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A Linear Threshold Model for Optimal Stopping Behavior

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In many real life decisions, options are distributed in space and time, making it necessary to search sequentially through them, often without a chance to return to a rejected option. The optimal strategy in these tasks is to choose the first option that is above a threshold that depends on the current position in the sequence. The implicit decision making strategies by humans vary but largely diverge from this optimal strategy. The reasons for this divergence remain unknown. We present a new model of human stopping decisions in sequential decision making tasks based on a linear threshold heuristic. The first two studies demonstrate that the linear threshold model accounts better for sequential decision making than existing models. Moreover, we show that the model accurately predicts participants' search behavior in different environments. In the third study, we confirm that the model generalizes to a real-world problem, thus providing an important step towards understanding human sequential decision making.

optimal stopping | cognitive modeling | sequential decision making | adaptive behavior

Decisions that arise in everyday life often have to be made when options are presented sequentially. For example, searching for a parking spot, deciding when to take a vacation day, or finding a partner, all require that the decision maker accepts or rejects an option without knowing if future options will be more attractive. Decisions in such problems involve a trade-off between accepting a possibly suboptimal option prematurely and rejecting the current offer out of false hopes for better options in the future.

Despite the importance of such decisions, relatively little work has been made toward characterizing the process by which humans decide to stop searching in natural settings of this task.

Earlier research has focused on a simplified version of optimal stopping problems, the so-called secretary problem, where only the rank of the option relative to those already seen is shown (1–3) and only the overall best alternative is rewarded. In the secretary problem, the optimal strategy is to ascertain the maximum of the first 37% options and choose the next option that exceeds this threshold (4). Empirical studies suggest that in general people follow a similar strategy but usually set the cut-off (i.e., from which point on they will accept an option that exceeds the previous options) earlier than the 37% prescribed by the optimal solution (1, 5).

Some studies have investigated tasks closer to real sequential choice problems by presenting the actual value of the option to the decision makers (6–10). In this version, the optimal is based on calculating the probability of winning on the later positions. From this probability, a threshold is calculated for each option in the sequence as described by Gilbert and Mosteller (4, Section 3). Lee (6) estimated a family of

threshold-based models and showed that most participants decreased their choice thresholds as sequences progress. Although people are overall quite heterogeneous in their search behavior, they tend to cluster around the optimal solution (7, 8). Importantly, these studies still kept the restriction that only the best alternative is rewarded—a payoff function that does not correspond well with everyday experiences. Humans do find a mate, an apartment to live, or a ticket to fly to their vacation destination, and thus receive some payoff, even if that may not be the highest possible payoff.

In the present research, we propose a model of human decision making in optimal stopping problems using payoffs that are based on the actual values. In this variant of the search problem, the optimal decision thresholds are calculated based on the expected reward of the remaining options ((4, Section 5b) and SI Appendix, Text A). This leads to a decision threshold that changes notably nonlinear over the sequence.

In contrast, we propose that people rely on a mental shortcut and adapt their thresholds linearly over the sequence. We show that a model with this linearity assumption accurately captures when people stop search and accept an option, even in a real-world setting. Furthermore, this model allows us to predict under which conditions people search more or less than the optimal model, making it a useful tool to understand human sequential decision making.

We first sketch a family of cognitive models for describing behavior in optimal stopping problems. We then present results from three behavioral experiments that provide evidence for the validity of the linear model in a laboratory setting as well as in a real-world scenario.

Significance Statement

Behavioral research has made rapid progress toward revealing the processes by which we make choices between options that are presented simultaneously. Decisions in everyday life are typically more complex. We often encounter choices where options are separated in space and time and therefore the question is: “When is the right time to stop searching?” We suggest that humans use a probabilistic threshold. A model in which this threshold changes linearly over time, where the optimal policy prescribes a non-linear change, provides an excellent account to the data, even in real-life settings.

C.B., B.v.H., H.S. and S.G. designed research; C.B. and B.v.H. performed research; C.B., B.v.H. and H.S. contributed analytic tools; C.B. analyzed data; and C.B., B.v.H., H.S. and S.G. wrote the paper.

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Computational models. We explain the computational models based on a typical optimal stopping problem that we also used in our first two experiments. The decision maker (here a customer) is planning a vacation and decides to buy the plane ticket online. Ticket prices vary randomly from day to day and the customer wants to find the cheapest ticket. The customer checks the ticket price every day and decides if she wants to accept or reject the ticket, without having the option to go back in time to a previously rejected offer. Search time is limited by her vacation schedule (i.e., 10 decisions per trial) and, once accepted, the search ends.

More formally, we consider a decision maker who encounters a sequence of tickets with values denoted by x_1, \dots, x_{10} and she wants to find the minimum value in the sequence. If the decision maker accepts ticket x_i , the sequence terminates and she has to pay x_i ; otherwise, she continues to the next ticket. When the last ticket is reached, it must be accepted.

All models assume that the decision maker relies on a probabilistic threshold to make the decision to accept or reject a ticket—i.e., ticket x_i on position i is compared to a position dependent threshold t_i . This comparison yields an acceptance probability θ_i based on a sigmoid choice function with sensitivity parameter β and

$$\theta_i = \frac{1}{1 + \exp\{\beta(x_i - t_i)\}}. \quad [1]$$

Small values of β produce more stochasticity in decisions, whereas the policy approaches determinism when $\beta \rightarrow \infty$.

We examine the setting of thresholds by comparing the performance of four different models.

- The *Independent Threshold Model (ITM)* serves as our baseline. It assumes no dependency between the thresholds. It entails N independent threshold parameters t_1, \dots, t_N , one for each position in the sequence, where the decision maker can decide to accept or reject an offer (at position $N + 1$ the ticket must be accepted). The thresholds can take any value across positions. The model maintains maximal flexibility and provides an upper limit how well any threshold model can describe a person's decision given the assumption of a probabilistic threshold.
- The *Linear Threshold Model (LTM)* postulates that the thresholds are constrained by a linear relation to each other and therefore are completely defined by the first threshold t_0 and the linear increase δ as the sequence progresses:

$$t_{i+1} = t_i + \delta \cdot i, \quad [2]$$

This model entails three free parameters, the first threshold t_0 , the increase of the threshold δ and the choice sensitivity β .

- The *The Biased Optimal Model (BOM)* is based on the Bias-from-Optimal threshold model proposed by Guan et al. (8), assuming that humans are using thresholds that deviate systematically from the optimal thresholds. The optimal thresholds t_i^* for each position i are derived by determining the expected reward of the remaining options (derivation in (4, Section 5b) and in SI Appendix, Text A). The model entails a systematic bias parameter γ that reflects the divergence of the human threshold from the optimal one. Additionally, the thresholds depend

on a parameter α that determines how much their bias increases or decreases as the sequence progresses.

$$t_i = t_i^* + \gamma + \alpha \cdot i, \quad [3]$$

When γ and α are set to 0, the thresholds represent the optimal thresholds that lead to best performance. This model is therefore defined by three free parameters, γ , α and the choice sensitivity β .

- The *Cut-off Model (CoM)* is inspired by the optimal decision rule for the rank information version of the secretary problem where the distribution of the prices is unknown. It assumes that the DM has a fixed cut-off value k that determines how long she explores in the beginning of the sequence. The highest value seen in that initial sample of k tickets is then set as her threshold, and the first value that exceeds this threshold in the remainder of the sequence is chosen. This model has two free parameters, the cut-off value k and the sensitivity parameter β .

Models were implemented in a *hierarchical-Bayesian statistical framework* using JAGS (11) (SI Appendix, Text B).

Experiment 1. We asked 129 participants to solve a computer-based optimal stopping problem following the ticket-shopping task described above. Tickets were normally distributed with a mean value of \$180 and a standard deviation of \$20. In the first phase, subjects learned the distribution using a graphical method proposed by (12) (*Methods*). SI Appendix, Fig. S1A shows that this procedure was successful in ensuring participants learned the distribution.

In the second phase, participants performed 200 trials of the ticket-shopping task. In each trial, participants searched through a sequence of ten ticket prices. For each ticket, they could decide to accept or reject it at their own pace. Participants were aware that they could see up to 10 tickets in each trial, and they were always informed about the actual position and the number of remaining tickets (SI Appendix, Fig. S2E for a screen shot). It was not possible to go back to an earlier option after it was initially declined. If they reached the last ticket (10th) they were forced to choose this ticket. When participants accepted the ticket, they received feedback about how much they could have saved if they had chosen the best ticket in the sequence. Performance was incentivized based on the value of the chosen ticket (*Methods*).

Behavioral results. Subjects earned on average 17.1 points (SD: 4.2) in each trial (maximum points = 20), which represents a 6% loss on optimal earnings. Participants' marginal accept probabilities steadily increased as the sequence progressed (Fig. 1A, black line), but differed systematically from the optimal agent's accept probability (Fig. 1A, yellow line). On the second-to-last (9th) position, participants accepted the ticket only with a 28%, 95%-CI [26%, 29%], probability, whereas following the optimal policy would result in a significantly higher acceptance rate of 50%.

Overall, subjects stopped earlier than optimal. The average position at which a ticket was accepted was 4.7 (SD: 2.9), whereas an optimal agent would have stopped at an average stopping position of 5.2 (SD: 2.8). However, a closer look at Fig. 1A reveals that whether subjects accept too early or too late depends on the position: on earlier positions they accept options although they should continue to search, whereas, if

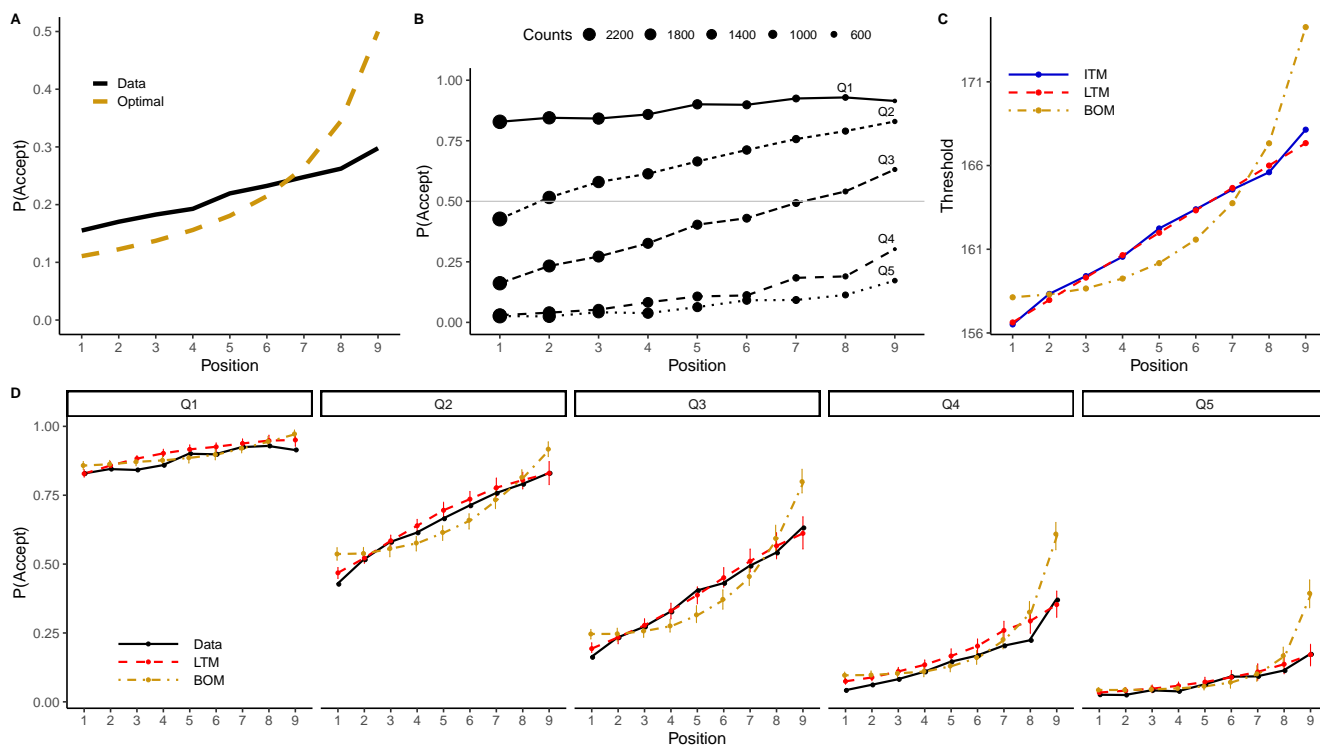


Fig. 1. (A) Probability to accept a ticket on each position across all prices. The dark line represents participant's frequency to accept, the dashed yellow line an optimal agent's probability to accept. (B) Participants' probability to accept. Each line represents ticket prices ranging from the first quantile to the fifth quantile. Q1: Tickets in first quantile, Q2: Tickets ranging from the first to the second quantile etc. The size of circles correspond to the number of data points on each position. (C) Estimated thresholds for the ITM with 9 free threshold parameters (solid blue line), the LTM with 2 free threshold parameters (dashed red line) and the BOM with 2 free threshold parameters (dash-dotted yellow line) (D) Posterior predictive mean and 95% HDI of the LTM (dashed red line) and the BOM (dash-dotted yellow line) for Q1 to Q5, as indicated in (B). Data: solid black lines

151 they get to position 7, they continue searching even for options
 152 that should be accepted according to the optimal policy.

153 Fig. 1B shows the accept probabilities conditional on ticket
 154 prices, split into the first five quantile ranges $Q_1 - Q_5$ (out
 155 of a total of ten quantile ranges). Q_i is defined as the range
 156 of ticket prices from the $0.i$ th to the $(0.i - 0.1)$ th quantile of
 157 the ticket price distribution. In this experiment, the ticket
 158 distribution corresponds to a Gaussian distribution with mean
 159 180 and standard deviation of 20. Accept probabilities for
 160 Q_4 and Q_5 did not reach 50% at position 9, in contrast to
 161 the optimal strategy that predicts much higher acceptance
 162 probabilities at this position.

163 Our models did not assume any learning over trials. This
 164 assumption was supported by an analysis of performance across
 165 trials. A linear mixed model on points per trial with trial
 166 number as fixed effect and by-participant random intercepts
 167 and random slopes for trial number showed no significant effect
 168 of trial number, $F(1, 64.00) = 0.02$, $p = 0.88$.

169 **Modeling results and discussion.** First, we checked whether
 170 the key assumptions of the modeling framework were supported.
 171 We calculated, per participant and model, posterior
 172 predictive p -values (p_{pp}) that compared misfit (i.e., deviance)
 173 of the observed data with misfit of synthetic generated data
 174 from the model. For the baseline model, ITM, this analysis
 175 indicated that the absolute fit was very good, and a probabilistic
 176 threshold adequately describes participants' responses;
 177 $p_{pp} < .05$ for only 8% of participants (SI Appendix, Fig. S3A).
 178 For the vast majority of participants the observed misfit was

consistent with the assumptions of the ITM plus sampling
 179 variability.

180
 181 The performance of the LTM was almost identical to the
 182 ITM, suggesting that the considerably more parsimonious
 183 LTM (three free parameters for LTM compared to ten for
 184 ITM) adequately describes behaviour in optimal stopping
 185 tasks. The distribution of p_{pp} -values of the LTM was almost
 186 identical to the ITM (SI Appendix, Fig. S3A-B). Fig. 1D
 187 provides qualitative evidence of the agreement between LTM
 188 and data; the LTM adequately predicts accept probabilities
 189 for each quantile at every position (see SI Appendix, Fig. S4
 190 for agreement between ITM and data). Fig. 1C compares the
 191 recovered thresholds of ITM and LTM and shows that the
 192 ITM thresholds essentially form a straight line lying exactly
 193 on top of the LTM thresholds.

194 The absolute fit of the BOM is clearly worse than for
 195 ITM/LTM; $p_{pp} < .05$ for 35% of participants (SI Appendix,
 196 Fig. S3C). The source for this increased misfit can be seen
 197 in Fig. 1D. Only for Q_1 and early positions of Q_4 and Q_5
 198 did the BOM provide an adequate account. Furthermore, the
 199 recovered thresholds (Fig. 1C) of the BOM clearly differ from
 200 the ITM in almost all positions. Results of the CoM are not
 201 shown explicitly as its performance was extremely poor. All
 202 $p_{pp} = 0$; there was not a single posterior sample for which the
 203 observed misfit of the CoM was smaller than for synthetic data
 204 generated from the CoM. Furthermore, choices were essentially
 205 random for CoM with $\beta_{CoM} = 0.02$ [0.01, 0.06] (for the other
 206 models, $\beta \approx 0.21$).

207 Participants differed in their first threshold and slope pa-

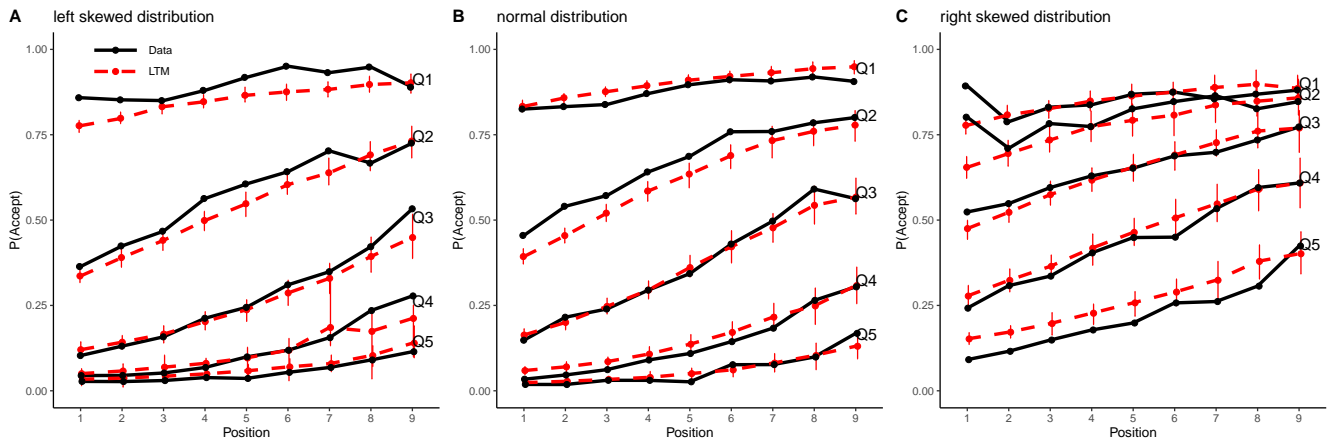


Fig. 2. Results of experiment 2: Empirical data appear in black lines and the posterior predictive means of the LTM in red lines. Bars represent the 95% HDI. The different lines represent the tickets ranging in from the Q1 to Q5. Q1: Tickets in first quantile, Q2: Tickets between the first and second quantile etc. (A) Condition 1: Tickets are left skewed distributed (PERT(40,195,200)) corresponding to a scarce environment. (B) Condition 2: Tickets are normally distributed (PERT(90,140,190)). (C) Condition 3: Tickets are right skewed distributed (PERT(120,125,400)) corresponding to a plentiful environment.

rameters estimated by the LTM. However, all slope parameters are larger than 0 indicating that all participants increased the thresholds over the sequence (see also SI Appendix, Text C).

These results suggest that humans use a linear threshold when searching for the best option. In the present tests we found that the human performance is only 6% off from the performance of an optimal agent, indicating that the linear strategy performs quite well. Therefore, using linear thresholds could be an ecologically sensible adaptation to sequential choice tasks. However, it could also mean that the LTMs good performance might not generalize to new task environments, in which the linear model performs less well – an ability that would be crucial for the LTM to be a useful model of human behavior.

Search behavior in Experiment 1 indicated that people deviate from the optimal model depending on the price structure of the sequence: In trials with good options in the beginning people tended to accept them too early. However, in trials with few or no good options they continued search longer than the optimal model prescribed (SI Appendix, Fig. S5). Accordingly, in tasks with plenty of good options people might search less than optimal. However, in tasks in which good options are rare they might be tempted to search too long.

To find out and further predict how people will adapt to the tasks, we conducted a simulation study comparing the optimal solution with a best performing linear model (using a grid search to find the best performing parameter values for the linear model) and an empirical study manipulating the distributions of ticket prices across three conditions: (1) a left skewed distribution simulating a scarce environment, (2) a normal distribution, (3) a right skewed distribution simulating an environment with plentiful desirable alternatives. As illustrated in SI Appendix, Fig. S6B, the simulation study showed that the optimal model predicts more search in a plentiful environment, whereas a linear model predicts more search in the scarce environment. Furthermore, the linear model predicts a stronger decline in performance in the scarce environment than the optimal model (SI Appendix, Fig. S6A).

Experiment 2. To show that the LTM can capture people's choice behavior across different tasks and allows us to predict

when people will search too much or too little we conducted a second experiment changing the distribution of options. We manipulated the different task environments by sampling tickets from (1) a left skewed (PERT*(40,195,200)), (2) a normal (PERT(90,140,190)) or (3) a right skewed distribution (PERT(120,125,400)), representing a scarce, a normal and a plentiful environment, respectively (SI Appendix, Fig. S1B-D, red lines). Each participant was assigned to only one condition. The final sample included 172 participants. The procedure was identical to Experiment 1, consisting of a learning phase, where participants got acquainted with the distribution (SI Appendix, Fig. S1B-D, participant's estimate in black lines), and a testing phase. In the testing phase, participants had to choose the lowest-priced ticket out of a sequence of 10 tickets with 200 trials (*Methods*).

Behavioral results. Participants' performance increased from the left-skewed (scarce) environment to the right-skewed (plentiful) environment ($F(2, 268) = 114, p < .0001$). As predicted by the best performing linear model, the loss compared to optimal performance was largest in the left-skewed condition, where only few good tickets occur (SI Appendix, Fig. S6A).

The average search length decreased from the left skewed scarce environment to the right skewed plentiful environment, $F(2, 268) = 11.5, p < .0001$. This pattern also follows the predictions of the best performing linear model in the simulation study but is in contrast to the optimal model's predictions (SI Appendix, Fig. S6B). Specifically, in the left skewed environment, where good tickets occur very rarely participants searched too long compared to an optimal agent, whereas in the environment where good tickets are abundant, participants ended their search too early compared to the optimal strategy.

Modeling Results and Discussion. Modeling results replicate the results from Experiment 1 and indicate that the LTM but not the BOM performed extremely well ($p_{pp} < .05$ for 7% to 10% of participants across the three conditions for LTM,

*The PERT distribution is a special case of the beta distribution defined by the minimum (a), most likely (b) and maximum (c) values that a variable can take and an additional assumption that its expected value is $\mu = \frac{a + 4b + c}{6}$.

283 but $p_{pp} < .05$ for 20% to 55% of participants for BOM, SI
 284 Appendix, Fig. S7). The observed accept probabilities (Fig. 2A-
 285 C, black lines, where each line represents a ticket price within
 286 the specified quantile range) are adequately described by LTM
 287 predictions (red lines) on almost all positions and in all three
 288 environments. Moreover, the threshold parameters for the
 289 ITM are again on top of the threshold parameters estimated
 290 by the LTM in all the three environmental conditions (SI
 291 Appendix, Fig. S8A-C).

292 These results indicate that humans use a linear threshold in
 293 optimal stopping problems, independent of the distributional
 294 characters of the task. However, this does not mean that people
 295 do not adapt to the task at all. Participants are responsive
 296 to task features and adapt their first threshold and the slope
 297 to the distributional characteristics of the task within the
 298 constraints of the linear model (SI Appendix, Fig. S8A-C).

299 Experiment 1 and 2 show that the linear model reflects a ro-
 300 bust psychological process when deciding between sequentially
 301 presented options. However, in both experiments deciders were
 302 explicitly trained on the distribution of options, something
 303 not common in real life decision making. The next experiment
 304 tests if the linear strategy can also explain choices in a realistic
 305 optimal stopping task where initial learning is omitted.

306 **Experiment 3.** The decision maker's goal is to buy online prod-
 307 ucts at the lowest rate where prices for this product are pre-
 308 sented sequentially. We selected commodity products from
 309 different categories (e.g food, leisure, kitchen tools) and col-
 310 lected for each product a set of prices from Amazon.com. Only
 311 products with approximately normal price distributions were
 312 selected for a final set of 60 products (SI Appendix, Table
 313 S1). In the experiment, prices were sampled from a normal
 314 distribution, with a mean and standard deviation estimated
 315 from the real prices. All participants worked on 120 trials,
 316 divided into two blocks of 60 trials. In these two blocks, the
 317 60 products were displayed in a random order (each product
 318 was encountered twice). Participants were aware that they
 319 could see up to 10 prices in each trial, and we indicated the
 320 average price of each product on the screen to reflect that
 321 people often have an idea of familiar products' prizes and to

minimize individual differences in these.

322 **Behavioral Results.** Data from 95 participants were analyzed
 323 and replicated the results from Experiments 1 and 2 (nor-
 324 mal distribution condition). Again, participants accepted too
 325 early, on average at position 4.6 (SD: 2.9). Comparing the
 326 performance in detail to the optimal strategy showed that (SI
 327 Appendix, Fig. S9) participants accepted too frequently at
 328 the beginning of the sequence (i.e., too low threshold) and
 329 searched too long towards the end of the sequence (i.e., too
 330 high threshold). We again found no evidence for learning
 331 across trials (linear mixed model on points per trial with trial
 332 number as fixed effect and by-participant random intercepts
 333 and random slopes for trial number showed no significant effect
 334 of trial number $F(1, 94) = 0.13, p = 0.72$).
 335

336 **Modeling Results.** To deal with the prices' variability we nor-
 337 malized all values using mean and SD prior to fitting our
 338 models. We could replicate the results from Experiment 1 and
 339 2, despite the fact that participants did not explicitly learn the
 340 product's prices beforehand: The LTM (10% of $p_{pp} < .05$, SI
 341 Appendix, Fig. S10A), but not the BOM (31% of $p_{pp} < .05$, SI
 342 Appendix, Fig. S10C), was able to capture the observed accept
 343 probabilities accurately on each position and for each quantile
 344 (Fig. 3B&C). Furthermore, threshold parameters estimated by
 345 the LTM were very similar to threshold parameters estimated
 346 by the ITM (SI Appendix, Fig. S11).

347 **Discussion.** In this paper, we designed a variant of an optimal
 348 stopping task that allowed us to quantitatively characterize
 349 the deviations of human behaviour from optimality. We found
 350 that humans apply a simplifying strategy, where thresholds are
 351 linearly increased over time. We implemented this assumption
 352 in a computational framework and demonstrated that this
 353 model not only provided an excellent fit to the data, it also
 354 outperformed other models found in the optimal stopping liter-
 355 ature. Furthermore, the linear threshold assumption makes a
 356 non-trivial prediction about search length, which we confirmed
 357 experimentally: Humans stop earlier in environments with
 358 many desirable alternatives compared to scarce environments.

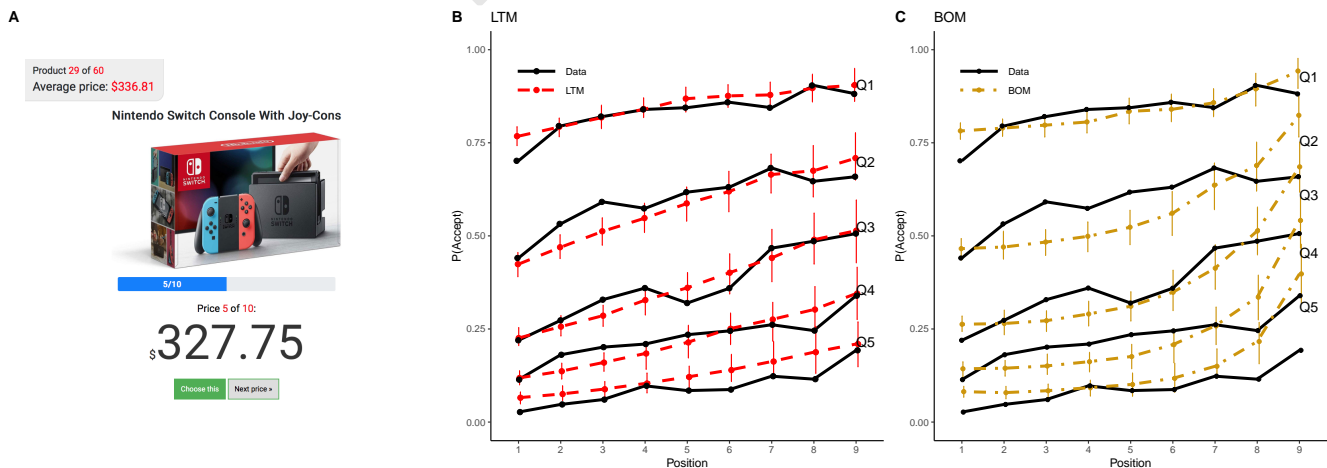


Fig. 3. (A) Screenshot of the product purchasing task. (B and C) Results of experiment 3: (B) Empirical data appear in solid black lines and the posterior predictive means of the LTM in dashed red lines. (C) Empirical data appear in solid black lines and the posterior predictive means of the BOM in dashed yellow lines. Bars represent the 95% HDI. The different lines represent the product prices ranging from the first quantile to the fifth quantile. Q1: Product prices in first quantile, Q2: Product prices between the first and second quantile, Q3: Product prices ranging from second to third quantile, etc.

359 These results contrast with the prediction from the optimal
360 model. Finally, in a online product purchase paradigm we
361 could show that our model generalizes to real-world sequential
362 choice problems. Understanding how humans make sequential
363 decisions will help quantify the conditions under which people
364 may succeed or fail in such tasks.

365 But why are humans relying on a linear strategy in adapt-
366 ing their thresholds when an optimal policy is nonlinear? For
367 one, our findings correspond well with recent studies demon-
368 strating that human choice behavior in related explore-exploit
369 paradigms is well described by a linear threshold rule (13, 14).
370 But a human linearity bias seems to be more general. Indeed,
371 a tendency to assume linear relationships has been reported
372 in a range of domains such as function learning (15, 16) and
373 reasoning (17–19). Crucially, simple strategies do not neces-
374 sarily perform badly. In particular in uncertain and complex
375 environments, simple heuristics can be efficient and powerful
376 tools if they are adapted to the structure of the environment
377 (20, 21). In this context, linearity could be considered as an
378 adaptation of the human mind to its environment.

379 Materials and Methods

381 **Participants.** We recruited 438 participants (272 females; age range:
382 18–62; $N_1 = 144$, $N_{2\text{left}} = 92$, $N_{2\text{normal}} = 110$, $N_{2\text{right}} = 92$,
383 $N_3 = 100$ in Experiments 1, 2 and 3, respectively) on Amazon
384 Mechanical Turk to participate in the experiments. Participants
385 gave informed consent, and the Harvard Committee on the Use
386 of Human Subjects approved the experiments. Participants were
387 excluded from analysis if they accepted the first option in a trial
388 in more than 95% of the trials. After applying these criteria, we
389 included data from 499 participants in the subsequent analysis
390 ($N_1 = 129$, $N_{2\text{left}} = 86$, $N_{2\text{normal}} = 102$, $N_{2\text{right}} = 84$, $N_3 = 95$).

391 **Task.** In Exp. 1 and 2, participants performed the same online ticket
392 shopping task that consisted of a learning and a testing phase. In the
393 learning phase, participants experienced the distribution from which
394 the ticket prices were drawn. In Exp. 1, the distribution from which
395 the values were sampled was normal with $\mathcal{N}(\mu = 180, \sigma = 20)$. The
396 procedure was as follows (SI Appendix, Fig. S2A–D): Participants
397 encountered sequentially 50 ticket prices drawn from the predefined
398 distribution. After every ten tickets, participants had to guess the
399 average value of the tickets seen so far. After each guess, participants
400 were told the correct response. At the end of the learning phase
401 participants were asked to complete a histogram (by dragging the
402 bars) for an additional 100 tickets that were drawn from the same
403 predefined distribution. Participants received feedback by observing
404 the correct distribution superimposed over their estimate (12).

405 In Exp. 2, we used three conditions to realize three dif-
406 ferent distributional environments, a left skewed distribution,
407 PERT(40,195,200), a normal distribution, PERT(90,140,190), and
408 a right skewed distribution, PERT(120,125,400). The procedure of
409 the learning phase was identical to Exp. 1, except that we removed
410 the section about reporting the mean for the skewed distributions
411 (SI Appendix, Fig. S2B). Visual inspection of the performance in
412 the histogram task suggested that participants learned the target
413 distributions well (SI Appendix, Fig. S1).

414 In the second phase of Exp. 1 and 2, participants performed the
415 ticket-shopping task. It started with a practice trial followed by 200
416 test trials. In each trial participants searched through a sequence of
417 10 ticket prices randomly drawn from the predefined distribution.
418 For each ticket, they could decide to accept or reject it at their own
419 speed. People were aware that they could see up to 10 tickets in
420 each trial and they were always informed about the actual position
421 and the number of remaining tickets (SI Appendix, Fig. S2E). It
422 was not possible to go back to an earlier option after it was initially
423 declined. If they reached the last (10th) ticket they were forced
424 to accept this ticket. When participants accepted the ticket, they
425 received explicit feedback about how much they could have saved

by choosing the lowest-priced ticket in the sequence (SI Appendix,
Fig. S2F).

Participants were paid according to their performance. In each
of the 200 trials there was a maximum of 20 points to earn. The
participants received the maximum number of 20 points if they
chose the lowest-priced ticket and 0 points for the worst ticket in
the sequence. The payoff for a ticket that lied between the lowest-
priced and the highest-priced was calculated proportional to the
distance to the lowest-priced ticket in the sequence. The exact
calculation for the points in each trial i was as follows:

$$points_i = \frac{20 \cdot (ticket_{max} - ticket_{chosen})}{ticket_{max} - ticket_{min}}, \quad [4]$$

where $ticket_{max}$ represents the most expensive ticket in the sequence
and $ticket_{min}$ the cheapest ticket in the sequence. Participants
received a base payment of \$4 and earned between \$0 and \$4
additionally depending on their performance.

In Exp. 3, participants performed an online product shopping
task that started with a practice trial followed by 120 test trials
divided into two blocks containing the same sixty products. In each
trial, they encountered a product and searched through a sequence of
ten prices. Prices were randomly drawn from a normal distribution
with a mean and standard deviation estimated from realistic prices
collected from Amazon.com. Participants received a base payment
of \$2 and a performance contingent bonus between \$0 and \$4.

Data Availability. Data and modeling scripts are available on the
Open Science Framework: <https://osf.io/wqth3/>.

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