Conventionalism in Reid's "Geometry of Visibles"

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The role of conventions in the formulation of Thomas Reid's theory of the geometry of vision, which he calls the "geometry of visibles", is the subject of this investigation. In particular, we will examine the work of N. Daniels and R. Angell who have alleged that, respectively, Reid's "geometry of visibles" and the geometry of the visual field are non-Euclidean. As will be demonstrated, however, the construction of any geometry of vision is subject to a choice of conventions regarding the construction and assignment of its various properties, especially metric properties, and this fact undermines the claim for a unique non-Euclidean status for the geometry of vision. Finally, a suggestion is offered for trying to reconcile Reid's direct realist theory of perception with his geometry of visibles.

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While Thomas Reid is well known as the leading exponent of the Scottish "common-sense" school of philosophy, his role in the history of geometry has only recently been drawing the attention of the scholarly community. In particular, several influential works, by N. Daniels and R. B. Angell, have claimed Reid as the discoverer of non-Euclidean geometry, an achievement, moreover, that pre-dates the geometries of Lobachevsky, Bolyai, and Gauss by over a half century. Reid's alleged discovery appears within the context of his analysis of the geometry of the visual field, which he dubs the "geometry of visibles". In summarizing the importance of Reid's philosophy in this area, Daniels is lead to conclude that "there can remain little doubt that Reid intends the geometry of visibles to be an alternative to Euclidean geometry";[1] while Angell, similarly inspired by Reid, draws a much stronger inference: "The geometry which precisely and naturally fits the *actual* configurations of the visual field is a non-Euclidean, two-dimensional, elliptical geometry. In substance, this thesis was advanced by Thomas Reid in 1764, . . ."[2] The significance of these findings has not gone unnoticed in mathematical and scientific circles, moreover, for Reid's name is beginning to appear more frequently in historical surveys of the development of geometry and the theories of space.[3]

Implicit in the recent work on Reid's "geometry of visibles", or GOV, one can discern two closely related, but distinct, arguments: first, that Reid did in fact formulate a non-Euclidean geometry, and second, that the GOV *is* non-Euclidean. This essay will mainly investigate the latter claim, although a lengthy discussion will be accorded to the first. Overall, in contrast to the optimistic reports of a non-Euclidean GOV, it will be argued that there is a great deal of conceptual freedom, or slack, in the construction of any geometry pertaining to the visual field. Rather than single out a non-Euclidean structure as *the only* geometry consistent with visual phenomena, an examination of Reid, Daniels, and Angell, will reveal the crucial role of geometric "conventions", especially of the metric sort, in the formulation of the GOV (where a "metric" can be simply defined as a system for determining distances, the measures of angles, etc.). Consequently, while a non-Euclidean geometry is consistent with Reid's GOV, it is only one of many different geometrical structures that a GOV can possess. Angell's theory, that the GOV can only be construed as non-Euclidean, is thus incorrect. After an exploration of Reid's theory and the alleged non-Euclidean nature of the GOV, in section 1 and 2 respectively, the focus will turn to the tacit role of conventionalism in Daniels' reconstruction of Reid's GOV argument, and in the contemporary treatment of a non-Euclidean visual geometry offered by Angell (sections 3 and 4). Finally, in the conclusion, a suggestion will be offered for a possible reconstruction of Reid's GOV that does not violate his avowed "direct realist" theory of perception, since this epistemological thesis largely prompted his formulation of the GOV.

1. The "Geometry of Visibles"

In the *Inquiry into The Human Mind,* Reid's first major work (from 1764), the GOV is put forward as the geometry of the visual field. The structure of Reid's argument can be briefly summarized: since human vision lacks the ability to determine the depth of all our “visible figures” (i.e., the figure of a body/thing as experienced visually), it follows that all visible figures appear to be equally distant. Based on this equality of our experience of distance, Reid infers that every visible figure has geometrical properties that are indistinguishable from a figure drawn on a sphere, or “spherical figure”, thus singling out the representation of visual figures by means of spherical figures. Reid then demonstrates that the properties of spherical figures differ from the properties associated with Euclidean geometry: e.g., the sum of the angles of a spherical triangle exceeds 180 degrees. Consequently, the geometry of visible figure, or GOV, is non-Euclidean spherical geometry.

As for the specific details of the GOV, Reid begins his exposition by first noting that the definitions of point, line, angle, and circle, in his new geometry “are the same as in common geometry” (Inq. 6.9)[4], where “common geometry” presumably denotes Euclidean (although he does not actually provide any of these definitions). He then argues, in principles 1 and 2, for the construction of a visible spatial geometry modeled on a sphere (doubly-elliptical, using the modern terminology) with the eye placed at its center:

> Supposing the eye placed in the centre of a sphere, every great circle of the sphere will have the same appearance to the eye as if it was a straight line; for the curvature of the circle being turned directly toward the eye, is not perceived by it. And, for the same reason, any line which is drawn in the plane of a great circle of the sphere, whether it be in reality straight or curve, will appear straight to the eye.
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> Every visible right line will appear to coincide with some great circle of the sphere, and the circumference of that great circle, even when it is produced until it returns into itself, will appear to be a continuation of the same visible right line, ... For the eye, perceiving only the position of objects with regard to itself, and not their distance, will see those points in the same visible place which have the same position with regard to the eye, how different soever their distances from it may be. (Inq. 6.9)

For Reid, since every visible right line “appears” to coincide with a great circle (which is a circle of greatest diameter on the sphere), he concludes, in principle 4 and 5, that the properties of visible angles and triangles are the same as the properties of spherical angles and triangles:

> Since the visible [right] lines appear to coincide with the great circles, the visible angle comprehended under the former must be equal to the visible angle comprehended under the latter. But the visible angle comprehended under the two great circles, when seen from the centre, is of the same magnitude with the spherical angle which they really comprehended, as mathematicians know; . . . .
>
> The properties, therefore, of visible right-lined triangles are not the same with the properties of plain triangles, but are the same with those of spherical triangles. (Inq. 6.9)

After procuring a list of propositions for the GOV, that spell out in more detail some of its specific features, he concludes his presentation by drawing a sharp distinction between the GOV and Euclidean geometry:

> Those figures and that extension which are the immediate objects of sight, are not the figures and the extension about which common geometry is employed; that the geometrician, while he looks at his diagram, and demonstrates a proposition, hath a figure presented to his eye, which is only a sign and representative of a tangible figure; . . . and that these two figures have different properties, so that what he demonstrates of the one, is not true of the other. (Inq. 6.9)

This passage also reveals Reid’s thoughts on the geometry of the sense of touch, which he dubs “tangible” figure; a view, moreover, that is obviously derived from Berkeley’s *New Theory of Vision.*[5] In contrast to the two-dimensional non-Euclidean GOV, tangible figure is governed by ordinary three-dimensional Euclidean geometry. In fact, Reid does appear to hold that Euclidean geometry *is* the geometry of the external, material world (considered apart from vision), while it is only the GOV that is non-Euclidean. In Essays, 2.14, for instance, he repeatedly affirms (Berkeley’s thesis) that a three-dimensional Euclidean structure is an intrinsic feature of physical objects: it is the “real” magnitude of a body, as opposed to the "apparent" magnitude discerned through vision: "The real magnitude of a line is measured by some known measure of length. . . . This magnitude is an object of touch only, and not of sight; . . . ." (We will return to this issue below).

2. Reid and Non-Euclidean Geometry.

Before beginning our examination of the use of conventions in the formulation of the GOV, a discussion of Reid's alleged "discovery" of non-Euclidean geometry is in order. Daniels insists that the GOV is a fully-fledged non-Euclidean geometry, and not merely a spherical Euclidean geometry, since "[Reid] did think that the [GOV] is a fully consistent alternative to Euclidean geometry, if only for two-dimensional visual space" (Daniels, 128). Although not explicitly stated, Daniels may believe that the similar geometrical work of many of Reid's predecessors, most notably, G. Saccheri and J. Lambert, does not fall under a non-Euclidean classification due to the simple fact that these mathematicians did not accept their alternative geometries as "real".[6] Saccheri, for instance, had seriously investigated the "obtuse angle hypothesis", which allows the angles of an quadrilateral to exceed 90 degrees, as a means of proving the parallel postulate of Euclidean geometry (i.e., by deriving an inconsistency from the obtuse angle hypothesis, which is incompatible with Euclidean geometry, he hoped to establish the truth of the parallel postulate). Saccheri did not believe that his non-Euclidean constructions were "real", in the mathematical sense, although he did obtain non-Euclidean geometrical results that far exceed the modest philosophically-based conclusion of Reid's GOV. Consequently, are Saccheri's results, which are so similar to Reid's, to be demoted to the status of "spherical Euclidean", rather than "non-Euclidean", simply because he thought that such geometries were (or would ultimately prove) inconsistent? If this is Daniels' main rationale for granting Reid an exclusive right to the non-Euclidean crown, then it is tantamount to claiming that a mathematician's *beliefs* about her work is itself sufficient to secure a mathematical classification of its *content*; in this case, either Euclidean or non-Euclidean. But, how can a mathematician's beliefs about a mathematical theory, an external feature of that theory, determine its classification? More Plausibly, any judgment concerning a theory's overall characteristics should depend on its internal mathematical properties, and not on the author's mere intentions (an external property).

Overall, Daniels, and the other proponents of a non-Euclidean GOV, assume that there exists a straightforward and unproblematic definition of what constitutes a non-Euclidean geometry. Yet, the development of modern mathematics reveals a much more complex and intricate story, as the "evolving" classification of Spherical geometry clearly demonstrates. In Reid's day, the failure of the parallel postulate on the surface of a sphere was not seen as heralding a new brand of geometry, since the peculiar properties of great circles on spheres had long been a part of the Euclidean tradition (and, indeed, extended back to Euclid himself)[7]; but, as J. Gray points out: "this geometry [i.e., spherical] is now given almost immediately in modern textbooks as an example of a non-Euclidean geometry" (Gray 1989, 169). A part of this classification problem resides in an over-dependence on Euclidean insights and definitions: that is, spherical Euclidean geometries, which admit the obtuse angle hypothesis, mark an advance on the path to the modern non-Euclidean notion, but such geometries still lean heavily on Euclidean definitions and metrical intuitions (as will be examined in the case of Reid below). The two most important conceptual breakthroughs on the road to a full-fledged non-Euclidean geometry had to await the nineteenth century, when the analytic approach to geometrical concepts was launched by the work of Taurinus and Gauss, and the investigation of the intrinsic structure of manifolds (differential geometry) was begun by Beltrami and Riemann. Analytic techniques allowed geometers to move away from the Euclidean understanding of geometrical objects, such as "surface" or "line", by defining these objects purely by means of algebraic equations, and not via their often Euclidean-biased geometrical representations (i.e., since algebraic equations are essentially neutral and uninterpreted as regards their geometric meaning, they do not uniquely favor a Euclidean interpretation). Differential geometry, which is largely based on the analytic achievement, introduced the characterization of surfaces in terms of their intrinsic, as opposed to extrinsic, curvature; where "intrinsic" refers to the determination of a surface's curvature from a perspective confined entirely to that surface, and "extrinsic" pertains to its calculation from outside (or off) the surface. As a result of Riemann's pioneering work, curvature could now be characterized intrinsically for each point on a surface (or, more precisely, for the infinitesimal neighborhood surrounding each point on a manifold) without having to embed that surface in a larger, Euclidean space. Using these procedures, geometry was freed from the necessity, or "tyranny", of a Euclidean backdrop for making measurements of curvature. (Determining the "radius of curvature" of a point on a curve by finding the circle that best approximates the curvature at that point is an example of an extrinsic approach, since the circle's radius lies outside the curve.)

Returning to Reid, there can be little doubt as to which geometrical category the GOV falls under: despite the pleas of Daniels and other commentators, Reid's theory clearly presupposes the non-analytic, global (i.e., non-local or non-infinitesimal), and extrinsic scaffolding of the older Euclidean classical tradition—and this conclusion would seem to place Reid's GOV in the "spherical Euclidean" tradition, rather than in the more modern "non-Euclidean". Like Lambert and Saccheri, for example, his investigation proceeds from an intractable Euclidean intuition concerning the definition of a "line", which in the *Inquiry* is taken to be "the same as in common geometry [i.e., Euclidean]" (Inq. 6.9); and, in his manuscripts, his final version proposes that "a right line is said to be parallel to a right line when being in the same plane it is in every point equally distant from it."[8] This last definition, moreover, betrays the influence of Euclidean metrical notions in Reid's overall approach, an influence that even extends to the handling of his most basic geometrical concepts. In fact, proposition 8 in the exposition of the GOV is based on this understanding of parallel lines: "a circle may be parallel to a right line—that is, may be equally distant from it in all its parts" (Inq. 6.9). In the modern theory, where the distance function takes on a local (infinitesimal), algebraic form, there are many lines which can be classified as "equally distant" to a given line, since there are many metrics (different algebraic functions) that can be employed. Reid, on the other hand, follows his Euclidean predecessors in confidently assuming that a unique global determination of "same distance from" can be applied to the entire surface of the sphere, thus picking out a privileged class of lines. Reid's metric, furthermore, is not based on any infinitesimal procedure (of which Reid was suspicious[9]), but is merely an extension of the common Euclidean understanding of length on a plane surface to measurements on a spherical surface. Just as the lines in plane geometry are projected onto the sphere, it would appear that the concepts of Euclidean distance are included in the transfer as well.

In addition, the description of the GOV places the observer, or eye, at the center of the sphere, and it is from this position that measurements of position, length, and angle are conducted. Since the eye is not *on* the spherical surface, the GOV correspondingly fails to count as an intrinsic theory of geometrical curvature—in fact, the center position of the eye constitutes (somewhat ironically) the origin of a spherical polar coordinate system, which is appropriately deemed a spherical geometry in most text books, and thus not necessarily a non-Euclidean geometry (as noted above).[10] Various claims of Reid tacitly expose this extrinsic characterization of the GOV, as: "I require no more knowledge in a blind man, in order to his being able to determine the visible figure of bodies, than that he can project the outline of a given body, upon the surface of a hollow sphere, whose centre is in the eye" (Inq. 6.7). Despite Daniels' appeals, such descriptions make would seem to suggest that the GOV is *best interpreted* as a sphere embedded in a larger Euclidean space, such that the figures of plane Euclidean geometry can be projected onto the sphere's surface.

As a possible rejoinder to this line of criticism, one might try to enlist Reid's many claims concerning the spherical "representation" of the visible figures, since a representation need not be interpreted as an actual spatial projection. Reid asserts, for example, that "visible figure will be represented by that part of the surface of the sphere, on which it might be projected, the eye being in the center" (Inq. 6.9). If one also includes Reid's denial of three-dimensional curvature for his visibles (as will be explored further below), then the extrinsic characterization of the GOV can be seen as merely a property of the particular model—namely, spherical geometry—that Reid employed to demonstrate the consistency of his alternative geometry. In other words, Reid used spherical Euclidean geometry to provide an intuitively comprehensible instantiation (model) of his bare geometric definitions and principles, and thus one should not impute an extrinsic classification to the entire GOV based on a simple confusion between the theory and its model. This response may go a long way towards clearing Reid of the charge of inconsistency in developing his account of visible figure (although not necessarily Daniels' reconstruction, as will be seen in the next section), but it raises a further issue that needs to be addressed if the GOV is to qualify as a genuine or valid geometry: Does Reid first provide a set of axioms/postulates for his geometry, and then proceed to construct a model? Or, has Reid simply reversed the process, and assembled a set of postulates that hold true of a model that he had picked out beforehand? Daniels regards the latter case as a more accurate portrayal of Reid's actual construction of the GOV:

> Reid begins with an interpretation or model, visible space [i.e., spherical geometry], and develops a geometry for it, which turns out to be non-Euclidean. Starting with a model may be what leads Reid to think of his geometry as specifically tied to this particular model. (Daniels 1989, 22)

Indeed, the GOV's eight basic principles (see section 1) directly refer to spherical geometry almost out of necessity, since they largely function as set of auxiliary or correspondence definitions that connect the more basic elements (lines, points, etc.) to Reid's theory of visible space. This realization does not necessarily restrict the GOV from a non-Euclidean classification, but it does raise serious doubts over its status as an "axiomatic" system, and thus in what sense it can stand comparison with the more geometrically formal and comprehensive results produced in the nineteenth (and eighteenth) century. (It should be noted, moreover, that Daniel never claims that the GOV is a complete and consistent axiomatic formulation of a non-Euclidean geometry.)

Consequently, although some of Reid's claims would seem to downplay the importance of projections on a spherical model, the very construction of the GOV (as well as many of Reid's own explanations, as above) puts the spherical Euclidean model, extrinsically considered, on the very ground floor of the GOV's construction, if not ultimate meaning.[11]

In conclusion, Reid's conception of the GOV—global, non-analytic, extrinsic, and heavily indebted to Euclidean ideas—falls naturally within the spherical Euclidean tradition in geometry, and this realization seriously weakens the GOV's claim to be regarded as the first non-Euclidean geometry (but, see endnote 11). Of course, if the distinction between Euclidean and non-Euclidean is not precise, and allows for a continuum of values ranging from, say, "clearly Euclidean" to "clearly non-Euclidean", then one could simply declare that all crude geometrical forays into the obtuse angle hypothesis qualify as non-Euclidean. If Daniels and company were to adopt this strategy, however, then nearly all previous work on the geometry of the sphere would have to be classified as "non-Euclidean." Yet, I whole-heartedly agree with J. Van Cleve's comment on this kind of geometric classification: "No one credits the ancient Greek astronomers who worked out the geometry of figures on the celestial sphere with being the first discoverers of non-Euclidean geometry."[12]

3. Daniels' Reconstruction of The GOV.

The presuppositions involved in the act of measurement, especially determinations of spatial distance, received their first important philosophical treatment by H. Poincaré at the turn of the twentieth century. Poincaré's views became the central doctrine of the "conventionalist" faction within the Logical Positivist program for the philosophy of science, as most persuasively defended by H. Reichenbach and A. Grünbaum. In brief, conventionalism as it pertains to the measurement of spatial distance, dubbed "metrical conventionalism", holds that the determination of distance, and thus spatial geometry, is always dependent upon certain stipulations concerning the properties of our measuring instruments and procedures. Stipulations about the behavior of a standard meter stick, for example, directly determine the type of geometry we ascribe to a space, since we can always retain a particular geometry if we adopt an appropriate "convention" for the measurements carried out with the meter stick: if one desires a flat space-time, in the modern setting of General Relativity, then one must posit the existence of "forces" in the space-time that distort the meter sticks (in order to account for their failure to obtain the standard Euclidean measures); but if one simply stipulates that the meter sticks retain an invariant magnitude throughout all space and time, then the measurements of distance will produce a non-Euclidean result, thus vindicating a curved space-time view. In either case, the properties we assign to our measuring apparatus, which are conventional, determine the geometry we assign to a given space.[13]

Although it need not affect our examination of the GOV, many of the criticisms aimed at the metric conventionalist school centered upon their implicit anti-realist conception of magnitude and distance: e.g., one might reasonably wonder if conventionalism allows any objective, non-conventional features of congruence or length. Because metric conventionalism in the realm of physical geometry raises these troubling realist/anti-realist worries, it is interesting to note that a better case for metric conventionalism could be made with respect to the geometry of vision, where the relevant geometry is purely conceptual, and not physical. That is, in a conceptual geometry constructed from our "visible figures", the realist difficulties are rendered largely irrelevant. With this in mind, we shall now proceed to explore the extent of conventionalist metrical doctrine in Daniels' and Angell's version of the GOV.

Daniels' exposition of a doubly-elliptical GOV closely follows Reid's theory, mainly for the reason that Daniels seems intent on merely laying out Reid's arguments for the GOV, rather than attempting to conclusively prove, as does Angell, that the geometry of vision is spherical. Accordingly, the starting point of Daniels reconstruction is the placement of the eye at center of a sphere, along with restriction that "the eye is capable of 360 degree rotation, but not translation" (point (e), 6). The case for a spherical GOV follows naturally, if not inevitably, from this stipulation, as revealed in Reid's characterization of "right-line" (see section 1).

Yet, what justifies the choice of a spherical model for vision? Is it due to the fact that the eye is, albeit roughly, a sphere? Daniels admits that "the anatomy of the human eye seems to have been one motivating consideration" (10). And, in more detail, he comments:

> Since the material impression on the retina is what "suggests" visible figure, Reid seems to feel we can preserve the properties of visible figure if we preserve the properties of the material impression. But the retina is treated by Reid as just a portion of the surface of a hollow sphere. To preserve properties of a material impression on such a surface, Reid projects the impression back through the center (focal point) of The Eye and out onto an arbitrary sphere. (10-11)

In this context, "material impression" refers to the physical effects (or event) of a ray of light striking the retina, where an impression "suggests two things to the mind—namely, the colour and position of some external object" (Inq. 6.8, but an impression is not a sensation). While Daniels' view may retain a degree of plausibility, he also claims that "visible surfaces are neither curved nor plane" (8), a judgment that closely follows Reid's own presentation. In the analysis of the hypothetical Idomenians, who only possess the two-dimensional sense of sight (and not three-dimensional tactile sense organs), Reid explains:

> The beings we have supposed, having no conception of a third dimension, his visible figures have length and breadth indeed; but thickness is neither included nor excluded, being a thing of which he has no conception. And, therefore, visible figures, although they have length and breadth, as surfaces have, yet they are neither plain surfaces nor curve surfaces. For a curve surface implies curvature in a third dimension, and a plain surface implies the want of curvature in a third dimension; and such a being can conceive neither of these, because he has no conception of a third dimension. (Inq. 6.9)

Unfortunately, the conjunction of these various claims brings to light an internal inconsistency in Daniels' reconstruction: (1) the eye projects the "material impressions" through the eye onto a sphere, but (2) the visible figures, which are "suggested" by these material impressions, do not possess *any* curvature—indeed, "visible surfaces are neither curved nor plane" (Daniels, 8). Yet, what point is there in singling out a spherical surface, or any surface possessing a determinate curvature, for that matter, if the eye *cannot* discern that surface's curvature? Daniels may simply have transferred the "sphericality" of the projection of the material impressions to their corresponding visible figures, but, once again, this move is unwarranted if the figures "suggested" by the impressions are only two-dimensional (i.e., not curved, which requires a third-dimension). Moreover, even if Daniels' division between material impression and visible figure is rejected, for it apparently constitutes a separate metaphysical problem of its own in need of substantial argument, one is still left with a projection onto a curved surface (the sphere) of a figure that does not possess curvature. (Daniels' attempt to meet this difficulty will be addressed below.)

If visible figure does not possess curvature, can visible figure be projected onto a different surface, such as a plane or cube? In response to this question, Daniels states:

> Anatomical considerations and Reid's theory of perception, as well as the special properties of The Eye make the sphere the "natural" representation of visible space. Projection onto no other surface preserves the properties of visible figure. Projection onto a cube with The Eye at its center would violate the symmetry considerations based on the anatomy of the eye (a sphere, in Reid's idealization). Similarly, it would seem arbitrary in view of property (g), the claim that The Eye sees all points as (if they are) equidistant. (11)

There are numerous objections that can be raised against this argument (and some we will have to postpone until the discussion of Angell), but we will focus on a few points. First, even granting the hemispherical shape of the retina, the choice of the sphere remains problematic since the retina does not span the entire 360 degrees of the eye (as mandated in Daniels' construction, point (e)—but more on this later). Second, Daniels reasons that the equidistant position of all points from the eye (point (g)) naturally favors a spherical GOV, which is based on his further contention that the eye cannot make depth discriminations among visible figures or points (point (b), Daniels 6; and Reid, Inq. 6.9). Now, it is not all that clear that one can legitimately move from the claim that "the relative distances *among* visible figures cannot be distinguished" to the conclusion that "*all* visible figures have the *same* distance from the eye". Analogous instances of this argument quickly leads to absurd results: for example, from the fact that "I cannot determine the relative colors among dogs" it does not necessarily follow that "I judge that all dogs have the same color": that is, I may not be able to discern a color difference between dog x and dog x′, and between dog x′ and dog x″, etc., but this does not entail that I judge that x, x′, x″, etc., all possess an identical color (since transitivity may fail—I may judge x≠x″, although x=x′, and x′=x″). What Daniels (and Reid) need to make their case are the further stipulations that, while depth discrimination fails between (or among) visible figures, (i) a depth discrimination can be discerned between the eye and each visible figure, and (ii) that the distance between the eye and each visible figure, in (i), is identical.

Quite possibly, a vague awareness of this dilemma may have prompted Daniels' thesis that visible figures are formed out of an equivalence class of real objects: "Visible points, lines, and figures can be thought of as the objects that result when Reid's equivalence-relation, *same position with regard to The Eye,* is treated as an identity relation (for The Eye in the construction)" (17-18). In other words, a visible point is created when all the real (i.e., physical) points that lie on the same (radial) line drawn from the center of the eye are regarded (by the mind?) as an identical point. Returning to our discussion from section 1, Daniels would appear to be offering a quasi-analytic interpretation of visible figure, since it eschews direct reference to geometric elements: i.e., it is just a class of real objects grouped under the property, "same position with respect to the eye". Daniels then infers that the GOV is a mathematical (geometric) *description* of these equivalence classes when so grouped, much like a geometric line is a model for an algebraic equation (in analytic geometry):

> Seeing through The Eye forces us to collapse the equivalence relation into an identity relation. . . . This now gives us a way of restating the sense in which the notions of point, line, figure, and surface are "less determined" for a being that (having vision for its only sense) cannot conceive of three dimensions: the equivalence relation collapses into an identity relation
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> We might say that the visible point is an "hypostatized" object. The Eye converts the equivalence class into an object, a visible point, which is the object we see when we see a visible point by means of The Eye. It is these objects, and not their projections onto a sphere, which are the visibles. Reid's geometry of visibles is developed in order to give a mathematical description of the properties of these special objects. (17-18)

Construed in this analytical fashion, Daniels aims to free the GOV of any Euclidean connotations (of a projection onto a sphere), as well as accommodate Reid's demand that visible figures do not possess any three-dimensional characteristics: the GOV, rather, is simply a model of Reid's special equivalence class of points. Nevertheless, the property that collapses the equivalence class into an identity claim, i.e., "same position with respect to the eye", is itself a *geometric, three-dimensional* property—and thus Daniels' analytic proposal is seriously compromised, if not outright falsified. This property makes reference to the eye as the center of (Daniels') unit sphere, thereby favoring the interpretation of the GOV as a Euclidean projection of the visible figures radially outward *in the third dimension* to the sphere's surface. Yet, even if we put aside this major obstacle, and straightforwardly accept Daniels' analytic thesis, there would still appear to be no special reason for adopting a non-Euclidean reading of the GOV over a spherical Euclidean interpretation. The equivalence class construction, since it is presumably analytic, can admit any geometric model that captures the GOV's collected principles and "same position" clause—but spherical Euclidean geometry clearly meets these criteria, as so many of Reid's and Daniel's "projective" descriptions of the GOV reveal. Consequently, why is the non-Euclidean model singled out as the only viable candidate?

If Daniels is to be faulted for an unjustified choice of a non-Euclidean GOV, much of the blame, however, should rightly be placed on Reid's own lack of clarity in his analysis of material impressions. Unlike Berkeley, who is rather clear in denying that any metrical (distance) information can be derived or obtained from our visual experience alone (prior to experience), Reid's development of the GOV leaves open the possibility that he might allow a certain degree of spatial information to be directly derived from material impressions. This spatial information would take the form of Daniel's point (g), of course, i.e., that all visibles are the same distance from the eye, thereby leading to the natural choice of a sphere for a geometric model of the GOV. The imaginary two-dimensional Idomenians would appear to support this inference, since Reid stipulates that the visual geometry of the Idomenians is, in fact, the (alleged) non-Euclidean spherical geometry of the GOV. In contrast to Reid, Berkeley clearly denies that one can move from the inability of the eye to make depth discriminations (Daniel's point (b)), to the further contention that all visibles are "seen" as having the same distance from the eye. This distinction, of mere depth (or "outness") from distance, arises within the context of a discussion of Molyneux's problem, which concerns the spatial and geometrical judgments that a previously blind person would make on first obtaining sight. Berkeley concludes that a person given sight would deem that all of his perceived visible figures were "in his eye, or rather in his mind", which, in our terms, amounts to a lack of depth discrimination ("outness") for visibles.[14] Nevertheless, Berkeley also contends that we do not perceive visible figures as either flat or curved; in other words, they are two-dimensional, rather than three-dimensional (*New Theory of Vision,* 158-159; compare with Reid, Inq. 6.9, as above). The absence of a third-dimensional component means that the only spatial information that the visible figures can impart to the perceiver will involve the relative positions of contiguity and non-contiguity in two dimensions (i.e., the chair is next to, or three feet to the left of, the table), whereas the visible figures cannot provide any information on the differences in depth among the figures (i.e., the chair is also two feet behind the table). Accordingly, Daniels' point (g) is absent from Berkeley's account of vision despite the fact that he (Berkeley) denies that the eye can perceive differences in depth.[15] Reid's failure to sufficiently separate the notion of the depth of visible figures, which provide no information on their distance relative to the eye, from the concept of the actual outward distance of the figures as judged by the eye, might thus constitute the basis of the problems plaguing both Reid's case for the GOV, as well as Daniels' reconstruction of that theory.[16]

Overall, if Reid and Daniels' move from a lack of depth discrimination to spherical distance is not a justified maneuver, then one must inevitably judge that the cornerstone of the GOV is a *conventional* stipulation—a stipulation, moreover, whose plausible rejection undermines the non-Euclidean status of the GOV. In the next section, further aspects of the conventionalist approach, especially of the metric variety, will be discussed with respect to Angell's (and Daniels') theory.

4. Angell's Case for The GOV.

Angell's theory of the geometry of vision closely parallels Reid's, although the construction of his system generally proceeds along independent lines. In order to establish a spherical non-Euclidean GOV, Angell puts forth a number of specific examples (of which we need only to consider a few). First, if a person stands between a set of railroad tracks and looks forward in the direction of the tracks, they will see the tracks converge at some point on the horizon, and thus form an angle of a determinate number of degrees (greater than zero). Yet, if the person gazes down at their feet, the wooden ties (that lie underneath the tracks) will form two 90 degree angles at the juncture where wooden ties and railroad track meet, thus forming a large triangle possessing a non-Euclidean measure of more than 180 degrees (90 + 90 + a value greater than 0 = a value greater than 180; Angell, 95). Second, consider a person measuring the angles formed by three stars: one situated due north on the horizon, another due east on the horizon, and the third directly overhead. As with the previous example, the combined measure of the angles will exceed 180 degrees (95). Finally, if one simply examines the four corners of a square ceiling, where two walls meet the ceiling, the combined total of the four angles will surpass 360 degrees, once again violating Euclidean geometric doctrine (95-96, although this example is adapted from J. R. Lucas).

As a means of measuring the angles in these (and other) examples, Angell develops a procedure that utilizes a ruler, protractor, and a stick, all held rigidly under the eye:

> Take a stick 14.35 inches long, attach a six-inch metal strip marked off in quarter inches to one end of it, and bend the metal strip so that each point on it is equidistant from the free end of the stick. When the free end is placed just below the eye, the quarter-inch marks on the metal strip at the other end each mark off just one degree. . . of visual distance.
>
> For objective measurements of seen *angles* among visibles . . . it suffices to attach a protractor perpendicularly to the same stick, with its center at the end where the metal strip is attached. When this device is held to the eye and the angles in the protractor are aligned with the angles in the visible, an accurate, objective measure of the angles in the visible is provided. (93)

Employing this device, Angell believes an unambiguous determination of the GOV can be provided, and that the results will favor a non-Euclidean geometric structure.

What is initially puzzling about Angell's theory is the inordinately central role that Euclidean measurements assume in the construction of his GOV. He begins by setting up a convention for the measurements of angle and distance using a set of devices that are presumed to retain Euclidean characteristics. Second, the examples afforded by Angell do not exhibit a non-Euclidean GOV from a *single* perspective, or "single view" (to borrow Van Cleve's term, 37); rather, as in the railroad track example, one needs first to look along the horizon, and then look down at one's feet (or at each successive star, or individual corner of the ceiling). There is no single view, in short, from which all the angles of the rail track (stars, ceiling corners) can be surveyed.[17] Now, combining these two observations, the classic conventionalist argument would focus on the arbitrary nature of Angell's measuring techniques as regards their, for lack of a better term, "global" veracity: that is, do the measuring devices retain an invariant measure of angle and distance as they are moved around in the visual field among different view points? Just as Poincaré's meter sticks can possess different expansion properties, and thereby provide measurements favoring many alternative geometries, it would seem that Angell's ruler and protractor could equally undergo different expansion or contraction rates. A non-Euclidean determination of, say, the ceiling corners could then be judged to be the result of a "funny" force in various parts of physical space that contracts the protractor (i.e., shrinks the distance between its calibrated angles) as it moves from measuring one visible figure to another—one could then conclude that the protractor provided a false non-Euclidean measurement of a "real" Euclidean visual space. It must be admitted, however, that this conventionalist strategy does not seem to work as successfully in the context of visual space as with physical space. It does not seem plausible to posit a real force in space that could change with the perspective of an observer alone, since this would quickly lead to contradictory ascriptions of the value of the force to the same visual object at the same time: for example, an observer A may judge that a force shrunk his protractor as he turned to look at an object X, but another observer B who maintained an unaltered perspective in observing X, would judge that no force interfered with her protractor measurements—so, how can the existence of a *real* force in space, which really distorts the physical apparatus, depend entirely on the relative measurements (conducted by different observers) with respect to the same object at the same time?[18]

Conventionalism does play a crucial role in the measurements of visual geometry, nevertheless, despite its somewhat different character. The first problem relates to Angell's aforementioned dependence on Euclidean measuring devices: in short, how does one guarantee that the stick and protractor system maintains, or even initially determines, a Euclidean measure? Even if the existence of external forces is dismissed, there appears to be no means of guaranteeing that the measuring stick is consistently and identically employed to all visible figures. For instance, if the measuring stick is tilted slightly left or right in measuring a particular angle, then a veritable host of diverse results could be obtained that will directly decide the overall geometry: a protractor that is tilted away from the eye might find more degrees in an angle than one slanted toward the eye, and the cumulative effect of such changing and variable uses of the system could make the difference between a Euclidean or non-Euclidean determination of the overall geometry of vision. Similar metric conventions are at work in Reid's and Daniels' version of the GOV, moreover. Daniels explains that the distances between visible points can be determined by fixing radial lines, or rays, from the points to the center of his non-rotating eye, and then calculating the angle of rotation between the rays: "In order to measure distance between the positions of [visible] points, the eye must keep track of (1) angles formed by rays projected from points to the center of the stable (non-rotating) eye and (2) angle of rotation" (Daniels, 7). But, Daniels must assume, as a convention, that the final ray remains fixed as the angle is measured off from the initial ray to the final ray. If the final ray changes it position relative to the first as the measurement is conducted, then nearly any length, and thus geometry, can be attributed to the system.[19]

Returning to Angell's version of the GOV, one might guard against the potential for varying applications of the measuring apparatus by claiming that there is a single viewpoint, and thus a unique visible figure, that reveals the Euclidean characteristics of the stick and protractor setup. This unique viewpoint would thereby ensure an invariant visible figure for all subsequent measurements with the system (i.e., the unique visible provides a method for checking the apparatus to guarantee its correct application in all measurements). Yet, what visual configuration could reveal an intrinsic Euclidean property if the GOV is, as Angell concludes, non-Euclidean? If the three-dimensional Euclidean geometry of Angell's measuring stick, a physical object, is *different* from the geometry of the corresponding visible figure (the *visible* stick and protractor), then there would appear to be no objective, *non-conventional* way of picking out a preferred visible figure to represent the physical object. Since the two geometries are not identical, any visible figure, even one provided by the variably-slanting stick perspectives, would appear to have as much claim to represent a Euclidean physical object as any other. Alternatively, if there is only one viewpoint that correctly represents the real three-dimensional stick and protractor, then this admission is tantamount to basing a non-Euclidean geometry on a Euclidean visible figure. Since the unique visible measuring stick is now deemed to accurately represent the real Euclidean physical object, and this visible figure (stick and protractor) occupies a large portion of any possible single visual experience (i.e., the measuring stick can accompany all single views), it would seem that the most natural interpretation of the GOV's geometry should now favor a Euclidean structure. In essence, Angell is admitting that (i) some visible figures are Euclidean, (ii) these Euclidean figures take up the majority of any single view ("momentary visual field"), but (iii) the geometry of vision is non-Euclidean—but, is this a coherent position? Furthermore, relying on the concept of an "approximate" Euclidean visible figure, as opposed to a real Euclidean visible figure, fails to avoid the dilemma just described: if visible figures are non-Euclidean approximations to Euclidean visible figures, then this assumes that one has knowledge of what constitutes a Euclidean visible figure; (a) either by conventional stipulation (which raises the same conventionalist worries as above—the first horn of the dilemma); or (b) by some visible figure that is virtually identical to a Euclidean visible figure (which implies that we have some independent, and mysterious, knowledge of what constitutes a Euclidean visible outside of our experience of only non-Euclidean visibles—the second horn of the dilemma). So, approximation runs afoul of the same difficulties, once more raising concerns over the consistency of Angell's GOV.[20]

Angell would likely contest the preceding judgment, insisting that the GOV is founded on the geometry of large visible figures, such as ceilings and stellar triangles, and not on the geometry of small visible figures that can be encompassed in an individual viewpoint (or what we labeled above, a "single view"). Large visible figures, in contrast to the visible figure of the measuring stick, cannot be included in a single view, but require several individual views that are pieced together "geometrically", thus revealing their overall non-Euclidean character. Reid argued a similar point, recognizing that small visible figures closely approximated Euclidean figures (Inq., 6.9). By declaring that the domain of visibles is exclusively devoted to large figures, however, Angell is moving the GOV inexorably away from the actual, single viewpoints that comprise our visual phenomena to a more hypothetical, generalized realm of visual "figure". In other words, these large-scale visual figures are not possible figures of visual experience since they cannot be included in a single view: they are largely hypothetical constructs formed from several distinct single views (along with their accompanying small scale visible figures).

It is in this move from the small scale to the large scale visibles that conventionalist worries of a different sort begin to creep in, for there are no non-conventional methods for constructing a large scale visible figure from the small scale visibles contained in several single views. Angell and Daniels, as well as Reid, seem content to construct these large visible figures by a process that only allows a fixed spatial position and the rotations about that fixed point. From a fixed position in the center of a room, it is indeed true that rotations about that point will disclose four obtuse ceiling corners, thereby sanctioning a non-Euclidean GOV. Yet, this is only one of many processes or formulas for constructing a large-scale visible figure. If one admits both rotations about a spatial point *and* translations (motions) of that point in space, then an entirely different species of large scale visible figure can be assembled from the small scale visibles: since the eye can now move to a perspective directly underneath each corner, a 90 degree measurement can be obtained as mandated by a Euclidean version of the GOV. Given that there are numerous potential candidates for constructing a GOV, what other grounds, besides *convention*, can Angell appeal to in justifying his choice (of fixed spatial position and rotations)?

At this point, Angell could invoke the general shape of the eye, which is spherical, along with the contention that a fixed point (plus rotation) enjoys a simplicity over any other design for the GOV (which is similar to Daniels' conclusions, as mentioned in section 3). More precisely, rotation about a fixed point is the easiest means of scanning the angles and distances contained in large visible figures. Unfortunately, there are numerous problems with this line of argument: not only does the portion of the eye actually responsible for vision, the retina, lack a completely spherical shape, but it is also physically impossible for an eye to rotate 360 degrees about a fixed point. A movement of the head is also required to view Angell's large visible figures—that is, for the eye to measure all of the angles that comprise the ceiling, a *translation* of the eye must occur (as the head moves up and down). Yet, once a translation is shown to be an integral component of any measurements involving large-scale figures (e.g., in the track example, first looking at the horizon, and then moving your head down to gaze at your feet), the rationale for Angell's (Daniels', Reid's) "fixed spatial position" construction is seriously weakened, if not outright repudiated. In short, why not admit measuring conventions that incorporate translations if Angell's own method is shown to rely upon them?

In addition, once the necessity of the eye's translation is acknowledged, another conventional component of Angell's system comes to the forefront: namely, the assumption that, once an angle is measured, it *retains the same value* as the eye is moved to a new position (to measure a new angle). When a translation of the eye to a new spatial position occurs, the visible angle of the object that is subtended at the eye will change, resulting in a change of value for the previously measured angles.[21] Hence, if Angell needs to stipulate that all of the previously measured angles maintain an identical value across the many translations that comprise the measurement process, not only is this a conventional assumption, but the rules of projective geometry would declare that is false, as well.

5. Final Assessment of the GOV.

Once the role of conventional assumptions in the construction of the GOV is taken fully into account, the claims for its unique non-Euclidean structure appear greatly exaggerated, and quite possibly untenable. This is not to say that a non-Euclidean interpretation of visible phenomena is necessarily false; rather, as mentioned at the outset, a non-Euclidean GOV is just one of the many different geometries that can be constructed from visible figures. Given a particular set of measuring conventions, the visible figures will favor a specific geometrical interpretation—yet, once a new set of conventions is adopted, a new geometric structure will likely prevail. The reason for this liberal tolerance of alternative geometries probably resides in the idiosyncratic, foundational role allotted to large-scale visible figures. Since these large visibles cannot be encompassed in a single view, the diversity of methods for stitching together several such views, to form a large-scale picture, inevitably leads to their metrically "amorphous" character. Reid, along with Daniels and Angell, have demonstrated one procedure for obtaining these large-scale visible figures, but it is only one of many such procedures.

Before concluding, it would useful to briefly examine the underlying intent or goal of Reid's GOV, since our preceding discussion may shed light on some of the interpretational difficulties associated with Reid's project.

Reid's GOV appears within the context of his overall analysis of vision, a philosophical (or natural philosophical) problem that largely prompted his "direct realist" theory of perception. Contrary to Locke, Hume, and nearly all of his Early Modern predecessors, Reid believes that sense experience does not consist in a three-part relationship between a perceiver, a perceived object, and a mental item that represents that perceived object (a theory often dubbed "representational realism"). In contrast, Reid holds that perception is a direct, two-part relation between the observer and observed object. As mentioned in section 1, he also accepts (following Berkeley) that the geometry of the external world is three-dimensional Euclidean, and that tactile experiences provide direct information in this format. Moreover, tactile experience discloses the "real", as opposed to "apparent", magnitude of bodies. Given this unpromising background, can a direct realist embrace Reid's GOV?

Luckily for the direct realist, there are several passages that suggest a somewhat different interpretation of the GOV: "the visible figure of bodies is a real and external object to the eye, as their tangible figure is to the touch, . . ." (Inq., 6.8). As explained in this context, the "reality" of the objects or figures of visible experience are equal to the "reality" of the corresponding figures of tangible experience. In a later work, he states:

> When I use the names of tangible and visible space [i.e., the space of tangible and visible figure], I do not mean to adopt Bishop Berkeley's opinion, so far as to think that they are really different things, and altogether unlike. I take them to be different conceptions of the same thing; the one very partial, and the other more complete; but both distinct and just, as far as they reach. (Essays, 2.29)

On the whole, the cumulative effect of these explanations would seem to raise doubts about the lower epistemological status accorded to visible experience. Reid is conferring a certain degree of independence to visible figures, as contained in his reference to their "distinct and just" conception. However, Reid is careful not to imply that these two experiences of spatial figure, "are really different things, and altogether unlike." What does this last assertion mean? To answer this question, it is important to observe that he qualifies his claim in the next sentence, concluding that they are "different conceptions of the same thing." The best way to understand these two claims, I believe, is to conjoin them so as to read: "the two figures (of spatial experience) are not different in that they are two different conceptions of two different things, rather they are two different conceptions of the same thing." The advantages of this translation of the passage will become apparent below. Next, it is important to resist the temptation to interpret Reid's mention of "partial" and "complete" conceptions, corresponding to visible and tangible figures respectively, as invoking some form of dependence of the visibles on the tangibles. Understood in this manner, inconsistencies soon arise, as Van Cleve has carefully argued (70): for example, if a non-Euclidean visible triangle (with angles totaling over 180 degrees) *is part* of a Euclidean tangible triangle (with angles totaling 180 degrees exactly), then the corresponding object in the external world possesses a contrary pair or predicates (180 degrees and not 180 degrees). Yet, by heeding Reid's advice, that the visible and tangible figures are "distinct" and "different conceptions", this interpretational option is excluded.

Returning to our previous discussion, if visible and tangible figures are "distinct and just", and "different conceptions of the same thing", then one can construe both figures as two different *perspectives* (or *constructions, frameworks*) of the same external, material object.[22] In other words, material objects are geometrically "amorphous" or non-unique (much like large scale visible figures), since they only manifest a particular geometric structure from within a perceptual perspective. The perspectives that brings about these individual geometric forms are the human tactile and visual sensory processes—and these sensory frameworks only allow specific geometric formulations of the same external object: they are, in effect, the "different conceptions of the same thing" (i.e., a two-dimensional non-Euclidean perspective for visibles, and a three-dimensional Euclidean perspective for tangibles). Therefore, both the non-Euclidean GOV and the Euclidean tangibles can be viewed as complementary, but not competing, conceptions of an underlying "object" (and the perceptual "space" dependent on those figures). These external objects only divulge their information within a chosen, and probably *conventional,* perspective, but neither framework is the "correct" framework (although Reid does state that tangible experience is more complete than the visible). More precisely, the conventional construction of the alternative GOVs parallels the conventional sensory representation of external objects, for an evolutionist would likely claim that a different development of the human species might have endowed our sensory organs with a different form of geometry.

Nevertheless, this rendition of Reid's theory suffers at the hands of the numerous passages that seem to equate external objects exclusively with tangible figure, and not with both tangible and visible figure; e.g., in the quote from section 1, where he asserts: "The real magnitude of a line is measured by some known measure of length. . . . This magnitude is an object of touch only, and not of sight; . . ." (Essays, 2.14). These types of descriptions make it difficult to accept a direct realist rendering of the GOV, needless to say, for the visible figures are once more being grouped under the seemingly non-realist category of "apparent" magnitudes. Accordingly, even granting the value of this interpretation, the nature of Reid's visible figures, and GOV, must remain a continuing source of scholarly investigation and debate.

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1 N. Daniels, *Thomas Reid's 'Inquiry': The Geometry of Visibles and the Case for Realism.* (Stanford: Stanford University Press, 1989), 13.

2 R. B. Angell, "The Geometry of Visibles", *Nous,* 8, 1974, 87.

3 For example, J. Gray, *Ideas of Space: Euclidean, Non-Euclidean, and Relativistic,* 2nd ed. (Oxford: Oxford University Press, 1989), 71. One of the earliest references can be found in the Presidential Address of J. Cockle, "On the Confluences and Bifurcations of Certain Theories", London Mathematical Society, Nov. 8, 1888.

4 I will follow the standard convention and signify the *Inquiry into the Human Mind,* by 'Inq.', and the *Essays on the Intellectual Powers,* by 'Essays'. The editions used are by D. Brookes (Edinburgh: Edinburgh University Press, 1996) for the *Inquiry,* while the *Essays* come from, *Philosophical Works, vols. 1 and 2,* W. Hamilton, ed. (Hildesheim: Georg Olms, 1983), 8th ed., originally published 1895.

5 Much of Reid's overall theory of vision stems directly from, and is a response to, Berkeley's influential work. However, the GOV is Reid's original conception. See, G. Berkeley, *An Essay towards a New Theory of Vision,* in *The Works of George Berkeley,* ed. by A. Luce and T. Jessop (Edinburgh: Nelson, 1957), vol. 1, 159-238.

6 See Daniels, 22-23, and 141, footnote 43, where he states that "[Lambert] was inclined to reject the possibility of this new [non-Euclidean] geometry." On the whole, it is difficult to determine from these passages whether or not Daniel's really does dismiss the work of early geometers on the grounds that they either attempted to disprove the possibility of a non-Euclidean geometry, or, as in the case of Lambert, never openly stated that a non-Euclidean geometry was consistent (although Lambert's work seemed to imply that such geometries were, in fact, consistent).

7 M. Kline, *Mathematical Thought from Ancient to Modern Times* (Oxford: Oxford University Press, 1972), 89, 119-121. Kline's opinion on the discovery non-Euclidean geometry likewise echoes our concerns over the difficulty of choosing the appropriate criteria for deciding this debate: "If one means by the creation of a non-Euclidean geometry the recognition that there can be geometries alternative to Euclid's then Klugel and Lambert deserve the credit. If non-Euclidean geometry means the technical development of the consequences of a system of axioms containing an alternative to Euclid's parallel axiom then most credit must be accorded to Saccheri and even he benefited by the work of many men who tried to find a more acceptable substitute axiom for Euclid's" (Kline 1972, 869).

8 Quoted in P. Wood, "Reid, Parallel Lines, and the Geometry of Visibles", *Read Studies,* 2 (1998), 35. This passage comes from the Aberdeen University Library manuscripts (AUL MS 3061/11, pp. 11, 16, 18, 23). Similarly, Wood also questions the alleged

discovery of a non-Euclidean on the grounds that Reid was devoted to upholding Euclid's geometry, and the parallel postulate, throughout his (fairly substantial) work on geometry (Wood, 40-41).

9 AUL MS 2131/5/II/47, f. 4r. Wood refers to this comment (33), as does S. Weldon, "Thomas Reid's Theory of Vision", Ph.D. diss., McGill University, 1978, 181.

10 See, for instance, D. Brannan, M. Esplen, and J. Gray, *Geometry* (Cambridge: Cambridge University Press, 1999) 327-328.

11 It may be possible to reformulate Reid's GOV into an axiomatic pattern, however: the propositions that Reid offers, after his eight basic principles, could be re-arranged as his basic axioms or postulates (since they contain the important non-Euclidean results), with the eight principles regarded as a particular instantiation of the propositions on a spherical surface (i.e., the model). If one rejects the arguments concerning the "Euclidean tradition" (as presented above), and places instead the right to a non-Euclidean status solely on a consistent axiomatic structure that rejects the parallel postulate, then it could be claimed that the GOV's rough outline of an axiomatic system comprises a non-Euclidean geometry, since the GOV also provides a consistent model and does *openly* reject the Euclidean parallel postulate—but, of course, this axiomatization is more philosophical than geometrical, and thus may not be deemed sufficiently rigorous.

12 J. Van Cleve, "Thomas Reid's Geometry of Visibles", unpublished manuscript, 30.

13 The classic texts on the conventionalist thesis of space and time are: H. Reichenbach, *The Philosophy of Space and Time* (New York: Dover, 1958); A. Grünbaum, *Philosophical Problems of Space and Time* (New York: Alfred A. Knopf, 1963); while Poincare's analysis is found in *Science and Hypothesis* (New York: Dover, 1952). Additionally, there is a sense in which even the measurements of abstract geometry are conventional, since we must make assumptions on the straightness or regularity of our rulers and compasses when measure geometric figures (and, indeed, it is often the case that our instruments are irregular).

14 G. Berkeley, *New Theory of Vision,* section 41. Here, I am indebted to the work of M. Atherton, *Berkeley's Revolution in Vision* (Ithaca: Cornell University Press, 1990), 72-76, 90-95. See, also D. M. Armstrong, *Berkeley's Theory of Vision* (Melbourne: Melbourne University Press, 1960), 2-8. For Berkeley, as for Reid, the ability to discern depth differences among physical objects is acquired by experience through various means: e.g., the muscular tensions involved in focusing the eye on distant objects becomes automatically and unconsciously connected with the various distances of the objects (*New Theory of Vision,* sec. 16-28; Inq. 6.22. It is only after many experiences of this sort become associated, often unconsciously, with visible figures that we begin to make judgements of depth discrimination, and distance, among bodies. There is a serious problem lurking in the background here: what are the muscular tensions associated with, or causally correlated to, in Reid's theory? If material impressions, and visible figures,

provide no distance information on the external objects that are their cause, why are there muscular tensions at all? That is, what accounts for the different muscular tensions associated with different visible figures? If they all equally fail to provide distance information, it would seem that the muscle tensions should be the same for all figures (in fact, there should probably be no muscle tensions). On the other hand, if the muscular tensions are keyed to the distance of the external objects, then this is tantamount to linking the perception of distance to our muscles (and not our visible figures)—but how can our muscles obtain distance information?!

15 Oddly enough, Reid does comment on Molyneux's problem (in the context of Porterfield's work), in section 18, chapter 6, of the *Inquiry:* "Our having a natural and original perception of the distance of objects from the eye, appears contrary to a well-attested fact: for the young gentleman couched by Mr. Cheselden imagined, at first, that whatever he saw touched his eye, as what he felt touched his hand." But, one may reasonably ask, how can Reid make this assertion without undermining, or contradicting, his own construction of the GOV (which relies so heavily on the "equal distance from the eye" spherical geometry)? Equally problematic, Reid later goes on, in the same section, to specifically deny that we judge all of the objects that we visually perceive to be the same distance away from our eyes (although here Reid may be referring to the judgments of distance after the tactile experience of real object distances has been conjoined to our more basic experience of visual figures, thereby eliminating any inconsistencies with the GOV—see endnote 15). See, also, W. Porterfield, *Treatise on the Eye: The Manner and Phenomena of Vision* (Edinburgh, 1759), vol. 1, 372. Porterfield's work was clearly influential in shaping Reid's views, and may have led to Reid's hypothesis of the equi-distant position of all visible figures (from the eye): See, Daniels, 40-46.

16 William Hamilton's footnote to Inq. 6.18, in his edition of the complete works, nicely draws this distinction: "We must be careful not, like Reid and philosophers in general, to confound the perceptions of mere *externality* or *outness,* and the knowledge we have of *distance,* through the eyes….The patient, though he had little or no perception of distance: *i.e.,* of the *degree of externality,* had still a perception of that externality absolutely" (*Philosophical Works,* 177; original italics). Hamilton fails to relate this point to the construction of the GOV, however, and thereby misses a perfect opportunity to raise serious criticisms for Reid (which Hamilton clearly loved to do). Besides Berkeley, Malebranche was another predecessor of Reid's who also acknowledged the distinction between mere "outness" and distance. See, N. Malebranche, *The Search after Truth,* ed. by T. Lennon and P. Olscamp (Cambridge: Cambridge U. Press, 1997), 40-47.

17 Angell does admit this point, moreover, noting that "we shall want to speak of a given person's visual field as more extended spatially than any momentary visual fields [which are limited in spatial extent to a small portion of the face]" (91). Unfortunately, he immediately plunges into conventionalist waters by reasoning that "in following a line with the eye (e.g., scanning the horizon), we take portions of the line previously scanned but no longer in the momentary visual field to be continuous with the portions later scanned" (91). But, why make this *assumption,* rather than another?

18 We are leaving aside, in this discussion, the relativity of various properties in the Special and General Theories of Relativity. Although temporal coordinates are relative to the observer, the time-like and space-like separation among events is not relative, of course, nor is the space-time interval. In addition, in the conventionalist model of spatiotemporal (intrinsic) measurements, it is assumed that all inhabitants on a given surface will agree as to the results of the measurement (but may disagree as to their interpretation), which is not the case as described above (since their would be no agreement on the results of a hypothetical measurement).

19 Reid does state, Inq.6.7., that two visible figures are the same size ("congruent") if they subtend an identical angle with the eye. Yet, as Daniels' openly admits (55-56), this does not equip the GOV with a metric (although Daniels' believes that Reid must have intended his comments to function as a metric). For example, a conformal space can determine if two distinct angles on a given surface possess an identical angle size, yet this information is not sufficient to determine the quantitative magnitude of these angles (i.e., which angle is bigger, and by how much—see, e.g., E. Kreyszig, *Differential Geometry* (New York: Dover, 1963), 193). Accordingly, Reid's brief mention of the congruence of angle size for identical visibles falls far short of specifying a distance function for measuring the angle subtended at the eye by any *single* visible figure. In addition, Reid never discusses the difficulties associated with estimating the size of visible figures, a subject that Berkeley treats at length in the *New Theory of Vision* (sec. 54-78). Berkeley was aware that the judgment of the size of bodies (visible figures) was subject to illusions and errors, as the "moon illusion" nicely demonstrates: i.e., the moon on the horizon appears bigger than the moon high overhead, although the visible figure is the same size. The measurement of non-adjacent, spatially separated visible figures is thus potentially prone to error; and this realization seriously threatens any system of measurement (of visible figure) that relies exclusively on fixing rays and gauging angles that subtend the eye. In short, how can we be certain that are estimates of the rays and angles are not subject to the same class of visual illusions and misperceptions?

20 Interestingly, Angell's attempt to supply the GOV with an invariant, non-conventional measuring apparatus is reminiscent of a similar maneuver by H. Reichenbach and A. Grünbaum as regards the measurement of physical geometry. Reichenbach, for example, put forward the notion that the metric of physical space could be unambiguously determined once the existence/non-existence of universal distorting forces had been stipulated (i.e., forces that distort all measuring apparatus to the same degree regardless of their material composition). Therefore, an *empirical* aspect of the measurement process—the overall behavior of the rigid measuring rods—was taken as fixed, and thus non-conventional (after the conventional choice of universal forces had been made). Yet, as Einstein later argued, the "rigidity" of the measuring rods also depends on the ability to detect, and correct for, the local forces of thermal expansion, mechanical stress, etc., that likewise distort the measuring rods (but the distortion effects vary for different materials). Thus, the physical laws that largely determine which rods are "rigid" *presuppose* metrical concepts (i.e., how heat changes the *length* of the rod). Even the

local rigidity of the measuring rods now appears conventional, since any geometry can be utilized in the local distortion laws—and the different geometries chosen for these laws can determine the overall spatial geometry. In a similar fashion, Angell believes that he can regard the visible figure of the measuring stick as an *empirically* unproblematic feature of his theory, and then proceed to ascertain the overall geometry of vision by direct measurement. The visible figures of the measuring apparatus thus play the same role as the local rigidity correction laws for physical geometry. Just as one must stipulate a metric for the local distortion of the measuring rods, one faces an equally conventional dilemma in choosing among a host of different visible figures to represent the physical measuring stick and protractor. And, like the physical case, the overall geometry of vision (or large visible figures) is underdetermined as a direct consequence of this local conventionalist choice: i.e., a different choices of visible figure to represent the measuring stick, such as a variably slanting or non-slanting visible figure, can result in a different overall geometry of vision. Einstein's comments appear in "Remarks on the Essays", in *Albert Einstein, Philosopher-Scientist,* ed. by P. Schilpp (La Salle, Ill.: Open Court, 1949), 663-688. See, also, M. Carrier, *The Completeness of Scientific Theories* (Dordrecht: Kluwer, 1994), 191-194, for a nice discussion of these issues.

21 See, for example, the discussion of lateral motion on perceived size and angle in M. Hershenson, *Visual Space Perception* (Cambridge, Mass.: MIT Press, 1999), 145-155. Here, it should also be mentioned that once the translations of the eye (or of the objects in the visual field) are allowed into the calculations of the visual geometry, the behavior of the visible figures will not be consistent with a spherical non-Euclidean geometry. Rather, the rapid increase and decrease in the size of bodies, as they move forward or backward respectively, will favor a hyperbolic non-Euclidean geometry. R. Luneburg's important work on the geometry of vision was largely based on this fact: "Luneburg argued…in favor of a hyperbolic space because only in a hyperbolic space would the visual size of a distant object (as measured by its projection on a nearby transparent screen) decrease…with the object's distance from the observer." (P. Heelan, *Space-Perception and the Philosophy of Science* (Berkeley: U. of California Press, 1983), 328, fn. 29. It should be noted, however, that if one confines their investigation of visual geometry to the non-translation case, then the visual data that Luneburg refers to is different from Reid's (and Angell's) data, so the GOV (and Angell's theory) need not conflict with Luneburg's theory.

22 "Perspectives" is not ideally the best choice of term, however, since it harbors visual connotations that would tend to downplay the potential for gathering information from the other sense organs.

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