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DEBT CRISES, FAST AND SLOW

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Drawing on the theory of sovereign risk, we show that, driven by self-fulfilling expectations of default, both slow-moving and rollover (fast) crises are generically possible in models with standard features, at intermediate and high levels of debt, respectively. This is without relying on the specification of debt auctions by Cole and Kehoe (2000). A necessary condition is that debt tolerance thresholds—the time- and state- contingent levels of debt above which default becomes the preferred action by the government—respond endogenously to shifts in investors' expectations. In a sunspot equilibrium, the threat of belief-driven crises may not be enough for the government to deleverage in a recession, and bring debt to default-free levels. Unless the initial debt is close enough to the critical threshold above which the country becomes vulnerable to such crises, the government will keep borrowing, gambling on economic recovery in the future.

Debt crises, fast and slow*

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Preliminary

Abstract

Drawing on the theory of sovereign risk, we show that, driven by self-fulfilling expectations of default, both slow-moving and rollover (fast) crises are generically possible in models with standard features, at intermediate and high levels of debt, respectively. This is without relying on the specification of debt auctions by [Cole and Kehoe \(2000\)](#). A necessary condition is that debt tolerance thresholds—the time- and state- contingent levels of debt above which default becomes the preferred action by the government—respond endogenously to shifts in investors' expectations. In a sunspot equilibrium, the threat of belief-driven crises may not be enough for the government to deleverage in a recession, and bring debt to default-free levels. Unless the initial debt is close enough to the critical threshold above which the country becomes vulnerable to such crises, the government will keep borrowing, gambling on economic recovery in the future.

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1 Introduction

The academic and policy literature has long reflected on the possibility that countries with relatively weak fundamentals face disruptive belief-driven runs on national debt, raising borrowing costs and creating unsustainable debt dynamics, when not forcing the government to default immediately on its liabilities. Different models have been developed to lend theoretical support to such a view, emphasizing that, once the stock of their debt is sufficiently high, the equilibrium is no longer unique.

One class of models draws on the seminal contribution by [Calvo \(1988\)](#), inspired by the persistent and unstable inflation experienced by Brazil in the 1980s, at times of large fiscal imbalances. Multiple equilibria emerge, featuring the possibility of belief driven outright default or inflationary debt debasement, as investors price government bonds depending on their expectations of future debt paths. The anticipation of a steep path leading to default causes interest rates to rise; higher borrowing costs in turn accelerate debt accumulation; with a high and growing stock of debt, default occurs as soon as the economy is hit by a sufficiently negative shock, validating investors' pessimistic expectations. In view of this dynamic, in notable recent work [Lorenzoni and Werning \(2019\)](#) dub these crises “slow-moving”.¹ Another class of models draw on the seminal work by [Cole and Kehoe \(2000\)](#), inspired by the experience of Mexico in the mid 1990s. As the government auctions off its debt, agents may coordinate their belief on an imminent default and decide not to participate in the auction—a rollover crisis then forces the government to default, again validating agents' expectations.² Since the switch across equilibrium coincides with a sudden loss of market access, these crises are “fast”. Technically, the two models differ in a key assumption concerning the timeline along which the government sets how much bonds to issue, and investors set their price. At a deeper level, they shed light on different ways in which a liquidity crisis may occur.

In this paper, we show that, in addition to slow-moving debt crises, fast ones, in the form of rollover crises, are also quite pervasive in models adopting a dynamic [Calvo \(1988\)](#) setting, without necessarily relying on the assumptions spelled out in [Cole and Kehoe \(2000\)](#). The reason is insightful from both a theoretical and a practical vantage point. When the regime of investors' expectations turns from optimistic to pessimistic, higher costs of debt either reduce the social utility of not defaulting, or force the government to issue a high volume of risky debt and gamble on future recoveries to avoid immediate default. In either case, the switch in investors' expectations causes the debt tolerance threshold of the government—the debt level above which default becomes the dominant action conditional on weak

¹The Calvo model has also been revived by [Corsetti and Dedola \(2016\)](#), in the context of an analysis of monetary backstops to government debt inspired by the launch of the Outright Monetary Transactions programme by the European Central Bank in 2012.

²Recent contributions in this class include [Bocola and Dovis \(2019\)](#), [Conesa and Kehoe \(2017\)](#), [Bianchi and Mondragon \(2018\)](#) and [Corsetti et al. \(2017b\)](#), among others.

fundamentals—to fall. Depending on the initial level of debt, it is possible that the best response of the government to deteriorating expectations does not deliver enough adjustment for its financing need to be satisfied at any equilibrium prices. As the market anticipates this, the government loses market access and the country faces a debt rollover crisis.

To gain insight on the root differences between fast and slow-moving debt crises, we set up a dynamic model where a discretionary government optimizes its fiscal policy, deciding in each period whether to adjust primary surpluses or default. Intentionally, the model borrows the setting of [Conesa and Kehoe \(2017\)](#), except that the “timing” of the auction is that of [Calvo \(1988\)](#)³—hence the model abstracts from rollover crises à la [Cole and Kehoe \(2000\)](#). For comparison, we also consider a version of the model closer to [Lorenzoni and Werning \(2019\)](#), where the decision to default is dictated by the debt limit implied by the country’s natural budget constraint, evaluated at the maximum acceptable primary surplus the economy can deliver across time and circumstances. We will refer to the two versions as the baseline and the “debt limit” model, respectively.

Our contributions to the literature build on the fact that, in either model, the debt tolerance threshold may depend on the regime of expectations. We show that, when debt thresholds are sensitive to expectations, depending on the initial level of debt, crises can be slow or fast. Slow crises are possible at intermediate levels of initial debt—in the numerical example using our baseline model, for debt levels between 59% and 121% of GDP. In this region, investors’ pessimism translates into high risk premia that in turn ignite a slow-moving debt crisis: the hike in interest rates accelerates the dynamic of debt accumulation, and leads to default when the economic conditions worsen or if they fail to improve early enough. Fast crises are possible at higher levels of debt—in our example, between 121% and 204%. In this region, as investors become pessimistic, they anticipate that the government will not be willing/able to undertake the required adjustment to sustain debt at any finite equilibrium risky interest rates. In other words, the market comes to believe that there is no finite bond price that satisfies the government’s (re-optimized) financing need and the pricing equilibrium conditions. When this happens, the country loses market access and the government simultaneously defaults. In this sense, fast crises take the form of rollover crises—analytically different from the one conceptualized by [Cole and Kehoe \(2000\)](#), but clearly appealing to the same economic logic.

We show that lengthening the maturity of government debt is not necessarily effective in ruling out equilibrium multiplicity leading to slow-moving debt crises—these remain pervasive in both our baseline and the debt-limit model. However, longer maturities may rule out fast rollover crises. Analytically, for the fast crises

³See the discussion in [Lorenzoni and Werning \(2019\)](#), [Corsetti and Dedola \(2016\)](#) and [Ayres et al. \(2018\)](#)

not to be an equilibrium outcome, a necessary condition is that the debt tolerance thresholds are not sensitive to expectations: thresholds must be the same regardless of whether investors are optimistic or pessimistic. In the debt limit scenario, this condition is verified for a debt maturity that is not too short and probabilities of recovery that are non-negligible. In our baseline, the parameter restrictions for ruling out fast crises are much more stringent.

Last but not least, when an arbitrarily small probability of switching from the good to the bad equilibrium is internalized by both investors and policymakers, the government may have an incentive to keep debt below the relevant debt thresholds, if necessary by deleveraging even during a recession. Different from [Cole and Kehoe \(2000\)](#), however, the incentive for the policymakers to deleverage drives fiscal policy only over a relative small range of debt levels around the debt thresholds. In a recession, for debt levels sufficiently higher than the thresholds, the consumption smoothing motive dominates governments optimal policy, causing deficits and debt accumulation—i.e., gambling on the recovery.

From a policy perspective, our results have key implications for debt sustainability analysis and the design of policies to enhance sustainability. Estimates of debt tolerance thresholds are a crucial input in assessing the extent to which a country can steer away from default. In this paper, we stress that these thresholds are not only contingent on the current and future state of the economy and/or preferences of the policymakers. They are also sensitive to investors' expectations. This introduces a further complication in debt sustainability analysis, hardly discussed by the existing literature.

Moreover, our analysis clarifies that pervasive rollover risk may not be enough of an incentive for implementing (even optimally smoothed) debt reduction strategies. We stress that this result is obtained independently of political economy considerations, with policymakers modelled as short-sighted or self-interested. In our framework, even a forward-looking benevolent government will generally find it optimal to raise debt in a recession, smoothing consumption at the cost of keeping the country in a state of vulnerability to self-fulfilling crises. This result may strengthen the case for an international compact, offering countries a combination of liquidity assistance and official loans favoring economically and politically acceptable policies of deleveraging. In both respects, our analysis contributes to ongoing policy and academic work reconsidering the modalities and structure of official lending and assistance by international organizations.

The literature. This paper draws on the seminal contributions by [Calvo \(1988\)](#) and [Conesa and Kehoe \(2017\)](#), in turn related to [Cole and Kehoe \(2000\)](#). [Calvo \(1988\)](#) introduced the feedback loop between self-fulfilling expectations and debt burdens in a two-period model, where the government's financing need is taken as

given, and the price and quantity of bonds are jointly determined in equilibrium. Self-fulfilling expectations of default generate market “runs” that manifest themselves in a surge in the interest rate charged by investors to the government—but no rollover crisis is modelled in the same context. Conversely, [Conesa and Kehoe \(2017\)](#) focus on liquidity crises whereby the market may suddenly become unwilling to roll over government’s debt in anticipation of a default. In our paper we aim at reconsidering the nature and dynamic of rollover crises—we do so specifying a model on the style of [Calvo \(1988\)](#) model, but adopting a dynamic setting using the same environment as [Conesa and Kehoe \(2017\)](#), except for the specification of auctions underlying their view of rollover crises.

In [Eaton and Gersovitz \(1981\)](#), [Arellano \(2008\)](#) and [Conesa and Kehoe \(2017\)](#), the government commits to bond issuance before the market sets bond prices. This, together with an assumption restricting equilibrium pricing if no rollover crisis materializes, allows these authors to abstract from the multiplicity problem stressed by [Calvo \(1988\)](#). Also assuming that the government preset bond issuance, [Auclert and Rognlie \(2016\)](#) expand on [Eaton and Gersovitz \(1981\)](#) and discuss conditions such that a unique equilibrium exists. Relative to these papers, we find it reasonable that the government adjusts its policies based on the equilibrium bond prices it observes in the market. The government chooses bond issuance and primary deficits (the government financing need) as a function of the (equilibrium) interest rate.

[Lorenzoni and Werning \(2019\)](#) reconsiders [Calvo \(1988\)](#) in a dynamic setting, stressing that the increase in the sovereign’s borrowing costs driven by self-fulfilling expectations of default leads a country to accumulate debt slowly but relentlessly over time. As the debt stock rises, at some point default occurs unless the conditions of the economy improve sufficiently. [Ayres et al. \(2018\)](#) adopts a framework similar to [Arellano \(2008\)](#) but for the timing assumption, to investigate the likelihood that a country becomes vulnerable to belief-driven crises. Also drawing on [Calvo \(1988\)](#), [Corsetti and Dedola \(2016\)](#) and [Bacchetta et al. \(2018\)](#) write monetary models and discuss how the central bank can backstop government debt, i.e. eliminate self-fulfilling crises by using, respectively, either unconventional (balance sheet) policy, or conventional (inflation) policy.⁴

Several paper have been developing the model with rollover crises of [Cole and Kehoe \(2000\)](#), into new directions. By way of example, [Bocola and Dovis \(2019\)](#) characterize how the maturity of sovereign debt can be structured to respond to rollover risk and fundamental risk. [Aguiar et al. \(2019\)](#) consider a variant of rollover crises modelling uncertainty in social utility upon defaulting. [Chamon \(2007\)](#) elaborates on the idea that the way in which sovereign bonds are underwritten and offered by investment banks may guard a country against rollover crises. Rollover crises are also modelled and discussed by [Giavazzi and Pagano \(1989\)](#), [Alesina et al. \(1992\)](#)

⁴See also [Aguiar et al. \(2013\)](#).

and [Cole and Kehoe \(1996\)](#)).

Finally, in writing this paper we draw extensively on previous work on debt bailout, especially on [Corsetti et al. \(2017a\)](#), which introduces official lending in a [Conesa and Kehoe \(2017\)](#) framework, but also on [Conesa and Kehoe \(2014\)](#) and [Roch and Uhlig \(2018\)](#).

This paper is organized as follows. Section 2 lays out the model similar to [Conesa and Kehoe \(2017\)](#) but with a different timing assumption. For our baseline, Sections 3 discusses equilibrium multiplicity with different type of crises, while Section 4 presents a calibrated numerical example and offer a discussion of the model equilibria with long and short-term debt. Section 5 analyzes whether the perceived threat of a belief-driven crisis would prevent a government from running deficits during recessions. Section 6 reconsiders the analysis in a debt limit framework. Section 7 carries out sensitive analysis focusing on debt maturity and probabilities of recovery. Section 8 concludes.

2 Model

In this section we specify our dynamic model of debt sustainability and default. The environment draws on [Conesa and Kehoe \(2017\)](#), except that we abstract from rollover risk envisioned by [Cole and Kehoe \(2000\)](#)—in particular, we set the timing of investors' and the government decisions in the style of models after [Calvo \(1988\)](#), such that a government with a given financing need would not be able to set the amount of bonds to be issued before investors set bond price.

The state of the economy in every period, $s = (B, z_{-1}, a)$, is (i) the level of government debt owed to the risk neutral investors B , (ii) whether default has occurred in the past $z_{-1} = 0$ or not $z_{-1} = 1$, (iii) whether the economy is in a recession $a = 0$ or not $a = 1$. As in [Conesa and Kehoe \(2017\)](#), the country's GDP is

$$y(a, z) = A^{1-a} Z^{1-z} \bar{y}$$

with $A \leq 1$, and $Z < 1$. The parameter A denotes the business cycle: a recession occurs when $A < 1$. When the government defaults, the penalty is a permanent drop in productivity by the factor Z .

The economy starts out with $a_0 = 0$ and $z = 1$. From period 1, the economy recovers with probability $p < 1$ and once recovered, it never falls back to recession again. If the government defaults, it stays at the state of default $z = 0$ forever.

The government issues non-contingent bonds to risk neutral investors. As is customary in [Hatchondo and Martinez \(2009\)](#), we model maturity of government

bonds as follows. The bonds have geometrically decreasing coupons: a bond issued at t pays the sequence of coupons

$$\kappa, (1 - \delta)\kappa, (1 - \delta)^2\kappa \dots$$

where $\delta \in [0, 1]$. Hence, assuming risk neutral investors whose discount factor is β , the bond price without any default risk is:

$$q = \frac{\beta\kappa}{1 - \beta(1 - \delta)}$$

We normalize bond price by setting $\kappa = 1 - \beta + \beta\delta$ so that default-free bond price is β . The parameter δ indexes the maturity of debt, where $\delta = 0$ corresponds to the case of “consol” and $\delta = 1$ corresponds to the case of short-term bond. A bond issued at $t - m$ is equivalent to $(1 - \delta)^m$ bonds issued at t . Hence, the outstanding market bonds can be summarized by a single state variable B .

Sovereign tax revenue is $\theta y(a, z)$. As in [Conesa and Kehoe \(2017\)](#), the tax rate θ is constant for the government. Government spending is g . We stipulate that there is some critical expenditure level \bar{g} , below which the normal functioning of the state becomes problematic—hence, in the preferences of the policymaker, disutility is high. The government’s budget constraint is given by

$$zq(B', s)(B' - (1 - \delta)B) = g + z\kappa B - \theta y(a, z)$$

where the right hand side defines the (endogenous) Gross Financing Need (GFN) of the government. As is well understood in the literature, timing and strategy of issuance are the key to modelling debt sustainability. We follow the criteria in [Calvo \(1988\)](#), see [Figure 1](#).⁵

1. The aggregate state $s = \{(B, z_{-1}, a)\}$ is known. Each of a continuum of measure one risk neutral investors set bond price $q(b', s)$.
2. The government decides to default or repay, which determines $y(a, z)$. If it defaults, it stays at the state of default forever. If it repays, it chooses how much to borrow from risk neutral investors B' and investors purchase government bonds. In equilibrium, $B' = b'$. This determines g .

A comment is in order concerning the difference between our timing assumption and the assumption by the literature after [Eaton and Gersovitz \(1981\)](#) and [Cole and Kehoe \(2000\)](#). In this literature, the government sets the total issuance of bonds at face value; investors set bond prices afterwards. In [Cole and Kehoe \(2000\)](#), this

⁵[Corsetti and Dedola \(2016\)](#), [Ayres et al. \(2018\)](#) and [Lorenzoni and Werning \(2019\)](#) adopt the same timing assumption.

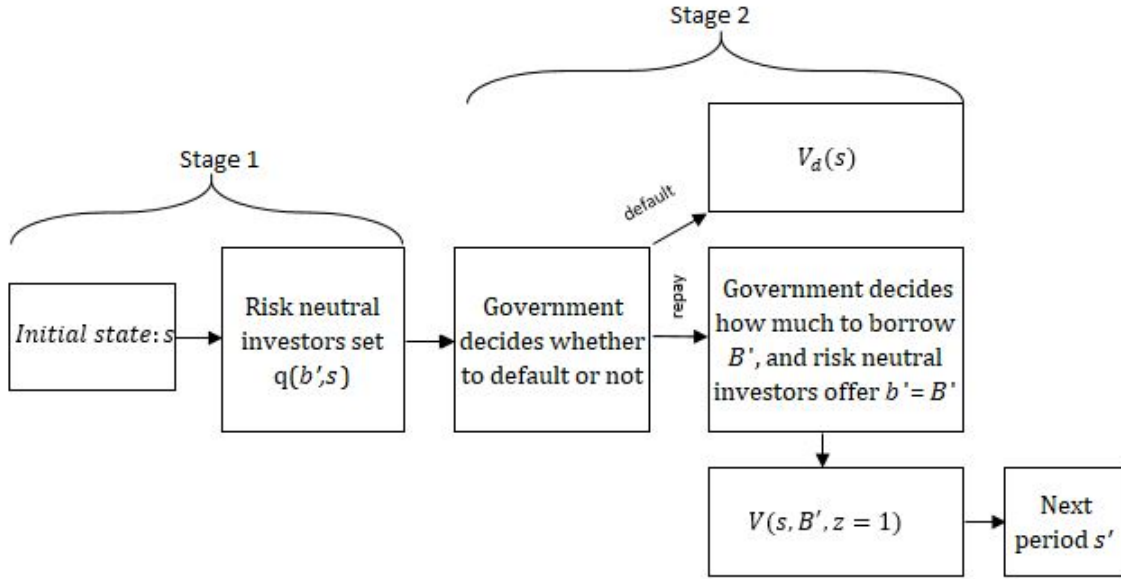


Figure 1: Timing description

timing assumption acts as a selection criterion that rules out crises of the type analyzed by Calvo (1988). Implicit in this selection criterion is that, provided investors are willing to finance the government, they offer the best price for the bonds. In practice, a credible commitment to a specific bond issuance requires that the current primary surplus must be adjusted enough if bond prices unexpectedly turn out to be lower. The point of departure of our analysis is to relax this assumption, so as to study the possibility of multiple values for the equilibrium bond prices $q(b', s)$ at stage 1 in Figure 1.

In our model, before announcing total issuance, the government will know the pricing schedule $q(b', s)$ on which market participants coordinate their expectations. The price at which investors are willing to buy newly issued bonds will change with issuance but, most importantly, will depend on investors' beliefs about government solvency. Given the prevailing bond pricing schedule $q(b', s)$, the government will adjust its fiscal decisions and issuance strategy in response to it. In line with the arguments by Lorenzoni and Werning (2019), we find this timing assumption a plausible working hypothesis.

2.1 Bond pricing

For tractability, we follow the literature and assume that investors are risk neutral and discount the future using the factor β . The bond prices $q(b', s)$ are therefore determined by the probability that investors assign to default in the future. Denoting by x the linear consumption of investors, their problem is:

$$\begin{aligned}
W(b, s) &= \max_{x, b'} x + \beta \mathbb{E}[W(b', s')] \\
\text{s.t.} \quad &x + q(b', s)b' = w + z(B'(q(b', s), s), s, q(b', s)) \times \\
&\quad (\kappa b + q(b', s)(b' - (1 - \delta)b)), \\
&x \geq 0, b \geq -\mathcal{A}
\end{aligned}$$

The constraint $b \geq -\mathcal{A}$ imposes the no-Ponzi condition, but \mathcal{A} is set large enough that the constraint never binds. By the same token, x , the linear consumption of investors, is large enough (deep pockets assumption) to rule out corner solutions.

As in [Conesa and Kehoe \(2017\)](#), there are two cutoff levels of debt each period, $\bar{B}(a)$ where $a = 0, 1$, with $\bar{B}(0) \leq \bar{B}(1)$:

1. If $B \leq \bar{B}(0)$, the government does not default, independently of the business cycle.
2. If $B \leq \bar{B}(1)$, the government does not default provided the economy is not in a recession.

For future reference, we find it insightful to dub these thresholds as the “debt tolerance” of the country.

Recall that, once the government defaults ($z = 0$), z stays at 0 forever. This assumption implies that the equilibrium bond price is zero in any history with past default:

$$q(b', (B, 0, a)) = 0$$

If the government had not defaulted in previous periods, the first order condition of risk neutral investors’ problem implies:

$$\begin{aligned}
q(b', s) &= \beta \mathbb{E} \left[z \left(B'(q(b'(s'), s'), s'), s', q(b'(s'), s') \right) (\kappa + (1 - \delta)q(b'(s'), s')) \right] \\
&\text{where } s' = s'(q(b'(s), s))
\end{aligned} \tag{1}$$

It is well understood that the source of equilibrium multiplicity in the model is rooted in the possibility that multiple values of $q(b', s)$ solve (1).

In equilibrium, $q(b', s)$ is consistent with market clearing condition $b' = B'$ which implies that one possible bond price function can be

$$q(B', (B, 1, 0)) = \begin{cases} \beta [\kappa + (1 - \delta)\mathbb{E}[q'(\cdot)]] & \text{if } 0 \leq B' \leq \bar{B}(0) \\ \beta p & \text{if } \bar{B}(0) < B' \leq \bar{B}(1) \\ 0 & \text{if } \bar{B}(1) < B' \end{cases}$$

in a recession and

$$q(B', (B, 1, 1)) = \begin{cases} 1 & \text{if } 0 \leq B' \leq \bar{B}(1) \\ 0 & \text{if } \bar{B}(1) < B' \end{cases}$$

in normal times.

2.2 Government optimization problem

Given the bond price function $q(b', s)$, if the government decides not to default, it chooses its fiscal deficit and issues $b' = B'$ at the equilibrium bond price $q(B', s)$. The government's problem can be reduced to choose B', z to solve

$$\begin{aligned} V(s) = \max \quad & u(c, g) + \beta \mathbb{E}[V(s')] \\ \text{s.t.} \quad & g + z\kappa = \theta y(a, z) + zq(B', s)(B' - (1 - \delta)B), \\ & c = (1 - \theta)y(a, z), \\ & z = 0 \text{ if } z_{-1} = 0 \end{aligned} \tag{2}$$

As in [Conesa and Kehoe \(2017\)](#), we posit that, for any B , the following condition holds

$$u_g((1 - \theta)A\bar{y}, \theta A\bar{y} - \kappa B) > u_g((1 - \theta)\bar{y}, \theta\bar{y} - \kappa B)$$

This ensures that, in a recession, the government always has an incentive to raise debt and gamble for redemption due to the high marginal benefit of government spending when the economy is in a bad state.

2.2.1 Baseline

In the framework presented above, the government defaults if and only if the utility of repaying debt V_n is smaller than the utility of defaulting V_d :

$$V_n < V_d$$

The value of defaulting is determined by assuming that, in case of debt repudiation, the country loses market access *and* experiences a discrete contraction in output by Z —output stays at $AZ\bar{y}$ in a recession and $Z\bar{y}$ in normal times. For tractability, both costs are assumed to be permanent. This will define our baseline default model.

2.2.2 Debt limit

Relative to the baseline model specified above, one may envision scenarios in which the government is averse to default, yet the prevailing conditions in the economy and the market may undermine its ability to honour its liabilities.

In Ghosh et al. (2013) and Lorenzoni and Werning (2019), default is decided against a given path of maximum (contingent) primary surpluses that the government can generate: default occurs if and only if the amount the government can borrow from the market is not enough to finance its interest bill, given this path. In our framework, the government defaults in a recession if and only if:

$$\max\{q(B', s)(B' - (1 - \delta)B)\} < \kappa B - \underbrace{(\theta A \bar{y} - \bar{g})}_{\text{max primary surplus}}$$

If the stock of initial debt is high, by borrowing, the government can smooth the adjustment in spending and taxation required to service debt across many periods. But given the path of maximum primary surpluses, bond prices may move against the government to such an extent that the required adjustment in the short run become intolerable. We will analyze this scenario in section 6.

2.3 Equilibrium

An equilibrium is a value function for the government $V(s, q)$ and policy functions $B'(s, q)$, $z(s, q)$ and $g(s, q)$, a value function for investors $W(b, s)$, policy function $b'(b, s)$, and an equilibrium bond price function $q(B', s)$ such that

1. Given policy function $z(s, q)$, $g(s, q)$, $V(s, q)$ and $B'(s, q)$, $b'(b, s)$ solves investors' problem at the beginning of the period and $q(B', s)$ is consistent with market clearing and rational expectations.
2. $V(s, q)$, $B'(s, q)$, $z(s, q)$ and $g(s, q)$ solve government's optimization problem in (2) given bond price function $q(B', s)$.

For tractability, the notion of equilibrium we consider follows a simple Markov structure.

3 Equilibrium multiplicity and debt crises with short-term debt

In this section, we specialize the baseline model laid out in the previous section, in that we consider short-term debt only. The economy can be plagued by multiple equilibria: debt crises can be driven by a switch in the regime of investors' expectations across these equilibria. Debt crises can be either "fast" or "slow". Intuitively, slow-moving crises occur for intermediate levels of debt, where a switch in expectations causes borrowing costs to rise, but the government can deliver enough adjustment for the market to keep satisfying its financing need at the higher equilibrium rates. Rollover crises occur at higher level of debt, because once investors

turn pessimistic, the government may not be willing (or able as in section 6) to contain its deficit enough to obtain financing at any finite equilibrium interest rates. Knowing this, investors simply refuse to rollover the government debt. If and when investors become pessimistic, they “run” and the government loses market access.

As discussed below, key to these results is that the government debt tolerance, i.e., the debt default thresholds, may change with investors’ expectations. If investors become “pessimistic” about debt sustainability, so that in equilibrium the cost of borrowing for the government rises, the government may default at lower level of debt, relative to equilibrium where investors are “optimistic”. The size of the shift relative to the initial stock of debt determines whether the crisis is slow-moving or fast. We should emphasize once more that the “rollover crises” in our model develop in a different institutional framework compared to the seminal model by [Cole and Kehoe \(2000\)](#).

We start by analyzing the debt tolerance thresholds in an expansion and in a recession. Then we study and interpret the equilibrium.

3.1 Debt tolerance thresholds in normal times and recessions

In our notation, debt tolerance thresholds are contingent on the state of the economy—we write $\bar{B}(a)$ $a = 0, 1$: $\bar{B}(0)$ is the maximum sustainable debt level in a recession and $\bar{B}(1)$ is the maximum sustainable debt level in normal times. Let $V_n(B, z_{-1}, a)$ and $V_d(a)$ denote, respectively, the government’s utility if the government repays its debt, and the government’s utility of defaulting, assessed either in normal times ($a = 1$) or in a recession ($a = 0$).

3.1.1 The debt tolerance threshold in normal times, $\bar{B}(1)$.

The derivation of $\bar{B}(1)$ is straightforward, since under our simplified assumption the government’s optimization problem is deterministic after the economy recovers (a stays at 1 forever). In this case, without loss of generality, we can abstract from multiplicity. If the government decides to repay existing debt, it pays $(1 - \beta)B$ each period to investors to obey no-Ponzi condition. The government utility conditional on repaying its debt is

$$V_n(B, 1, 1) = \frac{u((1 - \theta)\bar{y}, \theta\bar{y} - (1 - \beta)B)}{1 - \beta}$$

Write the utility of defaulting when the economy is not in a recession as

$$V_d(1) = \frac{u((1 - \theta)Z\bar{y}, \theta Z\bar{y})}{1 - \beta}$$

It follows that $\bar{B}(1)$ can be characterized by solving:

$$V_n(\bar{B}(1), 1, 1) = V_d(1)$$

The debt tolerance threshold $\bar{B}(1)$ is unique in that investors hold a unique consistent view, that the economy will remain in normal times forever (there is no output uncertainty any more). Fundamental risk will instead be crucial in generating multiple debt tolerance thresholds when the economy is in a recession.

3.1.2 The debt tolerance threshold(s) in a recession $\bar{B}(0)$.

To determine the threshold in a recession, it is useful to distinguish from the start the possibility that investors can hold multiple consistent beliefs that are either “pessimistic” or “optimistic”.

Suppose investors are pessimistic—in that, given the equilibrium financing need of the government, they attribute probability one to a default in the following period if a recession persists. In this case, investors offer a low bond price βp to the government. While the utility of defaulting in a recession is independent of expectations:

$$V_d(0) = \frac{u((1-\theta)AZ\bar{y}, \theta AZ\bar{y})}{1-\beta(1-p)} + \beta \frac{pu((1-\theta)Z\bar{y}, \theta Z\bar{y})}{(1-\beta)(1-\beta+\beta p)}$$

the utility of repaying debt given the low bond price (high cost of borrowing) is not. Denoting this utility with the subscript “pes”, to stress that this may depend on expectations, we have:

$$\begin{aligned} V_{pes}(B, 1, 0) = & \max_{0 \leq B' \leq \bar{B}(1)} \left(u(c, g) + \beta \left(\frac{p}{1-\beta} u((1-\theta)\bar{y}, \theta\bar{y} - (1-\beta)B') \right. \right. \\ & \left. \left. + (1-p)V_d(0) \right) \right) \quad (3) \\ \text{s.t.} \quad & g + B = \theta A\bar{y} + \beta p B', \\ & c = (1-\theta)A\bar{y} \end{aligned}$$

To determine the debt threshold $\bar{B}(0)_{pes}$, we solve the equation below:

$$V_{pes}(\bar{B}(0)_{pes}, 1, 0) = V_d(0)$$

To ensure $\bar{B}(0)_{pes}$ is self-fulfilling, the choice variable $B'(\bar{B}(0)_{pes})$ in the value function (3) must be larger than $\bar{B}(0)_{pes}$.

By the same token, we can derive the debt tolerance level in an optimistic scenario, $\bar{B}(0)_{opt}$. In an optimistic world, investors may presume that, given the equilibrium financing need of the government, this will be willing and able to service its debt even if the economy remains in a recession in the next period. Hence, we

write the utility of repaying debt assuming that the equilibrium bond prices are riskless: $V_{opt}(B, 1, 0) = \max\{V_{opt,1}(B, 1, 0), V_{opt,2}(B, 1, 0)\}$. Note that we allow for the possibility that the debt issuance capacity of the government is constrained by a debt threshold below the one conditional on the economic recovery. In particular, either of the following may be relevant:

$$\begin{aligned}
V_{opt,1}(B, 1, 0) = & \max_{0 \leq B' \leq \bar{B}(0)_{opt}} u(c, g) + \beta \left(\frac{p}{1-\beta} u((1-\theta)\bar{y}, \theta\bar{y} - (1-\beta)B') \right. \\
& \left. + (1-p)V_{opt}(B', 1, 0) \right) \\
\text{s.t.} & \quad g + B = \theta A\bar{y} + \beta B', \\
& \quad c = (1-\theta)A\bar{y}
\end{aligned}$$

$$\begin{aligned}
V_{opt,2}(B, 1, 0) = & \max_{\bar{B}(0)_{opt} < B' \leq \bar{B}(1)} u(c, g) + \beta \left(\frac{p}{1-\beta} u((1-\theta)\bar{y}, \theta\bar{y} - (1-\beta)B') \right. \\
& \left. + (1-p)V_d(0) \right) \\
\text{s.t.} & \quad g + B = \theta A\bar{y} + \beta p B', \\
& \quad c = (1-\theta)A\bar{y}
\end{aligned}$$

The debt threshold $\bar{B}(0)_{opt}$ is again the solution of the equation below.

$$V_{opt}(\bar{B}(0)_{opt}, 1, 0) = V_d(0)$$

The algorithm for computing an equilibrium in a recession is shown in Appendix A. To gain insight on the thresholds characterized above, in the next subsection we rely on a simple graphical apparatus.

3.2 An intuitive graphical analysis

In the two panels of Figure 2, we draw a debt Laffer curve for our model, illustrating the interaction between market expectations, bond issuance and vulnerability to default in a recession one period ahead. The x axis measures bond issuance at time 0 relative to the debt tolerance threshold. The y axis measures the resources that the government can obtain by issuing debt at the equilibrium price, qB' , relative to the (endogenous) Gross Financing Need of the government, GFN. In the panel to the left, the Laffer Curve is drawn as a solid blue line from the origin, which has slope β , the risk-free bond price, as long as new issuance does not increase the stock of debt above the threshold $\bar{B}(0)$ —it has a flatter slope βp for a debt stock between $\bar{B}(0)$ and $\bar{B}(1)$ due to default risk. The second panel in the figure shows that $\bar{B}(0)$

and the GFN of the government depends on the regime of expectations.⁶

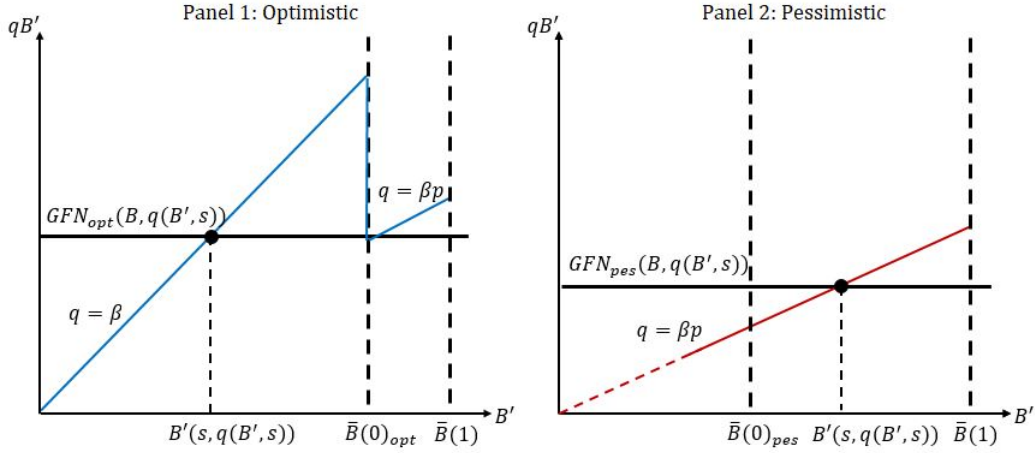


Figure 2: Bond pricing and Laffer curves with optimistic and pessimistic investors

In Figure 2 panel 1, we depict a world where investors develop an optimistic view. Interest rates on sovereign’s borrowing remain low, which have two key effects. On the one hand, low borrowing costs make the sovereign’s utility of repaying debt higher—this translates into a higher debt tolerance threshold in a recession ($\bar{B}(0) \uparrow$ to $\bar{B}(0)_{opt}$). Everything else equal, more debt is sustainable. On the other hand, cheaper debt may induce the government to relax its budget policy, hence raise spending and cut taxes. Given the initial debt, this translate into endogenously higher Gross Financing Need. A high price of bonds and a higher debt tolerance threshold allow the government to run a larger deficit without creating default risk. As shown in Figure 2, the new issuance of bonds remains well below the debt tolerance level. No default is expected to occur one period ahead—validating the optimistic expectations of investors.

Now, suppose investors develop a pessimistic view. This is shown in Figure 2 panel 2.⁷ In a pessimistic world, investors charge a higher interest rate on sovereign debt at a lower level of borrowing. Hence at the risky price ($q(b', s) = \beta p$ is lower), more debt needs to be issued against any given GFN.

As the higher borrowing costs reduce the government’s utility of repaying debt, the debt tolerance threshold in a recession will be low—lower than in the optimistic world ($\bar{B}(0)_{opt} > \bar{B}(0)_{pes}$). Facing a higher cost of borrowing, nonetheless, the government may consider containing its primary deficit, hence the GFN also falls. However, the endogenous fall in financing need may not be sufficient to rule out pessimistic expectations. Despite lower financing need, sovereign’s debt issuance

⁶Throughout this subsection, we will posit that the initial debt is low enough that immediate default is never optimal.

⁷Along the dotted line, investors’ pessimistic belief is not self-fulfilling, and thus βp is not an equilibrium price when B' is low.

$B'(s, q(B', s) = \beta p)$ surges with the costs of borrowing. As the government issues more debt, B' rises above the (lower) tolerance threshold $\bar{B}(0)_{pes}$. One period ahead, unless the economy recovers, the government will default, validating investors' pessimism.

3.3 Equilibrium runs on debt: none, slow and fast

Having clarified the logic of belief-driven runs on debt in the previous subsection, in what follows we shift focus on the role of the initial level of debt as driver of the equilibrium, and allow for the possibility that default occurs immediately, in response to a change in the regime of expectations. To do so, we will combine the two panels in Figure 2 into a single graph, depicting both the optimistic and the pessimistic regime of expectations together. Moreover, we will make the GFN an endogenous function of the initial debt and the price of bonds. In each graph, we will include debt thresholds and draw the Laffer curve.

Investors' pessimistic expectations may/may not be self-fulfilling, depending on the initial level of government debt. This is illustrated by the three panels of Figure 3, each depicting one of three equilibria that are possible for a government initial debt level that is, respectively, low, intermediate and high. The three panels illustrate, respectively, a scenario of no risk of crisis, one in which slow-moving debt crises are possible, and the third one in which rollover crises are possible.

In our model, the Laffer curves and the debt tolerance thresholds (for both the optimistic and pessimistic scenarios) are independent of the initial debt level. Hence they are exactly the same across the three panels of Figure 3. Different from Figure 2, the government financing needs (GFN) is plotted as a function of the level of initial debt and different bond prices. The distance from the origin to the GFN line is a function of the level of initial debt: the higher the stock of liabilities inherited by the government, the larger this distance. The slope of the GFN line is negative. To see why: moving down the line, think of each point on the GFN line as crossing a ray from the origin (not shown), corresponding to a lower bond price q . At lower q 's, the government faces higher borrowing costs. The government has thus an incentive to adjust its spending optimally, reducing its financing need. As discussed above, however, this optimal adjustment falls short of reducing the new issuance of debt, hence the GFN line is decreasing monotonically. In the panels, we also depict a new debt threshold, labelled B_N , which denotes the maximum amount of the *initial* debt level in a recession below which the country is immune to pessimism.

Panel 1 of Figure 3 illustrates a case in which the initial debt stock is so low that the equilibrium is unique and bonds are traded at default-free prices. In the panel, the GFN intersects the debt Laffer curve in two points, at L_{opt} and L_{pes} . In the first point, debt is issued at risk-free rate; in the second point, debt is default-risky. It is easy to verify that the latter cannot be an equilibrium. Even if investors become

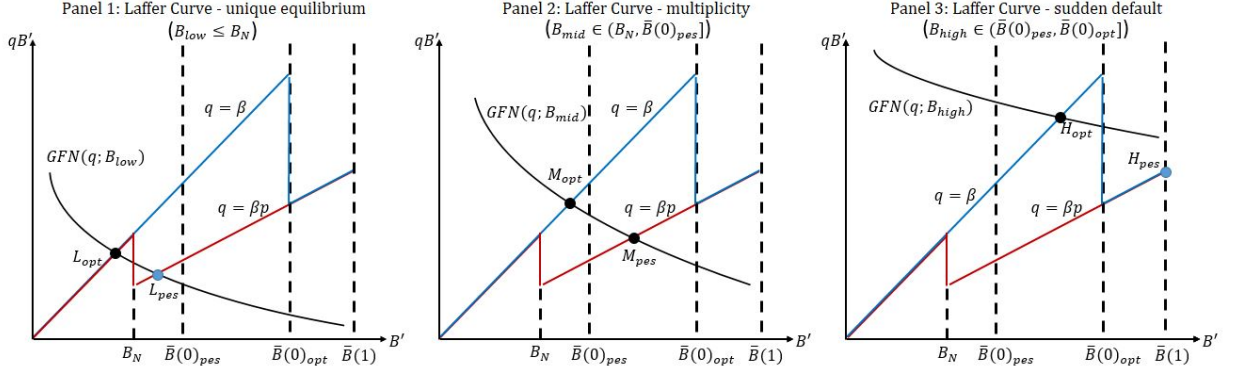


Figure 3: Multiple equilibria and unique equilibrium with $\delta = 1$

pessimistic, the government would still issue debt below the tolerance threshold $\bar{B}(0)_{pes}$, since its GFN is moderate. Investors' pessimistic view would not be validated ex-post. The only self-fulfilling equilibrium is at the point L_{opt} , with riskless pricing in equilibrium. In other words, in panel 1, $q = \beta p$ is not the solution to investors' first order condition (1), shown below:

$$\underbrace{q(b', s)}_{=\beta p} \neq \beta \mathbb{E} \left[\overbrace{z \left(\underbrace{B'(q(b'(s'), s'), s'), s', q(b'(s'), s')}_{< \bar{B}(0)} \right)}^{=1} \right] \quad (4)$$

Using this equation, we can determine at which level of debt the equilibrium is no longer unique. B_N can be found by solving the equation (5) below.

$$B_N = \sup_{b'} \left\{ \underbrace{q(b', s)}_{=\beta p} \neq \beta \mathbb{E} \left[\overbrace{z \left(\underbrace{B'(q(b'(s'), s'), s'), s', q(b'(s'), s')}_{< \bar{B}(0)} \right)}^{=1} \right] \right\} \quad (5)$$

For *initial debt levels* larger than B_N , if investors develop a pessimistic view on government solvency, an equilibrium with default can be self-validating: the government either borrows more than $\bar{B}(0)_{pes}$, or default immediately. We now turn to these cases, looking at panels 2 and 3.

In panel 2, the government initial debt B_{mid} is at intermediate level. Precisely, B_{mid} is larger than B_N but not too high—smaller than $\bar{B}(0)_{pes}$. Similar to panel 1, the GFN function intersects the debt Laffer curves at point M_{opt} and point M_{pes} , respectively. Now, both points can be an equilibrium. The economic intuition has already been discussed in Figure 2. When investors buy newly issued sovereign debt at the riskless price β , overall borrowing will remain below the relevant debt tolerance threshold, $\bar{B}(0)_{opt}$, validating ex-post the investors' optimistic view. The same logic applies to point M_{pes} . For both $q = \beta$ and $q = \beta p$ to be equilibrium prices, the initial stock of debt must be such that the equations (6) and (7) from

the investors' first order condition (1) are satisfied at once:

$$\underbrace{q(b', s)}_{=\beta} = \beta \mathbb{E} \left[z \left(\overbrace{B'(q(b'(s'), s'), s'), s', q(b'(s'), s'))}^{=1} \right) \right] \quad (6)$$

$\underbrace{\hspace{10em}}_{<\bar{B}(0)} \qquad \underbrace{\hspace{10em}}_{\bar{B}(0)=\bar{B}(0)_{opt}}$

$$\underbrace{q(b', s)}_{=\beta p} = \beta \mathbb{E} \left[z \left(\overbrace{B'(q(b'(s'), s'), s'), s', q(b'(s'), s'))}^{=p} \right) \right] \quad (7)$$

$\underbrace{\hspace{10em}}_{>\bar{B}(0)} \qquad \underbrace{\hspace{10em}}_{\bar{B}(0)=\bar{B}(0)_{pes}}$

The type of equilibrium with belief-driven default shown in Panel 2 of Figure 3 corresponds to a scenario in which, as stressed by Lorenzoni and Werning (2019), the debt crisis is ‘slow-moving’. Interest rates are high because investors expect the government to default if the recession persists. Because of high borrowing costs, the stock of government debt rises prior to default. But default only occurs if and only if the country remains in a recession in the future. In our parameterization of the model, the ‘slow-moving’ idea boils down to a one-period delay, but this is enough to capture the main message: the crisis is preceded by debt accumulation driven by a surge in borrowing costs reflecting self-validating, pessimistic views by investors.

In Panel 3 of Figure 3, we show that the model admits another type of belief-driven default in equilibrium, possible for a relatively high initial debt level, higher than $\bar{B}(0)_{pes}$. The GFN line now intersects the Laffer curves at the point H_{opt} . If investors buy government bonds at the riskless price β , despite the high stock of initial liabilities, new debt issuance remains below the relevant threshold, $\bar{B}(0)_{opt}$. However, if investors turn pessimistic, the hike in borrowing rates causes the government to become much less ‘tolerant’ of the adjustment required to service the debt. At the point H_{pes} , investors anticipate that the government will not be able and/or willing to adjust its primary needs enough to keep new issuance of debt below $\bar{B}(1)$ at the default-risk bond price, and thus “fast” debt crises occur.

We can interpret this “fast” debt crisis in two ways. On the one hand, when the market expects the government to default in a recession, the government can issue only at risky rates. But at these rates, the government is unable to satisfy its (adjusted) financing need by issuing debt within its maximum debt capacity in normal times—beyond which a default occurs with 100% probability. Even if investors were willing to charge high but finite interest rates conditional on the government to cut its deficit, immediate default would be the preferred option. On the other hand, anticipating the above, the market will not be willing to finance any government debt: the government loses market access. In a rollover crisis, the government has no alternative but to default. The condition is given by equation

(8) below.

$$\underbrace{q(b', s)}_{=0} = \beta \mathbb{E} \left[\overbrace{z \left(B'(q(b'(s'), s'), s'), s', \underbrace{q(b'(s'), s')}_{\bar{B}(0)=\bar{B}(0)_{pes}} \right)}^{=0} \right] \quad (8)$$

It is important to clarify why the point H_{pes} in the figure is not an equilibrium. This is shown in equation (9). Once investors turn pessimistic, the government optimally cuts its deficit and reducing its current financing need, moving down along the GFN line. In principle, the government could implement a further cuts in its deficit, but this would never be optimal given that spending and utility remains relative high after default, i.e., given that post default output remains sufficiently high relative to the critical expenditure level \bar{g} .

$$\underbrace{q(b', s)}_{=\beta p} \neq \beta \mathbb{E} \left[\overbrace{z \left(B'(q(b'(s'), s'), s'), s', \underbrace{q(b'(s'), s')}_{\bar{B}(0)=\bar{B}(0)_{pes}} \right)}^{=0} \right] \quad (9)$$

The Panel 3 in Figure 3 features the possibility of debt crises that we dub “fast”. There is no “slow-moving” increase in debt levels, leading up to a crisis. Upon the drop of the debt tolerance thresholds in response to changes in investors’ views on government solvency, a debt crisis arrives “fast” and “early”.

4 Sustainability and crises with long-term debt

For any given stock of debt, longer maturities may help sustainability, by reducing the exposure to rollover risk and the pass through of hikes on interest rates onto the total cost of servicing the outstanding debt. An important question is whether and under what circumstances maturity can rule out multiplicity leading to either slow-moving or to fast debt crises.

In this section, we show that multiplicity of equilibria remains pervasive. To do so, we rely on a numerical example, calibrating our model with standard parameter values.

4.1 Calibration

In solving the model with long-term debt, we posit the following functional forms for the utility function:

$$u(c, g) = \log(c) + \gamma \log(g - \bar{g}); \quad (10)$$

In our calibration, we set benchmark parameters following [Conesa and Kehoe \(2017\)](#). The parameter values are shown in [Table 1](#).

Table 1: Parameter values, baseline

\bar{y}	Output	100
Z	Cost of Default	0.95
β	Discount factor	0.98
γ	Relative weight of c and g in the utility function	0.20
θ	Government revenue as a share of output	0.36
\bar{g}	Level of the critical government expenditure	25
A	Fraction of output during recession	0.9
p	Probability of leaving the recession	0.2
δ	Amortization rate of market debt	0.2

As shown in the table, we normalize output \bar{y} to 100 so that the units in the model can be interpreted as percentage of GDP: e.g. $B = 50$ means that debt to GDP ratio is 50% in normal times. We set cost of default as 5% = $1 - Z$, and this cost of default is permanent. Our default cost is lower relative to the literature (e.g. [Alesina et al. \(1992\)](#)), on the grounds that we assume this cost to be permanent.⁸ We assume the relative weight of government utility is 0.2; sensitivity analysis shows that this parameter is unimportant for our result.

The severity of recession A is set at 0.9 so that a recession results in a decrease in output by 10% for the benchmark scenario. This parameter is crucial to generating gambling for recovery in an optimistic world. A more severe recession leads to a stronger smoothing motive for the government, which may induce the government to choose a high-debt risky-debt strategy—we report results for different A in sensitivity analysis.

We set the critical government expenditure \bar{g} at 25% of GDP: the higher this value, the smaller the room for discretionary spending. Government revenue as a fraction of output is set by θ . In normal times, the government’s income is 36, but in a recession, it drops to 32. We posit $\delta = 0.2$ to match average maturity from 2000-2009 for Greece, Italy and Spain, which is about 5 years. We set $p = 0.2$ so that the expected waiting time for recovery is 5 years.

The key novel result from our analysis is that, since the debt tolerance threshold in a recession moves with investors’ expectations, this might result in “fast” debt crises. This result holds also with long-term debt. [Figure 4](#) plots the policy functions conditional on a recession, together with the debt tolerance thresholds (in a recession and in normal times), in the optimistic world (left panel) and the pessimistic world

⁸Upon a default, in our baseline scenario Z cannot be too small in that the government spending cannot fall below \bar{g} . In other words, the conditions $\theta AZ\bar{y} > \bar{g}$ and $\theta Z\bar{y} > \bar{g}$ must be satisfied.

(the right panel).

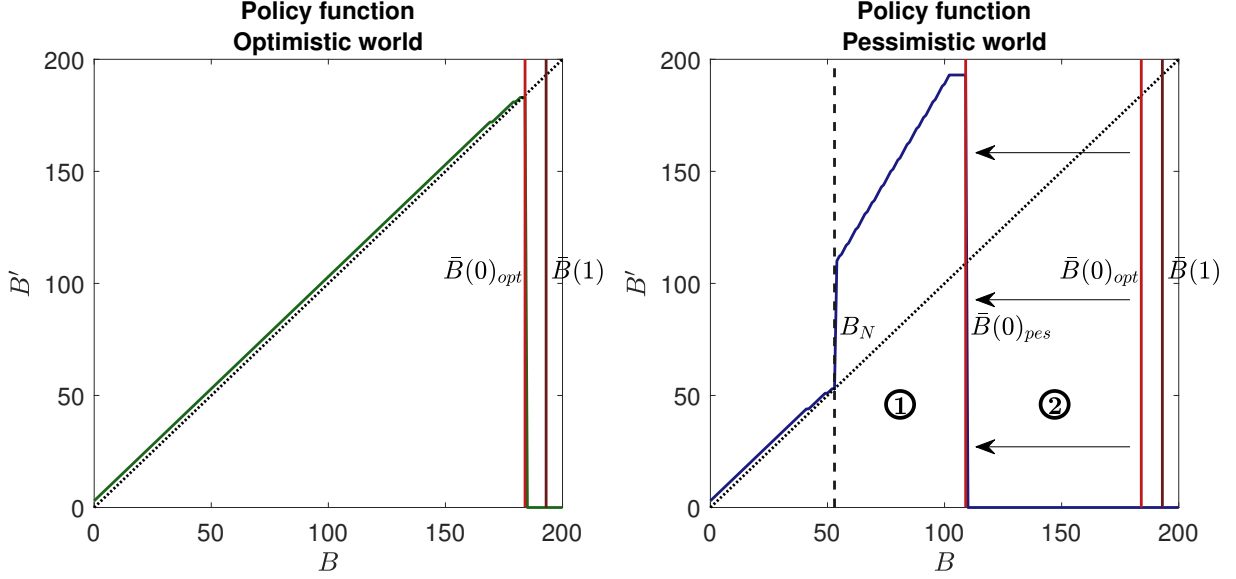


Figure 4: Policy functions in the optimistic world and the pessimistic world

A striking feature of the optimistic world—on the left panel of Figure 4—is the high value of the debt tolerance threshold in a recession, about 184% of the GDP in normal times, or about 204% of GDP when a recession occurs. Notably, in our exercises, we find that $\bar{B}(0)_{opt}$ is not sensitive to the probability of recovery p or debt maturity δ , but depends on A , as further discussed in section 7. In a recession, the government will smooth consumption by borrowing at risk-free rate until debt level reaches $\bar{B}(0)_{opt}$: the figure suggests that the dynamic of debt is mildly increasing.

The right panel of Figure 4 depicts a situation in which investors unexpectedly change their view on government solvency, from optimistic to pessimistic. While $\bar{B}(1)$ is not affected, by virtue of our assumption that, after recovering, the economy never falls back into a recession again, the consequences of such a change on the debt tolerance threshold in a recession are stark. There is a large drop from $\bar{B}(0)_{opt}$ to $\bar{B}(0)_{pes}$.

If the initial debt is in the region between 0 and B_N , the country is barely affected by the switch in expectations. The government is still able to borrow at risk-free rate, and, as a result, it keeps increasing the level of debt for smoothing purposes until the stock of debt reaches B_N , and then remains there, waiting for a recovery.

If the initial debt is anywhere above B_N but below $\bar{B}(0)_{pes}$ in a pessimistic world, the government will pay high rates of interest and its debt will start to accumulate at a faster pace. In the region labelled ① in the figure, default may then occur depending on whether the economy fails to recover in the next period. This is the scenario of a “slow-moving” crisis: a default is preceded by debt accumulation. Note that, under our parameterization, a slow-moving crisis can arise for a debt to GDP ratio as low as 53% of GDP in normal times (about 59% of GDP in a recession).

If debt is in the region between $\bar{B}(0)_{pes}$ and $\bar{B}(0)_{opt}$ —the region labelled ② in the figure—the crisis will be of the type that we dub “fast”: it will occur in response to the shift in expectations. Observe that in the “fast crisis” region, as long as investors are optimistic, the government can actually issue debt at risk-free rate. But once investors change their view and charge high risky rates, they understand that the sovereign will be unwilling to reduce its financing need enough to keep new issuance below $\bar{B}(1)$. The debt market dries out. Facing such a rollover crisis, the government defaults immediately. There is no “slow-moving” accumulation of debt. In our calibration, fast crises can occur with a debt to GDP ratio in a recession between about 121% and 204%.

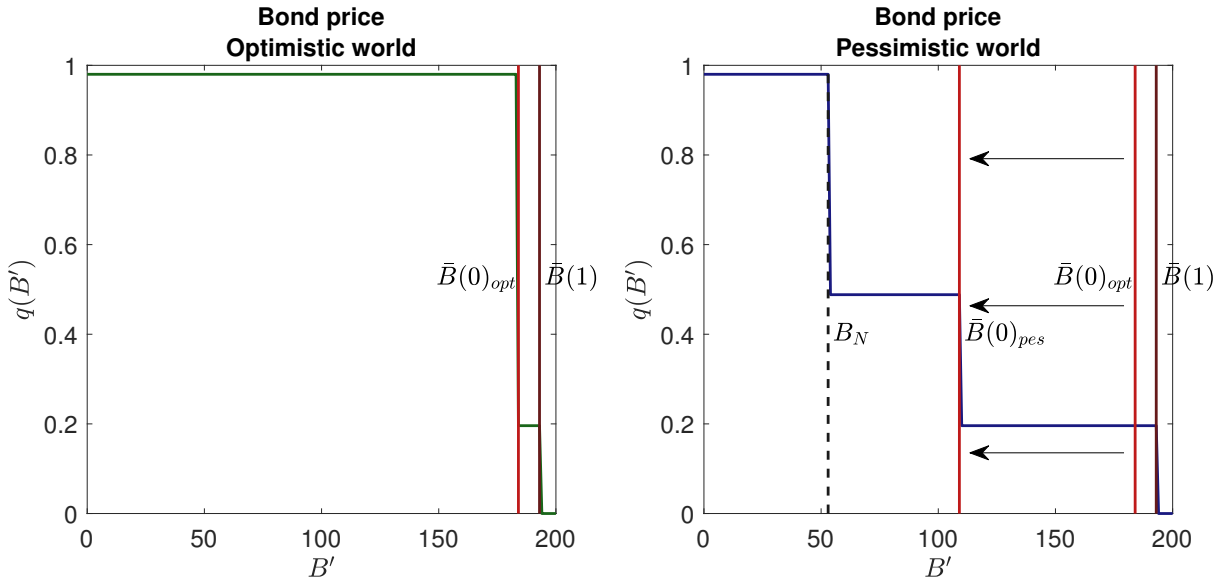


Figure 5: Bond prices in the optimistic and the pessimistic world

In Figure 5, we plot the price of government bonds in both the optimistic and the pessimistic worlds, contingent on a recession. The left panel in the figure shows that this price remains high for a wide range of debt-to-GDP ratio in the optimistic world. The price drops very markedly in the narrow region between $\bar{B}(0)_{opt}$ and $\bar{B}(1)$. The right panel of Figure 5 illustrates the impact of a change in investors’ expectations, from optimistic to pessimistic. If the amount of new issuances lies between B_N and $\bar{B}(0)_{pes}$, the government may be exposed to the probability of slow-moving debt crises next period. Thus, the bond price drops to 0.48. If it issues above $\bar{B}(0)_{pes}$ but below $\bar{B}(1)$, the “fast” debt crises may occur, which decreases the bond price even further.

For comparison, in Figure 6 we display policy functions conditional on a recession assuming one-period bonds ($\delta = 1.0$). A notable result from the comparison of this with Figure 4 is that, as δ converges to unity, $\bar{B}(0)_{pes}$ is much lower, while $\bar{B}(0)_{opt}$ stays constant. This is because, when investors hold an optimistic view on government solvency, they lend the government at risk-free rate: thus there is little

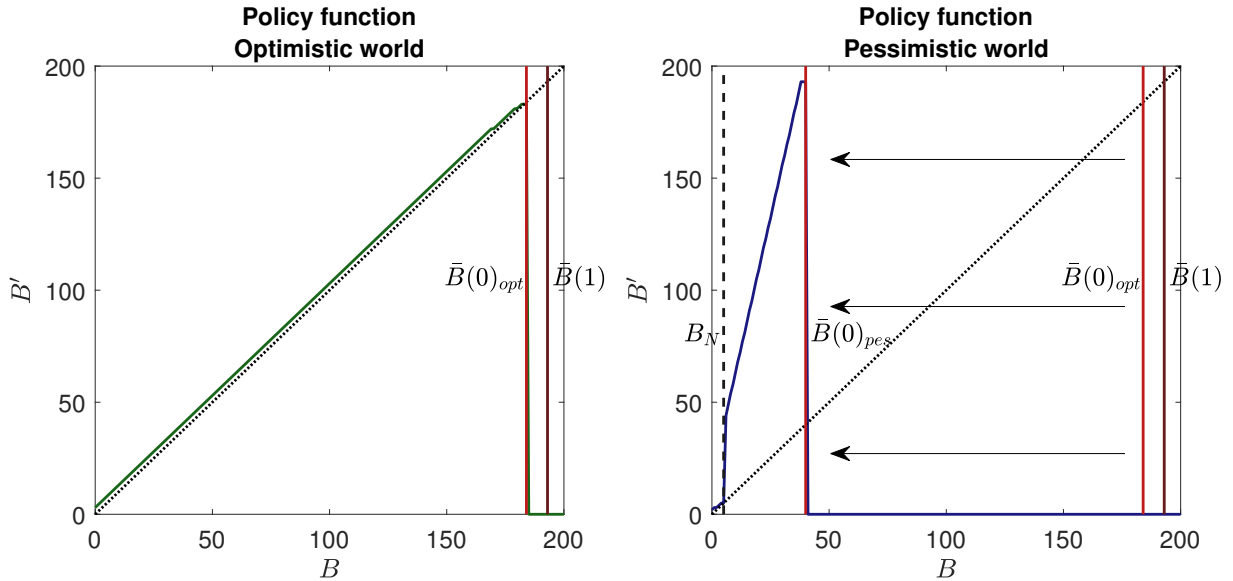


Figure 6: Policy functions for one-period bonds, $\delta = 1.0$

scope for maturity to make a difference. Indeed, the left panel in Figure 6 features exactly the same dynamics as the left panel of Figure 4.

The right panel of Figure 6 instead suggests that, with short-term debt, the region of “fast crises”, between $\bar{B}(0)_{pes}$ and $\bar{B}(0)_{opt}$, is much wider. The rollover crises might occur for low level of debt (above 40% of GDP in normal times). Moreover, an increase in the region of “fast crises” is not fully compensated by narrowing of the region of “slow crises”, as both B_N and $\bar{B}(0)_{pes}$ shrink when maturity is shorter. B_N falls from 53 to 5. When government bonds are all short-term, a country in a recession might suffer a “slow-moving” crisis even if it has negligible outstanding debt.

Summing up: a long debt maturity substantially improves government’s welfare by increasing debt thresholds in a pessimistic world, but it may not rule out any types of self-fulfilling crises. Threats of both “slow-moving” and “fast” debt crises are still pervasive with long-term debt. In section 6, we will show that the same does not necessarily apply to the debt limit framework.

5 Does the threat of self-fulfilling crisis motivate debt deleveraging?

So far we have carried out our analysis under the implicit assumption that, when in an optimistic mode, investors and the government attribute zero probability to the bad equilibrium. In this section, we relax this assumption and construct sunspot equilibria, heavily drawing on the approach by [Conesa and Kehoe \(2017\)](#). Namely, we posit that investors are initially optimistic on government solvency, but all agents

in the economy are aware that market views may turn pessimistic with probability π . If this pessimistic view is self-fulfilling (in that the government borrows more than $\bar{B}(0)_{pes}$ once the switch occurs), investors remain pessimistic forever afterwards. Debt tolerance threshold in a recession will now be denoted as $\bar{B}(0)_\pi$. We posit a small sunspot probability, equal to $\pi = 0.04$.

Our key result is that, in a sunspot equilibrium, the government may choose to decrease debt to safe levels in a recession, motivated by large gains in expected utility from either eliminating sunspot crises altogether (we dub this the welfare ‘cliff effect’ of belief-driven crises), or lowering borrowing costs (the ‘price effect’), or both. However, different from [Cole and Kehoe \(2000\)](#), deleveraging will be preferred over debt accumulation only for a small range of debt above the threshold at which slow-moving crises become a possibility. For a very wide range of debt levels, the government prefers to accumulate liabilities and smooth consumption, gambling on the prospective recovery.

Figure 7 displays the policy function (left) and the bond price function (right) in sunspot equilibrium with long-term debt. For debt levels in the region between 0 and B_N , the debt dynamics are the same as in the right panel of Figure 4, and the government is able to issue safe debt.

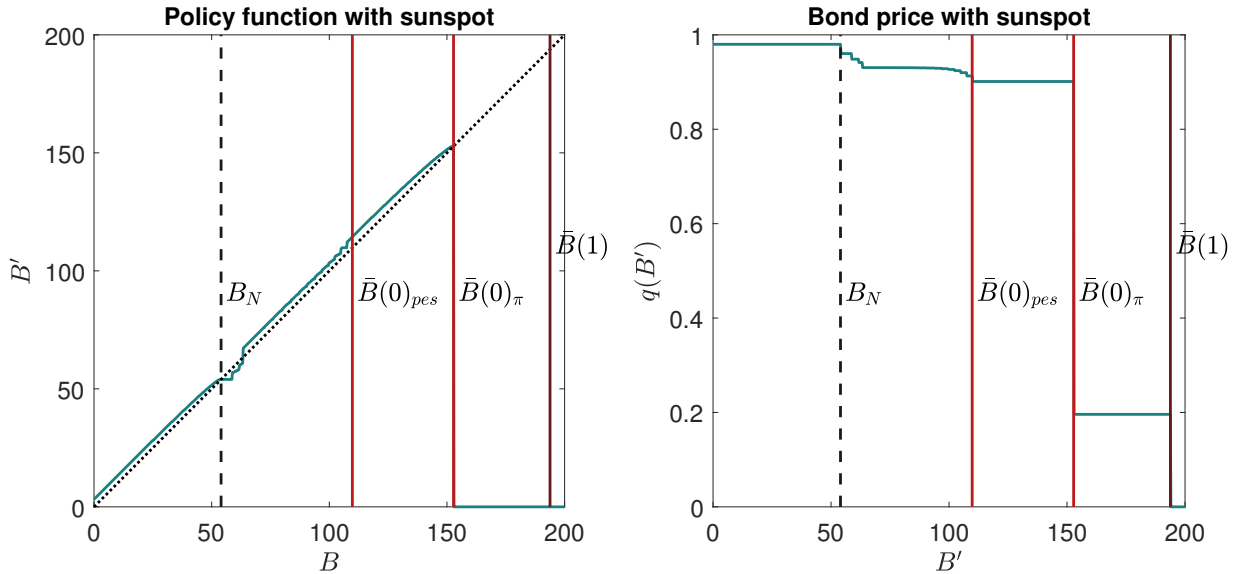


Figure 7: $\delta = 0.2$, $A = 0.9$, $p = 0.2$ with sunspot

In the region between B_N and $\bar{B}(0)_\pi$, where the economy is vulnerable to sunspot crises, the debt dynamics are different from what we have seen so far—it is no longer uniform. This region can indeed be split into two subregions. For an initial debt level close to B_N , the government chooses to run surpluses and reduce its borrowing. This allows the government to avoid high and increasing borrowing costs, as well as a large utility loss if self-fulfilling pessimistic expectations materialize. However, for a larger initial debt, the government prefers to keep borrowing. It will do so until

its debt level reaches $\bar{B}(0)_\pi$, even for debt levels above (but close to) $\bar{B}(0)_{pes}$, where self-fulfilling crises, if they occur, are “fast”.

Why is deleveraging optimal for debt levels just above B_N , but not so for debt levels just above $\bar{B}(0)_{pes}$? The key insight is that keeping debt below $\bar{B}(0)_{pes}$ shields the country from “fast” crises, but not from slow-moving ones. Hence, while the government may still have some advantage not to let debt trespass $\bar{B}(0)_{pes}$, this advantage is exclusively in terms of lower borrowing costs (as shown on the right panel of Figure 7), not in terms of eliminating the possibility of crises ‘tout-court’ (the ‘cliff effect’ is less relevant here).⁹ The borrowing costs advantage (about 1.4%)¹⁰ is not enough to offset the need to smooth consumption in a recession via borrowing. The government exposed to the risk of fast crises de facto accumulates debt faster than slow moving one. This is shown in Appendix C.

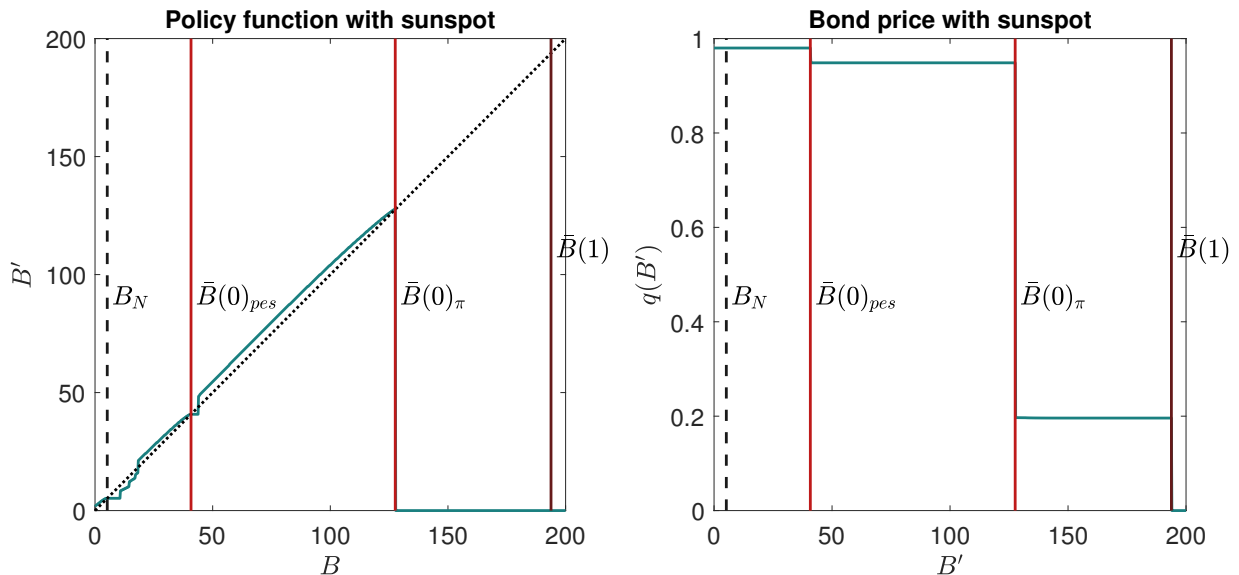


Figure 8: $\delta = 1.0$, $A = 0.9$, $p = 0.2$ with sunspot

If government debt is short-term, the ‘price’ and ‘cliff effects’ play a somewhat different role in shaping government decisions. The case of one-period bonds, with $\delta = 1.0$, is shown in Figure 8. Besides the fact that levels and shifts in thresholds are now substantially different from Figure 7, there is a subtle change in the policy function. As for the case of long-term debt, there is optimal deleveraging for a range of debt above B_N . But now we also have optimal deleveraging for a very small range of debt just above $\bar{B}(0)_{pes}$. A key insight can be learnt from the right panel of the graph. Note that, once debt rises above B_N , investors keep lending at risk-free rate, even if it is understood that a switch in expectations may end up igniting a

⁹Discountuity in value function, like a ‘cliff’ in a pessimistic world, motivates the government to deleverage. See Appendix B for details.

¹⁰This is obtained by $(1 - q(\bar{B}(0)_{pes} + 1))/q(\bar{B}(0)_{pes} + 1) - (1 - q(\bar{B}(0)_{pes}))/q(\bar{B}(0)_{pes})$. We use the same formula to derive yield difference of one-period bonds in our next simulation, which is 3.4%.

slow-moving debt crisis. There is no price effect in trespassing the threshold. The government’s deleveraging decision only reflects the prospective loss of welfare (the ‘cliff effect’). Conversely, although the cliff effect is absent at the threshold $\bar{B}(0)_{pes}$, the full pass-through of changes in market interest rates (about 3.4%) on government borrowing costs takes only one period. The aggravation of cost is a good enough incentive for the government to pursue some deleveraging for a stock of liabilities just above $\bar{B}(0)_{pes}$.

6 Equilibrium multiplicity and debt crises in a “debt limit” framework

In this section, we turn to economies in which the government is willing to exhaust all possibilities of adjustment before repudiating the debt and fall in a low-output financial autarky situation. Given the path of maximum adjustment in the primary surplus, the default condition can be written as:

$$\theta y(a, z) - \bar{g} + \max\{q(B', s)B'\} < B$$

Our debt limit default framework features similar dynamics to our baseline model. Debt tolerance thresholds may vary with investors’ view on solvency, and it affects debt paths and policy function.

6.1 Debt thresholds and crises with short term debt

In what follows, we will characterize the debt tolerance thresholds, and show how these are pinned down by the maximum adjustment in primary surpluses the government is willing/able to generate. They may be shifting in response to the regime of investors’ expectations.

6.1.1 The debt tolerance threshold in normal times $\bar{B}(1)$.

In normal times, the government budget constraint is

$$B = \theta \bar{y} - g + q(B', s)B'$$

Since, once the economy recovers, it never falls back to a recession again, the government optimization problem is deterministic—in normal times, there is no reason to borrow or lend for consumption smoothing purposes. If no default has occurred

in the past, the government will simply service its existing debt at the risk-free rate, paying $(1 - \beta)B$ to investors each period, to satisfy the no-Ponzi condition.

Given θ , the government will not default if and only if

$$B \leq \frac{\theta\bar{y} - \bar{g}}{1 - \beta} = \bar{B}(1)$$

where \bar{g} is the critical expenditure level.

6.1.2 The debt tolerance threshold in a recession $\bar{B}(0)$.

In a recession, the government budget constraint reflects the decline in tax revenue due to the downturn in activity ($A < 1$):

$$B = \theta A\bar{y} - g + q(B', s)B'$$

In a **pessimistic world**, investors are only willing to buy bonds at the low risky price. Given the definition of the debt tolerance threshold, the maximum the government can borrow is capped by the stock of debt that the government can service if the economy recovers, that is, $\max\{q(B', s)B'\} = \beta p\bar{B}(1)$. Now, to avoid default, the current debt must satisfy:

$$B < \theta A\bar{y} - \bar{g} + \beta p\bar{B}(1)$$

In other words, the government will not default if and only if

$$B \leq \theta A\bar{y} - \bar{g} + \beta p\bar{B}(1) = \bar{B}(0)_{pes}, \quad (11)$$

an expression that gives us the current debt tolerance threshold $\bar{B}(0)_{pes}$.

In an **optimistic world**, the government can actually choose between two debt issuance strategies. One consists of issuing a lot of debt, at a low, risky price—essentially this is the same strategy as described above, and is therefore associated to the same debt threshold (11). The other one consists of keeping new issuance in check, so to ensure that debt remains safe. This can be dubbed as a “low-risk low-debt” issuance strategy. By using the same steps above, we can derive the maximum sustainable debt conditional on the safe-debt strategy as:

$$B \leq \frac{\theta A\bar{y} - \bar{g}}{1 - \beta}$$

Thus, $\bar{B}(0)_{opt}$ can be characterized as follows:

$$\bar{B}(0)_{opt} = \max \left\{ \frac{\theta A\bar{y} - \bar{g}}{1 - \beta}, \bar{B}(0)_{pes} \right\}$$

Which strategy gives the government higher revenue in an optimistic world depends on parameters. If all government debt is short-term, we find that $\bar{B}(0)_{opt} > \bar{B}(0)_{pes}$, and thus a safe-debt strategy makes the government better off.

A notable implications is that, with short-term debt, the possible equilibria in a debt-limit framework are similar to the ones depicted in the three panels of Figure 3. The same analysis also applies: crises can be slow-moving, for intermediate level of debt, but can be fast, once the initial stock of debt is high enough (obviously the thresholds will be quite different in a debt-limit framework). However, in the next section, we will show that the crises in two frameworks may be quite different when debt is long-term.

6.2 Sustainability and crises

To study the debt limit model numerically, we find it instructive to use a variant of our baseline. In particular, instead of assuming (10), we posit that the government suffers a utility cost Γ if it cuts spending below \bar{g} , and rewrite the objective function as follows

$$u(c, g) = \log(c) + \mathbb{1}_{g > \bar{g}}(\gamma \log(g - \bar{g} + \epsilon)) - (1 - \mathbb{1}_{g > \bar{g}}) * \Gamma,$$

where $\mathbb{1}_{g > \bar{g}}$ is an indicator function equal to 0 if spending falls below is critical value, and we assume an arbitrary small ϵ to ensure that $u(c, g)$ is bounded below when $g \rightarrow \bar{g}$. This is the key implication: if the output cost of defaulting $1 - Z$ is large enough to bring spending below the critical level \bar{g} , and Γ is large enough, the value of repaying will never be below that of defaulting. Yet, as shown below, default is possible, depending on the initial conditions, the persistence of recessions and, as shown below, investors' expectations.

Using this new framework, we now set $Z = 0.8$, $\theta = 0.35$, $\bar{g} = 30$ such that government spending falls below the critical level \bar{g} upon a default.¹¹ For other parameters except for maturity indicator δ , we adopt the same values as in the baseline of Table 1. To save space, we concentrate on consistent debt paths before the economy recovers. This is equivalent to assuming no sunspot. Sunspots are discussed in Appendix E. We will show that, relative to our results so far, long-term debt tends to rule out “fast” debt crises more easily, but remains ineffective in ruling out “slow-moving” debt crises.

This result is shown in the two panels of Figure 9, which depicts policy functions with long-term bonds (left panel) and one-period bonds (right panel). Each panel illustrates both the optimistic and the pessimistic world.

¹¹We observe that the initial recessionary state can be quite adverse, i.e., A can be so low that the government cannot finance the critical level of spending \bar{g} without borrowing. In other words, $A\theta\bar{g} < \bar{g}$. We discuss this case in Appendix D.

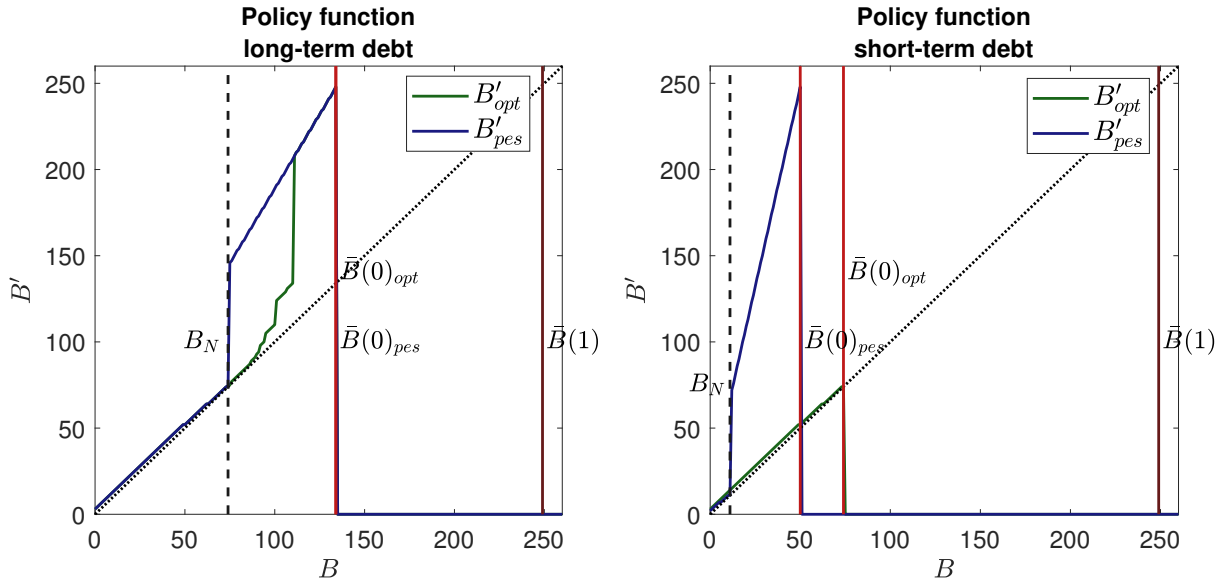


Figure 9: Policy functions where $\delta = 0.2$ (left) and $\delta = 1.0$ (right)

The right panel of Figure 9 depicts the policy functions with one-period bonds. The debt dynamics are very similar to Figure 6 but with a much lower threshold $\bar{B}(0)_{opt}$. In an optimistic world, the government accumulates debt over time to smooth consumption till it reaches $\bar{B}(0)_{opt}$. In a pessimistic world, the government issues safe debt at a slow pace in the region between 0 and B_N ; it starts to accumulate risky debt at a fast pace in the region between B_N and $\bar{B}(0)_{pes}$. Fast, rollover crises can nonetheless occur for debt levels between $\bar{B}(0)_{pes}$ and $\bar{B}(0)_{opt}$.

The debt dynamics shown by the left panel in Figure 9 are quite different. The equilibrium is unique for a low level of debt (in the region between 0 and 74) and for a high level of debt (in the region above 110). Multiplicity exists for intermediate levels of debt (in the region between 74 and 110). With long-term debt, $\bar{B}(0)_{pes}$ coincides with $\bar{B}(0)_{opt}$: “Fast” debt crises are no longer possible.

With respect to fast debt crises, debt maturity is much more consequential in the debt limit than in the baseline model. As discussed below, in our calibration, we find that “fast” debt crises are ruled out in the debt-limit version of our model for any δ below 0.57, corresponding to a debt maturity of seven quarters—for any longer debt maturity, “none” and “slow” are the only possible outcomes in debt limit framework. Multiplicity is not ruled out however.

7 What determines a debtor’s resilience to slow and fast self-fulfilling crises?

In this subsection we study how an economy may/may not be vulnerable to debt crises may differ, depending on the maturity of its debt and on the nature of the

recession—its depth and expected persistence. We first carry out some sensitivity analysis comparing our baseline with the debt limit framework. We then focus on conditions under which fast debt crises are ruled out in both.

7.1 Debt maturity and the persistence/depth of economic recessions

For our baseline model, in Figure 10 we plot debt tolerance thresholds in a recession as we vary debt maturity (left panel) and the probability of recovery (right panel).

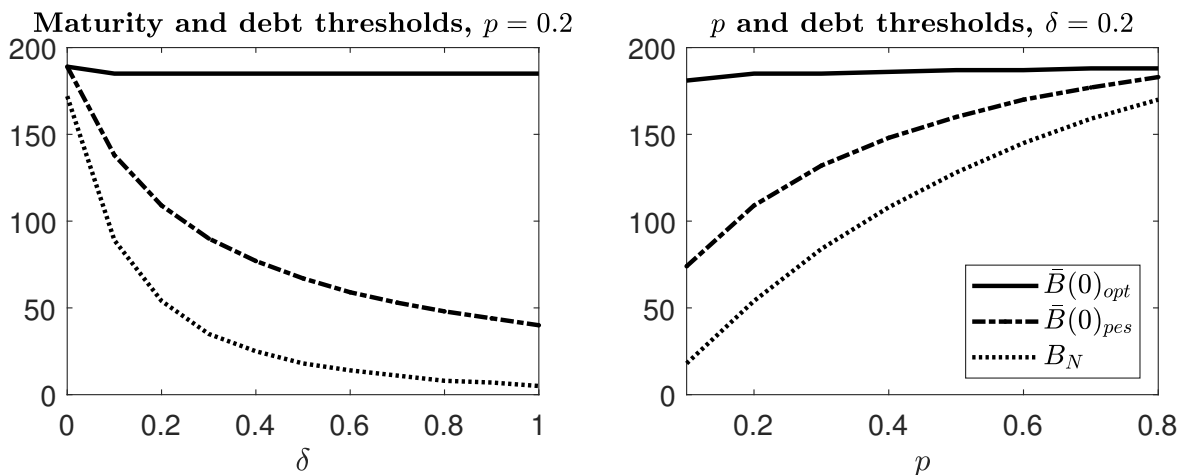


Figure 10: Debt thresholds in the baseline model given $A = 0.9$

Starting from the the left panel of Figure 10, we first note that, in an optimistic world, $\bar{B}(0)_{opt}$ is insensitive to debt maturity, but for extremely small values of δ ($\delta \rightarrow 0$) corresponding to very long maturities. As long as investors remain optimistic, the government can borrow at risk-free rate. Long-term debt and short-term debt are basically equivalent—a small effect can be detected only at extreme maturities, reflecting a lower incidence of debt rollover on the gross financing need.

In contrast, both B_N and $\bar{B}(0)_{pes}$ decrease sharply with δ , that is, they increase with a longer debt maturity of debt. To see why, consider the net bond revenue in a pessimistic world, $\beta p(B' - (1 - \delta)B) - \kappa B$, where $\beta p B'$, $\beta p(1 - \delta)B$ and κB denote, respectively, revenue from newly issued bonds, the value of the outstanding stock of bonds, and interest payment to investors. Maturity has two opposite effects on net bond revenue. As the maturity of bonds becomes longer ($\delta \downarrow$), the value of the outstanding stock of bonds $\beta p(1 - \delta)B$ rises but the interest payments due in the period κB fall. The first effect decreases, the second effect increases the net bond revenue. But now rearrange the net bond revenue equation, as follows: $\beta p B' - [1 - \beta(1 - p)(1 - \delta)]B$. It is apparent that the second effect always dominates the first one: a fall in δ unambiguously increases net debt revenue—explaining why $\bar{B}(0)_{pes}$ and B_N are larger as the debt maturity becomes longer. One could note

that, when investors hold a pessimistic view of the government, an official swap of short-term bonds for long-term bonds may improve the debt tolerance threshold of a country (a point discussed is detailed in [Corsetti et al. \(2017a\)](#)).

In [Figure 10](#), observe that the “fast” crisis zone, the distance between $\bar{B}(0)_{opt}$ and $\bar{B}(0)_{pes}$, becomes wider, the shorter debt maturity is. On the contrary, the “slow-moving” crisis zone, the distance between B_N and $\bar{B}(0)_{pes}$, remains approximately unchanged as maturity shortens— B_N as well as $\bar{B}(0)_{pes}$ basically falls at a similar pace.

Mirroring these results, the right panel of [Figure 10](#) shows that the probability of recovery p does not have much of an effect on $\bar{B}(0)_{opt}$, while it has a significant impact on both B_N and $\bar{B}(0)_{pes}$. Using once again the expression for the net bond revenue in a pessimistic world, $\beta p(B' - (1 - \delta)B) - \kappa B$, we can see that this is unambiguously increasing in p . By contrast, the net bond revenue in an optimistic world, $\beta(B' - (1 - \delta)B) - \kappa B$, does not vary with p . Observe that a higher probability of recovery p significantly narrows the “fast” crisis zone. It also narrows, but to a lesser extent, the “slow-moving” crisis zone.

We repeat this sensitivity analysis for the debt limit framework—results are shown in [Figure 11](#). There is at least one significant difference relative to our baseline. Focusing on the the left panel in [Figure 11](#), note that the debt threshold $\bar{B}(0)_{opt}$ is insensitive to debt maturity only for a δ higher than 0.57, that is, for relatively short maturities. With short-term debt, a low-debt safe-debt issuance strategy yields higher revenue than high-debt risky-debt issuance strategy—it is rationale for the government to remain on the good side of the Laffer curve. As explained in our comments to the previous figure, intuitively, the revenue from a low-risk issuance strategy is not affected by δ because investors lend at risk-free rate. However, once δ becomes smaller than 0.57, i.e., once debt maturity becomes sufficiently long, the government rationally switches to a high debt, risky-debt issuance strategy even in the optimistic world. Different from the left panel of [Figure 10](#), $\bar{B}(0)_{opt}$ is now the same as $\bar{B}(0)_{pes}$, both increasing in debt maturity (lower δ). The remarkable implication is that, with longer maturities, the “fast” crises zone no longer exists.

A similar picture is provided by the right panel of [Figure 11](#), which plots debt thresholds against the probability of recovery p . Different from the corresponding panel in [Figure 10](#), $\bar{B}(0)_{opt}$ now rises substantially with larger p , and coincides with $\bar{B}(0)_{pes}$ for any p larger than 0.08. Only for a very low probability of recovery, the low-debt safe-debt issuance strategy dominates the high-debt risky-debt issuance strategy in the optimistic world, causing $\bar{B}(0)_{opt}$ to diverge from $\bar{B}(0)_{pes}$, and to remain insensitive to p . For any non-negligible probability of recovery (p larger than 0.08), risky-debt strategy generates higher revenue and the “fast” crises zone disappears.

Overall, our sensitivity analysis confirms a striking difference between the base-

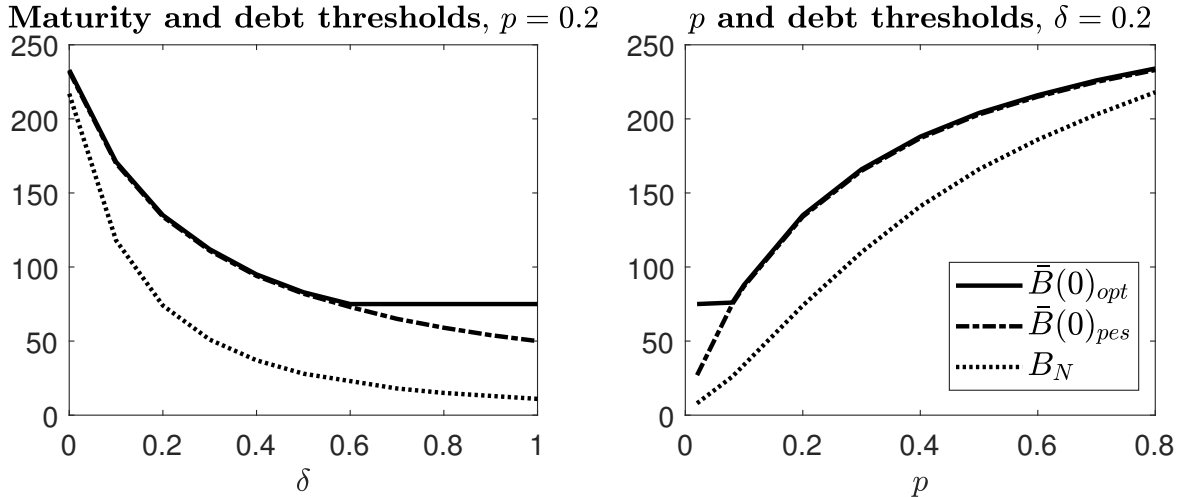


Figure 11: Debt thresholds in the debt limit framework given $A = 0.9$

line model and the debt limit framework. For a mildly long debt maturity and non-negligible probability of recovery, fast (rollover) crises are still possible in the former, but not in the latter. The reason is that in a debt limit framework, the government takes advantage of long debt maturities and good recovery prospect to smooth consumption issuing risky debt (de facto putting the economy in a slow-moving debt crisis mode) even when investors have optimistic expectations.

7.2 Ruling out “fast” debt crises

In the sensitivity analysis carried out above, we were unable to rule out “fast” debt crises in our baseline model. However, logically, it must be possible that “fast” debt crises also disappear in this framework—in circumstances in which the government has a strong incentive to pursue high-debt risky-debt issuance strategy even when investors’ expectations are optimistic. We now show that this may be the case if the country is in a very deep recession (A is very low), the probability of recovery is quite high, and debt maturity is sufficiently long, as to mute the pass-through of high interest rates on the total cost of debt servicing. The case is shown in Figure 12.

In Figure 12 we set $A = 0.8$ and $p = 0.6$: the current recession is exceptionally deep (with a loss of output equal to 20%), but the likelihood of exiting from in a period is larger than 50%. Figure 12 shows policy functions when debt is long-term ($\delta = 0.2$) in the left panel, and when debt is short ($\delta = 1.0$) in the right panel.

When government bonds are only short-term, we confirm the results in the previous subsection. On the right panel of Figure 12, $\bar{B}(0)_{opt}$ does not coincide with $\bar{B}(0)_{pes}$, and thus fast debt crises are possible. It is worth reiterating the reason. Despite the depth of the recession and the good prospects for a recovery, due to the short debt maturity (high δ), the government is de facto financially constrained—a

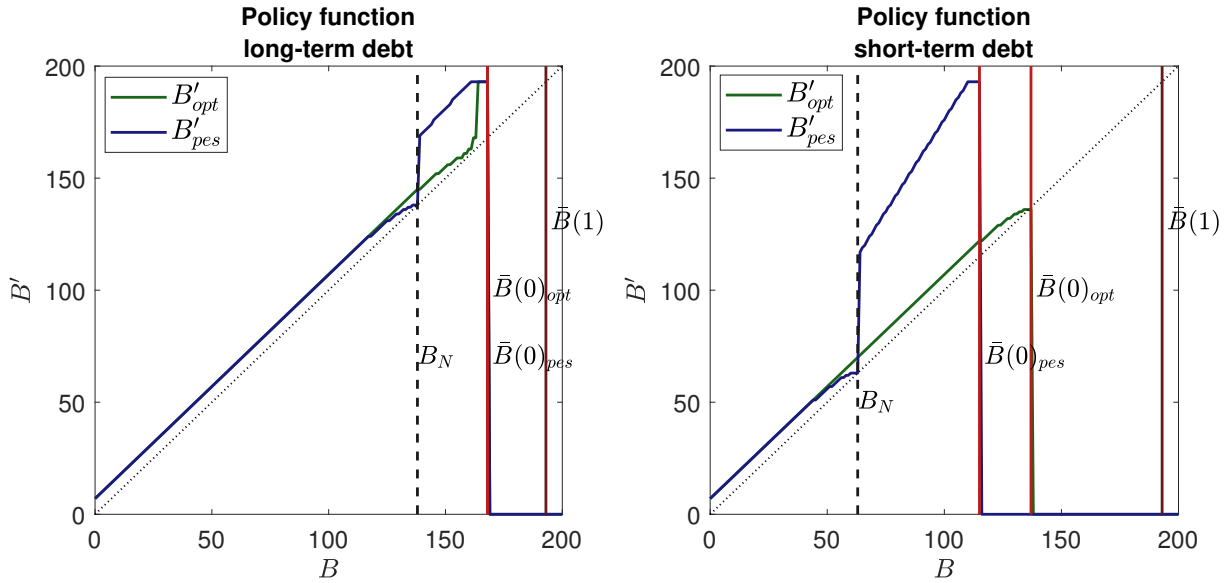


Figure 12: Policy functions in a very severe recession and high probability of recovery, $A = 0.8$ and $p = 0.6$, where $\delta = 0.2$ (left) and $\delta = 1.0$ (right)

risky issuance strategy would quickly raise its interest costs. In an optimistic world, the government has no incentive to gamble: it will stick to a low-debt safe-debt issuance strategy.

The picture is quite different on the left panel of Figure 12. As long as debt maturity is sufficiently long, in the optimistic world, a very severe recession makes borrowing very attractive for consumption smoothing motive. Because the probability of recovery is high and there is no need to roll over debt fully every period, below $\bar{B}(0)_{opt}$ the government keeps accumulating debt. If, unfortunately, the recession lasts many periods, the government ends up gambling for recovery.

Strikingly, the government is more conservative in a pessimistic world. For any debt level below B_N , the government slowly accumulates debt till it reaches B_N . Given that investors are pessimistic, the government finds it too expensive to smooth above B_N , as the interest rate would surge abruptly—a jump which would not occur in the optimistic world. In the region between B_N and $\bar{B}(0)_{pes}$, the government gambles on the recovery.

8 Conclusion

Are countries vulnerable to belief-driven sovereign debt crises? Leading answers in the literature emphasize that, once the stock of their debt is sufficiently high, the equilibrium is no longer unique. According to one approach, at any point in time a switch in market expectations from a good to a bad equilibrium may result in higher borrowing costs causing debt accumulation—the crisis then develops slowly, from a combination of an unsustainable build-up of debt and fundamental stress, as in

Calvo (1988) and Lorenzoni and Werning (2019). According to another approach, the switch causes a country to lose market access: a rollover crisis forces a sudden default, as in Cole and Kehoe (2000).

In this paper, we have shown both crises may occur in a dynamic Calvo (1988) setting. When the regime of investors' expectations turns from optimistic to pessimistic, higher costs of debt either reduce the social utility of not defaulting, or force the government to issue a high volume of risky debt and gamble on future recoveries. In either case, the debt tolerance threshold of the government—the debt level above which default becomes the dominant action—falls with the switch in investors' expectations. At this switch, depending on the initial level of debt, investors may anticipate that the government will not be willing/able to undertake the required adjustment to sustain debt at any finite equilibrium interest rates. At sufficiently high levels of debt, countries able to borrow at the risk-free rate may suddenly lose market access and, in the absence of external official support, default.

By characterizing a sunspot equilibrium, we are able to revisit debt dynamics and deleveraging under the threat of a rollover crisis. As in the existing literature, we find that a forward-looking benevolent government to reduce debt even during recessions, motivated by the prospective loss of welfare in a belief-driven crisis and high costs of borrowing. Different from the existing literature, however, in our new framework deleveraging is optimal only for a relatively small range of debt close to the debt threshold at which the country becomes exposed to slow-moving debt crisis. On the contrary, deleveraging is not generally optimal around the higher debt threshold at which the country becomes exposed to fast, rollover crises. Our results suggest the threat of a belief-driven debt run will not be generally effective in leading forward-looking benevolent governments to embrace precautionary fiscal policy of risk reduction.

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A The algorithm for computing value function in a recession

A.1 Baseline

The algorithm computes two debt thresholds in an optimistic world, and three debt thresholds in a pessimistic world.

A.1.1 Optimistic

1. Compute the debt tolerance threshold in normal times $\bar{B}(1)$ by solving the equation below

$$\frac{u((1-\theta)\bar{y}, \theta\bar{y} - (1-\beta)\bar{B}(1))}{1-\beta} = V_d(1)$$

After the economy recovers, the government's optimization problem is deterministic. Thus, the value function in normal times can be characterized by

$$V(B, a = 1) = \begin{cases} \frac{u((1-\theta)\bar{y}, \theta\bar{y} - (1-\beta)B)}{1-\beta} & \text{if } 0 \leq B \leq \bar{B}(1) \\ V_d(1) & \text{if } \bar{B}(1) < B \end{cases}$$

2. Guess initial values for threshold $\bar{B}(0)_{opt}$ and the bond price function $\tilde{q}_{opt}(B', 0)$ in a recession.
3. Given the bond price function $\tilde{q}_{opt}(B', 0)$ and $\bar{B}(0)_{opt}$, guess value function $\tilde{V}(B, 0)$ in an optimistic world. Perform value function iteration and update initial guess until it satisfies convergence criterion $\max_B |V(B, 0) - \tilde{V}(B, 0)| < \epsilon$.

$V(B, a = 0) = \max\{V_{opt,1}(B, 0), V_{opt,2}(B, 0), V_d(0)\}$, where

$$V_{opt,1}(B, 0) = \max_{0 \leq B' \leq \bar{B}(0)_{opt}} u(c, g) + \beta \left(pV(B', 1) + (1-p)\tilde{V}(B', 0) \right)$$

$$\text{s.t.} \quad \begin{aligned} g + \kappa B &= \theta A\bar{y} + \tilde{q}_{opt}(B', 0)(B' - (1-\delta)B), \\ c &= (1-\theta)A\bar{y} \end{aligned}$$

$$V_{opt,2}(B, 0) = \max_{\bar{B}(0)_{opt} < B' \leq \bar{B}(1)} u(c, g) + \beta \left(pV(B', 1) + (1-p)V_d(0) \right)$$

$$\text{s.t.} \quad \begin{aligned} g + \kappa B &= \theta A\bar{y} + \beta p(B' - (1-\delta)B), \\ c &= (1-\theta)A\bar{y} \end{aligned}$$

4. Derive a new value of $\bar{B}(0)_{opt}$ by solving equation below

$$V(\bar{B}(0)_{opt,new}, 0) = V_d(0)$$

5. Update bond price function and compute the error. New values of $q_{opt}(B', 0)$ are

$$q_{opt}(B', 0) = \begin{cases} \beta \left(p + (1-p)(\kappa + (1-\delta)\tilde{q}_{opt}(B'(B', 0), 0)) \right) & \text{if } 0 \leq B' \leq \bar{B}(0)_{opt} \\ \beta p & \text{if } \bar{B}(0)_{opt} < B' \leq \bar{B}(1) \\ 0 & \text{if } \bar{B}(1) < B' \end{cases}$$

6. If $\max_{B'} |\tilde{q}_{opt}(B', 0) - q_{opt}(B', 0)| > \epsilon$ or/and $|\bar{B}(0)_{opt} - \bar{B}(0)_{opt,new}| > \epsilon$, then $\tilde{q}_{opt}(B', 0) = q_{opt}(B', 0)$ and $\bar{B}(0)_{opt} = \bar{B}(0)_{opt,new}$, and go back to 3. Otherwise, we finish deriving $\bar{B}(0)_{opt}$, $q_{opt}(B', 0)$, and $V(B, 0)$ in an optimistic world.

A.1.2 Pessimistic

1. Repeat A.1.1 step 1.

2. Compute the government's utility and policy function given bond price $q(B', 0) = \beta p$, denoted as $V_{pes}(B, 0)$ and $B'_{pes}(B, 0)$, respectively.

$$\begin{aligned} V_{pes}(B, 0) = \max_{0 \leq B' \leq \bar{B}(1)} & u(c, g) + \beta \left(pV(B, 1) + (1-p)V_d(0) \right) \\ \text{s.t.} & g + \kappa B = \theta A\bar{y} + \beta p(B' - (1-\delta)B), \\ & c = (1-\theta)A\bar{y} \end{aligned}$$

3. Derive $\bar{B}(0)_{pes}$ by solving equation below.

$$V_{pes}(\bar{B}(0)_{pes}, 0) = V_d(0)$$

Check whether $B'_{pes}(\bar{B}(0)_{pes}, 0) > \bar{B}(0)_{pes}$. If not, pessimistic expectations are not self-fulfilling. Note that, different from $\bar{B}(0)_{opt}$, we can derive $\bar{B}(0)_{pes}$ without guessing.

4. Derive B_N by solving equation below.

$$B_N = \sup_B \{ B'_{pes}(B, 0) \leq \bar{B}(0)_{pes} \}$$

5. Guess the bond price function $\tilde{q}_{pes}(B', 0)$ in a recession.

6. Given the bond price function $\tilde{q}_{pes}(B', 0)$, guess value function in a pessimistic world $\tilde{V}(B, 0)$. Perform value function iteration and update initial guess until it satisfies convergence criterion $\max_B |V(B, 0) - \tilde{V}(B, 0)| < \epsilon$.

$$V(B, a = 0) = \begin{cases} \max \{V_{safe}(B, 0), V_{pes}(B, 0), V_d(0)\} & \text{if } 0 \leq B \leq B_N \\ V_{pes}(B, 0) & \text{if } B_N < B \leq \bar{B}(0)_{pes} \\ V_d(0) & \text{if } \bar{B}(0)_{pes} < B \end{cases}$$

where

$$\begin{aligned} V_{safe}(B, 0) = & \max_{0 \leq B' \leq B_N} u(c, g) + \beta \left(pV(B', 1) + (1-p)\tilde{V}(B', 0) \right) \\ \text{s.t.} & \quad g + \kappa B = \theta A\bar{y} + \tilde{q}_{pes}(B', 0)(B' - (1-\delta)B), \\ & \quad c = (1-\theta)A\bar{y} \end{aligned}$$

7. Update bond price function and compute the error. New values of $q_{pes}(B', 0)$ are

$$q_{pes}(B', 0) = \begin{cases} \beta \left(p + (1-p)(\kappa + (1-\delta)\tilde{q}_{pes}(B'(B', 0), 0)) \right) & \text{if } 0 \leq B' \leq B_N \\ \beta \left(p + (1-p)(\kappa + (1-\delta)\beta p) \right) & \text{if } B_N < B' \leq \bar{B}(0)_{pes} \\ \beta p & \text{if } \bar{B}(0)_{pes} < B' \leq \bar{B}(1) \\ 0 & \text{if } \bar{B}(1) < B' \end{cases}$$

8. If $\max_{B'} |\tilde{q}_{pes}(B', 0) - q_{pes}(B', 0)| > \epsilon$, then $\tilde{q}_{pes}(B', 0) = q_{pes}(B', 0)$, and go back to 6. Otherwise, we finish deriving $V(B, 0)$ in a pessimistic world and $q_{pes}(B', 0)$.

A.2 Debt limit

A.2.1 Optimistic

1. Derive the debt threshold in normal times. $\bar{B}(1)$ can be characterized by

$$\bar{B}(1) = \frac{\theta\bar{y} - \bar{g}}{1 - \beta}$$

2. Derive the debt threshold in a recession.

$$\bar{B}(0)_{opt} = \max \left\{ \frac{\theta A\bar{y} - \bar{g}}{1 - \beta}, \frac{\theta A\bar{y} - \bar{g} + \beta p \bar{B}(1)}{1 - \beta(1-p)(1-\delta)} \right\}$$

3. REWRITE The methodology of deriving the government's policy choice and

bond price function is similar to the baseline framework. The main difference here is that debt thresholds are already given, and the utility of every finite grid point must be larger than the utility of defaulting. For instance, we can simply set $V_d(0) = -99999999$ such that it is never optimal to default strategically in a simulation.

A.2.2 Pessimistic

1. Repeat A.2.1 step 1.
2. Derive the debt threshold in a recession.

$$\bar{B}(0)_{pes} = \frac{\theta A \bar{y} - \bar{g} + \beta p \bar{B}(1)}{1 - \beta(1-p)(1-\delta)}$$

3. Set $V_d(0) = -99999999$ and use the debt thresholds we derived at stage 1 and 2 to solve the equilibrium.

B The ‘Cliff effect’ in welfare due to self-fulfilling crises

Figure 13 depicts the government’s value function in a pessimistic world for $\delta = 0.2$. We dub ‘cliff’ the discontinuity in the value function at the debt threshold. A cliff is apparent at B_N . At this threshold, the possibility of self-fulfilling crises due to pessimistic expectations causes utility to deteriorate by a significant amount. In a sunspot equilibrium, this loss of utility may motivate the government to deleverage and keep debt below B_N . Note that, however, there is no cliff around the other threshold - a significant utility incentive for the government to deleverage only exists around B_N .

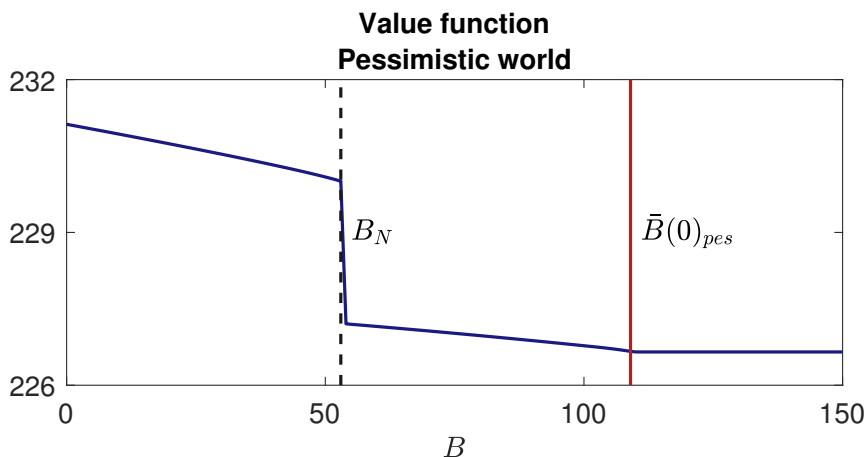


Figure 13: Cliff effect ($\delta = 0.2$)

C Debt evolution in baseline sunspot equilibria

Figure 14 displays debt path starting from $B = 75$ in sunspot equilibria. The government accumulates debt over time as long as the sunspot event does not occur. Debt accumulation is slower when debt level is below $\bar{B}(0)_{pes}$, whereas it accelerates after sovereign debt enters “fast” crises zone, above $\bar{B}(0)_{pes}$.

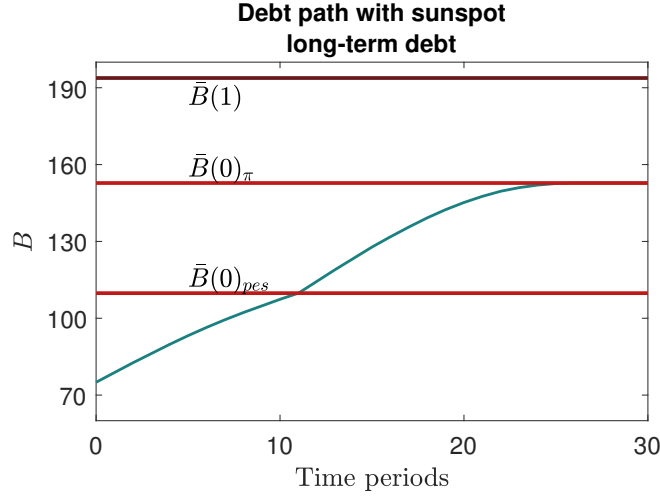


Figure 14: Debt path starting from $B = 75$

D Deep recessions in a debt limit framework

In a deep recession, the government may only be able to sustain \bar{g} via borrowing. In other words, $\bar{g} > \theta A \bar{y}$. This case is shown in Figure 15 where $A = 0.8$. The figure shows the path of optimal debt accumulation over time, contrasting the economy with long-term bonds (left) and one-period bonds (right). The initial debt level is set to 0 in both panels. In either case, the government keeps increasing its debt towards unsustainable levels (depending on the persistence of recession) until the economy recovers.

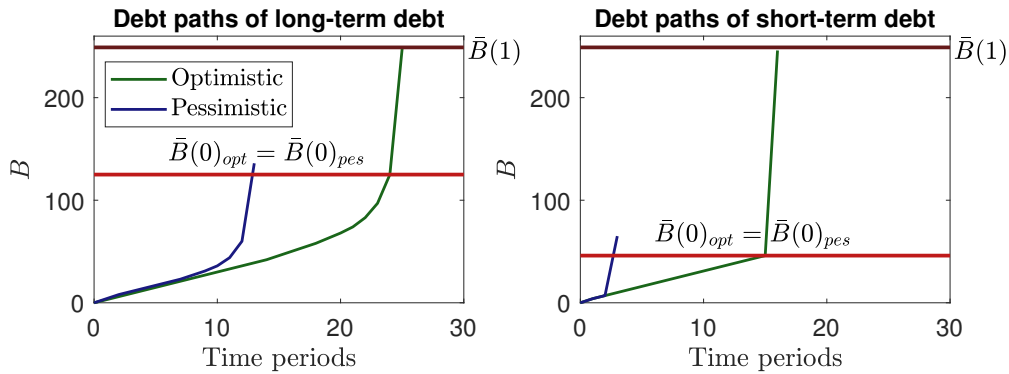


Figure 15: Deep recession $A = 0.8$

Note that the government accumulates debt faster, and defaults earlier, in a pessimistic world. Comparing the two panels also shows that, when debt is short-term, the debt tolerance threshold in a recession $\bar{B}(0)$ is lower and thus the government ends up gambling on the recovery earlier.

E Sunspot equilibria in a Debt Limit framework

In a debt limit framework, a sunspot equilibrium modifies our previous analysis in two respects. First, when government bonds are long-term, at intermediate level of debt, there is an acceleration of debt accumulation. Second, when debt is short-term, debt thresholds become sensitive to the probability attributed to the sunspot—they shift at low values of these probabilities.

In Figure 16, we display the bond price function and debt accumulation in the time domain for two different levels of debt in our economy with long-term bonds. Each panel illustrates both the optimistic equilibrium and the sunspot equilibrium. We omit the policy function from the graph because this is visually very close to an optimistic world in Figure 9.

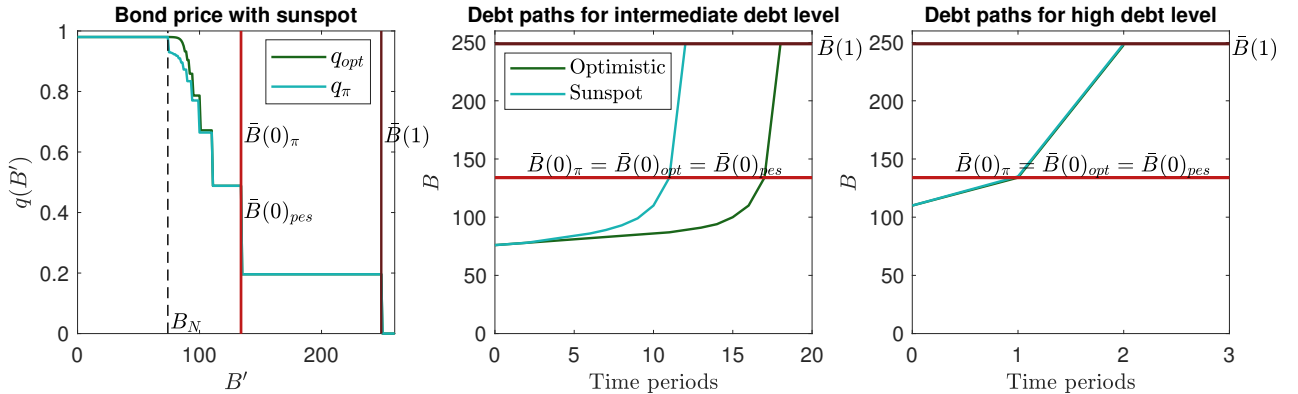


Figure 16: Policy function and bond price function ($\delta = 0.2$)

The center and right panels of Figure 16 clarify the main difference between the optimistic and the sunspot equilibria. In both panels, default may occur with positive probability, but the center panel starts from a moderate debt level ($B = 76$), while the right panel starts from a high debt level ($B = 110$).

The sunspot equilibrium makes a difference only for the case in the center panel of Figure 16: the government accumulates debt faster in a sunspot world. When the economy is exposed to sunspot crises, the government has to pay higher spread at intermediate level of debt. This accelerates debt accumulation: as debt crises arrive earlier, the spread rises even further, larger than $\pi = 4\%$ for B' close to B_N in the left panel of Figure 16.

When debt is high enough, pricing in the sunspot equilibrium is less crucial. The right panel of Figure 16 shows that the debt paths are identical in both the sunspot

and optimistic equilibria. Intuitively, investors may turn pessimistic at $T = 1$, but the government always chooses risky-debt high-debt issuance strategy at $T = 1$ regardless of investors' belief (multiplicity does not exist for high debt levels, see Figure 9). As a result, the sunspot is immaterial for the equilibrium.

The economy with one-period debt features different debt dynamics. Strikingly, $\bar{B}(0)_\pi$ coincides with $\bar{B}(0)_{pes}$: in the sunspot equilibrium, the debt tolerance threshold shrinks towards $\bar{B}(0)_{pes}$ with one-period debt. We find that for any π above 1% (consistent with the policy function), the government always issues default-free debt up to $\bar{B}(0)_{pes}$.