers use their understanding of students' perspectives and experiences to inform the actions they take in the classroom.

The Instructional Triangle

To better understand how empathy connects to teaching, I drew on the instructional triangle to unpack how pedagogical empathy might affect the relationships between instructors, students, and mathematical content. Building on Lampert's (2001) work, Cohen, Raudenbush, and Ball (2003) used the instructional triangle to define instruction as "interactions among teachers and students around content, in environments" (p. 122). Thus, the instructional triangle provides a model for studying teaching interactions. An adapted model of the instructional triangle is shown in Figure 2.

Figure 2

An Adapted Model of the Instructional Triangle (Cohen, Raudenbush, & Ball, 2003; Lampert, 2001)



In this model of instruction, the arrows emanating from the "Instructors" vertex represent that "the work of teaching is done in simultaneous relationships with students, with content, and with the student-content connection" (Lampert, 2001, p. 423). Furthermore, instructors work within constrained environments and must deal with issues of coordination, resource use, and incentives that might arise from these environments (Cohen et al., 2003). Outside influences such as department culture or the availability of different types of classrooms can affect how much energy instructors put into their teaching or what types of instructional strategies they choose to employ. Students also come from different economic and social backgrounds, which contribute to these environmental influences in the classroom. Thus, interactions between teachers, students, and content are mediated through their environments.

Lampert argued that the practice of teaching occurs across these arrows and that instructors' decisions are often influenced by different relationships within the triangle. However, in some instances, an instructor's interactions may be more dependent on one of the arrows than the others. In this study, I use the instructional triangle to analyze how different factors that might influence empathy align with the relationships depicted by the instructional triangle.

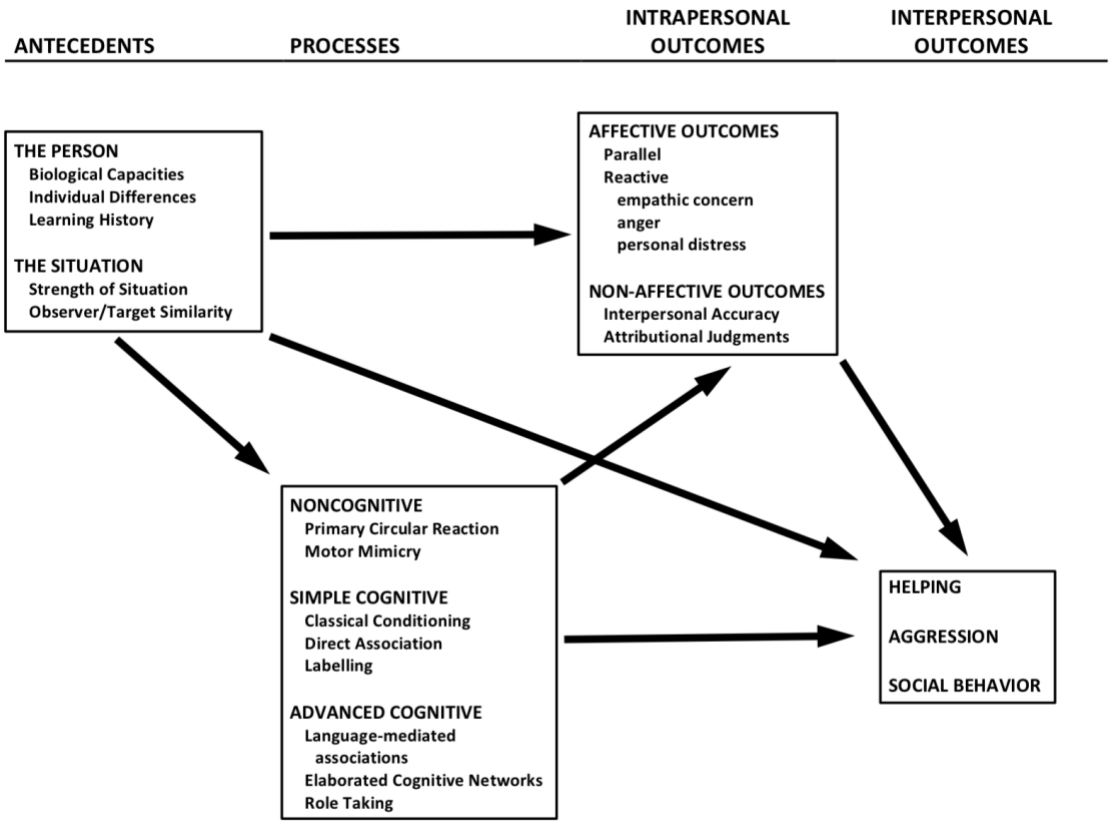
An Organizational Model of Empathy

After analyzing the data, I found that many of my results could be explained in terms of Davis's (1996) organizational model of empathy. Davis (1996) argued that this model emphasizes the connectedness of different constructs of empathy and can be used to examine situations where empathy is elicited. Davis defined empathy broadly as "a set of constructs having to do with the responses of one individual to the experiences of

another" (p. 12) and noted that a typical empathic experience involves an observer being exposed in some way to a target, after which the observer experiences a cognitive, affective, and/or behavioral response. The model identified four related constructs within this empathic experience: *antecedents*, *processes*, *intrapersonal outcomes*, and *interpersonal outcomes*. Figure 3 shows a diagram of this organizational model connecting the four constructs.

Figure 3

An Organizational Model of Empathy (Davis, 1996, p. 13)



Antecedents refer to characteristics of the observer, target, or situation (Davis, 1996). This construct illustrates how empathy might be influenced by characteristics of observers, including both their capacity for empathy and their previous learning histories.

Furthermore, situational factors, such as the strength of the situation and degree of similarity, may also influence how empathy is experienced.

The second construct in the organizational model refers to the processes by which empathic outcomes are produced (Davis, 1996). These processes occur within the observer and can require varying degrees of cognitive effort, ranging from what Davis termed as noncognitive processes to advanced cognitive process. In addition, the processes that the observer undergoes to understand the target's experience are distinct from the outcomes of these processes.

Intrapersonal outcomes are the affective and cognitive responses that result from experiencing empathy (Davis, 1996). Affective responses are defined as the emotional reactions experienced by an observer and may be either parallel or reactive with what the target is feeling. In general, cognitive outcomes occur due to more advanced cognitive processes. Two such outcomes deal with interpersonal accuracy, or successfully understanding another's thoughts or feelings, and attributional judgments, which refers to inferring the reasons behind another's behavior (Davis, 1996).

The final stage of the organizational model, interpersonal outcomes, refers to the behavioral responses directed toward the target (Davis, 1996). Both cognitive and affective components of empathy are thought to contribute to the likelihood of an empathic helping response, and these intrapersonal responses are viewed as having the most direct impact on interpersonal outcomes (Davis, 1996). Furthermore, "the effect of empathy on behaviors that occur within *social relationships,* a topic which has only recently begun to attract consistent research interest, also falls into this category" (Davis, 1996, p. 19).

After presenting my findings, I return to this organizational model of empathy in the discussion to consider how it relates to the context of teaching. I discuss connections between this model and factors that might influence pedagogical empathy. I also explain why some of the factors I found in my study may not be present in this organizational model and consider how this might extend research on empathy.

Purpose

The results presented in this chapter stem from a larger study that explores how pedagogical empathy might influence teaching practices. As a first step, it is important to consider how instructors conceptualize empathy and identify factors that have the potential to influence pedagogical empathy. In this study, I chose to examine how GSIs view pedagogical empathy because of their unique positioning. Because GSIs are both teachers and students, they have the opportunity to reflect on pedagogical empathy using their past and current experiences in both contexts. Previous work examines how GSIs conceptualize empathy (see Chapter 4). Thus, this chapter focuses on the following research question: *What factors might influence the ways in which mathematics graduate student instructors empathize with their students?*

Below I describe the methods used to gather and analyze data for this study. I then present my findings and discuss connections between the instructional triangle and factors that have the potential to influence pedagogical empathy. I also consider how these findings relate to Davis's (1996) organizational model of empathy and empathy induction. Finally, I consider how the insights gained from this study can be used to inform GSI professional development efforts and design activities to increase instructors' awareness of pedagogical empathy.

Methods

The data for this study came from qualitative interviews with 11 graduate student instructors (GSIs), in which they were asked about their views on pedagogical empathy. The purpose of this study was to better understand the phenomenon of pedagogical empathy using basic qualitative research methods as recommended by Merriam and Tisdell (2016). As Merriam and Tisdell (2016) note, constructivism underlies all qualitative research since "individuals construct reality in interaction with their social worlds" (p. 24). This research approach, therefore, aligns with the aims and framing of this study.

Participants

A total of 11 mathematics GSIs participated in this study. Each GSI was the instructor of record of a precalculus course—either Intermediate Algebra, College Algebra, Trigonometry, or Precalculus—and had varying levels of teaching experience. All participants were native English speakers who had completed their bachelors' degrees at American universities. Participants selected their own pseudonyms, which are used throughout this chapter. Table 6 provides descriptions of the GSIs who participated in my study, listed in alphabetical order by their chosen pseudonym.

The second year GSIs in my study were serving as the instructor of record for the first time and had two-semesters of previous experience as recitation leaders during the first two semesters of their graduate program. The third, fourth, and fifth year graduate students had one, two, and three years, respectively, of teaching experience as the instructor of record for a mathematics course. In addition, all of the second year GSIs were enrolled in a year-long pedagogy course focused on the teaching and learning of

mathematics. This purpose of this pedagogy course was to help GSIs become more reflective practitioners. Assignments for this course included reading and discussing mathematics education research articles, writing essays and reflecting on teaching practices, observing mathematics classes taught by other instructors in the department, and learning about various learning theories. The other GSIs in my study had also previously taken this course in their second year of the graduate program.

Table 6

Descriptions of Participants

Instructor	Course	Year in Graduate Program
Aaron	College Algebra	2
Amanda	Intermediate Algebra	2
Beth	Trigonometry	2
Bill	Precalculus	4
Chris	College Algebra	2
Henry	Trigonometry	3
Jack	Precalculus	3
Jane	College Algebra	2
Joe	Precalculus	4
Mark	Precalculus	5
Oliver	Trigonometry	4

Data

I collected data during the summer and fall 2018 semesters by conducting a total of four interviews and three classroom observations of each GSI. The findings in this chapter emerged primarily from the analysis of the initial interview with each participant. The initial interview was a semi-structured interview designed to explore how GSIs conceptualize empathy as it relates to teaching mathematics. At the start of the interview, I asked participants to write down their definition of empathy as it relates to teaching and provide an example that illustrated their definition. Participants then discussed their definitions and examples and answered other related questions such as: "Where do you think empathy comes from?" and "Is it necessary for math teachers to empathize with their students?" (A full list of interview questions can be found in Appendix A under the Initial Interview Protocol.)

Qualitative Analysis

I transcribed and analyzed the interviews using MAXQDA 2020 (VERBI Software, 2019). Since pedagogical empathy is a novel theoretical construct, I initially used inductive coding to generate a set of preliminary conceptual codes relating to how GSIs conceptualize empathy within the context of teaching and learning mathematics (Miles, Huberman, & Saldaña, 2014). I then tagged each interview using these preliminary codes. Coded segments were typically short responses or partial responses to interview questions that were approximately four to six sentences long. After tagging the interviews, I exported the coded segments corresponding to each preliminary code and summarized them using Excel. I then sorted these segments using pattern coding (Miles et al, 2014). During this process, I further refined the conceptual codes and developed

into themes. These themes related to two arrows on the instructional triangle: instructor-student interactions and instructor-student-content interactions.

Findings

Two main categories of themes emerged from open coding the initial interviews: factors influencing instructor-student interactions and factors influencing instructor-student-content interactions. The themes that fall under these categories highlight elements that have the potential to influence how GSIs experience pedagogical empathy. Table 7 provides an overview of the categories and related themes along with a short summary of each theme and a representative quote from a participant.

Table 7

Overview of Findings

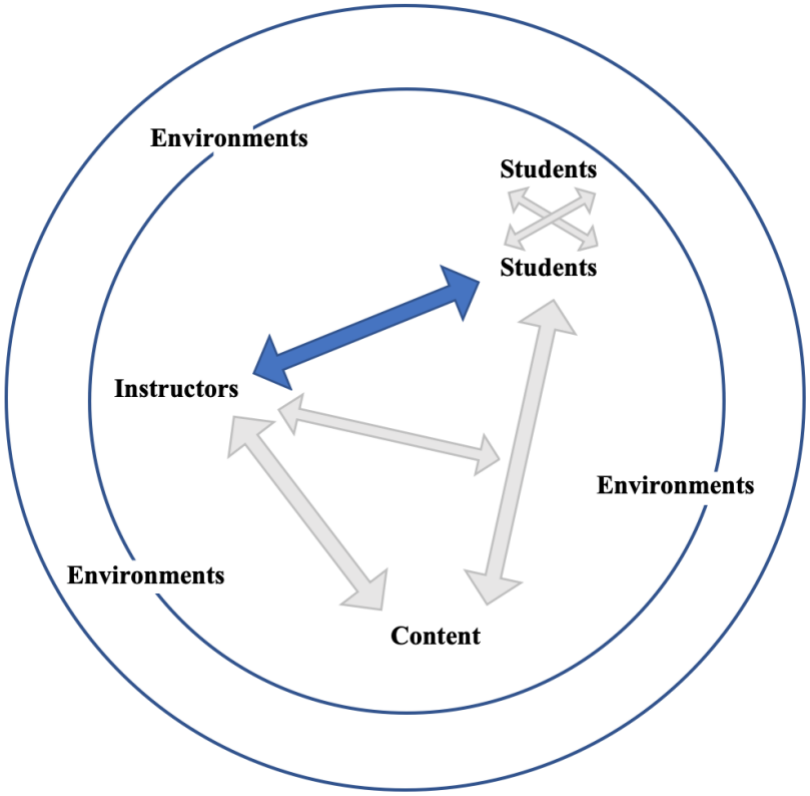
Category	Theme	Summary	Representative Quote
Factors Influencing Instructor-Student Interactions	Instructors' Past Experiences	Drawing on past experiences in order to empathize with students in their class	"I think I draw from those experiences, my own personal experiences as a student to kind of guide me in how I should interact with my own students." - Oliver
	Students' Mathematical Experiences	Considering students' current and past experiences with mathematics	"Students come in from a lot of backgrounds, many in which they're taught to hate math or to think of themselves as bad at math." - Mark
	Students' Experiences Outside the Classroom	Empathizing with students in ways that go beyond the scope of teaching and learning mathematics	"People have come up to me with problems about their life, just in general, and you have to be able to relate to that also, which isn't necessarily math-related." - Bill
Factors Influencing Instructor-Student-Content Interactions	Importance of Communicating Pedagogical Empathy to Students	Importance of communicating pedagogical empathy to students and whether that process occurs in the classroom	"I think if I make it clear that I'm concerned with what their perspective is and what their thinking is, then I imagine they would get the impression that I care what they're feeling and thinking, so that I'm having empathy for them." - Henry
	Course Structure	How logistical considerations such as class size and physical layout as well as teaching responsibilities and classroom norms influence empathetic interactions	"The way that our classes are set up, at least in the precalculus level with active learning, I think that also makes it easier to experience empathy because you have a lot more one-on-one interaction with your students in class." - Amanda

Factors Influencing Instructor-Student Interactions

My participants discussed several factors when considering how they either did or did not empathize with their students. These factors relate to interactions between instructors and students (see Figure 4) and are discussed under three main themes: instructors' past experiences, students' mathematical experiences, and students' experiences outside the classroom. Specifically, GSIs discussed how drawing on their past experiences, considering students' experiences with mathematics, and accounting for students' experiences outside of the classroom might affect their interactions with their students. These themes are described in depth below.

Figure 4

The Instructional Triangle: Instructor-Student Interactions



Drawing on Past Experiences

All of the participants discussed how empathizing with students can be impacted by past personal experiences. In some cases, these experiences helped GSIs relate to their students and were seen as a way to empathize with students. In other cases, GSIs saw having a lack of similar past personal experiences as a potential barrier to empathizing with their students. A few GSIs also reflected on being able to develop empathy over time by learning from their experiences as both teachers and students.

Finding Similarities in Experiences as a Student. Oliver was one of the GSIs who discussed relating to his students by drawing on his past experiences as an undergraduate student. He recalled being in a physics class and constantly needing to use sine and cosine, even though he "barely understood what they meant at the time." He went on to explain how "this moment created something" that he could reflect back on and use when trying to empathize with his students. Oliver also gave a specific example of how his experiences as a student have influenced his teaching practices:

> *I think often I don't click with instructors that don't have this [empathy]. I feel like certain instructors, they get frustrated. I feel like there's a sense of frustration with you when you don't understand something quickly, and I've never been the person to learn things really quickly. So I think I draw from those experiences, my own personal experiences as a student to kind of guide me in how I should interact with my own students and how not to be frustrated with their slowness or their inability to comprehend something that maybe seems trivial to me.*

Amanda was another GSI who talked about understanding what it feels like to be a student who is struggling with something. When asked where empathy might come

from, she reflected on her experiences as a first-year graduate student. In particular, Amanda said that she remembered how it felt to be "sitting in a classroom listening to a lecture and have no idea what's going on," and that sometimes she will see that same look of confusion in her students' eyes. Amanda went on to say that her empathy comes from wanting students "to understand that they can get to an understanding of the material," and that she does not "want anybody to leave a classroom thinking that they're bad at math."

Finding Dissimilarities in Experiences as a Student. While some GSIs found it relatively easy to connect with students in their precalculus courses, others discussed how their life experiences might be dissimilar to those students, making it harder to empathize with them. In particular, Jack talked about how it is easier to empathize with situations that are more "proximal" and how it "takes skill to empathize with people who are further away from you in life experience." He mentioned how it might be difficult for GSIs to empathize with their students:

> *I do think it's probably harder for people to empathize with students who take the lower math courses. There are a lot of experiential barriers between someone who's in graduate school in mathematics teaching an Intermediate Algebra student. So I do think that it is more difficult in that situation for the instructor to empathize.*

In contrast, Jack also discussed how it was easier for him to empathize with his students in a Calculus II course that he had recently taught because it was easier to "simulate the student's emotions" and "detect what then needs to be done."

Beth expressed similar thoughts about why it might be difficult for GSIs to empathize with their students: “From my experience with other grad students teaching courses, for some of them, they didn’t struggle with it. They don’t see a reason why their students are struggling with it.” She went on to explain that sometimes it is difficult for her to figure out how to explain something to a student because “it’s hard to come up with those sorts of reasonings for very basic things,” and that GSIs who are teaching lower level math courses “might struggle more because it’s been a long time since they’ve seen it and they think it’s easy.”

Developing Empathy Through Reflection. A few of the GSIs with more teaching experience discussed their belief that empathy could be developed over time because of their past experiences with teaching. Specifically, Mark discussed how he was less cognizant of what his students were experiencing when he first started teaching:

> *I think it’s fair to say I was less empathetic, less aware of my students’ feelings the first few semesters I was teaching than I am now because I just didn’t realize...at that point the extent to which people did actually come in dreading these math classes, and it’s not because they’re like lazy or something, but just because they’ve been taught to hate them.*

Mark also talked about how one can develop empathy by recognizing “something that you’ve seen a lot in other students.” Likewise, Aaron discussed not having had similar experiences to many of his students and developing empathy by being exposed to people’s stories, reading articles, and absorbing information about people’s college lives.

Joe was another GSI who talked about empathy as a skill that comes from "training and practice." He also explained how teachers might "start" in different places when it comes to experiencing empathy:

> *I think a lot of empathy comes from practice, but I think also I'd say people don't start in the same place. Like some people, the first class they ever teach, they're like really good at empathizing with students, and some people don't get that and maybe I'm guessing some people never develop that and some people develop that while they go on and practice more and work at it, if they work at it.*

In the past, Joe had been "less empathetic to students who didn't really care about their math class" because he had always liked math and thought it was interesting. He discussed trying to "contextualize" being unenthusiastic about a class by thinking about having to take a driving course, which he did not enjoy. After making this connection between his own experiences and his students' experiences, Joe described being more aware of how his students might feel and trying to use this newfound understanding in his interactions with them.

Considering Students' Mathematical Experiences

In addition to discussing how empathy can be influenced by past experiences, several participants reflected on how students' past experiences with mathematics might impact pedagogical empathy. Many GSIs talked about how students might feel or think about mathematics, often linking back to cultural stereotypes and students' prior experiences with mathematics. Participants also acknowledged that teachers and students could hold opposing attitudes about mathematics and that it is important to consider these

different perspectives as they have the potential to influence how teachers empathize with their students.

Acknowledging Students' Mathematical Backgrounds. A few GSIs discussed how students might feel or think about mathematics and the resulting impact that might have on their experience in the course. Aaron brought up empathizing with a student in his class who he described as "hesitant" and "scared." He talked about "trying to think of ways to figure out more about how this student is feeling about math" and how he could "bring her more confidence while she approaches problem solving." Another participant, Mark, emphasized the importance of knowing that "students come in from a lot of backgrounds, many in which they're taught to hate math or to think of themselves as bad at math" and suggested that in order to "reach all students" instructors need to consider students' perspectives of mathematics as a discipline.

Connecting Student Experiences to Instructors' Mathematical Backgrounds. In addition to considering student perspectives of mathematics, a few of the GSIs discussed their own perspectives and experiences with mathematics. Beth talked about how one of her goals as a teacher is to get her students to "appreciate math and appreciate that they can do it" because she wants them to feel the way that she feels about mathematics. She went on to explain how this view impacts her teaching:

> *I think if you close yourself off from how your students are feeling and just like this is the material I need to teach and I'm going to teach this material at this pace regardless of how my students are learning, you're going to cut a lot of people off from learning math and actually wanting to pursue it. I don't know. I mean people think I'm crazy for wanting to do math, and I want more people to*

> *think, "oh I could do this. I could go on to Calc 3 if I wanted to or even further than that."*

Mark also brought up his history with mathematics, although he discussed having mixed feelings about it: "I did go through a few years in high school where I didn't like my math classes, but even then, I didn't really feel like I disliked math. I just disliked the math class because it was boring." He went on to share how Calculus had changed his perspective and how that influenced his view of the course as an instructor:

> *Calculus, for me, had been just such a joyful class. It was sort of like what reignited my interest in math, reminded me that math class was a cool thing. So, when I was teaching it the first semester, it didn't really even occur to me that my students could legitimately be worried about this and frightened about this.*

Other GSIs openly discussed their negative experiences with math. Aaron explained how he shares a personal story with his students to tell them about his "perspective on the culture of math and how there's a fear-based culture of math in many cases." Aaron's story involves being a student in an undergraduate math class where he was handed a sheet of problems to solve:

> *I didn't know how to even start any of them because I hadn't taken the necessary class prerequisites. So, I was just sitting there waiting, not really able to do anything, and the professor stood in front of me and just started staring at me, at my blank sheet of paper for like five minutes. He finally broke the silence by asking, "Do you understand?" And it made me feel really bad.*

By sharing this story with his students, Aaron hopes that they will be able to recognize and acknowledge their own fears and anxieties about math and can better relate to him as an instructor.

Considering Students' Experiences Outside the Classroom

While many GSIs talked about how students' mathematical experiences might influence empathy, several of them also discussed how instructors are likely empathize with their students in ways that go beyond the scope of teaching and learning mathematics. Nine out of the 11 participants addressed this fact by acknowledging that students' experiences outside of the classroom can profoundly affect their learning and emphasized that it is important for instructors to consider non-mathematical factors when dealing with certain situations.

Dealing with Issues in Students' Lives. Bill brought up nonacademic factors during his interview and suggested that part of being a teacher is empathizing with students outside of the classroom:

> *There's also parts of it that aren't even about math, right, that are…I mean even just in the couple of years that I've been teaching, people have come up to me with problems about their life, just in general, and you have to be able to relate to that also, which isn't necessarily math-related. I guess you have to be able to be empathetic, have empathy for students even outside of an academic setting.*

Other GSIs echoed this sentiment. Mark talked about all of the "stuff students are going through, inside and outside of our classrooms that affects their ability to learn and get something out of the class." Similarly, Henry mentioned being aware of and

understanding that there might be lots of different reasons why a student may not be doing well in a class:

> *It might be something to do with their understanding of the material. It might be something to do with how hard they're working. It might be something completely outside of those things involving the student's life and keeping that in mind is probably beneficial.*

Considering Students' Responsibilities. Aaron also discussed the importance of understanding and relating to what students are going through outside of the classroom. Specifically, he noted the importance of considering a student's "history in a "holistic sense" and "conditions like what their schedule looks like, whether they have a supportive family environment or a demanding family environment, whether they have a commute to school that's long, all of these sorts of things." He went on to say that it is necessary for teachers to be aware of these external pressures and to understand "how they impact the student's learning outcomes."

Another GSI, Jane, talked about teaching first-year college students who might be adjusting to a new environment: "you're empathetic to the fact that they just got to college and that they're not only taking your course, but they're transitioning into this new experience, these new classes that are way different than their high school classes." She also clarified that experiencing pedagogical empathy in this way is "more general" and not "necessarily specific to math," but still important, especially for students in their first year of college.

Summary of Factors Influencing Instructor-Student Interactions

The three themes discussed above—instructors' past experiences, students' mathematical experiences, and students' experiences outside of the classroom—can influence interactions between instructors and students. Instructors' past experiences impact whether instructors see themselves as similar or dissimilar to their students, thus serving as an affordance or barrier to pedagogical empathy in instructor-student interactions. In addition, considering students' mathematical experiences can help instructors better relate to their students and understand students' perspectives. Finally, students encounter a variety of issues in their daily lives, which can influence their experiences in the classroom. Acknowledging students' experiences outside of the classroom can help instructors realize what students might be dealing with in their personal lives, so that they can make better informed decisions during instructor-student interactions.

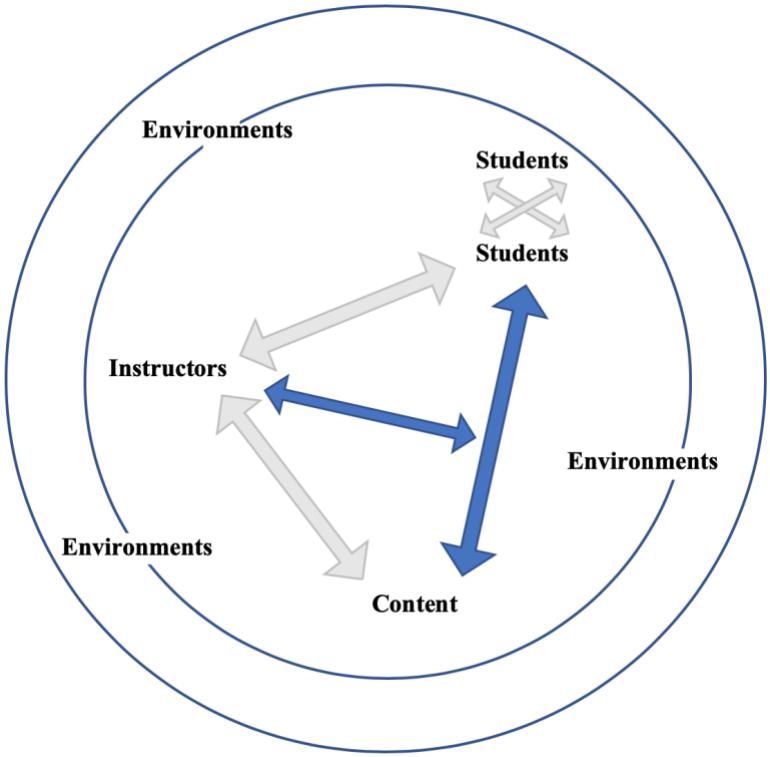
Factors Influencing Instructor-Student-Content Interactions

In addition to discussing factors that might influence instructor-student interactions, my participants also brought up factors related to students' interactions with mathematics, such as whether students felt comfortable asking questions and how much responsibility they felt for their students' learning. These factors can be seen as influencing the instructor-student-content arrow (see Figure 5) and are described below under two main themes: the importance of communicating pedagogical empathy to students and the course structure. Communicating pedagogical empathy to students involves how instructors mediate student-content interactions and whether that process occurs in the classroom. Course structure refers to logistical considerations, such as class

size and set up, as well as teaching responsibilities and classroom norms, which set the stage for classroom interactions involving content. These themes are discussed in depth below.

Figure 5

The Instructional Triangle: Instructor-Student-Content Interactions



The Importance of Communicating Pedagogical Empathy to Students

All of the participants mentioned the communication process involved in expressing empathy to students. Many of them considered whether students are able to perceive pedagogical empathy and discussed how verbal and non-verbal communication might affect classroom interactions around content. Some GSIs emphasized the role of empathy in creating an environment where students feel comfortable asking questions about mathematics, while others referenced back to their own experiences as students and

argued that empathy can make teachers seem more approachable to students who might need help with the course material.

Understanding How Students Perceive Empathy. Several GSIs addressed the question of whether students are able to perceive pedagogical empathy. Beth brought up an example where she had responded to student concerns about homework deadlines by having a group discussion with the class and listening to some of their suggestions. Afterwards, she "updated the schedule to fit the feedback" that her students gave her, while still managing to "fit everything in the class." Beth said that she understands "what it's like to be a student," and that making these adjustments was her way of communicating that empathy to her students. She went on to say,

> *I hope that they feel like I care about them because I do. I care that my students want to come to class and want to be there and want to learn. And I feel by making myself available to suggestions, they hopefully feel that I am empathetic towards them.*

Henry also had similar thoughts about the ways in which students are able to perceive pedagogical empathy:

> *I think if I make it clear that I'm concerned with what their perspective is and what their thinking is, then I imagine they would get the impression that I care what they're feeling and thinking, so that I'm having empathy for them.*

Like Henry and Beth, Joe also talked about how a "student's perception of teacher empathy is something they develop subconsciously" and speculated that empathy "usually comes across as something more specific like: I do or don't feel comfortable asking questions."

Creating a Comfortable Learning Environment. Other GSIs discussed being aware of the ways in which they responded to student questions and emphasized the importance of empathy in creating an environment where students feel comfortable contributing their thoughts. In particular, Chris said that "never talking down to a student" or "making them feel bad for not knowing the correct answer" were ways that he expressed empathy towards his students. Mark also spoke to this in the context of helping a student:

> *You can sort of ask a student where it is that this stops making sense. And you can ask it in such a way that makes them feel stupid, and you can ask it in such a way that makes them realize that you are just genuinely interested in helping them and you're not trying to judge them or something. And I feel like when it comes to empathy, that's the trick: making them feel like you do genuinely care about them and want them to succeed.*

Mark went on to say that "ideally" teachers do genuinely care about their students and want them to succeed, and "communicating that to them…is the art of empathy in the classroom."

Relating Empathy to Approachability. While many GSIs focused on expressing empathy from a teacher's perspective, some GSIs brought up their experiences as students and discussed how their perceptions of pedagogical empathy influenced whether or not they felt comfortable approaching a teacher. For example, Bill shared his experience of taking a class with a teacher who would respond to questions with a look of "how dare you ask this question or not know the answer to it already." As a result, Bill was hesitant to talk to his teacher about mathematics and didn't feel like he was "allowed

to approach that teacher with a question." Bill then stated that a large part of expressing empathy is "the way that you look or present yourself when asked [a] question" and that it is important to think about "your tone of voice" and "mannerisms" when responding to students who are confused.

Oliver was another GSI who discussed some of his experiences as a student and talked about two types of teachers: ones that are willing to start talking and figure out the source of confusion, and others that dismiss the question and tell the student to read the book again or do more homework problems. Oliver went on to say,

> *I'm putting my own experiences in my students, and I feel like they wouldn't come back to my office hours for future questions or wouldn't continue to try to have a dialogue with me if I didn't practice patience…so I think [empathy is] important because I think it gives a sense of approachability, like you're more likely to go back for further questions later on.*

Oliver also said that he expresses empathy to his students by "not showing frustration" and "spending enough time" with them in order to "digest where the confusion is and try to walk them through some concrete thought."

Course Structure

Several participants mentioned how structural components of a course can either support or deter opportunities for experiencing pedagogical empathy in the classroom. The structure of a course refers to logistical considerations, such as class size and physical layout, as well as teaching responsibilities and classroom norms that mediate instructor-student-content interactions. This structure influences the opportunities that instructors have to support student-content interactions, including how often teachers see

their students, how many students are in a course, and what types of interactions are possible. The position and level of authority that teachers have within the course structure can also affect how they view their role in the classroom and their perceived instructional responsibilities. Classroom norms also influence instructor-student-content interactions and have the potential to shape the ways in which teachers experience pedagogical empathy.

Interacting with Students Individually. A few participants discussed how it was easier to empathize with students when the structure of the course allowed for direct interactions with them. For example, Chris discussed how teachers are more likely to experience empathy "when they're not just lecturing at them [students] the whole time and then telling them to beat it." He went on to describe how empathy can be more easily elicited in courses where "two-way communication is heavily encouraged" and the teacher is not the only person who is allowed to talk about mathematics. Amanda felt similarly, stating:

> *The way that our classes are set up, at least in the precalculus level with active learning, I think that also makes it easier to experience empathy because you have a lot more one-on-one interaction with your students in class.*

Similarly, Mark discussed the affordances of being able to "sit down one-on-one" with a student who needs help or is feeling overwhelmed. However, he also mentioned how it is often difficult to "avoid time constraints" and ensure that all students understand a concept before moving on to the next topic and identified the number of students in his class as being a potential barrier to experiencing empathy:

> *In an ideal world where I have time to actually make sure each student gets the help they need, I would try to figure out where the disconnect begins and then work from there. But again, I say this in the context of a classroom where I've got time to do this. In a classroom where there's one teacher and 42 students, corners have to be cut unfortunately.*

Feeling Responsible for Students' Learning. In addition to contextual factors like the structure of a course and the number of students in a class, GSIs' roles and responsibilities might influence the way that they empathize with their students. In particular, Jane talked about the difference between being a recitation leader and being an instructor of record. As the instructor of record for a precalculus class, Jane felt "a lot more of an emotional burden on my students to do well and for me to teach them the material," whereas leading a recitation comes with less "responsibility for their education and what they learn." She also suggested that GSIs might have more opportunities to feel empathy when they are the instructor of record, as compared to being a recitation leader, because having that responsibility for helping students learn is "another way that you can feel empathy."

Managing Group Work. Another facet of the course structure that Jane touched on was the design of the course, where students are encouraged to work on problems in small groups. She discussed how her students might not like "the way that the class is run" with the group work, but that it was important to understand how they were feeling: "whether or not you agree or think that the way that they feel is justified or not…it's still understanding that they feel that way and then also why and how that's affecting them."

Jane also talked about trying to see both sides of the issue and empathizing with multiple students.

> *Sometimes I have students who don't want to work in groups, but then I have students who really want to work in groups. But then their groupmates don't want to work in groups, and so then you have to empathize with both sides of that and figure out what to do to help fix the situation or the issue.*

Mark also brought up group work as a factor that might influence empathy in the classroom. Specifically, Mark was concerned about group dynamics and what happens when "some of the group members more or less get what you're saying, but then there are others who feel left behind." He went on to say, "it's easy to move onward because maybe half the group or most of the group, even all but one student in the group feels comfortable." To address this issue, Mark tries to help individual students without "isolating" them in front of their peers. Often, he does this by including their group members in the conversation, which introduces another dimension to consider:

> *One thing I try and do in a situation where there's one out of the five or six who's struggling is, when possible, it's nice to have one of the other students walk us through something. But then the other students aren't necessarily thinking about how the things they say make their group members feel. Sometimes they are. Sometimes they're very much not.*

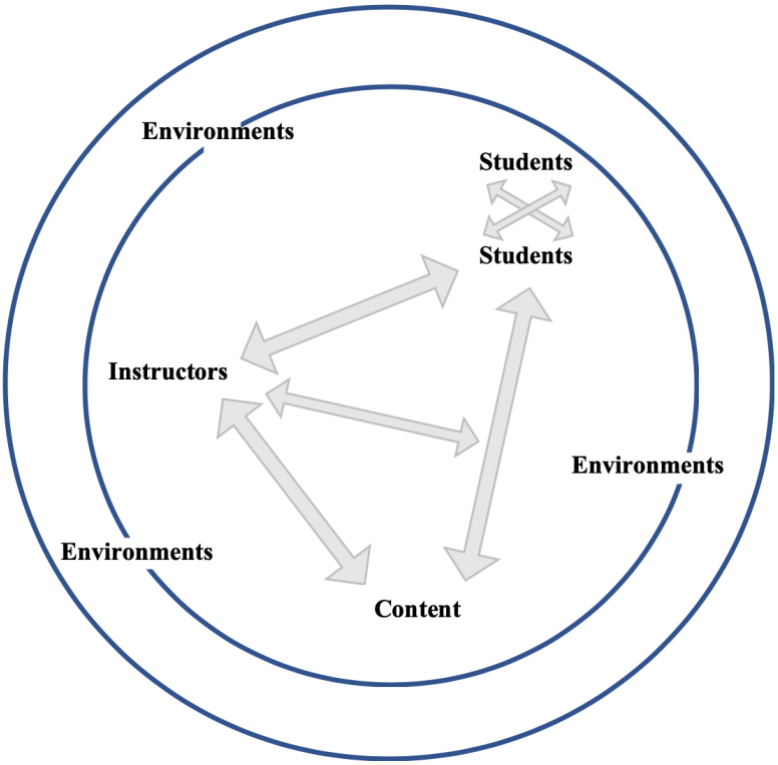
As Mark points out in this quote, encouraging students to interact with one another adds further complexity to the classroom and creates additional ways in which teachers might empathize with their students.

Environmental Influences

The factors discussed above can be seen as influencing instructor-student and instructor-student-content interactions. Since these interactions occur within different environments, these factors can also be seen as a function of the environment influences (see Figure 6).

Figure 6

Environmental Influences



The environment is arguably the least well-defined component of the instructional triangle (Neal & Neal, 2013). However, environmental influences can be seen in many of the factors that emerged from my data. For example, administrative logistics in a department such as mathematical course requirements for graduate students can influence GSIs' experiences as students and help them see similarities (or differences) between

them and their students. In addition, a department's culture can affect instructors' values and whether they perceive communicating pedagogical empathy to be important in their teaching. The environment also influences students' past and current experiences with mathematics as well as their experiences outside of the classroom. Students' experiences, in turn, influence how instructors might empathize with them. Finally, environmental influences dictate how courses are structured and what types of interactions are possible between instructors, students, and content. Thus, the environment mediates the ways in which instructors might empathize with their students.

Discussion and Conclusion

The themes described above highlight several factors that have the potential to influence how GSIs experience pedagogical empathy. These factors can greatly impact interactions between instructors and students. Table 8 shows each of the themes and the corresponding subthemes that emerged from my data. In this section, I discuss how these themes address my research question, connect my findings to Davis's (1996) organizational model of empathy, and pose further questions worth investigating in this area of research.

As suggested by my findings, instructors' past experiences can either support or hinder the ways in which they empathize with their students. It may be easier for instructors who see similarities between their own experiences and those of a student to empathize with that student, while lacking similar experiences might make it more difficult. These findings agree with other research that has examined the relationship between similar experiences and empathy (Davis, 1996; Eklund, Andersson-Stråberg, & Hansen, 2009). Environments that allow instructors to interact with their students can

help instructors make connections with their students and find similarities with them, thus supporting instructors' pedagogical empathy.

Table 8

Factors Influencing GSIs' Pedagogical Empathy: Themes and Subthemes

Categories	Themes	Subthemes
Factors Influencing Instructor-Student Interactions	Instructors' Past Experiences	Remembering back to experiences as a student
		Lacking similar experiences
		Developing empathy through reflection
	Students' Mathematical Experiences	Acknowledging students' mathematical backgrounds
		Reflecting on personal mathematical background
	Students' Experiences Outside the Classroom	Dealing with issues in students' lives
		Considering students' responsibilities
Factors Influencing Instructor-Student-Content Interactions	Importance of Communicating Pedagogical Empathy to Students	Understanding how students perceive empathy
		Creating a comfortable learning environment
		Relating empathy to approachability
	Course Structure	Interacting with students individually
		Feeling responsible for students' learning
		Managing group work

In addition to instructors' past experiences, students' mathematical experiences have the potential to influence how GSIs interact with their students. As Mark (and other GSIs) acknowledged, students "come from a lot of backgrounds, many in which they're

taught to hate math or to think of themselves as bad at math." Experiencing empathy can increase GSIs' awareness of these different backgrounds and help them understand how their students might feel towards mathematics. If GSIs are aware of these negative perceptions or fixed mindsets, they can then take steps to reframe them and encourage their students to view mathematics in a more positive light.

It is also important to recognize that empathy can be elicited in ways that are beyond the scope of teaching and learning mathematics. Students' experiences outside the classroom also have the potential to influence interactions between instructors and students. As many of the GSIs noted, students encounter a variety of issues in their daily lives, which can influence the learning that happens in the classroom. Pedagogical empathy can help instructors understand what students are experiencing both inside and better attend to individual student needs.

The importance of communicating pedagogical empathy to students deals with the interactions between instructors, students, and content. Many of the GSIs in my study were cognizant of how expressing empathy can help students feel more comfortable interacting with their instructors. As Oliver noted, empathy is important because it helps create "a sense of approachability" and students are more likely to ask questions to an instructor who they view as approachable. Drawing on empathy can also help teachers communicate to their students that they care about them and want them to succeed, which is, as Mark stated, "the art of empathy in the classroom." Thus, pedagogical empathy is an important component of responding to students.

Finally, the structure of a course is another major influence on interactions between instructors, students, and content. As some of the GSIs pointed out, instructors

are more likely to experience empathy in certain types of settings—small classes that allow for more "one-on-one" interactions with students. In particular, Amanda felt that the structure of the precalculus level courses provided her with opportunities for these types of interactions, thus making it "easier to experience empathy." Amanda's perspective contrasts with that of Mark, who was concerned about the number of students in his precalculus course and viewed this structure of "one teacher and 42 students" as a potential barrier to empathy. Even though Amanda and Mark were teaching in similar contexts, they had different perspectives on how the structure of the course might (or might not) afford opportunities to experience empathy. Thus, these examples suggest that the structure of the course can be seen as either an affordance or a barrier to empathy, depending on the view of the instructor.

Limitations

One limitation of this study is its primary focus on the instructor perspective of empathy in the classroom. While this perspective is important to understand, it is also necessary to understand how students conceptualize empathy in the classroom. Future work should include students' perspectives on pedagogical empathy. For example, researchers could study when and how students perceive pedagogical empathy in the classroom, and how that might impact their experience in the course overall. Understanding how students view empathy in the classroom would provide useful insights for both instructors and leaders of professional development.

Another limitation is the narrow population of instructors who participated in this study. The graduate student instructors who I interviewed likely have different views of empathy than more experienced instructors, who may not clearly remember their own

experiences as students. In addition, the culture of a department might influence instructors' views on empathy. For example, instructors in mathematics departments that value and incentivize teaching may have different opinions on pedagogical empathy compared to instructors in departments at other institutions. It would be worthwhile to extend this study to a broader range of participants in order to investigate similarities and differences across different populations of instructors.

Applying Davis's Model of Empathy to the Context of Teaching

Many of the themes that emerged from my data align with the components of Davis's (1996) organizational model of empathy. Instructors' past experiences are part of the antecedents in Davis's model. These past experiences contribute to an instructor's capacity for empathy and whether they see themselves as similar to their students or not. In addition, students' mathematical experiences and students' experiences outside of the classroom also relate to the antecedents construct. Students' learning histories and attitudes towards mathematics are antecedents of the target and could influence the strength of the situation in an empathic experience. As Davis (1996) noted, strong displays of emotion by targets often lead to powerful empathic responses in observers. Students who show their emotions in class, therefore, are more likely to evoke pedagogical empathy.

The processes construct from Davis's model was also apparent in several comments made by my participants. This construct involves attempts by instructors to imagine or understand their students' perspectives, an advanced cognitive process in the observer. An example of this came up in one of Henry's quotes about communicating empathy to students. In this conversation, Henry discussed considering students'

perspectives and trying to understand their thinking, which require advanced cognitive processes associated with empathy. Jack also referred to these processes when he stated that it "takes *skill* to empathize with people who are further away from you in life experience" (emphasis mine).

Intrapersonal outcomes were hard to discern from my findings, possibly because these outcomes relate to the emotional reactions within the observer that they may or may not consciously acknowledge. One of the few instances of intrapersonal outcomes in my findings can be seen in Jane's comment about understanding how her students were feeling and why they were feeling that way: "whether or not you agree or think that the way that they feel is justified or not…it's still understanding that they feel that way and then also why and how that's affecting them." In this quote, Jane mentions inferring why her students are feeling a certain way—an example of an attributional judgment or intrapersonal outcome.

The importance of communicating empathy to students relates to interpersonal processes, or the behavioral responses that are directed toward the target. My participants discussed how they might respond to their students in order to express their empathy and make students feel comfortable approaching them with questions. Thus, these responses involved with communicating empathy to students are interpersonal outcomes that might be visible to someone other than the observer and target.

Finally, while some of the factors discussed under course structure may fall under the antecedents construct, Davis's organizational model of empathy does not fully capture the essence of this theme. An underlying assumption of Davis's model is that observers and targets have opportunities for interactions. A potential reason why course

structure does not fit within this model is because teachers and students may not always have these opportunities for interaction. Therefore, it makes sense that instructors would consider how factors related to the course structure might make it more (or less) likely for them to interact and empathize with their students.

Implications and Future Research

One question prompted by my findings is whether GSIs might be able to *translate* their experiences from one context to another in order to empathize with students. During his interview, Joe discussed being able to "contextualize" what his students might feel by recalling his experience of taking a driving course that he did not enjoy. Joe's story suggests that this *translation* of empathy might be possible, although it is unclear what motivated it. Future research in this area should examine how empathy in one context can be translated to another and how to increase instructors' awareness of this process. In the meantime, leaders of GSI professional development should design activities that encourage GSIs to reflect on their past experiences as both teachers and students in a variety of contexts. Engaging in this reflection might help GSIs better understand how to leverage past experiences—which may or may not be related to mathematics—in order to empathize with a wider range of students in different ways.

A second question that remains unanswered is how pedagogical empathy might be related to equity in the classroom. Are GSIs more likely to empathize with students with whom they more closely identify? Could this tendency lead to inequitable teaching practices? Are there ways to identify and avoid this occurrence? If so, how might professional development be structured to acknowledge this fact and help expand GSIs' perspectives to include a broader awareness of student experiences? Future research in

this area could help shed light on some of these questions and better explain the connection between empathy and equity.

Conclusion

Many facets of pedagogical empathy remain unexplored. Further research is needed to better understand the concept of pedagogical empathy and its connections to different types of knowledge that inform teaching decisions. This work sheds light on factors that can influence instructors' pedagogical empathy and provides insights for leaders of GSI professional development to consider. By encouraging GSIs to reflect on both their own and their students' experiences, leaders of professional development can increase GSIs' awareness of pedagogical empathy and help them identify different ways to relate to their students, thus supporting their growth as undergraduate mathematics instructors.

CHAPTER 6: THE INFLUENCE OF PEDAGOGICAL EMPATHY ON TEACHING DECISIONS

Teaching is a complex practice. It involves various individuals (e.g., learners, educators, administrators) in various settings (e.g., classes, office hours, tutoring centers) undertaking various sorts of human endeavors (e.g., learning how to complete the square, understanding different perspectives). These components are all "multi-dimensional, non-linear, interconnected, far from equilibrium and unpredictable" (Kuhn, 2008). In this chapter, I examine one component of teaching practice—decision-making—through a lens of empathy. In what follows, I discuss an in-depth case study of four graduate student instructors (GSIs) in which I embrace the complexity of teaching interactions and analyze connections between teachers' conceptualizations of pedagogical empathy and their decision-making processes. This chapter builds on the work in the previous two chapters that discuss how GSIs conceptualize empathy. I now investigate how pedagogical empathy might influence GSIs' decision making in interactions with their students. I use the findings from this study to make the argument that pedagogical empathy can be conceptualized as a filter that connects pedagogical content knowledge and emotional knowledge to decision making.

Defining Pedagogical Empathy

Empathy is a complicated phenomenon to study, partially because it is difficult to define. Numerous definitions of empathy abound in research literature. In fact, de

Vignemont and Singer (2006) claimed that "there are probably nearly as many definitions of empathy as people working on the topic." Following the work done in Chapter 5, I define empathy as "a social interaction between any two individuals wherein one individual experiences the feelings of a second individual" (Feshbach & Feshbach, 2009, p. 85). In addition, I propose the use of the term *pedagogical empathy* to refer specifically to the empathy that a teacher experiences because of an interaction with a student or group of students. This pedagogical empathy has the potential to influence teaching decisions and involves how teachers use their understanding of students' perspectives to inform the actions they take in the classroom.

Pedagogical Empathy as a Filter

This chapter aims to expand on Zembylas's (2007) framework, which established a connection between pedagogical content knowledge (Shulman, 1986, 1987) and emotional knowledge (Zembylas, 2007). Zembylas (2007) contended that "teacher knowledge is a form of *knowledge ecology* – a system consisting of many sources and forms of knowledge in a symbiotic relationship" including "content knowledge, pedagogical knowledge, curriculum knowledge, knowledge of learners, emotional knowledge, knowledge of educational values and goals and so on" (p. 356, emphasis mine). Specifically, Zembylas (2007) defined emotional knowledge and used the term "emotional ecology" (p. 356) to explain the relationship between emotional knowledge and pedagogical content knowledge.

Pedagogical content knowledge refers to "the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented

for instruction" (Shulman, 1987, p. 8). In their review of pedagogical content knowledge in mathematics education research, Depaepe, Verschaffel, and Kelchtermans (2013) found that definitions of pedagogical content knowledge included the following characteristics: "it deals with teachers' knowledge, it connects content and pedagogy, it is specific to teaching particular subject matter, and content knowledge is an important and necessary prerequisite" (p. 22). In addition, most conceptualizations of pedagogical content knowledge involved knowledge of students' (mis)conceptions and knowledge of instructional strategies and representations (Depaepe et al., 2013).

Zembylas (2007) used the term emotional knowledge to refer to "a teacher's knowledge about/from his or her emotional experiences with respect to one's self, others (e.g. students, colleagues), and the wider social and political context in which teaching and learning takes place" (p. 356). He also identified various types of emotional knowledge and categorized these under individual, relational, and socio- political planes of emotional ecology (see Table 9).

Table 9

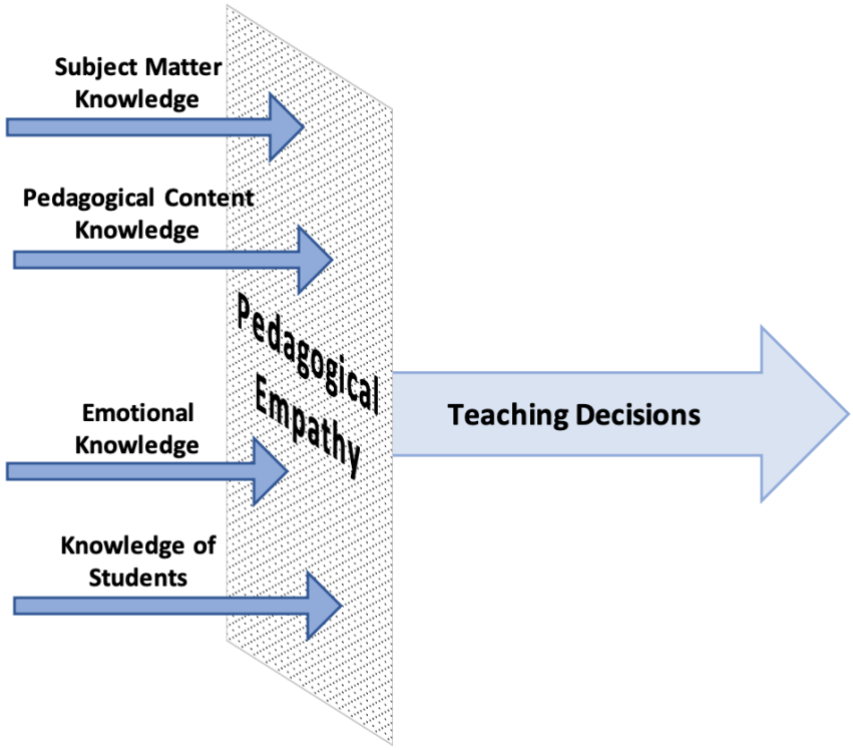
Planes of Emotional Ecology and Types of Emotional Knowledge (Zembylas, 2007, p. 358)

Planes	Types of emotional knowledge
Individual	Emotional connections to the subject matter; attitudes and beliefs about learning and teaching; educational vision and philosophy; emotional self-awareness
Relational	Emotional affiliations with students; students' own emotional experiences; caring; empathy; classroom emotional climate; knowledge of students' emotions; social-emotional interactions
Socio-political	Emotional knowledge of the institutional/cultural context (power relations); emotional understanding of curricular deliberations; emotional politics of pedagogies and subject matter discourses

Building on Zembylas's (2007) work, I conceptualize pedagogical empathy as a pedagogical filter that connects pedagogical content knowledge and emotional knowledge (and potentially other types of knowledge) to teachers' decisions (see Figure 7). By *filter*, I mean a construct that teachers use to determine what types of knowledge should inform their teaching decisions. Studying this filter can help researchers explain why teachers draw on different types of knowledge, such as emotional knowledge and pedagogical content knowledge, as they make decisions. The findings presented in this paper describe how GSIs might use this pedagogical filter in one-on-one interactions with their students.

Figure 7

Pedagogical Empathy as a Filter



Case Study

My argument is grounded in analysis of a collective case study guided by the question: *How might GSIs' pedagogical empathy influence their decision making in the context of interactions with their students?*

GSIs are an interesting population to study when exploring the phenomenon of pedagogical empathy because of their unique position as both teacher and student and because most GSIs are novice instructors. Asking them to reflect on pedagogical empathy has the potential to both induce empathy for their students and influence their actions in the classroom in ways that may not have occurred otherwise. Because they are novices, they may be likely to notice and point out situations where they might have responded differently to their students, which came out during interviews in this study.

The participants in this study were GSIs in a mathematics department with an extensive training program for GSIs. In addition to participating in pre-semester orientations geared towards teaching, all of the GSIs had taken or were enrolled in a year-long pedagogy course to help them become better instructors.

Literature Informing the Case Study Design

Decision Making

The analysis presented in this chapter draws on a framework developed by Schoenfeld (2008, 2010, 2011b). In his work, Schoenfeld maintains that teacher's in-the-moment decision making can be modeled as a function of their resources, orientations, and goals. Resources refer to an individual's knowledge as well as social and material resources that are available to them, while teachers' orientations include their beliefs, values, preferences, and tastes (Schoenfeld, 2011b). Goals are objectives that teachers

consciously or unconsciously set out to achieve and can include both broad and specific aims (Schoenfeld, 2011b). The decisions that teachers make can be viewed as choosing goals that are consistent with teachers' resources and orientations. Thus, it is important to examine how these three components of decision making are interconnected, rather than selecting one to study in isolation (Schoenfeld, 2011b).

Pillars of Active Learning

The context of this study involves precalculus courses that are highly coordinated (Rasmussen & Ellis, 2015) in part to promote the use of active learning methods. Active learning is a broad term that includes many different types of teaching methods and classroom norms. Laursen and Rasmussen (2019) used the term active learning to refer to the process of involving students in "higher order thinking tasks such as analysis, synthesis and evaluation" (p. 131). Furthermore, active learning requires that "students talk to each other about what they are doing and thinking, as conversations are powerful in clarifying, solidifying, and elaborating learners' ideas" (p. 131). Laursen and Rasmussen (2019) presented four pillars that make up the foundation of effective teaching using inquiry-based active learning methods. The design of this case study draws heavily on two of these pillars: instructors' inquiry into student thinking, which relates directly to how instructors empathize with students, and instructors' abilities to foster equity in their choices.

Study Design

In this study, I used empathy as a lens through which to look at teachers' in-the-moment decision making. By employing stimulated-recall interview methods (Calderhead, 1981) and analyzing specific teaching interactions, I break down how

teachers' pedagogical empathy might influence the decisions they choose to make while working with students.

Data

I collected data during the fall of 2018 through four interviews and three classroom observations. The analysis presented in this chapter draws on both interview and observation data. Before being observed, each GSI met with me for an initial interview to elicit their thoughts on pedagogical empathy. Prior to this interview, I did not prompt or encourage the GSIs to think about empathy. The purpose of this interview was to gain insights into how my participants conceptualized empathy in a teaching setting and make them more aware of their thoughts about empathy for future observations and interviews.

Following their initial interview, I observed the GSIs three times over the course of the semester. Each observation was separated by approximately one month, and all observations were video recorded with the instructor using a wireless microphone. Immediately after each observation, I would ask the GSI if they had noticed any particular interactions with a student or small group of students. Typically, the GSIs would point out one or two interactions they had noticed, which I used to narrow the focus of our post-observation interview. If GSIs did not point out any specific interactions, I would select one or two interactions that involved a meaningful exchange of feedback from the classroom observation. Before the post-observation interview, I would watch the portions of their video of their classroom observation that they had pointed out to me, transcribe the conversation between the instructor and student(s), and develop specific questions based on that interaction.

The post-observation interview typically occurred as soon after the observation as possible in order to increase the likelihood of GSIs accurately recalling their thoughts and feelings in the moment. Typically, I met with the participants within one to three days after their observation. I began each interview by reminding the participant of the definition of empathy they had given during previous interview and asking whether there was anything about that definition they wanted to change. We then watched a video of an interaction they had pointed out to me after their observation and went through a series of questions related to that observation. These questions typically involved asking GSIs what they had noticed or why they had thought the interaction was significant, whether they had experienced empathy, and what goals, knowledge, and beliefs they might have been thinking about during the interaction. A sample post-observation interview protocol can be found in Appendix B. The findings in this chapter come from observation data and interview responses related to selected interactions with each participant.

Participants

I conducted the analysis for this chapter from the data that I collected from four graduate student instructors who were involved in the larger qualitative study. These GSIs were graduate students in a mathematics PhD program, who were teaching a precalculus course as the instructor of record. GSIs selected their own pseudonyms, which are used throughout this chapter. Table 10 shows the courses that these four GSIs were teaching and their year in the graduate program. The GSIs are listed in alphabetical order by their chosen pseudonym.

Table 10

Participant Information

Instructor	Course	Year in Graduate Program
Aaron	College Algebra	2
Jane	College Algebra	2
Joe	Precalculus	4
Oliver	Trigonometry	4

Aaron and Jane were both second-year graduate students who were teaching College Algebra for their first time as the instructor of record. College Algebra was a 3-credit course which met for 75 minutes, three times a week. Aaron had approximately 40 students in his class, while Jane was teaching a special section of the course and had 11 students in her class. Jane's section of College Algebra was restricted to students in the William H. Thompson Scholars Learning Community. Students in this program must be in need of financial assistance in order to attend college, and a majority of them identify as first-generation college students. At the time of data collection, both Aaron and Jane were currently enrolled in a year-long pedagogy course designed to help GSIs become more reflective and informed teachers.

Oliver and Joe were fourth-year graduate students who had previously gone through the pedagogy course at this institution. Thus, Oliver and Joe were more experienced GSIs, who each had at least two years of teaching experience as the instructor of record for a precalculus course. At the time of data collection, Oliver was teaching Trigonometry, which was a 2-credit course that met for 50 minutes, twice a

week. Joe was teaching Precalculus, a 5-credit course which met every day for either 50 minutes (MWF) or 75 minutes (TR). Joe had approximately 40 students enrolled in his class, while Oliver had approximately 35 students enrolled in his class.

These four participants were selected based on their conceptualizations of empathy. Aaron and Jane were chosen because they were the only two GSIs who responded no when asked if behavioral outcomes or resulting actions are part of the process of empathizing with students. In addition, Aaron and Jane were both second year GSIs who were instructors of record for the first time so they could serve as a comparison for one another. Joe was selected as a case because he was the only GSI to profess to hold this view that empathy was a cognitive process, although it was clear from his post-observation interviews that the affective part of empathy was also important to Joe. Finally, Oliver was selected because he was the only GSI to say that congruency and self/other distinction or merging were not important dimensions of empathy. (See Chapter 4 for more information about GSIs' conceptualizations of empathy.) Furthermore, Oliver and Joe were both fourth year GSIs and had a comparable amount of teaching experience. By analyzing data collected from these four GSIs, I was able to compare how different views of empathy might affect the ways in which instructors use pedagogical empathy as a filter to make teaching decisions.

Analysis

I first transcribed all interviews using MAXQDA 2020 (VERBI Software, 2019). Next, I analyzed GSIs' definitions and conceptualizations of empathy to determine their views on different dimensions of empathy (see Chapter 4). After this preliminary

analysis, I selected four participants to use for comparison in this study. Participants were chosen based on their views of empathy and to provide a varying range of experiences.

After selecting these four cases, I went back through their post-observation interviews and rewatched videos of the student interactions that we had discussed in them. I then chose two specific interactions for each GSI to analyze further to narrow my focus. During this phase of analysis, I selected interactions where I could see explicit connections between different types of knowledge and GSIs' decision making. While going through the data, I wrote up detailed summaries of each interaction and then analyzed the corresponding interview questions about that interaction using thematic analysis. Finally, I developed descriptive narratives to explain how pedagogical empathy can be conceptualized as a filter that connects pedagogical content knowledge and emotional knowledge to decision making.

Unpacking GSIs' Decision Making Through Their Conceptualizations of Empathy

The findings presented below are separated into cases based on the four participants: Jane, Aaron, Joe, and Oliver. For each case, I first provide a summary of the GSI's conceptualization of pedagogical empathy. I then describe two teaching interactions that I observed and discuss the GSI's interpretations of the interactions. Finally, I unpack how the GSIs might have used pedagogical empathy as a filter to determine what knowledge to draw on while making teaching decisions.

Jane: Using Pedagogical Empathy to Navigate Group Dynamics

At the time of data collection, Jane was a second year GSI who was teaching College Algebra (75-minute classes, three times a week, 11 students enrolled) as the instructor of record for the first time. She had previously led Calculus I recitation sections

as a first-year graduate student. During her first observation, Jane had 11 students in attendance; and during her third observation, she had nine students in attendance. In the following sections, I discuss Jane's conceptualization of empathy and two of her interactions with students that I observed, one during her second observation and the other during her third observation. I then analyze how Jane's pedagogical empathy might have influenced her teaching decisions in these two interactions.

Jane's Conceptualization of Empathy

In her initial interview, Jane wrote down the following definition of empathy as it relates to a teaching context:

> *Being able to understand the class from the students' perspective and understand how students feel/why they feel the way they do, whether it's about the material, the class, environment, classmates, or teacher and how it affects their experience in the classroom.*

To illustrate her definition, Jane gave a couple of different examples of how an instructor could empathize with her students. The first example dealt with empathizing with students when they are struggling to understand a mathematical concept, while the second example dealt with understanding when and why students might be unhappy with the way the course is going. Jane also talked about differences between students and their preferences and how an instructor might have to "empathize with both sides" of a situation.

Throughout the interview process, Jane expanded on these examples and further discussed her view of pedagogical empathy. During her final post-observation interview, Jane had the opportunity to read through four other definitions of empathy written by

other participants in the study. From these definitions, Jane pointed out how empathizing with students includes understanding their "obligations" outside of class and predicting how they are going to feel about the material before going into class.

Jane also discussed her view on different dimensions of empathy during her final interview. When talking about whether empathy involves cognitive or affective processes, Jane thought that both were necessary. She remarked:

> *You have to think about the situation that [students] are in in order to understand how they're feeling, like the different factors of the situation and how they affect your student[s]. But then part of empathizing with others is feeling how they're feeling, so there definitely is an emotional aspect involved.*

When discussing the dimension of self/other distinction, Jane replied that it was important because an instructor's "thoughts and feelings could probably get in the way" and affect how they are perceiving students' thoughts and feelings. As Jane remarked, "you want the student to learn and feel comfortable doing math…so you want to put your own opinions and feelings aside in order to make sure that you're giving all of your students an equal opportunity to learn." She went on to say that "you have to put your own feelings aside in order to empathize with students in a reasonable way…to understand how they're feeling and what you can do to help."

Jane felt that behavioral outcomes were "separate" from empathy and gave a couple of examples to explain her reasoning. One example she brought up was working with a student one-on-one in class who is "really struggling." As an instructor, Jane said, "you have to decide how much time you can give to this one specific student compared to other students." She went on to say that, "you want to be fair with how you treat your

students, so that's part of what you have to take into account when you're reacting to your feelings of empathy." Thus, in Jane's view, resulting actions are influenced by more than just empathy and should be thought of separately.

Jane's Interaction with Maria[6]: Managing Group Dynamics

In her first observation, Jane helped one of her students, Maria, as she was working on Problem 4 in the student workbook shown in Figure 8. Maria was trying to figure out the context of the problem so that she could answer parts (a) through (e).

Figure 8

Exponential Functions: Problem 4

Problem 4. The grades of six students are given by the equations below, where time is measured in weeks since the first midterm exam.

(i) $P = 97(1.001)^t$	(ii) $P = 58(1.05)^t$	(iii) $P = 72(0.9)^t$
(iv) $P = 85(0.99)^t$	(v) $P = 79(1.03)^t$	(vi) $P = 85(0.89)^t$

(a) Alex's grade has been decreasing since the first exam; Which formula(s) could represent her grade?

(b) Which student's grade is falling the fastest?

(c) Umar's grade has been rising since the first exam; Which formula(s) could represent his grade?

(d) Which student's grade is rising the fastest?

(e) Karan came out of the first exam with a 85% in the class; which formula(s) could represent his grade?

Jane's interaction with Maria began when Jane sat down at Maria's table to check in with her group. Maria immediately asked Jane, "Is this the weeks?" as she pointed to one of the variables in her workbook. Jane read through the problem and explained to Maria that the variable *P* represented grades and the variable *t* represented time. Maria

[6] All of the student names that are used in this chapter are pseudonyms.

then asked how many weeks had passed, offering the answer of 0.9 from equation (iii) in Problem 4. Jane corrected her by pointing out that 0.9 represented the growth factor in the equation and referenced back to a previous example to explain that the growth factor is the amount being multiplied by the current value to get the next value. Maria then asked the question, "So I just have to do that to each one?" Jane's response to this question was to restate what part (a) of Problem 4 was asking and to bring Maria's three group members into the conversation.

Jane asked the rest of Maria's group "How did things go for Problem 4?" However, the other three members of the group had already moved on and thought Jane was asking them about the next problem. The three other group members began talking about how to solve Problem 5 before Jane reiterated that she was asking about the previous problem, Problem 4. Jane and Maria's group members discussed how to come up with answers for parts (a) and (b), before Maria's group members began talking about Problem 5 again. Before Jane left the table, she turned back to Maria, looked over her work on Problem 4, and asked, "Do you feel good about this one now?" Maria said, "yeah," to which Jane replied, "Okay. Awesome." Jane's interaction with Maria and her group lasted approximately six minutes, and a transcript of it can be found in Appendix C.

After watching through this interaction in her post-observation interview, Jane discussed the complicated group dynamics at this table. Jane remarked, "I felt kind of bad that I asked for her groupmates input, but then they had already moved on to the next problem." Jane went on to say that she was worried that Maria might be "embarrassed or something since she still hadn't figured out number four." To complicate matters further,

Jane's students had just taken the first exam, and Maria was the only student out of her group that had failed, while her other three group members had gotten an A on the exam. Jane was concerned that Maria "might have not felt comfortable asking" her group members for help and was unsure of how to encourage Maria to do so.

Jane also talked about being aware of how Maria might have felt and taking steps to try to avoid making her feel bad:

> *I was trying to not make [Maria] feel bad when her groupmates were ahead. I was trying to be careful about what I said so that she didn't feel bad about being on the previous problem because then I feel like she would have just gotten discouraged and maybe not paid as much attention or would've been too self-conscious to really think about the math.*

Jane went on to say that bringing attention to the fact that Maria was "behind" in front of her group members "would've been a distraction for her and it would have negatively affected her…even if it was unintentional to make her feel bad for being behind."

Jane also discussed the role that gender and race might have played in this interaction. Maria identified as a Latina, while the other three students in her group were white. Before watching this interaction, Jane had mentioned how Maria had taken her high school math class in Spanish and did not know what the acronym "FOIL" meant in a previous class. After realizing that, Jane "was more aware of how [Maria's] experience" might have been different from her own. Moreover, Jane was aware of how stereotypes in mathematics might affect how Maria felt in that situation:

> *The stereotypical mathematician is not a person of color or a woman, usually, and so I feel like people from underrepresented groups are predisposed to think that they aren't good at math to begin with, so I feel like I try to keep that in mind.*

Jane also pointed out that she had put Maria in a group with Zach, the only male student in the class who often contributed to group and class discussions. In a subsequent interview, Jane talked about how she considered who Zach would be working with in his small group: "I try to be careful of who I put him in groups with, especially like with [Maria], who's very quiet and might not speak up or could feel discouraged easily by being in a group with somebody" like Zach.

Even though Jane's discussion with the group turned to Problem 5, Jane returned back to Maria before she left the table. When asked why she did this, Jane replied,

> *I wanted to make sure that she knew that I cared that she understood it…especially because then I started talking with the rest of the group about the next problem…I felt like I needed to go back and check with her and make sure that she was doing okay.*

By returning to Maria and her work on Problem 4, Jane hoped to "tell [Maria] that she was doing a good job and that it was good that she got it" even though she had yet to start on Problem 5.

In this interaction, Jane used pedagogical empathy to filter her emotional knowledge of how Maria might feel and her pedagogical content knowledge of how to help students learn about exponential growth to guide the way she responded to Maria and her group members. During the interaction, Jane was careful to not draw attention to the fact that Maria was "behind" her other group members and instead tried to include

them in her conversation with Maria. Ultimately, Jane's pedagogical empathy helped filter her knowledge of her students' personalities and identities in order to navigate the group dynamics at play and decide how to help Maria solve the problem without making her feeling discouraged.

Jane's Interaction with Keisha: Identifying the Vertex of a Parabola

In her third and final observation, Jane helped one of her students named Keisha. We had previously discussed Keisha after Jane's second observation, and Jane had mentioned to me that she was more aware of her interactions with both Keisha and Maria because we had talked about them in previous interviews. Before watching her interaction with Keisha in Jane's third post-observation interview, Jane brought up a time when Keisha had begun "tearing up" halfway through a class. Jane told her that she could "go to the bathroom or just step outside" if she needed a minute, because Jane "knew that obviously, in that moment, [Keisha] wasn't in a spot where she could learn math." Jane also said, "I felt really bad that there was something going on, and I didn't know if [Keisha] was getting upset because of the material, if she was just getting really stressed out or if it was something going on outside of class." Jane thought about asking Keisha if she was okay after class but ended up not talking to her because she did not want to "overstep" or "make [Keisha] feel like she had to tell me what was going on."

In the interaction that I observed during Jane's third observation, Keisha was working on Problem 3 (a) shown in Figure 9, trying to find the vertex of a parabola. Jane sat down next to Keisha and spent approximately two minutes talking to two of Keisha's group members about the workbook problems. Keisha was silent during this time and did not contribute to this group conversation. The interaction between Keisha and Jane began

after Jane turned to Keisha and asked her directly how she was doing. Keisha shook her head and said, “I don’t get it.” Jane asked whether Keisha wanted to talk about a specific problem and pointed to Problem 3 (a) in Keisha’s workbook. Jane then started asking Keisha questions about the problem trying to help guide her to finding the vertex of the parabola from the equation given.

Figure 9

The Vertex of a Parabola: Problem 3

Problem 3. For each of the following, find the vertex and the axis of symmetry. Does each parabola open upward or downward?

(a) $f(x) = 0.5(x-3)^2 + 1$ (b) $g(x) = -(x+6.1)^2 - 8.5$

Throughout the conversation, Keisha made several statements that were incorrect. After first identifying the vertex correctly as (3,1), she later asked if the value 0.5 was the vertex. Keisha also seemed confused by the different parts of the equation and struggled to find the axis of symmetry of the parabola (see time mark (19:20) in the transcript). Jane’s conversation with Keisha lasted a total of eight minutes. A full transcript of Jane’s interaction with Keisha can be found in Appendix C.

In her post-observation interview, Jane again brought up the group dynamics at play in her interaction with Keisha. Jane talked about how she purposefully avoided bringing Keisha’s group members into the conversation because she was concerned about how Keisha might feel. Jane remarked, “I don’t want [Keisha] to feel bad if I’m just asking her groupmates to explain stuff because they understand it.” Jane went on to say that she did not know whether involving Keisha’s group members might have been

"more productive or more harmful for [Keisha]" and that she did not want to make Keisha feel "uncomfortable or self-conscious."

Jane also discussed trying to figure out what was confusing Keisha: "I feel like there were a lot of different things going on that were sort of keeping her from understanding the problem, and it was hard to pinpoint exactly what all of those were and address all of them." Jane noted that a goal that she should have when helping Keisha is "helping her figure out what she doesn't understand so that she can ask specific problems, even if the thing that she's not understanding is all of it."

When asked whether she experienced empathy in this interaction, Jane responded yes because she remembered trying to determine how Keisha felt while they were working through the problem. At one point, Jane felt "frustrated" because they "were going through it very carefully and taking a long time on it, but then even at the end of it, it didn't really seem like [Keisha] was understanding what we were doing." However, Jane said that she was "really trying to make sure that [her frustration] wasn't showing" because she did not want to make Keisha "feel worse if she was already feeling bad about not understanding" the problem. Jane also noted that she was not "angry" at Keisha for struggling with the problem, but Jane was frustrated because she "didn't know what else to do or say or how else to explain" the problem to Keisha.

Towards the end of the post-observation interview, Jane talked about how she felt after helping Keisha and about how Keisha might have felt after this interaction. Jane said that Keisha was probably still confused after their interaction. She also talked about how it was probably very discouraging for Keisha to feel that way after Jane had spent a lot of time trying to help her understand that problem. Jane was also discouraged after

their interaction and felt bad knowing that she was leaving Keisha's table with Keisha still "not totally understanding all of it." Thinking back, Jane said that if faced with a similar situation in the future, she would "try to think of something positive to say" before ending the conversation so that "the student maybe would feel a little bit better."

In this interaction, Jane tried to understand how Keisha might be feeling while helping her with this problem. Even though Keisha had failed the first two exams, Jane still felt compelled to help Keisha to try to improve "her self-confidence." Jane also thought that Keisha might feel "uncomfortable" asking her group members for help. Thus, Jane's emotional knowledge of Keisha and her knowledge of the group dynamics filtered through her pedagogical empathy, which led to her decision to talk to Keisha one-on-one, rather than bring her other three group members into the conversation. However, after watching this interaction, Jane was uncertain of whether she was able to help Keisha understand the mathematics at hand, and as Jane acknowledged, Keisha was probably still confused and discouraged after their conversation. Thus, while Jane's pedagogical empathy was informing her decision making, she felt like she was unable to help Keisha and questioned whether she should have encouraged Keisha's group members to help her.

Jane's Filter: Using Pedagogical Empathy to Navigate Group Dynamics

The decisions that Jane made in her interactions with Maria and Keisha can be seen through her filter of pedagogical empathy. While working with Maria, Jane used her pedagogical empathy to draw on her emotional knowledge of how Maria might feel working in a group with students who were working ahead without her. Although Jane classified resulting actions and empathy as two separate processes when discussing the

dimensions of empathy, Jane saw how her pedagogical empathy influenced her actions to avoid making Maria feel "embarrassed" about being behind her group members. Jane also thought this pedagogical empathy led her to the decision to check back in with Maria before leaving the table. Jane's last question to Maria: "Do you feel good about this one now?" helped Jane to assess how Maria was feeling after getting help on this problem.

In Jane's interaction with Keisha, Jane's pedagogical empathy filtered both her pedagogical content knowledge of how to help Keisha understand how to find the vertex of a parabola and her emotional knowledge of Keisha's feelings to guide the conversation. As a result of drawing on pedagogical empathy, Jane made the decision to talk with Keisha one-on-one, rather than involving her group members. Jane discussed both the cognitive and affective parts of this interaction, recalling how she was trying to figure out where Keisha was confused, while at the same time noting that if she was in Keisha's position she would feel "discouraged." Jane's view of the self/other dimension of empathy as being important can also be seen in her interaction with Keisha. While working with Keisha, Jane felt "frustrated" because they were spending a lot of time working through this problem, but at the end of their interaction Keisha still seemed confused. However, Jane discussed putting her own emotions aside to try to help Keisha feel more comfortable with the problem, which reflects Jane's view on the importance of distinguishing between her own thoughts and feelings and Keisha's thoughts and feelings.

Jane used her pedagogical empathy to filter her pedagogical content knowledge and emotional knowledge in both of these interactions. Since Jane only had 11 students enrolled in her class, she was able to spend more time helping individual students, which

might have allowed her to develop more emotional knowledge and knowledge about her students and the group dynamics between them. Jane was able to draw on this knowledge during her interactions with Maria and Keisha although this filter of pedagogical empathy led her to make different decisions in two similar situations. In Maria's interaction, Jane chose to draw Maria's group members into the conversation, while in Keisha's interaction Jane decided to help Keisha one-on-one. Thus, pedagogical empathy can influence decision making by filtering knowledge in different ways, depending on the situation.

Aaron: A Case of Blockage

Similar to Jane, Aaron was a second year GSI who was teaching College Algebra (75-minute classes, three times a week, approximately 40 students enrolled) as the instructor of record for the first time. He had previously led Calculus II and III recitation sections as a first-year graduate student. During his second observation, Aaron had 36 students in attendance; and during his third observation, he had 29 students in attendance. In the following sections, I describe Aaron's conceptualization of empathy and two teaching interactions that I observed, one during his second observation and the other during his third observation. I then discuss how Aaron's pedagogical empathy is reflected in his teaching decisions in these two interactions.

Aaron's Conceptualization of Empathy

In his initial interview, Aaron wrote down the following definition of pedagogical empathy: "having an understanding of a student's unique perspective, history, and conditions within the context of a learning environment." Over the semester, Aaron made a few modifications to his definition of empathy. At the start of his first post-observation

interview, Aaron noted that "usually the working definition of empathy includes the idea of putting yourself in someone else's shoes or imagining yourself as being in that situation in some way…so that probably ought to be added" to his definition of empathy. During his final post-observation interview, Aaron discussed "being able to predict and use that predictive power" in his teaching after looking at four other definitions of empathy provided by other participants. For Aaron, adding this predictive element to his definition of empathy helped to clarify how empathy "might be specifically used in a teaching context."

In addition to providing a written definition of empathy, Aaron also discussed his views on different dimensions of empathy. Aaron saw empathy as both a cognitive and affective process. He explained that instructors can experience cognitive empathy when preparing for class, "the process of thinking: What is the student going to think about this material? What are my students specifically going to think? How are they going to react?" He went on to explain that he thinks of this process as a "cognitive exercise where [an instructor] decides consciously to cast [their] brain." However, Aaron felt that empathizing with students "in the moment" was more of an affective process.

When asked whether it was important to maintain a self/other distinction when empathizing with students (i.e., whether it is important for instructors to distinguish between their own thoughts and feelings and a student's thoughts and feelings when empathizing with them), Aaron talked about compartmentalizing his thought processes. In response to this question, Aaron discussed a hypothetical situation where one of his students was frustrated by a problem, and Aaron was also becoming frustrated because he

was having a difficult time helping the student understand the concept behind the problem. Aaron replied:

> *If the thought in my head is, "this is a frustrating situation because I'm not explaining things well, and that makes me feel bad because I want to be an effective instructor," if that's my only thought, then I'm just not experiencing empathy. If I'm experiencing that and simultaneously I'm engaging in the whole empathetic process, thinking of what [my student is] experiencing, thinking about how I would feel in that situation, etcetera…I would interpret it as being just different processes on a CPU, and the more resources devoted to one, the less devoted to the other.*

Thus, in Aaron's opinion, "the ideal instructor, from an empathetic perspective, wouldn't actually be feeling a whole lot in the sense of their own frustration about the situation, as much as they'd be thinking about the other person's frustration and how that feels."

In comparison to all of the other participants in my study except for Jane, Aaron differed on his view of whether behavioral responses are a part of empathy. When asked whether resulting actions are a part of empathizing with students, Aaron responded no. The only other instructor who also responded "no" to this question was Jane, hence why I chose to study the interactions between these two instructors and their students. As Aaron remarked, "I think that you act on your empathy, but I don't think it is itself an action." While Aaron professed to have this view, he also discussed how he might "use empathy," for example by predicting how his students might feel about a topic and making adjustments while lesson planning. Thus, while Aaron argued that resulting actions are

separate from empathy, it is evident that he could see how empathy could potentially influence his teaching decisions.

Aaron's Interaction with Jessica: Avoiding Answer Seeking

During his second observation, Aaron helped one of his students, Jessica, who was working on Problem 9 from the student workbook shown in Figure 10.

Figure 10

Graphs of Exponential Functions: Problem 9

Problem 9. Use the function $g(x)$ as defined below to answer parts (a)-(d).

$$g(x) = \begin{cases} x + 6, & x < 0 \\ 3, & x = 0 \\ 6(0.5)^x, & x > 0 \end{cases}$$

(a) Sketch a graph of $g(x)$ on the interval $[-5, 5]$.

(b) What are the domain and range of $g(x)$?

(c) Over what intervals is $g(x)$ increasing? Decreasing?

(d) State the long-run behavior of $g(x)$, and give the equation(s) of any horizontal asymptote(s).

The interaction began when Jessica called Aaron over and asked the question, "Are exponential growths parabolas?" To answer Jessica's question, Aaron drew two graphs on the whiteboard, one of a quadratic function and the other of an exponential

function. He then pointed out the differences between the two graphs and discussed how exponential functions tend to "explode." After this explanation, Jessica asked Aaron how to find the domain and range of the exponential function. Aaron started talking about the domain of the function and how putting negative numbers into the function would produce fractions as outputs.

About halfway through the interaction, Aaron started talking about a concept that was irrelevant to the problem Jessica was working on and asked her about the difference between graphs of increasing exponentials and decreasing exponentials, and why they looked like they were "mirror images of each other." This question led to a discussion of how the sign on the exponent related to the graph of the function.

After talking about the difference between increasing and decreasing exponential functions, Jessica returned to her previous question about how to find the range of an exponential function. Aaron drew the graph of $f(t) = 6(0.5)^t$ restricted to the domain $t \geq 0$ on the whiteboard to help Jessica think about the range. Jessica first suggested the incorrect range of $[6, \infty)$. Aaron corrected her by pointing to the graph and writing the correct range of $(0,6]$ on the board. He ended the conversation by discussing how an exponential model might be a good model for a species dying out, except for the fact that the function would never get to zero so there would never be an extinction event. The entire conversation lasted approximately 12 minutes. A full transcript of Aaron's interaction with Jessica can be found in Appendix C.

After watching this interaction during his post-observation interview, Aaron noticed that there were some "very long chunks of time" where he was talking "uninterrupted." The other part of the interaction that stood out to Aaron was his attempt

to engage Jessica in thinking about how the graphs of increasing exponential functions were different from those of decreasing exponential functions. In his interview, Aaron characterized Jessica as "a student who is very demanding of particular mechanistic solutions and answers." By asking Jessica about the differences in the graphs, Aaron "was trying to intentionally ask a question that would be more open-ended and interesting to try and…tease that sort of thinking out a little bit more." As Aaron stated, "I would consider the whole q of negative t graph flipping thing to be an attempt at empathy, just because I was trying to tease something out of them based on how I understood their learning style." However, Aaron felt that his attempt to get Jessica to think abstractly was not very "effective."

When asked about his goals during the interaction, Aaron responded that his primary goal throughout the entire interaction was just to answer Jessica's question. He went on to say that "the graph flipping was more of its own thing, where I was like…here's an opportunity for me to talk about something that I think is more interesting." Thus, Aaron's understanding of Jessica's "learning style" led him to move away from her original question of how to find the domain and range, in an attempt to get her to think outside of the box.

Reflecting on this interaction also prompted Aaron to talk about teaching a different course where he "got to class early every day and included a little couple minute interesting math fact unrelated to class." At the end of the semester, one of his students complained that Aaron's math facts were confusing and a waste of time since they were not directly related to the course material. Aaron compared this to his interaction with

Jessica and said that his previous experience "made [him] understand that sometimes students don't want the cool, fun math because it makes their brain hurt or something."

Aaron also returned to his graph flipping question when asked about his beliefs that might be influencing the way he approached this situation. Aaron responded, "There's the implicit belief that just kind of textbook answer seeking is a bad thing…[but] if I'm a journalism student, for example, I might not really care, and I just want to pass the class." Aaron went on to say, "My belief is that that view is wrong…you should always be seeking the big idea…if you're not doing that, then you're not taking advantage of the course in an appropriate way." However, Aaron was able to imagine how some of his students might feel:

> *But I can see the flip side, where if you really, really just don't like the course, and it's a requirement that you resent, then it could be a poor use of your mental resources to do that. I can see that. I still disagree with it, but I can see it. And certainly, I can see that as being a potential collision with student interactions wherein I am looking at the big idea, and then they're just internally sighing.*

Ultimately, Aaron's view of Jessica as an "answer seeking" student led him to ask her an irrelevant question in an attempt to try to get her to think in a more conceptual way about the graphs of exponential functions. Thus, Aaron was using his pedagogical content knowledge, along with his knowledge about Jessica as a student and his belief that students should always be "seeking the big idea," to guide his decision-making. Even though Aaron was thinking about Jessica's point of perspective, he disagreed with her perceived sentiments. Thus, Aaron's filter of pedagogical empathy allowed him to draw on some types of knowledge but prevented him from using his emotional knowledge of

how Jessica might feel about the situation (i.e., disinterested or annoyed) to inform his decisions.

Aaron's Interaction with Shantel: Choosing the Right Points

In his third and final observation, Aaron had an interaction with one of his students, Shantel. Aaron had previously mentioned how Shantel had seemed "lost" and "afraid to reach out for fear of exposing [herself] to identity issues." Aaron had also stated that he was unsure of how to help Shantel "without making her become more self-conscious," and referenced back to a similar situation he had experienced as a student.

During his third observation, Shantel was working on the problems shown in Figure 11. She had just finished Problem 3 in the student workbook and was trying to decide which points she should use in Problem 4 in order to determine how different transformations would change the original graph of $h(x)$.

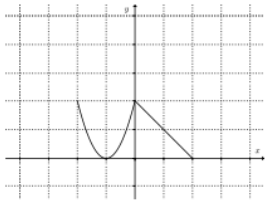
Aaron had just finished helping a student at a nearby table when Shantel raised her hand to get Aaron's attention. Aaron walked over to Shantel, sat down next to her, asked, "What's up?" Referring to Problem 4, Shantel replied, "How do you know what points to use? Because it doesn't say. Like, I just use one of these?" Aaron responded by saying that she could pick whichever points were the easiest or she could look at the whole graph to see what happens. He then told her that the points she had chosen on the graph of $h(x)$ were "really good points to use." Aaron went on to explain that one of the points Shantel had chosen would stay in the same place if a vertical stretch was applied since it was on the x-axis, and the other point she had chosen, which was not on the x-axis, would move.

Figure 11

Horizontal Stretches and Compressions: Problems 3 and 4

Problem 3. If the point $(3, 5)$ lies on the graph of $h(x)$, what point must lie on the graph of $h(3x)$? What point must lie on the graph of $3h(3x)$?

Problem 4. The function $h(x)$ is pictured below.

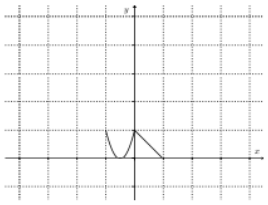


Match each of the following functions with the correct graph:

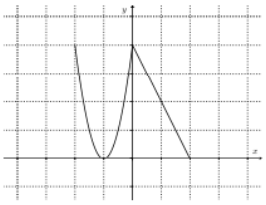
(a) $h(0.5x)$ _____ (c) $2h(0.5x)$ _____ (e) $h(2x)$ _____

(b) $2h(x)$ _____ (d) $0.5h(2x)$ _____ (f) $0.5h(x)$ _____

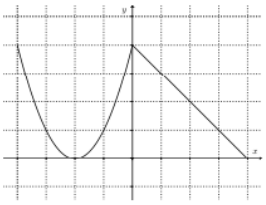
i)



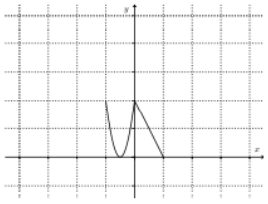
ii)



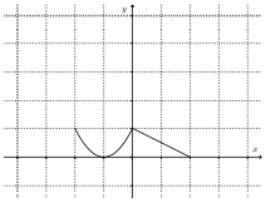
iii)



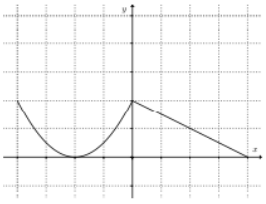
iv)



v)



vi)



Aaron's explanation seemed to confuse Shantel, and she asked him a second question, "It has to be on the x-axis?" Aaron responded by drawing the graph of $h(x)$ on the whiteboard and continuing to explain why the point Shantel had chosen, $(-1,0)$, was

useful in determining whether the graph had undergone either a vertical or horizontal shift. At one point he said, “If this point hasn’t moved in one of these graphs, then a vertical stretch might have happened, but a horizontal stretch can’t have happened.” Aaron’s conversation with Shantel lasted approximately four minutes, and a transcript of their interaction can be found in Appendix C.

In his post-observation interview, Aaron reflected on what he was trying to communicate to Shantel during this interaction. Initially, Aaron wanted to “make a teaching moment out of point selection” and explain why Shantel’s points were a “good choice, to kind of increase her confidence.” However, as the interaction progressed, Aaron realized that his explanation was confusing Shantel and “not making any sense at all.” After coming to this realization, Aaron’s focus shifted to what he was saying and how to revise his explanation.

When asked if he experienced empathy during his interaction with Shantel, Aaron pointed out how he was “trying to emphasize using her points and showing why her solution is a good solution, why her method is a good starting place. I was doing that intentionally because I wanted to improve her confidence.” Aaron went on to say that what was “less exemplary” about the interaction was that he was thinking about how to revise his explanation rather than trying to answer Shantel’s questions. Aaron summarized this interaction in the following quote:

> *I might actually characterize it in the way that we talked about earlier: as a situation where I'm empathizing, but then I start thinking about my explanation as being ineffective, and then focusing on revising as opposed to maintaining focus on the student.*

Aaron also discussed how he was confused by some of Shantel's questions and stated that instead of using these questions as "opportunities to pause and figure out more about what [his students] are thinking and where they are in the process," Aaron takes them as "evidence that [he's] not doing a good job," and that he needs to "repeat with a new explanation." When asked whether he would respond similarly to another student with the same questions, Aaron replied:

> *To be honest, probably, just because I think that this is a weak spot for me, in general. I'm used to questions being very well posed, I guess, as a mathematician. I'm used to people asking questions that are formulated carefully…so definitely my tendency when I see questions that I don't instantly understand, my reaction is not confident and polished at the moment.*

Aaron concluded by saying that thinking about how to engage students "in order to understand their current conception of the material" is a skill that he wants to improve.

In his interaction with Shantel, Aaron was drawing on his emotional knowledge of how Shantel might be feeling (i.e., unconfident, hesitant, self-conscious) to inform how he decided to respond to her. Aaron wanted "to improve her confidence" so he decided to explain why the points Shantel had chosen were good points. Thus, in the beginning of this interaction Aaron was using his filter of pedagogical empathy to draw on both emotional knowledge and pedagogical content knowledge to guide his decisions. However, Shantel became confused by Aaron's attempt to explain the problem. Instead of probing Shantel for what she was thinking, Aaron took her questions as a sign that he was not explaining the material clearly and focused his mental resources on revising his explanation. This shift away from Shantel's perspective made it difficult for Aaron to

draw on his emotional knowledge of what Shantel might be experiencing. Thus, his filter of pedagogical empathy "broke down" at some point, and the actions that Aaron decided to take were motivated by his need to revise his "ineffective" explanation instead of his emotional and pedagogical content knowledge.

Aaron's Filter: A Case of Blockage

Aaron's decision-making in the two interactions described above was informed by his pedagogical content knowledge and, to some extent, his emotional knowledge and knowledge of his students. In the first interaction, Aaron tried to get Jessica to think more conceptually about exponential functions, instead of directly answering a question she had asked him. Aaron classified this as an "attempt at empathy" since his understanding of Jessica as a student was motivating his actions. This scenario can be seen as falling under Aaron's original definition of empathy: "having an understanding of a student's unique perspective, history, and conditions within the context of a learning environment." However, because Aaron viewed Jessica as an "answer seeking" student and disagreed with this way of thinking, his filter of pedagogical empathy blocked him from using his emotional knowledge of the situation to guide his decision making.

In Aaron's second interaction, Aaron was trying to boost Shantel's confidence by explaining why the points she had picked on the graph were good points to use. However, Aaron's explanation ended up confusing Shantel, and Aaron became wrapped up in trying to revise his explanation instead of figuring out why Shantel was confused. Aaron's view of the self/other distinction dimension of empathy relates directly to this interaction. As Aaron stated, an "ideal instructor from an empathetic perspective" would be focused on what their students were feeling or thinking. At the start of this interaction,

Aaron was focused on how Shantel might be feeling and empathizing with her. During this part of the interaction, Aaron was using pedagogical empathy to draw on both his pedagogical content knowledge and his emotional knowledge regarding Shantel. However, as the interaction progressed, he became more focused on his explanation being "ineffective" and devoted "too much computer resources" to his own thoughts. His actions, therefore, reflected his own thoughts and feelings rather than Shantel's thoughts and feelings. Thus, Aaron's pedagogical empathy of pedagogical empathy broke down in this interaction as he was no longer drawing on his emotional knowledge in conjunction with his pedagogical content knowledge.

When asked about what he might do differently in future interactions, Aaron discussed the importance of making a distinction between "having empathy" and "acting on one's empathy." Aaron felt that "they are both important parts of the process, and if you aren't thinking about both of them as existing, you could be failing to have one but then think that you're okay because you have the other." This sentiment again shows how Aaron came to view resulting actions as an integral part of empathizing with students. Adopting this perspective throughout the interview process helped Aaron to examine his interactions with both Jessica and Shantel through a lens of empathy. As a result, Aaron was able to better understand what knowledge he had used to inform his teaching decisions and reflect on what he might do differently in the future when faced with a similar situation.

Joe: A Cognitive Version of Pedagogical Empathy

Unlike Jane and Aaron, Joe was a more experienced GSI who was in his fourth year in the graduate program. At the time of data collection, Joe was the instructor of

record for Precalculus (50-minute classes MWF and 75-minute classes TR, approximately 40 students enrolled). Joe had previously taught College Algebra, Precalculus, and a mathematics course for pre-service elementary school teachers as an instructor of record. During his second observation, Joe had 39 students in attendance; and during his third observation, he had 32 students in attendance. In the following sections, I discuss Joe's conceptualization of empathy and two teaching interactions that I observed, one during his second observation and the other during his third observation. I then analyze how Joe's pedagogical empathy might have influenced his teaching decisions in these two interactions.

Joe's Conceptualization of Empathy

In his initial interview, Joe provided the following written definition of empathy as it relates to teaching:

> *An awareness of student thoughts especially their understanding of content and comfort with content and the ability to "relate to" those feelings. Being able to predict what students are thinking and understand what they're thinking and use that to help your teaching.*

During his first post-observation interview, Joe added to this definition the phrase "understanding students' comfort with you" as an instructor. He also made it clear during his interviews that "part of being a good teacher is having this empathy" and that he had worked to develop empathy over time.

When discussing different dimensions of empathy, Joe took a different stance than every other GSI in my study on whether empathy is a cognitive or an affective process. Joe thought of empathy as a completely cognitive process:

> *For me, I'd say it's primarily cognitive, or I think about it in a cognitive way. I will often look at a student's face and say, "oh, I am seeing x emotion on their face. I should respond in this way." So, for me, it feels cognitive.*

Joe went on to say that he could imagine it being more "instinctual" or "emotional" for other people. This cognitive way of conceptualizing empathy also surfaced in post-observation interviews when Joe would analyze specific interactions with students. In fact, Joe took a cognitive approach to describing what motivated his decision-making in every interaction that Joe and I discussed.

Although Joe claimed to think about empathy in a "primarily cognitive" way, he also brought up the notion of affective empathy during one of his post-observation interviews. He described this affective empathy as "a special rare prize" that was "harder" for him to experience when interacting with his students on a day-to-day basis. This apparent dissonance between Joe's view of empathy as a cognitive process and his acknowledgement of empathy as an affective process was apparent in his characterizations of specific teaching interactions with his students. Even though Joe was able to identify cognitive components of empathy that he had experienced while interacting with his students, he did not think he had really empathized with them because he lacked this affective component of empathy.

Another dimension of empathy that Joe discussed was whether empathizing with students is an automatic or a controlled process. In this case, Joe thought empathy was both automatic and controlled. Joe distinguished between "the feeling of it and the acting on that feeling." As he stated, "feeling empathy, like this awareness of student thoughts, that is usually automatic," but with more conscious effort, instructors could probably

come to a better understanding of what their students are thinking and feeling. On the other side, Joe thought that "the acting on that feeling…is primarily active and chosen," but there might be "a few unconscious parts" of resulting actions.

Joe's Interaction with Dave: Decomposing Functions

During his second observation, Joe helped one of his students, Dave, with decomposing functions. Dave was working on Problem 8 in the student workbook shown in Figure 12 when Joe came over to his group and sat down to help them.

Figure 12

Function Composition: Problem 8

Problem 8. For the following composite functions, find possible formulas for $g(x)$, $h(x)$, and $f(x)$. Assume $g(x) \neq x$, $h(x) \neq x$, and $f(x) \neq x$.

(a) $g(h(x)) = (x+1)^2$

(b) $g(h(x)) = \sqrt{2x^2+5}$

(c) $g(h(x)) = \dfrac{1}{x-1} + x^2 - 2x + 1$

(d) $f(g(h(x))) = \dfrac{1}{\log(x^2+1)}$

(e) $g(h(x)) = \left(\sqrt[3]{\ln(x)}\right)^2$

(f) $g(h(x)) = \dfrac{1}{(x-3)^2}$

Joe's interaction with Dave began when Joe asked, "Can I help with anything?" One of Dave's group members, McKenzie, responded that another group member was helping their group with the workbook problems. Joe sat there for another second and noticed Dave staring down at his workbook, not doing anything. Joe then asked him directly, "Dave, got a question for me?" Dave responded back with, "Yeah. I don't know where to start."

Joe suggested that they look at the function $g(h(x)) = \sqrt{2x^2 - 5}$ in part (b) on Problem 8 to use as an example. He then tried to get Dave to give him an "inside part" that could be equal to $h(x)$ in the composed function. Dave misunderstood Joe's question of "What could you find first?" referring to the "inside part" of the composed function and responded with, "*x*. Is that right?" Dave then talked about squaring the function to "get rid" of the square root and subtracting the five. Joe replied with, "yes, so that's how you would like solve for it," and then pointed up to an example he had done earlier in class on the whiteboard. Joe then asked Dave again what he could find first as the "inside part" of the function, to which Dave replied, "I have no clue."

After this response from Dave, Joe decided to "phone a friend" and bring Dave's group member, McKenzie, into the conversation. Joe asked McKenzie if she could identify the "inside part" of the function and McKenzie responded with, "*x* squared plus five." Joe noticed that McKenzie's answer was incorrect, but said, "sure" and then corrected her by saying, "*two x* squared plus five."

Other group members then joined in the conversation and eventually the group got to a point where they had identified what $h(x)$ and $g(x)$ should be equal to in this problem. Before leaving the group, Joe recommended that Dave and his group members

try to “circle” parts of the equation that could be the inside part of the composed function. As Joe said, “the hard part is finding something to circle, and if you’re ever unsure just circle parts of the expression basically.” Joe’s conversation with Dave and his group members lasted approximately three and a half minutes, and a full transcript of it can be found in Appendix C.

Joe had pointed that interaction out to me as a memorable one immediately after his observation. When asked why Joe thought that was a significant interaction in his post-observation interview, he said, “looking at it again, it definitely sticks out to me as something where I was sort of like pulling all of my teaching tricks out.” Joe explained that he had tried explaining the problem in one way, and then when that was ineffective, he pointed up to the whiteboard to a previous example. After going back to the example, he tried explaining the problem in a different way, but Dave was still confused.

From there, Joe pulled in Dave’s group member, McKenzie, who then “said a wrong answer.” While discussing this interaction and looking at the transcript, Joe pointed to the ellipses and explained, “wherever there’s an ellipse, that’s me, the gears turning in my brain, and if I was a perfect supercomputer that would go super-fast.” He then replied that he “overrode” McKenzie’s wrong answer and responded instead with the correct answer. Joe noted that, “after that point, after I perceived [Dave] getting that first example, I think it basically clicked into place.” Joe also said that Dave “was entirely capable of the task,” but that “he didn’t understand what the task was asking.”

When asked whether he thought he experienced empathy in this interaction, Joe replied that he was unsure. Instead, Joe remarked that his “attitude” with this interaction was that, “this is a problem to be solved: the student not understanding is a problem to be

solved," which Joe felt like "is in some way at the opposite end of the spectrum from empathy." However, Joe also talked about drawing on his "knowledge of reading a student's reaction and seeing that [Dave] still didn't get it." Joe said he used this knowledge to "adapt" and "change [his] response based off of that." Therefore, even though Joe did not think he had empathized with Dave in this interaction, this situation could be characterized by Joe's classification of "cognitive" empathy, where he would "look at a student's face" and say, "oh, I am seeing *x* emotion on their face. I should respond in this way."

Joe also had a difficult time expressing his feelings during this interaction. When asked how he felt during the conversation with Dave, Joe responded "it was kind of a U-shaped feeling, where I was feeling fine going in, and then I was like, 'uh oh, uh oh, uh oh,' as [Dave] continued to not get it." Joe went on to say that "there was that inflection point where the student gets it for the first time" and that he experienced "a rising, more positive feeling after that." After responding in this way, I asked Joe specifically to name some of these feelings: was he "concerned", "worried", or "confused as to why the student was confused"? To this question, Joe replied that he usually just has "a general impression" of how he is feeling, and in this interaction, "things were getting worse. The conversation was going in a worse direction." Thus, Joe was unable to verbalize any specific feelings that he had experienced during this interaction.

In this interaction, Joe used his pedagogical empathy to draw on his knowledge of students and his pedagogical content knowledge, as well as his pedagogical knowledge about "teaching tricks," to guide his decision making. Even though he took a cognitive stance toward what he was thinking and feeling while talking with Dave, Joe was able to

"read" Dave's reactions and used his emotional knowledge to "judge if [Dave] felt confident." Thus, Joe's emotional knowledge was also filtered through his pedagogical empathy. After seeing that Dave still was confused about how to compose functions, Joe's emotional knowledge was filtered through his pedagogical empathy, which led to Joe's decision to "phone a friend" and ask Dave's group member, McKenzie, to join the conversation.

Joe's Interaction with Molly: Drawing the Wrong Triangle

During Joe's third and final observation, Joe helped one of his students, Molly, with Problem 4 shown in Figure 13. Molly had worked through part of this problem and wanted to verify that she was on the right track.

The interaction between Joe and Molly began when Joe walked over to Molly's table and asked, "Can I help out over here?" One of Molly's group members replied, "oh yeah," and Molly responded by saying, "Maybe. I don't know if we're doing this right. I'm probably not." Joe sat down next to Molly and asked if she could walk him through what she had done. Molly began explaining how she had reasoned through where a person would be on the Ferris wheel after 30 seconds. She explained, "cause you start at the three o'clock position…right here, and for a whole rotation it's two minutes. So, I thought after a minute you'd be there so half of that is right there." Joe pointed out that another assumption Molly had used is that the person on the Ferris wheel begins ascending at time $t = 0$, to which Molly replied, "yep, sorry."

Figure 13

The Sine & Cosine Functions: Problem 4

Problem 4. Sketch a Ferris wheel with a diameter of 30 meters and a boarding platform at ground level. The wheel completes one full revolution every 2 minutes, and at time $t = 0$ an individual is at the 3:00 position and ascending.

(a) Mark your position on the wheel after 30 seconds. What is your height after 30 seconds?

(b) Mark your position on the wheel after 10 seconds. What is the corresponding angle (from the 3:00 position) in degrees?

(c) What is your initial height, at time t=0?

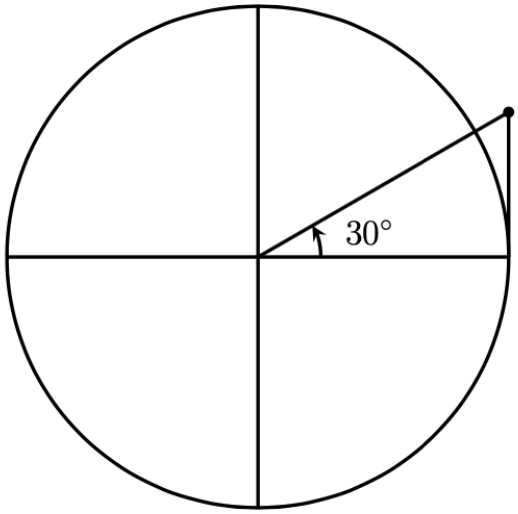
(d) What is your height after 10 seconds? [Hint: Look back at your work in Problem 3.]

After that exchange, Molly moved on to ask Joe about finding the height of a person on the Ferris wheel after ten seconds, which was the part of the problem she was most unsure of whether she was "doing it right." Joe confirmed that Molly had found the correct angle of 30 degrees corresponding to ten seconds and then suggested that Molly draw a triangle on her picture of the Ferris wheel to help find the height at that position. Joe tried to explain how Molly should draw the triangle, but Molly had a difficult time

understanding what Joe was saying and ended up drawing an incorrect triangle, shown in Figure 14.

Figure 14

Depiction of Molly's Triangle



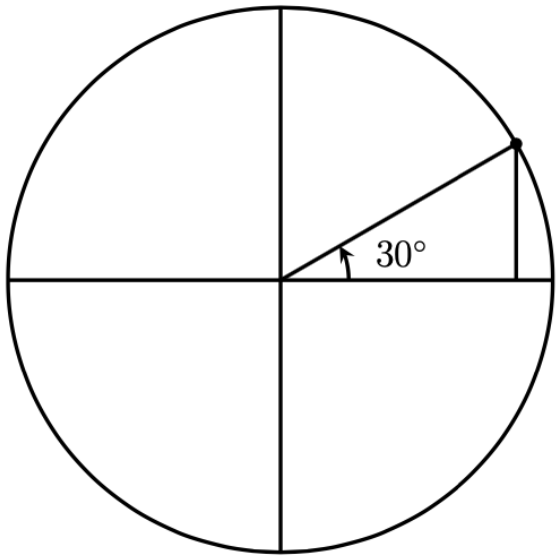
After Molly had drawn the incorrect triangle shown in Figure 14, Joe pointed out that the triangle "won't quite meet on the edge" of the Ferris wheel. Molly then said, "I should just like redraw it." Seeing that Molly was still incorrectly drawing her triangle, Joe replied, "So, I'm not supposed to do this as the instructor, but can I draw the picture for you?" Joe then drew a picture of the Ferris wheel with the correct triangle in it in Molly's workbook, as shown in Figure 15.

After Joe drew this picture, Molly said, "Gotcha. I see what you mean. Sorry. Thank you." Joe then asked Molly if she knew the length of the hypotenuse of the triangle, to which Molly responded with the correct length of 15. Molly then pointed to the vertical leg of the triangle and said, "so you're trying to find this right here?" Joe replied, "That's right. You're trying to find that opposite side." Molly then came up with the correct trigonometric function to use in order to find the length of that vertical side:

"So, it's opposite hypotenuse: sine," to which Joe replied, "Exactly. So, sine of 30 degrees is *x* over 15." Joe's conversation with Molly lasted approximately three minutes, and a transcript of it can be found in Appendix C.

Figure 15

Depiction of Joe's Triangle



In his post-observation interview, Joe noted that this interaction was memorable to him because Molly "was really struggling with getting the right picture" and Joe "ended up just sort of saying the right answer or drawing the right picture." Joe also pointed out that Molly said, "sorry" several times throughout the interaction, which Joe interpreted as her being "embarrassed that she wasn't getting it and sort of flustered that it wasn't making sense to her." Overall, Joe thought that he and Molly "weren't having as good of an interaction" as they could have because Joe was having difficulty "trying to communicate in words what picture to draw."

When asked what he was thinking or feeling during that interaction, Joe referenced back to the idea of "parallel emotions," a notion he had discussed while

talking about the self/other dimension of empathy. Joe remarked that Molly was probably "a little bit embarrassed that she wasn't getting the right answer," while Joe was "embarrassed that I wasn't able to communicate the right answer."

In this interaction, Joe used his emotional knowledge to interpret Molly's feelings. Joe's filter of pedagogical empathy allowed him to use both his emotional knowledge and his pedagogical content knowledge in order to decide how to help Molly work through this problem. Joe noted that "an important part of that interaction was me reading the student's responses to me." Something that Joe "distinctly" remembered was correcting Molly on her initial reasoning and pointing out the assumption that the person on the Ferris wheel begins by ascending, rather than descending. Joe took Molly's response of, "yep, sorry," to mean that Molly "thought she had something wrong…or thought she had made a mistake or was too careless or something." Joe went on to say that he did not "want [Molly] to be having that emotion for this part [of the problem] because she did good work there—that was correct, and I wanted her to feel good about that." To "correct any negative emotions" Molly might have had, Joe used pedagogical empathy to draw on his emotional knowledge and decided to respond by "more strongly praising her" than he would have otherwise.

Joe's Filter: A Cognitive Version of Pedagogical Empathy

Joe's cognitive view of pedagogical empathy was reflected in the decisions he made while working with his students. Joe perceived most of his interactions with students as "mechanical" transactions where a student had a problem and Joe needed to figure out how to solve it. In his interactions, Joe used pedagogical empathy to try to "read" his students emotions and decide what to do. Although Joe professed to have a

cognitive view of empathy, Joe's perception of his interaction with Dave shows that he also valued the affective part of empathy. Even though he was aware of Dave's understanding of the mathematical content, Joe did not feel like he related to Dave and therefore did not think he had empathized with him in this interaction. Thus, this ability to "relate to" a student's feelings was important to Joe. In his interaction with Dave, Joe used pedagogical empathy to filter his emotional knowledge, pedagogical content knowledge, and pedagogical knowledge, which helped guide his decision making.

Joe's interaction with Molly was the only interaction that we discussed where Joe felt like he had empathized with his student. Joe felt that he was able to empathize with Molly because of the "parallel emotions" that he felt with her. Hence, Joe's perception of this interaction also demonstrates the importance that he placed on the affective component of empathy. In addition, Joe's view of the automatic and controlled parts of empathy can also been seen in his interaction with Molly. While talking about the Ferris wheel problem, Joe empathized with Molly, "reading her responses" of "sorry" as her thinking she had done something wrong. For Joe, this part of the interaction was more subconscious, and his empathy was automatically elicited. Joe then used his pedagogical empathy to draw on his emotional knowledge, which led to his decision to praise Molly's work more enthusiastically than he might have otherwise in order to help Molly feel validated and confident about her work.

Overall, Joe tended to analyze his interactions in a more cognitive way, which led to his inability to describe emotional undertones that may or may not have been present in an interaction. It is also possible that Joe's view of empathy as a "rare prize" led him to categorize several of the interactions we discussed as him understanding his students'

thought processes but not "personally relating" to them. In most of these interactions, Joe's pedagogical empathy led him to make decisions based on his emotional knowledge of the situation, even though he analyzed his interactions in a cognitive way and described them as "mechanical" transactions. Thus, while Joe professed thinking about empathy as a cognitive process, his pedagogical empathy was able to filter both emotional knowledge and more cognitive forms of knowledge.

Oliver: Using Pedagogical Empathy to Push Students Forward

Similar to Joe, Oliver was a fourth year GSI who had a few years of experience as an instructor of record. Oliver had previously taught College Algebra, Business Calculus, Calculus II, and Precalculus as the instructor of record and, at the time of data collection, was teaching Trigonometry (50-minute classes, two times a week, approximately 35 students enrolled). During his second observation, Oliver had 34 students in attendance; and during his second observation, he had 33 students in attendance. In the following sections, I discuss Oliver's conceptualization of empathy and two teaching interactions that I observed, one during his first observation and the other during his second observation. I then analyze how Oliver's pedagogical empathy might have influenced his teaching decisions in these two interactions.

Oliver's Conceptualization of Empathy

In his initial interview, Oliver defined empathy in a teaching context as, "the ability to understand when something is difficult to learn." He expanded on that definition in his first post-observation interview, saying empathy is

> *Being able to put yourself in your students' shoes and think about how it would feel if everybody around you understands something and you don't—as an*

instructor, being able to help a student through that or at least attempting to help a student through that.

Oliver also noted that empathizing with students requires instructors to "practice patience and non-judgement" because it helps give them a "sense of approachability."

When discussing different dimensions of empathy, Oliver took a different stance than all of the other participants in two different categories: congruent or incongruent and self/other distinction or merging. Congruency deals with whether it is important for an instructor to accurately perceive the thoughts and feelings of a student when empathizing with them. Self/other distinction or merging refers to an instructor being able to distinguish between their thoughts and feelings and the thoughts and feelings of their students when empathizing with them.

Oliver responded to a question about the importance of congruency by saying it is "more important to attempt" to empathize with students than to have an accurate understanding of what they are experiencing. Oliver went on to describe a hypothetical scenario where a student is having some sort of difficulty with a problem. Oliver said that he would try to do his best "at not making [his student] feel bad and sort of helping them along the way," but stated that he could be "misunderstanding" the situation and perhaps the student is "unmotivated by the question" or does not want to engage with the class. From Oliver's perspective, "accuracy is kind of an irrelevant question" in this type of scenario. What is more important is that Oliver's attempt to help the student may "engage them" and motivate them to continue working on the mathematics at hand.

Similarly, Oliver responded that it was not important to distinguish between his feelings and his students' feelings when empathizing with them. Oliver stated that he

"hardly ever" thinks about how he is feeling while working with students. Therefore, Oliver found it unnecessary to think about the distinction between his students' feelings and his own. He also related the self/other distinction or merging dimension of empathy back to congruency and stated that, "there's not a good way to assess accuracy of how I feel, how [my students] are feeling. I'm only making a good estimation of that, and I'm sure my feelings also have to go into the calculation that I'm doing." Thus, while Oliver did think that empathy was an important part of teaching, he did not think these two dimensions of empathy were important in interactions with students.

Oliver's Interaction with Katie and Her Group: Finding the Period of Tangent

During his first observation, Oliver helped Katie and her group members who were working on Problem 1 shown in Figure 16. Oliver had just worked through how to sketch a graph of the tangent function on the whiteboard with the entire class, so Katie and her group members were focusing on part (b) of Problem 1.

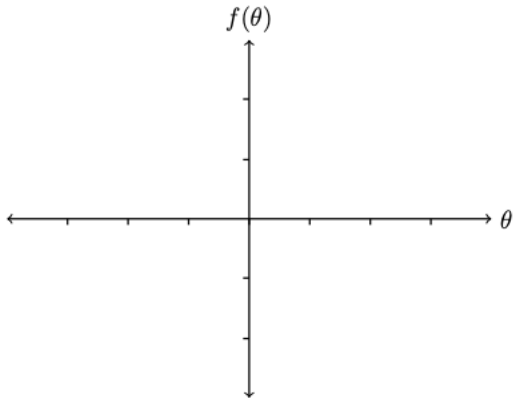
Oliver's interaction with this group began when Katie raised her hand as Oliver was walking by their table. Katie asked Oliver, "How do you know what the period of a periodic function is?" to which Oliver replied, "Well you tell me. What is the period of a periodic function?" Katie then asked whether the period was infinity. Oliver responded by saying, "careful," and then asked how she would define a function in her own words. Katie then said, "when you move it over and it's still the same." Oliver repeated this definition and then asked again, "What do you think the period is in that context?" referring to the graph of tangent. Katie replied by asking, "Does it not have one?" clearly confused by how to find the period of tangent.

Figure 16

The Tangent Function & Cofunctions: Problem 1

Problem 1.

(a) Sketch a graph of $f(\theta) = \tan(\theta)$. Your sketch should include appropriate labeling of coordinate values on the θ axis.



(b) What is the period of the tangent function?

(c) Identify three values of θ for which $\tan(\theta) = 0$. What is the value of $\sin(\theta)$ for each of these values of θ? Recalling that $\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)}$, can you explain why this is the case?

(d) Identify three values of θ for which $\tan(\theta)$ is undefined (where are there vertical asymptotes?). What is the value of $\cos(\theta)$ for each of these? Why does it make sense that these values for $\cos(\theta)$ would cause the tangent function to be undefined?

At this point, Oliver walked around to the other side of the table to bring the rest of the group into the conversation. He opened it up to Katie's group members by asking, "Anybody here at the table have an answer for that…what's the period of a function?" Katie's group member, Emily, joined in and tried to give a definition for the period of a function. Oliver eventually helped the group come to a definition of period as the

"shortest amount of time before the graph essentially starts repeating itself." Emily then pointed to the graph of tangent that Oliver had drawn on the whiteboard earlier in class and asked, "Is it the gap between those two?" Another group member, Tessa, asked, "Where does it start at? Like it's pi over two, but then what is it on the other side?" again referring to the graph of tangent on the board. To respond to this question, Oliver replied, "there's really no start…it's just one value where it doesn't work and anything to the left and to the right, you just get these huge drastic numbers…but I think this pattern's going to keep happening." Finally, Oliver encouraged the group to move on to parts (c) and (d) of Problem 1 and "think together" before coming back and trying to answer the period question in part (b).

In his post-observation interview, Oliver said that this interaction had stood out to him because he was surprised that his students were struggling to define the period of a function, especially since they had recently gone over this definition in class and had just completed a homework assignment on it. Oliver was also surprised that Katie had thought the period of a function could be infinity, but "rather than laugh" at that answer, Oliver chose to say, "careful" and help Katie rethink the definition of period.

When asked about whether he experienced empathy in that interaction, Oliver replied yes and referred back to his written definition of empathy. He said at that point in the class, finding the period "is a trivial question…you can see it in the graph." He went on to say, "but if I put myself in their shoes, it may be a terribly difficult question, especially if they don't understand what the period is." Oliver also noted that he tried to help Katie and her group members without "scolding them" or making them "feel bad," even though "it's always a struggle to know how to address very wrong answers." Thus,

in this interaction, Oliver was using pedagogical empathy to draw on his emotional knowledge and pedagogical content knowledge to decide how to respond to Katie and her group.

Oliver also discussed the way he decided to end the conversation and how he was trying to empathize with the group as a whole. At the end of the interaction, Oliver suggested that the group work on the following parts of the problem before coming back to part (b) to come up with the period of tangent. Oliver remarked that he thought it might be more helpful for the group to move on rather than "slamming their heads against the table" trying to answer that question. He also wanted to bring the conversation back to a question that Tessa had asked about "where does it start?" Oliver was unsure of what Tessa meant but tried to use this question as a springboard to help the group think about where the tangent function might be equal to zero. Thus, Oliver used his knowledge of how the entire group might be feeling after struggling with this problem and decided to try to move the group forward using their contributions and questions.

Oliver's Interaction with Danny: Working Through the Confuzzledness

During his second observation, Oliver helped one of his students, Danny, work through Problem 2 (a) shown in Figure 17. Oliver had pointed out this interaction to me after his observation because he just handed back the first exam at the beginning of this class and was surprised by Danny's low score. As a result, Oliver was trying to pay more attention to Danny in class.

The interaction began when Oliver walked over to Danny's table and asked how they were doing. Oliver then patted Danny's workbook and said, "I don't see a lot of graphs," referring to Problem 2. Danny replied, "Well I'm kind of confuzzled on the

graphs." Oliver repeated this back to the student, "Confuzzled?" and then said, "Let's see if we can work through the confuzzledness." Oliver then began helping Danny work through Problem 2 (a), sketching a graph of the function $f(t) = 7\sin(t) - 8$.

Figure 17

Generalized Sinusoidal Functions: Problem 2 (a)

Problem 2. Identify the period, midline, and amplitude of each of the trigonometric functions below. Include a sketch of each.

(a) $f(t) = 7\sin(t) - 8$

Danny had correctly written down that the period was 2π, the midline was -8, and the amplitude was 7. Oliver encouraged Danny to start by drawing the midline on his graph. Oliver then asked Danny, "If you start at negative eight, how low should you go, if your amplitude is seven?" At this point, Danny sat silently for about ten seconds before asking Oliver, "were you talking about from like up or down?" Oliver responded, "both," and proceeded to explain that the maximum value on the graph should be the midline plus the amplitude and the minimum value should be the midline minus the amplitude. Danny pointed to his graph and said, "so, like right here?" Oliver replied back and said, "But you should know the exact value, right? If you start at negative eight and you go lower by seven, how low should you go?" After another couple of exchanges, Danny said that the minimum value would be "negative nine," to which Oliver replied, "Careful, you start at negative eight, and you're going down by seven."

At this point in the conversation, one of Danny's group members chimed in and said the correct minimum value of negative 15. This group member then turned to Danny and explained how to get the minimum value based off knowing the midline and

amplitude. At this point in the interaction, Oliver began helping Danny's other group members with a different problem before leaving the group to wrap up the class period.

When talking about this interaction in his post-observation interview, Oliver noted that in the past Danny had been good at giving him a sense that he was understanding the material. Oliver went on to say, "I think this was one of the first times that I had a more close interaction with him, and I started to sort of see the disconnection." After watching through a video clip of his interaction with Danny, Oliver said that he "was surprised by [Danny's] answer" and that he was trying to "diagnose" how far behind Danny was in the class. Oliver also talked about the group dynamics at Danny's table:

> *That table talks a lot, and non-mathematically. So I just wanted to give [Danny] a little more attention so that perhaps I could figure out how to engage with him. How do I engage with this group so that they start being more productive?*

Oliver also mentioned that "based on [his] interactions with the group, which is usually with the other people, they know what they're doing." He went on to say that "everybody at that table scored in the high 90s" on the first exam while Danny "scored in the low 30s."

On the topic of empathy, Oliver referred back to his definition when discussing his interaction with Danny. Oliver said, "it can be hard to learn something," but in this case, he was uncertain that Danny was trying to learn. Oliver remarked, "I can empathize that some things are difficult to learn, but I feel like there's a lack of trying. Again, I feel bad for saying that, but I'm suspecting that that is the case, that there's not a lot of attempt at that." Oliver also had conflicted feelings over this sentiment and

acknowledged that maybe he was making a "bad judgment" or maybe there were "other factors" to consider but conceded that he was working with the knowledge that he had at that point in time.

Oliver also discussed how he thought Danny might be feeling during this interaction. During the interaction, Oliver "sensed that [Danny] was trying to push" him away. Oliver said that he though Danny "was feeling a little nervous" when Oliver was asking him what the minimum value should be, and Danny was unable to come up with the correct answer. Oliver also felt like Danny was trying to tell him to "go away" during the entire conversation even though "he didn't say it audibly." Oliver remarked, "[Danny] seemed nervous, and as soon as the other student said, 'oh negative 15,' [Danny] was like, 'oh, got it!' So again, I don't know if he got it….It felt like he just wanted me to step away."

After discussing his interaction with Danny, Oliver returned to his original written definition of empathy and said that he felt like his definition was "a little restrictive." Oliver explained,

> *I'm having situations here where in a way I feel like I'm being empathetic in my role as an instructor that I want [my students] to do well, but also like these interactions where I think he wanted me to go away, part of me is not allowing me to do this. Like, "I can't. I need to have this interaction with you. I'm your instructor."*

Oliver went on to say that, "in a sense that's not very empathetic…when I step away from my role as an instructor, it would be maybe a little humiliating if you thought that your classmates know that you're struggling." Thus, even though Oliver felt like he was

doing what was "best" for Danny, he still had conflicted emotions about how he might be making Danny feel.

In this interaction, Oliver's pedagogical empathy determined what knowledge he drew on to make his decisions. Even though Oliver sensed that Danny felt uncomfortable, Oliver chose not to act on this knowledge because he wanted to do what he thought was best for Danny. Thus, Oliver's pedagogical empathy allowed him to draw on his pedagogical content knowledge and prevented him from using his emotional knowledge.

Oliver's Filter: Using Pedagogical Empathy to Push Students Forward

The decisions that Oliver made in his interactions with Katie and Danny can be viewed through his filter of pedagogical empathy. In both of these interactions, Oliver used his pedagogical empathy to decide how to help his students carry on and push forward through their confusion. In Oliver's interaction with Katie and her group, Oliver was surprised by how the group struggled to find the period of tangent or even recall the definition of period. From Oliver's perspective, finding the period of tangent was "trivial," although he empathized with Katie and her group members and said that this question might be "terribly difficult" for them especially since they were struggling to come up with a definition for the period of a function. Instead of calling attention to Katie's "very wrong answer" of infinity, Oliver used his pedagogical empathy to draw on his emotional knowledge and pedagogical content knowledge to help Katie and her group members conceptualize the period of a function and encouraged them to keep working on the later parts of the problem as a group.

Oliver's stance on the congruency dimension of empathy can be seen in his interaction with Danny. Oliver was concerned about Danny because he had just failed the

first exam. Even though Oliver did not know how much effort Danny was putting into learning the material, Danny's perceived "lack of trying" did not prevent Oliver from empathizing with him. Oliver recognized that he might not accurately understand what Danny was experiencing and might have been making a "bad judgement" about Danny's situation. Regardless of whether Oliver was accurately understanding Danny's emotions, Oliver thought it was important to "continue to help him out" and show Danny that he "cared" about him as a student.

During his interaction with Danny, Oliver felt conflicted about his emotions and the decisions he was making. Oliver empathized with Danny and tried to show him that he was there to help by checking in on how Danny was doing during class. However, Oliver felt like he was putting "pressure" on Danny during his interaction with him, which might have made him feel bad. As Oliver noted, "I'm struggling with how I'm making [Danny] feel with what I know is best for him." This conflict was also reflected in Oliver's teaching decisions: even though he had emotional knowledge about what Danny might be feeling, this knowledge did not make it through his filter of pedagogical empathy. Instead, Oliver drew on his pedagogical content knowledge to decide how to respond to Danny. As a result, Oliver ignored Danny's attempts to get him to "go away" and encouraged him to push forward with the problem.

Discussion

These four cases illustrate how pedagogical empathy can be conceptualized as a filter that connects different types of knowledge to instructors' decision making. In Jane's interactions with Maria and Keisha, she was able to draw on her emotional knowledge and pedagogical knowledge to inform her decision making. However, Jane's filter of

pedagogical empathy led her to make different decisions when managing the group dynamics in two similar situations. While Jane was able to use her emotional knowledge, Aaron, on the other hand, was not. Aaron tried to draw on this type of knowledge in both of his interactions, but his filter of pedagogical empathy prevented him from using his emotional knowledge to guide his decision making. In Joe's interactions, he used his pedagogical empathy to draw on his emotional knowledge, even though he viewed empathy as a cognitive process. Finally, Oliver used his pedagogical empathy to decide how to help his students carry on and push forward through their confusion. In his interaction with Danny, Oliver's pedagogical empathy filtered out his emotional knowledge causing him to feel conflicted about the situation. However, Oliver did use his pedagogical empathy to draw on his pedagogical content knowledge and decide how to respond to Danny's attempts to push him away.

Limitations and Future Research

One limitation of this study was that I focused solely on instructors' perspectives of their interactions with students in the classroom. In future work, I plan to incorporate students' perspectives into a study on pedagogical empathy in order to provide another point of reference from which to study this phenomenon. Taking into account students' perspectives would also allow instructors to analyze their decision-making from an important viewpoint in the classroom. In addition, researchers have yet to explore how students in undergraduate mathematics courses perceive pedagogical empathy. Thus, examining students' perspectives on empathy would also begin to fill a large gap in the research literature.

Another limitation of this study was the limited number of observations and interviews that I conducted with each GSI. Although I observed each GSI three times, more observations and interviews would have provided me with more interactions to analyze. However, these additional observations and interviews were not feasible due to the number of participants and the scope of the study. In addition, while I did observe several interactions between my participants and their students, I was only able to ask in-depth questions about a few of these interactions during each post-observation interview. Future research could focus on one instructor and their interactions with their students over the course of an entire semester. This ethnographic case study could generate a deeper understanding of that individual instructor's views on empathy and the resulting influence on teaching decisions.

A final limitation of this study is its generalizability. The four instructors who participated in my study all taught in the same university and had gone through a similar GSI professional development program. Therefore, it is likely that their perspectives on empathy and teaching would be different from GSIs in other mathematics departments who did not receive this level of support or professional development. In future work, researchers could recruit a wider range of GSIs from multiple institutions that might create different cross case comparisons.

Conclusion

The purpose of this study was to examine how GSIs' pedagogical empathy might influence their teaching decisions in interactions with students. By analyzing specific teaching interactions with my participants, I was able to conceptualize pedagogical

empathy as a filter and examine the connection between different types of knowledge and teaching decisions.

Throughout the process of reflecting on their teaching decisions, the four GSIs in my study developed as instructors. All of the GSIs had different takeaways. Aaron discussed how he found it "valuable to think about the distinction between acting on one's empathy and having empathy." Jane noted how she was "more aware" of how she interacted with students and when she was empathizing with them. Oliver talked about incorporating empathy in his "mental model" and how it helped him think more about his students' perspectives. Joe discussed how watching videos of himself teaching had helped him develop an "awareness" of his decisions as an instructor.

By engaging in this process of reflection, instructors can develop an awareness of pedagogical empathy and use it as a lens through which to view their teaching decisions. Leaders of professional development can incorporate these types of reflective activities into their programs to help GSIs grow as empathic individuals involved in the complex process of teaching.

CHAPTER 7: CONCLUSION

Overview of Study

The purpose of this dissertation has been to examine how mathematics graduate student instructors conceptualize pedagogical empathy and analyze how that empathy might influence their teaching decisions. Mathematics education researchers have studied various components of affect, including emotion. Many researchers, however, have chosen to study either student affect or teacher affect, rather than examining their intersection where empathy is situated. In fact, little research has been done at the post-secondary level that focuses exclusively on how instructors empathize with their students. Thus, this dissertation sheds light on an area that has yet to be fully explored.

To investigate the phenomenon of pedagogical empathy, I conducted a qualitative research study involving numerous interviews and observations of 11 graduate student instructors. My role as an interviewer and observer was to help my participants reflect on their views of pedagogical empathy and use empathy as a lens through which to examine their teaching decisions. Throughout the process of gathering and analyzing data, I remained as objective as possible, often refraining from sharing my own views on empathy with my participants. I also frequently conferred with another researcher to discuss and verify my analysis and findings. Below, I summarize my findings and provide some broad implications for future research.

Summary of Findings

Since empathy is a difficult construct to define, I first focused on how instructors conceptualize empathy in a teaching context. Each of the graduate student instructors in my study constructed their own definition of empathy and provided examples to illustrate their definitions. These written definitions served as a reference point that my participants and I could look back on during each interview. Over the course of the semester, the graduate student instructors in my study were able to refine their ideas as we reflected together on how they conceptualized pedagogical empathy.

During the final interview, I asked my participants a series of questions relating to Cuff et al.'s (2016) framework that describes eight dimensions of empathy. GSIs' had differing views on these eight dimensions, some of which had changed throughout the study. The three dimensions of empathy where GSIs' showed the most variation in responses were viewing empathy as a cognitive or affective process, viewing empathy as a trait or state, and viewing empathy as an automatic or controlled process. A surprising finding from this part of the study was that I was unable to develop "profiles" of GSIs based off of their perspectives on Cuff et al.'s eight dimensions of empathy. Many GSIs would respond similarly in some categories but contradict each other in other categories. In addition, it is questionable whether I would have been able to develop profiles with a larger pool of participants. This finding, therefore, helps illustrate the nuances associated with defining empathy and the challenges inherent in studying this phenomenon.

After analyzing my participants' definitions of empathy, I expanded my focus to investigate factors associated with teaching and interacting with students that might affect how instructors empathize with students. The purpose of this analysis was to better

understand these factors so that leaders of professional development can develop activities that are specifically targeted to help GSIs think about empathy in the classroom. In this part of the study, I identified a total of five primary factors associated with GSIs' conceptualizations of pedagogical empathy: instructors' past experiences, students' mathematical experiences, students' experiences outside the classroom, the importance of communicating pedagogical empathy to students, and the course structure. These factors fell under two categories relating to the instructional triangle: factors influencing instructor-student interactions and factors influencing instructor-student-content interactions. Finally, I connected the factors that I found to Davis's (1996) organizational model of empathy. Overall, the GSIs in my study viewed these factors as both supports and hinderances to empathizing with their students. Thus, these factors can also be seen as areas to target in professional development for GSIs.

The final part of my analysis involved constructing detailed narratives to describe how pedagogical empathy can be conceptualized as a filter that connects different types of knowledge to decision making. For this chapter, I narrowed in on observations and interviews with four of my participants who I chose to study because of their stances on the different dimensions of empathy discussed in Chapter 4. In the post-observation interviews, I used stimulated recall methods to help these GSIs reflect on the decisions that they made in specific interactions with their students that they had noticed during the observation. I used these cases to show how pedagogical empathy connects emotional knowledge and pedagogical content knowledge to teachers' decisions.

Overall Implications

It is evident from this study that pedagogical empathy is an important factor in interactions between instructors and their students. As the GSIs in my study reflected on empathy and their teaching practices, their conceptualizations of empathy evolved with respect to different dimensions. These dimensions of empathy proved to be a useful way of analyzing instructors' views and connects this work to psychological research on empathy. In addition, unpacking these dimensions of empathy might be a worthwhile professional development activity in order to help GSIs better understand their own conceptualizations of empathy and how that influences their teaching.

The findings in this study also suggest that instructors can develop a more empathic disposition towards students over time through deliberate effort. Throughout the interview process, several of my participants brought up motivating factors of empathy and discussed how they had become more aware of how students might feel through various situations they had experienced. In some cases, this awareness had grown out of interactions with particular students or discussions with other GSIs about their students. In other cases, GSIs had reflected on their own experiences in different situations to try to understand what their students might be going through in their courses. For one instructor, this led to a *translation* of empathy, which helped him understand students who were uninterested in mathematics. It is encouraging to think that empathy can be cultivated and used as a positive way of relating to students. Accordingly, professional development activities should be designed to help GSIs develop a broader awareness of different student experiences.

Finally, empathy can be a useful lens through which to examine teaching decisions. Instructors who engage in deep reflection on specific interactions with their students may develop a better understanding of both their own teaching decisions and their students' perspectives. This process of reflection also has the potential to change teaching practices and can help instructors make more informed decisions in future interactions with their students. As many of my participants expressed in their final interviews, engaging in this process throughout the semester helped them think more carefully about their interactions with their students and made them more aware of when they were empathizing with students.

Future Research

In this study, I have sought to describe how my participants conceptualized pedagogical empathy and how pedagogical empathy can be conceptualized as a filter that connects emotional knowledge and pedagogical content knowledge to decision-making. There are several directions that researchers could expand on this work in future studies. First, future work should carefully examine students' perspectives on pedagogical empathy. A major limitation of this study was its one-sided approach to studying this phenomenon that involves both instructors and students (or observers and targets in the language of empathy research). Developing a better understanding of how students perceive empathy would be helpful for both instructors and researchers moving forward.

Future research should also build on this study to compare how different contexts might influence pedagogical empathy. There are several areas in which comparisons could be drawn: novice vs. more experienced instructors; small, liberal arts institutions vs. large, research-intensive institutions; first-year mathematics courses vs. upper-level,

advanced mathematics courses; etc. These comparisons would help illustrate how empathy might be elicited in different situations.

Finally, future studies on pedagogical empathy should draw connections between empathy and equity. It is possible that instructors are more likely to empathize with students who they view as similar to themselves. This circumstance brings up serious questions related to equity, especially since empathy has the potential to influence teaching practices. More research is needed to examine the relationship between empathy and equity to help show how empathy might be used to promote equity in the classroom, rather than engendering inequities.

This study highlights the importance of pedagogical empathy in interactions between GSIs and their students. Through this work, I aim to help the field of undergraduate mathematics education research move forward in a new direction of study. By investigating this phenomenon further, researchers can continue to develop a better understanding of the influence of pedagogical empathy on teaching practices and the intricacies involved in teaching interactions.

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APPENDIX A: DATA COLLECTION PROTOCOLS

Initial Interview Protocol

For this study, I'm interested in studying how math GTAs[7] experience empathy for their students and how that might influence the feedback they provide to their students. Since you are participating in this study, I believe it's important to understand your perspective of empathy and what you think it means to have or experience empathy within the context of teaching.

So to start off, I have a little activity for you and before you begin, I just want to let you know that there are no right or wrong answers to these questions. My main goal in having you do this is to try to start gaining an understanding of what you think about these things.

(Show them the definition of empathy handout and give them time to work on it. When they are finished, move on to the following questions.)

1. Could you explain your definition of empathy to me?

2. How does your definition of empathy relate to the example that you came up with?

    a. Where do you think that empathy is coming from? What might have caused you or this teacher to experience empathy for that student?

    b. Is empathy being communicated to the student in some way? How?

3. Do you think it's necessary for math teachers to have empathy for their students? Why or why not?

4. Can you describe a situation where someone teaching a college math class might be **more** likely to experience empathy for their students?

5. Can you describe a situation where someone teaching a college math class might be **less** likely to experience empathy for their students?

[7] In UNL's Mathematics Department, the term "graduate teaching assistant" or "GTA" is used for any graduate student on a teaching assistantship. The term GSI is more accurately used in the body of this dissertation as "GSI" is used in the research literature to describe graduate students who are the instructor of record for undergraduate mathematics courses.

6. Is there anything else about the way you conceptualize empathy as it relates to teaching math that you'd like me to know or that I haven't asked about?

To conclude, I have a few background questions that I'd like to ask you.

7. How many years have you been teaching here at UNL?

8. What courses have you taught?

9. Did you have any prior teaching experience before starting grad school or becoming an instructor at UNL? Where and what did you teach?

10. Why did you decide to pursue a graduate degree in mathematics?

11. Do you plan on continuing to teach mathematics in the future? In what capacity or setting?

Wrap Up (looking toward the observation):

What does a typical day look like in your classroom?

Definition of Empathy Activity

How would you define empathy as it relates to a teaching context?

Provide an example to illustrate the definition you came up with above. (Try to use an example from one of your own experiences as a teacher. If you can't think of one, describe a hypothetical situation that would fit your definition.)

Classroom Observation Protocol

General Information

Instructor ID: _____ Date: _____

Course ID: _____ Observer: _____

Start Time: _____ End Time: _____

Demographics

Number of Students

Total Enrolled: _____ Males: _____ Females: _____

Total in Attendance: _____ Males: _____ Females: _____

Room Setup

☐ Tables Number of Seats/Table: _____

☐ Individual Desks Arranged in Groups Number of Desks/Group: _____

☐ Individual Desks

Room Diagram: (note location of camera & observer)

Researcher Positioning

Description of relationship between researcher & instructor: _____

Perceived attitudes concerning the researcher's presence in the classroom: _____

Perceived effect of the video camera's presence in the classroom: _____

Description of researcher's thoughts, feelings, and experiences prior to observation: _____

Consideration of how these prior thoughts, feelings, and experiences may affect researchers' perception of the observation: _____

Purpose of Sampling

What is the exact purpose of observing this particular instructor, course, and/or lesson?

Class Context

Math topic covered in class:

Description of classroom norms:

Timeline of Class Lesson

Sequence of class activities including any important events or interactions:

Start Time	End Time	Class Activity

Criteria for Meaningful Interactions

Meaningful interactions must involve:

- Feedback
    - Conversation between a teacher and a student (or small group of students) about aspects of student performance during the time when students are working on problems in class

- Extended dialogue
    - Interaction consists of more than one verbal response from a teacher
    - Interaction could consist of a back and forth exchange between a teacher and a student for an extended period of time

- Personal component
    - Teacher uses some kind of personal pronoun when conversing with the student (i.e., you, I, we, us, they, etc.)

Descriptive Notes: Description of activities and individuals engaged in those activities. Note any interactions of importance between instructor and individual students or groups.	***Reflective Notes:*** Questions about interactions, observations of nonverbal behavior, interpretations of observations.

General Observation Notes **START VIDEO RECORDING!!!**

Immediately after the observation:

Were there any specific interactions between you and your students today that stood out to you?

Post – Observation Reflection

Post-Observation Identification of Important Interaction

For use in the post-observation interview, identify an important interaction between instructor and student(s): _____

Describe reasoning for choosing this interaction: _____

Reflection on Researcher Positioning

Changes in relationship between researcher & instructor: _____

Changes in perceived attitudes concerning the researcher's presence in the classroom: _____

Changes in perceived effect of the video camera's presence in the classroom: _____

Consideration of how the prior thoughts, feelings, and experiences may have affected researchers' perception of the observation: _____

Description of researcher's thoughts, feelings, and experiences after observation: _____

Consideration of how the thoughts, feelings, and experiences after observation may affect researchers' perception of the observation: _____

Reflection on Purpose of Sampling

Did the observation serve the intended purpose?

☐ Yes

☐ No

Comments: _____

Reflections on Interactions *Observation #_____*

Description of Interactions: _____

Sample Post-Observation Interview Protocol

(Joe Post-Observation Interview 2)

To begin, I want to go back to your definition of empathy like we did last time and revisit how you are defining it and if there are any other examples that might illustrate what you mean. Again, I want to remind you that there are no wrong answers, and that I am just interested in hearing your thoughts on this topic.

1. When I first interviewed you, you wrote down this definition and described empathy as

    - An awareness of student thoughts, especially their understanding of content and comfort with content and the ability to "relate to" those feelings

Last time I interviewed you, you wanted to add a little bit to that definition and said that empathy from a teacher's perspective is

- Understanding students' thoughts and their understanding of the content and their comfort with the content and then also their comfort with you

You also gave another definition:

- Being able to predict what students are thinking and understand what they're thinking and use that to help your teaching

So, thinking about it now, is there anything else that you want to change or add or take away from this definition?

Prompt: Are there any experiences that you've had since our last interview that might have caused this change?

Prompt: Are there any other situations or experiences that you would use to illustrate your definition of empathy?

Next, I want to talk about an interaction that you mentioned to me after your observation yesterday.

2. This interaction involved a student who was struggling with where to start on how to decompose functions. *(Watch brief video clip of interaction.)*

    - Why did you think that was a significant interaction?
    - Can you remember what you were thinking or feeling in that moment?
    - Do you think you experienced empathy in this interaction?
        - *Prompt (if either yes or no):* How can you tell?
        - *Prompt (if yes):* What was it about this interaction that allowed you to experience empathy?
        - *Prompt (if no or uncertain):* Do you think this was a "missed opportunity" for you to experience empathy? Why or why not?
    - Can you discuss a goal that you might have had going into this interaction?
    - What type of knowledge might you have been using during this interaction?
    - What beliefs might be influencing the way you think about a situation like this?

Other prompts:

- At the beginning of this interaction, you were trying to get Dave to find an inside part of the function. And you asked him of couple of times to do this, and he eventually said, "I have no clue." At that point you said, "let's phone a friend." Why did you decide to do that?
- How did you feel after having this conversation?
- Do you think you would have had a similar experience talking to another student about this? Why or why not?

Is there anything else that you'd like me to know about this interaction that I haven't yet asked about? *(Look at notes from after the observation.)*

Another interaction that I want to talk about was with a couple of students who were sitting on the other side of the room from the camera.

3. These students were working on this problem and were trying to figure out how to use the graphs. *(Show Joe the problem and watch brief video clip of interaction.)*

    - What do you notice about this interaction?
    - Can you remember what you were thinking or feeling in that moment?
    - Do you think you experienced empathy in this interaction?
        - *Prompt (if either yes or no):* How can you tell?
        - *Prompt (if yes):* What was it about this interaction that allowed you to experience empathy?
        - *Prompt (if no or uncertain):* Do you think this was a "missed opportunity" for you to experience empathy? Why or why not?
    - Can you discuss a goal that you might have had going into this interaction?
    - What type of knowledge might you have been using during this interaction?
    - What beliefs might be influencing the way you think about a situation like this?

Other prompts:

- Do you think you would have had a similar experience talking to another group of students about this? Why or why not?

Is there anything else that you'd like me to know about this interaction that I haven't yet asked about? *(Look at notes from after the observation.)*

Is there anything else that that I haven't asked you about that might help me to better understand how you experience or conceptualize empathy as a teacher?

Additional Questions for the Final Interview

To begin, I want to revisit your definition of empathy. Again, I want to remind you that there are no wrong answers, and that I am just interested in hearing your thoughts on this topic.

The last couple of times that I've interviewed you, you have defined/described empathy as

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  Is there anything that you would want to change about that definition or description?

  Prompt: Are there any experiences that you've had since our last interview that might have caused this change?

  Prompt: Are there any other situations or experiences that you would use to illustrate your definition of empathy?

Now I want to go through a few other definitions of empathy within the context of teaching that other GTAs have come up with and talk through some of them.

[Hand them the sheet of definitions and let them look through it.]

- Any there any definitions that you think are better or worse than others?
- Are there any definitions or parts of definitions that you strongly agree or disagree with?
- After looking through these definitions, do you want to change anything about your definition? Do you want to adopt or include a different definition from this list?

Other Questions: (Feel free to use examples from your own experiences).

Within the context of teaching…

- Do you think there's a difference between experiencing empathy for students and experiencing other related concepts like sympathy or compassion for students? Can you explain those differences?
- Do you think of experiencing empathy for students as more of an affective or cognitive process? Why?

- How important do you think it is to accurately perceive the feelings or thoughts of students when experiencing empathy for them?
    - Very important
    - Somewhat important
    - Somewhat unimportant
    - Not important at all
- Do you think a student has to be present in order for you to experience empathy for them? Why or why not?
- When experiencing empathy for students, how important is it that you distinguish between their thoughts or feelings and your own thoughts or feelings?
    - Very important
    - Somewhat important
    - Somewhat unimportant
    - Not important at all
- Do you think resulting actions or behavioral outcomes are part of experiencing empathy for students? Why or why not?
- Do you think of experiencing empathy for students as more of an ability or state of mind? Why?
- Do you think of experiencing empathy for students as more of an automatic or controlled process? Why?

Final Questions at the End of the Interview:

- Now that you've had the chance to reflect with me a few times, what might you do differently in the future?
- What have you learned about planning, teaching, and reflecting on teaching from this process?

APPENDIX B: GSIS' WRITTEN DEFINITIONS OF EMPATHY

Aaron

Initial written definition:

- Having an understanding of a student's unique perspective, history, and conditions within the context of a learning environment.

Revisions:

- Putting yourself in someone else's shoes or imagining yourself as being in that situation in some way. (Added during Post-Observation Interview 1.)
- Being able to predict and use that predictive power. (Added during Post-Observation Interview 3, after looking at other definitions.)

Amanda

Initial written definition:

- Trying to understand the class from the point of view of your students – like understanding when your students feel lost / upset / discouraged or when they feel strong and excited mathematically and relating to them through shared or perceived experiences.

Revisions:

- Predicting before you go into class how your students are going to feel about something instead of just responding to how your students are feeling in the moment. (Added during Post-Observation Interview 2.)
- Understanding why students feel the way they do. (Added during Post-Observation Interview 3, after looking at other definitions.)

Bill

Initial written definition:

- Being able to put yourself in the position of the student to better relate to their fears, anxieties, confusions, etc.

Revisions:

- Being patient like if you have a student who is really confused about something and you try to explain as best as you can, but it's still not getting through—trying to think of other ways to describe it, because it can be frustrating if you feel like you're saying the same thing—to be able to stick with that without outwardly showing frustration. (Added during Post-Observation Interview 1.)

Chris

Initial written definition:

- Understanding where a student is getting confused and trying your best to walk them toward a solution in a way that you know they'll understand.

Revisions:

- Thinking about their interactions with their fellow students and making adjustments. Trying to be aware of their understanding of the content and comfort and being able to relate to those feelings. (Added during Post-Observation Interview 3, after looking at other definitions.)

Henry

Initial written definition:

- Understanding and feeling to some extent what another person is feeling or experiencing. Within a teaching context, being able to interact with students in a way that is understanding of their experience of the material and classroom, which may be very different from that of the instructor.

Revisions:

- Considering things other than content, like cultural differences and difficult personal situations, and how you should take this into account when doing things like writing exam questions and making accommodations for students. (Added during Post-Observation Interview 1.)

Jack

Initial written definition:

- The ability of the instructor to simulate the emotions of their students and respond accordingly.

Revisions:

- (Made no revisions throughout the semester.)

Jane

Initial written definition:

- Being able to understand the class from the students' perspective and understand how students feel/why they feel the way they do, whether it's about the material, the class, environment, classmates, or teacher and how it affects their experience in the classroom.

Revisions:

- Understanding your students' obligations outside of class. Predicting before you go into class how your students are going to feel about the material. (Added during Post-Observation Interview 3, after looking at other definitions.)

Joe

Initial written definition:

- An awareness of student thoughts, especially their understanding of content and comfort with content and the ability to "relate to" those feelings. Being able to predict what students are thinking and understand what they're thinking and use that to help your teaching.

Revisions:

- Understanding students' comfort with you. (Added during Post-Observation Interview 1.)
- Empathy is something you should have and part of being a good teacher is having this empathy. (Added during Post-Observation Interview 3, after looking at other definitions.)

Mark

Initial written definition:

- Understanding the feelings of your students and how the circumstances affect them.

Revisions:

- And acting accordingly. (Added during Post-Observation Interview 3.)

Oliver

Initial written definition:

- The ability to understand when something is difficult to learn.

Revisions:

- Being able to put yourself in your students' shoes and think about how it would feel if everybody around you understands something and you don't—as an instructor, being able to help a student through that or at least attempting to help a student through that. (Added during Post-Observation Interview 1.)

APPENDIX C: TRANSCRIPTS OF INTERACTIONS BETWEEN GSIS AND THEIR STUDENTS

Aaron's Interaction with Jessica

(32:36)

Jessica: Aaron?

Aaron: Yeah.

Jessica: Are exponential growths parabolas?

Aaron: They are different from parabolas. If you look at a parabola...*[erases and draws on the whiteboard]* what this is saying is that – I'll do it in terms of t…

Jessica: Okay.

Aaron: …so that it matches what we usually use for exponentials. Uh…let's see, two times…so there are a lot of differences. One difference is that in general, parabolas increase in both directions. So, as t is really negative we get a large value. As we get closer and closer to zero, the value shrinks, that's because we're squaring the number, so we're always flipping it up into the positive. But when we are close to zero, we're squaring a very small number so it's something very small. Whereas, with an exponential, I'm increasing all along the t axis. As I start really far out here, I'm really small. As I move to the right, I get bigger, no matter where I am. Here, as I move to the right, I'm shrinking, shrinking, shrinking, shrinking, shrinking then growing, growing, growing, growing. So that's one difference. Another difference is that exponential functions tend to explode, a lot. And by that I mean, which is bigger – 100 squared or 2 to the 100th power? You can try computing this in your calculator, but you might actually start having – well I mean you can try computing it in your calculator. That might be a good way to see this.

Jessica: Yeah. It's – that one's bigger *[pointing to two to the 100th power]*.

Aaron: Yeah, but it's a lot bigger, right? It's like a huge number.

Jessica: It's raised to the number *x* to the 30th.

Aaron: x to the 30? Like it's some number…

Jessica: Like it was like point two zero two…

Aaron: …like one point two times ten to the 30th power. So one with 30 zeros after it, as opposed to one with four zeros after it. Yeah, so in general, these things explode by quite a lot more if they have a nice growth rate. I mean of course I could make this number, I could make this number really small and the growth would shrink down, but the fundamental thing that we're doing of taking high powers is like a stronger thing to do – just like how multiplying numbers tends to make bigger things than adding numbers. Exponentiating numbers tends to be…yeah.

Jessica: So I know that this one's right because that's pretty much what we just did…

Aaron: Yeah.

Jessica: …but this one, when they decrease, does it start and then kind of edge out or no? How would you do that, because it's *[inaudible]* and it's greater than y equals zero.

Aaron: Yeah. So you have the right idea, but the…can you see this? Or is it too

Jessica: Yeah.

Aaron: Okay. Whenever you're doing exponential in the other direction, like with zero less than b less than one. It's shrinking down, but importantly, it never crosses the t axis. It never goes negative. And that's because in the formula that I have, I'm always multiplying positive numbers together. For any positive number here, or actually even any negative number here, any number here, I'm multiplying positive numbers together, so I should always get something positive. And I'll never get something zero because I can never write zero as a product of nonzero numbers…

Jessica: Yeah.

Aaron: So it's never zero or negative.

Jessica: So how would you find the domain and range of that?

Aaron: Well, domain – I can put any number here, that's not too bad.

Jessica: So it's all real numbers?

Aaron: Yeah. For the range, if I have some negative number here, like I'm over here.

Jessica: Wait, hold on. For domain, you can put negatives in too?

Aaron: Yeah, yeah. I can put negatives – so we commonly like to think of these things as modeling a process that starts at time zero and then we just kind of ignore the left-hand side of the graph. But we can, if we want to, also have negative numbers, because if I want to find something like this *[writing two to the negative first power on the board]*, the rule that I know is that this is equal to the reciprocal. Or if I want to find one point two three or something, then it's *[writing on the board]*.

Jessica: Okay. Yeah. That would be like the reciprocal of that.

Aaron: So that's why I drew this part over here.

Jessica: So those ones are *[inaudible]*.

Aaron: Those are for negative values of t, like *[writing numbers along the x axis]*.

Jessica: So…

(39:21)

Aaron: So, I have a question for you, actually. Do you notice how these look very similar, except they're rotated? *[referring to the graphs of the increasing exponential and decreasing exponential]*

Jessica: Yeah.

Aaron: Why do you think that that is?

Jessica: This one's negative, that one's positive *[pointing to the board]*.

Aaron: Um…what do you mean by it's negative.

Jessica: Well like okay so that one the b is less than one, and then that one's b is greater, er above one.

Aaron: Yeah, but why does that lead to them being like mirror images of each other? If you imagine putting in negative t, what that does is it flips all of the numbers, for a given value of t, I'm now looking at negative t and then seeing that value. So when I do this, I basically take the resulting – the graph of this is going to be the graph of Q of t, but rotated around the y axis, like this *[motioning with hands]*. It's going to be flipped.

Jessica: Oh, so I could just always work with positive numbers and then if it's a negative, you could just flip it?

Aaron: Um…well I don't think that it makes as much physical sense to – like if you're thinking of a population that's decreasing because they're dying off…

Jessica: Uh huh.

Aaron: I still like to think of time as going in – forward, instead of thinking of putting in negative values of time and then time going backwards.

Jessica: Okay

Aaron: So I wouldn't advise thinking about it that way. I just used this as uh – I'm just using this as a way to explain why there are – there's this mirror image thing going on.

Jessica: How would you find the range *[inaudible]*?

Aaron: Um…for an exponential on the whole domain, like as long as they're not doing some piecewise thing,

Jessica: Well, okay.

Aaron: The answer is any…

Jessica: Well there is, but…

Aaron: Yeah.

Jessica: …I was looking, I like did each one individually and then I combined them.

Aaron: Yeah, so yours was one that was decreasing, right? Like it was uh…

Jessica: Yeah, so it started *[inaudible]*…

Aaron: …exponential decay?

Jessica: …at the y intercept was 6 and there was no, it was greater than zero, so…

Aaron: Yeah.

Jessica: …without the top part.

Aaron: Yeah, so – what was the, could you give me the function?

Jessica: It's six times um…point five t.

Aaron: So, I guess the question is: given any number…*[writing on the whiteboard]*

Jessica: Wouldn't it be…

Aaron: No, I don't think this is the right way to think about it.

Jessica: Wouldn't it be six would be the um…

Aaron: If you're only putting in positive numbers, we get smaller and smaller and smaller, but we never stop, I always keep shrinking…

Jessica: So it would be positive infinity. It'd be 6 to positive infinity?

Aaron: Six to positive infinity is this interval right here *[pointing to interval on board]*, but we're starting here, and then going down. So it's actually this, this chunk right here *[drawing on the graph]*. It's just *[writing the range (0,6] on board]*. I start at six, I get smaller from there, I never pass zero, but I keep getting smaller and smaller and smaller, and I never stop getting smaller.

Jessica: So it'd be six and then zero since…

Aaron: But I can never get zero, because I can never multiply things together to get zero so I get really close to zero, I get really, really small but never quite die out, which is kind of weird. It's kind of weird because like we talk about these things modeling bacteria populations and stuff like that, and of course in real life, there's an extinction event, right? There's a time when this species goes extinct. This is one reason why the exponential model is like a good model that makes a lot of sense, but isn't perfect. None of these functions are perfect representations of data, unfortunately.

(44:32)

Aaron's Interaction with Shantel

(15:35)

[Shantel has her hand raised. Aaron walks over and sits down next to her.]

Aaron: What's up?

Shantel: On this one, how do you know what points to use? Because it doesn't say like – like I just use one of these.

Aaron: Yeah. You can kind of pick whichever ones are easiest. Or you can kind of look at the whole thing and see what happens. Um…those look like really good points to use to me, because the way that you've set it up is, you've picked a point that is on the x-axis, so if any vertical stretching happens, it's going to stay where it is. You've picked a point that's not on the x-axis, so it's going to move.

Shantel: It has to be on the x-axis?

Aaron: So, they give you this graph *[drawing graph on the whiteboard]*, this funky graph, and you picked this point and you picked that point. What I'm saying is that, what's nice about the points that you chose is that if they vertically stretch, then this point is going to stay the same. So you can just look at these graphs and see that if the – if this point hasn't moved in one of these graphs, then a vertical stretch might have happened, but a horizontal stretch can't have happened.

Shantel: If it stays the same as vertically?

Aaron: Which point is that? Zero, negative one *[labeling the point on the graph on the whiteboard]*. So in this next one, the one that looks like this *[drawing another graph on the whiteboard]*.

[Blake comes back to the table.]

Aaron: *[to Blake]* Sorry *[Aaron gets up from Blake's chair]*.

Blake: You're fine.

Aaron: So in this one, this point hasn't moved. So *[writes a sentence on the whiteboard]* if a horizontal stretch had happened, then this point would have either shifted this way *[motioning with hands]* or shifted this way. So choosing this point is great, because it's now giving you a way to tell whether a horizontal shift has happened. If this point stays the same, then a horizontal shift didn't happen. If this point moved, then a horizontal shift

did happen. It kind of identifies whether one's taken place. So in this case, I know that a horizontal shift hasn't happened, but it looks like it's different. You also had this point, right? Negative two, two *[labeling the other point on the original graph on the whiteboard]*, and this is which graph – this is graph two, right? This is the original *[labeling the graphs on the whiteboard]*.

(19:32)

Jane's Interaction with Maria

(30:42)

[Jane sits down with a group next to Maria. Maria immediately asks her a question.]

Maria: Is this the weeks?

Jane: I'm sorry?

Maria: Is this the weeks? *[Pointing to her workbook]*

Jane: Mmm. Um. Okay so the problem says the grades of six students are given by the equations below where time is measured in weeks. Okay. So that means, so *P* is a function of *t*, okay? So that means, so *P* is the grade, right? That depends on how many weeks have passed. So we're actually, our *t*, our weeks is represented by *t*, we input *t*, we input the number of weeks that have passed, and then our output is the grade. Okay? So when they say time is measured in weeks they're saying well if five weeks have passed, then I input five in for *t*. And then whatever value I get out, that's the grade for that student.

Maria: Okay. So like, so how many weeks have passed then? 0.9?

Jane: Um…so 0.9, that represents, which part of our function is that? *[Pointing to notes written on the whiteboard]*

Maria: *[Looking at the whiteboard]* The *b*?

Jane: Mmhmm *[agreeing]* and what do we call that?

Maria: *[Looks back at workbook]* Um…I don't know.

Jane: *[Pointing at the whiteboard again]* We call *b* the…growth...

Maria: Oh the growth factor.

Jane: …growth factor. So we can think about the growth factor as the – so remember we, like in our examples before we were taking our amount and then we were multiplying it. So like for example, we were taking 500 and multiplying it by 1.045. Right? So *b*, our *b*, or in this case point nine *[pointing at equation in workbook]*, is the amount that I multiply by the value that I have to get the next value.

(32:33)

Maria: Okay.

Jane: Okay.

Maria: So…

Jane: So like we knew that we needed to multiply that 1.045 by our amount that we had, 500, to get to the next value. So same idea here.

Maria: So I just have to do that to each one?

Jane: Well so…so this is asking…this is asking Alex's grade has been decreasing, which of these functions could represent her grade.

Zach: *[Talking to Amy about next problem – Problem 5 in Section 7.1 worksheet]* Well I think since you want it to be decreasing *[inaudible]*…

Jane: So let's maybe like consult with the groupmates.

Zach: …because if you wanted it to go up, you'd have to have – it'd have to be greater than one.

Amy: That's what I was thinking.

Zach: But since it's going to decrease each year you'd just leave it as the .03.

Amy: And then *t* would be years?

Zach: Yeah. It'd be the years after.

Amy: I plugged in three for *t* and then solved it and it was 8.1. *[inaudible]*

Jane: So how did things go for Problem 4?

Amy: Like I put like you multiply like 300,000 by .03 because it's decreasing.

Jane: Okay.

Amy: And then *t* would be like the amount of years so I plugged in three.

Jane: Okay.

Zach: Yeah I did as…

Amy: Like I…

Zach: It was 8.1.

Jane: Okay. Okay. That's a good...that's a good point. So – which doesn't really make a lot of sense, right? Like eight people.

Amy: No.

Jane: Okay so maybe. I'll let you guys think about other things. Grace *[addressing fourth student at the table]*, how did number 4 go for you?

Grace: I don't get [inaudible]

Amy: Oh wait, for this one?

Jane: Yeah. How did number…

Amy: Oh sorry.

Jane: So I was asking about number 4. That's okay. How did number 4 go?

Amy: Pretty good. I just based all of them off of like this middle number.

Jane: The middle…okay, which we call – what do we call that?

Students: *[together]* Growth factor.

Jane: The growth factor. So how did you use the growth factor to determine if like the formula could represent Alex's grade, which is decreasing?

Amy: *[Inaudible]* Any less than one.

Jane: *[Nodding]* yeah so if your growth factor, if our *b* value, is less than one *[pointing to notes on the board]*, then we know that our function – that we're experiencing exponential decay, so our function is decreasing.

Maria: Oh okay.

Jane: Yeah. Good. And how did you determine which student's grade is falling the fastest?

Amy: Um…the lowest?

Grace: The lowest.

Zach: Yeah the lowest.

Jane: *[Nodding]* Yeah. That *b* value can also describe how…how quickly your function is decaying or growing. Yeah. Good. Okay.

Zach: *[inaudible]*

Amy: Where'd you get the 3,000?

Zach: Well I just wanted to see what 3% of 300,000 was.

Jane: So what does it mean to be decreasing by 3% a year?

Zach: The population's going down.

Jane: Okay. So if your population is decreasing by 3% each year, how much of the population is left after one year?

Amy: 3% less than the first year?

Zach: It'd be 97%.

Jane: Which would be – yeah so if 3%, if it decreased by 3%, then the amount – that means that like three percent of the population is removed after one year, so how much of the population is left?

Zach: So it would be…

Grace: .97. So you do like .97 to the…

Amy: Yeah.

Grace: You would put .03 into the equation like .16.

Zach: Oh…okay. That makes…

Jane: Cause the…

Grace: They want the population after three years *[inaudible]*

Zach: …that makes sense. Because if it was going to increase by 3% you would do the 1.03.

Amy: Oh wait. I put *[inaudible]*

[Jane looks over at Maria's workbook]

Jane: So Maria, are you…do you feel good about this one now?

Maria: Yeah.

Jane: Okay. Awesome.

(36:57)

Jane's Interaction with Keisha

(11:10)

[Jane walks over to the table, sits down, and asks group how things are going. She mainly talks to two students at the table before turning to Keisha.]

(13:16)

Jane: Keisha, how are you doing?

Keisha: *[Shakes her head.]* I don't get it.

Jane: Okay, which – do you want to talk about a specific problem? Let's look at Problem 3. How did that one go?

[Keisha flips to Problem 3 in her workbook.]

Jane: Okay, so – so it's giving us – let's just focus on part a. So it's giving you um the equation for a parabola. Um what form is it in – is this equation?

Keisha: Um…*[long pause]* standard form? Or…

Jane: What does standard form look like?

Keisha: *[Looks back in her workbook.]* Or vertex form?

Jane: Yeah. This is in vertex form. Okay, so um how do you – how do you know it's in vertex form?

Keisha: *[inaudible – something about parentheses]*

Jane: Okay. You can see these parentheses, and what's inside the parentheses?

Keisha: The *x*.

Jane: Okay, so there's an *x* and it's got like this expression *x* minus three, right? And then we're squaring it and then we're adding – we're adding one at the end. Okay, so since this is in vertex form, um…so they're asking us to find the vertex. So what is the vertex?

Keisha: Uh…the three?

Jane: Okay, three…

Keisha: And the plus one.

Jane: …and one. Yeah, so the vertex – so the vertex is the point three comma one. Okay? So this three here – this three here – that's like *h* – that's your *h* value, which is like your *x*-coordinate for your vertex. Yeah, so the vertex is this point three comma one. So like – like – like do you know where the vertex is on the graph? Like if – where is the vertex on this graph *[pointing to a graph in the workbook]*?

Keisha: Um…like between them?

Jane: So the vertex of a parabola is – if the parabola's facing up, then it's like the – the lowest point. So in this case, they're telling you like the parabola – the vertex is one, two. If it's facing down, then the vertex is like the highest point on the parabola. So here, we're getting the information that the vertex is the point three comma one. Is it – now is the parabola facing upward or downward?

Keisha: Upward.

Jane: How do you know it's upward?

Keisha: Because it's going up, or…it'd be going up. It's not going down.

Jane: Okay. How can we figure that out though based on the equation?

Keisha: The plus one.

Jane: Okay, so the plus one tells you what the *y*-coordinate is for your vertex. If we're facing up – maybe someone over here can help me out. How – how do we know if the parabola's facing up or down?

Gabby: Oh if it's like negative – if your *a* is negative, it's going to go – flip over the *x*-axis, so it's downward, but if it's positive you know that it's up *[motioning with her hands]*.

Jane: Mmhmm *[agreeing]*. Yep. So what is the *a* value in part (a)?

Keisha: The *x*…the one?

Jane: Mmm, so let's look back at our – at the equation. Did you write it in your notes? The equation?

[Keisha flips back in her workbook.]

Jane: Okay, so here is vertex form *[pointing to Keisha's workbook]*. So where – so *a* is right here, right? So what would *a* be in this equation?

Keisha: 0.5?

Jane: Yeah, the 0.5. And since point – like 0.5 is – it's like positive, then we know that…

Keisha: It's going up.

Jane: It's going – it's facing upward, yeah. So that's telling us, since it's facing upward, the vertex is like this – this like lowest point on our parabola. And so then, okay…

Keisha: So that's the vertex, 0.5, or…

Jane: The 0.5 is telling you – so, that's like telling you – what does the .5 tell you *[turning to the other students at the table]*?

Zach: Uh…that it's vertically stretched, right?

Jane: Mmhmm *[agreeing]*.

Zach: By that much.

Jane: Yeah, so that a in front tells you if you're vertically stretched or compressed. Okay. So that's just telling – and it's also telling you…

Zach: Or it's compressed since it's less than one.

Jane: …yeah since it's less than one, then you know that you – wait.

Zach: Right? Or is it…

Gabby: It's compressed.

Jane: It's compressed. Yeah.

Zach: Yeah compressed, yeah, yeah.

Jane: It's the opposite when it's horiz…okay, okay. So we know that it's going to be compressed, yeah. That always takes me a second to think about too. Okay. So um…so the a just tells you whether you're stretched or compressed and it also tells you whether you're facing up or down. In this case, since *a* is positive, then we're facing up. Okay? And then what is the axis of symmetry going to be?

Keisha: Um…*[long pause]* these two numbers?

(19:20)

Jane: Well so what did we say that the axis of symmetry was *[pointing to the board]*? Or maybe do you have it in your notes.

Keisha: It's *h*.

Jane: Yeah, so the axis of symmetry was just the vertical line going through the vertex. So it would be like the line going through – going right here *[pointing to Keisha's workbook]*. Okay? And that line is just *x* equals *h*. Okay? So in our problem over here, what was *h*?

Keisha: 0.5.

Jane: Well 0.5 was…

Keisha: *a*

Jane: …*a*. What is – what is *h*?

Keisha: Three?

Jane: Yeah. So I would maybe like just write an *h* above your – like above the three so that you know that that's your *h* and then you know that *a* is .5, and then one would be your…

Keisha: *[inaudible]*

Jane: Well so one would be, we called it *k* before.

Keisha: Oh…

Jane: So, since *h* is three, then what is your axis of symmetry going to be?

[Long pause.]

Jane: So it's the vertical line at *x* equals what?

Keisha: *h*

Jane: Mmhmm *[affirmation]*. And in this case?

Keisha: Negative three.

Jane: Well *h* is just three here, since we're subtracting in our equation, *h* is just going to be the three.

Keisha: *[inaudible – something like, "So it's where this goes?"]*

Jane: Which part?

Keisha: *[inaudible – something like, "The axes? The x?"]*

Jane: So – so we would just write like *x* equals and then whatever *h* is – is the vertical line *[moving hand up and down]*. And since *h* is three, we said, then we know that our axis of symmetry is going to be…

Keisha: Three?

Jane: Mmhmm *[affirmation]*. Yep, and we would write *x* equals three to represent that it's a line *[moving hand up and down]*. Okay. So go ahead and try part *b*. *[Turning to other students]* did you all try number 4?

(21:36)

Joe's Interaction with Dave

(7:22)

Joe: Can I help with anything?

McKenzie: John is helping us.

Joe: Okay. Dave, got a question for me?

Dave: Yeah. I don't know where to start.

Joe: Yeah.

Dave: To be honest.

Joe: So maybe let's start on b because that one is good for an example. What's sort of – what's an inside part of that function or like what's something that you could find at first and then do other stuff to it. Like if you're finding that whole expression, what could you find first?

Dave: *x*. Is that right?

Joe: Uh so if you, if you – like if I give you an *x*, what's like eh…let me frame it this way. You're looking for some part of that function that you can sort of separate from the rest of it. So…

Dave: Well, um…if you square it all, can't you get rid of this?

Joe: Yeah. Um, but…

Dave: Or get rid of the minus five or something?

Joe: Mhmm *[agreeing]*. Yes, so that's how you would like solve for it. Um, but sort of like up there in that example, my inside part was the nine *x* plus seven. Cause like when I was finding – if I wanted to find an inside part, if I wanted to find *e* to the nine *x* plus seven, I could find the nine *x* plus seven first, and then do *e* to that as like a completely separate, as its own thing.

Dave: Okay.

Joe: So, something here, what could you find first? Or that's sort of like is doing its own thing.

Dave: I have no clue.

Joe: All right, let's phone a friend. McKenzie?

McKenzie: All right.

Joe: So, on part (b), what's some part of this that's like an inside part of the expression?

McKenzie: *x* squared plus five.

Joe: Sure. Um…uh so the two *x* squared plus five. Try circling that.

Emma: Circling the *[inaudible]*

Joe: Yep. Yep.

Emma: *[inaudible]*

Joe: Mhmm *[agreeing]*.

Emma: So, like the *h* of *x* is like the inside thing, and the *g* of *x* is like the outside. So, the stuff on the inside is being plugged into the stuff on the outside. Does that make sense?

Hunter: It's like an onion in Shrek, you know.

Joe: Yeah. Many layers.

Dave: Well like I know how to do these when they're like…

McKenzie: Like I got a, but…

Dave: …when it's like a *f* of *x* equals whatever. I know how to like write these equations.

Joe: So maybe some more examples would help. Like down here – so usually a good thing to look for is just something that's in parenthesis. So like down here, you could take your inside part to be *x* minus three. Or down here, you could take your inside part to be the natural log of *x*. Or up here you could take it to be…I don't know like *x* squared plus one or even the log of all that stuff.

Dave: So, all we're doing is like this would be *h*…

Joe: Yep. Yep.

Dave: …and then *g* would be like square root of *x*.

Joe: Yes. That's all we're doing there.

Dave: Oh, okay.

Emma: This makes it so much easier.

Dave: All right.

Joe: So, the hard part is finding something to circle, and if you're ever unsure just circle parts of the expression basically.

Dave: Okay. Thank you.

(10:48)

Joe's Interaction with Molly

(2:39)

Joe: Can I help out here?

Veronica: Oh yeah.

Joe: Great.

Molly: Maybe. I don't know if we're doing this right. I'm probably not.

Joe: All right. Can you walk me through it?

Veronica: I don't know if we're doing anything right.

Molly: Yeah. Okay so let's start with the first part…

Joe: Yes.

Molly: …because that's – I don't know how…

Joe: Yes.

Molly: …I think I did that right, but not that *[pointing to her workbook]*.

Joe: Okay.

Molly: Because – it's a circle, right?

Joe: Yes.

Molly: So, wouldn't that be 30?

Joe: So…

Molly: That seems too easy.

Joe: Where – where – can you point on the circle to where you are after 30 seconds.

Molly: Right there.

Joe: Yeah. And why did you know that's where you'd be?

Molly: Well, cause you start at the three o'clock position…

Joe: Yes.

Molly: …right here and for a whole rotation it's two minutes so I thought after a minute you'd be there so half of that is right there.

Joe: Yeah. And the one thing to point out is that it's also ascending…

Molly: Oh yeah.

Joe: …so you went up at first.

Molly: Yep. Sorry.

Joe: If it had said descending, you would be at the bottom, but you're – you're absolutely right.

Molly: Yeah.

Joe: Okay. And since you're up there, it's just the diameter of the circle, which is 30.

Molly: Yeah.

Joe: So perfect. All right.

Molly: Cool. Okay so for this…

Joe: Yes.

Molly: This is the part where I don't know if I'm doing it right. So the height after ten seconds – so for this, did I do that part right *[pointing to her workbook]*?

Joe: Okay.

Molly: Because after ten seconds you'd be at 30.

Joe: So, you said three degrees per second and times ten seconds is 30 degrees, I agree.

Veronica: Yes.

Molly: Okay. So, then your height after ten seconds – oh, so I'd have to say 30 degrees. So, I take the cosine of 30 degrees…

Joe: Let's draw our triangle.

Molly: …and then…yeah, okay.

Joe: So, we're going to have a triangle, whose center is the center of the Ferris wheel – or one point is the center of the Ferris wheel.

Molly: Okay.

Joe: One point is where you are, and then you draw the vertical line down.

Molly: So, this is…wait. I don't know. What did you…

Joe: So, let's draw it here – can you draw…

Molly: Okay. Yeah.

Joe: …from the center of the Ferris wheel out to the edge.

Molly: Okay.

Joe: Yeah. Like that – where it's a 30-degree angle.

Molly: Oh okay. Gotcha. Like that *[drawing in her workbook]* and then down?

Joe: Uh can you move your hand for a second? Yeah. Yeah. Out to that point and then down.

Molly: Like that? I don't know really how to – and then that's a 90-degree angle right there? Is that what you mean?

Joe: Sorry. I mean from the center out to that point there *[pointing to her workbook]*.

Molly: Oh, sorry I don't know why I drew it from my 30 *[inaudible]*…

Joe: And then straight down.

Molly: Straight down.

Joe: All right. Good. Um…and it won't quite meet on the edge there.

Molly: Oh okay. Sorry.

Joe: Okay.

Molly: It's hard on this. I should just like redraw it.

Joe: So, I'm not supposed to do this as the instructor, but can I draw the picture for you?

Molly: Yes. You can do it.

Joe: All right. So, you have – you have the circle.

Molly: Yeah.

Joe: From the center.

Molly: Yeah.

Joe: So, here's – here's the horizontal…

Molly: Yeah.

Joe: At 30 degrees, you go out there.

Molly: Okay. Oh okay.

Joe: And then straight down there. And it cuts down right there.

Molly: Gotcha. I see what you mean. Sorry. Thank you. Okay.

Joe: Okay. So, that's where we are.

Molly: Yes.

Joe: Now with this picture, what's the hypotenuse of that?

Molly: Um…is that 15?

Joe: Yeah.

Molly: So that's your radius.

Joe: Exactly, it's the radius. Hypotenuses are always radii for us.

Molly: Okay.

Joe: So that's going to be 15.

Molly: And so, you're trying to find this right here *[pointing to her workbook]*.

Joe: That's right. You're trying to find that opposite side.

Molly: So, it's opposite hypotenuse. Sine.

Joe: Exactly. So, sine of 30 degrees is *x* over 15.

Molly: Awesome. Thank you.

Joe: And you can solve for *x* that way.

(5:40)

Oliver's Interaction with Katie and Her Group

(08:20)

[Katie raises her hand, and Oliver walks over.]

Oliver: Yes?

Katie: How do you know what the period of a periodic function is?

Oliver: Well you tell me. What is the period of a periodic function?

Katie: Wait. Is it infinity?

Oliver: Careful. So, this is from week one right? So, the period – what defines a periodic function in your own words?

Katie: Like when you move it over and it's still the same.

Oliver: Okay. When you can move it over and it's still the same. So, it's like essentially repeating itself, right? So again, what do you think the period is in that context?

Katie: Does it not have one?

Oliver: Say that again.

Katie: Does it not have one?

Oliver: Mmm maybe. *[Walks around to the other side of the table]* Umm…anybody here at the table have an answer for that? Or like can illuminate us? What do you guys think? Just on a function, what's the period of a function?

Emily: It's like the first set.

Oliver: The first set, what do you mean by that?

Emily: So, like when you have a pattern of numbers. I don't know how to explain it. It's just how it works in my head.

Oliver: No, no, you're doing a good job. Go ahead.

Emily: Okay so you know like when we had the graph or like the table? Like you could see the numbers?

Oliver: You mean on the *x*-axis?

Emily: Kind of.

Tessa: Like in a table.

Emily: Yeah like in a table like that kind of.

Oliver: Okay.

Emily: So, like when I'm looking at it, there's like the first number and then the last number of that before the pattern repeats.

Oliver: Before the pattern repeats, right. So, it's sort of like, maybe like the smallest time, yeah? Before it starts doing it again?

Emily: Yes.

Oliver: Yeah. I think that's a good way to say what the period is right? So, it's the shortest amount of time before the graph essentially starts repeating itself. Okay. So here I drew one…

Emily: Is it the gap between those two? *[pointing to the graph on the board]*

Oliver: The gap?

Emily: Is that a second one that you drew?

Oliver: Yeah. I think I did a poor job at drawing it, but…

Tessa: Where does it start at? Like it's pi over two but then what is it on the other side?

Oliver: Where does it start? What do you mean by where it starts?

Tessa: The next one.

Oliver: The next one? Well, um, there's really no start, really. So, as you can see, like it's just one value where it doesn't work and anything to the left and to the right, you just get these huge drastic numbers. So, there's really no start. But I think this pattern's going to keep happening, right? So, if this question about the period is a little difficult think about (c) and (d). (c) asks you where is tangent zero? So then maybe that gives you those anchors. Like, when does it cross the *x*-axis, and maybe that'll help you. Think together, okay, about this, and then come back to the period question.

(11:15)

Oliver's Interaction with Danny

(12:59)

Oliver: How're you guys doing?

Students: Doing really good. Good. Doing all right.

Oliver: *[Patting Danny's workbook]* I don't see a lot of graphs. Okay.

Danny: Well I'm kind of confuzzled on the graphs.

Oliver: Confuzzled?

Danny: Yeah, I'm kind of confuzzled.

Oliver: Let's see if we can work through the confuzzledness.

Danny: Cause like, okay so I'm not like understanding *[inaudible]*

Oliver: Okay. So, the period here you said is two pi. The midline is negative eight and the amplitude is seven. So maybe start by drawing where the midline is. Careful, you wrote negative eight, right? So, it shouldn't be on the…yeah there you go. Okay. Yeah, and now figure out how high should that go and how low should that go.

Danny: *[inaudible – hard to tell if he is talking or not]*

Oliver: Uh huh *[positive]*.

Danny: *[inaudible – hard to tell if he is talking or not]*

Oliver: If you start at negative eight, how low should you go, if your amplitude is seven?

(14:05)

[Long pause while Danny is hunched over his workbook.]

(14:14)

Danny: Wait, were you talking about from like…were you talking about from like up or down?

Oliver: Both, right? The amplitude should be the same from the midline. Your max should be your midline plus your amplitude and your minimum should be

your midline minus your amplitude. So how low should it go? *[Answering another student's question.]* At least a few of them.

Danny: So, like right here?

Oliver: But you should know the exact value, right? If you start at negative eight and you go lower by seven, how low should you go?

Danny: Like you said, but you still have to find the bottom part. It needs to be the min minus the amp, so that should just be like right here?

Oliver: Sure, but what's that value?

Danny: It'd be like negative nine?

Oliver: Careful, you start at negative eight, and you're going down by seven.

Danny: So, then it'd be like negative

Oliver: What's that?

Max: Negative 15.

Oliver: Negative 15, yeah. So you're starting at negative eight—go ahead explain.

Max: The midline is at negative eight and the amplitude is seven *[inaudible]*.

Danny: Oh so *[inaudible – motioning with hands]*.

Oliver: Yeah. You start there and you go down by seven, so you're at negative 15. *[Oliver begins talking to other students at the table about a different problem.]*

Tom: The period on these are correct, right?

Oliver: Uh…yes. I don't agree with your amplitudes though. Think about the definition of amplitude.

Max: Isn't it just two.

Students: *[inaudible]*

Oliver: Yeah, but why?

Garrett: Because amplitude can't be negative.

Oliver: Right. It's just a distance, right.

Max: I understand. When I see that, I know it.

Oliver: Always address the definition. Right? The definition tells you whether it's right or not.

(15:53)