An Improved Multiobjective Optimization Evolutionary Algorithm Based on Decomposition with Hybrid Penalty Scheme

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ABSTRACT

The multiobjective evolutionary algorithm based on decomposition (MOEA/D) decomposes a multiobjective optimization problem (MOP) into a number of single-objective subproblems. Penalty boundary intersection (PBI) in MOEA/D is one of the most popular decomposition approaches and has attracted significant attention. In this paper, we investigate two recent improvements on PBI, i.e. adaptive penalty scheme (APS) and subproblem-based penalty scheme (SPS), and demonstrate their strengths and weaknesses. Based on the observations, we further propose a hybrid penalty sheme (HPS), which adjusts the PBI penalty factor for each subproblem in two phases, to ensure the diversity of boundary solutions and good distribution of intermediate solutions. HPS specifies a distinct penalty value for each subproblem according to its weight vector. All the penalty values of suboroblems increase with the same gradient during the first phase, and they are kept unchanged during the second phase.

CCS CONCEPTS

• **Theory of computation** → **Evolutionary algorithms**; • **Computing methodologies** → **Optimization algorithms**;

KEYWORDS

decomposition, multiobjective evolutionary algorithm, penalty boundary intersection, adaptive penalty scheme, subproblem-based penalty scheme, hybrid penalty scheme

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1 INTRODUCTION

Some problems in real-world applications from manufacturing to economics are multi-objective optimization problems (MOPs). Due to the nature of parallelism, evolutionary algorithms (EAs) can result in multiple Pareto solutions in a single run. Three traditional decomposition approaches were proposed in MOEA/D [2], namely weighted sum (WS), the Tchebycheff (TCH), penalty-based boundary intersection (PBI). In PBI, the penalty parameter θ is intended to balance convergence and diversity. A small θ favours convergence whereas a large one favours diversity. The original PBI approach used a fixed penalty parameter θ, which is not always suitable for different MOPs. Yang et al. [1] proposed two newly penalty schemes, i.e., adaptive penalty scheme(APS) and subproblem-based penalty scheme(SPS). In APS, every subproblem is assigned the identical penalty value, and the penalty value θ is adapted by a generationally increasing function. Conversely, in SPS the penalty value θ for each subproblem is set differently, and it does not change in the course of evolution. The observations of SPS and APS clearly demonstrate that 1) subproblems should be treated differently regarding the penalty parameter and 2) the penalty parameter should be adaptively adjusted to balance between diversity and convergence at different search stages. Therefore, it is naturally straightforward to put forward a two-phase hybrid penalty scheme (HPS) that inherits the advantages of both APS and SPS.

2 HYBRID PENALTY SCHEME FOR MOEA/D WITH PBI

2.1 Hybrid Penalty Scheme (HPS)

The new HPS we propose is a hybridization of APS and SPS. HPS intends to treat the penalty parameters for all the subproblems differently, and increase the value for each penalty parameter gradually until the resulting penalties are sufficiently enough to guarantee good distribution of solutions to all the subproblems.

For simplicity, we use the same change of penalty (i.e. δ, which is recommended to be 10 in this paper based on our preliminary tests) for all subproblems. In addition, we allow the penalty parameter θ_i to undergo such change for at most two thirds of the total computation resources. This is because we do not want very big penalty values at the end which leave little to none room for convergence. This is particularly true for difficult problems where algorithms converge very slowly. It may be argued that a small change of penalty θ can avoid large penalty values in the end. However, a small θ value is not a good choice as it may result in insufficient penalties for subpropblems to keep population well diversified at some point of evolution.

For these reasons, HPS is implemented in two phases during the evolution. In the first phase, each subproblem i is initialized with a different penalty parameter θ_i ($\theta_i = 0.9e^{\alpha\beta_i}$) in which β_i is associated wtih the corresponding scalar vector w_i. Then, the penalty parameter θ_i increases linearly. In the second phase, the penalty value (θ_i) of each subproblem i is fixed at a certain value. θ_i is defined as

$$\theta_i = \begin{cases} 0.9e^{\alpha\beta_i} + \frac{t}{\frac{2}{3}Iter_{max}}\delta & t \leq \frac{2}{3}Iter_{max} \\ 0.9e^{\alpha\beta_i} + \delta & t > \frac{2}{3}Iter_{max} \end{cases} \tag{1}$$

$$\beta_i = \max_{1=<j<=M} w_i^j - \min_{1=<j<=M} w_i^j$$

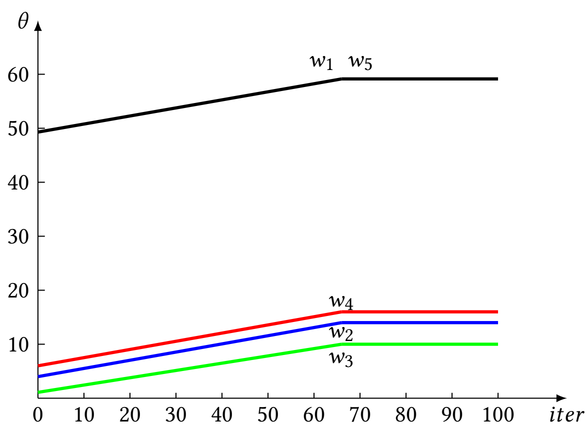

Figure 1: Illustration of the change of penalty θ for different subproblems.

To demonstrate how their corresponding penalty values change in the new scheme HPS, we randomly choose five scalar vectors, w_1=(1, 0), w_2=(0.6970, 0.3030), w_3=(0.5050, 0.4949), w_4=(0.2929, 0.7071), w_5=(0, 1). The results calculated by formula (1) are plotted in Fig.1. The penalty parameter for each subproblem increases linearly in the first phase and then levels out. This means the evolution for each subproblem favours convergence at first and gradually emphasises diversity. It is also easy to see the difference between the boundary subproblems (w_1 and w_5) and the intermediate subproblems (w_2 –w_4). The boundary subproblems are always given bigger penalties in order to ensure true boundary POF points can be correctly identified. Compared with boundary subproblems, the intermediate subproblems have lower penalty values in the hope of faster approximation to the POF.

2.2 The Framework of MOEA/D with HPS

For completeness, we present the framework of MOEA/D with HPS in **Algorithm 1**. This framework extends the MOEA/D algorithm by adding a step of penalty calculation for each subproblem in every generation.

Algorithm 1 MOEA/D-HPS

Input:
- $MaxIteration$: the stopping criterion
- N: the number of subproblems considered in MOEA/D
- T: the neighbourhood size

Output: approximated Pareto-optimal set

Initialization: Generate a uniform spread of N weight vectors: $w^1, \ldots, w^N$ and then compute the T closest weight vectors to each weight vector by the Euclidean distance. For each $i = 1, \ldots, N$, set $B(i) = \{i_1, \ldots, i_T\}$ where $w^{i_1}, \ldots, w^{i_T}$ are the T closest weight vectors to w^i

Generate an initial population $P = \{x^1, \ldots, x^N\}$ by uniformly randomly sampling from the decision space

$t = 0$

while gen := 1 to $MaxIteration$ **do**
  for i := 1 to N **do**
    Calculate θ_i by formula (1)
  end for
  for i := 1 to N **do**
    Randomly select two indexes r_1 and r_2 from $B(i)$
    Apply genetic operators on individuals r_1, r_2 to produce a new solution **y**
    Calculate PBI values of **y** and each individual **x** in $B(i)$, respectively.
    If **y** is better than any individual **x** in $B(i)$ ($g^{pbi}(\mathbf{y}|w^j, z^*) \leq g^{pbi}(\mathbf{x}|w^j, z^*)$), then **x** is replaced by **y**
  end for
end while
Output P

3 CONCLUSION

In this paper, we propose a two-phase hybrid penalty scheme (HPS). HPS not only specifies distinct penalty values for different subproblems but also adaptively adjust the penalty values in the first phase. Then in the second phase, HPS keeps slightly large penalty values for all the subproblems so that better diversity is obtained.

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