THE CONSTRUCTION AND USE OF MATHEMATICAL PROGRAMMING MODELS FOR THE ANALYSIS OF THE INTEGRATED INVESTMENT AND FINANCING DECISION WITHIN A FIRM

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DECLARATION

It is anticipated that some of the work in this thesis may be published as papers joint with D.R. Atkins. Work carried out by D.R. Atkins which has been included in this thesis is contained in separate sections and this fact is acknowledged in a footnote. The substance of Chapter two has already been published as "A re-examination of the Baumol-Quandt paradox" *The Engineering Economist*, Vol. 21, No. 3 while a summary version of Chapter five has been accepted for publication in the *Journal of Business Finance and Accounting*.

In memory of Peter

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ABSTRACT

This thesis examines the contribution that mathematical programming models can make to the solution of the joint problem of investment and financing within a firm. In particular it contrasts the performance of rules for investment appraisal which are based on discounting methodologies with the solutions which are obtainable from linear programming models. Using a method of analysis which exploits the relationship between the primal and dual solutions in such models, it argues that there are strong theoretical reasons why linear programming models will not generate solutions which are radically different from those which can be arrived at by simple discounting procedures. It concludes that linear programming models in their current form add little to the practice of investment appraisal. It shows however, that such models provide a powerful framework for the development of normative decision rules for project appraisal within the broader context of the firm's operating environment. The impact of alternative measures of debt capacity and the effect of finite and irregular cash flow patterns on the investment decision are all considered using this framework. These ideas are then applied to the specific problem of the valuation of a financial lease contract. The final Chapter returns to the problem of using linear programming models for investment appraisal and explores one way in which they might be restructured to be of practical assistance to corporate financial planners.

CHAPTER 1

The Power and limitations of Mathematical Programming Models for the Appraisal of Capital Expenditure Decisions - A Survey.

1.1 Introduction.

The last few years have seen an increasing acceptance by business analysts of the appraisal of capital expenditure by discounting the cash flows estimated to be generated by such proposals. Yet despite its theoretical superiority over other more traditional methods of investment appraisal it still remains open to a great deal of criticism both of a theoretical and practical nature (see for example * Adelson(70)).

A parallel development of recent years has been the exploration and implementation of computerised financial planning models. These have been developed partially to provide subsidiary analysis to discounted cash flow (D.C.F.) appraisal and partially as a tool in their own right. In their simplest versions they take the form of a simulation model or rather, to use a more correct title, a financial statement generator in which the effect of various decisions on selected financial indicators can be readily assessed. Their virtue lies merely in the speed and power of computation rather than any inherent mathematical sophistication and it is probably for this reason that they have been fairly widely accepted in industry. Mathematical models in their more sophisticated versions usually take the form of linear programming (L.P.)

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* References give author(s) and date of publication. Where two or more articles are referenced by the same author in the same year then additional distinguishing symbols will be used.

formulations of some aspect of the companies operations. The particular aspect of a company's operations that has attracted most attention is the capital expenditure decision. However, in spite of the fact that the initial formulation of the problem is now over ten years old, the survey work by Grinyer and Wooller (75), Higgins and Finn (77) in the United Kingdom and the work of Naylor and Schauland(76) in the States indicate that the instances of its implementation are still relatively few.

The intention of this thesis is to evaluate the theoretical shortcomings of existing mathematical programming models* of the investment and financing decision, to analyse and extend their contribution to financial theory as normative frameworks for decision making and to indicate one possible future direction of development that might enhance their managerial acceptability.

The purpose of this chapter is merely to survey the relevant background material and to summarize and underline the inter-linking nature of the ideas which will be developed in the subsequent arguments. The main themes of the research will be introduced initially in the following sections with no attempt at a detailed analysis. They will then be investigated more thoroughly in a corresponding later chapter.

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* The discussion will in general be restricted to deterministic or certainly equivalent formulations, though considerations of uncertainty also affect other aspects of the formulation. Many of the constraints included in the models (e.g. restrictions the number of times fixed interest payments must be covered) and designed purposely to cope with uncertainty in future cash flows. However, specific discussion of stochastic financial models as exemplified in the work of Byrne, Charnes, Cooper and Kootanek (67) or Nasland (66) will be excluded.

1.2 Programming Models, Capital Budgeting and Interdependencies.

The main weakness of the early work carried out in the field of normative models for capital investment selection is the assumption of independency in project selection. Lloyd Amey(72) classifies the interdependencies that do arise in practice into four main categories and it is these interdependencies that will form the subject matter of this thesis. These categories are not mutually exclusive but form convenient groups giving rise to particular problems. Briefly they are:

(1) Physical dependence where feasibility and profitability of accepting any set affects the feasibility and profitability of accepting any different set.

(2) Dynamic and intertemporal dependence arising from the timing of a particular investment.

(3) Serial dependence in that each investment may affect all future investments.

(4) Capital Market imperfections which cause the non-separability of the firms investment decisions and the stockholders consumptions preferences.

Lloyd Amey considers two projects and shows under conditions of perfect capital markets and with a cost of capital which is invariant with time then the internal rate of return criterion and net present value criterion with suitable modifications are able to cope with problems of mutual exclusivity, contingency and intertemporal dependence. He argues that modifications to such rules become impractical if the number of such interdependencies is large and in these circumstances mathematical programming formulations become necessary. Hence mathematical programming is introduced primarily to provide an efficient combinatorial search procedure over feasible subsets. It is the practicality of the search which causes us to resort the mathematical programming and not any inherent superiority of the solution. Indeed as we shall see in certain cases alternative and equally efficient search procedures exist. Nevertheless mathematical programming remains a powerful tool for dealing with interactive financial decisions.

Unfortunately, the nature of the interactions in the case of financial models can cause severe problems of formulation. A particularly apposite example is afforded by the paradox associated with the choice of objective functions; in that if one tries to maximize the net present value of a set of projects which are subjected to budget constraints then since the dual values associated with the budget constraints give the correct discount rates to use in the objective function, one cannot specify the objective function until the dual is solved, and one cannot solve the dual until the objective function is known. It is worthwhile tracing the development of this problem historically since its affect on later workers in the field has been profound and a few subsequent writers* have not quoted the original paper by Baumol and Quandt (65) in which they first highlighted this paradox.

The problem of the selection of an optimal subset of projects when the firm is precluded from undertaking all projects with a positive net present value at its cost of capital was discussed first by Lorie and Savage (55). Unfortunately, they arrived at a solution largely by trial and error and hence their method was

*See for example Lustig and Schwab (68), Carleton (69), Elton (70), Myers (72), Merville and Tavis (73), Burton and Damon (74).

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unsatisfactory from a computational point of view.

It was Weingartner (62) who was the first to demonstrate that the Lorie-Savage problem could be formulated as

$$\text{Max } Z = \sum_{j=1}^{n} \hat{c}_j x_j \qquad (1.2.1)$$

$$\text{subject to} \quad \sum_{j=1}^{n} C_{tj} x_j \le F_t \quad t=0,1,2, H \quad . . (1.2.2.)$$

$$0 \le x_j \le 1, \; x_j \text{ integer} \qquad (1.2.3.)$$

$$= \sum_{t=0}^{\infty} c_{tj}/(1+r)^t$$

where $\hat{c}_j$ is the net present value of project j

c_{tj} is the net cash flow from project j in time t

C_{tj} is the capital requirement of project j in time t

F_t is the total capital available in t

r is the borrowing rate

and x_j takes the value 1 if the project is accepted and zero otherwise.

In this form the solution to the problem can be found by integer programming methods. By relaxing the integer constraint on project selection and regarding the components of the vector X as the scale of acceptance of an individual project the problem can be reformulated as a linear programming problem.

The importance of this step is twofold. The first and most important consequence is that powerful algorithms exist for computational solutions of linear programs. As we shall see this relatively basic model can be extended easily to cope with constraints other than simple cash balance constraints. A second aspect of the formulation is one which we shall exploit extensively later. The formulation of the capital rationing problem by Weingartner is in terms of budgets and quantities of resources.

In this thesis formulations of this type will be referred to as the primal problem. Closely related to this primal problem is a dual problem which is in terms of prices and values of those resources. The mathematical relationship between the primal and dual problems is discussed extensively in the standard works on mathematical programming (see for example Beale(62), Dantzig(63), Hadley (62)). Of more immediate concern is the economic interpretation of the dual which gives information on the marginal value of additional funds.

The use of dual values to evaluate the cost of funds was considered first by Charnes, Cooper and Miller(59). They were concerned with the problem of a warehouse in which the primal objective function was undiscounted cumulative profits. In this case the corresponding dual variables took on the dimensions of interest rates. It was left to Baumal and Quandt *(op cit)* to point out that the dual solution of the Weingartner problem gives the opportunity value of an extra £1 in each of the constraint years. Thus this dual solution gives information on the marginal efficiency of capital and hence the appropriate discount rate to be used in each time period. However, in the formulation of an objective function we have already assumed a particular discount rate. We thus have the Baumal and Quandt (65) paradox referred to above. They claimed that a more correct form of the discount factor would be ρ_t/ρ_o where ρ_t, ρ_o are the dual values associated with the budget constraints in year t and now respectively. This discount factor is the proposed replacement for $^1/_{(1+r)^t}$ in the computation of $\hat{c}_j$. In addition they argued that the capital outlays were merely the net cash outflows from the project selection. Thus their formulation of the problem is as follows:

$$\text{Max } Z = \sum_{t=1}^{T} \sum_{j=1}^{n} \rho_t/\rho_o \, c_{tj} x_j \qquad (1.2.4)$$

$$\text{s.t. } -\sum_{j=1}^{n} c_{jt} x_j \leq F_t \qquad t=0,\ldots\ldots T \qquad (1.2.5)$$

They proved* that the corresponding dual formulation implied $\sum c_{jt}\rho_t \leq 0$ and concluded that the only solution to this problem was the trivial one with x and ρ identically zero.

Baumal and Quandt suggested a way out of this dilemma by reformulating the objective function such that it maximised the utility of withdrawals from the firm. The revised formulation is:

$$\text{Max } Z = \sum_{t} U_t W_t \qquad (1.2.6)$$

$$\text{s.t. } -\sum_{j} c_{jt} x_j + W_t \leq F_t \qquad (1.2.7)$$

where U_t denotes the utility of a withdrawal in time t. The obvious criticism of this particular formulation is the difficulty of determining a utility function and as such it is essentially a non-operational model.

Weingartner's own response to the Baumol-Quandt criticism was the terminal value model in which he maximises the post-horizon cash flows of the chosen project set. Thus the model is:

$$\text{Max } Z = \sum_{j=1}^{n} \hat{c}_j x_j \text{ where } \hat{c}_j = \sum_{t=H}^{\infty} \frac{c_{jt}}{(1+r)^{t-T}} \qquad (1.2.8)$$

$$\text{subject to } -\sum c_{jt} x_j \leq F_t \qquad t=0,\ldots\ldots H-1 \qquad (1.2.9)$$

$$\text{and} \qquad 0 \leq x_j \leq 1 \qquad j=1,\ldots\ldots n \qquad (1.2.10)$$

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* Their proof appears to contain an elementary error (Section 2.3).

This is an interesting way of avoiding the problem since the constraints run over this period* 0 to H-1, and the objective function over the non-overlapping period H to ∞. In this case the duals are unrelated to the post-horizon discount rates. The model assumes in fact that the post-horizon discount rate is sufficiently distant to be approximated by a constant.

Ironically, it also implies that sufficient uncertainty surrounds the objective function to make the problem of the appropriate discount rate immaterial. Thus we are trying to maximise a linear function of the subset of the information about which we are least certain! An additional criticism is that it can also be shown that it is equivalent to the maximisation of net present value under assumptions of a perfect market. However, such assumptions would preclude any rationing of funds and under such conditions there is no need to resort to linear programming models since conventional discounting techniques are adequate.

A modified form of this objective function is that used by Chambers (67), in which he maximises the net present value of the dividend stream. His objective function is:

$$\text{Max } Z = \sum_{t=1}^{H-1} \frac{D_t}{(1+i)^t} + \frac{V_H}{(1+i)^H} \qquad (1.2.11)$$

Here D_t is the dividend in time period t and V_H is the terminal value of the firm. The discount rate i in this case reflects the shareholders' time preference. It should be noted that the last two mentioned objective functions are both variants on a

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* In the original formulation the constraint set ran from periods 0 to H. The particular formulation here follows a modification by Bernhard (69) who pointed out that the Baumol and Quandt paradox then occurred in period H.

more generalised form of maximising $f(D_1, D_2, \ldots . D_{H-1}, V_H)$ where f is a linear function. They are in essence of the same structure as Baumol and Quandt's maximisation of the utility of withdrawals. Many authors* have adopted this approach to the problem. They have resorted to the utility formulation of the problem and argued that the presence of capital markets imposes a well defined form for this utility function. While such an approach has strong theoretical justifications under assumptions of free access to capital markets it avoids, rather than resolves, the paradox and in a later paper when Chambers (72) modifies his model specifically to include financing opportunities as well as investment projects again resorts to a terminal value model to avoid interdependencies between discount rates and objective function valuation.

It is interesting at this stage to review one further** approach which has been suggested in the financial literature in which its origin lies in the original Lorie-Savage approach to the problem. Its aim is to find a solution to the one period capital budgeting problem of choosing a set of projects when the outlay in the first period is subject to a cash constraint. The projects are initially ranked at the firms cost of capital and the internal rate of return of the marginally rejected project is determined. The projects are re-ranked at this rate of return and the new marginally rejected project is determined. This process is continued until such time as there is no change in the accepted project list (i.e. those projects

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* For example, see Myers (72) or Elton (70)

** See Quirin (67) or Lustig and Schwab (68).

with positive Net Present Value at the two discount rates). The idea behind this process is that the correct discount rate under capital rationing is the marginal productivity of capital or the internal rate of return of the marginally rejected project. This is in fact only a partial truth. The idea behind it appears to be based on an important paper by Hirschleifer (59) in which he discusses discount criteria and the appropriate discount rates to be used. One of his conclusions was that in cases where we are in a borrowing situation then the borrowing rate is the appropriate rate, in cases where we are in a lending situation then it is the lending rate, but in a capital rationing situation it is the marginal productivity of capital. While in itself this might appear an obvious result it does have very important ramifications, much of the theoretical basis of the Baumol-Quandt paper rests in their interpretation of the Hirschleifer paper. Atkins (72) has shown how this last approach can be reformulated as a mathematical programming problem. Thus the model is:

$$\text{Max } Z = \sum_{j=1}^{n} \sum_{t=1}^{H} \frac{c_{jt} x_j}{(1+r)^t} \qquad (1.2.12)$$

$$-\sum_j c_{jo} x_j \le F_o \qquad (1.2.13)$$

$$\text{and} \qquad 0 \le x_j \le 1 \qquad (1.2.14)$$

where the subscript zero denotes a budget constraint in the first period only. In addition it is required that the discount factor $^1/_{(1+r)^H}$ should equal ρ_o, the dual of the budget constraint. With this formulation the problem is solvable by the methods of parametric programming. The important point that this formulation shows is that this assumes that the discount rate is a constant for all periods and its value is determined by the constraint only in the first year. There is no reason to assume that the discount factor applicable to the first year should persist beyond that year. The work of Hirschleifer shows that the correct discount factor applicable from year to year depends upon the budget constraints and lending or borrowing opportunities in each of the years up to the horizon. This brings us full circle back to the problem of the relationship between the discount rates and the duals, and an understanding of this relationship is a vital preprequisite to many of the ideas to be developed in later chapters.

In chapter two a simple numerical example is chosen and it is shown that it is possible to generate a solution in which the primal values are consistent with the dual values. By respecifying the problem, with greater attention being paid to a rigorous definition of the variables, it is shown that while Baumol and Quandt managed to identify correctly one solution, they succeeded in identifying merely one solution of many. Moreover, the solution they identified was unfortunately the null vector solution. It is shown further by introducing projects which enable funds to be carried between periods then the solution is both unique and non-zero.

In this way it is possible to generate an internal price-vector or generalized discount rate measuring the intrinsic profitability of a project set. This idea is readily seen to be an extension of the internal rate of return concept applied to individual projects to encompass the multiproject multi-period constrained case. The requisite ideas to interpret this extension can be found in the paper by Hirschleifer(58) which has already been referred to, while the more general nature of this solution provides us with a mechanism for analysing the multi-period case. Moreover, it is argued that Hirschleifer's work was a natural forerunner of the later mathematical programming approaches and such is the fundamental nature of his results that they form a recurrent theme throughout this thesis.

1.3 Profitability Indices, Rules of Thumb and Approximate Solutions to Capital Budgeting Models.

It has been argued in the previous section that conventional discounting techniques will in general break down under problems of capital rationing because of the interdependencies that arise. This view of the inadequacy of discounting under such circumstances and the consequent need for mathematical programming models is widely accepted by academics.

Thus Amey (72) in the paper already cited states

> "in general a programming formulation is indispensable when there are interdependencies."

While Bromwich (70) in a survey of capital budgeting states

> "The application of programming methods to capital rationing situations yields the set of investments, for each year, which maximises total net present worth in the face of scarce funds in the future. No rule of thumb criteria can do this satisfactorily because of the vast number of possible combinations of projects which could be involved."

Nevertheless, despite their undoubted theoretical superiority, the rigid structure and prohibitive data requirements of many LP models is a severe limitation on their practical usefulness. Their implicit assumption of shareholder wealth maximization may well attribute too much weight to this single criterion* and a naive

* This use of a single objective function to describe the organisational goals of the firm is discussed more fully in section 1.6.

description of the planning process of the firm. A more likely description of the planning process is the view argued by such authors as Simon (57) or Hopwood (74) where profitability is merely one of many criteria which need to be considered, albeit an important one and as such acts as a constraint on the decision making process rather than the overriding purpose of any decision. In this sense discounting is a very effective tool since it attributes a numerical value to the profitability of a project. The decision to accept or reject any project can then be made against other criteria with a knowledge of the consequent impact on profitability. The other great restriction on the use of mathematical programming as a practical method of project selection is the need for a complete specification together with a centrally coordinated analysis of all project opportunities upto some planning horizon. Not only does such a process appear to have prohibitive data requirements but may well cut across existing organisational responsibilities.

Hence although it could be argued that mathematical programming is shunned merely because of organisational and data problems, a rather more disquieting observation is where this is not the case and authors cite numerical examples obtained from their models then their solutions appear to differ little from solutions which could be obtained by fairly simple rules of thumb.* In fact, not only is there often a large measure of agreement but also the difference usually seems to occur only among projects which are marginally acceptable and for the very projects which the decision to go ahead is most likely to be made on criteria other than the purely financial anyway.

* A rule of thumb here is used as an "umbrella" term to include any form of analysis based on a discounting methodology.

Examination of the published solutions of two of the major contributions to this field provide confirmatory evidence of this point. Weingartner (*op cit* p. 183) in an attempt to illustrate the misleading nature of discounting techniques uses a modified form of his basic horizon model for the selection of the optimal subset from a set of 30 projects whose cash flows span twenty-six years. In the case where the decisions are made subject to a simple upper bound on the amount of debt available in a period, 11 projects are included in this optimal set. However, out of the twelve projects ranked highest by an internal rate of return criterion eleven of them appear in the selected set. The only exception to this is a project ranked 9 with an internal rate of return of 11.03% compared with a cost of capital (rate on interest on debt) or 10%. In fact the solution would tend to suggest that even this project is only just excluded since it has the largest reduced cost of the excluded set.

In an attempt to integrate the investment and financing decisions, Chambers (71) develops a complex and realistic model which consists of the selection from a set of thirteen projects available in each of five years up to the planning horizon. The projects can be financed by combinations of debentures and rights issues. Also available as options to the firm are the possibilities of investing either in the equity of other companies or short term government securities. The model incorporates the current United Kingdom tax system and selection is made subject to cash availability with debt availability restricted by the book level of gearing. Thus the constraints impose a great deal of interdependency between project decisions since any project investment decision is likely to affect any future investment decision because of the impact made by its retained earnings on the book value of the equity and hence the debt capacity of the firm. The results quoted by Chambers are that the same ten out of the thirteen available projects are chosen in each year. The remaining projects which are sometimes included and sometimes not, are ranked 10, 12, 13 by an internal rate of return criterion. Chambers calculates the weighted average cost of capital assuming a fully geared position as 9.8%, while the internal rate of return of these latter projects are 10.4%, 9.6% and 9.2% respectively. He finds also that this investment strategy is largely independent of the firms initial cash position and level of gearing.

While both authors correctly point out the inconsistences of conventional discounting methods and analyse the dissimilarities of their solutions from those obtained by such methods, both gloss over the remarkable degree of similarity. Thus it would appear that in the case of Weingartner's model a simple ranking by internal rate of return would have yielded a satisfactory, near optimal, solution and Chambers would have lost little if he had chosen projects with a positive net terminal value at the computed cost of capital. Thus neither model seems to offer a substantial improvement over elementary rules of thumb.

The question now arises whether these and similar results obtained on other models are simply freaks of particular data sets, or are inherent structural feature of such models. It is this task that occupies most of the third chapter but it must be pointed out that by concentrating on Weingartner's and Chambers' models, two models are being studied that have essentially the same basic structure. Both models are characterized by an objective function which is the maximization of the value of a firm where the restriction placed on its investment schedule, apart from a cash balance requirement, is a limitation on the amount of debt it may incur by a debt capacity constraint. Despite considerable development and elaboration of the constraint set by the various writers* in the field this characteristic structure of maximizing a measure of the value of a firm subject to cash availability and restrictions on the level of debt remains a basic subset. Hence an understanding of the relationship between the mathematical programming solutions of the Weingartner and Chambers models and the discounting formulae should be illuminating of more complex formulations.

In Weingartner's case the debt capacity restriction takes the form of a simple upper bound and in Chamber's case it is related to the book value of the assets. Another model apparently of the same form is where the restriction is a times interest covered on the debt. However, this constraint does differ significantly from the other constraints in that the limitation on the amount of debt here is solely a function of the (profitability) of the investment decision.† These three models cover the most commonly used accounting restrictions of the level of debt and the extension of the work to include theoretical financial market measures of debt capacity proves to be mathematically fairly simple.

The approach to be taken can be best illustrated by the contrast of the Lorie-Savage(55) method of solving the capital rationing problem with that proposed by Weingartner. Lorie-Savage

* See for example Bernhard (69), Hamilton and Moses (73) or Myers and Pogue (74).

† In Weingartner's model the debt capacity is clearly independent of the investment decision. While in the Chamber's model although dependent in part on the investment decision the debt capacity can always be increased by a further equity issue.

solved the one period case by a simple ranking procedure. Their solution to the two period case was also a type of ranking by using two indices, in this case the appropriate Lagrange multipliers for the budget constraints, which they arrived at by trial and error. Weingartner cast the problem into a mathematical programming form and showed that the general n-period problem is capable of systematic solution. Both techniques are search procedures; however Lorie and Savage were looking for the price-vector of the cash balances and could thus be considered to be a search of the dual spaces. On the other hand Weingartner and other writers who rely on linear programming formulations could be considered to be searching the primal space for the appropriate value of the decision vector.

The idea of searching for a constant price vector against which projects can be screened is not new. It has long been recognised that under conditions of capital rationing it is necessary to introduce a modification to the simple rule of thumb of accepting all projects with a positive net present value at the lending rate and the most appropriate modifications have been debated extensively in the literature.* The main weakness of much of this discussion is that it centres around fairly simple numerical examples, which are chosen mainly to illustrate a particular point rather than to provide a general analysis.†

In the Weingartner case the dual search proves particularly revealing. The model incorporates almost the same set of assumptions as simple discounting methods, the only difference being the imposition

* See for example Quirin (67), Schwab and Lustig (69), Bernhard (69) Beenhacker (73), Hoskins (74).

† Two exceptions to this are the papers by Bernard (69), who uses a framework for analysis similar to that which will be developed and the one by Lustig or Shwab (69), though in the end both of these successfully deal with situations where the capital rationing applies to the first time period only.

of a 'hard' constraint on debt availability and the dual analysis provides a rigorous framework for an examination of the necessary modification to discounting formulae in such situations.

In the Chambers' case, the dual search shows that a general analytical solution to the linear programming model is possible. Hence in both these cases the dual search procedure proves to be more efficient and more useful than the primal search procedure. While an analytical treatment suffices to determine the dual feasible region for Chambers'and Weingartners' models it is difficult to extend this idea to more complex models. Nevertheless computational evidence will be cited to show the robustness of discounting indices even in complex models. However, the purpose of chapter three is not to develop rules of thumb to different kinds of models but rather to emphasise the power of conventional methods of appraisal,to gain insight into the nature of the solutions to these models and to attempt to define more clearly their role in practical decision-making situations.

1.4 Economic Objective functions, the valuation of investment opportunities and the finite horizon problem.

Although evidence was cited in the previous section that the numerical impact of interactions between the investment and financing decisions may be less intractable than that suggested by many authors, the existence of this interaction where there are significant degrees of market imperfection* remains unquestioned. In any rigorous treatment of the theory of valuation of the firm the interaction needs to be treated explicitly.

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* The most significant arguments to the contrary embodied in the work of Modigliani and Miller (58) specifically assume perfect market conditions.

A mathematical programming framework affords a potentially very powerful analytical tool for this. The advantage of mathematical programming models in this area is their representation of the economic value of the firm as the objective function and their explicit treatment of market imperfections as constraints. Many interesting and economically meaningful deductions can be made from these models by use of the Kuhn-Tucker conditions* for optimality.

The problem of valuation of the firm, within or without the context of mathematical programming, is a core problem of financial theory. The purpose of this part of the thesis is not to tackle directly any of the fundamental issues but to show the contribution that mathematical programming can make to exploring the consequences and logical consistencies of a particular formulation.

This contribution will be discussed more extensively in chapter four. In this section the background material and the nature of one particular problem will be discussed - the horizon truncation problem.

Of necessity any linear programming model of the firm must have a finite horizon. The properties required of this finite horizon focus precisely on the substance of chapter four - the conceptual problems arising from the interactions of capital market imperfections and the impact on the valuation formula used for the objective function. This aspect is best seen in a historical context and once again the work of Weingartner provides the most

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* Simple expositions of the Kuhn-Tucker conditions can be found in standard operational research texts such as Hillier and Lieberman (67)

appropriate vehicle.

As we have seen his original approach of maximizing the net present value of the project set was subject to Baumol-Quandt's criticism of inconsistency. Their suggested way out of the paradox of maximizing the utility of withdrawals from the firm by a model of the form*

$$\text{Max} \quad \sum U_t W_t \tag{1.4.1}$$

$$\text{Subject to} \quad - \sum c_{tj} x_j + W_t \leq F_t \tag{1.4.2}$$

was rejected by Weingartner because of the problems of specification of an appropriate utility function.† Instead he resorted to a horizon valuation model.

* The notation is the same as in section 1.2

† As mentioned, later writers such as Myers (72) have identified U_t with the relative utility of total funds in time period t and thus with interest rates exogenously determined by the capital market. Thus Myers rewrites the model in the form

$$\text{Max} \quad U_t \left[F_t + \sum c_{tj} x_j \right] \tag{1.4.3}$$

$$= \sum U_t F_t + \text{Max} \sum_t \sum_j U_t c_{tj} x_j \tag{1.4.4}$$

$$\text{subject to} \quad \sum - c_{tj} x_j \leq F_t \tag{1.4.5}$$

He argues that in a certain world, investors facing a prevailing interest rate K will all adjust their portfolios so that the following conditions hold

$$U_t / U_{t+1} = \frac{1}{(1+K)} \quad > \quad U_t / U_o = \frac{1}{(1+K)^t} \tag{1.4.6}$$

Defining $U_o = 1$ means that the firm can use the observed rate K to infer the marginal utilities required by the Baumol and Quandt formulation.

Thus Weingartner's reformulated model was

$$\text{Max } Z = \sum_{j=1}^{n} \hat{c}_j x_j + v_T - w_T \qquad (1.4.7)$$

$$\text{s.t.} \quad -\sum_j c_{1j} x_j + v_1 - w_1 \leq F_1 \qquad (1.4.8)$$

$$-\sum_j c_{tj} x_j + (1+r_L)w_{t-1} - (1+r_B)v_{t-1} - w_t + v_t \leq F_t$$

$$t = 1, \; H - 1 \qquad (1.4.9)$$

$$w_t \leq B_t \qquad t=1,\ldots\ldots H-1 \qquad (1.4.10)$$

$$0 \leq x_j \leq 1 \qquad \forall j \qquad (1.4.11)$$

$$r_t, \; w_t \geq 0. \qquad \forall t \qquad (1.4.12)$$

with the additional notation

w_t borrowing in period t.

v_t lending in period t.

r_L is the interest rate on lending.

r_B is the interest rate on borrowing.

B_t is a limit on the borrowing in t.

The scalar quantity $\hat{c}_j$ representing the post horizon value of cash flows is given by

$$\hat{c}_j = \sum_{t=H}^{\infty} c_{tj} x_j \Big/ \prod_{\tau=H+1}^{t} (1+i_\tau) \qquad (1.4.13)$$

This approach gives rise to three important questions.

In what sense is the pursuit of optimal wealth at some future time compatible with maximization of the value of the firm now?

What is the significance of and the determinants of the choice of horizon?

What is the appropriate post-horizon valuation procedure?

A dual analysis of this model provides a foundation for the answers to these questions, the dual of the cash balance equation gives the marginal value of an extra £1 of earnings and thus the ratio of the duals in successive periods gives the interperiod discount rate at which projects ought to be screening. In effect the dual is the opportunity cost of capital. The relationship between ρ_t, the lending rate and the borrowing rate and the dual on the debt capacity (λ_t) is

$$(1+r_L)\ \rho_{t+1} \leq \rho_t \leq (1+r_B)\ \rho_{t+1} + \lambda_t \qquad (1.4.16)$$

It should be emphasised that these duals are outputs from the optimum linear programming solutions. Thus where the firm is lending, the left hand inequality becomes an equality and $\rho_t = (1+r_L)\rho_{t+1}$; where the firm is borrowing with spare debt capacity $\rho_t = (1+r_B)\rho_{t+1}$ and where the firm is borrowing upto its limit $\rho_t = (1+r_B)\rho_{t+1} + \lambda_t$. Thus the opportunity cost of capital may be the lending rate, the borrowing rate or the marginal productivity of capital. The marginal value of project j is given by $\mu_j = \hat{c}_j - \sum_{t=1}^{H} c_{tj}\rho_t$. Here $\hat{c}_j$ represents the value at H of post horizon cash flows and $c_{tj}\rho_t$ is the value of the cash flow from project j valued at H. Hence μ_j is a generalization* of the net terminal value concept.

In answer to the first of these problems, Weingartner concluded that where borrowing and lending rates were equal with $r_L = r_B$ and borrowing is unrestricted† then maximazation of the terminal value

* See Weingartner (74) p.164 et seq. Page numbers refer to the 1974 edition, though the 1962 reference will be given where the historical context of the work is important.

† The inequality then implies $\rho_t = (1+r)\rho_{t+1}$

or $\rho_t = (1+r)^{H-t}$ with $\mu_j = \hat{c}_j - \sum_{t=1}^{H} c_{tj}(1+r)^{H-t}$

of the firm was equivalent to the maximization of the net present value. However, this set of assumptions implies perfect capital markets and under such conditions a linear programming formulation of the capital investment problem is unnecessary. In conditions of capital rationing Weingartner concluded that maximization of the net terminal value was not equal to maximization of the net present value of the project set.

The dual analysis reveals also the difficulty associated with a specification of a suitable valuation function for post horizon cash flows. As Weingartner states

> "The rate taken to be appropriate in computing the horizon values $\hat{c}_j$ is the lending-borrowing rate used in the models. However, this rate is not the proper one if there are effective limits on borrowing."

Thus Weingartner admits that while the correct discount rate is effectively incorporated into the valuation in the pre-horizon period it is not clear which of the borrowing, lending or marginal rates is the correct one in the post horizon period. Weingartner does provide a clear discussion of the requirements of an horizon, though little further guidance as to how one might determine such an horizon. Thus he states

> "In order to unhook the infinite chains of actions and their consequences in the model of the firm's investment decisions, we seek a point in time such that the decisions which call for implementation before this date will be exactly the same, whether or not events past that moment are treated explicitly or implicitly (and hence partially ignored). More concretely, and in terms of our model,

we seek a value of H such that the set of accepted projects having outlays or revenues in year H or sooner are exactly the same whether the model makes use of an infinite horizon or a horizon set at H.

In dynamic Models in general such a horizon does not necessarily exist or there may be many of them. If there are several the earliest having this property may be designated as the preferred one."

The discussion gives rise to a definition of a suitable horizon valuation - which shall be termed the fundamental horizon valuation principle.

The horizon valuation is a satisfactory valuation model if, for all optimal feasible solutions, the set of pre-horizon decisions with respect to that horizon would be unaltered for any other choice of horizon.

The existence of such a horizon will be discussed in Chapter Four.

Weingartner's approach to the horizon truncation problem implies that the horizon is an intrinsic property of the model and its determinants are found from within the model. The alternative approach is to regard the horizon as a function of the firm's planning. Such an approach is exemplified by Chambers (67) in his paper 'The allocation of funds subject to restrictions on reported results'; he states that

"the horizon is chosen as a date beyond which opportunities cannot be predicted with any confidence, no information is lost by ignoring interactions between projects after that date, or assuming that funds are reinvested at the standard rate. In this approach in which the aim was to develop a model to assist management with planning it was convenient to adopt the same planning horizon".

While this may not be totally satisfactory from a theoretical viewpoint it may well prove necessary in practice.

In the later paper (71) on 'The joint porblem of investment and finance', he adopts a terminal valuation approach since

"This allows the marginal cost of capital in each year upto a planning horizon to be determined within the model"......

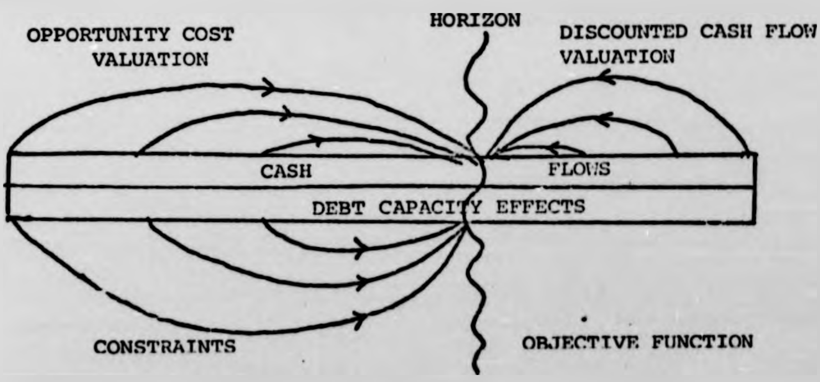
He suggests that at the horizon the net value of post horizon cash flows (NPVH)

"takes no account of any prospects for reinvesting some or part of the capital at more than the marginal rate of return. In fact managers would normally expect to be able to invest substantial sums after the horizon at better than marginal rates, and this expectation would normally be shared by shareholders. It would seem to follow that NPVH understates the true value of funds available after the horizon."

Chambers recognises that the interactive nature of post horizon decisions may affect the opportunity cost of cash flows and hence the valuation.

The investment valuation method implicit with both Weingartner and Chambers models can be represented by Figure 1.4.1.

FIGURE 1.4.1.



In order to avoid problems associated with the Baumol and Quandt paradox the horizon is used artificially to separate out the constraint set and the valuation flows. The reduced cost associated with a project decision produced by a conventional linear programming analysis is a generalised net present value which is equal to the post-horizon cash flow contribution less the use of capital and debt capacity valued at their opportunity rates in the pre-horizon period. It should be noted that in both models the post horizon cash flow valuation is approximated by using a pre-determined average discount rate and the debt capacity effects are totally ignored. It can be seen that neither of these models can satisfy the fundamental horizon principle. The implications and limitations of such models will be discussed more extensively in chapter four, while the next section will introduce the idea of using a mathematical programming framework for the evaluation of a particular financing instrument - a financial lease.

1.5 A mathematical programming framework for Lease* evaluation.

A financial lease is a noncancellable contractual commitment on the part of the lessee to make a series of payments to a lessor for the use of an asset. The lessee acquires most of the economic benefits resulting from the use of the asset though the lessor retains title to it. The payments made by the lessee to the lessor are such as to reimburse the lessor for the assets and the financing costs associated with the assets, plus any administration costs and to give him a return on his financial investment. Hence the decision to lease a piece of equipment is at one and the same time the decision to acquire that same piece of equipment. The contractual nature of

* In the ensuing discussion it is assumed that a lease refers to a financial lease rather than an operating lease.

a lease repayment schedule means that the firm is undertaking a form of debt financing while simulaneously it is acquiring an asset which will alter the future cash revenues patterns of the organisation. Thus by its very nature the lease contract is a prime example of an investment and financing instrument.

It would appear that the most suitable method for the evaluation is to include it within a mathematical programming model of the firm in which all the available investment and financing opportunities are considered simultaneously. While such an approach obviously offers a mechanism for integrating the lease decision into a formal planning system, the analytical framework afforded by mathematical programming theory can make a major contribution to the development of appropriate valuation formulae. The requisitive analysis is carried out in Chapter Five. In this section the relevant background and survey of some of the approaches suggested in the financial literature will be discussed.

The initial work of Vancil(63) was followed by a lull but more recently the attention of academics has refocussed on the lease-buy problem as is evidenced by a spate* of papers purporting to solve the lease-buy decisions. This revival in interest in the evaluation of financial leases would appear to stem in part from its increasing prominence in the planned financing structure of U.K. firms. As Fawthrop and Terry (76) point out:

> "The growing prominence in the U.K. capital market is made clear by a recent estimate from the Equipment Leasing Association which suggests that the industry now provides equipment with an initial cost of approximately £1,000 million."

* Of the 75 articles cited by Terry (76) the majority of these have been published between 1973-1976.

While the numerous writing of academics has resulted in little consensus as to the correct method of analysis of a lease. A common but by no means universally accepted approach is to compare the merits of lease financing with that of debt financing via a discounted present value method of the cash flows resulting from these alternatives.

This gives rise to two particular measures of the cost of a lease which will prove of great value in our analysis. They are the interest rate on the lease and the after tax cost of the lease. The interest rate implied in a lease is just that rate of interest which when applied to the outstanding capital on the lease is such that the lease repayments meet both capital and interest. In order to make precise this definition and to facilitate the subsequent discussions it is convenient to write down the algebraic expressions for the lease-buy decision from the point of view of the lessee, using the following notations:

P_t = Lease payment at the end of year t (t=1,2,H)

b_t = Tax allowance on the assets during year t (t=1,2,H)

I_t = Interest payment on debt at end of year t (t=1,h)

R_t = Repayment of principal at the end of year (t=1,H)

r = Debt interest rate

A_o = Cost of asset

w_t = Debt outstanding at the end of t

T = Marginal tax rate on corporate net income

H = Length of the lease contract

Hence we have the lease interest rate i_L defined by the equation

$$A_o = \sum_{t=1}^{H} \frac{Pt}{(1+i_L)t} \qquad (1.5.5)$$

and the after tax cost of the lease r_L defined by the equation

$$A_o = \sum_{t=1}^{H} \frac{P_t(1-T) + b_t T}{(1+r_L)^t} \qquad (1.5.6)$$

In general, academics* tend to reject such measures as internal rates of return in favour of net present value methods, though in this particular case under the most rigorous analysis the former measure provides a very good decision parameter.

Mao's analysis(69) exemplifies the more usual net present value approach. The discounted cost of a lease financing is:

$$= \sum_{t=1}^{H} \frac{P_t(1-T)}{(1+K)^t} \qquad (1.5.7)$$

while the corresponding cost of debt financing is

$$\sum_{t=1}^{H} \frac{P_t - (I_t+b_t)T}{(1+K)^t} + w_o \qquad (1.5.8)$$

In the first expression only the lease payments are allowable against tax while in the second expression both depreciation charges and interest charges are allowable against tax. Hence from this analysis it can be seen that the value of the lease-buy decision is:

$$\sum_{t=1}^{H} \frac{P_t(1-T)}{(1+K)^t} - \sum_{t=1}^{H} \frac{P_t - (I_t+b_t)T}{(1+K)^t} - w_o \qquad (1.5.9)$$

So far nothing has been said about the appropriate discount rate K to use and this remains the centre of much of the controversy about lease analysis.

Mao suggests that K is the firms marginal investment return; an assumption which would imply that the lease is being considered under some state of capital rationing. The use of the marginal

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* See Van Horne (77) p. 88.

investment return as the appropriate discount rate is subject to much dispute.* Other writers such as Vancil adopt an average cost of capital discount rate. Vancil, recognising that other sources of money are available, argues that it is desirable to eliminate the differences in the amounts of financing when comparing specific proposals. Since leasing provides more financing than debt the company will have more fixed charges under the lease plan than under the debt plan. These higher fixed charges may prompt investors to discount earnings (or dividends) at different rates. Vancil's (61) approach is to compare leasing with borrowing only after the difference in the amounts of funds provided have been removed. At a particular time t, of a lease repayment P_t, rw_t represents the imputed interest expense while the remaining $P_t - rw_t$ represents repayment of the principal. In order to remove the difference in the amount of financing provided by leasing and borrowing the Basic Interest approach focuses on the tax savings associated with the non-interest portion of the lease payments. Hence the cost of leasing under this approach is given by the difference between the price of the assets and the present value of the tax savings associated with the non-interest portion of the lease payments. This is given by the expression:

$$A_o = \sum_{t=1}^{H} \frac{(P_t - r\,w_t)}{(1+K)^t} \qquad (1.5.10)$$

For the purpose of comparison, the present value of the alternative which is that of debt financing is just given by:

$$A_o = \sum_{t=1}^{H} \frac{w_t T}{(1+K)^t} \qquad (1.5.11)$$

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* See Bower (73)

In this case the cost of interest charges on debt financing have been eliminated already and do not appear in the expression. Leasing here is viewed as an alternative to debt. One of the difficulties of such an approach is that of comparing difrering amounts of debt financing and loan repayment schedules.

Using a variation of Vancil's algorithm, Bower, Herringer and Williamson (66) specifically tackle this problem by assuming that the loan payment schedule is the same configuration as the lease repayment schedule to 'wash out' this difference. The remaining details of their approach is of less interest to this brief survey than their choice of discount rate - both Vancil and Bower, Herringer and Williamson chose the weighted average cost of capital.

It can be argued that conceptually it is wrong to use the cost of capital in making decisions between methods of financing. The cash flows under consideration are contractually fixed or are associated with tax savings and involve very little risk. It thus seems erroneous to use a cost of capital, which emobodies a risk premium for the firm as a whole. The counter-arguments of Vancil and BHW is that investors and creditors, in their valuation of the firm, recognise the difference in tax savings between the two methods. Because both investors and creditors determine the overall cost of capital the average cost of capital is the appropriate rate. A cynic might well remark that the debt rate is avoided because discounting at a debt rate would in general cause leasing to be sufficiently unattractive and that neither of these algorithms would yield results which would explain its popularity. The use of the average cost of capital gives rise to one further problem. Where there is significant portion of lease finance, which will be usually more expensive than debt finance, then this fact ought to be reflected in the cost of capital rate. Thus the discount rate used in the above algorithms is dependent upon the decision to lease. Such an interdependency would appear to be insolvable, at least within the current framework.

A lease clearly alters the pattern of future cash flows available for reinvestment purposes. All the approaches discussed so far concentrate on the lease as a financing instrument and make no attempt to analyse the investment consequences of the lease decision. Fawthrop and Terry (76) attempt to redress this omission by introducing the concept of residual balances. Their argument is that the cash inflows, net of tax and dividend payments, associated with the lease decision become a primary source of finance in the undertaking of further capital expenditure.

Any evaluation of a lease should attribute to the lease the value of this additional capital. The resulting analysis separates the cash flows associated with a lease into component cash flows and the resultant expression for the value of a lease takes the form:

PV (Lease) = PV (Net of tax operating cash flows)
+PV (Lease interest payments)
+PV (Repayments of Lease capital)
+PV (Earnings on Residual Capital Balances)

where PV stands for the present value, evaluated at the weighted average cost of capital. In common with Vancil the interest cost component of the lease is separated out. The significant difference between this expression and the other expressions is the inclusion of revenue flows in the evaluation of the lease, via the earnings on the residual capital balances.

The residual capital balances need further explanation. The authors define these as:

"The residual amount of capital outstanding (on the lease) after successive cumulative repayments have been made".

They argue that these balances represent funds which can be reinvested so that they earn the average return on assets enjoyed by the firm. This return is assumed to be at a rate above the cost of capital of the firm and as such the assumption is tacitly made that the firm is operating in a capital rationing situation. It is interesting to compare this last approach, in which the investment alternatives are elucidated and valued at the marginal reinvestment rate before discounting at the weighted average cost of capital, with the first approach by Mao in which the financing flows are elucidated and valued at the debt rate before discounting at the marginal reinvestment rate. Thus the emphasis has shifted from the lease-buy option as a financing decision to that of an investment decision, while the intervening discussion concentrated on the differing amounts of debt available under the alternatives.

In summary, the debate on lease evaluation centres on two key issues.

The first of these is the appropriate discount rate to use in the evaluation. Clearly the lease involves an investment decision which implies the use of funds at the appropriate reinvestment rates. It is also a financing decision which because of its riskless nature is very similar to a debt instrument and suggests discounting at a debt rate.

The second major issue is the impact that a lease may have on the debt capacity of the firm. Since it has been argued that a lease is an alternative form of debt it will presumably affect the perceived capital structure of the firm. This change in capital structure should be reflected in any cost of capital used.

Both of these problems would seem intractable within the current framework.

The advantage of a mathematical programming framework is in its ability to cope with these issues. Within such a framework the appropriate discount rate is determined by the decision set and the debt displacement is reflected in the debt capacity constraint.

In chapter five a generalized expression which clearly defines the relative roles of the various discount rates and the debt capacity effects will be developed. The strength of this expression is in its ability to ensure a logical consistency between sets of assumptions about the nature of the capital markets and the resulting valuation formulae. Hence it is relatively easy to explore alternative beliefs about the operation of the capital markets. It will be seen that under the most rigorous assumptions of perfect capital markets leasing is an unattractive proposition. While as imperfections, in either the capital markets or accounting measures of debt, are introduced into the assumptions then situations in which leasing would be an attractive proposition can be discerned.

1.6 Towards a practical planning system.

As was indicated in the introduction the survey work of Grinyer (72) and Higgins and Finn (77) in the United Kingdom and that of Gershefski (70) and Naylor and Schauland (76) in the States has shown that while there exists many corporate financial models very few are of the mathematical programming type.*

*Grinyer found only one optimising model out of fifty models in his survey while Gershefski suggests that 95% of the models he surveyed were of the simulation type - a result confirmed in the later survey of Naylor & Schauland

The reasons for this soon emerge if we examine current ideas on the nature of the objectives and of the planning process within an organisation and contrast these with the structure of the objectives and planning process implicit in the two types of financial models.

The objective function normally chosen in most corporate financial mathematical programming models found in the literature is the maximisation of the value of the firm. This valuation criterion is in accordance with traditional economic thinking which assumes that the objective of the firm is the maximisation of the long run profits. However, the inadequacies of classical economic theories in accounting for the behaviour of the firm has led to a series of revisions of the concept of the firm as a profit maximiser.

One of the first major revisions was by Baumal(59) whose observations led him to conclude that firms do not devote all their energies to maximising profits but rather that,as long as a satisfactory level of profits is attained,a company will seek to maximise its sales revenue. The importance of this hypothesis is that the firm is no longer working towards a single objective but must balance two competing and not necessarily consistent goals. Baumol's idea is still primarily a description of the behaviour of the firm in the market place.

A more comprehensive and directly challenging attack on the economic theory of the firm arises from the work of organisational theorists. H.A. Simon(57) argues very persuasively that the omniscient rationality attributed to economic man bears little resemblance to reality. A more accurate description of the behaviour of decision making within an organisation is that of a search for satisfactory solutions. In this model of behaviour the objective function becomes a two valued utility function: good enough or not good enough.

While most of the models that we have already discussed appear in part to incorporate these ideas by the inclusion of policy constraints such as a minimum level of return on capital. Simons' (64) interpretation of these constraints is somewhat different. In his view decisions are not directed towards a single goal but with discovering courses of action that help to satisfy a whole series of constraints. It is these constraints that motivate the decision maker and guide his search. In this sense the constraints are more 'goal like' than binding limits on the possible actions. Any planning mechanism ought thus to aid the decision maker to find 'satisfactory' plans with respect to these constraints or goals rather than to maximise a single criterion and regard the constraints as inviolate.

The foregoing discussion provides a key for the understanding of the high degree of acceptability of simulation models. The characteristic feature of these simulation models is that they examine the consequence of a decision by producing a series of financial indicators. These indicators range from projected profit and loss statements, balance sheets sources and use of funds statements to merely a few financial ratios. Hence, by having an immediate analysis of the consequence of any decision, the decision maker can search rapidly through a series of alternative plans hopefully to arrive at a satisfactory solution. Hence, the computer is merely performing, albeit many times faster, analysis traditionally carried by the accountant. Although their high degree of managerial acceptability may well stem from this emulation of traditional accounting methodologies it imposes a severe limitation upon their power. In particular they are unable to provide much guidance in searches for alternative and possibly better solutions. Thus if a particular plan is unacceptable it is left to the user to input another series of decisions in the hope that this will improve the general level of performance. While it is true that certain models do incorporate decision rules*. These rules are usually simple pre-emptive lists such that if a particular restriction is not satisfied in a period then the restriction is overcome by searching through a pre-ordered list of alternatives. A more sophisticated variation of this is the method of backward iteration (Grinyer and Woller(75)) when previous decisions can be altered to overcome a restriction in a particular time period. Though again this can be seen as a limited search through a pre-ordered list.

In contrast mathematical programming is a very powerful tool. Its main limitation is that before the search is commenced it is necessary to specify a minimum set of conditions which any plan must satisfy together with a single measure of the value of this plan. This prior specification of minimum conditions and a single criterion introduces an unfamiliar and, possibly unacceptable, rigidity into the planning system.

A further contrast between financial simulation models and mathematical programming models is in the nature and quantity of the information flows between the model and the user.

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*See, for example, Chambers, Singhai, Taylor and Wright (71).

Financial simulation models are characterised by requiring decision inputs from the user and output the information in the form of the consequent impact on the value of selected financial policy variables (e.g. return on capital, earnings per share). In linear programming models the information input is merely the data relating to the benefits and costs of various alternatives and the plan is output in the form of a set of decisions. In this case the impact on financial policy variables has to be determined separately. Hence, as currently used mathematical programming models search through decision space for a plan which maximizes a scalar measure of company performance whereas simulation models are used to search, even though that search is unstructured, over a vector of policy variables.

It is the contention of this section of the thesis that the acceptability of simulation models stems largely from their ability to provide an interactive search mechanism over a vector of policy variables. It is thus the aim of the final Chapter of this thesis to illustrate one method whereby mathematical programming algorithms may be used to enhance this search.

The remainder of this section concentrates on the approaches proposed so far in the literature in order to understand why they have failed to provide a viable alternative to either simulation or LP models.

The work of Simon (57 and 64) Cyert and March (63) in developing a behavioural theory of the firm finds its recognition in operational research methodology in the recent development of multi-criteria methods. These methods accept the multicriteria nature of many planning systemsand attempt to explore the various alternatives in a systematic fashion. Although this approach at first sight would appear to provide the appropriate planning mechanism a closer examination of the two mainstreams of research in this area indicate quite daunting implications for the management user.

The first of these approaches originates from the early work of Charnes, Cooper and Ijiri (63) in goal programming. In this approach the objective function usually takes the form of a weighted linear combination of deviations from a set of goals. While their formulation is intuitively appealing, its rather simplistic structure can give rise to anomalies caused by solution instabilities*. Another major difficulty is the

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* A particularly apposite example is the case of attempting to maximise profits in each of two years where total profits are limited to a fixed quantity. If the problem is formulated as

$$\min \ (1+\varepsilon)\, z_1 + z_2$$

$$\text{s.t.} \quad p_1 + z_1 \geq 1$$

$$p_2 + z_2 \geq 1$$

$$p_1 + p_2 \leq 1$$

where p_1, p_2 denote profits in each of the two consecutive years and z_1, z_2 are shortfalls from target. The ratio $1 + \varepsilon : 1$ expresses a preference for profits in year one over year two. Then the solution if $p = (0,1)$ for a positive value of ε and $p = (1,0)$ for a negative value. Thus an infinitesimal charge in the weights can completely alter the form of the solution. While this example may seem trivial and unlikely to occur in practice the reverse appears to be true.

specification of a trade-off function between conflicting goals. This difficulty is compounded in the case of financial planning models because the goals are usually ratios introducing a non-linearity into the problem.

The importance of ratios is fairly clear from the extent to which they are discussed in standard texts on financial analysis*. In addition there have been various publications which give ratio norms for various industrial categories. Although there is a plethora of ratios and their definitions vary widely (Perrin(66)) certain key ratios can be identified as particularly significant in corporate financial planning. Obvious examples are measures of profitability such as return on capital, earnings per share, measurement of debt levels such as gearing and times covered together measures of growth of sales and profit.

The idea of incorporating financial ratios into mathematical programming methods is not new. Chambers(67) in his paper 'Programming the allocation of funds subject to restrictions on reported results' concludes:

> "It became evident in discussions of the first aspect - the effect on published results - that at least in the short run, managers were using several overlapping but distinct criteria to measure the firms' performance and the success of capital budgeting. On the other hand, they did not question the fundamental importance of cash flows which a project could be expected to generate. On the other hand, they were unwilling altogether to neglect the

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*See Lev(74), Van Horne (77)

> changes which the project would bring about in other parts of the published accounts, derived on the basis not of cash flows but of accruals. They regard the accounting convention of assigning costs and revenues to the periods judged to give rise to them as defining rules of a game in which they wanted a good score."

However, in his particular model these ratios were hard constraints and could not be violated. A more appropriate model according to the organisational theorists would be one where constraints were not hard and could be broken if it seemed beneficial. While goal programming certainly affords such a structure, the quantification of constraint violations is a fundamental problem associated with the weights used in goal programming. These weights are the relative value that the decision maker attaches to deviation from one criterion as opposed to another and the difficulty of attaching sensible values to these weights in any realistic planning model has led many authors to abandon goal programming formulations for financial models. Such an attitude is characterised by Carleton, Dick and Downes(73).

> "If the objective function in a goal programme has more than one argument, absolute priorities have to be imposed arbitrarily. Consequently, nonachievability of all the goals, when such is the programme solutions, leaves unanswered the important economic question of how objectives trade off against one another. In other words, finance theory, even applied gently, has something to contribute to management's undertaking of how financial policy requirements fit together. And goal programming is a substantially less powerful tool than linear programming for accomplishing this."

It would appear that if an operationally viable search tool is to be developed goal programming as it currently stands falls some way short.

The second mainstream of multi-objective research is the development of algorithms for the generation of efficient solutions. A solution is said to be efficient if the performance on a particular criterion can only be improved to the detriment of the performance on some other criterion.* Clearly the decision maker need only consider efficient solutions in his search for the most acceptable one. For linearly independent criteria Benayoun and Tergny(70) have shown that these efficient solutions are situated on the boundary of the feasible region. If the efficient solution lies at a vertex, it is referred to as an extreme efficient solution, otherwise it is referred to as a non-extreme efficient solution. Every multi-criteria LP problem has only a finite number of extreme efficient solutions but an infinite number of non-extreme solutions. Non-extreme efficient solutions can be expressed as convex combinations of extreme efficient solutions, but not all such combinations yield non-extreme efficient solutions.†

While a fairly comprehensive survey of algorithms for the determination of sets of efficient solutions can be found in

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* Mathematically, if $\gamma_i(x)$, $i \in I$ denotes the criteria on which decision x is judged. Then solution x is efficient if and only if there is no other solution y such than

$$\gamma_i(y) \geqslant \gamma_i(x) \quad \forall_i$$

and

$$\gamma_i(y) \geqslant \gamma_i(x) \quad \text{for some } i \in I$$

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† See Yu and Zeleny (73) for a further discussion of this.

Thanassouilis (76) the basic limitation of the approach is self evident on consideration of the details of just two such algorithms. This limitation is a natural consequence of the fact that efficiency of solutions is a very weak form of comparison, leaving a large number of solutions to be considered before the final compromise solution can be selected. For example an algorithm which has been proposed by Yu and Zeleny (73) centres on the determination of all non-dominated faces. Although strictly speaking such an approach should not be termed an algorithm, since it offers no guidance to the determination of a final solution even given that the 'best' face has been determined, a more disturbing feature is the computational implications of the approach. Thus the method essentially requires consideration of some 2^{m+n} systems of equations where m is the number of constraints specifying the feasible region and n the number of structural variables. Since the problem posed for solution in the last chapter consists of some 77 structural variables and 48 constraints, this method is seen as computationally infeasible.

Another algorithm, which has been proposed by Evans and Steuer (75) involves the determination of extreme efficient solutions. Briefly the method relies on the connectedness* of the efficient vertices and generates the complete series of efficient vertices by moving from vertex to vertes. A check for efficiency of vertex needs to be carried out at each stage and this itself requires the solution of a linear program. Again, such an algorithm proves computationally

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*Connectedness in this context means that a neighbouring efficient vertex can be obtained from the current efficient vertex in only one simplex iteration.

prohibitive* for most realistically sized problems.

It would seem that on the one hand goal programming methods confront the decision maker with a non determinable prior specification of trade-offs while the algorithmic searches of efficient solutions present the decision maker with a superabundance of alternatives. Thus,until the informational inputs required of the decision maker in goal programming can be reduced,or,until the algorithmic approach can be modified to produce appropriate and order subsets of possible efficient solutions,neither method can be considered as practical.

In chapter four a utility framework for goal programming is examined. This framework provides a powerful and insightful mechanism for the development of the tools necessary for carrying out an interactive search of the set of efficient solutions. In the next section a realistically sized planning problem is proposed to provide the discipline of a precise contextual setting for a thorough test of these search procedures. It will be seen while a natural strategy evolves the essence of the method developed is in its flexibility of response to the decision makers preferences. In this way, a model is developed which may in the end begin to bridge the gap between mathematical programming models and simulation models.

1.7 A financial planning model.

The central theme of this thesis is the nature, impact and

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* Steuer reports that a sample of 25 constraints, 50 variable problems with three objective functions had an average of 605 extreme efficient vertices and required 152 seconds of CPU time on an IBM 370/165 computer. However, the time required appears to increase exponentially with the size of the problem and he was unable to obtain complete sets of solutions in any reasonable time to problems much larger than this.

resolution of the interdependencies that arise between the investment and the financing decision. In previous sections various aspects of these interdependencies have been introduced, though most of the subsequent discussion of necessity has centred around fairly simple models. Thus in section 1.3 it was suggested that models which consist only of a cash constraint and a debt capacity constraint may be 'solved' by a relatively straight forward application of discounting principles, though it is certainly far from clear how such discounting approaches might behave in more complex models. Section 1.4 introduced some of the problems that arose out of interdependencies between the form of the valuation model, the financing options and the constraint set. In particular it concentrated upon the effect of a finite horizon time. The extent to which this poses a problem in practice for large scale planning models remains unknown. A similar question emerges in the theory of lease valuation. While analytical methods suffice for the development of valuation formulae in most of the models mentioned so far, such methods have proved inadequate when it comes to dealing with more elaborate models. This is of course a major weakness in the analysis since the leasing decision appears to be a result of a complex interaction of tax laws and debt availability determined largely by reporting standards. Finally the work of the last section suggests that the firm operates in an environment where its courses of action are constrained by consideration of the impact that they might have on a whole multiplicity of criteria. An exploration of this idea requires a model rich in detail but much less rigid in structure than the conventional linear programming models hitherto discussed.

Unfortunately, many of the models which have been used to illustrate the various aspects of the above problems are relatively trivial in nature and fail to provide adequate test material. In order to provide for a more comprehensive examination of the ideas developed in this thesis a realistically sized* programming model of the firm was developed.

The model was developed in two distinct forms. The first of these follows the traditional economic valuation approach where the objective is the maximisation of shareholder wealth. In this model all the constraints are hard constraints in that a plan is infeasible unless it simultaneously satisfies all the constraints. The same data and basic structure is also used to generate a parallel version which takes the form of a 'goal' programming model. In this model all the financial restrictions or constraints are 'soft' constraints and hence it is possible that all or any of these restrictions may not be met in an acceptable plan.

The model provides a central test bed for the computational evaluation of the main ideas of this thesis and despite its size and complexity it plays a contributing rather than leading role in this thesis. In order to emphasise the nature of this role and avoid breaking up the theoretical arguments, a detailed statement of the model is reserved for the appendices with a discussion of the structure of the objective function in the appropriate chapters. A short summary of the main features of the model should suffice at this point, while a detailed mathematical statement can be found in appendix I.

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*The final form of the model had over 360 variables with over 180 constraining and defining equations excluding simple bounds.

As already stated, the model is a linear programming representation of the investment opportunities over time facing an organisation together with corresponding a set of financing alternatives. Briefly, it contains four groups of variables representing accounting quantities, financing and investment opportunities and variables associated with goals and targets. The accounting variables have been chosen at a level of detail that gives sufficient richness for the purpose in hand without an excessive amount of detail. Hence, while current assets are included at an aggregate level, capital assets are grouped into two categories to allow for different tax treatments. Also overdraft, dividends and tax payable are identified as separate elements composing the short term liabilities because of their importance as financing elements. For the same reason long term capital and shareholders capital are separately identified also. The model has two main groups of constraints.

(a) A technological set consisting of the cash balance equations and accounting definitions.

(b) A financial policy set associated with the performances on certain key financial criteria such as return on capital, times interest covered, earnings and dividend per share.

Apart from the financing alternatives the firm is faced with a series of decisions to be made about investments in projects. There are 16 different projects in all, though since some of these projects are available in more than one year there are in fact up to 45 projects available over the eight year planning period. The projects are specified in terms of their contribution to sales, earnings, current assets and liabilities together with a statement of their capital requirements in both building and land and plant and equipment. The internal rate of return of these projects varies between 7.5% and 15.5%. A complete summary of the projects occurs in appendix IV.

It is further assumed that the organisation at the start has already a series of on-going operations and future financing commitments such as planned long term debt requirements. Apart from these projections resulting from its current operations, the firm has a series of policy targets, for instance a minimum return dividend payout and capital in each year, and sales targets which it hopes to achieve over the planning period. A statement of these targets together with the other base data appears in appendix III. Also contained in appendix III is a statement of the taxation allowances which the organisation may claim and details of the assumptions made about the timing of the cash inflows and outflows during a year.

It will be seen that the model in itself is not original, indeed it would be difficult to generate a model which is completely different from all the many other models produced in this field. Clearly, the antecendents in the literature on whose ideas the model is based can be found in the pioneering work of Weingartner (62), the work of Chambers (67,71) on the incorporation of financial constraints and equity issues, the share price valuation approach of Carleton(70) and the complexity and output procedures by Hamilton and Moses (73). The model is little more than a synthesis and extension of the features considered best in these models. Any unique nature lies in the use and emphasis of the model and the structure of the objective functions necessary for goal programming.

CHAPTER 2.

Interdependencies, Hirschleifer, Baumol and Quandt.

2.1 Introduction.

In this chapter the nature of the interdependencies that arise between the set of investment decisions and the discount rate at the optimum in capital budgeting models is examined in detail. As was indicated in section 1.2 Baumol and Quandt (65) suggested that the dual values gave the correct discount rate or opportunity cost of funds to use in the formulation of the problem. The subsequent attempt to solve the problem reformulated in this way led them to suggest that there was no solution other than the null solution. The following section shows how it is possible by paying particular attention to the assumptions and by careful definition of the mathematical variables to cite numerical counter examples with non-trivial solutions in which the discount rate is consistent with the dual value. Section 2.3 then provides a formal mathematical treatment of the problem in which it is shown that in general there exists many consistent solutions and the numerical example is merely one of a particular subset of these solutions. Section 2.4 identifies the economic meaning of these solutions and the implications for discount methodology by relating the solutions to the fundamental paper by Hirschleifer (58) on the theory of optimal investment decisions. It is argued in conclusion that this paper forms a basis for the development of mathematical programming approaches to the capital investment problem.

2.2 A Respecification and Numerical Counter-Examples.

The Baumol and Quandt model is, as already stated:

$$\text{Max } B = \sum_j \sum_t c_{jt}(\rho_t/\rho_o)\ x_j \qquad 2.2.1$$

$$\text{s.t.} \qquad -\sum c_{jt}x_j \le F_t \qquad t=0,1,2....T \qquad 2.2.2$$

The factor ρ_t/ρ_o which is the discount rate is arrived at by the following argument. If we were indifferent to either £100 now or £110 in one year's time, it would imply that we were discounting funds at 10%. In general indifference between an amount S_o now and S_1 in time period 1 where $S_o = KS_1$, implies a discount rate of K.

Thus briefly, Baumol and Quandt argue that within the mathematical programming framework the value in year zero of an additional S_o pounds is $S_o \frac{\partial Z}{\partial F_o}$ since each pound will add $\frac{\partial F}{\partial F_o}$ to the capitalised present value of the earning stream, where Z denotes the discounted value of the firms earnings and F_o is the budget constraint in year zero. This indifference between S_o in year zero and S_1 in year 1 implies a discount factor applying between year 0 and year 1 of $\frac{\partial Z}{\partial F_1} / \frac{\partial Z}{\partial F_o}$ since S_o in year zero adds $S_o \frac{\partial Z}{\partial F_o}$ to the discounted value of the earnings stream and S_1 in year 1 adds $S_1 \frac{\partial Z}{\partial F_1}$ to the discounted value of the earning stream. Now $\frac{\partial Z}{\partial F_t}$ is equal to the dual price (denoted by ρ_t) corresponding to the t-th constraint. Thus writing D_t as the corresponding (one period) discount factor we have $D_t = \rho_t/\rho_{t-1}$ and the present value of S_t (discounting for all t periods up to the present) as

$$S_o = D_1D_2....D_tS_t = (\rho_t/\rho_o)\ S_t \qquad 2.2.3$$

Thus the discounting factor for funds in period t is ρ_t/ρ_o.

As already stated, Baumol and Quandt form the dual of their model and conclude that the only solution to the problem is the trivial one. However, their particular form of the model has some rather strange assumptions and by modifying and clarifying these assumptions the model takes on a form which has a non-trivial solution.

The three main modifications that need to be made to the Baumol and Quandt model are:

(i) An upper limit needs to be placed on the amount that can be invested in any one project. This is rather more realistic than Baumol and Quandt's projects because even if a particular project was unbounded it is unlikely to have a linear return to scale. The imposition of upper bounds allows a piece-wise linear approximation to the returns to the project to be made. It is in fact a generalisation or extension of the model rather than an additional restriction. The other point about this restriction is that many of the conceptual ideas behind this formulation are contained in Hirschleifer's paper on the theory of optimal investment and in this paper he introduced the idea of ranking projects to enable the generation of a production function with diminishing returns to scale. While one could generalise or rather restrict the arguments to infinite linear projects it is mathematically of much less interest.

It should perhaps be noted that under this modification Baumol and Quandt's conclusion no longer follows.

Thus the dual of the formulation is:

$$v_j - \sum_t c_{jt}\rho_t \geq 1/\rho_o \sum_t a_{jt}\rho_t \quad 2.2.4.$$

where v_j is the dual associated with $o \leq x_j \leq 1$.

Then $(-1 - 1/\rho_o) \sum_t c_{jt}\rho_t \geq - v_j$ 2.2.5.

and this no longer implies that $\sum_j c_{jt}\rho_t \leq 0$ 2.2.6.

for all j, a necessary condition in Baumol and Quandt's proof.

(ii) Another clarification which is necessary is that there exists a 'market' mechanism for carrying money from one period to the next. It is thus convenient to make the simplest assumption that there exists an unbounded project with the cash flow characteristics of -1 in period t and 1 + i in period t + 1 for all t. This is perfectly general, provided that if necessary i may be zero. The original Baumol and Quandt model provides no explicit mechanism for carrying forward money from one period to the next and they fail to clarify the position of any surplus funds.

(iii) If we adopt the same arguments for relating the duals to the discount rates as Baumol and Quandt, in that indifference between

$$s_{t-1} \frac{\partial z}{\partial F_{t-1}} \quad \text{and} \quad s_t \frac{\partial z}{\partial F_t}$$

effectively determines the discount rate. The only difference is the specific problem of when one of the duals vanishes. If $\frac{\partial z}{\partial F_t} = 0$ then it does not mean that £1 in year t is worthless since at least in the proposed model the pound can be loaned to the money market at an interest i until required nor does a $\frac{\partial z}{\partial F_{t-1}} = 0$ imply that the prevailing one period rate is infinite. To avoid this problem of dividing by zero we can use the equivalent form in the model that $\rho_t = u_t\rho_o$ where u_t is the discount factor as defined above.

The reformulated model is :

$$\text{Max } Z = \sum_{t}\sum_{j\epsilon J} c_{jt}\, u_t\, x_j \qquad 2.2.7.$$

$$\text{s.t.} \quad -\sum_{j\epsilon J} c_{jt}\, x_j \leqslant F_t \qquad t=0,1,\ldots.T \qquad 2.2.8.$$

$$0 \leqslant x_j \leqslant 1 \qquad j\epsilon J' \qquad 2.2.9.$$

where J refers to the total investment opportunity set and J' is the production subset.

In addition the constraint relating the duals to the discount rates are written $\rho_t = u_t\rho_o$ $t=1,\ldots.T$ and where $u_o = 1$.

Before attempting to solve the general problem it would seem prudent to look at particular examples in order to gain some insight into the structure of any solutions.

The examples are so framed that the optimal investment schedule would appear to be fairly obvious. The rationale behind this intuitive example is then examined in order to relate it to the previous analysis.

TABLE 2.2.1 Net Cash Inflows from Investments.

Project \ Time	t=0	t=1	t=2	Upper Bound
x_1	-1	1.1		
x_2	-1	2		1
x_3	-1	0	2.5	1
x_4		-1	2	1
x_5		-2	3	1
x_6		-1	1.1	
Budget	½	1	0	

. Consider the investment opportunity set in Table 2.2.1 where

(a) X_1 and X_6 represents investment in finance markets of 10%. These projects are assumed unbounded.

(b) u_1 and u_2 denote the 1 and 2 period discount rates.

Then the objective function is

$$\text{Max} = (-1 + 1.1u_1)X_1 + (-1 + 2u_1)X_2 + (11 + 2.5u_2)X_3 + (-u_1 + 2u_2)X_4 + (-2u_1 + 3u_2)X_5 + (-u_1 + 1.1u_2)X_6 \quad 2.2.10.$$

subject to

$$X_1 + X_2 + X_3 \leq \tfrac{1}{2} \quad 2.2.11.$$

$$X_4 + 2\,X_5 + X_6 - 2\,X_2 - 1.1\,X_1 \leq 1 \quad 2.2.12.$$

$$-\,2.5\,X_3 - 2\,X_4 - 3\,X_5 - 1.1\,X_1 \leq 0 \quad 2.2.13.$$

$$0 \leq X_2,\ X_3,\ X_4,\ X_5 \leq 1 \quad 2.2.14.$$

If we look at the first year investment opportunities then clearly X_2 is superior to X_1 and a combination of X_2 with either X_4 or X_5, provided one has not already exhausted these projects, is superior to X_3. Thus it would seem the rational investment is to accept X_2 at scale ½, which exhausts the budget. This leaves 2 available for investment in period 1, being the 1 from the budget and ½ × 2 being the return in period 1 from X_2. Again it would seem that the rational investment schedule is to take X_4 at full scale and X_5 at scale ½ which exhausts the budget. Thus the optimal solution would appear to be

$$X_2 = \tfrac{1}{2},\ X_4 = 1,\ X_5 = \tfrac{1}{2} \text{ with } X_1 = X_3 = X_6 = 0. \quad 2.2.15.$$

What implications has this for the discount rates? Returning to the original arguments of Baumol and Quandt presumably the investor would be indifferent between δ_0 in year zero or $2\delta_0$ in year 1 since an additional δ_0 in year zero could be invested in project 2 to give in year 1. Thus it would appear that the appropriate discount factor is ½ i.e. $u_1 = \tfrac{1}{2}$. In year 1 the investor is presumably indifferent between an extra δ_1 in year 1 or an extra $3/2\delta_1$ in year 2 since the best available opportunity is that of X_5 where for each 2 units of investments in year 1, 3 units are returned in year 2. Thus the appropriate discount factor between 1 and 2 is 2/3 and $u_2 = 1/3$ ($= 2/3 \times u_1$).

An obvious but fairly important point is the way in which the discount rate is determined by the marginally rejected projects. There are several other features to note.

TABLE 2.2.2 The cash flows associated with the optimal investment schedule.

Projects	t=0	t=1	t=2
X_2	-½	+1	
X_4		-1	2
X_5		-1	1.5
Totals	-½	-1	3.5
Discount rates	1	½	1/3
Discount values	-½	-½	1.1/6

∴ Total N.P.V. is 0.167

If we look at the individual projects that constitute the optimum solution then we find that the N.P.V. of X_4, which is the only project accepted in full, is positive while the N.P.V. of X_2 and X_5, which are the partially accepted projects, is zero and the N.P.V. of the rejected projects X_1, X_3, X_6 is of course negative. Thus the discount rates as determined sort out the projects into the fully accepted, partially accepted and totally rejected projects which groupings then satisfy the budget constraints.

The discount rates can be more formally related to the duals by examining the effects of small increases in cash to the formulated linear programming problem, when the optimal solution is assumed to be $X_1 = X_5 = \frac{1}{2}$ and $X_4 = 1$. Then an extra δ_o in year 1 increases the objective function by an amount $\rho_o \delta_o$. If an extra δ_o were available it would alter the cash flow pattern since presumably it would be invested in X_2 to increase the objective function value by an amount $(-1 + 2\,u_1)\delta_o$. In addition the extra $2\delta_o$ then made available in year 1 would then be invested in X_5 to yield an extra $3\delta_o$ in year 2 and make a net increase in the objective function value of $(-2\,u_1 + 3\,u_2)\delta_o$.

We can thus write $\rho_o\delta_o = (-1 + 2u_1)\delta_o + (-2u_1 + 3u_2)\delta_o$. 2.2.16.

A similar argument for an extra δ_1 available in year 1 gives

$$\rho_1\delta_1 = \tfrac{1}{2}\,(-2u_1 + 3u_2)\delta_1 \qquad 2.2.17.$$

Since in year 2 there are effectively no more investment opportunities facing the firm

$$\rho_2 = 0$$

We have also assumed that $\rho_2 = u_2\rho_o$ and that $\rho_1 = u_1\rho_o$.

These five equations have the unique solution

$$u_1 = 1/2 \quad u_2 = 1/3 \qquad \rho_2 = \rho_1 = \rho_o = 0.$$

If we substitute for u_1 and u_2 in the objective function then indeed we confirm that

$$Z = -0.45\,X_1 + 0\,X_2 - 0.167\,X_3 + 0.167\,X_4 + 0\,X_5 - 0.3\,X_6 \qquad 2.2.18.$$

subject to the same constraints as before has the solution that the objective maximum is 0.167 which occurs when $X_4 = 1$ and the associated duals of each of the constraints is zero. This point is not surprising since the discount rate is determined by the marginally rejected projects X_2 and X_5 which thus have zero N.P.V. and extra funds would merely be available for investment in these projects and would contribute nothing to the objective function. These results could well be expected to hold for all cases and the following example provides further evidence of this.

Consider the effect of project X_3 having the following cash flow pattern. -1,1,2.5., and the budget constraint in year 1 being reduced to 0, while being increased to 1 unit in year 0. The complete investment opportunities are as in Table 2.2.3.

TABLE 2.2.3

Time / Project	t=0	t=1	t=2	Upper Bound
X_1	-1	1.1		
X_2	-1	2		1
X_3	-1	1	2.5	1
X_4		-1	2	1
X_5		-2	3	1
X_6		-1	1.1	
Budget	1	0	1	

If we choose to invest in project 2 then we could re-invest the 2 units made available in year 1 in projects X_4 and X_5 to give a total of 3.5 in year 2. If we undertook project X_3 in year 0 then the 1 unit made available for re-investment in year 1 could be invested in X_4 to give a combined cash flow of 4.5 in year 2. The two investment schedules give resulting cash flows of -1, 0, 3.5 and -1, 0, 4.5, the latter being preferable, assuming re-investment, to other alternatives and clearly then the optimal solution appears to be $X_3 = 1$ $X_4 = 1$ with $X_1 = X_4 = X_5 = X_6 = 0$. What discount rates do these projects imply? If we formulate our L.P. model and carry out the procedure outlined previously then the following results apply:

$$\text{Max } Z = (-1 + 1.1u_1)X_1 + (-1 + 2u_1)X_2 + (-1 + u_1 + 2.5u_2)X_3$$

$$+ (-u_1 + 2u_2)X_4 + (-2u_1 + 3u_2)X_5 + (-u_1 + 1.1u_2)X_6 \quad 2.2.19.$$

subject to

$$X_1 + X_2 + X_3 \leq 1 \quad 2.2.20.$$

$$X_4 + 2X_5 + X_6 - X_3 - 2X_2 - 1.1X_1 \leq 1 \quad 2.2.21.$$

$$-2.5X_3 - 2X_4 - 3X_5 - 1.1X_4 \leq 0 \quad 2.2.22.$$

$$0 \leq X_2, X_3, X_4, X_5 \leq 1 \quad 2.2.23.$$

Again consider the effect of an additional δ_o, δ_1 in years 1 and 2, where $\delta_o, \delta_1 \geq 0$.

Then

$$\rho_o\delta_o = (-1 + 2u_1)\delta_o + (-2u_1 + 3u_2)\delta_o \quad 2.2.24.$$

i.e. $$\rho_o = (-1 + 2u_1) + (-2u_1 + 3u_2) \quad 2.2.25$$

and $$\rho_1 = \tfrac{1}{2}(-2u_1 + 3u_2) \quad 2.2.26.$$

$$\rho_2 = 0 \quad 2.2.27.$$

with $$\rho_2 = u_2\rho_o, \rho_1 = u_1\rho \quad 2.2.28.$$

The solution is

$$u_1 = 1/2, u_2 = 1/3, \rho_o = \rho_1 = \rho_2 = 0 \quad 2.2.29.$$

If we discount the project cash flows at these rates then the N.P.V. of projects X_1 and X_6 is negative while the N.P.V. of projects X_3 and X_4 is positive and the N.P.V. of X_2 and X_5 are zero. However, in this case we have integer solutions and in effect we have no marginally rejected projects. Now previously the discount rates were determined by the marginally rejected projects. If we examine the argument more closely we see that in evaluating the duals the additional assumption was made that δ_o, δ_1 were positive. If one were to ask the question what is the value of K such that one would be indifferent between paying out δ_1 in year one or $K\delta_1$ in year 2 the answer, again assuming S_1 is positive would not be a value of $K = 2/3$ since a reduction of S_1

in the budget availability in year one would reduce the amount of money available in year 2 by $2\delta_1$. In the first year the problem is even more complicated since a reduction in the current budget of δ_o reduces the amount of money available in year 1 by δ_o and in year 2 by $2\delta_o$. Again if we relate these to the duals the appropriate discount rates are:

with $\rho_o = (-1 + u_1 + 2.5u_2) + (-u_1 + 2u_2)$ 2.2.30.

$\rho_1 = -u_1 + 2u_2$ 2.2.31.

$\rho_2 = 0 \quad \rho_2 = u_2\rho_o, \rho_1 = u_1\rho_o$ 2.2.32.

The solution in this case is $u_1 = 4/9, u_2 = 2/9$. Discounting the projects at these rates then projects X_1, X_2, X_5, X_6 are negative while X_3 and X_4 are zero. There are of course two other solutions; these are associated with either relaxing the constraint at zero while tightening the constraint in period one, or alternatively, tightening the constraint at zero while relaxing the constraint in 1. Thus as a generalisation where the linear programming model results in a solution where the accepted projects in a particular period have integral values then the interperiod discount rate between that period and the following has two values depending on whether one is considering increments or decreases to the budgets constraints.

2.3 The Mathematical Theory*

Having considered some examples it is now appropriate to draw together the mathematical theory. There are really two cases to consider depending on whether funds can be 'carried forward' or not. For the sake of completeness both cases will be considered.

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*This analysis was developed by Atkins in the paper by Ashton and Atkins (76), it is included here for completeness.

Using the same notation as above the problem is of finding u, p and x, such that

$$\text{Max } \pi = \sum_j \sum_t u_t c_{jt} x_j = u'Cx \qquad 2.3.1.$$

$$\text{and} \quad -Cx \leq F \qquad 2.3.2.$$

$$x_j \leq 1 \quad \text{all } j \qquad 2.3.3.$$

$$x_j \geq 0 \quad u_t \geq 0, \quad \text{and } p_t \geq 0 \text{ for all } j \text{ and } t \qquad 2.3.4.$$

and ρ_t is the dual of budget constraint F_t

$$\text{and} \quad \rho_t = u_t \rho_o \qquad 2.3.5.$$

A solution (x,u) to this will be termed a *consistent* solution. That is, an investment schedule along with a set of discount rates that are in the correct relationship to the value of marginal budget changes and which together maximise the present value with respect to those discount rates is a *consistent* solution.

Taking the general case first, if we can find x, u, ρ, v, w such that the following equations are satisfied, then by Kuhn-Tucker theory (x,u) is consistent.

$$\sum_t c_{jt} u_t + \sum_t c_{jt} \rho_t - v_j + w_j = 0 \quad \text{all } j \qquad 2.3.6.$$

$$\rho_t (F_t + \sum_j c_{jt} x_j) = 0 \quad \text{all } t \qquad 2.3.7.$$

$$v_j \ (1 - x_j) = 0 \quad w_j x_j = 0 \quad \text{all } j \qquad 2.3.8.$$

$$\rho_t \geq 0 \quad \text{etc.} \qquad 2.3.9.$$

$$\rho_t = u_t \rho_o \qquad 2.3.10.$$

$$-Cx \leq F \text{ and } x_j \leq 1 \quad \text{all } j \qquad 2.3.11.$$

Simplifying and using our knowledge that we would expect ρ_t to be zero we have:

$$(1 + p_o)\ C'u - Iv + Iw = 0 \qquad 2.3.12.$$

$$F + Cx \geq 0 \qquad 2.3.13.$$

$$v_j(1-x_j) = 0 \qquad w_j x_j = 0 \qquad 1 \geq x_j \geq 0 \qquad 2.3.14.$$

Putting $y_j = 1-x_j$ for convenience, we know that any zero solution to the problem

$$\text{Min} \quad y' . v + x' . w \qquad 2.3.15.$$

such that $(1 + \rho_o)\, C'u - Iv + Iw = 0$ 2.3.16.

$$F + Cx > 0 \qquad 2.3.17.$$

$$x_j + y_j = 1 \qquad x_j, y_j \geq 0 \text{ etc.} \qquad 2.3.18.$$

is also a solution to the above, and vice-versa.

Lemma: The minimum of the quadratic $p . q$ where each set of variables satisfy some linear equations $Cp \leq c$ and $Dq \leq d$ occurs at a point p^*, q^* which are vertices of their respective convex regions. Proof is trivial, e.g. write each as a linear combination of their vertices.

Thus in order to ensure that all consistent solutions have been found it is only necessary to inspect the vertices.

As an example consider the simple case of two projects over three years.

TABLE 2.3

		Time			Upper Bound
		0	1	2	1
project	1	-1	1	1	1
	2	-1	0	2.5	1
	M_t	1.5	0	0	

In this case spare cash in time period 1 is lost as it cannot be used. Problem

$$\max\ (-x_1 - x_2) + u_1(x_1) + u_2(x_1 + 2.5x_2) \qquad 2.3.19.$$

$$x_1 + x_2 \leq 1.5 \qquad 2.3.20.$$

$$y_1 + x_1 = 1 \qquad 2.3.21.$$

$$y_2 + x_2 = 1 \qquad 2.3.22.$$

(for simplicity we have set $u_o = 1$ and $\rho_o = 0$)

the extra optimality conditions are

$$-1 + u_1 + u_2 - v_1 + w_1 = 0 \qquad 2.3.23.$$

$$-1 + 2.5u_2 - v_2 + w_2 = 0 \qquad 2.3.24.$$

and we wish to minimise $z = x_1w_1 + x_2w_2 + y_1v_1 + y_2v_2$ 2.3.25.

The vertices are shown in figures 2.41 below and each combination investigated in table 2.4.2.

Figures 2.3.1

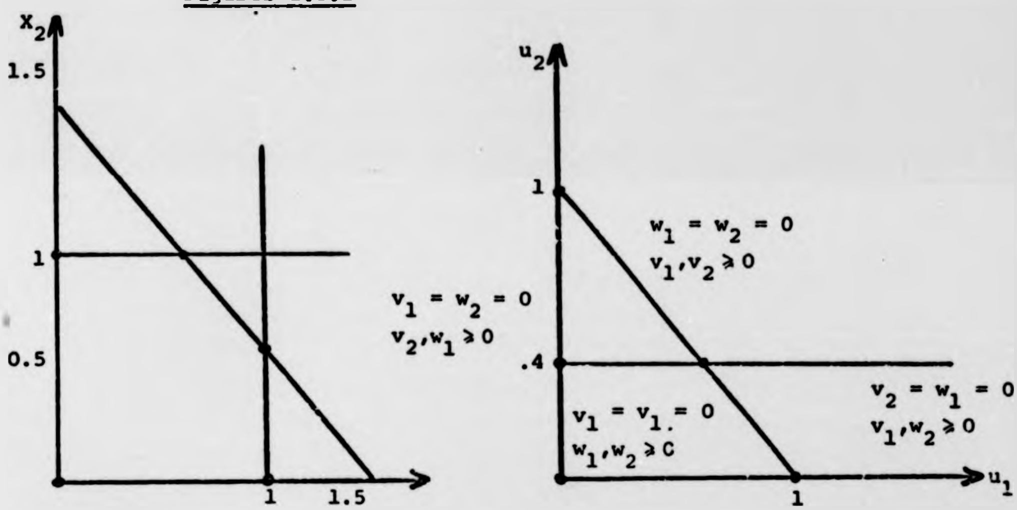

Table 2.3.2. The Values of z.

x	y		(w,v) (u) (1,1,0,0) (0,0)	(0,1,0,0) (1,0)	(0,0,0,0) (.6,.4)	(0,0,0,1.5) (0,1)	(.6,0,0,0) (0,.4)
(0,0)	(1,1)		0	0	0	1.5	0
(1,0)	(0,1)		1	0	0	1.5	.6
(1,½)	(0,½)	*	1.5	.5	0	.75	.6
(½,1)	(½,0)	*	1.5	1	0	0	.3
(0,1)	(1,0)		1	1	0	0	0

Thus we see that there exist many consistent solutions in general, remembering that the relevant linear combinations of the above are also consistent. Even if only those schedules are considered which exhaust the initial budget (marked *) there are three consistent solutions. To be complete the condition $u_o = 1$ ought to be released and u_2 set equal to 1 (say) so that solutions with $u_o = 0$ can be generated. There is thus also a straightforward way of generating such consistent solutions if required. A dual vertex is chosen e.g. $u = (1,0,1)$ and the relevant L.P. e.g.

$$\text{Max } 1.(-x_1-x_2) + 0(x_1) + 1(x_1 + 2.5x_2) = 1.5x_2 \qquad 2.3.26.$$

$$\text{s.t.} \quad x_1 + x_2 \leq 1.5 \qquad x_1 + y_1 = 1 \qquad x_2 + y_2 = 1 \qquad 2.3.27.$$

is solved to give consistent solutions as above. This existence of consistent solutions is not guaranteed of course for each dual vertex as is shown in the example above when $u = (1,1,0)$ and the projects must be chosen to exhaust the first year budget.

If the practically more interesting case when excess finds can be 'carried forward' at a minimum market interest rate of i (>0) is considered, the use of net present value criteria in general assume the existence of such financial opportunities, so it would seem reasonable to include them initially as part of the project set. We thus have a new project associated with each year with cash flows of -1 and 1+i in succeeding years. This implies that the budgets are entirely used in each except the last period and the objective function becomes

$$\max. \; u_T \sum_j a_{jT} x_j + \sum_{t=0}^{T-1} u_t M_t \qquad 2.3.27.$$

Apart from the added constant this is very similar to the horizon value or in this case just terminal cash problem

$$\text{Max} \sum_j c_{jT} x_j \qquad 2.3.29.$$

and the two solutions are identical apart from the duals differing by a fixed proportion. As the horizon value problem has a unique solution, apart from alternative neighbouring optima, it can be used to find consistent solutions of the Baumol and Quandt model with project bounds added. Thus in the 'carryover' case, not only can consistent solutions exist, but also can be found by the solution of a single horizon value maximisation linear programme. This theory can be illustrated by adding to the simple example two further 'carry forward' projects with i = 0. The data is now as shown in Table 2.3.3.

TABLE 2.3.3.

		Time			Upper bound
		0	1	2	
project	1	-1	1	0	
	2	-1	1	1	1
	3	-1	0	2.5	1
	4	0	-1	1	
Budget M_t		1.5	0	0	

The problem is

$$\max \; u_2 (x_2 + 2.5x_3 + x_4) \qquad 2.3.30.$$

such that

$$x_1 + x_2 + x_3 \leq 1.5 \qquad 2.3.31.$$

$$-x_1 - x_2 + x_4 \leq 0 \qquad 2.3.32.$$

$$x_2 \leq 1 \quad x_3 \leq 1 \qquad 2.3.33.$$

The solution to this is

$$x_1 = 0 \; x_2 = .5 \; x_3 = 1 \; x_4 = .5 \qquad 2.3.34.$$

with $u_o = 2u_T$ and $u_1 = u_2$ e.g. $u = (1,.5,.5)$ 2.3.35.

If the previous analysis of enumerating all vertices was undertaken, it would be seen that with five primal vertices and seven dual vertices only one of the corresponding thirty-five combinations was consistent. The use of this value of u in calculating the net present value of projects predicts correctly which projects would or would not be undertaken. They are also the dual values of the budget constraints.

As important point must be noted with regard to this analysis. The dual equations, $C'U - IV + IW = 0 \quad U, V, W \geq 0$, strictly define an unbounded cone, all the equations passing through the origin. This means that the only dual vertex is the origin. This certainly is a solution in general just as Baumol and Quandt claim in their paper, but in order to span the dual space rays are needed as cone generators, and it is these latter that have provided the solutions.

2.4. An Economic Interpretation.

It is now worthwhile recapping the main ideas and seeing what conclusions can be drawn. The starting point was the same objective as Baumol and Quandt, that of attempting to find a solution to the problem of maximizing the net present value of the projects we accept, subject to budget constraints, when the discount rate is determined by the dual evaluators. The first point to make is that as soon as we impose upper bounds to project investments and rewrite the relationship between the duals and the discount rates in the form $\rho_t = u_t \rho_o$ then the Baumol and Quandt analysis breaks down. In fact the logic breaks down even without the extension to include upper bounds. Thus Baumol and Quandt used the fact that the dual equation

$$\sum_t c_{jt} \rho_t \leq 0 \qquad 2.4.1.$$

being also the coefficient of x_j in the objective function would force $x_j = 0$ and hence obtain the trivial solution. But it has been argued that because of what the model is trying to do one would expect that partially accepted projects would have

$$\sum_t c_{jt} u_t = 0 \qquad 2.4.2.$$

and as an unbounded project will always be partially accepted if at all then x_j equal to anywhere from zero to infinity would also be a solution, and hence solutions other than the trivial one exist. As has been shown it is possible to find solutions to such formulations which satisfy the above conditions. Such solutions have been called consistent solutions and it has been proved that these solutions lie at the vertices of the project space. Where there are no specific projects for carrying cash forward from one period to the next if has been found that there may be several quite different alternative solutions. In the cases where there are carry forward projects then the problem can simply be reduced to the problem of maximizing the horizon value* which will in general have a unique linear programming solution.

It is interesting to note that the set of discount rates generated in this last case, which is of course the most frequently occurring in practice, removes some of the problems surrounding the re-investment assumption in discounting techniques, (see for example Fawthrop (71)), since the re-investment assumption is stated explicitly and the future discount rates automatically reflect the re-investment assumptions. It is also worth noting at this stage another property of this set of discount rates. If we find the net present value of each of our projects at this set of discount rates, then our decision rule is quite simple. We reject projects with a negative net present value and accept those with a positive net present value. Such a decision rule will automatically

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* See Freeland and Rosenblatt (77) for a general proof of this result.

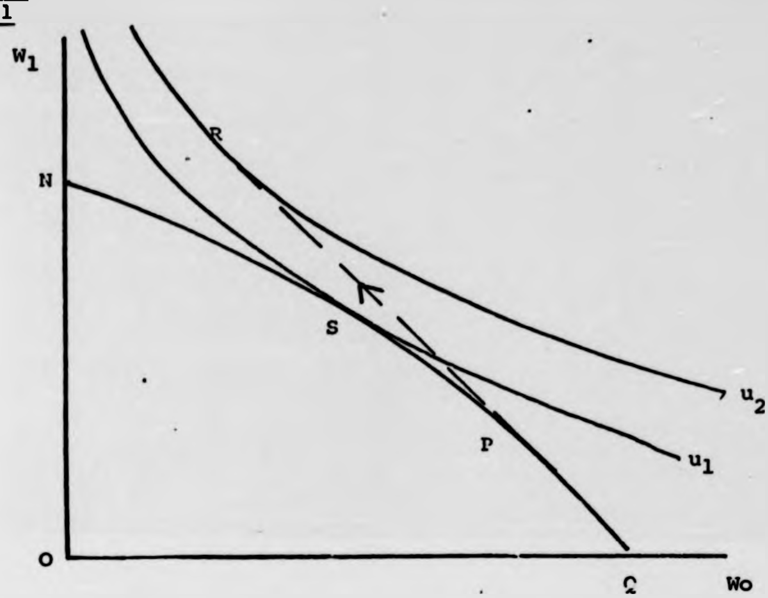
satisfy our budget constraints and maximise our net present value. It should be added that this set of discount rates causes the dual evaluators to be zero, since the interperiod discount rates are determined by indifference to small increments in the budget constraints at the optimum. Thus the zero of the dual evaluators would seem to be an inherent feature of the model.

While these properties of our discount rates are all very satisfying as regards their internal consistency it does not prove the validity of the model when judged by external criteria and the implications of the findings as regards the specification of a theoretically correct objective function have yet to be discussed. Much of the theoretical underpinning of these models rests on Hirschleifer's (58) original analysis.

If we return to this analysis we find that he was concerned with decision rules which maximised utility of consumption and among the rules he considered were the net present value criterion and the internal rate of return criterion. His methodology was to use an isoquant framework to develop a theoretical understanding of the problem and it is worthwhile repeating here some of that analysis. Initially two particular cases will be cited, one in which the optimum is achieved by a mixture of investment in production opportunities followed by investment in capital markets,and the other in which the optimum is achieved by a combination of investment in production opportunities coupled with borrowing from the capital market.

Figure 2.4.1. illustrates the first of these cases

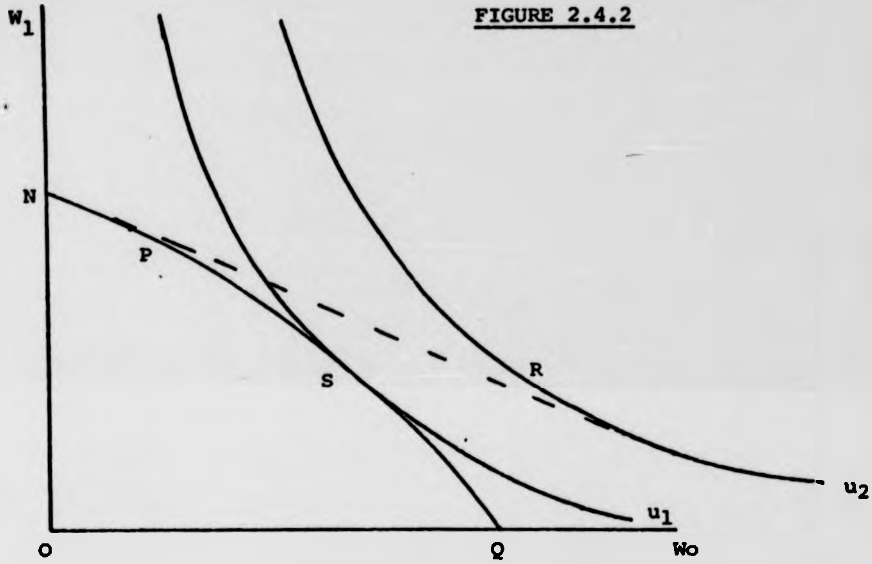
FIGURE
2.4.1



The axis W_o, W_1 represent the amount of income available for consumption in time period 0 and time period 1. Income available for consumption at time period 0 may be transformed into income available for consumption in time period 1 by investing in the production opportunities Q P S N. The dashed line represents the market line and it is assumed that funds can be borrowed or lent at a constant interest rate i - the slope of the market line is -(1+i).U_1, U_2 represent increasing utilities of W_0, W_1. In the absence of market opportunities the decision would be starting with initial income OQ at time now to invest in productive opportunities upto the point S, when the utility of (W_o, W_1) would be maximised. In the presence of market opportunities then the decision would be to invest in production upto point P and then to lend to the market to point R when a position on the utility isoquant U_2 which is higher than U_1 could be achieved.

In the second case illustrated by Figure 2.4.2. the decision in the absence of market opportunities would be invested in production opportunities upto level QS. The availability of the market line enables production to be carried out until P followed by borrowing from the market along PR enabling R to be reached which is on isoquant U_2.

FIGURE 2.4.2

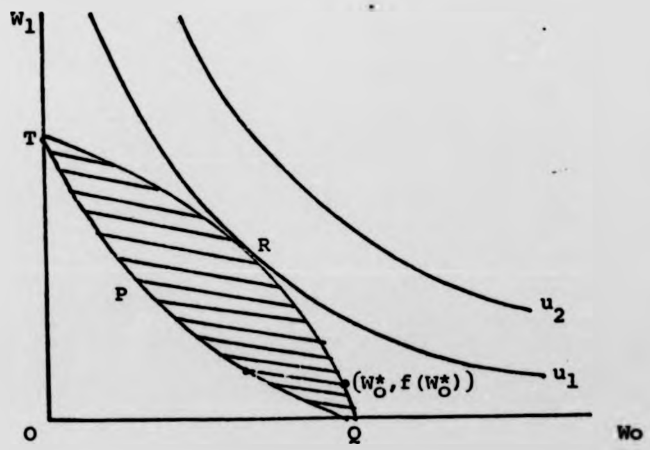


It should be noted that in order to define a suitable production opportunity set the projects are ranked according to diminishing returns to scale. The criterion is the net increase in period 1 for unit sacrifice now. Mathematically it can be represented by $\Delta W_1/(-\Delta W_o)-1$. At the optimum the slope of the productive function $-\frac{\partial W_1}{\partial W_o}$ gives the marginal productivity of capital. One particular rule that Hirschleifer considers is that the firm should adopt all projects with a positive net present value at the market rate of interest. This is equivalent to choosing all projects such that $\Delta Wo + \Delta W_1/(1+i)$ is positive or equivalently $-\partial W_1/\partial Wo \geq 1+i$. In the two cases discussed so far such a rule would cause selection of all production opportunities PQ, though it would indicate nothing directly about capital market decisions.

Hence the rule of accepting all projects with a positive net present value at the market rate is a correct one in such circumstances. Such a rule it should also be noted maximises the net present value of the chosen project set. Indeed the criterion maximisation of the net present value of income from the investment would give also the correct production investment decision, since this involves maximisation of $Wo + W_1/(1+i)$ which is a series of isoquants parallel to the market line, though as we shall see this criterion is not the correct one in general. It does not give the correct solution where the firm does not have access to market opportunities or at least has only limited access.

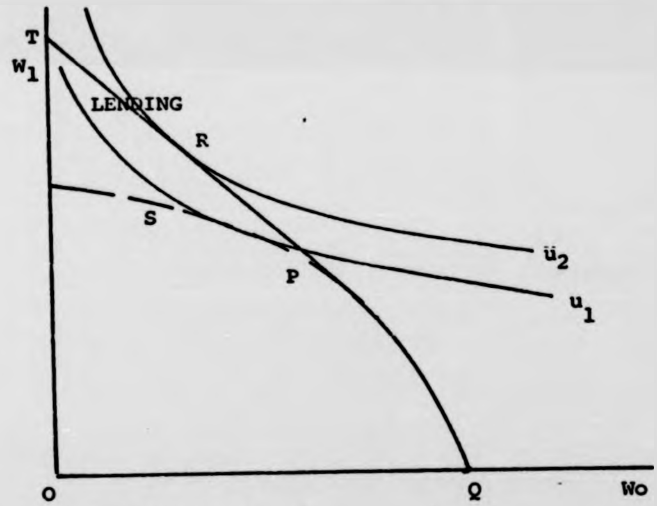
The particular case in which the firm does not have access to the capital is illustrated in Figure 2.4.3. shown below and it is convenient at this stage to relate these diagrams more directly to the mathematical programming approach.

Figure 2.4.3.



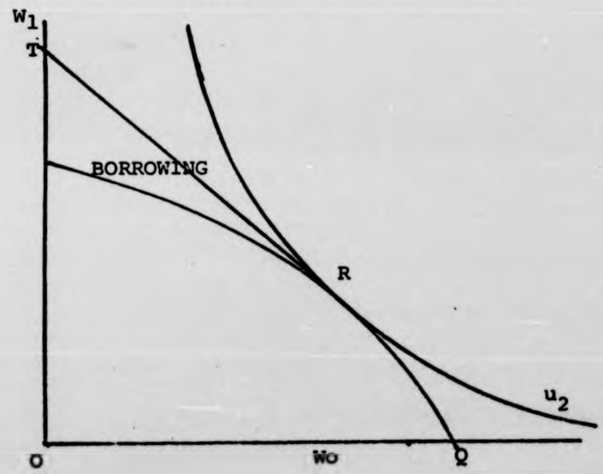
Hirschleifer's analysis indicates that the production set QR should be undertaken. A few preliminary remarks enables us to identify easily the correspondence between this analysis and the mathematical programming approaches to this problem. The first point to note is that the ranking of projects is merely a device for finding efficient boundaries. Thus if the projects were ranked according to decreasing returns to scale we get the curve QPT and different choices of projects give the various points within the feasible region TRPQ. If we allowed further the possibility of not requiring income to be invested then the set of feasible alternatives is the area in the positive quadrant defined by TRQO. The effect of introducing market opportunities is merely to alter the feasible regions. Thus Figure 2.4.1. can now be redrawn as Figure 2.4.4.

Figure 2.4.4.



and Figure 2.4.2. redrawn as Figure 2.4.5.

Figure 2.4.5.



In these last figures the same letters are used to refer to the corresponding points in Figures 2.4.1 and 2.4.2. Thus the straight portion (PT) of the efficient opportunity set represents capital market transactions. The slope of this line is of course just $-(1+i)$.

At the optimum the appropriate discount rate is given in Hirschleifer's case by $-\frac{dW_1}{dW_o}$. In the mathematical programming case the ratio of the duals is $\frac{\partial u}{\partial W_o} \Big/ \frac{\partial u}{\partial W_1}$ which is equal to $-\frac{dW_1}{dW_o}$. Thus there is a one to one correspondence between the mathematical programming formulations and Hirschleifer's isoquant analysis.

Such an observation provides us with a means for identifying the various resolutions of the paradox.

Myers (*op cit*) postulates that the external market imposes a well defined structure on the utility surfaces U_1, U_2. It causes the isoquants to take the form $Wo + W_1/(1+i)$. Under such conditions the maximisation of the present value of withdrawals is then the correct solution. Thus Myers assumes that although the firm does not have access to the market, the owners of the firm do. This approach effectively avoids the paradox since it still leaves unresolved the situation when the owners of the firm do not have access or at least only limited access to the capital market.

It should now begin to be clear how if we try to maximise the net present value of the opportunity set where the discount rate is the marginal productivity of capital then the net present value, in general, is positive and finite.

If we assume that the discount rate is determined by the marginal productivity of capital then at any particular point $(W_o^*, f(W_o^*))$

where $W_1 = f(W_o)$ defines the production function, the net present value of the adopted project set is

$$\int_{W_Q}^{W_o^*} dW_o + \frac{1}{f'(W_o^*)} \int_{W_Q}^{W_1^*} dW_1 . \qquad 2.4.1$$

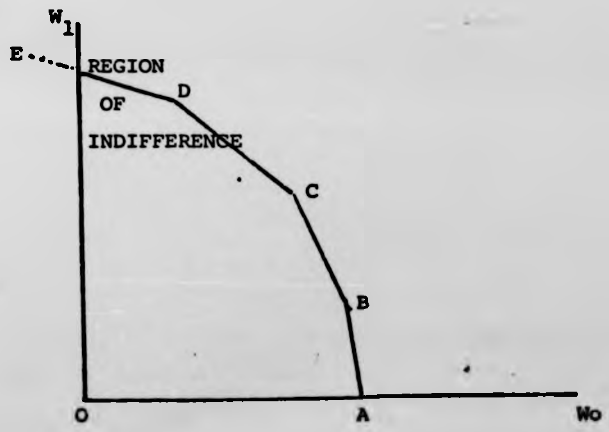
Hence W_Q is the potential income at time now. This expression becomes

$$\frac{f(W_o^*)}{f'(W_o^*)} - (W_Q - W_o^*) \qquad 2.4.2$$

For the strictly convex monotonically decreasing function that we have postulated in our analysis such a function has its maximum value at point T, where $W_o^* = 0$. At this point the magnitude of the slope or discount rate is smallest and the included project set is the largest. This is the solution that we have identified in which all available income is reinvested. In Figure 2.4.3. it corresponds with the adoption of all productive investments QT.

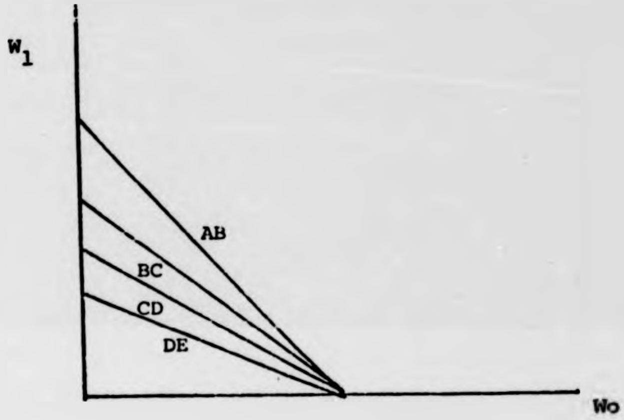
In the case where the production function is piece-wise linear the solution is not necessarily unique in that we may be indifferent to the scale of a project. Such a two-period solution is illustrated in Figure 2.4.6.

Figure 2.4.6.



Hence AB, BC; CD, DE represent projects, we are indifferent to the scale of project DE and the remaining projects when evaluated at the slope of DE make positive contributions to the net present value.

Figure 2.4.7



If Baumols and Quandt's original formulation of the two period case is considered in these terms (see Figure 2.4.7.), then since there are no scale constraints a particular project AB (say) would dominate all other projects. The discount rate would be determined by the slope AB and the net present value would be zero since we are indifferent to all points on AB - the line of zero net present value when discounted at the gradient of AB.

2.5 Conclusion.

In the end perhaps none of this analysis now seems very profound. In reformulating the Baumol and Quandt model we have defined a closed system whereby all cash generated in a period must be used in that period or carried forward to later periods. The only exception to this is the last period when the carry over mechanism does not apply. We can hardly expect such a model to make statements about our consumption preference since consumption is never an alternative that we provide to the model. Nevertheless such is the nature of the analysis that it defines clearly the various roles played by the productive and market investment, our utility function and the emergent discount rates We see that the appropriate investment criterion is not the maximization of net present value of the project set, but rather, that of finding the appropriate discount rate which are determined by the gradients at the points of tangency between the highest isoquant and the production-investment-financing opportunity set. Such a decision rule divides the project investment set into those which have positive present value, those which have negative present value and those which have zero net present value. The adoption then of all projects with a positive net present value will result in the highest isoquant being attained and while such a decision rule obviously maximises the net present value of the accepted set at that rate, the converse is patently not true. The maximisation of the net present value will not automatically generate discount rates which will lead us to operate so that our utility is maximised.

The foregoing discussion contains several important ideas which will be examined in some detail in later chapters. In chapter three, consideration will be given to methods of identifying the set of discount rates which correctly partitions projects into totally accepted, rejected and partially accepted subsets. It will be seen that frequently it is considerably easier to search for this set of discount rates first, and hence compute project acceptability, rather than to attempt to find the investment schedule directly. It is also clear from the discussion that if, as we presumably are, interested in the firm as a means of generating income for consumption in future periods, then we must be prepared to state explicitly our time preference for consumption. In chapter four an attempt is made to consider the impact of capital market opportunities on this preference function. It will be seen that the existence of capital markets largely enables the consumption decisions to be uncoupled from the investment decision, though the extent of the achieved independency between the investment and consumption decisions is determined by the degree of perfection assumed in capital markets. In order to facilitate this discussion it is necessary to examine explicitly the impact of uncertainty on the valuation of income streams by introducing parameters specifying the degree of uncertainty of these streams. While quite an elaborate normative framework for decision making can be constructed by the introduction of a single measure of the risk of an income stream, in practice, the capital markets estimate the size and risk of income streams by consideration of a whole series of indicators. The final chapter of the thesis shows how it is possible to develop an algorithm where the investment and financing decisions are made in a pareto optimal fashion with regard to this set of indicators.

In summary the Baumol and Quandt paradox appears to stem from a misconception of the nature of the net present value criterion. Nevertheless its resolution is an essential prerequisite to the discussion of the various models proposed in subsequent chapters of this thesis. Its resolution reassures us of the validity of the formulation, and the deductions made from, these models and an understanding of the paradox in terms of Hirschleifer's analysis provides us with a useful overview of some of the core issues facing mathematical programming in the development of models of the capital investment decision.

CHAPTER 3.

Discounting Methods and Rule of Thumb Solutions to the Capital Budgeting Problem.

3.1 Introduction

An appealing and potentially very powerful idea was identified in the last Chapter. If by some method we could discover the correct discount vector, then this vector would lead us immediately to the optimal investment schedule since it could be used to partition the project set into three categories consisting of accepted, rejected and marginal projects. Where the firm is operating in a perfect capital market under conditions of certainty then the prevailing market rate provides the single parameter necessary for the computation of this vector. In this case the rule project selection reduces to the familiar discounted net present value criterion at the market rate. In the more realistic case when assumptions of certainty in future operating income do not hold then restrictions are normally imposed on the amount of borrowing (or debt financing) that a firm may undertake. In such circumstances the discount vector is no longer simply related to a single market rate and it would seem necessary to employ some method for seeking out the appropriate vector. In mathematical programming formulations of the capital budgeting problem restrictions on the amount of debt financing that may be used are incorporated into the model in the form of explicit constraints and the search for a discount vector is nothing more than a search of the corresponding dual space.

If the only concern were the gaining of optimal solutions then the search of the dual space is usually no simpler than the direct determination of the investment schedule by the more normal search of the primal space and the foregoing observation is trivial. If however, a major concern in the appraisal of capital expenditure decisions is the generation of methods which can be used to filter or preselect projects for further scrutiny then the contrast between the primal and dual search is far from trivial. In fact a case will be argued that reasonably good and robust approximations or rules of thumb can be generated more easily, and their strengths and weaknesses can be analysed more readily, through the medium of the dual formulation than through the primal. In the models which will be investigated the success of the search over the dual feasible region rests on the existence of an exterior financial market which provides either sources of capital or investments for surplus funds. It will be seen that the dual equations associated with these market instruments confine the dual feasible region so that it is sufficiently 'small', with relatively well defined boundaries, that an optimum or near optimum can be found with a minimum of computational effort.

In this Chapter consideration will be given to numerical solutions to the capital budgeting problem which can be achieved by simple rules of thumb derived from an analysis of the dual space. These solutions will be compared and contrasted with the formal solutions of the corresponding primal linear programming problem. In particular three models will be discussed in some detail. These are the Weingartner (63) model, the Chambers (71) model and the model proposed in section 1.7 of this thesis.

The basic horizon model of Weingartner forms a natural starting point for such an analysis. Not only does it occupy a central place in the literature but it incorporates the same set of assumptions as conventional discounting methodologies; differing only in the introduction of an additional, though crucial, assumption, of a 'hard' constraint on capital availability. Because of this it has become a yardstick against which rules of thumb may be measured. In section 3.2 the dual analysis is carried out for the Weingartner model. This analysis leads to a natural ranking of the projects for each particular time period. It is these rankings that form the basis of the search procedure proposed and a framework for the analysis of other rules of thumb.

In the section following, the dual analysis is used as a framework for the examination of some of the other rules of thumb proposed in the literature. It is argued that while all are capable of giving the correct (optimal) solution under certain circumstances, none of the other methods can guarantee an optimal solution. However, it is further argued that the structure of the investment project set is such that most of these rules will give reasonably close approximates to the optimal solution.

In section 3.4, the method stemming from this dual analysis is applied to Weingartners basic horizon model. The particular problem chosen is the one employed by Weingartner to illustrate the use of linear programming for the optimal choice of projects subject to a hard rationing constraint. It is seen that the Weingartner problem does not really provide an adequate test of the method since its solution can be virtually determined immediately by inspection of the rankings generated. A more testing problem is proposed where there are many attractive projects competing for very limited funds. In all there are forty-five projects available spread over eight time periods where capital rationing occurs in five of these periods. Nevertheless the method is able to generate the optimal solution to this problem without too much difficulty.

The Chambers model (*op cit*) is a different order of complexity from the Weingartner basic horizon model. Its restriction on debt is related to the book value of the assets and would thus appear inextricably tied up with the investment decisions. Despite this the dual analysis in section 3.5 of the market instruments, although algebraically tedious, yields a particularly simple decision rule which enables the project set to be classified into the three basic categories discussed earlier. Moreover, it is seen that this analysis proves considerably more insightful into the structure and nature of the solution than the straightforward application of a conventional linear programming algorithm.

The model proposed in section 1.7 is of a different order of complexity again from the Chambers model. Not only does a times interest covered constraint more intimately link* the investment and the financing decision but there are in addition many other constraints on the investment and financing decisions. As one might anticipate the incorporation of these additional constraints prevents a rigorous analytical treatment of the dual structure. Nevertheless it will be seen that a fairly crude approximation still leads to an acceptable decision rule. The implications of these observations for possible future directions of work in mathematical programming formulations of the capital budgeting problem are examined in the concluding section.

3.2 <u>The Weingartner Model</u>

The basic horizon model of Weingartner with 'hard' constraints on the level of debt can be written as

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* See footnote page 18 of this thesis.

$$\text{Max} \sum_{j=1}^{N} \hat{c}_j x_j + v_T - w_T \qquad 3.2.1.$$

subject to

$$-\sum_j c_{1j}x_j + r_1 - w_1 \leq F_1$$

$$-\sum_j c_{tj}x_j - (1+r_L)v_{t-1} + v_t + (1+r_B)w_{t-1} - w_t \leq F_t \quad \text{for } t=2,\ldots,T$$

3.2.2.

$$w_t \leq B_t \quad \text{for } t=1,\ldots,T-1$$

3.2.3

$$0 \leq x_j \leq 1 \quad \text{all } j=1,\ldots,N \text{ and } v_t, w_t \geq \text{ for all } t \qquad 3.2.4$$

where x_j denotes the scale of acceptance of project j

c_{tj} is the cash inflow from project j in time period t

w_t, v_t denote borrowing and lending respectively in t

F_t is the cash flow available from existing 'old' projects

B_t is the upper limit on borrowing in period t

r_B, r_L are the borrowing and lending rate of interest respectively

and $\hat{c}_j = \sum_{t=T+1}^{\infty} \frac{c_{tj}}{(1+r_B)^{t-T}}$ is the post horizon value 3.2.5

The dual equations corresponding to lending and borrowing are:

$$\rho_t - (1+r_L)\rho_{t+1} \geq 0 \quad \text{for } t=1,\ldots,T-1 \qquad 3.2.6$$

$$-\rho_t + (1+r_B)\rho_{t+1} + \beta \geq 0 \qquad 3.2.7$$

$$\text{and} \quad \rho_T \geq 1 \qquad 3.2.8$$

$$-\rho_T \geq -1 \qquad 3.2.9$$

where ρ_t is the dual on the cash balance constraint and β_t is the dual on the borrowing constraint. Inequalities 3.2.6 and 3.2.7 give

$$(1+r_L)\rho_{t+1} \leq \rho_t \leq (1+r_B)\rho_{t+1} + \beta_t \qquad 3.2.10$$

If we consider first the slightly simpler case where borrowing and lending rates are both equal to the single rate r. Then inequality 3.2.10 implies

$$\rho_t = (1+r)\rho_{t+1} + \beta_t \quad t=1,\ldots,T-1 \text{ and } \rho_T=1 \qquad 3.2.11$$

or

$$\rho_t = (1+r)^{T-t} + \sum_{s=t}^{T-1} (1+r)^{s-t}\beta_s \qquad 3.2.12$$

The reduced cost associated with project j is thus

$$\hat{c}_j + \sum_{t=1}^{T} c_{tj}\rho_t = \hat{c}_j + \sum_{t=1}^{T} c_{tj}(1+r)^{T-t} + \sum_{s=1}^{T-1}\sum_{t=1}^{s} c_{tj}(1+r)^{s-t}\beta_s \qquad 3.2.13$$

and the decision rule is accept project j at full scale if the reduced cost is positive, reject if negative and partially accept when the reduced cost is zero. In the absence of capital budgeting constraints then $\beta_t = 0$ for all t, and the rule becomes the familiar net terminal value rule.

If the net terminal value of project j is denoted by

$$NTV_j = \hat{c}_j + \sum_{t=1}^{T} c_{tj}(1+r)^{T-t} \qquad 3.2.14$$

the discounted cost in time period t of expenditures to date on project j by

$$TV_j(t) = -\sum_{s=1}^{t} c_{sj}(1+r)^{t-s} \qquad 3.2.15$$

and the effective budget limit formed from the debt limit in that year plus accumulated funds from 'old' projects by

$$L_t = B_t + \sum_{s=1}^{t} F_s(1+r)^{t-s} \qquad 3.2.16$$

then the dual of the original horizon model can be written as

$$\text{MIN} \sum_j \mu_j + \sum_t L_t \beta_t \qquad 3.2.17$$

such that

$$\mu_j \geq NTV_j - \sum_{t=1}^{T-1} TV_j(t)\beta_t \quad \text{all } j \qquad 3.2.18$$

$$\mu_j \geq 0 \quad \text{all } j \quad \beta_t \geq 0 \quad \text{all } t \qquad 3.2.19$$

where μ_j are the duals on the $x_j \leq 1$ constraints.

Furthermore if a project is accepted then the right hand side of inequality 3.2.18 is positive. If the project is rejected the right hand side is negative. Whereas if the project is partially accepted then the right hand side is zero. Thus the problem of choosing the optimal project set can be reduced to one of finding the appropriate β-values. Once these β-values are known we can find those which will be accepted at their upper bounds, those that will be rejected, together with the partially accepted projects. A convenient way of looking at this is to consider the (hyper) planes in the β-space associated with each project defined by the equality

$$\sum_{t=1}^{T-1} TV_j(t)\beta_t = NTV_j \qquad 3.2.20$$

This can be illustrated in figure 3.2.1 for the two dimensional case by the simple example of the eight projects shown in Table 3.2.1. Thus project A requires cash outlays of £100 in year one, £50 in year two and £30 in year three. The horizon is coterminous with year three and the post horizon value of cash flows for project A is £246 at 10%. Hence for project A we have the linear function

$$40 - 100\beta_1 - 160\beta_2$$

The equation defined by equating this expression to zero defines a line in the β-space (figure 3.2.1). Projects G,F do not begin until period 2, hence their vertical plot, while project H has a negative net terminal value at 10% and can be rejected without further consideration*.

Table 3.2.1 A simple example: PROJECT DATA

Project	Capital Outlays Year 1	Year 2	Year 3	$\hat{c}_j$	TV(1)	TV(2)	NTV	IRR(%)
A	100	50	30	246	100	160	40	24
B	100	50	40	256	100	160	30	22
C	100	100	100	351	100	210	20	16
D	50	10	10	89	50	65	8	16
E	50	50	50	162	50	105	2	11
F	-	50	40	100	0	50	5	20
G	-	100	60	175	0	115	5	15
H	100	50	20	193	100	160	-3	9
D_i	180	100	100					
B_i	100	100	∞	all figures in £				
L_i	280	398	428					

The positive quadrant is divided into two regions by each project line, one region away from the origin where

$$NTV_j - \sum_{t=1}^{T-1} TV_j(t)\beta_t < 0 \qquad 3.2.22$$

representing rejection and the other region where

$$NTV_j - \sum_{t=1}^{T-1} TV_j(t)\beta_t > 0 \qquad 3.2.23$$

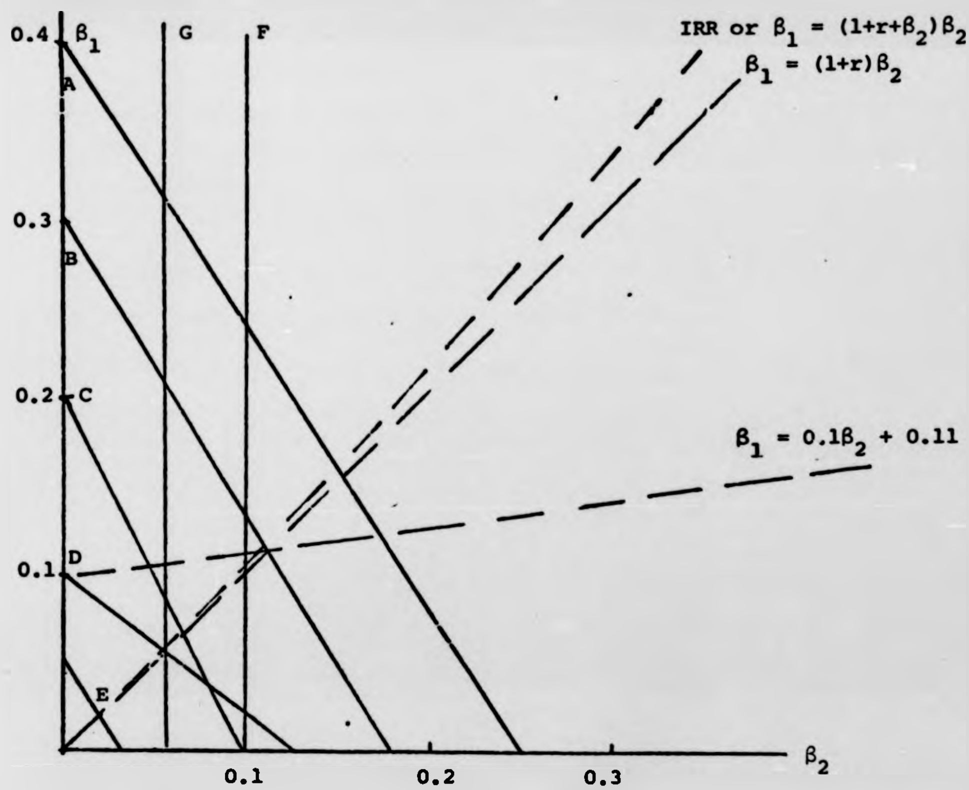
representing acceptance. Hence in general for any set of β-coordinates

---

* It is of course preferable to lend money to the capital market at 10% than to invest in project H, *ceteris paribus*:

lines passing to the left of that coordinate represent rejected projects and lines passing to the right of the coordinate represent accepted projects. It follows that any continuous monotonic non-decreasing function in the positive quadrant passing through the origin represents a ranking of the projects. As the origin in the β-space is approached along this curve the list of projects accepted at full scale increases.

FIGURE 3.2.1 The β-space for the projects in Table 3.2.1.



3.3 A Re-examination of Rule of Thumb Solutions to the Hard Rationing Problem†

The previous discussion provides us with the necessary framework for the rigorous examination of the various rules of thumb proposed in the literature.

Take for example the case where the only significant budget constraint is in the first year. This implies that all the duals β_t are zero apart from β_1 and the solution to the dual L.P is given by merely accepting projects in the ranked order of $\frac{NTV_j}{TV_j(1)}$ which is the familiar ratio of terminal value to initial outlay, the Lorie-Savage (55) solution*. In figure one this rank is generated by descending the β_1 axis.

On the other hand the ratio of discounted benefits to discounted benefits to discounted costs might be considered more appropriate for cash flows spread over several years**. This is equivalent to setting $\beta_1 = 0$ and $\beta_2 > 0$ in the example and can be achieved by the rank $\frac{NTV_j}{TV_j(2)}$ or equivalently by using the rank defined by descending the β_2 axis.

A third familiar rule of thumb is ranking projects by internal rate of return. This is equivalent to making another approximation to the dual, namely by putting $\beta_t = \beta_{t+1}(1+i) \quad t=1,2,\ldots,T-2$ 3.3.1

and

$$\beta_{T-1} = (i-r) \tag{3.3.2}$$

and using i as a parameter. In terms of figure 3.2.1. this is equivalent to ranking along the parameterised curve

---

* Bernhard (71) correctly analysed this ratio using a method of analysis similar to the one developed here, though he failed to extend his analysis to the case where the binding constraint was other than in the first year.

** See Quirin (67)

† The section is based on analysis carried by Atkins in the paper by Ashton and Atkins (74).

$$\beta_t = (i-r)(1+i)^{T-1-t} \quad t=1,\dots,T-1 \qquad 3.3.3$$

more simply for the two dimensional case under discussion

$$\beta_1 = \beta_2(1+r+\beta_2) \qquad 3.3.4$$

Another frequently suggested rule is to rank by some measure as discounted benefits/discounted costs, that is by $\frac{NTV_j}{TV_j(T-1)}$ and to calculate the IRR of the marginally accepted project. The suggestion is now to rerank projects again by NTV/TV(T-1) but using the internal rate of return of the marginally rejected project as the new discount rate, in this case r = 20% as the project is F. The idea behind this is that this rate is a better approximation to the 'true' opportunity cost of funds. The assumptions behind this idea were discussed in section 1.2. This is equivalent to a second approximation to the dual by making

$$\beta_{T-1} = \beta + (i-r) \qquad 3.3.6$$

$$\beta_{T-2} = (i-r)(\beta+1+i) \qquad 3.3.7$$

$$\beta_t = (1+i)\beta_{t+1} \quad \text{for } t=1,2,\dots,T-3 \qquad 3.3.8$$

or

$$\beta_t = (i-r)(\beta+1+i)(1+i)^{T-2-t} \quad \text{for } t=1,2,\dots,T-3 \qquad 3.3.9$$

where i is now a constant, the internal rate of return of the marginally rejected project, in this case 20% and β is the parameter.*

In the example i=0.2, r=0.1 and the reranking is equivalent to ranking along the line defined by $\beta_2 = \beta + 0.1$ 3.3.10

*The proof of this was first derived by Atkins in the paper by Ashton and Atkins (74). It is reproduced in appendix XIV.

$$\beta_1 = 0.1(\beta+1.2) \qquad 3.3.11$$

or equivalently the line

$$\beta_1 = 0.1\beta_2 + 0.11 \qquad 3.3.12$$

which is shown dotted in the diagram. The new ranks, which could be calculated from the original data as being in the order A B F G D C E, corresponds to the ranks along this line. The implication behind this approach is of course to continue to rerank until no further changes occur.

It should now be plain that not only can many of the traditional rules of thumb be investigated by means of the approximations that they imply to the dual, but also conversely that almost any continuous monotonic non-decreasing function of the β_t's has an implication as some form of ranking procedure. Now such an observation would be of practical significance only if rankings obtained from the various rules of thumb were roughly similar.

In this type of model, this is likely to be true since the rankings in each period are computed from the relative values of $NTV_j/TV_j(t)$ where

$$TV_j = -c_{tj} + (1+r)TV_j(t-1) \text{ with } TV_j(t) = -c_{oj} \qquad 3.3.13$$

Now TV_j is a weighted average of all previous cash flows where the least weight is given to the most recent. This smoothes the relative values of $TV_j(t)$ and results in a stable ranking of projects. Further simplification occurs because we need only to consider the ranking of a project whilst $TV_j(t) > 0$, i.e. whilst the project is a net absorber of funds. Typically this is for only the first few years of a project's life.

All these factors help to reduce the number of intersections of the lines and hence to reduce the number of alternative possible rankings. In this context it can be noted that the axis-ranks play a very special role in that they really define extreme project ranks and hence span all possible rankings. Thus if the axial ranks are quite similar so also will be any other rank, including such 'average' ranks as internal rate of return. This result alone can often simplify problems.

Take the example above, and accept projects in the ranked order along the axes β_1 and β_2.

TABLE 3.2.2

	NTV/TV(1)	NTV/TV(2)
Totally accepted	A,B,G,F	A,B,D
Partially accepted	C	F
Rejected	D,E,(H)	C,E,G,(H)

Thus immediately A and B can be accepted, E and H rejected, leaving just C,D,F and G as possible marginal projects. In fact more than this can be claimed as can be seen by inspection of the actual NTV/TV(t) ratios as below.

	Year 1	Year 2
Project C	0.20	0.09
D	0.16	0.12
F	∞	0.10
G	∞	0.05

Project F clearly dominates both C and G in the sense of having a higher rank in each year and will always be chosen in preference, which leaves the principal choice to be between D and F or even both. In this way mere inspection of the axis ranks can often reduce the number of likely combinations down to very few. In this case only two real options remain, either to accept D completely and F partially at 0.26 or F completely with D at 0.43, the latter being also the IRR solution incidently. This simple case also illustrates a point worthy of further consideration. Once the marginal projects have been identified, a task which it is argued is not laborious for most financial models, then the final choice is most likely to be made on the grounds of criteria other than the purely financial. Thus the two remaining options above differ by about ½% in the final plan value, which is likely to be of much less practical significance than many other features of projects D and F that have not been considered in this simple model.

A further observation supports the claim that in practice the number of plausible rankings might be quite small. In the large number of experiments carried out on these types of models in the development of this thesis seldom were there solutions in which the β_t are non-zero in more than two or three years. In fact, Weingartner's own result, in which a twenty-six year horizon model ultimately had only one β_t non-zero is by no means untypical. It is, of course, simple enough to artificially generate a project set in which every β_t is positive, it need only contain as many projects as years. The point is that this seldom seems to occur on real project sets. This will be returned to below, but its practical importance will be emphasized here.

Firstly, knowing which β_t are likely positive means that the dominance analysis above need only be done in those years. Secondly, and somewhat conversely, the dominance analysis usually helps to highlight the years in which $\beta_t > 0$ anyway. Thus in the example above, the two options of D or F partially accepted both imply year two as the bottleneck. In which case only the NTV/TV(2) ranking is relevant, leading to the optimal solution below

	A	B	D	F	v_1	w_1	v_2	w_2	D	
Year 1	-100	-100	-50	0	0	70	0	0	180	
Year 2	- 50	- 50	-10	-14	0	-77	0	100	100	
Return	246	256	89	28				-110		= 407.6

where project A, B, D are fully accepted, with project F partially accepted at 28%. Any deficit or surplus funds result in borrowing and lending decisions.

3.4 A rule of thumb solution to Weingartner's Horizon Model

A claim has been made above that the number of different plausible rankings is likely to be quite small and hence that many 'rules of thumb' such as IRR would be fairly robust in the sense of giving near optimal solutions for many different project sets. As such a claim must ultimately depend on the particular types of project sets under consideration no exact proof can be offered, only a case can be argued as has been done. This case has only been illustrated by a small example so far, so this section concludes with two large examples.

TABLE 3.4.1 DISCOUNTED TERMINAL VALUES OF OUTLAYS

Projects \ Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	100	90	79	66.9	54.6	41.0	27.1	13.8	1.2	-9.7	-17.7	-10.3	-11.4	-12.5	-13.8	-15.1	-16.6	-18.3	-20.1	-22.2	-24.4
2	100	90	81	71.1	60.2	48.2	39.0	28.9	17.8	5.6	-7.8	-18.6	-30.5	-43.5	-57.9	-70.7	-87.0	-101.8	-117.9	-135.7	-153.3
3	100	95	89.5	83.6	76.8	69.5	63.4	56.8	49.4	41.4	32.5	24.8	16.3	6.9	-3.4	-14.8	-25.2	-36.8	-49.4	-63.4	-78.7
4	100	90	93	91.3	93.4	86.8	90.5	85.5	76.0	80.6	68.7	73.6	58.9	56.8	52.5	39.7	37.7	32.5	21.7	0.12	-0.4
5	100	170	247	191.7	136.9	84.5	37.0	-3.3	-33.6	-51.0	-56.0	-61.7	-67.8	-74.6	-82.1	-90.3	-99.3	-109.2	-120.2	-132.2	-145.4
6	200	195	189.5	183.4	176.5	169.4	161.4	152.6	142.8	132.1	120.3	107.3	93.1	77.4	60.1	41.1	20.2	-2.7	-28.0	-55.8	-86.4
7							80	68	54.8	40.3	25.3	10.9	-2.0	-12.2	-19.4	-23.4	-25.7	-28.3	-31.3	-34.2	-37.6
8									60	96	115.7	92.2	56.4	37.0	24.7	15.2	8.7	29.6	11.6	-3.3	-15.6
9			120	107	92.7	72.0	44.2	18.6	-4.6	-25.0	-42.5	-56.7	-67.4	-74.1	-81.6	-89.8	-98.7	-108.6	-119.5	-131.4	-144.6
15	100	260	386	415	436	449	454	440	420	406	387	365	342	317	288	257	223	185	143	98	48
16								95	164	134	105	78.5	55.3	369	33.6	11.8	4.0	-1.58	-5.7	-9.3	-10.2
23		270	397	311	227	146	80	28	-40	-29.3	-47.2	-61.9	-68.1	-74.8	-82.3	-90.6	-99.7	-109.7	-120.6	-132.7	-14.6
24											40	29	18.9	11.7	6.0	1.6	-0.3	-0.3	-0.4	-0.4	-0.4

TABLE 3.4.2 The β-values for Weingartner's data.

Project	Year 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	IRR
1	.244	.271	.301	.364	.467	.595	.900	1.77	20.3													11.03
2	1.55	1.73	1.92	2.18	2.58	3.22	3.98	5.37	8.72	27.7												13.94
3	0.79	0.83	0.88	0.94	1.02	1.13	1.24	1.39	1.59	1.90	2.42	3.17	4.82	11.4								11.90
4	0	0	.002	.002	.001	.002	.002	.002	.002	.002	.002	.002	.002	.003	.004	.004	.004	.004	.007	1.18		10.02
5	1.45	0.86	0.59	0.76	1.06	1.72	3.93															12.26
6	0.91	0.93	0.96	0.99	1.03	1.07	1.12	1.19	1.27	1.37	1.51	1.69	1.95	2.34	3.02	4.40	9.02					11.75
7							0.47	0.55	0.69	0.94	1.48	3.45										13.84
8									0.63	0.39	0.32	0.46	0.67	1.01	1.52	2.47	4.32	1.27	3.24			12.57
9			1.21	1.35	1.56	2.01	3.27	7.77														15.66
15	1.78	0.69	0.46	0.43	0.41	0.40	0.39	0.40	0.42	0.44	0.46	0.49	0.52	0.56	0.62	0.69	0.80	0.96	1.24	1.82	4.2	10.86
16								0.11	0.06	0.08	0.10	0.13	0.18	0.28	0.45	0.86	2.55					10.24
23		0.54	0.37	0.47	0.64	1.00	1.82	5.2														11.94
24											.01	.02	.02	.04	.07	.23						10.19

TABLE 3.4.2 The β-values for Weingartner's data.

Project	Year 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	IPR
1	.244	.271	.301	.364	.467	.595	.900	1.77	20.3													11.03
2	1.55	1.73	1.92	2.18	2.58	3.22	3.98	5.37	8.72	27.7												13.94
3	0.79	0.83	0.88	0.94	1.02	1.13	1.24	1.39	1.59	1.90	2.42	3.17	4.82	11.4								11.90
4	0	0	.002	.002	.001	.002	.002	.002	.002	.002	.002	.002	.002	.003	.004	.004	.004	.004	.007	1.18		10.02
5	1.45	0.86	0.59	0.76	1.06	1.72	3.93															12.26
6	0.91	0.93	0.96	0.99	1.03	1.07	1.12	1.19	1.27	1.37	1.51	1.69	1.95	2.34	3.02	4.40	9.02					11.75
7							0.47	0.55	0.69	0.94	1.48	3.45										13.84
8									0.63	0.39	0.32	0.46	0.67	1.01	1.52	2.47	4.32	1.27	3.24			12.57
9			1.21	1.35	1.56	2.01	3.27	7.77														15.66
15	1.78	0.69	0.46	0.43	0.41	0.40	0.39	0.40	0.42	0.44	0.46	0.49	0.52	0.56	0.62	0.69	0.80	0.96	1.24	1.82	4.2	10.86
16								0.11	0.06	0.08	0.10	0.13	0.18	0.28	0.45	0.86	2.55					10.24
23		0.54	0.37	0.47	0.64	1.00	1.82	5.2														11.94
24											.01	.02	.02	.04	.07	.23						10.19

(i) Weingartners Horizon Model

This model considers 30 projects over twenty six years, although the horizon is drawn in year 21. The cash flows associated with projects are displayed in Table 9A.1 on page 180 of Weingartner* (74) and this table is reproduced in appendix XV. Many of the projects can be eliminated from further consideration since they are simple investments returning less than the cost of capital. Thus only projects 1 to 9 inclusive and 15, 16, 23 and 24 warrant further consideration. Tables 3.4.1 and 3.4.2 show the value of $TV_j(t)$ and the β-values** respectively for these projects. Where the project begins to make a net contribution to the firm having repaid the debt the ratio is not calculated. The final Σ row gives the sum of the TV_j for the projects. It should be noted that the net funds required by projects exceeds those available only during the first three years. Hence the constraints on project selection need only be considered for years 1, 2 and 3. The square boxes indicate the first partially rejected project ranked individually in each of these years, so that all projects are immediately accepted except for projects 1, 4 and 23. The relevant ranks for these are summarized in Table 3.4.3.

TABLE 3.4.3.

		Year	
	1	2	3
Project 1	6	7	8
Project 4	7	8	9
Project 23	-	6	7

* References to page numbers are those of the Kershaw edition (published in 1974) of 'Mathematical Programming and the analysis of capital investment problems" by H.M. Weingartner.

** Attention is drawn to the relative stability of the rankings implicit in these β-ratios.

Project 23 dominates the others and is therefore accepted at its maximum scale of 0.7. The other two are rejected. This is identical to the LP solution shown in Table 9A.6 on page 183 of Weingartner's text.

It is also worth noting that the internal rate of return solution, apart from upgrading project 23 to full scale would simply interchange projects 1 and 15, bringing the former in and taking the latter out. This would affect the total plan by only about 1.4%. Hence while it has not been difficult to generate the optimal solution to the Weingartner model by a simple search of the dual space though more important to notice is that even a solution obtained by a simple ranking by IRR would have given reasonable results.

(ii) Example Two

The project data in appendix IV was generated from summary statistics of actual company operations. The primary purpose of this data was to provide realistic test material for the discussion in chapter six on the problems of large scale financial planning models in practice, though the irregularity of the resulting cash flow patterns makes it appropriate data for a more thorough testing of the ideas put forward in this chapter. A simultaneous reduction of both cash availability from existing projections and the cost of additional funds was made to ensure that borrowing was forced to its limit in most years. Appendices IV & XV containsall the relevant cash flow data and the results of a particular LP solution* to the Weingartner horizon model with''hard' upper bounds on debt availability can also be found in Appendix XV. In this solution the cost of borrowing was 9%

* This solution and all the other LP solutions quoted in this chapter were found using the ICL package XDLA on the University of Birmingham's ICL1906 machine.

TABLE 3.4.5.

PROJECT	TV(t) 1	2	3	4	5	6	7	8	NTV	RANKINGS FOR EACH YEAR 1	2	3	4	5	6	7	8	NTV/TV(t) 2	RATIOS MARGINAL PROJECTS 3	4	5	6	7	STEP II a	b	c	STEP III	[illegible]
PR04Y1	350	254	205	126	42	-57	-130	-189	133.3	3	1	1	1	2	1=	1=	1=							✓	✓	✓	✓	✓
PR13Y1	432	409	212	173	6	-32	-119	-132	131.5	4	4	2	2	1	1=	1=	1=							✓	✓	✓	✓	✓
PR01Y1	100	217	362	339	247	44	- 80	-209	94.8	1	2	(6)	8	5	6	1=	1=	0.44	0.26	0.28	0.39	2.14	- 1.18	✓	✓	✗	✓	0.18
PR12Y1	260	318	304	303	292	152	- 23	-186	100.4	2	(5)	4	6	8	7	1=	1=	0.32	0.33	0.33	0.34	0.66	- 4.42	✓	✗	✓	✗	✗
PR16Y1	305	462	464	356	228	113	8	- 96	16.4	(5)	9	12	18	[illegible]	18	9	1=	0.04	0.04	0.05	0.07	0.14	1.98	✗	✗	✗	✗	✓
PR22Y1	121	140	202	134	81	76	-1	- 55	7.6	6	8	11	15	18	21	1=	1=	0.06	0.04	0.06	0.09	0.10	-10. 4	✗	✗	✗	✗	✓
PR04Y2		350	254	205	126	42	- 57	-130	123.4		3	3	4	3	5	1=	1=							✓	✓	✓	✓	✓
PR03Y2		369	399	423	326	239	57	- 43	87.9		7	7	10	7	9	10	1=	0.24	0.22	0.21	0.27	0.37	1.56	✗	✗	✗	✗	✗
PR13Y2		432	409	212	173	6	- 32	-119	121.8		6	5	5	4	4	1=	1=	0.28	0.30	0.57	0.70	20.03	- 3.76	✗	✓	✓	0.58	✓
PR14Y2		180	223	118	24	-53	-117	-125	3.0		11	14	20	14	1=	1=	1=	0.02	0.01	0.03	0.13	0.05	0.03	✗	✗	✗	✗	✗
PR24Y2		463	403	436	306	194	95	- 35	13.5		10	13	19	22	24	21	1=	0.03	0.03	0.03	0.04	0.07	0.14	✗	✗	✗	✗	✗
PR02Y3			196	261	203	163	100	71	13.3			10	16	20	22	25	24		0.07	0.05	0.07	0.08	0.13	✗	✗	✗	✗	✗
PR11Y3			353	364	292	206	148	46	68.0			8	11	11	10	13	14		0.19	0.19	0.23	0.33	0.46	✗	✗	✗	0.17	0.69
PR15Y3			167	273	229	162	107	84	25.8			9	14	16	17	17	17		0.16	0.09	0.11	0.16	0.24	✗	✗	✗	✗	✗
PR01Y4				100	217	362	339	247	75.2				3	6	(13)	18	18			0.75	0.35	0.21	0.22	✓	✓	✓	✓	✓
PR12Y4				260	318	304	303	292	79.7				7	9	11	15	19							✓	✓	✓	✓	✓
PR11Y4				355	364	292	206	148	63.0				12	13	15	16	16			0.18	0.17	0.22	0.31	✗	✗	✗	✗	✗
PR13Y4				432	409	212	173	6	104.4				(9)	(10)	8	11	13			0.24	0.26	0.49	0.60	✓	✓	✓	✓	✓
PR22Y4				121	140	202	134	81	6.0				17	23	27	29	27			0.05	0.04	0.03	0.05	✗	✗	✗	✗	✗
PR14Y4				180	223	118	24	- 53	2.6				21	24	28	24	1=			0.02	0.01	0.02	0.11	✗	✗	✗	✗	✗
PR25Y4				270	332	182	269	95	37.9				[illegible]	15	14	22	30			0.10	0.12	0.21	0.14	✗	✗	✗	✗	✗
PR02Y5					196	261	203	163	11.4					21	25	28	29				0.06	0.04	0.06	✗	✗	✗	✗	✗
PR03Y5					369	399	423	326	69.8					12	16	19	22				0.19	0.18	0.17	✓	✓	✓	✓	✗
PR11Y5					355	364	292	206	58.3					17	19	23	23				0.17	0.16	0.20	✗	✗	✗	0.06	✗
PR04Y6						350	254	205	90.7						12	14	15							✓	✓	✓	✓	✓
PR11Y6						355	364	292	54.0						23	26	25					0.15	0.15	✗	✗	✗	0.04	✗
PR14Y6						180	223	118	2.2						30	32	34					0.01	0.01	✗	✗	✗	✗	✗
PR15Y6						167	273	229	20.5						20	27	26					0.12	0.08	✗	✗	✗	✗	✗
PR16Y6						305	462	464	11.2						26	31	33					0.03	0.04	✗	✗	✗	✗	✗
PR01Y7							100	217	59.7							12	20							✓	✓	✓	✓	✓
PR04Y7							350	254	83.0							(20)	21						0.24	✓	✓	✓	✓	✓
PR22Y7							121	140	4.8							30	32						0.04	✓	✓	✓	✓	✓
PR14Y7							180	223	2.0							33	36						0.01	✗	✗	✗	0.46	0.48
PR02Y8								196	9.0								31							✓	✓	✓	✓	✓
PR15Y8								167	17.5								28							✓	✓	✓	✓	✓
PR22Y8								121	4.4								37							✓	✓	✓	✓	✓
PR25Y8								370	27.9								35							✓	✓	✓	✓	✓

Notes: The 1= Ranks are given to projects which have a net contribution towards cash flows in that period and must therefore be accepted.

The O refers to the marginally accepted project for that years ranking alone.

and the borrowing constraint was active in five out of the seven possible years.

The data necessary for a solution via the method outlined above is summarized in Table 3.4.5. Because of the volume of the data it is convenient to break up the analysis into three distinct phases.

Phase I

In this phase the projects which will definitely be accepted and those that will definitely be rejected are identified. Thus projects which return less than the lending rate can be eliminated from further consideration. Hence project PR05 available in years 2, 4, 6, project PR21 available in years 2, 5 and 6 and project PR23 available in years 1, 5 and 6 are rejected immediately. Whilst from the axial NTV/TV rankings in each year, 10 projects can be accepted without further analysis. This leaves 27 projects as possible contenders for marginal acceptance. The remaining funds available for each year from 2 to 7, the only likely bottlenecks are shown in Table 3.4.6.

TABLE 3.4.6 (In £1000's) Total net capital available in each year.

		1	2	3	4	5	6	7	8
B_t		750	750	750	750	750	450	450	450
D_t		400	300	200	0	0	0	0	0
L_t		1150	1482	1741	1820	1906	1698	1798	1905
Capital Available for further investments	PHASE I		469	1070	1056	1414	1091	1447	
	PHASE II (a)		-66	404	-118	-120	-78	144	
	PHASE II (b)		-180	299	-27	-1	68	153	
	PHASE II (c)		-281	357	9	-46	-40	96	
	PHASE III		0	310	0	0	0	0	

Phase II

Of the eight remaining projects competing for the available 469,000 in the second year, the three projects PR01Y1, PR12Y1 and PR13Y2 dominate the others. As funds will only cover the acceptance of at most two of these three, the remaining five can be rejected. Phase II(a) continues with the alternatives of choosing PR01Y1 and PR12Y1. Because of the dominance existing between projects available for starting in periods three and four PR0174 and PR13Y3 are chosen and the others rejected. Phases II(b) and II(c) correspond to the other two alternatives. The remaining cash balances for the three alternatives are shown in Table 3.4.6, the negative balances can always be removed at a later stage by accepting partial rather than whole projects. The critical years are now seen to be 2, 4, 5 and 6 and the ratios for the three projects under consideration are shown in Table 3.4.7.

TABLE 3.4.7.

	Years			
	2	4	5	6
PR01Y1	0.44	0.28	0.39	2.14
PR12Y1	0.32	0.33	0.34	0.66
PR13Y2	0.28	0.57	0.70	20.03

It can be seen that PR12Y1 is almost dominated by the other two in these crucial years. Thus alternative II(b) is seen to be the most appropriate choice and the negative balances can be removed by accepting partial projects. Project PR01Y1 is chosen at full scale rather than PR13Y2 because it has the higher ratio in the most crucial year, year two. The result is shown as step III. This result is in fact *identical to the optimal linear programming* *solution.* The final column indicates the IRR solution, in which for projects available in more than one year, preference is given to earlier years and it should be noted that this also differs little from the optimal solution for this particularly severe example. The value of the program loading by IRR is £2664 compared with the true optimal of £2671. It is perhaps this observation which is more disturbing than the fact that the trueoptimum has been obtained by a simple rule of thumb. While this exercise was carried out for further models with different initial cash flows and different interest rates, similar results were obtained and it is not worth repeating the analysis here. When differences were allowed between the borrowing and lending rates then this introduced a certain degree of 'fuzziness' into the investment decision consisting of those projects whose β rankings differed at these two rates. The differences in fact were quite small and further were only relevant for those marginal projects whose investment decision overlapped a transition between a budget surplus and a budget deficit. Again in all the cases examined the IRR provided a good ranking method for projects and this can be illustrated if once more we return to an example from Weingartners original work* Here Weingartner assumes at 10% borrowing rate and a 5% lending rate. The optimal project subset according to the LP solution consists of 19 projects from the available 30, though selection by an IRR ranking would have produced only one error in project selection and would have been within 2% of the value contributed by the optimal project set.

The observations of this section at present are merely discomforting for the proponents of linear programming models but

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* See Weingartner (74) p. 189.

against this it should be pointed out that this model represents pioneering work in the field and is relatively unsophisticated. It would now seem appropriate to examine the decision power of linear programming models of a more complex and sophisticated nature.

3.5 The Chambers Model

In the previous section a claim was made, based on a straight forward analysis, that for Weingartners horizon model many simple rules of thumb give tolerably close solutions to the optimal. In this section a similar claim, albeit in a slightly different form, is made about another major class of models. Whereas the Weingartner model considered debt capacity to be determined by fixed upper bounds, these models limit debt by restricting its value to be less than a fixed fraction of the value of equity in each year upto the horizon. The example chosen is the well known model by Chambers (71) which was introduced briefly in section 1.3. In Chambers model both debt and equity are measured in terms of book (accounting) values*. Since a detailed discussion of the structure and results of the model is readily available in the original article it is not repeated here, though summary data relevant to the subsequent analysis can be found in appendix XVIII.

The model may be stated as **

$$\text{Max} \sum_{t=1}^{H} \sum_{j=1}^{19} v_{tj} x_{tj} \qquad (3.5.1.)$$

* Myers and Pogue (74) develop a similar model where debt and equity values are in accord with capital market valuations. An analysis of this will be postponed until the next chapter.

** See Chambers (71), p. 272

subject to $L^t/E^t \leq g$ $\quad$ $t=1,2,\ldots.H$ $\quad$ (3.5.2)

$$F_o^t + \sum_{s=1}^{t} \sum_j F_{sj}^t X_{sj} = D_o^t + N_o^t \qquad t=1,2,\ldots.H \qquad (3.5.2)$$

$$0 \leq X_{tj} \leq 1 \text{ for } j=1,\ldots.14 \qquad 0 \leq X_{tj} \text{ for } j=15,17,18,19^* \quad (3.5.4)$$

where L^t and E^t represent the total value of debt and the book value of equity at the end of period t; g is the specified leverage. The constraints F_o^t, D_o^t and N_o^t represent respectively funds flow from 'old' projects already on the books, dividend payments and debt repayments as planned at the outset. The X_{tj} refer to the scale of project j begun in period t for j=1,.....14. The projects labelled j=15,17,18,19 will be considered in more detail below. The constants V_{tj} and F_{sj}^t refer respectively to the horizon valuation and the cash flows of each project. The dual equations associated with the financing and investment instruments of rights, debentures, market investments and government securities will be analysed individually.

The case of rights (Project 17)**

$$V_{t,17} = -S_t(1+i) \qquad \text{for } t=H \qquad (3.5.5)$$

$$= -S_t(1+i)^{H+1-t} + \sum_{1=t+1}^{H} d_s(1+i)^{H+1-S} \qquad \text{for } t=1,2,\ldots H-1 \quad (3.5.6)$$

This term should be adjusted slightly to allow for flotation costs but this will be ignored below. S_t is the issue price in period t, i represents the return available to shareholders on comparable equity investments elsewhere and d_t is the dividend per share. The cash flow stream is given by

*Project 16 is an aquisition and will be omitted here for simplicity.

** The project numbers refer to the original article.

$$F^{*}_{t,17} = (-S_t, d_{t+1}, d_{t+2}, \ldots\ldots) \qquad (3.5.7)$$

and the impact on equity by

$$E^{*}_{t,17} = (S_t, S_t - d_{t+1}, S_t - d_{t+1} - d_{t+2}, \ldots\ldots\ldots) \qquad (3.5.8)$$

Thus the dual equation, with ρ_t and ℓ_t as the duals on the cash balance and the gearing constraints respectively, is

$$(-S_t P_t + \sum_{s=t=1}^{H} d_s \rho_s) - g\left(S_t \ell_t + \sum_{s=t+1}^{H} (S_t - \sum_{k=t+1}^{s} d_r)\ell_s \right)$$

$$\geq - S_t(1+i)^{H+1-t} + \sum_{s=t+1}^{H} d_s (1+i)^{H+1-S} \qquad (3.5.9)$$

$$\text{Defining } L_t = \sum_{k=t}^{H} \ell_k \text{ and } \psi_t = (1+i)^{H+1-t} - \rho_t - gL_t \qquad (3.5.10)$$

Equation 3.5.9. simplifies to

$$S_t \psi_t \geq \sum_{s=t+1}^{H} d_s \psi_s \qquad (3.5.11)$$

or

$$S_H \psi_H \geq 0 \qquad (3.5.12)$$

$$S_{H-1} \psi_{H-1} \geq d_H \psi_H \text{ etc.} \qquad (3.5.13)$$

which implies that all $\psi_t \geq 0$ or equivalently that

$$\rho_t + gL_t \leq (1+i)^{H+1-t} \qquad (3.5.14)$$

<u>Investment in common stock (project 19)</u>

The dual equation for investment in common stock with a return of r_e is straightforward, affecting as it does only the first cash equation and the debt capacity permanently.*

$$\rho_t + g \sum_{s=t}^{H} \ell_t \geq (1+i)^{H+1-t} \qquad (3.5.15)$$

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* But see Chambers, the comment below Table 3 page 275.

This combined with the previous result for rights issue gives

$$\rho_t + gL_t = (1+i)^{H+1-t} \qquad (3.5.16)$$

Confirmation of this result and encouraging evidence of the correctness of the above analysis can be made by reference to the results in Table 5, page 277 of Chamber's article. The point is illustrated in Table 3.5.1. although a discussion will be postponed until after debentures have been considered.

TABLE 3.5.1 A Comparison of the Theoretical and Computed values of ρ_t+gL_t.

Year	ρ_t	l_t^*	L_t	$\rho_t+\frac{1}{2}L_t$	$(1.12)^{6-t}$
1	1.507	0.262	0.535	1.774	1.762
2	1.437	0.070	0.273	1.573	1.573
3	1.308	0	0.203	1.409	1.405
4	1.153	0.038	0.203	1.254	1.254
5	1.038	0.165	0.165	1.120	1.120

Debentures (project 18)

Because of considerations of tax lags, flotation costs, the impact of interest payments on retained profits, the dual equations for debentures are algebraically tedious; nevertheless they 'respond' to the same approach. The cash stream associated with a unit (£100,000) debenture issue is

$$F_{t},18 = (100 - f, - 100r\ (1-T), \ldots\ldots) \qquad (3.5.17)$$

where r is the debt rate, f the flotation costs, and T the corporation tax rate. The effect on equity is

$$E_{t,18} = (f, r(1-T),\ r(1-T),\ \ldots\ldots) \qquad (3.5.18)$$

while the debt is permanently changed by 100.

* The sign change is due to an inequality reversal.

The dual equations are

$$-\rho_t(100-f) + \rho_{t+1}100r + \rho_{t+2}100r(1-T) \ldots\ldots \rho_H 100r(1-T)$$

$$+ (100+gr)\ell_t + \sum_{k=t+1}^{H} (100+gr+g100+(1-T)(k-t))\ell_k$$

$$\geq \begin{cases} -100 + f + 100\,rT & \text{for } t=1,\ldots.H-1 \quad (3.5.19) \\ -100 + f & \text{for } t=H \quad (3.5.20) \end{cases}$$

or on rearrangement

$$f(\rho_t+gL_t) - 100(\rho_t-L_t) + \rho_{t+1}100rT \qquad (3.5.21)$$

$$+ 100r(1-T) \sum_{s=t+1}^{H} \{\rho_s+gL_s\} \geq \begin{cases} -100 + f + 100rT \\ -100 + f \end{cases} \qquad (3.5.22)$$

Using the result in equation 3.5.16 that

$$\rho_t + gL_t = (1+i)^{H+1-t}$$

$$\rho_t - L_t \leq 1 - \frac{f}{100} - rT + r(1-T) \sum_{s=t+1}^{H} (1+i)^{H+1-s}$$

$$+ \frac{f}{100}(1+i)^{H+1-t} + \rho_{t+1}rT \quad \text{for } t=1,\ldots H-1 \qquad (3.5.23)$$

or with $\frac{f}{100} + \frac{r(1-T)}{r_e} = K$

$$\rho_t - L_t \leq K \left\{ (1+12)^{H+1-t} - 1 \right\} + (1-r) + \rho_{t+1}rT \qquad (3.5.24)$$

which gives on substituting the numerical values of the various parameters.

$$\rho_t - L_t \leq 1.002 \quad \text{for } t = 1 \qquad (3.5.25)$$

$$\leq 1.073 \quad \text{for } t = 2 \qquad (3.5.26)$$

$$\leq 1.114 \quad \text{for } t = 3 \qquad (3.5.27)$$

$$\leq 1.164 \quad \text{for } t = 4 \qquad (3.5.28)$$

$$\leq 1.221 \quad \text{for } t = 5 \qquad (3.5.29)$$

One year government securities (project 15)

With the interest rate on securities as r_L and the corporate tax rate of T, with a one year lag in payment then

$$F^{*}_{t,15} = (1, -(1+r_L), r_L T) \quad (3.5.30)$$

and the impact on equity is

$$E_{t,15} = (0, r_L(1-T), r_L(1-T), r_L(1-T), \ldots.) \quad (3.5.31)$$

Then for t=1,2,....H-3

$$\rho_t \geq (1+r_L)\rho_{t+1} - r_L T\rho_{t+2} + gL_{t+1} \quad (3.5.32)$$

Using the fact* that $L_t \geq 0$ and that $\rho_t \geq \rho_{t+1}$ then

$$\rho_t \geq \left[\frac{1+r_L}{2} + \frac{1+r_L}{2}\left\{1 - \frac{4r_L T}{(1+i)^2}\right\}^{\frac{1}{2}}\right]^{H+1-t} \quad (3.5.33)$$

or approximately

$$\rho_t \geq (1+r_L(1-T))^{H+1-t} \quad (3.5.34)$$

The results so far have been generated purely algebraically, but an economic interpretation gives some insight. For rights issue the total contribution of an extra £1 of rights to the objective value must be less than or equal to 12 per cent, since otherwise rights would be issued until it was no longer profitable to make further issues. The contribution of £1 of rights in relaxing the cash balance constraint is ρ_t and the contribution to relaxing the debt capacity constraint** is $\frac{1}{2}L_t$. Hence the inequality $\rho_t + \frac{1}{2}L_t \leq (1.12)^{H+1-t}$.

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* In fact the equality $\rho_t + gL_t = (1+i)^{H+1-t}$ could be used to impose a stronger lower bound of ρ_t, but the size of the correction scarcely warrants it.

** It should be noted that the right hand side of the leverage constraint is $0.5E^t_o - L^t_o$ where E^t_o, L^t_o represents the Equity and Debt at time t resulting from the initial decisions. Thus an additional £1 of equity relaxes this constraint by 0.5

Similarly the company can get a return of at least 12% by investing £1 in the equity of other companies. The opportunity cost of such an investment, which is $\rho_t + \frac{1}{2}L_t$, is thus at least 12% or as an inequality $\rho_t + \frac{1}{2}L_t \geq (1.12)^{H+1-t}$.

These last two results imply that $\rho_t + \frac{1}{2}L_t = (1.12)^{H+1-t}$. This is because the firm can be considered in equilibrium with other firms in the market. The value of funds to the investor whether they are payments to the firm for rights or whether they are receipts in the form of dividends from other companies is 12%. The precise division of the value of these funds between their effect on the leverage constraints depends on the other financing/investment decisions of the firm.

Since investment in 1 year government securities does not have a substantial impact on the debt capacity, the interperiod discount rate should be no less than 4% or $\rho_t \geq 1.04\rho_{t+1}$. By similar reasoning to the case of rights issues, the total contribution of an extra £1 of debt must be less than 4%.* The contribution of £1 of debt in relaxing the cash balance constraint is ρ_t and its impact through a permanent reduction in debt capacity is L_t. Thus $\rho_t - L_t \leq (1.04)^{H+1-t}$. These inequalities differ from those derived earlier, but this intuitive approach ignores transaction costs, tax-lags and the effect of interest payments on retained profits (and hence equity reserves). The difference is fairly slight and it is convenient to use this intuitive approximation** to obtain just one more result. When debt is being issued, the inequality becomes an equality and so $\rho_t - L_t = (1.04)^{H+1-T}$. Combining this result

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* To be more accurate, for government securities $\rho_t \geq 1.036\rho_{t+1}$.

** The debenture equation is a fairly crude approximation which works reasonably well over the limited range considered. It should be noted that the 4% used in this approximation is the IRR of the after tax flows and not the after tax nominal cost.

with $\rho_t + \frac{1}{2}L_t = (1.12)^{H+1-t}$ and solving for ρ_t and L_t gives

$$\rho_t = 2/3(1.12)^{H+1-t} + 1/3(1.04)^{H+1-t} \quad 3.5.35$$

$$L_t = 2/3(1.12)^{H+1-t} - (1.04)^{H+1-t} \quad 3.5.36$$

The first equation implies that where the firm is raising debt, even though the firm may not necessarily be at its leverage limit, then the appropriate discount rate is just a 'weighted average cost of capital', with the equity rate of 12% and the debt of 4% weighted in the ratio of 2:1.

The expression for L_t may be rewritten in the form $L_t = \rho_t - (1.04)^{H+1-t}$ and the leverage dual is seen as the difference between the weighted average cost of capital and the debt rate. Thus although the pure debt appears cheaper, there is an opportunity cost associated with debt which is just equal to this difference.

Returning to the previous inequalities they may be summarized as below

The equity inequality (3.5.16) $\rho_t + \frac{1}{2}L_t = (1.12)^{H+1-t}$

The debt inequalities (3.5.25 - 29)

$$\rho_t - L_t \leq \begin{matrix} 1.002 \\ 1.073 \\ 1.114 \\ 1.164 \\ 1.221 \end{matrix}$$

The inequality (3.5.34) for government securities

$$\rho_t \geq (1.04)\rho_{t+1}$$

This dual feasible region can be represented as shown in Figure 3.5.1.

FIGURE 3.5.1.

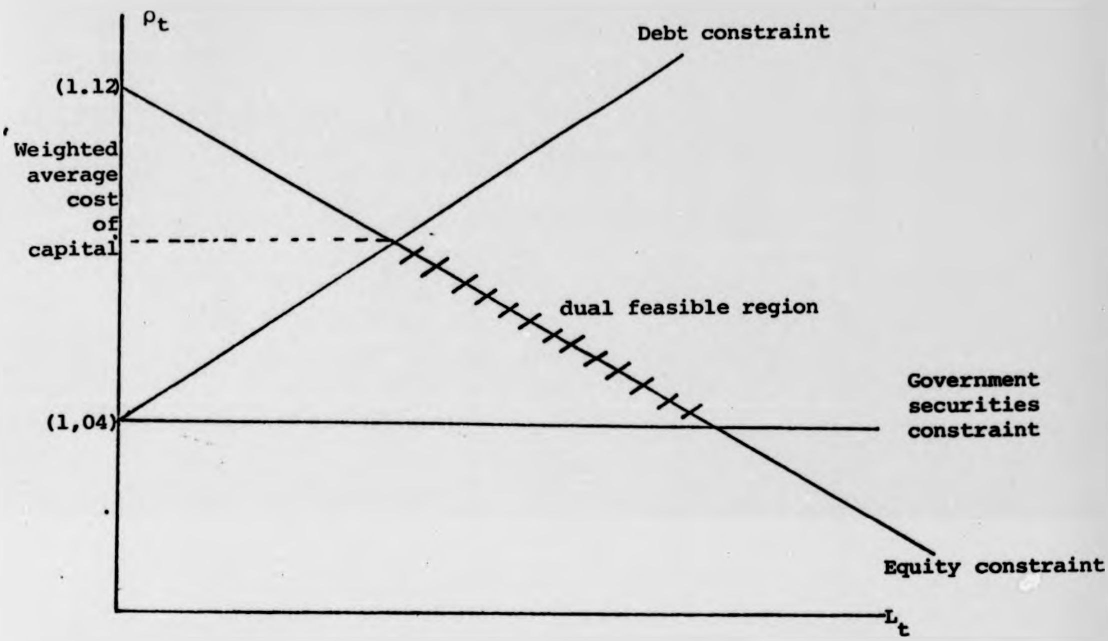

Figure 3.5.1. represents just the t'th section of the dual space. The feasible region is just the hatched line. This enables a complete set of rectangular bounds corresponding to the end points of this 'truncated line' representing the dual feasible space to be calculated. The upper bounds on ρ_t together with the lower bounds on L_t arise from when the firm is raising debt. In the figure this occurs when the firm is 'operating' at the upper left-hand end of the hatched line. The lower bounds on L_t arise from when the firm is in a cash-surplus situation or operating at the lower right-hand end of the line. These bounds are shown in Table 3.5.2. together with the results obtained with the data in appendix XVI and the results quoted from the original paper.*

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* The program results (see appendix XVIII) differ slightly from those published by Chambers due to some slight discrepancies in the source data. Chambers' results are in parentheses.

TABLE 3.5.2.

Year	ρ_t lower	ρ_t actual	ρ_t upper	L_t lower	L_t actual	L_t upper
1	1.573	1.576 (1.507)	1.582	0.361	0.367 (0.535)	1.138
2	1.155	1.408 (1.437)	1.437	0.273	0.339 (0.273)	0.843
3	1.114	1.308 (1.308)	1.308	0.194	0.197 (0.203)	0.362
4	1.075	1.157 (1.153)	1.194	0.121	0.197 (0.203)	0.362
5	1.036	1.036 (1.038)	1.081	0.077	0.169 (0.165)	0.169

These results are encouraging evidence of the correctness of the analysis. In particular it should be noted that in periods 2 and 3 when the firm is raising debt the value of ρ_t is precisely that given by 'the weighted average cost of capital.' The importance of these results is that they give an upper and lower bound on the 'value' of an individual project. This value is an adjusted net present value in that it consists of project cash flows valued at the horizon plus an estimate of the contribution that these project cash flows make to the debt capacity. With these bounds, projects can be screened into those which will definitely be accepted (i.e. those whose lower bound is positive), those which may or may not be accepted, (these will have a negative lower bound but a positive upper bound) and those which will definitely be rejected (i.e. those with negative upper bounds). The result of this analysis is shown in Table 3.5.3.

TABLE 3.5.3.

Project No.	D.C.F. Ranking	Value of Project						Actual* Reduced Cost	Decision
		Unfavourable			Favourable				
		Cash Flows	Debt Capacity	Net	Cash Flows	Debt Capacity	Net		
1	2	43.8	0.6	44.4	-	-	-	52	Accept
2	3	67.3	0.7	67.0	-	-	-	84	Accept
3	4	102.5	2.5	105.0	-	-	-	128	Accept
4	1	79.6	6.6	86.2	-	-	-	110	Accept
5	6	9.2	1.8	11.0	-	-	-	23	Accept
6	10	-30.2	22.2	-8.0	10.5	-7.5	3.0	-2	?
7	8	-18.9	9.7	-9.2	18.4	-4.0	14.4	3	?
8	4	32.7	4.3	37.0	-	-	-	52	Accept
9	7	11.4	1.6	13.0	-	-	-	20	Accept
10	13	-26.4	-0.3	26.7	17.9	-0.1	17.8	-2	?
11	12	-27.0	-6.5	33.5	14.3	1.9	12.4	-7	?
12	11	-11.7	0.1	-11.6	22.4	0.2	22.2	4	?
13/14	9	1.1	-0.4	0.7	-	-	-	9	Accept

* These actual reduced costs have not been published in Chambers(71) but can either be calculated from the duals or, as these were, from the program,on data made available by Chambers.

In effect Table 3.5.3 presents a formal solution to the Chambers model for all possible combinations of initial cash flow positions and debt commitments.* The final investment decision of course still depends upon the initial state of the firm and linear programming is a readily available mechanism for determining an optimum with respect to this initial state. The dual analysis yields little more than a sophisticated version of Hirchleifers (59) rule discussed in section 1.2 - 'where the firm is borrowing funds then the appropriate discount rate is the borrowing rate, where the firm is in surplus then the appropriate rate is the lending rate' - though in this case the actual discount rates were adjusted for the impact of the project on debt capacity.**

Clearly however, it would be presumptious to draw general conclusions from an examination of just two simple*** models and further discussion of the issues raised by the foregoing analysis will be postponed until a further and more complex model has been examined.

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* Linear programming solutions corresponding to extreme configurations of initial cash flows and debt commitments can be found in appendix XVI.

** It is perhaps worth emphasising that the linear programming solution was achieved only by the somewhat artificial device of assuming identical projects where available in each year. Such an assumption is not necessary for the solution arrived at by the dual analysis. Furthermore this dual analysis illustrates the relatively minor impact of assumptions made about future investment opportunities on current investment decisions.

*** In fairness to the authors it should be mentioned that Chambers in an unpublished working paper and Weingartner in his book have proposed extensions to their basic models. The impact of these extensions will not be analysed here in detail since many of them have been incorporated into the next model to be discussed.

3.6 The impact of additional constraints

This chapter has so far dealt with models which consisted only of cash balance constraints and debt capacity constraints. The particular model proposed in section 1.7, while maintaining this basic structure, has many additonal constraints. While some of these can be considered primarily as restrictions on the investment set - for example the restriction on the return of capital is in this category - some of these such as dividend policy restrictions can be considered as a restriction on the financial market opportunities. Moreover, certain restrictions such as the times covered constraint, intimately connect investment profitability to the debt raising potential. The effect of these additional constraints is effectively to preclude a rigorous analytical treatment of the investment schedule in a manner similar to that carried out on the Weingartner and Chambers models. This is confirmed by a cursory examination of the impact of the non-debt capacity constraints. Clearly if the initial level of debt were very high it would be impossible to cover debt by the available projects; equally if the required minimum return on capital were pitched too high, again there would be no feasible solution and the dual space would be unbounded. Thus the impact on the non-debt capacity constraints can be major. Hence, in this case no rule of thumb (excluding the simplex algorithm and its variants) readily gives the correct solution. The question remains however, whether the use of such simple rules as selection by net present value or internal rates of return would break down completely, or whether they still remain fairly good rules and produce, if not optimal, at least reasonably good solutions. Such an answer would, by its very nature be specific to the model under discussion. Nevertheless it may provide us with a justification for the use of financial linear programming models, equally it could well provide further evidence of the power of discounting methods.

The complexity of the duals leaves us with little alternative but to begin the analysis on a simplified model which retains the same basic structure. Such a model would consist of a cash balance equation plus a times covered constraint.* There are two forms of debt in the model developed. Examination of the runs included in the Appendices suggests that it is long term debt (which incidentally has the lower nominal rate) that is generally preferred. So it is to this that our attention is turned first. An immediate problem is that the restriction on debt is related solely to project profitability no amount of equity can relieve this constraint unless a profitable project exists. The implication of this for our analysis is that we start by considering intersections between investment opportunities and the debt opportunities in our dual analysis.

Consider a project beginning in time t which returns a constant infinite income stream with an internal rate of return m. If we further assume that the tax rate on earnings is 50% with no tax lag, then the associated dual equation*** is

$$\rho_t - m\rho_{t+1} - 2m\lambda_{t+1} - m\rho_{t+2} - 2m\lambda_{t+1} \ldots\ldots \geq 0 \qquad 3.6.1.$$

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* Analysis of the dual inequalities associated with debt, equity and market investments can be found in Appendix XVII. Results from this analysis are merely quoted in this chapter.

** This constraint is assumed to be of the form that the earnings before tax and after depreciation must be at least K times the interest on debt. See equation 2.03 of Appendix I.

*** This dual equation could be regarded as arising in two ways. One is that all projects can be analysed in terms of a constant earning stream and that this equation is related to a particular project. The alternative and probably more realistic analysis, is that the equations are average equations over all of the projects which commence in a particular time period.

where λ_t is the dual on the times covered constraint .

The dual inequality associated with a non-repayable debt instrument of nominal rate r is of the form

$$\rho_t + \left(\frac{r}{2}\right) \lambda_{t+1} + Kr_{\ t+1} + \left(\frac{r}{2}\right)\rho_{t+1} + Kr\lambda_{t+2} \ldots\ldots \geq 0 \qquad 3.6.2.$$

If we consider the situation in which debt is being raised and the limitation on the times covered constraint results on the marginal project having an internal rate of return m, inequalities 3.6.1 and 3.6.2. then become equalities.

We can eliminate the debt duals by multiplying 3.6.4. by Kr and 3.6.5. by 2m and adding these two equations. The resulting equation is of the form:

$$\rho_t(Kr - 2m) - \rho_{t+1}\ (Krm - mr) - \rho_{t+2}\ (Krm - mr) \ldots\ldots = 0 \qquad 3.6.3.$$

If we assume that a solution* to the equation exists in the form $\rho_{t+1} = \pi\rho_t$ then the following characteristic equation results.

$$\pi^t - a\pi^{t+1} - a\pi^{t+2} \ldots\ldots\ldots = 0 \qquad 3.6.4.$$

where

$$a = \frac{m(K-1)r}{(Kr-2m)}$$

or

$$\pi^t\ (1 - az - az^2 \ldots\ldots\ldots\ldots) = 0 \qquad 3.6.5.$$

---

* Methods for the solution of such difference equations are discussed extensively in standard texts (See for example Goldberg (58))

which reduces to

$$\pi^t \left(1 - \frac{a\pi}{1-a\pi}\right) = 0 \qquad 3.6.6.$$

Ignoring the trivial solution $\pi = 0$ then

$$\pi = \frac{1}{1 + a} \qquad 3.6.7.$$

which implies that

$$\rho_t = \frac{\rho_{t+1}}{\pi} = (1+a)\rho_{t+1}$$

$$= \left[1 + \frac{m(K-1)r}{(Kr-2m)}\right]\rho_{t+1} \qquad 3.6.8.$$

If we ignore the impact of the non-debt capacity constraints then the dual inequatlites for the issue of dividends and rights lead to the single equality*

$$\rho_t = (1+i)\rho_{t+1} \qquad 3.6.9.$$

where i denotes the equity rate.

Hence if the above analysis is correct, it would suggest that the internal rate of return of the marginal project (m) is given by the solution of

$$1 + i = 1 + \frac{m(K-1)r}{(Kr-2m)} \qquad 3.6.10$$

or

$$m = \frac{Kr}{(K-1) + 2i} \qquad 3.6.11$$

Projects with an internal rate of return above this value would be accepted, while projects with a lower internal rate of return would be rejected.

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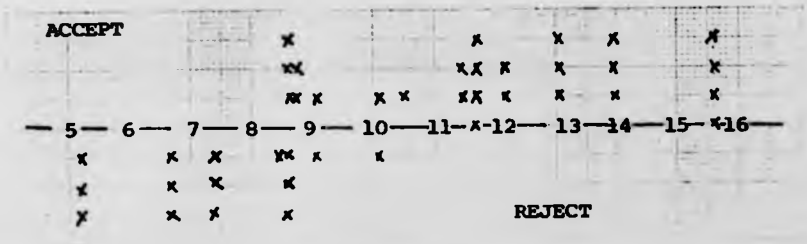
* See appendix XVII. This equality can be deduced easily by equating the non-debt capacity duals in the appropriate inequalities to zero.

Since the derivation of this formula has been intuitive rather than rigorous, before proceeding it is worthwhile examining whether this fairly crude approach has any validity in practice. In the model under discussion the values of the parameters in the formula are i = 12%, r = 8%, K = 10%. This gives a value of 10% as the appropriate cutoff rate. The model was run with all the financial reporting constraints suppressed except the times covered constraint. The results are illustrated in Figure 3.6.1. The horizontal axis is the internal rate of return of the project. A cross above the line denotes an accepted project, a cross below the line denotes a rejected project, marginal projects are marked on the line. A vertical line of crosses arises because the projects are repeated in later years and thus there is more than one project with the same internal rate of return. The cutoff rate is in fact quite sharply defined at 8.7%; there being only projects PRO2Y5 with an internal rate of return of 9.08% and PR25Y5 with an internal rate of return of 10.06% in direct violation of this cutoff rule.* It should be noted that there are several marginally accepted projects with relatively high internal rates of return. The reasons for this will be examined in detail later.

FIGURE 3.6.1.



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* It should be noted that only the initial outlay from project PR25Y8 occurs in the pre-horizon period. Hence, it hardly constitutes a valid counter-example since the accept-reject decision is largely a function of the post-horizon discount rate.

The results are sufficiently encouraging that it is worthwhile extending this model to cover short term debt or overdraft. The dual inequalities associated with overdraft (nominal rate r_s) are

$$-\rho_t + (1+r_{s/2})\, \rho_{t+1} + K\, r_s \lambda_{t+1} \geq 0 \qquad t=1,\ldots.T-1 \qquad (3.6.12)$$

If overdraft is being used as a financing instrument in time period t then

$$Kr_s \lambda_{t+1} = \rho_t - (1+r_{s/2})\, \rho_{t+1} \qquad (3.6.13)$$

while for the marginal ("infinite") project inequality still holds. If we find the relationship between the discount rates in consecutive years then the same functional form as before holds with

$$\rho_t = \left[1 + \frac{m(K-1)r}{(Kr_s - 2m)} \right] \rho_{t+1} \qquad (3.6.14)$$

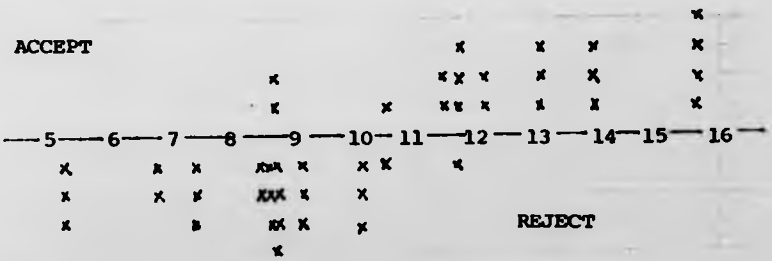
with r this time replaced by r_s.

This gives an expression for the internal rate of return of the marginal project of

$$m = \frac{Kr_s i}{(K-1)r + 2i} \qquad (3.6.15)$$

since r_s = 12%, the cut off rate for projects selected by overdraft only would be m = 10.9%. Figure 4.6.2. shows projects selected by overdraft alone. Again it shows a fairly well defined cutoff rate of around 10.5%.

FIGURE 3.6.2.



This analysis also suggests that where the firm has both long term debt and overdraft available, most of the debt financing will take place by the one with the lower nominal rate. Such a result is confirmed readily by inspection of any of the linear programming solutions included in the appendices.

In general the introduction of other 'balance sheet' constraints will distant the cut off rates and may well blur its sharpness. Figures 3.6.3 and 3.6.4 show project selection subject to all the constraints. The first of these is selction with normal earnings from existing projects and the second illustrates selection where there is a 10% reduction in earnings from existing projects.*

FIGURE 3.6.3.

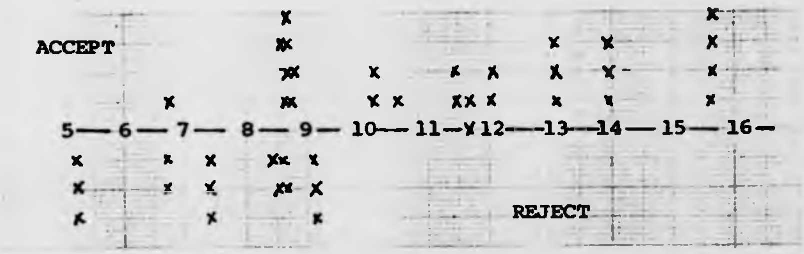
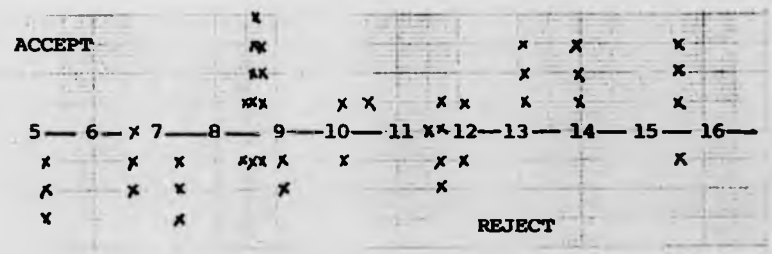


FIGURE 3.6.4.



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* Normal here is a convenient reference term for the case where earnings from existing projects are as in appendix III. It was chosen as a base case since it represented the lowest level of earnings for which a feasible solution existed in the absence of any investment projects. The usefulness of this as a base point will become self evident in the next chapter. Parametric analysis further showed that if earnings from existing projects were reduced by 21.2% there was no feasible solution even with all project opportunities present.

While the other constraints do have some distant effect, it is much less than might be expected and it is worthwhile trying to explain this. As in the previous models, discounting indices are merely ranking devices on the desirability of projects. The power of ranking methods in generating approximate LP solutions has been used by others, notably Senju and Toyopa (68) for the solution of integer programming problems. Fogler (72) has directly exploited this algorithm for the selection of optimal investment portfolios. He carried out a series of experiments using ranking procedures on an integer problem with 60 projects and 30 constraints. His conclusion was that the portfolio selected gave a 'total profit impressively high' (when compared with the true optimum). One of the key assumptions made by Fogler in explaining this, was that there was some degree of linear dependence between the constraining equations. Thus he argued that a project's use of a particular resource was roughly proportional to its use of other resources.

In the case under discussion here the development of the analysis so far has rested largely on the fact that the cash flows are proportional to the pre-tax earnings. Examination of the other constraints shows that in the case of return on capital constraint the 'numerator' is also proportional to the pre-tax earnings. This is also true of the earnings per share and the dividend cover constraints since here the numerator is proportional to the net profit after tax which in turn is roughly proportional to the pre-tax earnings. Thus the model here satisfies this condition* of the constraint set being linearly

* Of the remaining constraints, clearly the dividend per share constraint is in no way proportional to the pre-tax earnings. However, this independence does mean that it has a minimal effect on project selection since it largely determines the cash disbursements from the firm and as such its major effect is in the financing strategy. The remaining constraint does exhibit a 'loose' relationship with pre-tax earnings since both the current liabilities and pre-tax earnings are each roughly proportional to the level of sales.

dependent in some approximate way - though in the end the power of this single parameter of internal rate of return is still most impressive. Table 3.6.1. provides a further illustration of this.

TABLE 3.6.1. VALUE OF FIRM USING DIFFERENT IRR CUT-OFF RATES

Level of Earnings from Existing Projects	IRR CUT-OFF RATE								
	Optimum Value	6%	7%	8%	9%	10%	11%	12%	13%
Normal	1984	1760	1790	1848	1861	1877	1842	1753	1752
Reduction by 5%	1720	1523	1548	1583	1520	1534	1478	1497	1504
Reduction by 10%	1435	1186	1117	1327	1206	1235	1214	1202	1206
Increase by 5%	2232	1986	2010	2101	2124	2138	1220	1994	1994

The projects are selected with different internal rates of return used as a cut-off* and at different levels of earnings from existing projects. The maximum values as the cut-off rates are varied are indicated by the boxed entries. In the case of normal earnings and a 5% increase in normal earnings, the maximum value does in fact occur at a cut-off rate of 10%. Thus the predicted rate indeed minimises the difference in value between the optimum solution and the solution arrived at by a simple IRR cut-off rule. We can look at another measure of difference between the solutions by looking at the size of the error in project selection. This error (D_{IRR}) can be defined as

$$D_{IRR} = \sum_j \left| x_j^{(OPT)} - x_j^{(IRR)} \right|$$

* In the linear programme, the post horizon value of a project in the objective function was increased by a large positive value for an internal rate of return greater than the cut-off rate and a large negative value if the internal rate of return was less than the cut-off value. Thus selection was by feasibility than by internal rate of return then optimal financing and investment in the usual way. A statement of the objective function can be found in appendix V.

where $x_j^{(OPT)}$ is the scale at which project j is undertaken in the optimum (LP) solution;

$x_j^{(IRR)}$ is the scale of acceptance of project j for a particular IRR cut of rate.

This error norm is shown in Table 3.6.2. for various internal rates of return used as the cutoff.

TABLE 3.6.2. ERROR NORM FOR SCALE OF PROJECTS*

Level of Earnings from Existing Projects	IRR Cut-Off Rate		
	8%	10%	12%
Normal Earnings	5.79	11.73	17.73
Decrease by 5%	8.74	12.93	15.01
Decrease by 10%	9.61	10.95	11.35
Increase by 5%	6.06	6.66	17.28

From table 3.6.2. it can be seen that minimising the error in the scale of project selection does not necessarily give the optimum solution with respect to maximisation of the value of the firm. The error in the scale of project selection tends to be minimised around 8% while the loss in value arising out of imperfect selection tends to minimised** at around 10%. Thus the dual analysis of the simplified which predicts that the appropriate internal rate of return cut-off rate is 10% appears to be well justified.

While selection by a simple IRR cut-off gives satisfactory solutions once the appropriate cut-off rate has been determined. The prior determination of this cut-off rate may be considered to be not an easy

* The final year (year 8) was omitted from the analysis since their selection was largely just an NPV criterion at 10%. This implied an upper bound of 41.0 for the D - statistics.

** When earnings from existing projects are reduced more new projects need to be introduced to maintain optimality. This accounts for the lower cut-off rate.

task. The theme of this chapter has been that fairly simple rules of thumb give good ranking methods for use in a preliminary screening of projects. The projects can be then further scrutinized against other criteria before a final selection is made. It is possible to simulate such a decision procedure on the LP model. This is done by ranking the projects according to the internal rate of return and then including in the objective function a large positive multiple of this rank.* Since the simplex algorithm proceeds by including in the solution the non-basic variable with the largest reduced cost, this device ensures** that projects are loaded sequentially by their IRR ranks. Table 4.6.3. shows the results of such a procedure. If the stopping criterion adopted is that the

TABLE 3.6.3. LOADING BY IRR RANKINGS

NORMAL EARNINGS		EARNINGS INCREASED by 10%		EARNINGS REDUCED by 10%	
Iteration No.***	Objective Value	Iteration No.	Objective Value	Iteration No.	Objective Value
64	1841	56	2029	56	1289
65	1851	57	2050	57	1295
66	1855	58	2097	58	1296
67	1858	59	2097	59	1296
68	1866	60	2141	60	1287
69	1870	61	2146	61	1274
70	1868	62	2164	62	1277
71	1838	63	2190	63	1269
72	1840	64	2203	64	1272
73	1839	65	2203	65	1275
74	1834	66	2202	66	1253
75	1827	67	2195	67	1238
76	1824	68	2142	68	1233
78	1800	69	2138		
		70	2122		

* The precise formulation of this problem can be found in appendix

** In actual fact, the optimisation algorithms XDLA are considerably more sophisticated than this with block pricing, major and minor iterations plus many similar facilities incorporated as standard. It is possible

(continued on page 123)

loading of project ceases when the objective value falls in two consecutive iterations then the following results are obtained.

TABLE 4.6.4 THE ADOPTION OF PROJECTIONS BY IRR RANKINGS

Earnings Level	Optimum Level (LP solution)	Value obtained in Loading
Normal Earnings	1984	1870
Earnings increased by 10%	2471	2203
Earnings reduced by 10%	1435	1296

Table 3.6.4. illustrates the sort of results that might be achieved using a financial statement generator, where a preliminary screening or ordering of the projects is carried out by an IRR criterion and final selection is made subject to a satisfactory performance on a whole host of other criteria. It further emphasises the power of discounting indices particularly when used in conjunction with a financial statement generator.

In fairness the results look better than they really are. A more correct measure of the power of the methodology is in a comparison of the additional contribution to the value of the firm made by the adopted projects in each case. Considerations of feasibility make estimations of the value of the firm in the base case of no additional projects available difficult

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however, by careful parameter specification to ensure that the optimisation procedure accords with this simple description.

*** The iteration number is the iteration number of primal dual algorithm used by XDLA. The initial basis is the optimal solution (not necessarily feasible) of the linear programme with all projects excluded.

to determine. However, in the case of normal earnings an optimum feasible solution without projects does exist and the corresponding value of the firm is £1.30m. Thus the optimal selection of projects increases the net present value of the firm by £0.68m whereas the rule of thumb selection just discussed only increases its value by £0.57m. If we assume that the base value of the firm in the case of above normal earnings and below normal earnings is £1.43m and £1.17m respectively.* Then the rule of thumb added value is £0.77m and £0.13m compared with possible values of £1.04m and £0.26m respectively. Whilst such a rule may be considered adequate at normal earnings and above, it performs fairly badly under conditions of low earnings.

If a comparison is made between optimum project selection and internal rates of return at differing levels of earnings then apparent anomalies are observed.

TABLE 3.6.5. OPTIMAL PROJECT SELECTION and INTERNAL RATES OF RETURN

PROJECT	IRR	DECISION			PROJECT	IRR	DECISION		
		Normal Earnings	+10%	-10%			Normal Earnings	+10%	-10%
PR01Y1	13.04	✓	✓	✓	PR02Y5	9.08	X	X	X
PR04Y1	15.59	✓	✓	✓	PR03Y5	11.47	✓	✓	0.1
PR12Y1	12.13	✓	✓	X	PR11Y5	11.68	0.45	0.54	X
PR13Y1	13.97	✓	✓	✓	PR21Y5	5.22	X	X	X
PR16Y1	8.62	X	X	X	PR23Y5	6.73	X	X	X
PR22Y1	8.75	✓	✓	✓	PR04Y6	15.59	✓	✓	✓
PR23Y1	6.73	X	X	X	PR05Y6	7.41	X	X	X
PR03Y2	11.47	✓	✓	0.64	PR11Y6	11.68	✓	✓	✓
PR04Y2	15.59	✓	✓	X	PR14Y6	8.7	✓	✓	✓
PR05Y2	7.41	X	X	X	PR15Y6	10.06	✓	✓	✓
PR13Y2	13.97	✓	✓	✓	PR16Y6	8.62	✓	✓	✓
PR14Y2	8.7	✓	✓	✓	PR21Y6	5.22	X	X	X
PR21Y2	5.22	X	X	X	PR23Y6	6.73	✓	0.81	0.34
PR24Y2	8.57	X	X	X	PR01Y7	13.04	✓	✓	✓
PR02Y3	9.08	✓	✓	X	PR04Y7	15.59	✓	✓	✓
PR11Y3	11.68	0.49	0.39	0.63	PR14Y7	8.7	✓	✓	✓
PR15Y3	10.06	✓	✓	X	PR22Y7	8.75	X	X	X
PR01Y4	13.04	✓	✓	✓	PR02Y8	9.08	X	X	X
PR05Y4	7.41	X	X	X	PR15Y8	10.06	X	X	X
PR11Y4	11.68	0.33	0.39	X	PR22Y8	8.75	X	X	X
PR12Y4	12.13	✓	✓	✓	PR25Y8	10.51	X	X	X
PR13Y4	13.97	✓	✓	✓					
PR14Y4	8.7	✓	✓	✓					
PR22Y4	8.75	✓	✓	✓					
PR25Y4	10.5	✓	✓	✓					

* The figures are estimates arrived at by taking values 10% above and below the £1.30m figure.

Inspection of Table 3.6.5. shows that not only is the investment profile relatively stable over this range of earnings but also project 22 available in years 1, 4 and 8 with an internal rate of return of only 8.75% and project 14 available in years 2, 4 and 6 with an internal rate of return of 8.7% always tend to be included. On the other hand project 11 which is available in years 3, 4, 5 and 6 with an internal rate of return of 11.68% is marginal in years 3, 4 and 5. Clearly our analysis is inadequate unless we can explain these anomalies.

Returning to the dual analysis and ignoring all but the cash balance contribution and the debt capacity effects the reduced cost * (μ_j) of project j beginning at time t is given by

$$\mu_j = c_{jt}\rho_t - c_{jt+1}\rho_{t+1} \;\ldots\ldots\ldots + \lambda_t e_{jt} + \lambda_{t+1} e_{jt+1} \;\ldots\ldots \quad (3.6.16)$$

where in addition to the usual notations e_{jt} is the (book) earnings of project j in period t and λ_t here denotes the dual of the times covered constraint.

In the absence of constraints other than that on debt capacity then the equity relationship

$$\rho_t = (1+i)\rho_{t+1} \quad \text{(equation (3.6.9))}$$

and the dual equality (equation (3.6.2.)) associated with the raising of long term debt still hold. Hence

$$-\rho_t + {}^{r}\!/_{2}\rho_{t+1} + Kr\lambda_{t+1} + {}^{r}\!/_{2}\rho_{t+2} + Kr\lambda_{t+2} \ldots\ldots = 0$$

If we make the assumption** that λ_t is proportional to ρ_t (i.e. $\lambda_t = f\rho_t$ where f is a constant) then the value of f is given by the

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* Again this reduced cost has been calculated within the context of an "infinite" horizon model.

** This assumption is made purely on intuitive grounds. In the end the justification for it rests on the results that it is able to generate.

solution of the equation

$$p_t \left[-1 + \frac{r}{2i} + \frac{Krf}{i}\right] = 0 \qquad (3.6.17)$$

Thus f is given by the expression

$$f = \left(\frac{2i - r}{2Kr}\right) \qquad (3.6.18)$$

Substitution of this result into the expression for the reduced cost* of the project (equation 3.6.16) gives

$$\mu_j = NPV_j + f \times E_j \qquad (3.6.19)$$

where NPV_j is the net present value of cash flows associated with project j at the equity rate i and E_j is the value of the pre-tax earnings also discounted at i. Hence the present value (reduced cost) of a project is partially its net present value at the equity rate and partially a discounted earnings premium. The cash flow contribution can be adjusted for the finite horizon by rewriting NPV_j as $NPV_j + NPVH_j$ where NPV_j is the pre-horizon cash flows discounted at the equity rate and $NPVH_j$ is the net present value of the post-horizon cash flows discounted at the appropriate rate.**

If numerical values*** of expression (3.6.19) are calculated they can be seen to accord fairly well with the actual values of the reduced cost produced by an LP solution. Table 3.6.6. illustrates this point for various solutions reflecting assumptions about the level of earnings from existing projects and the presence or otherwise of non-debt capacity constraints.

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* Some confidence is gained in the correctness of the analysis by a comparison of the structure of this expression with the corresponding but more rigorously derived expression for the reduced cost which will be found in 5.5.

** A rate of 10% coinciding with the theoretical IRR cut off rate, was assumed throughout most of these computational experiments.

*** Since debt capacity effects are ignored in the post-horizon LP solution for the sake of comparison E_j consisted of only the pre-horizon earnings.

TABLE 3.6.6. Observed and Predicted Reduced Costs of Projects.

PROJECT	CASH FLOW CONTRI-BUTION	EARNINGS DISCOUNTED at 12% X	PREDICTED REDUCED COSTS	OBSERVED REDUCED COSTS					
				Normal Earnings		Earnings reduced by 10%		Earnings increased by 10%	
				ALL CONSTRAINTS	TIMES COVERED ONLY	ALL CONSTRAINTS	TIMES COVERED ONLY	ALL CONSTRAINTS	TIMES COVERED ONLY
PR01Y1	9.1	60.1	69.2	20.9	33.2	5.6	33.5	35.1	51.7
PR04Y1	30.4	44.9	75.1	61.0	67.9	56.5	63.1	72.7	66.9
PR12Y1	1.5	48.0	49.5	11.4	28.0	- 2.4	26.7	28.7	36.6
PR13Y1	20.7	52.8	73.5	67.2	76.7	58.4	73.2	89.9	70.8
PR16Y1	-41.8	56.1	14.3	-30.5	-14.5	-48.4	-16.4	- 4.5	0
PR22Y1	-15.3	70.1	54.8	2.2	6.3	16.1	7.4	5.2	9.6
PR23Y1	-45.1	47.9	2.8	-15.7	- 2.1	-11.9	- 2.6	- 3.4	- 5.3
PR03Y2	- 6.3	63.7	57.4	2.1	7.9	0	7.1	22.0	35.2
PR01Y2	27.2	36.3	63.5	8.3	0	- 3.7	0	28.6	42.9
PR05Y2	-59.5	47.6	-11.9	-49.5	-56.9	-61.9	-58.7	-30.5	-26.9
PR13Y2	18.5	47.0	65.0	12.2	24.6	35.2	21.5	32.3	51.2
PR14Y2	- 8.5	63.3	54.8	21.2	39.6	46.8	37.4	27.4	44.6
PR21Y2	-103.0	62.4	-40.6	-81.1	-80.9	-51.1	-89.1	-59.9	-57.3
PR24Y2	-38.2	53.8	15.6	-35.5	-20.6	-14.9	-26.8	-17.3	0
PR02Y3	-14.5	19.2	4.3	1.8	0.5	- 2.4	0.5	1.8	0.5
PR11Y3	- 2.[illegible]	26.8	24.4	0	15.9	0	16.0	0	16.0
PR15Y3	-10.6	19.2	9.1	1.2	3.9	- 6.6	4.0	1.5	3.9
PR01Y4	9.0	16.3	25.3	22.6	17.7	14.5	17.7	22.7	17.8
PR05Y4	-42.0	24.4	-17.6	-17.2	-26.6	-39.1	-26.6	-16.7	-26.6
PR11Y4	1.8	16.4	18.2	0	7.3	-17.1	7.3	0	7.3
PR12Y4	4.7	17.8	21.4	15.8	12.5	1.9	12.5	15.4	12.6
PR13Y4	15.9	34.9	49.6	33.3	46.7	18.5	43.6	34.2	43.4
PR14Y4	- 2.0	10.1	8.1	7.4	21.3	5.8	21.3	7.2	21.2
PR22Y4	-10.1	16.7	6.6	5.8	2.1	6.6	2.1	5.2	2.1
PR25Y4	- 8.7	32.2	23.5	17.3	16.8	10.0	16.8	16.3	16.9
PR02Y5	-10.1	9.6	- 0.5	- 1.5	- 4.1	- 6.0	- 4.1	- 1.1	- 4.[illegible]
PR03Y5	5.3	21.6	22.4	6.5	8.6	0	8.6	6.9	8.7
PR11Y5	1.3	7.0	8.3	0	1.9	- 6.7	1.9	0	2.1
PR21Y5	-68.3	29.8	-38.5	-39.5	-48.4	-44.4	-48.4	-39.9	-48.3
PR23Y5	-21.4	16.3	- 5.1	-13.9	-14.9	-18.1	-14.9	-14.3	-14.8
PR04Y6	18.6	9.1	27.7	-40.5	21.9	39.3	21.9	39.1	22.0
PR05Y6	-33.4	8.5	-24.9	- 1.3	-28.3	- 4.2	-28.3	- 2.6	-28.1
PR11Y6	1.3	- 0.6	0.7	27.8	0	37.8	0	23.8	0.1
PR14Y6	5.3	11.4	16.7	9.5	3.2	8.8	3.2	8.8	3.3
PR15Y6	- 4.7	2.1	15.4	11.1	- 3.5	9.4	- 3.5	10.4	- 3.5
PR16Y6	-19.6	2.4	-17.2	7.0	-19.2	4.3	-19.2	5.5	-18.9
PR21Y6	-62.0	19.3	-47.7	-17.7	-48.8	-20.5	-48.8	-20.4	-48.7
PR23Y6	-22.6	10.6	-12.0	1.2	-15.6	0	-15.6	0	-15.6
PR01Y7	13.3	- 0.9	12.4	13.1	9.2	12.5	9.1	13.2	9.1
PR04Y7	18.9	1.4	20.3	31.3	18.9	28.8	18.9	31.4	19.0
PR14Y7	- 3.4	6.6	2.6	3.2	-0.3	2.1	- 0.3	3.2	- 0.2
PR22Y7	- 7.2	6.2	- 7.2	- 8.9	-10.1	- 9.4	-10.1	- 8.9	-10.1
PR02Y8	- 9.0	0.2	- 4.7	- 4.9	- 4.7	- 4.9	- 4.8	- 4.9	- 4.7
PR15Y8	- 2.0	- 0.4	- 2.4	- 2.0	- 2.2	- 2.0	- 2.2	- 2.0	- 2.2
PR22Y8	- 4.0	3.2	- 0.8	- 4.9	- 3.0	- 1.9	- 3.0	- 4.5	- 2.9
PR25Y8	- 1.6	3.5	- 1.6	- 1.7	- 0.1	- 1.7	- 0.1	- 1.6	0

This is seen to be particularly true where all the constraints other than the cash balance and debt capacity constraints are suppressed. In general the observed reduced cost is less than the predicted reduced cost. This is because in calculating the reduced cost it is assumed that the debt capacity constraint was always binding. In this analysis the debt capacity premium is normally large (and positive) compared with the net present value.

In fact the net present value of the cash flows serves as a rough estimate of the lower bound of the reduced cost. The inter-period cash discount rate is approximately 12% whether the firm is borrowing or lending. Hence if the firm is always in a surplus position implying a zero value for the projects contribution to debt capacity then the reduced cost of the project is simply the discounted value of its cash flows at the (relatively high) 12% rate. Whereas if the firm is always in a deficit situation and forced to raise debt finance then the earnings premium makes a large positive contribution to the reduced cost of the projects. Therefore the predicted reduced costs of Table 3.6.6. which gives a full weighting to the earnings premium are effectively rough estimates* of the upper bounds of the reduced costs. Again this accords well with observation. In particular this analysis explains the relative attraction of certain projects with a low IRR (e.g. projects 14 and 22), since for both of these projects the debt capacity premium makes a substantial contribution to their overall value.

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* The estimates are not precise as in the Chambers case because it is assumed that the debt capacity constraint is either binding in every year or binding in non precise estimate would require consideration of the debt capacity constraint in each year independently.

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These predicted reduced costs should be a useful index of project profitability since they provide a measure of the attractiveness of a project with respect to the basic constraints of cash availability and debt capacity. In order to simulate the use of this index as a preliminary screening device, the linear programme was set up with a large weighting in the objective function proportional to the rank of the predicted reduced cost. Hence a term of the form $1000\sum_j (RANK_j - N)$ was included in the objective function where $RANK_j$ denotes the rank of project j according to its predicted reduced cost (see Table 3.6.6.) By varying the size of N using objective function parameterization the cut-off rank for project acceptability was altered. Again such a process of including projects into the investment schedule subject to a satisfactory performance on other financial criteria simulates an approach frequently adapted by users of financial statement generators. Table 3.6.7. shows the results of such an experiment.

TABLE 3.6.7. VALUE OF THE FIRM SELECTION ON REDUCED COST RANKING

N	Normal Earnings	Earnings plus 10%	Earnings less 10%
10	N/A		
11		2335	
12	1881	2338	N/A
13	1883	2348	
14	1887	2341	
15	1888	2346	1349
16	1883	2346	1349
17	1858	2334	1323
18	1854	2308	1320
19	1874	2312	1322
20	1870	2320	1322
21	1873	2310	1305
22	1869		1321
23	1862		1328
24	1847		1327
25	1833		1314
26			1320
27			1316
28		N/A	1312
29	N/A		1294
30			

It can be seen that in general the peak value of the firm occurs for a value of N around 15. Consideration of Table 3.6.6. shows that this solution corresponds to the adoption of all projects with a positive value for the predicted reduced cost. It can be seen also that this index is an improvement over the IRR index* in that the additional contribution to the value of the firm of the adopted project set is now £0.58M, £0.91M and £0.18M for the case of normal earnings from original projects and earnings 10% above and below this figure respectively. This figure compares with the corresponding optimal LP solution of £0.68M, £1.04M and £0.26M. The question still remains whether such a solution is acceptable.

The suggested approach here is typical of that of financial statement generators It is simple to use and understand and generates good, rather than optimal, project sets which satisfy general restrictions imposed on financial policy variables. These financial policy constraints themselves carry a cost of course and a further increase in the value of the firm is theoretically possible if the project set were chosen ignoring all but the restriction on debt capacity. In fact this cost is £0.12M, £0.15M and £0.09M in the particular cases considered here. Thus the loss due to using a non optimal but feasible solution method is less than the loss incurred because of considerations given to financial policy.** Because of adverse reactions by the financial markets it is not usually possible to ignore restrictions on financial policy variables. Under such circumstances it is vitally important to have a method which is

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* There are subsidiary peaks which roughly coincide with the adoption of an identical number of projects to that of the optimal LP solution. There are however slight discrepancies between the adopted sets in the two cases.

** In the case where financial policy considerations do not play a significant part in project selection we can revert to fairly simple models and of course rule of thumb solutions.

capable of exploring fully these constraints and here financial statement generators in conjunction with simple rules of thumb are frequently more flexible and more acceptable tools than complex and rigid LP models.

3.7 Conclusion

In this chapter just three models have been examined in detail. They all have the same basic structure, being concerned with the optimal selection from a set of investment projects according to a discounted cash flow criterion modified by restrictions on debt availability. In addition, the last model discussed includes many further restrictions on the possible investment and financing strategies. Optimal or near optimal solutions to each of these models were generated by an analysis of the dual inequalities associated with the financing instruments. Apart from providing numerical solutions to these models the analysis provided an insight into the impact on project selection of different structures for the restriction on debt, thus establishing a formal correspondence between the solutions generated by LP algorithms and those based on a discounting methodology. In the case of the Weingartner model it was seen that many of the rules of thumb proposed in the literature are merely attempts at approximations to the dual solution; while for the Chambers model, the existence of a general (and economically sensible) analytical solution was determined. The greater complexity of the last model discussed meant that the analysis was intuitive and lacking in rigour. The aim here was to identify the principal determinants of project selection that might serve as a preliminary screening device. Mathematical niceties were largely ignored and while some of the loose ends will be picked up in the next chapter the justification for the analysis must rest with the results generated.

However, the purpose of this chapter is certainly not to suggest that the methods of analysis developed here should replace linear programming approaches and a discussion of the relative merits of the two methods is an irrelevant side issue which diverts attention away from more important points.

The first of these is that for all three models there existed a class of projects whose acceptance is not doubted on purely economic grounds. Equally there existed a class of projects whose inclusion could not be justified on purely economic grounds. In this sense none of the proposed methods, whether simple discounting procedures or formal mathematical programming treatments can really claim superiority. The identification of good and poor projects with respect to a net present value criterion is not really a problem. Any of the methods mentioned in this chapter will readily identify these two sets. If there is any superiority in mathematical programming solutions it is in their ease with which they can make decisions about projects whose inclusion or otherwise may make a marginal impact on the value of the firm. Thus it would seem that in their current form, linear programming models of the capital investment decision provide the proverbial sledgehammer with which to crack the capital investment nut.

It must, though, be stressed that this in no way denies the contribution of the models of Weingartner and Chambers to the development of the subject. Weingartner's work, apart from forming a basis of all subsequent models, provides the framework within which the methods of discounting can be examined. Chambers model makes a valuable theoretical contribution to the problem of the treatment of equity financing, as well as of project valuation under restrictions on the book value of debt by providing a means of valuing a project's effect on debt capacity. From this discussion, emerges a recognition of the important role which can be played by mathematical programming models in the theory of normative decision making. In particular this brings us to the second main point of this chapter - that a major contribution of these capital investment models is in the provision of a framework for a rigorous analytical treatment of the impact on project valuation of capital market imperfections.

Weingartner (62) was aware of the analytical power of these models and he discussed at some length the effect on the optimal investment schedule of hard capital rationing. Bernhard (69) also makes extensive use of the analysis of the dual inequalities in his survey of capital budgeting models and more recently Myers (74) has used this approach for the valuation of projects in the light of modern developments in financial theory.

In chapter four this work will be drawn together in an attempt to move towards a more consistent theory of investment project appraisal. In particular two important and complementary ideas will be looked at within this primal-dual framework. These are, firstly a generalization of the Modigliani Miller cost of capital formula to deal with finite horizon projects and secondly an extension of the MM fundamental principle of valuation to deal with optimal growth paths in infinite horizon planning situations.

Continuing in the same vein, chapter five extends the analysis on the impact of various debt capacity constraints on project valuation by looking in detail at the valuation of one specific type of project opportunity - the financial lease contract. Whereas the impact of debt capacity restrictions may be marginal for many capital investment opportunities this will certainly not be true for the leasing decision, which is a simultaneous investment and (debt) financing opportunity. A mathematical programming formulation affords a natural framework within which such analysis can be carried out.

While the foregoing discussion clearly reveals an important role for LP models in finance, the original intention of this research and the explicit intention of most other workers in this field is the provision of management decision tools. If we now redirect our attention back to this issue and reconsider what was the intended primary role of LP models in finance we can discern two distinct lines of approach.

The first is to cling to the belief that the practical complexities of an actual planning situation are such that a LP formulation is still the only realistic way of determining an optimal plan and the existence of analytical solutions to simple models in no way invalidates the methodology.

There are two main objections to this belief. The first is the lack of evidence that complex LP models yield radically different results from fairly simple models. Certainly the evidence of this chapter suggests that even for relatively complex models simple discounting rules still remain very useful indicators of the attractiveness of projects. Moreover the more elaboration of the constraint set may well detract from, rather than enhance, their usefulness. Such models require an *a priori* specification of a minimum set of conditions which must be fulfilled by any plan and would seem an inadequate reflection of the planning process. In fact the whole approach seems far too rigid and naive ever to gain managerial acceptance.

The alternative approach is to regard LP as merely one aid in the battery of tools which are available to financial planners. Thus Chambers (72) in a follow-up paper to the 'Joint problem of Investment and Finance' discusses how his particular model might be implemented. He suggests that his LP model is best used in conjunction with a financial statement generator. Here the financial statement generator is used to explore alternative dividend policies while the LP model is used to select the optimal set of investment projects with respect to a particular dividend policy. While such a procedure is clearly a more acceptable use of LP models than attempts to use them as all embracing central decision processes, it does subscribe to the notion of LP models as preselection devices and it is their superiority in this role which has been subject to most questioning in this chapter.

In summary, in their current form LP models of the investment and financing decision would appear at best to perform inadequately their intended primary prupose of being a major decision tool of corporate financial planners. The central problems surrounding their usage in such a role arises from their inability to address directly the main issues in the financial planning process. The need is for methods of identifying and of exploring alternative financial strategies. While LP models are very effective in identifying feasible (and optimal) plans with respect to a particular criterion, they are far too rigid for the exploration of alternative strategies. In contrast simulation models have proved very effective for the exploration of alternative strategies, though their main weakness is their inability to give direct guidance to other and possibly improved strategies. In chapter six the issue of developing a mechanism which directly tackles the problem of identifying and exploring efficient financial strategies is discussed. The aim here is the development of a corporate financial planning model with the flexibility and managerial acceptability of a financial statement generator, yet, which retains the powerful decision logic of a mathematical programming model.

CHAPTER 4

Economic Objective functions, the Valuation of Investment Opportunities and the Finite Problem.

4.1 Introduction

In the previous chapter the discussion concentrated on the generation of approximate numerical solutions to LP models of the investment decision. The method of approximation was to take the discounted cash flow valuation of an investment project and, using Lagrangian multipliers, compute an adjustment for the projects contribution to debt capacity. This adjusted net present value incorporated the impact of the interactions which arise between the investment and financing decision under conditions of imperfect capital market.

Now among the core problems of modern corporate financial theory is the valuation of individual projects within the broader context of the firms total operating environment. While the problem is usually broached within the framework of perfect capital markets in equilibrium, it is the extent, and impact, of market imperfections that lead to severe analytical difficulties. Thus the emergence of mathematical programming models as a means of integrating the investment and financing decision, and of rigorously exploring the resultant interactions, provides a very powerful analytical tool for the development of more rigorous theories of valuation. In particular unlike more traditional methodologies of financial theory, where the arguments are developed in terms of infinite and,frequently constant,non-interacting income streams and financing outflows, mathematical programming provides a means of dealing with irregular and finite transaction patterns.

In this Chapter the discussion will corcentrate on the contribution of mathematical programming financial models to the development of normative models for project valuation.

The starting point for such a discussion is a brief resumé of approaches to the valuation of uncertain cash flows via the use of risk adjusted discount rates together with the implications of different capital structures for the investment decisions. The next section uses these ideas, which are central to financial theory,for the development of alternative formulations of objective functions which can be incorporated into mathematical programming models for financial planning. Within such a framework the formulations of Carleton (69), Weingartner (63) Chambers (71) and other authors are examined and from this framework a general theory of the sequential valuation of individual investment projects is developed. These ideas are extended to a more rigorous analysis of the cost of capital formula first derived by Modigliani and Miller and it is shown that their formula breaks down in general for the appraisal of finite or irregular investment cash flows. The remainder of the Chapter is devoted to various aspects of the horizon problem. Thus section 4.6 examines the way in which an analysis of the dual equations in the prehorizon period facilitates a consistent formulation of the horizon valuation, while the final two sections look at possible applications in financial modelling situations of the recently developed theory on the solutions to infinite time horizon LP models. Here both the nature of long run equilibrum solutions and the practical implications of using finite horizon approximations is examined in detail.

4.2 The Cost of Capital and Risk Adjusted Discount Rates.

Differenceswhich arise in the form of the expression for the cost of capital result from the two different approaches* taken towards the valuation of total corporate cash flows. The first approach, the net operating income (NI) approach computes the value of the firm by capitalizing the income (dividend) stream accruing to the shareholders and adding to this the value of debt. The alternative approach, the net operating income (NOI) approach computes the value of the firm by directly capitalizing the net operating income of the firm. Both these approaches to the derivation of a cost of capital will be considered here since they provide an insight into the structure of valuation formulae commonly used in financial planning models. It is also a convenient point at which to define a notation which will be used throughout this Chapter.

Let V_t = Value**of the firm at time t

ω_t = Value of debt at time t

ψ_t = Value of equity.

The relationship between these values at any point in time is

$$V_t = \omega_t + \psi_t \qquad 4.2.1.$$

i = Equity discount rate

r = (pretax) rate on the firms debt

* See Durand (59) or any modern standard text on financial management such as Van Horne (77) or Weston and Brigham (78).

** At this point the term 'value' has not been defined precisely; for instance whether it is book value or market value. The usage will be defined within the context of a particular argument. Neither will any precise interpretation of the various interest rates be offered until much later in the Chapter

In the simple* analysis that follows it is assumed for convenience that the income generated by the firm will persist at its current level for all future time.** This implies no net new investment on the part of the firm. In addition there are no corporate taxes and all income is redistributed as dividends.

Let x = The expected income from operations in each year

Then $$x = r\omega + i\psi \qquad 4.2.2.$$

and $$\frac{x}{V} = \frac{r\omega}{V} + \frac{i\psi}{V} \qquad 4.2.3.$$

If further K denotes the fraction of debt in the capital structure, then the weighted average cost of capital -a can be defined as $\frac{x}{V}$.

Hence $$a = Kr + (1-K)i \qquad 4.2.4.$$

The debate on the cost of capital centres on the way in which i, r and consequently a varies with the proportion of debt in the capital structure. Modigliani and Miller (58, 59) argue that under assumptions of a perfect capital market and no corporate taxes then the value of the firm and hence the value of the average cost of capital is independent of the degree and leverage. Their argument rests on the ability of an investor to undo corporate leverage by personal borrowing or lending. Against this Durand (59) questions the extent to which arbitrage can take place because of perceived differences between personal and corporate gearing, while Baxter (67) and Stiglitz (72) argue that non linear effects of bankruptcy costs will in the end imply an optimum debt equity

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* It should be pointed out that the analysis presented here follows one of the accepted patterns of analysis of financial theorists and is presented as a vehicle for introducing the concept of a cost of a capital. It will be argued that because of the excessively restricted assumptions made it is an inadequate theoretical framework for analysing the impact on investment decisions of debt financing.

** Hence the t subscript will be omitted in the remainder of this section.

structure.

In contrast the traditionalists* argue that both the returns required by debt holders and by equity holders vary as the degree of 'financial risk' or gearing varies, but in such a way that at some stage the weighted average cost of capital has a minimum.

With the introduction of corporate taxes the value of the company is altered because of the tax deductability of the interest payments and thus the after tax earning where T is the tax rate then

$$\begin{aligned} x^T &= (x - r\omega)(1 - T) + r\omega \\ &= x(1 - T) + rT\omega \end{aligned} \qquad 4.2.5.$$

According to the NOI approach, so vigorously argued by Modigliani and Miller (63,69), the expected income stream $x(1 - T)$ should be capitalised at the constant rate a_o where a_o is the rate of capitalisation of a pure equity stream from the firm and the 'certain' tax savings stream should be capitalised at the 'risk-free' rate r. Thus the value of the firm is given by $v_o = \frac{(1 - T)\bar{x}}{a_o} + T\omega$ 4.2.6. and consists of the firm market value under all equity financing plus the present value of tax generated capital allowances.

In contrast the NI approach, adopted by traditional theorists argues that the after tax residual earnings should be capitalised at i, being that portion of the income attributable to shareholders, with the interest component capitalised at r.

This gives

$$v = w + \frac{(\bar{x} - rT\omega)(1 - T)}{i} \qquad 4.2.7.$$

Using as a definition** of the cost of capital $x(1-T)/v$ then these

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* See for example Solomon (63)

**It should be emphasised that this is merely one of many possible definitions (see Nantell and Carlson (75). An alternative definition will be provided in section 4.4.

two approaches give a value for the cost of capital of

$$a = a_o(1 - KT) \qquad 4.2.8.$$

in the Modigliani and Miller (MM) case and

$$a = i(1 - K) + Kr(1-T) \qquad 4.2.9.$$

for the traditional case. While it can be seen that both of these costs of capital are functionally dependent on K, the traditionalist further argues that the variation of the equity rate and the debt rate with K is such as to result in a minimisation of the cost of capital and a consequent maximisation of the value of the firm.

In the MM case the average cost of capital appears to decrease uniformly as the amount of debt increases to the point at which there would be no equity financing. A resolution of this paradox of bankruptcy is offered by Robichek and Myers (65) who argue that the possibility such that there is some limit on the proportion of debt in the capital structure. Thus both approaches introduce a debt capacity restriction which takes the form of a target limit on the percentage of debt in the total capital structure. The incorporation of such a restriction on debt turns out to have had a profound influence on the structure of mathematical programming models used for financial planning.

One further point which has been largely glossed over so far in this thesis and is of particular relevance to this Chapter is the implicit assumptions made in the approach to the valuation of returns from risky investments. In essence, the approach adopted throughout this thesis has been to value a risky investment by discounting the expected cash flow stream resulting from the investment at a rate adjusted for the 'risk' of that stream. The theoretical justification for such an approach lies in the work of Sharpe (64), Litner (65) and Black (72) all of whom examined the determinants of the market price of a security or risky asset under equilibrium portfolio conditions. Their work, on the capital asset pricing model (CAPM) uses a two parameter specification of risk in which it assumed that the investor is economically rational preferring more expected wealth to less expected wealth, and is risk averse, measuring risk by the standard deviation of the return from an investment. The development of the theory assumes perfect capital markets, homogeneous expectations of returns from investors and equality of borrowing and lending rates. The impact of relaxing some of the assumptions in capital asset pricing theory are discussed by, among others, Mao (71), Jensen (72). In the original, Sharpe, Litner and Black treatment of the CAPM a one-period horizon model is assumed. The extension to a multi-period model has been carried out by Brennan (73) and Farma (77) and it is this last mentioned author who provides both a detailed analysis of, and deviation of, the form of the valuation of risky investments used in this thesis.

Farma's starting point is that according to CAP theory, the excess one-period return required of a risky investment over that of a risk free asset* is proportional to the excess market return** over a risk-free asset, where the constant of proportionality depends on the covariance of the return from the individual risky asset with that of the market portfolio. He then extends and generalizes this one-period model into a multi-period valuation model. A sufficient condition for the multi-period model to take the form of a discounted sum of expected cash flows at a constant

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* Typically it is argued that Government stocks provide such a risk-free asset.

** This is the return expected from a portfolio of all the risky securities of the market held in proportion to their market value.

discount rate is that the risk-free rate is constant and known and that the future covariances of the cash flows from the investment with the market returns are also constant and known. In terms of most of the valuation models discussed in this thesis this condition is satisfied provided that the operating environment of the firm is stable and provided that the firm has a fixed (as measured in terms of risk) investment policy with respect to this environment.

4.3 Valuation models and the structure of Objective functions

As was discussed in the last section, two different approaches have been taken to the valuation of the firm. The approaches are the net income approach and the net operating income approach. Both of these need to be considered here since they give different, though not necessarily contradictory, forms for the cost of capital. In view of the furore created by the debate on the effects of capital structure on the cost of capital, the non-contradictory nature of the results emerging from some of the analysis to be presented in this chapter might seem surprising. The reasons for such results arise from the restrictive nature of the assumptions which are necessary in most linear programming models which are to be used for financial planning.

In essence such models are required to attribute a value row to a decision z_t taken at time t in the future. Thus it is necessary to identify a mapping $PV : z_t \to \nu_o$ where ν_o is the value now of the decision z_t. The requirement that such a function is linear implies that the discount rate is a constant and independent of any decisions taken in the intervening period, including decisions taken about the capital structure. While such an assumption might seem prohibitively restrictive if LP models are being used for the development of financial valuation theory, it must be viewed within the context of such models. The basic structure of these models is that they consist of a valuation (objective) function and a debt capacity constraint. In general the relative cheapness of debt results in this debt capacity constraint being binding in most periods. Thus this produces the stable capital structure necessary for the assumption of constant discount rates.

The lack of contradiction between NI and NOI approaches is further a consequence of defining only two of the three interlinked discount rates which relate to debt, equity and operating flows. The third discount rate is a deduction from the model and is dependent on the structure of the model. It is important to stress this difference between discount rates which are deductions from a model and those which are either implicitly or explicitly prior specifications to a model, since this problem is a constant source of misunderstanding.

In particular the early attempts at the formulation of appropriate objective functions for use in mathematical programming models were subjected to the severe criticism of Baumol and Quandt of primal-dual inconsistency*. The result of this was that the objective functions of the early models ** were solely horizon valuations. Thus the constraint set was specified over the pre-horizon period while the objective function merely valued the post-horizon effects. Hence the valuation of individual projects was such that the pre-horizon period valuation was carried out via a dual pricing mechanism while the post-horizon valuation was carried out using a predetermined constant equilibrium cost of capital.

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* See section 1.2

** See for example the discussion of Weingartner's and Chambers' models in section 1.4

Clearly such a methodology is unsatisfactory from a theoretical point of view, since the valuation is arbitrarily dependent on the horizon, and from the practical point of view, since here net present value methods are in general preferable to net terminal valuations. Fortunately it was possible in Chapter two to identify the source of this paradox and to provide a satisfactory resolution of it.

The paradox stemmed largely from a misinterpretation of the model where an attempt was made to make statements about consumption preferences from a model which specifically excluded the consumption alternative. This error was further compounded by the assumptions that the firm and/or individual investors were excluded access to the capital market. Thus in order to make progress it is necessary to specify the nature of possible consumption functions* and market discount rates and to take cognizance of role of the capital market in the determination of these rates.

If we thus adopt the alternative model proposed by Baumol and Quandt of maximising the value of the utility of withdrawals, then we have in the notation introduced earlier

$$\text{MAX}\,\psi = \psi_o(D,E) \qquad 4.3.1.$$

A well defined mathematical structure can be imposed on the function ψ_o by reference to the fundamental principle of valuation as expounded by MM (61). The return to equity (i) can be defined in terms of the net increase in share price plus any dividends flows in relation to the initial share price. The relationship is thus

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* In fact as we shall see, under assumptions of perfect capital markets then the consumption decision is irrelevant to the investment decision.

$$i = \frac{d_t + p_{t+1} - p_t}{p_t} \qquad 4.3.2.$$

where p_t price of shares at the start of period t and d_t is the net dividend per share paid at the end of period t.

Equation 4.3.2. can be rewritten in the form

$$p_t = \frac{d_t + p_{t+1}}{(1+i)} \qquad 4.3.4.$$

If in addition the number of shares outstanding at the start of t is n_t.

Then $$\psi_t = n_t p_t = \frac{n_t d_t + n_t p_{t+1}}{(1+i)} \qquad 4.3.5.$$

and

$$n_t p_{t+1} = n_{t+1} p_{t+1} - (n_{t+1} - n_t) p_{t+1} \qquad 4.3.6.$$

$$= \psi_t - E_t \qquad 4.3.7.$$

giving

$$\psi_t = \frac{D_t - E_t + \psi_{t+1}}{(1+i)} \quad \text{where } D_t = n_t d_t \qquad 4.3.8.$$

Now while equation 4.3.2. defines the return on equity mathematically, the actual value of i is exogenously determined by the capital market forces. These take into account the business* risk in the firms operating income and financial risk involved in the firm's capital structure. With the assumptions that the firm continues to invest in projects with the same degree of business risk and that the debt capacity constraint ensures a stable capital structure then i can be regarded as a constant. If it is further assumed that all new issues are in the form of rights which are totally taken up by existing shareholders,** then the recursive use of

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* In keeping with the discussion in section 4.2, business risk must now be defined in terms of the covariance of the returns on the firm's project with the return on the market portfolio.

** This simplifying assumption is necessary since we are concerned with maximisation of shareholder wealth. If the body of shareholders were allowed to change, it is no longer clear how the possible conflicting interests of existing and future shareholders can be catered for.

expression 4.3.8 gives a value for the equity of the form

$$\psi_o = \sum_{t=1}^{H-1} \frac{D_t - E_t}{(1+i)^t} + \frac{\psi_H}{(1+i)^H} \qquad 4.3.9.$$

In essence the capital market has imposed the necessary structure on the utility function of equation 4.3.1. This generates with a very convenient form of objective function for incorporation into linear programming models. The first two variables D_t, E_t present no problems in evaluations since they are readily incorporated as decision variables within the model though of course ψ_H does present the now familiar horizon value problems.

This valuation formula was first developed by Carleton (70) and most of the models discussed so far can be considered to be specific examples of it. The derivation presented here is to clarify the assumptions and the context of the formula and to enable the limitation of any conclusions drawn from such a model to be clearly seen.

If we refer back to some of the models already discussed in the first Chapter, the objective function in the Weingartner model is trivially maximize $\psi_H = V_H(X) - \omega_H$ or the equity proportion of the post-horizon cash flows. Here $V_H(X)$ is the post-horizon value of the firms net operating income valued at the debt rate. The implications of such a model are that there is a predetermined dividend policy and that the firm is operating under conditions of perfect certainty.

Of more interest is the Chambers (71) model. In this model the net present value at the horizon (NPVH) of rights, as well as debt and project cash flows, are treated explicitly. In the valuation of equity and debt Chambers argues:

'Managers should be led to make a new rights issue only if there is some increase in the value of the firm to existing shareholders after giving subscribers to the new issue a return (in this example of 12 per cent). A NPVH of the new rights issue is, therefore, defined as that amount at the horizon which, taken together with the dividends to which they will be entitled over the planning period, gives a return of 12 per cent to new investors.'

A similar argument is used to obtain NPVH for debentures issued in the planning period by discounting post-horizon cash flows associated with these at 4 per cent, while post-horizon cash flows from investment projects are discounted at a weighted average cost of capital.

Thus for rights the post-horizon value is the alternative cost to the shareholders of money they subscribe in t. This is given by

$$- p_t(1+i) \qquad \text{for} \quad t = H - 1$$

$$- \left[p_t(1+i)^{H-t} - \sum_{\tau=t+1}^{H-1} d_t(1+i)^{H-\tau} \right] \quad \text{for } t = 1, H - 2 \qquad 4.3.10.$$

where p_t is the price of a unit of equity issued at t and d_t is the dividend per share which the firm plans to pay in t.

The whole valuation model in fact can readily be seen as an example of the analysis developed above. The model relating the value of future equity streams to the shareholders is

$$\psi = \sum_{t=1}^{H-1} \frac{1}{(1+i)^t} (D_t - n_t p_t) + \frac{\psi_H}{(1+i)^H} \qquad 4.3.11.$$

where the additional notation n_t is the number of rights issued at time t. Since in the Chambers' model the dividend policy is predetermined then D_t can be written in the form*

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* There appears to be a slight anomaly in the treatment of t=1 with $D_1 \equiv D_1^0$. The reason for the form chosen should be apparent from the result.

$$D_t = D_t^{\,0} + d_t \sum_{\tau=2}^{t} n_\tau \qquad 4.3.12.$$

where $D_t^{\,0}$ is the dividend planned on the existing shares.

Hence

$$\psi_o = \sum_{t=1}^{H-1} \frac{1}{(1+i)^t} \left[D_t^{\,o} + d_t \sum_{\tau=2}^{t} n_\tau - n_t p_t \right] + \frac{\psi_H}{(1+i)^H} \qquad 4.3.13.$$

$$= \sum_{t=1}^{H-1} \frac{D_t^{\,0}}{(1+i)^t} + \frac{1}{(1+i)^H} \left\{ \sum_{t=1}^{H-1} \left[d_t (1+i)^{H-t} \sum_{\tau=2}^{t} n_\tau - n_t p_t (1+i)^{H-t} \right] + \psi_H \right\} \qquad 4.3.14$$

$$= \sum_{t=1}^{H-1} \frac{D_t 0}{(1+i)^t} + \frac{1}{(1+i)^H} \left\{ \sum_{t=1}^{H-1} n_t \left(\sum_{\tau=t+1}^{H-1} d_\tau (1+i)^{H-\tau} - p_t (1+i)^{H-t} \right) + \psi_H \right\} \qquad 4.3.15.$$

The first term is just a constant and the expression in curly brackets is just a constant times the following expression

$$= \sum_{t=1}^{H-1} n_t \left[- p_t (1+i)^{H-t} + \sum_{\tau=t+1}^{H-1} d_\tau (1+i)^{H-\tau} \right] + \psi_H \qquad 4.3.16$$

We still need a valuation for ψ_H the value of the equity portion of the firms terminal value. Referring back to the earlier work we have the equation 4.2.1.

$$\psi_H = V_H - \omega_H$$

V_H is related to the value of the firms future after tax cash flows and ω_H refers to the debt servicing and repayment streams. By discounting the former at the weighted average cost of capital and the latter at the debt rate, Chambers is essentially adopting a traditional approach to valuation. It will be argued that such a valuation is consistent in the sense that all the valuation procedures used within this model are consistent with the explicit and implicit assumptions made about the behaviour of the capital markets. To justify this statement it will be necessary to develop a more general framework for the analysis of the relationship between individual investment and financing projects which is implied by the structure of an LP. While this occupies most of the next two sections it is worth examining briefly a paper by Bhaskar (74) which illustrates some of the pitfalls involved in attempting to devise a consistent formulation of a financial linear programming model. In this paper Bhaskar attempts a rigorous analysis of the way in which borrowing and lending instruments might be incorporated into a capital budgeting model in the light of modern financial theory. However, his choice of a modified Weingartner model as the analytical framework is singularly unfortunate.

The model* as presented is

$$\text{Max} \quad \sum_{j=1}^{n} \hat{c}_j x_j + \hat{c}_v V_t \qquad 4.3.17.$$

subject to

$$-\sum_{j=1}^{n} c_{tj} x_j + V_t - (1+r_L)\, V_{t-1} - \omega_t + (1+r_B)\omega_{t-1} \leq F_t \qquad 4.3.18$$

$$\omega_t \leq B_t \qquad 4.3.19$$

$$0 \leq x_j \leq 1$$

The notation used is the standard one adopted in this thesis with the caveat that the interpretation of the coefficients in the objective function is slightly different. Here $\hat{c}_j = \sum_{t=0}^{\infty} \frac{c_{tj}}{(1+a)^t}$

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* This includes the minor modification of considering 1 year debt only. Bhaskar incorporates debt with a longer repayment period.

is the net present value (as opposed to net terminal value) at a weighted average cost of capital (a) while $\hat{c}_v = \frac{rL - a}{(1+a)^{t+1}}$ is similarly a net present value of £1 of lending at the same weighted average cost of capital. Two immediate problems present themselves within such a valuation framework. One is the choice of discount rates and the other is the implication that such discount rates carry about the capital markets.

The use of a weighted average cost of capital is motivated by the MM argument that the weighted average cost of capital (unlike the equity rate) is independent of debt decisions in a perfect market. While the use of a constant weighted average cost of capital facilitates a linear structure, the MM hypothesis specifically assumes perfect capital markets. As Bhashar himself admits in a postscript

> 'is it valid to assume an MM type world in a (hard) capital rationing situation? The problem here is that it may not be possible for arbitrage to take place because of capital rationing.'

This gives rise to the second main problem. Bhashar in using a constant cost of capital has actually assumed that while the firm has limited access to the capital market (as implied by the constraint on debt) the shareholders themselves have perfect access (i.e. unlimited personal borrowing or lending at the debt rate r). While this assumption might not seem totally unacceptable, Bhashar has made no provision in the model for the firm to raise further equity capital. Thus he has included shareholder investment/consumption preferences within the valuation procedure but omitted from the model the necessary mechanism whereby these preferences might be exercised.

There is also an inconsistency in the incorporation of lending into the model. The implication of the NOI income approach assumed by MM is that projects should be valued at a rate appropriate to their risk. The implication of this is that the lending project which consists of cash flows of -1 and $1 + r_L$ should be discounted at r_L the lending rate. It would thus disappear from the objective function. While Bhashar argues that such a solution is suboptimal using a two project counterexample*, he misses the point that the lending project has altered the business risk and thus the equity return require by shareholders.

Clearly if the intention is to use LP models for developing financial theory, or indeed, as a decision making aid, then a great deal of care must be taken in structuring the model. In the next section a framework is developed which allows for a more thorough analysis of the implicit assumptions made with LP models for financial planning and in section 4.6 illustrates the methodology applied to the model introduced in section 1.7.

4.4 The Cost of Capital : a General Framework

While empirical evidence on the cost of capital debate has proved inconclusive** the resolution of the issue is of less immediate importance to this thesis than the shortcomings of the theoretical analysis. In particular two of the assumptions which were made in the analysis presented in section 4.2 are sufficiently restrictive to invalidate the application of the cost of capital

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* A rigorous development of a model incorporating the assumptions of modern financial theory will be presented and analysed in section 4.5.

** See Durand (59); Weston (63).

formula in nearly all capital investment decisions. Clearly, a cost of capital formula whose derivation is subject to the assumption of no net new investment is not the most appropriate method for screening new capital investment projects. Further the assumption that the net contribution of the set of investment projects will be a constant income stream in perpetuity must be considered at best a very poor approximation to reality. Thus the task of this section is to present a method of analysis which does not require these assumptions.

With this aim in mind, consider the following model of the set of investment and financing decisions facing the firm

$$\text{MAX} \quad \psi(\mathbf{X},\mathbf{D},\mathbf{E},\boldsymbol{\omega}) \qquad 4.4.1$$

subject to cash balance restrictions

$$-\sum c_{oj}X_j - \omega_o + D_o - E_o = F_o \qquad 4.4.2$$

$$-\sum_j c_{tj}X_j - \omega_t + (1+r(1-T))\omega_{t-1} + D_t - E_t = F_t \quad t=1,H \qquad 4.4.3$$

and a restriction on the level of debt finances

$$\omega_t \le \phi_t\,(\mathbf{X},\mathbf{D},\mathbf{E}) \qquad 4.4.4$$

plus a scale constraint on the project

$$0 \le X_j \le 1 \qquad 4.4.5$$

and the usual non-negativity conditions where the heavy type denotes vectors whose individual components are interpreted* as follows:

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* Again the notation is presented here for convenience and complete list of the mathematical notation used throughout this thesis can be found in appendix II.

D_t - dividend paid in time period t

E_t - equity issued in time period t

ω_t - debt financing in period t

X_j - scale of acceptance of investment j

and the other symbols are:

c_{jt} - cash inflow from project j in t

F_t - funds from existing project

T - corporate tax rate

ϕ_t - debt capacity in time period t

If we further denote by:

ρ_t - shadow price on additional cash

λ_t - shadow price on debt capacity

μ_j - shadow price on the scale of acceptance of project j

H - the planning horizon

then Kuhn-Tucker optimality conditions give for dividends

$$\rho_t - \sum_{t=0}^{H} \lambda_t \frac{\partial \phi_t}{\partial D_t} \geq \frac{\partial \psi}{\partial D_t} \qquad 4.4.6$$

and for equity issues

$$-\rho_t + \sum_{t=0}^{H} \lambda_t \frac{\partial \phi_t}{\partial E_t} \geq \frac{\partial \psi}{\partial E_t} \qquad 4.4.7$$

while for debt the relevant inequality is

$$-\rho_t + (1+r(1-T))\rho_{t+1} + \lambda_t \geq \frac{\partial \psi}{\partial \omega_t} \qquad 4.4.8$$

and that for the scale of acceptance of a project is

$$\mu_j \geq \frac{\partial \psi}{\partial X_j} - \sum_{t=0}^{H} \rho_t c_{tj} - \sum_{t=0}^{H} \lambda_t \frac{\partial \phi}{\partial X_j} \qquad 4.4.9$$

The right hand side of inequality 4.4.9 can be considered to be a generalisation* of the net present value concept to include the project contribution to debt capacity. Hence if the expression on the right hand side is positive then the project is included in the optimal solution, if the expression is negative then the project is rejected whereas a zero value results from partial acceptance. The project decision is thus dependent on its own direct contribution to the value of the firm $\frac{\partial\psi}{\partial x_j}$ and to the debt capacity $\frac{\partial\phi_t}{\partial x_j}$ as well as the marginal value of funds ρ_t and the marginal value of extra debt λ_t.

In general both ρ_t and λ_t can be determined by consideration of the financing opportunities. Thus if we assume that equity issues can be treated as negative dividends** then this implies

$$\frac{\partial\phi_t}{\partial D_t} = - \frac{\partial\phi_t}{\partial E_t} \qquad 4.4.10$$

and

$$\frac{\partial\psi}{\partial D_t} = - \frac{\partial\psi}{\partial E_t} \qquad 4.4.11$$

giving

$$\rho_t - \sum_{t=0}^{H} \lambda_t \frac{\partial\phi_t}{\partial D_t} = \frac{\partial\psi}{\partial D_t} \qquad 4.4.12$$

while if debt financing is being undertaken, inequality 4.4.8 becomes an equality and we have

$$- \rho_t + (1+r(1-T))\rho_{t+1} + \lambda_t = \frac{\partial\psi}{\partial\omega_t} \qquad 4.4.13$$

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*See Weingartner (74) or Peterson (69) p. 446 for a fuller discussion of this point. Extensive use has already been made of this idea in chapter III.

**This assumption requires no difference in effective tax rates between dividends and retained earnings, no transaction costs and that all rights are taken up by existing shareholders. These assumptions are less restrictive than they might appear at first sight.

Equations 4.4.12 and 4.4.13 are usually sufficient to define ρ_t and λ_t from this we can deduce a value for the right hand side of inequality and hence the appropriate valuation formulae for the contribution from a potential investment.

These observations provide for a more rigorous definition of the term 'cost of capital' than that which is to be found in standard texts for use in capital investment appraisal.

If f is a function $1 \Big/ \prod_{\tau=1}^{t} (1+u(\tau))$ of the parameters u,t where $u = g(\rho,\lambda,\psi',\phi')$ such that for project j involving net cash inflows c_{jt} in t

$$\sum_{t=0}^{\infty} f(u,t)\, c_{jt} \begin{cases} < 0 & u(t) > \rho_t^* \\ = 0 & u(t) = \rho_t^* \\ < 0 & u(t) < \rho_t^* \end{cases} \qquad 4.4.14$$

then ρ_t^* is the cost of capital. In the type of model being considered here u and thus ρ_t^* is in general a function g of ρ,λ,ψ',ϕ' where both the dual vectors ρ,λ and the vectors of derivatives ψ',ϕ' can in turn usually be expressed in terms of the interest and tax rates supplied to the model. There are two important points to be made about the valuation formula and cost of capital formula developed here.

The first is one to which frequent reference has already been made and will be only briefly mentioned again here. In finite horizon linear programming models where the impact of any decision extends beyond the horizon period the valuation formula, and hence the cost of capital, may depend on the choice of horizon. The second is that any investment project valuation formula is critically dependent on the assumptions of the impact of debt and equity on the value of the firm and the value of debt capacity. Once these assumptions have been made then the appropriate formula for valuation of an investment project follows as a logical consequence. Such an observation provides a method of checking the consistency of the formulation, since presumably the resultant valuation of an investment project will reveal the nature of any implicit assumptions made about investment cash flows. Both these points will be briefly illustrated for the Chambers (71) model.

The valuation model used by Chambers has already been discussed in some detail in the previous section. The objective function ψ is a discounted value of the cash flows associated with the issue of rights, debentures and investment projects. In contrast the debt capacity ϕ is a constant multiple g of the book value of new equity and retained earnings and is thus affected by the issue of rights, profits retained from investments and such expenses as flotation costs of new financing.

The analysis of the dual equations associated with equity issues and with debt financing has already been carried out in section 3.5. It was shown that when debt financing was being used then the dual on the cash balance equation (equation 3.5.35) could be approximated by

$$\rho_t = \frac{(1+i)^{H-t} + g(1+r(1-T))^{H-t}}{1 + g}$$

while if the firm was in a cash surplus situation in the sense that it was lending money to the fixed interest market, the cash balance dual was given by the expression (equation 3.5.34)

$$\rho_t = [1+r(1-T)]\rho_{t+1}$$

In both these cases, equation 3.5.36 , gave the debt capacity constraint dual as $\lambda_t = \frac{1}{g}[(1+i)^{H-t} - \rho_t]$.

The resulting project valuation, as represented by the reduced cost, thus values cash flows at the appropriate borrowing or lending rate upto the horizon. Moreover, the borrowing rate in the pre-horizon period turns out to be a weighted average cost of capital rate where the weighting factor is based on a book value figure. In the post horizon period, there is no information as to whether the firm is in a cash surplus situation or a cash deficit situation. Chambers in fact chooses a weighted average cost of capital figure, where the weighting is again in terms of book values. Thus the model gives a consistency at least in the approach adopted, if not in the precise functional form, to valuation.

Some inconsistencies do arise but these are of a technical nature, arising from the finite horizon, rather than inconsistencies arising from the assumptions implied by the choice of valuation model and the nature of the restrictions on the use of debt. These do result in project valuation being horizon dependent, though this dependency is not critical.* Thus in the pre-horizon period the weighting is done after the 'time factor' has been applied to the equity and debt rates whereas in the post-horizon period the weighting is done prior to the application of a time factor. In addition while a project's (small) contribution to debt capacity is valued at shadow price on debt in the prehorizon period, this contribution is ignored post horizon.

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* Thus in Chapter three it was seen that valuing a project merely by discounting at the weighted average cost of capital, which is equivalent to a zero horizon time, did not lead to major distortions in the investment decision.

Although the Chambers' model provides a specific illustration of some of the conclusions that can be drawn from a general approach to project valuation, this analysis alone does not justify the rather elaborate framework which has been developed in this section. The justification presented in the introduction to this section for the development of the framework was that the resulting valuation formulae,and hence any cost of capital deduced from it, are not dependent upon any assumed regularity of perpetuity of cash flows. In the next section it will be shown using a dual analysis, that the widely accepted MM cost of capital formula as represented by equation 4.2.8. does not hold for finite or non-constant cash flows.

4.5 The MM cost of capital formula for finite and irregular flows.

Myers and Pogue (74) developed a model to be used for practical financial planning which they argue is in accordance with modern financial theory. In particular they sepcifically assume two basic postulates of capital market theory to hold namely*

"1. That the risk characteristics of a capital investment opportunity can be evaluated independently of the risk characteristics of the firm's existing assets or other opportunities.

2. The Modigliani-Miller result that the total market value of the firm is equal to its unlevered value plus the net present value of taxes saved due to debt financing."

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* ibid p. 580

For practical planning purposes, Myers and Pogue admit to a certain degree of market imperfections, introducing constraints on liquidity and dividend policy. However, in a separate paper Myers (74) considers only the impact of a constraint on debt capacity on the rules for project selections. Myers' main attention is on the theoretical structure of the model and his subsequent mathematical analysis is both obtuse and incomplete. In this subsection the model will be cast into a more convenient conceptual form and its implications will be explored using the ideas and methodology of section 4.4.

In essence Myers' model can be written

$$\text{MAX } V_0 = \sum_{t=0}^{H} \left| \sum_{j=1}^{n} \frac{c_{tj}X_j}{(1+a_0)^t} + \frac{rT}{(1+r)^{t+1}}\,\omega_{t+1} \right| \qquad 4.5.1$$

subject to

$$-\sum_{j=1}^{n} c_{0j}X_j - \omega_0 + D_o - E_0 = F_0 \qquad (\rho_0) \qquad 4.5.2.$$

$$-\sum_{j=1}^{n} c_{tj}X_j - \omega_t + (1+r(1-T))\,\omega_{t-1} + D_t - E_t = 0 \qquad (\rho_t) \qquad 4.5.3.$$

$$(t=1,H)$$

$$\omega_t \le K(V_t^X + V_t^\omega) \qquad (t=0,H) \qquad (\lambda_t) \qquad 4.5.4.$$

$$(1+a_0)V_{t-1}^X = \sum_j c_{tj}X_j + V_t^X \qquad (t=1,H)\,(\theta_t^X) \qquad 4.5.5.$$

$$(1+r)V_{t-1}^\omega = rT\omega_{t-1} + V_t^\omega \quad (t=1,H)\,(\theta_t^\omega) \qquad 4.5.6.$$

$$0 \le X_j \le 1 \qquad (\mu_j) \qquad 4.5.7.$$

plus the usual non-negativity conditions, except for θ_t^X, θ_t^ω which are free-variables, Some preliminary comments on the structure of the model are necessary prior to any mathematical analysis. The objective function 4.5.1. is to maximize the market value of the firm where the market value according to MM is the market value of the unlevered firm plus the present value of tax-savings. The market value of the unlevered firm is just the sum of the after tax cash flows from projects $-c_{tj}$ discounted at a rate a_0, which is assumed to be the appropriate rate for the particular risk of that project assuming a base-case of all equity financing.* The present value of tax savings is just the after tax cash flows on one year debt discounted at the rate r and thus consists of the sum of terms like $-\frac{\omega_t}{(1+r)^t} + \frac{(1+r(1-T))}{(1+r)^{t+1}}\omega_t$. Hence the objective function is a direct consequence of the two postulates ennunciated at the beginning of this section.

Equations 4.5.2 and 4.5.3 just represent the familiar cash balance equations and do not present any particular problems. The restriction on the level of debt - equation 4.5.4. - is such that the debt at time t must be less than some fraction K of the total market value of the firm at time t. Hence it is assumed that the firm readjusts its debt level at the end of every period in terms of its total market value at that time and that this level is maintained during the next period. Equations 4.5.5. and 4.5.6. and thus merely convenient definitions of V_t^X, V_t^W for carrying out the necessary revaluation of the firm in each period. In terms of the discussion of the previous section the total market**value of the firm (V) is defined in terms of X and ω while the debt capacity ϕ is also

*See Myers and Pogue ibid p. 587.

**The use of total market value V rather than the value of equity ψ implies that the function V such be substituted for ψ in the analysis of the previous section.

functionally dependent on the decision vectors X and ω. Since this analysis to be presented shortly is in terms of net present values the finite horizon does not present any problems in theoretical project valuations since H can be defined to occur after the last of the projects cash flows. However, a full understanding of the model does not emerge until the effect of H tending to infinity is considered, and this will be done in section 4.7.

The assumption of dividend irrelevancy is reflected in the fact that the inclusion of the terms D_t, E_t in the cash balance equations do not affect the value of the firm, hence $\frac{\partial v}{\partial D_t} = \frac{\partial v}{\partial E_t} = 0$. An immediate consequence of this is that $\rho_t = 0$ for all t and this observation simplifies the analysis considerably.

However, since the mathematical analysis becomes algebraically complex it is perhaps easiest to illustrate the approach by considering investments lasting over periods 0 and 1 only. Now the dual inequalities for the initial debt and the initial value of the debt stream give respectively

$$\lambda_0 - rT\theta_1^W \geq \frac{rT}{1+r} \qquad 4.5.8.$$

$$-K\lambda_0 + (1+r)\theta_1^W \geq 0 \qquad 4.5.9.$$

The solution of this system is

$$\lambda_0 = \frac{rT}{1+r(1-KT)} = \frac{rT}{1+r'} \quad \text{where } r' = r(1-KT) \qquad 4.5.10$$

and

$$\theta_1^W = \frac{K\lambda_0}{1+r} \qquad 4.5.11$$

The positive value to λ_0 tells us that the debt constraint is binding, as indeed one would expect in an MM world, since the tax shield on debt results in debt being relatively cheap.

Consider the case of a single project with investment $c_{0j} = -1$ to be made now (time t=0) and an after tax cash flow of $c_{1j} = 1 + x'$ one year later*. The analysis of the dual inequalities associated with the scale of acceptance of a project gives the (generalized) net present value of this one period project taken at full scale** as

$$\mu_0 = \left(-1 + \frac{1+x'}{1+a_0}\right) + (1+x')\theta_1^X \qquad 4.5.12.$$

while the dual inequality associated with the value of the project income stream is

$$-K\lambda_0 + (1+a_0)\theta_1^X \geq 0 \qquad 4.5.13$$

Since this implies $\theta_1^X > 0$ and thus $V^X > 0$ the inequality becomes an equality from which

$$\theta_1^X = \frac{K\lambda_0}{1+a_0} = \frac{KrT}{(1+a_0)(1+r')} \qquad 4.5.14$$

Substitution of the values of λ_0 and θ_1^X into equation 4.5.12 gives the generalized net present value of the project as

$$-1 + \left[\frac{1}{1+a_0} + \frac{KrT}{(1+r')(1+a_0)}\right] \qquad 4.5.15$$

which implies in accordance with the definition of cost of capital in the last section, a screening rate for the one period project of

$$\rho^* = a_0 - rKT\left(\frac{1+a_0}{1+r}\right) \qquad 4.5.16$$

---

* This implies an after tax return of x'. Thus if x is the pre-tax return then $x' = x(1-T)$ and the pre-tax cash inflow is $\frac{1+x(1-T)}{1-T}$.

The analysis further assumes that there are also sufficient profits to take full advantage of tax allowances.

** Since in this case and in the subsequent analysis, the results apply to any project the distinguishing j subscript is omitted.

This is identical to the formula deduced by Myers (74). It should be noted that in general that $\rho^* = a_0 - rKT\left(\frac{1+a_0}{1+r}\right) > a_0 - a_0KT = \hat{\rho}_{MM}$, the discount rate postulated by Modigliani and Miller. This analysis can be extended to determine the total value of income in any time period. Thus the dual inequalities for debt and the debt income stream at time t are

$$\lambda_t - rT\,\theta_{t+1}^W \geq \frac{rT}{(1+r)^{t+1}} \qquad 4.5.17$$

$$-K\lambda_t + (1+r)\theta_{t+1}^W - \theta_t^W \geq 0 \qquad 4.5.18$$

The first of these implies that $\lambda_t > 0$ and the debt constraint is always binding. This in turn implies that $V_t^W > 0$ and thus both of these inequalities become equalities.

A little algebraic manipulation yields the simple recurrence relationship

$$\lambda_t[1+r(1-KT)] = \lambda_{t-1} \qquad 4.5.19$$

from which it can be deduced that the shadow price on debt capacity is given by

$$\lambda_t = \frac{rT}{(1+r')^{t+1}} \qquad 4.5.20$$

The contribution to debt capacity of an additional £1 of income (i.e. after tax cash flow) in period t is given by the dual θ_t^X to the equation 4.5.5. which values the income stream. Consideration of this dual equality* gives

$$-K\lambda_t + (1+a_0)\theta_{t+1}^X - \theta_t^X = 0 \qquad 4.5.21$$

---

*Since $V_t^X > 0$ the inequality is in fact an equality.

Hence

$$\theta_t^X = \frac{KrT}{(1+a_0)(1+r')^t} + \frac{\theta_{t-1}^X}{(1+a_0)} \quad (t=2,\infty) \qquad 4.5.22$$

Using the result of equation 4.5.14

$$\theta_1^X = \frac{KrT}{(1+a_0)(1+r')}$$

then we get

$$\theta_t^X = \frac{KrT}{(1+a_0)(1+r')^t} + \frac{KrT}{(1+a_0)^2(1+r')^{t-1}} \cdots \quad \cdots \frac{KrT}{(1+r')(1+a_0)^t} \qquad 4.5.23$$

$$= \frac{KrT}{(1+a_0)(1+r')^t} \quad \frac{\left[1 - \left(\frac{1+r'}{1+a_0}\right)^t\right]}{1 - \frac{1+r'}{1+a_0}}$$

$$= \frac{KrT}{a_0-r'}\left[\frac{1}{(1+r')^t} - \frac{1}{(1+a_0)^t}\right] \qquad 4.5.24$$

Thus an extra £1 of income in period t makes a direct contribution of $\frac{1}{(1+a_0)^t}$ to the value of the firm and an indirect contribution via its impact on the debt capacity of $\frac{KrT}{(a_0-r')}\left[\frac{1}{(1+r')^t} - \frac{1}{(1+a_0)^t}\right]$.

Hence the appropriate discount factor for the cash flow c_t in t is

$$\frac{1}{(1+a_0)^t} + \frac{KrT}{a_0 - r'}\left[\frac{1}{(1+r')^t} - \frac{1}{(1+a_0)^t}\right] \qquad 4.5.25$$

This is the closed function form of the adjusted present value formula (APV) of Myers, and hence for convenience his nomenclature will be used. Myers(74) suggests the APV of a project can be computed from the recursive definition

$$APV_{t-s} = A_{t-s} + \frac{rTK}{1+r} (APV_{t-s} - c_{t-s}) + \frac{rTK}{1+r} \left[\sum_{\tau=t-s+1}^{\tau-1} \frac{(APV_\tau - c_\tau)}{(1+r)^{\tau-t+s}} \right] \quad 4.5.26$$

with

$$A_{t-s-1} = c_{t-s-1} + \frac{A_{t-s}}{1+a_o} \quad 4.5.27$$

However, Myers states that the method of calculating the adjusted present value is tedious to work through manually and that objections to the practical use of APV might be made on both the basis of increased (mathematical) complexity and the difficulty of any easy interpretation of the formula. In Ashton and Atkins (78), Atkins shows that expression 4.5.25 is a consequence of equations 4.5.26 and 4.5.27, This alternative derivation is reproduced in appendix XVIII.

If α is defined by

$$\alpha = \frac{a_o - r}{a_o - r'}, \; = 1 - \frac{rTK}{a_o - r'}, \quad 4.5.28$$

then expression 4.5.25 can be rewritten

$$APV_o = (1-\alpha) \sum_{t=0}^{H} \frac{c_t}{(1+r')^t} + \alpha \sum_{t=0}^{H} \frac{c_t}{(1+a_o)} \quad 4.5.29$$

for a project of length H. While this last form is probably easiest for computational purposes its simplicity obscures any interpretation. If we use the expression for the projects contribution to debt capacity as represented by equation 4.5.23 then APV_o can be written as

$$\frac{1}{(1+a_o)^t} + \frac{KrT}{(1+r')^t} \cdot \frac{1}{(1+a_o)} + \frac{KrT}{(1+r')^{t-1}} \cdot \frac{1}{(1+a_o)^2} \cdots\cdots \frac{KrT}{(1+r')} \cdot \frac{1}{(1+a_o)^t} \quad 4.5.30$$

Clearly the first term represents the value now of an uncertain cash flow c_t, t-periods hence. The second term can be interpreted as follows. The uncertain cash flow in t, has a value of $\frac{1}{(1+a_o)}$ in period t-1 and knowledge of this cash flow makes a contribution to the value of debt capacity now of $\frac{KrT}{(1+r')^t} \times \frac{1}{(1+a_o)}$. Hence the general term results from an uncertain cash flow in t having value $1/(1+a_o)^{t-s}$ at an interim period s and knowledge of this cash flow increases the present value of debt capacity now by an amount

$$\frac{KrT}{(1+r')^{s+1}} \times \frac{1}{(1+a_o)^{t-s}} \qquad 4.5.31$$

This concept of the value of knowledge of future cash flows is a natural extension of the windfall gain concept of income discussed in Robichek and Myers (65).

By reverting to the form for the APV originally derived in equation 4.5.29 several results follow almost immediately. In the case where the project is a perpetuity with constant cash inflow stream where $C_t = \tilde{C}$ (say) t=1,2....∞

$$APV_o = \left[\frac{1}{r'} - \left(\frac{a_o - r}{a_o - r'}\right)\frac{1}{r} + \left(\frac{a_o - r}{a_o - r'}\right)\frac{1}{\rho_o}\right]\tilde{C} - I_o \qquad 4.5.32$$

where I_o is the initial investment

$$= \frac{\tilde{C}r}{\rho_o r'} - I_o = \frac{\tilde{C}}{\rho_o(1-KT)} - I_o \qquad 4.5.33$$

$$= \sum_{t=1}^{\infty} \frac{\tilde{C}}{(1+\hat{\rho})^t} - I_o \qquad 4.5.34$$

where $\hat{\rho} = \rho_o(1-KT)'$ is the weighted average cost of capital according to Modigliani and Miller.

Where the cash flows can be regarded as equivalent in risk to that of borrowing, as in leasing cash flows then a_o can be set equal to r. In this case for an asset costing C_o

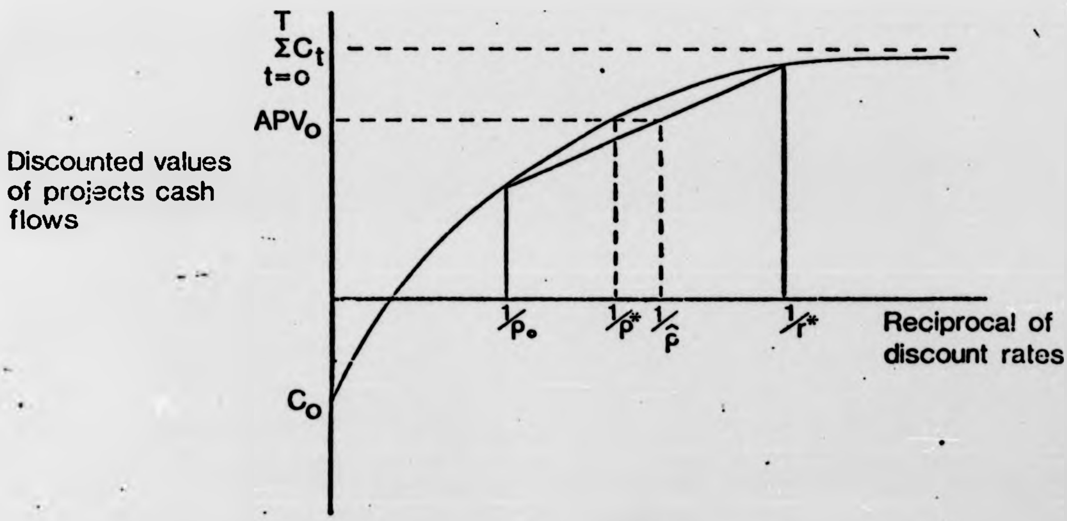
$$APV_o = C_o - \sum_{t=1}^{H} \frac{C_t}{(1+r')^t} = C_o - \sum_{t=1}^{H} \frac{C_t}{[1+r(1-TK)]^t}$$

Here C_t is the case flows associated with the leasing decision consisting of after-tax lease repayments and loss of tax allowances. This is the result obtained by Myers et al (74) using a variation on the APV approach and will be rederived more directly in the next chapter by solving the appropriate particular case of the recurrence relationship 4.5.26 and 4.5.27.

The discount rate p* at which shareholders ought to screen cash flows from projects can be defined by the solution of equation (22) such that

$$\sum_{t=0}^{H} \frac{C_t}{(1+\rho^*)^t} = APV_o = (1-\alpha) \sum_{t=0}^{H} \frac{C_t}{(1+r')^t} + \alpha \sum_{t=0}^{H} \frac{C_t}{(1+a_o)^t} \qquad 4.5.36$$

FIGURE 4.5.1 The relationship between the APV cut-off rate and the MM cost of capital



While no general algebraic expression exists for the solution of such an expression it is relatively easy to show that for most investment projects such a solution does exist. Moreover, the computation of the solution is relatively trivial. The only additional notation necessary for this discussion is to define a net present value function by the equations

$$f(x) = \sum_{t=0}^{T} \frac{C_t}{(1+\frac{1}{x})^t} \qquad (x > o) \qquad 4.5.37$$

$$f(0) = C_o \qquad (x = 0) \qquad 4.5.38$$

It is clear from the above definition that the net present value of the cash flows at a discount rate y, NPV(y), is just $f(\frac{1}{y})$. It follows also from the above definition that

$$APV_o = (1-\alpha)\, f(\frac{1}{r'}) + \alpha\, f(\frac{1}{a_o}) \qquad 4.5.39$$

The function $f(\frac{1}{y})$ is a continuous function of y for non-negative values of y. Moreover for simple investment* it will be a concave monotonically increasing function. As y increases (i.e. the discount rate tends to zero) the function will tend asymptotically to the positive value $\sum_{t=0}^{T} C_t$. Typically the shape of the function is as in figure 4.5.1. where the two axes are the discounted values of the cash flow and the reciprocals of the discount rates.

The desired results follow almost immediately. APV_o is a weighted linear combination of $f(\frac{1}{a_0})$, $f(\frac{1}{r'})$ where the weights α, $1-\alpha$ respectively are both positive and sum to unity. Hence $f(\frac{1}{r'}) > APV_o > f(\frac{1}{a_o})$ and once $f(\frac{1}{y})$ has been computed for appropriate values of y in the range $(\frac{1}{a_o} \leq y \leq \frac{1}{r'})$, $\frac{1}{\rho^*}$ which is the abscissa value** for which the function is

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* See Mao (69) for a discussion of simple investments.

** The continuity of f ensures the existence of such a discount rate.

equal to APV_o can be found by interpolation.

Now

$$\frac{1-\alpha}{r^*} + \frac{\alpha}{a_o} = \left(1 - \frac{a_o - r}{a_o - r'}\right)\frac{1}{r'} + \left(\frac{a_o - r}{a_o - r'}\right)^1\!/_{a_o} \qquad 4.5.40$$

$$= \frac{1}{a_o(1-TK)} = \frac{1}{\hat{\rho}} \qquad 4.5.41$$

Hence, $1/\hat{\rho}$ is also a linear combination of $1/\rho_o$, $1/r*$ with weights α, $1-\alpha$. We can use the concavity of the function $f(y)$ to deduce the following result.

$$f(\frac{1}{\rho*}) = APV_o \qquad 4.5.42$$

$$= (1-\alpha)f(\frac{1}{r'}) + \alpha f(1/\rho_o) \qquad 4.5.43$$

$$< f\left(\frac{1-\alpha}{r'} + \frac{\alpha}{a_o}\right) = f(\frac{1}{\hat{\rho}}) \qquad 4.5.44$$

While the increasing monotonicity of f further implies

$$\frac{1}{\rho*} < \frac{1}{\hat{\rho}} \qquad 4.5.45$$

or

$$\rho^* > \hat{\rho} \qquad 4.5.46$$

The result is a generalisation of the result observed by Myers for the one period case. All these results are illustrated in figure 4.5.1.

Thus it is seen that the MM cost of capital formula will in general break down when applied to projects whose cash flows are not constant perpetuities. There remains the problems of how one might compute or observe the rate a_o and of its relationships to the equity and other rates. These issues will be discussed at some length in section 4.7, while the following section looks in detail at how the dual analysis might be extended to examine the 'consistency' of formulations of financial programming models in practice.

4.6 Consistency and the elimination of Formulation Errors.

The model introduced in section 1.7 and detailed in appendices I & III is fairly complex, and as such is liable to formulation errors. There are at least two different and very distinct methods of checking the consistency of the formulation. The first of these is the 'traditional' double entry form. Here the consistency of the formulation is checked by constructing balance sheets and cash flow statements from the structural variables in the model. Such a method is primarily a method of checking the consistency of the technological set of equations (equations A1.1.1 to A1.1.14 of appendix I). In effect the report writer computes independently from the LP model the increase in shareholders' equity and the increase in liabilities together with the change in the net cash balance position which are brought about by the year on year decisions. The change in the capital provided is compared with the increase in the total assets of the firm as represented by the LP variable $ASSETS_t$ while the change in the cash balance position is compared with the net change in the LP variables $MARK_t - OVDR_t$. Appendix IX illustrates such a check.

In addition to any errors that might arise in the constraining equations, errors can, and do, arise in the formulation of the objective functions. Here the dual system of equations provide an interesting means of 'audit'.

The theoretical justification for the approach adopted again arises from the work of Hirschleifer. He showed that given free access to the capital markets the appropriate discount rate for investment appraisal was determined by the return required on the particular capital market instrument which was utilised in arriving at the investment decision. Further the discussion of chapter two showed how this ciscount rate was simply related to the ratio of successive cash balance duals*. These ideas can be used to check that the single economic criterion used for valuing the firm (appendix V) is consistent with the technological set of equations defining the accounting and cash flow relationships (equations A1.1.1 to A1.1.14 of Appendix I)

The methodology is to find the relationship between the objective function coefficients and the duals on the cash balance equations on the financial policy constraints. The dual variables associated with the financial policy constraints (equation A1.2.1 to A1.2.6 of appendix I) are then set to zero thereby simulating free access to the capital markets. The resulting relationship between the dual values on the cash balance constraints and the coefficient in the objective function provides a check on the consistency of the model structure.

The single economic criterion used in all but the penultimate chapter of this thesis is the maximization of the value of the net equity stream (i.e. dividends less rights issues) upto the horizon plus that portion of the horizon value of the firm which is attributable to the holders of equity. Thus the objective function takes the form

$$\text{MAX } \psi_o = \sum_{t=1}^{H-1} \frac{DV_t}{(1+i)^{t+1}} - \sum_{t=1}^{H} \frac{P.RG_t}{(1+i)^t} + \frac{\psi_H}{(1+i)^H} \qquad 4.6.1$$

which for convenience of the subsequent discussion will be written

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* This statement does not contradict any of the arguments of this chapter. Here the assumption is being made of free access to one particular financial instrument. The rest of analysis presented in this chapter examines the interacting roles of various types of financial instruments where restrictions are placed on the use of these instruments.

$$MAX\ \psi_o = \sum_{t=1}^{H} ZDV_t.DV_t\ ZRG_t.RG_t) + \frac{ZOVDR_H.OVDR_H}{(1+i)^H}$$

$$\frac{ZLL_{H-1}.ZLL_{H-1}}{(1+i)^H} + \frac{ZOVDR_{H-1}.OVDR_{H-1}}{(1+i)^H} + \frac{ZMARK_{H-1}.ZMARK_{H-1}}{(1+i)}$$

$$+ \frac{ZMARK_H.MARK_H}{(1+i)^H} + \frac{ZDE_H.DE_H}{(1+i)^H} + \frac{ZX_j.X_j}{(1+i)^H} \qquad 4.6.2.$$

where the individual contributions of the various investment and financing instruments are individually identified and are denoted by the prefix Z.

Examination of equation 4.6.1. immediately reveals some apparent anomalies in the valuation of pre-horizon equity flows. Dividends, but not rights, are omitted from the expression in the final year. Furthermore the dividend variable in t is discounted by $^1/_{(1+i)}{}^{t+1}$ whereas the rights stream is discounted by $^1/_{(1+i)}{}^{t}$. An examination* of the dual equation soon reveals the reasons.

The dual equality corresponding to the issue of dividends (DV_t) for periods 1 to H is

$$- CL_t + \rho_t - DTARG_t - DCOV_t = ZDV_t \qquad 4.6.3.$$

and the equality (A17.5) for CL_t in periods 1 to H is

$$CL_t = \rho_t - \rho_{t+1} - \alpha\, ROCE_t + \beta\, LQDY_t$$

Hence $\rho_{t+1} = ZDV_t - \alpha\, ROCE_t + \beta\, BLQDY_t \quad (t=1,H)$ 4.6.4.

For time period t=H, the corresponding equations are

$$\rho_H - CL_H - DTARG_H - \varepsilon\, DCOV_H = ZDV_H \qquad 4.6.5.$$

and

$$CL_H = \rho_H - \alpha\, ROCE_H + \beta\, LQDY_H$$

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* The work of this section rests heavily on the preliminary dual analysis which is carried out in appendix XVII.
It is thus assumed in the subsequent discussion that $H \leq 8$.

from which

$$ZDV_H = \alpha\, ROCE_H - \beta\, LQDY_H - DTARG_H - \varepsilon\, DCOV_H \qquad 4.6.6.$$

It follows from the initial discussion that we are interested in valuing dividends given free access to the capital markets. In effect this means that we can ignore the financial policy constraints and set their dual values to zero.

Thus $ROCE_t = LQDY_t = DTARG_t = DCOV_t = ERPS_t = 0 \quad (t=1,H)$ 4.6.7.

This implies that

$$\rho_t = ZDV_{t-1} \quad (t=1,H) \qquad 4.6.8.$$

and

$$ZDV_H = 0 \qquad 4.6.9.$$

Since the implications of these last two equations are best considered in conjunction with the raising of equity capital, further discussion of equations 4.6.8 and 4.6.9 will be temporarily postponed.

For rights issued at price P the dual inequality is

$$-P\rho_t + EQ_t \geq ZRG_t \quad (t=1,H) \qquad 4.6.10$$

while the dual equation associated with the number of shares outstanding (NUM_t) is

$$EQ_t - EQ_{t+1} + \delta\, ERPS_t - \omega\, DTARG_t = 0 \quad (t=1,H)$$

and

$$EQ_H + \delta\, ERPS_H - \omega\, DTARG_H = 0$$

In the absence of financial policy considerations then the last two equations taken together with equation 4.6.10 lead to

$$-\rho_t \leq \frac{ZRG_t}{P} \quad (t=1,H) \qquad 4.6.11$$

from which it follows that

$$ZRG_t \leq -P.ZDV_{t-1} \qquad (t=1,H) \qquad 4.6.12$$

Where the inequality is an equality if rights are issued.

Consider first the apparently anomalous result that the objective function coefficient valuing dividends at the horizon is zero. The reason for this becomes clear if the definition of the dividend variable is re-examined. DV_H represents the declared dividend at the horizon which is to be paid one year later in the post-horizon period - it does not represent a cash flow. Its contribution to the value of the firm will be represented via an increase in short term market investments, being money set aside for dividends declared but not paid. The accrual nature of the dividend variable is reflected in the time lags between the two sides of equality 4.6.8. Here the objective function coefficient for dividends in t is actually related to the cash balance dual in t+1. In the case of rights issues, RG_t represents the number of rights issued in time period t and gives rise to an actual cash flow. As a consequence, there are no such time lags in the corresponding equations (equations 4.6.11 and 4.6.12) for rights.

It follows from the analysis of Hirschleifer that given the firm is actually issuing dividends or rights the interperiod discount factor is just 1+i, i.e.

$$\rho_t / \rho_{t+1} = 1+i \qquad 4.6.13$$

Since RG_1 represents rights issued at the end of the first period it follows that

$$ZRG_1 = -\frac{P}{1+i} \qquad 4.6.14$$

from which

$$ZRG_t = - \frac{P}{(1+i)^t} \qquad (t=1,H) \qquad 4.6.15$$

and

$$ZDV_t = \frac{1}{(1+i)^{t+1}} \qquad (t=1,H-1) \qquad 4.6.16$$

with $ZDV_H = 0$ as before.

The foregoing analysis takes care of the equity streams and detaile consideration must now be given to the term ψ_H. The portion of the horizon value which is attributable to the equity holders consists of the post-horizon operating cash flows from projects adopted in the prehorizon period, less the horizon value of debt. The horizon value of the investment projects is just the net post horizon cash flow from projects discounted back to the horizon at 10%. A rate of 10% was chosen in keeping with the earlier analysis of section where it was shown that a reasonable cut off rate for the screening of projects was 10%. This would appear to be the most suitable rate since the model is largely concerned with accept/reject decisions. This rate, of course, was deduced from a dual analysis of the cash balance and debt constraints and itself is illustrative of another example of the use of cash balance duals in the valuation procedure.

If the objective function value of the short term investments is now considered, then for the variable $OVDR_t$, the following dual inequalities hold

$$-RS.EA_t + CL_t + \gamma ECOV_t \geq ZOVDR_t \qquad (t=1,H-1) \qquad 4.6.17$$

and

$$CL_H + \gamma ECOV_H \geq ZOVDR_H \qquad 4.6.18$$

In addition we can use the equalities A17.9 and A17.17 for CL_t and EA_t to deduce

from which

$$ZRG_t = -\frac{P}{(1+i)^t} \qquad (t=1,H) \qquad 4.6.15$$

and

$$ZDV_t = \frac{1}{(1+i)^{t+1}} \qquad (t=1,H-1) \qquad 4.6.16$$

with $ZDV_H = 0$ as before.

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and

$$CL_H + \gamma ECOV_H \geq ZOVDR_H \qquad 4.6.18$$

In addition we can use the equalities A17.9 and A17.17 for CL_t and EA_t to deduce

$$\rho_t \geq -ZOVDR_t + (1+RS)\rho_{t+1} + TRS\rho_{t+2} - RS(1+\alpha T)ROCE_{t+1}$$

$$+ \alpha ROCE_t - RS.T.\beta LQDY_{t+1} + RS(1-T)\ ERPS_{t+1}$$

$$+ RS[1-T].DCOV_{t+1} - RS.ECOV_{t+1} - \gamma ECOV_t (t=1,H-2) \qquad 4.6.19$$

$$\rho_{H-1} \geq -\ ZOVDR_{H-1} + (1+RS)\rho_H$$

$$+ \alpha ROCE_{H-1} - RS.T.\beta LQDY_{t+1} - \beta.LQDY_t$$

$$+ RS(1-\ )\ ERPS_H + RS\ [1-T]\ DCOV_H - RS.ECOV_H - \gamma ECOV_{H-1} \qquad 4.6.20$$

$$\rho_H \geq -\ ZOVDR_H + \alpha ROCE_H - \beta.LQDY_H - \gamma ECOV_H \qquad 4.6.21$$

In the absence of financial policy constraints, then the relationship between the cash balance duals for t=1, H-2 when overdraft is being used is

$$\rho_t = (1+RS)\rho_{t+1} - T.RS.\rho_{t+2} \qquad 4.6.22$$

The interpretation of this equality is fairly simple. The $-(1+RS)\rho_{t+1}$ represents the repayment of debt plus interest in time t+1 of the debt taken out in t. The term $+ T.RS.\rho_{t+2}$ represents the tax relief which occurs in time period t+2. If we are interested in the interperiod discount rate π where $\rho_t = \pi\rho_{t+1}$ then π is given by the solution of

$$\pi^2 - (1+RS)\pi + T.RS = 0$$

or

$$\pi = \frac{(1+RS) \pm \sqrt{(1+RS)^2 - 4TRS.}}{2} \qquad 4.6.23$$

Now RS and T. are small, being 0.12 and 0.5 respectively and we can approximate π ignoring terms the order of $(RS)^3$ by

$$\pi = \frac{(1+RS) \pm \left[1+\frac{RS^2}{2} + 2RS(1-2T) - \frac{4(RS)^2}{8}(1-2T)^2\right]}{2} \qquad 4.6.24$$

Considering only the positive square root* we have

$$\pi = 1 + (1-T)RS + T.RS^2(1-T) \qquad 4.6.25$$

Here the principal term in the interperiod discount rate is (1-T)RS, the after tax rate on debt, while the term T(1-T)RS is a 'correction' due to lagged tax allowances. With RS = 0.12 and T = 0.5 then the interperiod discount rate is 1.0636. The validity of the expression can be checked by examining the ratio of the cash balance duals when overdraft is being used in the absence of other constraints. Thus in the sample printout - Figure 4.6.1. - overdraft is being raised in period three and four. The cash balance cuals are 0.7105 and 0.6682 giving an interperiod discount factor of 1.0633.

FIGURE 4.6.1.

NAME	R.H.S.	DUAL PRICE
CB1	-4580.0000	0.8943
CB2	0	0.8546
CB3	0	0.7105
CB4	0	0.6682
CB5	1000.0000	0.5687
CB6	0	0.5087
CB7	0	0.4525
CB8	0	0.4040

This offers confirmation of the correctness of our analysis. The analysis now affords us with a mechanism for correctly determining the value of $ZOVDR_{H-1}$ and $ZOVDR_{H-2}$. If we assume that at the planning horizon the value of the firm must be reduced by the value of outstanding debt to give the value of the equity portion then we have $ZOVDR_H = -1$ in equation 4.6.2.

*The boundary conditions are chosen so that only this root appears in practice.

In order to ensure consistency in the valuation then $ZOVDR_{H-1}$ needs to be defined so that

$$\rho_{H-1} = \pi \rho_H \qquad 4.6.26$$

or

$$\pi = - ZOVDR_{H-1} - (1+RS) \qquad 4.6.27$$

Thus

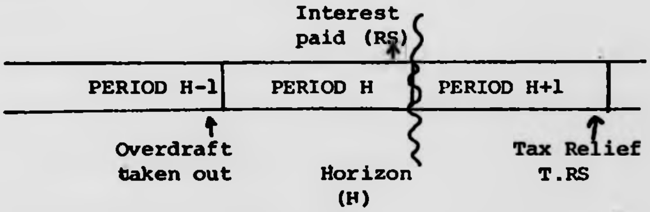
$$ZOVDR_{H-1} = T.RS - T(1-T)RS^2 \qquad 4.6.28$$

Hence with $RS = 0.12$ and $T = 0.5$ then

$$ZOVDR_{H-1} = 0.0572 \qquad 4.6.29$$

Although this term might seem somewhat peculiar it arises from the following cash flows shown schematically in the Figure 4.6.2.

FIGURE 4.6.2.



Thus £1 borrowed at the end of year H-1 results in a cash outflow of RS in year H consisting of interest payment with a reduction in the tax paid in the post horizon period (i.e. at the end of year H+1) of T.RS. This tax relief when valued at the horizon by discounting at the effective rate on debt π contributes

$$\frac{T.RS}{1 + (1-T)RS^2 + T.RS\ (1-T)}$$

or ignoring terms of order $(RS)^3$. $T.RS - T(1-T)RS^2$ to the horizon value. A similar piece of analysis can be carried out for market investments. Here the interperiod discount rate in general is given by

$$1 + (1-T)RI + R(1-T)RI^2 \qquad 4.6.30$$

Again this result can be confirmed by examination of the cash balance duals where market investments are being raised. Such a result is shown in figure 4.6.3. for H = 8 where the ratio of the duals is 1.0 36 22 against a theroretical value of 1.0 36 25. This gives a value for the horizon value of short term investments taken out in period H - 1 of -0.0338 where this term again arises from tax payments made post horizon on the interest received in period H.

FIGURE 4.6.3.

	NAME		VALUE	REDUCED COST
B	MARK1	+	209.2234	0
	MARK2	+	0	-0.0749
B	MARK3	+	361.3706	0
B	MARK4	+	249.0143	0
B	MARK5	+	275.8134	0
B	MARK6	+	1597.3600	0
B	MARK7	+	3860.3429	0
B	MARK8	+	7390.7397	0

NAME	R.H.S.	PRICE
CL8	-902.0000	1.0000
CB1	-4580.0000	1.3642
CB2	0	1.2605
CB3	0	1.1935
CB4	0	1.1516
CB5	1000.0000	1.1113
CB6	0	1.0725
CB7	0	1.0350
CB8	0	1.0000

It should perhaps be further emphasised that not only does this analysis provide a mechanism for determining the appropriate value of the objective function for short term investments and loan, it also provides a method of structuring the technological constraint set. In particular a great deal of difficulty was encountered because the variable specification contained both transactions which were accruals and actual cash flows. The 'reasonableness' of equations 4.6.25 and 4.6.30 suggest that the current asset and current liabilities were indeed correctly incorporated into the model.

Finally the impact of long term debt must be considered. For long term debt (LL_t) issued in period t we have

$$-\rho_t + D_t = ZLL_t \qquad (t=1,H) \qquad 4.6.31$$

where ZLL_t is included since a 15 year debenture taken out in any of the eight prehorizon periods will always have some post-horizon cash flows.

For long term debt outstanding in t, (DE_t) we have the equality

$$-RL\rho_{t+1} + (1-T)RL\ PR_{t+1} + D_t - D_{t+1} + \gamma ECOV_t = 0 \qquad (t=1,H-1)$$

and

$$D_H + \gamma ECOV_H = \frac{ZDE_H}{(1+i)^H}$$

We can substitute in this for PR_{t+1} from equation A17.14 and D_t from equation 4.6.31 to deduce

$$\rho_t - (1+RL)\rho_{t+1} + TRL\rho_{t+2} + \alpha T.RL\ ROCE_{t+1} + \beta T.RL\,LQDY_{t+1}$$

$$+ (1-\tau)RL\,ERPS_{t+1} + (1-T)RL\ DCOV_{t+1}$$

$$= ZLL_t - ZLL_{t+1} \qquad (t=1,H-2) \qquad 4.6.32$$

with

$$\rho_{H-1} - (1+RL)\rho_H + T.RL\rho_H + (1-T)RL.ERPS_H + (1-T)RL\,DCOV_H$$

$$= ZLL_{H-1} - ZLL_H \qquad 4.6.33$$

and

$$\rho_H = \frac{ZDE_H}{(1+i)^H} + \gamma ECOV_H \qquad 4.6.34$$

From which in the absence of other financial policy constraints we have

$$\rho_t - (1+RL)\rho_{t+1} + TRL\rho_{t+2} = ZLL_t - ZLL_{t+1} \qquad 4.6.35$$

$$\rho_{H-1} - (1+RL)\rho_H = ZLL_{H-1} - ZLL_H \qquad 4.6.36$$

$$\rho_H = \frac{ZDE_H}{(1+i)^H} + \gamma ECOV_H \qquad 4.6.37$$

Again at the horizon it is convenient* to define $Z(DE_H) = -1$. We also make the assumption that while the firm is borrowing the interperiod discount rate is a constant π. The solution to the homogeneous part of equation 4.6.35 gives a value for π of

$$\pi = 1 + (1-T)RL + T(1-T)RL^2 \qquad 4.6.38$$

Thus for consistency

$$ZLL_{H-1} = T.RL - T(1-T)RL^2 \qquad 4.6.39$$

and

$$ZLL_t = 0 \qquad (t=1,H-2 \text{ and } t=H) \qquad 4.6.40$$

Again the non-zero objective function coefficient arises from post-horizon tax relief on debt interest payments made in the pre-horizon period. In particular for the model under discussion with RL = 0.08 and T = 0.05

$$ZLL_{H-1} = 0.0384 \qquad 4.6.41$$

4.7 Infinite Time Horizon Linear Programmes and Long Run Equilibrium Solutions

All the models referenced so far in this thesis have been finite horizon models where the investment and financing decisions are considered jointly over some finite planning period. As has already been demonstrated the net result of such an approach is that projects are valued at an internally determined opportunity cost of capital

*Strictly speaking the outstanding net of tax interest stream and the final repayment should be capitalized at the internal rate of return of the stream. In general, and in this case in particular, the correction is negligible.

in the pre-horizon period but are valued by a pre-determined cost of capital in the post-horizon period. Hence in the case of Weingartner's horizon models and the Chambers' model where analytical solutions to the implied pre-horizon opportunity cost of capital were available it was possible to compare this solution with the post-horizon discount rate and to examine the nature of the post-horizon approximation in some detail.

Chambers is aware of the approximate nature of his post-horizon valuation procedures and addresses directly the way in which it might be improved both theoretically and practically. Thus he states*

> "Managers would normally expect to be able to invest substantial sums after the horizon at better than marginal rates, ... NPVH understates the true value to the firm of funds available after the horizon"

He gives a careful analysis of the extra information needed to avoid such an undervaluation. He states that if details were available of the likely returns on future investment opportunities then an appropriate adjustment could be made providing of course that the capital market parameters do not change. Carleton (70) also considers in some detail how he might provide a reasonable post-horizon valuation. His solution to the horizon value problem is to assume that the firm enters a steady state growth situation and values the anticipated dividend stream using Gordon's (62) model. He further suggests several ways in which the growth rate can be extrapolated from, and made to be consistent with, the pre-horizon performance.

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* Ibid p. 286

A formalization of these two approaches would lead to the following infinite linear model:

$$\text{Max } c'.z^o + \pi c'z^1 + \pi^2 c'.z^2 \qquad 4.7.1.$$

subject to

$$[B]z^1 \leq f_o$$

$$-[A]z^1 + [B]z^2 \leq (1+g)f$$

$$- [A]z^1 + [B]z^2 \leq (1+g)^2 f \qquad 4.7.2.$$

where in this notation

z^t - is a non-negative (column vector) of decisions (including financing decisions) taken at time t.

c' - is a (transposed) valuation vector

[B] - is the pre-horizon matrix of resources uses

[A] - is the matrix of post-horizon consequences

f - vector of flows from existing operations, f_o being the first period values.

π - is a discount factor

and g - is a growth factor

Thus the set of decisions facing the firm now can be considered as part of a set of decisions from a repeating set* of opportunities. The total decision set then can be seen as the first of a set of infinite decisions. The linear program to be solved can be thought of as an infinite ladder as represented by figure 4.7.1.

*The period of this repeating set may of course be longer than a year. In the subsequent discussion it is convenient to consider the period as a year for simplicity of argument.

It should be emphasised that the above structure is merely an explicit formalization of the implicit assumptions of finite horizon valuation models. Thus implicit in the valuation models of Weingartner, Chambers, Carleton and other writers is the continuing existence of both the firm, future investments and the capital markets. Furthermore it is generally assumed that there are no major changes at the horizon in the parameters describing the behaviour of these markets. Hence no radically new assumptions have been incorporated into the generalization of the existing approaches.

The infinite system of equations 4.7.1. and 4.7.2 can be written in the more compact form

$$\text{Max} \quad \sum_{t=0}^{\infty} \pi \, C' z^t \qquad 4.7.3$$

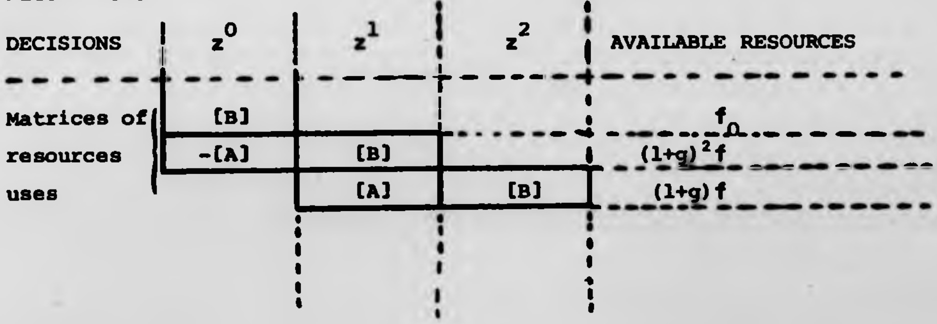
such that

$$[B]z^0 \le f_0 \qquad 4.7.4.$$

$$[B]z^t - [A]z^{t-1} \le (1+g)^t f \qquad 4.7.5.$$

One immediate observation is that if η is the dual vector associated with period 1 resource allocation, then the reduced finite LP

FIGURE 4.7.1



$$\underset{z^o}{\text{MAX}} \left\{ c'z^o + \eta_1'[A]z^o \right\} \qquad 4.7.6.$$

$$[B]z^o \leq f_o \qquad 4.7.7.$$

gives the same decision set for the first period as that of the infinite LP describe by equations 4.7.3., 4.7.4 and 4.7.5. Such a valuation model would satisfy the postulated horizon principle. The difficulty remains, of course, of actually computing η_1. Most authors have approximated η_1 by a constant one-parameter cash discount vector. For example, Chambers uses the approximation $\eta_1(a) = \left(\frac{1}{1+a}, \frac{1}{(1+a)^2} \ldots. \mid 0 \right)$ where the weight average cost of capital a is used as a single parameter for valuating cash flows and the null vector is the valuation vector for the post horizon debt capacity effects.

The theory of infinite LP systems which can be represented by equations 4.7.3., 4.7.4. and 4.7.5. have been explored extensively by Evers (73, 74, 75, 76, 77) who discusses the existence of long run equilibrium solutions as well as methods of generating horizon valuations such that the infinite model can be truncated in a way that satisfies the horizon principle.

Evers shows that under certain conditions* one of which is $\pi(1+g) < 1$ then the decision vector z^t and the dual vector η^t to the infinite LP system converge in the sense that

$$z^t \rightarrow (1+g)^t \tilde{z} \qquad 4.7.8.$$

$$\eta^t \rightarrow (1+g)^t \tilde{\eta} \qquad 4.7.9.$$

where $\tilde{z}$ and $\tilde{\eta}$ are the equilibrium primal and dual vectors given by the solution to the system.

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*For a discussion of these conditions see Evers (June 73 p. 13). It will be assumed in the subsequent discussions that these conditions are satisfied.

$$\left([B] - [A]/_{(1+g)}\right)\tilde{z} + \tilde{y} = f \qquad 4.7.10$$

$$\left([B] - [A]\right)\tilde{\eta} - v = c \qquad 4.7.11$$

where

$$\tilde{v}'.\tilde{z} + \tilde{\eta}'.\tilde{y} = 0 \qquad 4.7.12$$

and

$$\tilde{v},\tilde{z},\tilde{\eta},\tilde{y} \geq 0 \qquad 4.7.13$$

The application of the theory to financial planning models can be illustrated by constructing a simple example. Thus with the objective function the maximation of the present value* of the future dividend stream the infinite horizon model is

$$\text{Max } \psi_o = \sum_{t=0}^{\infty} \frac{1}{(1+i)} D_t \qquad 4.7.14$$

with a cash balance constraint**

$$X_o + D_o - \omega_o \leq F_o \qquad (\rho_o)$$

$$X_t - (1+\kappa')X_{t-1} + D_t + [1+r(1-T)]\omega_{t-1} - \omega_t \leq 0 \quad (\rho_t) \qquad 4.7.15$$

plus a debt capacity constraint where debt is limited by the value of the equity

$$\omega_t \leq K\psi_t \qquad (t=0,1,\infty) \qquad (\lambda_t) \qquad 4.7.16$$

In addition MM's fundamental principle of valuation gives

$$D_t + \psi_t - (1+i)\psi_{t-1} = 0 \qquad (t=0,1,\infty) \qquad (\theta_t) \qquad 4.7.17$$

If it is assumed that the firm has a growing set of opportunities then

$$X_t \leq (1+g)^t \qquad (t=0,\infty) \qquad (\mu_t) \qquad 4.7.18$$

---

* It is, of course, necessary to assume that the shareholder requires a constant return from the firm. This assumption will be discussed in more detail later.

**The symbols in brackets represent the dual variables.

Here it is assumed that projects consist of an investment of 1 followed by a return of 1+x(1-T) = 1+x' the following year. Thus the return from the project is constant, in keeping with the earlier discussion, but the scale of opportunities is increasing. Then it is relatively easy to identify that

$$[B]=\begin{bmatrix}1 & -1 & 0 & 1\\ 0 & 1 & -K & 0\\ 1 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \quad \text{and } [A]=\begin{bmatrix}0 & -(1+r) & 0 & 1\\ 0 & 0 & -K & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \qquad 4.7.19$$

The equilibrium combination* for this system is

$$\tilde{D} = \left(\frac{i-g}{1+g}\right)\psi \qquad 4.7.20$$

$$\tilde{\omega} = K\psi \qquad 4.7.21$$

$$\tilde{\psi} = \frac{x - g}{i - g + Kr(1-T) - Kg} \qquad 4.7.22$$

$$\tilde{X} = 1$$

when it is assumed i > x' > r(1-T) > g giving the equilibrium path on multiplying by $(1+g)^t$. In addition to the equilibrium solution identified above, there is the possibility that the original system may have a homogeneous solution which satisfies

$$-[A]z^t + [B]z^{t-1} = 0 \qquad 4.7.24$$

and the general solution to the system is then the equilibrium solution plus the homogeneous solution. In this case the equation 4.7.14 - 18 give

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*For the models discussed here the equilibrium solutions are relatively easy to find. A general algorithm based on complementarity theory for the numbrical computation of equilibrium is to be found in Evers (June 73, Nov. 73., July 77).

$$D_t - \omega_t + X_t = -(1+r(1-T))\omega_t + (1+x')X_t \qquad 4.7.25$$

$$\omega_t - K\psi_t = 0 \qquad 4.7.26$$

$$D_t + \psi_t = (1+i)\psi_{t-1} \qquad 4.7.27$$

$$X_t = 0 \qquad 4.7.28$$

A non-trivial solution to this system exists where

$$\psi_t = \left[\frac{1 + i + K(1+r(1-T))}{1 + K}\right] \psi_{t-1} \qquad 4.7.29$$

The expression in brackets is similar to the conventional weighted average cost of capital formula and will be denoted by a. The complete homogeneous solution then becomes

$$\begin{aligned} \psi_t &= (1+a)\psi_{t-1} \\ D_t &= (i-a)\psi_t \\ \omega_t &= K\psi_t \\ X_t &= 0 \end{aligned} \qquad 4.7.30$$

The difficulty here is obvious. This solution implies that the firm is growing at a rate 1+a which is greater than its growth in opportunities which are only growing at the rate g. Clearly such a situation is not acceptable. Also the debt is growing at the rate 1+a and would soon far outstrip the value of realisable assets (i.e. assets in place) which are only growing at the rate 1+g. – a situation which would not be permitted in practice. The reason for this anomalous behaviour of the debt capacity constraint arises because the firm is able to borrow large amounts of funds at a rate r against a 'promise' of increased future dividends and then to distribute these funds to shareholders with a preference rate i. The increased future dividends are then met by further borrowing.

Evers explores the conditions under which it is possible to produce a valuation model such that the truncated LP gives the same solution as the infinite LP. He concludes that such a valuation is possible for systems where the square matrix H defined by

$$[H] = ([\widetilde{B}] - \pi[\widetilde{A}])^{-1} [\widetilde{B}] \qquad 4.7.31$$

has no eigenvalues e_i, such that

$$1 \geq \left| \frac{e_i - 1}{e_i} \right| > (1+g)\pi \qquad 4.7.32$$

Here $[\hat{A}]$, $[\hat{B}]$ are the matrices formed from [A] and [B] but with the column and rows associated with non-basic components of the (z,η) equilibrium combination deleted. In the example under discussion the eigenvalues of H are 1 (three times) and $\frac{1+i}{1+r}\left(. \, 1 + \frac{1}{K}\right)$.

Condition 4.7.32 holds for e=1 in which case $\frac{e-1}{e} = 0$ but the condition applied to last eigenvalue requires $\frac{i+Kr(1-T)}{1+K} > a$ which contradicts the initial assumptions. The requisite truncation condition is always satisfied for systems for which the homogeneous solution is the null solution.

The problem is thus is to attempt to identify systems with trivial homogeneous solutions which have non-trivial equilibrium solutions. It was suggested that the 'South-Sea bubble' phenomenumwould not have occurred in practice because the debt would have been restricted by the value of assets in place.* Thus consider the model

$$\text{MAX} \sum_{t=0}^{\infty} \frac{D_t}{(1+i)^t}$$

such that $X - (1+x)X_{t-1} + D_t - \omega_t + (1+r)\omega_t \leq 0$

$$X_t \leq (1+g)^t \qquad 4.7.33$$

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* Myers (78) in a paper 'The Determinants of Corporate Borrowing' comes to a similar conclusion about the importance of the value of existing assets, using what appears to be a completely different approach. In fact, there is some similarity in that both approaches make assumptions about how the providersof debt capital view promises of future income streams.

where the debt capacity is limited by the current value of assets, i.e.

$$\omega_t \leq K X_t \qquad 4.7.34$$

This system gives rise to the following equilibrium equations with primal

$$\tilde{D} + \frac{r-g}{1+g}\tilde{\omega} - \tilde{X}\frac{x-g}{1+g} + y_1 = 0$$

$$\tilde{\omega} + y_2 = K\tilde{X}$$

$$\tilde{X} + y_3 = 1 \qquad 4.7.35$$

and dual

$$\tilde{\rho} - v_1 = 1$$

$$-\tilde{\rho}\left(\frac{1-r}{1+i}\right) + \tilde{\lambda} - v_2 = 0$$

$$\tilde{\rho}\left(\frac{i-x}{1+i}\right) - K\tilde{\lambda} + \tilde{\mu} - v_3 = 0 \qquad 4.7.36$$

with complementarity and non-negativity conditions holding, the solution of this system is

$$\tilde{D} = \frac{x - Kr + g(1-K)}{1 + g} \qquad \tilde{\rho}=1$$

$$\tilde{\omega} = K \qquad \tilde{\lambda}=\left(\frac{i-r}{1+i}\right)$$

$$\tilde{X} = 1 \qquad \tilde{\mu}= \frac{K(i-r) - (i - x)}{1 + i} \qquad 4.7.37$$

provided $i > x > r > g$ and $\frac{i-x}{i-r}, \frac{x-g}{r-g} > K$ 4.7.38

It is worthwhile interpreting these solutions in some detail. If an extra £1 is available then since there are no further investment opportunities the correct decision is to distribute that £1, hence $\tilde{\rho}=1$, ignoring the discount factor.

In contrast if an extra £1 of debt capacity becomes available then this results in an extra £1 now with interest £r and capital repayable one year later giving a net present value of $\left(1 - \frac{1+r}{1+i}\right) = \frac{i-r}{1+i}$.

Whereas if the scale of a project can be increased then the net present value of the investment is worth $-1 + \frac{1+x}{1+i}$ but gives an increase in debt capacity valued at $K\left(1 - \frac{1+r}{1+i}\right)$ with a net benefit of $\frac{K(i-r) - (i-x)}{1 + i}$

It should be noted that if $r > \frac{x - i(1-K)}{K}$ or if $x < Kr - g(1-K)$ then the equilibrium solution would be $\tilde{D} = \tilde{X} = \tilde{\omega} = 0$. In these cases the firm would either not be able to raise loans sufficiently cheaply or the return on the assets would be insufficient to support debt finance. Under such circumstances the firm would quickly redistribute earnings from its existing assets to shareholders without making further investment and cease trading.

The model as represented by 4.7.33 and 4.7.34 is in fact considerably more general than might appear at first sight. While the debt capacity is restricted by the value of the assets in places, other restrictions on debt take a similar mathematical form. Thus if the restriction on debt was such that debt interest was to be more than K_t times covered by income then the restriction would have taken the form

$$\omega_t \leq (1+x)X_t \big/ K_t r = K_t{}'X_t \leq K_t{}'(1+g)^t$$

Alternatively, for a simple upper bound on debt the form would have been $\omega_t \leq (1+g)^t B$.

This last form is essentially Weingartner's model with the possibility of a uniform growth in both opportunities and debt availability. Hence all these three models have a restriction on debt capacity of the form $\omega_t \leq (1+g)^t z$ and as such if $i > x > r$ will have a solution of the form above. These models are also well-behaved in the sense that they have trivial homogeneous solutions thus the homogeneous solutions to the system defined by 4.7.33 and 4.7.34 is

$$D_t - \omega_t = -(1+r)\omega_{t-1}$$

$$\omega_t = 0$$

$$x_t = 0 \qquad 4.7.39$$

with solution $D_t = \omega_t = x_t = 0$. 4.7.40

Thus such a model is capable of truncation in accordance with the horizon principle propounded.

This model as originally introduced by equation 4.7.33 and 4.7.34 related debt to the value of assets and could be considered a simplified version of the Chambers' model with taxation and depreciation ignored. It should be noted that in this case the shadow price on debt $\frac{i-r}{1+i}$ is proportional to the difference between the equity and debt rates - a structure very similar to that deduced in section 3.5 for the shadow price on debt in the pre-horizon period.

In Weingartner's version of 4.7.33 and 4.7.34 $i=r$ and the equilibrium solution is $\tilde{\omega} = 0$, $\tilde{D} = \frac{x-r}{1+g}$, $\tilde{X} = 1$ 4.7.41

with corresponding duals $\tilde{\rho} = 1$, $\tilde{\lambda} = 0$, $\tilde{\mu} = \frac{x-r}{1+r}$ (assuming that the investment returns more than the debt rate). It should be noted that under such circumstances capital rationing no longer exists since the debt capacity dual is zero. This fact on reflection is not surprising. If the firm is capable of generating surplus funds after servicing its debt then given sufficient time in a stable operating environment the firm will move into a permanent funds surplus situation.

Since in the Weingartner model debt has no intrinsic value,* once this point is reached no further debt will be raised.

A cursory glance at the literature on capital budgeting will reveal that nearly all Weingartner type models where numerical examples are included display a short run rationing phenomena.

The method of analysis discussed so far has concerned itself with long run equilibrium conditions whereas the real power of linear programming models is in the planning over relatively short time periods where the firm is essentially in a disequilibrium condition. In such cases the cost of capital as defined in section 4.4 may take completely different forms under such conditions of equilibrium and disequilibrium.

The following model should clarify the problem. Assume that the objective of the firm is the maximisation of the net present value of the dividend stream, where the upper limits on the level of debt is imposed by the suppliers of capital (i.e. both equity and loan capital). Therefore assume that debt must be less than a fixed percentage of the value of the firm as measured in terms of its (current) asset level, and, as measured by its market value. Here the market value is again simply the projected future dividend stream. It is further assumed that these restrictions are such that the resulting debt structure means that the lender of debt finance and the shareholders are happy with a constant return.** Hence the model is

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* A consequence of no taxes and a world of certainty.

** It would be easy to extend the model to cover an increased step function for the debt rate as debt increased but for illustration purposes it is not considered necessary here.

$$\psi_o = \sum_{t=0}^{\infty} \frac{1}{(1+i)^t} D_t \qquad 4.7.42$$

$$X_o + D_o - \omega_o \leq F_o \qquad 4.7.43$$

$$X_t - (1+x)X_{t-1} + D_t - \omega_t + (1+r)\omega_{t-1} \leq 0 \quad t=1,\infty \qquad 4.7.44$$

$$\psi_t + D_t - (1+i)\psi_{t-1} = 0 \qquad 4.7.45$$

$$X_t \leq (1+g)^t \qquad 4.7.46$$

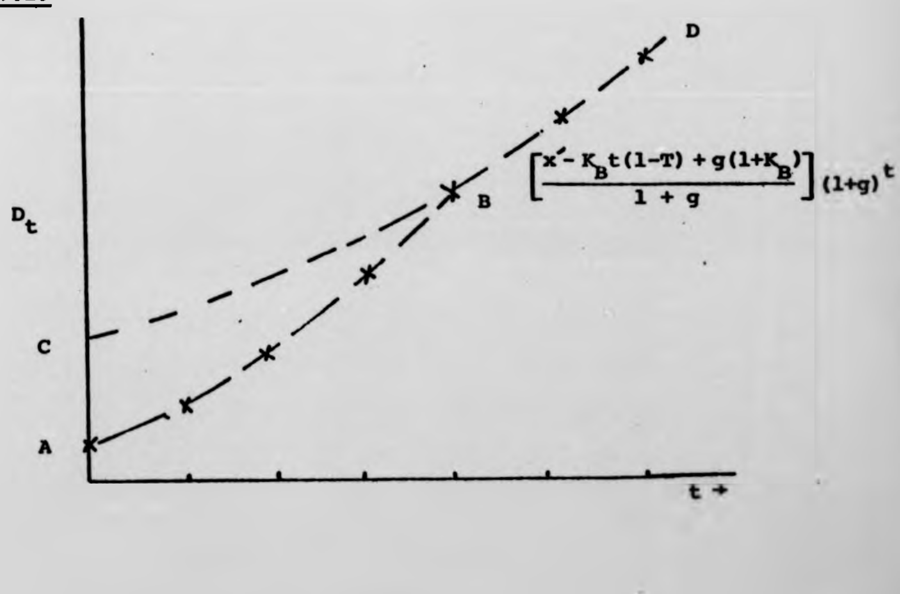
$$\omega_t \leq K_m\psi_t \qquad 4.7.47$$

$$\omega_t \leq K_B X_t \qquad 4.7.48$$

plus non-negativity conditions.

In the long run the debt will grow at the rate of growth of investment opportunities i.e. at 1+g. The equilibrium conditions result from the debt restriction (inequality 4.7.48) on the value of the assets and will be given by the equations of the last section. Thus in the long run the dividends will grow at (1+g), the rate of growth of opportunities.

FIGURE 4.7.2.



The equilibrium path for dividends thus is CD in Figure 4.7.2. and the dividend payment is given by

$$\left[\frac{x' - K_B\, r(1-T) + g(1+K_B)}{(1+g)}\right](1+g)^t \qquad 4.7.49$$

If the initial flow of funds into the firm is such that it is unable to pay out the initial (equilibrium) dividend then the firm will use debt to grow at a rate faster than the growth in opportunities, provided that this does not violate the restriction imposed by its level of assets, until it reaches the equilibrium path. Hence if the initial optimum dividend payment is represented by the point A, the firm will move along the path AB until it meets the equilibrium path CD at B. Thus the complete solution of the firm's dividend decision is represented by the path ABD. Once the firm has reached its equilibrium path then the value of a project commenced in time period t is

$$\left(-1 + \frac{1+x'}{1+i}\right)\frac{(1+g)^t}{(1+i)^t} + \frac{K_B(i-r)}{(1+i)^{t+1}} \qquad 4.7.50$$

which consists of its discounted cash flow value at the equity rate plus its debt capacity contribution.

However, while the firm is on the portion AB of its path then the above cost of capital formula do not apply. If we assume that the firm is using debt financing then the dual analysis yields

$$-\rho_t + (1+r(1-T))\rho_{t+1} + \lambda_t^B + \lambda_t^M = 0 \qquad 4.7.51$$

$$\rho_t - (1+i)\rho_{t+1} - K_B\lambda_t^B + K_M\lambda_t^M = 0 \qquad 4.7.52$$

Now under the assumption that the level of debt is determined by the market constraint $\lambda_t^B = 0$ and the solution of 4.7.51 and 4.7.52 gives a value for ρ_t of

$$\rho_t = \left(\frac{1+K_M r(1-T)+i}{1 + K_M}\right)\rho_{t+1}$$

or

$$\rho_t = \frac{1}{(1+a)^t} \quad \text{where } a = \frac{K_M r(1-T)+i}{1 + K_M} \qquad 4.7.53$$

Here a is the traditional weighted average cost of capital. In this case the generalised NPV of the project is simply of the one period project is

$$\mu_t = \left[- \frac{1}{(1+a)^t} + \frac{1 + x'}{(1+a)^{t+1}}\right] \qquad 4.7.54$$

Equation 4.7.54 is of course the standard text book formula.

Hence it is seen not only is the cost of capital critically* dependent on the restriction on debt capacity but the actual form that such a restriction takes may vary over the life cycle of the firm. In this particular case initially the firm is able to use debt financing to grow at a rate faster than its growth in opportunities but in the long run the firm must be restricted to grow at the same rate as its opportunities. It should also be noted that in the early phase of its growth the weighted average cost of capital is actually independent of the precise debt equity ratios but is the appropriate valuation rate for projects provided that the firm is using debt finance. This is a consequence of assuming that the equity and debt rates themselves are constant up to a fixed level of gearing and inelastic thereafter. Clearly such an assumption does place severe limitations on the conclusions that can be drawn from such models and this is a point which must be re-addressed in the final chapter. In addition the model just discussed was developed in a framework which does not strictly accord with modern financial theory and must therefore be considered as merely illustrative of the problems involved in long term and short term financial planning.

* Elton, Gruber and Leiber (75) explore the long run cost of capital in continuous time using control theory. However, they erroneuously assume the MM cost of capital formula to hold under different forms of debt capacity restrictions.

The model developed by Myers and Pogue (74) and represented by the systems of equations 4.5.1 to 4.5.7 is in accord with modern financial theory and it would thus seem appropriate to explore the nature of any long run equilibrium solutions. For the convenience of the analysis it is convenient to assume only one investment project consisting of a unit outlay and a return of 1 + x' the following year. The model is represented by 4.5.1 to 4.5.7 can be then conveniently rewritten in the form

$$\text{Max } V_o = \sum_{t=0}^{\infty} \left(\frac{x-a}{(1+a)^{t+1}} X_t + \frac{rT}{(1+r)^{t+1}} \omega_t \right) \qquad 4.7.55$$

subject to

$$X_o - \omega_o + D_o - E_o = F_o \qquad (\rho_o)$$

$$X - (1+x')X_t - \omega_t + (1+r(1-T))\omega_{t-1} + D_t - E_t = 0 \quad (t=1,\infty) \quad (\rho_t) \qquad 4.7.56$$

$$\omega_t \le K(V_t^X + V_t^{\omega}) \qquad (t=0,\infty) \quad (\lambda_t) \qquad 4.7.57$$

$$V_{t-1}^X - (1+x')X_{t-1} + X_t - V_t^X = 0 \quad (t=1,\infty) \quad (\theta_t^X) \qquad 4.7.58$$

$$(1+r)V_{t-1}^{\omega} - rT\omega_{t-1} - V_t^{\omega} = 0 \quad (t=1,\infty) \quad (\theta_t^{\omega}) \qquad 4.7.59$$

$$X_t \le (1+g)^t \qquad 4.7.60$$

plus the usual non-negativity conditions, except for θ_t^X, θ_t^{ω} which are free variables.

The model as formulated in equations 4.7.55 to 4.7.60 differs significantly from the other models discussed in this section in that there exists two separate and non equal discount factors in the objective function. Thus the theory developed by Evers cannot be applied directly. However, if feasible solutions exist such that for the primal solution $z^t \to (1+g)^t \tilde{z}$ and for the dual solution

$\tilde{\eta}^t \to \pi^t \tilde{\eta}$ with complementarity holding then such a solution is optimal. Starting with the dual system such a solution turns out to be relatively easy to find. Thus we are seeking a solution with non-negative values for $\tilde{\lambda}, \tilde{\theta}^x, \tilde{\theta}^w, \tilde{\mu}$ such that ratio of successive dual values in a constant.

Now equation 4.5.19 for λ_t gives

$$\lambda_t = \frac{\lambda_{t-1}}{[1+r(1-KT)]} = \frac{rT}{[1+r(1-KT)]^{t+1}}$$

Thus we must take $\tilde{\lambda} = \frac{rT}{1+r'}$, and the ratio of successive duals as $^1/_{1+r'}$, for the equilibrium solution to be asymptotically consistent with the solution of section 4.5.

This implies $\tilde{\theta}^w = 1$ 4.7.61

and

$$\tilde{\theta}^x = \frac{KrT}{a_o - r'} \qquad 4.7.62$$

with

$$\tilde{\mu} = \left(\frac{x'-r}{1+r'}\right) \frac{KrT}{a_o - r'}, \qquad 4.7.63$$

For such a system to satisfy complementarity then all the primal inequalities must be equalities and thus

$$\tilde{x} = 1 \qquad 4.7.64$$

$$\tilde{\omega} = K\left(\frac{r-g}{r'-g}\right)\left(\frac{x'-g}{a_o-g}\right) \qquad 4.7.65$$

while

$$\tilde{V} = \tilde{V}^x + \tilde{V}^\omega \qquad 4.7.66$$

$$= \left(\frac{r-g}{r'-g}\right)\left(\frac{x'-g}{a_o-g}\right) \qquad 4.7.67$$

Since this solution is primal-dual feasible and complementarity holds then it represents the long-run equilibrium path.

Apart from the $(1+g)^t$ growth factor the long run value of the firm is

$$\tilde{V} = \left(\frac{x'-g}{a_o-g}\right)\left(\frac{r-g}{r'-g}\right) \qquad 4.7.68$$

Now the net operating income in time period t is $(1+g)^t(x'-g)$ or a stream $x'-g$ growing at the rate $1+g$ in perpetuity. Hence the implication is that to value to total income of the firm this stream should be discounted at a rate a where a is given by the solution to

$$\tilde{V} = \left(\frac{x'-g}{a_o-g}\right)\left(\frac{r-g}{r'-g}\right) = \sum_{t=0}^{\infty} \frac{(1+g)^t(x'-g)}{(1+a)^{t+1}} \qquad 4.7.69$$

This gives a value to a of

$$a = a_o(1-KT) - KTg\left(\frac{a_o-r}{r-g}\right) \qquad 4.7.70$$

If the income stream is constant, with $g = 0$, then the above expression reduced to $a_o(1-KT)$ which is of course the MM formula. As was observed in section 4.5 the MM cost of capital is not correct for a non-constant stream, though except in simple cases analytical expressions do not exist for the cost of capital.

This pleothera of rates might appear somewhat confusing and so far the analysis has not indicated how or even whether it is possible to compute these rates from readily available data. Fortunately, it is relatively easy to relate the above rates to the return on equity and the cost of debt.

MM's fundamental principle of valuation as present in equation 4.3.8 defines the return on equity as

$$i = \frac{D_t - E_t + \psi_t - \psi_{t-1}}{\psi_{t-1}}$$

Then from the cash balance equation for the long run equilibrium path

$$D_t - E_t = \left[(x'-g) + \frac{K(r-g}{(r'-g)}(g - r(1-T)\left(\frac{x'-g}{a_o-g}\right)\right](1+g) \qquad 4.6.71$$

Also

$$V_{t-1} = \left(\frac{r-g}{r'-g}\right)\left(\frac{x'-g}{a_o-g}\right)(1+g)^{t-1} \qquad 4.6.72$$

where

$$\psi_{t-1} = V_{t-1} - \omega_{t-1}$$

in accordance with the earlier definition contained in equation 4.2.1. This gives for i the expression

$$i = \frac{(x'-g) + \left[\frac{K(r-g)}{r'-g}\,[g - r(1-KT)]\left(\frac{x'-g}{a_o-g}\right)\right]}{\left(\frac{r-g}{r'-g}\right)\left(\frac{x'-g}{a_o-g}\right)(1-K)} \qquad 4.6.73$$

After some further algebraic manipulation, the following relationship emerges

$$i(1-K) + Kr\ (1-T) = a_o - \frac{rKT}{r-g}(a_o-g) \qquad 4.6.74$$

$$= a_o(1-KT) - \frac{KTg}{r-g}(a_o-r) \qquad 4.6.75$$

$$= a \qquad 4.6.76$$

Hence the conventional weighted average cost of capital formula still holds in this growth case provided the inadequacies of the MM cost of capital formula are accepted. Thus a_o is computable from measurements of the equity return and the formulae as presented in this section are consistent. This provides some justification for the comment made in the introduction to section 4.3 that the different forms for the cost of capital are not necessarily contradictory provided they arise from different but consistent approaches to the valuation problem.

4.8 The practical implications of a finite horizon

Frequent reference has already been made to the horizon problem. In particular two aspects have been of prime concern in this chapter. The first has been the impact of a finite horizon on the use of LP models in the development of theories of valuation. The second is the practical implications of using a finite horizon in financial planning models. It is this latter aspect which is now of immediate concern.

Two possible approaches to determining that horizon has already been discussed in section 1.5. These are the pragmatic approach adopted by Chambers (71) who argues that the planning horizon in practice is largely determined by the firms forecasting ability and natural planning cycle and the theoretically appealing, though possibly non-implementable approach of Weingartner (63) who suggests that it is the point at which increasing the horizon yields no net benefit. The questions to be addressed in this section are two-fold. What are the potential dangers in the Chambers approach and what are the problem of devising a practical methodology which conforms with Weingartner's definition of horizon?

It is assumed that in any implementation, whatever approach is adopted in determining the horizon, the model would be used on a rolling-horizon basis, whereby decisions would be tentatively made in all years upto some horizon but only the year-one decisions would be implemented. At the end of year-one all data would be updated and tentative decisions again would be made upto the horizon advanced by one year. This time year two decisions would be implemented. The planning process would thus continue on this rolling-horizon basis, with planning being over several years, though with only immediate decisions being implemented. While such a process overcomes in part the static nature of LP planning models, it does not in itself solve the problem of how distant the horizon should be

In this section the suggestion of Weingartner that different horizon dates should be tried until one is found which does not (materially) affect the implemented decisions is explored. The exploration is carried out using the model proposed in section 1.7 and detailed in appendices I to V.

Such an exploration of course must be specific to this model and to the horizon valuation used; however, if this model is accepted as being of realistic complexity then the result of such experiments might give some indication as to the seriousness or otherwise of finite horizons in practical planning situations. It is also perhaps worth noting that although Weingartner's ideas on the determinants of the horizon have been widely accepted by other writers, there appears to have been no actual experimentation to determine its viability.

In order to simulate the rolling-horizon planning process the following set of experiments were carried out on the LP model.

The horizon was fixed successively at times upto eight years ahead* in steps of one year. The post-horizon valuation ψ_H at each of these horizonswas just that described in section 4.6 and took the objective function took the form

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* This did of course assume that the initial decisions did become independent of the horizon within the eight year period. The somewhat arbitrary and expedient assumption is justified by the results later in this section.

$$\text{Max} \sum_{t=1}^{H-1} \left\{ \frac{D_t}{(1+i)^{t+I}} - \frac{P.Rg_t}{(1+i)^t} \right\} + \frac{\psi_H}{(1+i)^{H+I}} \qquad 4.8.1$$

The LP model was set up so that it was possible to suppress any constraints occurring in the periods t=H+1 to t=8. Thus the constraint set was operative only over the pre-horizon period.

With the model set up as described and the horizon set at the value H, the optimal decision set for the periods t=1 to H was found. The first year (t=1) investment and financing opportunities were fixed at their solution values using a simple bounding procedure and the horizon was advanced one year. A new optimal solution with respect to both the horizon H + 1 and the existing (or 'implemented') year-one decisions was found. The projects and investments for the second year (t=2) were fixed at their optimal values. The process was repeated until the planning covered the whole eight year span hence simulating a 'rolling-horizon' decision procedure The experiment was repeated for values of H ranging from 1 to 8 in integer steps and for varying levels of earnings from existing projects.

It should be emphasised that the D-statistic* in Table 4.8.1. applies only to the first six years, since projects selected in years 7 and 8 are largely on a NPV criterion in any case. Further the results are strictly only true when the planning horizon in H years for projects implemented in time period t such that $t + H \leq 8$.

Tables 4.8.1. and 4.8.2 shows the effect of various planning horizons on the error in project selection, as measured by the D-statistic and by the value of the plan. These results are displayed graphically in figures 4.8.1. and 4.8.2. Six years was chosen since

* See section 3.6.

TABLE 4.8.1. Error in project selection for different horizons

LEVEL OF EARNINGS \ HORIZON TIME (H)	1	2	3	4	5	6	7
Normal Earnings	7.71	7.01	2.69	1.05	0.86	0	0
Above Average Earnings	7.44	7.44	2.51	1.1	0.74	0.74	0
Below Average Earnings	10.36	7.82	4.85	3.63	2.40	2.40	0

TABLE 4.8.2. Value of plan (£'000's)

LEVEL OF EARNINGS \ HORIZON TIME (H)	1	2	3	4	5	6	7	8
Normal Earnings	1915	1921	2022	2051	2063	2063	2063	2063
Above Average Earnings	2399	2430	2534	2556	2557	2557	2561	2561
Below Average Earnings	1213	1287	1430	1456	1473	1473	1478	1478

FIGURE 4.8.1.

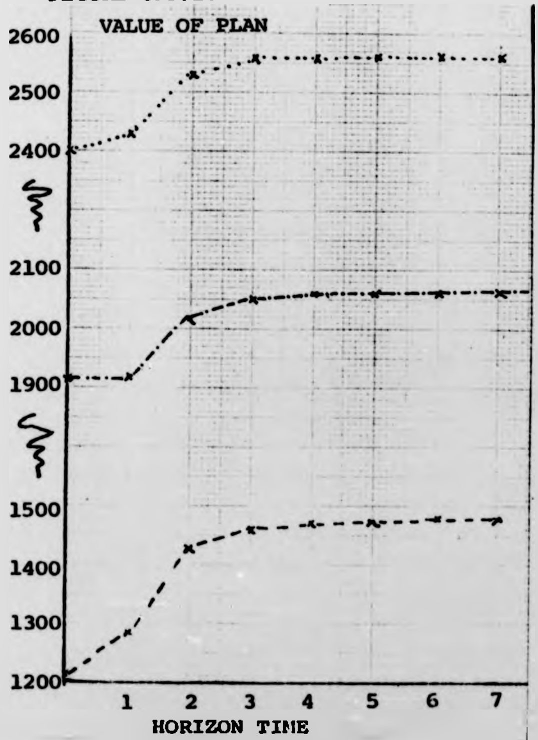
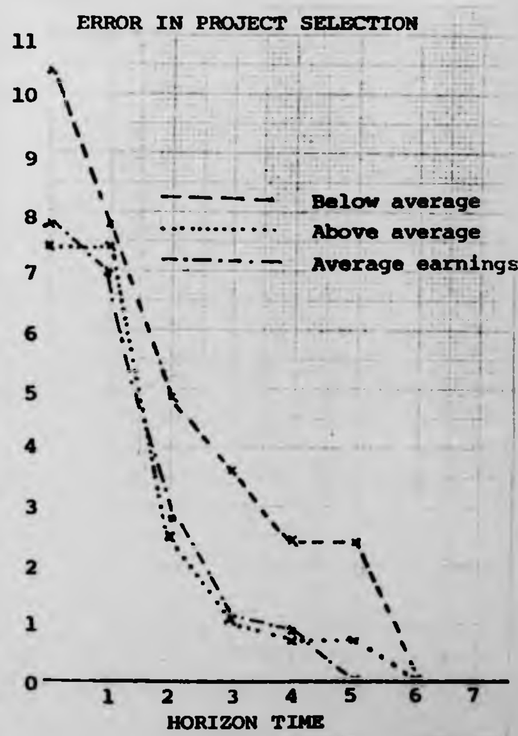


FIGURE 4.8.2.



for H - 6 errors in project selection occurred only in the first two years, whereas for H = 7 errors occurred in all the years upto the horizon. Full details of these results can be found in appendix XIX.

It is possible to examine how far ahead planning must take place before a particular years decisions are unaffected and before a particular year's decisions are only marginally affected. This is shown in Table 4.8.3.

4.8.3. Planning Horizon* (H) necessary before a particular year's decisions are unchanged

LEVEL OF EARNINGS / YEAR OF DECISION (t)	Normal		Above Average		Below Average	
	Identical	Marginal	Identical	Marginal	Identical	Marginal
1	4	4	7	4	7	4
2	6	1	3	3	N/A	5
3	6	1	7	3	5	1
4	N/A	3	5	2	2	2
5	N/A	1	1	1	N/A	3

Many of the conclusions to be drawn from these results are fairly obvious though it is worth speculating on possible explanations of these results to see if any general statements about financial planning models can be made.

As can be easily seen from figures 4.8.1. and 4.8.2. the more distant the planning horizon the greater the accuracy. In fact the indication is that in this particular case a horizon of four to five

* In this table, N/A (not available) means that the horizon time H is such that t+H is certainly greater than 8 years, while a 'marginal' difference in solutions means that the total size of the errors in the scale of project selection is less than unity.

years is sufficient and that information about other projects beyond this point is of no further value.

In this context it is important to stress the change in the nature of the information which takes place at the horizon. The assumption is that the expected value of a projects contribution to the firm does not change as the planning period unfolds and the horizon time recedes. All that changes is the information available about new and alternative opportunities. Thus the risk profile as measured by the expected return and the variance of the returns* does not alter, but rather, the uncertainty surrounding alternative opportunities is removed. Hence risk is differentiated from uncertainty by the existence or otherwise of knowledge about the probability distribution of returns (see Luce and Raiffa (57)). Using this terminology, the conclusion is that for this particular model the project decisions are largely independent of the actual planning horizon and the uncertainty implied by that horizon, provided that the planning horizon is more than five years hence. While this conclusion is of course specific to this model these results when considered in conjunction with those of chapter three suggest that certain more general conclusions might be drawn.

In chapter three, it was argued that simple discounting rules break down only slowly as the complexity of models is increased. Now discounting techniques are horizon-independent valuation procedures using pre-determined interest rates. The model being discussed here is a straightforward extension of such a procedure. The investment project is valued using interest rates and resource shadow prices internally determined by investment and financing interactions in the pre-horizon

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* This is implied by the use of a constant discount factor to value the project cash flows

period, while a simple discounting procedure at some pre;determined rate is used for post-horizon valuation. The evidence of Chapter three suggested that the internally determined interest rates were relatively stable and could be approximated by easily computable constant parameter vectors. Thus, simple discounting rules were able to generate solutions whose value was in excess of 90% of the optimal value. This result is bettered by using a horizon of only three years within an LP model and lends further support to the argument developed in that chapter that it is only during the first few years of a project's life, while the project remains a net investment to the firm, that the accept-reject decision is doubtful. Once this initial investment period has been fully analysed any decision made about the project is unlikely to be revised in the light of further information about other opportunities. Hence a horizon of three to four years should suffice under such circumstances. One strength of LP models of course lies in their ability to rigorously analyse this initial period of a project's life.

Finally the increase in the value of the firm's plan is not proportionately reflected in the decrease in the error in the D-statistic. This suggests that the extension of the planning horizon merely enables a more accurate analysis of projects whose contribution to the firm is increasingly marginal. This merely re-echoes a point made throughout chapter three about the role of LP in discriminating between marginal projects.

While investment projects exhibit a remarkable degree of stability with respect to choice of horizon, the financing projects exhibit no such stability. Table 4.8.4. summarizes the change in the use of financing instruments between a one-year and an eight-year planning horizon at a normal level of earnings from existing projects.

TABLE 4.8.4. Effect of planning horizon on financing

INSTRUMENT	HORIZON \ YEAR	1	2	3	4	5	6	7	8
OVERDRAFT	ONE-YEAR HORIZON	-	-	223	166	249	88	240	-
	EIGHT-YEAR HORIZON	-	-	-	26	-	-	-	-
RIGHTS	ONE-YEAR HORIZON	88	794	75	1	1	-	-	-
	EIGHT-YEAR HORIZON	232	800	-	-	-	-	-	-
LONG TERM DEBT	ONE-YEAR HORIZON	374	2	11	1000	459	1000	-	-
	EIGHT-YEAR HORIZON	483	4	619	1000	812	945	592	-

The most obvious comment concerns the relative use of overdraft and long term debt facilities. Short term financial planning, as represented by the one-year horizon model, requires much more use of the comparatively expensive overdraft financing. Planning over a longer term horizon results in the use of the cheaper long term debt. This particular point is perhaps the most crucial problem in the use of finite horizons. Although project selection remains robust with respect to the choice of horizon, financing alternatives appear not to. While it could be argued that the difference in costs between alternative forms of finance is small, this argument ignores the hidden cost of bankruptcy. Thus the incorporation of restrictions on possible financing alternatives in the pre-horizon period is largely to eliminate the possibility of such an occurrence and the choice of a financing strategy which is acceptable in the pre-horizon period could lead the firm into serious difficulties in the post-horizon period.

One approach to this problem is to use horizon posture constraints which ensure that the firm's financial structure at the horizon is such that difficulties are unlikely to occur in the post-horizon period. This approach suffers from being somewhat arbitrary, attaching no costs or benefits to deviations from the target structure, and does not directly tackle the essentially static nature of such a planning model. The alternative approach is to incorporate the effect of post-horizon constraints into the terminal valuation procedure. In fact this is the approach which is adopted in a joint and unpublished piece of research by the author in conjunction with Atkins, and as such, only the method and results will be outlined here.

The particular problem is whether the valuation algorithm can be devised which satisfied the fundamental horizon principle. While Evers provides an existence proof of such a valuation procedure he gives no indication as to how such a valuation formula might be devised in practice.

The infinite LP system of equations can be recast into the mathematically equivalent form

$$\psi_o(f_o) = \max_{z^o} \left\{ c'z^o + \pi\psi_1(s) \right\}$$

such that

$$[B]\, z^o \le f_o$$

and where

$$s = (1+g)f_o - [A]z^o.$$

where $\psi_o(f_o)$ emphasises the dependence of the plan on the initial resource vector f_o and $\psi_1(s)$ denotes the horizon valuation which is dependent on the initial decisions. This structure is in effect nothing more than an extension of MM's fundamental principle of valuation. Thus the dividend maximization model when recast into such a form becomes

$$\psi_o(F_o) = \underset{\substack{D_1 \\ D_1 \varepsilon \Gamma}}{\text{Max}} \left\{ \frac{D_1}{1+i} + \frac{\psi_1(F_o(D_1))}{1 + i} \right\}$$

where the vector $F_o(D_1)$ denotes the resources available in period one depending on the dividend paid out at the beginning of the period Γ denotes the feasible set of dividends.

The above system is a dynamic programming formulation with a multidimensional state vector (Bellman and Drefus (62)). While such systems are frequently computationally intractable (Morin (77)) several factors enable reasonably good approximations to the solution to be generated for only a small increase in the computing time. Firstly for most financial models the solution, or at least the investment set, is relatively stable with respect to the vector s. Furthermore the solution derived using more conventional valuation formulae as well as the equilibrium solution of the infinite horizon model provide a series of good starting points. The procedure thus depends on the fact that ψ is piecewise linear and convex and that initial approximations can be generated using the conventional valuation procedures and the equilibrium solutions to span the space of s. The algorithm then uses these values to generate new (ψ,s) combination improving the approximation to the function ψ. In effect the combinations (ψ,s) represent states on the possible path as the firm moves towards the equilibrium combination $(\tilde{\psi},\tilde{s})$. Once the firm reaches its equilibrium path it will remain on it.

Using such an algorithm, convergence turns out to be fairly rapid. Thus for a problem consisting of 10 projects and 8 financing opportunities subject to 5 pre-horizon constants only 25% was added to the computational time using this algorithm as opposed to a conventional terminal valuation procedure.

Although the set of investment decisions was only marginally affected using this horizon valuation procedure, the level of debt financing was altered by a factor of six. In the conventional form of the model the level of debt was restricted in the pre-horizon period by a times interest cover, though clearly there were no restrictions on the times covered factor in the post horizon period. Thus the incorporation of a 'post-horizon constraint' directly into the valuation procedure avoided the potential difficulty arising from a failure to cover interests payments adequately in the post-horizon period. The danger therefore of using finite horizon models in practice lies not in the investment opportunities foregone, but rather, from the possibility of accepting (financing) commitments which might seriously jeopardize the long term viability of the firm.

4.9 Conclusion

In this chapter the role of mathematical programming models in analysing the interactions between the investment and financing decision have been examined. Within this mathematical programming framework it has been possible to develop normative rules for the appraisal of investment projects. The main conclusions to be drawn from such an analysis are that any cost of capital formula used for project appraisal is critically dependent on the nature of the restrictions placed on the level of debt and that the conventionally accepted MM cost of capital formula breaks down for finite or irregular cash flows. The methodology adopted further provided insight into the consistency and structuring of financial planning models. Also examined were the practical implications of a finite horizon in LP financial planning models. It was argued that the investment decision is largely independent of the planning horizon and that the use of finite horizons does not pose any severe limitations on the use of LP models for such purposes. However, the real problem in using such models appears to lie in the danger of undertaking financing commitments which might seriously jeopardize the future profitability of the enterprise. Fortunately, relatively minor changes to the computational procedures enable this particular problem to be overcome.

In summary this chapter has explored the contribution that linear programming models can make to the extension of discounting techniques into situations where the capital markets impose restrictions on the access to borrowing. In the following chapter these ideas are further applied to the analysis of one particular financing instrument - a lease contract.

CHAPTER 5

THE VALUATION OF A FINANCIAL LEASE - A MATHEMATICAL PROGRAMMING FRAMEWORK.

5.1 Introduction.

One possible method of evaluating a lease in practice is to incorporate the lease as a project into a mathematical programming model of the firm in which all investment and financing decisions are considered simultaneously. While such an approach is certainly valid, the work of chapter 3 suggests that the solutions generated by many of these models show little or no improvement over discounting approaches. Moreover, this work, together with that of the last chapter, has shown that many linear programming formulations of the investment decision are equally capable of analytical or semi-analytical solutions. Thus mathematical programming models of the investment and financing decisions provide more than a mere computational tool for lease evaluation; they provide a generalised framework in which analytical expressions for the value of a lease may be derived.

The derivation of these analytical expressions is by the use of the Kuhn-Tucker optimality conditions for constrained optimisation. In this chapter a general mathematical programming model of the firm will be developed and by the use of the Kuhn-Tucker conditions an expression for the value of a lease will be deduced. The mathematical programming approach is similar to that developed in the recent paper by Myers, Dill and Batisto (76) and this paper owes much to their excellent exposition.

The particular valuation model generated by Myers will be examined in some detail. Myers' work assumes that the correctness of Modigliani and Millers (MM) contention that the only value of debt is in its tax shield, that dividend policy is irrelevant and that the assumptions of the capital asset pricing model holds.

In contrast the section following adopts a traditional approach to valuation and uses an analogous expression resting on the different assumptions of traditional financial theory. These two expressions are contrasted with "naive discounting" measures of the value of a lease and it is seen that the relative 'pureness' of the assumptions of the economic theory underlying the MM and traditional valuation models fail to provide an adequate rationale for leasing.

In the following sections various accounting measures of debt capacity are introduced. Thus the next section uses a mathematical programming model of the firm developed by Chambers (71) in which debt is measured in book value terms and the restriction of the use of debt in a restriction on the (book) level of leverage. This accounting valuation introduces sufficient imperfection into the measurement of debt that situations are identified when it is preferable to lease even though the after tax interest rate on lease finance is higher than that on debt finance. Of course, it could be argued that the financial markets are unlikely to use such a "naive" measure of debt such as book values preferring to relate the amount of debt to expected future earnings. The next section therefore modifies the Chambers' model so that the debt capacity is related to the future cash inflows. It is shown that rather than removing the arbitrariness from the book measures of debt capacity such a step compounds the problem and, depending on the precise nature of the times cover constraint, situations arise when leasing can seem very attractive indeed. The next model of valuation examined in Weingartner's basic horizon model (74) with simple bounds on debt availability. This model enables the impact of "hard" capital rationing on the lease evaluation problem to be determined.

All these models have the common basic structure where the only financial restriction is on debt availability. The model outlined in section 1.7 has many other constraints imposed on its investment and financing strategies and it is worthwhile examining the determinants of the lease decision in such circumstances. Although a rough analytical treatment of project selection was produced in chapter three it is preferable here to identify *post ante* the precise role played in the valuation by the various constraints. In section 5.8 a methodology is developed which separates out the contribution to the lease value of the various constraint sets. The final section draws together the conclusions arising from the various models and suggests that the economic analysis of the lease evaluation problem may well view leasing in too simplistic a framework. The main reasons for leasing that emerge from this chapter are the imperfections of accounting measures of debt, the non-availability of medium term financing opportunities and the need for balance sheet management.

5.2 An Analytical Framework.

We assume that the objective of the firm's management is the maximisation of the value ψ of the firm at some, as yet, unspecified time, i.e. Max $\psi(x, L, v, w, D, E)$ subject to a cash balance constraint

$$C_t(x,L,v,w,D,E) \leq F_t \qquad 5.2.1.$$

and a debt capacity constraint

$$w_t \leq \phi_t(x,L,v,D,E) \qquad 5.2.2.$$

plus the scale constraints

$$0 \leq L_j \leq x_j \leq 1 \qquad 5.2.3.$$

Here L denotes a vector of leasing opportunities, where the individual components of L are associated with the scale of a particular lease opportunity. The rest of the notation is as before and is summarized for convenience in appendix II.

The Kuhn-Tucker condition for optimality when applied to the leasing variable give

$$\frac{\partial \psi}{\partial L_j} - \sum_{t=0}^{H} \rho_t \frac{\partial C_t}{\partial L_j} + \sum_{t=0}^{H} \lambda_t \frac{\partial \phi_t}{\partial L_j} - \mu_j^L \leq 0 \qquad 5.2.4.$$

If project j is leased then the inequality becomes an equality and the reduced cost (μ_j^L) of the lease is given by

$$\mu_j^L = \frac{\partial \psi}{\partial L_j} - \sum_{t=0}^{H} \rho_t \frac{\partial C_t}{\partial L_j} + \sum_{t=0}^{H} \lambda_t \frac{\partial \phi_t}{\partial L_j} \qquad 5.2.5.$$

If we look at the terms in more detail then we see that $\partial\psi/\partial L_j$ is the direct marginal increase in the value of the company for each unit of leasing. $\frac{\partial C_t}{\partial L_j}$ is the cash flow associated with a unit of the lease and ρ_t is the discounting or compounding factor, depending on whether the model is a net present value model or a terminal horizon model. Hence, the first two terms represent the 'pure' cash flow effects of the lease.

The term $\frac{\partial \phi_t}{\partial L_j}$ is the amount of debt capacity used up by the lease and λ_t is the value associated with the debt capacity. Hence the role of μ_j^L is akin to the role played by the dual on the project constraint and can be interpreted as the generalised net present value of the lease. If the right hand side of equation 5.2.5. is negative the lease ought not to be taken on, while if it is positive the lease ought to be adopted. The only real problems are the values $\frac{\partial\psi}{\partial L_j}$, ρ_t, λ_t, and $\frac{\partial\phi_t}{\partial L_j}$. Their values are intimately linked to the valuation model adopted and the measure of debt capacity chosen. The remainder of this chapter is concerned with this problem.

5.3 Lease evaluation in a Modiligani - Miller World.

As already stated Myers assumes the correctness of MM's contention that the only value of debt is its tax shield, that dividend policy is irrelevant and that the assumptions of the capital was pricing model holds. He shows in a separate paper (Myers (74)) that the implications of such assumptions are that $\rho_t = 0$. In addition the marginal value*of debt, where debt is one-year renewable, is given by $\lambda_t = \frac{rT}{(1+r)^{t+1}}$ for all years in which debt is raised. The 'cheapness' of debt in the MM world would ensure that debt is always used to its limit and hence that the debt capacity constraint is always binding. The MM idea is that an upper limit on the amount of debt is imposed by the existence of a target ratio of the market value of debt to the market value of the firm.

A complication arises because the market value of assets have differing risks attached to them. Thus we could identify the debt capacity ϕ_t with $K\psi_t$ where K is the firms overall target debt ratio or with $\sum_j K_j \psi_{jt}$ where K_j is the debt ratio associated with a particular asset risk stream. For the time being we shall assume the latter more general form and discuss the problem again when we come to interpret our solution. It should also be noted that Myer's restriction on debt applied solely to 'pure' debt; his measure of debt does not include leasing - a point which is not at all clear from Myers' owen analysis. The impact of the lease on debt is via its impact on the market value of the firm. If we denote the value of £1 leasing at

*See equation 4.5.20

time t by V_t then we can use the adjusted present value approach, discussed in the last chapter.

$$V_t = \frac{\partial \psi_t}{\partial L_j} + \sum_{\tau=t}^{H} \lambda_\tau \frac{\partial \phi_{\tau t}}{\partial L_j} \qquad 5.3.1.$$

$$= A_t + \sum_{\tau=t}^{H} \frac{rT}{(1+r)^{\tau+1-t}} K_L V_\tau \qquad 5.3.2.$$

Here $\frac{\partial \phi_t}{\partial L_j} = K_L V_\tau$ where K_L is the debt value ratio for the lease

and $\frac{\partial \psi_t}{\partial L_j} = A_t$ where A_t denotes just the net present value per £1 of leasing of the lease cash flows. The discount rate according to the capital asset pricing model would be the riskless rate r. This implies that

$$A_t = \frac{A_{t+1}}{(1+r)} - \frac{b_t T + P_t(1-T)}{(1+r)} \qquad 5.3.3.$$

and

$$\sum_{\tau=t}^{H} \frac{f V_\tau}{(1+r)^{\tau-t}} = f V_t + \frac{f}{(1+r)} \sum_{\tau=t+1}^{H} \frac{V_\tau}{(1+r)^{\tau-t-1}} \qquad 5.3.4.$$

where

$$f = \frac{K_L \, r \, T}{(1+r)} \qquad 5.3.5.$$

Hence combining equations 5.3.2 - 3 - 4 and 5 gives

$$V_t = \frac{-(b_t T + P_t(1-T))}{(1+r)} + fV_t + \frac{V_{t+1}}{(1+r)} \qquad 5.3.6.$$

Hence

$$V_t = \frac{-(b_t T + P_t(1-T))}{[1+r(1-K_L T)]} + \frac{V_{t+1}}{[1+r(1-K_L T)]} \qquad 5.3.7.$$

Hence 5.3.7. relates the value of the lease at time t to its value at time t+1 and the cash flows incurred by the lease contract in the intervening period. Now on termination of the lease the value of the lease is zero hence $V_H = 0$.

Also

$$V_o = c_o - \left[\frac{b_1T + P_1(1-T)}{1+r(1-K_LT)}\right] + \frac{V_1}{[1+r(1-K_LT)]} \qquad 5.3.8.$$

where c_o is the cost of the asset.

We can thus use the recurrence relationship (5.3.7.) together with the boundary conditions to generate the value of the lease* as:

$$V_o = c_o - \sum_{t=1}^{H} \frac{P_t(1-T) + b_tT}{[1+r(1-K_LT)]^t} \qquad 5.3.9.$$

which is Myers' formula, though Myers' own derivation is somewhat more complicated.** The issue still remains as to the appropriate value of K_L.

If it is assumed that because of the contractural nature of lease repayments that a lease is associated with cash flows which are certain then it could be argued that the value of K_L should be riskless value and be equal to unity. This approach then gives a value for the lease of

$$V_o = c_o - \sum_{t=1}^{H} \frac{P_t(1-T) + b_tT}{[1+r(1-T)]^t} \qquad 5.3.10.$$

This is, of course, just the net present value of the after tax cash flows associated with the lease discounted at the after tax debt rate. Hence, the lease decisions with these assumptions would appear to be quite simple. If the after tax rate on debt is

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*Strictly speaking this is the value per unit of leasing. For ease of reference the term value will be used.

** The formula could have been derived as a special case of the adjusted present value formula. The certain nature of the cash flows implies that $a_o = r$ and substitution of this value into expression 4.5.29. gives the desired form immediately. The formula was derived from first principles here in order that the various assumptions made could be cited explicitly.

greater than this after tax cost of the lease it is preferable to lease rather than to use debt finance. If the cost of the lease is greater than the after tax debt rate then debt finance is cheaper. Since in general as Vancil (61) observes debt is usually a cheaper form of finance than a lease; in a Modigliani-Miller world leasing is unattractive. Perhaps this rather simplistic result from a relatively sophisticated piece of analysis is disappointing: though on reflection, it is not surprising. Indeed it would perhaps be surprising if the assumptions of market perfections subsumed within this model led to any result other than the value of a lease is just the after tax cash flows discounted at the after tax debt rate. In a strict economic market view it is difficult to see that leasing is anything other than a relatively unattractive alternative*.

5.4 A Traditional Approach.

While a few authors have used the after tax cost of debt as the appropriate discount rate, many authors have used a weighted average cost of capital formula, where the weighting factor is a debt equity ratio. It is possible to redefine ψ, ϕ such that the mathematical programming formulation accords with this 'traditional' approach.

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* Myers thoroughly explores the problem of the effects of differing tax rates on the lessee and lessor as well as the effects of different depreciation patterns and shows that this may give rise to circumstances when leasing is attractive. This chapter assumes throughout that the firm is paying tax at a standard rate on all its earnings and uses for the sake of numerical illustration on straight line depreciation. The purpose of the chapter is to identify reasons for leasing which do not arise solely because of particular advantageous tax situations.

The particular model chose is the one of section*

$$\text{Max } \psi_o = \sum_{t=0}^{\infty} \frac{D_t - E_t}{(1+i)^t}$$

s.t. Project cash flows + $D_t - E_t - (w_t - w_{t-1}) + w_{t-1} r(1-T) \leq F_t$

$$w_t \leq K\psi_t$$

$$(1+i)\psi_{t-1} = D_t - E_t + \psi_t$$

where $\psi_t, D_t, E_t, w_t \geq 0$

The objective function in this case is the maximization of the net cash flows to the shareholders discounted at the (equity) rate i where maximization is carried out subject to a restriction on the market values of debt and equity.

As we saw in section 4.7 a dual analysis of this model yields a value for $\rho_t = \frac{1}{(1+a)^t}$ where $a = \frac{i+Kr(1-T)}{1+K}$ and plays the role of the weighted average cost of capital. The shadow price on debt is given here by $\lambda_t = \frac{w - r(1-T)}{(1+a)^{t+1}}$

It should be noted that the lease interest rate plays no role in the weighted average cost of capital. This is because we have made the implicit assumption that while the lease may effect the value of the firm by affecting the future dividend streams it is assumed that it does not affect the perceived risk of that stream. In other words we have assumed that the return required by the

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* It may seem strange to choose a model which has been subject to such severe criticism in the last chapter but it is a convenient vehicle for the analysis. We must of course assume that the firm is in a disequilibrium state and currently using debt to grow faster than the growth rate of opportunities. It must be further assumed that there are other restrictions on debt which are currently non binding but which will be the eventual determinants of the equilibrium values.

holders of equity and debt is not materially affected by the lease decision. While this assumption may not be strictly justified it is difficult to incorporate alternative assumptions.

A more intractable difficulty is the effect that the lease has on the debt capacity. If the lease has no effect, then clearly the impact of the lease is merely via its effect on the cash balance equations and $\frac{\partial\phi}{\partial L_j} = 0$. Also in this case $\frac{\partial\psi}{\partial L_j} = 0$ since the firm is valued in terms of its net equity flows. The term $\frac{\partial C_t}{\partial L_j}$ $(=C_t)$ are just the depreciation tax shields and the lease repayments. They are given by the expression

$$C_t = P_t(1-T) + b_tT \qquad 5.4.1.$$

This produces a value for the lease of $V_o = c_o - \sum_{t=1}^{H} \frac{P_t(1-T) + b_tT}{(1+a)^t}$ 5.4.2.

or just the incremental cash flows associated with the lease evaluated at a weighted average cost of capital. This analysis is, of course, equivalent merely to treating the lease as another project and as such is somewhat unsatisfactory since a lease may be viewed, in part, as an alternative to debt. If we assume that this is so and incorporate the value of the lease into the debt capacity constraint, so that this constraint now reads:

$$w_t + V_t \leq k\psi_t \qquad 5.4.3.$$

Then the value of the lease at time t is given by

$$V_t = -\sum_{\tau=t}^{H} \frac{C_t}{(1+a)^{\tau-t}} + f\sum_{\tau=t}^{T} \frac{V_\tau}{(1+a)^{\tau-t}} \qquad 5.4.4.$$

where

$$f = \frac{a - r(1-T)}{(1+w)} \qquad 5.4.5.$$

Then

$$V_t = C_t + fV_t - \frac{1}{(1+a)} \sum_{\tau=t+1}^{H} \frac{C_\tau + fV_\tau}{(1+w)^{t-(\tau+1)}} \qquad 5.4.6.$$

$$= C_t + fV_t + \frac{f}{(1+a)} V_{t+1} \qquad 5.4.7.$$

$$= V_t(1-f) = C_t + \frac{f}{(1+a)} V_{t+1} \qquad 5.4.8.$$

Again with the same initial conditions and end conditions as in section 5.3 the value of the lease is given by

$$V_o = c_o - \sum_{t=1}^{H} \frac{P_t(1-T) + b_t T}{[1+r(1-T)]^t} \qquad 5.4.9.$$

Thus in this case the value of the lease is just the after tax cash flows discounted at the debt rate. Neither result is surprising, if the lease makes no impact on debt capacity then it is merely another project and its value is just the incremental cash flows of the lease evaluated at the weighted average cost of capital. If the lease is treated as an alternative to debt then it must also be valued at the debt rate. While the latter treatment would seem preferable it is clear that in general the relative cheapness of debt would make the lease unattractive. Again within a framework of theoretical market valuations of assets and liabilities leasing is an unattractive instrument.

The foregoing analysis uses market values for measuring debt capacity in which the ability of the firm to support debt is related to future income streams. In general* financial markets actually

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*See for example Barges (63).

impose restrictions on the use of debt which are more closely related to accounting valuations. In these debt capacity is related to current income levels and existing asset-liability structures. The remainder of this chapter looks at lease evaluation methods where debt capacity is measured in more conventional accounting terms.

5.5 Leasing in an Accounting Framework.

The model of Chambers (71) is eminently suitable for the analysis of the impact of the accounting treatment of leases. The model incorporates the main features of the current U.K. tax system and the restriction placed on the level of debt is the book (accounting) value of gearing or leverage.

Moreover, as was shown in section 3.5, the linear programming has a well defined dual feasible region which is capable of an analytical treatment. However, one of the difficulties of this particular model is that the algebraic expressions for the duals associated with the cash balance constraints and the debt capacity constraints are cumbersome. Thus, the dual on the cash balance constraint* is given by the solution 3.5.14 and 3.5.24

$$\rho_t = \frac{g}{1+g}(1+i)^{H-t} + \frac{1}{(1+g)}\left\{\frac{1-f}{100} - r\ T+r(1-T)\sum_{\tau=t+1}^{H}(1+i)^{H+1-\tau}\right\}$$

while the dual on the debt capacity λ_t is given by equation 3.5.14

$$\lambda_t = \frac{[(1+i)^{H-t} - \rho_t]}{(1+g)}$$

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* This is the dual where the firm is actually raising debt. It will be assumed temporarily that if the firm is leasing it remains in a deficit (debt raising) state implying that the funds required for investments in fixed assets exceed that generated by on-going operations. This restriction will be relaxed later.

Here g denotes the level of gearing, i the equity rate and f the flotation costs associated with equity.

Since this (and most of the subsequent models to be discussed) are terminal valuation models in which the objective function is the maximization of the horizon value of the firm, it can be assumed, without loss of generality, that the horizon is coterminous with (or post dates) the last lease payment. This ensures that

$$\frac{\partial \psi}{\partial L_j} = 0$$

The cash flows* associated with the lease repayments are $P_t(1-T) + b_t T$ and the book value of the lease at time t is given by:

$$\sum_{\tau=t+1}^{H-1} \frac{P_\tau}{(1+i_L)^{H-\tau}} \qquad 5.5.1.$$

where i_L is the implied pre-tax interest rate on the lease (see section 1.5). In addition the decision to lease would affect the book value of retained earnings arising from the difference in lease repayment and depreciation† expenses, b_t. The value of this at time t is:

$$\sum_{\tau=1}^{t} (P_t-b_t)(1-T) \qquad 5.5.2.$$

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* This is a minor inconsistency here since the debt dual is calculated assuming that there is a one year lag in tax payments while this calculation on the lease repayments assumes no tax lag. However, since the purpose of this section is to identify the circumstances under which leasing takes place it was not thought necessary to change this assumption for this section only. It will be seen that this difference is not crucial.

† For the sake of convenience it is assumed that book and tax depreciation rates coincide.

Thus the net debt capacity effect is:

$$\frac{\partial \phi_t}{\partial L_j} = \sum_{\tau=t+1}^{H-1} \frac{P_\tau}{(1+i_L)^{H-\tau}} + g \sum_{\tau=1}^{t} (P_t - b_t)(1-T) \qquad 5.5.3.$$

This defines all the terms of equation 5.2.4. though the resulting algebraic expression conveys little insight into the impact of such a valuation system on the lease decision. In order to gain some idea of the order of magnitude of the various effects, some numerical computations were carried out.

In Table 5.5.1. the net present value /£100 of lease is shown assuming a 12% equity rate, a limit on debt to equity of 50% and a 40% tax rate. The equity flotation costs were 3% and the lease was repaid in 5 equal annual instalments. Straight line depreciation over 5 years was assumed throughout.

Table 5.5.1 Net Present Values* /£100 Lease Finance in the Chamber Model.

Lease Interest Rate		Debt (debenture) rate (%)								
Before Tax(%)	After Tax(%)	Nominal rate before tax	5	6	7	8	9	10	11	12
		Effective after tax	3.3	4.0	4.6	5.3	5.9	6.5	7.1	7.7
6	3.7		-0.3	1.1	2.6	4.0	5.5	6.9	8.4	9.8
8	5.0		-3.1	-1.6	-0.1	1.3	2.8	4.3	5.7	7.2
10	6.2		-5.9	-4.4	-2.9	-1.4	0.1	1.6	3.1	4.6
12	7.5		-8.7	-7.2	-5.7	-4.2	-2.7	-2.3	0.3	1.9
14	8.7		-11.6	-10.1	-8.6	-7.0	-5.5	-4	-2.4	-0.9
16	9.9		-14.5	-13.0	-11.5	-9.9	-8.4	-6.8	-5.3	-3.7
18	11.2		-17.5	-16.0	-14.4	-12.8	-11.3	-9.7	-8.1	-6.6
20	12.4		-20.6	-19.0	-17.4	-15.8	-14.2	-12.6	-11.0	-9.5

*The net present value is related to the net terminal value by the factor $(1+i)^H$.

In table 5.5.1. it is assumed that the firm is always in a deficit situation. With such an assumption it can be seen that a lease is only attractive where its after tax rate is comparable with the after tax rat on debt. Thus in the original article where Chambers used a 6% before tax (4% after tax rate on debt) a lease does not become attractive until its after tax rate is down to 4.3%.

At first sight even this may seem somewhat puzzling. Thus a firm finds it more attractive to lease a project at an after tax cost of 4.3% when debt is available at only 4%. This point is immediately clarified if we write down the net cash flows together with the effects on debt capacity of an 'acquire plus buy with debt' as against on 'acquire via a lease' decision for £100 of assets. These are shown in Table 5.5.2.

Table 5.5.2. Comparison of cash flows and capacity effects.

DECISION \ YEAR	1	2	3	4	5	POST HORIZON
Buy with debt						
Debt servicing flows	100	(6)	(3.6)	(3.6)	(3.6)	(85.3)
Use of debt capacity	101.5	103.3	105.6	106.9	108.7	-
Acquire via lease						
Lease servicing flows	100	(23)	(23)	(23)	(23)	(23)
Use of debt capacity	101.5	86	69.1	50.8	30.9	-

While the net present values of the two cash flows streams differ little, both having an internal rate of return of about 4%, the debt capacity effects differ markedly. If debentures are issued to fund the project the use of debt capacity increase over time.

This is because apart from the debt being assumed non-redeemable during the life of the project the servicing of this debt reduces profits and thus the book value of equity via retained earnings. In the case of a lease, the lease repayments reduce the book value of the outstanding debt and hence release debt capacity, the reduction in retained earnings caused by the lease playing only a minor role.

The debt capacity is even more marked if it is assumed that the firm is in a cash deficit position for the first three years and a cash surplus for years 4 and 5. This case is shown in table 5.5.3.

<u>Table 5.5.3.</u> Net present values/£100 Lease
Assuming the firm is in a cash deficit* position during first three years of the lease.

Lease Interest Rate										
Before Tax (%)	After Tax (%)	Nominal rate before tax	5	6	7	8	9	10	11	12
		Effective after tax	3.3	4.0	4.6	5.3	5.9	6.5	7.1	7.7
6	3.7		7.6	8.1	8.6	9.1	9.6	10.1	10.6	11.1
8	5.0		5.0	5.5	6.0	6.5	6.9	7.4	7.9	8.4
10	6.2		2.3	2.8	3.3	3.8	4.2	4.7	5.2	5.7
12	7.5		-0.4	0.1	0.6	1.0	1.5	2.0	2.4	2.9
14	8.7		-3.1	-2.7	-2.2	-1.7	-1.3	-0.8	-0.4	0.1
16	9.9		-5.9	-5.5	-5.0	-4.6	-4.1	-3.7	-3.2	-2.7
18	11.2		-8.8	-8.3	-7.9	-7.4	-7.0	-6.5	-6.1	-5.6
20	12.4		-11.7	-11.2	-10.8	-10.4	-9.9	-9.5	-9.0	-8.6

* The interperiod discount rate when the firm has surplus funds is calculated from $\rho = (1+i_G(1-T))\rho_{t+1}$ where i_G is the rate on Government stock. In this case i_G was assumed to be 6%. The dual on the debt capacity is calculated as before from the formula $\lambda_t = \frac{[(1+i)^{H+1-t} - \rho_t]}{(1+g)}$

Under such circumstances, for instance, the rate at which leasing fails to be attractive when the after tax debt rate is 4% is now as high as 7.5%.

5.6 The Times Covered Constraint.

The Chambers model discussed in the previous section uses as its restriction on debt capacity the level of the firm's gearing. One other frequently used restriction on the level of debt which we have identified, is the extent to which the interest costs are covered by the earnings of the firm. It is relatively easy to modify the Chamgers model so that the restriction on debt is in the form of a times interest covered. If we assume that the after tax interest payable is covered K times by the net after tax operating cash inflows in that period, the dual inequalities for debenture issues at time t are of the form:

$$-\rho_t + \sum_{\tau=t+1}^{H-1} r(1-T)\rho_\tau + \sum_{\tau=t}^{H} Kr(1-T)\lambda_\tau \leq -1 \qquad t=1,\ldots\ldots,H-1 \qquad 5.6.1.$$

$$-\rho_H + K_r(1-T)\lambda_H \leq -1 \qquad t=H \qquad 5.6.2.$$

The dual inequalities for rights issues lead us to the conclusion that:

$$\rho_t \geq (1+i)^{H+1-t} \qquad 5.6.3.$$

If we assume for the ease of analysis that the firm is in a deficit situation* throughout the period of the lease and it is raising both debt and equality in each year, then the above set of inequalities become equalities. We can deduce that:

* A similar assumption, which was later relaxed, was made in the previous section

$$\rho_t = (1+i)^{H+1-t} \qquad 5.6.4.$$

$$\lambda_t = \left[\frac{i - r(1-T)}{Kr(1-T)}\right](1+i)^{H-t} \quad t=1,2....,H-1 \qquad 5.6.5.$$

$$\lambda_H = \frac{i}{Kr(1-T)} \qquad 5.6.6.$$

There remains the problem of the measurement of the 'debt' associated with the lease. One possible alternative would be to examine the cover of the imputed interest portion of the lease. However, such an analysis is rejected here for two reasons. The first is that the purpose of this constraint is to relate more directly the ability of a firm to meet contractual payments out of its operating income. Under such circumstances the partitioning of one such payment into two cash flow streams (which are to be analysed differently) would seem nonsensical. The second objection is that such a treatment is in effect largely an accounting approach, apportioning repayment into interest plus repayment of principal, and as such is similar to the analysis already carried out on the Chambers model.

Equally, since a lease repayment is in part an interest payment and in part a repayment of capital it should not be unfairly treated (in comparison with debt) by assuming that the total lease payment must be covered K times by the net cash inflows.

The approach adopted is that the operating cash flows after tax and after lease repayments have been made must cover interest payable K times. Such a restriction clearly suffers from fairly obvious drawbacks. The main one is the rather arbitrary division into a risky 'adjusted' income stream and fixed debt interest payments.

It has the very great advantage of computational simplicity. The analysis carried out, therefore, must be considered as illustrative of the approach rather than definitive.

The value of the lease becomes with $\frac{\partial \psi}{\partial L_j} = 0$

$$c_0(1+i)^{H+1} - \sum_{\tau=1}^{H}(1+i)^{H+1-\tau}\left[\rho_\tau(1-T) + b_\tau T\right] - \sum_{\tau=1}^{H-1}(1+i)^{H-\tau}\left[\rho_\tau(1-T)+b_\tau T\right]$$

$$\left[\frac{i-Kr(1-T)}{Kr(1-T)}\right] + \left[\rho_H(1-T) + b_H T\right]\frac{i}{Kr(1-T)} \qquad 5.6.7.$$

which, ignoring the anomalous 'end effects' term for debt capacity of $\frac{i}{Kr(1-T)}$ instead of $\frac{i-r(1-T)}{Kr(1-T)}$, the expressions for the net terminal value of lease can be written as:

$$(1+i)^{H+1}\left[c_0 - \sum_{\tau=1}^{H}\left(\frac{P_\tau(1-T)+b_\tau T}{(1+i)^\tau}\right)\left(1+i + \frac{i-\tau(1-T)}{Kr(1-T)}\right)\right] \qquad 5.6.8.$$

The expression in the square brackets is the net present value of the lease. It can be seen that this value is the net present value of the cash flows associated with the cost of the lease discounted at the equity rate plus a premium proportional to this net present value. Hence, while the lease repayment cash flows are evaluated at a relatively high equity rate, making the lease attractive, cognisance must be taken of the penalty associated with the use of the debt capacity.

Table 5.6.1. summarises the effect* of various debt rates, leasing rates together with times-interest covered factors on the net present value of £100 of leasing. The lease again is assumed repayable in equal instalments over five years.

TABLE 5.6.1. Value† of a lease under a times covered restriction on debt.

DEBT TIMES COVERED FACTOR (F) / TIMES COVERED	0.20	0.24	0.28	0.32	0.36	0.40	0.44	0.48
	AFTER TAX DEBT RATES							
1	11.1	10.7	10.3	10.0	9.6	9.6	9.3	8.8
5	8.5	7.5	6.6	6.0	5.4	5.0	4.6	4.3
10	6.6	5.4	4.6	4.0	3.5	3.1	2.8	2.6
15	5.4	4.2	3.5	3.0	2.6	2.3	2.0	1.8
20	4.6	3.5	2.8	2.4	2.0	1.8	1.6	1.4
LEASE RATES AFTER TAX (before tax in brackets)	£NPV / £100 OF LEASE							
3.7(6)	14.0	11.2	8.3	5.4	2.6	-0.3	-3.1	-6.0
5.0(8)	11.0	8.0	5.1	2.1	-0.8	-3.8	-6.8	-9.7
6.2(10)	7.9	4.9	1.8	-1.3	-4.3	-7.4	-10.5	-13.6
7.5(12)	4.8	1.6	-1.6	-4.8	-7.9	-11.1	-14.3	-17.4
8.7(14)	1.6	-1.7	-5.0	-8.3	-11.6	-14.8	-18.1	-21.4
9.9(16)	-1.7	-5.1	-8.5	-11.9	-15.3	-18.7	-22.1	-25.4
11.2(18)	-5.1	-8.6	-12.1	-15.6	-19.1	-22.6	-26.1	-29.6
12.4(20)	-8.4	-12.0	-15.7	-19.3	-22.9	-26.5	-30.1	-33.6

* The Table relates the NPV/£100 of lease to the debt times covered factor and the after tax lease rate. Thus a debt times covered factor of 0.28 (column 3) is equivalent to a times covered value of 5 and an after tax debt of 6.6% or to a times covered of 10 and an after tax debt rate of 4.6%. At an after tax lease rate of 5% both of these combinations give a positive net present value to the lease of £5.1/£100 leased.

† Equity Rates (i) = 12% Tax Rate (T) = 40%

$$F = i + \frac{i-r(1-T)}{Kr(1-T)}$$ denotes the debt times-covered factor.

The results are not surprising. If the debt rate is low or the times interest cover is low, then the structure of the debt capacity constraint favours leasing and leasing becomes quite an attractive proposition. Thus it is marginally worth leasing (£1.6 NPV/£100 leasing) even if the after tax lease rate is 8.7% when the after tax debt is only 4.6% provided the cover required is 20. With a debt rate at 8.5% after tax, the cover needs to fall to 5 times for leasing still to be attractive. Again it is worth emphasising that such an analysis is merely illustrative of the problems and possible results of using a times interest cover restriction of debt. It must be remembered that the actual values computed rest heavily on the definition of 'times covered'.

5.7 The Weingartner Model and Leasing.

It has been seen that under certain circumstances leasing may be attractive, though this attraction would appear to stem from the ability of a lease to meet a medium term debt requirement or from a particularly favourable method of accounting for the impact of a lease on debt capacity. Even under such circumstances the attraction of a lease is frequently marginal. Two authors who have suggested that leasing may be particularly attractive where there is some form of hard capital rationing are Fawthrop and Terry (75). In this case Weingartner's basic horizon model provides the requisite analytical framework and it is thus appropriate to attempt a formal treatment of the lease decision within this model.

An immediate problem is the way in which the lease affects the debt capacity. For the sake of convenience it is assumed that the value of the one-year renewable debt plus the value of the after tax lease repayments should not exceed the borrowing limit in any one period. Thus the model adopted with the usual notation is:

$$\text{MAX } \psi = \sum_j \hat{c}_j \, x_j + v_H - \omega_H \tag{5.7.1.}$$

subject to

$$-\sum_j c_{oj} \, L_j + \sum_j c_{oj} \, x_j + v_o - \omega_o \leq D_o \tag{5.7.2.}$$

$$\sum_j c_{tj} \, x_j - \big(1+r(1-T)\big)v_{t-1} + v_t + \big(1+r(1-T)\big)\omega_{t-1} - \omega_t$$

$$+ \sum_j \big(P_{jt}(1-T) + b_j T\big) L_j \leq D_t \qquad t=1,2,\ldots\ldots H \tag{5.7.3.}$$

$$\omega_t + \sum_j P_{jt}(1-T)L_j \leq B_t \qquad t=1,2,\ldots\ldots H-1 \tag{5.7.4.}$$

$$0 \leq L_j \leq x_j \leq 1 \tag{5.7.5.}$$

Again, for ease of analysis, the lease is assumed to start in the first year and the last lease payment terminates prior to the horizon.

The dual analysis of lending and borrowing instruments give

$$\rho_t = \big(1+r(1-T)\big)\rho_{t+1} + \lambda_t \tag{5.7.6.}$$

$$\rho_t = \big(1+r(1-T)\big)^{H-t} + \sum_{\tau=t}^{H-1} \big(1+r(1-T)\big)^{\tau-t}\lambda_\tau \tag{5.7.7.}$$

Hence the value of the lease (dropping the j subscript) is given by:

$$c_o \left[\big[1+r(1-T)\big]^H + \sum_{t=0}^{H-1} \big(1+r(1-T)\big)^t \lambda_t \right] - \sum_{t=1}^{H} \big(P_t(1-T)+b_t T\big) \big(1+r(1-T)\big)^{H-t}$$

$$-\sum_{t=1}^{H-1} \sum_{\tau=t}^{H-1} \big(P_t(1-T) + b_t T\big) \big(1+r(1-T)\big)^{\tau-t}\lambda_\tau - \sum_{t=1}^{H-1} P_t(1-T)\lambda_t \tag{5.7.8.}$$

Again these equations are somewhat cumbersome and in order to gain insight it is convenient to discuss the simplified situation where the debt constraint is binding only in the first period when the lease contract is made. In this case the lease is undertaken specifically to relieve the capital rationing in this year. The value of the lease then becomes:

$$c_o\left[\left(1+r(1-T)\right)^H + \lambda_o\right] - \sum_{t=1}^{H}\left(P_t(1-T) + b_tT\right)(1+r)^{H-t}$$

$$= \left[c_o\left(1+r(1-T)\right)^H - \sum_{t=1}^{H}\left(P_t(1-T) + b_tT\right)\left(1+r(1-T)\right)^{H-t}\right] + c_o\lambda_o \qquad (5.7.9.)$$

Examination of this expression shows it to be the net terminal value of the lease cash flows at the market rate plus a premium, $c_o\lambda_o$. This premium is the funds made available by the use of a lease time the debt capacity shadow price. This shadow price is the net terminal value per unit of outlay on the marginal project. Thus the value of λ_o represents the (above average) return on a project which is only marginally accepted because of restrictions on funds.* It can be seen, therefore, that this premium plays a similar role to the residual capital balances suggested by Fawthrop and Terry.

Again the unwieldiness of the resulting algebraic expression for the value of a lease affords little in the way of a general understanding of the impact of the various parameters. One further complicating factor is that for an accurate computational analysis to be carried a detailed specification of all project cash flows and capital availability is necessary, and no simple general analysis is achievable.

However, the magnitude of the shadow price on debt in any year is intimately linked to the existence of marginal projects with above average rates of return and it is possible to produce a reasonable computational analysis, without the details specified above, by averaging out the debt capacity effects. Thus although in any full

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* In the case where the marginal project is the lease project then this expression further simplifies and the value of the lease becomes the net terminal value of the lease cash flows plus the net terminal value of the project cash flows. This is merely because the lease enables this project which would then be rejected because of lack of funds to be undertaken.

analysis the inter-period discount rate varies from year to year depending on whether the debt capacity constraint is binding or not, we can assume* that $\rho_t = (1+i_m)\rho_{t+1}$ where i_m denotes the marginal reinvestment rate. This gives a value for ρ_t of:

$$\rho_t = (1+i_m)^{H-t} \tag{5.7.10.}$$

and a value for λ_t of:

$$\lambda_t = \big(i-r(1-T)\big)\rho_{t+1} = \big(i-r(1-T)\big)(1+i_m)^{H-t-1} \tag{5.7.11.}$$

with these assumptions the net terminal value of the lease becomes:

$$c_o(1+i_m)^H - \sum_{t=1}^{H}\big(P_t(1-T) + b_tT\big)(1+i_m)^{H-t} - \sum_{t=1}^{H} P_t(1-T)\big(i-i_r(1-T)\big)(1+i_m)^{H-t-1} \tag{5.7.12.}$$

and the net present value is obtained by dividing this last expression by $1+r(1-T)^H$ giving

$$\left[\frac{1+i_m}{1+r(1-T)}\right]^H \left\{c_o - \sum_{t=1}^{H}\frac{P_t(1-T)+b_tT}{(1+i_m)^t} - \sum_{t=1}^{H}\frac{P_t(1-T)(i-r(1-T))}{(1+i_m)^{t+1}}\right\} \tag{5.7.13.}$$

This expression relates the value of the lease to the repayments, capital allowances, debt rates and the above average return on projects.

Various values of this expression were computed for differing lease rates and a debt rate tax of 10%. The results are shown in Table 5.7.1.

TABLE 5.7.1.** The Net Present Value/£100 of Lease at various marginal reinvestment rates and various lease interest rates.

Lease Before Tax	Interest After Tax	Reinvestment Rates (after tax)						
		11	12	13	14	15	16	17
12	6.2	10.6	12.7	15	17.4	20	22.6	25.4
14	7.3	8.3	10.4	12.6	14.9	17.4	20	22.6
16	8.4	5.9	8	10.1	12.4	14.8	17.3	19.9
18	9.5	3.5	5.5	7.6	9.8	12.1	14.5	17.1
20	10.6	1.1	3	5	7.1	9.4	11.7	14.2
22	11.7	-1.4	0.5	2.4	4.5	6.6	8.9	11.3
24	12.8	-3.9	-2.1	-0.2	1.7	3.8	6	8.4
26	13.9	-6.4	-4.7	-2.9	-1	1	3.1	5.4
28	15.0	-9	-7.4	-5.6	-3.8	-1.9	0.2	2.4
30	16.0	-11.6	-10	-8.4	-6.6	-4.8	-2.8	-0.7
32	17.0	-14.2	-12.8	-11.2	-9.5	-7.7	-5.8	-3.8
34	18.1	-16.9	-15.5	-14	-12.4	-10.7	-8.9	-6.9
36	19.2	-19.6	-18.3	-16.8	-15.3	-13.7	-11.9	-10.1

* The implications and validity of this approximation to the dual solution was discussed in section 3.3. See also appendix XIV.

** The assumptions made in drawing up this table were:
(1) the lease repayments are in 5 equal instalments
(2) The tax rate is 50% with no tax lag
(3) Straight line tax depreciation over the life of the lease.

It can be observed that the use of debt capacity by the lease means that the reinvestment rate must be slightly higher than the after tax cost of the lease before it is worthwhile leasing. Fairly clearly the higher this reinvestment rate the greater is the value of the lease. Fawthrop and Terry illustrate their algorithm with a reinvestment rate of 15% after tax and an after tax lease rate of 12%.

Attention must be drawn to the reason for leasing. It may seem somewhat puzzling that the lease is not dominated by debt in that fairly clearly since the debt is renewable on a one year basis a 'debt package' could be put together which should be cheaper. However, the assumptions made are that for a lease taken out at time t the impact of the lease on the debt capacity is not recognised until time t+1. While this may appear to invalidate the analysis since leasing is only made attractive by a favourable and somewhat arbitrary 'accounting' convention, this is only partially true. It may well be that one year (short-term) debt would not be available for the financing of a medium term project. Where this is so and the lease is used to overcome a medium term financing difficulty then the foregoing analysis is substantially correct.

5.8 Leasing and Financial Policy Considerations.

It would seem worthwhile to conclude this chapter with an analysis of lease projects in the model proposed in section 1.7. Here the presence of a whole multitude of other constraints precludes a rigorous analytical solution and the approach adopted is somewhat different. The aim of the method developed is a post ante analysis of the impact of the various constraint sets on the value of a lease. This can be achieved by allowing lease financing to be available to a few of the projects. Because of the different tax allowances available on building and machinery four projects were chosen, two where the capital investment was in buildings and, two where the capital investment was in machinery. Thus projects PRO4Y1, PRO3Y2, PRO4Y2, PR11Y3, were assumed available for leasing and the relevant costs per £100 of lease are shown in Table 5.8.1. together with the nominal (implied) interest rate.

TABLE 5.8.1.

Name of Lease	Repayments/£100 (5 year contract)	Nominal Interest Rate
LO4Y1	£27.8	12%
LO3Y2	£28.0	15.5%
LO4Y2	£26.4	10.0%
L11Y3	£30.2	8.0%

As in section 5.6 it was assumed that the lease forms a prior claim on earnings and that the (pre-tax) earnings after lease payments must adequately cover interest charges before tax (in this case the cover is assumed 10 times). An analysis of the leases is shown in Table 5.8.2. The repayments represent a reduction in the (pre-tax) earnings of the projects while the capital cost causes a corresponding change in the book value of assets. Hence since the leases are specified in terms of these accounting variables it is necessary to develop a methodology by which the impact of changes in accounting variables can be translated into their cash flow contribution, their debt capacity contribution and their impact on the other financial policy constraints. This can be achieved by partitioning the dual vectors associated with earnings and changes in assets, into a cash flow component, a debt capacity component and a financial policy component. Calculation of the reduced cost of the lease projects then gives its net present value with these three components clearly identified.

In order to carry out such an analysis, the following identities which are derived from the dual analysis in appendix XVII are necessary.

$$BL_t = \frac{1}{1.03}\left[BL_{t+1} - 0.0313\ EA_t - 0.2\ TP_t + 0.191\ TP_{t+1} + 0.04\ TA_{t+1}\right.$$

$$\left. - 0.04\ TA_t - \rho_t + \rho_{t+1} + \alpha\ ROCE_t\right\} \quad (t=1,7) \qquad (5.8.1.)$$

$$TA_t = TA_{t+1} - 0.5TP_t \quad (t=1,7) \qquad (5.8.2.)$$

$$PE_t = 0.75\ PE_{t+1} - 0.25\ EA_t - 0.75\ (\rho_t - \rho_{t+1}) + 0.5(TP_t - TP_{t+1}) \quad (t=1,7) \qquad (5.8.3.)$$

$$EA_t = \rho_t - T\rho_{t+1} - [1+\alpha T]\ ROCE_t - T\beta LQDY_y$$

$$- (1-T)\ ERPS_t + (1-T)DCOV_t - ECOV_t \quad (t=1,7) \qquad (5.8.4.)$$

together with the boundary conditions

$$BL_8 = (-\rho_8 \ \vdots\ 0 \ \vdots\ 0) \qquad (5.8.5.)$$

$$PE_8 = (-\rho_8 \ \vdots\ 0 \ \vdots\ 0) \qquad (5.8.6.)$$

$$TA_8 = (0 \ \vdots\ 0 \ \vdots\ 0) \qquad (5.8.7.)$$

$$EA_8 = (\rho_8 \ \vdots\ 0 \ \vdots\ 0) \qquad (5.8.8.)$$

where the partitions refer to the cash balance dual, the times cover dual and all other constraints respectively. Using backward recursion starting at t=7 the following partitioned dual vectors can be computed for the run in appendix XX , where the rows refer to the time periods.

$$\underline{EA} = \begin{pmatrix} 0.4937 & 0.0735 & 0 \\ 0.4484 & 0.0840 & -0.0042 \\ 0.4001 & 0.1014 & -0.0080 \\ 0.3142 & 0 & 0 \\ 0.3400 & 0.1253 & +0.0326 \\ 0.2269 & 0.0133 & - \\ 0.2149 & 0 & - \\ 0.4039 & 0 & - \end{pmatrix}$$

$$\underline{BL} = \begin{pmatrix} -0.6579 & 0.0009 & -0.0111 \\ -0.5982 & 0.0009 & -0.0092 \\ -0.5321 & 0.0010 & -0.0068 \\ -0.4494 & 0.0015 & -0.0039 \\ -0,4533 & 0.0133 & -0.0039 \\ -0.3494 & 0.0065 & -0.0004 \\ -0.3286 & 0 & 0 \\ -0.4043 & 0 & 0 \end{pmatrix}$$

$$BL_t = \frac{1}{1.03}\left\{BL_{t+1} - 0.0313\ EA_t - 0.2\ TP_t + 0.191\ TP_{t+1} + 0.04\ TA_{t+1} - 0.04\ TA_t - \rho_t + \rho_{t+1} + \alpha\ ROCE_t\right\}\ (t=1,7) \qquad (5.8.1.)$$

$$TA_t = TA_{t+1} - 0.5TP_t \quad (t=1,7) \qquad (5.8.2.)$$

$$PE_t = 0.75\ PE_{t+1} - 0.25\ EA_t - 0.75\ (\rho_t - \rho_{t+1}) + 0.5(TP_t - TP_{t+1})\ (t=1,7) \qquad (5.8.3.)$$

$$EA_t = \rho_t - T\rho_{t+1} - [1+\alpha T]\ ROCE_t - T\beta LQDY_y - (1-T)\ ERPS_t + (1-T)DCOV_t - ECOV_t\ (t=1,7) \qquad (5.8.4.)$$

together with the boundary conditions

$$BL_8 = (\ -\rho_8 \ \vdots\ 0\ \vdots\ 0\ ) \qquad (5.8.5.)$$

$$PE_8 = (\ -\rho_8 \ \vdots\ 0\ \vdots\ 0\ ) \qquad (5.8.6.)$$

$$TA_8 = (\ 0 \ \vdots\ 0\ \vdots\ 0\ ) \qquad (5.8.7.)$$

$$EA_8 = (\ \rho_8 \ \vdots\ 0\ \vdots\ 0\ ) \qquad (5.8.8.)$$

where the partitions refer to the cash balance dual, the times cover dual and all other constraints respectively. Using backward recursion starting at t=7 the following partitioned dual vectors can be computed for the run in appendix XX , where the rows refer to the time periods.

$$\underline{EA} = \begin{pmatrix} 0.4937 & 0.0735 & 0 \\ 0.4484 & 0.0840 & -0.0042 \\ 0.4001 & 0.1014 & -0.0080 \\ 0.3142 & 0 & 0 \\ 0.3400 & 0.1253 & +0.0326 \\ 0.2269 & 0.0133 & - \\ 0.2149 & 0 & - \\ 0.4039 & 0 & - \end{pmatrix}$$

$$\underline{BL} = \begin{pmatrix} -0.6579 & 0.0009 & -0.0111 \\ -0.5982 & 0.0009 & -0.0092 \\ -0.5321 & 0.0010 & -0.0068 \\ -0.4494 & 0.0015 & -0.0039 \\ -0,4533 & 0.0133 & -0.0039 \\ -0.3494 & 0.0065 & -0.0004 \\ -0.3286 & 0 & 0 \\ -0.4043 & 0 & 0 \end{pmatrix}$$

$$\underline{PE} = \begin{pmatrix} -0.4944 & 0.0591 & -0.0015 \\ -0.4496 & 0.0543 & -0.0019 \\ -0.4005 & 0.0448 & -0.0014 \\ -0.3157 & -0.0259 & 0.0025 \\ -0.3360 & -0.0346 & 0.0267 \\ -0.2401 & -0.0033 & 0.0145 \\ -0.2148 & 0 & 0 \\ -0.4039 & 0 & 0 \end{pmatrix}$$

$$ECOV = \begin{pmatrix} 0 & 0.0735 & 0 \\ 0 & 0.0830 & 0 \\ 0 & 0.1014 & 0 \\ 0 & 0 & 0 \\ 0 & 0.1253 & 0 \\ 0 & 0.0133 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

TABLE 5.8.2. An analysis of the lease data.

Project LO4Y1 POST HORIZON VALUE*= 0 AFTER TAX COST = 11.9%

	1	2	3	4	5	6	7	8
Plant & Equipment	250	130						
Repayments		(69.2)	(105)	(105)	(105)	(105)	(36)	
Tax Relief			34.6	52.5	52.5	52.5	52.5	18
Loss of Allowances		(125)	(65)					
Net Cash Flow	250	(64.2)	(135.4)	(52.5)	(52.5)	(52.5)	16.5	18

Project LO3Y2 POST HORIZON VALUE* = -24.6 AFTER TAX COST 4.8%

	1	2	3	4	5	6	7	8
Building & Land		200						
Repayments			(56)	(56)	(56)	(56)	(56)	
Tax Relief				28	28	28	28	28
Loss of Allowances			(40)	(4)	(4)	(4)	(4)	(4)
Net Cash Flow		200	(96)	(32)	(32)	(32)	(32)	24

Project LO4Y2 POST HORIZON VALUE* = 11 AFTER TAX COST 10.5%

	1	2	3	4	5	6	7	8
Plant & Equipment		250	130					
Repayments			(66)	(100)	(100)	(100)	(100)	(34)
Tax Relief				33	50	50	50	50
Loss of Allowances			125	(65)				
Net Cash Flow		250	(61)	(132)	(50)	(50)	(50)	16

Project L11Y3 POST HORIZON VALUE = 6.5 AFTER TAX COST 7.6%

	1	2	3	4	5	6	7	8
Building & Land			225					
Repayments				(68)	(68)	(68)	(68)	(68)
Tax Relief					34	34	34	34
Loss of Allowances				45	(4.5)	(4.5)	(4.5)	(4.5)
Net Cash Flow			225	(113)	(38.5)	(38.5)	(38.5)	(38.5)

*Post horizon value is the post horizon cash flows associated with the lease discounted at 10%. In general because the leases occur relatively early in the planning period these values are fairly small. The after tax cost is the internal rate of return of the after tax cash flows as defined by equation 1.5.6. in section 1.5

Examination of the vector for instance $\underline{PE}$ shows that £1 increase in the cost of plant and equipment say in year 5 decreases the net present value of the programme by £0.3439. This is in part of a change in the net present value of direct cash contribution £0.3360 which arises mainly out of the discounted cost of the asset less tax reliefs. In addition the effect of depreciation is to decrease the debt capacity by £0.0346 because of the consequent reduction in the reported earning. Finally the alteration in the capital base and to the reported earnings makes a net contribution to relaxing the other constraints of £0.0267. It is now easy to ascertain the individual components of a lease decision. If the vector $\underline{BL}^L$, $\underline{PE}^L$, denote the amount of building and land leased and the amount of plant and equipment leased respectively over the planning period, while $\underline{P}$ denotes the repayment schedule and $NPVH_L$ the NPV of post horizon cash flows associated with the leased then the value of the lease (reduced cost) is

$$V^L = NPVH_L - \underline{BL}^L . \underline{BL} - \underline{PE}^L . \underline{PE} - \underline{P} . \underline{EA} - \underline{P} . \underline{ECOV}$$

Computation of this expression using the partitioned vectors give the individual contributions of the various constraints. These are shown in table 5.8.3.

TABLE 5.8.3.

SOURCE	NET PRESENT VALUES			
	LO4Y1	LO3Y2	LO4Y2	L11Y3
CASH	8.7	25.8	19.1	11.9
EARNINGS COVER	(40.1)	(27.1)	(22.0)	(19.7)
OTHERS	2.2	2.8	2.9	3.6
NET	(29.2)	1.6	0	(4.2)

The result of the computer run is that only project PRO3Y2 with a positive reduced cost is leased at full scale while leases on projects PRO4Y1 and PR11Y3 which would make negative net contributions are rejected. Project LO4Y2 is partially leased. Thus only the cheapest lease, as measured by after tax cost is adopted. The after tax cost of this lease is 4.8%. Lease LO4Y2 with an after tax cost of 10.5% breaks even. It is interesting to compare this with the after tax cost of long term debt at 4% and the after tax cost of overdraft at around 6%. The above analysis gives some indication of the effect of the financial policy constraints on the lease decision. In both the case of the lease adopted at full scale and in the case of the partial lease it is their positive contribution to relaxing other constraints that prevent them from being rejected. While the above analysis is specific to this run, it is illustrative of a general methodology in which a lease is considered within the total planning framework. The particular analysis presented here shows how it is possible to ascertain the impact of any particular subset of financial policy considerations on the value of the lease.

5.9 Conclusion.

The purpose of this chapter has been twofold. The first has been to present a general framework for the analysis of the lease-buy decision. The value of such a framework lies not in its ability to innovate new financial theory but rather in its ability to rigorously explore the ramifications of existing theory. It assumes a consistency in the lease valuation process by ensuring that the valuation is a direct and logical consequence of any initial set of assumptions. Hence within this framework it has been possible to explore the conventional discounted cash flow approaches to the lease-buy decision by looking at economic measures of debt capacity where debt capacity is measured in terms of future income or dividend streams. The relatively uncomplicated discount structures that emerge from such an analysis is not really surprising on reflection. The underlying assumptions of such approaches are essentially simplistic in nature. A mathematical programming framework merely adds sophistication in the rigour of the analysis and not in any refinement of the assumptions made. Within such a framework leasing tends to be a relatively unattractive proposition. Of course, such a conclusion is reached without reference to the possible impact of differing tax rates on lessee and lessor or any discussion on the possible impact of the various patterns of capital allowances. It is acknowledged that these can have profound influences on the lease decision, a point which is thoroughly investigated by Myers *et al* (76). The emphasis of the discussion here has been to concentrate rather on other forms of market imperfections and this fulfills the second purpose of the chapter.

The two particular market imperfections that were discussed in detail were concerned with the problems associated with 'accounting' measures of debt and with the term of the loan not coinciding with a temporary shortage of capital.

In both the case of the Chambers' model, where debt was measured in terms of book 'accounting' values, and in the Weingartner model, where the debt limit was a 'hard' limit on fixed commitments, situations were identified where despite the relatively higher after tax cost of a lease when compared with the alternative debt financing, leasing still proved to be attractive. The subsequent analysis showed that this situation arose because the term of the lease was more suitable to the particular financing requirements of the firm. In the Chambers' model the lease was most attractive when used as an instrument to overcome a temporary rationing situation. In the Weingartner model, which exemplified discounting approaches in a 'hard' capital rationing situation, the attraction of a lease rested in its ability to expand the pool of available finance. While in the latter a situation with hindsight it may seem obvious that leasing would prove to be attractive, the algorithm developed are essentially simplistic in nature. A mathematical programming framework merely adds sophistication in the rigour of the analysis and not in any refinement of the assumptions made. Within such a framework leasing tends to be a relatively unattractive proposition. Of course, such a conclusion is reached without reference to the possible impact of differing tax rates on lessee and lessor or any discussion on the possible impact of the various patterns of capital allowances. It is acknowledged that these can have profound influences on the lease decision, a point which is thoroughly investigated by Myers *et al* (76). The emphasis of the discussion here has been to concentrate rather on other forms of market imperfections and this fulfills the second purpose of the chapter.

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In both the case of the Chambers' model, where debt was measured in terms of book 'accounting' values, and in the Weingartner model, where the debt limit was a 'hard' limit on fixed commitments, situations were identified where despite the relatively higher after tax cost of a lease when compared with the alternative debt financing, leasing still proved to be attractive. The subsequent analysis showed that this situation arose because the term of the lease was more suitable to the particular financing requirements of the firm. In the Chambers' model the lease was most attractive when used as an instrument to overcome a temporary rationing situation. In the Weingartner model, which exemplified discounting approaches in a 'hard' capital rationing situation, the attraction of a lease rested in its ability to expand the pool of available finance. While in the latter a situation with hindsight it may seem obvious that leasing would prove to be attractive, the algorithm developed a formal analysis of this situation and clarified the roles played by the debt interest rates and the marginal reinvestment rates.

Although the times covered constraint was introduced primarily as a method of relating more closely the income streams to future contractual obligations, the rather arbitrary from of the reuslting restriction negated this aim. Certainly situations were readily observable when leasing was very attractive but this depended very clearly on our measurement of the times covered factor. Thus in the end this section merely served to emphasise the severe limitations of deterministic or certainty equivalent analysis of the lease problem. A theoretically correct analysis of the impact of uncertainty would require a full specification of the variances and covariances of future income streams together with the costs of default on contractual commitments. At present such an analysis is not within the ambit of this thesis.

In the final section the analysis was extended to examine the impact of general financial policy consideration on the leasing decision. It was shown that within the context of a fairly realistic planning situation leasing may prove a valuable strategy - though this value arises from the informational content of the company's accounts and in such circumstances leasing presents a very useful 'window-dressing' mechanism which can mitigate in its favour.

In summary it is difficult to see the attraction of leasing within a rational economic market framework. Certainly situations under which leasing should be taken have been identified in the paper but these stem from imperfect capital markets and what in effect amounts to a sort of 'off-balance sheet' financing caused by imperfections in accounting measurements. The irony is that the academic debates on leasing have concentrated upon attempts to 'purify' existing algorithms. This much sought after promised land may well turn out to be a desert.

Chapter 6. Towards a practical planning system.

6.1 Introduction.

So far this thesis has concentrated entirely on the structural interdependencies and their relationship to financial theory which arise in the use of corporate financial mathematical programming models. Hopefully it has been shown that such models can make major contributions to our theoretical understanding of the capital investment decision. However, such a contribution is purely normative and the models discussed so far have clearly failed to fulfil their original purpose of providing a comprehensive methodology for tackling the intricasies of corporate financial planning.

In this last context the only computer based models to have achieved any degree of success have been fairly simplistic financial statement generators. From the point of view of the Operational Research scientist, the comparative failure of mathematical programming models must be viewed with some disquiet. Operational research scientists have been unable, in effect, to provide Corporate Financial Management with a more sophisticated decision aid than the use of the computer as a consolidator of projected accounting and financial transactions.

A possible key reason for the comparative failure of the programming approach has already been identified. In section 1.6 attention was drawn to the difference between the nature of the search procedure in financial statement generators and mathematical programming models; mathematical programming models search through decision space for a plan which maximizes a scalar measure of the firm's financial performance whereas simulation models are used to search over a vector of projected financial policy variables. The central hypothesis of this Chapter is that a large degree of the managerial acceptability of financial statement generators stems from this ability to explore a vector of financial policy variable. The basic intention of this last Chapter is to present one approach which shows how mathematical programming algorithms can be adapted to enhance the efficiency of this search over the vector of policy variables.

In section 6.2 a set of financial policy variables will be defined over which a search is to be carried out together with a model which enables the search to be accomplished. The section following then discusses the problems that are likely to arise within such a model structure and the limitation of the currently proposed methods of vector optimisation. Section 6.4 develops a theory of the nature of multicriteria decision making and section 6.5 suggests how such a theory might be implemented. The remainder of the Chapter is concerned with possible approaches to the implementation of these ideas. However, because many of the problems identified remain unsolved,and their solutions would appear to require major extensions to the theory of multicriteria programming,the procedure is presented as a case study. Here, the various difficulties encountered are identified and discussed though in the end they have frequently to be circumvented by *ad hoc* procedures. In spite of the obvious shortcomings of the methods devised it is hoped that this final Chapter opens up a new direction for further research rather than closes a hitherto promising avenue.

6.2 The Structure of the Model.

The model introduced in section 1.7 is a linear programming representation of the investment opportunities together with a set of financing alternatives facing an organisation over an eight year period. It has been used extensively in the earlier Chapters in a conventional linear programming format where the optimization was carried out using a scalar measure for evaluating the set of decisions. In this latter form restrictions imposed on the value of financial policy variables were minimum conditions that any plan must meet and plans which did not belong to this feasible set were rejected from further consideration. Clearly the use of the model in this way does not conform to the nature of the planning process as elucidated by organisational theorists.* In their description of the planning process decisions are not directed towards a single goal but are rather concerned with discovering courses of action which help satisfy a whole series of targets or constraints. These targets are not set *a priori* and constraints other than the technological set are not inviolate. Hence if we are to modify the current model in line with this description of the planning process, then the modifications should try to facilitate the identification and ordering of sets of satisfactory plans with respect to the financial policy constraints rather than to search out a single optimum.

It is relatively easy to adapt the model to try out these ideas. The existing constraints can be viewed as falling into one of two disjoint sets. These are a technological set consisting of cash balances and accounting definitions, and a policy variable set consisting of various financial criteria. This policy variable set is constructed from criteria measuring return on capital employed, earnings per share, dividend per share, liquidity, times interest covered, dividend cover, sales and profits. Only sales

---

* See section 1.6

and profits are new; the remaining policy variables have always been included but with minimum bounds imposed on their possible values. This minimum bound must now be removed and a more realistic mechanism for controlling their possible values be introduced since these eight policy variables in each of the eight years up to the planning horizon now constitute sixty-four criteria over which a search is to be carried out.

6.3 The choice of multicriteria method.

Even a brief reflection of this model serves to highlight some of the potential difficulties that are faced in the development of a comprehensive multicriteria methodology.

(a) Firstly, there is simply the problem of size, especially the number of criteria. Where numerical solutions to multi-objective problems are quoted, the actual problem tends to have a relatively small number of criteria (Geoffrion, Dyer and Feinberg (72), Evans and Steuer (73)). The large number of criteria in financial planning stems from the decision makers desire to maintain control over both short term (liquidity) and longer term (sales growth) criteria and to be able to differentiate this control at a year by year level of detail.

(b) Many of the criteria are ratios, in fact six out of the eight basic criteria are making 48 ratios in all. The difficulties are obvious. The various approaches suggested in the literature were rejected; fractional programming methods (See Kornbluth (73)) were considered too cumbersome and expensive on computer time, neither are such methods readily available to practioners, substituting a surrogate fractional function (Hannan (77)) appeared to wildly inaccurate. In the end a somewhat *ad hoc* approximation was substituted whose justification must rest in the results it produced. It did enable a linear search to be carried out over the efficient set of non linear solutions.

(c) Many of the criteria are interdependent, in the sense that regardless of the actual feasible investment set, criteria are functionally related via the accounting definitions. An extreme example of this is that earnings per share equals dividends per share times dividend cover. This type of problem is not removed by redefining the criteria set to remove any mathematical redundancies (Shubik (61)). For one thing such independencies are not always so explicit, being frequently related through timelags associated with tax and dividend payments; and secondly, that would be to withdraw a step from the decision makers involvement. He has typically specified that the set of criteria is a minimum set with which he is prepared to interact and he wants to be able to explore preferences with respect to them all.

(d) Finally these criteria are meaningful in financial and company terms. This comment is not as trite as it seems. The criteria cannot be handled as a homogenous group; at different stages of the exploration process different criteria will assume more significance and varied levels of aggregation or disaggregation will be appropriate.

This final source of difficulty is worth exploring further because it will greatly influence the choice of a successful methodology. Thus, particularly in the early stages, the growth factor and average level of criteria such as sales, profit and dividend per share

are liable to be much more important than an individual year's figures. In later stages when the overall plan strategy has been roughly determined, consideration may then be given to individual criteria in particular years. Further an examination of the criteria shows that they can be classified conveniently into three main sets. These are:

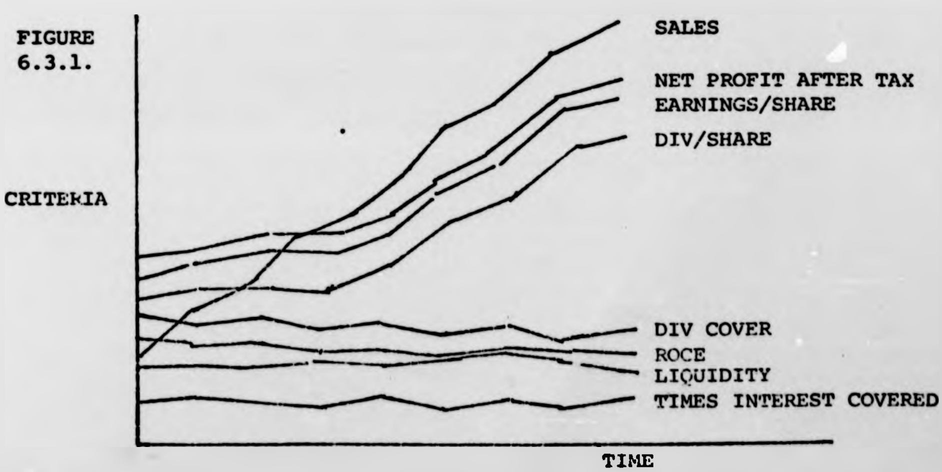
(i) Profitability Indices - return on capital employed, net profit after tax, sales, earnings per share.

(ii) Dividend Policy Variables - dividend per share and dividend cover.

(iii) Safety Ratios - times interest covered, and liquidity.

The final form of the solution could be schematically represented by figure 6.3.1. where the criteria have been characterized by level, growth and stability of growth. Thus experiments with alternatives might cluster around broad strategies such as profitability indices versus safety indices or, within the setting of dividend policy; dividend per share against the risk measure implied by dividend cover. It is within such characteristics that the search for efficient solutions should be carried out. This observation provides both a structure to the search and a corresponding reduction in the size of the search space.

FIGURE 6.3.1.



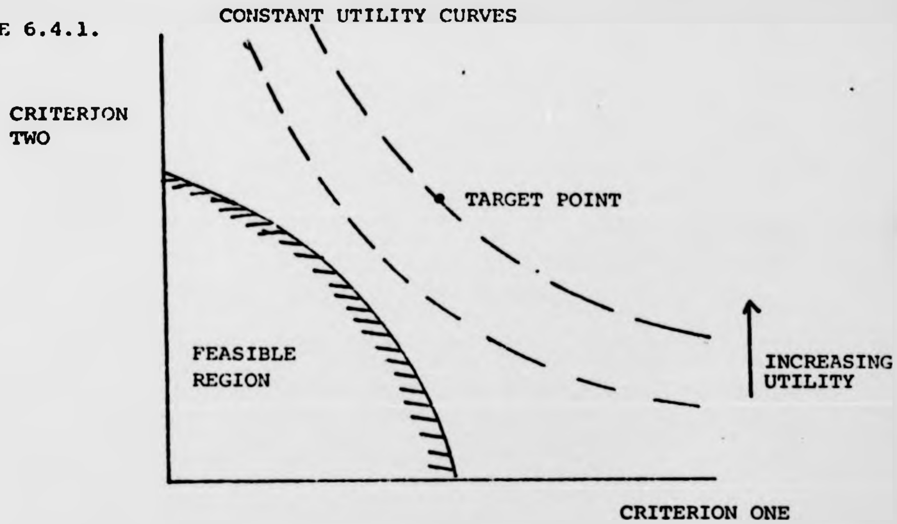
The problems of size, ratios, interdependencies and intelligibility not only exposes the deficiencies of existing methodologies but has a profound influence on the actual choice of methodology. Two of the three main methodologies available were dismissed almost immediately as being, as of this date, unable to deal with realistically sized financial planning problems. These were, firstly, the *a priori* calibration of a utility function over the criteria (Bristin (66), Keeney (75)). This was not solely because of the notorious difficulties involved in extracting and analysing appropriate data but equally because it went against the philosophy of interactive methods which financial management find attractive - that preferences are developed during the process of comparing alternative, not *a priori*. Secondly, methods which involved enumerating efficient vertices (Evans and Stueur (73) Zeleny (74)) were also dismissed. With so many extreme vertices the methods for reducing them to a workable number seemed too crude and primitive for the type of structured search which was aimed for. Thus only the third type of methodology remained - that of interactively searching the decision maker's preferences.

It is natural in financial planning to speak in terms of targets or goals; company performance as measured by such criteria as dividend cover, liquidity, or return on capital employed have target ratios adopted by custom and practice. In this sense as has been argued before, many of the constraints typically used in mathematical programming formulations are more goal-like than binding limits on possible courses of action. Hence an obvious choice of methodology is a goal programming formulation where it is understood that both weights on goals and goal deviations are available to be modified interactively. The goal constraints referred to here are, of course, just the financial policy variable set discussed earlier in this section.

6.4 A Utility Theory Framework for Goal Programming.

In order to carry out effectively the above search procedure it is necessary to re-examine the various goal programming formulation within a coherent and comprehensive framework - such a framework is found in utility theory. To this end consider the diagram below showing the position of targets in criteria space. For simplicity of exposition the argument is restricted to just two criteria, though generalization to the multi-criteria case is fairly easy.

FIGURE 6.4.1.



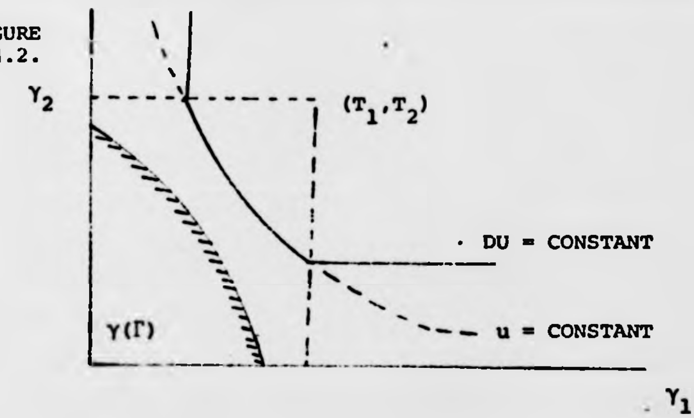
The dotted curves represent the decision makers utility indifference curves and prior to experimentation we have very scant knowledge of this indifference map. In general we know only his prior guesses of the target values and the relative importance attached to meeting those targets. In addition we can make certain further assumptions about the general nature of indifference curves such as convexity, continuity and differentiability.

Having decided to use target programming we have already implied an approximation to the indifference curves $U(Y_1, Y_2)$ = constant, by concentrating attention on the disutility of under-achievement given by

$$DU(Y_1,Y_2) = MAX \{0, U(T_1,T_2) - U(Y_1,Y_2)\} \qquad (6.4.1.)$$

within the target region defined by $Y_i \leq T_i$ for all. Outside the target region the form of DU changes, a typical illustration is given in Figure 6.4.2.

FIGURE 6.4.2.



At any point on the curve DU = constant, the slope of the tangent gives the relative tradeoffs between the criteria that the decision maker would accept at that level of the criteria. Thus a linear approximation to DU of the form $\sum_i u_i(T_i - Y_i)$ would indicate that a reduction of $\frac{1}{u_i}$ in the underachievement of goal i would be just compensated by an increase of $\frac{1}{u_j}$ in the underachievement of goal j. Goal programming concentrates on this type of approximation where the u's play the part of goal weighting.

An alternative linear approximation to the function $u(T_1-\gamma_1,T_2-\gamma_2)$ is the Chebychev norm of minimizing $\max\{u_1(T_1-\gamma_1),u_2(T_2-\gamma_2),0\}$. This will be referred to as minimax programming where the aim is to minimize the maximum shortfall from target over all the criteria. Both of the approximations discussed so far can be considered specific examples of the general model

$$\text{MIN} \left(\sum_i (z_i)^p\right)^{1/p} \qquad (6.4.2.)$$

Such that

$$\gamma_i(x) + {}^{z_i}/_{u_i} \geq T_i \qquad (6.4.3.)$$

$$x \in \Gamma$$

Or alternatively

$$\text{MIN} \sum \left(\text{MAX}\left[0,u_i(T_i-\gamma_i(x))\right]^p\right)^{1/p} \qquad (6.4.4.)$$

$$x \in \Gamma$$

Where Γ denotes the feasible region and target overachievements are ignored. The goal programming formulation corresponds to the p=1 norm when we have

$$\text{MIN} \sum_i z_i \qquad (6.4.5.)$$

s.t.

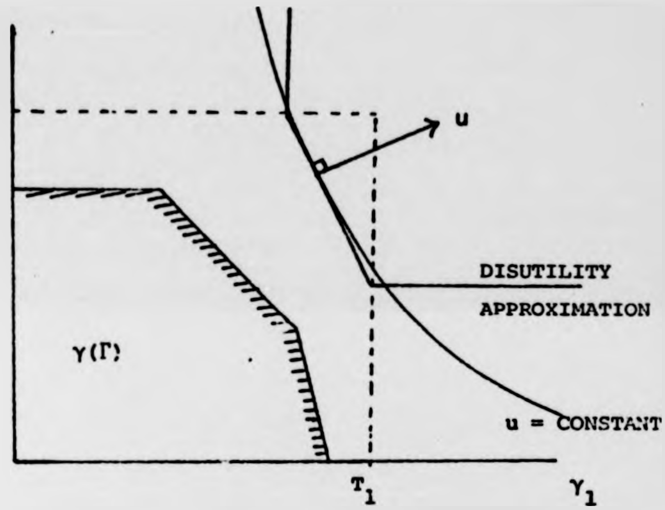
$$\gamma_i(x) + {}^{z_i}/_{u_i} \geq T_i \quad \text{all } i \quad x \in \Gamma \qquad (6.4.6.)$$

---

* The work of this section was carried out in early 1974. In fact prior to this date it would appear that Lane (72) in an unpublished Ph.D. thesis also correctly identified the nature of the linear approximation implied by goal and minimax programming. However, he failed to identify the hybrid linear approximation to be discussed shortly or to find an effective method for determining the parameters T and u.

This form is unsuitable for many uses, but with financial models it is positively misleading. To explain this consider the isoquants or this utility approximation shown in figure 6.4.3. with $\gamma(\Gamma)$ representing the image of Γ under γ_i for all i=1,2

FIGURE 6.4.3.



The linear approximation within the target region implies that only vertices are possible contenders for 'solutions' and that slight changes in the u_i causes jumps, often major, in such solutions. If we return to the simple example of section 1.6 where an attempt was made to maximize profits (p_1,p_2) in two consecutive years then the p=1 metric gives the following goal program to be solved for p_1,p_2

$$\text{MIN } (1+\varepsilon)z_1 + z_2 \qquad (6.4.7.)$$

$$\text{s.t.} \quad p_1 + z_1 \geq 1 \qquad (6.4.8.)$$

$$p_2 + z_2 \geq 1 \qquad (6.4.9.)$$

$$p_2 + p_2 \leq 1 \qquad (6.4.10.)$$

The solution is f = (1,0) for positive values of ε and f = (0,1) for negative values. Hence with this metric, infinitesimal changes in the preferences can completely alter the solution.

While an attractive choice for p might appear to be the $p=2$ metric in which the objective is to minimize the weighted sum of the squares of deviations such a choice would lead to quadratic programming with a prohibitive increase in computer time. The other obvious choice resulting in a linear function is the $p=\infty$ metric, and this is the minimix formulation

$$\text{MIN } z \qquad (6.4.11.)$$

such that

$$\gamma_i(x) + \frac{z}{u_i} \geq T_i \quad \text{all } i \qquad (6.4.12.)$$

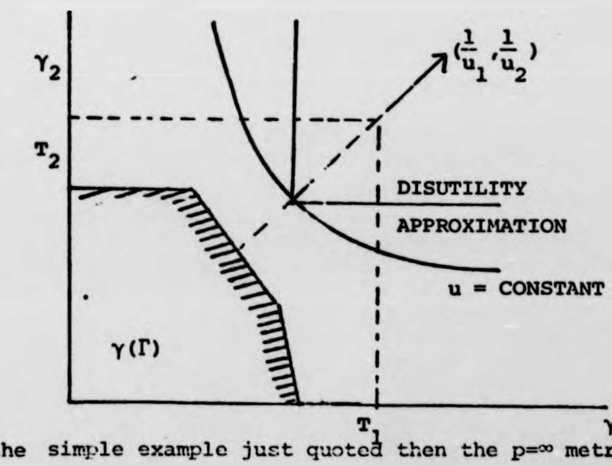
$$x \in \Gamma$$

or alternatively

$$\min_{x \in \Gamma} \max_i \left\{ 0,\ u_i(T_i - \gamma_i(x)) \right\} \qquad (6.4.13.)$$

The isoquants are now right angles 'corners' anchored to a line through the target point of slope $(\ldots.,\ \frac{1}{u_i}\ \ldots..)$. This makes the solution point a continuous function of both the weights u_i and the targets T_i. This is illustrated in Figure 6.4.4.

FIGURE 6.4.4.



Referring to the simple example just quoted then the $p=\infty$ metric results in the division of profits in the two years being in the ratio of

$1+\varepsilon : 1$. This ratio is both intuitively reasonable and is also continuous with respect to ε, where ε is in $[0,\infty)$.

Unfortunately, the existence of upper bounds in $\gamma(\Gamma)$ on the value of a criterion, as with $\gamma_2(x)$ in Figure 6.4.4. causes multiple solutions and the possibility of nonefficient solutions such as $\gamma=(0,b_2)$ above.

To overcome the inherent problems of both values of $p=1$ and $p=\infty$ extensive use of a linear hybrid formulation was made. This was

$$\text{MIN} \quad \alpha_H z + \sum z_i \qquad (6.4.14.)$$

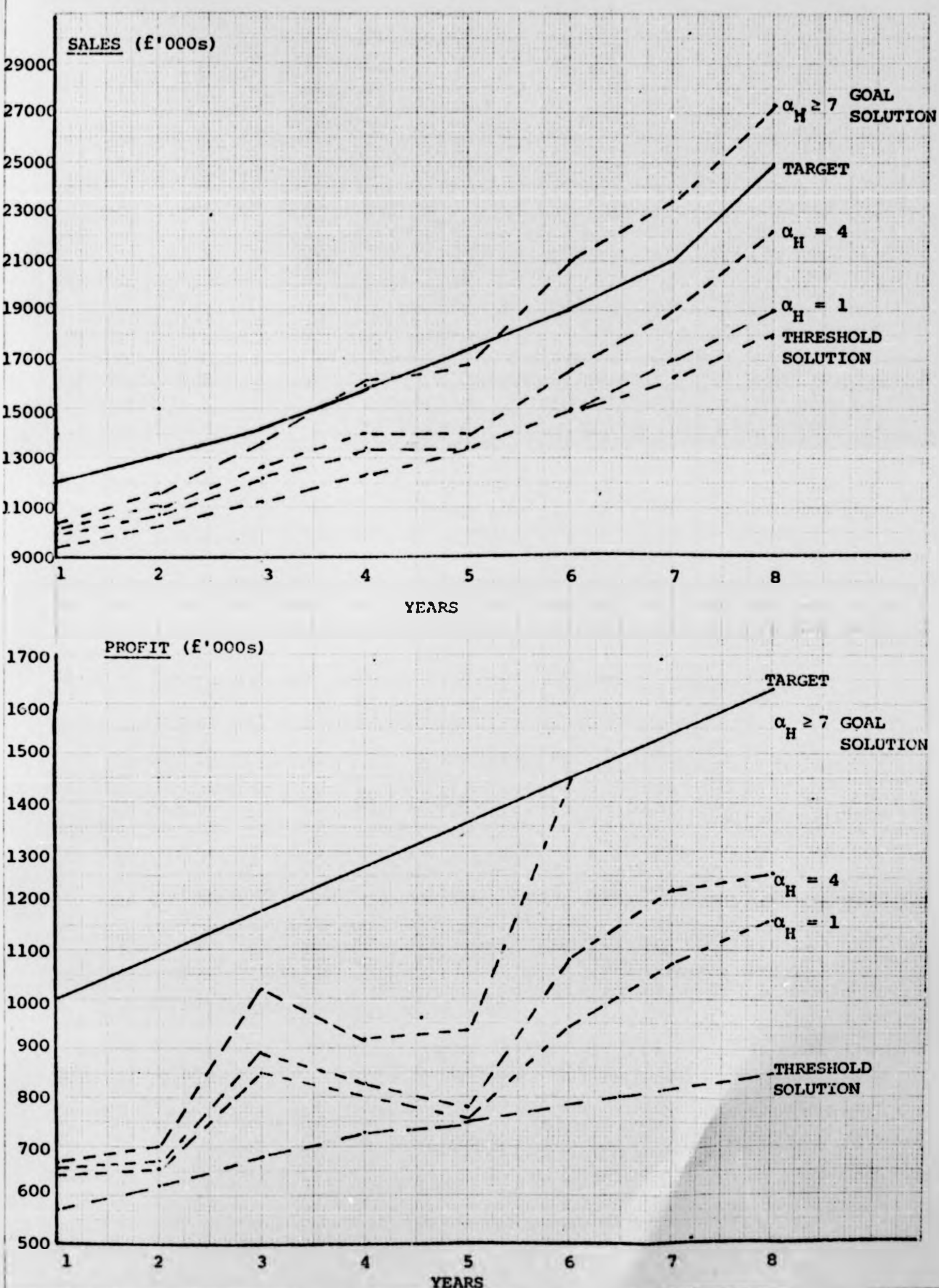
so that

$$\gamma_i(x) + (z+z_i)/u_i \geq T_i \text{ . all } i \qquad (6.4.15.)$$

$$x \in \Gamma$$

In this formulation if z were to decrease by δ within the target region then each z_i needs to increase by δ and hence the objective changes by $-\alpha_H\delta + N^*\delta$ where N^* is the number of the z_i that need to increase to allow z to decrease. When $\alpha_H < 1$ we have minimix programming when $\alpha_H \geq N$ where N is the number of criteria, goal programming. As α_H varies between these extremes the isoquants associated with each value of α_H varies also. The effect of this is to 'smooth' the solution in that as α_H decreases the solution changes from a typical $p=1$ goal programming form where weighted deviations from targets are at extreme value, to one whereby the weighted deviations tend to equal one another as with $p=\infty$ or minimax programming. This is a particularly useful property when an attempt is being made to plan overtime, since the degree of stability of a solution can be controlled by the single parameter α_H. Figure 6.4.5. shows the effect of varying the α_H parameter in the model for the profit and sales target. In this experimental run, where only these last two policy variables were being considered, increasing the value of α_H

FIGURE 6.4.5.


SALES (£'000s)
29000
27000
25000
23000
21000
19000
17000
15000
13000
11000
9000
1
2
3
4
5
6
7
8
YEARS
$\alpha_H \geq 7$ GOAL SOLUTION
TARGET
$\alpha_H = 4$
$\alpha_H = 1$
THRESHOLD SOLUTION
PROFIT (£'000s)
1700
1600
1500
1400
1300
1200
1100
1000
900
800
700
600
500
1
2
3
4
5
6
7
8
YEARS
TARGET
$\alpha_H \geq 7$ GOAL SOLUTION
$\alpha_H = 4$
$\alpha_H = 1$
THRESHOLD SOLUTION

meant that the number of non-zero $z_{SALES,t}$ and $z_{PROFIT,t}$ increased resulting in a wider year by year variation.*

6.5 The Implementation.

It is clear that computer and software manufacturers have developed very efficient and sophisticated linear programming algorithms, matrix generators and report writers. To throw away this accumulated experience and develop specific computer codes would have been to step away from implementation. However, the decision to use existing software** does place limitations on the interactive process and the structuring of the model.

The interactive process used with the goal programming formulation was to adjust weights and targets parametrically. Thus, for illustration treating only the extremes of goal programming with p=1 and minimax programming with p=∞ we have objective function and right hand side parametrics respectively.

p=1

$$\text{MIN} \sum_i u_i(1+\lambda_i)y_i$$

$$\gamma_i(x) + y_i \geq T_i$$

$$x \in \Gamma \qquad (6.5.1.)$$

p=∞

$$\text{MIN } z$$

$$\gamma_i(x) + z/u_i \geq T_i(1+\lambda_i)$$

$$x \in \Gamma \qquad (6.5.2.)$$

---

* In the actual run, their objective function employed goal programming for all the policy variables except sales and profit. The substructure of the model relating only to sales and profit was

$$\text{Min } \alpha_H I + \sum_t z_{SALES,t} + \sum_t z_{PROFIT,t}$$

where

$$I = \text{MAX } [z_{SALES}, z_{PROFIT}]$$

subject to

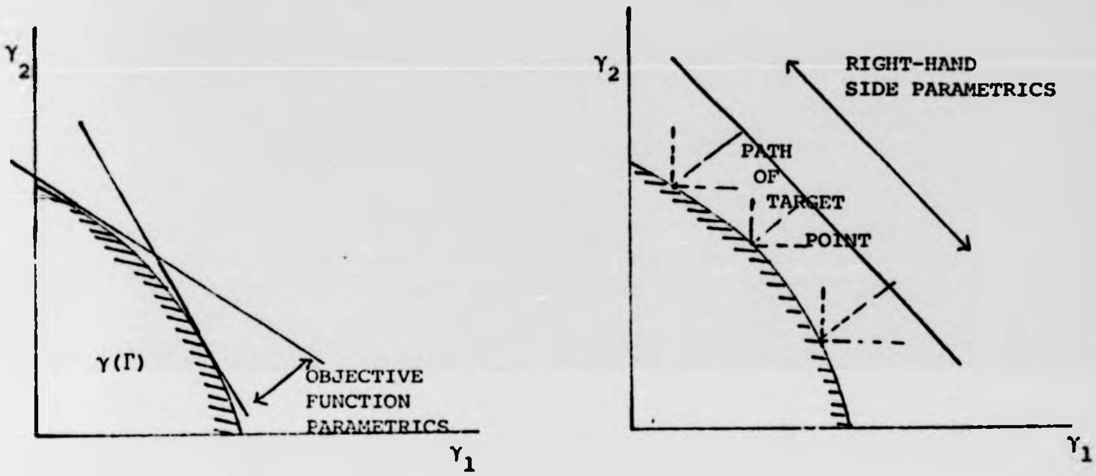
$$\text{SALES}_t + \frac{(z_{SALES} + z_{SALES,t})}{u_{SALES,t}} \geq \text{SALES TARGET}_t$$

$$\text{PROFIT}_t + \frac{(z_{PROFIT} + z_{PROFIT,t})}{u_{PROFIT,t}} \geq \text{PROFIT TARGET}_t$$

** The initial work was carried out using the ICL linear programming package XDLA at the University of Birmingham, England. The work was completed using the IBM linear programming package MPSX/370 at the University of British Columbia, Canada.

where y_i has replaced ${}^{z_i}/_{u_i}$ in 6.5.1.

Figure 6.5.1.



Usually eleven steps were taken in the parametric direction and the results filed for subsequent analysis. A hierarchical structure of information was then made available from each run so that the decision maker was able to see the consequences of a decision in any detail required. This hierarchy consisted of:

(a) Average values over time of levels and growth (where relevant) of each of the criteria for each value of the parameter.

(b) The value of each criterion in each year for a particular parameter value.

(c) Balance sheets, cash flow statements and profit and loss statements corresponding to any solution.

(d) The unanalysed linear programming solution.

This information was available on a visual display unit as required, though facilities existed for immediate hard copies of any or all of this information. Some sample print-outs are included in appendix XIII . As far as the decision maker was concerned this output was comparable to the results of a cash flow simulation for eleven choices of options. The inputs required of course were much different.

The problem of ratio criterion in this application was only overcome by an *ad hoc* procedure which needs to be replaced by further theoretical research. Full details are given as they occur in the next section but the principle is to convert the constraint

$$\frac{N_{it}(x)}{D_{it}(x)} + z_{it} \geq T_{it} \qquad (6.5.3.)$$

where N_{it}, D_{it} are linear functions on x to

$$\{N_{it}(x) - D_{it}(x).T_{it}\} + z_{it}L\{D_{it}(x)\}\frac{T_{it}}{100} \geq 0 \qquad (6.5.4.)$$

Where the suffixes refer to criterion i in time period t, z_{it} conveniently denotes a percentage shortfall from target T_{it} and $L\{D_{it}(x)\}$ is the estimated likely value of the denominator $D_{it}(x)$ in the region of the 'optimal' solution. For many financial criteria such denominators display regular growth and a reasonably accurate prediction of their values over the planning period is not too difficult. Thus for the model proposed in this thesis the denominator is always one of the following: total net assets, current liabilities, number of shares issued, interest payable or dividend payments. The first three have a fairly large starting base and accumulate steadily though the last two can be more volatile and are more troublesome especially if a significant tranche of long term debt is repayable. The values of course of these denominators can be updated as the search progresses. Furthermore the search will tend to become concentrated on a particular part of the efficient surface when fairly accurate predictions of their likely values are possible.

It should be noted however, that errors in the predicted value of the denominator merely affected the interval at which these solutions were filed for further analysis by the report writer. The report writer computed the precise value of the criteria at these solution points from the actual value of the denominators and not their expected value. In this way a non-linear search was controlled by a linear search procedure with a consequent gain in processing time but without loosing any of the structure inherent in the non-linearities.

6.6 The Search Strategy.

The search procedure was originally envisaged as taking place over three distinct phases, which are described below:

(i) Phase I

The primary purpose of this phase was to obtain rapidly a region of the efficient surface over which a more detailed search could be carried out. To achieve this a goal programming approach was adopted between criteria but with a minimax approach within each criterion over time. Clearly while stability between criteria was not important, stability over time within a particular criterion was considered important. There was also a secondary purpose to this phase which was to determine rough orders of magnitude of the criteria weights. While the final solution over the other phases does not depend on the weights, the speed at which convergence is obtained clearly does. Throughout the search it was found to be important to keep the relative deviations of all the criteria from the targets roughly balanced.

(ii) Phase II

While phase I was concerned with determining the appropriate region over which to continue the search, phase II was concerned with a more detailed search of the average levels and the growth rates of an individual criterion. The method used was minimax programming between criteria and within criteria. This was necessary to remove some serious instabilities that had been observed in phase I.

(iii) Phase III

Ideally on exit from the second phase both levels of stability of growth rates should be satisfactory and there remains the possibility of trading-off stability in growth for a criteria against the actual growth rate for the criteria. This was to be achieved in the third phase.

As it turned out even this three phase approach was too rigid. While phase I proved relatively straightforward the second and third phases were less so. This was largely because the next most appropriate step to take in a search procedure is a response to the current solution. In this case it depends on how the decision maker perceives the weaknesses of the current solution. Thus the essence of any multi-objective method is a flexibility in response.

6.7 The Phase I search

This search was carried out using objective parametrics on the eight weights relating to the criteria. Stability between time period within a particular criterion was maintained by using the minimax metric over time within a criterion. Hence the model was recast into the form

$$\text{MIN} \sum_{i \in I} u_i(1+\lambda_i)z_i \qquad (6.7.1.)$$

so that

$$\{N_{it}(x) - T_{it}\, D_{it}(x)\} + L\left\{\frac{D_{it}(x)T_{it}}{100}\right\} z_i \geq 0 \quad \text{all } i,t \quad (6.7.2.)$$
$$x \in \Gamma$$

where u_i is the weighting on z_i and T_{it} is the target for criterion i in time period t. Thus although the criteria themselves are the ratio of the linear forms $N_{it}(x)$, $D_{it}(x)$, where trivially $D_{it}(x) \equiv 1$ for sales and profit, they have been recast into a linear form by multiplying throughout by $D_{it}(x)$ and using $L\{D_{it}(x)\}$ as the coefficient of z_i. It should also be noted that by including T_{it} in the coefficient of z_i the individual z_i's represent maximum percentage deviates* from target for a particular criterion.

Initial values for u_i and T_{it} were set from discussions with the decision maker and values of $L\{D_{it}(x)\}$ were available from preliminary experimentation with the model. The set of criteria I was also partitioned into $\{J,K,L\}$ and the initial parametrics defined:

$$\lambda_i = 0 \quad i \in L$$
$$\lambda_i = \lambda \quad i \in J$$
$$\lambda_i = -\lambda \quad i \in K \qquad (6.7.3.)$$

and λ was made to vary from -1.0 to +1.0 in steps of 0.2. Three experiments were run for different choices of $\{J,K\}$, a typical result is shown in Appendix XIII and the results summarised showing the decision maker's 'best' choice over λ for each run are shown in Table 6.7.1.

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*This resulted in the deviations having all the same order of magnitude while numerical values of the targets actually range from 0.11 (dividend/share in year 1) to over 30,000 (sales in year 8).

Table 6.7.1.

CRITERIA	RESULTING OBJECTIVE WEIGHTS AT 'BEST' λ		
Sales	3.40	2.04	0.82
Return on Capital	3.80	2.28	3.65
Earnings per Share	4.00	2.40	2.40
Liquidity	3.40	4.20	4.20
Interest Cover	2.50	3.50	3.50
Dividend Cover	3.40	3.40	3.40
Dividend/Share	2.50	2.50	2.50
Net Profit	3.60	2.16	3.46
J: Set	Times interest covered Liquidity	Sales	Sales
K: Set	Sales net profit return on capital earnings/share	Return on Capital Net Profit	
'Best' value of parameter	$\lambda = -0.4$	$\lambda = 0.6$	$\lambda = 0$

Each succeeding run of these three started from the previous 'best' choice of weights.

A serious difficulty, which illustrates the problem of instabilities in goal programming formulation arises if an attempt is made to explore alternative dividend policies with this particular formulation. The results of taking the set J to consist only of dividend over and K as dividend per share with the remaining criteria held constant in L is shown in Table 6.7.2. with an increased resolution of λ near the critical point. Thus between $\lambda = 0.59$ and 0.60 a 2% variation in weights causes a 500% change on dividend cover and dividend per share doubles. This difficulty arises because the two dividend policy variables dividend per share and dividend cover are in direct opposition and are functionally related to a third criterion (viz. earnings per share). Thus depending upon the relative weights an attempt is made to meet completely either the dividend/share target or the dividend cover target.

The effect is in fact even more exaggerated than in Table 6.7.2. which gives only average criteria values rather than values in individual years. In this particular case the problem is made worse because we are trying to find the minimum of the weighted sum of maximum deviations from the target of each criterion, i.e.

$$\text{MIN} \sum_{i \in I} \ \underset{t}{\text{MAX}} \left[\frac{u_i D_{it}(x)}{L\{D_{it}(x)\}} \cdot \left(\frac{N_{it}(x)}{D_{it}(x)} - T_{it} \right) \Big/ T_{it} \right] \qquad (6.7.4.)$$

Because of the linear nature of the trade-offs assumed in goal programming it may be preferable to continue to reduce the maximum deviation for a single criterion on a target in one period, even though a satisfactory average for that criterion has already been achieved and irrelevant of the fact that the levels on other criteria are no longer acceptable. It should be emphasised though that this difficulty is merely illustrative of the general problem of instabilities in goal programming formulation. For this reason the remaining searches were carried out using a minimax structure.

Table 6.7.2. CRITERIA VALUES
(AVERAGES OVER THE EIGHT YEARS)

λ	1.0	0.8	0.6	0.59	0.58	0.56	0.54	0.52	0.4	0.2	0.0
Earnings/Share	43.4	44.1	45.05	42.64	42.57	42.57	42.57	42.57	42.34	42.34	41.92
Dividend Cover	24.20	22.6	18.41	3.68	3.63	3.63	3.46	3.45	3.09	3.09	3.10
Dividend/Share	6.85	6.90	6.77	11.50	12.26	12.65	12.65	12.65	13.66	13.66	13.62
Dividend	166	163	153	299	303	303	315	315	347	345	345

6.8 The Phase II Search

One of the initial tasks of this phase was to explore possible dividend policies - throughout phase I a minimum dividend per share of £0.127* was imposed as a hard constraint. This constraint was merely an expedient to maintain a degree of stability in the dividend payout and to overcome the difficulties discussed in the preceeding few paragraphs while the weightings attached to the other criteria were explored. For this second phase the problem was reformulated as

$$\text{MIN} \quad \alpha_H z + \sum_{i \in I} z_i \qquad (6.8.1.)$$

such that

$$\{N_{it}(x) - T_{it} \quad D_{it}(x)\} + L\left\{\frac{D_{it}T_{it}}{100\ u_i}\right\}[z+z_i] \geq \lambda\delta_{it} \qquad (6.8.2.)$$

where the notation is as before, with the additional variables defined as

α_H - degree of hybridisation

δ_{it} - defines the range of parametric variation.

A smooth transition from phase I to phase II was obtained by defining the new target levels to be equal to the final 'best' point of phase I i.e.

$$T_{it} = \frac{N_{it}^{I}(x^*)}{D_{it}^{I}(x^*)}$$

This means that the initial target point is on the efficient surface and the subsequent parameterisation is such that it moves this target away from the efficient surface. The effect of this is illustrated schematically in Figure 6.8.1.

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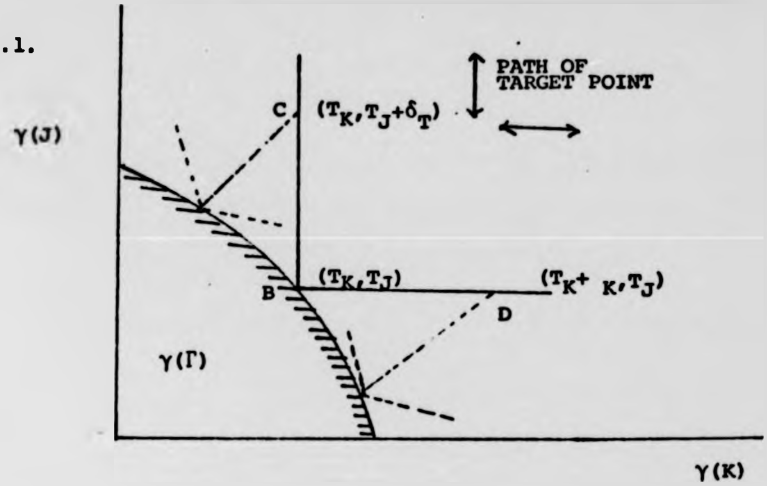
* This figure was 10% below the current dividend per share level

Figure 6.8.1.



This represents a search over the {J,K} subjects of I with two runs

$$\text{(i)} \quad \delta_{it} = T_{it} \cdot L\{D_{it}(x)\} \qquad i \in J \qquad (6.8.3.)$$

$$= 0 \qquad i \in K \cup L$$

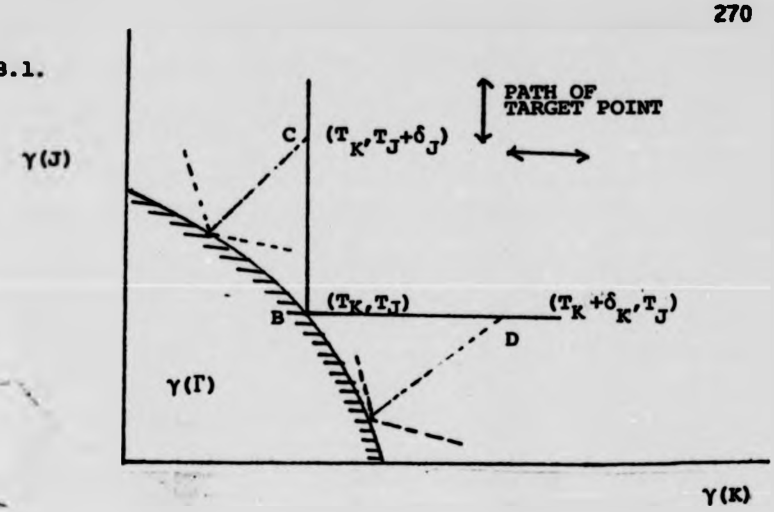
$$\text{(ii)} \quad \delta_{it} = T_{it} \cdot L\{D_{it}(x)\} \qquad i \in K \qquad (6.8.4.)$$

$$= 0 \qquad i \in J \cup L$$

In each case λ varied from 0 to +1 in steps of 0.2 and the effect of this is to move the target first along the line BC and then along the line BD. This enables a region of the efficient surface centred upon B to be explored.

As it turned out one of the most important factors governing the search was the choice of α_H. Too low a value meant that the search was subject to the implicit bound problem, while too high a value resulted in the solution being insensitive to further changes in target levels. See figure 6.8.2.

Figure 6.8.1.



This represents a search over the {J,K} subjects of I with two runs

(i) $\delta_{it} = T_{it} \cdot L\{D_{it}(x)\}$ $\quad i \in J$ (6.8.3.)

$= 0$ $\quad i \in K \cup L$

(ii) $\delta_{it} = T_{it} \cdot L\{D_{it}(x)\}$ $\quad i \in K$ (6.8.4.)

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FIGURE 6.8.2.

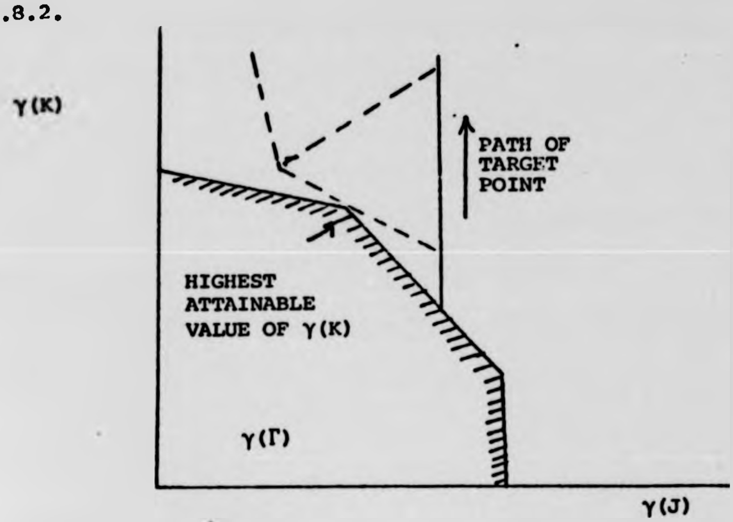


Table 6.8.3. shows the effect of such a parametric search on the dividend policy variables. The criteria not listed in the table were roughly constant over the range of parameterisation. The first half of the table shows the effect of increasing the target on dividend per share while the second half shows the effect of increasing the target on dividend cover. A value for α_H of 1.5 was chosen, following an unsuccessful run in which a value of 3.5 for α_H resulted in the solution being insensitive to changes in the dividend per share target.

There are several points to note. The first and most important is that the instabilities associated with the previous goal programming formulation have been avoided. However, a secondary problem has arisen: the previously acceptable levels for the times interest cover and liquidity ratios fall to unacceptable levels as the dividend per share is increased. The cause of this is that the dividends in the early years are largely paid for by long term borrowing, with the subsequent effect that the times interest covered falls to very low levels (-10.0) during this period before a growth in earnings begins to ease the situation.

At this stage of the search, it would be clearly preferable to explore the two dividend policy variables in isolation, with the exploration making a minimal impact on the values of the other policy variables. This can be achieved by the method of sub-space hybridisation in which the hybridisation is carried out only on the non-parameterised criteria. Further defining α_H to be sufficiently large (greater than 6 in this case) we can obtain goal programming on the non-dividend policy variables with minimax on the dividend policy variables. The advantage of this is that goal programming is insensitive to changes in target levels, an alternative and equivalent view is that the parametric search is restricted to the dividend/share - dividend cover planes. The effect of such a parametric procedure is shown in Table 6.8.4.

As now can be seen from the table the liquidity and times interest covered ratios do not fall to unacceptable levels as the dividend/share and dividend cover figures are altered. It also indicates the extent to which the dividend per share can be increased before changes start to take place in other ratios. Having chosen a suitable dividend policy it turned out that while many of the criteria had satisfactory average levels, there remain unsatisfactory time trends in some of the criteria. Again the liqudity and times covered constraints were a major problem. These ratios in each of the eight time period for the previous best solution (λ=0.2 for dividend cover) are shown in Table 6.8.5.

Table 6.8.3.	CRITERIA VALUES (AVERAGES OVER TIME)										
	Dividend/Share						Dividend cover				
λ	1	0.8	0.6	0.4	0.2	0.0	0.2	0.4	0.6	0.8	1.0
Liquidity	1.89	1.9	1.93	1.94	1.96	2.01	2.00	2.04	2.09	2.11	2.16
Times covered	10.62	11.22	11.89	12.97	13.60	14.79	14.36	13.62	12.98	13.29	15.26
Dividend cover	1.74	1.84	1.95	2.09	2.23	2.40	2.77	3.27	4.06	5.27	7.09
Dividend/Share	23.81	24.06	22.39	20.69	19.02	17.22	15.81	13.90	11.71	9.49	7.24
Dividend	619	585	551	516	480	441	383	315	245	190	145

Table 6.8.4.	CRITERIA VALUES (AVERAGES OVER TIME)										
	Dividend/Share						Dividend cover				
λ	1	0.8	0.6	0.4	0.2	0.0	0.2	0.4	0.6	0.8	1.0
Liquidity		No		1.98	2.00	2.01	2.02	2.05	2.07	2.09	2.12
Times covered		Change		14.50	14.60	14.90	16.13	18.02	20.22	22.83	27.82
Dividend cover				2.13	2.21	2.40	2.69	3.10	3.66	4.43	5.56
Dividend/share		in		19.06	18.41	17.14	15.46	13.56	11.59	9.66	7.78
Dividend		Values		508	485	439	389	340	287	237	189

Table 6.8.5.	CRITERIA VALUES								
Time Period	1	2	3	4	5	6	7	8	Average
Liquidity	2.42	2.08	2.03	1.98	1.89	1.92	1.91	1.94	2.02
Times Interested Covered	13.25	12.98	13.33	14.50	18.77	16.86	19.16	20.28	16.14

Clearly in the early years the times interest covered ratio is on the low side with the liquidity ratio on the low side in later years. The response to this is to define new average targets of 2.25 and 18.0 for liquidity and times interest covered respectively and then to redefine δ_{jt} for j corresponding to these two criteria, such that $\delta_{jt} > 0$ in years t when this average is not reached and $\delta_{jt} < 0$ in the years when the averages were exceeded. Furthermore the size of δ_{jt} can be made proportional to shortfall (excess) and $\sum_t \delta_{jt} = 0$. This has the effect of shifting the two end points inwards towards the average. Table 6.8.6. shows the results of such a run.

Table 6.8.6.	CRITERIA VALUES								
Time Period	1	2	3	4	5	6	7	8	Average
Liquidity	2.60	2.25	2.10	2.08	2.03	2.05	2.05	2.04	2.15
Times interest covered	14.23	14.26	14.31	14.71	18.99	21.25	22.79	20.10	17.57

The "jump" in time interest covered between years 4 and 5 is because a large repayment of long term debt occurs during period 5.

While this procedure does not fit nearly into the search scheme as originally proposed for phase II it illustrates the necessity of being able to respond to a particular feature of the solution rather than to straight jacket the responses of the decision maker. Thus it may be necessary to introduce a smoothing option at this stage as well as the 'de-smoothing' effect of phase III.

6.9 The Phase III Search*

The motivation behind the phase III search was that by the end of phase II the decision maker will have had opportunity to experiment at some length with average levels and growth rates and presumably would now like to relax certain criteria in certain years away from the 'smoothed' solution of phase II. Thus it is considered part of the skill of financial management to know when it is worth the risk of relaxing say liquidity or profit in early years in order to improve the medium term position or perhaps earnings cover the year before a major debt repayment. The essence of the multiple criteria approach adopted here is not to pre-determine which years and which criteria to relax but allow the most advantageous relaxations to be demonstrated by the algorithm. To do this the minimax metric

*The experiments in this section were carried out by D.R. Atkins.

between years must be dropped and individual deviations z_{it} for each criteria in each year re-introduced. There are two major difficulties with attempting this. Firstly care must be taken to prevent the problems of instability associated with the p=1 goal programming model entering again. Hence a formulation of the type

$$\text{MIN} \quad \sum_i \sum_t z_{it} \tag{6.9.1.}$$

$$Y_{it}(x) + z_{it}/u_{it} \geq T_{it} \quad \text{all } i,t \tag{6.9.2.}$$

$$x \in \Gamma$$

cannot be contemplated for this reason. In addition such a formulation would lose control over the trade-offs established between criteria.

The second difficulty arises because of the need to continue the phase III search from the previous 'best' solution found at the end of phase II. This will not happen automatically if the structure of the objective function is changed to permit this new exploration. Again, as in the previous phase, this type of issue is one that is going to need to be addressed by all large scale applications of multicriteria optimisation in which some structured search is attempted. The methods adopted for this thesis are unlikely to be of general applicability but it is hoped they are instructive in representing one particular approach to these two difficulties.

The first step of phase III is the same as with phase II, to define the new target levels to be equal to their final 'best' point of phase II.

$$T_{it} = \frac{N_{it}^{II}(x^*)}{D_{it}^{II}(x^*)} \tag{6.9.3.}$$

This means that the target is achievable and efficient hence any objective function structure would reoptimise to the same point. The problem was then reformulated as:

$$\text{MINIMIZE } (\alpha_H^J z^J + \sum_{i \in J} z_i) + (\alpha_H^{J'} z^{J'} + \lambda \sum_{i \in J'} z_{it}) \qquad (6.9.4.)$$

so that

$$\left\{N_{it}(x) - T_{it} \cdot D_{it}(x)\right\} + E\left\{\frac{D_{it}T_{it}}{100u_i}\right\}(z^J + z_i) \geq 0, \quad i \in J \qquad (6.9.5.)$$

$$\left\{N_{it}(x) - T_{it}D_{it}(x)\right\} + E\left\{\frac{D_{it}T_{it}}{100u_i}\right\}(z^{J'} + z_{it}) \geq 0 \quad i \in J' \qquad (6.9.6.)$$

$$x \in \Gamma$$

$$I = J \cup J' \qquad J \cap J' = \emptyset$$

This structure needs some explanation. Initially we could have $J' = \emptyset$ and $J = I$ and the structure would be identical to that of phase II (equations (6.8.1.) and (6.8.2.)), though the value of $\lambda\delta_{it}$ in equation (6.8.2.) now has been incorporated into the phase III target of equations 6.9.5-6. If a series of experiments were done with different choices for J and J' then those criteria left within J would still be dealt with as in phase II, while those criteria in J' could improve their average values over time at the expense of introducing additional deviations z_{it} in particular years. The number of such deviations introduced can be controlled by the hybrid parameter λ. As λ is increased in integer steps by objective function parameterisation more and more z_{it} can enter leading to a less 'smooth' though hopefully, a more attractive solution. The criteria chosen for inclusion in J' must be those with the least liability to cause instability. Thus this procedure does not really allow for further exploration in the dividend policy criteria.

One last technical note needs to be added for completeness before presenting the results. The parameterisation with respect to λ cannot begin immediately because the target point is now on the efficient surface and there is no reason for changes in λ to affect the solution. The target point has therefore to be altered again and 'lifted away' from the efficient surface. To do this with minimal disruption of the current 'best' solution, the right-hand side parametrics of phase II were again used but with δ_{jt} chosen so that the value of the weighted deviations on all criteria remained balanced. Minor changes did in fact occur following redefinition of the criteria remaining in set J but these changes were not considered serious.

Three principal experiments were made with the choice of J' as in Table 6.9.1. Each experiment started from the phase II 'best' solution adjusted by 'lifting off' as above.

Table 6.9.1.

Experiment	Criteria in J'
A	Earnings per share in each year. Sales in each year. Profit in each year
B	Those in A plus return on capital employed in each year.
C	Those in B plus the liquidity ratio

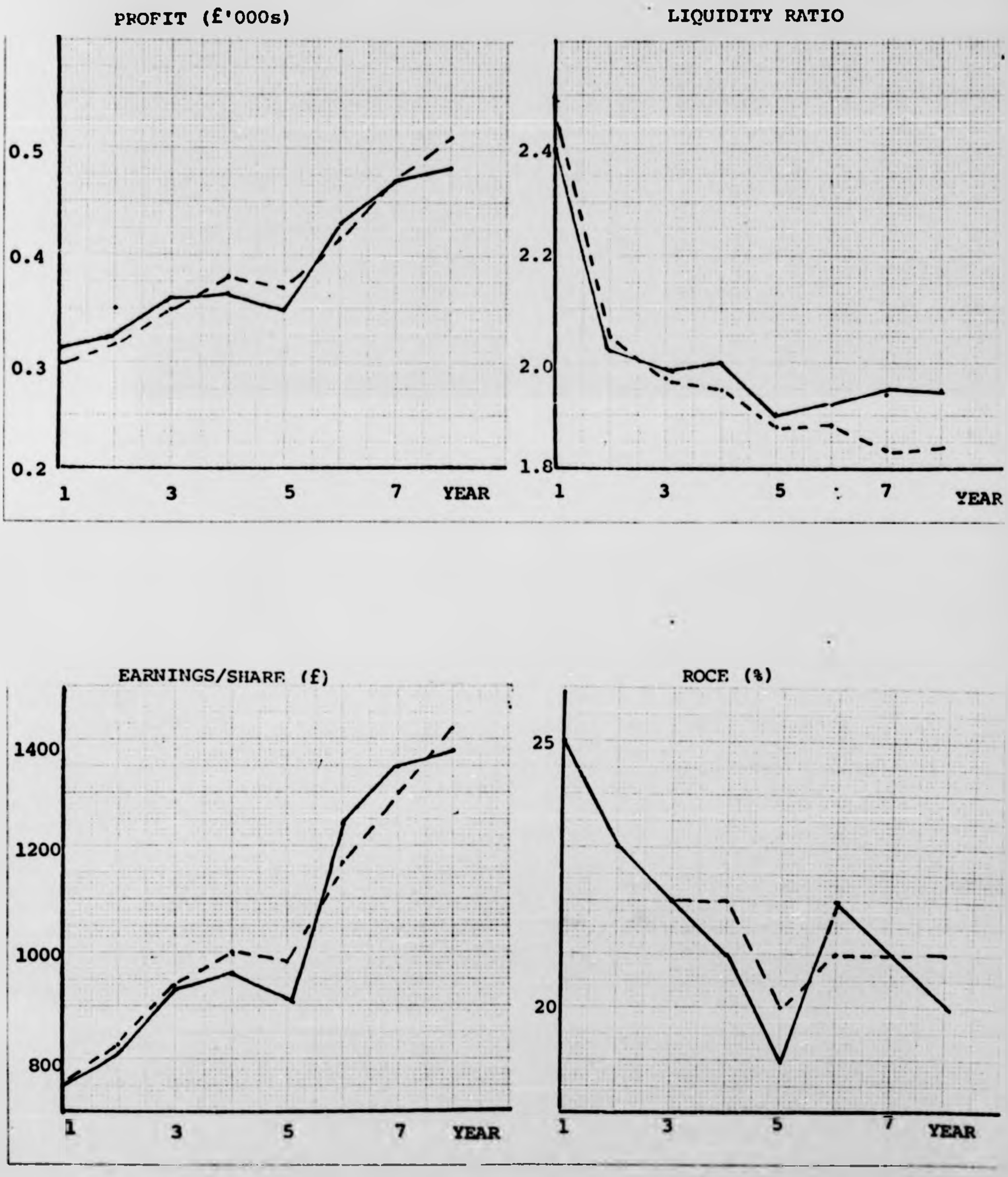
In Table 6.9.2. the number of individual deviations $z_{it}(i \in J')$ that entered as λ decreased over six steps is shown for each experiment. The last five columns show the number of years in which z_{it} entered for each criterion.

Table 6.9.2.

λ Step	Value of z_J	Value of $z_{J'}$	Number of deviations entered				
			Sales	Profit	Earnings per share	Return on Capital	Liquidity
A 1	30.85	22.56	0	0	0		
2	30.15	12.53	0	0	2		
3	30.99	10.40	0	0	2		
4	33.76	0	0	3	3		
5	30.38	0	0	3	4		
6	28.26	0	1	4	4		
B 1	30.22	18.93	0	0	2	0	
2	30.77	16.00	0	0	2	0	
3	30.77	15.00	0	0	2	0	
4	35.27	1.71	0	1	3	1	
5	34.47	0	1	2	4	1	
6	27.06	0	1	5	4	5	
C 1	26.47	20.76	0	0	0	0	0
2	26.96	19.53	0	0	2	0	0
3	28.09	15.38	0	0	2	0	1
4	28.09	15.38	0	0	2	0	1
5	31.67	0	0	2	4	1	1
6	22.77	0	1	6	4	5	5

The years in which it is attractive to relax earnings per share are years two and five. Both are difficult years; year two because initial outlays have still to generate adequate returns and year five because of the need to make a substantial debt repayment schedules in that year whilst maintaining growth. Heavy initial investment also explains a poor liquidity position in year five and the below average earnings in year five results in a poor return on capital employed. Other deviations arise because the sales target in year four and the profit targets in years two and eight are also difficult to meet. As a comparison of the 'smoothed' and 'unsmoothed' nature of the results, figure 6.9.1. shows the yearly values for profit, earnings, return and liqudity ratio for initial and final λ-values for experiment C. While Table 6.9.3. shows for this experiment the average values over time for λ-values of 1, 5 and 6. In

FIGURE 6.9.1.


PROFIT (£'000s)
0.5
0.4
0.3
0.2
1
3
5
7
YEAR
LIQUIDITY RATIO
2.4
2.2
2.0
1.8
1
3
5
7
YEAR
EARNINGS/SHARE (£)
1400
1200
1000
800
1
3
5
7
YEAR
ROCE (%)
25
20
1
3
5
7
YEAR

particular, a comparison of the values 1 and 5 for λ shows the pivotal role played by the earnings cover constraint.

Table 6.9.3. Average Values for λ-values of 1, 5 and 6 in Experiment C.

λ	1	5	6
ROCE(%)	21.77	22.17	21.82
LQDY	1.98	2.02	2.01
ECOV (in p)	17.20	16.81	17.56
ERPS	39.30	40.88	38.60
DCOV	2.62	2.64	2.70
DVPS (in p)	15.28	15.81	14.38
SALES (in £1000)	18844	19091	18453
PROFIT (in £1000)	1056	1103	1049

A comparison of the actual investment decisions corresponding to the initial and final λ-values for experiment C shows substantial differences. Out of the 22 totally or partially accepted projects 10 have a change in scale of 25% or more, 5 of these having a change of 75% or more. Particularly noticeable is increased investment in year three with only a modest increase in financing. This damages the firms performance in the middle years but allows the benefits to be reaped in the closing years.

6.10 Conclusions

In this Chapter an attempt has been made to look in detail at just one particular way in which mathematical programming methods might be modified to produce a more managerially acceptable decision tool for corporate financial planning. The essence of the strategy devised was that it should be responsive to the decision makers preferences. Hence the idea emerged that the decision maker should not only be able to indicate the currently most satisfactory solution but should also be able to give guidance to the desirable features of any improved solution and so direct the search into the appropriate region of the efficient surface. This strategy of responding to the perceived weaknesses of the existing solution ensured a rapid convergence to the final solution. Another feature of the method was the avoidance of inquiring directly into the decision makers trade-off preferences between criteria relying instead on the decision maker to indicate the preferred alternative of an ordered set of efficient solutions.

Clearly there remain many weaknesses in the method outlined here; for example the objective function structures devised are frequently cumbersome, though here a matrix generator would have helped considerably. Only one solution strategy and a limited set of search tools were considered. There remain many other plausible strategies and additional multicriteria tools which might prove useful. In the absence of any coherent and comprehensive framework on the properties of linear multicriteria structures the methods developed were of an ad hoc nature. Also in the end the important problem of controlling the intertemporal stability was resolved unsatisfactorily.

While clearly the methodology as presented here is still a long way from implementation and in need of considerable refinement in the search procedures. A comprehensive solution to all the weaknesses identified and the problems raised in this Chapter may not be necessary prior to trial implementation. A firm currently using a financial statement generator could have this type of multiobjective programme built on to the front so that the statement generator would now be a report writer to an LP. In the initial stages of development and implementation most of the investment and financing opportunities could be fixed at specific values within the model and the resulting tool would be indistinguishable as but should also be able to give guidance to the desirable features of any improved solution and so direct the search into the appropriate region of the efficient surface. This strategy of responding to the perceived weaknesses of the existing solution ensured a rapid convergence to the final solution. Another feature of the method was the avoidance of inquiring directly into the decision makers trade-off preferences between criteria relying instead on the decision maker to indicate the preferred alternative of an ordered set of efficient solutions.

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In summary the contribution of the work of this Chapter to such a process is that it begins to address some of the practical and procedural issues involved in the use of a multicriteria approach to financial planning.

The contribution of this Chapter to the thesis is that it illustrates one possible avenue for the future construction and use of a mathematical programming model for corporate financial planning.

CHAPTER 7

Conclusion

This thesis has been so structured that the detailed conclusions have already been presented at the end of each chapter. However, it is perhaps worthwhile to take a more global perspective of the work and to see the relationships between, and the limitations of, these conclusions. To this end a brief review of the development of the thesis would seem appropriate.

In chapter two the theoretical foundations of much of the subsequent analysis were laid down. Here the nature of the relationship between the primal formulation of the investment and financing decision and the dual formulation was reexamined and clarified. The third chapter was then able to exploit the structure of the dual solution to impose bounds on the primal solution. These bounds showed that for many models, whose objective function is based on a discounting methodology and where decisions are constrained by debt capacity (and possibly other) considerations, the chosen set of investment projects is not radically different from that which could be obtained by the use of a simple rule of thumb.

It was this conclusion which directed the research into the exploration of two different,and quite distinct roles, which could be played by L.P. models in financial planning.

The first of these roles was the development of analytical tools for financial theory. Chapter four looked in detail at how L.P. models could be used as a framework for the normative appraisal of individual projects within the broader context of the firms total investment and financing opportunity set. These ideas were extended in chapter five to the analysis of a simultaneous investment and financing decision - that of a financial lease.

In contrast the last section explored a very different role for L.P. models and considered how they could be restructured to become more relevant and more effective decision tools for use by corporate financial planners. This analysis led to the formulation of an interactive goal programming system.

Both of these uses for L P. models have their obvious limitations and attendant unsolved problems and it would be inappropriate to conclude without drawing attention to these issues and indicating where future research might be directed.

The first of these problem areas is the development of an algorithm for solving horizon truncated financial planning L.P. models in accordance with the horizon principle enunciated in the introductory chapter. The outlines of such an algorithm were briefly reported in chapter four and although it appeared to work reasonably efficiently for the example cited in that section, a great deal of involved programming would be necessary prior to a more general implementation.

The second problem area is the structuring of suitable objective functions for use in the goal programming search. Here it has been possible to develop a primitive algebra for the classification of objective functions in multi-criteria programming. Such an algebra provides alternative linear models which might be used in multi-criteria programming for the generation of solution with particular structural features. As both of these developments form part of joint ongoing research with Atkins their details have not been included in this thesis.

The limitations on the use of linear models, as normative frameworks for financial theory or as interactive goal programming devices, are sufficiently serious that the particular formulations, though not necessarily the methodology, adopted in this paper would appear to afford little future until they can be overcome.

In using the model for the development of a normative theory of investment appraisal it was necessary to adopt without further question many results based on a two moment equilibrium theory of capital markets. Thus the incorporation of uncertainty was principally via a risk adjusted discount rate coupled with restrictions on the level of debt. While such an approach maintains linearity it does require the return on debt to be perfectly elastic upto some predetermined limit and perfectly inelastic thereafter, while the return on other financing instruments were required to be constant through this range. This simplification contradicts many of the assumptions of capital market theory and places a severe limitation on the validity of the conclusions which can be drawn from such models. The mere adoption of a step wise linear approximation to the risk return schedule is too crude for theoretical, though not necessarily for practical, purposes.

In using the restructured model for multi-criteria programming the non-linearities introduced by financial ratios were largely glossed over. Further research has shown that, for the fractional criteria necessary to financial planning, the efficient region is not necessarily closed and might also include interior points. Such findings severely limit the use of the interactive goal-programming models presented here and indicate that until the topology of the feasible region in multi-criteria fractional programming formulations is better understood, it would be unwise to continue with the development of this particular programming methodology.

These two major limitations would thus seem to block, though hopefully only temporarily, further progress. It would thus seem an appropriate point at which to formally present the findings so far and to submit this thesis.

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APPENDIX I

A Mathematical Statement of the Model.

The following set of equations constitute the model for periods 1 to 8. The source of the equation is given. The symbols in brackets give the corresponding row names which serve as variables in the computer model. The notation is defined in appendix II.

There are three distinct constraint sets. The first set consists of the accounting and technological constraints which are common to both the single criterion and the multicriteria model. The second set consists of the 'hard' constraints on financial policy variables used in the single criterion model only while the third set constitutes part of the goal programming structure used for exploring alternative financial strategies, and is exclusive to the multicriteria model.

1 Accounting and Technological Constraints

1.01 Sales (Total sales equals the sales from existing projects plus sales from new projects)

$$\sum_j S_{jt}X_j - SALES = - SO_t \qquad [TS_t$$

1.02 Building and Land (Book value of building and land equals new investments from projects plus existing investment less depreciation)

$$\sum_j CBL_{jt}X_j - 1.03\ FABL_t + FABL_{t-1} = - CBLO_t \qquad [BL_t$$

1.03 Plant and Equipment (As for building and Land)

$$\sum_j CPE_{jt}X_j + FAPE_{t-1} - 1.3333\ FAPE_t = - CPEO_t \qquad [PE_t$$

1.04 Earnings (Earnings equals the earnings from existing and new projects less net short term interest payments less depreciation

$$\sum_j EA_{jt}X_j - 0.0310\ FABL_t - 0.3333\ FAPE_t - EARN_t - RS.OVDR_{t-1} + RIMARK_{t-1} = - EO_t$$

$$[EA_t$$

1.05 Current Assets (Total current assets equals current assets from existing and new projects plus short term deposits)

$$\sum_j (AP_{jt}+ST_{jt})X_j + MARK_t - CURA_t = - CAO_t \qquad [CA_t$$

1.06 Current Liabilities (Total current liabilities equals liabilities from old and new projects plus overdraft, dividends and taxation payable)

$$\sum_j (AR_{jt}X_j) + OVDR_t - CURL_t + TAX_t = - CLO_t \qquad [CL_t$$

1.07 Number of Shares (Increase in the total number of shares outstanding equals shares plus rights issues)

$$- \sum_{\tau \leq t} RG_\tau + NUM_t = N_o \qquad [EQ_t$$

1.08 Debt (Increase in debt outstanding equals new debt less any debt repayments)

$$- LL_t + DE_t - DE_{t-1} = - DERPO_t \qquad [D_t$$

1.09 Net Profit after Tax (Net book profit after tax equals (1 - Tax rate) times taxable earnings)

$$- 0.5(E_t - RL.DE_{t-1}) + NPAT_t = 0 \qquad [PR_t$$

1.10 Tax payable (Tax payable equals (1 - tax rate) times gross earnings less actual tax allowances)

$$TAX_t - NPAT_t - 0.2FABL_{t-1} + 0.191FABL_t - 0.5FAPE_{t-1}$$

$$+ 0.5FAPE_t + 0.5BLTA_t = TAO_t \qquad [TP_t$$

1.11 Tax allowances (Tax allowances on buildings and land equals existing allowances plus any new allowances)

$$0.04FABL_{t-1} - 0.041FABL_t + BLTA_t - BLTA_{t-1} = 0 \qquad [TA_t$$

1.12 Cash Balance (Total cash inflows equals total cash outflows)

$$E_t - FABL_t + FABL_{t-1} - FAPE_t + FAPE_{t-1} - CA_t + CA_{t-1}$$

$$+ CL_t - CL_{t-1} + LL_t + 1.6RG_t - TAX_t - DV_t - RLDE_{t-1} = 0 \qquad [CB_t$$

1.13 Scale Constraint (A project can be taken on at any level up to full scale)

$$0 \le X_j \le 1$$

1.14 Non-negativity (All primal variables are constrained to be positive or zero)

2 Single Criterion Model - Financial Policy Variables

2.01 Return of Capital Employed (Earnings after depreciation and short term interest should be greater than α-times net book value of assets after depreciation)

$$- E_t + \alpha(FABL_t + FAPE_t + CA_t - CL_t) \le 0 \quad [ROCE_t$$

2.02 Current Ratio (Ratio of current assets to current liabilities should be greater than β)

$$- CA_t + \beta CL_t \le 0 \quad [LQDY_t$$

2.03 Times Covered (Earnings after depreciation and short term interest should be greater than γ times total interest payments on debt)

$$- E_t + \gamma(RL.DE_{t-1} + RS.OVDR_{t-1}) \le 0 \quad [ECOV_t$$

2.04 Earnings per share (Net book profit after tax should be greater than δ_t times the number of shares)

$$- NPAT_t + \delta_t NUM_t \le 0 \quad [ERPS_t$$

2.05 Dividend Cover (Dividends should be covered ε times by distributable profit)

$$- NPAT_t + \varepsilon DIV_t \le 0 \quad [DCOV_t$$

2.06 Dividend Target (Planned dividend/share should be met)

$$DIV_t - DTARG_t.NUM_t \le 0 \quad [DTARG_t$$

3 Multicriteria-Model - Financial Policy Variables

3.01 Return on Capital Employed (Where possible earnings after depreciation and short term interest should be greater than α_t-times net book value of assets after depreciation)

$$- E_t + \alpha_t(FABL_t + FAPE_t + CA_t - CL_t) - u_{ROCE}.ZED \leq 0 \quad [ROCE_t$$

3.02 Current Ratio (Where possible the ratio of current assets to current liabilities should be greater than β_t)

$$- CA_t + \beta_t CL_t - u_{LQDY}.ZED \leq 0 \quad [LQDY_t$$

3.03 Times Covered (Where possible, earnings after depreciation and short term interest should be greater than γ_t times total interest payments on debt)

$$- E_t + \gamma_t(RL.DE_{t-1} + RS.OVDR_{t-1}) - u_{ECOV}.ZED \leq 0 \quad [ECOV_t$$

3.04 Earnings per share (Where possible the book profit after tax should be greater than δ_t times the number of shares)

$$- NPAT_t + \delta_t.NUM_t - u_{ERPS}.ZED \leq 0 \quad [ERPS_t$$

3.05 Dividend Cover (Where possible dividends should be covered ε times by distributable profit)

$$- NPAT_t + \varepsilon_t DIV_t - u_{DCOV_t}.ZED \leq 0 \quad [DCOV_t$$

3.06 Sales Target (Planned sales target should be aimed for)

$$SALES_t + u_{ST}.ZED \geq SALES\ TARGET_t \quad [STARG_t$$

3.07 Profit Target (Planned profit target should be aimed for)

$$NPAT_t + u_{PT}.ZED \geq PROFIT\ TARGET_t \quad [PTARG_t$$

3.08 Dividend Target (Planned dividend/share should be met)

$$DIV_t - DTARG_t.NUM_t + u_{DPS}.ZED \geq 0 \quad [DTARG_t$$

3.09 Upper limit on Growth (In the multicriteria model the total growth factor measured in terms of fixed and current assets should not be more than three times over the eight-year planning period)

$$FABL_8 + FAPE_8 + CURA_8 - CURL_8 \le 14100$$ [NW8

APPENDIX II

Definitions and Notation

For ease of reference the definitions and notation are divided into two sections. The first section explains the notation which is used in the formulation of the model to be found in Appendix 1 and the variable names used in the computer program of which sample printouts are also to be found in the Appendices. The second section deals with the mathematical notation which is used in the main body of the thesis for the development of theoretical arguments.

Definitions and Notation of Variables used in the computer model.

AP_{jt} - accounts payable on project j in time period t.

AR_{jt} - accounts receivable on project j in time period t.

$BLTA_t$ - accumulated tax allowances on building and land at time period t.

CAO_t - value of working capital in time period t resulting from operations already undertaken.

CA_t - total value of current assets at the end of time period t.

CBL_{jt} - capital expenditure on building and land on project j in time period t.

$CBLO_t$ - capital expenditure on building and land from commitments already undertaken.

CLO_t - value of creditors in time period t resulting from operations already undertaken.

$CPEO_t$ - capital expenditure on building and land from commitments already undertaken.

DE_t - total value of long term in time period t.

$DEPRO_t$ - planned debt repayment in time period t.

$DTARG_t$ - the dividend per share target in time period t.

DV_t - actual dividend declared in time period t. Paid in time period t + 1.

E_t - earnings in time period t after depreciation and short term interest payments/receipts.

EA_{jt} - gross earnings in time period t from project j.

$FABL_t$ - book value of building and land at time period t.

$FAPE_t$ - book value of plant and equipment at time period t.

LL_t - new long term debt taken out in time period t.

$MARK_t$ - short term deposits at the end of time period t.

N_o - number of shares outstanding at the beginning of the planning period.

$NPAT_t$ - net book profit after tax in time period t.

NUM_t - number of shares outstanding at the end of time period t.

$OVDR_t$ - overdraft at the end of time period t.

PR_nY_t - denotes project n undertaken at time t.

$PTARG_t$ - profit target for time period t.

RG_t - number of rights issued in time period t.

RI - interest rate on short term deposits.

RL - interest rate on long term debt.

RS - interest rate on overdraft facilities.

S_{jt} - sales generated by project j in time period t.

$SALES_t$ - total sales in time period t.

SO_t - sales in period t from existing operations.

$STARG_t$ - sales target for period t.

TAX_t - tax payable on operations for period t.

TAO_t - tax allowances in period t from existing building and land.

u	- weighting vector on deviations from targets.
x_j	- scale at which project j is undertaken.
ZED	- vector representing deviations from targets on the policy variables.
α_H	- 'degree' of hybridisation of goal programming model.
α_t	- return on capital employed required in period t.
β_t	- minimum value of current ratio in period t.
γ_t	- nmmber of times earnings cover debt in period t.
δ_t	- earnings/share required in period t.
ε_t	- value of dividend cover in period t.

Definitions and Notation used in the development of the theoretical arguments in the main text

Below are two alphabetical lists (English and Greek) of the notation used in the main text. This summary is provided mainly for quick reference; the precise definition may vary with the context of the argument and any ambiguities should be resolved by reference to the local definition. Heavy type used in the text indicates a vector and the list below should be interpreted as the components of these vectors where appropriate.

a	- weighted average cost of capital.
a_o	- risk adjusted discount rate for the valuation of project cash flows assuming a base case of all equity financing.
A_o	- cost of an asset to be leased.
[A]	- matrix of resource outputs from the adoption of a set of decisions.
APV_t	- adjusted present value in t.
[B]	- matrix of resource input from the adoption of a set of decisions.

b_t - capital allowances available in period t per unit of project adoption.

B_t - borrowing limit in Weingartner model.

c_{tj} - cash inflow from project j in time period t.

$\hat{c}_j$ - net present (terminal) value of project j.

C_{tj} - capital required by project j in time period t.

d_t - dividend/share in time period t.

D_t - total dividends paid by the firm in period t.

$D_{it}(x)$ - denominator ratio criterion i in period t.

D-statistic - Chebychev error norm for project selection.

e_{jt} - earnings from project j in period t.

E_t - total value of equity issued in time period t.

f - flotation costs associated with equity issues or occasionally a constant multiplier.

F_t, F_o^t - funds available from existing projects in time period t.

g - level of gearing.

H - planning horizon

i - interest rate (usually of equity capital).

i_L - implied interest rate in lease financing.

I_t - total interest paid in t.

IRR - internal rate of return.

j - subscript used to denote project number.

K - constant defining limit on capital structure.

L{ } - likely or estimated value of a function.

L_j - scale of leasing project j.

ℓ_t - duals on leverage (gearing) in Chambers model.

$L_t \left(= \sum_{\tau=t}^{H} \ell_\tau\right)$ - duals on leverage (gearing) in Chambers model.

m - marginal reinvestment rate.

MM - attributable to Modigliani and Miller.

$N_{it}(x)$	- numerator of ratio criterion i in time period t.
n_t	- number of shares issue in t.
NPV	- net present value.
NTV	- net terminal value.
P_t	- issue price of a share in t.
P_t	- (pre-tax) lease payment to be made in t.
r	- interest rate (usually debt).
r'	- = r(1-KT).
r_B	- borrowing interest rate.
r_L	- lending interest rate.
R_t	- repayment of principal of a loan or lease.
S_t	- issue price in t of a share.
T	- rate of Corporation tax.
T_{it}	- target for criterion i in time period t.
$TV_j(t)$	- compounded value of net funds to project j at time t.
u_i	- relative utility attached to criterion i.
U	- utility function.
v_t	- fixed interest investment in t.
V_t	- value of firm at time t.
W_t	- level of debt in t.
x_t	- income per unit investment in period t.
x'	- after tax income per unit investment.
X_j	- scale of acceptance of project j.
Z	- objective function.
Z_i, Z_t, Z_{it}	- deviations (from criterion i) (in time period t) from target.
z^t	- vector of decisions taken in t.
β_t	- dual on the borrowing limit in time period t.
δ_{it}	- incremental step on target i in time period t.

γ_{it}	- criterion function for criterion i in time period t.
Γ	- feasible region.
ε	- used to denote small increment.
η	- vector of dual variables.
θ_t^W	- dual on debt valuation stream at time t.
θ_t^X	- dual on income valuation stream at time t.
λ_t	- dual on the debt capacity constraint at time t.
μ_j	- dual on the scale of acceptance of project j.
π	- discount factor.
ρ_t	- dual on cash balance constraint in time period t.
ρ^*	- cost of capital rate for the screening of projects.
ϕ_t	- function denoting debt capacity at time t.
ψ_t	- function denoting the value of equity at time t.
ω_t	- level of debt at time t.

Appendix III

The Initial Balance Sheet, Operating projections and the background Environment

Initial Balance Sheet (£'000s)

SHARE CAPITAL AND LONG-TERM DEBT		ASSETS		
Share Capital		Fixed Assets		
(2,000 @ £1)	2,000	Land and Buildings	1,634	
Reserves	1,200	Plant and machinery	881	
Long-Term Debt	1,500		2,515	2,515
		Current Assets		
		Short-Term Deposits	800	
		Debtors	1,560	
		Stock	1,700	
			4,060	4,060
		Less Current Liabilities		
		Creditors	1,120	
		Tax	370	
		Overdraft	100	
		Dividend payable	285	
			1,875	1,875
		Net Current Assets		2,185
	4,700	TOTAL ASSETS		4,700

INITIAL PROJECTIONS (£'OOO)

	YEAR-1	YEAR-2	YEAR-3	YEAR-4	YEAR-5	YEAR-6	YEAR-7	YEAR-8
Sales from existing projects	11000	10000	9500	8800	8000	7500	7500	7000
Gross earnings from existing projects	1950	1850	1600	1560	1240	1220	1200	1000
Planned expenditure Building and Land	400	600	500	200	-	-	-	-
Tax allowances	130	130	130	120	120	120	120	100
Plant and equipment	600	500	400	400	400	400	400	400
Working capital (Debtors and Stock)	3510	3588	3705	3432	3120	2925	2925	2730
Creditors	1120	1145	1170	1095	1020	946	948	902

Additional Data

(i) The initial market value of sharesis £2 and the rights issue price is £1.6. No more than £0.8m may be raised in the form of rights at any one time.

(ii) Long-term debt is available at 8% over 25 years upto £1m in any year.

(iii) There is a planned debt repayment of £1m in year 5.

(iv) Overdraft is available upto £0.25m in any year at 12% before tax.

(v) Excess funds may be placed on 1 year deposit at 7%.

(vi) The sales during the current financial year were £10,550,000 producing a net profit after tax of £900,000.

(vii) The initial value of tax allowances on building and land is £130,000.

<u>The internally imposed financial constraints under which the firm operates* are as follows</u>

(a) Return on capital imployed in any year must be greater than 18%.

(b) The dividend cover should be greater than 1.5.

(c) The ratio of current assets to current liabilities must be greater than 1.8.

(d) The number of times that debt is covered should be greater than 10.

(e) The earnings/share and dividend per share figures in each year are:

	YEAR-1	YEAR-2	YEAR-3	YEAR-4	YEAR-5	YEAR-6	YEAR-7	YEAR-8
Earnings per share (£)	0.20	0.20	0.21	0.21	0.22	0.22	0.24	0.24
Dividend per share (£)	0.130	0.135	0.140	0.145	0.150	0.155	0.160	0.165

<u>Treatment of Taxation</u>

There are two categories of capital assets. These are:

(a) Building and Land.

(b) Plant and Equipment.

<u>Tax Allowances available</u>

Building and land receive a first year allowance of 40% with 4% of the initial total cost allowed on a straight-line basis thereafter.

Plant and equipment receive a first year allowance of 100%.

<u>Book Depreciation Rates</u>

The book depreciation rates are 3% on building and land and 25% on plant and plant and equipment both on a reducing balance.

It is assumed that the rate of corporation tax is 50% and that there is a time lag of one year in payment.

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* These constraints only apply to the single criterion model.

Timing of Cash Flows

One of the problems associated with programming models is the mapping of continuous time into discrete time. It was decided because of:

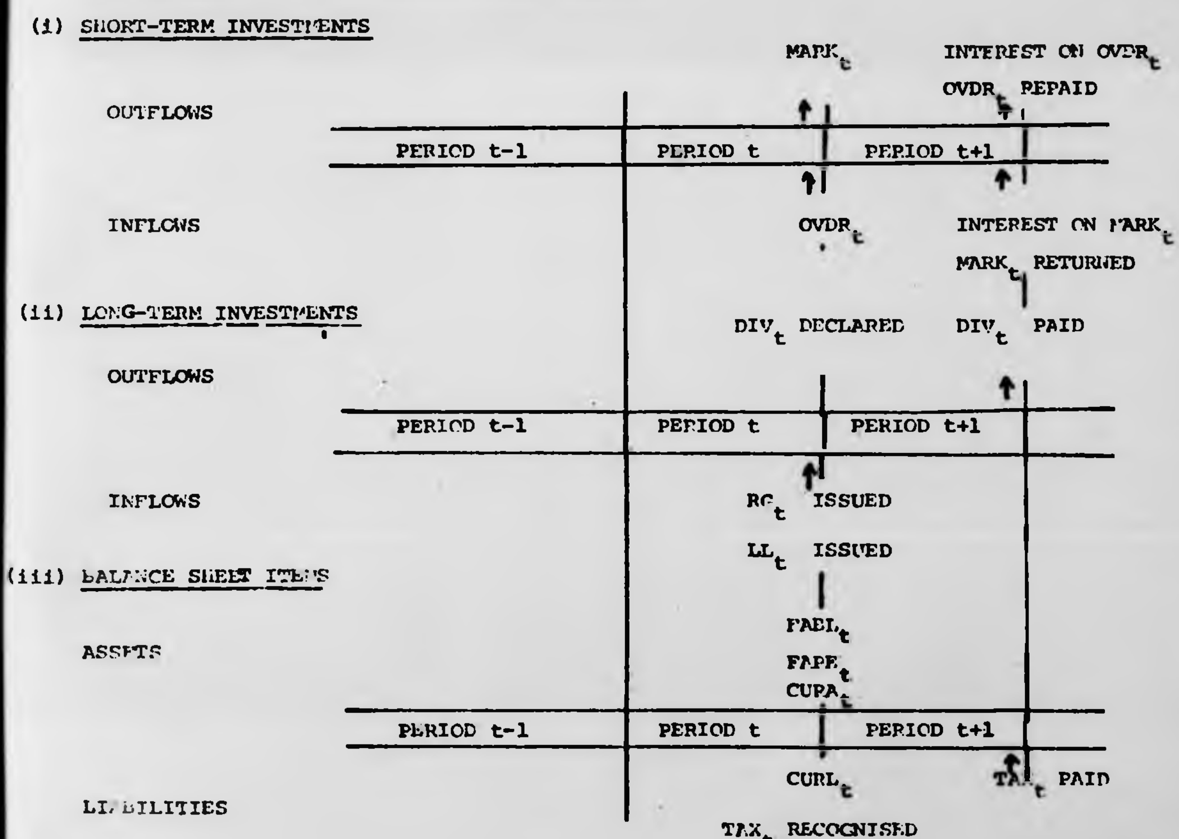
(a) Projects had been developed in which all cash flows were recognised at the end of a period.

(b) The model was to be used for valuation and as such it was necessary to have well defined recognition points.

(c) The model was to be used to generate Balance Sheet information.

that the simplifying assumption of recognising all transactions at the end of a period was adopted. Figure A3.1 illustrate the implications of this approach.

Figure A3.1 THE TIMING OF CASH FLOWS



Appendix IV

Project Data

Table A4.1 Project Specification

The projects were specified by the following accounting data over their eight year lives.

PROJECT NO. PR 1

SALES	0	0	500	800	1000	1200	1200	1100
BUILDING/LAND	100	50	50	0	0	0	0	0
PLANT/EQUIPMENT	0	80	70	0	0	0	0	0
EARNINGS	0	0	100	184	240	360	300	253
CURRENT ASSETS	0	0	200	357	393	429	424	404
CURRENT LIABS	0	0	40	67	71	85	84	79

PROJECT NO. PR 2

SALES	310	670	700	690	650	620	590	530
BUILDING/LAND	50	0	0	0	0	0	0	0
PLANT/EQUIPMENT	90	45	0	0	0	0	0	0
EARNINGS	30	30	105	105	105	87	80	64
CURRENT ASSETS	123	234	297	294	266	261	251	195
CURRENT LIABS	37	74	73	78	73	70	69	65

PROJECT NO. PR 3

SALES	410	620	1800	1680	1740	1520	1310	1020
BUILDING/LAND	200	0	0	0	0	0	0	0
PLANT/EQUIPMENT	100	80	0	0	0	0	0	0
EARNINGS	20	62	270	324	312	274	238	153
CURRENT ASSETS	149	257	714	692	718	629	607	497
CURRENT LIABS	60	103	285	200	185	176	153	121

PROJECT NO. PR 4

SALES	300	760	980	910	830	760	710	690
BUILDING/LAND	75	25	0	0	0	0	0	0
PLANT/EQUIPMENT	250	130	0	0	0	0	0	0
EARNINGS	27	130	226	200	174	152	134	124
CURRENT ASSETS	97	139	317	305	278	244	232	224
CURRENT LIABS	45	108	123	118	109	110	107	83

PROJECT NO. PR 5

SALES	510	830	1250	1330	1350	1310	1280	1200
BUILDING/LAND	145	0	0	0	0	0	0	0
PLANT/EQUIPMENT	180	150	90	0	0	0	0	0
EARNINGS	54	116	224	266	270	250	230	192
CURRENT ASSETS	123	362	535	595	591	550	498	451
CURRENT LIABS	38	90	124	130	141	135	126	127

PROJECT NO. PR 11

SALES	120	270	750	1250	1300	1250	1000	1000
BUILDING/LAND	225	0	0	0	0	0	0	0
PLANT/EQUIPMENT	120	100	75	0	0	0	0	0
EARNINGS	0	15	135	250	260	250	190	200
CURRENT ASSETS	30	75	115	240	340	380	360	375
CURRENT LIABS	20	60	95	105	140	170	195	200

PROJECT NO. PR 12

SALES	120	390	580	760	1000	1010	1100	950
BUILDING/LAND	190	0	0	0	0	0	0	0
PLANT/EQUIPMENT	50	80	70	50	30	10	10	0
EARNINGS	4	36	108	160	230	220	232	190
CURRENT ASSETS	41	134	229	341	486	434	354	283
CURRENT LIABS	17	53	124	168	200	198	185	193

PROJECT NO. PR 13

SALES	600	940	1560	1100	430	660	210	90
BUILDING/LAND	250	0	0	0	0	0	0	0
PLANT/EQUIPMENT	140	120	70	0	0	0	0	0
EARNINGS	72	160	468	242	48	112	36	9
CURRENT ASSETS	174	236	518	451	166	238	61	39
CURRENT LIABS	60	102	180	121	85	103	25	10

PROJECT NO. PR 14

SALES	500	1000	1250	1500	1500	1500	1250	1000
BUILDING/LAND	100	100	0	0	0	0	0	0
PLANT/EQUIPMENT	50	50	50	50	50	50	50	50
EARNINGS	50	150	250	300	300	300	250	200
CURRENT ASSETS	120	210	300	390	480	550	630	670
CURRENT LIABS	40	80	120	160	200	200	200	120

PROJECT NO. PR 15

SALES	230	680	720	710	730	680	720	710
BUILDING/LAND	50	25	0	0	0	0	0	0
PLANT/EQUIPMENT	60	40	0	0	0	0	0	0
EARNINGS	7	34	72	142	146	123	137	142
CURRENT ASSETS	84	188	218	272	292	329	318	294
CURRENT LIABS	20	25	40	73	85	102	100	101

PROJECT NO. PR 16

SALES	200	600	1000	1200	1200	1200	1200	1000
BUILDING/LAND	160	60	0	0	0	0	0	0
PLANT/EQUIPMENT	100	100	50	0	0	0	0	0
EARNINGS	0	42	160	240	264	264	240	180
CURRENT ASSETS	65	215	365	448	460	463	451	400
CURRENT LIABS	20	70	100	140	160	160	150	140

PROJECT NO. PR 21

SALES	1200	2000	2000	2000	1800	1700	1400	1000
BUILDING/LAND	300	0	0	0	0	0	0	0
PLANT/EQUIPMENT	250	400	200	0	0	0	0	0
EARNINGS	192	360	360	340	288	233	182	120
CURRENT ASSETS	372	640	645	702	690	672	585	475
CURRENT LIABS	181	279	290	285	263	241	219	189

PROJECT NO. PR 22

SALES	500	500	500	500	500	500	500	500
BUILDING/LAND	100	0	0	0	0	0	0	0
PLANT/EQUIPMENT	40	100	150	40	0	40	0	0
EARNINGS	100	100	110	110	105	100	110	105
CURRENT ASSETS	150	154	163	169	178	176	173	166
CURRENT LIABS	69	73	69	67	68	68	66	61

PROJECT NO. PR 23

SALES	700	750	780	800	810	800	770	750
BUILDING/LAND	125	65	0	0	0	0	0	0
PLANT/EQUIPMENT	250	100	0	0	0	0	0	0
EARNINGS	140	165	156	152	137	128	103	105
CURRENT ASSETS	210	244	284	310	318	313	282	257
CURRENT LIABS	100	100	112	115	121	121	120	100

PROJECT NO. PR 24

SALES	1200	1700	1560	1490	1495	1520	1530	1510
BUILDING/LAND	150	0	100	0	0	0	0	0
PLANT/EQUIPMENT	250	200	100	50	0	0	0	0
EARNINGS	156	255	265	209	224	258	275	242
CURRENT ASSETS	406	506	567	503	510	537	555	570
CURRENT LIABS	187	250	270	270	265	255	260	250

PROJECT NO. PR 25

SALES	1000	1200	1250	1260	1290	1300	1310	1300
BUILDING/LAND	125	25	0	50	0	0	0	0
PLANT/EQUIPMENT	180	150	0	150	0	0	0	0
EARNINGS	140	220	230	240	220	190	150	120
CURRENT ASSETS	367	419	455	451	446	467	450	445
CURRENT LIABS	162	190	200	196	198	221	210	205

Table A4.2 The Cash Flows and IRR of the Projects.

Basic Project Type		Years 1	2	3	4	5	6	7	8	9	IRR
	PR1	-100	-109	-128	52	120	222	128	122	-123	13.04
	PR2	-196	-49	79	56	76	37	46	77	-31	9.08
	PR3	-369	0	8	130	113	202	104	116	-73	11.47
	PR4	-350	124	69	96	94	102	69	48	-60	15.59
	PR5	-361	-122	14	147	154	152	150	127	-94	7.41
	PR11	-355	19	102	109	74	114	114	99	-96	11.68
	PR12	-260	-37	39	26	35	163	187	161	-92	12.13
	PR13	-432	58	229	56	181	39	84	3	0	13.97
	PR14	-180	-29	123	104	79	59	-1	-66	-71	8.7
	PR15	-167	-93	66	86	68	31	86	100	-70	10.06
	PR16	-305	-133	35	146	156	133	114	105	-86	8.62
	PR21	-649	-116	192	204	114	96	134	115	-54	5.22
	PR22	-121	-9	-51	84	64	11	83	54	-51	8.75
	PR23	-345	48	112	55	61	68	77	59	-49	6.73
	PR24	-463	97	-1	165	137	114	138	84	-116	8.57
	PR25	-370	68	176	-72	195	86	65	49	-56	10.51

(all cash flows in £1000's)

Table A4.3 The Availability of Projects

A tick indicates that a particular project was available in that year.

YEAR / PROJECT	1	2	3	4	5	6	7	8
PR1	✓			✓			✓	
PR2			✓		✓			✓
PR3		✓			✓			
PR4	✓	✓				✓	✓	
PR5		✓		✓		✓		
PR11			✓	✓	✓	✓		
PR12	✓			✓				
PR13	✓	✓		✓				
PR14		✓		✓		✓	✓	
PR15			✓					✓
PR16	✓					✓		
PR21		✓			✓	✓		
PR22	✓			✓			✓	✓
PR23	✓				✓	✓		
PR24	✓							
PR25				✓				✓

APPENDIX V

(i) The Structure of the Objective Function - Single Criterion Model.

When the model was being used for the maximization of the value of the firm (chapters 3 - 5) the form of the objective function was

$$\text{MAX } \psi_o = \sum_{t=1}^{H-1} \frac{DV_t}{(1+i)^{t+1}} - \sum_{t=1}^{H} 1.6 \frac{RG_t}{(1+i)^t}$$

$$+ \frac{0.572\ OVDR_{H-1} + 0.0384\ LL_{H-1} - 0.0338\ MARK_{H-1}}{(1+i)^H}$$

$$- \left\{ \frac{DE_H + OVDR_H}{(1+i)^H} \right\} + \frac{MARK_H}{(1+i)^H}$$

$$+ \sum_{j \varepsilon \{PR_n Y_t\}} X_j \left(\sum_{\tau=H+1}^{\tau=t+8} \frac{C_{\tau j}}{(1+a)^{t-H}} \right) \qquad \text{A5.1}$$

where i, the equity rate was 12% and a, the cost of capital for discounting project cash flows was 10% or 10.5% as detailed in the text.

The first two terms in A5.1 represent the net dividend flow to the equity holders. At the horizon the portion of the value of the firm attributable to the equity holders consists of the after tax cash flows less adjustments for the value of the outstanding fixed interest instruments. The value of the former is just the post-horizon after tax cash flows discounted at a, while the latter consists of the market value of the fixed interest instruments plus adjustments for unpaid taxation in H-1. The details of the derivation of the form of A5.1 are to be found in section 4.6.

(ii) Selection by internal rate of return - structure of objective function

In section 3.6 the single criterion model was used to consider selection by IRR, the objective function took the form

$$\psi_o + \sum_{j \in \{PR_n Y_t\}} (1000 \times IRR_j) X_j \qquad \text{A5.2}$$

where ψ_o is as defined in equation 5.1, with H = 8 and

$$\left.\begin{aligned} IRR_j &= +1 \quad \text{If IRR of project } j > i \\ &= -1 \quad \text{If IRR of project } j < i \end{aligned}\right|$$

where i denotes the cut-off rate for selection.

(iii) Selection by internal rate of return ranking

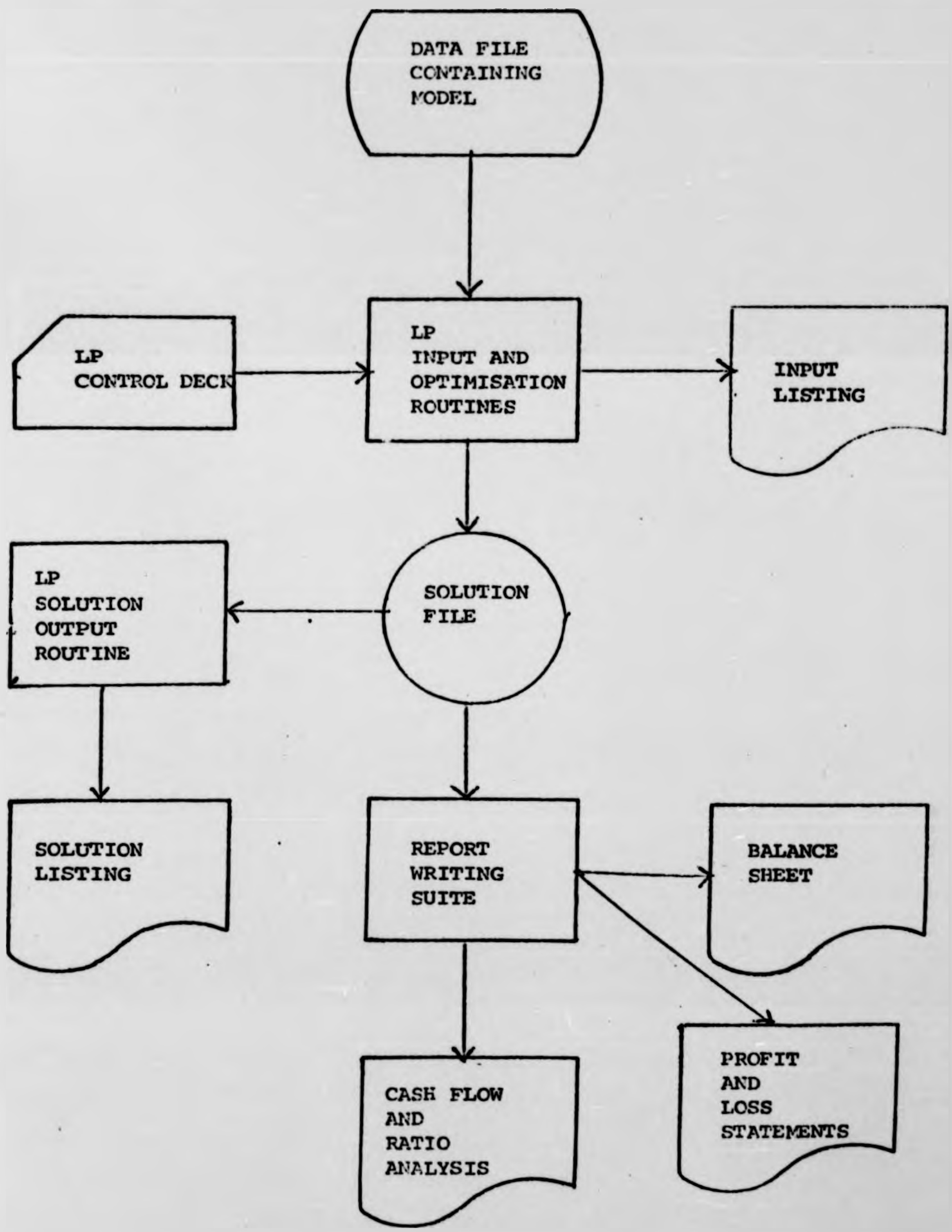
In this case the objective function took the form

$$\psi_o + \sum_{j \in \{PR_n Y_t\}} (1000 \times RANK_j) X_j$$

where $RANK_j$ was a number in the range 1 to 45, corresponding to the ranking of project j by an internal rate of return criterion. For projects which were available in more than one year, the project occurring first was given the highest ranking.

Appendix VI

A Systems Flow Chart - The Single Criterion Model



APPENDIX VII

INPUT PARAMETERS FOR THE SINGLE CRITERION MODEL

A7.1 A Sample Control Deck

A7.2 LP Input Data Listing

EXHIBIT A7.1 A SAMPLE CONTROL DECK

```
PROGRAM DESCPIPTION

            'OBJECT'("ED"("OBJECTFILE"))

PROGRAM
            'BEGIN'
            OPEN(PROB,"ED"("PROBFIMMODEL"),2)
            PROBFILE(PROB)
            PROBLEM("OPTIMODEL")
            WORK("ED"("WORKFILE A2"))
            WORK("ED"("WORKFILE A3"))
            WORK("ED"("WORKFILE A4"))
            WORK("ED"("WORKFILE A5"))
            WORK("ED"("WORKFILE A6"))
            CORE(50000)
            OPEN(TEMP,"ED"("WORKFILE A1"),2)
            REVISE("OPTIMODEL-86",,TEMP)
            £LRSTL:="ROWYR".10
            £LCSTL:="COLYR".10
            LIST(1)
            REINSTATE
            SCALE(1,,,,)
            £LRHS0:="RHS1"
            £LRST11:=("DUMMY")
            £LCST11:=("DUMMY")
            £LOBJ0:="COBJ"
            PRIMAL
            SOLUTION
            REINSTATE
            £LRHS1:="RHS2"
            £LCMULT1:=0.1
            PRIMAL
            SOLUTION
            REINSTATE
            £LCMULT1:=-0.1
            PRIMAL
            SOLUTION
            'END'
            'FINISH'
```

EXHIBIT A7.2 LP INPUT DATA LISTING

```
                         ROW SET (########)          COLUMN SET (########)

PROBLEM SIZE

NUMBER OF ROWS:-

   COLUMN SET BATCHES                         1
   COLUMN SCALING FACTORS                     0
   BOUNDS                                     3
   KERNEL ROWS                              155
      OBJECTIVES                             11
      EQUALITY CONSTRAINTS                   96
      CONVEXITY CONSTRAINTS                   0
      LESS OR EQUAL CONSTRAINTS              40
      GREATER OR EQUAL CONSTRAINTS            8
      TOTAL NUMBER OF CONSTRAINTS           144

NUMBER OF COLUMNS:-

   ROW SET BATCHES                            1
   ROW SCALING FACTORS                        0
   VARIABLES                                174
      CONTINUOUS VARIABLES                  174
         NON-NEGATIVE VARIABLES             174
         NON-POSITIVE VARIABLES               0
         FREE VARIABLES                       0
         ZERO VARIABLES                       0
      INTEGER VARIABLES                       0
         NON-NEGATIVE VARIABLES               0
         NON-POSITIVE VARIABLES               0
         FREE VARIABLES                       0
         ZERO VARIABLES                       0
         SPECIAL VARIABLES                    0
      SPECIAL ORDERED SET VARIABLES           0
         TYPE 1 VARIABLES                     0
         TYPE 2 VARIABLES                     0
   RIGHT HAND SIDES                          16
   RANGES                                     0
```

ROW SET (########) COLUMN SET (########)

COLUMN NAMES AND NUMBER OF NON-ZERO ELEMENTS

VARIABLES

SALES1	+	1	SALES2	+	1	SALES3	+	1	SALES4	+	1	SALES5	+	1	SALES6	+	1
SALES7	+	1	SALES8	+	1	EARN1	+	5	EARN2	+	5	EARN3	+	5	EARN4	+	5
EARN5	+	5	EARN6	+	5	EARN7	+	5	EARN8	+	5	NPAT1	+	4	NPAT2	+	4
NPAT3	+	4	NPAT4	+	4	NPAT5	+	4	NPAT6	+	4	NPAT7	+	4	NPAT8	+	4
TAX1	+	3	TAX2	+	3	TAX3	+	3	TAX4	+	3	TAX5	+	3	TAX6	+	3
TAX7	+	3	TAX8	+	6	FABL1	+	10	FABL2	+	10	FABL3	+	10	FABL4	+	10
FABL5	+	10	FABL6	+	10	FABL7	+	10	FABL8	+	7	FAPE1	+	8	FAPE2	+	8
FAPE3	+	8	FAPE4	+	8	FAPE5	+	8	FAPE6	+	8	FAPE7	+	8	FAPE8	+	6
CURA1	+	5	CURA2	+	5	CURA3	+	5	CURA4	+	5	CURA5	+	5	CURA6	+	5
CURA7	+	5	CURA8	+	5	CURL1	+	5	CURL2	+	5	CURL3	+	5	CURL4	+	5
CURL5	+	5	CURL6	+	5	CURL7	+	5	CURL8	+	5	OVDR1	+	3	OVDR2	+	3
OVDR3	+	3	OVDR4	+	3	OVDR5	+	3	OVDR6	+	3	OVDR7	+	5	OVDR8	+	4
MARK1	+	2	MARK2	+	2	MARK3	+	2	MARK4	+	2	MARK5	+	2	MARK6	+	2
MARK7	+	4	MARK8	+	3	DV1	+	6	DV2	+	6	DV3	+	6	DV4	+	6
DV5	+	6	DV6	+	6	DV7	+	6	DV8	+	4	NUM1	+	4	NUM2	+	4
NUM3	+	4	NUM4	+	4	NUM5	+	4	NUM6	+	4	NUM7	+	4	NUM8	+	3
RG1	+	4	RG2	+	4	RG3	+	4	RG4	+	4	RG5	+	4	RG6	+	4
RG7	+	4	RG8	+	4	DE1	+	5	DE2	+	5	DE3	+	5	DE4	+	5
DE5	+	5	DE6	+	5	DE7	+	7	DE8	+	6	LL1	+	3	LL2	+	3
LL3	+	3	LL4	+	3	LL5	+	3	LL6	+	3	LL7	+	3	LL8	+	4
BLTA1	+	3	BLTA2	+	3	BLTA3	+	3	BLTA4	+	3	BLTA5	+	3	BLTA6	+	3
BLTA7	+	3	BLTA8	+	2	PR01Y1	+	34	PR04Y1	+	41	PR12Y1	+	45	PR13Y1	+	39
PR16Y1	+	41	PR22Y1	+	43	PR23Y1	+	41	PR03Y2	+	36	PR04Y2	+	37	PR05Y2	+	37
PR15Y2	+	37	PR14Y2	+	42	PR21Y2	+	37	PR24Y2	+	39	PR02Y3	+	32	PR11Y3	+	32
PR15Y3	+	33	PR01Y4	+	22	PR05Y4	+	29	PR11Y4	+	28	PR12Y4	+	31	PR13Y4	+	29
PR14Y4	+	32	PR22Y4	+	30	PR25Y4	+	31	PR02Y5	+	24	PR03Y5	+	24	PR11Y5	+	24
PR21Y5	+	25	PR23Y5	+	25	PR04Y6	+	21	PR05Y6	+	21	PR11Y6	+	20	PR14Y6	+	22
PR15Y6	+	21	PR16Y6	+	21	PR21Y6	+	21	PR23Y6	+	21	PR01Y7	+	8	PR04Y7	+	17
PR14Y7	+	17	PR22Y7	+	16	PR02Y8	+	11	PR15Y8	+	11	PR22Y8	+	11	PR25Y8	+	11

RIGHT HAND SIDES

RHS2	8	RHS3	16	RHS4	1	RHS5	0	NIL	0	RHS2	8
RHS3	16	RHS4	1	RHS5	0	NIL	0	RHS2	8	RHS3	16
RHS4	1	RHS5	0	NIL	0	RHS1	59				

```
                    ROW SET (########)          COLUMN SET (########)

ROW NAMES AND NUMBER OF NON-ZERO ELEMENTS

COLUMN SETS

COLYR   .02    23   COLYR   .03    42   COLYR   .04    66   COLYR   .05    87   COLYR   .06   111   COLYR   .07   131
COLYR   .08   151   COLYR   .10    42

BOUNDS

RANGE          16   UPBND          70   LOBND           0

KERNEL ROWS

GOAL     Z      0   ORJ      Z     15   OBJ1     Z     48   ORJ2     Z     47   OBJ3     Z     55   OBJ4     Z      5
OBJ5     Z      1   ORJ6     Z     51   ORJ7     Z      0   ORJ8     Z      0   TS1      =      7   TS2      =     14
TS3      =     18   TS4      =     25   TS5      =     30   TS6      =     39   TS7      =     42   TS8      =     46
EA1      =      8   EA2      =     18   FA3      =     21   EA4      =     28   EA5      =     33   EA6      =     41
EA7      =     46   EA8      =     50   BL1      =      8   BL2      =     13   BL3      =      8   BL4      =     12
BL5      =     10   BL6      =     12   BL7      =     12   BL8      =      9   PE1      =      7   PE2      =     16
PE3      =     17   PE4      =     19   PE5      =     19   PE6      =     25   PE7      =     21   PE8      =     18
CA1      =      8   CA2      =     15   CA3      =     19   CA4      =     26   CA5      =     31   CA6      =     40
CA7      =     43   CA8      =     47   CL1      =     10   CL2      =     17   CL3      =     21   CL4      =     28
CL5      =     33   CL6      =     42   CL7      =     45   CL8      =     49   CB1      =      9   CB2      =     14
CB3      =     14   CB4      =     14   CB5      =     14   CB6      =     14   CB7      =     14   CB8      =     14
TP1      =      5   TP2      =      7   TP3      =      7   TP4      =      7   TP5      =      7   TP6      =      7
TP7      =      7   TP8      =      7   TA1      =      2   TA2      =      4   TA3      =      4   TA4      =      4
TA5      =      4   TA6      =      4   TA7      =      4   TA8      =      4   PR1      =      2   PR2      =      3
PR3      =      3   PR4      =      3   PR5      =      3   PR6      =      3   PR7      =      3   PR8      =      3
EQ1      =      2   EQ2      =      3   EQ3      =      3   EQ4      =      3   EQ5      =      3   EQ6      =      3
EQ7      =      3   EQ8      =      3   D1       =      2   D2       =      3   D3       =      3   D4       =      3
D5       =      3   D6       =      3   D7       =      3   D8       =      3   ROCE1    +      5   ROCE2    +      5
ROCE3    +      5   ROCE4    +      5   ROCE5    +      5   ROCE6    +      5   ROCE7    +      5   ROCE8    +      5
LODY1    +      2   LODY2    +      5   LODY3    +      2   LODY4    +      2   LODY5    +      2   LODY6    +      2
LODY7    +      2   LODY8    +      2   ECOV1    +      3   ECOV2    +      3   ECOV3    +      3   ECOV4    +      3
ECOV5    +      3   ECOV6    +      3   ECOV7    +      3   ECOV8    +      3   ERPS1    +      2   ERPS2    +      2
ERPS3    +      2   ERPS4    +      2   ERPS5    +      2   ERPS6    +      2   ERPS7    +      2   ERPS8    +      2
DTARG1   -      2   DTARG2   -      2   DTARG3   -      2   DTARG4   -      2   DTARG5   -      2   DTARG6   -      2
DTARG7   -      2   DTARG8   -      2   DCOV1    +      2   DCOV2    +      2   DCOV3    +      2   DCOV4    +      2
DCOV5    +      2   DCOV6    +      2   DCOV7    +      2   DCOV8    +      2   COBJ     Z     66

COLUMN NAMES AND NUMBER OF NON-ZERO ELEMENTS

ROW SETS

ROWYR   .02    15   ROWYR   .03    23   ROWYR   .04    39   ROWYR   .05    56   ROWYR   .06    72   ROWYR   .07    88
ROWYR   .08   104   ROWYR   .10    50   ROWYR   .24    36
```

```
                                        ROW SET (########)          COLUMN SET (########)

LISTING BY COLUMNS

ROW SETS

          ERPS2    000000000*0000000000000     ERPS3    000000000*0000000000000
          ERPS4    000*****0*0000000000000     ERPS5    0000****0*0000000000000
          ERPS6    00000***0*0000000000000     ERPS7    000000**0*0000000000000
          ERPS8    0000000*0*0000000000000     DTARG1   000000000*0000000000000
          DTARG2   000000000*0000000000000     DTARG3   00******0*0000000000000
          DTARG4   000*****0*0000000000000     DTARG5   0000****0*0000000000000
          DTARG6   00000***0*0000000000000     DTARG7   000000**0*0000000000000
          DTARG8   0000000*0*0000000000000     DCOV1    000000000*0000000000000
          DCOV2    000000000*0000000000000     DCOV3    00******0*0000000000000
          DCOV4    000*****0*0000000000000     DCOV5    0000****0*0000000000000
          DCOV6    00000***0*0000000000000     DCOV7    000000**0*0000000000000
          DCOV8    0000000*0*0000000000000

VARIABLES

SALES1    TS1          -1.0000

SALES2    COLYR    0*******000000000000000
          TS2          -1.0000

SALES3    COLYR    00******000000000000000
          TS3          -1.0000

SALES4    COLYR    000*****000000000000000
          TS4          -1.0000

SALES5    COLYR    0000****000000000000000
          TS5          -1.0000

SALES6    COLYR    00000***000000000000000
          TS6          -1.0000

SALES7    COLYR    000000**000000000000000
          TS7          -1.0000

SALES8    COLYR    0000000*000000000000000
          TS8          -1.0000

EARN1     EA1          -1.0000  CB1          1.0000  PR1          -0.5000  ROCE1         -1.0000
          ECOV1        -1.0000

EARN2     COLYR    0*******000000000000000
          EA2          -1.0000  CB2          1.0000  PR2          -0.5000  ROCE2         -1.0000
          ECOV2        -1.0000

EARN3     COLYR    00******000000000000000
          EA3          -1.0000  CB3          1.0000  PR3          -0.5000  ROCE3         -1.0000
          ECOV3        -1.0000

EARN4     COLYR    000*****000000000000000
```

```
                         ROW SET (########)              COLUMN SET (########)

LISTING BY COLUMNS

VARIABLES

         EA4          -1.0000  CR4        1.0000  PR4      -0.5000  ROCE4     -1.0000
         ECOV4        -1.0000

EARN5    COLYR     0000****0000000000000000
         EA5          -1.0000  CB5        1.0000  PR5      -0.5000  ROCE5     -1.0000
         ECOV5        -1.0000

EARN6    COLYR     00000***0000000000000000
         EA6          -1.0000  CR6        1.0000  PR6      -0.5000  ROCE6     -1.0000
         ECOV6        -1.0000

EARN7    COLYR     000000**0000000000000000
         EA7          -1.0000  CB7        1.0000  PR7      -0.5000  ROCE7     -1.0000
         ECOV7        -1.0000

EARN8    COLYR     0000000*0000000000000000
         EA8          -1.0000  CB8        1.0000  PR8      -0.5000  ROCE8     -1.0000
         ECOV8        -1.0000

NPAT1    TP1          -1.0000  PR1        1.0000  ERPS1    -1.0000  DCOV1     -1.0000

NPAT2    COLYR     0*******0000000000000000
         TP2          -1.0000  PR2        1.0000  ERPS2    -1.0000  DCOV2     -1.0000

NPAT3    COLYR     00******0000000000000000
         TP3          -1.0000  PR3        1.0000  ERPS3    -1.0000  DCOV3     -1.0000

NPAT4    COLYR     000*****0000000000000000
         TP4          -1.0000  PR4        1.0000  ERPS4    -1.0000  DCOV4     -1.0000

NPAT5    COLYR     0000****0000000000000000
         TP5          -1.0000  PR5        1.0000  ERPS5    -1.0000  DCOV5     -1.0000

NPAT6    COLYR     00000***0000000000000000
         TP6          -1.0000  PR6        1.0000  ERPS6    -1.0000  DCOV6     -1.0000

NPAT7    COLYR     000000**0000000000000000
         TP7          -1.0000  PR7        1.0000  ERPS7    -1.0000  DCOV7     -1.0000

NPAT8    COLYR     0000000*0000000000000000
         TP8          -1.0000  PR8        1.0000  ERPS8    -1.0000  DCOV8     -1.0000

TAX1     CL1           1.0000  CR1       -1.0000  TP1       1.0000

TAX2     COLYR     0*******0000000000000000
         CL2           1.0000  CR2       -1.0000  TP2       1.0000

TAX3     COLYR     00******0000000000000000
         CL3           1.0000  CR3       -1.0000  TP3       1.0000
```

```
                                        ROW SET (########)                COLUMN SET (########)

LISTING BY COLUMNS

VARIABLES

TAX4     COLYR    000*****00000000000000000
         CL4           1.0000   CR4          -1.0000  TP4          1.0000

TAX5     COLYR    0000****00000000000000000
         CL5           1.0000   CR5          -1.0000  TP5          1.0000

TAX6     COLYR    00000***00000000000000000
         CL6           1.0000   CR6          -1.0000  TP6          1.0000

TAX7     COLYR    000000**00000000000000000
         CL7           1.0000   CR7          -1.0000  TP7          1.0000

TAX8     COLYR    0000000*00000000000000000
         OBJ1         -1.0000   OBJ2         -1.0000  OBJ3        -1.0000  CL8          1.0000
         CB8          -1.0000   TP8           1.0000

FABL1    COLYR    000000000*000000000000000
         EA1          -0.0310   BL1          -1.0300  BL2          1.0000  CB1         -1.0000
         CB2           1.0000   TP1           0.1910  TP2         -0.2000  TA1         -0.0410
         TA2           0.0400   ROCE1         0.1800

FABL2    COLYR    0*******0*000000000000000
         EA2          -0.0310   BL2          -1.0300  BL3          1.0000  CB2         -1.0000
         CB3           1.0000   TP2           0.1910  TP3         -0.2000  TA2         -0.0410
         TA3           0.0400   ROCE2         0.1800

FABL3    COLYR    00******0*000000000000000
         EA3          -0.0310   BL3          -1.0300  BL4          1.0000  CB3         -1.0000
         CB4           1.0000   TP3           0.1910  TP4         -0.2000  TA3         -0.0410
         TA4           0.0400   ROCE3         0.1800

FABL4    COLYR    000*****0*000000000000000
         EA4          -0.0310   BL4          -1.0300  BL5          1.0000  CB4         -1.0000
         CB5           1.0000   TP4           0.1910  TP5         -0.2000  TA4         -0.0410
         TA5           0.0400   ROCE4         0.1800

FABL5    COLYR    0000****0*000000000000000
         EA5          -0.0310   BL5          -1.0300  BL6          1.0000  CB5         -1.0000
         CB6           1.0000   TP5           0.1910  TP6         -0.2000  TA5         -0.0410
         TA6           0.0400   ROCE5         0.1800

FABL6    COLYR    00000***0*000000000000000
         EA6          -0.0310   BL6          -1.0300  BL7          1.0000  CB6         -1.0000
         CB7           1.0000   TP6           0.1910  TP7         -0.2000  TA6         -0.0410
         TA7           0.0400   ROCE6         0.1800

FABL7    COLYR    000000**0*000000000000000
         EA7          -0.0310   BL7          -1.0300  BL8          1.0000  CB7         -1.0000
         CB8           1.0000   TP7           0.1910  TP8         -0.2000  TA7         -0.0410
         TA8           0.0400   ROCE7         0.1800
```

```
                                 ROW SET (########)                  COLUMN SET (########)

        LISTING BY COLUMNS

        VARIABLES

        FABL8     COLYR      0000000+0+0000000000000000
                  OBJ4            1.0000  EA8           -0.0310  BLA           -1.0300  CB8           -1.0000
                  TP8             0.1910  TA8           -0.0410  ROCE8          0.1800

        FAPE1     EA1            -0.3300  PE1           -1.3300  PE2            1.0000  CB1           -1.0000
                  CB2             1.0000  TP1            0.5000  TP2           -0.5000  ROCE1          0.1800

        FAPE2     COLYR      0*******0000000000000000
                  EA2            -0.3300  PE2           -1.3300  PE3            1.0000  CB2           -1.0000
                  CB3             1.0000  TP2            0.5000  TP3           -0.5000  ROCE2          0.1800

        FAPE3     COLYR      00******000000000000000000
                  EA3            -0.3300  PE3           -1.3300  PE4            1.0000  CB3           -1.0000
                  CB4             1.0000  TP3            0.5000  TP4           -0.5000  ROCE3          0.1800

        FAPE4     COLYR      000*****000000000000000000
                  EA4            -0.3300  PE4           -1.3300  PE5            1.0000  CB4           -1.0000
                  CB5             1.0000  TP4            0.5000  TP5           -0.5000  ROCE4          0.1800

        FAPE5     COLYR      0000****000000000000000000
                  EA5            -0.3300  PE5           -1.3300  PE6            1.0000  CB5           -1.0000
                  CB6             1.0000  TP5            0.5000  TP6           -0.5000  ROCE5          0.1800

        FAPE6     COLYR      00000***000000000000000000
                  EA6            -0.3300  PE6           -1.3300  PE7            1.0000  CB6           -1.0000
                  CB7             1.0000  TP6            0.5000  TP7           -0.5000  ROCE6          0.1800

        FAPE7     COLYR      000000**000000000000000000
                  EA7            -0.3300  PE7           -1.3300  PE8            1.0000  CB7           -1.0000
                  CB8             1.0000  TP7            0.5000  TP8           -0.5000  ROCE7          0.1800

        FAPE8     COLYR      0000000*000000000000000000
                  OBJ4            1.0000  EA8           -0.3300  PE8           -1.3300  CB8           -1.0000
                  TP8             0.5000  ROCE8          0.1800

        CURA1     CA1            -1.0000  CB1           -1.0000  CB2            1.0000  ROCE1          0.1800
                  LQDY1          -1.0000

        CURA2     COLYR      0*******000000000000000000
                  CA2            -1.0000  CB2           -1.0000  CB3            1.0000  ROCE2          0.1800
                  LQDY2          -1.0000

        CURA3     COLYR      00******000000000000000000
                  CA3            -1.0000  CB3           -1.0000  CB4            1.0000  ROCE3          0.1800
                  LQDY3          -1.0000

        CURA4     COLYR      000*****000000000000000000
                  CA4            -1.0000  CB4           -1.0000  CB5            1.0000  ROCE4          0.1800
                  LQDY4          -1.0000

        CURA5     COLYR      0000****000000000000000000
```

```
                              ROW SET (########)              COLUMN SET (########)

      LISTING BY COLUMNS

      VARIABLES

                CA5        -1.0000  CB5        -1.0000  CB6         1.0000  ROCE5      0.1800
                LQDY5      -1.0000

      CURA6     COLYR    00000**000000000000000
                CA6        -1.0000  CB6        -1.0000  CB7         1.0000  ROCE6      0.1800
                LQDY6      -1.0000

      CURA7     COLYR    000000**000000000000000
                CA7        -1.0000  CB7        -1.0000  CB8         1.0000  ROCE7      0.1800
                LQDY7      -1.0000

      CURA8     COLYR    0000000*000000000000000
                OBJ4        1.0000  CA8        -1.0000  CB8        -1.0000  ROCE8      0.1800
                LQDY8      -1.0000

      CURL1     COLYR    00000000*0000000000000
                CL1        -1.0000  CB1         1.0000  CB2        -1.0000  ROCE1     -0.1800
                LQDY1       1.8000

      CURL2     COLYR    0*******0*000000000000
                CL2        -1.0000  CB2         1.0000  CB3        -1.0000  ROCE2     -0.1800
                LQDY2       1.8000

      CURL3     COLYR    00******0*000000000000
                CL3        -1.0000  CB3         1.0000  CB4        -1.0000  ROCE3     -0.1800
                LQDY3       1.8000

      CURL4     COLYR    000*****0*000000000000
                CL4        -1.0000  CB4         1.0000  CB5        -1.0000  ROCE4     -0.1800
                LQDY4       1.8000

      CURL5     COLYR    0000****0*000000000000
                CL5        -1.0000  CB5         1.0000  CB6        -1.0000  ROCE5     -0.1800
                LQDY5       1.8000

      CURL6     COLYR    00000***0*000000000000
                CL6        -1.0000  CB6         1.0000  CB7        -1.0000  ROCE6     -0.1800
                LQDY6       1.8000

      CURL7     COLYR    000000**0*000000000000
                CL7        -1.0000  CB7         1.0000  CB8        -1.0000  ROCE7     -0.1800
                LQDY7       1.6000

      CURL8     COLYR    0000000*0*000000000000
                OBJ4       -1.0000  CL8        -1.0000  CB8         1.0000  ROCE8     -0.1800
                LQDY8       1.3000

      OVDR1     COLYR    00000000O*000000000000
                RANGE     100.0000  UPBND     250.0000  LOBND             0
                EA2        -0.1200  CL1         1.0000  ECOV1        1.2000
```

```
                    ROW SET (########)          COLUMN SET (########)

LISTING BY COLUMNS

VARIABLES

OVDR2     COLYR    0*******0000000000000000
          RANGE     100.0000  UPBND      250.0000
          EA3        -0.1200  CL2          1.0000  ECOV2        1.2000

OVDR3     COLYR    00******000000000000000
          RANGE     100.0000  UPBND      250.0000
          EA4        -0.1200  CL3          1.0000  ECOV3        1.2000

OVDR4     COLYR    000*****000000000000000
          RANGE     100.0000  UPBND      250.0000
          EA5        -0.1200  CL4          1.0000  ECOV4        1.2000

OVDR5     COLYR    0000****000000000000000
          RANGE     100.0000  UPBND      250.0000
          FA6        -0.1200  CL5          1.0000  ECOV5        1.2000

OVDR6     COLYR    00000***000000000000000
          RANGE     100.0000  UPBND      250.0000
          EA7        -0.1200  CL6          1.0000  ECOV6        1.2000

OVDR7     COLYR    000000**000000000000000
          RANGE     100.0000  UPBND      250.0000
          OBJ6        0.0572  EA8         -0.1200  CL7          1.0000  ECOV7        1.2000
          COBJ        0.0231

OVDR8     COLYR    0000000*000000000000000
          RANGE     100.0000  UPBND      250.0000
          OBJ6       -1.0000  CL8          1.0000  ECOV8        1.2000  COBJ        -0.4039

MARK1     COLYR    00000000*0000000000000
          LOBND            0
          EA2         0.0700  CA1          1.0000

MARK2     COLYR    0*******0000000000000000
          LOBND            0
          EA3         0.0700  CA2          1.0000

MARK3     COLYR    00******000000000000000
          LOBND            0
          EA4         0.0700  CA3          1.0000

MARK4     COLYR    000*****000000000000000
          LOBND            0
          EA5         0.0700  CA4          1.0000

MARK5     COLYR    0000****000000000000000
          LOBND            0
          EA6         0.0700  CA5          1.0000

MARK6     COLYR    00000***000000000000000
          LOBND            0
```

```
                    ROW SET (########)              COLUMN SET (########)
LISTING BY COLUMNS
VARIABLES

          EA7        0,0700  CA6          1,0000

MARK7     COLYR    000000**0000000000000000
          LOBND          0
          UBJ6      -0,0380  EA8          0,0700  CA7          1,0000  COBJ        -0,0153

MARK8     COLYR    0000000*0000000000000000
          LOBND          0
          OBJ6       1,0000  CA8          1,0000  COBJ         0,4039

DV1       COLYR    000000000*00000000000000
          OBJ        0,7972  CL1          1,0000  CB1         -1,0000  DTARG1       1,0000
          DCOV1      1,5000  COBJ         0,7972

DV2       COLYR    0********0*0000000000000
          OBJ        0,7118  CL2          1,0000  CB2         -1,0000  DTARG2       1,0000
          DCOV2      1,5000  COBJ         0,7118

DV3       COLYR    00******0*00000000000000
          OBJ        0,6355  CL3          1,0000  CB3         -1,0000  DTARG3       1,0000
          DCOV3      1,5000  COBJ         0,6355

DV4       COLYR    000*****0*00000000000000
          OBJ        0,5674  CL4          1,0000  CB4         -1,0000  DTARG4       1,0000
          DCOV4      1,5000  COBJ         0,5674

DV5       COLYR    0000****0*00000000000000
          OBJ        0,5066  CL5          1,0000  CB5         -1,0000  DTARG5       1,0000
          DCOV5      1,5000  COBJ         0,5066

DV6       COLYR    00000***0*00000000000000
          OBJ        0,4523  CL6          1,0000  CB6         -1,0000  DTARG6       1,0000
          DCOV6      1,5000  COBJ         0,4523

DV7       COLYR    000000**0*00000000000000
          OBJ        0,4039  CL7          1,0000  CB7         -1,0000  DTARG7       1,0000
          DCOV7      1,5000  COBJ         0,4039

DV8       COLYR    0000000*0*00000000000000
          CL8        1,0000  CB8         -1,0000  DTARG8       1,0000  DCOV8        1,5000

NUM1      COLYR    000000000*00000000000000
          EQ1        1,0000  EQ2         -1,0000  ERPS1        0,2000  DTARG1      -0,1300

NUM2      COLYR    0*******0*00000000000000
          EQ2        1,0000  EQ3         -1,0000  ERPS2        0,2000  DTARG2      -0,1350

NUM3      COLYR    00******0*00000000000000
          EQ3        1,0000  EQ4         -1,0000  ERPS3        0,2100  DTARG3      -0,1400
```

```
                         ROW SET (########)           COLUMN SET (########)

LISTING BY COLUMNS

VARIABLES

NUM4     COLYR    000*****0*00000000000000
         EQ4          1.0000  EQ5          -1.0000  ERPS4         0.2100  DTARG4        -0.1450

NUM5     COLYR    0000****0*00000000000000
         EQ5          1.0000  EQ6          -1.0000  ERPS5         0.2200  DTARG5        -0.1500

NUM6     COLYR    00000***0*00000000000000
         EQ6          1.0000  EQ7          -1.0000  ERPS6         0.2200  DTARG6        -0.1550

NUM7     COLYR    000000**0*00000000000000
         EQ7          1.0000  EQ8          -1.0000  ERPS7         0.2400  DTARG7        -0.1600

NUM8     COLYR    0000000*0*00000000000000
         EQ8          1.0000  ERPS8         0.2400  DTARG8        -0.1650

RG1      UPBND      800.0000
         OBJ         -1.4290  CB1           1.6000  EQ1          -1.0000  COBJ          -1.4280

RG2      COLYR    0*******0000000000000000
         UPBND      800.0000
         OBJ         -1.2760  CB2           1.6000  EQ2          -1.0000  COBJ          -1.2760

RG3      COLYR    00******0000000000000000
         UPBND      800.0000
         OBJ         -1.1390  CB3           1.6000  EQ3          -1.0000  COBJ          -1.1339

RG4      COLYR    000*****0000000000000000
         UPBND      800.0000
         OBJ         -1.0170  CB4           1.6000  EQ4          -1.0000  COBJ          -1.0170

RG5      COLYR    0000****0000000000000000
         UPBND      800.0000
         OBJ         -0.9080  CB5           1.6000  EQ5          -1.0000  COBJ          -0.9070

RG6      COLYR    00000***0000000000000000
         UPBND      800.0000
         OBJ         -0.8110  CB6           1.6000  EQ6          -1.0000  COBJ          -0.8110

RG7      COLYR    000000**0000000000000000
         UPBND      800.0000
         OBJ         -0.7280  CB7           1.6000  EQ7          -1.0000  COBJ          -0.7240

RG8      COLYR    0000000*0000000000000000
         UPBND      800.0000
         OBJ         -0.6470  CB8           1.6000  EQ8          -1.0000  COBJ          -0.6464

DE1      COLYR    00000000*00000000000000
         CB2         -0.0800  PR2           0.0400  D1            1.0000  D2            -1.0000
         ECOV1        0.8000

DE2      COLYR    0*******0*00000000000000
```

```
                    ROW SET (#########)                    COLUMN SET (#########)

    LISTING BY COLUMNS

    VARIABLES

              CB3        -0.0800  PR3          0.0400  D2        1.0000  D3       -1.0000
              ECUV2       0.8000

    DE3       COLYR   00******0*0000000000000
              CB4        -0.0800  PR4          0.0400  D3        1.0000  D4       -1.0000
              ECUV3       0.8000

    DE4       COLYR   000*****0*0000000000000
              CB5        -0.0800  PR5          0.0400  D4        1.0000  D5       -1.0000
              ECUV4       0.8000

    DE5       COLYR   0000****0*0000000000000
              CB6        -0.0800  PR6          0.0400  D5        1.0000  D6       -1.0000
              ECUV5       0.8000

    DE6       COLYR   00000***0*0000000000000
              CB7        -0.0800  PR7          0.0400  D6        1.0000  D7       -1.0000
              ECUV6       0.8000

    DE7       COLYR   000000**0*0000000000000
              OBJ6        0.0364  CB8         -0.0800  PR8       0.0400  D7        1.0000
              D8         -1.0000  ECOV7        0.8000  COBJ      0.0155

    DE8       COLYR   0000000*0*0000000000000
              OBJ1       -1.0000  OBJ4        -1.0000  OBJ6     -1.0000  D8        1.0000
              ECOVR       0.8000  COBJ        -0.4039

    LL1       RANGE     100.0000  UPBND     1000.0000
              OBJ3       -0.6749  CB1          1.0000  D1       -1.0000

    LL2       COLYR   0*******00000000000000000
              RANGE     100.0000  UPBND     1000.0000
              OBJ3       -0.6425  CB2          1.0000  D2       -1.0000

    LL3       COLYR   00******00000000000000000
              RANGE     100.0000  UPBND     1000.0000
              OBJ3       -0.6137  CB3          1.0000  D3       -1.0000

    LL4       COLYR   000*****00000000000000000
              RANGE     100.0000  UPBND     1000.0000
              OBJ3       -0.5889  CB4          1.0000  D4       -1.0000

    LL5       COLYR   0000****00000000000000000
              RANGE     100.0000  UPBND     1000.0000
              OBJ3       -0.5650  CB5          1.0000  D5       -1.0000

    LL6       COLYR   00000***00000000000000000
              RANGE     100.0000  UPBND     1000.0000
              OBJ3       -0.5444  CB6          1.0000  D6       -1.0000
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                    ROW SET (########)              COLUMN SET (########)

LISTING BY COLUMNS

VARIABLES

LL7       COLYR     000000**0000000000000000
          RANGE       100.0000  UPBND      1000.0000
          OBJ3         -0.5261  CA7           1.0000  D7         -1.0000

LL8       COLYR     0000000*0000000000000000
          RANGE       100.0000  UPBND      1000.0000
          OBJ3         -0.5097  OBJ5         -1.0000  CBR         1.0000  DR         -1.0000

BLTA1     TP1           0.5000  TA1           1.0000  TA2        -1.0000

BLTA2     COLYR     0*******0000000000000000
          TP2           0.5000  TA2           1.0000  TA3        -1.0000

BLTA3     COLYR     00******0000000000000000
          TP3           0.5000  TA3           1.0000  TA4        -1.0000

BLTA4     COLYR     000*****0000000000000000
          TP4           0.5000  TA4           1.0000  TA5        -1.0000

BLTA5     COLYR     0000****0000000000000000
          TP5           0.5000  TA5           1.0000  TA6        -1.0000

BLTA6     COLYR     00000***0000000000000000
          TP6           0.5000  TA6           1.0000  TA7        -1.0000

BLTA7     COLYR     000000**0000000000000000
          TP7           0.5000  TA7           1.0000  TA8        -1.0000

BLTA8     COLYR     0000000*0000000000000000
          TP8           0.5000  TA8           1.0000

PRO1Y1    UPBND         1.0000
          OBJ1       -122.5000  OBJ2       -122.5000  OBJ3      -122.5000  OBJ6      -110.0000
          TS3         500.0000  TS4         800.0000  TS5       1000.0000  TS6      1200.0000
          TS7        1200.0000  TS8        1100.0000  EA3        100.0000  EA4       184.0000
          EA5         240.0000  EA6         360.0000  EA7        300.0000  EA8       253.0000
          BL1         100.0000  BL2          50.0000  BL3         50.0000  PE2        80.0000
          PE3          70.0000  CA3         200.0000  CA4        359.0000  CA5       392.0000
          CA6         429.0000  CA7         424.0000  CA8        404.0000  CL3        40.0000
          CL4          67.0000  CL5          71.0000  CL6         85.0000  CL7        84.0000
          CL8          79.0000  COBJ        -44.4290

PRO4Y1    UPBND         1.0000
          OBJ1        -60.0000  OBJ2        -60.0000  OBJ3       -60.0000  OBJ6       -54.0000
          TS1         300.0000  TS2         760.0000  TS3        980.0000  TS4       910.0000
          TS5         830.0000  TS6         760.0000  TS7        710.0000  TS8       690.0000
          EA1          27.0000  EA2         130.0000  EA3        226.0000  EA4       200.0000
          EA5         174.0000  EA6         152.0000  EA7        134.0000  EA8       124.0000
          BL1          75.0000  BL2          25.0000  PE1        250.0000  PE2       130.0000
          CA1          97.0000  CA2         139.0000  CA3        317.0000  CA4       305.0000
          CA5         278.0000  CA6         244.0000  CA7        232.0000  CA8       224.0000
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ROW SET (********) COLUMN SET (********)

LISTING BY COLUMNS

VARIABLES

	CL1	45.0000	CL2	108.0000	CL3	123.0000	CL4	118.0000
	CL5	109.0000	CL6	110.0000	CL7	107.0000	CL8	88.0000
	COBJ	-21.8104						
PR12Y1	UPBND	1.0000						
	OBJ1	-91.2000	OBJ2	-91.2000	OBJ3	-91.2000	OBJ6	-82.000
	TS1	120.0000	TS2	390.0000	TS3	580.0000	TS4	760.0000
	TS5	1000.0000	TS6	1010.0000	TS7	1100.0000	TS8	950.0000
	EA1	4.0000	EA2	36.0000	EA3	108.0000	EA4	160.0000
	EA5	230.0000	EA6	222.0000	EA7	232.0000	EA8	190.0000
	BL1	190.0000	PE1	50.0000	PE2	80.0000	PE3	70.0000
	PE4	50.0000	PE5	30.0000	PE6	10.0000	PE7	10.0000
	CA1	41.0000	CA2	134.0000	CA3	220.0000	CA4	341.0000
	CA5	486.0000	CA6	438.0000	CA7	354.0000	CA8	283.0000
	CL1	17.0000	CL2	53.0000	CL3	124.0000	CL4	168.0000
	CL5	200.0000	CL6	198.0000	CL7	185.0000	CL8	193.0000
	COBJ	-33.1198						
PR13Y1	UPBND	1.0000						
	OBJ1	-5.0000	OBJ2	-5.0000	OBJ3	-5.0000	TS1	600.0000
	TS2	940.0000	TS3	1560.0000	TS4	1100.0000	TS5	480.0000
	TS6	660.0000	TS7	210.0000	TS8	90.0000	EA1	72.0000
	EA2	160.0000	EA3	468.0000	EA4	247.0000	EA5	48.0000
	EA6	112.0000	EA7	36.0000	EA8	9.0000	BL1	250.0000
	PE1	140.0000	PE2	120.0000	PE3	70.0000	CA1	174.0000
	CA2	286.0000	CA3	518.0000	CA4	451.0000	CA5	164.0000
	CA6	238.0000	CA7	61.0000	CA8	39.0000	CL1	60.0000
	CL2	102.0000	CL3	180.0000	CL4	121.0000	CL5	85.0000
	CL6	103.0000	CL7	25.0000	CL8	10.0000		
PR16Y1	UPBND	1.0000						
	OBJ1	-85.7000	OBJ2	-85.7000	OBJ3	-85.7000	OBJ6	-77.3000
	TS1	200.0000	TS2	600.0000	TS3	1000.0000	TS4	1200.0000
	TS5	1200.0000	TS6	1200.0000	TS7	1200.0000	TS8	1000.0000
	EA2	42.0000	EA3	160.0000	EA4	240.0000	EA5	264.0000
	EA6	264.0000	EA7	240.0000	EA8	180.0000	BL1	160.0000
	BL2	60.0000	PE1	100.0000	PE2	100.0000	PE3	50.0000
	CA1	65.0000	CA2	215.0000	CA3	365.0000	CA4	446.0000
	CA5	460.0000	CA6	464.0000	CA7	451.0000	CA8	400.0000
	CL1	20.0000	CL2	70.0000	CL3	100.0000	CL4	140.0000
	CL5	160.0000	CL6	160.0000	CL7	150.0000	CL8	140.0000
	COBJ	-31.2215						
PR22Y1	UPBND	1.0000						
	OBJ1	-50.5000	OBJ2	-50.5000	OBJ3	-50.5000	OBJ6	-45.4000
	TS1	500.0000	TS2	500.0000	TS3	500.0000	TS4	500.0000
	TS5	500.0000	TS6	500.0000	TS7	500.0000	TS8	500.0000
	EA1	100.0000	EA2	100.0000	EA3	110.0000	EA4	110.0000
	EA5	105.0000	EA6	100.0000	EA7	110.0000	EA8	105.0000
	BL1	100.0000	PE1	40.0000	PE2	100.0000	PE3	150.0000

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                    ROW SET (########)          COLUMN SET (########)

     LISTING BY COLUMNS

     VARIABLES

               PE4         40.0000  PE6         40.0000  CA1        150.0000  CA2        154.0000
               CA3        163.0000  CA4        169.0000  CA5        178.0000  CA6        176.0000
               CA7        173.0000  CA8        166.0000  CL1         69.0000  CL2         73.0000
               CL3         69.0000  CL4         67.0000  CL5         68.0000  CL6         68.0000
               CL7         66.0000  CL8         61.0000  COBJ       -18.3371

     PR23Y1    UPBND        1.0000
               OBJ1       -48.7000  OBJ2       -48.7000  OBJ3       -48.7000  OBJ6       -43.8000
               TS1        700.0000  TS2        750.0000  TS3        780.0000  TS4        800.0000
               TS5        810.0000  TS6        800.0000  TS7        770.0000  TS8        750.0000
               FA1        160.0000  EA2        165.0000  EA3        156.0000  EA4        152.0000
               FA5        137.0000  EA6        128.0000  EA7        108.0000  EA8        105.0000
               BL1        125.0000  BL2         65.0000  PE1        250.0000  PE2        100.0000
               CA1        210.0000  CA2        244.0000  CA3        284.0000  CA4        310.0000
               CA5        318.0000  CA6        313.0000  CA7        282.0000  CA8        257.0000
               CL1        100.0000  CL2        100.0000  CL3        112.0000  CL4        116.0000
               CL5        121.0000  CL6        121.0000  CL7        120.0000  CL8        100.0000
               COBJ       -17.6908

     PRO3Y2    COLYR     0*******000000000000000
               UPBND        1.0000
               OBJ1        48.8000  OBJ2        46.2000  OBJ3        51.2000  OBJ6        45.5000
               TS2        410.0000  TS3        620.0000  TS4       1800.0000  TS5       1680.0000
               TS6       1740.0000  TS7       1520.0000  TS8       1310.0000  EA2         20.0000
               EA3         62.0000  EA4        270.0000  EA5        324.0000  EA6        312.0000
               EA7        274.0000  EA8        238.0000  BL2        200.0000  PE2        100.0000
               PE3         80.0000  CA2        149.0000  CA3        257.0000  CA4        714.0000
               CA5        692.0000  CA6        718.0000  CA7        629.0000  CA8        607.0000
               CL2         40.0000  CL3        103.0000  CL4        285.0000  CL5        200.0000
               CL6        185.0000  CL7        176.0000  CL8        153.0000  COBJ        18.3774

     PRO4Y2    COLYR     0*******000000000000000
               UPBND        1.0000
               OBJ1        -7.6000  OBJ2        -9.7000  OBJ3        -5.6000  OBJ6        -5.3000
               TS2        300.0000  TS3        760.0000  TS4        980.0000  TS5        910.0000
               TS6        830.0000  TS7        760.0000  TS8        710.0000  EA2         27.0000
               FA3        130.0000  EA4        226.0000  EA5        200.0000  EA6        174.0000
               EA7        152.0000  EA8        134.0000  BL2         75.0000  BL3         25.0000
               PF2        250.0000  PF3        130.0000  CA2         97.0000  CA3        139.0000
               CA4        317.0000  CA5        305.0000  CA6        278.0000  CA7        244.0000
               CA8        232.0000  CL2         45.0000  CL3        108.0000  CL4        123.0000
               CL5        118.0000  CL6        109.0000  CL7        110.0000  CL8        107.0000
               COBJ        -2.1407

     PRO5Y2    COLYR     0*******000000000000000
               UPBND        1.0000
               OBJ1        41.6000  OBJ2        38.3000  OBJ3        44.7000  OBJ6        39.5000
               TS2        510.0000  TS3        830.0000  TS4       1250.0000  TS5       1330.0000
               TS6       1350.0000  TS7       1310.0000  TS8       1280.0000  EA2         54.0000
               FA3        116.0000  EA4        224.0000  EA5        246.0000  EA6        270.0000
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ROW SET (########) COLUMN SET (########)

LISTING BY COLUMNS

VARIABLES

	EA7	250.0000	EA8	230.0000	BL2	145.0000	PE2	180.0000
	PE3	150.0000	PE4	90.0000	CA2	128.0000	CA3	362.0000
	CA4	535.0000	CA5	595.0000	CA6	591.0000	CA7	550.0000
	CA8	498.0000	CL2	38.0000	CL3	90.0000	CL4	124.0000
	CL5	130.0000	CL6	141.0000	CL7	135.0000	CL8	126.0000
	COBJ	15.9540						
PR13Y2	COLYR	0*******0000000000000000						
	UPBND	1.0000						
	OBJ1	3.4000	OBJ2	3.4000	OBJ3	3.4000	OBJ6	3.0000
	TS2	600.0000	TS3	940.0000	TS4	1560.0000	TS5	1100.0000
	TS6	480.0000	TS7	660.0000	TS8	210.0000	EA2	72.0000
	EA3	160.0000	EA4	468.0000	EA5	242.0000	EA6	48.0000
	EA7	112.0000	EA8	36.0000	BL2	250.0000	PE2	140.0000
	PE3	120.0000	PE4	70.0000	CA2	174.0000	CA3	286.0000
	CA4	518.0000	CA5	451.0000	CA6	166.0000	CA7	238.0000
	CA8	61.0000	CL2	60.0000	CL3	102.0000	CL4	180.0000
	CL5	121.0000	CL6	85.0000	CL7	103.0000	CL8	25.0000
	COBJ	1.2117						
PR14Y2	COLYR	0*******0000000000000000						
	UPBND	1.0000						
	OBJ1	-131.8000	OBJ2	-134.3000	OBJ3	129.4000	OBJ6	-117.0000
	TS2	500.0000	TS3	1000.0000	TS4	1250.0000	TS5	1500.0000
	TS6	1500.0000	TS7	1500.0000	TS8	1250.0000	EA2	50.0000
	EA3	150.0000	EA4	250.0000	EA5	300.0000	EA6	300.0000
	EA7	300.0000	EA8	250.0000	BL2	100.0000	BL3	100.0000
	PE2	50.0000	PE3	50.0000	PE4	50.0000	PE5	50.0000
	PE6	50.0000	PE7	50.0000	PE8	50.0000	CA2	120.0000
	CA3	210.0000	CA4	300.0000	CA5	390.0000	CA6	480.0000
	CA7	550.0000	CA8	630.0000	CL2	40.0000	CL3	80.0000
	CL4	120.0000	CL5	160.0000	CL6	200.0000	CL7	200.0000
	CL8	200.0000	COBJ	-47.2563				
PR21Y2	COLYR	0*******0000000000000000						
	UPBND	1.0000						
	OBJ1	65.0000	OBJ2	63.0000	OBJ3	66.7000	OBJ6	59.3000
	TS2	1200.0000	TS3	2000.0000	TS4	2000.0000	TS5	2000.0000
	TS6	1800.0000	TS7	1700.0000	TS8	1400.0000	EA2	192.0000
	EA3	360.0000	EA4	360.0000	EA5	340.0000	EA6	288.0000
	EA7	238.0000	EA8	182.0000	BL2	300.0000	PE2	250.0000
	PE3	400.0000	PE4	200.0000	CA2	372.0000	CA3	640.0000
	CA4	645.0000	CA5	702.0000	CA6	690.0000	CA7	672.0000
	CA8	585.0000	CL2	181.0000	CL3	279.0000	CL4	290.0000
	CL5	285.0000	CL6	263.0000	CL7	241.0000	CL8	219.0000
	COBJ	23.9513						
PR24Y2	COLYR	0*******0000000000000000						
	UPBND	1.0000						
	OBJ1	-23.0000	OBJ2	-27.1000	OBJ3	-19.1000	OBJ6	-18.0000

ROW SET (########) COLUMN SET (########)

LISTING BY COLUMNS

VARIABLES

	TS2	1200.0000	TS3	1700.0000	TS4	1560.0000	TS5	1490.0000
	TS6	1495.0000	TS7	1520.0000	TS8	1530.0000	EA2	156.0000
	EA3	255.0000	EA4	265.0000	EA5	209.0000	EA6	224.0000
	EA7	258.0000	EA8	275.0000	BL2	150.0000	BL4	100.0000
	PE2	250.0000	PE3	200.0000	PE4	100.0000	PE5	50.0000
	CA2	406.0000	CA3	506.0000	CA4	567.0000	CA5	513.0000
	CA6	510.0000	CA7	537.0000	CA8	555.0000	CL2	187.0000
	CL3	250.0000	CL4	270.0000	CL5	270.0000	CL6	265.0000
	CL7	255.0000	CL8	260.0000	COBJ	-7.2702		
PR02Y3	COLYR	00******0000000000000000						
	UPBND	1.0000						
	OBJ1	91.2000	OBJ2	91.8000	OBJ3	90.5000	OBJ6	81.7000
	TS3	310.0000	TS4	670.0000	TS5	700.0000	TS6	690.0000
	TS7	650.0000	TS8	620.0000	EA3	30.0000	EA4	80.0000
	EA5	105.0000	EA6	105.0000	EA7	105.0000	EA8	87.0000
	BL3	50.0000	PE3	90.0000	PE4	45.0000	CA3	123.0000
	CA4	284.0000	CA5	297.0000	CA6	294.0000	CA7	266.0000
	CA8	261.0000	CL3	37.0000	CL4	74.0000	CL5	78.0000
	CL6	78.0000	CL7	73.0000	CL8	70.0000	COBJ	32.9986
PR11Y3	COLYR	00******0000000000000000						
	UPBND	1.0000						
	OBJ1	124.7000	OBJ2	121.8000	OBJ3	127.1000	OBJ6	115.8000
	TS3	120.0000	TS4	270.0000	TS5	750.0000	TS6	1250.0000
	TS7	1300.0000	TS8	1250.0000	EA4	15.0000	EA5	135.0000
	EA6	250.0000	EA7	260.0000	EA8	250.0000	BL3	225.0000
	PE3	120.0000	PE4	100.0000	PE5	75.0000	CA3	30.0000
	CA4	75.0000	CA5	115.0000	CA6	240.0000	CA7	340.0000
	CA8	380.0000	CL3	20.0000	CL4	60.0000	CL5	95.0000
	CL6	105.0000	CL7	140.0000	CL8	170.0000	COBJ	46.7716
PR15Y3	COLYR	00******0000000000000000						
	UPBND	1.0000						
	OBJ1	119.0000	OBJ2	117.8000	OBJ3	119.8000	OBJ6	107.6000
	TS3	230.0000	TS4	680.0000	TS5	720.0000	TS6	710.0000
	TS7	730.0000	TS8	660.0000	EA3	7.0000	EA4	34.0000
	EA5	72.0000	EA6	142.0000	EA7	146.0000	EA8	123.0000
	BL3	50.0000	BL4	25.0000	PE3	60.0000	PE4	40.0000
	CA3	84.0000	CA4	188.0000	CA5	218.0000	CA6	272.0000
	CA7	292.0000	CA8	329.0000	CL3	20.0000	CL4	25.0000
	CL5	40.0000	CL6	73.0000	CL7	85.0000	CL8	102.0000
	COBJ	43.4596						
PR01Y4	COLYR	000*****0000000000000000						
	UPBND	1.0000						
	OBJ1	347.8000	OBJ2	348.9000	OBJ3	346.3000	OBJ6	312.0000
	TS6	500.0000	TS7	800.0000	TS8	1000.0000	EA6	100.0000
	EA7	184.0000	EA8	240.0000	BL4	100.0000	BL5	50.0000
	BL6	50.0000	PE5	80.0000	PE6	70.0000	CA6	200.0000

ROW SET (########) COLUMN SET (########)

LISTING BY COLUMNS

VARIABLES

	CA7	359.0000	CA8	392.0000	CL6	40.0000	CL7	67.0000
	CL8	71.0000	COBJ	126.0168				
PR05Y4	COLYR	000*****000000000000000						
	UPRND	1.0000						
	OBJ1	328.3000	OBJ2	333.4000	OBJ3	323.3000	OBJ6	292.0000
	TS4	510.0000	TS5	830.0000	TS6	1250.0000	TS7	1330.0000
	TS8	1350.0000	EA4	54.0000	EA5	116.0000	EA6	224.0000
	EA7	266.0000	EA8	270.0000	BL4	165.0000	PE4	180.0000
	PE5	150.0000	PF6	90.0000	CA4	128.0000	CA5	362.0000
	CA6	535.0000	CA7	595.0000	CA8	591.0000	CL4	38.0000
	CL5	90.0000	CL6	124.0000	CL7	130.0000	CL8	141.0000
	COBJ	117.9388						
PR11Y4	COLYR	000*****000000000000000						
	UPRND	1.0000						
	OBJ1	230.0000	OBJ2	231.6000	OBJ3	278.0000	OBJ6	206.0000
	TS4	120.0000	TS5	270.0000	TS6	750.0000	TS7	1250.0000
	TS8	1300.0000	EA5	15.0000	EA6	135.0000	EA7	250.0000
	FA8	260.0000	BL4	225.0000	PE4	120.0000	PE5	100.0000
	PE6	75.0000	CA4	30.0000	CA5	75.0000	CA6	115.0000
	CA7	240.0000	CA8	340.0000	CL4	20.0000	CL5	60.0000
	CL6	95.0000	CL7	105.0000	CL8	140.0000	COBJ	83.2034
PR12Y4	COLYR	000*****000000000000000						
	UPRND	1.0000						
	OBJ1	404.0000	OBJ2	412.8000	OBJ3	395.5000	OBJ6	357.0000
	TS4	120.0000	TS5	390.0000	TS6	580.0000	TS7	760.0000
	TS8	1000.0000	EA4	4.0000	EA5	36.0000	EA6	108.0000
	FA7	160.0000	EA8	230.0000	BL4	190.0000	PE4	50.0000
	PE5	80.0000	PF6	70.0000	PE7	50.0000	PE8	30.0000
	CA4	41.0000	CA5	134.0000	CA6	279.0000	CA7	341.0000
	CA8	486.0000	CL4	17.0000	CL5	53.0000	CL6	124.0000
	CL7	168.0000	CL8	200.0000	COBJ	144.1923		
PR13Y4	COLYR	000*****000000000000000						
	UPRND	1.0000						
	OBJ1	119.7000	OBJ2	122.9000	OBJ3	176.5000	OBJ6	106.0000
	TS4	600.0000	TS5	940.0000	TS6	1560.0000	TS7	1100.0000
	TS8	480.0000	EA4	72.0000	EA5	160.0000	EA6	468.0000
	FA7	242.0000	EA8	48.0000	BL4	250.0000	PF4	140.0000
	PE5	120.0000	PF6	70.0000	CA4	174.0000	CA5	286.0000
	CA6	518.0000	CA7	451.0000	CA8	166.0000	CL4	60.0000
	CL5	102.0000	CL6	180.0000	CL7	121.0000	CL8	85.0000
	COBJ	42.8134						
PR14Y4	COLYR	000*****000000000000000						
	UPRND	1.0000						
	OBJ1	-54.9000	OBJ2	-66.2000	OBJ3	-45.5000	OBJ6	-42.3000
	TS4	500.0000	TS5	1000.0000	TS6	1250.0000	TS7	1500.0000

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                         ROW SET (########)          COLUMN SET (########)

LISTING BY COLUMNS

VARIABLES

          TS8      1500.0000   EA4        50.0000   EA5       150.0000   EA6       250.0000
          FA7       300.0000   EA8       300.0000   BL4       100.0000   BL5       100.0000
          PE4        50.0000   PF5        50.0000   PE6        50.0000   PE7        50.0000
          PE8        50.0000   CA4       120.0000   CA5       210.0000   CA6       300.0000
          CA7       390.0000   CA8       480.0000   CL4        40.0000   CL5        80.0000
          CL6       120.0000   CL7       160.0000   CL8       200.0000   COBJ      -17.0850

 PR22Y4   COLYR     000*****000000000000000
          UPBND       1.0000
          OBJ1       94.5000   OBJ2       96.3000   OBJ3       92.7000   OBJ6       83.6000
          TS4       500.0000   TS5       500.0000   TS6       500.0000   TS7       500.0000
          TS8       500.0000   EA4       100.0000   EA5       100.0000   EA6       110.0000
          FA7       110.0000   EA8       105.0000   BL4       100.0000   PE4        40.0000
          PE5       100.0000   PF6       150.0000   PE7        40.0000   CA4       150.0000
          CA5       154.0000   CA6       163.0000   CA7       169.0000   CA8       178.0000
          CL4        69.0000   CL5        73.0000   CL6        69.0000   CL7        67.0000
          CL8        68.0000   COBJ       33.7660

 PR25Y4   COLYR     000*****000000000000000
          UPBND       1.0000
          OBJ1      143.7000   OBJ2      144.0000   OBJ3      143.2000   OBJ6      128.0000
          TS4      1000.0000   TS5      1200.0000   TS6      1250.0000   TS7      1260.0000
          TS8      1290.0000   EA4       140.0000   EA5       220.0000   EA6       230.0000
          EA7       240.0000   EA8       220.0000   BL4       175.0000   BL5        25.0000
          BL7        50.0000   PE4       180.0000   PE5       150.0000   PE7       150.0000
          CA4       367.0000   CA5       419.0000   CA6       455.0000   CA7       451.0000
          CA8       446.0000   CL4       162.0000   CL5       190.0000   CL6       200.0000
          CL7       196.0000   CL8       198.0000   COBJ       51.6992

 PR02Y5   COLYR     0000****000000000000000
          UPBND       1.0000
          OBJ1      189.4000   OBJ2      197.5000   OBJ3      201.0000   OBJ6      165.0000
          TS5       310.0000   TS6       670.0000   TS7       700.0000   TS8       690.0000
          FA5        30.0000   EA6        80.0000   EA7       105.0000   EA8       105.0000
          BL5        50.0000   PF5        90.0000   PE6        45.0000   CA5       123.0000
          CA6       280.0000   CA7       297.0000   CA8       294.0000   CL5        37.0000
          CL6        74.0000   CL7        78.0000   CL8        78.0000   COBJ       66.6435

 PR03Y5   COLYR     0000****000000000000000
          UPBND       1.0000
          OBJ1      427.9000   OBJ2      444.5000   OBJ3      412.7000   OBJ6      374.0000
          TS5       410.0000   TS6       620.0000   TS7      1800.0000   TS8      1680.0000
          FA5        20.0000   EA6        62.0000   EA7       270.0000   EA8       324.0000
          BL5       200.0000   PF5       100.0000   PE6        80.0000   CA5       149.0000
          CA6       257.0000   CA7       714.0000   CA8       692.0000   CL5        60.0000
          CL6       103.0000   CL7       285.0000   CL8       200.0000   COBJ      151.0586

 PR11Y5   COLYR     0000****000000000000000
          UPBND       1.0000
          OBJ1      287.4000   OBJ2      297.2000   OBJ3      278.1000   OBJ6      252.0000
```

ROW SET (########) COLUMN SET (########)

LISTING BY COLUMNS

VARIABLES

	TS5	120.0000	TS6	270.0000	TS7	750.0000	TS8	1250.0000
	EA6	15.0000	EA7	135.0000	EA8	250.0000	BL5	225.0000
	PE5	120.0000	PE6	100.0000	PE7	75.0000	CA5	30.0000
	CA6	75.0000	CA7	115.0000	CA8	240.0000	CL5	20.0000
	CL6	60.0000	CL7	95.0000	CL8	105.0000	COBJ	101.7828
PR21Y5	COLYR	0000***00000000000000000						
	UPBND	1.0000						
	OBJ1	309.3000	OBJ2	386.2000	OBJ3	354.0000	OBJ6	322.0000
	TS5	1200.0000	TS6	2000.0000	TS7	2000.0000	TS8	2000.0000
	EA5	192.0000	EA6	360.0000	EA7	360.0000	EA8	360.0000
	BL5	300.0000	PE5	250.0000	PE6	400.0000	PE7	200.0000
	CA5	372.0000	CA6	640.0000	CA7	645.0000	CA8	702.0000
	CL5	181.0000	CL6	279.0000	CL7	290.0000	CL8	285.0000
	COBJ	130.0558						
PR23Y5	COLYR	0000***00000000000000000						
	UPBND	1.0000						
	OBJ1	203.4000	OBJ2	210.9000	OBJ3	196.4000	OBJ6	178.0000
	TS5	700.0000	TS6	750.0000	TS7	780.0000	TS8	800.0000
	EA5	140.0000	EA6	165.0000	EA7	156.0000	EA8	152.0000
	BL5	125.0000	BL6	65.0000	PE5	250.0000	PE6	100.0000
	CA5	210.0000	CA6	244.0000	CA7	284.0000	CA8	310.0000
	CL5	100.0000	CL6	100.0000	CL7	112.0000	CL8	116.0000
	COBJ	71.8942						
PR04Y6	COLYR	00000***000000000000000000						
	UPBND	1.0000						
	OBJ1	319.7000	OBJ2	333.7000	OBJ3	306.8000	OBJ6	279.0000
	TS6	300.0000	TS7	760.0000	TS8	980.0000	EA6	27.0000
	EA7	130.0000	EA8	226.0000	BL6	75.0000	BL7	25.0000
	PE6	250.0000	PE7	130.0000	CA6	97.0000	CA7	139.0000
	CA8	317.0000	CL6	45.0000	CL7	108.0000	CL8	123.0000
	COBJ	112.6881						
PR05Y6	COLYR	00000***000000000000000000						
	UPBND	1.0000						
	OBJ1	572.8000	OBJ2	605.1000	OBJ3	543.9000	OBJ6	495.0000
	TS6	510.0000	TS7	830.0000	TS8	1250.0000	EA6	54.0000
	EA7	116.0000	EA8	224.0000	BL6	145.0000	PE6	180.0000
	PE7	150.0000	PE8	90.0000	CA6	128.0000	CA7	362.0000
	CA8	535.0000	CL6	38.0000	CL7	90.0000	CL8	124.0000
	COBJ	199.9305						
PR11Y6	COLYR	00000***000000000000000000						
	UPBND	1.0000						
	OBJ1	375.6000	OBJ2	395.3000	OBJ3	357.8000	OBJ6	325.0000
	TS6	120.0000	TS7	270.0000	TS8	750.0000	EA7	15.0000
	EA8	135.0000	BL6	225.0000	PE6	120.0000	PE7	100.0000
	PE8	75.0000	CA6	30.0000	CA7	75.0000	CA8	115.0000

ROW SET (########)   COLUMN SET (########)

LISTING BY COLUMNS

VARIABLES

	CL6	20.0000	CL7	60.0000	CL8	95.0000	COBJ	131.2675
PR14Y6	COLYR	00000***0000000000000000						
	UPBND	1.0000						
	OBJ1	130.1000	OBJ2	118.8000	OBJ3	138.6000	OBJ6	122.0000
	TS6	500.0000	TS7	1000.0000	TS8	1250.0000	EA6	50.0000
	EA7	150.0000	EA8	250.0000	BL6	100.0000	BL7	100.0000
	PE6	50.0000	PE7	50.0000	PE8	50.0000	CA6	120.0000
	CA7	210.0000	CA8	300.0000	CL6	40.0000	CL7	80.0000
	CL8	120.0000	COBJ	49.2758				
PR15Y6	COLYR	00000***0000000000000000						
	UPBND	1.0000						
	OBJ1	271.4000	OBJ2	286.2000	OBJ3	258.1000	OBJ6	236.0000
	TS6	230.0000	TS7	680.0000	TS8	720.0000	EA6	7.0000
	EA7	34.0000	EA8	72.0000	BL6	50.0000	BL7	25.0000
	PE6	60.0000	PE7	40.0000	CA6	84.0000	CA7	188.0000
	CA8	218.0000	CL6	20.0000	CL7	25.0000	CL8	40.0000
	COBJ	95.3204						
PR16Y6	COLYR	00000***0000000000000000						
	UPBND	1.0000						
	OBJ1	515.6000	OBJ2	541.5000	OBJ3	492.2000	OBJ6	448.0000
	TS6	200.0000	TS7	600.0000	TS8	1000.0000	EA7	42.0000
	EA8	160.0000	BL6	160.0000	BL7	60.0000	PE6	100.0000
	PE7	100.0000	PE8	50.0000	CA6	65.0000	CA7	215.0000
	CA8	365.0000	CL6	20.0000	CL7	70.0000	CL8	100.0000
	COBJ	180.9472						
PR21Y6	COLYR	00000***0000000000000000.						
	UPBND	1.0000						
	OBJ1	546.0000	OBJ2	575.4000	OBJ3	520.1000	OBJ6	473.0000
	TS6	1200.0000	TS7	2000.0000	TS8	2000.0000	EA6	192.0000
	EA7	360.0000	EA8	360.0000	BL6	300.0000	PE6	250.0000
	PE7	400.0000	PE8	200.0000	CA6	372.0000	CA7	640.0000
	CA8	645.0000	CL6	181.0000	CL7	279.0000	CL8	290.0000
	COBJ	191.0447						
PR23Y6	COLYR	00000***0000000000000000						
	UPBND	1.0000						
	OBJ1	244.1000	OBJ2	258.6000	OBJ3	231.1000	OBJ6	211.0000
	TS6	700.0000	TS7	750.0000	TS8	780.0000	EA6	140.0000
	EA7	165.0000	EA8	156.0000	BL6	125.0000	BL7	65.0000
	PE6	250.0000	PE7	100.0000	CA6	210.0000	CA7	244.0000
	CA8	284.0000	CL6	100.0000	CL7	100.0000	CL8	112.0000
	COBJ	85.2229						
PR01Y7	COLYR	000000**0000000000000000						
	UPBND	1.0000						
	OBJ1	301.1000	OBJ2	345.1000	OBJ3	262.5000	OBJ6	244.0000

ROW SET (########) COLUMN SET (########)

LISTING BY COLUMNS

VARIABLES

	RL7	100.0000	BL8	50.0000	PE8	80.0000	COBJ	98.5516
PR04Y7	COLYR	000000**000000000000000000							
	UPBND	1.0000							
	OBJ1	360.0000	OBJ2	390.9000	OBJ3	343.9000	OBJ6	315.0000
	TS7	300.0000	TS8	760.0000	EA7	27.0000	EA8	130.0000
	RL7	75.0000	BL8	25.0000	PE7	250.0000	PE8	130.0000
	CA7	97.0000	CA8	139.0000	CL7	45.0000	CL8	108.0000
	COBJ	127.2285						
PR14Y7	COLYR	000000**000000000000000000						
	UPBND	1.0000						
	OBJ1	246.4000	OBJ2	238.2000	OBJ3	247.7000	OBJ6	222.0000
	TS7	500.0000	TS8	1000.0000	EA7	50.0000	EA8	150.0000
	RL7	100.0000	BL8	100.0000	PE7	50.0000	PE8	50.0000
	CA7	120.0000	CA8	210.0000	CL7	40.0000	CL8	80.0000
	COBJ	89.6658						
PR22Y7	COLYR	000000**000000000000000000						
	UPBND	1.0000						
	OBJ1	156.7000	OBJ2	174.5000	OBJ3	141.0000	OBJ6	111.0000
	TS7	500.0000	TS8	500.0000	EA7	100.0000	EA8	100.0000
	RL7	100.0000	PE7	40.0000	PE8	100.0000	CA7	150.0000
	CA8	154.0000	CL7	69.0000	CL8	73.0000	COBJ	44.8329
PR02Y8	COLYR	0000000*000000000000000000						
	UPBND	1.0000						
	OBJ1	224.0000	OBJ2	256.2000	OBJ3	197.6000	OBJ6	184.0000
	TS6	310.0000	EA8	30.0000	BL8	50.0000	PE8	90.0000
	CA8	125.0000	CL8	37.0000	COBJ	74.3176		
PR15Y8	COLYR	0000000*000000000000000000						
	UPBND	1.0000						
	OBJ1	201.7000	OBJ2	236.1000	OBJ3	172.6000	OBJ6	162.0000
	TS8	230.0000	EA8	7.0000	BL8	50.0000	PE8	60.0000
	CA8	84.0000	CL8	20.0000	COBJ	65.4318		
PR22Y8	COLYR	0000000*000000000000000000						
	UPBND	1.0000						
	OBJ1	137.1000	OBJ2	159.8000	OBJ3	117.9000	OBJ6	110.0000
	TS8	500.0000	EA8	100.0000	BL8	100.0000	PE8	40.0000
	CA8	150.0000	CL8	69.0000	COBJ	44.4290		
PR25Y8	COLYR	0000000*000000000000000000						
	UPBND	1.0000						
	OBJ1	431.8000	OBJ2	469.4000	OBJ3	513.4000	OBJ6	366.0000
	TS8	1000.0000	EA8	140.0000	BL8	125.0000	PE8	180.0000
	CA8	367.0000	CL8	162.0000	COBJ	147.8274		

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ROW SET (********) COLUMN SET (********)

LISTING BY COLUMNS

RIGHT HAND SIDES

RHS2	EA1	-1953.0000	EA2	-1850.0000	EA3	-1600.0000	EA4	-1560.0000
	EA5	-1240.0000	EA6	-1220.0000	EA7	-1200.0000	EA8	-1000.0000
RHS3	CA1	-3510.0000	CA2	-3588.0000	CA3	-3705.0000	CA4	-3432.0000
	CA5	-3120.0000	CA6	-2925.0000	CA7	-2925.0000	CA8	-2730.0000
	CL1	-1120.0000	CL2	-1145.0000	CL3	-1170.0000	CL4	-1095.0000
	CL5	-1020.0000	CL6	-946.0000	CL7	-948.0000	CL8	-902.0000
RHS4	FQ1	250.0000						
RHS2	EA1	-1953.0000	EA2	-1850.0000	FA3	-1600.0000	FA4	-1560.0000
	EA5	-1240.0000	EA6	-1220.0000	EA7	-1200.0000	EA8	-1000.0000
RHS3	CA1	-3510.0000	CA2	-3588.0000	CA3	-3705.0000	CA4	-3432.0000
	CA5	-3120.0000	CA6	-2925.0000	CA7	-2925.0000	CA8	-2730.0000
	CL1	-1120.0000	CL2	-1145.0000	CL3	-1170.0000	CL4	-1095.0000
	CL5	-1020.0000	CL6	-946.0000	CL7	-948.0000	CL8	-902.0000
RHS4	EQ1	250.0000						
RHS2	EA1	-1953.0000	EA2	-1850.0000	FA3	-1600.0000	EA4	-1560.0000
	EA5	-1240.0000	EA6	-1220.0000	FA7	-1200.0000	EA8	-1000.0000
RHS3	CA1	-3510.0000	CA2	-3588.0000	CA3	-3705.0000	CA4	-3432.0000
	CA5	-3120.0000	CA6	-2925.0000	CA7	-2925.0000	CA8	-2730.0000
	CL1	-1120.0000	CL2	-1145.0000	CL3	-1170.0000	CL4	-1095.0000
	CL5	-1020.0000	CL6	-946.0000	CL7	-948.0000	CL8	-902.0000
RHS4	FQ1	250.0000						
RHS1	TS1	-11000.0000	TS2	-10000.0000	TS3	-9500.0000	TS4	-8800.0000
	TS5	-8000.0000	TS6	-7500.0000	TS7	-7500.0000	TS8	-7000.0000
	FA1	-1953.0000	EA2	-1850.0000	EA3	-1600.0000	FA4	-1560.0000
	EA5	-1240.0000	EA6	-1220.0000	EA7	-1200.0000	EA8	-1000.0000
	BL1	-2034.0000	BL2	-600.0000	BL3	-500.0000	BL4	-200.0000
	PE1	-1481.0000	PE2	-500.0000	PE3	-400.0000	PE4	-600.0000
	PE5	-400.0000	PE6	-400.0000	PE7	-400.0000	PE8	-400.0000
	CA1	-3510.0000	CA2	-3588.0000	CA3	-3705.0000	CA4	-3432.0000
	CA5	-3120.0000	CA6	-2925.0000	CA7	-2925.0000	CA8	-2730.0000
	CL1	-1120.0000	CL2	-1145.0000	CL3	-1170.0000	CL4	-1095.0000
	CL5	-1020.0000	CL6	-946.0000	CL7	-948.0000	CL8	-902.0000
	CB1	-4580.0000	CA5	1600.0000	TP1	702.0000	TP2	-65.0000
	TP3	-65.0000	TP4	-60.0000	TP5	-60.0000	TP6	-60.0000
	TP7	-60.0000	TP8	-55.0000	TA1	-65.0000	PR1	-60.0000
	EQ1	2000.0000	D1	1500.0000	D3	-1000.0000		

APPENDIX VIII THE REPORT WRITER-SINGLE CRITERION MODEL

```
12/25/42          27/10/75          COMPILED BY XALV MK. 3A
LINE STATEMENT
   0   'TRACE'1
   0   'BEGIN'
   1   'INTEGER' I,J,K,M;
   1   'REAL'RES,RI,PL,RS;
   2
   2     'PROCEDURE' VARRTEXT(A,N);  'VALUE' N;
   5       'ARRAY' A;  'INTEGER' N;  'EXTERNAL';
   7     'INTEGER''PROCEDURE' INSTRARR(S,A);
   9       'STRING' S;  'ARRAY' A;  'EXTERNAL';
  11     'BOOLEAN''PROCEDURE' TEST(N);  'VALUE' N;  'INTEGER' N;  'EXTERNAL';
  15     'PROCEDURE' READTRAP(P);  'PROCEDURE' P;  'EXTERNAL';
  15
  18     'PROCEDURE' READERR(N);  'INTEGER' N;
  21     'BEGIN'
  21       'INTEGER' J;
  21       NEWLINE(1);
  23       WRITETEXT('('***%READ%FAIL%JUST%BEFORE%/')');
  24       'FOR' J:= 1 'STEP' 1 'UNTIL' 360 'DO' PRINTCH(READCH);
  26       PAUSE(99);
  27     'END' READERR;
  27
  27     'PROCEDURE' INTILL(STR);  'STRING' STR;
  30     'COMMENT' SKIPS OVER CURRENT INPUT STREAM UNTIL CHARACTER IMMEDIATELY
  30     FOLLOWING STRING STR, SWITCH 1 ON GIVES DIAGNOSTIC PRINT OF FIRST 120
  30     CHARACTERS SKIPPED;
  30     'BEGIN'
  30       'INTEGER''ARRAY' BUF[1:30];  'INTEGER' COUNT;
  31       COUNT:= INSTRARR(STR,BUF);
  33       'IF' TEST(1) 'THEN'
  33       'BEGIN'
  33         NEWLINE(1);  WRITETEXT('('***%INTILL%%')');  PRINT(COUNT,8,0);
  37         NEWLINE(1);  VARRTEXT(BUF,COUNT);
  39       'END';
  40     'END' INTILL;
  40
  40     'PROCEDURE' DATAIN(VAR,STR1,STR2);  'REAL' VAR;  'STRING' STR1,STR2;
  44     'COMMENT' SEARCHES IN TURN FOR STRINGS STR1, STR2 AND THEN AT LEAST
  44     ONE SPACE BEFORE READING A REAL VARIABLE VAR, SWITCH 2 ON GIVES
  44     DIAGNOSTIC PRINT OF VALUE READ;
  44     'BEGIN'
  44       INTILL(STR1);  INTILL(STR2);
  47   LOOP:
  47       'IF' NEXTCH'NE'CODE('('%')') 'THEN'
  47         'BEGIN'  SKIPCH;  'GOTO' LOOP;  'END';
  51       VAR:= READ;
  52       'IF' TEST(2) 'THEN'
  52       'BEGIN'
  52         NEWLINE(1);  WRITETEXT('('***%DATAIN%%')');  PRINT(VAR,0,8);
  56       'END';
  57     'END' DATAIN;
  57
  57
  57     'PROCEDURE' ARRAYIN(ARR,SIZE,STR1,STR2);  'VALUE' SIZE;
  60       'REAL''ARRAY' ARR;  'INTEGER' SIZE;  'STRING' STR1,STR2;
  63     'BEGIN'
  63       'INTEGER' J;
  63       'FOR' J:= 1 'STEP' 1 'UNTIL' SIZE 'DO' DATAIN(ARR[J],STR1,STR2);
  66     'END' ARRAYIN;
  66
  66     'PROCEDURE' INPUT(ARR,SIZE,STR);  'VALUE' SIZE;
  69       'REAL''ARRAY' ARR;  'INTEGER' SIZE;  'STRING' STR;
  72     'BEGIN'
  72       ARRAYIN(ARR,SIZE,STR,'('*')');
  74     'END' INPUT;
  74
```

```
'PROCEDURE'NEXTDUMP;
    'BEGIN' 'INTEGER'  N;INTILL('('DUMP;DUMP')');
    N:=READ;
    'IF' N 'EQ'M'THEN' NEXTDUMP 'ELSE' M:=N;
    'END';
'PROCEDURE' OUTPUT(A);
'REAL''ARRAY'A;
'BEGIN'
      COPYTEXT('('*')');
      'FOR'I:=1'STEP'1'UNTIL'8 'DO'PRINT(((A[I]+0.5)*10)'/'10,7,0);
'END';
'PROCEDURE' PERIOD;
'BEGIN'
      NEWLINE(1);
  WRITETEXT('(''('1C 34S')'PERIOD-1%%PERIOD-2%%PERIOD-3%%PERIOD-4')');
      WRITETEXT('(''('2S')'PERIOD-5%%PERIOD-6%%PERIOD-7%%PERIOD-8')');
      NEWLINE(1);
'END';
'PROCEDURE'ACHIEVEMENT(A,B);
'REAL''ARRAY'A,B;
'BEGIN'
      COPYTEXT('('*')');
      'FOR'I:=1'STEP'1'UNTIL'8'DO'PRINT(A[I],5,1);
      NEWLINE(1);
      COPYTEXT('('*')');
      'FOR' I:=1'STEP'1'UNTIL'8'DO'PRINT(B[I],5,1);
      NEWLINE(1);
      COPYTEXT('('*')');
      'FOR'I:=1'STEP'1'UNTIL' 8'DO'PRINT((B[I]-A[I])/A[I]*100,5,1);
'END';
'BEGIN'
'REAL''ARRAY'SALES,EARN,NPAT,RG,LL,CB,ROCE,LOPY,ECOV,ERPS,DCOV,DTARG,
PTARG ,STARG,A2,A3,A4,A5,A6,A7,A8[1:8]
,AA,A0,TAX,FABL,FAPE,CURA,CURL,OVDR,MARK,DV,NUM,DE,DLTA,A1[0:8];

   SELECTINPUT(1);
   READTRAP(READERR);
INTILL('('LISTING%BY%COLUMN')');
INTILL('('VARIABLES')');
ARRAYIN(ROCE,8,'('FABL')','('ROCE')');
ARRAYIN(LOPY,8,'('CURL')','('LOPY')');
DATAIN(RS,'('OVDR1')','('EA2')');RS:=-RS;
DATAIN(R1,'('MARK1')','('EA2')');
ARRAYIN(DCOV,8,'('DV')','('DCOV')');
'FOR' J:=1 'STEP' 1 'UNTIL' 8 'DO'
    'BEGIN'
DATAIN(ERPS[J],'('NUM')','('ERPS')');
INTILL('('DTARG')');
SKIPCH;
DTARG[J]:=READ;
    'END';
DATAIN(RL,'('DE1')','('CR2')');RL:=-RL;
INTILL('('ECOV1')');
ECOV[1]:=READ;
'FOR'J:=2'STEP'1'UNTIL'8 'DO' DATAIN(ECOV[J],'('DE')','('ECOV')');
'FOR' I:=1 'STEP' 1 'UNTIL' 8 'DO' 'BEGIN'
                                   DTARG[I]:=DTARG[I]*(-100.0);
                                   ERPS[I]:=100.0*ERPS[I];
                                   ROCE[I]:=ROCE[I]*100.0;
                                 ECOV[I]:=ECOV[I]/RL;
                                        'END';
 INTILL('('SOLUTION')');
 INTILL('('DUMP')');
 INTILL('('DUMP')');
 N:=READ;PAPERTHROW;
```

```
START:
INPUT(SALES,M,'('SALES')');
INPUT(EARN,M,'('EARN')');
INPUT(NPAT,M,'('NPAT')');
INPUT(TAX,M,'('TAX')');
INPUT(FABL,M,'('FABL')');
INPUT(FAPE,M,'('FAPE')');
INPUT(CURA,M,'('CURA')');
INPUT(CURL,M,'('CURL')');
INPUT(OVDR,M,'('OVDR')');
INPUT(MARK,M,'('MARK')');
INPUT(DV,M,'('DV')');
INPUT(NUM,B,'('NUM')');
INPUT(RG,M,'('RG')');
INPUT(DE,M,'('DE')');
INPUT(LL,M,'('BLTA')');
INTILL('('ROWXINFORMATION')');
INTILL('('PROBLEM')');
COPYTEXT('('ROWXINFORMATION')');
  SELECTINPUT(2);  'COMMENT' GEO3 SHOULD PROVIDE *CRO;

NUM[0]:=READ;
RES:=READ;
DE[0]:=READ;
FABL[0]:=READ;FAPE[0]:=READ;
MARK[0]:=READ;CURA[0]:=READ;
TAX[0]:=READ;OVDR[0]:=READ;DV[0]:=READ;CURL[0]:=READ;

'COMMENT'     BALANCE SHEET;

COPYTEXT('('*')');            ;PERIOD;
COPYTEXT('('*')');
OUTPUT(NUM);A1[0]:=RES;
A0[0]:=0.0;AA[0]:=0.0;
'FOR'I:=1'STEP'1'UNTIL' 8 'DO''BEGIN'
                              A1[I]:=A1[I-1]+NPAT[I]-DV[I];
                              A0[I]:=A0[I-1]+NPAT[I]-TAX[I];
                              AA[I]:=0.6*RG[I]+AA[I-1];
                              A2[I]:=A1[I]+NUM[I]+DE[I]+AA[I]+A0[I];
                              A3[I]:=CURA[I]-MARK[I];
                              A4[I]:=CURL[I]-TAX[I]-OVDR[I]-DV[I];
                              A5[I]:=CURA[I]-CURL[I];
                              A6[I]:=FABL[I]+FAPE[I]+CURA[I]-CURL[I];
                             'END';
OUTPUT(AA);
OUTPUT(A0);
OUTPUT(A1);
OUTPUT(DE);
OUTPUT(A2);
NEWLINE(2);
COPYTEXT('('*')');NEWLINE(1);
COPYTEXT('('*')');NEWLINE(1);
OUTPUT(FABL);
OUTPUT(FAPE);
COPYTEXT('('*')');NEWLINE(1);
OUTPUT(MARK);
OUTPUT(A3);
OUTPUT(CURA);
COPYTEXT('('*')');NEWLINE(1);
OUTPUT(A4);
OUTPUT(TAX);
OUTPUT(OVDR);
OUTPUT(DV);
OUTPUT(CURL);
OUTPUT(A5);
OUTPUT(A6);

'COMMENT'      PROFIT AND LOSS;

PAPERTHROW;WRITETEXT('('XTDUMP')');PRINT(M,3,0);
           NEWLINE(2);COPYTEXT('('*')');PERIOD;
OUTPUT(SALES);
```

```
'FOR'I:=1'STEP'1'UNTIL'N'DO''BEGIN'
                                    A1[I]:=SALES[I]-EARN[I]-RS*OVDR[I-1]
                                                          +RI*MARK[I-1];
                                    A2[I]:=EARN[I]+RS*OVDR[I-1]-RI*MARK[I-1];
                                    A3[I]:=RI*MARK[I-1];
                                    A4[I]:=RS*OVDR[I-1];
                                    A5[I]:=RL*DE[I-1];
                                    A6[I]:=EARN[I]-A5[I];
                                    A7[I]:=NPAT[I]-DV[I];
                                    'END';
OUTPUT(A1);
OUTPUT(A2);
OUTPUT(A3);
OUTPUT(A4);
OUTPUT(A5);
OUTPUT(A6);
OUTPUT(NPAT);
OUTPUT(DV);
OUTPUT(A7);
NEWLINE(3);

'COMMENT'   CASH FLOW STATEMENT;

COPYTEXT('('*')');NEWLINE(1);
COPYTEXT('('*')');
'FOR' I:=1'STEP'1 'UNTIL' N 'DO''BEGIN'
                                       A3[I]:=OVDR[I]-OVDR[I-1];
                                       A4[I]:=DE[I]-DE[I-1];
                                           A1[I]:=EARN[I]+0.0303*FABL[I]
                                     +0.3333*FAPF[I]+CURL[I]-CURL[I-1]
                                            -CURA[I]+CURA[I-1]+RS*OVDR[I-1]
                                          +MARK[I]-MARK[I-1]-DV[I]+DV[I-1]
                                     -TAX[I]+TAX[I-1]-A3[I]-RI*MARK[I-1];
                                       A2[I]:=RI*MARK[I-1];
                                       A5[I]:=1.6*RG[I];
                                       A6[I]:=A1[I]+A2[I]       +A4[I]+A5[I];
                                       'END';
OUTPUT(A1);
OUTPUT(A2);
OUTPUT(A4);
OUTPUT(A5);
OUTPUT(A6);
COPYTEXT('('*')');
'FOR' I:=1'STEP'1'UNTIL' N'DO''BEGIN'
                                    A1[I]:=1.0303*FABL[I]-FABL[I-1];
                                    A2[I]:=1.3333*FAPF[I]-FAPF[I-1];
                                    A3[I]:=RS*OVDR[I-1];
                                    A4[I]:=RL*DE[I-1];
                                    A6[I]:=MARK[I]-MARK[I-1]-OVDR[I]+OVDR[I-1];
                                    A7[I]:=TAX[I-1];
                                    A8[I]:=DV[I-1];
                                    A5[I]:=A1[I]+A2[I]+A3[I]+A4[I]+A7[I]+A8[I];
                                    'END';
OUTPUT(A1);
OUTPUT(A2);
OUTPUT(A3);
OUTPUT(A4);
OUTPUT(A8);
OUTPUT(A7);
OUTPUT(A5);
OUTPUT(A6);

'COMMENT'    INDICATORS;

PAPERTHROW;WRITETEXT('('XXDUMP')');PRINT(M,3,0);
NEWLINE(1);
'FOR' I:=1'STEP' 1 'UNTIL' N'DO''BEGIN'
                                    A1[I]:=EARN[I]/(FABL[I]+FAPE[I]+CURA[I]
                                                                   -CURL[I]);
                                     A1[I]:=A1[I]*100.0;
                                    A2[I]:=CURA[I]/CURL[I];
                                    A3[I]:=EARN[I]/(RL*DE[I]+RS*OVDR[I]);
                                    A4[I]:=NPAT[I]/NUM[I]*100.0;
                                    A5[I]:=NPAT[I]/DV[I];
                                    A6[I]:=DV[I]/NUM[I]*100.0;
                                    'END';
              COPYTEXT('('*')');PERIOD;
ACHIEVEMENT(ROCE,A1);
ACHIEVEMENT(LODV,A2);
ACHIEVEMENT(ECOV,A3);
ACHIEVEMENT(EPPS,A4);
ACHIEVEMENT(DCOV,A5);
ACHIEVEMENT(DTARG,A6);
COPYTEXT('('*')');
  FREEINPUT;  'COMMENT' RELEASING CRM ON CHANNEL 2;
SELECTINPUT(1);
NEXTDUMP: PAPERTHROW; 'GOTO' START;
'END';
'END';
```

APPENDIX IX

SOLUTIONS TO THE SINGLE CRITERION MODEL

EXHIBIT A9.1 LP SOLUTION AT NORMAL EARNINGS

PROBLEM OPTIMODEL-04 SOLUTION DATE 04/08/76 TI

DUMP:DUMP 111

RIGHT HAND SIDE RHS1
OBJECTIVE COBJ +0*OBJ6
LOWER BOUND LOBND
UPPER BOUND UPBND

ROW SET -(DUMMY) COL SET

COLUMN INFORMATION

NAME		VALUE	LOWER BOUND	UPPER BOUND	OBJECTIVE	REDUCED COST
B SALES1	◆	12632.6805	0		0	0
B SALES2	◆	14400.0000	0		0	0
B SALES3	◆	17538.5030	0		0	0
B SALES4	◆	22701.5050	0		0	0
B SALES5	◆	23369.1617	0		0	0
B SALES6	◆	26869.6714	0		0	0
B SALES7	◆	31065.3995	0		0	0
B SALES8	◆	31601.8174	0		0	0
B EARN1	◆	1566.0992	0		0	0
B EARN2	◆	1590.3866	0		0	0
B EARN3	◆	2109.5797	0		0	0
B EARN4	◆	2999.4366	0		0	0
B EARN5	◆	2795.6114	0		0	0
B EARN6	◆	3506.5499	0		0	0
B EARN7	◆	3960.5015	0		0	0
B EARN8	◆	4182.7636	0		0	0
B NPAT1	◆	733.3496	0		0	0
B NPAT2	◆	715.8586	0		0	0
B NPAT3	◆	975.2705	0		0	0
B NPAT4	◆	1394.2393	0		0	0
B NPAT5	◆	1252.2267	0		0	0
B NPAT6	◆	1613.5044	0		0	0
B NPAT7	◆	1806.3384	0		0	0
B NPAT8	◆	1917.5195	0		0	0
B TAX1	◆	166.1524	0		0	0
B TAX2	◆	1.0426	0		0	0
B TAX3	◆	512.4289	0		0	0
B TAX4	◆	967.8164	0		0	0
B TAX5	◆	976.2739	0		0	0
B TAX6	◆	1069.5633	0		0	0
B TAX7	◆	1479.1344	0		0	0
B TAX8	◆	1948.6801	0		0	0
B FABL1	◆	2668.9320	0		0	0
B FABL2	◆	3855.3321	0		0	0
B FABL3	◆	4600.0244	0		0	0
B FABL4	◆	5596.9805	0		0	0
B FABL5	◆	5895.8650	0		0	0
B FABL6	◆	6486.2901	0		0	0
B FABL7	◆	6879.8992	0		0	0
B FABL8	◆	6849.4167	0		0	0
B FAPE1	◆	1474.4361	0		0	0
B FAPE2	◆	2274.0121	0		0	0
B FAPE3	◆	2723.6955	0		0	0
B FAPE4	◆	2982.9484	0		0	0
B FAPE5	◆	3207.9276	0		0	0
B FAPE6	◆	3832.8455	0		0	0
B FAPE7	◆	4087.5733	0		0	0
B FAPE8	◆	3798.9273	0		0	0
B CURA1	◆	4190.0560	0		0	0
B CURA2	◆	4841.0000	0		0	0
B CURA3	◆	6245.6257	0		0	0
B CURA4	◆	8276.5327	0		0	0
B CURA5	◆	8419.4370	0		0	0
B CURA6	◆	10036.6037	0		0	0
B CURA7	◆	11324.8956	0		0	0
B CURA8	◆	13581.4326	0		0	0
B CURL1	◆	1767.4659	0		0	0
B CURL2	◆	2164.2815	0		0	0
B CURL3	◆	3328.3598	0		0	0
B CURL4	◆	4598.0737	0		0	0
B CURL5	◆	4677.4650	0		0	0
B CURL6	◆	5575.8909	0		0	0
B CURL7	◆	5864.1063	0		0	0
B CURL8	◆	7193.5023	0		0	0

	OVDR1	+	0	0	250.0000	0	-0.0436
	OVDR2	+	0	0	250.0000	0	-0.0931
	OVDR3	+	0	0	250.0000	0	-0.0599
	OVDR4	+	0	0	250.0000	0	-0.0238
	OVDR5	+	0	0	250.0000	0	-0.1304
	OVDR6	+	0	0	250.0000	0	-0.0156
	OVDR7	+	0	0	250.0000	0.0231	-0.0124
	OVDR8	+	0	0	250.0000	-0.4039	0
B	MARK1	+	218.0560	0		0	0
	MARK2	+	0	0		0	-0.1296
	MARK3	+	0	0		0	-0.0670
	MARK4	+	0	0		0	-0.00086
	MARK5	+	0	0		0	-0.0690
B	MARK6	+	461.7588	0		0	0
B	MARK7	+	476.8291	0		-0.0153	0
B	MARK8	+	2421.5923	0		0.4039	0
B	DV1	+	290.2535	0		0.7972	0
B	DV2	+	477.2389	0		0.7118	0
B	DV3	+	850.1806	0		0.6355	0
B	DV4	+	805.3603	0		0.5674	0
B	DV5	+	797.9723	0		0.5066	0
B	DV6	+	1075.6696	0		0.4523	0
B	DV7	+	485.2351	0		0.4039	0
B	DV8	+	1278.3463	0		0	0
B	NUM1	+	2232.7193	0		0	0
B	NUM2	+	3032.7193	0		0	0
B	NUM3	+	3032.7193	0		0	0
B	NUM4	+	3032.7193	0		0	0
B	NUM5	+	3032.7193	0		0	0
B	NUM6	+	3032.7193	0		0	0
B	NUM7	+	3032.7193	0		0	0
B	NUM8	+	3032.7193	0		0	0
B	RG1	+	232.7193	0	800.0000	-1.4280	0
U	RG2	+	800.0000	0	800.0000	-1.2760	0.0826
	RG3	+	0	0	800.0000	-1.1339	-0.0376
	RG4	+	0	0	800.0000	-1.0170	-0.0629
	RG5	+	0	0	800.0000	-0.9070	-0.0089
	RG6	+	0	0	800.0000	-0.8110	-0.1104
	RG7	+	0	0	800.0000	-0.7240	-0.0571
	RG8	+	0	0	800.0000	-0.6464	-0.00016
B	DE1	+	1983.3740	0		0	0
B	DE2	+	1987.9433	0		0	0
B	DE3	+	2436.9747	0		0	0
B	DE4	+	3436.9747	0		0	0
B	DE5	+	3494.2643	0		0	0
B	DE6	+	4346.5581	0		0	0
B	DE7	+	4346.5581	0		0.0155	0
B	DE8	+	4346.5581	0		-0.4039	0
B	LL1	+	483.3740	0	1000.0000	0	0
B	LL2	+	4.6093	0	1000.0000	0	0
B	LL3	+	848.9914	0	1000.0000	0	0
U	LL4	+	1000.0000	0	1000.0000	0	0.0103
B	LL5	+	857.2896	0	1000.0000	0	0
B	LL6	+	852.2439	0	1000.0000	0	0
	LL7	+	0	0	1000.0000	0	-0.0039
	LL8	+	0	0	1000.0000	0	0
B	BLTA1	+	44.4262	0		0	0
B	BLTA2	+	95.6555	0		0	0
B	BLTA3	+	130.1233	0		0	0
B	BLTA4	+	175.5944	0		0	0
B	BLTA5	+	193.4505	0		0	0
B	BLTA6	+	223.5532	0		0	0
B	BLTA7	+	246.1772	0		0	0
B	BLTA8	+	251.8073	0		0	0
U	PR01Y1	+	1.0000	0	1.0000	-44.4299	20.9754
U	PR04Y1	+	1.0000	0	1.0000	-21.8106	60.9994
U	PR12Y1	+	1.0000	0	1.0000	-33.1198	11.4430
U	PR13Y1	+	1.0000	0	1.0000	0	67.2091
	PR16Y1	+	0	0	1.0000	-31.2215	-30.4693
U	PR22Y1	+	1.0000	0	1.0000	-18.3371	2.2580
	PR23Y1	+	0	0	1.0000	-17.6908	-15.7094
U	PR03Y2	+	1.0000	0	1.0000	18.3774	2.1875
U	PR04Y2	+	1.0000	0	1.0000	-2.1407	8.2901
	PR05Y2	+	0	0	1.0000	15.9540	-49.5435
U	PR13Y2	+	1.0000	0	1.0000	1.2117	12.2017
U	PR14Y2	+	1.0000	0	1.0000	-47.2563	21.1587
	PR21Y2	+	0	0	1.0000	23.9513	-81.0567

PROBLEM OPTIMODEL-M6 SOLUTION DATE 04/08/76 111

DUMP;DUMP 111

RIGHT HAND SIDE RHS1
OBJECTIVE COBJ +0*OBJ6
LOWER BOUND LOBND
UPPER BOUND UPBND

ROW SET -(DUMMY) COL SET

COLUMN INFORMATION

	NAME		VALUE	LOWER BOUND	UPPER BOUND	OBJECTIVE	REDUCED COST
	PR24Y2	*	0	0	1.0000	-7.2702	-35.7877
U	PR02Y3	*	1.0000	0	1.0000	32.9986	1.7773
B	PR11Y3	*	0.4875	0	1.0000	46.7716	0
U	PR15Y3	*	1.0000	0	1.0000	43.4596	1.1691
U	PR01Y4	*	1.0000	0	1.0000	126.0168	22.6666
	PR05Y4	*	0	0	1.0000	117.9388	-17.2978
B	PR11Y4	*	0.3323	0	1.0000	83.2034	0
U	PR12Y4	*	1.0000	0	1.0000	144.1923	15.7933
U	PR13Y4	*	1.0000	0	1.0000	42.8134	33.3094
U	PR14Y4	*	1.0000	0	1.0000	-17.0850	7.4108
U	PR22Y4	*	1.0000	0	1.0000	33.7660	5.8577
U	PR25Y4	*	1.0000	0	1.0000	51.6992	17.2760
	PR02Y5	*	0	0	1.0000	66.6435	-1.5181
U	PR03Y5	*	1.0000	0	1.0000	151.0586	6.5843
B	PR11Y5	*	0.4484	0	1.0000	101.7828	0
	PR21Y5	*	0	0	1.0000	130.0558	-39.4934
	PR23Y5	*	0	0	1.0000	71.8942	-13.9222
U	PR04Y6	*	1.0000	0	1.0000	112.6881	40.4800
	PR05Y6	*	0	0	1.0000	199.9305	-1.3103
U	PR11Y6	*	1.0000	0	1.0000	131.2675	25.7888
U	PR14Y6	*	1.0000	0	1.0000	49.2758	9.5258
U	PR15Y6	*	1.0000	0	1.0000	95.3204	11.0532
U	PR16Y6	*	1.0000	0	1.0000	180.9472	7.0064
	PR21Y6	*	0	0	1.0000	191.0447	-17.7150
U	PR23Y6	*	1.0000	0	1.0000	85.2229	1.2545
U	PR01Y7	*	1.0000	0	1.0000	98.5516	13.1655
U	PR04Y7	*	1.0000	0	1.0000	127.2285	31.3665
U	PR14Y7	*	1.0000	0	1.0000	89.6658	3.2261
	PR22Y7	*	0	0	1.0000	44.8329	-8.8970
	PR02Y8	*	0	0	1.0000	74.3176	-4.8664
	PR15Y8	*	0	0	1.0000	65.4318	-2.0391
	PR22Y8	*	0	0	1.0000	44.4290	-4.4821
	PR25Y8	*	0	0	1.0000	147.8274	-1.6646
	OBJECTIVE		1984.0158				

PROBLEM OPTIMODEL-86 SOLUTION DATE 04/08/76 TIM

DUMP:DUMP 111 RIGHT HAND SIDE RHS1

OBJECTIVE COBJ +0*OBJ6

LOWER BOUND LOBND

UPPER BOUND UPBND

ROW SET -(DUMMY) COL SET

ROW INFORMATION

	NAME		SLACK	R.H.S.	PRICE
#	OBJ6	Z	2178.3596	0	
	TS1		0	-11000.0000	0
	TS2		0	-10000.0000	0
	TS3		0	-9500.0000	0
	TS4		0	-8800.0000	0
	TS5		0	-8000.0000	0
	TS6		0	-7500.0000	0
	TS7		0	-7500.0000	0
	TS8		0	-7000.0000	0
	EA1		0	-1953.0000	0.4802
	EA2		0	-1850.0000	0.6803
	EA3		0	-1600.0000	0.4928
	EA4		0	-1560.0000	0.3126
	EA5		0	-1240.0000	0.4391
	EA6		0	-1220.0000	0.2323
	EA7		0	-1200.0000	0.2149
	EA8		0	-1000.0000	0.4039
	BL1		0	-2034.0000	-0.6620
	BL2		0	-600.0000	-0.6635
	BL3		0	-500.0000	-0.5253
	BL4		0	-200.0000	-0.4505
	BL5		0	0	-0.4387
	BL6		0	0	-0.3369
	BL7		0	0	-0.3286
	BL8		0	0	-0.4043
	PE1		0	-1481.0000	-0.5323
	PE2		0	-500.0000	-0.5839
	PE3		0	-400.0000	-0.4325
	PE4		0	-400.0000	-0.3382
	PE5		0	-400.0000	-0.3421
	PE6		0	-400.0000	-0.2261
	PE7		0	-400.0000	-0.2149
	PE8		0	-400.0000	-0.4039
	CA1		0	-3510.0000	-0.0476
	CA2		0	-3588.0000	-0.1639
	CA3		0	-3705.0000	-0.0889
	CA4		0	-3432.0000	-0.0316
	CA5		0	-3120.0000	-0.0853
	CA6		0	-2925.0000	-0.0150
	CA7		0	-2925.0000	-0.0129
	CA8		0	-2730.0000	-0.4039
	CL1		0	-1120.0000	0.0476
	CL2		0	-1145.0000	0.1639
	CL3		0	-1170.0000	0.0889
	CL4		0	-1095.0000	0.0289
	CL5		0	-1020.0000	0.0547
	CL6		0	-946.0000	0.0102
	CL7		0	-948.0000	0.0129
	CL8		0	-902.0000	0.4039
	CB1		0	-4580.0000	0.8967
	CB2		0	0	0.8491
	CB3		0	0	0.6852
	CB4		0	0	0.5963
	CB5		0	1000.0000	0.5613
	CB6		0	0	0.4379
	CB7		0	0	0.4168
	CB8		0	0	0.4039
	TP1		0	702.0000	0.8491
	TP2		0	-65.0000	0.6852
	TP3		0	-65.0000	0.5963
	TP4		0	-60.0000	0.5674
	TP5		0	-60.0000	0.5066
	TP6		0	-60.0000	0.4277
	TP7		0	-60.0000	0.4039
	TP8		0	-55.0000	0

	TA1		0	-65.0000	-2.0181
	TA2		0	0	-1.5935
	TA3		0	0	-1.2509
	TA4		0	0	-0.9528
	TA5		0	0	-0.6691
	TA6		0	0	-0.4158
	TA7		0	0	-0.2019
	TA8		0	0	0
	PR1		0	-60.0000	0.8491
	PR2		0	0	0.6674
	PR3		0	0	0.5701
	PR4		0	0	0.5674
	PR5		0	0	0.5066
	PR6		0	0	0.4113
	PR7		0	0	0.4039
	PR8		0	0	0
	EQ1		0	2000.0000	0.0067
	EQ2		0	0	0
	EQ3		0	0	0
	EQ4		0	0	0
	EQ5		0	0	0
	EQ6		0	0	0
	EQ7		0	0	0
	EQ8		0	0	0
	D1		0	1500.0000	0.8967
	D2		0	0	0.8491
	D3		0	0	0.6852
	D4		0	0	0.5860
	D5		0	-1000.0000	0.5613
	D6		0	0	0.4379
	D7		0	0	0.4207
	D8		0	0	0.4039
B	ROCE1	+	404.8159	0	0
B	ROCE2	+	5.6553	0	0
B	ROCE3	+	266.2023	0	0
B	ROCE4	+	792.9448	0	0
B	ROCE5	+	483.1702	0	0
B	ROCE6	+	846.1761	0	0
B	ROCE7	+	1003.4144	0	0
B	ROCE8	+	1116.7342	0	0
B	LODY1	+	1008.7253	0	0
B	LODY2	+	945.2936	0	0
B	LODY3	+	254.5781	0	0
	LODY4	+	0	0	-0.0034
	LODY5	+	0	0	-0.0382
	LODY6	+	0	0	-0.0060
B	LODY7	+	769.5042	0	0
B	LODY8	+	633.1285	0	0
	ECOV1	+	0	0	-0.0080
	ECOV2	+	0	0	-0.1649
	ECOV3	+	0	0	-0.0927
B	ECOV4	+	69.8569	0	0
	ECOV5	+	0	0	-0.1311
B	ECOV6	+	29.3033	0	0
B	ECOV7	+	483.2550	0	0
B	ECOV8	+	705.5171	0	0
B	ERPS1	+	286.6057	0	0
B	ERPS2	+	109.3145	0	0
B	ERPS3	+	338.3995	0	0
B	ERPS4	+	757.3683	0	0
B	ERPS5	+	585.0285	0	0
B	ERPS6	+	946.3001	0	0
B	ERPS7	+	1078.5358	0	0
B	ERPS8	+	1189.6666	0	0
	DTARG1	-	0	0	0.0519
B	DTARG2	-	67.8218	0	0
B	DTARG3	-	225.5996	0	0
B	DTARG4	-	363.6160	0	0
B	DTARG5	-	343.0644	0	0
B	DTARG6	-	605.5981	0	0
	DTARG7	-	0	0	0
B	DTARG8	-	727.9476	0	0
B	DCOV1	+	297.9693	0	0
	DCOV2	+	0	0	-0.0176
	DCOV3	+	0	0	-0.0261
B	DCOV4	+	189.1989	0	0
B	DCOV5	+	55.2682	0	0.000000013
	DCOV6	+	0	0	-0.0164
B	DCOV7	+	1078.5358	0	0
	DCOV8	+	0	0	0
#	COBJ	Z	1986.0158	0	

EXHIBIT A9.2 FINANCIAL STATEMENTS AND RATIO ANALYSIS AT NORMAL EARNINGS

BALANCE SHEET (£'000S)

	PERIOD-1	PERIOD-2	PERIOD-3	PERIOD-4	PERIOD-5	PERIOD-6	PERIOD-7	PERIOD-8
CAPITAL								
SHARE CAPITAL	2233	3033	3033	3033	3033	3033	3033	3033
SHARE PREMIUM	140	620	620	620	620	620	620	620
TAXATION EQUALISATION	567	1282	1745	2171	2447	2991	3318	3287
RESERVES	1643	1882	2207	2798	3252	3790	5111	5750
LONG TERM DEBT	1983	1988	2637	3637	3494	4347	4347	4347
TOTAL LIABILITIES	6566	8804	10241	12258	12846	14780	16428	17036
ASSETS								
FIXED ASSETS								
LAND AND BUILDINGS	2669	3853	4600	5597	5896	6486	6880	6849
PLANT AND MACHINERY	1474	2274	2724	2983	3208	3833	4088	3799
CURRENT ASSETS								
SHORT TERM DEPOSITS	218	0	0	0	0	462	477	2422
DEBTORS AND STOCK	3972	4841	6246	8277	8419	9575	10848	11160
CURRENT ASSETS	4190	4841	6246	8277	8419	10037	11325	13581
CURRENT LIABILITIES								
CREDITORS	1311	1686	2166	2827	2903	3431	3900	3967
TAX	166	1	512	968	976	1070	1479	1949
OVERDRAFT	0	0	0	0	0	0	0	0
DIVIDEND PAYABLE	290	477	650	803	798	1076	485	1278
CURRENT LIABILITIES	1767	2164	3328	4598	4678	5576	5864	7194
NET CURRENT ASSETS	2423	2677	2917	3679	3742	4461	5461	6388
TOTAL ASSETS	6566	8804	10241	12258	12846	14780	16428	17036

DUMP 111

PROFIT AND LOSS STATEMENT (£'000S)

	PERIOD-1	PERIOD-2	PERIOD-3	PERIOD-4	PERIOD-5	PERIOD-6	PERIOD-7	PERIOD-8
SALES	12633	14400	17539	22702	23369	26870	31005	31402
LESS COST OF SALES	11090	12825	15429	19702	20574	23363	27077	27252
TRADING PROFIT	1543	1575	2110	2999	2795	3507	3928	4149
INTEREST ON INVESTMENTS	56	15	0	0	0	0	32	33
LESS INTEREST BANK OVERDRAFTS	12	0	0	0	0	0	0	0
LESS INTEREST LONG TERM DEBT	120	159	159	211	291	280	348	348
PROFIT BEFORE TAX	1467	1432	1951	2789	2505	3227	3613	3835
PROFIT AFTER TAX	733	716	975	1394	1252	1614	1806	1918
DIVIDEND	290	477	650	803	798	1076	485	1278
ADDED TO RESERVES	443	239	325	591	454	538	1321	639

CASH FLOW STATEMENT

	PERIOD-1	PERIOD-2	PERIOD-3	PERIOD-4	PERIOD-5	PERIOD-6	PERIOD-7	PERIOD-8
SOURCES								
TRADING INCOME	1594	1956	2232	2794	3977	4353	4695	5378
INVESTMENT INCOME	56	15	0	0	0	0	32	33
INCREASE IN LONG TERM DEBT	483	5	649	1000	-142	852	0	0
RIGHTS ISSUES	372	1280	0	0	0	0	0	0
TOTAL CASH INFLOW	2506	3256	2881	3794	3834	5205	4727	5411
USES								
BUILDING AND LAND	1116	1301	886	1166	478	787	602	177
PLANT AND EQUIPMENT	1085	1558	1358	1254	1294	1902	1617	978
INTEREST OVERDRAFT	12	0	0	0	0	0	0	0
INTEREST LONG TERM DEBT	120	159	159	211	291	280	348	348
DIVIDEND PAYMENTS	285	290	477	650	803	798	1076	485
TAX PAID	370	166	1	512	968	976	1070	1479
TOTAL CASH OUTFLOW	2988	3474	2881	3794	3834	4743	4712	3467
NET CHANGE IN CASH POSITION	-481	-217	0	0	0	462	15	1945

DUMP 111

ACHIEVEMENT LEVELS

		PERIOD-1	PERIOD-2	PERIOD-3	PERIOD-4	PERIOD-5	PERIOD-6	PERIOD-7	PERIOD-8
RETURN	TARGET	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0
ON	ACHIEVEMENT	24.2	18.1	20.6	24.5	21.8	23.7	24.1	24.6
CAPITAL	PERCENT DEVIATION	34.3	0.4	14.4	35.9	20.9	31.8	33.9	36.6
	TARGET	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8
LIQUIDITY	ACHIEVEMENT	2.4	2.2	1.9	1.8	1.8	1.8	1.9	1.9
	PERCENT DEVIATION	31.7	24.3	4.2	0.0	0.0	0.0	7.3	4.9
TIMES	TARGET	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
COVERED	ACHIEVEMENT	10.0	10.0	10.0	10.3	10.0	10.1	11.4	12.0
	PERCENT DEVIATION	0.0	-0.0	-0.0	3.1	-0.0	0.8	13.9	20.3
EARNINGS	TARGET	20.0	20.0	21.0	21.0	22.0	22.0	24.0	24.0
PER	ACHIEVEMENT	32.8	23.6	32.2	46.0	41.3	53.2	59.6	63.2
SHARE	PERCENT DEVIATION	64.2	18.0	53.1	118.9	87.7	141.8	148.2	163.4
DIVIDEND	TARGET	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
COVER	ACHIEVEMENT	2.5	1.5	1.5	1.7	1.6	1.5	3.7	1.5
	PERCENT DEVIATION	68.4	0.0	-0.0	15.7	4.6	0.0	148.2	0.0
DIVIDEND	TARGET	13.0	13.5	14.0	14.5	15.0	15.5	16.0	16.5
PER	ACHIEVEMENT	13.0	15.7	21.4	26.5	26.3	35.5	16.0	42.2
SHARE	PERCENTAGE DEVIATION	-0.0	16.6	53.1	82.7	75.4	128.8	0.0	155.5

EXHIBIT A9.3 LP SOLUTION AT A TEN PER CENT INCREASE IN EARNINGS

PROBLEM OPTIMODEL-MA SOLUTION DATE 04/08/76 TIME

DUMP:DUMP 112 RIGHT HAND SIDE RHS1 +0.1000*RHS2
OBJECTIVE COBJ +0*OBJ6
LOWER BOUND LOBND
UPPER BOUND UPBND
ROW SET -(DUMMY) COL SET

COLUMN INFORMATION

	NAME		VALUE	LOWER BOUND	UPPER BOUND	OBJECTIVE	REDUCED COST
B	SALES1	+	12639.5533	0		0	0
B	SALES2	+	14400.0000	0		0	0
B	SALES3	+	17527.2264	0		0	0
B	SALES4	+	22682.6068	0		0	0
B	SALES5	+	23323.8619	0		0	0
B	SALES6	+	26683.0103	0		0	0
B	SALES7	+	30873.6983	0		0	0
B	SALES8	+	31316.1939	0		0	0
B	EARN1	+	1781.9992	0		0	0
B	EARN2	+	1779.8980	0		0	0
B	EARN3	+	2273.0134	0		0	0
B	EARN4	+	3157.1092	0		0	0
B	EARN5	+	2907.0864	0		0	0
B	EARN6	+	3595.8995	0		0	0
B	EARN7	+	4071.9014	0		0	0
B	EARN8	+	4787.7293	0		0	0
B	NPAT1	+	830.9996	0		0	0
B	NPAT2	+	800.9541	0		0	0
B	NPAT3	+	1047.5118	0		0	0
B	NPAT4	+	1464.9039	0		0	0
B	NPAT5	+	1299.6935	0		0	0
B	NPAT6	+	1652.9953	0		0	0
B	NPAT7	+	1856.1557	0		0	0
B	NPAT8	+	1964.0697	0		0	0
B	TAX1	+	263.8024	0		0	0
B	TAX2	+	86.1383	0		0	0
B	TAX3	+	593.7696	0		0	0
B	TAX4	+	1036.1583	0		0	0
B	TAX5	+	1015.4838	0		0	0
B	TAX6	+	1126.6172	0		0	0
B	TAX7	+	1532.9854	0		0	0
B	TAX8	+	1991.8022	0		0	0
B	FABL1	+	2668.9320	0		0	0
B	FABL2	+	3853.3321	0		0	0
B	FABL3	+	4579.5003	0		0	0
B	FABL4	+	5588.7314	0		0	0
B	FABL5	+	5907.2869	0		0	0
B	FABL6	+	6474.2167	0		0	0
B	FABL7	+	6856.1340	0		0	0
B	FABL8	+	6826.3637	0		0	0
	FAPE1	+	1474.4361	0		0	0
	APE2	+	2274.0121	0		0	0
	E3	+	2715.2186	0		0	0
	4	+	2974.3747	0		0	0
	P	+	3208.2138	0		0	0
		+	3806.8934	0		0	0
	NAME	+	4058.7042	0		0	0
B	FAPE8	+	3777.2212	0		0	0
B	CURA1	+	4754.5035	0		0	0
B	CURA2	+	4841.0000	0		0	0
B	CURA3	+	6242.8071	0		0	0
B	CURA4	+	8271.1035	0		0	0
B	CURA5	+	8615.3281	0		0	0
B	CURA6	+	10152.8854	0		0	0
B	CURA7	+	11472.5454	0		0	0
B	CURA8	+	13811.6365	0		0	0
B	CURL1	+	1834.4024	0		0	0
B	CURL2	+	2306.1077	0		0	0
B	CURL3	+	3455.4622	0		0	0
B	CURL4	+	4595.0575	0		0	0
B	CURL5	+	4675.1623	0		0	0
B	CURL6	+	5640.4919	0		0	0
B	CURL7	+	5846.2905	0		0	0
B	CURL8	+	7247.1530	0		0	0

	OVDR1	*	0	0	250.0000	0	-0.0283
	OVDR2	*	0	0	250.0000	0	-0.0708
	OVDR3	*	0	0	250.0000	0	-0.0633
	OVDR4	*	0	0	250.0000	0	-0.0200
	OVDR5	*	0	0	250.0000	0	-0.1159
	OVDR6	*	0	0	250.0000	0	-0.0400
	OVDR7	*	0	0	250.0000	0.0231	-0.0124
	OVDR8	*	0	0	250.0000	-0.4039	0
B	MARK1	*	282.5035	0		0	0
	MARK2	*	0	0		0	-0.0767
	MARK3	*	0	0		0	-0.0658
	MARK4	*	0	0		0	-0.0062
	MARK5	*	0	0		0	-0.0633
B	MARK6	*	627.8159	0		0	0
B	MARK7	*	679.8595	0		-0.0153	0
B	MARK8	*	2702.1240	0		0.4039	0
B	DV1	*	260.0000	0		0.7972	0
B	DV2	*	535.9694	0		0.7118	0
B	DV3	*	698.3412	0		0.6355	0
B	DV4	*	736.5612	0		0.5674	0
B	DV5	*	760.4026	0		0.5066	0
B	DV6	*	1101.7302	0		0.4523	0
B	DV7	*	431.7367	0		0.4039	0
B	DV8	*	1309.3798	0		0	0
B	NUM1	*	2000.0000	0		0	0
B	NUM2	*	2698.3542	0		0	0
B	NUM3	*	2698.3542	0		0	0
B	NUM4	*	2698.3542	0		0	0
B	NUM5	*	2698.3542	0		0	0
B	NUM6	*	2698.3542	0		0	0
B	NUM7	*	2698.3542	0		0	0
B	NUM8	*	2698.3542	0		0	0
	RG1	*	0	0	800.0000	-1.4280	-0.0887
B	RG2	*	698.3542	0	800.0000	-1.2760	0
	RG3	*	0	0	800.0000	-1.1339	-0.0359
	RG4	*	0	0	800.0000	-1.0170	-0.0595
	RG5	*	0	0	800.0000	-0.9070	-0.0096
	RG6	*	0	0	800.0000	-0.8110	-0.0934
	RG7	*	0	0	800.0000	-0.7240	-0.0571
	RG8	*	0	0	800.0000	-0.6464	-0.00016
B	DE1	*	2224.8724	0		0	0
B	DE2	*	2224.8724	0		0	0
B	DE3	*	2841.2668	0		0	0
B	DE4	*	3841.2668	0		0	0
B	DE5	*	3633.8604	0		0	0
B	DE6	*	4494.8743	0		0	0
B	DE7	*	4494.8743	0		0.0155	0
B	DE8	*	4494.8743	0		-0.4039	0
B	LL1	*	724.8724	0	1000.0000	0	0
	LL2	*	0	0	1000.0000	0	-0.0025
B	LL3	*	616.3943	0	1000.0000	0	0
U	LL4	*	1000.0000	0	1000.0000	0	0.0129
B	LL5	*	792.5937	0	1000.0000	0	0
B	LL6	*	861.0139	0	1000.0000	0	0
	LL7	*	0	0	1000.0000	0	-0.0039
	LL8	*	0	0	1000.0000	0	0
B	BLTA1	*	44.4262	0		0	0
B	BLTA2	*	95.6555	0		0	0
B	BLTA3	*	129.2818	0		0	0
B	BLTA4	*	175.2597	0		0	0
B	BLTA5	*	193.8893	0		0	0
B	BLTA6	*	223.0407	0		0	0
B	BLTA7	*	245.1735	0		0	0
B	BLTA8	*	250.8082	0		0	0
U	PR01Y1	*	1.0000	0	1.0000	-44.4290	35.1393
U	PR04Y1	*	1.0000	0	1.0000	-21.8106	72.7414
U	PR12Y1	*	1.0000	0	1.0000	-33.1198	28.7206
U	PR13Y1	*	1.0000	0	1.0000	0	84.8943
	PR16Y1	*	0	0	1.0000	-31.2215	-4.5409
U	PR22Y1	*	1.0000	0	1.0000	-18.3371	5.2108
	PR23Y1	*	0	0	1.0000	-17.6908	-3.4211
U	PR03Y2	*	1.0000	0	1.0000	18.3774	22.0489
U	PR04Y2	*	1.0000	0	1.0000	-2.1407	28.6743
	PR05Y2	*	0	0	1.0000	15.9546	-30.5334
U	PR13Y2	*	1.0000	0	1.0000	1.2117	32.2450
U	PR14Y2	*	1.0000	0	1.0000	-47.2563	27.3591
	PR21Y2	*	0	0	1.0000	23.9513	-59.9380

PROBLEM OPTIMODEL-80 SOLUTION DATE 04/08/76 TI

DUMP:DUMP 112

RIGHT HAND SIDE RHS1 +0.1000*RHS2
OBJECTIVE COBJ +0*OBJ6
LOWER BOUND LOBND
UPPER BOUND UPBND

ROW SET -(DUMMY) COL SET

COLUMN INFORMATION

	NAME		VALUE	LOWER BOUND	UPPER BOUND	OBJECTIVE	REDUCED COST
	PR24Y2	+	0	0	1.0000	-7.2702	-17.5288
U	PR07Y3	+	1.0000	0	1.0000	32.9986	1.8609
B	PR11Y3	+	0.5936	0	1.0000	46.7716	0
U	PR15Y3	+	1.0000	0	1.0000	43.4596	1.5429
U	PR01Y4	+	1.0000	0	1.0000	126.0168	22.6861
	PR05Y4	+	0	0	1.0000	117.9388	-16.6616
B	PR11Y4	+	0.3862	0	1.0000	83.2034	0
U	PR12Y4	+	1.0000	0	1.0000	144.1923	15.3927
U	PR13Y4	+	1.0000	0	1.0000	42.8134	34.2314
U	PR14Y4	+	1.0000	0	1.0000	-17.0856	7.1791
U	PR22Y4	+	1.0000	0	1.0000	33.7660	5.1967
U	PR25Y4	+	1.0000	0	1.0000	51.6992	16.3370
	PR02Y5	+	0	0	1.0000	66.6435	-1.1578
U	PR03Y5	+	1.0000	0	1.0000	151.0586	6.8958
B	PR11Y5	+	0.5368	0	1.0000	101.7828	0
	PR21Y5	+	0	0	1.0000	130.0558	-39.9860
	PR23Y5	+	0	0	1.0000	71.8942	-14.3091
U	PR04Y6	+	1.0000	0	1.0000	112.6881	39.1241
	PR05Y6	+	0	0	1.0000	199.9305	-2.6612
U	PR11Y6	+	1.0000	0	1.0000	131.2675	23.7982
U	PR14Y6	+	1.0000	0	1.0000	49.2758	8.7840
U	PR15Y6	+	1.0000	0	1.0000	95.3204	10.4422
U	PR16Y6	+	1.0000	0	1.0000	180.9472	5.5012
	PR21Y6	+	0	0	1.0000	191.0447	-20.4008
B	PR23Y6	+	0.8093	0	1.0000	85.2229	0
U	PR01Y7	+	1.0000	0	1.0000	98.5516	13.1655
U	PR04Y7	+	1.0000	0	1.0000	127.2285	31.3665
U	PR14Y7	+	1.0000	0	1.0000	89.6658	3.2261
	PR22Y7	+	0	0	1.0000	44.8329	-8.8970
	PR02Y8	+	0	0	1.0000	74.3176	-4.8664
	PR15Y8	+	0	0	1.0000	65.4318	-2.0391
	PR22Y8	+	0	0	1.0000	44.4290	-4.4821
	PR25Y8	+	0	0	1.0000	147.8274	-1.6646
OBJECTIVE			2471.5725				

EXHIBIT A9.4 FINANCIAL STATEMENTS AND RATIO ANALYSIS FOR A TEN PER CENT INCREASE IN EARNINGS

BALANCE SHEET (£'000S)

	PERIOD-1	PERIOD-2	PERIOD-3	PERIOD-4	PERIOD-5	PERIOD-6	PERIOD-7	PERIOD-8
CAPITAL								
SHARE CAPITAL	2000	2698	2698	2698	2698	2698	2698	2698
SHARE PREMIUM	0	419	419	419	419	419	419	419
TAXATION EQUALISATION	567	1282	1736	2165	2449	2975	3299	3271
RESERVES	1771	2038	2387	3116	3655	4206	5430	6285
LONG TERM DEBT	2225	2225	2841	3841	3634	4495	4495	4495
TOTAL LIABILITIES	6563	8662	10082	12239	12856	14794	16541	17168
ASSETS								
FIXED ASSETS								
LAND AND BUILDINGS	2669	3853	4580	5589	5907	6474	6856	6826
PLANT AND MACHINERY	1474	2274	2715	2974	3208	3807	4059	3777
CURRENT ASSETS								
SHORT TERM DEPOSITS	283	0	0	0	0	628	680	2702
DEBTORS AND STOCK	3972	4841	6243	8271	8415	9525	10793	11110
CURRENT ASSETS	4255	4841	6243	8271	8415	10153	11473	13812
CURRENT LIABILITIES								
CREDITORS	1311	1686	2164	2822	2899	3412	3882	3946
TAX	264	86	593	1036	1016	1127	1533	1992
OVERDRAFT	0	0	0	0	0	0	0	0
DIVIDEND PAYABLE	260	536	698	737	760	1102	432	1309
CURRENT LIABILITIES	1835	2306	3456	4595	4675	5641	5846	7247
NET CURRENT ASSETS	2420	2535	2787	3676	3740	4512	5626	6565
TOTAL ASSETS	6563	8662	10082	12239	12856	14794	16541	17168

DUMP 112

PROFIT AND LOSS STATEMENT (£'000S)

	PERIOD-1	PERIOD-2	PERIOD-3	PERIOD-4	PERIOD-5	PERIOD-6	PERIOD-7	PERIOD-8
SALES	12640	14400	17527	22683	23324	26683	30874	31316
LESS COST OF SALES	10902	12640	15254	19526	20417	23087	26846	27076
TRADING PROFIT	1738	1760	2273	3157	2907	3596	4028	4240
INTEREST ON INVESTMENTS	56	20	0	0	0	0	44	48
LESS INTEREST BANK OVERDRAFTS	12	0	0	0	0	0	0	0
LESS INTEREST LONG TERM DEBT	120	178	178	227	307	291	360	360
PROFIT BEFORE TAX	1662	1602	2095	2930	2600	3305	3712	3928
PROFIT AFTER TAX	831	801	1048	1465	1300	1653	1856	1964
DIVIDEND	260	534	698	737	760	1102	432	1309
ADDED TO RESERVES	571	267	349	728	540	551	1424	655

CASH FLOW STATEMENT

	PERIOD-1	PERIOD-2	PERIOD-3	PERIOD-4	PERIOD-5	PERIOD-6	PERIOD-7	PERIOD-8
SOURCES								
TRADING INCOME	1789	2141	2393	2948	4088	4464	4790	5454
INVESTMENT INCOME	56	20	0	0	0	0	44	48
INCREASE IN LONG TERM DEBT	725	0	616	1000	-206	861	0	0
RIGHTS ISSUES	0	1117	0	0	0	0	0	0
TOTAL CASH INFLOW	2570	3278	3009	3948	3881	5325	4834	5501
USES								
BUILDING AND LAND	1116	1301	865	1179	498	763	590	177
PLANT AND EQUIPMENT	1085	1558	1346	1251	1303	1868	1605	978
INTEREST OVERDRAFT	12	0	0	0	0	0	0	0
INTEREST LONG TERM DEBT	120	176	178	227	307	291	360	360
DIVIDEND PAYMENTS	285	260	534	698	737	760	1102	432
TAX PAID	370	264	86	593	1036	1016	1127	1533
TOTAL CASH OUTFLOW	2988	3561	3009	3948	3881	4697	4782	3479
NET CHANGE IN CASH POSITION	-417	-282	0	0	0	628	52	2022

DUMP 112

ACHIEVEMENT LEVELS

		PERIOD-1	PERIOD-2	PERIOD-3	PERIOD-4	PERIOD-5	PERIOD-6	PERIOD-7	PERIOD-8
RETURN	TARGET	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0
ON	ACHIEVEMENT	27.2	20.5	22.5	25.8	22.6	24.3	24.6	25.0
CAPITAL	PERCENT DEVIATION	50.8	14.2	25.3	43.3	25.6	35.0	36.8	38.8
	TARGET	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8
LIQUIDITY	ACHIEVEMENT	2.3	2.1	1.8	1.8	1.8	1.8	2.0	1.9
	PERCENT DEVIATION	28.8	16.6	0.4	0.0	-0.0	-0.0	9.0	5.9
TIMES	TARGET	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
COVERED	ACHIEVEMENT	10.0	10.0	10.0	10.3	10.0	10.0	11.3	11.9
	PERCENT DEVIATION	0.1	0.0	-0.0	2.7	0.0	0.0	13.2	19.2
EARNINGS	TARGET	20.0	20.0	21.0	21.0	22.0	22.0	24.0	24.0
PER	ACHIEVEMENT	41.5	29.7	38.8	54.3	48.2	61.2	68.8	72.8
SHARE	PERCENT DEVIATION	107.7	48.4	84.9	158.5	119.0	178.4	186.6	203.3
DIVIDEND	TARGET	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
COVER	ACHIEVEMENT	3.2	1.5	1.5	2.0	1.7	1.5	4.3	1.5
	PERCENT DEVIATION	113.1	-0.0	0.0	32.6	14.0	0.0	186.6	0.0
DIVIDEND	TARGET	13.0	13.5	14.0	14.5	15.0	15.5	16.0	16.5
PER	ACHIEVEMENT	13.0	19.8	25.9	27.3	28.2	40.8	16.0	48.5
SHARE	PERCENTAGE DEVIATION	0.0	46.6	84.9	88.3	87.9	163.4	0.0	194.1

PROBLEM OPTIMODEL-86 SOLUTION DATE 06/08/76 TIME 1

DUMP:DUMP 114

RIGHT HAND SIDE RHS1 -0.1000*RHS2
OBJECTIVE COBJ +0*OBJ6
LOWER BOUND LOBND
UPPER BOUND UPBND

ROW SET -(DUMMY) COL SET

COLUMN INFORMATION

	NAME		VALUE	LOWER BOUND	UPPER BOUND	OBJECTIVE	REDUCED COST
B	SALES1	+	12482.7500	0		0	0
B	SALES2	+	13560.0120	0		0	0
B	SALES3	+	15448.9047	0		0	0
B	SALES4	+	18952.0791	0		0	0
B	SALES5	+	19021.0031	0		0	0
B	SALES6	+	21781.4578	0		0	0
B	SALES7	+	24530.0278	0		0	0
B	SALES8	+	25123.7078	0		0	0
B	EARN1	+	1405.5257	0		0	0
B	EARN2	+	1430.1018	0		0	0
B	EARN3	+	1827.1583	0		0	0
B	EARN4	+	2420.0022	0		0	0
B	EARN5	+	2137.5184	0		0	0
B	EARN6	+	2690.3053	0		0	0
B	EARN7	+	2832.8759	0		0	0
B	EARN8	+	3036.2934	0		0	0
B	NPAT1	+	662.7018	0		0	0
B	NPAT2	+	664.7747	0		0	0
B	NPAT3	+	865.3050	0		0	0
B	NPAT4	+	1121.4632	0		0	0
B	NPAT5	+	967.4591	0		0	0
B	NPAT6	+	1261.2757	0		0	0
B	NPAT7	+	1265.5200	0		0	0
B	NPAT8	+	1375.5029	0		0	0
B	TAX1	+	133.3702	0		0	0
B	TAX2	+	94.6094	0		0	0
B	TAX3	+	499.8206	0		0	0
B	TAX4	+	710.5763	0		0	0
B	TAX5	+	744.4364	0		0	0
B	TAX6	+	774.1972	0		0	0
B	TAX7	+	935.8309	0		0	0
B	TAX8	+	1343.8622	0		0	0
B	FABL1	+	2604.4600	0		0	0
B	FABL2	+	3550.5094	0		0	0
B	FABL3	+	4190.6206	0		0	0
B	FABL4	+	5106.3755	0		0	0
B	FABL5	+	5149.2878	0		0	0
B	FABL6	+	5680.8598	0		0	0
B	FABL7	+	6056.0047	0		0	0
B	FABL8	+	6069.5191	0		0	0
B	FAPE1	+	1436.8421	0		0	0
B	FAPE2	+	1970.1200	0		0	0
B	FAPE3	+	2223.0526	0		0	0
B	FAPE4	+	2485.8828	0		0	0
B	FAPE5	+	2660.8098	0		0	0
B	FAPE6	+	3702.2130	0		0	0
B	FAPE7	+	3530.6796	0		0	0
B	FAPE8	+	3380.2102	0		0	0
B	CURA1	+	3931.0000	0		0	0
B	CURA2	+	4555.4922	0		0	0
B	CURA3	+	5580.9346	0		0	0
B	CURA4	+	6866.1800	0		0	0
B	CURA5	+	6704.6681	0		0	0
B	CURA6	+	7748.0265	0		0	0
B	CURA7	+	8576.4375	0		0	0
B	CURA8	+	10205.6381	0		0	0
B	CURL1	+	1713.0615	0		0	0
B	CURL2	+	2110.9915	0		0	0
B	CURL3	+	2846.6598	0		0	0
B	CURL4	+	3798.0313	0		0	0
B	CURL5	+	3701.0412	0		0	0
B	CURL6	+	4304.4592	0		0	0
B	CURL7	+	4394.1936	0		0	0
B	CURL8	+	5376.2975	0		0	0
	OVDR1	+	0	0	250.0000	0	-0.0845
B	OVDR2	+	20.4818	0	250.0000	0	0
	OVDR3	+	0	0	250.0000	0	-0.0424
	OVDR4	+	0	0	250.0000	0	-0.0526
	OVDR5	+	0	0	250.0000	0	-0.0594
	OVDR6	+	0	0	250.0000	0	-0.0128
	OVDR7	+	0	0	250.0000	0.0231	-0.0100
	OVDR8	+	0	0	250.0000	-0.4039	0

	MARK1	+	0	0		0	-0.0209
	MARK2	+	0	0		0	-0.1607
	MARK3	+	0	0		0	-0.0418
	MARK4	+	0	0		0	-0.0412
	MARK5	+	0	0		0	-0.1024
B	MARK6	+	125.3335	0		0	0
	MARK7	+	0	0		-0.0153	-0.0068
B	MARK8	+	1348.6906	0		0.4039	0
B	DV1	+	285.6652	0		0.7972	0
B	DV2	+	429.8498	0		0.7118	0
B	DV3	+	504.8848	0		0.6355	0
B	DV4	+	747.7621	0		0.5674	0
B	DV5	+	631.6394	0		0.5066	0
B	DV6	+	827.5172	0		0.4523	0
B	DV7	+	655.6533	0		0.4039	0
B	DV8	+	917.0019	0		0	0
B	NUM1	+	2197.4249	0		0	0
B	NUM2	+	2847.8332	0		0	0
B	NUM3	+	2847.8332	0		0	0
B	NUM4	+	2847.8332	0		0	0
B	NUM5	+	2847.8332	0		0	0
B	NUM6	+	2847.8332	0		0	0
B	NUM7	+	2847.8332	0		0	0
B	NUM8	+	2847.8332	0		0	0
B	RG1	+	197.4249	0	800.0000	-1.4280	0
B	RG2	+	650.4082	0	800.0000	-1.2760	0
	RG3	+	0	0	800.0000	-1.1339	-0.0037
	RG4	+	0	0	800.0000	-1.0170	-0.00020
	RG5	+	0	0	800.0000	-0.9070	-0.0106
	RG6	+	0	0	800.0000	-0.8110	-0.1049
	RG7	+	0	0	800.0000	-0.7240	-0.0462
	RG8	+	0	0	800.0000	-0.6464	-0.00016
B	DE1	+	1756.9046	0		0	0
B	DE2	+	1756.9046	0		0	0
B	DE3	+	2283.9479	0		0	0
B	DE4	+	3032.5027	0		0	0
B	DE5	+	2671.9980	0		0	0
B	DE6	+	3277.9475	0		0	0
B	DE7	+	3541.0948	0		0.0155	0
B	DE8	+	3541.0948	0		-0.4039	0
B	LL1	+	756.9046	0	1000.0000	0	0
	LL2	+	0	0	1000.0000	0	-0.0190
B	LL3	+	527.0433	0	1000.0000	0	0
B	LL4	+	748.5549	0	1000.0000	0	0
B	LL5	+	639.3953	0	1000.0000	0	0
B	LL6	+	606.0645	0	1000.0000	0	0
B	LL7	+	263.1473	0	1000.0000	0	0
	LL8	+	0	0	1000.0000	0	0
B	BLTA1	+	36.8631	0		0	0
B	BLTA2	+	82.2504	0		0	0
B	BLTA3	+	115.0765	0		0	0
B	BLTA4	+	154.0549	0		0	0
B	BLTA5	+	161.4407	0		0	0
B	BLTA6	+	188.3840	0		0	0
B	BLTA7	+	209.4462	0		0	0
B	BLTA8	+	215.2363	0		0	0
U	PR01Y1	+	1.0000	0	1.0000	-44.4290	5.6210
U	PR04Y1	+	1.0000	0	1.0000	-21.8106	56.5381
	PR12Y1	+	0	0	1.0000	-33.1198	-2.4798
U	PR13Y1	+	1.0000	0	1.0000	0	58.4561
	PR16Y1	+	0	0	1.0000	-31.2215	-48.4146
U	PR22Y1	+	1.0000	0	1.0000	-18.3371	16.1243
	PR23Y1	+	0	0	1.0000	-17.6908	-11.9470
B	PR03Y2	+	0.6342	0	1.0000	18.3774	0
	PR04Y2	+	0	0	1.0000	-2.1407	-3.7291
	PR05Y2	+	0	0	1.0000	15.9540	-61.9259
U	PR13Y2	+	1.0000	0	1.0000	1.2117	35.1857
U	PR14Y2	+	1.0000	0	1.0000	-47.2563	46.7496
	PR21Y2	+	0	0	1.0000	23.9513	-51.0586

PROBLEM OPTIMODEL-[illegible]　　SOLUTION　　DATE [illegible]

DUMP:DUMP 114

RIGHT HAND SIDE　RHS1 -0.1000*RHS2
OBJECTIVE　COBJ +0*OBJ6
LOWER BOUND　LOBND
UPPER BOUND　UPBND

ROW SET　-(DUMMY　)　COL SET

COLUMN INFORMATION

	NAME		VALUE	LOWER BOUND	UPPER BOUND	OBJECTIVE	REDUCED COST
	PR24Y2	*	0	0	1.0000	-7.2702	-14.8844
	PR02Y3	*	0	0	1.0000	32.9986	-2.3615
B	PR11Y3	*	0.6317	0	1.0000	46.7716	0
	PR15Y3	*	0	0	1.0000	43.4596	-6.6178
U	PR01Y4	*	1.0000	0	1.0000	126.0168	14.4810
	PR05Y4	*	0	0	1.0000	117.9388	-39.1422
	PR11Y4	*	0	0	1.0000	83.2036	-17.1277
U	PR12Y4	*	1.0000	0	1.0000	144.1923	1.9441
U	PR13Y4	*	1.0000	0	1.0000	42.8134	18.5232
U	PR14Y4	*	1.0000	0	1.0000	-17.0850	5.8786
U	PR22Y4	*	1.0000	0	1.0000	33.7660	6.6165
U	PR25Y4	*	1.0000	0	1.0000	51.6992	10.0112
	PR02Y5	*	0	0	1.0000	86.6435	-5.9873
B	PR03Y5	*	0.1020	0	1.0000	151.0586	0
	PR11Y5	*	0	0	1.0000	101.7828	-8.6786
	PR21Y5	*	0	0	1.0000	130.0558	-44.4106
	PR23Y5	*	0	0	1.0000	71.8942	-18.0508
U	PR04Y6	*	1.0000	0	1.0000	112.6881	39.2808
	PR05Y6	*	0	0	1.0000	199.9305	-4.1634
U	PR11Y6	*	1.0000	0	1.0000	131.2675	23.7719
U	PR14Y6	*	1.0000	0	1.0000	49.2758	8.8082
U	PR15Y6	*	1.0000	0	1.0000	95.3204	9.4177
U	PR16Y6	*	1.0000	0	1.0000	180.9472	4.2780
	PR21Y6	*	0	0	1.0000	191.0447	-20.5052
B	PR23Y6	*	0.3359	0	1.0000	85.2229	0
U	PR01Y7	*	1.0000	0	1.0000	98.5516	12.4758
U	PR04Y7	*	1.0000	0	1.0000	127.2285	28.8576
U	PR14Y7	*	1.0000	0	1.0000	89.6658	2.1304
	PR22Y7	*	0	0	1.0000	44.8329	-9.4021
	PR02Y8	*	0	0	1.0000	74.3176	-4.8664
	PR15Y8	*	0	0	1.0000	65.4318	-2.0391
	PR22Y8	*	0	0	1.0000	44.4290	-4.4821
	PR25Y8	*	0	0	1.0000	147.8274	-1.6646

OBJECTIVE　　1435.6682

EXHIBIT A9.6 FINANCIAL STATEMENTS FOR A TEN PER CENT INCREASE IN EARNINGS

BALANCE SHEET (£'000S)

	PERIOD-1	PERIOD-2	PERIOD-3	PERIOD-4	PERIOD-5	PERIOD-6	PERIOD-7	PERIOD-8
CAPITAL								
SHARE CAPITAL	2197	2848	2848	2848	2848	2848	2848	2848
SHARE PREMIUM	119	509	509	509	509	509	509	509
TAXATION EQUALISATION	509	1060	1403	1808	2011	2478	2828	2859
RESERVES	1557	1772	2110	2464	2800	3214	4044	4502
LONG TERM DEBT	1757	1757	2284	3033	2672	3278	3541	3541
TOTAL LIABILITIES	6139	7945	9154	10682	10840	12327	13769	14259
ASSETS								
FIXED ASSETS								
LAND AND BUILDINGS	2485	3530	6197	5108	5149	5681	6056	6050
PLANT AND MACHINERY	1437	1970	2223	2486	2687	3202	3531	3380
CURRENT ASSETS								
SHORT TERM DEPOSITS	0	0	0	0	0	125	0	1349
DEBTORS AND STOCK	3931	4556	5581	6886	6705	7623	8576	8857
CURRENT ASSETS	3931	4556	5581	6886	6705	7748	8576	10206
CURRENT LIABILITIES								
CREDITORS	1294	1566	1842	2335	2325	2703	3003	3115
TAX	133	95	500	717	744	774	936	1344
OVERDRAFT	0	21	0	0	0	0	0	0
DIVIDEND PAYABLE	286	430	505	748	632	828	456	917
CURRENT LIABILITIES	1713	2111	2847	3799	3701	4305	4394	5376
NET CURRENT ASSETS	2218	2445	2734	3087	3004	3444	4182	4829
TOTAL ASSETS	6139	7945	9154	10682	10840	12327	13769	14259

DUMP 113

ACHIEVEMENT LEVELS

		PERIOD-1	PERIOD-2	PERIOD-3	PERIOD-4	PERIOD-5	PERIOD-6	PERIOD-7	PERIOD-8
RETURN	TARGET	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0
ON	ACHIEVEMENT	22.9	18.0	20.0	22.7	19.7	21.9	20.6	21.3
CAPITAL	PERCENT DEVIATION	27.2	-0.0	10.9	26.7	9.6	21.5	14.3	18.2
	TARGET	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8
LIQUIDITY	ACHIEVEMENT	2.3	2.2	2.0	1.8	1.8	1.8	2.0	1.9
	PERCENT DEVIATION	27.5	19.9	8.9	0.7	0.6	-0.0	8.4	5.5
TIMES	TARGET	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
COVERED	ACHIEVEMENT	10.0	10.0	10.0	10.0	10.0	10.3	10.0	10.7
	PERCENT DEVIATION	0.0	-0.0	-0.0	0.0	0.0	2.8	0.0	7.1
EARNINGS	TARGET	20.0	20.0	21.0	21.0	22.0	22.0	24.0	24.0
PER	ACHIEVEMENT	29.3	22.6	29.6	39.4	33.3	43.6	45.1	48.3
SHARE	PERCENT DEVIATION	46.3	13.2	41.0	87.6	51.2	98.1	88.1	101.2
DIVIDEND	TARGET	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
COVER	ACHIEVEMENT	2.3	1.5	1.7	1.5	1.5	1.5	2.8	1.5
	PERCENT DEVIATION	50.0	0.0	11.4	0.0	0.0	-0.0	88.1	0.0
DIVIDEND	TARGET	13.0	13.5	14.0	14.5	15.0	15.5	16.0	16.5
PER	ACHIEVEMENT	13.0	15.1	17.7	26.3	22.2	29.1	16.0	32.2
SHARE	PERCENTAGE DEVIATION	-0.0	11.8	26.6	81.1	47.9	87.5	-0.0	95.2

PROFIT AND LOSS STATEMENT (£'000S)

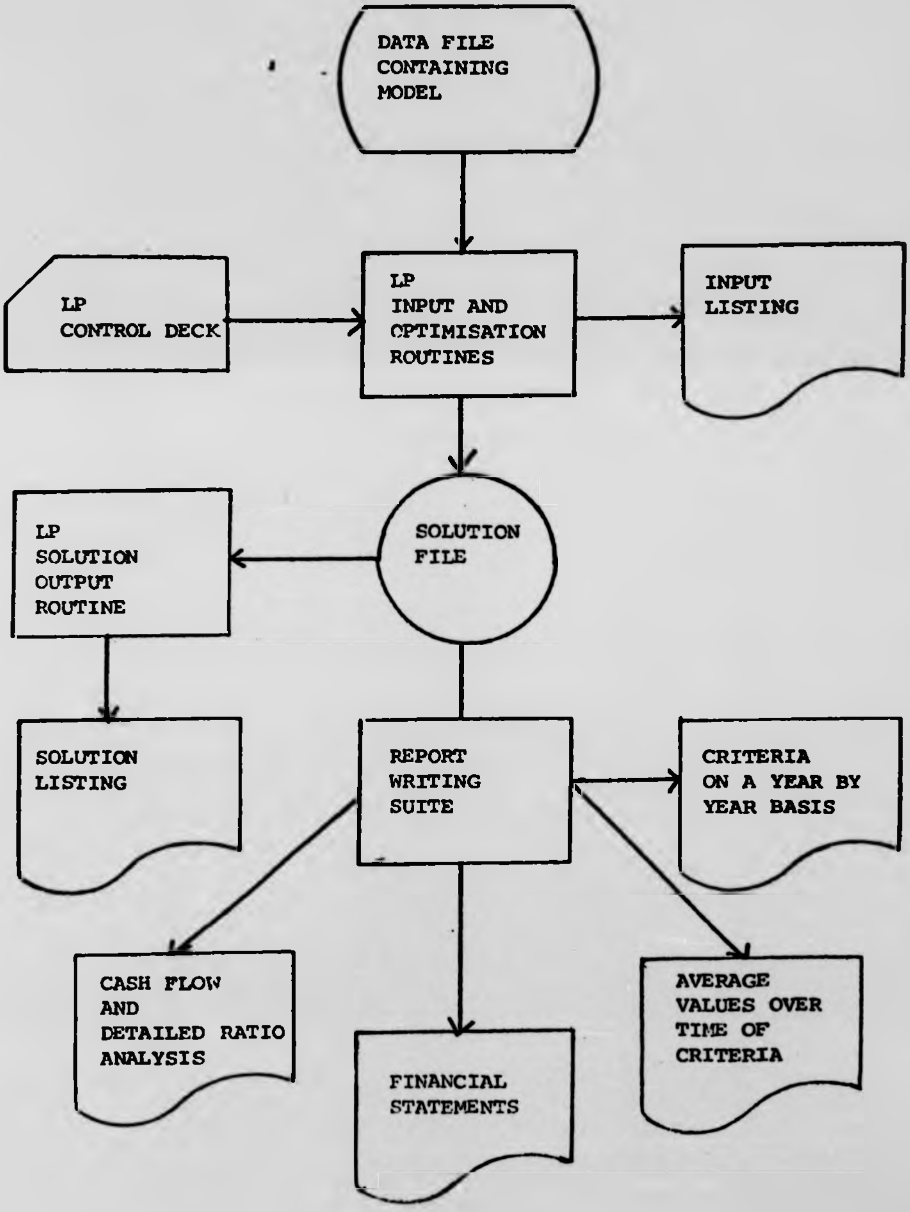
	PERIOD-1	PERIOD-2	PERIOD-3	PERIOD-4	PERIOD-5	PERIOD-6	PERIOD-7	PERIOD-8
SALES	12482	13560	15449	18952	19021	21782	24531	25124
LESS COST OF SALES	11121	12130	13619	16526	16884	19085	21707	22089
TRADING PROFIT	1362	1430	1830	2426	2138	2696	2824	3034
INTEREST ON INVESTMENTS	56	0	0	0	0	0	9	0
LESS INTEREST BANK OVERDRAFTS	12	0	3	0	0	0	0	0
LESS INTEREST LONG TERM DEBT	120	141	141	183	243	214	262	283
PROFIT BEFORE TAX	1286	1290	1687	2243	1895	2483	2571	2751
PROFIT AFTER TAX	643	645	843	1122	948	1241	1285	1376
DIVIDEND	286	430	505	748	632	828	456	917
ADDED TO RESERVES	357	215	338	374	316	414	830	459

CASH FLOW STATEMENT

	PERIOD-1	PERIOD-2	PERIOD-3	PERIOD-4	PERIOD-5	PERIOD-6	PERIOD-7	PERIOD-8
SOURCES								
TRADING INCOME	1419	1841	1948	2597	3361	3396	3531	4176
INVESTMENT INCOME	56	0	0	0	0	0	9	0
INCREASE IN LONG TERM DEBT	257	0	527	749	-360	606	263	0
RIGHTS ISSUES	316	1041	0	0	0	0	0	0
TOTAL CASH INFLOW	2048	2882	2475	3345	3000	4002	3803	4176
USES								
BUILDING AND LAND	926	1153	793	1067	197	704	559	177
PLANT AND EQUIPMENT	1035	1190	994	1091	1096	1583	1505	976
INTEREST OVERDRAFT	12	0	3	0	0	0	0	0
INTEREST LONG TERM DEBT	120	141	141	183	243	214	262	283
DIVIDEND PAYMENTS	285	286	430	505	748	632	828	456
TAX PAID	370	133	95	500	717	744	774	936
TOTAL CASH OUTFLOW	2748	2902	2455	3345	3000	3876	3928	2828
NET CHANGE IN CASH POSITION	-699	-20	21	0	0	125	-124	1349

Appendix X

A Systems Flow Chart - The Multicriteria Model



APPENDIX XI INPUT DATA-MULTICRITERIA MODEL

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PROBLEM' GOALMODEL-05          LIST(1)                          DATE 20/07/76    TIME 12/11/22     PAGE 0001

                               ROW SET (########)               COLUMN SET (########)

PROBLEM SIZE

NUMBER OF ROWS:-

   COLUMN SET BATCHES                        1
   COLUMN SCALING FACTORS                    0
   BOUNDS                                    2
   KERNEL ROWS                             238
       OBJECTIVES                           10
       EQUALITY CONSTRAINTS                112
       CONVEXITY CONSTRAINTS                 0
       LESS OR EQUAL CONSTRAINTS            46
       GREATER OR EQUAL CONSTRAINTS         72
       TOTAL NUMBER OF CONSTRAINTS         228

NUMBER OF COLUMNS:-

   ROW SET BATCHES                           1
   ROW SCALING FACTORS                       0
   VARIABLES                               198
       CONTINUOUS VARIABLES                198
           NON-NEGATIVE VARIABLES          198
           NON-POSITIVE VARIABLES            0
           FREE VARIABLES                    0
           ZERO VARIABLES                    0
       INTEGER VARIABLES                     0
           NON-NEGATIVE VARIABLES            0
           NON-POSITIVE VARIABLES            0
           FREE VARIABLES                    0
           ZERO VARIABLES                    0
           SPECIAL VARIABLES                 0
       SPECIAL ORDERED SET VARIABLES         0
           TYPE 1 VARIABLES                  0
           TYPE 2 VARIABLES                  0
   RIGHT HAND SIDES                          1
   RANGES                                    0
```

ROW SET (########) COLUMN SET (########)

ROW NAMES AND NUMBER OF NON-ZERO ELEMENTS

KERNEL ROWS

BNDDCOV7 -	2	BNDDCOV8 -	2	BNDDVPS1 -	2	BNDDVPS2 -	2	BNDDVPS3 -	2	BNDDVPS4 -	2
BNDDVPS5 -	2	BNDDVPS6 -	2	BNDDVPS7 -	2	BNDDVPS8 -	2				

COLUMN NAMES AND NUMBER OF NON-ZERO ELEMENTS

ROW SETS

ROWYR	.02	15	ROWYR	.03	25	ROWYR	.04	43	ROWYR	.05	62	ROWYR	.06	79	ROWYR	.07	97
ROWYR	.08	115	ROWYR	.10	56	ROWYR	.24	36									

VARIABLES

SALES1	+	2	SALES2	+	2	SALES3	+	2	SALES4	+	2	SALES5	+	2	SALES6	+	2
SALES7	+	2	SALES8	+	2	EARN1	+	7	EARN2	+	7	EARN3	+	7	EARN4	+	7
EARN5	+	7	EARN6	+	7	EARN7	+	7	EARN8	+	7	NPAT1	+	7	NPAT2	+	7
NPAT3	+	7	NPAT4	+	7	NPAT5	+	7	NPAT6	+	7	NPAT7	+	7	NPAT8	+	7
TAX1	+	3	TAX2	+	3	TAX3	+	3	TAX4	+	3	TAX5	+	3	TAX6	+	3
TAX7	+	3	TAX8	+	3	FABL1	+	10	FABL2	+	10	FABL3	+	10	FABL4	+	10
FABL5	+	10	FABL6	+	10	FABL7	+	10	FABL8	+	7	FAPE1	+	8	FAPE2	+	8
FAPE3	+	8	FAPE4	+	8	FAPE5	+	8	FAPE6	+	8	FAPE7	+	8	FAPE8	+	6
CURA1	+	6	CURA2	+	6	CURA3	+	6	CURA4	+	6	CURA5	+	6	CURA6	+	6
CURA7	+	6	CURA8	+	6	CURL1	+	6	CURL2	+	6	CURL3	+	6	CURL4	+	6
CURL5	+	6	CURL6	+	6	CURL7	+	6	CURL8	+	6	OVDR1	+	3	OVDR2	+	3
OVDR3	+	3	OVDR4	+	3	OVDR5	+	3	OVDR6	+	3	OVDR7	+	3	OVDR8	+	2
MARK1	+	2	MARK2	+	2	MARK3	+	2	MARK4	+	2	MARK5	+	2	MARK6	+	2
MARK7	+	2	MARK8	+	1	ASSETS1	+	3	ASSETS2	+	3	ASSETS3	+	3	ASSETS4	+	3
ASSETS5	+	3	ASSETS6	+	3	ASSETS7	+	3	ASSETS8	+	3	INTR1	+	3	INTR2	+	3
INTR3	+	3	INTR4	+	3	INTR5	+	3	INTR6	+	3	INTR7	+	3	INTR8	+	3
DV1	+	6	DV2	+	6	DV3	+	6	DV4	+	6	DV5	+	6	DV6	+	6
DV7	+	6	DV8	+	6	NUM1	+	6	NUM2	+	6	NUM3	+	6	NUM4	+	6
NUM5	+	6	NUM6	+	6	NUM7	+	6	NUM8	+	5	RG1	+	2	RG2	+	2
RG3	+	2	RG4	+	2	RG5	+	2	RG6	+	2	RG7	+	2	RG8	+	2
DE1	+	5	DE2	+	5	DE3	+	5	DE4	+	5	DE5	+	5	DE6	+	5
DE7	+	5	DE8	+	2	LL1	+	2	LL2	+	2	LL3	+	2	LL4	+	2
LL5	+	2	LL6	+	2	LL7	+	2	LL8	+	2	BLTA1	+	3	BLTA2	+	3
BLTA3	+	3	BLTA4	+	3	BLTA5	+	3	BLTA6	+	3	BLTA7	+	3	BLTA8	+	2
RET	+	13	PRF	+	12	SAL	+	12	EPS	+	12	LOD	+	12	INT	+	13
DVP	+	12	DVC	+	11	PR01Y1	+	29	PR04Y1	+	36	PR12Y1	+	40	PR13Y1	+	36
PR16Y1	+	36	PR22Y1	+	38	PR23Y1	+	36	PR03Y2	+	31	PR04Y2	+	32	PR05Y2	+	32
PR13Y2	+	32	PR14Y2	+	37	PR21Y2	+	32	PR24Y2	+	34	PR02Y3	+	27	PR11Y3	+	27
PR15Y3	+	28	PR01Y4	+	17	PR05Y4	+	24	PR11Y4	+	23	PR12Y4	+	26	PR13Y4	+	24
PR14Y4	+	27	PR22Y4	+	25	PR25Y4	+	26	PR02Y5	+	19	PR03Y5	+	19	PR11Y5	+	19
PR21Y5	+	20	PR23Y5	+	20	PR04Y6	+	16	PR05Y6	+	16	PR11Y6	+	15	PR14Y6	+	17
PR15Y6	+	16	PR16Y6	+	16	PR21Y6	+	16	PR23Y6	+	16	PR01Y7	+	3	PR04Y7	+	12
PR14Y7	+	12	PR22Y7	+	11	PR02Y8	+	6	PR15Y8	+	6	PR22Y8	+	6	PR25Y8	+	6

ROW SET ROWYR .10 COLUMN SET COLYR .10

PROBLEM SIZE

NUMBER OF ROWS:-

COLUMN SET BATCHES	1
COLUMN SCALING FACTORS	0
BOUNDS	2
KERNEL ROWS	56
OBJECTIVES	0
EQUALITY CONSTRAINTS	2
CONVEXITY CONSTRAINTS	0
LESS OR EQUAL CONSTRAINTS	40
GREATER OR EQUAL CONSTRAINTS	14
TOTAL NUMBER OF CONSTRAINTS	56

NUMBER OF COLUMNS:-

ROW SET BATCHES	1
ROW SCALING FACTORS	0
VARIABLES	58
CONTINUOUS VARIABLES	58
NON-NEGATIVE VARIABLES	58
NON-POSITIVE VARIABLES	0
FREE VARIABLES	0
ZERO VARIABLES	0
INTEGER VARIABLES	0
NON-NEGATIVE VARIABLES	0
NON-POSITIVE VARIABLES	0
FREE VARIABLES	0
ZERO VARIABLES	0
SPECIAL VARIABLES	0
SPECIAL ORDERED SET VARIABLES	0
TYPE 1 VARIABLES	0
TYPE 2 VARIABLES	0
RIGHT HAND SIDES	0
RANGES	0

ROW SET (########) COLUMN SET (########)

ROW NAMES AND NUMBER OF NON-ZERO ELEMENTS

KERNEL ROWS

BNDDCOV7 - 2	BNDDCOV8 - 2	BNDDVPS1 - 2	BNDDVPS2 - 2	BNDDVPS3 - 2	BNDDVPS4 - 2
BNDDVPS5 - 2	BNDDVPS6 - 2	BNDDVPS7 - 2	BNDDVPS8 - 2		

COLUMN NAMES AND NUMBER OF NON-ZERO ELEMENTS

ROW SETS

ROWYR .02 15	ROWYR .03 25	ROWYR .04 43	ROWYR .05 62	ROWYR .06 79	ROWYR .07 97
ROWYR .08 115	ROWYR .10 56	ROWYR .24 36			

VARIABLES

SALES1 + 2	SALES2 + 2	SALES3 + 2	SALES4 + 2	SALES5 + 2	SALES6 + 2
SALES7 + 2	SALES8 + 2	EARN1 + 7	EARN2 + 7	EARN3 + 7	EARN4 + 7
EARN5 + 7	EARN6 + 7	EARN7 + 7	EARN8 + 7	NPAT1 + 7	NPAT2 + 7
NPAT3 + 7	NPAT4 + 7	NPAT5 + 7	NPAT6 + 7	NPAT7 + 7	NPAT8 + 7
TAX1 + 3	TAX2 + 3	TAX3 + 3	TAX4 + 3	TAX5 + 3	TAX6 + 3
TAX7 + 3	TAX8 + 3	FABL1 + 10	FABL2 + 10	FABL3 + 10	FABL4 + 10
FABL5 + 10	FABL6 + 10	FABL7 + 10	FABL8 + 7	FAPE1 + 8	FAPE2 + 8
FAPE3 + 8	FAPE4 + 8	FAPE5 + 8	FAPE6 + 8	FAPE7 + 8	FAPE8 + 6
CURA1 + 6	CURA2 + 6	CURA3 + 6	CURA4 + 6	CURA5 + 6	CURA6 + 6
CURA7 + 6	CURA8 + 6	CURL1 + 6	CURL2 + 6	CURL3 + 6	CURL4 + 6
CURL5 + 6	CURL6 + 6	CURL7 + 6	CURL8 + 6	OVDR1 + 3	OVDR2 + 3
OVDR3 + 3	OVDR4 + 3	OVDR5 + 3	OVDR6 + 3	OVDR7 + 3	OVDR8 + 2
MARK1 + 2	MARK2 + 2	MARK3 + 2	MARK4 + 2	MARK5 + 2	MARK6 + 2
MARK7 + 2	MARK8 + 1	ASSETS1 + 3	ASSETS2 + 3	ASSETS3 + 3	ASSETS4 + 3
ASSETS5 + 3	ASSETS6 + 3	ASSETS7 + 3	ASSETS8 + 3	INTR1 + 3	INTR2 + 3
INTR3 + 3	INTR4 + 3	INTR5 + 3	INTR6 + 3	INTR7 + 3	INTR8 + 3
DV1 + 6	DV2 + 6	DV3 + 6	DV4 + 6	DV5 + 6	DV6 + 6
DV7 + 6	DV8 + 6	NUM1 + 6	NUM2 + 6	NUM3 + 6	NUM4 + 6
NUM5 + 6	NUM6 + 6	NUM7 + 6	NUM8 + 5	RG1 + 2	RG2 + 2
RG3 + 2	RG4 + 2	RG5 + 2	RG6 + 2	RG7 + 2	RG8 + 2
DE1 + 5	DE2 + 5	DE3 + 5	DE4 + 5	DE5 + 5	DE6 + 5
DE7 + 5	DE8 + 2	LL1 + 2	LL2 + 2	LL3 + 2	LL4 + 2
LL5 + 2	LL6 + 2	LL7 + 2	LL8 + 2	BLTA1 + 3	BLTA2 + 3
BLTA3 + 3	BLTA4 + 3	BLTA5 + 3	BLTA6 + 3	BLTA7 + 3	BLTA8 + 2
RET + 13	PRF + 12	SAL + 12	EPS + 12	LOD + 12	INT + 13
DVP + 12	DVC + 11	PR01Y1 + 29	PR04Y1 + 36	PR12Y1 + 40	PR13Y1 + 36
PR16Y1 + 36	PR22Y1 + 38	PR23Y1 + 36	PR03Y2 + 31	PR04Y2 + 32	PR05Y2 + 32
PR13Y2 + 32	PR14Y2 + 37	PR21Y2 + 32	PR24Y2 + 34	PR02Y3 + 27	PR11Y3 + 27

ROW SET (########) COLUMN SET (########)

ROW NAMES AND NUMBER OF NON-ZERO ELEMENTS

COLUMN SETS

COLYR	.02	23	COLYR	.03	42	COLYR	.04	66	COLYR	.05	87	COLYR	.06	111	COLYR	.07	131
COLYR	.08	151	COLYR	.10	58												

BOUNDS

UPBND	78	LOBND	0

KERNEL ROWS

NW8	+	4	GOAL	Z	8	OBJ	Z	4	OBJ1	Z	6	OBJ2	Z	4	OBJ3	Z	2
OBJ4	Z	1	OBJ5	Z	2	OBJ6	Z	2	OBJ7	Z	2	OBJ8	Z	2	TS1	=	7
TS2	=	14	TS3	=	18	TS4	=	25	TS5	=	30	TS6	=	39	TS7	=	42
TS8	=	46	EA1	=	8	EA2	=	18	EA3	=	21	EA4	=	28	EA5	=	33
EA6	=	41	EA7	=	46	EA8	=	50	BL1	=	8	BL2	=	13	BL3	=	8
BL4	=	12	BL5	=	10	BL6	=	12	BL7	=	12	BL8	=	9	PF1	=	7
PF2	=	16	PF3	=	17	PF4	=	19	PF5	=	19	PF6	=	25	PF7	=	21
PF8	=	18	CA1	=	8	CA2	=	15	CA3	=	19	CA4	=	26	CA5	=	31
CA6	=	40	CA7	=	43	CA8	=	47	CL1	=	10	CL2	=	17	CL3	=	21
CL4	=	28	CL5	=	33	CL6	=	42	CL7	=	45	CL8	=	49	CB1	=	9
CB2	=	14	CB3	=	14	CB4	=	14	CB5	=	14	CB6	=	14	CB7	=	14
CB8	=	14	TP1	=	5	TP2	=	7	TP3	=	7	TP4	=	7	TP5	=	7
TP6	=	7	TP7	=	7	TP8	=	7	TA1	=	2	TA2	=	4	TA3	=	4
TA4	=	4	TA5	=	4	TA6	=	4	TA7	=	4	TA8	=	4	PR1	=	2
PR2	=	3	PR3	=	3	PR4	=	3	PR5	=	3	PR6	=	3	PR7	=	3
PR8	=	3	EQ1	=	2	EQ2	=	3	EQ3	=	3	EQ4	=	3	EQ5	=	3
EQ6	=	3	EQ7	=	3	EQ8	=	3	D1	=	2	D2	=	3	D3	=	3
D4	=	3	D5	=	3	D6	=	3	D7	=	3	D8	=	3	ROCE1	+	3
ROCE2	+	3	ROCE3	+	3	ROCE4	+	3	ROCE5	+	3	ROCE6	+	3	ROCE7	+	3
ROCE8	+	3	LODY1	+	3	LODY2	+	3	LODY3	+	3	LODY4	+	3	LODY5	+	3
LODY6	+	3	LODY7	+	3	LODY8	+	3	ECOV1	+	3	ECOV2	+	3	ECOV3	+	3
ECOV4	+	3	ECOV5	+	3	ECOV6	+	3	ECOV7	+	3	ECOV8	+	3	ERPS1	+	3
ERPS2	+	3	ERPS3	+	3	ERPS4	+	3	ERPS5	+	3	ERPS6	+	3	ERPS7	+	3
ERPS8	+	3	STARG1	-	2	STARG2	-	2	STARG3	-	2	STARG4	-	2	STARG5	-	2
STARG6	-	2	STARG7	-	2	STARG8	-	2	PTARG1	-	2	PTARG2	-	2	PTARG3	-	2
PTARG4	-	2	PTARG5	-	2	PTARG6	-	2	PTARG7	-	2	PTARG8	-	2	DTARG1	-	3
DTARG2	-	3	DTARG3	-	3	DTARG4	-	3	DTARG5	-	3	DTARG6	-	3	DTARG7	-	3
DTARG8	-	3	DCOV1	+	3	DCOV2	+	3	DCOV3	+	3	DCOV4	+	3	DCOV5	+	3
DCOV6	+	3	DCOV7	+	3	DCOV8	+	3	PSEQ	+	0	PPEQ	+	0	PDEQ	+	0
TOTAL1	=	5	TOTAL2	=	5	TOTAL3	=	5	TOTAL4	=	5	TOTAL5	=	5	TOTAL6	=	5
TOTAL7	=	5	TOTAL8	=	5	IP1	=	3	IP2	=	3	IP3	=	3	IP4	=	3
IP5	=	3	IP6	=	3	IP7	=	3	IP8	=	3	BNDROCE1	-	2	BNDROCE2	-	2
BNDROCE3	-	2	BNDROCE4	-	2	BNDROCE5	-	2	BNDROCE6	-	2	BNDROCE7	-	2	BNDROCE8	-	2
BNDLODY1	-	2	BNDLODY2	-	2	BNDLODY3	-	2	BNDLODY4	-	2	BNDLODY5	-	2	BNDLODY6	-	2
BNDLODY7	-	2	BNDLODY8	-	2	BNDECOV1	-	2	BNDECOV2	-	2	BNDECOV3	-	2	BNDECOV4	-	2
BNDECOV5	-	2	BNDECOV6	-	2	BNDECOV7	-	2	BNDECOV8	-	2	BNDERPS1	-	2	BNDERPS2	-	2
BNDERPS3	-	2	BNDERPS4	-	2	BNDERPS5	-	2	BNDERPS6	-	2	BNDERPS7	-	2	BNDERPS8	-	2
BNDDCOV1	-	2	BNDDCOV2	-	2	BNDDCOV3	-	2	BNDDCOV4	-	2	BNDDCOV5	-	2	BNDDCOV6	-	2

```
                                ROW SET (#########)              COLUMN SET (#########)

LISTING BY COLUMNS

ROW SETS

         ERPS2     000000000*000000000000000      ERPS3     000000000*000000000000000
         ERPS4     000*****0*000000000000000      ERPS5     0000****0*000000000000000
         ERPS6     00000***0*000000000000000      ERPS7     000000**0*000000000000000
         ERPS8     0000000*0*000000000000000      STARG3    00******0*000000000000000
         STARG4    000*****0*000000000000000      STARG5    0000****0*000000000000000
         STARG6    00000***00000000000000000      STARG7    000000**00000000000000000
         STARG8    0000000*00000000000000000      PTARG3    00******0*000000000000000
         PTARG4    000*****0*000000000000000      PTARG5    0000****0*000000000000000
         PTARG7    000000**00000000000000000      PTARG8    0000000*00000000000000000
         DTARG1    000000000*000000000000000      DTARG2    000000000*000000000000000
         DTARG3    00******0*000000000000000      DTARG4    000*****0*000000000000000
         DTARG5    0000****0*000000000000000      DTARG6    00000***0*000000000000000
         DTARG7    000000**0*000000000000000      DTARG8    0000000*0*000000000000000
         DCOV1     000000000*000000000000000      DCOV2     000000000*000000000000000
         DCOV3     00******0*000000000000000      DCOV4     000*****0*000000000000000
         DCOV5     0000****0*000000000000000      DCOV6     00000***0*000000000000000
         DCOV7     000000**0*000000000000000      DCOV8     0000000*0*000000000000000

VARIABLES

SALES1   UPBND       14000.0000
         TS1            -1.0000   STARG1        1.0000

SALES2   COLYR     0*******000000000000000000
         UPBND       16000.0000
         TS2            -1.0000   STARG2        1.0000

SALES3   COLYR     00******000000000000000000
         UPBND       18000.0000
         TS3            -1.0000   STARG3        1.0000

SALES4   COLYR     000*****000000000000000000
         UPBND       23000.0000
         TS4            -1.0000   STARG4        1.0000

SALES5   COLYR     0000****000000000000000000
         UPBND       27000.0000
         TS5            -1.0000   STARG5        1.0000

SALES6   COLYR     00000***000000000000000000
         UPBND       30000.0000
         TS6            -1.0000   STARG6        1.0000

SALES7   COLYR     000000**000000000000000000
         UPBND       33000.0000
         TS7            -1.0000   STARG7        1.0000

SALES8   COLYR     0000000*000000000000000000
         UPBND       36000.0000
         TS8            -1.0000   STARG8        1.0000
```

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```
PROBLEM   GOALMODEL-05          LIST(1)                         DATE  20/07/76     TIME  12/11/26      PAGE  0007
                                ROW SET (########)              COLUMN SET (########)

LISTING BY COLUMNS

VARIABLES

EARN1     EA1           -1.0000  CR1            1.0000  PR1            -0.5000  ROCE1         -1.0000
          ECOV1         -1.0000  BNDROCE1      -1.0000  BNDECOV1       -1.0000

EARN2     COLYR     0*******000000000000000000
          EA2           -1.0000  CR2            1.0000  PR2            -0.5000  ROCE2         -1.0000
          ECOV2         -1.0000  BNDROCE2      -1.0000  BNDECOV2       -1.0000

EARN3     COLYR     00******000000000000000000
          EA3           -1.0000  CR3            1.0000  PR3            -0.5000  ROCE3         -1.0000
          ECOV3         -1.0000  BNDROCE3      -1.0000  BNDECOV3       -1.0000

EARN4     COLYR     000*****000000000000000000
          EA4           -1.0000  CR4            1.0000  PR4            -0.5000  ROCE4         -1.0000
          ECOV4         -1.0000  BNDROCE4      -1.0000  BNDECOV4       -1.0000

EARN5     COLYR     0000****000000000000000000
          EA5           -1.0000  CR5            1.0000  PR5            -0.5000  ROCE5         -1.0000
          ECOV5         -1.0000  BNDROCE5      -1.0000  BNDECOV5       -1.0000

EARN6     COLYR     00000***000000000000000000
          EA6           -1.0000  CR6            1.0000  PR6            -0.5000  ROCE6         -1.0000
          ECOV6         -1.0000  BNDROCE6      -1.0000  BNDECOV6       -1.0000

EARN7     COLYR     000000**000000000000000000
          EA7           -1.0000  CR7            1.0000  PR7            -0.5000  ROCE7         -1.0000
          ECOV7         -1.0000  BNDROCE7      -1.0000  BNDECOV7       -1.0000

EARN8     COLYR     0000000*000000000000000000
          EA8           -1.0000  CR8            1.0000  PR8            -0.5000  ROCE8         -1.0000
          ECOV8         -1.0000  BNDROCE8      -1.0000  BNDECOV8       -1.0000

NPAT1     UPBND       1000.0000
          TP1           -1.0000  PR1            1.0000  ERPS1          -1.0000  PTARG1         1.0000
          DCOV1         -1.0000  BNDERPS1      -1.0000  BNDDCOV1       -1.0000

NPAT2     COLYR     0*******000000000000000000
          UPBND       1200.0000
          TP2           -1.0000  PR2            1.0000  ERPS2          -1.0000  PTARG2         1.0000
          DCOV2         -1.0000  BNDERPS2      -1.0000  BNDDCOV2       -1.0000

NPAT3     COLYR     00******000000000000000000
          UPBND       1300.0000
          TP3           -1.0000  PR3            1.0000  ERPS3          -1.0000  PTARG3         1.0000
          DCOV3         -1.0000  BNDERPS3      -1.0000  BNDDCOV3       -1.0000

NPAT4     COLYR     000*****000000000000000000
          UPBND       1400.0000
          TP4           -1.0000  PR4            1.0000  ERPS4          -1.0000  PTARG4         1.0000
          DCOV4         -1.0000  BNDERPS4      -1.0000  BNDDCOV4       -1.0000

NPAT5     COLYR     0000****000000000000000000
```

```
                                ROW SET (########)          COLUMN SET (########)

LISTING BY COLUMNS

VARIABLES

          UPBND     1500.0000
          TP5         -1.0000  PR5          1.0000  ERPS5      -1.0000  PTARG5      1.0000
          DCOV5       -1.0000  BNDERPS5    -1.0000  BNDDCOV5   -1.0000

NPAT6     COLYR    00000***000000000000000
          UPBND     1800.0000
          TP6         -1.0000  PR6          1.0000  ERPS6      -1.0000  PTARG6      1.0000
          DCOV6       -1.0000  BNDERPS6    -1.0000  BNDDCOV6   -1.0000

NPAT7     COLYR    000000**000000000000000
          UPBND     2100.0000
          TP7         -1.0000  PR7          1.0000  ERPS7      -1.0000  PTARG7      1.0000
          DCOV7       -1.0000  BNDERPS7    -1.0000  BNDDCOV7   -1.0000

NPAT8     COLYR    0000000*000000000000000
          UPBND     2400.0000
          TP8         -1.0000  PR8          1.0000  ERPS8      -1.0000  PTARG8      1.0000
          DCOV8       -1.0000  BNDERPS8    -1.0000  BNDDCOV8   -1.0000

TAX1      CL1          1.0000  CB1         -1.0000  TP1         1.0000

TAX2      COLYR    0*******000000000000000
          CL2          1.0000  CB2         -1.0000  TP2         1.0000

TAX3      COLYR    00******000000000000000
          CL3          1.0000  CB3         -1.0000  TP3         1.0000

TAX4      COLYR    000*****000000000000000
          CL4          1.0000  CB4         -1.0000  TP4         1.0000

TAX5      COLYR    0000****000000000000000
          CL5          1.0000  CB5         -1.0000  TP5         1.0000

TAX6      COLYR    00000***000000000000000
          CL6          1.0000  CB6         -1.0000  TP6         1.0000

TAX7      COLYR    000000**000000000000000
          CL7          1.0000  CB7         -1.0000  TP7         1.0000

TAX8      COLYR    0000000*000000000000000
          CL8          1.0000  CB8         -1.0000  TP8         1.0000

FABL1     COLYR    00000000*00000000000000
          EA1         -0.0310  BL1         -1.0300  BL2         1.0000  CB1        -1.0000
          CB2          1.0000  TP1          0.1910  TP2        -0.2000  TA1        -0.0410
          TA2          0.0400  TOTAL1       1.0000

FABL2     COLYR    0*******0*00000000000000
          EA2         -0.0310  BL2         -1.0300  BL3         1.0000  CB2        -1.0000
          CB3          1.0000  TP2          0.1910  TP3        -0.2000  TA2        -0.0410
```

LISTING BY COLUMNS

VARIABLES

	TA3	0.0400	TOTAL2	1.0000				
FABL3	COLYR	00******0*00000000000000						
	EA3	-0.0310	BL3	-1.0300	BL4	1.0000	CB3	-1.0000
	CB4	1.0000	TP3	0.1910	TP4	-0.2000	TA3	-0.0410
	TA4	0.0400	TOTAL3	1.0000				
FABL4	COLYR	000*****0*00000000000000						
	EA4	-0.0310	BL4	-1.0300	BL5	1.0000	CB4	-1.0000
	CB5	1.0000	TP4	0.1910	TP5	-0.2000	TA4	-0.0410
	TA5	0.0400	TOTAL4	1.0000				
FABL5	COLYR	0000****0*00000000000000						
	EA5	-0.0310	BL5	-1.0300	BL6	1.0000	CB5	-1.0000
	CB6	1.0000	TP5	0.1910	TP6	-0.2000	TA5	-0.0410
	TA6	0.0400	TOTAL5	1.0000				
FABL6	COLYR	00000***0*00000000000000						
	EA6	-0.0310	BL6	-1.0300	BL7	1.0000	CB6	-1.0000
	CB7	1.0000	TP6	0.1910	TP7	-0.2000	TA6	-0.0410
	TA7	0.0400	TOTAL6	1.0000				
FABL7	COLYR	000000**0*00000000000000						
	EA7	-0.0310	BL7	-1.0300	BL8	1.0000	CB7	-1.0000
	CB8	1.0000	TP7	0.1910	TP8	-0.2000	TA7	-0.0410
	TA8	0.0400	TOTAL7	1.0000				
FABL8	COLYR	0000000*0*00000000000000						
	NW8	1.0000	EA8	-0.0310	BL8	-1.0300	CB8	-1.0000
	TP8	0.1910	TA8	-0.0410	TOTAL8	1.0000		
FAPE1	EA1	-0.3300	PE1	-1.3300	PE2	1.0000	CB1	-1.0000
	CB2	1.0000	TP1	0.5000	TP2	-0.5000	TOTAL1	1.0000
FAPE2	COLYR	0*******000000000000000000						
	EA2	-0.3300	PE2	-1.3300	PE3	1.0000	CB2	-1.0000
	CB3	1.0000	TP2	0.5000	TP3	-0.5000	TOTAL2	1.0000
FAPE3	COLYR	00******000000000000000000						
	EA3	-0.3300	PE3	-1.3300	PE4	1.0000	CB3	-1.0000
	CB4	1.0000	TP3	0.5000	TP4	-0.5000	TOTAL3	1.0000
FAPE4	COLYR	000*****000000000000000000						
	EA4	-0.3300	PE4	-1.3300	PE5	1.0000	CB4	-1.0000
	CB5	1.0000	TP4	0.5000	TP5	-0.5000	TOTAL4	1.0000
FAPE5	COLYR	0000****000000000000000000						
	EA5	-0.3300	PE5	-1.3300	PE6	1.0000	CB5	-1.0000
	CB6	1.0000	TP5	0.5000	TP6	-0.5000	TOTAL5	1.0000

```
                              ROW SET (########)              COLUMN SET (########)

LISTING BY COLUMNS

VARIAPLES

FAPE6    COLYR      00000***000000000000000
         EA6           -0.3300  PE6           -1.3300  PE7           1.0000  CB6          -1.0000
         CB7            1.0000  TP6            0.5000  TP7          -0.5000  TOTAL6        1.0000

FAPE7    COLYR      000000**000000000000000
         EA7           -0.3300  PF7           -1.3300  PEA           1.0000  CB7          -1.0000
         CRA            1.0000  TP7            0.5000  TPA          -0.5000  TOTAL7        1.0000

FAPE8    COLYR      0000000*000000000000000
         NWR            1.0000  EA8           -0.3300  PEA          -1.3300  CB8          -1.0000
         TP8            0.5000  TOTAL8         1.0000

CURA1    CA1           -1.0000  CR1           -1.0000  CB2           1.0000  LQDY1        -1.0000
         TOTAL1         1.0000  BNDLQDY1      -1.0000

CURA2    COLYR      0*******000000000000000
         CA2           -1.0000  CR2           -1.0000  CB3           1.0000  LQDY2        -1.0000
         TOTAL2         1.0000  BNDLQDY2      -1.0000

CURA3    COLYR      00******000000000000000
         CA3           -1.0000  CR3           -1.0000  CB4           1.0000  LQDY3        -1.0000
         TOTAL3         1.0000  BNDLQDY3      -1.0000

CURA4    COLYR      000*****000000000000000
         CA4           -1.0000  CR4           -1.0000  CB5           1.0000  LQDY4        -1.0000
         TOTAL4         1.0000  BNDLQDY4      -1.0000

CURA5    COLYR      0000****000000000000000
         CA5           -1.0000  CR5           -1.0000  CB6           1.0000  LQDY5        -1.0000
         TOTAL5         1.0000  BNDLQDY5      -1.0000

CURA6    COLYR      00000***000000000000000
         CA6           -1.0000  CR6           -1.0000  CB7           1.0000  LQDY6        -1.0000
         TOTAL6         1.0000  BNDLQDY6      -1.0000

CURA7    COLYR      000000**000000000000000
         CA7           -1.0000  CR7           -1.0000  CB8           1.0000  LQDY7        -1.0000
         TOTAL7         1.0000  BNDLQDY7      -1.0000

CURA8    COLYR      0000000*000000000000000
         NWR            1.0000  CA8           -1.0000  CB8          -1.0000  LQDY8        -1.0000
         TOTAL8         1.0000  BNDLQDY8      -1.0000

CURL1    COLYR      000000000*0000000000000
         CL1           -1.0000  CR1            1.0000  CB2          -1.0000  LQDY1         2.5000
         TOTAL1        -1.0000  BNDLQDY1       3.0000

CURL2    COLYR      0*******0*0000000000000
         CL2           -1.0000  CR2            1.0000  CB3          -1.0000  LQDY2         2.5000
         TOTAL2        -1.0000  BNDLQDY2       3.0000
```

```
PROBLEM   GOALMODEL-05          LIST(1)                              DATE  20/07/76      TIME  12/11/27      PAGE  0011
                                ROW SET (########)                   COLUMN SET (########)

LISTING BY COLUMNS

VARIABLES

CURL3      COLYR      00******0*000000000000000
           CL3            -1.0000  CB3             1.0000  CB4             -1.0000  LODY3            2.5000
           TOTAL3         -1.0000  BNDLODY3        3.0000

CURL4      COLYR      000*****0*000000000000000
           CL4            -1.0000  CB4             1.0000  CB5             -1.0000  LODY4            2.5000
           TOTAL4         -1.0000  BNDLODY4        3.0000

CURL5      COLYR      0000****0*000000000000000
           CL5            -1.0000  CB5             1.0000  CB6             -1.0000  LODY5            2.5000
           TOTAL5         -1.0000  BNDLODY5        3.0000

CURL6      COLYR      00000***0*000000000000000
           CL6            -1.0000  CB6             1.0000  CB7             -1.0000  LODY6            2.5000
           TOTAL6         -1.0000  BNDLODY6        3.0000

CURL7      COLYR      000000**0*000000000000000
           CL7            -1.0000  CB7             1.0000  CB8             -1.0000  LODY7            2.5000
           TOTAL7         -1.0000  BNDLODY7        3.0000

CURL8      COLYR      0000000*0*000000000000000
           NW8            -1.0000  CL8            -1.0000  CB8              1.0000  LODY8            2.5000
           TOTAL8         -1.0000  BNDLODY8        3.0000

OVDR1      COLYR      000000000*000000000000000
           UPBND         250.0000  LOBND                0
           EA2            -0.1200  CL1             1.0000  IP1              0.1200

OVDR2      COLYR      0*******0000000000000000
           UPBND         250.0000
           EA3            -0.1200  CL2             1.0000  IP2              0.1200

OVDR3      COLYR      00******00000000000000000
           UPBND         250.0000
           EA4            -0.1200  CL3             1.0000  IP3              0.1200

OVDR4      COLYR      000*****00000000000000000
           UPBND         250.0000
           EA5            -0.1200  CL4             1.0000  IP4              0.1200

OVDR5      COLYR      0000****00000000000000000
           UPBND         250.0000
           EA6            -0.1200  CL5             1.0000  IP5              0.1200

OVDR6      COLYR      00000***00000000000000000
           UPBND         250.0000
           EA7            -0.1200  CL6             1.0000  IP6              0.1200

OVDR7      COLYR      000000**00000000000000000
           UPBND         250.0000
           EA8            -0.1200  CL7             1.0000  IP7              0.1200
```

```
LISTING BY COLUMNS

VARIABLES

OVDR8     COLYR    0000000*0000000000000000
          UPBND      250.0000
          CL8          1.0000  IP8          0.1200

MARK1     COLYR    000000000*0000000000000
          LOBND            0
          FA2          0.0700  CA1          1.0000

MARK2     COLYR    0*******0000000000000000
          LOBND            0
          EA3          0.0700  CA2          1.0000

MARK3     COLYR    00******0000000000000000
          LOBND            0
          EA4          0.0700  CA3          1.0000

MARK4     COLYR    000*****0000000000000000
          LOBND            0
          FA5          0.0700  CA4          1.0000

MARK5     COLYR    0000****0000000000000000
          LOBND            0
          EA6          0.0700  CA5          1.0000

MARK6     COLYR    00000***0000000000000000
          LOBND            0
          EA7          0.0700  CA6          1.0000

MARK7     COLYR    000000**0000000000000000
          LOBND            0
          EA8          0.0700  CA7          1.0000

MARK8     COLYR    0000000*0000000000000000
          LOBND            0
          CA8          1.0000

ASSETS1   COLYR    000000000*0000000000000
          ROCE1        0.2500  TOTAL1      -1.0000  BNDROCE1     0.3500

ASSETS2   COLYR    000000000*0000000000000
          ROCE2        0.2500  TOTAL2      -1.0000  BNDROCE2     0.5000

ASSETS3   COLYR    000000000*0000000000000
          ROCE3        0.2500  TOTAL3      -1.0000  BNDROCE3     0.3000

ASSETS4   COLYR    000000000*0000000000000
          ROCE4        0.2500  TOTAL4      -1.0000  BNDROCE4     0.3000

ASSETS5   COLYR    000000000*0000000000000
          ROCE5        0.2500  TOTAL5      -1.0000  BNDROCE5     0.3000
```

ROW SET (########) COLUMN SET (########)

LISTING BY COLUMNS

VARIABLES

```
OVDR8     COLYR     0000000*0000000000000000
          UPBND      250.0000
          CL8          1.0000  IP8          0.1200

MARK1     COLYR     000000000*0000000000000
          LOBND            0
          FA2          0.0700  CA1          1.0000

MARK2     COLYR     0*******0000000000000000
          LOBND            0
          EA3          0.0700  CA2          1.0000

MARK3     COLYR     00******0000000000000000
          LOBND            0
          EA4          0.0700  CA3          1.0000

MARK4     COLYR     000*****0000000000000000
          LOBND            0
          FA5          0.0700  CA4          1.0000

MARK5     COLYR     0000****0000000000000000
          LOBND            0
          EA6          0.0700  CA5          1.0000

MARK6     COLYR     00000***0000000000000000
          LOBND            0
          EA7          0.0700  CA6          1.0000

MARK7     COLYR     000000**0000000000000000
          LOBND            0
          EA8          0.0700  CA7          1.0000

MARK8     COLYR     0000000*0000000000000000
          LOBND            0
          CA8          1.0000

ASSETS1   COLYR     000000000*0000000000000
          ROCE1        0.2500  TOTAL1      -1.0000  BNDROCE1     0.3500

ASSETS2   COLYR     000000000*0000000000000
          ROCE2        0.2500  TOTAL2      -1.0000  BNDROCE2     0.5000

ASSETS3   COLYR     000000000*0000000000000
          ROCE3        0.2500  TOTAL3      -1.0000  BNDROCE3     0.3000

ASSETS4   COLYR     000000000*0000000000000
          ROCE4        0.2500  TOTAL4      -1.0000  BNDROCE4     0.3000

ASSETS5   COLYR     000000000*0000000000000
          ROCE5        0.2500  TOTAL5      -1.0000  BNDROCE5     0.3000
```

ROW SET (########) COLUMN SET (########)

LISTING BY COLUMNS

```
VARIABLES

ASSETS6   COLYR      000000000*0000000000000
          ROCEA          0.2500  TOTAL6       -1.0000  BNDROCE6     0.3000

ASSETS7   COLYR      000000000*0000000000000
          ROCE7          0.2500  TOTAL7       -1.0000  BNDROCE7     0.3000

ASSETS8   COLYR      000000000*0000000000000
          ROCE8          0.2500  TOTAL8       -1.0000  BNDROCE8     0.3000

INTR1     COLYR      000000000*0000000000000
          ECOV1         15.0000  IP1          -1.0000  BNDECOV1    25.0000

INTR2     COLYR      000000000*0000000000000
          ECOV2         15.0000  IP2          -1.0000  BNDECOV2    25.0000

INTR3     COLYR      000000000*0000000000000
          ECOV3         15.0000  IP3          -1.0000  BNDECOV3    25.0000

INTR4     COLYR      000000000*0000000000000
          ECOV4         15.0000  IP4          -1.0000  BNDECOV4    25.0000

INTR5     COLYR      000000000*0000000000000
          ECOV5         15.0000  IP5          -1.0000  BNDECOV5    25.0000

INTR6     COLYR      000000000*0000000000000
          ECOV6         15.0000  IP6          -1.0000  BNDECOV6    25.0000

INTR7     COLYR      000000000*0000000000000
          ECOV7         15.0000  IP7          -1.0000  BNDECOV7    25.0000

INTR8     COLYR      000000000*0000000000000
          ECOVR         15.0000  IP8          -1.0000  BNDECOV8    25.0000

DV1       COLYR      000000000*0000000000000
          CL1            1.0000  CR1          -1.0000  DTARG1       1.0000  DCOV1        3.5000
          BNDDCOV1       5.0000  BNDDVPS1     -1.0000

DV2       COLYR      0*******U*0000000000000
          CL2            1.0000  CB2          -1.0000  DTARG2       1.0000  DCOV2        3.5000
          BNDDCOV2       5.0000  BNDDVPS2     -1.0000

DV3       COLYR      00******U*0000000000000
          CL3            1.0000  CB3          -1.0000  DTARG3       1.0000  DCOV3        3.5000
          BNDDCOV3       5.0000  BNDDVPS3     -1.0000

DV4       COLYR      000*****0*0000000000000
          CL4            1.0000  CR4          -1.0000  DTARG4       1.0000  DCOV4        3.5000
          BNDDCOV4       5.0000  BNDDVPS4     -1.0000

DV5       COLYR      0000****0*0000000000000
          CL5            1.0000  CB5          -1.0000  DTARG5       1.0000  DCOV5        3.5000
```

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ROW SET (########) COLUMN SET (########)

LISTING BY COLUMNS

VARIABLES

```
         BNDDCOV5      5.0000  BNDDVPS5   -1.0000

DV6      COLYR     00000***0*0000000000000
         CL6           1.0000  CR6        -1.0000  DTARG6    1.0000  DCOV6     3.5000
         BNDDCOV6      5.0000  BNDDVPS6   -1.0000

DV7      COLYR     000000**0*0000000000000
         CL7           1.0000  CR7        -1.0000  DTARG7    1.0000  DCOV7     3.5000
         BNDDCOV7      5.0000  BNDDVPS7   -1.0000

DV8      COLYR     0000000*0*0000000000000
         CL8           1.0000  CR8        -1.0000  DTARG8    1.0000  DCOV8     3.5000
         BNDDCOV8      5.0000  BNDDVPS8   -1.0000

NUM1     COLYR     000000000*0000000000000
         EQ1           1.0000  EQ2        -1.0000  ERPS1     0.3000  DTARG1   -0.2000
         BNDERPS1      0.3600  BNDDVPS1    0.2400

NUM2     COLYR     0*******0*0000000000000
         EQ2           1.0000  EQ3        -1.0000  ERPS2     0.3350  DTARG2   -0.2150
         BNDERPS2      0.4200  BNDDVPS2    0.2800

NUM3     COLYR     00******0*0000000000000
         EQ3           1.0000  EQ4        -1.0000  ERPS3     0.3600  DTARG3   -0.2400
         BNDERPS3      0.4800  BNDDVPS3    0.3200

NUM4     COLYR     000*****0*0000000000000
         EQ4           1.0000  EQ5        -1.0000  ERPS4     0.3900  DTARG4   -0.2600
         BNDERPS4      0.5400  BNDDVPS4    0.3600

NUM5     COLYR     0000****0*0000000000000
         EQ5           1.0000  EQ6        -1.0000  ERPS5     0.4450  DTARG5   -0.2900
         BNDERPS5      0.6000  BNDDVPS5    0.4000

NUM6     COLYR     00000***0*0000000000000
         EQ6           1.0000  EQ7        -1.0000  ERPS6     0.4800  DTARG6   -0.3200
         BNDERPS6      0.6600  BNDDVPS6    0.4400

NUM7     COLYR     000000**0*0000000000000
         EQ7           1.0000  EQ8        -1.0000  ERPS7     0.5400  DTARG7   -0.3600
         BNDERPS7      0.7200  BNDDVPS7    0.4800

NUM8     COLYR     0000000*0*0000000000000
         EQ8           1.0000  ERPS8       0.6000  DTARG8   -0.4000  BNDERPS8  0.7800
         BNDDVPS8      0.5200

RG1      UPBND       800.0000
         CB1           1.6000  EQ1        -1.0000

RG2      COLYR     0*******000000000000000
```

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```
                                         ROW SET (########)                COLUMN SET (########)

LISTING BY COLUMNS

VARIABLES

                UPBND        800.0000
                CB2            1.6000  EQ2            -1.0000

        RG3     COLYR     00******0000000000000000
                UPBND        800.0000
                CB3            1.6000  EQ3            -1.0000

        RG4     COLYR     000*****00000000000000000
                UPBND        800.0000
                CB4            1.6000  EQ4            -1.0000

        RG5     COLYR     0000****00000000000000000
                UPBND        800.0000
                CB5            1.6000  EQ5            -1.0000

        RG6     COLYR     00000***00000000000000000
                UPBND        800.0000
                CB6            1.6000  EQ6            -1.0000

        RG7     COLYR     000000**00000000000000000
                UPBND        800.0000
                CB7            1.6000  EQ7            -1.0000

        RG8     COLYR     0000000*00000000000000000
                UPBND        800.0000
                CB8            1.6000  EQ8            -1.0000

        DE1     COLYR     000000000*0000000000000
                CB2           -0.0800  PR2             0.0400  D1             1.0000  D2           -1.0000
                IP1            0.0800

        DE2     COLYR     0*******0*0000000000000
                CB3           -0.0800  PR3             0.0400  D2             1.0000  D3           -1.0000
                IP2            0.0800

        DE3     COLYR     00******0*0000000000000
                CB4           -0.0800  PR4             0.0400  D3             1.0000  D4           -1.0000
                IP3            0.0800

        DE4     COLYR     000*****0*0000000000000
                CB5           -0.0800  PR5             0.0400  D4             1.0000  D5           -1.0000
                IP4            0.0800

        DE5     COLYR     0000****0*0000000000000
                CB6           -0.0800  PR6             0.0400  D5             1.0000  D6           -1.0000
                IP5            0.0800

        DE6     COLYR     00000***0*000000000000000
                CB7           -0.0800  PR7             0.0400  D6             1.0000  D7           -1.0000
                IP6            0.0800
```

ROW SET (########) COLUMN SET (########)

LISTING BY COLUMNS

VARIABLES

```
DE7     COLYR   000000**0*00000000000000
        CB8        -0.0800  PR8       0.0400  D7       1.0000  D8      -1.0000
        IP7         0.0800
DF8     COLYR   0000000*0*00000000000000
        D8          1.0000  IP8       0.0800
LL1     CB1         1.0000  D1       -1.0000
LL2     COLYR   0*******0000000000000000
        CB2         1.0000  D2       -1.0000
LL3     COLYR   00******0000000000000000
        CB3         1.0000  D3       -1.0000
LL4     COLYR   000*****0000000000000000
        CB4         1.0000  D4       -1.0000
LL5     COLYR   0000****0000000000000000
        CB5         1.0000  D5       -1.0000
LL6     COLYR   00000***0000000000000000
        CB6         1.0000  D6       -1.0000
LL7     COLYR   000000**0000000000000000
        CB7         1.0000  D7       -1.0000
LL8     COLYR   0000000*0000000000000000
        CB8         1.0000  D8       -1.0000
BLTA1   TP1         0.5000  TA1       1.0000  TA2     -1.0000
BLTA2   COLYR   0*******0000000000000000
        TP2         0.5000  TA2       1.0000  TA3     -1.0000
BLTA3   COLYR   00******0000000000000000
        TP3         0.5000  TA3       1.0000  TA4     -1.0000
BLTA4   COLYR   000*****0000000000000000
        TP4         0.5000  TA4       1.0000  TA5     -1.0000
BLTA5   COLYR   0000****0000000000000000
        TP5         0.5000  TA5       1.0000  TA6     -1.0000
BLTA6   COLYR   00000***0000000000000000
        TP6         0.5000  TA6       1.0000  TA7     -1.0000
BLTA7   COLYR   000000**0000000000000000
        TP7         0.5000  TA7       1.0000  TA8     -1.0000
BLTA8   COLYR   0000000*0000000000000000
```

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PROBLEM   GOALMODEL-05        LIST(1)                                  DATE  20/07/76      TIME  12/11/28      PAGE  0017
                              ROW SET (########)                COLUMN SET (########)

LISTING BY COLUMNS

VARIABLES

          TP8          0.5000  TA8          1.0000

RET       GOAL         0.0348  ORJ          0.0348  OBJ1         0.0348  OBJ5         0.0348
          OBJ8         0.0348  ROCE1       -1.0000  ROCE2       -1.0000  ROCE3       -1.0000
          ROCE4       -1.0000  ROCE5       -1.0000  ROCE6       -1.0000  ROCE7       -1.0000
          ROCE8       -1.0000

PRF       GOAL         0.0258  ORJ          0.0258  OBJ1         0.0258  OBJ3         0.0258
          PTARG1       1.0000  PTARG2       1.0000  PTARG3       1.0000  PTARG4       1.0000
          PTARG5       1.0000  PTARG6       1.0000  PTARG7       1.0000  PTARG8       1.0000

SAL       GOAL         0.0141  ORJ          0.0141  OBJ1         0.0141  OBJ6         0.0141
          STARG1       1.0000  STARG2       1.0000  STARG3       1.0000  STARG4       1.0000
          STARG5       1.0000  STARG6       1.0000  STARG7       1.0000  STARG8       1.0000

EPS       GOAL         0.1373  ORJ          0.1373  OBJ1         0.1378  OBJ4         0.1378
          ERPS1       -1.0000  ERPS2       -1.0000  ERPS3       -1.0000  ERPS4       -1.0000
          ERPS5       -1.0000  ERPS6       -1.0000  ERPS7       -1.0000  ERPS8       -1.0000

LOD       GOAL         0.0981  ORJ1        -0.0981  OBJ2        -0.0981  OBJ8        -0.0891
          LODY1       -1.0000  LODY2       -1.0000  LODY3       -1.0000  LODY4       -1.0000
          LODY5       -1.0000  LODY6       -1.0000  LODY7       -1.0000  LODY8       -1.0000

INT       GOAL         1.1279  ORJ1        -1.1279  OBJ2        -1.1279  OBJ3        -1.1279
          OBJ5        -1.1279  ECOV1       -1.0000  ECOV2       -1.0000  ECOV3       -1.0000
          ECOV4       -1.0000  ECOV5       -1.0000  ECOV6       -1.0000  ECOV7       -1.0000
          ECOV8       -1.0000

DVP       GOAL         0.0879  ORJ2         0.0879  OBJ6        -0.0879  OBJ7        -0.0879
          DTARG1       1.0000  DTARG2       1.0000  DTARG3       1.0000  DTARG4       1.0000
          DTARG5       1.0000  DTARG6       1.0000  DTARG7       1.0000  DTARG8       1.0000

DVC       GOAL         0.3625  ORJ2         0.3620  OBJ7         0.3620  DCOV1       -1.0000
          DCOV2       -1.0000  DCOV3       -1.0000  DCOV4       -1.0000  DCOV5       -1.0000
          DCOV6       -1.0000  DCOV7       -1.0000  DCOV8       -1.0000

PRO1Y1    UPRND        1.0000
          TS3        500.0000  TS4        800.0000  TS5       1000.0000  TS6       1200.0000
          TS7       1200.0000  TS8       1100.0000  EA3        100.0000  EA4        184.0000
          EA5        240.0000  EA6        360.0000  EA7        300.0000  EA8        253.0000
          BL1        100.0000  BL2         50.0000  BL3         50.0000  PE2         80.0000
          PE3         70.0000  CA3        200.0000  CA4        359.0000  CA5        392.0000
          CA6        429.0000  CA7        424.0000  CA8        404.0000  CL3         40.0000
          CL4         67.0000  CL5         71.0000  CL6         83.0000  CL7         84.0000
          CL8         79.0000

PRO4Y1    UPRND        1.0000
          TS1        300.0000  TS2        760.0000  TS3        980.0000  TS4        910.0000
          TS5        830.0000  TS6        760.0000  TS7        710.0000  TS8        690.0000
          EA1         27.0000  EA2        130.0000  EA3        226.0000  EA4        200.0000
```

ROW SET (########) COLUMN SET (########)

LISTING BY COLUMNS

VARIABLES

	EA5	174.0000	EA6	152.0000	EA7	134.0000	EA8	124.0000
	BL1	75.0000	BL2	25.0000	PE1	250.0000	PE2	130.0000
	CA1	97.0000	CA2	139.0000	CA3	317.0000	CA4	305.0000
	CA5	278.0000	CA6	244.0000	CA7	232.0000	CA8	224.0000
	CL1	45.0000	CL2	108.0000	CL3	123.0000	CL4	118.0000
	CL5	109.0000	CL6	110.0000	CL7	107.0000	CL8	88.0000
PR12Y1	UPBND	1.0000						
	TS1	120.0000	TS2	390.0000	TS3	580.0000	TS4	760.0000
	TS5	1000.0000	TS6	1010.0000	TS7	1100.0000	TS8	950.0000
	EA1	4.0000	EA2	36.0000	EA3	108.0000	EA4	160.0000
	EA5	230.0000	EA6	222.0000	EA7	232.0000	EA8	190.0000
	BL1	190.0000	PE1	50.0000	PE2	80.0000	PE3	70.0000
	PE4	50.0000	PE5	30.0000	PE6	10.0000	PE7	10.0000
	CA1	41.0000	CA2	134.0000	CA3	229.0000	CA4	341.0000
	CA5	486.0000	CA6	438.0000	CA7	354.0000	CA8	283.0000
	CL1	17.0000	CL2	53.0000	CL3	174.0000	CL4	168.0000
	CL5	200.0000	CL6	198.0000	CL7	185.0000	CL8	193.0000
PR13Y1	UPBND	1.0000						
	TS1	600.0000	TS2	940.0000	TS3	1560.0000	TS4	1100.0000
	TS5	480.0000	TS6	660.0000	TS7	210.0000	TS8	90.0000
	EA1	72.0000	EA2	160.0000	EA3	468.0000	EA4	242.0000
	EA5	48.0000	EA6	112.0000	EA7	36.0000	EA8	9.0000
	BL1	250.0000	PE1	140.0000	PE2	120.0000	PE3	70.0000
	CA1	174.0000	CA2	286.0000	CA3	518.0000	CA4	451.0000
	CA5	166.0000	CA6	238.0000	CA7	61.0000	CA8	39.0000
	CL1	60.0000	CL2	102.0000	CL3	180.0000	CL4	121.0000
	CL5	85.0000	CL6	103.0000	CL7	25.0000	CL8	10.0000
PR16Y1	UPBND	1.0000						
	TS1	200.0000	TS2	600.0000	TS3	1000.0000	TS4	1200.0000
	TS5	1200.0000	TS6	1200.0000	TS7	1200.0000	TS8	1000.0000
	EA2	42.0000	EA3	160.0000	EA4	240.0000	EA5	264.0000
	EA6	264.0000	EA7	2[illegible]0.0000	EA8	180.0000	BL1	160.0000
	BL2	60.0000	PE1	100.0000	PE2	100.0000	PE3	50.0000
	CA1	65.0000	CA2	215.0000	CA3	365.0000	CA4	446.0000
	CA5	460.0000	CA6	464.0000	CA7	451.0000	CA8	400.0000
	CL1	20.0000	CL2	70.0000	CL3	100.0000	CL4	140.0000
	CL5	160.0000	CL6	160.0000	CL7	150.0000	CL8	140.0000
PR22Y1	UPBND	1.0000						
	TS1	500.0000	TS2	500.0000	TS3	500.0000	TS4	500.0000
	TS5	500.0000	TS6	500.0000	TS7	500.0000	TS8	500.0000
	EA1	100.0000	EA2	100.0000	EA3	110.0000	EA4	110.0000
	EA5	105.0000	EA6	100.0000	EA7	110.0000	EA8	105.0000
	BL1	100.0000	PF1	40.0000	PE2	100.0000	PE3	150.0000
	PE4	40.0000	PE6	40.0000	CA1	150.0000	CA2	154.0000
	CA3	163.0000	CA4	169.0000	CA5	178.0000	CA6	176.0000
	CA7	173.0000	CA8	166.0000	CL1	69.0000	CL2	73.0000

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LISTING BY COLUMNS

VARIABLES
                 CL5          69.0000  CL4          67.0000  CL5          68.0000  CL6          68.0000
                 CL7          66.0000  CL8          61.0000

     PR23Y1      UPBND         1.0000
                 TS1         700.0000  TS2         750.0000  TS3         780.0000  TS4         800.0000
                 TS5         810.0000  TS6         800.0000  TS7         770.0000  TS8         750.0000
                 EA1         140.0000  EA2         165.0000  EA3         156.0000  EA4         152.0000
                 EA5         137.0000  EA6         128.0000  EA7         108.0000  EA8         105.0000
                 BL1         125.0000  BL2          65.0000  PE1         250.0000  PE2         100.0000
                 CA1         210.0000  CA2         244.0000  CA3         284.0000  CA4         310.0000
                 CA5         318.0000  CA6         313.0000  CA7         282.0000  CA8         257.0000
                 CL1         100.0000  CL2         100.0000  CL3         112.0000  CL4         116.0000
                 CL5         121.0000  CL6         121.0000  CL7         120.0000  CL8         100.0000

     PR03Y2      COLYR    0*******000000000000000000
                 UPBND         1.0000
                 TS2         410.0000  TS3         620.0000  TS4        1800.0000  TS5        1680.0000
                 TS6        1740.0000  TS7        1520.0000  TS8        1310.0000  EA2          20.0000
                 EA3          62.0000  EA4         270.0000  EA5         324.0000  EA6         312.0000
                 EA7         274.0000  EA8         238.0000  BL2         200.0000  PE2         100.0000
                 PE3          80.0000  CA2         149.0000  CA3         257.0000  CA4         714.0000
                 CA5         692.0000  CA6         718.0000  CA7         629.0000  CA8         607.0000
                 CL2          60.0000  CL3         103.0000  CL4         285.0000  CL5         200.0000
                 CL6         185.0000  CL7         176.0000  CL8         153.0000

     PR04Y2      COLYR    0*******000000000000000000
                 UPBND         1.0000
                 TS2         300.0000  TS3         760.0000  TS4         980.0000  TS5         910.0000
                 TS6         830.0000  TS7         760.0000  TS8         710.0000  EA2          27.0000
                 EA3         130.0000  EA4         226.0000  EA5         200.0000  EA6         174.0000
                 EA7         152.0000  EA8         134.0000  BL2          75.0000  BL3          25.0000
                 PE2         250.0000  PE3         130.0000  CA2          97.0000  CA3         139.0000
                 CA4         317.0000  CA5         305.0000  CA6         278.0000  CA7         244.0000
                 CA8         232.0000  CL2          45.0000  CL3         108.0000  CL4         123.0000
                 CL5         118.0000  CL6         109.0000  CL7         110.0000  CL8         107.0000

     PR05Y2      COLYR    0*******000000000000000000
                 UPBND         1.0000
                 TS2         510.0000  TS3         830.0000  TS4        1250.0000  TS5        1330.0000
                 TS6        1350.0000  TS7        1310.0000  TS8        1280.0000  EA2          54.0000
                 EA3         116.0000  EA4         224.0000  EA5         266.0000  EA6         270.0000
                 EA7         250.0000  EA8         230.0000  BL2         145.0000  PE2         180.0000
                 PE3         150.0000  PE4          90.0000  CA2         128.0000  CA3         362.0000
                 CA4         535.0000  CA5         595.0000  CA6         591.0000  CA7         550.0000
                 CA8         498.0000  CL2          38.0000  CL3          90.0000  CL4         124.0000
                 CL5         130.0000  CL6         141.0000  CL7         135.0000  CL8         124.0000

     PR13Y2      COLYR    0*******000000000000000000
                 UPBND         1.0000
                 TS2         600.0000  TS3         940.0000  TS4        1560.0000  TS5        1100.0000
```

ROW SET (########) COLUMN SET (########)

LISTING BY COLUMNS

VARIABLES

	TS6	480.0000	TS7	660.0000	TS8	210.0000	EA2	72.0000
	EA3	160.0000	EA4	468.0000	EA5	242.0000	EA6	48.0000
	FA7	112.0000	EA8	36.0000	BL2	250.0000	PE2	140.0000
	PE3	120.0000	PE4	70.0000	CA2	174.0000	CA3	286.0000
	CA4	518.0000	CA5	451.0000	CA6	166.0000	CA7	238.0000
	CA8	61.0000	CL2	60.0000	CL3	102.0000	CL4	180.0000
	CL5	121.0000	CL6	85.0000	CL7	103.0000	CL8	25.0000
PR14Y2	COLYR	0*******000000000000000						
	UPBND	1.0000						
	TS2	500.0000	TS3	1000.0000	TS4	1250.0000	TS5	1500.0000
	TS6	1500.0000	TS7	1500.0000	TS8	1250.0000	EA2	50.0000
	EA3	150.0000	EA4	250.0000	EA5	300.0000	EA6	300.0000
	FA7	300.0000	EA8	250.0000	BL2	100.0000	BL3	100.0000
	PE2	50.0000	PE3	50.0000	PE4	50.0000	PE5	50.0000
	PE6	50.0000	PE7	50.0000	PE8	50.0000	CA2	120.0000
	CA3	210.0000	CA4	300.0000	CA5	390.0000	CA6	480.0000
	CA7	550.0000	CA8	630.0000	CL2	40.0000	CL3	80.0000
	CL4	120.0000	CL5	160.0000	CL6	200.0000	CL7	200.0000
	CL8	200.0000						
PR21Y2	COLYR	0*******000000000000000						
	UPBND	1.0000						
	TS2	1200.0000	TS3	2000.0000	TS4	2000.0000	TS5	2000.0000
	TS6	1800.0000	TS7	1700.0000	TS8	1400.0000	EA2	192.0000
	FA3	360.0000	EA4	300.0000	EA5	340.0000	EA6	288.0000
	FA7	238.0000	EA8	182.0000	BL2	300.0000	PE2	250.0000
	PE3	400.0000	PE4	200.0000	CA2	372.0000	CA3	640.0000
	CA4	645.0000	CA5	702.0000	CA6	690.0000	CA7	672.0000
	CA8	585.0000	CL2	181.0000	CL3	279.0000	CL4	290.0000
	CL5	285.0000	CL6	263.0000	CL7	241.0000	CL8	214.0000
PR24Y2	COLYR	0*******000000000000000						
	UPBND	1.0000						
	TS2	1200.0000	TS3	1700.0000	TS4	1560.0000	TS5	1490.0000
	TS6	1495.0000	TS7	1520.0000	TS8	1530.0000	EA2	156.0000
	FA3	255.0000	EA4	265.0000	EA5	209.0000	EA6	224.0000
	EA7	258.0000	EA8	275.0000	BL2	150.0000	BL4	100.0000
	PE2	250.0000	PE3	200.0000	PE4	100.0000	PE5	50.0000
	CA2	406.0000	CA3	506.0000	CA4	567.0000	CA5	513.0000
	CA6	510.0000	CA7	537.0000	CA8	555.0000	CL2	187.0000
	CL3	250.0000	CL4	270.0000	CL5	270.0000	CL6	265.0000
	CL7	255.0000	CL8	260.0000				
PR02Y3	COLYR	00******000000000000000						
	UPBND	1.0000						
	TS3	310.0000	TS4	670.0000	TS5	700.0000	TS6	690.0000
	TS7	650.0000	TS8	620.0000	EA3	30.0000	EA4	80.0000
	FA5	105.0000	EA6	105.0000	EA7	105.0000	EA8	87.0000
	BL3	50.0000	PE3	90.0000	PE4	45.0000	CA3	123.0000

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PROBLEM   GOALMODEL-05          LIST(1)                          DATE  20/07/76      TIME  12/11/29      PAGE  0021
                                ROW SET (########)               COLUMN SET (########)

LISTING BY COLUMNS

VARIABLES

                 CA4        284.0000   CA5        297.0000   CA6        294.0000   CA7        266.0000
                 CA8        261.0000   CL3         37.0000   CL4         74.0000   CL5         78.0000
                 CL6         78.0000   CL7         73.0000   CL8         70.0000

        PR11Y3   COLYR      00******0000000000000000
                 UPBND        1.0000
                 TS3        120.0000   TS4        270.0000   TS5        750.0000   TS6       1250.0000
                 TS7       1300.0000   TS8       1250.0000   EA4         15.0000   EA5        135.0000
                 EA6        250.0000   EA7        260.0000   EA8        250.0000   BL3        225.0000
                 PE3        120.0000   PE4        100.0000   PE5         75.0000   CA3         30.0000
                 CA4         75.0000   CA5        115.0000   CA6        240.0000   CA7        340.0000
                 CA8        380.0000   CL3         20.0000   CL4         60.0000   CL5         95.0000
                 CL6        105.0000   CL7        140.0000   CL8        170.0000

        PR15Y3   COLYR      00******0000000000000000
                 UPBND        1.0000
                 TS3        230.0000   TS4        680.0000   TS5        720.0000   TS6        710.0000
                 TS7        730.0000   TS8        680.0000   EA3          7.0000   EA4         34.0000
                 EA5         72.0000   EA6        142.0000   EA7        146.0000   EA8        123.0000
                 BL3         50.0000   BL4         25.0000   PE3         60.0000   PE4         60.0000
                 CA3         86.0000   CA4        188.0000   CA5        218.0000   CA6        272.0000
                 CA7        292.0000   CA8        329.0000   CL3         20.0000   CL4         25.0000
                 CL5         40.0000   CL6         73.0000   CL7         85.0000   CL8        102.0000

        PRO1Y4   COLYR      000*****0000000000000000
                 UPBND        1.0000
                 TS6        500.0000   TS7        800.0000   TS8       1000.0000   EA6        100.0000
                 EA7        184.0000   EA8        240.0000   BL4        100.0000   BL5         50.0000
                 BL6         50.0000   PE5         80.0000   PE6         70.0000   CA6        200.0000
                 CA7        359.0000   CA8        392.0000   CL6         40.0000   CL7         67.0000
                 CL8         71.0000

        PRO5Y4   COLYR      000*****0000000000000000
                 UPBND        1.0000
                 TS4        510.0000   TS5        830.0000   TS6       1250.0000   TS7       1330.0000
                 TS8       1350.0000   EA4         54.0000   EA5        116.0000   EA6        224.0000
                 EA7        266.0000   EA8        270.0000   BL4        145.0000   PE4        180.0000
                 PE5        150.0000   PE6         90.0000   CA4        128.0000   CA5        367.0000
                 CA6        535.0000   CA7        595.0000   CA8        591.0000   CL4         38.0000
                 CL5         90.0000   CL6        124.0000   CL7        130.0000   CL8        141.0000

        PR11Y4   COLYR      000*****0000000000000000
                 UPBND        1.0000
                 TS4        120.0000   TS5        270.0000   TS6        750.0000   TS7       1250.0000
                 TS8       1300.0000   EA5         15.0000   EA6        135.0000   EA7        250.0000
                 EA8        260.0000   BL4        225.0000   PE4        120.0000   PE5        100.0000
                 PE6         75.0000   CA4         30.0000   CA5         75.0000   CA6        115.0000
                 CA7        240.0000   CA8        340.0000   CL4         20.0000   CL5         60.0000
                 CL6         95.0000   CL7        105.0000   CL8        140.0000
```

ROW SET (########) COLUMN SET (########)

LISTING BY COLUMNS

VARIABLES

PR12Y4	COLYR	000*****0000000000000000							
	UPBND	1.0000							
	TS4	120.0000	TS5	390.0000	TS6	580.0000	TS7	760.0000
	TS8	1000.0000	EA4	4.0000	EA5	36.0000	EA6	108.0000
	EA7	160.0000	EA8	230.0000	BL4	190.0000	PE4	50.0000
	PE5	80.0000	PE6	70.0000	PE7	50.0000	PE8	30.0000
	CA4	41.0000	CA5	134.0000	CA6	279.0000	CA7	341.0000
	CA8	486.0000	CL4	17.0000	CL5	53.0000	CL6	124.0000
	CL7	168.0000	CL8	200.0000				
PR13Y4	COLYR	000*****0000000000000000						
	UPBND	1.0000						
	TS4	600.0000	TS5	940.0000	TS6	1560.0000	TS7	1100.0000
	TS8	480.0000	EA4	72.0000	EA5	160.0000	EA6	468.0000
	EA7	242.0000	EA8	48.0000	BL4	250.0000	PE4	140.0000
	PE5	120.0000	PE6	70.0000	CA4	174.0000	CA5	286.0000
	CA6	518.0000	CA7	451.0000	CA8	166.0000	CL4	60.0000
	CL5	102.0000	CL6	180.0000	CL7	121.0000	CL8	85.0000
PR14Y4	COLYR	000*****0000000000000000						
	UPBND	1.0000						
	TS4	500.0000	TS5	1000.0000	TS6	1250.0000	TS7	1500.0000
	TS8	1500.0000	EA4	50.0000	EA5	150.0000	EA6	250.0000
	EA7	300.0000	EA8	300.0000	BL4	100.0000	BL5	100.0000
	PE4	50.0000	PE5	50.0000	PE6	50.0000	PE7	50.0000
	PE8	50.0000	CA4	120.0000	CA5	210.0000	CA6	300.0000
	CA7	390.0000	CA8	480.0000	CL4	40.0000	CL5	80.0000
	CL6	120.0000	CL7	160.0000	CL8	200.0000		
PR22Y4	COLYR	000*****0000000000000000						
	UPBND	1.0000						
	TS4	500.0000	TS5	500.0000	TS6	500.0000	TS7	500.0000
	TS8	500.0000	EA4	100.0000	EA5	100.0000	EA6	110.0000
	EA7	110.0000	EA8	105.0000	BL4	100.0000	PE4	40.0000
	PE5	100.0000	PE6	150.0000	PE7	40.0000	CA4	150.0000
	CA5	154.0000	CA6	163.0000	CA7	169.0000	CA8	178.0000
	CL4	69.0000	CL5	73.0000	CL6	69.0000	CL7	67.0000
	CL8	68.0000						
PR25Y6	COLYR	000*****0000000000000000						
	UPBND	1.0000						
	TS4	1000.0000	TS5	1200.0000	TS6	1250.0000	TS7	1260.0000
	TS8	1290.0000	EA4	140.0000	EA5	220.0000	EA6	230.0000
	EA7	240.0000	EA8	220.0000	BL4	125.0000	BL5	25.0000
	BL7	50.0000	PE4	180.0000	PE5	150.0000	PE7	150.0000
	CA4	367.0000	CA5	419.0000	CA6	455.0000	CA7	451.0000
	CA8	446.0000	CL4	162.0000	CL5	190.0000	CL6	200.0000
	CL7	196.0000	CL8	198.0000				
PR02Y5	COLYR	0000****0000000000000000						
	UPBND	1.0000						

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PROBLEM   GOALMODEL-05          LIST(1)                                DATE  20/07/76      TIME  12/11/30        PAGE  0023

                                ROW SET (########)                   COLUMN SET (########)

LISTING BY COLUMNS

VARIABLES
                TS5          310.0000  TS6          670.0000  TS7          700.0000  TS8          690.0000
                EA5           30.0000  EA6           80.0000  EA7          105.0000  EA8          105.0000
                BL5           50.0000  PE5           90.0000  PE6           65.0000  CA5          123.0000
                CA6          280.0000  CA7          297.0000  CA8          294.0000  CL5           37.0000
                CL6           74.0000  CL7           78.0000  CL8           78.0000

      PR03Y5    COLYR     0000****000000000000000
                UPBND          1.0000
                TS5          410.0000  TS6          620.0000  TS7         1800.0000  TS8         1640.0000
                EA5           20.0000  EA6           62.0000  EA7          270.0000  EA8          324.0000
                BL5          200.0000  PE5          100.0000  PE6           80.0000  CA5          149.0000
                CA6          257.0000  CA7          714.0000  CA8          692.0000  CL5           60.0000
                CL6          103.0000  CL7          285.0000  CL8          200.0000

      PR11Y5    COLYR     0000****000000000000000
                UPBND          1.0000
                TS5          120.0000  TS6          270.0000  TS7          750.0000  TS8         1250.0000
                EA6           15.0000  EA7          135.0000  EA8          250.0000  BL5          225.0000
                PE5          120.0000  PE6          100.0000  PE7           75.0000  CA5           30.0000
                CA6           75.0000  CA7          115.0000  CA8          240.0000  CL5           20.0000
                CL6           60.0000  CL7           95.0000  CL8          105.0000

      PR21Y5    COLYR     0000****000000000000000
                UPBND          1.0000
                TS5         1200.0000  TS6         2000.0000  TS7         2000.0000  TS8         2000.0000
                EA5          192.0000  EA6          360.0000  EA7          340.0000  EA8          340.0000
                BL5          300.0000  PE5          250.0000  PE6          400.0000  PE7          200.0000
                CA5          372.0000  CA6          640.0000  CA7          645.0000  CA8          707.0000
                CL5          181.0000  CL6          279.0000  CL7          290.0000  CL8          285.0000

      PR23Y5    COLYR     0000****000000000000000
                UPBND          1.0000
                TS5          700.0000  TS6          750.0000  TS7          780.0000  TS8          800.0000
                EA5          140.0000  EA6          165.0000  EA7          156.0000  EA8          152.0000
                BL5          125.0000  BL6           65.0000  PE5          250.0000  PE6          100.0000
                CA5          210.0000  CA6          244.0000  CA7          284.0000  CA8          310.0000
                CL5          100.0000  CL6          100.0000  CL7          112.0000  CL8          116.0000

      PR04Y6    COLYR     00000***000000000000000
                UPBND          1.0000
                TS6          300.0000  TS7          760.0000  TS8          980.0000  EA6           27.0000
                EA7          130.0000  EA8          226.0000  BL6           75.0000  BL7           25.0000
                PE6          250.0000  PE7          130.0000  CA6           97.0000  CA7          139.0000
                CA8          317.0000  CL6           65.0000  CL7          108.0000  CL8          123.0000

      PR05Y6    COLYR     00000---000000000000000
                UPBND          1.0000
                TS6          510.0000  TS7          830.0000  TS8         1250.0000  EA6           54.0000
                EA7          116.0000  EA8          224.0000  BL6          145.0000  PE6          180.0000
                PE7          150.0000  PE8           90.0000  CA6          128.0000  CA7          362.0000
```

ROW SET (#########) COLUMN SET (#########)

LISTING BY COLUMNS

VARIABLES

	CA8	535.0000	CL6	38.0000	CL7	90.0000	CL8	124.0000
PR11Y6	COLYR	00000***0000000000000000						
	UPBND	1.0000						
	TS6	120.0000	TS7	270.0000	TS8	750.0000	EA7	15.0000
	EA8	135.0000	BL6	225.0000	PE6	120.0000	PE7	100.0000
	PE8	75.0000	CA6	30.0000	CA7	75.0000	CA8	115.0000
	CL6	20.0000	CL7	60.0000	CL8	95.0000		
PR14Y6	COLYR	00000***0000000000000000						
	UPBND	1.0000						
	TS6	500.0000	TS7	1000.0000	TS8	1250.0000	EA6	50.0000
	EA7	150.0000	EA8	250.0000	BL6	100.0000	BL7	100.0000
	PE6	50.0000	PE7	50.0000	PE8	50.0000	CA6	120.0000
	CA7	210.0000	CA8	300.0000	CL6	40.0000	CL7	80.0000
	CL8	120.0000						
PR15Y6	COLYR	00000***0000000000000000						
	UPBND	1.0000						
	TS6	230.0000	TS7	680.0000	TS8	720.0000	EA6	7.0000
	EA7	34.0000	EA8	72.0000	BL6	50.0000	BL7	25.0000
	PE6	60.0000	PE7	40.0000	CA6	84.0000	CA7	188.0000
	CA8	218.0000	CL6	20.0000	CL7	25.0000	CL8	40.0000
PR16Y6	COLYR	00000***0000000000000000						
	UPBND	1.0000						
	TS6	200.0000	TS7	600.0000	TS8	1000.0000	EA7	42.0000
	EA8	160.0000	BL6	160.0000	BL7	60.0000	PE6	100.0000
	PE7	100.0000	PE8	50.0000	CA6	65.0000	CA7	215.0000
	CA8	365.0000	CL6	20.0000	CL7	70.0000	CL8	100.0000
PR21Y6	COLYR	00000***0000000000000000						
	UPBND	1.0000						
	TS6	1200.0000	TS7	2000.0000	TS8	2000.0000	EA6	192.0000
	EA7	360.0000	EA8	360.0000	BL6	300.0000	PE6	250.0000
	PE7	400.0000	PE8	200.0000	CA6	377.0000	CA7	640.0000
	CA8	645.0000	CL6	181.0000	CL7	279.0000	CL8	290.0000
PR23Y6	COLYR	00000***0000000000000000						
	UPBND	1.0000						
	TS6	700.0000	TS7	750.0000	TS8	780.0000	EA6	140.0000
	EA7	165.0000	EA8	156.0000	BL6	125.0000	BL7	65.0000
	PE6	250.0000	PE7	100.0000	CA6	210.0000	CA7	244.0000
	CA8	284.0000	CL6	100.0000	CL7	100.0000	CL8	112.0000
PR01Y7	COLYR	000000**0000000000000000						
	UPBND	1.0000						
	BL7	100.0000	BL8	50.0000	PE8	80.0000		
PR04Y7	COLYR	000000**0000000000000000						

```
PROBLEM   GOALMODEL-05        LIST(1)                                DATE  20/07/76      TIME  12/11/30      PAGE  0025
                              ROW SET (########)             COLUMN SET (########)

LISTING BY COLUMNS

VARIABLES

                UPBND         1.0000
                TS7         300.0000   TS8         760.0000   EA7          27.0000   EA8         130.0000
                BL7          75.0000   BL8          25.0000   PE7         250.0000   PE8         130.0000
                CA7          97.0000   CA8         139.0000   CL7          45.0000   CL8         108.0000

     PR14Y7     COLYR     000000**000000000000000
                UPBND         1.0000
                TS7         500.0000   TS8        1000.0000   EA7          50.0000   EA8         150.0000
                BL7         100.0000   BL8         100.0000   PE7          50.0000   PE8          50.0000
                CA7         120.0000   CA8         210.0000   CL7          40.0000   CL8          80.0000

     PR22Y7     COLYR     000000**000000000000000
                UPBND         1.0000
                TS7         500.0000   TS8         500.0000   EA7         100.0000   EA8         100.0000
                BL7         100.0000   PE7          40.0000   PE8         100.0000   CA7         150.0000
                CA8         154.0000   CL7          69.0000   CL8          73.0000

     PR02Y8     COLYR     0000000*00000000000000000
                UPBND         1.0000
                TS8         310.0000   EA8          30.0000   BL8          50.0000   PE8          90.0000
                CA8         123.0000   CL8          37.0000

     PR15Y8     COLYR     0000000*00000000000000000
                UPBND         1.0000
                TS8         230.0000   EA8           7.0000   BL8          50.0000   PE8          60.0000
                CA8          84.0000   CL8          20.0000

     PR22Y8     COLYR     0000000*00000000000000000
                UPBND         1.0000
                TS8         500.0000   EA8         100.0000   BL8         100.0000   PE8          40.0000
                CA8         150.0000   CL8          69.0000

     PR25Y8     COLYR     0000000*00000000000000000
                UPBND         1.0000
                TS8        1000.0000   EA8         140.0000   BL8         125.0000   PE8         180.0000
                CA8         367.0000   CL8         162.0000

     RIGHT HAND SIDES

     RHS1       NW8       14100.0000   TS1      -11000.0000   TS2      -10000.0000   TS3        -9500.0000
                TS4       -8800.0000   TS5       -8000.0000   TS6       -7500.0000   TS7        -7500.0000
                TS8       -7000.0000   EA1       -1953.0000   EA2       -1850.0000   EA3        -1600.0000
                EA4       -1560.0000   EA5       -1240.0000   EA6       -1220.0000   EA7        -1200.0000
                EA8       -1000.0000   BL1       -2034.0000   BL2        -600.0000   BL3         -500.0000
                BL4        -200.0000   PE1       -1481.0000   PE2        -500.0000   PE3         -400.0000
                PE4        -400.0000   PE5        -400.0000   PE6        -400.0000   PE7         -400.0000
                PE8        -400.0000   CA1       -3510.0000   CA2       -3588.0000   CA3        -3705.0000
                CA4       -3432.0000   CA5       -3120.0000   CA6       -2925.0000   CA7        -2925.0000
                CA8       -2730.0000   CL1       -1120.0000   CL2       -1145.0000   CL3        -1170.0000
                CL4       -1095.0000   CL5       -1020.0000   CL6        -946.0000   CL7         -948.0000
```

ROW SET (########) COLUMN SET (########)

LISTING BY COLUMNS

RIGHT HAND SIDES

CL8	-902.0000	CR1	-4580.0000	CB5	1000.0000	TP1	702.0000
TP2	-65.0000	TP3	-65.0000	TP4	-60.0000	TP5	-60.0000
TP6	-60.0000	TP7	-60.0000	TP8	-55.0000	TA1	-65.0000
PR1	-60.0000	EQ1	2000.0000	D1	1500.0000	D5	-1000.0000
STARG1	16000.0000	STARG2	17920.0000	STARG3	20000.0000	STARG4	22480.0000
STARG5	25167.0000	STARG6	28197.0000	STARG7	31580.0000	STARG8	35370.0000
PTARG1	900.0000	PTARG2	1017.0000	PTARG3	1140.0000	PTARG4	1275.0000
PTARG5	1428.0000	PTARG6	1600.0000	PTARG7	1790.0000	PTARG8	2000.0000

A P P E N D I X XII

THE REPORTING SUITE-MULTICRITERIA MODEL

A12.1 PROGRAM FOR COMPLETE FINANCIAL STATEMENT ANALYSIS

A12.2 PROGRAM FOR SUMMARY STATISTICS

EXHIBIT A12.1 PROGRAM FOR COMPLETE FINANCIAL STATEMENT ANALYSIS

```
*LISTING OF :WORKFILE.AAA100061234(1/) PRODUCED ON 27OCT75 AT 22.43.29

*OUTPUT BY LISTFILE IN ':UWPSPU05.GOALANALYS' ON 28OCT75 AT 08.10.19

DOCUMENT    1-1


22/43/00          27/10/75         COMPILED BY XALV MK. 3A
LINE STATEMENT
       0          'INPUT' 0 = CR1
       0          'INPUT' 1 = TR0
       0          'INPUT' 2 = CR0
       0          'OUTPUT' 0 = LP0
       0          'CONTINUE'

22/43/00          27/10/75         COMPILED BY XALV MK. 3A
LINE STATEMENT
       0         'TRACE'1
       0         'BEGIN'
       1         'INTEGER' I,J,K,M;
       1         'REAL'RES,RI,PL,RS;
       2
       2           'PROCEDURE' VARRTEXT(A,N);  'VALUE' N;
       5             'ARRAY' A;  'INTEGER' N;  'EXTERNAL';
       7           'INTEGER''PROCEDURE' INSTRARR(S,A);
       9             'STRING' S;  'ARRAY' A.  'EXTERNAL';
      11           'BOOLEAN''PROCEDURE' TEST(N);  'VALUE' N;  'INTEGER' N;  'EXTERNAL';
      15           'PROCEDURE' READTRAP(P);  'PROCEDURE' P;  'EXTERNAL';
      18
      18           'PROCEDURE' READERR(N);  'INTEGER' N;
      21           'BEGIN'
      21             'INTEGER' J;
      21             NEWLINE(1);
      23             WRITETEXT('('***%READ%FAIL%JUST%BEFORE%/')');
      24             'FOR' J:= 1 'STEP' 1 'UNTIL' 360 'DO' PRINTCH(READCH);
      26             PAUSE(99);
      27           'END' READERR;
      27
      27           'PROCEDURE' INTILL(STR);  'STRING' STR;
      30           'COMMENT' SKIPS OVER CURRENT INPUT STREAM UNTIL CHARACTER IMMEDIATELY
      30           FOLLOWING STRING STR. SWITCH 1 ON GIVES DIAGNOSTIC PRINT OF FIRST 120
      30           CHARACTERS SKIPPED;
      30           'BEGIN'
      30             'INTEGER''ARRAY' BUF[1:30];  'INTEGER' COUNT;
      31             COUNT:= INSTRARR(STR,BUF);
      33             'IF' TEST(1) 'THEN'
      33             'BEGIN'
      33               NEWLINE(1);  WRITETEXT('('***%INTILL%')');  PRINT(COUNT,8,0);
      37               NEWLINE(1);  VARRTEXT(BUF,COUNT);
      39             'END';
      40           'END' INTILL;
      40
      40           'PROCEDURE' DATAIN(VAR,STR1,STR2);  'REAL' VAR;  'STRING' STR1,STR2;
      44           'COMMENT' SEARCHES IN TURN FOR STRINGS STR1, STR2 AND THEN AT LEAST
      44           ONE SPACE BEFORE READING A REAL VARIABLE VAR. SWITCH 2 ON GIVES
      44           DIAGNOSTIC PRINT OF VALUE READ;
      44           'BEGIN'
      44             INTILL(STR1);  INTILL(STR2);
      47         LOOP:
      47             'IF' NEXTCH'NE'CODE('('X')') 'THEN'
      47               'BEGIN'  SKIPCH;  'GOTO' LOOP;  'END';
      51             VAR:= READ;
      52             'IF' TEST(2) 'THEN'
      52             'BEGIN'
      52               NEWLINE(1);  WRITETEXT('('***%DATAIN%%')');  PRINT(VAR,0,8);
      56             'END';
      57           'END' DATAIN;
      57
      57           'PROCEDURE' RNSIN(ARR,SIZE,STR);  'VALUE' SIZE;
      60             'REAL''ARRAY' ARR;  'INTEGER' SIZE;  'STRING' STR;
      63           'COMMENT' LOOP SIZE TIMES: SEARCH FOR STRING STR AND THEN ONE SPACE
      63           BEFORE READING TWO REAL VARIABLES, THE SECOND IS STORED IN ARRAY ARR.
      63           SWITCH 2 ON GIVES DIAGNOSTIC PRINT OF THE ARRAY;
      63           'BEGIN'
      63             'REAL' DUMMY;  'INTEGER' J;
```

```
      'FOR' J:= 1 'STEP' 1 'UNTIL' SIZE 'DO'
      'BEGIN'
        INTILL(STR);
LOOP: 'IF' NEXTCH 'NE' CODE('('*')') 'AND'NEXTCH 'NE' CODE('('-')')
'THEN'
          'BEGIN'  SKIPCH;  'GOTO' LOOP;  'END';
SKIPCH;
        DUMMY:= READ;  ARR[J]:= READ;
      'END' J LOOP;
      'IF' TEST(2) 'THEN'
      'BEGIN'
        NEWLINE(1);  WRITETEXT('('***%RHSIN%%')');
        'FOR' J:= 1 'STEP' 1 'UNTIL' SIZE 'DO' PRINT(ARR[J],0,8);
      'END';
    'END' RHSIN;

    'PROCEDURE' ARRAYIN(ARR,SIZE,STR1,STR2);  'VALUE' SIZE;
      'REAL''ARRAY' ARR;  'INTEGER' SIZE;  'STRING' STR1,STR2;
    'BEGIN'
      'INTEGER' J;
      'FOR' J:= 1 'STEP' 1 'UNTIL' SIZE 'DO' DATAIN(ARR[J],STR1,STR2);
    'END' ARRAYIN;

    'PROCEDURE' INPUT(ARR,SIZE,STR);  'VALUE' SIZE;
      'REAL''ARRAY' ARR;  'INTEGER' SIZE;  'STRING' STR;
    'BEGIN'
      ARRAYIN(ARR,SIZE,STR,'('*')');
    'END' INPUT;

'PROCEDURE'NEXTDUMP;
      'BEGIN' 'INTEGER'  N;INTILL('('DUMP;DUMP')');
      N:=READ;
      'IF' N 'EQ'M'THEN' NEXTDUMP 'ELSE' M:=N;
      'END';
'PROCEDURE' OUTPUT(A);
'REAL''ARRAY'A;
'BEGIN'
       COPYTEXT('('*')');
       'FOR'I:=1'STEP'1'UNTIL'8 'DO'PRINT(((A[I]+0.5)*10)'/'10,7,0);
'END';
'PROCEDURE' PERIOD;
'BEGIN'
       NEWLINE(1);
   WRITETEXT('('''('1C 34S')'PERIOD-1%%PERIOD-2%%PERIOD-3%%PERIOD-4')');
       WRITETEXT('('''('2S')'PERIOD-5%%PERIOD-6%%PERIOD-7%%PERIOD-8')');
       NEWLINE(1);
'END';
'PROCEDURE'ACHIEVEMENT(A,B);
'REAL''ARRAY'A,B;
'BEGIN'
       COPYTEXT('('*')');
       'FOR'I:=1'STEP'1'UNTIL'M'DO'PRINT(A[I],5,1);
       NEWLINE(1);
       COPYTEXT('('*')');
       'FOR' I:=1'STEP'1'UNTIL'M'DO'PRINT(B[I],5,1);
       NEWLINE(1);
       COPYTEXT('('*')');
       'FOR'I:=1'STEP'1'UNTIL' M'DO'PRINT((B[I]-A[I])/A[I]*100,5,1);
'END';
'BEGIN'
'REAL''ARRAY'SALES,EARN,NPAT,NG,LL,CR,ROCE,LQDY,ECUV,EPPS,DCUV,DTARG,
PTARG ,STARG,A2,A3,A4,A5,A6,A7,A8[1:8]
,AA,AD,TAX,FABL,FAPF,CURA,CURL,OVDR,MARK,DV,NUM,DF,RLTA,A1[0:8];

   SELECTINPUT(1);
   READTHAP(HEADERR);
INTILL('('LISTING%BY%COLUMN')');
INTILL('('VARIABLES')');
ARRAYIN(ROCE,M,'('FABL')','('ROCE')');
ARRAYIN(LQDY,M,'('CURL')','('LQDY')');
DATAIN(RS,'('OVDR1')','('FA2')');PS:=-RS;
DATAIN(RI,'('MARK1')','('FA2')');
ARRAYIN(DCUV,M,'('DV')','('DCUV')');
'FOR' J:=1 'STEP' 1 'UNTIL' 8 'DO'
     'BEGIN'
DATAIN(EPPS[J],'('NUM')','('EPPS')');
INTILL('('DTARG')');
SKIPCH;
DTARG[J]:=READ;
     'END';
```

```
DATAIN(RL,'('DE1')','('CR2')');RL:=-RL;
INTILL('('ECOV1')');
ECOV[1]:=READ;
'FOR'J:=2'STEP'1'UNTIL'8 'DO' DATAIN(ECOV[J],'('DE')','('ECOV')');
'FOR' I:=1 'STEP' 1 'UNTIL' 8 'DO' 'BEGIN'
                                  ROCE[I]:=ROCE[I]*100.0;
                                ECOV[I]:=ECOV[I]/RL;
                                  ERPS[I]:=100.0*ERPS[I];
                                  DTARG[I]:=DTARG[I]*(-100.0);
                                         'END';
INTILL('('SOLUTION')');
INTILL('('DUMP')');
INTILL('('DUMP')');
M:=READ;PAPERTHROW;
START:
INPUT(SALES,8,'('SALES')');
INPUT(EARN,8,'('EARN')');
INPUT(NPAT,5,'('NPAT')');
INPUT(TAX,8,'('TAX')');
INPUT(FABL,8,'('FABL')');
INPUT(FAPE,8,'('FAPE')');
INPUT(CURA,8,'('CURA')');
INPUT(CURL,8,'('CURL')');
INPUT(OVDR,8,'('OVDR')');
INPUT(MARK,8,'('MARK')');
INPUT(DV,8,'('DV')');
INPUT(NUM,8,'('NUM')');
INPUT(RG,8,'('RG')');
INPUT(DE,8,'('DE')');
INPUT(LL,8,'('BLTA')');
INTILL('('ROWXINFORMATION')');
INTILL('('PROBLEM')');
COPYTEXT('('ROWXINFORMATION')');
RHSIN(STARG,8,'('STARG')');
RHSIN(PTARG,8,'('PTARG')');
  SELECTINPUT(2);  'COMMENT' GEO3 SHOULD PROVIDE *CRO;

NUM[0]:=READ;
RES:=READ;
DE[0]:=READ;
FABL[0]:=READ;FAPE[0]:=READ;
MARK[0]:=READ;CURA[0]:=READ;
TAX[0]:=READ;OVDR[0]:=READ;DV[0]:=READ;CURL[0]:=READ;

'COMMENT'     BALANCE SHEET;

COPYTEXT('('*')');           ;PERIOD;
COPYTEXT('('*')');
OUTPUT(NUM);A1[0]:=RES;
A0[0]:=0.0;AA[0]:=0.0;
'FOR'I:=1'STEP'1'UNTIL' 8 'DO''BEGIN'
                                     A1[I]:=A1[I-1]+NPAT[I]-DV[I];
                                     A0[I]:=A0[I-1]+NPAT[I]-TAX[I];
                                     AA[I]:=0.6*RG[I]+AA[I-1];
                                     A2[I]:=A1[I]+NUM[I]+DE[I]+AA[I]+A0[I];
                                     A3[I]:=CURA[I]-MARK[I];
                                     A4[I]:=CURL[I]-TAX[I]-OVDR[I]-DV[I];
                                     A5[I]:=CURA[I]-CURL[I];
                                     A6[I]:=FABL[I]+FAPE[I]+CURA[I]-CURL[I];
                                    'END';
 OUTPUT(AA);
 OUTPUT(A0);
 OUTPUT(A1);
 OUTPUT(DE);
 OUTPUT(A2);
 NEWLINE(2);
 COPYTEXT('('*')');NEWLINE(1);
 COPYTEXT('('*')');NEWLINE(1);
 OUTPUT(FABL);
 OUTPUT(FAPE);
 COPYTEXT('('*')');NEWLINE(1);
 OUTPUT(MARK);
 OUTPUT(A3);
 OUTPUT(CURA);
 COPYTEXT('('*')');NEWLINE(1);
 OUTPUT(A4);
 OUTPUT(TAX);
 OUTPUT(OVDR);
 OUTPUT(DV);
 OUTPUT(CURL);
 OUTPUT(A5);
 OUTPUT(A6);


 PAPERTHROW;WRITETEXT('('XYDUMP')');PRINT(M,3,0);
             NEWLINE(2);COPYTEXT('('*')');PERIOD;
 OUTPUT(SALES);
```

```
'FOR' I:=1'STEP'1'UNTIL' 8'DO''BEGIN'
                                    A1[I]:=SALES[I]-EARN[I]-RS*OVDR[I-1]
                                                             +RI*MARK[I-1];
                                    A2[I]:=EARN[I]+RS*OVDR[I-1]-RI*MARK[I-1];
                                    A3[I]:=RI*MARK[I-1];
                                    A4[I]:=RS*OVDR[I-1];
                                    A5[I]:=RL*DE[I-1];
                                    A6[I]:=EARN[I]-A5[I];
                                    A7[I]:=NPAT[I]-DV[I];
                                    'END';
OUTPUT(A1);
OUTPUT(A2);
OUTPUT(A3);
OUTPUT(A4);
OUTPUT(A5);
OUTPUT(A6);
OUTPUT(NPAT);
OUTPUT(DV);
OUTPUT(A7);
NEWLINE(3);

'COMMENT'   CASH FLOW STATEMENT;

COPYTEXT('('*')');NEWLINE(1);
COPYTEXT('('*')');
'FOR' I:=1'STEP'1 'UNTIL' 8 'DO''BEGIN'
                                       A3[I]:=OVDR[I]-OVDR[I-1];
                                       A4[I]:=DE[I]-DE[I-1];
                                           A1[I]:=EARN[I]+0.0303*FABL[I]
                                      +0.3333*FAPE[I]+CURL[I]-CURL[I-1]
                                        -CURA[I]+CURA[I-1]+RS*OVDR[I-1]
                                       +MARK[I]-MARK[I-1]-DV[I]+DV[I-1]
                                     -TAX[I]+TAX[I-1]-A3[I]-RI*MARK[I-1];
                                       A2[I]:=RI*MARK[I-1];
                                       A5[I]:=1.0*RG[I];
                                       A6[I]:=A1[I]+A2[I]       +A4[I]+A5[I];
                                       'END';
OUTPUT(A1);
OUTPUT(A2);
OUTPUT(A4);
OUTPUT(A5);
OUTPUT(A6);
COPYTEXT('('*')');
'FOR' I:=1'STEP'1'UNTIL' 8'DO''BEGIN'
                                    A1[I]:=1.0303*FABL[I]-FABL[I-1];
                                    A2[I]:=1.3333*FAPE[I]-FAPE[I-1];
                                    A3[I]:=RS*OVDR[I-1];
                                    A4[I]:=RL*DE[I-1];
                                    A6[I]:=MARK[I]-MARK[I-1]-OVDR[I]+OVDR[I-1];
                                    A7[I]:=TAX[I-1];
                                    A8[I]:=DV[I-1];
                                    A5[I]:=A1[I]+A2[I]+A3[I]+A4[I]+A7[I]+A8[I];
                                    'END';
OUTPUT(A1);
OUTPUT(A2);
OUTPUT(A3);
OUTPUT(A4);
OUTPUT(A6);
OUTPUT(A7);
OUTPUT(A5);
OUTPUT(A6);

'COMMENT'    INDICATORS;

PAPERTHROW;WRITETEXT('('XXDUMP')');PRINT(M,3,0);
NEWLINE(1);
'FOR' I:=1'STEP' 1 'UNTIL' 8'DO''BEGIN'
                                    A1[I]:=EARN[I]/(FABL[I]+FAPE[I]+CURA[I]
                                                                   -CURL[I]);
                                     A1[I]:=A1[I]*100.0;
                                    A2[I]:=CURA[I]/CURL[I];
                                    A3[I]:=EARN[I]/(RL*DE[I]+RS*OVDR[I]);
                                    A4[I]:=NPAT[I]/NUM[I]*100.0;
                                    A5[I]:=NPAT[I]/DV[I];
                                    A6[I]:=DV[I]/NUM[I]*100.0;
                                    'END';
                 COPYTEXT('('*')');PERIOD;
ACHIEVEMENT(ROCE,A1);
ACHIEVEMENT(LQDY,A2);
ACHIEVEMENT(ECOV,A3);
ACHIEVEMENT(EPPS,A4);
ACHIEVEMENT(DCOV,A5);
ACHIEVEMENT(DTARG,A6);
ACHIEVEMENT(STARG,SALES);
ACHIEVEMENT(PTARG,NPAT);
COPYTEXT('('*')');
  FREEINPUT;  'COMMENT' RELEASING CRD ON CHANNEL 2;
SELECTINPUT(1);
NEATDUMP; PAPERTHROW; 'GOTO' START;
'END';
'END';
```

EXHIBIT A12.2 PROGRAM FOR SUMMARY STATISTICS

```
#LISTING OF :WORKFILE.AAA00071641(1/) PRODUCED ON 31OCT75 AT 17.21.29
#OUTPUT BY LISTFILE IN 'SUMMSPOUS.AHSPARAMET' ON 31OCT75 AT 17.21.32
DOCUMENT     I-1

17/20/26          31/10/75          COMPILED BY XALV MK.  3A
LINE STATEMENT
       0          'INPUT' 0 = CR1
       0          'INPUT' 1 = TR0
       0          'OUTPUT' 0 = LP0
       0          'CONTINUE'

17/20/26          31/10/75          COMPILED BY XALV MK.  3A
LINE STATEMENT
       0          'BEGIN'
       1          'INTEGER' I,J,K,M;
       1          'REAL'RES,RI,RL,RS;
       2
       2            'PROCEDURE' VARPTEXT(A,N);  'VALUE' N;
       5              'ARRAY' A;  'INTEGER' N;  'EXTERNAL';
       7            'INTEGER''PROCEDURE' INSTRARR(S,A);
       9              'STRING' S;  'ARRAY' A;  'EXTERNAL';
      11            'BOOLEAN''PROCEDURE' TEST(N);  'VALUE' N;  'INTEGER' N;  'EXTERNAL';
      15            'PROCEDURE' READTRAP(P);  'PROCEDURE' P;  'EXTERNAL';
      18
      18            'PROCEDURE' READERR(N);  'INTEGER' N;
      21            'BEGIN'
      21              'INTEGER' J;
      21              NEWLINE(1);
      23              WRITETEXT('('***%READ%FAIL%JUST%BEFORE%/')');
      24              'FOR' J:= 1 'STEP' 1 'UNTIL' 360 'DO' PRINTCH(READCH);
      26              PAUSE(99);
      27            'END' READERR;
      27
      27            'PROCEDURE' INTILL(STR);  'STRING' STR;
      30            'COMMENT' SKIPS OVER CURRENT INPUT STREAM UNTIL CHARACTER IMMEDIATELY
      30            FOLLOWING STRING STR. SWITCH 1 ON GIVES DIAGNOSTIC PRINT OF FIRST 120
      30            CHARACTERS SKIPPED;
      30            'BEGIN'
      30              'INTEGER''ARRAY' BUF[1:30];  'INTEGER' COUNT;
      31              COUNT:= INSTRARR(STR,BUF);
      33              'IF' TEST(1) 'THEN'
      33              'BEGIN'
      33                NEWLINE(1);  WRITETEXT('('***%INTILL%%')');  PRINT(COUNT,A,0);
      37                NEWLINE(1);  VARPTEXT(BUF,COUNT);
      39              'END';
      40            'END' INTILL;
      40
      40            'PROCEDURE' DATAIN(VAR,STR1,STR2);  'REAL' VAR;  'STRING' STR1,STR2;
      44            'COMMENT' SEARCHES IN TURN FOR STRINGS STR1, STR2 AND THEN AT LEAST
      44            ONE SPACE BEFORE READING A REAL VARIABLE VAR. SWITCH 2 ON GIVES
      44            DIAGNOSTIC PRINT OF VALUE READ;
      44            'BEGIN'
      44              INTILL(STR1);  INTILL(STR2);
      47          LOOP:
      47              'IF' NEXTCH'NE'CODE('('%')') 'THEN'
      47                'BEGIN'  SKIPCH;  'GOTO' LOOP;  'END';
      51              VAR:= READ;
      52              'IF' TEST(2) 'THEN'
      52              'BEGIN'
      52                NEWLINE(1);  WRITETEXT('('***%DATAIN%%')');  PRINT(VAR,0,8);
      56              'END';
      57            'END' DATAIN;
      57
```

```
'PROCEDURE' ARRAYIN(ARR,SIZE,STR1,STR2);  'VALUE' SIZE;
  'REAL''ARRAY' ARR;  'INTEGER' SIZE;  'STRING' STR1,STR2;
 'BEGIN'
  'INTEGER' J;
  'FOR' J:= 1 'STEP' 1 'UNTIL' SIZE 'DO' DATAIN(ARR[J],STR1,STR2);
 'END' ARRAYIN;

 'PROCEDURE' INPUT(ARR,SIZE,STR);  'VALUE' SIZE;
  'REAL''ARRAY' ARR;  'INTEGER' SIZE;  'STRING' STR;
 'BEGIN'
  ARRAYIN(ARR,SIZE,STR,'('+')');
 'END' INPUT;

'PROCEDURE'NEXTDUMP;
   'BEGIN' 'INTEGER'  N;INTILL('('DUMP;DUMP')');
   N:=READ;
   'IF' N 'EQ'M'THEN' NEXTDUMP 'ELSE' M:=N;
   'END';
'PROCEDURE' DUAL(ARR,SIZE,STR);'VALUE' SIZE;
   'REAL''ARRAY' ARR;  'INTEGER' SIZE;  'STRING' STR;
  'BEGIN'
   'REAL' DUMMY;  'INTEGER' J;
   'FOR' J:= 1 'STEP' 1 'UNTIL' SIZE 'DO'
   'BEGIN'
     INTILL(STR);
LOOP: 'IF' NEXTCH 'NE' CODE('('+')') 'AND'NEXTCH 'NE' CODE('('-')')
'THEN'
       'BEGIN'  SKIPCH;  'GOTO' LOOP;  'END';
   'END' J LOOP;
SKIPCH;
DUMMY:=READ;DUMMY:=READ;ARR[J]:=READ;
   'IF' TEST(2) 'THEN'
   'BEGIN'
    NEWLINE(1);  WRITETEXT('('***%RHSIN%%')');
    'FOR' J:= 1 'STEP' 1 'UNTIL' SIZE 'DO' PRINT(ARR[J],0,8);
   'END';
  'END' BNDUAL;
'PROCEDURE' PUTOUT(A,B);'REAL''ARRAY' A,B;
    'BEGIN' 'INTEGER' L,N,V;
    'FOR' I:=1'STEP' 1 'UNTIL' 6 'DO'
        'BEGIN'
         N:=ENTIER(LN(ABS(A[I])+0.05)/LN(10.0));
        L:=1;N:=N+1;
         'IF' N 'LT' 3 'THEN' L:=1 'ELSE' L:=0;
         'IF' ABS(B[I]) 'LT' 0.0001 'THEN''BEGIN'V:=7-2*L-N;
                                 'BEGIN' SPACE(V); PRINT(A[I],N,L);'END';
                                         'END'
                                     'ELSE''BEGIN' V:=6-2*L-N;
                               'BEGIN' SPACE(V);  PRINTCH(CODE('('*')'));
                                           PRINT(A[I],N,L);'END';
                                      'END';
        'END';
NEWLINE(1);
SPACE(6);
    'END';
'PROCEDURE' PERIOD;
'BEGIN'
    NEWLINE(1);
  WRITETEXT('('('1C 34S')'PERIOD-1%%PERIOD-2%%PERIOD-3%%PERIOD-4')');
    WRITETEXT('('('2S')'PERIOD-5%%PERIOD-6%%PERIOD-7%%PERIOD-8')');
    NEWLINE(1);
'END';
'BEGIN'
'REAL''ARRAY'          FORN,RG,LL,CB,ROCE,LODY,ECOV,EPPS,DCOV,DTARG,
PTARG ,STARG,A2,A3,A4,A5,A6,A7,A8[1:8],SALES,NPAT
,AA,A0,TAX,FARL,FAPF,CURA,CURL,OVDR,MARK,DV,NUM,DF,RLTA,A1[0:8];

  SELECTINPUT(1);
  READTRAP(READERR);
INTILL('('LISTING%BY%COLUMN')');
INTILL('('VARIABLES')');
ARRAYIN(ROLE,8,'('FAGI')','('ROCE')');
ARRAYIN(LODY,8,'('CURI')','('LODY')');
DATAIN(RS,'('OVDR1')','('FA2')');RS:=-RS;
DATAIN(RL,'('DE1')','('CU2')');RL:=-RL;
INTILL('('SOLUTION')');
INTILL('('DUMP')');
INTILL('('DUMP')');
M:=READ;PAPERTHROW;
K:=0;
SALES[0]:=11000;NPAT[0]:=700;
```

```
START:
NEWLINE(2);
'FOR' I:=1 'STEP' 1 'UNTIL' 8 'DO' 'BEGIN'
                                 DATAIN(SALES[I],'('SALES')','('*')');
'FOR' J:= 1 'STEP' 1 'UNTIL' 50 'DO' SKIPCH;
STARG[I]:=READ;
                                     'END';
INPUT(EARN,8,'('EARN')');
'FOR' I:=1 'STEP' 1 'UNTIL' 8 'DO' 'BEGIN'
                                 DATAIN(NPAT[I],'('NPAT')','('*')');
'FOR' J:= 1 'STEP' 1 'UNTIL' 50 'DO' SKIPCH;
PTARG[I]:=READ;
                                     'END';
INPUT(EARL,8,'('EARL')');
INPUT(EAPE,8,'('EAPE')');
INPUT(CURA,8,'('CURA')');
INPUT(CURL,8,'('CURL')');
INPUT(OVDR,8,'('OVDR')');
INPUT(DV,8,'('DV')');
INPUT(NUM,8,'('NUM')');
INTILL('('RG')');
INPUT(DE,8,'('DE')');
INPUT(LL,8,'('PLTA')');
INTILL('('ROW%INFORMATION')');
INTILL('('PROBLEM')');
COPYTEXT('('LOW')');
'GOTO' LABEL;
DUAL(ROCE,8,'('RNDROCE')');
DUAL(LQDY,8,'('RNDLQDY')');
DUAL(ECOV,8,'('RNDECOV')');
DUAL(ERPS,8,'('RNDERPS')');
DUAL(DCOV,8,'('RNDDCOV')');
DUAL(DTARG,8,'('BSDDVPS')');
LABEL: 'FOR' I:=1 'STEP' 1 'UNTIL' 8 'DO'
ROCE[I]:=LQDY[I]:=ECOV[I]:=ERPS[I]:=DCOV[I]:=DTARG[I]:=0.0;
ROCE[1]:=ERPS[3]:=DCOV[5]:=LQDY[2]:=DTARG[7]:=0.1;
ECOV[8]:=PTARG[5]:=STARG[4]:=-0.01;
'COMMENT'    INDICATORS;

'FOR' I:=1'STEP' 1 'UNTIL' 8'DO''BEGIN'
                                 A1[I]:=EARN[I]/(EARL[I]+EAPE[I]+CURA[I]
                                                          -CURL[I]);
                                  A1[I]:=A1[I]*100.0;
                                 A2[I]:=CURA[I]/CURL[I];
                                 A3[I]:=EARN[I]/(RL*DE[I]+PS+OVDR[I]);
                                 A4[I]:=NPAT[I]/NUM[I]*100.0;
                                 A5[I]:=NPAT[I]/DV[I];
                                 A6[I]:=DV[I]/NUM[I]*100.0;
A7[I]:=(SALES[I]-SALES[I-1])/SALES[I-1]*100.0;
A8[I]:=(NPAT[I]-NPAT[I-1])/NPAT[I-1]*100.0;
                                 AA[I]:=0.0;
                                  'END';
PERIOD;
SPACE(6);
WRITETEXT('('EARNINGS%PER%SHARE')');SPACE(8);
PUTOUT(A4,ERPS);
WRITETEXT('('RETURN%ON%CAPITAL')');SPACE(9);
PUTOUT(A1,ROCE);
WRITETEXT('('PROFIT')');SPACE(20);
PUTOUT(NPAT,PTARG);
WRITETEXT('('(PROFIT%GROWTH)')');SPACE(11);
PUTOUT(A8,AA);
WRITETEXT('('DIVIDEND%PER%SHARE')');SPACE(8);
PUTOUT(A6,DTARG);
WRITETEXT('('DIVIDEND%COVER')');SPACE(12);
PUTOUT(A5,DCOV);
WRITETEXT('('TIMES%COVERED')');SPACE(13);
PUTOUT(A3,ECOV);
WRITETEXT('('LIQUIDITY')');SPACE(17);
PUTOUT(A2,LQDY);
WRITETEXT('('SALES')');SPACE(21);
PUTOUT(SALES,STARG);
WRITETEXT('('(SALES%GROWTH)')');SPACE(12);
PUTOUT(A7,AA);
K:=K+1;NEXTDUMP;
'IF'(K'/'3)*3 'EQ' K 'THEN' PAPERTHROW;'GOTO' START
'END';
'END';
```

A P P E N D I X   X l l l

A HIERARCHY OF INFORMATION

EXHIBIT A13.1 AVERAGE VALUES OF CRITERIA OVER TIME

(THIS REPRESENTS AN OBJECTIVE PARAMETRIC RUN OF SAFETY INDICES AGAINST PROFITABILITY INDICES)

		L= 6	L= 5	L= 4	L= 3	L= 2	L= 1	L= 7	L= 8	L= 9	L=10	L=11
ROCE	AVER	22.64	22.47	22.39	22.22	22.15	22.21	22.30	22.08	21.98	21.67	20.05
LODY	AVER	1.96	2.01	1.97	2.03	2.08	2.06	2.04	2.03	2.04	2.05	2.30
ECOV	AVER	9.06	10.51	10.49	10.47	10.61	10.70	10.66	13.93	13.18	13.64	13.22
ERPS	AVER	56.81	55.11	52.55	52.13	51.89	51.48	49.60	40.58	40.73	39.64	31.44
	GROWTH	10.98	10.58	9.97	10.14	9.96	9.86	9.45	6.54	6.45	5.70	1.68
DCOV	AVER	2.29	2.95	2.95	2.96	2.91	2.92	2.79	2.49	2.54	2.65	2.25
DVPS	AVER	24.88	18.73	17.86	17.69	17.88	17.64	17.74	16.28	16.07	15.03	14.04
	GROWTH	12.16	8.42	7.81	7.96	7.96	7.89	7.73	6.52	6.27	5.19	3.30
DIV	AVER	498.	375.	357.	354.	358.	353.	355.	397.	359.	342.	355.
	GROWTH	12.05	8.38	7.77	7.92	7.92	7.85	7.72	9.24	7.81	7.30	6.44
SALES	AVER	20833.	19875.	18960.	18727.	18458.	18354.	17770.	17357.	15986.	15967.	13688.
	GROWTH	12.72	12.07	12.02	11.89	11.62	11.60	11.58	11.37	10.11	9.99	6.19
PROFIT	AVER	1136.	1102.	1051.	1043.	1038.	1030.	994.	990.	909.	903.	794.
	GROWTH	10.98	10.58	9.97	10.14	9.96	9.86	9.48	9.34	8.05	7.93	4.68
GOAL		797.	693.	604.	578.	565.	563.	570.	616.	632.	684.	894.

EXHIBIT A13.2 THE CRITERIA FOR A PARTICULAR (L = 4) PARAMETRIC VALUE

L= 4

ROCE		0.24	0.21	0.22	0.23	0.22	0.23	0.22	0.22
LODY		2.56	2.04	1.86	1.90	1.83	1.83	1.85	1.92
ECOV		8.80	7.46	7.90	10.49	10.13	11.62	12.40	15.91
ERPS		0.39	0.37	0.42	0.51	0.52	0.61	0.65	0.73
	GROWTH	6.3	-3.7	12.1	22.2	2.3	17.0	7.3	12.2
DCOV		3.04	2.93	2.95	2.94	2.93	2.93	2.93	2.93
DPS		0.13	0.13	0.14	0.17	0.18	0.21	0.22	0.25
	GROWTH	-10.6	-0.0	11.4	22.6	2.5	16.9	7.4	12.2
DIV		254.	254.	283.	347.	356.	416.	447.	501.
	GROWTH	-10.9	-0.0	11.4	22.6	2.5	16.9	7.4	12.2
SALES		12890.	13763.	15117.	17468.	19134.	21668.	24457.	27184.
	GROWTH	25.1	6.8	9.8	15.6	9.5	13.2	12.9	11.1
PROFIT		773.	744.	834.	1019.	1042.	1219.	1309.	1468.
	GROWTH	6.3	-3.7	12.1	22.2	2.3	17.0	7.3	12.2

EXHIBIT A.13.3 A SUMMARY OF LP SOLUTION FOR L = 4

I = 4

SALES	12890,	13763,	15117,	17468,	19134,	21668,	24457,	27184,
EARN	1665,	1696,	1876,	2247,	2298,	2636,	2814,	3133.
NPAT	773,	744,	834,	1019,	1042,	1219,	1309,	1468.
TAX	184,	237,	598,	847,	677,	807,	933,	1235,
FABL	2500,	3455,	3975,	4342,	4780,	5114,	5422,	5701.
FAPF	1508,	2073,	2175,	2207,	2582,	3081,	3505.	3654.
CURA	4630,	4658,	5528,	6481,	6807,	7655,	8629,	9907.
CURL	1811,	2282,	2967,	3417,	3722,	4188,	4656.	5162.
OVDR	0,	162,	233,	0,	250,	250,	250,	0,
MARK.	550,	0,	0,	0,	0,	0,	35,	140,
ASSETS	6907,	7905,	8711,	9613,	10447,	11662,	12900.	14100.
INTR	208,	228,	238,	214,	227,	227,	227,	197,
DV	254,	254,	283,	347,	356,	416,	447,	501,
NUM	2000,	2000,	2000,	2000,	2000,	2000,	2000,	2000.
RG	0,	0,	0,	0,	0,	0,	0,	0,
DF	2601,	2601,	2620,	2678,	2461,	2461,	2461,	2461.
LI	1101,	0,	19,	58,	783,	0,	0,	0.
BETA	58.	79.	104.	123.	145.	164.	181.	198.
RET		12,9538649						
LQD		33,6323761						
INT		83,3063465						
EPS		11,1083544						
DVC		1,7662328						
SAI		20,0485135						
PRF		17,3154390						
DVP		20,6235250						
PR01Y1		0,7891179						
PR04Y1		1,0000000						
PR13Y1		0,6501274						
PR22Y1		1,0000000						
PR23Y1		1,0000000						
PR03Y2		1,0000000						
PR14Y2		1,0000000						
PR24Y2		0,1928829						
PR14Y4		1,0000000						
PR22Y4		0,5712643						
PR25Y4		1,0000000						
PR03Y5		0,1583592						
PR21Y5		1,0000000						
PR23Y5		1,0000000						
PR05Y6		1,0000000						
PR14Y6		1,0000000						
PR15Y6		1,0000000						
PR16Y6		0,3515726						
PR21Y6		0,2368026						
PR04Y7		1,0000000						
PR14Y7		1,0000000						
PR22Y7		1,0000000						
PR02Y8		1,0000000						
PR15Y8		1,0000000						
PR22Y8		1,0000000						
PR25Y8		1.0000000						

EXHIBIT A13.4 A COMPLETE LP SOLUTION

PROBLEM BETAGOAL-01 SOLUTION DATE 21/09/76 TIME 1

DUMP:DUMP 116

RIGHT HAND SIDE RHS1
OBJECTIVE GOAL
LOWER BOUND LOBND
UPPER BOUND UPBND

ROW SET COL SET

COLUMN INFORMATION\

NAME		VALUE	LOWER BOUND	UPPER BOUND	OBJECTIVE	REDUCED COST
B SALES1	*	11764.0367	0	14000.0000	0	0
B SALES2	*	12618.4400	0	16000.0000	0	0
B SALES3	*	14596.1591	0	18000.0000	0	0
B SALES4	*	17419.2955	0	23000.0000	0	0
B SALES5	*	18669.9616	0	27000.0000	0	0
B SALES6	*	21250.8905	0	30000.0000	0	0
B SALES7	*	24881.1579	0	33000.0000	0	0
B SALES8	*	26969.4631	0	36000.0000	0	0
B EARN1	*	1507.0192	0		0	0
B EARN2	*	1555.8243	0		0	0
B EARN3	*	1826.3634	0		0	0
B EARN4	*	2203.8656	0		0	0
B EARN5	*	2168.7514	0		0	0
B EARN6	*	2530.4306	0		0	0
B EARN7	*	2987.0912	0		0	0
B EARN8	*	3365.4017	0		0	0
B NPAT1	*	693.5096	0	1000.0000	0	0
B NPAT2	*	698.7902	0	1200.0000	0	0
B NPAT3	*	829.3971	0	1300.0000	0	0
B NPAT4	*	1005.6943	0	1400.0000	0	0
B NPAT5	*	973.2950	0	1500.0000	0	0
B NPAT6	*	1167.5252	0	1800.0000	0	0
B NPAT7	*	1616.2856	0	2100.0000	0	0
B NPAT8	*	1605.0108	0	2400.0000	0	0
B TAX1	*	243.2549	0		0	0
B TAX2	*	256.5693	0		0	0
B TAX3	*	537.2891	0		0	0
B TAX4	*	767.1190	0		0	0
B TAX5	*	643.3707	0		0	0
B TAX6	*	734.7032	0		0	0
B TAX7	*	1144.9176	0		0	0
B TAX8	*	1500.2808	0		0	0
B FABL1	*	2332.3773	0		0	0
B FABL2	*	3182.7308	0		0	0
B FABL3	*	3807.8247	0		0	0
B FABL4	*	4256.3108	0		0	0
B FABL5	*	4708.8158	0		0	0
B FABL6	*	4933.9690	0		0	0
B FABL7	*	5179.7656	0		0	0
B FABL8	*	5366.4327	0		0	0
B FAPE1	*	1382.9137	0		0	0
B FAPE2	*	1786.6791	0		0	0
B FAPE3	*	1967.7525	0		0	0
B FAPE4	*	2098.2906	0		0	0
B FAPE5	*	2400.2552	0		0	0
B FAPE6	*	3129.0686	0		0	0
B FAPE7	*	3276.1297	0		0	0
B FAPE8	*	3223.3992	0		0	0
B CURA1	*	4153.5091	0		0	0
B CURA2	*	4288.1846	0		0	0
B CURA3	*	5339.9225	0		0	0
B CURA4	*	6807.4767	0		0	0
B CURA5	*	6634.7809	0		0	0
B CURA6	*	7568.4858	0		0	0
B CURA7	*	9013.6453	0		0	0
B CURA8	*	11061.4857	0		0	0
B CURL1	*	1543.3605	0		0	0
B CURL2	*	1864.2099	0		0	0
B CURL3	*	2450.5257	0		0	0
B CURL4	*	3087.6686	0		0	0
B CURL5	*	3481.0733	0		0	0
B CURL6	*	3965.6905	0		0	0
B CURL7	*	4620.7070	0		0	0
B CURL8	*	5929.3176	0		0	0

	OVDR1	◆	0	0	250.0000	0	0.0115
	OVDR2	◆	0	0	250.0000	0	0.0356
	OVDR3	◆	0	0	250.0000	0	0.0111
	OVDR4	◆	0	0	250.0000	0	0.0062
U	OVDR5	◆	250.0000	0	250.0000	0	-0.0150
U	OVDR6	◆	250.0000	0	250.0000	0	-0.0130
	OVDR7	◆	0	0	250.0000	0	0.0188
U	OVDR8	◆	250.0000	0	250.0000	0	0
B	MARK1	◆	411.0385	0		0	0
	MARK2	◆	0	0		0	0.0518
	MARK3	◆	0	0		0	0.0412
B	MARK4	◆	373.0917	0		0	0
	MARK5	◆	0	0		0	0.0152
	MARK6	◆	0	0		0	0.0123
B	MARK7	◆	289.0009	0		0	0
B	MARK8	◆	1274.6501	0		0	0
B	ASSETS1	◆	6275.6394	0		0	0
B	ASSETS2	◆	7393.3865	0		0	0
B	ASSETS3	◆	8659.0740	0		0	0
B	ASSETS4	◆	10074.6097	0		0	0
B	ASSETS5	◆	10284.7786	0		0	0
B	ASSETS6	◆	11565.8329	0		0	0
B	ASSETS7	◆	12868.8336	0		0	0
B	ASSETS8	◆	14100.0000	0		0	0
B	INTR1	◆	158.2638	0		0	0
B	INTR2	◆	167.5791	0		0	0
B	INTR3	◆	192.6569	0		0	0
B	INTR4	◆	222.1613	0		0	0
B	INTR5	◆	185.3801	0		0	0
B	INTR6	◆	185.3801	0		0	0
B	INTR7	◆	155.3801	0		0	0
B	INTR8	◆	185.3801	0		0	0
B	DV1	◆	138.7019	0		0	0
B	DV2	◆	139.7580	0		0	0
B	DV3	◆	165.8786	0		0	0
B	DV4	◆	201.1389	0		0	0
B	DV5	◆	278.0843	0		0	0
B	DV6	◆	339.2929	0		0	0
B	DV7	◆	404.6530	0		0	0
B	DV8	◆	458.5745	0		0	0
B	NUM1	◆	2057.7062	0		0	0
B	NUM2	◆	2057.7062	0		0	0
B	NUM3	◆	2057.7062	0		0	0
B	NUM4	◆	2057.7062	0		0	0
B	NUM5	◆	2057.7062	0		0	0
B	NUM6	◆	2057.7062	0		0	0
B	NUM7	◆	2057.7062	0		0	0
B	NUM8	◆	2057.7062	0		0	0
B	RG1	◆	57.7062	0	800.0000	0	0
	RG2	◆	0	0	800.0000	0	0.0184
	RG3	◆	0	0	800.0000	0	0.1118
	RG4	◆	0	0	800.0000	0	0.1683
	RG5	◆	0	0	800.0000	0	0.1821
	RG6	◆	0	0	800.0000	0	0.1093
	RG7	◆	0	0	800.0000	0	0.1273
	RG8	◆	0	0	800.0000	0	0.1513
B	DE1	◆	1978.0674	0		0	0
B	DE2	◆	2094.7391	0		0	0
B	DE3	◆	2405.7119	0		0	0
B	DE4	◆	2777.0168	0		0	0
B	DE5	◆	1942.2506	0		0	0
B	DE6	◆	1942.2506	0		0	0
B	DE7	◆	1942.2506	0		0	0
B	DE8	◆	1942.2506	0		0	0
B	LL1	◆	478.0476	0		0	0
B	LL2	◆	116.6917	0		0	0
B	LL3	◆	310.9727	0		0	0
B	LL4	◆	371.3049	0		0	0
B	LL5	◆	165.2338	0		0	0
	LL6	◆	0	0		0	0.0117
	LL7	◆	0	0		0	0.0241
	LL8	◆	0	0		0	0.0362
B	BLTA1	◆	30.6275	0		0	0
B	BLTA2	◆	67.8243	0		0	0
B	BLTA3	◆	96.6359	0		0	0
B	BLTA4	◆	118.9317	0		0	0
B	BLTA5	◆	161.6407	0		0	0

PROBLEM BETAGOAL-01 SOLUTION DATE 21/09/76 TIME

DUMP:DUMP 116

RIGHT HAND SIDE RHS1
OBJECTIVE GOAL
LOWER BOUND LOBND
UPPER BOUND UPBND

ROW SET COL SET

COLUMN INFORMATION

	NAME		VALUE	LOWER BOUND	UPPER BOUND	OBJECTIVE	REDUCED COST
B	BLTA6	*	155.5808	0		0	0
B	BLTA7	*	170.5924	0		0	0
B	BLTA8	*	182.5235	0		0	0
B	RET	*	119.1032	0		0.0826	0
B	PRF	*	207.1367	0		0.0571	0
B	SAL	*	3955.3592	0		0.0200	0
	EPS	*	0	0		0.3083	0.0390
B	LOD	*	2246.1258	0		0.0117	0
B	INT	*	1140.3127	0		0.0666	0
B	DVP	*	364.5080	0		0.0879	0
	DVC	*	0	0		0.3620	0.2871
U	PR01Y1	*	1.0000	0	1.0000	0	-5.8396
U	PR04Y1	*	1.0000	0	1.0000	0	-15.5628
	PR12Y1	*	0	0	1.0000	0	3.8086
B	PR13Y1	*	0.7734	0	1.0000	0	0
	PR16Y1	*	0	0	1.0000	0	6.7027
	PR22Y1	*	0	0	1.0000	0	1.7705
	PR23Y1	*	0	0	1.0000	0	3.8685
B	PR03Y2	*	0.6178	0	1.0000	0	0
	PR04Y2	*	0	0	1.0000	0	6.9369
	PR05Y2	*	0	0	1.0000	0	12.9681
	PR13Y2	*	0	0	1.0000	0	16.7361
U	PR14Y2	*	1.0000	0	1.0000	0	-31.2207
	PR21Y2	*	0	0	1.0000	0	12.5269
B	PR24Y2	*	0.3151	0	1.0000	0	0
U	PR02Y3	*	1.0000	0	1.0000	0	-1.5449
	PR11Y3	*	0	0	1.0000	0	4.9734
B	PR13Y3	*	0.7866	0	1.0000	0	0
	PR01Y4	*	0	0	1.0000	0	0.2508
	PR05Y4	*	0	0	1.0000	0	1.5339
	PR11Y4	*	0	0	1.0000	0	2.6751
	PR12Y4	*	0	0	1.0000	0	3.3325
	PR13Y4	*	0	0	1.0000	0	9.5572
U	PR14Y4	*	1.0000	0	1.0000	0	-19.7382
U	PR22Y4	*	1.0000	0	1.0000	0	-0.1317
U	PR25Y4	*	1.0000	0	1.0000	0	-8.4825
B	PR02Y5	*	0.0894	0	1.0000	0	0
U	PR03Y5	*	1.0000	0	1.0000	0	-7.2053
	PR11Y5	*	0	0	1.0000	0	3.7188
B	PR21Y5	*	0.8810	0	1.0000	0	0
	PR23Y5	*	0	0	1.0000	0	0.0443
U	PR04Y6	*	1.0000	0	1.0000	0	-4.1769
U	PR05Y6	*	1.0000	0	1.0000	0	-0.8360
	PR11Y6	*	0	0	1.0000	0	5.0241
U	PR14Y6	*	1.0000	0	1.0000	0	-10.9055
U	PR15Y6	*	1.0000	0	1.0000	0	-3.3359
B	PR16Y6	*	0.0198	0	1.0000	0	0
	PR21Y6	*	0	0	1.0000	0	3.5518
	PR23Y6	*	0	0	1.0000	0	3.3526
	PR01Y7	*	0	0	1.0000	0	4.7038
	PR04Y7	*	0	0	1.0000	0	0.9507
U	PR14Y7	*	1.0000	0	1.0000	0	-6.9206
U	PR22Y7	*	1.0000	0	1.0000	0	-2.0813
	PR02Y8	*	0	0	1.0000	0	1.1876
	PR15Y8	*	0	0	1.0000	0	1.8072
U	PR22Y8	*	1.0000	0	1.0000	0	-4.6619
U	PR25Y8	*	1.0000	0	1.0000	0	-2.6730
	OBJECTIVE		235.0135				

EXHIBIT A13.5 A COMPLETE ANALYSIS OF AN LP SOLUTION

```
BETAGOAL-01          SOLUTION                          DATE  21/09/76      TIME  12/57/20      PAGE  0007

DUMP:DUMP 114                  RIGHT HAND SIDE   RHS1
                               OBJECTIVE         GOAL
                               LOWER BOUND       LOBND
                               UPPER BOUND       UPBND
                   ROW SET                                    COL SET                        -(ZED      )
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BALANCE SHEET (£'000S)

	PERIOD-1	PERIOD-2	PERIOD-3	PERIOD-4	PERIOD-5	PERIOD-6	PERIOD-7	PERIOD-8
CAPITAL								
SHARE CAPITAL	2058	2058	2058	2058	2058	2058	2058	2058
SHARE PREMIUM	35	35	35	35	35	35	35	35
TAXATION EQUALISATION	450	893	1185	1423	1753	2206	2477	2587
RESERVES	1755	2314	2977	3782	4477	5325	6337	7483
LONG TERM DEBT	1978	2095	2406	2777	1942	1942	1942	1942
TOTAL LIABILITIES	6275	7393	8660	10074	10265	11566	12849	14100
ASSETS								
FIXED ASSETS								
LAND AND BUILDINGS	2332	3183	3808	4256	4709	4934	5180	5366
PLANT AND MACHINERY	1383	1787	1963	2098	2400	3029	3276	3223
CURRENT ASSETS								
SHORT TERM DEPOSITS	412	0	0	374	0	0	290	1275
DEBTORS AND STOCK	3742	4288	5340	6434	6637	7569	8724	9787
CURRENT ASSETS	4154	4288	5340	6808	6637	7569	9014	11062
CURRENT LIABILITIES								
CREDITORS	1211	1468	1747	2119	2310	2642	3071	3321
TAX	243	257	537	767	643	735	1145	1500
OVERDRAFT	0	0	0	0	250	250	0	250
DIVIDEND PAYABLE	139	140	166	201	278	339	405	459
CURRENT LIABILITIES	1593	1864	2451	3088	3481	3966	4621	5529
NET CURRENT ASSETS	2560	2424	2889	3720	3156	3603	4393	5532
TOTAL ASSETS	6275	7393	8660	10074	10265	11566	12849	14100

DUMP 114

PROFIT AND LOSS STATEMENT (£'000S)

	PERIOD-1	PERIOD-2	PERIOD-3	PERIOD-4	PERIOD-5	PERIOD-6	PERIOD-7	PERIOD-8
SALES	11764	12618	14596	17419	18670	21251	24881	26570
LESS COST OF SALES	10301	11092	12770	15215	16527	18691	21863	23224
TRADING PROFIT	1463	1527	1826	2204	2143	2560	3018	3345
INTEREST ON INVESTMENTS	56	29	0	0	26	0	0	20
LESS INTEREST BANK OVERDRAFTS	12	0	0	0	0	30	30	0
LESS INTEREST LONG TERM DEBT	120	158	168	193	222	155	155	155
PROFIT BEFORE TAX	1387	1398	1659	2011	1947	2375	2833	3210
PROFIT AFTER TAX	694	699	829	1006	973	1188	1416	1605
DIVIDEND	139	140	166	201	278	339	405	459
ADDED TO RESERVES	555	559	664	805	695	848	1012	1146

CASH FLOW STATEMENT

	PERIOD-1	PERIOD-2	PERIOD-3	PERIOD-4	PERIOD-5	PERIOD-6	PERIOD-7	PERIOD-8
SOURCES								
TRADING INCOME	1604	1929	1824	2311	3072	3120	3541	3768
INVESTMENT INCOME	56	29	0	0	26	0	0	20
INCREASE IN LONG TERM DEBT	478	117	311	371	-834	0	0	0
RIGHTS ISSUES	92	0	0	0	0	0	0	0
TOTAL CASH INFLOW	2231	2074	2135	2682	2264	3120	3541	3788
USES								
BUILDING AND LAND	769	947	741	578	595	375	403	327
PLANT AND EQUIPMENT	963	999	830	835	1102	1638	1339	1022
INTEREST OVERDRAFT	12	0	0	0	0	30	30	0
INTEREST LONG TERM DEBT	120	158	168	193	222	155	155	155
DIVIDEND PAYMENTS	285	139	140	166	201	278	339	405
TAX PAID	370	243	257	537	767	643	735	1145
TOTAL CASH OUTFLOW	2519	2486	2135	2308	2888	3120	3001	3053
NET CHANGE IN CASH POSITION	-287	-411	0	374	-623	0	540	735

DUMP 114

ACHIEVEMENT LEVELS

		PERIOD-1	PERIOD-2	PERIOD-3	PERIOD-4	PERIOD-5	PERIOD-6	PERIOD-7	PERIOD-8
RETURN	TARGET	22.3	22.3	22.3	22.3	22.3	22.6	23.9	25.0
ON	ACHIEVEMENT	24.0	21.0	21.1	21.9	21.1	21.9	23.3	23.9
CAPITAL	PERCENT DEVIATION	7.7	-5.6	-5.4	-1.9	-5.3	-3.2	-2.7	-4.5
	TARGET	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
LIQUIDITY	ACHIEVEMENT	2.6	2.3	2.2	2.2	1.9	1.9	2.0	2.0
	PERCENT DEVIATION	4.3	-8.0	-12.8	-11.8	-23.7	-23.7	-22.0	-20.0
TIMES	TARGET	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0
COVERED	ACHIEVEMENT	9.5	9.3	9.5	9.9	11.7	13.6	19.2	18.2
	PERCENT DEVIATION	-36.5	-38.1	-36.7	-33.9	-22.0	-9.0	28.2	21.0
EARNINGS	TARGET	32.2	33.5	39.0	44.5	47.3	53.7	65.9	78.0
PER	ACHIEVEMENT	33.7	34.0	40.3	48.9	47.3	57.7	68.8	78.0
SHARE	PERCENT DEVIATION	4.7	1.4	3.4	9.8	-0.0	7.5	4.4	-0.0
DIVIDEND	TARGET	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5
COVER	ACHIEVEMENT	5.0	5.0	5.0	5.0	3.5	3.5	3.5	3.5
	PERCENT DEVIATION	42.9	42.9	42.9	42.9	-0.0	0.0	0.0	0.0
DIVIDEND	TARGET	20.0	21.5	24.0	26.0	29.0	32.0	36.0	40.0
PER	ACHIEVEMENT	6.7	6.8	8.1	9.8	13.5	16.5	19.7	22.3
SHARE	PERCENTAGE DEVIATION	-66.3	-68.4	-66.4	-62.4	-53.4	-48.5	-45.4	-44.3
SALES	TARGET	12800	15000	17800	20900	22500	25400	30300	32000
TARGET	ACHIEVEMENT	11764	12618	14596	17419	18670	21251	24881	26569
	PERCENT DEVIATION	-8.1	-15.9	-18.0	-16.7	-17.0	-16.3	-17.9	-17.0
PROFIT	TARGET	800	850	950	1050	1150	1250	1400	1600
TARGET	ACHIEVEMENT	694	699	829	1006	973	1188	1416	1605
	PERCENT DEVIATION	-13.3	-17.8	-12.7	-4.2	-15.4	-5.0	1.2	0.3

APPENDIX XIV

Proof of the fomulae for reranking by discounted benefits/discounted costs at the internal rate of return of the marginally rejected project.

Let this rate of return be i, and define the notation

$$TV_j(t,i) = -\sum_{s=1}^{t} c_{sj}(1+i)^{t-s} \qquad \text{A14.1}$$

Then the required reranking by parameter β means the approximation to the dual equations of

$$\mu_j + \beta TV_j(T-1,i) \geq \hat{c}_j - TV_j(T,i) \qquad \text{A14.2}$$

or alternatively

$$\mu_j + \beta TV_j(T-1,i) \geq \hat{c}_j + c_{Tj} - (1+i)TV_j(T-1,i) \qquad \text{A14.3}$$

Using the identity that

$$TV_j(T-1,i) = TV_j(T-1,r) + (i-r)\sum_{t=1}^{T-2} TV_j(t,i)(1+i)^{T-2-t} \qquad \text{A14.4}$$

the equation can be rewritten as

$$\mu_j + (\beta+i-r)\ TV_j(T-1,r) + (i-r)\sum_{t=1}^{T-2}(1+i)^{T-2-t}(\beta+1+i)TV_j(t,r)$$

$$\geq a_j - TV_j(T,r) = NTV_j \qquad \text{A14.5}$$

Thus this is seen to be equivalent to having

$$\beta_{T-1} = \beta + i - r$$

$$\beta_{T-2} = (i-r)(\beta+1+i)$$

$$\beta_t = (i-r)(\beta+1+i)(1+i)^{T-2-t} \quad \text{for } t=1,2,\ldots T-3 \qquad \text{A14.6}$$

The internal rate of return approximation.

Using the identity

$$(1+i)^{T-t} = (1+r)^{T-t} + (i-r)\sum_{s=0}^{T-t-1}(1+i)^{T-t-s-1}(1+r)^{s} \qquad \text{A14.7}$$

and putting $\beta_t = (i-r)(1+i)^{T-1-t}$ the equations

$$\sum_{t=1}^{T-1} TV_j(t)\beta_t = NTV_j \qquad \text{A14.8}$$

become

$$-\sum_{t=1}^{T} c_{tj}(1+r)^{T-t} - (i-r)\sum_{t=1}^{T-1}(1+i)^{T-1-t}\sum_{s=1}^{t} c_{sj}(1+r)^{t-s} = \hat{c}_j \qquad \text{A14.9}$$

or

$$-\sum_{t=1}^{T} c_{tj}\left\{(1+r)^{T-t} + (i-r)\sum_{s=o}^{T-t-1}(1+i)^{T-t-s-1}(1+r)^{s}\right\} = \hat{c}_j \qquad \text{A14.10}$$

or

$$-\sum_{t=1}^{T} c_{tj}(1+i)^{T-t} = \hat{c}_j \qquad \text{A14.11}$$

and i is seen to be the internal rate of return of project j.

A P P E N D I X   X V

THE WEINGARTNER MODEL

A15.1 CASH FLOW DATA USED BY WEINGARTNER

A15.2 THE LP INPUT DATA LISTING FOR THE MODEL USING THE PROJECTS DETAILED IN APPENDIX IV

A15.3 THE OPTIMAL SOLUTION IN THE CASE OF SEVERE CAPITAL RATIONING

EXHIBIT A15.1 CASH FLOW DATA AND IRR OF PROJECTS USED BY WEINGARTNER (EXTRACTED FROM "MATHEMATICAL PROGRAMMING AND THE ANALYSIS OF CAPITAL BUDGETING PROBLEMS" BY H M WEINGARTNER, KERSHAW EDITION, 1974, p.181-182)

CASH FLOWS ASSOCIATED WITH THIRTY HYPOTHETICAL INVESTMENT PROJECTS*

Project Number	Flow in Year 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1	−100	20	20	20	19	19	18	16	14	11	6	−8														
2	−100	20	18	18	18	18	14	14	14	14	14	10	10	10	10	10	6	6	6	6	6					
3	−100	15	15	15	15	15	13	13	13	13	13	11	11	11	11	11	9	9	9	9	9					
4	−100	20	6	11	7	16	5	14	18	3	20	2	22	8	10	18	6	9	14	24						
5	−100	−60	−60	80	74	66	56	44	30	14																
6	−200	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25
7							−80	20	20	20	19	17	14	10	6	2										
8									−60	−30	−10	45	34	25	16	12	8	−20	21	16	12	9	7	5	3	
9			−120	25	25	30	35	30	25	20	15	10	5													
10		−100	18	17	15	12	8	−10	18	17	15	12	8	−10	18	17	15	12	8							
11	−150	20	20	20	20	20	20	20	20	20	20	20														
12	−100	20	18	16	14	12	10	4	−20	20	18	16	14	12	10	4										
13	−150	−75	−75	60	60	55	50	44	38	36	35	34	33	30	25	17	9									
14	−50	−100	−175	50	55	60	65	60	50	40	30	20	10	−25	50	41	35	25	15	5						
15	−100	−150	−100	10	20	30	40	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60
16								−95	−60	47	42	37	31	24	18	13	9	6	4	3						
17										−175	50	45	35	25	10	−60	45	35	25	10						
18	−250	45	45	40	30	25	20	15	10	−40	40	32	25	19	14	10	7	5								
19	−75	−75	−40	40	40	40	35	35	30	25	15	5														
20	−180	20	12	16	13	11	19	17	12	15	19	13	14	17	20	14	11	15	17	12						
21												−80	18	16	14	12	10	4	16	14	10	6				
22		−85	20	20	16	15	13	10	7	3																
23		−270	−100	125	115	105	80	60	35	25	15	10														
24										−40	15	13	9	7	5	2										
25								−50	10	10	9	7	4	−14	9	9	8	6	3	−16	8	8	4			
26		−200	60	40	30	15	−25	−25	50	40	30	20	10													
27											−70	15	13	11	10	9	7	6	4	3	2					
28		−355	60	70	80	70	55	40	25	15	5															
29	−275	40	45	45	40	35	30	25	20	15	−75	35	30	25	20	15	10	5								
30	−140	20	20	18	16	14	11	8	−25	18	18	16	13	10	6	−25	16	16	14	11	8	5	2			

*$(-a_{tj})$

TABLE 9A.4. INTERNAL RATES OF RETURN AND RANKS FOR THIRTY INVESTMENT PROJECTS*

Project No.	1	2	3	4	5	6	7	8
Rate (%)	11.03	13.94	11.90	10.02	12.26	11.75	13.84	12.57
Rank	9	2	7	13	5	8	3	4
Project No.	9	10	11	12	13	14	15	16
Rate (%)	15.66	9.07	7.00	8.55	8.76	9.22	10.86	10.24
Rank	1	16	19	18	17	15	10	11
Project No.	17	18	19	20	21	22	23	24
Rate (%)	5.81	5.76	6.75	5.19	9.35	6.11	11.94	10.19
Rank	22	23	20	24	14	21	6	12
Project No.	25	26	27	28	29	30		
Rate (%)	4.52	4.25	3.50	4.71	4.64	4.93		
Rank	28	29	30	26	27	25		

*These rates are computed by using the horizon values at 10% of Table 9A.2.

EXHIBIT A15.2 THE LP INPUT DATA LISTING FOR THE MODEL USING THE PROJECTS DETAILED IN APPENDIX 1V.

```
#ROWS

OBJ(Z)
#GEN(OBJ(Z),6,1,1)
#GEN(CASH(-),8,1,1)
#GEN(LIMIT(+),8,1,1)
#GEN(L(+),9,1,1)

#COLS

PR01Y1,OBJ1,-122.5
PR01Y1,OBJ2,-122.5
PR01Y1,OBJ3,-122.5
PR01Y1,OBJ6,-110
PR01Y1,CASH1,-100,-100,-128,52,120,222,128,122
    ,UPBND,1
PR04Y1,OBJ1,-60
PR04Y1,OBJ2,-60
PR04Y1,OBJ3,-60
PR04Y1,OBJ6,-54
PR04Y1,CASH1,-350,124,69,96,94,107,69,48
    ,UPBND,1
PR12Y1,OBJ1,-91.2
PR12Y1,OBJ2,-91.2
PR12Y1,OBJ3,-91.2
PR12Y1,OBJ6,-82
PR12Y1,CASH1,-260,-37,39,26,35,163,187,161
    ,UPBND,1
PR13Y1,CASH1,-432,58,229,56,181,39,84,3
    ,UPBND,1
PR16Y1,OBJ1,-85.7
PR16Y1,OBJ2,-85.7
PR16Y1,OBJ3,-85.7
PR16Y1,OBJ6,-77.3
PR16Y1,CASH1,-305,-133,35,146,156,133,114,105
    ,UPBND,1
PR22Y1,OBJ1,-50.5
PR22Y1,OBJ2,-50.5
PR22Y1,OBJ3,-50.5
PR22Y1,OBJ6,-45.4
PR22Y1,CASH1,-121,-9,-51,84,64,11,83,54
    ,UPBND,1
PR23Y1,OBJ1,-48.7
PR23Y1,OBJ2,-48.7
PR23Y1,OBJ3,-48.7
PR23Y1,OBJ6,-43.8
PR23Y1,CASH1,-345,48,112,55,61,68,77,59
    ,UPBND,1
PR03Y2,OBJ1,48.8
PR03Y2,OBJ2,46.2
PR03Y2,OBJ3,51.2
PR03Y2,OBJ6,45.5
PR03Y2,CASH2,-369,0,8,130,113,202,104
    ,UPBND,1
PR04Y2,OBJ1,-7.6
PR04Y2,OBJ2,-9.7
PR04Y2,OBJ3,-5.6
PR04Y2,OBJ6,-5.3
PR04Y2,CASH2,-350,124,69,96,94,107,69
    ,UPBND,1
    ,L1,-1
L04Y2,CASH2,350,-97,-97,-97,-97,-97
    ,LIMIT3,62,62,62,62,62
     ,L1,+1
```

```
PR05Y2,OBJ1,41.6
PR05Y2,OBJ2,38.3
PR05Y2,OBJ3,44.7
PR05Y2,OBJ6,39.5
PR05Y2,CASH2,-361,-122,14,147,154,152,150
     ,UPBND,1
PR13Y2,OBJ1,3.4
PR13Y2,OBJ2,3.4
PR13Y2,OBJ3,3.4
PR13Y2,OBJ6,3.0
PR13Y2,CASH2,-432,58,229,56,181,39,84
     ,UPBND,1
     ,L2,-1
L13Y2,CASH2,432,-108,-108,-108,-108,-108,-108
     ,LIMITS,72,72,72,72,72,72
     ,L2,1
PR14Y2,OBJ1,-131.8
PR14Y2,OBJ2,-134.3
PR14Y2,OBJ3,129.4
PR14Y2,OBJ6,-117
PR14Y2,CASH2,-180,-29,123,106,79,59,-1
     ,UPBND,1

PR21Y2,OBJ1,65
PR21Y2,OBJ2,63
PR21Y2,OBJ3,66.7
PR21Y2,OBJ6,59.3
PR21Y2,CASH2,-549,-116,192,206,114,96,136
     ,UPBND,1
PR24Y2,OBJ1,-23
PR24Y2,OBJ2,-27.1
PR24Y2,OBJ3,-19.1
PR24Y2,OBJ6,-16.0
PR24Y2,CASH2,-463,97,-1,165,137,116,138
     ,UPBND,1
PR02Y3,OBJ1,91.2
PR02Y3,OBJ2,91.8
PR02Y3,OBJ3,90.5
PR02Y3,OBJ6,81.7
PR02Y3,CASH3,-196,-49,79,56,76,37
     ,UPBND,1
PR11Y3,OBJ1,126.7
PR11Y3,OBJ2,121.8
PR11Y3,OBJ3,127.1
PR11Y3,OBJ6,114.0
PR11Y3,CASH3,-355,19,102,109,76,116
     ,UPBND,1
PR15Y3,OBJ1,119
PR15Y3,OBJ2,117.8
PR15Y3,OBJ3,119.8
PR15Y3,OBJ6,107.6
PR15Y3,CASH3,-167,-93,66,86,68,31
     ,UPBND,1
PR01Y4,OBJ1,347.8
PR01Y4,OBJ2,348.9
PR01Y4,OBJ3,346.3
PR01Y4,OBJ6,312
PR01Y4,CASH4,-100,-109,-128,52,120
     ,UPBND,1
PR05Y4,OBJ1,328.3
PR05Y4,OBJ2,333.4
PR05Y4,OBJ3,323.3
PR05Y4,OBJ6,292
PR05Y4,CASH4,-361;-122,14,147,154
     ,UPBND,1
PR11Y4,OBJ1,230
PR11Y4,OBJ2,231.6
PR11Y4,OBJ3,228
PR11Y4,OBJ6,205.8
PR11Y4,CASH4,-355,19,102,109,76
     ,UPBND,1
PR12Y4,OBJ1,406
PR12Y4,OBJ2,412.8
PR12Y4,OBJ3,395.5
PR12Y4,OBJ6,357
PR12Y4,CASH4,-260,-37,39,76,35
     ,UPBND,1
PR13Y4,OBJ1,119.7
PR13Y4,OBJ2,122.9
```

PROBLEM WEINGARTNER INPUT DATE 13/01/76

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PR13Y4,OBJ3,126.5
PR13Y4,OBJ6,106
PR13Y4,CASH4,-432,52,229,56,181
     ,UPBND,1
     ,13,-1
L13Y4,OBJ6,52
     ,CASH4,528,-268,-52,-52,-52
     ,CT114,104,52,52,52,52
     ,13,1
PR14Y4,OBJ1,-54.9
PR14Y4,OBJ2,-60.2
PR14Y4,OBJ3,-45.5
PR14Y4,OBJ6,-45
PR14Y4,CASH4,-180,-29,123,104,79
     ,UPBND,1
PR22Y4,OBJ1,94.5
PR22Y4,OBJ2,96.3
PR22Y4,OBJ3,92.7
PR22Y4,OBJ6,83.6
PR22Y4,CASH4,-121,-9,-51,84,66
     ,UPBND,1
PR25Y4,OBJ1,143.7
PR25Y4,OBJ2,144
PR25Y4,OBJ3,143.2
PR25Y4,OBJ6,126
PR25Y4,CASH4,-370,68,176,-72,195
     ,UPBND,1
PR02Y5,OBJ1,189.4
PR02Y5,OBJ2,197.5
PR02Y5,OBJ3,201
PR02Y5,OBJ6,165
PR02Y5,CASH5,-196,-49,79,56
     ,UPBND,1
PR03Y5,OBJ1,427.9
PR03Y5,OBJ2,444.5
PR03Y5,OBJ3,412.7
PR03Y5,OBJ6,375.2
PR03Y5,CASH5,-369,0,8,130
     ,UPBND,1
PR11Y5,OBJ1,287.4
PR11Y5,OBJ2,297.2
PR11Y5,OBJ3,276.1
PR11Y5,OBJ6,252.6
PR11Y5,CASH5,-355,19,102,109
     ,UPBND,1
PR21Y5,OBJ1,369.3
PR21Y5,OBJ2,380.2
PR21Y5,OBJ3,354
PR21Y5,OBJ6,322
PR21Y5,CASH5,-549,-116,192,204
     ,UPBND,1
PR23Y5,OBJ1,203.4
PR23Y5,OBJ2,210.9
PR23Y5,OBJ3,196.4
PR23Y5,OBJ6,176
PR23Y5,CASH5,-345,48,112,55

     ,UPBND,1
PR04Y6,OBJ1,319.7
PR04Y6,OBJ2,335.7
PR04Y6,OBJ3,306.8
PR04Y6,OBJ6,279
PR04Y6,CASH6,-350,124,69
     ,UPBND,1
PR05Y6,OBJ1,572.8
PR05Y6,OBJ2,605.1
PR05Y6,OBJ3,543.9
PR05Y6,OBJ6,503
PR05Y6,CASH6,-361,-122,14
     ,UPBND,1
PR11Y6,OBJ1,375.6
PR11Y6,OBJ2,395.3
PR11Y6,OBJ3,357.8
PR11Y6,OBJ6,326.3
PR11Y6,CASH6,-355,19,102
     ,UPBND,1
PR14Y6,OBJ1,130.1
PR14Y6,OBJ2,118.8
PR14Y6,OBJ3,138.6
PR14Y6,OBJ6,122.95
PR14Y6,CASH6,-180,-29,123
     ,UPBND,1
```

```
PR15Y6,OBJ1,271.4
PR15Y6,OBJ2,286.2
PR15Y6,OBJ3,258.1
PR15Y6,OBJ6,230
PR15Y6,CASH6,-167,-93.66
      ,UPPND,1
PR16Y6,OBJ1,515.6
PR16Y6,OBJ2,541.5
PR16Y6,OBJ3,492.2
PR16Y6,OBJ6,448
PR16Y6,CASH6,-305,-133,35
      ,UPPND,1
PR21Y6,OBJ1,546
PR21Y6,OBJ2,575.4
PR21Y6,OBJ3,520.1
PR21Y6,OBJ6,473
PR21Y6,CASH6,-549,-116,192
      ,UPPND,1
PR23Y6,OBJ1,244.1
PR23Y6,OBJ2,258.6
PR23Y6,OBJ3,231.1
PR23Y6,OBJ6,211
PR23Y6,CASH6,-345,48,112
      ,UPPND,1
PR01Y7,OBJ1,301.1
PR01Y7,OBJ2,345.1
PR01Y7,OBJ3,262.5
PR01Y7,OBJ6,244
PR01Y7,CASH7,-100,-109
      ,UPPND,1
PR04Y7,OBJ1,366
PR04Y7,OBJ2,390.9
PR04Y7,OBJ3,343.9
PR04Y7,OBJ6,315
PR04Y7,CASH7,-350,124
      ,UPPND,1
PR14Y7,OBJ1,244.4
PR14Y7,OBJ2,236.2
PR14Y7,OBJ3,247.7
PR14Y7,OBJ6,222.4
PR14Y7,CASH7,-180,-29
      ,UPPND,1
PR22Y7,OBJ1,156.7
PR22Y7,OBJ2,174.5
PR22Y7,OBJ3,141
PR22Y7,OBJ6,111
PR22Y7,CASH7,-121,-9
      ,UPPND,1
PR02Y8,OBJ1,224
PR02Y8,OBJ2,256.2
PR02Y8,OBJ3,197.6
PR02Y8,OBJ6,184
PR02Y8,CASH8,-196
      ,UPPND,1
PR15Y8,OBJ1,201.7
PR15Y8,OBJ2,230.1
PR15Y8,OBJ3,172.6
PR15Y8,OBJ6,162
PR15Y8,CASH8,-167
      ,UPPND,1
PR22Y8,OBJ1,137.1
PR22Y8,OBJ2,159.8
PR22Y8,OBJ3,117.9
PR22Y8,OBJ6,110
PR22Y8,CASH8,-121
      ,UPPND,1
PR25Y8,OBJ1,431.8
PR25Y8,OBJ2,469.4
PR25Y8,OBJ6,367.6
PR25Y8,CASH8,-370
```

EXHIBIT A15.3 THE OPTIMAL SOLUTION IN THE CASE OF SEVERE CAPITAL RATIONING

LOOP INFORMATION

NAME		VALUE	LOWER BOUND	UPPER BOUND	OBJECTIVE	REDUCED COST
PR01Y1	+	1.0000		1.0000	-113.4258	5.3001
PR04Y1	+	1.0000		1.0000	-55.5555	72.3454
PR12Y1	+	0		1.0000	-84.4444	-25.3849
PR13Y1	+	1.0000		1.0000	0	27.1548
PR16Y1	+	0		1.0000	-79.3516	-142.5132
PR22Y1	+	0		1.0000	-66.7592	-48.6215
PR23Y1	+	0		1.0000	-45.0025	-133.4464
PR03Y2	+	0		1.0000	45.1451	-75.4381
PR04Y2	+	1.0000		1.0000	-7.0370	16.6383
PR05Y2	+	0		1.0000	-38.5183	-202.5055
PR13Y2	+	0.5844		1.0000	3.1481	0
PR14Y2	+	0		1.0000	-122.0369	-40.6843
PR21Y2	+	0		1.0000	60.1851	-330.6660
PR24Y2	+	0		1.0000	-21.2963	-166.7415
PR02Y3	+	0		1.0000	64.4444	-37.8561
PR11Y3	+	0.1650		1.0000	115.4628	0
PR15Y3	+	0		1.0000	110.1851	-26.8227
PR01Y4	+	1.0000		1.0000	322.6367	6.8552
PR05Y4	+	0		1.0000	303.6812	-134.0117
PR11Y4	+	0		1.0000	212.9628	-17.6123
PR12Y4	+	1.0000		1.0000	374.6737	6.2228
PR13Y4	+	1.0000		1.0000	110.8332	26.5308
PR14Y4	+	0		1.0000	-50.8333	-34.6871
PR22Y4	+	0		1.0000	-87.4999	-36.9836
PR25Y4	+	0		1.0000	133.6554	-30.9191
PR02Y5	+	0		1.0000	175.3792	-29.7317
PR03Y5	+	1.0000		1.0000	396.2033	2.6774
PR11Y5	+	0.0064		1.0000	266.1108	0
PR21Y5	+	0		1.0000	341.9441	-189.3318
PR23Y5	+	0		1.0000	188.3331	-67.9478
PR04Y6	+	1.0000		1.0000	298.0182	36.9584
PR05Y6	+	0		1.0000	530.3498	-67.9160
PR11Y6	+	0.0339		1.0000	347.7774	0
PR14Y6	+	0		1.0000	120.4628	-26.9394
PR15Y6	+	0		1.0000	251.2960	-6.0226
PR16Y6	+	0		1.0000	477.4069	-37.6939
PR21Y6	+	0		1.0000	505.5551	-157.3491
PR23Y6	+	0		1.0000	226.0183	-66.7161
PR01Y7	+	1.0000		1.0000	276.7963	60.1871
PR04Y7	+	1.0000		1.0000	338.8886	79.2573
PR14Y7	+	0.4690		1.0000	226.2961	0
PR22Y7	+	1.0000		1.0000	145.0924	3.4656
PR02Y8	+	1.0000		1.0000	207.4072	11.4072
PR15Y8	+	1.0000		1.0000	186.7591	19.7591
PR22Y8	+	1.0000		1.0000	176.9443	5.9443
PR25Y8	+	1.0000		1.0000	390.8144	29.8144
MARK1	+	[illegible]			0	0
MARK2	+	0			-0	-0.2312
MARK3	+	0			0	0
MARK4	+	0			0	-0.0047
MARK5	+	0			0	-0.0108
MARK6	+	0			0	-0.1419
MARK7	+	0			0	-0.0161
MARK8	+	0			1.0000	0
[illegible]	+	750.0000		750.0000	0	0
[illegible]	+	750.0000		750.0000	0	0.2312
[illegible]	+	[illegible]		750.0000	0	0
[illegible]	+	[illegible]		750.0000	0	0.0047
[illegible]	+	[illegible]		750.0000	0	0.0108
[illegible]	+	[illegible]		450.0000	0	0.1419
[illegible]	+	[illegible]		450.0000	0	0.0161
[illegible]	+	[illegible]			-1.0000	0
[illegible]		[illegible]				

APPENDIX XVI

The closed function form for the Adjusted Present Value

The general formula† for calculating the adjusted present value of a project is

$$APV_o = A_o + \sum_{t=1}^{T-1} \frac{TrL}{(1+r)^{t+1}} \times (APV_t - C_t) \qquad A16.1$$

where

$$A_t = \sum_{i=t}^{T} \frac{C_t}{(1+\rho_o)^{i-t}} \qquad A16.2$$

Equations can be redefined by the recursive system A16.1 and A16.2.

$$(1-f)APV_{T-S-1} - \frac{APV_{T-s}}{(1+r)} = A_{T-S-1} - \frac{A_{T-S}}{(1+r)} - fC_{T-S-1} \qquad A16.3$$

$$A_{T-S-1} = C_{T-S-1} + \frac{A_{T-S}}{(1+\rho_o)} \qquad A16.4$$

Hence using equation A16.4 to eliminate A_{T-S-1} from A16.3 gives

$$APV_{T-S-1} - \frac{APV_{T-S}}{(1+r)(1-f)} = C_{T-S-1} + \frac{A_{T-S}(r-\rho_o)}{(1+f)(1+\rho_o)(1+r)} \qquad A16.5$$

and using the result

$$(1+r)(1-f) = 1 + r - r\tau L = 1 + r(1-\tau L) \equiv 1 + r^*, \qquad A16.6$$

---

† S.C. Myers "Interactions of Corporate Financing and Investment Decisions - Implications for Capital Budgeting" Journal of Finance XXIX no. 2 (March 1974) pp. 1-25. The mathematical notation is the same as in Myers' original paper except where stated.

Then in general

$$APV_{T-S-1} = \sum_{t=0}^{t=S+1} \frac{C_{T-S-1+t}}{(1+r^*)^t} + \sum_{t=0}^{S} \frac{A_{T-S+t}(r-\rho_o)}{(1+\rho_o)(1+r^*)^{t+1}} \qquad A16.7$$

and in particular by letting S = T-1

$$APV_o = \sum_{t=0}^{t=T} \frac{C_t}{(1+r^*)^t} + \sum_{t=0}^{T} \frac{A_t(r-\rho_o)}{(1+\rho_o)(1+r^*)^t} - \frac{A_o(r-\rho_o)}{(1+\rho_o)} \qquad A16.8$$

where $r^* = r(1-\tau L)$.

Now

$$A_t = \sum_{i=t}^{T} \frac{C_t}{(1+\rho_o)^{i-t}}$$

and this expression can be substituted into the second term on the right hand side of A16.8 to give

$$APV_o = \sum_{t=0}^{t=T} \frac{C_t}{(1+r^*)^t} + \sum_{t=0}^{T} \sum_{i=t}^{T} \frac{C_t(r-\rho_o)}{(1+\rho_o)^{i-t+1}(1+r^*)^t} - \frac{A_o(r-\rho_o)}{(1+\rho_o)} \qquad A16.9$$

which on eliminating the second summation sign can be reduced to

$$APV_o = \sum_{t=0}^{t=T} C_t \left[\frac{1}{(1+r^*)^t} + \frac{r-\rho_o}{(1+r^*)} \left(\frac{\frac{1}{(1+\rho_o)^t} - \frac{1}{(1+r^*)^t}}{1 - \frac{1+\rho_o}{1+r^*}} \right) \right] \qquad A16.10$$

Equation A16.10 on rearrangement gives

$$APV_o = \sum_{t=0}^{t=T} C_t \left[\frac{1}{(1+r^*)^t} \left(1 - \frac{r-\rho_o}{r^*-\rho_o} \right) + \frac{(r-\rho_o)}{(r^*-\rho_o)} \frac{1}{(1+\rho_o)^t} \right] \qquad A16.11$$

If we put

$$\alpha = \frac{\rho_o - r}{\rho_o - r^*} = 1 - \frac{r\tau L}{\rho_o - r^*} \qquad A16.12$$

Then

$$APV_o = (1-\alpha) \sum_{t=0}^{T} \frac{C_t}{(1+r^*)^t} + \alpha \sum_{t=0}^{T} \frac{C_t}{(1+\rho_o)^t} \qquad A16.13$$

APPENDIX XVII

An analysis of the Dual Equation associated with the accounting variables

The following dual relationships apply t=1,8. The sourve of the equation is defined in the opening bracket.

$EARN_t)\quad - EA_t + \rho_t - (1-T)PR_t - ROCE_t - ECOV_t = 0$ A17.1

$NPAT_t)\quad - \frac{T.TP_t}{(1-T)} + PR_t - ERPS_t - DCOV_t = 0$ A17.2

$TAX_t)\quad CL_t - \rho_t + TP_t = 0$ A17.3

while the following equations apply t=1,7.

$CURA_t)\quad - CA_t - \rho_t + \rho_{t+1} + \alpha_t.ROCE_t = 0$ A17.4

$CURL_t)\quad - CL_t + \rho_t - \rho_{t+1} - \alpha_t-ROCE_t + \beta_t.LQDY_t = 0$ A17.5

and finally

$CURA_8)\quad - CA_8 - \rho_8 + \alpha.ROCE_8 = 0$ A17.6

$CURL_8)\quad - CL_8 + \rho_8 - \alpha_8.ROCE_8 + \beta_8.LQDY_t = 0$ A17.7

From which the following identities can be deduced

$CL_t = \rho_t - \rho_{t+1} - \alpha_t.ROCE_t + \beta_t.LQDY_t \quad (t=1,7)$ A17.8

$CL_8 = \rho_8 - \alpha_8.ROCE_8 + \beta_8.LQDY_8$ A17.9

$CA_t = \rho_{t+1} - \rho_t + \alpha_t.ROCE_t \quad (t=1,7)$ A17.10

$CA_8 = - \rho_8 + \alpha_8.ROCE_8$ A17.11

$TP_t = \rho_{t+1} + \alpha_t.ROCE_t + \beta_t LQDY_t \quad (t=1,7)$ A17.12

$TP_8 = \alpha_8.ROCE_8 + \beta_8.LQDY_8$ A17.13

$PR_t = \frac{T}{1-T}\left[\rho_{t+1} + \alpha_t ROCE_t + \beta_t LQDY_t\right] + ERPS_t + DCOV_t \quad (t=1,7)$ A17.14

$PR_8 = \frac{T}{1-T}\left[\alpha_8\ ROCE_8 + \beta_8.LQDY_8\right] + ERPS_8 + PCOV_8$ A17.15

$EA_t = \rho_t - T.\rho_{t+1} - [1+\alpha T].ROCE_t - T.\beta_t.LQDY_t$

$+ (1-T)ERPS_t + (1-T)DCOV_t - ECOV_t \quad (t=1,7)$ A17.16

$EA_8 = \rho_8 - [1+\alpha T].ROCE_8 - T.\beta_8.LQDY_8$

$+ (1-T)ERPS_8 + (1-T)DCOV_8 - ECOV_8$ A17.17

APPENDIX XVIII

THE CHAMBERS (71) MODEL

A18.1 LP INPUT DATA LISTING AND SOLUTION

A18.2 THE LP SOLUTION WHERE
THE FIRM IS IN A
DEFICIT STATE IN EACH YEAR

A18.3 THE LP SOLUTION WHERE THE
FIRM IS IN A SURPLUS
STATE IN EACH YEAR

EXHIBIT A18.1 LP INPUT DATA LISTING AND SOLUTION

```
PROBLEM    CHAMBERS                 INPUT                                      DATE  04/03/76      TIM
X106,COST,0,30,7,-4.5,-4.5,-4.5,-100,16,24,9,9
X107,UPBND,1
X107,COST,-9,30,23,9.5,-6,-11,-100,-37,-1,26,31
X107,DC2,-65,-36,-27,-5
X108,UPBND,1
X108,COST,177,,,,-7,-17.5,,-200,-160,-114,-66
X108,DC3,-40,-40,-46
X109,UPBND,1
X109,COST,74,,,,,-2.5,,-100,-80,-52,-25
X109,DC3,-20,-28,-27
X110,UPBND,1
X110,COST,188,,,,3.5,-1,,-300,-240,-172,-100
X110,DC3,-60,-68,-72
X111,UPBND,1
X111,COST,105,,,7.5,7,1.5,-200,-160,-123,-74,-33
X111,DC2,-40,-37,-49,-41
X112,L3,-1
X112,UPBND,1
X112,COST,189,,,0.5,,-1.5,-200,-160,-140,-119,-97
X112,DC2,-40,-20,-21,-27
L112,COST,-46,203,172,138.3,101.6,61.8,200,154,108,62,16
    ,DC1,     ,46,46,46,46
L112,L3,1
X113,UPBND,1
X113,COST,19,,,,9.5,15,,-100,-80,-80,-67
X113,SALES1,-1
X113,DC3,-20,0,-13
X114,UPBND,1
X114,COST,48,,,,-7,-14.5,,,,23,39
X114,SALES1,1
X114,DC4,-23,-16
X115,COST,,,-1.8,-1.8,-1.8,-1.8,-100,6,3.6,3.6,3.6
X115,DC2,-6,2.4
    ,UPBND,2000
X116,UPBND,1
X116,COST,870,-125,-183.6,-278.5,-353,-437,78,255,391,556,737
X116,DC1,73,177,130,163,183
X117 ,COST,-157,9,-78.5,-68,-57,-45.5,-33.5,157,136,114,91,67
    ,UPBND,2000
X118,COST,-95.3,101.5,103.3,105.1,106.9,108.7,97,91,87.4,83.8,80.2
X118,DC1,57.6,57.6,57.6,57.6,57.6
    ,UPBND,2000
X119,COST,176,50,50,50,50,50,-100,-100,-100,-100,-100
    ,UPBND,2000
X201,UPBND,1
X201,COST,130,,,,,-0.5,,,-100,-80,-58
X201,DC4,-20,-22
X202,UPBND,1
X202,COST,243,,,,,,,,,-200,-160,-118
X202,DC4,-40,-42
X203,UPBND,1
X203,COST,367,,,,,-4,,,-300,-240,-179
X203,DC4,-60,-61
X204,UPBND,1
X204,COST,139,,,,-3.5,-13,,-200,-160,-92,-29
X204,DC3,-40,-68,-63
```

```
X205,UPBND,1
X205,COST,76,,,,,-3,,-100,-80,-52,-24
X205,DC3,-20,-28,-28
X206,UPBND,1
X206,COST,0,,30,7,-4.5,-4.5,,-100,16,24,9
X206,DC3,-116,-8,+15
X207,UPBND,1
X207,COST,   -3,,30,23,9.5,-6,,-100,-37,-1,26
X207,DC3,-63,-36,-27
X208,UPBND,1
X208,COST,208,,,,,-7,,,-200,-160,-114
X208,DC4,-40,-46
X209,UPBND,1
X209,COST,95,,,,,,,,,-100,-80,-52
X209,DC4,-20,-28
X210,UPBND,1
X210,COST,244,,,,,3.5,,,-300,-240,-177
X210,DC4,-60,-68
X211,UPBND,1
X211,COST,137,,,,7.5,7,,-200,-160,-123,-74
X211,DC3,-40,-37,-49
X212,UPBND,1
X212,COST,195,,,,0.5,,,-200,-160,-140,-119
X212,DC3,-40,-20,-21
X213,UPBND,1
X213,COST,31,,,,,9.5,,,-100,-80,-80
X213,SALES2,-1
X213,DC4,-20
X214,UPBND,1
X214,COST,60,,,,,-7,,,,,,23
X214,SALES2,1
X214,DC5,-23
X215,COST,0,,,-1.8,-1.8,-1.8,, -100,6,3.6,3.6
X215,DC3,-6,2.4
    ,UPBND,2000
X216,UPBND,1
X216,COST,365.0,-325,-326.5,-314.8,-286.9,0,95,238,284.5,280.6
X217,COST,-160.9,,-78.5,-67.5,-56,-44,,157,135,112,88
    ,UPBND,2000
X218,COST,-95.3,,101.5,103.3,105.1,106.9,,97,91,87.4,83.8
X218,DC2,57.6,57.6,57.6,57.6
    ,UPBND,2000 .
X219,COST,157,,50,50,50,50,,-100,-100,-100,-100
     ,UPBND,200
X301,UPBND,1
X301,COST,140,,,,,,,,,,-100,-80
X301,DC5,-20
X302,UPBND,1
X302,COST,264,,,,,,,,,,-200,-160
X302,DC5,-40
X303,UPBND,1
X303,COST,397,,,,,,,,,,-300,-240
X303,DC5,-60
X304,UPBND,1
X304,COST,236,,,,,-3.5,,,-200,-160,-92
X304,DC4,-40,-68
```

PROBLEM CHAMBERS INPUT DATE 04/03/76 TIME

```
X305,UPRAND,1
X305,COST,97,........ -100,-80,-52
X305,RC4,-20,-28
X306,UPRAND,1
X306,COST,-15,,,30.7,-4.5,,,-100,16,24
X306,RC4,-116,-8
X307,UPRAND,1
X307,COST,24,,,30,23,9.5,,,-100,-57,-1
X307,RC4,-65,-36
X308,UPRAND,1
X308,COST,236,.........,-200,-160
X308,RC5,-40
X309,UPRAND,1
X309,COST,114,.........,-100,-80
X309,RC5,-20
X310,UPRAND,1
X310,COST,290,.........,-300,-240
X310,RC5,-60
X311,UPRAND,1
X311,COST,174,,,,,7,5,,,-200,-160,-123
X311,RC4,-40,-37
X312,UPRAND,1
X312,COST,199,,,,,0.5,,,-200,-160,-140
X312,RC4,-40,-20
X313,UPRAND,1
X313,COST,23,.........,-100,-80
X313,SALES3,-1
X313,RC5,-20
X314,UPRAND,1
X314,COST,28,...........
X314,SALES3,1
X315,COST,0,,,,-1.8,-1.8,,,-100,6,3.6
X315,RC4,-6,2.4
     ,UPRAND,2000
X317 ,COST,-165.1,,,-76.5,-67,-55,,,157,134,110
     ,UPRAND,2000
X318,COST,-95.2,,,101.5,103.3,105.1,,,97,91,87
X318,RC3,57.6,57.6,57.6
     ,UPRAND,2000
X319,COST,140,,,50,50,50,,,-100,-100,-100
X401,UPRAND,1
X401,COST,148,..........,-130
X402,UPRAND,1
X402,COST,281,..........,-210
X403,UPRAND,1
X403,COST,422,..........,-300
X404,UPRAND,1
X404,COST,253,.........,-201,-160
X405,UPRAND,1
X405,COST,117,.........,-100,-80
X406,UPRAND,1
X406,COST,-5,,,,30,7,,,,-100,16
X407,UPRAND,1
X407,COST,54,,,,30,23,,,,-100,-37
X408,UPRAND,1
X408,COST,255,..........,-200
```

```
PROBLEM    CHAMBERS              INPUT                                       DATE  04/03/76

X409,UPRND,1
X409,COST,124,..........,-100
X410,UPRND,1
X410,COST,325,..........,-300
X411,UPRND,1
X411,COST,196,.........,-200,-160
X412,UPRND,1
X412,COST,201,.........,-200,-160
X413,UPRND,1
X413,COST,65,..........,-100
X413,SALES4,-1
X414,UPRND,1
X414,COST,71,..........
X414,SALES4,1
X415,COST,-2.4,....,-1.8,...,-100,6
X415,DC5,-6
     ,UPRND,2000
X417,COST,-170.2,...,-78.5,-66.5,...,157,133
     ,UPRND,2000
X418,COST,-95.2,...,101.5,103.3,...,97,91
X418,DC4,57.6,57.6
     ,UPRND,2000
X419,COST,125,..,50,50,...,-100,-100
X501,UPRND,1
X501,COST,35,..........
X502,UPRND,1
X502,COST,57,..........
X503,UPRND,1
X503,COST,86,..........
X504,UPRND,1
X504,COST,299,..........,-200
X505,UPRND,1
X505,COST,127,..........,-100
X506,UPRND,1
X506,COST,110,....,30,....,-100
X507,UPRND,1
X507,COST,116,....,30,....,-100
X508,UPRND,1
X508,COST,33,..........
X509,UPRND,1
X509,COST,14,..........
X510,UPRND,1
X510,COST,-3,..........
X511,UPRND,1
X511,COST,219,..........,-200
X512,UPRND,1
X512,COST,224,..........,-200
X513,UPRND,1
X513,COST,-58,..........
X513,SALES5,-1
X514,UPRND,1
X514,COST,64,..........
X514,SALES5,1
X515,COST,103.6,..........,-100
     ,UPRND,2000
X517,COST,-175.9,....,-78.5,....,157
     ,UPRND,2000
X518,COST,-97.0,....,101.5,....,97
X518,DC5,57.6
     ,UPRND,2000
X519,COST,112,....,50,....,-100


#RHS


RHS1,LEV1,-83,-278,953,665,275,-834,-1844,-1426,-2272,-2840,0,0,0,0,0
RHS2,LEV1,500,50,500,-550,-500,900,900,1000,-2500,-2500
RHS3,LEV1,-83,-278,-953,-665,-275,434,1844,1426,2272,2840
RHS4,CASH1,434,1844,1426,2272,2840


#ENDATA
```

PROBLEM C[illegible]ERS   SOLUTION   DATE 06/03/76   TIME

[illegible]   RIGHT HAND SIDE RHS1
OBJECTIVE   COST
UPPER BOUND   UBND
ROW SET (LEVEL -L9)   COL SET

COLUMN INFORMATION

	NAME		VALUE	LOWER BOUND	UPPER BOUND	OBJECTIVE	REDUCED COST
U	X101	*	1.0000		1.0000	117.0000	52.2113
U	X102	*	1.0000		1.0000	218.0000	83.6791
U	X103	*	1.0000		1.0000	341.0000	127.7042
U	X104	*	1.0000		1.0000	169.0000	110.4482
U	X105	*	1.0000		1.0000	59.0000	22.6867
	L105	*	0			-23.0000	-5.4296
	X106	*	0		1.0000	0	-2.2012
U	X107	*	1.0000		1.0000	-9.0000	3.4008
U	X108	*	1.0000		1.0000	177.0000	51.3652
U	X109	*	1.0000		1.0000	74.0000	20.1987
	X110	*	0		1.0000	186.0000	-2.4342
	X111	*	0		1.0000	105.0000	-6.8453
U	X112	*	1.0000		1.0000	189.0000	3.5351
B	L112	*	0.4496			-46.0000	0
U	X113	*	1.0000		1.0000	19.0000	8.9896
B	X114	*	1.0000		1.0000	48.0000	0
	X115	*	0		2000.0000	0	-10.9648
U	X116	*	1.0000		1.0000	870.0000	1911.6063
	X117	*	0		2000.0000	-157.9000	-1.4109
	X118	*	0		2000.0000	-95.3000	-2.3367
B	X119	*	4.[illegible]512		2000.0000	176.0000	0
U	X201	*	1.0000		1.0000	130.0000	45.1768
U	X202	*	1.0000		1.0000	243.0000	71.1129
U	X203	*	1.0000		1.0000	367.0000	107.7772
U	X204	*	1.0000		1.0000	189.0000	106.0532
U	X205	*	1.0000		1.0000	76.0000	23.3191
U	X206	*	1.0000		1.0000	0	1.3452
U	X207	*	1.0000		1.0000	-3.0000	6.4316
U	X208	*	1.0000		1.0000	208.0000	61.4381
U	X209	*	1.0000		1.0000	95.0000	16.3046
	X210	*	0		1.0000	244.0000	-0.2415
B	X211	*	0.4161		1.0000	137.0000	0
U	X212	*	1.0000		1.0000	195.0000	10.7068
U	X213	*	1.0000		1.0000	31.0000	6.7065
B	X214	*	1.0000		1.0000	60.0000	0
	X215	*	0		2000.0000	0	-4.4914
U	X216	*	1.0000		1.0000	365.0000	545.3144
B	X217	*	2.2196		2000.0000	-160.9000	0
	X218	*	0		2000.0000	-95.3000	-9.9421
	X219	*	0		200.0000	157.0000	-0.7140
U	X301	*	1.0000		1.0000	140.0000	65.0230
U	X302	*	1.0000		1.0000	264.0000	76.1460
U	X303	*	1.0000		1.0000	397.0000	112.0649
U	X304	*	1.0000		1.0000	236.0000	91.6393
U	X305	*	1.0000		1.0000	97.0000	18.3086
	X306	*	0		1.0000	-15.0000	-19.4472
U	X307	*	1.0000		1.0000	26.0000	1.0002
U	X308	*	1.0000		1.0000	236.0000	66.0660

PROBLEM CHAMBERS SOLUTION DATE 04/03/76 TIME 1

DUMP:DUMP 4

RIGHT HAND SIDE RHS1
OBJECTIVE COST
UPPER BOUND UPBND

ROW SET (LEVI -L9) COL SET

COLUMN INFORMATION

	NAME		VALUE	LOWER BOUND	UPPER BOUND	OBJECTIVE	REDUCED COST
U	X309	*	1.0000		1.0000	114.0000	19.0230
U	X310	*	1.0000		1.0000	290.0000	5.0689
	X311	*	0		1.0000	174.0000	-6.3330
U	X312	*	1.0000		1.0000	199.0000	4.2364
U	X313	*	1.0000		1.0000	28.0000	11.0230
B	X314	*	1.0000		1.0000	78.0000	0
	X315	*	0		2000.0000	0	-10.3324
B	X317	*	0.3100		2000.0000	-165.1000	0
B	X318	*	12.3784		2000.0000	-95.2000	0
	X319	*	0			140.0000	-0.6765
U	X401	*	1.0000		1.0000	148.0000	44.4000
U	X402	*	1.0000		1.0000	281.0000	73.8000
U	X403	*	1.0000		1.0000	422.0000	111.2000
U	X404	*	1.0000		1.0000	283.0000	93.0600
U	X405	*	1.0000		1.0000	117.0000	22.0230
	X406	*	0		1.0000	-6.0000	-3.5419
U	X407	*	1.0000		1.0000	56.0000	2.8493
U	X408	*	1.0000		1.0000	255.0000	47.8000
U	X409	*	1.0000		1.0000	124.0000	20.4000
U	X410	*	1.0000		1.0000	325.0000	14.2000
U	X411	*	1.0000		1.0000	196.0000	6.8640
U	X412	*	1.0000		1.0000	201.0000	11.0460
U	X413	*	1.0000		1.0000	45.0000	17.4000
B	X414	*	1.0000		1.0000	71.0000	0
	X415	*	0		2000.0000	-2.4000	-7.9772
B	X417	*	2.1732		2000.0000	-170.2000	0
	X418	*	0		2000.0000	-95.2000	-9.4638
	X419	*	0			125.0000	-0.5345
U	X501	*	1.0000		1.0000	35.0000	35.0000
U	X502	*	1.0000		1.0000	57.0000	57.0000
U	X503	*	1.0000		1.0000	86.0000	86.0000
U	X504	*	1.0000		1.0000	299.0000	91.8000
U	X505	*	1.0000		1.0000	127.0000	73.4000
U	X506	*	1.0000		1.0000	110.0000	1.3371
U	X507	*	1.0000		1.0000	116.0000	7.3371
U	X508	*	1.0000		1.0000	33.0000	33.0000
U	X509	*	1.0000		1.0000	14.0000	14.0000
	X510	*	0		1.0000	-3.0000	-3.0000
U	X511	*	1.0000		1.0000	219.0000	11.8000
U	X512	*	1.0000		1.0000	224.0000	16.8000
U	X513	*	1.0000		1.0000	-58.0000	6.0000
B	X514	*	1.0000		1.0000	64.0000	0
B	X515	*	5.8200		2000.0000	-103.6000	0
B	X517	*	7.4459		2000.0000	-175.9000	0
	X518	*	0		2000.0000	-97.0000	-13.6376
	X519	*	0			112.0000	-0.0382
	OBJECTIVE		4625.3400				

```
PROBLEM    CHAMBERS                SOLUTION                          DATE  04/03/76        TIME  1

DUMP:DUMP    4                     RIGHT HAND SIDE   RHS1
                                   OBJECTIVE         COST
                                   UPPER BOUND       UPBND
                        ROW SET (LEV1     -L9        )                         COL SET

ROW INFORMATION

     NAME                   SLACK                    R.H.S.                   PRICE
 # COST       Z          8625.3400                        0
   LEV1       +                  0                 -93.0000                 -0.0281
   LEV2       +                  0                -278.0000                 -0.1424
 B LEV3       +            69.0505                 953.0000                       0
   LEV4       +                  0                 666.0000                 -0.0280
   LEV5       +                  0                 275.0000                 -0.1688
   CASH1                         0                -834.0000                  0.1688
   CASH2                         0               -1846.0000                  0.0992
   CASH3                         0               -1426.0000                  0.1514
   CASH4                         0               -2272.0000                  0.1210
   CASH5                         0               -2840.0000                  1.0360
   SALES1     +                  0                        0                -93.8293
   SALES2     +                  0                        0                -85.0024
   SALES3     +                  0                        0                -78.0000
   SALES4     +                  0                        0                -71.0000
   SALES5     +                  0                        0                -64.0000
 B L1         +             1.0000                        0                       0
 B L2         +                  0                        0                       0
 B L3         +             0.5596                        0                       0
 B L4         +                  0                        0                       0
 B L5         +                  0                        0                       0
 B L6         +                  0                        0                       0
 B L7         +                  0                        0                       0
 B L8         +                  0                        0                       0
 B L9         +                  0                        0                       0
```

TABLE A18.1 CASH BALANCE AND DEBT CAPACITY DUALS

YEAR	ρ_t	L_t	$\rho_t + \frac{1}{2}L_t$
1	1.5764	0.3673	1.7600
2	1.4076	0.3392	1.5772
3	1.3084	0.1968	1.4068
4	1.1570	0.1968	1.2554
5	1.0360	0.1688	1.1204

Here

$$L_t = \sum_{\tau=t}^{5} LEV_t$$

$$\rho_t = \sum_{\tau=t}^{5} CASH_t$$

and where LEV_t, $CASH_t$ are dual variables in the computer solution above and ρ_t, L_t are as defined in section 3.5.

EXHIBIT 18.2 THE LP SOLUTION WHERE THE FIRM IS IN A DEFICIT STATE IN EACH YEAR

PROBLEM CHAMBERS SOLUTION DATE 06/03/76 TIM

DUMPIDUMP A RIGHT HAND SIDE RHS1 +1.7500*RHS3
OBJECTIVE COST
UPPER BOUND UPBND
ROW SET (LEVEL -LO) COL SET

COLUMN INFORMATION

NAME		VALUE	LOWER BOUND	UPPER BOUND	OBJECTIVE	REDUCED COST
U X101	*	1.0000		1.0000	117.0000	50.3917
U X102	*	1.0000		1.0000	218.0000	80.0319
U X103	*	1.0000		1.0000	341.0000	121.4762
U X104	*	1.0000		1.0000	169.0000	112.1865
U X105	*	1.0000		1.0000	59.0000	23.2559
L105	*	0			-23.0000	-9.5336
X106	*	0		1.0000	0	-1.4692
U X107	*	1.0000		1.0000	-4.0000	3.5279
U X108	*	1.0000		1.0000	177.0000	67.3869
U X109	*	1.0000		1.0000	76.0000	18.7843
X110	*	0		1.0000	188.0000	-7.1010
X111	*	0		1.0000	105.0000	-5.6547
U X112	*	1.0000		1.0000	189.0000	3.5424
L112	*	0			-46.0000	-8.2176
U X113	*	1.0000		1.0000	19.0000	7.6316
B X114	*	1.0000		1.0000	60.0000	0
X115	*	0		2000.0000	0	-8.9453
U X116	*	1.0000		1.0000	870.0000	1914.4053
B X117	*	3.7465		2000.0000	-157.9000	0
B X118	*	4.7867		2000.0000	-95.3000	0
X119	*	0		2000.0000	176.0000	-0.8986
U X201	*	1.0000		1.0000	130.0000	46.7380
U X202	*	1.0000		1.0000	243.0000	76.2352
U X203	*	1.0000		1.0000	367.0000	112.0013
U X204	*	1.0000		1.0000	189.0000	103.3839
U X205	*	1.0000		1.0000	76.0000	21.9947
X206	*	0		1.0000	0	-1.1189
U X207	*	1.0000		1.0000	-3.0000	3.6085
U X208	*	1.0000		1.0000	208.0000	44.1060
U X209	*	1.0000		1.0000	95.0000	18.1678
X210	*	0		1.0000	246.0000	-6.0123
X211	*	0		1.0000	137.0000	-7.7746
U X212	*	1.0000		1.0000	195.0000	5.6641
U X213	*	1.0000		1.0000	31.0000	8.5859
B X214	*	1.0000		1.0000	60.0000	0
X215	*	0		2000.0000	0	-7.8699
U X216	*	1.0000		1.0000	365.0000	562.9574
B X217	*	10.7954		2000.0000	-160.9000	0
B X218	*	2.0590		2000.0000	-95.3000	0
X219	*	0		200.0000	157.0000	-9.7140
U X301	*	1.0000		1.0000	140.0000	47.7412
U X302	*	1.0000		1.0000	264.0000	69.4824
U X303	*	1.0000		1.0000	397.0000	105.2236
U X304	*	1.0000		1.0000	236.0000	95.6256
U X305	*	1.0000		1.0000	97.0000	20.1876
X306	*	0		1.0000	-15.0000	-25.3985
U X307	*	1.0000		1.0000	26.0000	3.0592
U X308	*	1.0000		1.0000	236.0000	41.4824

PROBLEM CHAMBERS SOLUTION DATE 04/03/76 TIME

DUMP:DUMP 6 RIGHT HAND SIDE RHS1 +1.7500*RHS3
OBJECTIVE COST
UPPER BOUND UPBND
ROW SET (LEVEL -LO) COL SET

COLUMN INFORMATION

NAME		VALUE	LOWER BOUND	UPPER BOUND	OBJECTIVE	REDUCED COST
U X309	*	1.0000		1.0000	116.0000	16.7612
X310	*	0		1.0000	290.0000	-1.7766
X311	*	0		1.0000	174.0000	-0.7559
U X312	*	1.0000		1.0000	199.0000	6.4042
U X313	*	1.0000		1.0000	28.0000	8.7412
B X316	*	1.0000		1.0000	78.0000	0
X315	*	0		2000.0000	0	-7.2056
B X317	*	4.2637		2000.0000	-165.1000	0
B X318	*	2.8360		2000.0000	-95.2000	0
X319	*	0			140.0000	-0.6765
U X401	*	1.0000		1.0000	148.0000	39.8541
U X402	*	1.0000		1.0000	281.0000	64.7083
U X403	*	1.0000		1.0000	422.0000	97.5624
U X404	*	1.0000		1.0000	283.0000	88.4824
U X405	*	1.0000		1.0000	117.0000	19.7412
X406	*	0		1.0000	-6.0000	-1.6362
U X407	*	1.0000		1.0000	58.0000	3.8009
U X408	*	1.0000		1.0000	255.0000	38.7083
U X409	*	1.0000		1.0000	126.0000	15.8541
U X410	*	1.0000		1.0000	325.0000	0.5424
U X411	*	1.0000		1.0000	196.0000	1.4824
U X412	*	1.0000		1.0000	201.0000	6.4824
U X413	*	1.0000		1.0000	45.0000	7.8541
B X414	*	1.0000		1.0000	71.0000	0
X415	*	0		2000.0000	-2.4000	-6.5132
B X417	*	7.7983		2000.0000	-170.2000	0
B X418	*	6.7263		2000.0000	-95.2000	0
X419	*	0			125.0000	-0.5345
U X501	*	1.0000		1.0000	35.0000	35.0000
U X502	*	1.0000		1.0000	57.0000	57.0000
U X503	*	1.0000		1.0000	86.0000	86.0000
U X504	*	1.0000		1.0000	299.0000	82.7083
U X505	*	1.0000		1.0000	127.0000	18.8541
X506	*	0		1.0000	110.0000	-0.6813
U X507	*	1.0000		1.0000	116.0000	5.5187
U X508	*	1.0000		1.0000	33.0000	33.0000
U X509	*	1.0000		1.0000	14.0000	14.0000
X510	*	0		1.0000	-3.0000	-3.0000
U X511	*	1.0000		1.0000	219.0000	2.7083
U X512	*	1.0000		1.0000	224.0000	7.7083
U X513	*	1.0000		1.0000	-58.0000	6.0000
B X514	*	1.0000		1.0000	66.0000	0
X515	*	0		2000.0000	103.6000	-6.5659
B X517	*	9.8629		2000.0000	-175.9000	0
B X518	*	5.6719		2000.0000	-97.0000	0
X519	*	0			112.0000	-0.0582
OBJECTIVE		1470.2645				

```
PROBLEM   CHAMBERS                 SOLUTION                                            DATE  04/03/78        TIME

DUMP:DUMP   6                       RIGHT HAND SIDE   RHS1 +1.7500*RHS3
                                    OBJECTIVE         COST
                                    UPPER BOUND       UPBND
                        ROW SET (LEV1     -L9       )                              COL SET

ROW INFORMATION

     NAME                    SLACK                  R.H.S.                      PRICE
                                                          0
# COST       Z            1470.2645             -278.2500                     -0.0861
   LEV1      *                    0             -764.5000                     -0.0767
   LEV2      *                    0             -714.7500                     -0.0632
   LEV3      *                    0             -484.5000                     -0.0551
   LEV4      *                    0             -206.2500                     -0.0778
   LEV5      *                    0              625.5000                      0.1498
   CASH1                          0             1383.0000                      0.1320
   CASH2                          0             1069.5000                      0.1198
   CASH3                          0             1706.0000                      0.1074
   CASH4                          0             2130.0000                      1.0815
   CASH5                          0                     0                    -96.1619
   SALES1    *                    0                     0                    -85.4185
   SALES2    *                    0                     0                    -78.0000
   SALES3    *                    0                     0                    -71.0000
   SALES4    *                    0                     0                    -64.0000
   SALES5    *                    0                     0                           0
B  L1        *               1.0000                     0                           0
B  L2        *                    0                     0                           0
B  L3        *               1.0000                     0                           0
B  L4        *                    0                     0                           0
B  L5        *                    0                     0                           0
B  L6        *                    0                     0                           0
B  L7        *                    0                     0                           0
B  L8        *                    0                     0                           0
B  L9        *                    0                     0
```

TABLE A18.3 CASH BALANCE AND DEBT CAPACITY DUALS.

YEAR	ρ_t	L_t	$\rho_t + \frac{1}{2}L_t$
1	1.5905	0.3569	1.7689
2	1.4407	0.2728	1.5771
3	1.3087	0.1961	1.4067
4	1.1889	0.1329	1.2553
5	1.0815	0.0778	1.1204

EXHIBIT A18.3 THE LP SOLUTION WHERE THE FIRM IS IN A SURPLUS STATE IN EACH YEAR

PROBLEM CHAMBERS SOLUTION DATE 04/03/76 TIME

DUMP:DUMP 5 RIGHT HAND SIDE RHS1 -3.5000*RHS4
OBJECTIVE COST
UPPER BOUND UPBND
ROW SET (LEVEL -LP) COL SET

COLUMN INFORMATION

	NAME		VALUE	LOWER BOUND	UPPER BOUND	OBJECTIVE	REDUCED COST
U	X101	*	1.0000		1.0000	117.0000	70.1053
U	X102	*	1.0000		1.0000	218.0000	119.4591
U	X103	*	1.0000		1.0000	341.0000	182.1538
U	X104	*	1.0000		1.0000	149.0000	155.0016
U	X105	*	1.0000		1.0000	59.0000	45.4470
	L105	*	0			-23.8000	-82.6193
	X106	*	0		1.0000	0	-0.9744
U	X107	*	1.0000		1.0000	-9.0000	4.8876
U	X108	*	1.0000		1.0000	177.0000	88.4509
U	X109	*	1.0000		1.0000	74.0000	37.5343
U	X110	*	1.0000		1.0000	188.0000	50.2906
U	X111	*	1.0000		1.0000	105.0000	39.2039
U	X112	*	1.0000		1.0000	182.0000	57.4576
	L112	*	0			-46.0000	-153.0523
U	X113	*	1.0000		1.0000	19.0000	26.2452
B	X114	*	1.0000		1.0000	49.0000	0
B	X115	*	24.5200		2000.0000	0	0
U	X116	*	1.0000		1.0000	870.0000	1964.1045
	X117	*	0		2000.0000	-157.9000	-1.2421
	X118	*	0		2000.0000	-95.3000	-106.8867
B	X119	*	6.2600		2000.0000	176.0000	0
U	X201	*	1.0000		1.0000	130.0000	62.0824
U	X202	*	1.0000		1.0000	243.0000	106.9241
U	X203	*	1.0000		1.0000	367.0000	158.4942
U	X204	*	1.0000		1.0000	189.0000	140.1256
U	X205	*	1.0000		1.0000	76.0000	40.6547
	X206	*	0		1.0000	0	-1.3555
U	X207	*	1.0000		1.0000	-3.0000	3.9140
U	X208	*	1.0000		1.0000	208.0000	75.2495
U	X209	*	1.0000		1.0000	95.0000	33.2141
U	X210	*	1.0000		1.0000	244.0000	61.4755
U	X211	*	1.0000		1.0000	137.0000	34.9902
U	X212	*	1.0000		1.0000	195.0000	48.1639
U	X213	*	1.0000		1.0000	31.0000	23.6122
B	X214	*	1.0000		1.0000	60.0000	0
B	X215	*	57.9455		2000.0000	0	0
U	X216	*	1.0000		1.0000	365.0000	686.4744
B	X217	*	1.4492		2000.0000	-160.9000	0
	X218	*	0		2000.0000	-95.3000	-79.0845
	X219	*	0		200.0000	157.6000	-0.6192
U	X301	*	1.0000		1.0000	140.0000	53.0002
U	X302	*	1.0000		1.0000	264.0000	90.0064
U	X303	*	1.0000		1.0000	397.0000	136.0007
U	X304	*	1.0000		1.0000	236.0000	125.4504
U	X305	*	1.0000		1.0000	97.0000	55.2169
	X306	*	0		1.0000	-15.0000	-23.5250
U	X307	*	1.0000		1.0000	24.0000	4.9967
U	X308	*	1.0000		1.0000	[illegible]	[illegible]

PROBLEM CHAMBERS SOLUTION DATE 04/03/76 TIME

BUMP:BUMP 1

RIGHT HAND SIDE RHS1 -3.5000*RHS6
OBJECTIVE COST
UPPER BOUND UPBND
ROW SET (LEVEL -L9) COL SET

COLUMN INFORMATION

NAME		VALUE	LOWER BOUND	UPPER BOUND	OBJECTIVE	REDUCED COST
U X309	*	1.0000		1.0000	114.0000	27.0002
U X310	*	1.0000		1.0000	290.0000	29.0007
U X311	*	1.0000		1.0000	174.0000	29.4786
U X312	*	1.0000		1.0000	109.0000	38.6677
U X313	*	1.0000		1.0000	28.0000	12.0002
B X314	*	1.0000		1.0000	78.0000	0
B X315	*	1.2707		2000.0000	0	0
X317	*	0		2000.0000	-165.1000	-1.0621
X318	*	0		2000.0000	-95.2000	-53.9306
B X319	*	27.7212			140.0000	0
U X401	*	1.0000		1.0000	148.0000	44.4000
U X402	*	1.0000		1.0000	281.0000	73.0000
U X403	*	1.0000		1.0000	422.0000	111.2000
U X404	*	1.0000		1.0000	283.0000	109.0006
U X405	*	1.0000		1.0000	117.0000	39.0602
X406	*	0		1.0000	-6.0000	-2.3510
U X407	*	1.0000		1.0000	58.0000	6.0207
U X408	*	1.0000		1.0000	255.0000	67.8000
U X409	*	1.0000		1.0000	124.0000	20.4000
U X410	*	1.0000		1.0000	325.0000	16.2000
U X411	*	1.0000		1.0000	196.0000	22.0004
U X412	*	1.0000		1.0000	201.0000	27.0004
U X413	*	1.0000		1.0000	45.0000	12.6000
B X414	*	1.0000		1.0000	71.0000	0
B X415	*	34.7127		2000.0000	-2.4000	0
B X417	*	3.2645		2000.0000	-170.2000	0
X418	*	0		2000.0000	-95.2000	-33.3955
X419	*	0			125.0000	-0.5565
U X501	*	1.0000		1.0000	35.0000	35.0000
U X502	*	1.0000		1.0000	57.0000	57.0000
U X503	*	1.0000		1.0000	86.0000	86.0000
U X504	*	1.0000		1.0000	299.0000	91.8000
U X505	*	1.0000		1.0000	127.0000	23.4000
U X506	*	1.0000		1.0000	110.0000	1.3371
U X507	*	1.0000		1.0000	116.0000	7.3371
U X508	*	1.0000		1.0000	33.0000	33.0000
U X509	*	1.0000		1.0000	16.0000	14.0000
X510	*	0		1.0000	-3.0000	-3.0000
U X511	*	1.0000		1.0000	219.0000	11.0000
U X512	*	1.0000		1.0000	226.0000	16.8000
U X513	*	1.0000		1.0000	-58.0000	6.0000
B X514	*	1.0000		1.0000	64.0000	0
B X515	*	65.3652		2000.0000	103.6000	0
B X517	*	6.6493		2000.0000	-175.0000	0
X518	*	0		2000.0000	-97.0000	-13.0376
X519	*	0			112.0000	-3.0382
OBJECTIVE		20711.5117				

PROBLEM CHAMBERS SOLUTION DATE 04/03/76 TIME

DUMP:DUMP 1

RIGHT HAND SIDE RHS1 -3.5000*RHS4
OBJECTIVE COST
UPPER BOUND UPBND
ROW SET (LEV) -L9) COL SET

COLUMN INFORMATION

NAME		VALUE	LOWER BOUND	UPPER BOUND	OBJECTIVE	REDUCED COST
U X309	*	1.0000		1.0000	114.0000	27.9002
U X310	*	1.0000		1.0000	290.0000	29.0007
U X311	*	1.0000		1.0000	174.0000	79.4786
U X312	*	1.0000		1.0000	199.0000	38.6477
U X313	*	1.0000		1.0000	28.0000	19.0002
B X314	*	1.0000		1.0000	78.0000	0
B X315	*	1.2797		2000.0000	0	0
X317	*	0		2000.0000	-165.1000	-1.0621
X318	*	0		2000.0000	-95.2090	-53.9346
B X319	*	27.7712			140.0000	0
U X401	*	1.0000		1.0000	168.0000	44.4000
U X402	*	1.0000		1.0000	281.0000	73.4000
U X403	*	1.0000		1.0000	422.0000	111.2000
U X404	*	1.0000		1.0000	283.0000	109.0006
U X405	*	1.0000		1.0000	117.0000	39.0002
X406	*	0		1.0000	-6.0000	-7.3510
U X407	*	1.0000		1.0000	58.0000	4.0407
U X408	*	1.0000		1.0000	255.0000	47.8000
U X409	*	1.0000		1.0000	124.0000	20.6000
U X410	*	1.0000		1.0000	325.0000	14.2000
U X411	*	1.0000		1.0000	194.0000	22.0004
U X412	*	1.0000		1.0000	201.0000	27.0004
U X413	*	1.0000		1.0000	45.0000	12.4000
B X414	*	1.0000		1.0000	71.0000	0
B X415	*	34.7187		2000.0000	-2.4000	0
B X417	*	3.2645		2000.0000	-170.2000	0
X418	*	0		2000.0000	-95.2000	-33.3955
X419	*	0			125.0000	-0.5565
U X501	*	1.0000		1.0000	35.0000	35.0000
U X502	*	1.0000		1.0000	57.0000	57.0000
U X503	*	1.0000		1.0000	86.0000	86.0000
U X504	*	1.0000		1.0000	249.0000	91.8000
U X505	*	1.0000		1.0000	127.0000	23.4000
U X506	*	1.0000		1.0000	110.0000	1.3371
U X507	*	1.0000		1.0000	116.0000	7.3371
U X508	*	1.0000		1.0000	33.0000	33.0000
U X509	*	1.0000		1.0000	14.0000	14.0000
X510	*	0		1.0000	-3.0000	-3.0000
U X511	*	1.0000		1.0000	219.0000	11.8000
U X512	*	1.0000		1.0000	224.0000	16.8000
U X513	*	1.0000		1.0000	-58.0000	6.0000
B X514	*	1.0000		1.0000	64.0000	0
B X515	*	64.3632		2000.0000	103.6000	0
B X517	*	6.7493		2000.0000	-175.0000	0
X518	*	0		2000.0000	-97.0000	-13.6376
X519	*	0			112.0000	-3.0382
OBJECTIVE		20711.5117				

```
PRO[illegible]    [illegible]ERS              SOLUTION                               DATE  04/05/76       TIME  [illegible]

[illegible]   5                     RIGHT HAND SIDE  RHS1 -3.5000RHS4
                                    OBJECTIVE        COST
                                    UPPER BOUND      UPBND
                     ROW SET (LEV1    -L9        )                      COL SET

ROW INFORMATION

    [illegible]             SLACK                 R.H.S.            PRICE
# COST       Z       20711.5117                       0
  L[illegible]1       +               0              -83.0000          -0.2517
  L[illegible]2       +               0             -278.0000          -0.2494
  L[illegible]3       +               0              953.0000          -0.1969
  L[illegible]4       +               0              666.0000          -0.1875
  L[illegible]5       +               0              275.0000          -0.1688
  CASH1                               0            -3753.0000           0.0580
  CASH2                               0            -8296.0000           0.0515
  CA[illegible]                                0            -6417.0000           0.0462
  CAS[illegible]                               0           -10226.0000           0.0412
  CA[illegible]5                               0           -127[illegible]0.0000           1.0360
  S[illegible]1      +               0                    0          -93.1113
  S[illegible]2      +               0                    0          -85.0094
  SALES3     +               0                    0          -78.0000
  SALES4     +               0                    0          -71.0000
  SALES5     +               0                    0          -64.0000
B L1         +          1.0000                    0                0
B L2         +               0                    0                0
B L3         +          1.0000                    0                0
B L4         +               0                    0                0
B L5         +               0                    0                0
B L6         +               0                    0                0
B L7         +               0                    0                0
B L8         +               0                    0                0
B L9         +               0                    0                0
```

TABLE A18.2 CASH BALANCE AND DEBT CAPACITY DUALS

YEAR	ρ_t	L_t	$\rho_t + \frac{1}{2}L_t$
1	1.2329	1.0543	1.7600
2	1.1749	0.8026	1.5762
3	1.1234	0.5532	1.400
4	1.0772	0.3563	1.2554
5	1.0360	0.1688	1.1204

APPENDIX XlX

THE IMPACT OF THE CHOICE OF HORIZON ON THE SET OF INVESTMENT AND FINANCING DECISIONS.

TABLE A 19.1 NORMAL LEVEL OF EARNINGS

PROJECTS	HORIZON 1	2	3	4	5	6	7	8
PRO1Y1	✓	✓	✓	✓	✓	✓	✓	✓
PRO4Y1	✓	✓	✓	✓	✓	✓	✓	✓
PR12Y1	✓	✓	✓	✓	✓	✓	✓	✓
PR13Y1	✓	✓	✓	✓	✓	✓	✓	✓
PR16Y1	×	×	×	×	×	×	×	×
PR22Y1	×	×	×	✓	✓	✓	✓	✓
PR23Y1	×	×	×	×	×	×	×	×
PRO3Y2	0.66	0.80	0.35	0.39	0.41	✓	✓	✓
PRO4Y2	✓	✓	✓	✓	✓	✓	✓	✓
PRO5Y2	×	×	×	×	×	×	×	×
PR13Y2	✓	✓	✓	✓	✓	✓	✓	✓
PR14Y2		✓	✓	✓	✓	✓	✓	✓
PR21Y2	×	×	×	×	×	×	×	×
PR24Y2	×	×	×	×	×	×	×	×
PRO2Y3	×	×	0.67	×	×	×	×	×
PR11Y3	✓	✓	✓	✓	✓	✓	✓	✓
PR15Y3	×	×	×	0.95	✓	0.90	0.90	0.90
PRO1Y4	✓	✓	✓	✓	✓	✓	✓	✓
PRO5Y4	×	×	×	6.01	×	×	×	×
PR11Y4	✓	✓	0.77	0.24	0.37	0.63	0.63	0.63
PR12Y4	✓	✓	✓	✓	✓	✓	✓	✓
PR13Y4	✓	✓	✓	✓	✓	✓	✓	✓
PR14Y4	×	×	✓	✓	✓	✓	✓	✓
PR22Y4	×	×	✓	✓	✓	✓	✓	✓
PR25Y4	✓	✓	✓	✓	✓	✓	✓	✓
PRO2Y5	×	×	×	×	×	×	×	×
PRO3Y5	✓	✓	✓	✓	✓	✓	✓	✓
PR11Y5	✓	0.87	×	0.37	0.32	0.33	0.33	0.33
PR21Y5	×	×	×	×	×	×	×	×
PR23Y5	×	×	×	×	×	×	×	×
PRO4Y6	✓	✓	✓	✓	✓	✓	✓	✓
PRO5Y6	×	×	×	×	×	×	×	×
PR11Y6	✓	✓	✓	✓	✓	✓	✓	✓
PR14Y6	×	×	✓	✓	✓	✓	✓	✓
PR15Y6	×	×	✓	✓	✓	✓	✓	✓
PR16Y6	×	×	✓	✓	✓	✓	✓	✓
PR21Y6	×	×	×	×	×	×	×	×
PR23Y6	×	×	×	×	×	×	×	×

FINANCIAL INSTRUMENTS	HORIZON 1	2	3	4	5	6	7	8
OVDR 1		249	249	4				
OVDR 2		206						
OVDR 3	223							
OVDR 4	166					26	26	26
OVDR 5	249							
OVDR 6	88							
OVDR 7	249							
OVDR 8								
MARK 1	1					218	218	218
MARK 2								
MARK 3	16	338	2					
MARK 4	301	346						
MARK 5	14	55						
MARK 6	1619	1453	1206	1187	1277	1329	1329	1329
MARK 7	2162	2353	2044	2099	2172	2438	2438	2438
MARK 8	3151	3590	3440	3557	3651	3979	3979	3979
DV 1	271	281	281	272	273	290	290	290
DV 2	408	432	620	474	474	477	477	477
DV 3	414	415	403	638	585	642	642	642
DV 4	428	431	768	746	636	745	745	745
DV 5	504	621	668	699	699	699	699	699
DV 6	869	839	870	870	870	870	870	870
DV 7	463	474	472	477	464	485	485	485
DV 8	1041	1002	1148	1181	1191	1251	1251	1251
RG 1	88	167	167	98	161	232	232	232
RG 2	794	799	713	800	800	800	800	800
RG 3	75				5			
RG 4	1			1				
RG 5	1		76	87				
RG 6								
RG 7								
RG 8								
LL 1	374	1		476	477	483	483	483
LL 2	2		358			4	4	4
LL 3	11	871	680	616	615	619	619	619
LL 4	1000	729	875	893	891	1000	1000	1000
LL 5	459	538	707	732	737	812	812	812
LL 6	1000	172	894	897	919	945	945	945
LL 7		517	556	550	567	592	592	592
LL 8								
VALUE OF PLAN	1915	1921	2022	2051	2063	2063	2063	2063

TABLE A19.2 AN INCREASE IN THE LEVEL OF EARNINGS FROM EXISTING PROJECTS OF TEN PERCENT

PROJECTS	HORIZON 1	2	3	4	5	6	7	8
PR01Y1	✓	✓	✓	✓	✓	✓	✓	✓
PR04Y1	✓	✓	✓	✓	✓	✓	✓	✓
PR12Y1	✓	✓	✓	✓	✓	✓	✓	✓
PR13Y1	✓	✓	✓	✓	✓	✓	✓	✓
PR16Y1	×	×	×	×	×	×	0.33	0.33
PR22Y1	×	×	×	✓	✓	✓	✓	✓
PR23Y1	×	×	×	×	×	×	×	×
PR03Y2	✓	✓	✓	✓	✓	✓	✓	✓
PR04Y2	✓	✓	✓	✓	✓	✓	✓	✓
PR05Y2	×	×	×	×	×	×	×	×
PR12Y2	✓	✓	✓	✓	✓	✓	✓	✓
PR14Y2	×	×	✓	✓	✓	✓	✓	✓
PR21Y2	×	×	×	×	×	×	×	×
PR24Y2	×	×	×	×	×	×	×	×
PR02Y3	×	×	0.57	×	×	×	0.11	0.11
PR11Y3	✓	✓	✓	✓	✓	✓	✓	✓
PR15Y3	×	×	✓	0.70	0.70	0.70	✓	✓
PR01Y4	✓	✓	✓	✓	✓	✓	✓	✓
PR05Y4	×	×	×	×	×	×	×	×
PR11Y4	✓	✓	0.65	0.65	✓	✓	✓	✓
PR12Y4	✓	✓	✓	✓	✓	✓	✓	✓
PR13Y4	✓	✓	✓	✓	✓	✓	✓	✓
PR14Y4	×	×	✓	✓	✓	✓	✓	✓
PR22Y4	×	×	0.43	0.89	✓	✓	✓	✓
PR25Y4	✓	✓	✓	✓	✓	✓	✓	✓
PR02Y5	×	×	×	×	×	×	×	×
PR03Y5	✓	✓	✓	✓	✓	✓	✓	✓
PR11Y5	✓	✓	✓	✓	✓	✓	✓	✓
PR21Y5	×	×	×	×	×	×	×	×
PR22Y5	×	×	×	×	×	×	×	×
PR04Y6	✓	✓	✓	✓	✓	✓	✓	✓
PR05Y6	×	×	×	×	×	×	×	×
PR11Y6	✓	✓	✓	✓	✓	✓	✓	✓
PR14Y6	×	×	✓	✓	✓	✓	✓	✓
PR15Y6	×	×	✓	✓	✓	✓	✓	✓
PR16Y6	×	×	×	×	×	×	×	×
PR21Y6	×	×	×	×	×	×	×	×
PR22Y6	×	×	×	×	×	×	×	×

FINANCIAL INSTRUMENTS	HORIZON 1	2	3	4	5	6	7	8
OVDR 1	160	12	249					
OVDR 2								
OVDR 3	201							
OVDR 4	137				8	8	42	42
OVDR 5								
OVDR 6								
OVDR 7								
OVDR 8								
MARK 1	217	290			282	282	170	170
MARK 2								
MARK 3	1	289						
MARK 4	317	434						
MARK 5	164	125						
MARK 6	1622	1540	1414	1363	1403	1406	1491	1491
MARK 7	2061	2503	2428	2478	2589	2594	2718	[illegible]
MARK 8	3424	3875	4057	4146	4308	4314	4469	[illegible]
DV 1	260	260	265	260	260	260	260	[illegible]
DV 2	513	368	515	533	533	533	530	530
DV 3	376	360	624	683	584	578	623	623
DV 4	390	455	496	688	460	451	570	570
DV 5	675	676	776	733	742	742	754	754
DV 6	895	893	1097	1095	1102	1102	1141	1141
DV 7	429	412	433	455	431	431	446	[illegible]
DV 8	1060	1057	1308	1324	1345	1346	1377	1377
RG 1			44					
RG 2	594	577	662	698	698	698	800	800
RG 3	91							
RG 4								
RG 5					148			
RG 6								
RG 7								
RG 8								
LL 1	378	599		724	724	724	711	711
LL 2	212		580					
LL 3		508	671	584	584	58	639	639
LL 4	1000	744	1000	1000	1000	1000	1000	1000
LL 5	751	519	740	709	705	705	785	785
LL 6	800	766	973	940	971	972	1000	1000
LL 7		525	672	706	749	750	763	763
LL 8								
VALUE OF PLAN	2399	2430	2534	2556	2557	2557	2562	2562

TABLE A 19.3 A DECREASE IN THE LEVEL OF EARNINGS FROM EXISTING PROJECTS OF TEN PERCENT

PROJECTS	HORIZON 1	2	3	4	5	6	7	8
PR01Y1	✓	✓	0.21	0.10	✓	✓	✓	✓
PR04Y1	✓	✓	✓	✓	✓	✓	✓	✓
PR12Y1	✓	0.11	×	×	×	×	0.93	0.93
PR13Y1	✓	✓	✓	✓	✓	✓	✓	✓
PR16Y1	×	×	×	×	×	×	×	×
PR22Y1	×	×	×	✓	✓	✓	✓	✓
PR23Y1	×	×	×	×	×	×	×	×
PR03Y2	×	×	×	0.02	×	×	×	×
PR04Y2	×	✓	0.80	✓	0.47	0.47	×	×
PR05Y2	×	×	×	×	×	×	×	×
PR13Y2	0.71	✓	✓	✓	✓	✓	✓	✓
PR14Y2	×	×	✓	✓	✓	✓	✓	✓
PR21Y2	×	×	×	×	×	×	×	×
PR23Y2	×	×	×	×	×	×	×	×
PR02Y3	×	×	0.59	0.42	×	×	×	×
PR11Y3	✓	✓	✓	✓	✓	✓	✓	✓
PR15Y3	×	×	×	×	×	×	×	×
PR01Y4	✓	✓	✓	✓	✓	✓	✓	✓
PR03Y4	×	×	×	×	×	×	×	×
PR11Y4	✓	×	×	×	×	×	×	×
PR12Y4	✓	✓	✓	✓	✓	✓	✓	✓
PR13Y4	✓	✓	✓	✓	✓	✓	✓	✓
PR14Y4	×	✓	✓	✓	✓	✓	✓	✓
PR20Y4	×	✓	✓	✓	✓	✓	✓	✓
PR23Y4	✓	✓	✓	✓	✓	✓	✓	✓
PR02Y5	×	×	×	×	×	×	×	×
PR03Y5	×	×	0.46	0.64	✓	✓	✓	✓
PR11Y5	×	×	×	×	×	×	×	×
PR21Y5	✓	×	×	×	×	×	×	×
PR23Y5	✓	×	×	×	×	×	×	×
PR04Y6	✓	✓	✓	✓	✓	✓	✓	✓
PR05Y6	×	×	×	×	×	×	×	×
PR11Y6	✓	✓	✓	✓	✓	✓	✓	✓
PR14Y6	×	×	✓	✓	✓	✓	✓	✓
PR15Y6	×	×	✓	✓	✓	✓	✓	✓
PR16Y6	×	×	×	×	×	×	×	×
PR21Y6	×	×	×	×	×	×	×	×
PR23Y6	×	×	×	×	×	×	×	×

FINANCIAL INSTRUMENTS	HORIZON 1	2	3	4	5	6	7	8
OVDR 1								
OVDR 2	22	42		11	11	12	38	38
OVDR 3	249							
OVDR 4	249	249						
OVDR 5	1728	157						
OVDR 6								
OVDR 7								
OVDR 8								
MARK 1								
MARK 2								
MARK 3								
MARK 4		619						
MARK 5		350						
MARK 6	66	1647	566	525	782	782	853	853
MARK 7	665	1849	723	823	1110	1100	1309	1309
MARK 8	1649	2621	1658	1830	2209	2207	2516	2516
DV 1	307	364	290	279	286	286	306	306
DV 2	399	411	413	426	426	426	432	432
DV 3	430	394	332	408	432	432	415	415
DV 4	506	667	484	465	412	412	487	481
DV 5	525	551	562	579	601	602	634	634
DV 6	565	754	747	759	822	823	855	855
DV 7	583	528	446	449	456	456	467	461
DV 8	838	745	852	892	966	966	999	999
RG 1	364	799	231	139	197	197	358	358
RG 2	336	5	498	668	613	613	526	526
RG 3	171	13			37	37	38	38
RG 4	443	483	59					
RG 5					2	2		
RG 6	335							
RG 7								
RG 8								
LL 1	130	146		259	256	256	240	240
LL 2			199					
LL 3		462	489	488	526	526	583	585
LL 4	248	606	621	643	641	641	634	634
LL 5	139	54	577	570	624	624	715	715
LL 6	1000	1000	652	631	792	792	800	799
LL 7		284	286	361	424	424	457	456
LL 8								
VALUE OF PLAN	1213	1287	1430	1456	1473	1473	1478	1478

A P P E N D I X X X

LEASE ANALYSIS IN THE SINGLE CRITERION MODEL

EXHIBIT A20.1 THE LP PRIMAL SOLUTION (INVESTMENT PROJECTS ONLY)

PROBLEM OPTIMODEL-87 SOLUTION DATE 03/03/76 TIME

DUMP1DUMP 96

RIGHT HAND SIDE RHS1
OBJECTIVE OBJ +0.4039+0816
LOWER BOUND LOBND
UPPER BOUND UPBND

COLUMN INFORMATION

	NAME		VALUE	LOWER BOUND	UPPER BOUND	OBJECTIVE	REDUCED COST
U	PR01Y1	*	1.0000	0	1.0000	-44.4290	33.9999
U	PR04Y1	*	1.0000	0	1.0000	-21.8106	52.8975
	L04Y1	*	0	0		0	-29.1606
U	PR12Y1	*	1.0000	0	1.0000	-33.1198	15.7959
U	PR13Y1	*	1.0000	0	1.0000	0	64.7660
	PR16Y1	*	0	0	1.0000	-31.2215	-19.7017
U	PR22Y1	*	1.0000	0	1.0000	-16.5371	3.6199
	PR23Y1	*	0	0	1.0000	-17.6908	-16.0806
U	PR03Y2	*	1.0000	0	1.0000	18.3774	27.7811
B	L03Y2	*	1.0000	0		-9.9359	0
U	PR04Y2	*	1.0000	0	1.0000	-2.1407	33.5713
B	L04Y2	*	0.5329	0		4.4429	0
	PR05Y2	*	0	0	1.0000	15.9560	-29.7663
U	PR13Y2	*	1.0000	0	1.0000	1.2117	36.9172
U	PR14Y2	*	1.0000	0	1.0000	-47.2553	29.7574
	PR21Y2	*	0	0	1.0000	23.9515	-60.7666
	PR24Y2	*	0	0	1.0000	-7.2702	-15.5042
U	PR02Y3	*	1.0000	0	1.0000	32.9986	6.4389
B	PR11Y3	*	0.5243	0	1.0000	46.7716	0
B	L11Y3	*	0.5243	0		2.5850	0
	PR15Y3	*	0	0	1.0000	63.4596	-0.0436
U	PR01Y4	*	1.0000	0	1.0000	126.0160	22.3515
	PR05Y4	*	0	0	1.0000	117.9358	-16.4773
	PR11Y4	*	0	0	1.0000	83.2034	0.0919
U	PR12Y4	*	1.0000	0	1.0000	144.1925	15.4876
U	PR13Y4	*	1.0000	0	1.0000	62.8134	55.6260
U	PR14Y4	*	1.0000	0	1.0000	-17.0850	8.1538
U	PR22Y4	*	1.0000	0	1.0000	33.7660	6.6655
U	PR25Y4	*	1.0000	0	1.0000	51.6992	14.4262
	PR02Y5	*	0	0	1.0000	66.6435	-1.7749
U	PR03Y5	*	1.0000	0	1.0000	151.0586	5.8018
	PR11Y5	*	0	0	1.0000	101.7828	-1.5137
	PR21Y5	*	0	0	1.0000	159.0556	-41.2916
	PR23Y5	*	0	0	1.0000	71.8942	-15.1361
U	PR04Y6	*	1.0000	0	1.0000	112.6881	39.1241
	PR05Y6	*	0	0	1.0000	199.9305	-2.6612
U	PR11Y6	*	1.0000	0	1.0000	131.2675	23.7982
U	PR14Y6	*	1.0000	0	1.0000	69.2756	8.7846
U	PR15Y6	*	1.0000	0	1.0000	95.3796	10.6422
U	PR16Y6	*	1.0000	0	1.0000	189.9472	5.5012
	PR21Y6	*	0	0	1.0000	191.0447	-20.6008
B	PR23Y6	*	0.7196	0	1.0000	85.2229	0
U	PR01Y7	*	1.0000	0	1.0000	96.5516	13.1655
U	PR04Y7	*	1.0000	0	1.0000	127.2285	31.3665
U	PR14Y7	*	1.0000	0	1.0000	89.6658	3.2261
	PR22Y7	*	0	0	1.0000	44.8329	-8.8970
	PR02Y8	*	0	0	1.0000	74.3176	-4.8664
	PR15Y8	*	0	0	1.0000	65.4318	-2.0391
	PR22Y8	*	0	0	1.0000	44.4290	-4.4621
	PR25Y8	*	0	0	1.0000	147.8274	-1.6760
	OBJECTIVE		2002.0548				

EXHIBIT A20.2 THE LP DUAL SOLUTION

PROBLEM OPTIMODEL-A7 SOLUTION DATE 03/05/76 TIME

DUMPINDEP 96

RIGHT HAND SIDE RHS1
OBJECTIVE OBJ +0.4039*OBJ6
LOWER BOUND LOBND
UPPER BOUND UPBND

ROW INFORMATION

	NAME		SLACK	R.H.S.	PRICE
B	L1	+	1.0000	0	0
	L2	+	0	0	-1.6090
B	L3	+	0.6671	0	0
	L4	+	0	0	-4.2104
#	OBJ	Z	1320.3096	0	
#	OBJ6	Z	1687.9058	0	
	TS1		0	-11000.0000	0
	TS2		0	-10000.0000	0
	TS3		0	-9500.0000	0
	TS4		0	-8400.0000	0
	TS5		0	-8000.0000	0
	TS6		0	-7500.0000	0
	TS7		0	-7500.0000	0
	TS8		0	-7000.0000	0
	EA1		0	-1953.0000	0.5673
	EA2		0	-1850.0000	0.5356
	EA3		0	-1600.0000	0.5095
	EA4		0	-1560.0000	0.3216
	EA5		0	-1240.0000	0.4327
	EA6		0	-1220.0000	0.2402
	EA7		0	-1200.0000	0.2149
	EA8		0	-1000.0000	0.4039
	BL1		0	-2034.0000	-0.6683
	BL2		0	-600.0000	-0.6063
	BL3		0	-500.0000	-0.5384
	BL4		0	-200.0000	-0.4503
	BL5		0	0	-0.4411
	BL6		0	0	-0.3455
	BL7		0	0	-0.3266
	BL8		0	0	-0.4043
	PE1		0	-1481.0000	-0.5551
	PE2		0	-500.0000	-0.5057
	PE3		0	-400.0000	-0.4476
	PE4		0	-400.0000	-0.3596
	PE5		0	-400.0000	-0.3459
	PE6		0	-400.0000	-0.2289
	PE7		0	-400.0000	-0.2149
	PE8		0	-400.0000	-0.4039
	CA1		0	-3510.0000	-0.0950
	CA2		0	-3538.0000	-0.0992
	CA3		0	-3715.0000	-0.1020
	CA4		0	-3432.0000	-0.0305
	CA5		0	-3120.0000	-0.0856
	CA6		0	-2925.0000	-0.0150
	CA7		0	-2925.0000	-0.0129
	CA8		0	-2730.0000	-0.4039
	CL1		0	-1120.0000	0.0950
	CL2		0	-1145.0000	0.0992
	CL3		0	-1176.0000	0.1020
	CL4		0	-1095.0000	0.0249
	CL5		0	-1020.0000	0.0576
	CL6		0	-945.0000	0.0117
	CL7		0	-948.0000	0.0127
	CL8		0	-902.0000	0.4039
	CR1		0	-6543.0000	0.8925
	CR2		0	0	0.7975
	CR3		0	0	0.6985
	CR4		0	0	0.5963
	CR5		0	1000.0000	0.5662
	CR6		0	0	0.4485
	CR7		0	0	0.4105
	CR8		0	0	0.4039
	TP1		0	792.0000	0.7975
	TP2		0	-65.0000	0.6985
	TP3		0	-65.0000	0.5965
	TP4		0	-60.0000	0.5674
	TP5		0	-60.0000	0.5066
	TP6		0	-60.0000	0.4465
	TP7		0	-60.0000	0.4039
	TP8		0	-55.0000	0

	TA1		0	-65.0000	-2.0083
	TA2		0	0	-1.6096
	TA3		0	0	-1.2605
	TA4		0	0	-0.9425
	TA5		0	0	-0.6766
	TA6		0	0	-0.4355
	TA7		0	0	-0.2019
	TA8		0	0	0
	PR1		0	-60.0000	0.7975
	PR2		0	0	0.6892
	PR3		0	0	0.5701
	PR4		0	0	0.5676
	PR5		0	0	0.5066
	PR6		0	0	0.4431
	PR7		0	0	0.4039
	PR8		0	0	0
	EQ1		0	2000.0000	0.000039
	EQ2		0	0	0
	EQ3		0	0	0
	EQ4		0	0	0
	EQ5		0	0	0
	EQ6		0	0	0
	EQ7		0	0	0
	EQ8		0	0	0
	D1		0	1500.0000	0.8925
	D2		0	0	0.7975
	D3		0	0	0.6943
	D4		0	0	0.5963
	D5		0	-1000.0000	0.5642
	D6		0	0	0.4465
	D7		0	0	0.4207
	D8		0	0	0.4039
B	ROCF1	+	460.8764	0	0
B	ROCF2	+	78.0144	0	0
B	ROCF3	+	333.1711	0	0
B	ROCF4	+	810.8346	0	0
B	ROCF5	+	509.6713	0	0
B	ROCF6	+	775.1707	0	0
B	ROCF7	+	839.9006	0	0
B	ROCF8	+	977.5130	0	0
B	LQDY1	+	822.5000	0	0
B	LQDY2	+	797.0369	0	0
B	LQDY3	+	145.4723	0	0
	LQDY4	+	0	0	-0.0018
	LQDY5	+	0	0	-0.0323
	LQDY6	+	0	0	-0.0166
B	LQDY7	+	681.0427	0	0
B	LQDY8	+	545.3257	0	0
	ECOV1	+	0	0	-0.0735
	ECOV2	+	0	0	-0.0827
	ECOV3	+	0	0	-0.0963
	ECOV4	+	0	0	-0.0090
	ECOV5	+	0	0	-0.1219
	ECOV6	+	0	0	-0.0133
B	ECOV7	+	336.3648	0	0
B	ECOV8	+	650.1074	0	0
B	ERPS1	+	314.6677	0	0
B	ERPS2	+	152.5356	0	0
B	ERPS3	+	360.0901	0	0
B	ERPS4	+	754.3000	0	0
B	ERPS5	+	573.8342	0	0
B	ERPS6	+	872.7065	0	0
B	ERPS7	+	955.3017	0	0
B	ERPS8	+	1076.1762	0	0
	DTARG1	-	0	0	0.0030
B	DTARG2	-	96.9645	0	0
B	DTARG3	-	240.0601	0	0
B	DTARG4	-	359.1600	0	0
B	DTARG5	-	312.2455	0	0
B	DTARG6	-	558.1455	0	0
	DTARG7	-	0	0	0
B	DTARG8	-	703.2251	0	0
B	DCOV1	+	324.5644	0	0
	DCOV2	+	0	0	-0.0090
	DCOV3	+	0	0	-0.0262
B	DCOV4	+	194.3615	0	0
B	DCOV5	+	91.2483	0	0
	DCOV6	+	0	0	-0.0037
B	DCOV7	+	955.3017	0	0
	DCOV8	+	0	0	0