Feel like writing a review for The Mathematical Intelligencer? *Contact the column editor if you wish to submit an unsolicited review, or if you would welcome being assigned a book to review.*

➢ Submissions should be uploaded to http://tmin.edmgr.com or to be sent directly to **Osmo Pekonen**, osmo.pekonen@jyu.fi

An Aristotelian Realist Philosophy of Mathematics

by James Franklin

LONDON: PALGRAVE MACMILLAN, 2014, 320 PP., 63.00 £, ISBN 9781137400727

REVIEWED BY ALEX KOO

What is mathematics actually about? To the non-philosopher, this question can seem somewhat absurd. Mathematics is about things like numbers, sets, and functions. But what, exactly, are these entities? Do they exist in space–time like protons and electrons? Are they manmade creations? How do we come to know anything about them? When pressed further, two more sophisticated positions tend to emerge. Platonism is the view that mathematical entities exist in a non–spatio-temporal realm sometimes referred to as "Plato's heaven." Things such as numbers and sets are abstract, mind-independent, and have no causal powers. The biggest problem facing the Platonist is their inability to clearly explicate what Plato's heaven actually is, and how we could come to know anything about the abstract entities that exist there without any causal interaction.

The second position is mathematical nominalism. Nominalists deny that mathematical entities are real or that they exist in any deep sense beyond the way in which we utilize them, such as ink-strokes on a page or verbal utterances. There are many different gradations of nominalism, but all agree that mathematics does not have a subject matter that is abstract or mind-independent. The biggest challenges for nominalists are that their positions seem to conflict with the beliefs of most practicing mathematicians, and nominalism does not seem able to explain why applied mathematics is so successful.

In his book *An Aristotelian Realist Philosophy of Mathematics*, James Franklin argues that Platonism and nominalism represent a false dichotomy in the philosophy of mathematics. There is (at least) one more potential answer to fundamental questions about mathematics that can evade the drawbacks of the two leading views. Franklin's position draws from a classical Aristotelian philosophy of mathematics. Unlike Platonism and nominalism, classical Aristotelianism says that mathematics is not about particular entities such as the number 2 or the set of natural numbers, but instead is about universals that are instantiated in the actual physical world. Science is fundamentally about universals. Although scientists study particular objects, what they are actually investigating are universal

DOI 10.1007/s00283-016-9627-4

properties and relations. Physics is interested in universals such as charge or colour, whereas psychology investigates mental universals like mental states. So what is mathematics actually about? Aristotelians believe that mathematics is just another science. Mathematics investigates the real world to learn about mathematical universals such as symmetry or ratio. These universals are mathematical properties and relations that are as real and physical as any other scientific universal.

Platonism and nominalism can also accommodate the view that the subject matter of mathematics is about universals and not particulars, but each position has a different take on what these universals actually are. For the Platonist, mathematical universals are real and they exist, but they are purely abstract and noncausal, and do not exist in the physical world. For the nominalist, universals are not real at all and have no actual properties. The classical Aristotelian position claims that mathematical universals are real, have causal powers, and exist in the physical world. Their existence is due to their instantiation in the physical world and not in some abstract Platonic realm. In this way, the Aristotelian is able to maintain a realist interpretation of mathematics but avoid any sort of commitment to an abstract, highly metaphysical realm that we have no causal access to.

Classical Aristotelianism has faced two famous and crippling challenges. If mathematics is about real physical things, then what about mathematical universals that are uninstantiated in the physical world? Consider extremely large finite numbers, certain complex or real numbers, or the empty set. It is possible that none of these are instantiated in the world. The second challenge is similar but appeals to infinity. Higher-level mathematics makes use of infinite sets of sizes far larger than the set of natural numbers. Even if we allow for an infinite physical universe, or perhaps an infinitely divisible and continuous space–time, the infinite sets with larger cardinality used in mathematics do not seem to be instantiated (or instantiatable) in the physical world. Classical Aristotelianism says that mathematics is about instantiated universals. If this is so, then all of these uninstantiated mathematical universals do not exist. This clearly does not mesh well with mathematical practice, and has thus led to the abandonment of the classical Aristotelian view.

Franklin's book is divided into two parts. Part I is a demonstration that Aristotelianism can make sense of all mathematical objects. In Chapters 1 and 2, Franklin introduces his own brand of Aristotelianism (Franklin introduces his position as "semi-Platonism" and "modal-Aristotelianism" but eventually abandons these terms in favour of just "Aristotelianism") that draws from both Platonist and classical Aristotelian philosophies in order to tackle the problem of the uninstantiated. Franklin claims that mathematics is not just about instantiated universals, but also about universals that are uninstantiated as well. For the most part, the universals that are actually instantiated in the world are a contingent matter. With two notable exceptions, there is nothing about an uninstantiated universal that necessarily rules it out from being instantiated. Franklin argues that we can still know about these uninstantiated universals and study their necessary interrelations. Whereas classical Aristotelianism says that universals are exactly those that are instantiated in the physical world, Franklin's Aristotelianism says that universals are those that *could* be instantiated in the physical world. Franklin's slogan for Aristotelianism makes this clear: "Instantiation is possible, but not necessary".

The central aim of the remainder of Part I is to demonstrate that almost all of mathematics can be understood as universals that *could* be instantiated in the physical world. Franklin goes through entities such as quantity, ratio, finite sets, geometry, and more to justify his position. The bulk of this presentation does not depart from a traditional classical Aristotelian take. The interesting and novel sections are where Franklin tackles uninstantiated universals that have posed difficulty for classical Aristotelianism, such as highly abstract mathematical objects (Chapter 4), and objects so large that they cannot be realized in anything physical (Chapter 8).

In Part II, Franklin argues that Aristotelianism can account for mathematical practice. Franklin goes through many classic and contemporary issues of mathematical practice such as proof, structure, explanation, visualization, and idealization, and claims that all these mathematical techniques can explain how we come to know about mathematical universals that either are or could be instantiated in the physical world. Franklin draws from a wealth of modern philosophy and science including theories of scientific explanation, artificial intelligence, infant learning, and imagination. Part II is far more speculative compared to Part I, which is grounded on concrete examples from both abstract and applied mathematics.

In what follows I will present three specific examples from *An Aristotelian Realist Philosophy of Mathematics* that are important cases for the Aristotelian. First are highly abstract objects that are not instantiated in the physical world. Second are objects of extremely large size, such as infinite sets. Third are zero and the empty set—two familiar objects that pose a unique challenge and solution. Franklin's treatment of these three issues provides a clear illustration of how his Aristotelianism is different from classical Aristotelianism, Platonism, and nominalism.

Franklin uses groups as an exemplar for an Aristotelian handling of highly abstract objects. Consider the cyclic group of order 2. This group has many abstract instantiations, but it also has physical instantiations such as a Caps Lock button, or any other simple toggle. The group SO(3) is a bit more complicated, but it too can be realized in a system of all physical rotations in three-dimensional space. But what about the infinite system of all possible rotations? For this to be physical, then all of SO(3) would need to be realized at once. But this is impossible because of the (supposed) finite nature of the universe. Franklin notes that the universe's finite size is a contingent fact that could have been otherwise. There is nothing about the system of all possible rotations itself that necessitates it being uninstantiated physically. If the world had been different, then this system could have a real and physical instantiation. Aristotelianism "emphasizes the realizability, rather than actual realization, of universals". Given this, we can still study and learn about uninstantiated universals by analyzing things such as their structure and relations despite not having a particular physical instantiation to study.

Franklin's treatment of infinity is a bit less clear. It is certainly possible that there are physical instantiations of some sizes of infinity such as infinitely many points in space, but Franklin does not believe that higher-order infinite sets can be instantiated in our finite universe. As mentioned earlier, the finiteness of the universe is a contingent fact, and thus higher-order infinite sets could be instantiated so these sets are real mathematical objects. Quizzically, Franklin seems to be somewhat standoffish in his commitment to infinite sets. He argues at length that the bulk of mathematics can be done without any infinite concepts. The infinite is a "luxury" that makes our calculations easier and smoother, but can be done without. Another assertion Franklin makes is that it is fine for one to be a realist regarding infinities of countable size, but to reject higher-order infinities. A different argument for the reality of larger infinite sets is needed, at which Franklin only gestures. So, although it seems that the Aristotelian should count infinite sets as real mathematical objects, Franklin has laid the groundwork for the Aristotelian to survive if we choose to reject infinite sets as well.

Unlike the highly abstract or extremely large objects considered earlier that could be instantiated, zero and the empty set are different in that, according to Franklin, they are both necessarily uninstantiatable. Aristotelianism thus dictates that zero and the empty set are not real mathematical objects, but this seems counterintuitive as both objects are ubiquitous in mathematical practice. Franklin knows that although it is certainly convenient to use zero and the empty set, and they may even be indispensable to our understanding of mathematics, this does not mean that they are real. Franklin bites the bullet and adopts a nominalist interpretation toward zero and the empty set. No one doubts their utility, but "[n]o amount of mathematical convenience… can alter metaphysical reality". It is possible and actual instantiation in the physical world that dictates mathematical reality, and zero and the empty set do not meet this requirement. Franklin goes to great lengths to demonstrate that all the standard objects of mathematics such as the natural numbers, abstract groups, and more are all real according to Aristotelianism, but despite this his realism does not extend over zero and the empty set.

Franklin does a nice job of developing the Aristotelian position. He blends a rich philosophical history of Aristotelianism with modern case studies in both applied and pure mathematics to illustrate how we can conceive of mathematical objects as physically possibly instantiated universals. Although he easily could have stopped there, Franklin recognizes that an important issue for contemporary philosophy of mathematics is to give an account of how philosophy fits with actual mathematical practice. The attention paid to the many techniques of mathematicians in Part II provides a nice rounding out of Aristotelianism and is certainly a strength of the book. It is one thing to carve out a unique and consistent philosophical position, but it is an entirely other matter to show it to be both tenable and attractive in practice. Franklin has succeeded in making significant inroads toward both of these goals.

Ultimately I find myself somewhat unconvinced. The most important idea behind Aristotelianism is that of the potentially instantiable universal. If a mathematical universal is instantiated, then we know that it is real and exists in virtue of this instantiation. We also are able to investigate the universal just as a scientist would investigate universals like charge or spin. As stated in the opening line of the book, "mathematics is a science of the real world" (1). Mathematical objects are "mind-independent objects which are spatio-temporal and causal, namely relations such as ratios". But how does this fit with Franklin's assertion that uninstantiated universals are real? They are clearly not in the "real world," nor are they spatio-temporal and causal, so what is the nature of their existence? Franklin gets around this by claiming that these questions are not meaningful. Uninstantiated universals are not granted any sort of "existence." But why are questions of existence meaningful for instantiated universals, yet at the same time meaningless for the uninstantiated? This is certainly a convenient move in order to avoid the existence of some sort of nonphysical Platonic realm, but I find it suspiciously ad hoc and unsatisfying. There is a clear conflict here between the physical properties of instantiated universals and those of the uninstantiated. To make this clear, what can it possibly mean for uninstantiated mathematical objects to have causal powers? If mathematics contains both instantiated and uninstantiated objects, then we cannot say that mathematics is a science of the real world at all.

Franklin has created a two-class system of mathematical objects based around instantiation. The dividing line between the two tiers is contingent, as which universals are instantiated and which are not is entirely contingent according to Aristotelianism. For those that fall on the instantiated side of the line we adopt a physical realist interpretation, whereas for those that are uninstantiated we seem to adopt a quasirealist interpretation. Even if this is somehow consistent, I have concerns that this quasirealist understanding of uninstantiated universals is simply nominalism in disguise. Franklin insists that Aristotelians are still realist regarding uninstantiated universals, but I struggle to see how his realism works in these cases beyond mere lip-service.

Franklin attempts to evade these problems by claiming that what really matters is that we "insist on the reality of relations between universals, instantiated or not" (29). Unfortunately, avoiding existence questions of mathematical universals by invoking the reality of relations simply shifts the problem back one level. Suppose I grant that relations of instantiated universals are real, such as the ratio of my height to that of my mother. What exactly does it mean for the relations of uninstantiated universals to be real? What does it mean for these relations to exist in something other than the physical world? Do these uninstantiated relations have causal powers? Instead of having a two-class system of mathematical universals, we now have a two-class system of mathematical relations where the exact same problems emerge.

Despite my skepticism, *An Aristotelian Realist Philosophy of Mathematics* is an interesting and challenging book. Franklin is certainly correct in stating that Platonism and nominalism unfairly dominate the landscape in the philosophy of mathematics, and developing a rival position is no small achievement. Franklin goes through a wealth of material in philosophy, mathematics, and science to motivate and justify his position. Although this is certainly a strength of the book in that it leaves no stone unturned, it is also a weakness as a vast amount of prior knowledge is required to understand all of Franklin's arguments. Whereas *An Aristotelian Realist Philosophy of Mathematics* may not be entirely accessible to the novice philosopher of mathematics, there will still be something for all readers to enjoy due to its large scope and variety of interesting and detailed examples.

Department of Philosophy
University of Toronto
Toronto, ON M5R 2M8
Canada
e-mail: alex.koo@utoronto.ca