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WORLD MARITIME UNIVERSITY
MALMö, SWEDEN

NAVIGATION, SHIP’S STABILITY AND DYNAMICS:

A COMPUTATIONAL APPROACH

by

Jean Louis Taty-Boussiana

Congo

A paper submitted to the Faculty of the World Maritime University in partial satisfaction of the requirements for the award of a

MASTER OF SCIENCE DEGREE
in
MARITIME EDUCATION AND TRAINING (NAUTICAL).

The contents of this paper reflect my personal views and are not necessarily endorsed by the UNIVERSITY.

Signature:

Date: 30 October 1990

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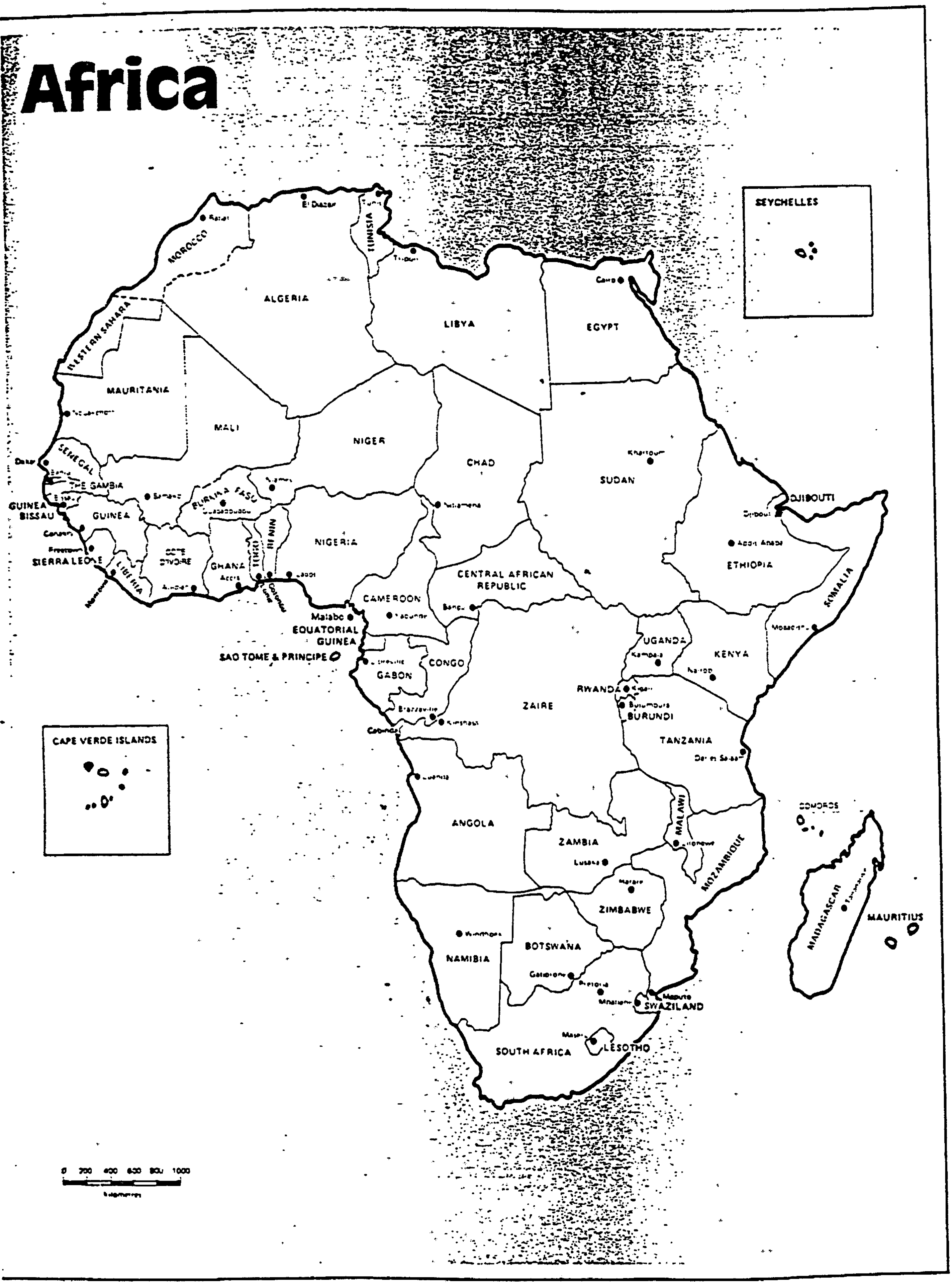
Lecturer
World Maritime University

Co-assessed by:

B. Berking

Prof. Dr. Bernhard Berking
Dept. of Maritime Studies
Hamburg Polytechnic
FRG

Africa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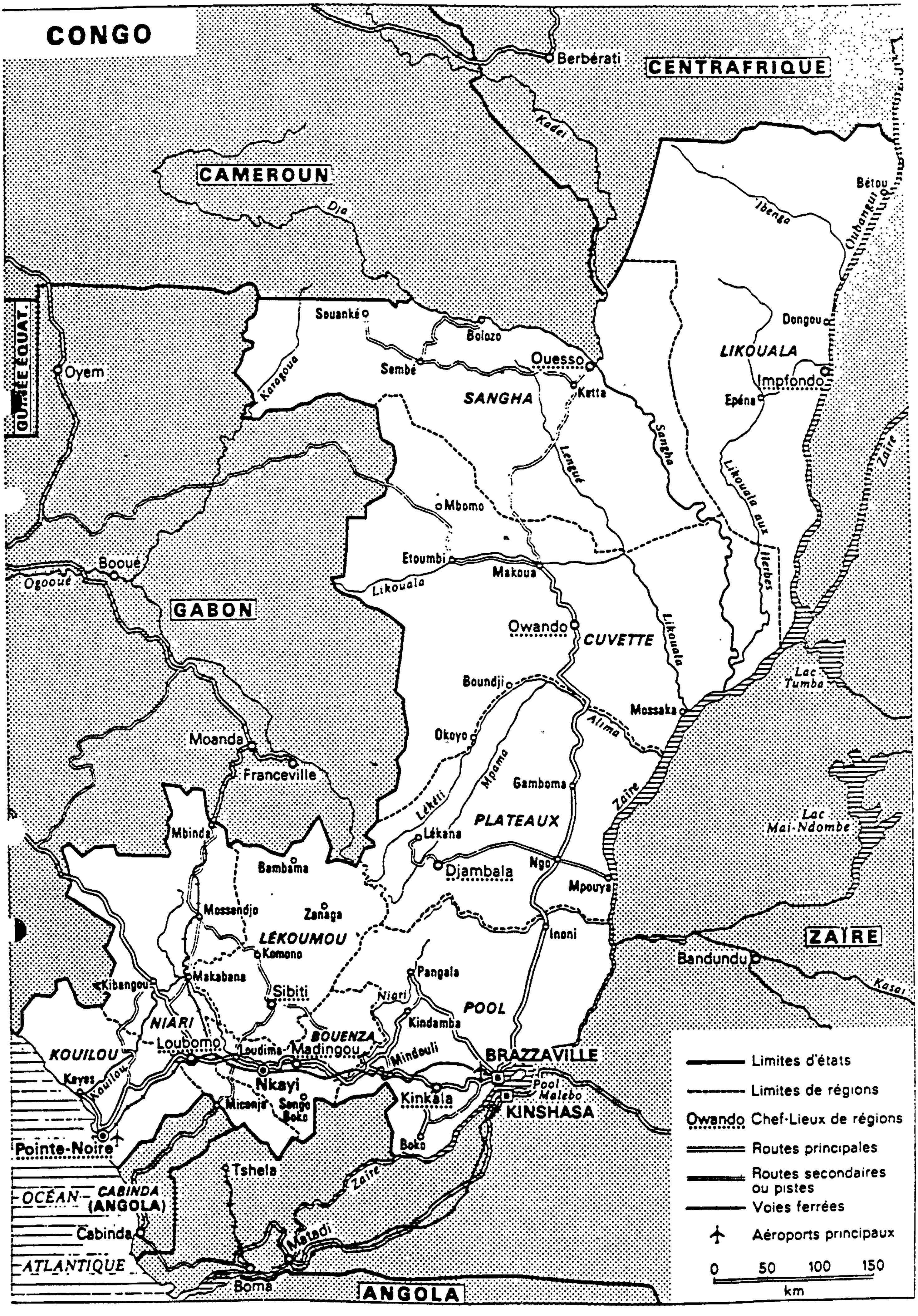
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Likouala aux Herbes
Zaïre
Mbomo
Etoumbi
Makoua
Booué
Ogooué
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CUVETTE
Likouala
Boundji
Lac Tumba
Mossaka
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Franceville
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Mpama
Lékéti
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Mbinda
Lékana
PLATEAUX
Bambama
Djambala
Ngo
Mpouya
Mossendjo
Zanaga
LÉKOUMOU
Inoni
ZAÏRE
Komono
Bandundu
Kibangou
Makabana
Pangala
Kasaï
Sibiti
Niari
POOL
NIARI
Kindamba
KOUILOU
Loubomo
BOUENZA
Loudima
Madingou
Mindouli
BRAZZAVILLE
Kayes
Kouilou
Nkayi
Pool Malebo
Kinkala
Miconje
Sengo Boko
KINSHASA
Pointe-Noire
Boko
Tshela
Zaïre
OCÉAN
CABINDA (ANGOLA)
Cabinda
Matadi
ATLANTIQUE
Boma
ANGOLA
Limites d'états
Limites de régions
Owando Chef-Lieux de régions
Routes principales
Routes secondaires ou pistes
Voies ferrées
Aéroports principaux
0 50 100 150
km

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CHAPTER 5 : STABILITY AT LARGE ANGLES.

CHAPTER 6 : ROLL MOTION.

CHAPTER 7 : VOYAGE PLANNING.

DEDICATION.

This work is dedicated to :

- My father Jean Félix TATHY
- My mother Célestine FOUTOU
- My unique brother Jean de Dieu and my seven sisters Cathérine, Joséphine, Cécile, Claire, Germaine, Yvette, Hortense
- My two sons Charisma and Hélios.

ACKNOWLEDGEMENTS.

The completion of this study would not have been possible without contributions from many individuals and organizations

I am indebted to the United Nations Development Programme UNDP for its generosity in sponsoring my studies at the World Maritime University.

My sincere and profond gratitude also goes to the Congolese Ministry of Transport and Civil Aviation for nominating me for this course.

Special thanks to:

- Professor Zade for his encouragement and advice
- Professor Mulders and Lecturer van Walen for their guidance and constructive remarks
- Professor Berking my co-assessor for the final structure of this work
- Visiting Professors
- The staff of World Maritime University.

ABSTRACT.

This paper deals with the use of LOTUS 1-2-3 as a tool for calculations with particular reference to rhumb line sailing, great circle sailing, satellites for navigation, navigational errors, stability at large angles and roll motion.

The introduction deals with the purpose of this paper, its intentions and limitations and a chronological categorization of the tools used to perform navigational, ship stability and dynamics calculations.

This paper is divided into two parts :

- Part I which is an general overview of LOTUS 1-2-3.

- Part II which deals with applications

Chapter 1 of Part I includes a brief description of LOTUS, the general requirements of the system, the hardware and software assumptions, how to write programs, how to load LOTUS into a computer and the structure of the LOTUS worksheet.

Chapter 2 of Part I deals with the advantages and benefits of LOTUS, its limitations and shortcomings, its applications in maritime education and training, the didactical philosophy and the pedagogical features of this paper.
LOTUS commands may be found in Appendix A.

In Part II,

Chapter 1 deals with calculations for rhumb line track, distance and the coordinates of the point of destination.
Appendices B and C give the listing of the programs .

Chapter 2 deals with calculations related to great circle distance, track, coordinates of the vertex and composite sailing.
Appendix D gives the listing of the program.

Chapter 3 deals with calculations for the user position, the diameter of circle of coverage, time of satellite's passage above the observer position, maximum angle of elevation, number of passes of the same satellite over the stationary observer's position during 24 hours, observed pass duration, distance from the user's position to satellite, position determination and accuracy by the Least Square Method, Transit accuracy when practicing satellite navigation.
The listing of the program is given in Appendix E.

Chapter 4 deals with navigational errors.
Calculations are related to variances on the ground course, ground speed, Dead Reckoning Position and Transferred LOP accuracies, time allowed for Dead Reckoning, safe distance from

danger and accuracy of the Possible Point of Collision PPC in ARPA .
IMO requirements.
The listing of the program is given in Appendix F.

Chapter 5 deals with stability at large angles.
Calculations are made for the effect of vertical and transverse corrections on the righting arm, metacentric height , free surface correction, righting arm from KN curves and area under the stability curve.
IMO requirements.
The listing of the program may be found in Appendix G.

Chapter 6 deals with roll motion .
Calculations are made for roll period , roll angle and encounter period of waves versus speed.
IMO requirements.
Appendix H gives the listing of the program.

Chapter 7 deals with voyage planning.
Exercises are proposed for voyage planning with emphasis on use of documents, analysis and decisions to be undertaken according to the IMO requirements.

A description of the organizational structure of the programs is included.

The results are compared to those obtained from the listed references.

The appendices give more details and information on the mathematical models and the practical requirements implemented in the programs.

INTRODUCTION.

For training purposes, the author proposes an approach for navigational, ship stability and dynamics calculations with particular reference to rhumb line sailing, great circle sailing, satellites for navigation, navigational errors, stability at large angles and roll motion.

Navigational and ship stability and dynamics calculations are two important elements of the curricula of maritime academies, regardless the implemented maritime education and training systems.

This importance finds its source in the required high accuracy of the tools (or methods) used for such calculations.

1. Categorization of the computational tools.

The abovementioned tools can be chronologically categorized into four groups :

- traditional mathematical method
- scientific and programmmable calculators
- non intelligent computers
- " intelligent " computers

The development and the improvement of such tools are the best illustration of the need of high accuracy for a safer navigation as far as position determination and stability are concerned.

1.1. The traditional mathematical method.

The problems solved in many engineering applications are mathematically defined problems.

The formulae are rather complex, the calculations are laborious, time consuming and source of unimaginable errors.

For position determination and stability calculations, tables of navigation and logarithms and graphs based on a mathematical formulation were developped by mathematicians navigators and naval architects.

The advantage offered by the mathematical solution of the problems involved in maritime education and training is essentially the development of intellectual skills added to the sea experience of the user.

In this context, the computer based method should be preferred to the traditional mathematical method because no mathematical background is required for the user in other words, the user doesn't have to be concerned about the algorithm.

The deficiencies of this method are mainly due to the psychological state of the user when handling long formulae and intrapolating between numerical values from tables and graphs. Loss of concentration, boredom, lack of motivation, time constraints and other similar disturbances are source of bad accuracy.

Despite the variety of pedagogical procedures used for the training of the students in that method, the human intelligence has its limits and the performances of the artificial methods (calculators and computers) fight in favour of the phase out of the traditional method.

1.2. Scientific and Programmable calculators.

With the introduction of these tools, the work and the performances of the student have been facilitated and the accuracy of the results has been improved.

But the number of decimals of the numerical results give a wrong idea of the accuracy for practical use mainly in case of scientific (non programmable) calculators. The user is obliged to make extrapolation which leads to errors.

The progammable calculators offer a better accuracy than their predecessors but they have been progressively replaced by computers because of their lower capabilities in terms of completeness, user-friendliness, documentation data base, reliability and portability.

These artificial methods have also their deficiencies characterized by erroneous outputs when the programs are designed by non sea experienced programmers.

The accuracy of the outputs is also dependent upon inputs from the user and psychological state of the user as mentioned for the traditional mathematical method.

Decision making is not a feature of the present non intelligent computers despite their higher level of sophistication and the software packages used.

But the benefits from the computers are without any doubt considerable as far as accuracy and time saving are concerned.

Their performances are higher than those which can be achieved by the human intelligence.

1.3. Computers.

Computer applications in maritime education and training can be listed as follows :

- Aid for the management of maritime academies

  The computer assists in the management of the maritime academy and in the management of the learning: student information, scheduling, wordprocessing and management information.

- Computer Supported Instruction

  The computer is used as a tool for calculations, for visualizing results, with simulation languages and with Computer Aided Design / Computer Aided Manufacture.

- Computer Assisted Testing

  The computer is used for item generation, creation of tests and scoring and analysis.

- Computer Managed Instruction

  The computer is used for registration, testing, prescription and scheduling

- Computer Assisted Instruction

  The computer is used for programmed instruction, tutorial, drill and practice, dialogue and socratic dialogue.

- Computer Assisted Learning

  The computer is used for simulation, modelling, gaming and problem solving.

- Intelligent Computed Assisted Instruction with the aid of expert systems.

An extensive use of the mathematical procedure programmed in BASIC, FORTRAN and other sophisticated languages has already been made in maritime education and training and on board merchant ships.

This paper will also show that it is possible to use an simple and unsophisticated computer based method as a tool to perform navigational, ship's stability and dynamics calculations.

This tool is called LOTUS 1-2-3 Release 2 .

PART I :

GENERAL OVERVIEW OF LOTUS 1-2-3.

CHAPTER 1 :

REVIEW OF LOTUS 1-2-3.

2.1. What is LOTUS 1-2-3 ?

LOTUS 1-2-3 is a user friendly integrated software package offering three applications: electronic spreadsheets, graphics and database management.

LOTUS 1-2-3 offers a thorough variety of simple mathematical functions and advanced functions which are used in engineering problems.

2.2. General system requirements.

To work with LOTUS 1-2-3 the user will need the following microcomputer hardware and software:

- an IBM or IBM compatible microcomputer

- dual floppy disk drives, or one dual floppy with a hard disk

- monitor and keyboard

- printer (optional)

- a LOTUS 1-2-3 system disk (dual floppy disk drive systems only; it is assumed that hard disk users have LOTUS 1-2-3 installed on the hard disk).

- one data diskette (dual floppy disk drive systems only; it is assumed that hard disk users will store LOTUS 1-2-3 files on the hard disk).

2.3. Hardware/Software assumptions.

In this paper, the following microcomputer hardware and software are used:

- IBM PC with dual floppy disk drives

- Educational version of LOTUS 1-2-3.

2.4. How to write programs ?

The flow of problem solving and program design is simply:

Step 1. Definition of the task

Step 2. Mathematical model
Check the solution strategy and make changes if necessary

Step 3. Write the program on paper
Check the proposed program and make changes if necessary

Step 4. Enter the program to the computer
Compare the results to those obtained by other methods.

This procedure is also applied when programming with other computer based methods (PASCAL, TURBO PASCAL, FORTRAN...)

2.5. How to load LOTUS 1-2-3 into the computer ?

To load LOTUS 1-2-3 into the computer, the following steps are recommended:

Step 1. Insert a DOS disk in drive A

Step 2. Turn on the monitor, computer and printer

Step 3. At the DOS - prompt, remove the DOS disk from drive A and replace it with the LOTUS 1-2-3 system disk

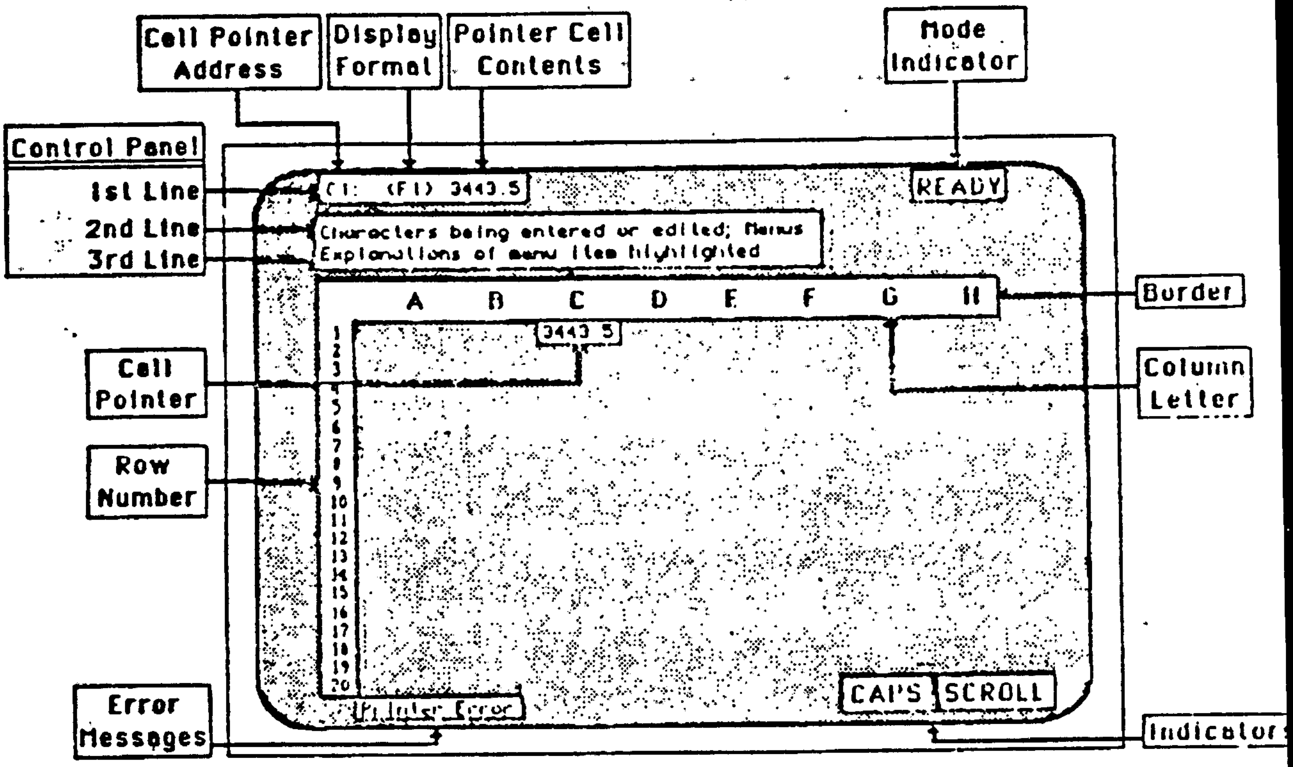
Step 4. Dual floppy disk users only, position a formatted diskette for data files in drive B

Step 5. At the operating system prompt to access a blank LOTUS 1-2-3 worksheet, type 123 and press Enter. To obtain the menu, press / then Enter.

Details on the various commands of LOTUS 1-2-3 may be found in Appendix A.

2.6. Structure of LOTUS worksheet. Ref 19.

Parts of the
Lotus 1-2-3 Screen



CHAPTER 2 . :

WHY LOTUS 1-2-3 ?

3.1. Advantages and benefits.

The advantages of the LOTUS 1-2-3 method may be seen in

- its simplicity compared to the most used languages for engineering problems such as BASIC, FORTRAN, PASCAL.

  From the listing of the program, the user can easily understand the mathematical theory implemented in the algorithm in other words the cells formulae.

- the user interface

  The program is self explanatory, the student understands the on screen presentation and can proceed without confusion or frustation.

  By its readability, the program provides opportunities to be enhanced and evaluative criteria to assess results

  The worksheets are organized so that the orientation of the user is facilitated : the inputs , calculations outputs are located and easily retrievable by their respective cells.

- the clarity of procedural and instructional statements

- the run time errors and syntax errors are also features of the proposed software.

- the availability of mathematical @ functions and @ string functions used for solving engineering problems

  Differential equations can be solved by using numerical integration methods mainly used in engineering applications.

  Simple and multiple integrals and their applications (areas, volumes, moments of inertia, center of gravity, shear forces and bending moments, power series and Fourier series), statistics and probability distributions, matrices can be calculated.

  The abovementioned list of mathematical applications is not exhaustive.

  Errors are avoided by using the copy command for long mathematical formulae.

- the reliability of the software package is illustrated by the results. A choice of number of decimals is possible by using the RFF command

- the cost of the program which is negligible

- the program can be used on several computer systems with different hardware characteristics
- large spreadsheets can be built with limited Random Access Memory RAM
- more information in spreadsheets and databases can be put by adding up to 4 Mb of RAM
- time can be saved with increased recalculation speed
- many new print features can be used
- files are saved and classified in an alphabetic order and can be password protected.

During a voyage, the different files can easily be retrieved by the watchkeeping officer.

3.2. What are the limitations and shortcomings ?

The angles used in trigonometric functions should be expressed in radians.

The software package gives erroneous results when the conversion of degrees to radians is omitted.

Image generation is not a feature of the software package in the field of electronic sea charts, stability plan, marine simulation, for instance.

For that purposes, the high level languages such as FORTRAN, PASCAL and TURBO PASCAL are recommended.

Decision making is the feature of expert systems.

3.3. What are the applications in maritime education and training ?

Because of the availability of mathematical functions, LOTUS 1-2-3 can be used in:

- Navigation
- Naval architecture
- Ship dynamics
- Cargo securing
- Marine Statistics
- Shipping Economics
- Meteorology
- Oceanography
- Geodesy
- Automation

- Electrotechnology

- Electronics

- Hydrodynamics

- Thermodynamics

- Applied Mechanics

- Strength of materials
  etc...

3.4. What is the didactical philosophy ?

It is not the intention of this paper

- to fight for the promotion of LOTUS 1-2-3

- to dissuade students from using other computer based methods or high level languages

- to transform the students into expert programmers in LOTUS

This paper has not the pretention to introduce a new method in teaching but it does have two aims.

Firstly, as the equipment of modern ships becomes more and more sophisticated and as the ship's officers become more and more involved in computer applications, so they are presented with more and more information in computerized form.

An important purpose of this paper is to develop some useful programs in order to enable the future officer to accept such information critically and to use it with understanding and confidence for making his professional decisions.

Secondly, the intention is that the student should become sufficiently acquainted with a simple method that he can apply it to make his own analysis of the more complicated problems with which he will be faced during his seafaring career .

At the same time, the limitations and shortcomings of the proposed method have been emphasised so that the student will appreciate where more advanced methods may be required and, may be, where the advice of an expert in programming may be necessary.

This approach is the first link in the chain of lectures that will be given to the students of the Académie Régionale des Sciences et Techniques de la Mer d'Abidjan (Côte d'Ivoire) in the field of navigation, ship's stability and dynamics.

3.5. Pedagogical features.

The best way to demonstrate the appropriateness of the proposed method for calculations related to navigation , ship stability and dynamics is to consider some practical topics relevant to maritime education and training studies : this is the subject matter of the following chapters.

Programs are written for calculations related to rhumb line and great circle sailings, satellites, navigational errors, stability at large angles and roll motion.

At the end of each chapter, a print out of the results of the programs is included for comparison to the results from the listed references.

The last chapter gives a list of exercises to enable the student to make his own programs assuming that the adequate theoretical background is provided in the previous chapters.

APPENDIX A :

LOTUS COMMANDS

Ref. 19

APPENDIX A: Lotus 1-2-3

Methods to Access Lotus 1-2-3

You may type either LOTUS or 123 from the DOS prompt to access the Lotus spreadsheet, database, and graphics capabilities.

Typing LOTUS from the DOS prompt produces a version of the Access menu as shown below. Once the Access Menu is displayed, select the 1-2-3 option to produce a blank spreadsheet on screen.

Lotus Version 1A

```
1-2-3  File Mgr  Disk Mgr  PrintGraph  Translate  Exit
```

Lotus Release 2 or 2.01

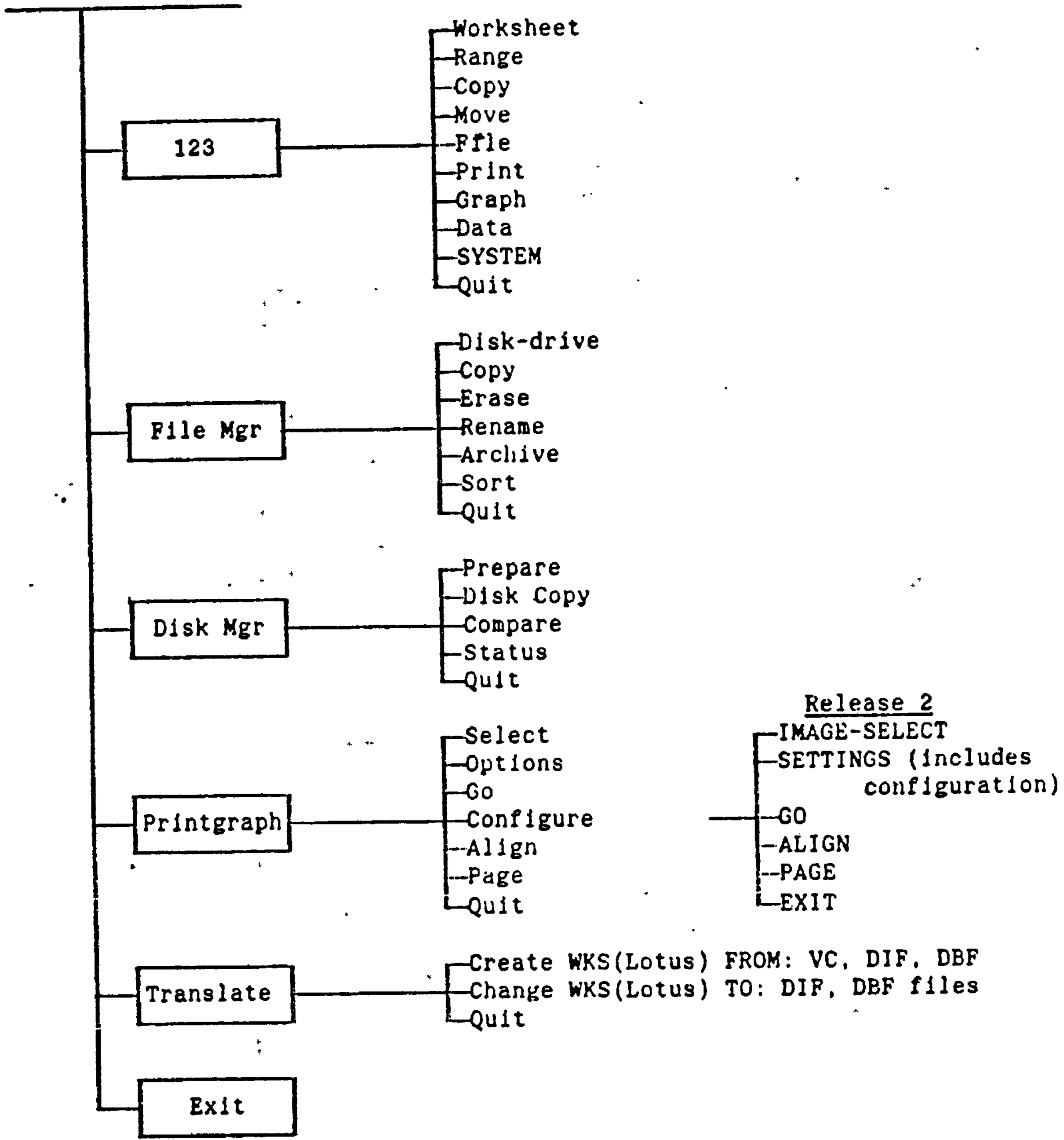
```
1-2-3  PrintGraph  Translate  Install  View  Exit
```

Typing 123 from the DOS prompt bypasses the Access menu and produces a blank spreadsheet on screen.

Lotus 1-2-3 Menu Structure

Menu options new to Release 2 are shown in CAPITAL letters throughout the menu structures displayed in this appendix; notes are provided concerning other Release 2 (2.01) changes.

LOTUS ACCESS MENU (Version 1A display; see notes below for Release 2.)



Release 2 Notes: Compare the Access menu displays for Version 1A and Release 2 on the previous page. For Release 2, the File-Manager and Disk-Manager options are deleted (the actions are now performed through DOS). A VIEW option is added to provide instant access to an on screen tutorial. An INSTALL option is added to provide step-by-step on screen instructions to install the program.

OVERVIEW OF THE 123 MAIN MENU

Worksheet	Initiate command sequence with impact on entire worksheet.
Range	Initiate command sequence with impact on specified portion of worksheet.
Copy	Duplicate specified range.
Move	Reposition specified range.
File	Initiate command sequence related to disk storage.
Print	Print now (or store for later printing) specified portion of worksheet or database.
Graph	Create, display, and store graphs.
Data	Create, maintain, and interpret database information.
SYSTEM	TEMPORARILY EXIT LOTUS TO THE DISK OPERATING SYSTEM (DOS).
Quit	Exit Lotus (no automatic save).

LOTUS 123 COMMAND SEQUENCES

123	
Worksheet	
Global	Controls overall settings.
Format	Appearance of data:
Fixed	Fixed # decimal places
Scientific	Exponential
Currency	$ and commas
,	Lacks $
General	Drop 0s after decimal pt
+/-	Pictograph of ++ or --
Percent	Times 100, show %
Date	
1	Day-Month-Year
2	Day-Month
3	Month-Year
4	MONTH-DAY-YEAR
5	MONTH-DAY
TIME	
1	HH:MM:SS AM/PM
2	HH:MM AM/PM
3	HH:MM:SS 24 HR
4	HH:MM 24 HR
Text	Show formula text
HIDDEN	SUPPRESS ALL DATA DISPLAY
Label-Prefix	Cell label alignment:
Left	Precede with '
Right	Precede with "
Center	Precede with ^
Column-Width	1-72 characters wide (1-240 in Release 2)
Recalculation	Formula recalc options:
Natural	In dependency order
Columnwise	Down columns/left to rt
Rowwise	Across rows/top to bottom
Automatic	Every worksheet change
Manual	Only when <Calc> key used
Iteration	# times recalc occurs

123 Commands Continued

Worksheet Commands Continued

Global Commands Continued

Protection	Access to protected cells:
Enable	Put protection on
Disable	Put protection off

Default	Printer and disk settings:
Printer	
Interface	Serial or parallel
Auto-LF	Line feed after <Return>
Left	Margin from left edge
Right	Rt. margin from left edge
Top	Margin from top
Bottom	Margin from bottom
Pg-Length	Total # lines/page
Wait	Pause ea. pg. change paper
Setup	Send start-up ctrl codes
NAME	Specifies printer to use
Quit	Return to DEFAULT submenu
Directory	Set current DIR @ startup
Status	Current default settings
Update	Record chg. default sets

OTHER

INTERNATIONAL (Non-USA formats.)
- PUNCTUATION
- CURRENCY
- DATE
- TIME
- QUIT

HELP (Onscreen availability.)
- INSTANT
- REMOVABLE

CLOCK (Date and time on screen.)
- STANDARD
- INTERNATIONAL
- NONE

Quit	Return to GLOBAL submenu.

123	Commands Continued			
	Worksheet	Commands Continued		
		Global	Concluded	
			ZERO	SPECIFY SUPPRESSION OF ZEROS.
		Insert		Insert blank rows or columns:
			Row	Insert row(s)
			Column	Insert column(s)
		Delete		Remove rows or columns:
			Row	Delete entire row(s)
			Column	Delete entire column(s)
		Column-width	(COLUMN in Release 2.)	Sets width of single column.
			Set	(SET-WIDTH in Release 2.)
			Reset	(RESET-WIDTH in Release 2.)
			HIDE	HIDE SPECIFIED COLUMN(S)
			DISPLAY	REDISPLAY HIDDEN COLUMN(S)
		Erase		Erase entire worksheet.
			No	
			Yes	
		Titles		"Lock-in" portion of screen:
			Both	Freeze rows and columns
			Horizontal	Rows above pointer
			Vertical	Columns left of pointer
			Clear	"Unlock" screen display
		Window		Split display screen:
			Horizontal	Split above pointer row
			Vertical	Split left of pointer column
			Sync	Windows scroll together
			Unsync	Independent scrolling
			Clear	Return to single screen

123						Commands Continued
	Worksheet					Commands Concluded
		Status				Display global settings.
		PAGE				PAGE-BREAK MARK FOR PRINTING.
	Range					
		Format				Appearance of data:
			Fixed			Fixed # decimal places
			Scientific			Exponential
			Currency			$ and commas
			,			Lacks $
			General			Drop 0s after decimal pt
			+/-			Pictograph of ++ or --
			Percent			Times 100, show %
			Date			
				1		Day-Month-Year
				2		Day-Month
				3		Month-Year
				4		MONTH-DAY-YEAR
				5		MONTH-DAY
				TIME		
					1	HH:MM:SS AM/PM
					2	HH:MM AM/PM
					3	HH:MM:SS 24 HR
					4	HH:MM 24 HR
			Text			Show formula text
			HIDDEN			SUPPRESS DATA IN A RANGE
			Reset			Restore default format
		Label-Prefix				Align existing labels - left, right, center options.
		Erase				Erase specified range.

123 Commands Continued

Range Commands Concluded . . . .

Name	Control named ranges:
Create	Define range address
Delete	Eliminate a range name
Labels	Name adjacent cell label (Right, Left, Up, and Down options.)
Reset	Eliminate ALL range names
TABLE	LISTS CURRENT RANGE NAMES
Justify	Limit width of text to specified column(s).
Protect	Restore capability for cell protection.
Unprotect	Remove capability for cell protection.
Input	Allows pointer movement to unprotected cells only.
VALUE	CONVERTS FORMULAS TO FIGURES.
TRANSPOSE	CONVERTS ROWS TO COLUMNS OR COLUMNS TO ROWS.
Copy	Duplicate range contents in another location.
Move	Move range contents to a new location.

123 Commands Continued

File			
	Retrieve		Call up worksheet (wks) from disk file.
	Save		Store current wks to disk.
	Combine		Bring file to current wks:
		Copy	Incoming cells replace
		Add	Overlapping values added
		Subtract	Overlapping values subtracted
	Xtract		Store PART of current wks:
		Formulas	Save formulas
		Values	Save current formula value
	Erase		Erase disk file(s):
		Worksheet	Erase (.WKS) file(s) (.WK1) in Release 2
		Print	Erase (.PRN) file(s)
		Graph	Erase (.PIC) file(s)
		OTHER	ERASE ANY FILE(S)
	List		List files/memory left:
		Worksheet	List (.WKS) files
		Print	List (.PRN) files
		Graph	List (.PIC) files
		OTHER	LIST ALL FILE NAMES
	Import		Read print file from disk:
		Text	Each line as a label
		Numbers	Numbers and "quoted" text
	Directory		Set current directory.

123 Commands Continued

Print

If a printer is on line: **Printer** OR To print later using a DOS TYPE command (Creates .PRN file): **File**

Range	Range to print.
Line	Advance printer a line.
Page	Advance to top next page.
Options	
—Header	Set up header line.
—Footer	Set up footer line.
—Margins	Space at paper edges.
Left	
Right	
Top	
Bottom	
—Borders	Portion printed on each page.
Columns	
Rows	
—Setup	Control codes to printer.
—Page-Length	# print lines/page.
—Other	
As-displayed	
Cell-formulas	Cell-by-cell listing for documentation.
Formatted	
Unformatted	Prints without page breaks, headers, or footers.
—Quit	Return main print menu.
Clear	Reset print options:
—All	All settings to default.
—Range	"Forget" print range.
—Borders	Eliminate borders.
—Format	Restores default settings.
Align	After manual paper positioning, tells printer top of page.
Go	Execute Print command.
Quit	Return to READY mode.

123	Commands Continued
Graph	
Type	Select graph style.
Line	
Bar	
XY	
Stacked-bar	
Pie	
X	Specify X range in XY graph or position labels below x-axis.
ABCDEF	Define up to six data-ranges.
Reset	Cancels graph ranges:
Graph	Clear ALL settings.
XABCDEF	Clear choice of settings.
Quit	Return to main graph menu.
View	Display current graph.
Save	Save current as (.PIC).
Options	Set graph options.
Legend	Assigns description of ABCDEF color or crosshatching.
Format	Display data connects/points.
Graph or ABCDEF	
Lines	Use lines only.
Symbols	Use symbols only.
Both	Both lines/symbols.
Neither	Neither lines/symbols.
Quit	Return to graph options menu.
Titles	

123 Commands Continued

Graph Continued

Options Concluded

Titles	Set graph/axes titles.
—First	Top middle of page graph title.
—Second	Allows 2nd line in title.
—X-axis	Title horizontal axis.
—Y-axis	Title vertical axis.
Grid	Display grid over graph.
—Horizontal	
—Vertical	
—Both	
—Clear	
Scale	Set display scale of axes.
—X-axis OR Y-axis	
Automatic	Restores default setting of Lotus-determined scale.
Manual	User-determined scale.
Lower	With manual, set lower scale.
Upper	With manual, set upper scale.
Format	Same options as Range FORMAT.
INDICATOR	DISPLAY/NOT DISPLAY SCALE INDICATORS ASSIGNED BY LOTUS.
Quit	Return options submenu.
—Skip	Skip factor-X labels.
Color	Display in color.
B&W	Display in black & white.
Data-Labels	Label data points within the graph for ABCDEF.
—Center	
—Left	
—Above	
—Right	
—Below	
Quit	Return to main graph menu.

Name

123				Commands Continued
	Graph			Concluded
		Name		Name graph settings to recall to screen.
			Use	Display named graph.
			Create	Name/save current sets.
			Delete	Forget name/graph sets.
			Reset	Eliminate ALL named graph.
		Quit		Return to READY mode.
	Data			
		Fill		Automatically fill a range with numbers incrementing in a fixed amount.
		Table		Generates table based on a formula.
			1	Vary one input to the formula.
			2	Vary two inputs to the formula.
			Reset	
		Sort		Sort a range:
			Data-Range	Set range to sort
			Primary-Key	1st field name to sort on
			Secondary-Key	2nd field name to sort on
			Reset	Cancel all sort settings
			Go	Activate the sort
			Quit	Return to main data menu
		Query		

123	Commands Continued
Data	Continued
Query	Search the database:
Input	Set database input range
Criterion	Set criterion range
Output	Set output range for Find and Extract commands
Find	Highlight requested records
Extract	Copy requested records to designated output range
Unique	Extract, deleting duplicates
<u>Delete</u>	
Cancel	Cancel the delete command
Delete	Execute, deleting specified records
Reset	Cancel I/O, criterion
Quit	Return to data main menu
Distribution	Calculate frequency distribution.
MATRIX	
INVERT	INVERT SQUARE MATRIX.
MULTIPLY	MULTIPLY MATRICES.
REGRESSION	RELATES INDEPENDENT AND DEPENDENT VARIABLES; TESTS STATISTICAL ACCURACY.
X-RANGE	COLUMNS INDEPENDENT VARIABLES.
Y-RANGE	COLUMNS DEPENDENT VARIABLES.
OUTPUT-RANGE	RANGE TO DISPLAY RESULTS.
<u>INTERCEPT</u>	
COMPUTE	COMPUTE Y INTERCEPT.
ZERO	FORCE Y INTERCEPT TO ZERO.
RESET	CANCEL REGRESSION SETTINGS.
GO	EXECUTE REGRESSION ANALYSIS: Constant Std Err of Y Est R Squared Number of Observations X Coefficient(s) Std Err of Coef.
QUIT	

123	Commands Concluded	
Data	Commands Concluded	
PARSE		CONVERTS COLUMN OF LONG LABELS (e.g., ASCII text file) TO COLUMNS OF NUMBERS/LABELS.
FORMAT-LINE CREATE EDIT		SET DATA TYPE AND BLOCK WIDTH.
INPUT-COLUMN		RANGE OF COLUMN FORMAT LINES.
OUTPUT-RANGE		UPPER LEFT CORNER OF BLANK RANGE TO HOLD INCOMING DATA.
RESET		ELIMINATE PARSE SETTINGS.
GO		EXECUTE PARSE OPERATION.
SYSTEM		TEMPORARILY EXIT LOTUS TO THE DISK OPERATING SYSTEM (DOS).
Quit		Exit Lotus(no automatic save).

PART II :

APPLICATIONS.

CHAPTER 1 :

RHUMB LINE SAILING.

1.1. Introduction.

In the present chapter, a LOTUS 1-2-3 program is designed to compute the calculations related to rhumb line distance,track and coordinates of the point of destination

An overview of the theory and computational model, an overall description of the computer program, its functions, structure and operation and a set of calculations and numerical results are included.

The computational model implemented in the computer program is based on a mathematical formulation.

1.2. Definition of the tasks.

The rhumb line is the curve described by a vessel with a constant true track.
A rhumb line makes the same angle with all meridians it crosses and appears as a straight line on a Mercator chart.

Task 1 : The coordinates of the points of departure and point of arrival are (L,G) and (L′,G′), respectively.

Calculate the rhumb line track and the distance between the two given points.

Task 2 : The coordinates of the point of departure (L1,G1), the distance d and the track C are given. Calculate the coordinates of the point of arrival (L2,G2).

1.3. The Mathematical Model.

Solutions to rhumb line problems can be obtained by the traditional mathematical method.

1.3.1. Rhumb line track .

The track C may be calculated by the following formula:

$$C = \arctan[\,|dlong|\,/(m2-m1)] \tag{1}$$

where

dlong = the difference of longitude = G′-G
m2 = the meridional part related to the latitude L′
m1 = the meridional part related to the latitude L
Meridional parts may be calculated for any spheroid from the formula :

$$m = [(180/\pi)*60/\pi)*(1- e^{2}) \int_{0}^{L} \sec x*(1-e^{2}*\sin^{2} x)^{-1} dx]$$

$$m = 10800/\pi *(\mathrm{Ln}[\tan(\pi/4 + L/2)-e^{2}*\sin L-1/3*e^{4}*\sin^{3} L -1/5*e^{6}*\sin^{5} L-......)$$

L = the latitude in radians

e = the eccentricity = 0.081999189 (International
Spheroid 1924)

1.3.2. Rhumb line distance.

The distance d may be calculated by the following formulae:

(2) $d = |60*(l2-l1)*\sec C|$, if C is less than 89 degrees.

where l2 = the length of the meridional arc related to the latitude L'

l1 = the length of the meridional arc related to the latitude L

The length l of the meridional arc is given by the formula

$$l = \int_0^L r\,dx = a*(1-e^2)\int_0^L (1-e^2*\sin^2 x)^{-3/2}\,dx$$

where

$r = a*(1-e^2)*(1-e^2*\sin^2 L)^{-3/2}$ = the radius of curvature in the meridian

L = the geodetic latitude of the place

e = the eccentricity of the ellipse
$= (a^2 - b^2 / a^2) = (2*f - f^2)^{1/2}$

a = the major semi-axis

b = the minor semi-axis

f = the flattening or the ellipticity of the earth
$= a - b / a$

Such a formula is expanded in the form

$$(2) \quad l = a(Ao*L - A2*\sin 2L + A4*\sin 4L - A6*\sin 6L + \ldots.)$$

where

$$Ao = 1 - 1/4*e^{2} - 3/64*e^{4} - 5/256*e^{6} - \ldots.$$

$$A2 = 3/8*(e^{2} + 1/4*e^{4} + 15/128*e^{6} + \ldots)$$

$$A4 = 15/256*(e^{4} + 3/4*e^{6} + \ldots)$$

$$A6 = 35/3072*e^{6} + \ldots.$$

The distance may be calculated by the following formula :

$$(3) \quad d = \left| 60*dlong*\cos Lm/\sin C \right| ,$$

if C is more than 89 degrees ($C \neq k*180$, $k = 1, 2$).

Lm = the mid latitude = $1/2*(L'+L)$

1.3.3. Coordinates of the point of destination.

The latitude L2 of the point of destination is expressed as:

$$(4) \quad L2 = L1 + dlat ,$$

where

$$dlat = d*\cos C$$

The longitude G2 of the point of destination is expressed as:

(5) $G2 = G1 + dlong$

Expression of the difference of longitude dlong.

Let (L1,G1) and (L2,G2) be the coordinates of the point of departure and the point of arrival, respectively.
$Lm = (L1 + L2)/2$ = the mid latitude.
$l = L2 - L1$ = the difference of latitude.
Let $f(L1) = f(Lm - l/2) = Ln[\tan(\pi/4 + L1/2)]$
$\quad f(L2) = f(Lm + l/2) = Ln[\tan(\pi/4 + L2/2)]$

Expanding f(L1) and f(L2) in Taylor -MacLaurin series, we obtain:

$$f(Lm + l/2) = f(Lm) + l/2*f'(Lm) + 1/2! *(l/2)^2 *f''(Lm) + + 1/3! *(l/2)^3 *f'''(Lm) + \ldots\ldots$$

and

$$f(L2) - f(L1) = l*f'(Lm) + 1/24*l^3 *f'''(Lm)$$

where
$\quad f'(Lm) = \sec Lm$ and $f'''(Lm) = (1 + \sin^2 Lm)/ \cos^3 Lm$

Finally,

(6) $dlong = d * \sin C * \sec Lm$, $p = d * \sin C$ = departure

after neglecting the terms above first order and according to the formula of the difference of latitude.

1.4. The flow chart.

The diagrams 1 and 2 in pages 24 and 25 show the details on the formulation of the problem and the steps needed for the computer program.

Point of departure (L , G)

Point of destination (L' , G')

Difference of latitude = dlat = l

Difference of longitude = dlong = g

Meridional parts $m1$ and $m2$

Difference of Meridional Parts DMP = $m2 - m1 = m$

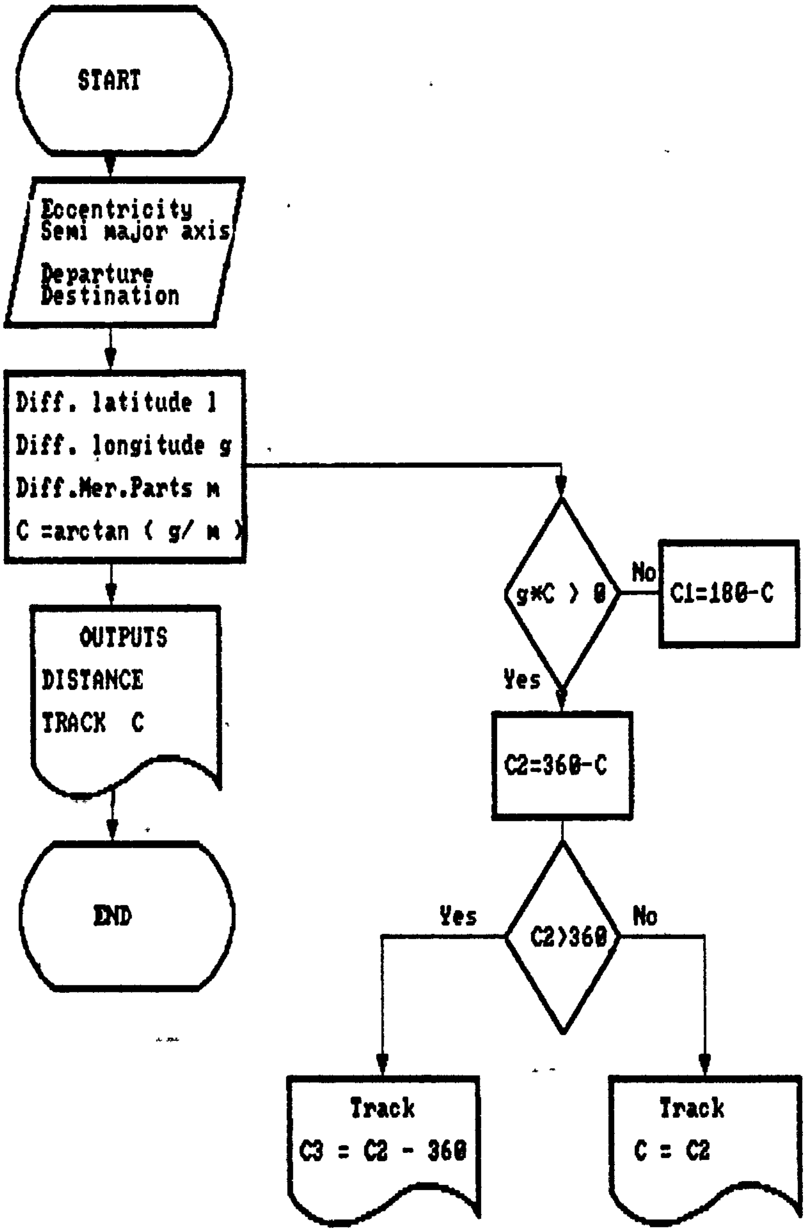
Mid latitude $Lm = 1/2 * (L + L')$

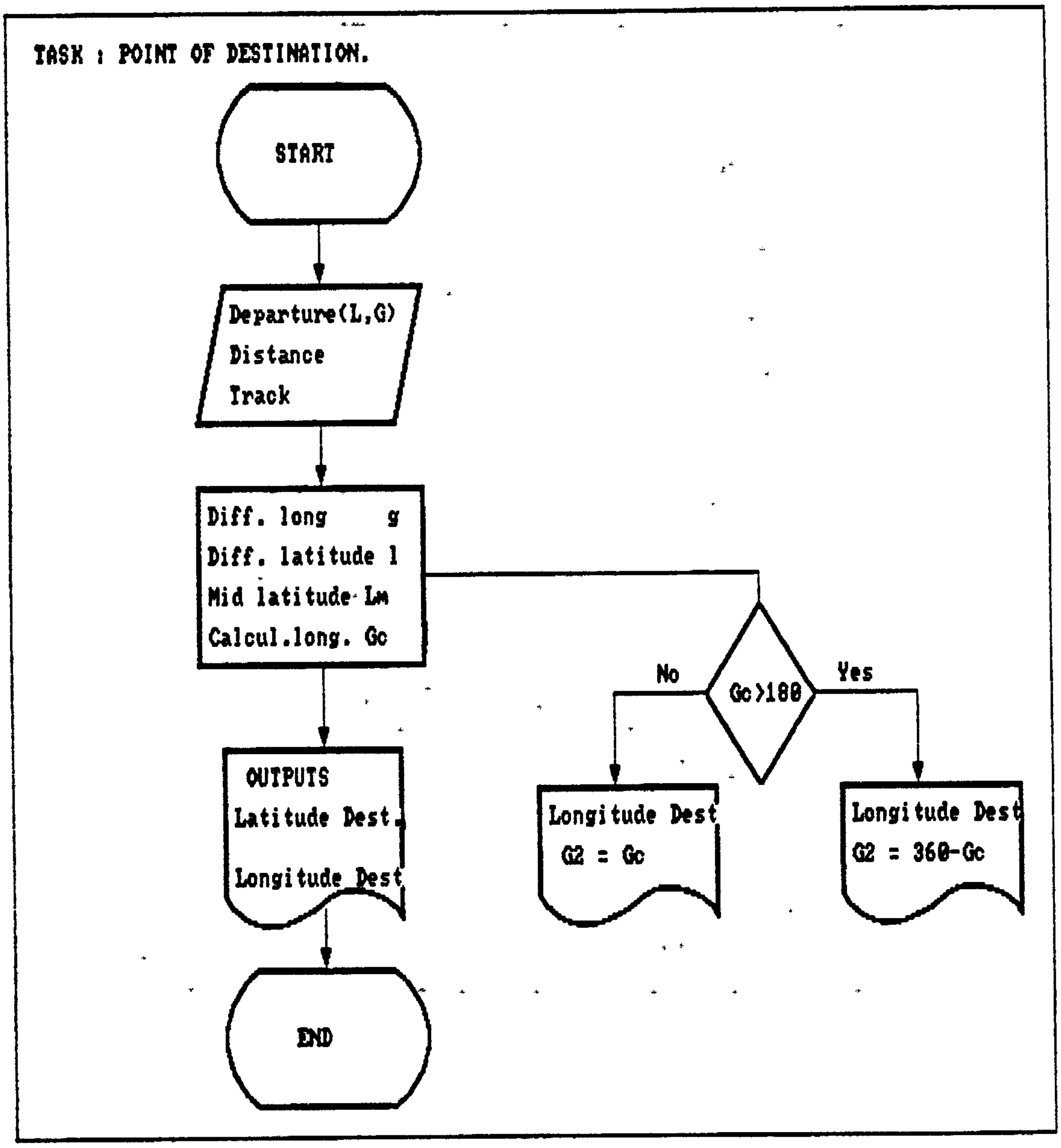
Rhumb line Track $C = \arctan (g / m)$

Distances $d = | 60 * l * \sec C |$

$d' = | 60 * g * \cos Lm / \sin C |$

TASK : RHUMB LINE DISTANCE AND TRACK.





TASK : POINT OF DESTINATION.

RHUMB LINE SAILING.

1.5. ORGANIZATION AND STRUCTURE OF THE LOTUS PROGRAM.

RHUMB 1 A.

	INPUTS	CELLS LOCATION
Eccentricity	e	B 13
Semi major axis	a	B 14
POINT OF DEPARTURE		
Latitude	North / South	B 15
	Degrees	B 18
	Minutes	B 19
Longitude	East / West	B 50
	Degrees	B 51
	Minutes	B 52
POINT OF DESTINATION		
Latitude	North / South	B 28
	Degrees	B 31
	Minutes	B 32
Longitude	East / West	B 53
	Degrees	B 54
	Minutes	B 55

CALCULATIONS (See listing)

OUTPUTS	CELLS LOCATION
DISTANCE	B 83 and C 83
TRACK (COURSE)	B 87 and C 87

RHUMB LINE SAILING.

1.5. ORGANIZATION AND STRUCTURE OF THE LOTUS PROGRAM.

RHUMB 2 A.

INPUTS			CELLS LOCATION
POINT OF DEPARTURE			
	Latitude	North / South	B 16
		Degrees	B 17
		Minutes	B 18
Distance			B 23
Track (Course)			B 24

CALCULATIONS (See listing)

OUTPUTS		CELLS LOCATION
POINT OF DESTINATION :		
	LATITUDE	B 37 and C 37
	LONGITUDE	B 41 and C 41

1.6. Examples.

Example 1 : Ref 1 pages 95 and 96.

Departure : L = 40 43 N
G = 74 00 W

Destination L′= 55 45 S
G′= 37 37 E

Example 2 : Ref 3, Vol 2, page 583.

L = 8 48.9 S
G = 89 53.3 W

L′= 17 06.9 S
G′= 104 51.6 W

Example 3 : Ref 3, Vol 2, page 585.

L = 32 14.7 N
G = 66 28.9 W

L′= 36 58.7 N
G′= 75 42.2 W

Example 4 : Ref 3, Vol 2, page 586.

L = 75 31.7 N
G = 79 08.7 W
Distance = 263.5 M
Track = 155

Example 5 : Ref 3, Vol 2, page 596.

L = 15 17.4 N
G = 151 37.8 E
Distance = 1253.4 M
Track = 70

1.7. LOTUS 1-2-3 results.

The results of LOTUS 1-2-3 may be found from page 30 to page 39.

```
.1: [W35] 'RHUMB LINE CALCULATIONS                                         REA

                      A                                         B
.   RHUMB LINE CALCULATIONS               PROGRAM RHUMB1A.TATY-BOUSSIANA J.L.
                                                        MET ( N ) 90
    TASK 1
    CONDITIONS

    Latitude is < 90 degrees
    Longitude is <180 deg.
    Lat. North is positive
    Lat. South is negative
.0  Longit. East is negative
.1  Longit. West is positive
.2
.3  The eccentricity e is                                        0.081818812
.4  The semi major axis a is                                        3444.054
    Hemisphere                                          N
.6  Point of departure (L1,G1)
.7
.8        Degrees of latitude L1                                          40
          Minutes of latitude L1                                          43
                           0.998324314
22-Sep-90  05:24 PM




21                         0.0025145837
22                          0.000002639              CALCULATIONS
23                         0.0000000034
24                         0.7069625554
                           0.0000007805
                                              Length of mer. arc l1 in rd
27                                                              0.7069633359
28  Hemisphere                                          S
29  Point of arrival (L2,G2)

          Degrees of latitude L2                                          55
32        Minutes of latitude L2                                          45
33                        -0.9690509673
34                         0.0000017983
35                                            Length of mer. arc l1 in rd
36                                                              -0.969049169
37              CALCULATIONS
38                         2.1799995314
39                         0.3081178617
40                         0.7793246619
22-Sep-90  05:25 PM
```

```
40                        0.7793246619
41                       -1.1772729014
42                        0.0043668302
43                        0.0000041464
44                        0.0000000071
45                       -0.0055334546
46                       -0.0000084365
47                       -0.0000034585
4                                              Meridional part m1
49                                                           2664.0945046
50                                                        W
51        Degrees of longitude G1                                      74
52        Minutes of longitude G1                                       0
5                                                         E
54        Degrees of longitude G2                                      37
55        Minutes of longitude G2                                      37
56
57  The dlong in min of arc is                                      -6697
58  The dlong in radians is                                 -1.9480783334
59  The meridional differ. (m2-m1) is                       -6692.1971117
22-Sep-90  05:38 PM
```

```
80                 OUTPUTS                   ACCURATE RESULTS  PRACTICAL RESULTS
81
8
83  The required distance in n.miles is        8151.2080571            8151.2
84
85
86
87  The required track in degrees is           134.97944722             135.0
88
89
90
91
92
93
94
95
96
97
98
99
22-Sep-90  05:40 PM
```

```
A1: [W35] 'RHUMB LINE CALCULATIONS                                            REA

                    A                                           B
1   RHUMB LINE CALCULATIONS                 PROGRAM RHUMB1A.TATY-BOUSSIANA J.L.
2                                                        MET ( N ) 90
3   TASK 1
4   CONDITIONS
5
6   Latitude is < 90 degrees
7   Longitude is <180 deg.
8   Lat. North is positive
9   Lat. South is negative
10  Longit. East is negative
11  Longit. West is positive
1
13  The eccentricity e is                                           0.081818812
14  The semi major axis a is                                           3444.054
15  Hemisphere                                          S
16  Point of departure (L1,G1)
1
18        Degrees of latitude L1                                              8
19        Minutes of latitude L1                                           48.9
20                          0.998324314
22-Sep-90  05:44 PM
```

```
2                          0.0025145837
22                          0.000002639             CALCULATIONS
23                         0.0000000034
24                        -0.1528313787
25                        -0.0000015207
2                                          Length of mer. arc l1 in rd
27                                                         -0.1528328994
28  Hemisphere                                          S
29  Point of arrival (L2,G2)
30
31        Degrees of latitude L2                                             17
32        Minutes of latitude L2                                            6.9
33                        -0.2967980576
34                        -0.0000024514
35                                         Length of mer. arc l1 in rd
36                                                          -0.296800509
37             CALCULATIONS
38                         0.8568766244
39                         0.7384091686
40                        -0.1544613329
22-Sep-90  05:45 PM
```

```
                                      -0.1544613329
                                       -0.303257179
                                      -0.0010258677
                                      -0.0000000538
                                -5.0707731598E-12
                                      -0.0019700745
                                      -0.0000003807
                                      -0.0000000198
                                                              Meridional part m1
                                                                       -527.47209002
                                                                     W
         Degrees of longitude G1                                                   89
         Minutes of longitude G1                                                 53.3
                                                                     W
         Degrees of longitude G2                                                  104
         Minutes of longitude G2                                                 51.6

   The dlong in min of arc is                                                   898.3
   The dlong in radians is                                               0.2613048778
   The meridional differ. (m2-m1) is                                    -508.27530377
22-Sep-90  05:46 PM
```

```
                      OUTPUTS                     ACCURATE RESULTS   PRACTICAL RESULTS

   The required distance in n.miles is         1005.01585969           1005.0


   The required track in degrees is             240.49798925            240.5










22-Sep-90  05:48 PM
```

```
A1: [W35] 'RHUMB LINE CALCULATIONS                                         REAI

                    A                                         B
1   RHUMB LINE CALCULATIONS               PROGRAM RHUMB1A.TATY-BOUSSIANA J.L.
2                                                        MET ( N ) 90
3   TASK 1
4   CONDITIONS
5
6   Latitude is < 90 degrees
7   Longitude is <180 deg.
8   Lat. North is positive
9   Lat. South is negative
10  Longit. East is negative
11  Longit. West is positive
12
13  The eccentricity e is                                          0.081818812
14  The semi major axis a is                                          3444.054
15  Hemisphere                                             N
16  Point of departure (L1,G1)
17
18        Degrees of latitude L1                                            32
19        Minutes of latitude L1                                          14.7
20                          0.998324314
22-Sep-90  05:56 PM




21                         0.0025145837
22                          0.000002639              CALCULATIONS
23                         0.0000000034
24                         0.5595689351
25                         0.0000020523
26                                           Length of mer. arc 11 in rd
27                                                            0.5595709873
28  Hemisphere                                            N
29  Point of arrival (L2,G2)
30
31        Degrees of latitude L2                                            36
32        Minutes of latitude L2                                          58.7
33                         0.6418955433
34                         0.0000014041
35                                           Length of mer. arc 11 in rd
36                                                            0.6418969474
37             CALCULATIONS
38                         1.8131794213
39                         2.0047403723
40                         0.5950818907
22-Sep-90  05:57 PM
```

```
0                           0.5950818907
1                           0.6955145622
2                           0.0035716912
3                           0.0000022688
4                           0.0000000026
5                           0.0040267191
6                           0.0000032511
7                           0.0000007058
8                                                Meridional part m1
9                                                              2033.4544697
0                                                          W
1        Degrees of longitude G1                                         66
         Minutes of longitude G1                                       28.9
                                                           W
4        Degrees of longitude G2                                         75
5        Minutes of longitude G2                                       42.2
6
   The dlong in min of arc is                                         553.3
o  The dlong in radians is                                     0.1609484459
9  The meridional differ. (m2-m1) is                           343.69202742
2-Sep-90  05:59 PM
```

```
30                 OUTPUTS                  ACCURATE RESULTS  PRACTICAL RESULTS

32
33  The required distance in n.miles is        536.36432367           536.4
34
35

37  The required track in degrees is           301.84721027           301.8
38
39
90
91
92
93
94
95
96
97
98
99
22-Sep-90  06:02 PM
```

```
A2: [W35] 'RHUMB LINE CALCULATIONS                                       REA

                    A                                  B
1                                      PROGRAM RHUMB2A.TATY-BOUSSIANA J. L
2   RHUMB LINE CALCULATIONS                          MET ( N ) 90
3
4   CONDITIONS
5
6   Latitude North is positive
7   Latitude South is negative
8   Latitude is less than 90 degrees
9   Longitude is less than 180 deg.
10  Longitude West is positive
11  Longitude East is negative
12
13  TASK 2
14                                                      INPUTS
15  POINT OF DEPARTURE (L1,G1)
16  Hemisphere                                            N
17        Degrees of latitude L1                                       75
18        Minutes of latitude L1                                     31.7
19                                                        W
20        Degrees of longitude G1                                      79
22-Sep-90  06:27 PM
```

```
15  POINT OF DEPARTURE (L1,G1)
16  Hemisphere                                            N
17        Degrees of latitude L1                                       75
18        Minutes of latitude L1                                     31.7
19                                                        W
20        Degrees of longitude G1                                      79
21        Minutes of longitude G1                                     8.7
22
23  The distance d in n.miles is                                    263.5
24  The track C in degrees is                                         155
25                                       CALCULATIONS
26  The diff. of latitude dlat is                            -3.9802016981
27
28  The departure p is                                        1.8559985328
29
30  The mid latitude Lm is                                    73.538232484
31
32  The diff. of longitude dlong is                          -6.5496088373
33
34  POINT OF ARRIVAL (L2,G2)
22-Sep-90  06:29 PM
```

```
34  POINT OF ARRIVAL (L2,G2)
35                                            ACCURATE RESULTS   PRACTICAL RESULT
36
37  Latitude L2 is                                 71.548131635            71.55
38
39                                                 72.595391163
40                                                 72.595391163
41  Longitude G2 is                                72.595391163            72.60
42
43
44
45
46
47
48
49
50
51
52
53
2[illegible]-Sep-90  06:31 PM
```

```
2: [W35] 'RHUMB LINE CALCULATIONS                                               REA

                       A                                  B
1                                          PROGRAM RHUMB2A.TATY-BOUSSIANA J. L
2   RHUMB LINE CALCULATIONS                            MET ( N ) 90
3
4   CONDITIONS
5
6   Latitude North is positive
7   Latitude South is negative
8   Latitude is less than 90 degrees
9   Longitude is less than 180 deg.
10  Longitude West is positive
11  Longitude East is negative
12
1   TASK 2
14                                                        INPUTS
15  POINT OF DEPARTURE (L1,G1)
16  Hemisphere                                             N
17        Degrees of latitude L1                                           15
1         Minutes of latitude L1                                           17
19                                                         E
20                                                                        151
22-Sep-90  06:07 PM
```

```
15  POINT OF DEPARTURE (L1,G1)
    Hemisphere                                             N
1         Degrees of latitude L1                                           15
18        Minutes of latitude L1                                           17
19                                                         E
20        Degrees of longitude G1                                         151
7         Minutes of longitude G1                                          57
2
23  The distance d in n.miles is                                         1253
24  The track C in degrees is                                              70
25                                     CALCULATIONS
26  The diff. of latitude dlat is                                7.1425206598
27
28  The departure p is                                           19.623914231
29
30  The mid latitude Lm is                                       18.854593663
31
32  The diff. of longitude dlong is                             -20.736603833
33
34  POINT OF ARRIVAL (L2,G2)
22-Sep-90  06:13 PM
```

```
POINT OF ARRIVAL (L2,G2)
                                        ACCURATE RESULTS   PRACTICAL RESULT

Latitude L2 is                               22.425853993          22.43


                                            -172.68660383
                                             172.68660383
Longitude G2 is                             -172.68660383        -172.69

22-Sep-90  06:14 PM
```

1.8. Results from the listed references.

Example 1 : Distance = 8166.5 M
Track = 135

Example 2 : Distance = 1007.1 M
Track = 240.4

Example 3 : Distance = 538.2 M
Track = 301.8

Example 4 : Latitude = 71 32.9 N
Longitude = 72 34.1 W

Example 5 : Latitude = 22 26 N
Longitude = 172 23.1 E

1.9. Comparisons and critical remarks.

The description of the computer program presented in the previous paragraph, demonstrates its organization and logical structure. It is important for a user of any computer program to be able to utilize the capabilities of the program without too much intellectual effort. It is also important that the results produced by a computer program are accurate, within the limits of the theory implemented in the program and the practical requirements and that the user understands the input requirements and the limitations of the program.

The discussion presented in this section covers:

- the testing and validation of the computer program
- the comparison of the results with those obtained from the listed references.

The following remarks can be formulated :

- the calculated distance for example 1 from Ref. 1 is not accurate because the LOTUS results for the other examples are almost similar to those obtained from the listed references.

- the advantages of LOTUS are time saving and more accuracy compared to the tables of navigation and logarithms used in the listed references.

The attention of the reader is drawn on the fact that the decimal part of the number of degrees of latitude or longitude should be multiplied by 60 in order to get the number of minutes (LOTUS results).

This should be considered as an exercise for the student when designing his own programs.

1.10. Listing of the program.

The appendix B gives the program for the calculations of the track and distance when practicing rhumb line sailing in both North hemisphere and South hemisphere.

The listing of the program for task 2 is given in Appendix C.

APPENDIX B :

PROGRAM : RHUMB 1 A.

```
A1: [W35] 'RHUMB LINE CALCULATIONS
B1: [W35] 'PROGRAM RHUMB1A. TATY-BOUSSIANA J.L.
C1: [W35] 'WORLD MARITIME UNIVERSITY
D1: [W35] ^MALMO, SWEDEN.
B2: [W35] ^MET ( N ) 90
A3: [W35] 'TASK 1
A4: [W35] 'CONDITIONS
A6: [W35] 'Latitude is < 90 degrees
A7: [W35] 'Longitude is <180 deg.
A8: [W35] 'Lat. North is positive
A9: [W35] 'Lat. South is negative
A10: [W35] 'Longit. East is negative
A11: [W35] 'Longit. West is positive
A13: [W35] 'The eccentricity e is
B13: [W35] @N(A13..A13)
A14: [W35] 'The semi major axis a is
B14: [W35] @N(A14..A14)
A15: [W35] 'Hemisphere
B15: [W35] ^S
C15: [W35] +B18+B19/60
A16: [W35] 'Point of departure (L1,G1)
C17: [W35] ^Lat. L in degrees
D17: [W35] ^Lat. L in radians
A18: [W35] 'Number of deg of L1 is
B18: [W35] @N(A18..A18)
C18: [W35] @IF(B15="S",-C15,C15)
D18: [W35] +C18*@PI/180
A19: [W35] 'Number of min of L1 is
B19: [W35] @N(A19..A19)
A20: [W35] (1-1/4*$B$13^2-3/64*$B$13^4-5/256*$B$13^6)
A21: [W35] 3/8*($B$13^2+1/4*$B$13^4+15/128*$B$13^6)
A22: [W35] 15/256*($B$13^4+3/4*$B$13^6)
A23: [W35] 35/3072*$B$13^6
A24: [W35] (A20*D18-A21*@SIN(2*D18))
A25: [W35] (A22*@SIN(4*D18)-A23*@SIN(6*D18))
B26: [W35] ^Length of mer. arc l1 in rd
C26: [W35] ^Length of mer. arc l1 in naut. miles
B27: [W35] +A24+A25
C27: [W35] +B27*(180/@PI)*60
A28: [W35] 'Hemisphere
B28: [W35] ^S
C28: [W35] +B31+B32/60
A29: [W35] 'Point of arrival (L2,G2)
C30: [W35] ^Lat. L2 in degrees
D30: [W35] ^Lat. L2 in radians
A31: [W35] 'Number of deg of L2 is
B31: [W35] @N(A31..A31)
C31: [W35] @IF(B28="S",-C28,C28)
D31: [W35] +C31*@PI/180
A32: [W35] 'Number of min of L2 is
B32: [W35] @N(A32..A32)
A33: [W35] (A20*D31-A21*@SIN(2*D31))
```

```
A34: [W35] (A22*@SIN(4*D31)-A23*@SIN(6*D31))
B35: [W35] ^Length of mer. arc 12 in rd
C35: [W35] ^Length of mer. arc 12 in naut.miles
B36: [W35] +A33+A34
C36: [W35] +B36*(180/@PI)*60
A38: [W35] @TAN(@PI/4+D18/2)
A39: [W35] @TAN(@PI/4+D31/2)
A40: [W35] @LN(A38)
A41: [W35] @LN(A39)
A42: [W35] +$B$13^2*@SIN(D18)
A43: [W35] 1/3*$B$13^4*(@SIN(D18))^3
A44: [W35] 1/5*$B$13^6*(@SIN(D18))^5
A45: [W35] +$B$13^2*@SIN(D31)
A46: [W35] 1/3*$B$13^4*(@SIN(D31))^3
A47: [W35] 1/5*$B$13^4*(@SIN(D31))^5
B48: [W35] ^Meridional part m1
C48: [W35] ^Meridional part m2
D48: [W35] +B51+B52/60
B49: [W35] 10800/@PI*(A40-A42-A43-A44)
C49: [W35] 10800/@PI*(A41-A45-A46-A47)
D49: [W35] +B54+B55/60
B50: [W35] ^W
A51: [W35] 'Number of deg of G1 is
B51: [W35] @N(A51..A51)
C51: [W35] ^Longitude G1 in degrees
D51: [W35] ^Longitude G1 in rd
A52: [W35] 'Number of min of G1 is
B52: [W35] @N(A52..A52)
C52: [W35] @IF(B50="W",D48,-D48)
D52: [W35] +C52*@PI/180
B53: [W35] ^W
C53: [W35] ^Longitude G2 in degrees
D53: [W35] ^Longitude G2 in rd
A54: [W35] 'Number of deg of G2 is
B54: [W35] @N(A54..A54)
C54: [W35] @IF(B53="W",D49,-D49)
D54: [W35] +C54*@PI/180
A55: [W35] 'Number of min of G2 is
B55: [W35] @N(A55..A55)
A57: [W35] 'The dlong in min of arc is
B57: [W35] (C54-C52)*60
A58: [W35] 'The dlong in radians is
B58: [W35] +D54-D52
A59: [W35] 'The meridional differ. (m2-m1) is
B59: [W35] +C49-B49
A60: [W35] @ABS(B57)
A62: [W35] 'The calculated track in degrees is
B62: [W35] 180/@PI*@ATAN(A60/B59)
A64: [W35] 'The dlat in radians is
B64: [W35] +B36-B27
A65: [W35] 'The dlat in min of arc is
B65: [W35] +B64*180*60/@PI
```

```
A71: [W35] @IF(B62>=0,B62,B62+180)
A74: [W35] +B65/@COS(B62*@PI/180)
A75: [W35] @ABS(A74)
A76: [W35] +B57*@COS(A77*@PI/180)/@SIN(B62*@PI/180)
A77: [W35] +C31-B65/120
A78: [W35] @ABS(A76)
B80: [W35] ^RESULTS
C80: [W35] ^RESULTS
A81: [W35] 'RHUMB LINE DISTANCE in naut. miles
A83: [W35] 'The required distance in n.miles is
B83: [W35] @IF(B62<89,A75,A78)
C83: (F2) [W35] +B83
A85: [W35] 'RHUMB line TRACK in degrees
A87: [W35] 'The required track in degrees is
B87: [W35] @IF(B57<=0,A71,360-A71)
C87: (F2) [W35] +B87
```

APPENDIX C :

PROGRAM : RHUMB 2 A.

```
B1: [W35] 'PROGRAM RHUMB2A. TATY-BOUSSIANA J.L.
C1: [W35] 'WORLD MARITIME UNIVERSITY
D1: [W35] ^MALMO, SWEDEN.
A2: [W25] 'RHUMB LINE CALCULATIONS
B2: [W35] ^MET ( N ) 90
A4: [W25] 'CONDITIONS
A6: [W25] 'Latitude North is positive
A7: [W25] 'Latitude South is negative
A8: [W25] 'Latitude is less than 90 degrees
A9: [W25] 'Longitude is less than 180 deg.
A10: [W25] 'Longitude West is positive
A11: [W25] 'Longitude East is negative
A13: [W25] 'TASK 2
B14: [W35] ^INPUTS
C14: [W35] +B17+B18/60
A15: [W25] 'POINT OF DEPARTURE (L1,G1)
C15: [W35] +B20+B21/60
A16: [W25] 'Hemisphere
B16: [W35] ^N
C16: [W35] ^Latitude L1 in degrees
A17: [W25] 'Number of degrees of L1 is
B17: [W35] @N(A17..A17)
C17: [W35] @IF(B16="N",C14,-C14)
A18: [W25] 'Number of min of L1 is
B18: [W35] @N(A18..A18)
B19: [W35] ^E
C19: [W35] ^Longitude G1 in degrees
A20: [W25] 'Number of degrees of G1 is
B20: [W35] @N(A20..A20)
C20: [W35] @IF(B19="E",-C15,C15)
A21: [W25] 'Number of min of G1 is
B21: [W35] @N(A21..A21)
A23: [W25] 'The distance d in n.miles is
B23: [W35] @N(A23..A23)
A24: [W25] 'The track C in degrees is
B24: [W35] @N(A24..A24)
B25: [W35] 'CALCULATIONS
A26: [W25] 'The diff. of latitude dlat is
B26: [W35] +B23*@COS(B24*@PI/180)/60
A28: [W25] 'The departure p is
B28: [W35] +B23*@SIN(B24*@PI/180)/60
A30: [W25] 'The mid latitude Lm is
B30: [W35] +C17+B26/2
A32: [W25] 'The diff. of longitude dlong is
B32: [W35] -B28/@COS(B30*@PI/180)
A34: [W25] 'POINT OF ARRIVAL (L2,G2)
B35: [W35] ^RESULTS
C35: [W35] ^RESULTS
A37: [W25] 'Latitude L2 is
B37: [W35] @IF((C17+B26)>90,90-(C17+B26),C17+B26)
C37: (F2) [W35] +B37
B39: [W35] +C20+B32
```

```
B40: [W35] @ABS(B39)
A41: [W25] 'Longitude G2 is
B41: [W35] @IF(@ABS(B39)>180#AND#B39>0,-360+B39,B39)
C41: (F2) [W35] +B41
B42: [W35] @IF(@ABS(B39)>180#AND#B39<0,360+B39,B39)
```

CHAPTER 2 :

GREAT CIRCLE SAILING.

2.1. Introduction.

A great circle is the intersection of the surface of a sphere and a plane through the center of the sphere. It is the largest circle that can be drawn on the surface of the sphere and is the shortest distance, along the surface, between any two points on the sphere.
In this chapter, a LOTUS 1-2-3 program is designed which enables the user to calculate the great circle distance and track, the coordinates of the vertex and to solve problems related to composite sailing.

On a Mercator chart a great circle appears as a sine curve extending equal distances each side of the equator.

2.2. Definition of the task.

Task 1 : The coordinates of two points M and M′ of the great circle are given. Calculate the distance d = MM′.

Task 2 : The coordinates of the points M and M′and the distance between them are given. Calculate the great circle track V (in point M).

Task 3 : The coordinates of the point of departure M and the track V are given. Calculate the coordinates of the vertex (Lv,Gv) which is the point of greatest latitude.

Task 4 : Composite sailing.

The composite sailing is used when the great circle would carry a vessel to a higher latitude than desired. The composite track consists of a great circle from the point of departure and tangent to the limiting parallel, a track line along the parallel, and a great circle tangent to the limiting parallel and through the destination. Calculate the longitude at which the limiting parallel is reached, the longitude at which the limiting parallel should be left and the total distance which is the sum of the great circle distances and along the parallel distance

2.3. The Mathematical Model.

2.3.1. Great circle distance.

Let M(L,G), M'(L',G'), be the point of departure and destination,respectively, g the difference of longitude G'- G.

According to the law of cosines, the distance d = MM' is given by:

cosd = sinL'*sinL + cosL'*cosL*cosg

(1) d = 60*(180/)*[arccos(sinL'*sinL+cosL'*cosL*cosg)]

d = distance in nautical miles.

2.3.2. Great circle track.

The great circle track is obtained from the law of cosines

(2) $\cos V = (\sin L' - \sin L * \cos MM')/(\cos L * \sin MM')$, in degrees.

2.3.3. Coordinates of the vertex.

2.3.3.1. Latitude Lv.

According to the law of sines,

(3) $\cos Lv = \sin V * \cos L$.

2.3.3.2. Longitude Gv.

Let $g1 = Gv - G$, then, according to the law of cotangents,

$\sin L = \cotan g1 * \cotan V$,

(4) $Gv = G + g1$, $0 < g1 < 180$ degrees.

2.3.4. Composite sailing.

2.3.4.1. Longitude at which the limiting parallel is reached.

The longitude Gb at which the limiting parallel is reached is given by :

(5) cos(Gb-G) = tanL*cotanLmax, Lmax = limiting latitude.

2.3.4.2. Longitude at which the limiting parallel is reached.

The longitude Gc at which the limiting parallel is reached is given by

(6) cos(Gc-G') = tanL'*cotanLmax

2.3.4.3. Total distance of the composite sailing.

The total distance is the sum of d1, d2 and d3.

(7) d = (d1 + d2 + d3)*60

where

d1 = arccos(sinL/sinLmax)

d2 = arccos(sinL'/sinLmax)

d3 = | Gc-Gb |* cosLmax

Then the Great circle track V', the conversion angle a (a = v*t/120*sinV'*tanL) and the Rhumb line track C' (C'= V'+-a) are calculated from the point of departure (L,G) and the point B (Lmax, Gb)

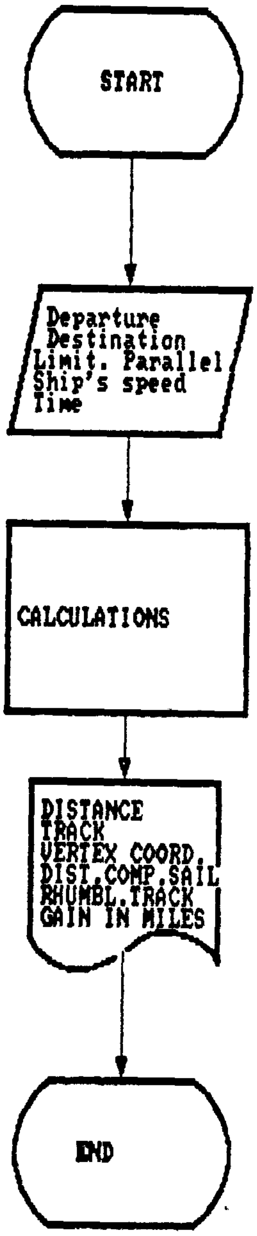
2.4. The flow chart.

The diagram in page 46 shows the steps for the computation of the distance d , the track V, the coordinates of the vertex (Lv, Gv), the rhumb line track, etc...

If the difference of longitude is West (g > 0), the great circle track is 360 - Vi where Vi = the calculated track .

If the difference of longitude is East (g < 0), the great circle track is Vi.

TASK : GREAT CIRCLE TRACK, DISTANCE, VERTEX, COMPOSITE SAILING.



GREAT CIRCLE SAILING.

2.5. ORGANIZATION AND STRUCTURE OF THE LOTUS PROGRAM.

GREAT 1 A.

INPUTS			CELLS LOCATION
POINT OF DEPARTURE			
	Latitude	North / South	B 11
		Degrees	B 13
		Minutes	B 14
	Longitude	East / West	B 17
POINT OF DESTINATION			
	Latitude	North / South	B 25
		Degrees	B 27
		Minutes	B 28
	Longitude	East / West	B 31
		Degrees	B 33
		Minutes	B 34
LIMITING PARALLEL Lmax			
		North / South	B 159
		Degrees	B 161
		Minutes	B 162
Ship's speed			B 215
Time			B 217

CALCULATIONS (See listing)

OUTPUTS	CELLS LOCATION
DISTANCE	B 47 and C 47
TRACK	B 83 and C 83
LATITUDE OF THE VERTEX	B 110 and C 110
LONGITUDE OF THE VERTEX	B 137 and C 137
DISTANCE COMPOSITE SAILING	B 197 and C 197
CONVERSION ANGLE	B 225
RHUMB LINE TRACK	B 227 and C 227

2.6. Examples.

Example 1 : Ref 1 pages 39, 40, 89, 91 and 93.

Departure : L = 45 00 N
G = 140 00 E

Destination : L' = 65 00 N
G' = 110 00 W

Limiting parallel = 67 00 N

Example 2 : Ref 3, Vol 2, page 608.

L = 12 45.2 N
G = 124 20.1 E

L' = 33 48.8 N
G' = 120 07.1 W

Limiting parallel (chosen by the author) = 40 N.

2.7. LOTUS 1-2-3 results.

The results are given from page 49 to page 59.

```
1: [W35] 'GREAT CIRCLE SAILING                                                   READ

                    A                                      B
  GREAT CIRCLE SAILING                   PROGRAM GREAT1A. TATY-BOUSSIANA J.L
                                                    MET ( N ) 90.
  TASK 1 :

  GREAT CIRCLE DISTANCE d

  INPUTS

  POINT OF DEPARTURE (L,G)
0
1 HEMISPHERE                                              N
2
3        Degrees of latitude L                                             45
4        Minutes of latitude L                                              0
5 The Latitude L in degrees is                                             45
6
7                                                         E
8
9       Degrees of longitude G                                            140
0       Minutes of longitude G                                              0
2-Sep-90  11:28 PM
```

```
1  The Longitude G in degrees is                  -140
2
3  POINT OF DESTINATION (L',G')
4
5  HEMISPHERE                                N
6
7        Degrees of latitude L'                  65              65
8        Minutes of latitude L'                   0
9  The Latitude L' in degrees is                 65
0
1                                            W
2
3        Degrees of longitude G'                110             110
4       Minutes of longitude G'                   0
5  The Longitude G' in degrees is               110
6
7  CALCULATIONS
8
9                                      0.6408563821
0                                      0.2988362387             250
22-Sep-90  11:30 PM
```

```
45                                      ACCURATE RESULTPRACTICAL RESULT
46
47  The required distance d in M is       3444.4995133        3444.5
48
49
50  TASK 2 :
51
52  GREAT CIRCLE TRACK V
53
54  INPUTS
55
56  POINT OF DEPARTURE LATITUDE L
57
58  HEMISPHERE                          N
59
60         Degrees of latitude L                 45               45
61         Minutes of latitude L                  0
62  The Latitude L in degrees is                 45
63
64
22-Sep-90  11:34 PM
```

```
64  POINT OF DESTINATION LATITUDE L'
65
66  HEMISPHERE                          N
67
68        Degrees of latitude L'                 65               65
69        Minutes of latitude L'                  0
70  The Latitude L' in degrees is                65
71
72  The distance d in degrees is          57.408325222
73
74  CALCULATIONS
75
76                                         0.906307787
77                                        0.3808819143
78                                        0.5957591521
79                                        0.8819434344   28.122305224
80
81                                      ACCURATE RESULTPRACTICAL RESULT
82
83  The required track V is               28.122305224         28.1
22-Sep-90  11:35 PM
```

```
91
92  POINT OF DEPARTURE LATITUDE L
93
94  HEMISPHERE                         N
95
96         Degrees of latitude L                  45             45
97         Minutes of latitude L                   0
98  The Latitude L is                             45
99
100 The track V is                       28.122305224
101
102 CALCULATIONS
103
104                                      0.4713552571
105                                      0.7071067812
106                                      0.3332984987
107                                      70.530896301
108                                    ACCURATE RESULTPRACTICAL RESULT
109
110 The Latitude Lv of the vertex is     70.530896301          70.53
22-Sep-90  11:36 PM
```

```
118
119 HEMISPHERE                         N
120
121        Degrees of latitude L                  45             45
122        Minutes of latitude L                   0
123 The latitude L in degrees is                  45
124
125 The track V is                       28.122305224        E
126                                             28.12
127 CALCULATIONS
128
129                                      0.7071067812
130                                      0.5344506674
131                                       2.646106832
132                                      1.2094736378
133
134                                                   PRACTICAL RESULT
135
136                                     -209.29773488
137 The Longitude of the vertex  Gv is   150.70226512         150.70
22-Sep-90  11:37 PM
```

```
180                                                       CALCULATIONS
181                                                   1
182                                        2.3558523658   0.4244748162
183                                        2.1445069205   0.9102891809
184                                                       0.7071067812
185 RESULTS                                               0.9205048535
186                                       -204.88257448    0.906307787
187 The longitude Gb is                    155.11742552   0.4226182617
188                                        134.45465517
189 The longitude Gc is                    134.45465517
190                                   ACCURATE RESULTPRACTICAL RESULTS
191 The length of arc MB is                2388.5946413         2388.6
192
193 The length of arc CM' is               604.55374112          604.6
194
195 The length of arc BC is                484.41525449          484.4
196
197 The total length of arc MBCM' is       3477.5636369         3477.6
198
199
22-Sep-90  11:42 PM
```

```
208                                                      PRACTICAL RESULTS
209 Initial great circle track M to B      33.544004146          33.54
210
211 Rhumb line track to be followed
212
213 Inputs
214
215 Speed of the ship in knots                       18
216
217 Time t                                           20
218
219 Latitude L                                       45
220
221 Calculations
222
223 Distance m1                                     360
224
225 Correction a in degrees                1.6577317835
226
227 The Rhumb line track is                31.886272363          31.89
22-Sep-90  11:44 PM
```

```
40
41 COMPOSITE SAILING
42
43 INPUTS
44
45 POINT OF DEPARTUTRE
46
47 HEMISPHERE                          N
48
49        Degrees of latitude L                       45
50        Minutes of latitude L                        0
51 The latitude L in degrees is                       45
52
53                                     E
54
55       Degrees of longitude G                      140
56       Minutes of longitude G                        0
57 The longitude G in degrees is                    -140
58
59 L -
2-Sep-90  11:39 PM
```

```
160
161      Degrees of latitude Lmax                     67
162      Minutes of latitude Lmax                      0
163 The latitude Lmax in degrees is                   67
164
165 POINT OF DESTINATION
166
167 HEMISPHERE                         N
168
169       Degrees of latitude L'                      65
170       Minutes of latitude L'                       0
171 The latitude L' in degrees is                     65
172
173                                    W
174
175       Degrees of longitude G'                    110
176       Minutes of longitude G'                      0
177 The longitude G' in degrees is                   110
178
179
22-Sep-90  11:40 PM
```

```
                    A                                          B
  GREAT CIRCLE SAILING                      PROGRAM GREAT1A. TATY-BOUSSIANA J.L
                                                      MET ( N ) 90.
  TASK 1 :

  GREAT CIRCLE DISTANCE d

  INPUTS

  POINT OF DEPARTURE (L,G)
0
1  HEMISPHERE                                             N
2
)         Degrees of latitude L                                          12
4         Minutes of latitude L                                        45.2
5  The Latitude L in degrees is                                12.7533333333
6
7                                                         E

9         Degrees of longitude G                                        124
0         Minutes of longitude G                                       20.1
2-Sep-90  11:51 PM
```

```
21  The Longitude G in degrees is                  -124.335
22
2   POINT OF DESTINATION (L',G')
24
25  HEMISPHERE                               N
26
27        Degrees of latitude L'                         33    33.813333333
2         Minutes of latitude L'                       48.8
29  The Latitude L' in degrees is               33.813333333
30
31                                           W
32
33        Degrees of longitude G'                       120   120.118333333
34       Minutes of longitude G'                        7.1
35  The Longitude G' in degrees is             120.118333333
36
37  CALCULATIONS
38
39                                              0.1228472671
40                                              0.8103573656    244.45333333
22-Sep-90  11:54 PM
```

```
5                                              ACCURATE RESULT   PRACTICAL RESULT
6
7  The required distance d in M is               6185.8760314         6185.9
8.
9
0  TASK 2 :
1
2  GREAT CIRCLE TRACK V
3
4  INPUTS
5
6  POINT OF DEPARTURE LATITUDE L
)
8  HEMISPHERE                             N
9
0          Degrees of latitude L                        12   12.7533333333
1          Minutes of latitude L                      45.2
   The Latitude L in degrees is              12.7533333333
3
4  POINT OF DESTINATION LATITUDE L'
2-Sep-90  11:55 PM
```

```
4  POINT OF DESTINATION LATITUDE L'
5
   HEMISPHERE                             N
67
68         Degrees of latitude L'                       33    33.813333333
69         Minutes of latitude L'                     48.8
70 The Latitude L' in degrees is              33.813333333
7
72 The distance d in degrees is              103.097933857
73
74 CALCULATIONS
75
76                                            0.5564889793
77                                           -0.0500264695
78                                            0.9499554439
79                                            0.6384672594    50.322378387
80
81                                        ACCURATE RESULT   PRACTICAL RESULT
82
83 The required track V is                    50.322378387          50.3
22-Sep-90  11:56 PM
```

```
1
2  POINT OF DEPARTURE LATITUDE L
3
4  HEMISPHERE                         N
5
6         Degrees of latitude L                              12  12.7533333333
7         Minutes of latitude L                            45.2
8  The Latitude L is                              12.7533333333
9
00 The track V is                                  50.322378387
01
02 CALCULATIONS
03
04                                                 0.7696489841
05                                                 0.9753294792
06                                                 0.7506613428
07                                                  41.35230204
08                                       ACCURATE RESULT   PRACTICAL RESULT
09
10 The Latitude Lv of the vertex is                 41.35230204          41.35
2-Sep-90  11:57 PM
```

```
19 HEMISPHERE                         N
20
21        Degrees of latitude L                              12  12.7533333333
22        Minutes of latitude L                            45.2
23 The latitude L in degrees is                   12.7533333333
24
25 The track V is                                  50.322378387        E
26                                                        50.32
27 CALCULATIONS
28
29                                                 0.2207541778
30                                                 1.2054635109
31                                                 3.7578288799
32                                                 1.3107126836
33
34                                                                 PRACTICAL RESULT
35
36                                                -199.43330492
37 The Longitude of the vertex  Gv is              160.56669508         160.57
2-Sep-90  11:58 PM
```

```
40
41 COMPOSITE SAILING
42
43 INPUTS
44
45 POINT OF DEPARTUTRE
46
47 HEMISPHERE                                N
48
49        Degrees of latitude L                             12
50        Minutes of latitude L                           45.2
   The latitude L in degrees is                   12.7533333333
52
53                                            E
54
55       Degrees of longitude G                            124
         Minutes of longitude G                           20.1
57 The longitude G in degrees is                      -124.335
58
59                                                       N
2-Sep-90  11:59 PM
```

```
60
         Degrees of latitude Lmax                            40
62       Minutes of latitude Lmax                             0
63 The latitude Lmax in degrees is                           40
64
65 POINT OF DESTINATION

67 HEMISPHERE                                N
68
69        Degrees of latitude L'                             33
70        Minutes of latitude L'                           48.8
71 The latitude L' in degrees is                    33.813333333
72
73                                           W
74
75        Degrees of longitude G'                           120
76        Minutes of longitude G'                           7.1
77 The longitude G' in degrees is                  120.118333333
78
79
23-Sep-90  12:00 AM
```

```
                                                                         CALCULATIONS
30
31                                                        0.2263380555
32                                                        0.8390996312   0.2697391907
33                                                        0.6697787043   0.7982111771
34                                                                       0.2207541778
35 RESULTS                                                               0.6427876097
36                                                       -198.68625222   0.5564889793
37 The longitude Gb is                                    161.31374778   0.8308549909
38                                                        157.15871305
39 The longitude Gc is                                    157.15871305
90                                                  ACCURATE RESULTS  PRACTICAL RESULTS
 ) The length of arc MB is                                4194.8315302          4194.8
92
93 The length of arc CM' is                               1801.9395869          1801.9
94
95 The length of arc BC is                                190.97647553           191.0

97 The total length of arc MBCM' is                       6187.7475926          6187.7
98
99
3-Sep-90  12:02 AM
```

```
                    [illegible] track is


                                                                     PRACTICAL RESULTS
09 Initial great circle track M to B                       51.75965026           51.76
10
11 Rhumb line track to be followed
12
   Inputs
14
15 Speed of the ship in knots                                       18
16
17 Time t                                                           20
18
19 Latitude L                                             12.7533333333
20
21 Calculations
22
23 Distance m1                                                     360
24
25 Correction a in degrees                                 0.5333121166
26
27 The Rhumb line track is                                 51.226338143           51.23
23-Sep-90  12:03 AM
```

2.8. Results from the references.

Example 1 :

Distance in nautical miles = 3444.5

Track in degrees = 28

Vertex latitude = 70 31.85 N

Vertex longitude = 150 42.14 W

Longitude Gb = 155 07 W

Longitude Gc = 134 27.3 W

Distances d 1 = 2388.6 M
d 2 = 604.6 M
d 3 = 484.4 M
Total distance d = 3477.6 M

Example 2 :

Distance = 6185.9 M

Track = 50.3

Vertex latitude = 41 21.2 N

Vertex longitude = 160 34.4 W

2.9. Comparisons and critical remarks.

- The LOTUS results are almost similar to those obtained from the listed references. The small difference is acceptable.

- The attention of the student is drawn on the fact that when the first vertex is found, the coordinates of the second vertex are expressed as follows :

Lv2 = - Lv1 and Gv2 = Gv1 + / - 180 degrees.

- The difference between the rhumb line and the great circle distances gives the number of miles gained when practicing great circle sailing in other words. The great circle sailing gives the shortest distance.

2.10. Listing of the LOTUS program.

Appendix D gives the listing of the program.

APPENDIX D :

PROGRAM : GREAT 1 A.

```
A1: [W35] 'GREAT CIRCLE SAILING
B1: [W35] 'PROGRAM GREAT1A. TATY-BOUSSIANA J.L.
C1: [W35] 'WORLD MARITIME UNIVERSITY
D1: [W35] ^MALMO, SWEDEN.
B2: [W35] ^MET ( N ) 90.
A3: [W35] 'TASK 1 :
A5: [W35] 'GREAT CIRCLE DISTANCE d
A7: [W35] 'INPUTS
A9: [W35] 'POINT OF DEPARTURE (L,G)
A11: [W35] 'HEMISPHERE
B11: [W35] ^N
A13: [W35] 'Number of degrees of latitude L
B13: [W35] @N(A13..A13)
C13: [W35] +B13+B14/60
A14: [W35] 'Number of minutes of latitude L
B14: [W35] @N(A14..A14)
A15: [W35] 'The Latitude L in degrees is
B15: [W35] @IF(B11="N",C13,-C13)
B17: [W35] ^E
A19: [W35] 'Number of degrees of longitude G
B19: [W35] @N(A19..A19)
C19: [W35] +B19+B20/60
A20: [W35] 'Number of minutes of longitude G
B20: [W35] @N(A20..A20)
A21: [W35] 'The Longitude G in degrees is
B21: [W35] @IF(B17="E",-C19,C19)
A23: [W35] 'POINT OF DESTINATION (L',G')
A25: [W35] 'HEMISPHERE
B25: [W35] ^N
A27: [W35] 'Number of degrees of latitude L'
B27: [W35] @N(A27..A27)
C27: [W35] +B27+B28/60
A28: [W35] 'Number of minutes of latitude L'
B28: [W35] @N(A28..A28)
A29: [W35] 'The Latitude L' in degrees is
B29: [W35] @IF(B25="N",C27,-C27)
B31: [W35] ^W
A33: [W35] 'Number of degrees of longitude G'
B33: [W35] @N(A33..A33)
C33: [W35] +B33+B34/60
A34: [W35] 'Number of minutes of longitude G'
B34: [W35] @N(A34..A34)
A35: [W35] 'The Longitude G' in degrees is
B35: [W35] @IF(B31="E",-C33,C33)
A37: [W35] 'CALCULATIONS
B39: [W35] @SIN(B15*@PI/180)*@SIN(B29*@PI/180)
B40: [W35] @COS(B15*@PI/180)*@COS(B29*@PI/180)
C40: [W35] +B35-B21
B41: [W35] @COS((B35-B21)*@PI/180)
C41: [W35] @IF(@ABS(C40)>180#AND#C40>0,-360+@ABS(C40),C40)
B42: [W35] +B40*B41
B43: [W35] +B39+B42
```

```
A45: [W35] 'RESULT
A47: [W35] 'The required distance d in M is
B47: [W35] 60*(180/@PI)*@ACOS(B43)
C47: (F1) [W35] +B47
A50: [W35] 'TASK 2 :
A52: [W35] 'GREAT CIRCLE TRACK V
A54: [W35] 'INPUTS
A56: [W35] 'POINT OF DEPARTURE LATITUDE L
A58: [W35] 'HEMISPHERE
B58: [W35] +B11
A60: [W35] 'Number of degrees of latitude L
B60: [W35] +B13
C60: [W35] +B60+B61/60
A61: [W35] 'Number of minutes of latitude L
B61: [W35] +B14
A62: [W35] 'The Latitude L in degrees is
B62: [W35] @IF(B58="N",C60,-C60)
A64: [W35] 'POINT OF DESTINATION LATITUDE L'
A66: [W35] 'HEMISPHERE
B66: [W35] +B25
A68: [W35] 'Number of degrees of latitude L'
B68: [W35] +B27
C68: [W35] +B68+B69/60
A69: [W35] 'Number of minutes of latitude L'
B69: [W35] +B28
A70: [W35] 'The Latitude L' in degrees is
B70: [W35] @IF(B66="S",-C68,C68)
A72: [W35] 'The distance d in degrees is
B72: [W35] +B47/60
A74: [W35] 'CALCULATIONS
B76: [W35] @SIN(B70*@PI/180)
B77: [W35] @SIN(B62*@PI/180)*@COS((B47/60)*@PI/180)
B78: [W35] @COS(B62*@PI/180)*@SIN(B72*@PI/180)
B79: [W35] (B76-B77)/B78
C79: [W35] 180/@PI*@ACOS(B79)
A81: [W35] 'RESULT
A83: [W35] 'The required track V is
B83: [W35] @IF(C40<0,360-180/@PI*@ACOS(B79),180/@PI*@ACOS(B79))
C83: (F1) [W35] +B83
A86: [W35] 'TASK 3 : COORDINATES OF THE VERTEX
A88: [W35] 'LATITUDE OF THE VERTEX Lv
A90: [W35] 'INPUTS
A92: [W35] 'POINT OF DEPARTURE LATITUDE L
A94: [W35] 'HEMISPHERE
B94: [W35] +B11
A96: [W35] 'Number of degrees of latitude L
B96: [W35] +B13
C96: [W35] +B96+B97/60
A97: [W35] 'Number of minutes of L
B97: [W35] +B14
A98: [W35] 'The Latitude L is
B98: [W35] @IF(B94="S",-C96,C96)
```

```
A100: [W35] 'The track V is
B100: [W35] +B83
A102: [W35] 'CALCULATIONS
B104: [W35] @SIN(B100*@PI/180)
B105: [W35] @COS(B98*@PI/180)
B106: [W35] +B104*B105
C106: [W35] 180/@PI*@ACOS(B106)
C107: [W35] @IF(C106>90,C106-90,C106)
A108: [W35] 'RESULT
A110: [W35] 'The Latitude Lv of the vertex is
B110: [W35] @IF(B83<90,C107,-C107)
C110: (F2) [W35] +B110
A113: [W35] 'LONGITUDE OF THE VERTEX Gv
A115: [W35] 'INPUTS
A117: [W35] 'POINT OF DEPARTURE LATITUDE L
A119: [W35] 'HEMISPHERE
B119: [W35] +B11
A121: [W35] 'Number of degrees of latitude L
B121: [W35] +B13
C121: [W35] +B121+B122/60
A122: [W35] 'Number of minutes of latitude L
B122: [W35] +B14
A123: [W35] 'The latitude L in degrees is
B123: [W35] @IF(B119="S",-C121,C121)
A125: [W35] 'The track V is
B125: [W35] +B83
C125: [W35] ^E
B126: (F2) [W35] +B125
C126: [W35] 'Number of degrees of longitude G
D126: [W35] +B19
E126: [W35] +D126+D127/60
A127: [W35] 'CALCULATIONS
C127: [W35] 'Number of minutes of longitude G
D127: [W35] +B20
C128: [W35] 'The Longitude G in degrees is
D128: [W35] @IF(C125="E",-E126,E126)
B129: [W35] @SIN(B123*@PI/180)
B130: [W35] @TAN(B125*@PI/180)
B131: [W35] 1/(B129*B130)
B132: [W35] @ATAN(B131)
A134: [W35] 'RESULT
B136: [W35] @IF(C41<0,D128-180/@PI*@ATAN(B131),D128+180/@PI*@ATAN(B131))
A137: [W35] 'The Longitude of the vertex  Gv is
B137: [W35] @IF(@ABS(B136)>180#AND#B31="E",-B136,360-@ABS(B136))
C137: (F2) [W35] @IF(@ABS(B137)>180#AND#B137>0,-360+B137,B137)
C138: (F2) [W35] @IF(@ABS(B137)>180#AND#B137<0,360+B137,B137)
A139: [W35] 'TASK 4
A141: [W35] 'COMPOSITE SAILING
A143: [W35] 'INPUTS
A145: [W35] 'POINT OF DEPARTUTRE
A147: [W35] 'HEMISPHERE
B147: [W35] +B11
```

```
A149: [W35] 'Number of degrees of latitude L
B149: [W35] +B13
A150: [W35] 'Number of minutes of latitude L
B150: [W35] +B14
A151: [W35] 'The latitude L in degrees is
B151: [W35] @IF(B147="S",-(B149+B150/60),B149+B150/60)
B153: [W35] +B17
A155: [W35] 'Number of degrees of longitude G
B155: [W35] +B19
A156: [W35] 'Number of minutes of longitude G
B156: [W35] +B20
A157: [W35] 'The longitude G in degrees is
B157: [W35] @IF(B153="W",B155+B156/60,-(B155+B156/60))
A159: [W35] 'LIMITING LATITUDE Lmax
B159: [W35] ^N
A161: [W35] 'Number of degrees of latitude Lmax
B161: [W35] @N(A161..A161)
A162: [W35] 'Number of minutes of latitude Lmax
B162: [W35] @N(A162..A162)
A163: [W35] 'The latitude Lmax in degrees is
B163: [W35] @IF(B159="S",-(B161+B162/60),B161+B162/60)
A165: [W35] 'POINT OF DESTINATION
A167: [W35] 'HEMISPHERE
B167: [W35] +B11
A169: [W35] 'Number of degrees of latitude L'
B169: [W35] +B27
A170: [W35] 'Number of minutes of latitude L'
B170: [W35] +B28
A171: [W35] 'The latitude L' in degrees is
B171: [W35] @IF(B167="S",-(B169+B170/60),B169+B170/60)
B173: [W35] +B31
A175: [W35] 'Number of degrees of longitude G'
B175: [W35] +B33
A176: [W35] 'Number of minutes of longitude G'
B176: [W35] +B34
A177: [W35] 'The longitude G' in degrees is
B177: [W35] @IF(B173="E",-(B175+B176/60),B175+B176/60)
A179: [W35] 'CALCULATIONS
B181: [W35] @TAN(B151*@PI/180)
B182: [W35] @TAN(B163*@PI/180)
C182: [W35] +B181/B182
B183: [W35] @TAN(B171*@PI/180)
C183: [W35] +B183/B182
D183: [W35] @COS(B163*@PI/180)
C184: [W35] @SIN(B151*@PI/180)
D184: [W35] @COS(B151*@PI/180)
A185: [W35] 'RESULTS
C185: [W35] @SIN(B163*@PI/180)
D185: [W35] +C184/C185
B186: [W35] +B157-180/@PI*@ACOS(C182)
C186: [W35] @SIN(B171*@PI/180)
D186: [W35] +C186/C185
```

```
E186: [W35] ^Longitude Gb
A187: [W35] 'The longitude Gb is
B187: [W35] @IF(@ABS(B186)>180,360-@ABS(B186),B186)
C187: [W35] @COS(B171*@PI/180)
E187: (F2) [W35] +B187
B188: [W35] +B177+180/@PI*@ACOS(C183)
E188: [W35] ^Longitude Gc
A189: [W35] 'The longitude Gc is
B189: [W35] @IF(@ABS(B188)>180,360-@ABS(B188),B188)
A191: [W35] 'The length of arc MB is
B191: [W35] 60*180/@PI*@ACOS(D185)
C191: (F1) [W35] +B191
A193: [W35] 'The length of arc CM' is
B193: [W35] 60*180/@PI*@ACOS(D186)
C193: (F1) [W35] +B193
A195: [W35] 'The length of arc BC is
B195: [W35] 60*@ABS(B189-B187)*@COS(B163*@PI/180)
C195: (F1) [W35] +B195
A197: [W35] 'The total length of arc MBCM' is
B197: [W35] @SUM(B191..B195)
C197: (F1) [W35] +B197
A200: [W35] 'Great circle track from M to B
A202: [W35] 'Calculations
B202: [W35] @SIN(B163*@PI/180)
B203: [W35] @SIN(B15*@PI/180)*@COS(B191*@PI/(180*60))
B204: [W35] @COS(B15*@PI/180)*@SIN(B191*@PI/(180*60))
C204: [W35] (B202-B203)/B204
B206: [W35] ^RESULT
A209: [W35] 'Initial great circle track M to B
B209: [W35] 180/@PI*@ACOS(C204)
C209: (F2) [W35] +B209
A211: [W35] 'Rhumb line track to be followed
A213: [W35] 'Inputs
A215: [W35] 'Speed of the ship in knots
B215: [W35] @N(A215..A215)
A217: [W35] 'Time t
B217: [W35] @N(A217..A217)
A219: [W35] 'Latitude L
B219: [W35] +B151
A221: [W35] 'Calculations
A223: [W35] 'Distance m1
B223: [W35] +B215*B217
A225: [W35] 'Correction a in degrees
B225: [W35] +B223/120*@SIN(B209*@PI/180)*@TAN(B151*@PI/180)
A227: [W35] 'The Rhumb line track is
B227: [W35] @IF(B11="N",B209+B225,B209-B225)
C227: (F2) [W35] +B227
```

CHAPTER 3 :

SATELLITES FOR NAVIGATION.

3.1. Introduction.

Many electronic position fixing systems in current operation have been developed.
Research and development continue towards improving the accuracy of the existing systems with most attention being directed towards satellite systems mainly the Navstar Global Positioning System which is a very important navigation aid of the future.

The existing Navy Navigation Satellite System (Transit) will be phased out in 1996.

3.2. Definition of the tasks.

In this chapter, calculations are related to the

- User Position expressed in Earth centered coordinates,
- diameter of a circle of coverage by satellite which can be visible over the elevation angle ELm
- time of satellite's passage above the observer's position, number of passes of the same satellite over the stationary observer during 24 hours
- maximum elevation angle of the satellite
- distance from the user to satellite
- Observed pass duration tobs , curve tobs = f(EL)
- satellite coverage in function of range from the user's position to satellite

- User position expressed in matrix form and accuracy

A description of the LOTUS 1-2-3 program used for these calculations and numerical results is given.

3.3. The Mathematical Model.

3.3.1. The user position Pu (Xu , Yu , Zu) expressed in Earth centered coordinates.

The coordinates of the user position in function of geodetic latitude , longitude and radius Rn are given by the following equations :

(1) $Xu = (Rn + H)*\cos Lu * \sin Gu$

(2) $Yu = (Rn + H)*\cos Lu* \cos Gu$

(3) $Zu = [(Rn (1- e^2) + H]* \sin Lu$

where Lu = the geodetic latitude
Gu = the geodetic longitude
H = the ellipsoid height
e = the eccentricity = 0.081818812
Rn = the radius of curvature of the prime vertical

3.3.2. Diameter of the circle of coverage.

The diameter of a circle of coverage by satellite which can be visible over ELm elevation angle is given by :

$$D = 2*(90 - ELm - arcsin[Re / (Re + H)*cosELm]*secL)$$

where Re = the earth radius

H = the altitude of the satellite above the Earth surface.

3.3.3. Time of satellite's passage above the observer's position.

The maximum period of time when satellite passes the area of coverage is given by :

$$tmax =4*\pi*(Re + H)/(360*Vs)*[90 - arcsin Re /(Re + H)]$$

where Vs = the satellite's speed.

3.3.4. Maximum elevation angle of a satellite.

The maximum elevation angle of the satellite is given by:

$$\tan ELm = (Re + H)*\cos a - Re / (Re + H)*\sin a ,$$

or

$$(6) \quad ELm = arctan(\cos a - \cos amax / \sin a)$$

since $\cos amax = Re /(Re + H)$, a = the angle of coverage.

The coordinates of the user (Lu,Gu), the nodal precession RAs are given. The angle a in C.A. is given by

$$cosa = (sinLu)^2 + (cosLu)^2*cos(RAs-Gu)$$

(Closest Approach : C. A.). Then ELm.

3.3.5. Number of passes of the same satellite over the stationary observer's position during 24 hours.

The number of passes of the same satellite over the stationary observer during 24 hours is given by :

Np = 4*{90-ELm-arcsin[Re/(Re+H)*cosELm]}/(T*360/24*cosL)

where L = the latitude of the observer.
T = the period of completing the orbit in hours.

3.3.6. Observed pass duration tobs.

The observed pass duration of a satellite can be calculated by the following formula:

(8) tobs = Ts{arccos[Re*cosELm/(Re+H)*cosa-ELm}/180

where
Ts = 84.4*[(Re+H)/Re]^1.5 = the satellite period.

3.3.7. Distance from the user to satellite.

The distance from the user to satellite is given by :

(10) $Rs = Re * [(r^{2} / Re^{2} - \cos EL)^{1/2} - \sin EL)]$

where r = the distance from the center of the Earth to satellite = Re + H
EL= the elevation angle

$$(11)\quad EL = \arcsin [(r^2 - Re^2 - Rs^2) / 2*Re*Rs].$$

3.3.8. Satellite coverage in function of range from the user's position to satellite.

The angle of coverage is given by :

$$(12)\quad a = \arccos [1 - (Rs^2 - H^2) / 2*(Re + H)*Re]$$

3.3.9. Position Equations.

If it is assumed that the relation between True Range R and Measured Range Rm can be written as :

R = Rm + b (US clock bias) + r (Random Range Error)

then the LOP equation is

$$X*\sin a*\cos EL + Y*\cos a*\cos EL + h*\sin EL + b - (Rdr - Rm) = r$$

If four satellites are observed simultaneously, then the set of 4 equations can be written in matrix form :

$$\begin{bmatrix} \sin a1*\cos EL1 & \cos a1*\cos EL1 & \sin EL1 & 1 \\ \sin a2*\cos EL2 & \cos a2*\cos EL2 & \sin EL2 & 1 \\ \sin a3*\cos EL3 & \cos a3*\cos EL3 & \sin EL3 & 1 \\ \sin a4*\cos EL4 & \cos a4*\cos EL4 & \sin EL4 & 1 \end{bmatrix} * \begin{bmatrix} X \\ Y \\ h \\ b \end{bmatrix} - \begin{bmatrix} Rdr1-Rm1 \\ Rdr2-Rm2 \\ Rdr3-Rm3 \\ Rdr4-Rm4 \end{bmatrix} = \begin{bmatrix} r1 \\ r2 \\ r3 \\ r4 \end{bmatrix}$$

which can be written

$$M * X - R = r$$

where M = the geometrical matrix

X = (X Y h b) = the position vector

R = (Rdr - R) = the observation vector

r = the random error vector.

3.3.10. Least Square solution of the position equation.

In the position equations , the geometrical matrix M and the observation vector R are known. The random error vector r is unpredictable.
From Least Square theory , it follows that the optimal estimators

X , Y , h , b are obtained by solving the equation

$$M * X = R$$

which gives (14) $X = M^{-1} * R$ (4 observations),

X is the optimal estimator for the position and user clock bias.

3.3.11. Error in X . Accuracy.

The accuracy of the estimators X , Y , h , b depends on UERE and the square root of the trace of $(MT * M)^{-1}$ which is called the Geometric Dilution of the (of Accuracy) Position GDOP in GPS system.

Under certain conditions , it can be shown that

$$\begin{bmatrix} sdx^2 & 0 & 0 & 0 \\ 0 & sdy^2 & 0 & 0 \\ 0 & 0 & sdh^2 & 0 \\ 0 & 0 & 0 & sdb^2 \end{bmatrix} = (MT * M)^{-1} * (UERE)^2$$

(15) $GDOP * UERE = (sdx^2 + sdy^2 + sdh^2 + sdb^2)^{1/2}$

(16) $PDOP * UERE = (sdx^2 + sdy^2 + sdh^2)^{1/2}$, $sdb = 0$

(17) $HDOP * UERE = (sdx^2 + sdy^2)^{1/2}$, $sdh = sdb = 0$

(18) $VDOP * UERE = sdh$, $sdx = sdy = sdb = 0$

(19) $TDOP * UERE = sdb$, $sdx = sdy = sdh = 0$

(20) The R 95 = 2 * HDOP * UERE in GPS .

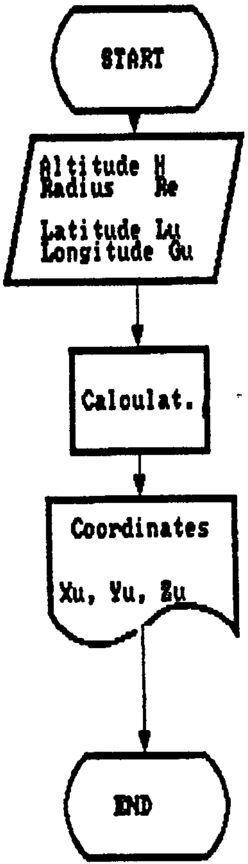
UERE = User Equivalent Range Error.

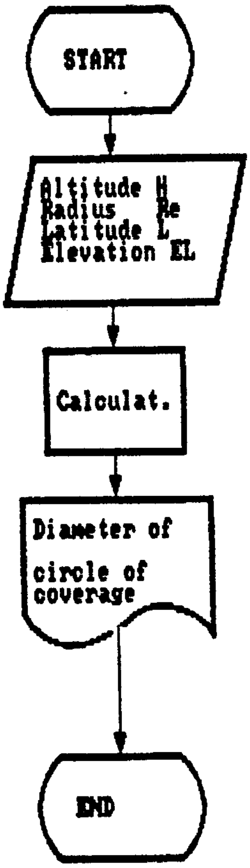
3.4. The flow chart.

The flow charts from page 69 to page 72 show the the different steps of calculations required for the computer program.

SATELLITES FOR NAVIGATION.

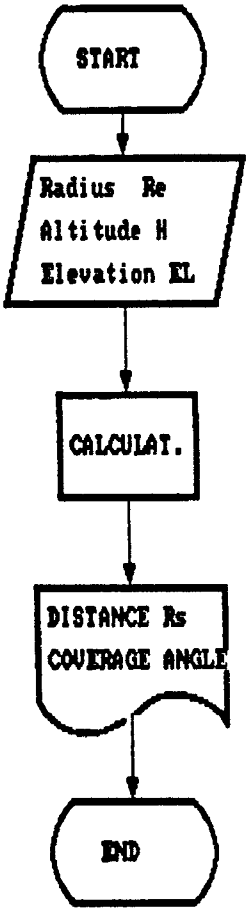
TASK : USER POSITION, DIAMETER OF CIRCLE OF COVERAGE.





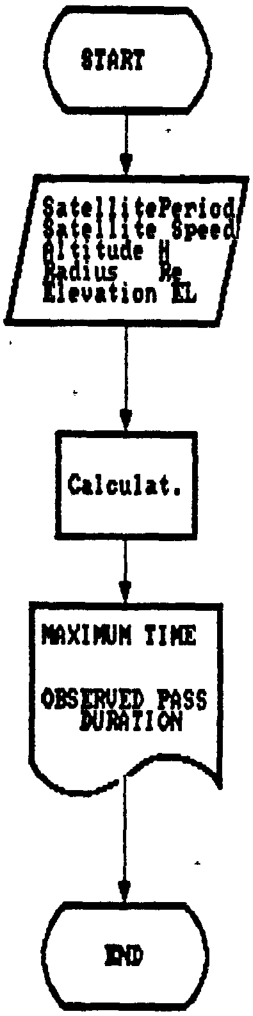
SATELLITES FOR NAUIGATION.

TASK : DISTANCE FROM USER TO SATELLITE, ANGLE OF COUERAGE.



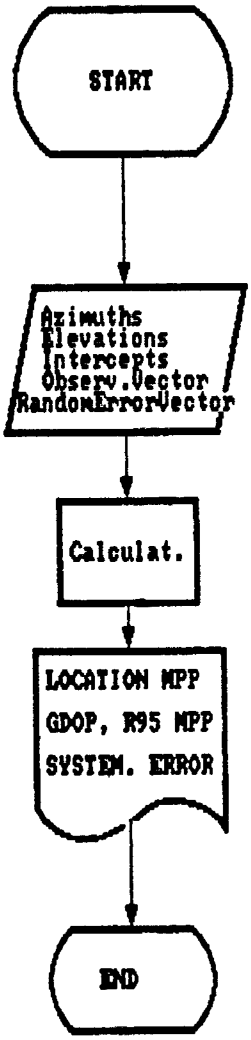
SATELLITES FOR NAVIGATION.

TASK : MAXIMUM TIME OF PASSAGE, OBSERVED PASS DURATION.



SATELLITES FOR NAVIGATION.

TASK : POSITION DETERMINATION USING MATICES. LEAST SQUARE METHOD.



SATELLITES FOR NAVIGATION.

3.5. ORGANIZATION AND STRUCTURE OF THE LOTUS PROGRAM.

SAT 1 A.

INPUTS		CELLS LOCATION
User latitude	North / South	B 9
	Degrees	B 10
	Minutes	B 11
User longitude	East / West	A 14
	Degrees	B 15
	Minutes	B 16
Ellipsoid heigth		C 18
Eccentricity		C 20
Earth radius		C 22
Elevation angle		B 37
Satellite speed		B 72
Angle of coverage in Closest Approach Pt		B 88
Right Ascension		F 96
Distance from the Earth center to satellite		B 123
Azimuths		B 183 .. B 188
Elevation angles		C 183 .. C 188
Intercepts		B 222 .. B 227
H 95 (1 LOP)		C 252
TRANSIT ACCURACY VERSUS ANGLE OF ELEVATION		
	Elevation angle	B 268 .. B 276
	R 68	C 268 .. C 276
OBSERVED PASS DURATION VERSUS ANGLE OF ELEVATION		
	Angle of coverage	F 117 .. F 124
	Angle of elevation	G 117 .. G 124

CALCULATIONS (See listing)

OUTPUTS	CELLS LOCATION
USER POSITION (Xu, Yu, Zu)	C 26, C 28, C 30
DIAMETER OF CIRCLE OF COVERAGE	C 59 and D 59
MAXIMUM TIME	C 81 and D 81
OBSERVED PASS DURATION	C 115 and D 115
NUMBER OF PASSES	F 82 And G 82
MAXIMUM ANGLE OF ELEVATION	C 100 and D 100
DISTANCE USER-SATELLITE	B 139
ANGLE OF COVERAGE	C 167 and D 167
LOCATION MPP (X , Y)	C 241 and D 244
GDOA and R 95	C 249 and D 259

3.6. Examples. Ref 5 pages 21, 24, 34, Appendix 1.

Example 1 :

Latitutde Lu = 45 00 N

Longitude Gu = 45 00 E

Radius Rn = 6370 km

Height H = 1100 km

Eccentricity e = 0.081818812

Example 2 :

Elevation EL = 10 (for the diameter of the circle of coverage)

Satellite speed Vs = 7 km/s

Satellite period = 107 min

Elevation EL = 5 (for the maximum period of time tmax)

Angle of coverage C.A. = 16

Example 3 :

Latitude Lu = 10 N

(for the number of passes , tobs)

Example 4 :

Latitude Lu = 50 N

Longitude Gu = 60 E

Righting Ascension RAs = 70

(for the maximum angle of elevation)

Example 5 :

Angle of elevation EL = 10
(for the distance Rs from the user to satellite,chosen by the author)

Example 6 :

Distance Rs = 2949.2 km
(for the satellite coverage)

Example 7 : Ref 7, pages 11.13 and 11.17

Azimuths	Intercepts
309.5	- 2.2
067	+ 0.1
153	+ 1.8
345.5	- 2.3
123.9	+ 3.2
000.8	- 1.6

3.7. LOTUS 1-2-3 results.

The results may be found from page 76 to page 94.

```
1: [W35] 'SATELLITES FOR NAVIGATION                                           READ

  SATELLITES FOR NAVIGATION             PROGRAM SAT1. TATY-BOUSSIANA J.L.
                                                      TASK 1

  USER POSITION ( Xu , Yu , Zu )
                    in
  Earth centered coordinates.

                INPUTS

  HEMISPHERE                                              N
        Degrees of latitude Lu                                             45
1       Minutes of latitude Lu                                              0
2                                                                          45
3
4                                      E
       Degrees of longitude Gu                                             45
6      Minutes of longitude Gu                                              0
7                                                                          45
8                                      The ellipsoid heigth H in km is
9
0                                      The eccentricity e is
3-Sep-90  12:22 AM
```

```
24: [W15] 'PRAC[illegible]

8  The ellipsoid heigth H in km is                1100
9
0  The eccentricity e is                   0.081818812
1
   The radius of curvature Rn in km is            6370
3
4              CALCULATIONS               ACCURATE RESULTPRACTICAL RESULTS
5
6  The coordinate Xu is                          -3735          -3735.0
7
8  The coordinate Yu is                  3735.00000000           3735.0
9
0  The coordinate Zn is                   5251.9346384           5251.9
1
2                TASK 2
3
4
5
6
7                                  21
23-Sep-90  12:25 AM
```

```
                                                     TASK 2
 DIAMETER OF CIRCLE OF COVERAGE

                 INPUTS

 The elevation angle ELm of the sat.                          10

 The earth radius Re in km is                               6370

 The altitude of the sat. H in km is                        1100
 The period T is                                    1.7833333333
 HEMISPHERE                                                    N
4        Degrees of latitude L                                45
5        Minutes of latitude L                                 0
6  The latitude L in degrees is                               45
7
              CALCULATIONS
9
0                                                    0.984807753
1                                                   0.8527443106
3-Sep-90  10:26 AM
```

```
                          1100
                  1.7833333333
13                           N
14                          45
15                           0
                            45

18
19
50                 0.984807753
51                0.8527443106
52                57.117867239
53
54                22.882132761
55
56                                                         PRACTICAL
57                                     ACCURATE RESULT       RESULT
58
59  Diameter of circle of coverage         64.72044497         64.7
60
23-Sep-90  10:30 AM
```

```
TIME OF SATELLITE'S PASSAGE


                 INPUTS

The radius Re in km is                                            6370

The ellipsoid heigth H in km is                                   1100

The satellite speed Vs in km / s is                                  7

             CALCULATIONS

                                                          37.250312893
                                                          31.488577749




3-Sep-90  12:29 AM
```

```
                                 6370

                                 1100

                                    7



                         37.250312893
                         31.488577749

                                       ACCURATE RESULTPRACTICAL RESULT

     The time tmax in minutes is       19.549322895           19.5

23-Sep-90  12:31 AM
```

```
NUMBER OF PASSES OF THE SAME
SATELLITE OVER THE STATIONARY
OBSERVER DURING 24 HOURS, Np

                 INPUTS
Hemisphere                                    N
       Degrees of latitude L                        10
       Minutes of latitude L                         0
The Latitude of the observer L                      10
The angle of elevation ELm in deg.                  10
The Earth radius Re in km is                      6370
The altitude of the sat. in km is                 1100
The period Ts in min                         107.18011

CALCULATIONS

                                              57.117867
                                              26.387950

3-Sep-90  12:37 AM
```

```
SATELLITE OVER THE STATIONARY
OBSERVER DURING 24 HOURS, Np

                 INPUTS
Hemisphere                                    N
       Degrees of latitude L                        10
       Minutes of latitude L                         0
The Latitude of the observer L                      10
The angle of elevation ELm in deg.                  10
The Earth radius Re in km is                      6370
The altitude of the sat. in km is                 1100
The period Ts in min                         107.18011

CALCULATIONS

                                              57.117867
                                              26.387950
                                                RESULT  PRACTICAL RESULT
The number of passes Np is                    3.4685728         3.47

23-Sep-90  12:39 AM
```

```
3                                                    TASK 4
4  MAXIMUM ELEVATION ANGLE
5
6                INPUTS
7  The radius Re in km is                                                   6370
8  The angle of coverage a in C.A.is                                          16
9  The ellipsoid height H in km is                                          1100
0
1             CALCULATIONS
2
3                                                                  0.8527443106
4                                                                  0.9612616959
5                                                                  0.2756373558


8
9
00                                      The max. angle of elevation ELm is

02
3-Sep-90  12:35 AM
```

```
3
4

6
7                                  6370
8                                    16
9                                  1100

1
2
3                          0.8527443106
4                          0.9612616959
5                          0.2756373558
6
7                                        ACCURATE RESULTPRACTICAL RESULT
8
9
100 The max. angle of elevation ELm is   21.489372276            21.5
101
102
23-Sep-90  12:36 AM
```

```
MAXIMUM ELEVATION ANGLE ELmax
                                        INPUTS
The ellipsoid height H is                  1100
The radius Re is                           6370
HEMISPHERE                               N
      Degrees of latitude Lu                 50
      Minutes of latitude  Lu                 0
The Lat. Lu of the User is                   50
                                         E
      Degrees of longitude G                 60
      Minutes of longitude G                  0
The long. Gu of the User is                 -60
The Rigth Ascension RAs is                   70          10
T
CALCULATIONS                          0.9848077
                                      0.5868240
                                      0.4068988
                                      0.9937229
                                      0.1118692
                                      0.8527443
3-Sep-90  12:41 AM
```

```
      Degrees of latitude Lu                 50
      Minutes of latitude  Lu                 0
The Lat. Lu of the User is                   50
                                         E
      Degrees of longitude G                 60
      Minutes of longitude G                  0
The long. Gu of the User is                 -60
The Rigth Ascension RAs is                   70          10
T
CALCULATIONS                          0.9848077
                                      0.5868240
                                      0.4068988
                                      0.9937229
                                      0.1118692
                                      0.8527443

                                      RESULT     PRACTICAL RESULT

The maximum angle of elev. ELmax is51.567311        51.6

3-Sep-90  12:42 AM
```

```
01                                                    TASK 5
02 OBSERVED PASS DURATION tobs in min
03
04 INPUTS
05
06 The radius Re in km is                                           6370
07 The ellipsoid height H in km is                                  1100
08 The angle of elevation Elm is                                       5
09 The angle a is                                                     16
10 CALCULATIONS
11 The Satellite period Ts in min is                       107.180110597
12                                                          6345.7602268
13                                                          7180.6248687
14                                                                  7470
15                                     The observ.pass dur.tobs in min is
16
17
18
19
20              INPUTS
3-Sep-90  12:43 AM
```

```
101
102
103
104
105
106                              6370
107                              1100
108                                 5
109                                16
110
111                     107.180110597          107.2
112                      6345.7602268
113                      7180.6248687
114                              7470 ACCURATE RESULTPRACTICAL RESULT
115 The observ.pass dur.tobs in min is  13.6380192621          13.6
116
117
118
119
120
23-Sep-90  12:45 AM
```

```
                                                        TASK 6
DISTANCE FROM THE USER TO SATELLITE

                INPUTS

Distance r from the Earth's center
to satellite is                                                          7470

The Earth's radius Re is                                                 6370

The angle of elevation EL is                                               10

             CALCULATIONS
                                                                     55800900
                                                                     40576900
                                                                 0.9698463104
                                                                 0.1736481777


3-Sep-90  12:46 AM
```

```
                INPUTS

Distance r from the Earth's center
to satellite is                                  7470

The Earth's radius Re is                         6370

The angle of elevation EL is                       10

             CALCULATIONS
                                             55800900
                                             40576900     1.375188839
                                         0.9698463104
                                         0.1736481777



                                 ACCURATE RESULTS  PRACTICAL RESULTS

The distance from the User to
satellite Rs in km is                    2949.4181906         2949.42
23-Sep-90  12:47 AM
```

```
                                              TASK 7
 SATELLITE COVERAGE IN FUNCTION
 OF RANGE FROM THE USER'S POSITION
            TO SATELLITE

                INPUTS

 The distance Rs in km is                     2949.4181906

 The ellipsoid heigth Hs is                           1100

 The Earth's radius Re is                             6370

                                           CALCULATIONS

                                              7489067.6628
                                                  95167800
                                               0.921306706

                                                              ACCURATE RESULT
Sep-90   12:50 AM
```

```
9


1
2
3          2949.4181906
4
                    1100
56
57                  6370
58
59      CALCULATIONS
60
61         7489067.6628
62             95167800
163         0.921306706
164
165                         ACCURATE RESULTPRACTICAL RESULT
166
167 Angle of coverage     22.882132761             22.9
168
23-Sep-90  12:51 AM
```

```
69                TASK 8
70
71 POSITION DETERMINATION USING MATRIX
72              CALCULATION
73 LEAST SQUARE METHOD
74
75 SIX SATELLITES ARE OBSERVED
76 SIMULTANEOUSLY
77
78 INPUTS
79
80 SATELLITES                                        ANGLE a          ELEVATION EL
81
82
83 Satellite 1                                             309.5                 0
84 Satellite 2                                                67                 0
85 Satellite 3                                               153                 0
86 Satellite 4                                             345.5                 0
87 Satellite 5                                             123.9                 0
88 Satellite 6                                               0.8                 0
3-Sep-90  12:52 AM
```

```
91 GEOMETRICAL MATRIX M                   -0.7716245834   0.6360782203
92                                         0.9205048535   0.3907311285
93                                         0.4539904997  -0.8910065242
94                                        -0.2503800041   0.9681476404
95                                         0.8300122851   -0.557745109
96                                         0.0139621803    0.999902524
97
98                                        -0.7716245834   0.6360782203
99                                         0.9205048535   0.3907311285
200                                        0.4539904997  -0.8910065242
201                                       -0.2503800041   0.9681476404
202                                        0.8300122851   -0.557745109
203                                        0.0139621803    0.999902524
204
205
206 TRANSPOSED MATRIX MT
207
208                                       -0.7716245834   0.9205048535
209                                        0.6360782203   0.3907311285
210
23-Sep-90  12:55 AM
```

```
)
[
2 PRODUCT MT * M
3                                                        2.4006465391   -1.2270314719
4                                                       -1.2270314719    3.5993534609
5
6 INVERSE OF ( MT * M )                                  0.504452433     0.171969499
7                                                        0.171969499     0.3364526437
8
9
0
) INTERCEPT ( Rdr - Rm )
2                                                                 -2.2
3                                                                  0.1
4                                                                  1.8
5                                                                 -2.3
                                                                   3.2
7                                                                 -1.6
8
9
-Sep-90   12:57 AM
```

```
28
  PRODUCT MT * INTERCEPT (Rdr-Rm)
30
31                                                       5.8163813014
32                                                      -8.5754786753
33
  PRODUCT(INV.( MT*M ))*MT*INTERCEPT
35
36 *                                                     1.4593669274
37                                                      -1.8850022928
38
39 LOCATION OF MPP                                              ACCURATE RESULT
40
41                                  The abscisse X of MPP is    1.4593669274
42
43
44                                  The ordinate Y of MPP is   -1.8850022928
45
46
47
3-Sep-90   12:58 AM
```

```
28
29
30
31               5.8163813014
32              -8.5754786753
33
34 INTERCEPT
35
36               1.4593669274
37              -1.8850022928

39                                    ACCURATE RESULTPRACTICAL RESULTS
40
41 The abscisse X of MPP is   1.4593669274             1.5
42

44 The ordinate Y of MPP is  -1.8850022928            -1.9
45
46
47
3-Sep-90  12:59 AM
```

```
46
47 GEOMETRICAL DILUTION OF ACCURACY
48
49                                    The GDOA is                    0.9170087659

51
52 THE M 95 ( 1 LOP ) is    The M 95 ( 1 LOP ) is                      2
53
54
55
56
```

```
228
229       )
230
231               5.8163813014
232              -8.5754786753
233
234 INTERCEPT
235
236               1.4593669274
237              -1.8850022928
238
239                            ACCURATE RESULTPRACTICAL RESULTS
240
2   The abscisse X of MPP is   1.4593669274            1.5
242
243
244 The ordinate Y of MPP is  -1.8850022928           -1.9
245
2
247
23-Sep-90  01:02 AM
```

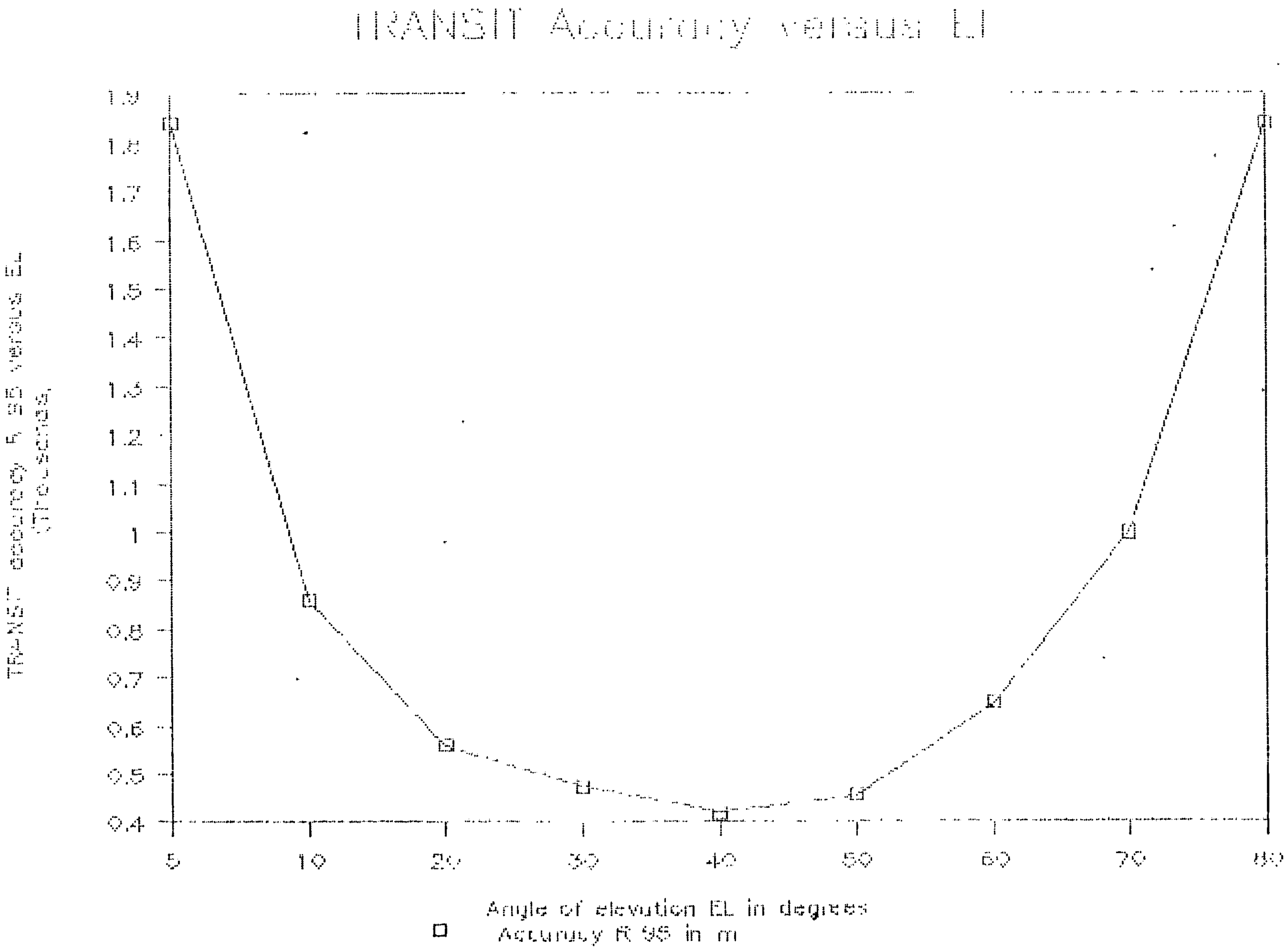
```
246
247 GEOMETRICAL DILUTION OF ACCURACY
248
2                             The GDOA is                0.9170087659
250
251
252                           The M 95 ( 1 LOP ) is                 2
253
2
255
256                                                   ACCURATE RESULT
257
258
259                           The R 95 required is       1.8340175318
260
261
262
263
264
265
23-Sep-90  01:03 AM
```

```
246
247
248
249 The GDOA is                    0.9170087659           0.9
250
251
252 The M 95 ( 1 LOP ) is                    2
253
254
255
256                              ACCURATE RESULTPRACTICAL RESULT
257
2
259 The R 95 required is         1.8340175318           1.8
260
261
262
2
264
265
23-Sep-90  01:06 AM
```

```
63
64 TRANSIT ACCURACY R 95 = f ( EL )
65
66                                  Angles of elevation EL inR 68 in meters
67
68                                                               5          1105.3
69                                                              10           515.8
70                                                              20           336.8
71                                                              30           284.2
72                                                              40           252.6
73                                                              50           273.7
                                                                60           389.5
75                                                              70             600
76                                                              80          1105.3
77
78

80
81
82
23-Sep-90  01:07 AM
```

```
2
264
265
266 Angles of elevation EL inR 68 in meters R 95 in meters
267
2                                  5         1105.3   1842.1666667
269                               10          515.8   859.66666667
270                               20          336.8   561.33333333
271                               30          284.2   473.66666667
272                               40          252.6            421
273                               50          273.7   456.16666667
274                               60          389.5   649.16666667
275                               70            600           1000
276                               80         1105.3   1842.1666667
277
278
279
280
281
282
23-Sep-90  01:08 AM
```

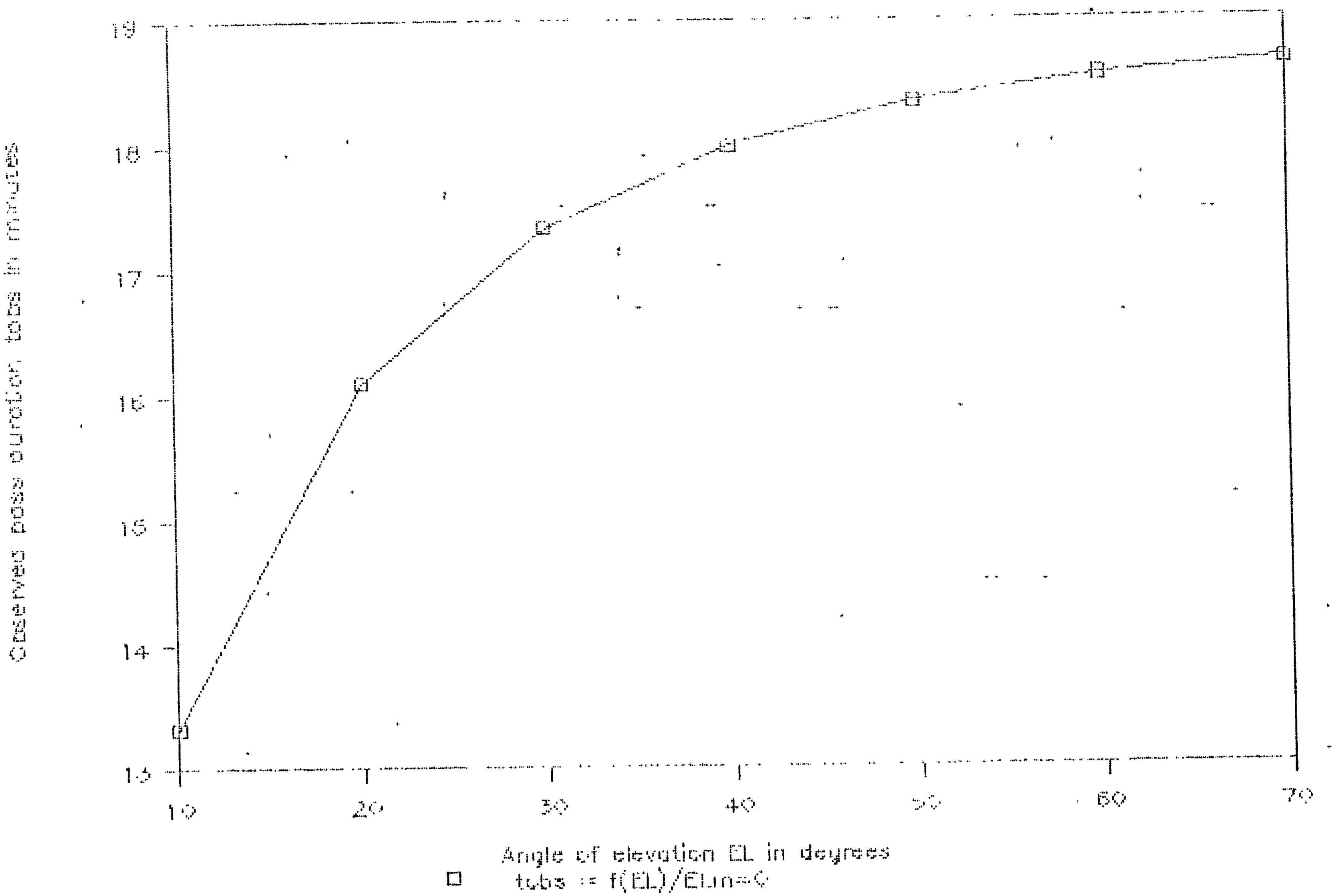

TRANSIT Accuracy versus EL
TRANSIT accuracy R 95 versus EL
(Thousands)
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
5
10
20
30
40
50
60
70
80
Angle of elevation EL in degrees
Accuracy R 95 in m

```
109
110
111
112
113
114
115          Angle a in C.A.                        Angle of elevation EL
116
117                                  31.5                                           0
118                                 22.75                                          10
1                                   16.75                                          20
120                                  12.5                                          30
121                                  9.25                                          40
122                                  6.75                                          50
123                                  4.75                                          60
1                                       3                                          70
125
126
127
128
23-Sep-90  01:11 AM
```

```
1
110
111
112
113
1
115        X              Y              X/Y
116
117
118          6370   6888.8412584   0.9246838127   0.3905920356
119          6370   7153.0580652   0.8905282107   0.4722913871
120          6370   7292.9311732   0.8734485283   0.5085559942
121          6370   7372.8628582   0.8639791791   0.5276768338
122          6370   7418.2213735   0.8586964016   0.5380757839
123          6370   7444.3443037   0.8556831522   0.5439268238
124          6370   7459.7626246   0.8539145708   0.5473347764
125
126
127
128
23-Sep-90  01:13 AM
```

```
109
110
111
112
113
114
115      Observed pass duration tobs
116            tobs = f ( EL )
117
118                                      13.3
119                                      16.1
120                                      17.4
121                                      18.0
122                                      18.4
123                                      18.6
124                                      18.7
125
126
127
128
23-Sep-90  01:16 AM
```

Observed pass duration of the satellite



Ref. 5

Table 1

NAVIGATION SATELLITE MOTION PARAMETERS

Abreviations used	Formulae and explanation	Earth surface	Transit system - - satellites	GPS/Navstar-satellites	Geostationary satellites /Inmarsat/
H-altitude above Earth surface [km]	R_E - earth radius	0	1100	20,200	35,800
Orbit's inclination i [°]		-	90	63	00
Period T_S [min]	$T_S = 2\pi \left(\frac{r^3}{MG}\right)^{1/2}$	84,4	107,3	719	1440
Radius of Earth coverage [o]	$\cos \alpha_{max} = 1/(1 + \frac{H}{R_E})$	-	31,5	76,1	81,5
Velocity [Km/s]	$V_S = V_o\left(1 + \frac{H}{R_E}\right)^{1/2}$	$V_o = 7.905$	7.301	3.873	3.06
Angular velocity [rad/s]	$W_S = W_o\left(1 + \frac{H}{R_E}\right)^{3/2}$	$W_o = 1.2539 \times 10^{-3}$	0.9763×10^{-3}	0.1457×10^{-3}	0.0729×10^{-3}

3.8. Results from the references.

User Position (Xu, Yu, Zu)

Xu = - 3735 km
Yu = 3735 km
Zu = 5251.9 km

Diameter of circle of coverage	D = 64.7 km
Maximum period of time	tmax = 18.7 min
Number of passes	Np = 3.1
Observation pass duration	tobs = 13.7 min
Maximum elevation	ELmax = 50.7 (RAs = 70)

Curves tobs = f (EL)
R 68 = g (EL)

Least Square Method . Matrix calculations.

Abscisse MPP X = 1.5 M

Ordinate MPP Y = - 1.9 M

G D O A = 0.9

R 95 = 1.8 M

3.9. Comparisons and critical remarks.

- The comparison shows the similarity between the LOTUS results and those obtained from the listed references.

- The attention of the reader is drawn on the fact that in practice, a satellite is visible for angles of elevation higher or equal to 5 degrees. The example given in Task 7, position determination using matrices and least square method is normally designed for star observations. It is not realistic when referring to satellites.

3.10. Listing of the LOTUS program.

The listing is given in Appendix E.

APPENDIX E :

PROGRAM : SAT 1 A.

```
A1: [W35] 'SATELLITES FOR NAVIGATION
B1: [W35] 'PROGRAM SAT1A.TATY-BOUSSIANA J.L.
C1: [W35] ^MET ( N ) 90
D1: [W35] 'WORLD MARITIME UNIVERSITY
E1: [W35] ^MALMO, SWEDEN.
B2: [W35] ^TASK 1
A3: [W35] 'USER POSITION ( Xu , Yu , Zu )
A4: [W35] ^in
A5: [W35] 'Earth centered coordinates.
A7: [W35] ^INPUTS
C8: [W35] ^Latitude Lu in degrees
A9: [W35] 'HEMISPHERE
B9: [W35] ^N
A10: [W35] 'The number of degrees of Lu is
B10: [W35] @N(A10..A10)
C10: [W35] @IF(B9="N",B12,-B12)
A11: [W35] 'The number of minutes of Lu is
B11: [W35] @N(A11..A11)
B12: [W35] +B10+B11/60
B14: [W35] ^E
C14: [W35] 'Longitude Gu in degrees
A15: [W35] 'The number of degrees of Gu is
B15: [W35] @N(A15..A15)
C15: [W35] @IF(B14="E",-B17,B17)
A16: [W35] 'The number of minutes of Gu is
B16: [W35] @N(A16..A16)
B17: [W35] +B15+B16/60
B18: [W35] 'The ellipsoid heigth H in km is
C18: [W35] @N(B18..B18)
B20: [W35] 'The eccentricity e is
C20: [W35] @N(B20..B20)
B22: [W35] 'The radius of curvature Rn in km is
C22: [W35] @N(B22..B22)
B24: [W35] ^CALCULATIONS
C24: [W35] ^RESULTS
B26: [W35] 'The coordinates Xu is
C26: [W35] (C22+C18)*@COS(C10*@PI/180)*@SIN(C15*@PI/180)
D26: (F1) [W35] +C26
B28: [W35] 'The coordinate Yu is
C28: (F8) [W35] (C22+C18)*@COS(C10*@PI/180)*@COS(C15*@PI/180)
D28: (F1) [W35] +C28
B30: [W35] 'The coordinate Zn is
C30: [W35] (C22*(1-C20^2)+C18)*@SIN(C10*@PI/180)
D30: (F1) [W35] +C30
B32: [W35] ^TASK 2
A33: [W35] 'DIAMETER OF CIRCLE OF COVERAGE
A35: [W35] ^INPUTS
A37: [W35] 'The elevation angle ELm of the sat.
B37: [W35] @N(A37..A37)
A39: [W35] 'The earth radius Re in km is
B39: [W35] +C22
A41: [W35] 'The altitude of the sat. H in km is
```

```
B41: [W35] +C18
A42: [W35] 'The period T is
B42: [W35] @N(A42..A42)
A43: [W35] 'HEMISPHERE
B43: [W35] +B9
A44: [W35] 'Number of degrees of the latitude
B44: [W35] @N(A44..A44)
A45: [W35] 'Number of minutes of latitude is
B45: [W35] @N(A45..A45)
A46: [W35] 'The latitude L in degrees is
B46: [W35] @IF(B43="N",B44+B45/60,-(B44+B45/60))
A48: [W35] ^CALCULATIONS
B50: [W35] @COS(B37*@PI/180)
B51: [W35] +B39/(B39+B41)
B52: [W35] 180/@PI*@ASIN(B51*B50)
B54: [W35] 90-B37-B52
B57: [W35] ^RESULT
B59: [W35] 'The diameter D is
C59: (F8) [W35] 2*B54/@COS(B46*@PI/180)
D59: (F1) [W35] +C59
A61: [W35] 'TASK 3
A63: [W35] 'TIME OF SATELLITE'S PASSAGE
E63: [W35] 'NUMBER OF PASSES OF THE SAME
E64: [W35] 'SATELLITE OVER THE STATIONARY
E65: [W35] 'OBSERVER DURING 24 HOURS, Np
A66: [W35] ^INPUTS
E67: [W35] ^INPUTS
A68: [W35] 'The radius Re in km is
B68: [W35] +C22
E68: [W35] 'Hemisphere
F68: [W35] +B43
E69: [W35] 'Number of degrees of latitude L
F69: [W35] +B44
A70: [W35] 'The ellipsoid heigth H in km is
B70: [W35] +C18
E70: [W35] 'Number of minutes of latitude L
F70: [W35] +B45
E71: [W35] 'The Latitude of the observer L
F71: [W35] @IF(F68="N",F69+F70/60,-(F69+F70/60))
A72: [W35] 'The satellite speed Vs in km / s is
B72: [W35] @N(A72..A72)
E72: [W35] 'The angle of elevation ELm in deg. is
F72: [W35] @N(E72..E72)
E73: [W35] 'The Earth radius Re in km is
F73: [W35] +C22
A74: [W35] ^CALCULATIONS
E74: [W35] 'The altitude of the sat. in km is
F74: [W35] +C18
E75: [W35] 'The period Ts in min
F75: [W35] +B111
B76: [W35] 4*@PI*(B68+B70)/(360*B72)
B77: [W35] (90-180/@PI*@ASIN(B68/(B68+B70)))
```

```
E77: [W35] 'CALCULATIONS
B79: [W35] ^RESULT
F79: [W35] 180/@PI*@ASIN(F73/(F73+F74)*@COS(F72*@PI/180))
F80: [W35] 360/24*(F75/60)*@COS(F71*@PI/180)
B81: [W35] ^The time tmax in minutes is
C81: [W35] +B76*B77/60
D81: (F1) [W35] +C81
F81: [W35] ^RESULT
E82: [W35] 'The number of passes Np is
F82: [W35] 4*(90-F72-F79)/F80
G82: (F2) [W35] +F82
B83: [W35] ^TASK 4
F83: [W35] ^TASK 4
A84: [W35] 'MAXIMUM ELEVATION ANGLE
E84: [W35] 'MAXIMUM ELEVATION ANGLE ELmax
F85: [W35] ^INPUTS
A86: [W35] ^INPUTS
E86: [W35] 'The ellipsoid height H is
F86: [W35] +C18
A87: [W35] 'The radius Re in km is
B87: [W35] +C22
E87: [W35] 'The radius Re is
F87: [W35] +C22
A88: [W35] 'The angle of coverage a in C.A.is
B88: [W35] @N(A88..A88)
E88: [W35] 'HEMISPHERE
F88: [W35] ^N
A89: [W35] 'The ellipsoid height H in km is
B89: [W35] +C18
E89: [W35] 'Number of degrees of lat. Lu
F89: [W35] @N(E89..E89)
E90: [W35] 'Number of minutes of lat. Lu
F90: [W35] @N(E90..E90)
A91: [W35] ^CALCULATIONS
E91: [W35] 'The Lat. Lu of the User is
F91: [W35] @IF(F88="N",+F89+F90/60,-(F89+F90/60))
F92: [W35] ^E
B93: [W35] +B87/(B87+B89)
E93: [W35] 'Number of degrees of long. G
F93: [W35] @N(E93..E93)
B94: [W35] @COS(B88*@PI/180)
E94: [W35] 'Number of minutes of long. G
F94: [W35] @N(E94..E94)
B95: [W35] @SIN(B88*@PI/180)
E95: [W35] 'The long. Gu of the User is
F95: [W35] @IF(F92="E",-(F93+F94/60),F93+F94/60)
E96: [W35] 'The RAs is
F96: [W35] @N(E96..E96)
G96: [W35] +F96-@ABS(F95)
B97: [W35] ^RESULT
E98: [W35] 'CALCULATIONS
F98: [W35] @COS(G96*@PI/180)
```

```
F99: [W35] (@SIN(F91*@PI/180))^2
B100: [W35] 'The max. angle of elevation ELm is
C100: [W35] 180/@PI*@ATAN((B94-B93)/B95)
D100: (F1) [W35] +C100
F100: [W35] (@COS(F91*@PI/180))^2*F98
B101: [W35] ^TASK 5
F101: [W35] @SUM(F99..F100)
A102: [W35] 'OBSERVED PASS DURATION tobs in min
F102: [W35] @SQRT(1-F101^2)
F103: [W35] +F87/(F86+F87)
A104: [W35] 'INPUTS
E105: [W35] 'RESULT
A106: [W35] 'The radius Re in km is
B106: [W35] +C22
A107: [W35] 'The ellipsoid height H in km is
B107: [W35] +C18
E107: [W35] 'The maximum angle of elev. ELmax is
F107: [W35] 180/@PI*@ATAN((F101-F103)/F102)
G107: (F1) [W35] +F107
A108: [W35] 'The angle of elevation Elm is
B108: [W35] @N(A188..A188)
A109: [W35] 'The angle a is
B109: [W35] @N(A109..A109)
A110: [W35] 'CALCULATIONS
A111: [W35] 'The Satellite period Ts in min is
B111: [W35] 84.4*((B106+B107)/B106)^(3/2)
C111: (F1) [W35] +B111
B112: [W35] +B106*@COS(B108*@PI/180)
B113: [W35] (B106+B107)*@COS(B109*@PI/180)
B114: [W35] +B106+B107
C114: [W35] ^RESULT
B115: [W35] 'The observ.pass dur.tobs in min is
C115: [W35] +B111*(180/@PI*@ACOS(B112/B113)-B108)/180
D115: (F1) [W35] +C115
F115: [W35] ^Angle a in C.A.
G115: [W35] ^Angle of elevation EL
H115: [W35] ^X
I115: [W35] ^Y
J115: [W35] ^X/Y
L115: [W35] ^Observed pass duration tobs
B117: [W35] ^TASK 6
F117: [W35] @N(F117..F117)
G117: [W35] @N(G117..G117)
A118: [W35] 'DISTANCE FROM THE USER TO SATELLITE
F118: [W35] @N(F118..F118)
G118: [W35] @N(G118..G118)
H118: [W35] +$B$106
I118: [W35] +$B$114*@COS(F118*@PI/180)
J118: [W35] +H118/I118
K118: [W35] @ACOS(J118)
L118: (F1) [W35] +$B$111*(180/@PI*K118)/180
F119: [W35] @N(F119..F119)
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```
G119: [W35] @N(G119..G119)
H119: [W35] +$B$106
I119: [W35] +$B$114*@COS(F119*@PI/180)
J119: [W35] +H119/I119
K119: [W35] @ACOS(J119)
L119: (F1) [W35] +$B$111*(180/@PI*K119)/180
A120: [W35] ^INPUTS
F120: [W35] @N(F120..F120)
G120: [W35] @N(G120..G120)
H120: [W35] +$B$106
I120: [W35] +$B$114*@COS(F120*@PI/180)
J120: [W35] +H120/I120
K120: [W35] @ACOS(J120)
L120: (F1) [W35] +$B$111*(180/@PI*K120)/180
F121: [W35] @N(F121..F121)
G121: [W35] @N(G121..G121)
H121: [W35] +$B$106
I121: [W35] +$B$114*@COS(F121*@PI/180)
J121: [W35] +H121/I121
K121: [W35] @ACOS(J121)
L121: (F1) [W35] +$B$111*(180/@PI*K121)/180
A122: [W35] 'Distance r from the Earth's center
F122: [W35] @N(F122..F122)
G122: [W35] @N(G122..G122)
H122: [W35] +$B$106
I122: [W35] +$B$114*@COS(F122*@PI/180)
J122: [W35] +H122/I122
K122: [W35] @ACOS(J122)
L122: (F1) [W35] +$B$111*(180/@PI*K122)/180
A123: [W35] 'to satellite is
B123: [W35] +C22+C18
F123: [W35] @N(F123..F123)
G123: [W35] @N(G123..G123)
H123: [W35] +$B$106
I123: [W35] +$B$114*@COS(F123*@PI/180)
J123: [W35] +H123/I123
K123: [W35] @ACOS(J123)
L123: (F1) [W35] +$B$111*(180/@PI*K123)/180
F124: [W35] @N(F124..F124)
G124: [W35] @N(G124..G124)
H124: [W35] +$B$106
I124: [W35] +$B$114*@COS(F124*@PI/180)
J124: [W35] +H124/I124
K124: [W35] @ACOS(J124)
L124: (F1) [W35] +$B$111*(180/@PI*K124)/180
A125: [W35] 'The Earth's radius Re is
B125: [W35] +C22
A127: [W35] 'The angle of elevation EL is
B127: [W35] @N(A127..A127)
A129: [W35] ^CALCULATIONS
B130: [W35] +B123^2
B131: [W35] +B125^2
```

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C131: [W35] +B130/B131
B132: [W35] (@COS(B127*@PI/180))^2
B133: [W35] @SIN(B127*@PI/180)
A136: [W35] ^RESULT
A138: [W35] 'The distance from the User to
A139: [W35] 'satellite Rs in km is
B139: [W35] +B125*(@SQRT(C131-B132)-@SIN(B127*@PI/180))
C139: (F2) [W35] +B139
C141: [W35] ^RESULT
B146: [W35] ^TASK 7
A147: [W35] 'SATELLITE COVERAGE IN FUNCTION
A148: [W35] 'OF RANGE FROM THE USER'S POSITION
A149: [W35] ^TO SATELLITE
A151: [W35] ^INPUTS
A153: [W35] 'The distance Rs in km is
B153: [W35] +B139
A155: [W35] 'The ellipsoid heigth Hs is
B155: [W35] +C18
A157: [W35] 'The Earth's radius Re is
B157: [W35] +C22
B159: [W35] ^CALCULATIONS
B161: [W35] +B153^2-B155^2
B162: [W35] 2*(B157+B155)*B157
B163: [W35] 1-(B161/B162)
C165: [W35] ^RESULT
B167: [W35] 'The angle a in degrees is
C167: [W35] 180/@PI*@ACOS(B163)
D167: (F1) [W35] +C167
A169: [W35] ^TASK 8
A171: [W35] 'POSITION DETERMINATION USING MATRIX
A172: [W35] ^CALCULATION
A173: [W35] 'LEAST SQUARE METHOD
A175: [W35] 'SIX SATELLITES ARE OBSERVED
A176: [W35] 'SIMULTANEOUSLY
A178: [W35] 'INPUTS
A180: [W35] 'SATELLITES
B180: [W35] ^ANGLE a
C180: [W35] ^ELEVATION EL
D180: [W35] ^h
E180: [W35] ^b
F180: [W35] ^INTERCEPT (Rdr-Rm)
G180: [W35] ^RANDOM VECTOR  r
A183: [W35] 'Satellite 1
B183: [W35] @N(B183..B183)
C183: [W35] @N(C183..C183)
D183: [W35] @N(D183..D183)
E183: [W35] @N(E183..E183)
F183: [W35] @N(F183..F183)
G183: [W35] @N(G183..G183)
A184: [W35] 'Satellite 2
B184: [W35] @N(B184..B184)
C184: [W35] @N(C184..C184)
```

```
D184: [W35] @N(D184..D184)
E184: [W35] @N(E184..E184)
F184: [W35] @N(F184..F184)
G184: [W35] @N(G184..G184)
A185: [W35] 'Satellite 3
B185: [W35] @N(B185..B185)
C185: [W35] @N(C185..C185)
D185: [W35] @N(D185..D185)
E185: [W35] @N(E185..E185)
F185: [W35] @N(F185..F185)
G185: [W35] @N(G185..G185)
A186: [W35] 'Satellite 4
B186: [W35] @N(B186..B186)
C186: [W35] @N(C186..C186)
D186: [W35] @N(D186..D186)
E186: [W35] @N(E186..E186)
F186: [W35] @N(F186..F186)
G186: [W35] @N(G186..G186)
A187: [W35] 'Satellite 5
B187: [W35] @N(B187..B187)
C187: [W35] @N(C187..C187)
D187: [W35] @N(D187..D187)
E187: [W35] @N(E187..E187)
F187: [W35] @N(F187..F187)
G187: [W35] @N(G187..G187)
A188: [W35] 'Satellite 6
B188: [W35] @N(B188..B188)
C188: [W35] @N(C188..C188)
D188: [W35] @N(D188..D188)
E188: [W35] @N(E188..E188)
F188: [W35] @N(F188..F188)
G188: [W35] @N(G188..G188)
A191: [W35] 'GEOMETRICAL MATRIX M
B191: [W35] @SIN(B183*@PI/180)*@COS(C183*@PI/180)
C191: [W35] @COS(B183*@PI/180)*@COS(C183*@PI/180)
B192: [W35] @SIN(B184*@PI/180)*@COS(C184*@PI/180)
C192: [W35] @COS(B184*@PI/180)*@COS(C184*@PI/180)
B193: [W35] @SIN(B185*@PI/180)*@COS(C185*@PI/180)
C193: [W35] @COS(B185*@PI/180)*@COS(C185*@PI/180)
B194: [W35] @SIN(B186*@PI/180)*@COS(C186*@PI/180)
C194: [W35] @COS(B186*@PI/180)*@COS(C186*@PI/180)
B195: [W35] @SIN(B187*@PI/180)*@COS(C187*@PI/180)
C195: [W35] @COS(B187*@PI/180)*@COS(C187*@PI/180)
B196: [W35] @SIN(B188*@PI/180)*@COS(C188*@PI/180)
C196: [W35] @COS(B188*@PI/180)*@COS(C188*@PI/180)
B198: [W35] @N(B191..B191)
C198: [W35] @N(C191..C191)
B199: [W35] @N(B192..B192)
C199: [W35] @N(C192..C192)
B200: [W35] @N(B193..B193)
C200: [W35] @N(C193..C193)
B201: [W35] @N(B194..B194)
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C201: [W35] @N(C194..C194)
B202: [W35] @N(B195..B195)
C202: [W35] @N(C195..C195)
B203: [W35] @N(B196..B196)
C203: [W35] @N(C196..C196)
A206: [W35] 'TRANSPOSED MATRIX MT
B208: [W35] @N(B208..B208)
C208: [W35] @N(C208..C208)
D208: [W35] @N(D208..D208)
E208: [W35] @N(E208..E208)
F208: [W35] @N(F208..F208)
G208: [W35] @N(G208..G208)
B209: [W35] @N(B209..B209)
C209: [W35] @N(C209..C209)
D209: [W35] @N(D209..D209)
E209: [W35] @N(E209..E209)
F209: [W35] @N(F209..F209)
G209: [W35] @N(G209..G209)
A212: [W35] 'PRODUCT MT * M
B213: [W35] @N(B213..B213)
C213: [W35] @N(C213..C213)
B214: [W35] @N(B214..B214)
C214: [W35] @N(C214..C214)
B216: [W35] @N(B216..B216)
C216: [W35] @N(C216..C216)
B217: [W35] @N(B217..B217)
C217: [W35] @N(C217..C217)
A221: [W35] 'INTERCEPT ( Rdr - Rm )
B222: [W35] +F183
B223: [W35] +F184
B224: [W35] +F185
B225: [W35] +F186
B226: [W35] +F187
B227: [W35] +F188
A229: [W35] 'PRODUCT MT * INTERCEPT (Rdr-Rm)
B231: [W35] @N(B231..B231)
B232: [W35] @N(B232..B232)
A234: [W35] 'PRODUCT(INV.( MT*M ))*MT*INTERCEPT
B236: [W35] @N(B236..B236)
B237: [W35] @N(B237..B237)
A239: [W35] 'LOCATION OF MPP
B241: [W35] 'The abscisse X of MPP is
C241: [W35] +B236
D241: (F1) [W35] +C241
B244: [W35] 'The ordinate Y of MPP is
C244: [W35] +B237
D244: (F1) [W35] +C244
A247: [W35] 'GEOMETRICAL DILUTION OF ACCURACY
B249: [W35] 'The GDOA is
C249: [W35] @SQRT(B216+C217)
D249: (F1) [W35] +C249
A252: [W35] 'THE M 95 ( 1 LOP ) is
```

```
B252: [W35] 'The M 95 ( 1 LOP ) is
C252: [W35] @N(C252..C252)
C256: [W35] ^RESULT
B259: [W35] 'The R 95 required is
C259: [W35] +C249*C252
D259: (F1) [W35] +C259
A264: [W35] 'TRANSIT ACCURACY R 95 = f ( EL )
B266: [W35] ^Angles of elevation EL in degr.
C266: [W35] ^R 68 in meters
D266: [W35] ^R 95 in meters
B268: [W35] @N(B268..B268)
C268: [W35] @N(C268..C268)
D268: [W35] 5/3*C268
B269: [W35] @N(B269..B269)
C269: [W35] @N(C269..C269)
D269: [W35] 5/3*C269
B270: [W35] @N(B270..B270)
C270: [W35] @N(C270..C270)
D270: [W35] 5/3*C270
B271: [W35] @N(B271..B271)
C271: [W35] @N(C271..C271)
D271: [W35] 5/3*C271
B272: [W35] @N(B272..B272)
C272: [W35] @N(C272..C272)
D272: [W35] 5/3*C272
B273: [W35] @N(B273..B273)
C273: [W35] @N(C273..C273)
D273: [W35] 5/3*C273
B274: [W35] @N(B274..B274)
C274: [W35] @N(C274..C274)
D274: [W35] 5/3*C274
B275: [W35] @N(B275..B275)
C275: [W35] @N(C275..C275)
D275: [W35] 5/3*C275
B276: [W35] @N(B276..B276)
C276: [W35] @N(C276..C276)
D276: [W35] 5/3*C276
```

CHAPTER 4 :

NAVIGATIONAL ERRORS.

4.1. Introduction.

Error is the difference between a specific value and the correct or standard value.

In general, errors can be described as being either systematic or random.

- Systematic errors are those which follow some law by which they can be predicted. The accuracy with which a systematic error can be predicted depends upon the accuracy with which the governing law is understood. An error which can be predicted , can be eliminated or compensation can be made for it.

- Random errors are chance errors unpredictable in magnitude or sign. They are governed by the laws of probability.

In this chapter, a LOTUS 1-2-3 program is designed to compute the calculations related to navigational random errors. The program is based on a mathematical formulation.

4.2. Definition of the tasks.
The Mathematical Model.

4.2.1. Task 1 : Variances.

The random errors involved in navigation are assumed to be normally distributed.
The mathematical expression of the normal or Gaussian distribution for one dimensional errors is :

$$f(x) = 1/ sd*(\sqrt{2*\pi})* \exp (- (x-m)^2 / 2*sd^2)$$

where m = the mean
sd = the standard deviation
sd = the variance.

4.2.1.1. Errors in the Estimated Ground Course.

$$GrC = GC + tc + dr + c$$

where GC = the gyro course
tc = the total correction of the gyrocompass
dr = the drift angle mainly caused by wind pressure on the ship
c = the current angle caused by the set

The GRC which is followed by the ship and the estimated GRC differ by a random course error with :

$$(1) \quad sd^2 GRC = 0.68 + (w/v*\sqrt{Au/A1}*sina)^2 + (20*curr/v)^2$$

where $sd1^2 = 0.68$ = the sum of the variance in the Gc and the variance in the tc.

$sd^2 2 = (w/v*\sqrt{Au/A1}*sina)^2$ = the variance in the dr

$sd^2 3 = (20*curr/v)^2$ = the variance in the c.

The general expression for the variance of the error in the estimated Ground course is valid for :

- constant course and speed during the last 2 hours
- latitude up to 60 degrees
- Master compass and repeaters aligned as well as possible
- tc observed at regular intervals not longer than 4 hours
- value of coefficient cw ($dr = cw*w/v*\sqrt{Au/A1}*sina$) for the ship not differing too much from 3
- speed of the ship at least 4 times as much as the CTcurrent rate.

- The Cross Track Error CTE

The CTE is the magnitude of the deviation between the estimated GrC and the intended GrC.
The variance is

(2) $$sd^2 CTE = (v*t^{0.68} *sd\ GrC /57.3)^2$$

4.2.1.2. Errors in the Estimated Groundspeed.

$$Vgr = v + ATcurrent$$

where v = the speed of the ship through the water
ATcurrent = the along track component of the current

ATcurrent result in an along track speed error having a normal distribution and a variance

$$(3)\qquad sd^{2}vgr = 0.04 + 1/9*(ATcurrent)^{2} \text{ or}$$
$$= (0.02*v)^{2} + 1/9*(ATcurrent)^{2}$$

- The Along Track Error ATE

The ATE is a random variable with a normal distribution and variance amounting to the greater one of

$$(4)\qquad sd^{2}ATE = 0.04*t^{1.36} + 1/9*(ATcurr)^{2} * t^{1.36} \text{ or}$$
$$= (0.02*v)^{2} * t^{1.36} + 1/9*(ATcurrent)^{2} * t^{1.36}$$

4.2.1.3. Task 2. Accuracies.

The accuracy is the error with respect to the mean having a 95 % probability which implies that it is represented by a 95 % Confidence Area.

The accuracy of any system is denoted by the 95 % Confidence Margin M95 or by the 95 % Confidence Radius R95 when the accuracy of an Most Probable Position is involved.

$$(5)\qquad M95 = 2*sd$$

The tables (Ref.7 ,pages 3.40 and 9.2), give the accuracies in Dead Reckoning for the following navigational quantities:

- Groundcourse
- Groundspeed
- DR-Cross Track Error
- DR-Along Track Error
- DR-Position
- Transferred Line of Position

4.2.1.4. Task 3 : Time allowed for Dead Reckoning.

The time allowed for Dead Reckoning is obtained from the formula :

$$dR95^{2}(transfer)=4*t^{1.36}*[(v/40)^{2} + (w/60*\sqrt{Au/Al}*sina)^{2} + c^{2}/9]$$

$$R95^{2}DR - R95^{2}MPP = dR95^{2}(transfer) = k*t^{1.36}$$

Then ,

$$t = exp(1.36/[Ln(R95^{2}DR - R95^{2}MPP)/ k])$$

4.2.1.5. Task 4 : Safe Distance from danger.

The safe distance D from danger is given by

$$(4/100*D)^{2} = R95^{2}MPP + dR95^{2}(transfer).$$

4.2.1.6. Task 5 : R95 of the Point of Possible Collision. Automatic Radar Plotting Aids ARPA.

The R68 PPC is mathematically expressed by (under certain conditions) :

$$R68^{2}PPC = (R68^{2}nplots + R68^{2}own) * (TCPA'/PlotInt + 1)^{2}$$

Then R95 PPC = 5/3*R68 PPC

where $R68^2$nplots = k1*(Distance)^2 + k2*(Range)^2

k1 = 941/n , k2 = 1104/n , n = number of plots.

$R68^2$own = k3 * v^2 * t^2 , k3 = 1.28 , v in knots

TCPA' = the Time of Closest Point of Approach

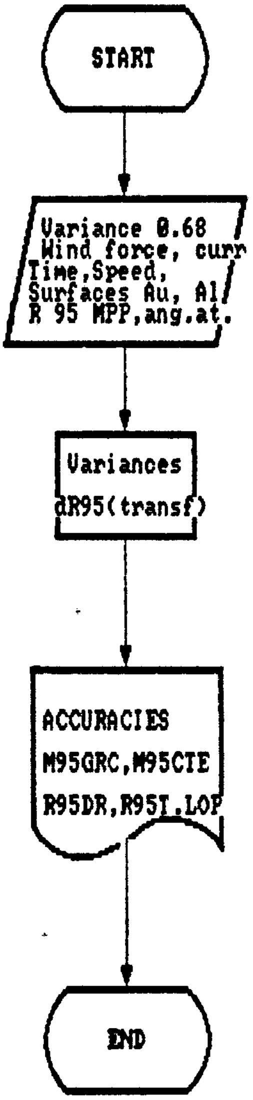
PlotInt = the Plotting Interval in minutes.

4.3. The flow chart.

The diagrams 1, 2, 3, 4 from page 104 to page 107 show the details on the formulation of the problems which are required for the LOTUS 1-2-3 program.

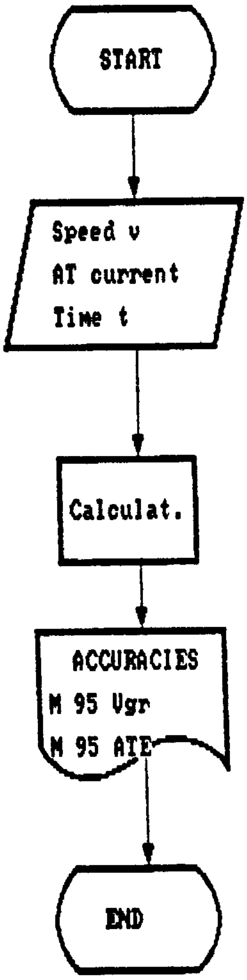
NAVIGATIONAL ERRORS.

TASK : ACCURACIES ON GROUND COURSE, CROSS TRACK ERROR, TRANSFERRED LOP.



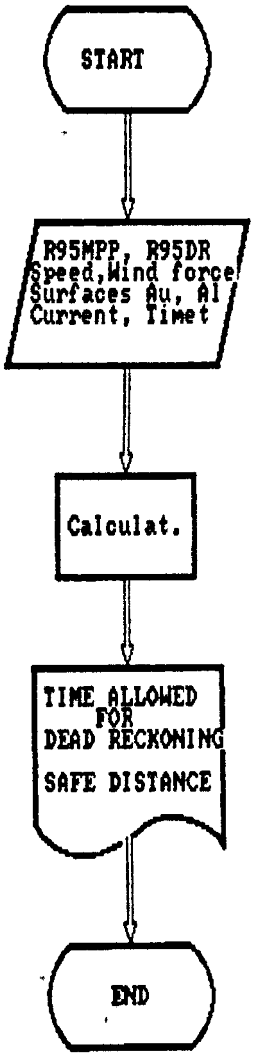
NAVIGATIONAL ERRORS.

TASK : ACCURACIES ON GROUND SPEED AND ALONG TRACK ERROR.



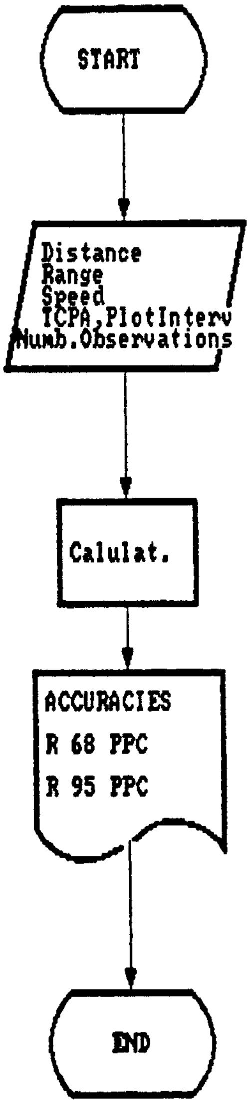
NAVIGATIONAL ERRORS.

TASK : TIME ALLOWED FOR DEAD RECKONING, SAFE DISTANCE FROM DANGER.



NAVIGATIONAL ERRORS.

TASK : R 95 OF THE POSSIBLE POINT OF COLLISION.



NAVIGATIONAL ERRORS.

4.4. ORGANIZATION AND STRUCTURE OF THE LOTUS PROGRAM.

ERROR 1 A.

INPUTS	CELLS LOCATION
Variance in Gc	C 28
Variance in tc	C 29
Ship's speed	B 31
Wind force	B 32
Lateral surface Au	B 33
Lateral surface Al	B 34
Angle a	B 36
CT - current	B 41
AT - current	B 53
Time	B 48
R 95 MPP	B 60
Range	B 171
Own ship's speed	B 165
TCPA'	B 170
Plot Interval	B 167
Number of plots	B 168
Number of observations	C 152
M 95 (1 LOP)	C 153

CALCULATIONS (See listing)

OUTPUTS	CELLS LOCATION
VARIANCE GROUND COURSE	C 46 and C 47
ACCURACY M 95	E 46 and F 46
VARIANCE GROUND SPEED	C 56
ACCURACY M 95	E 56 and F 56
VARIANCE CTE and M 95 CTE	C 51 and E 51
VARIANCE ATE and M 95 ATE	C 54 and E 54
dR 95 (transfer)	B 62
R 95 DR	C 67 and D 67
R 95 (transferred LOP)	C 73 and D 73
TIME ALLOWED FOR DEAD RECKONING	B 105 and C 105
SAFE DISTANCE FROM DANGER	C 134 and D 134
GDOA and R 95	C 155 and D 157
R 95 POSSIBLE POINT OF COLLISION	C 218

4.5. Examples.

Ref. 7 ,pages 5.39, 6.3 and 11.15.

Example 1 :

Wind w = 20 knots
Au/Al = 1.4
Angle a = 70 degrees
Current = 1.1 knot
Ship's speed = 12 knots
Time t = 1.5 hours
ATcurr = 0

Example 2 :

Current = 1.5 knot
Wind w = 20 knots
Angle a = - 135 degrees
cw = 3
Ship's speed = 18 knots
Au = Al
R 95 MPP = 0.5 M
dR 95 (transfer) = $2.0322*t^{0.68}$
R 95 DR = 1.2 M

Example 3 :

Time t = 1.5 hours
Speed = 18 knots
w*(Au/Al)^(1/2)*sina = 1.4

Example 4 :

Speed = 20 knots
Range = 12 M
Plot Interval = 3 min , n = 60

4.6. LOTUS results.

The print outs from page 111 to page 118 give the outputs of the LOTUS 1-2-3 program.

```
A1: [W35] 'NAVIGATIONAL ERRORS                                             REA

                  A                                   B
1   NAVIGATIONAL ERRORS              PROGRAM ERROR1A. TATY-BOUSSIANA J.L
2                                              MET ( N ) 90
3   INPUTS
4
5   CONDITIONS
6
7   NORMAL ( GAUSSIAN ) DISTRIBUTION
8
9   Gyrocompass and log obey to
10  IMO Performance Standards
11
12  Latitude is less than 60 degrees
13  Total corr. tc of the Gyrocompass
14  is observed regularly
    Coefficient cw = 3
16  Speed v of the ship through the
17  water is >=4*CTcurrent
18  v >= 10 knots
19  v <= 30 knots

22-Sep-90  06:42 PM
```

```
21
22  Disturbances fluctuate not
23  very fast nor are they constant
24  In an environment with constant
    disturbances the factor t^0.68
26  should be replaced by t                                 CALCULATIONS
27
28                                   The variance in the Gc is     0.08
29                                   The variance in the tc is      0.6

31  The speed v of the ship is                        12
32  The wind force w is                               20
33  The lateral surface Au is                        1.4
34  The lateral surface Al is                          1
35  The calculated sqrt(Au/Al) is           1.1832159566
36  The angle a is                                    70
37                                          0.9396926208
38  The given w*sina is                     18.793852416
39  The calculated w*sina is                18.793852416
40  The given sqrt(Au/Al) is                1.1832159566
22-Sep-90  06:44 PM
```

```
41  The CTcurrent is                                   1.1
42                       The variance in the dr is               3.4339753
43                       The variance in the CTcurrent is        3.3611111
44                                    Variance                    RESULTS
45
46                       The variance in the GRC is              7.4750864
47                                                                     7.5
48  The time t in hours                                1.5
49                                            1.3174740087
50                                            1.7357377637
51                       The variance  of the CTE is             0.5690541
52
53  The ATcurrent is                                     0
54                       The variance of the ATE is              0.0999784
55
56                       The variance in the Vgr is                 0.0576
57
58
59
60  The R95 MPP is                                     0.5          0.25
22-Sep-90  06:54 PM
```

```
41                RESULTS                     ACCURATE RESULTS  PRACTICAL RESULTS
42
43          Standard Deviation                ACCURACIES IN DR  ACCURACIES IN DR
44                 sd                               M95
45
46                         2.7340604268         5.4681208535         5.5
47                                  2.7
48
49
50
51                         0.7543567645          1.508713529
52
53
54                         0.3161937621         0.6323875242
55
56                                 0.24                 0.48         0.5
57
58
59
60
22-Sep-90  07:05 PM
```

```
57
58  DR POSITION
59
60  The R95 MPP is                                          0.5         0.25
61
62  dR95(transfer)                                 1.5849238755 2.5119836
63
64                                                              2.7619836
65                                                              RESULTS
66
67                    The R95 Dead Reckoning Position is 1.6619216   1.7
68
(   TRANSFERRED LOP
70
71  The  M95 MPP is                                         0.4
72
73                    The M95 (transferred LOP ) is        1.1793523   1.2

75
76
22-Sep-90   07:13 PM
```

```
76
77  TIME ALLOWED FOR DEAD RECKONING

79  INPUTS                                                            20
80                                                                    18
81  The speed of the ship is                                        -135
82  The angle a is                                        -14.1421356237
83  The product w*sina is                                            1.4
84  The surface Au is                                                  1
85  The sqrt of Au/A1 is                                             1.4
86  The surface A1 is                                                1.5
87  The current is
88                                                                   0.5
89  The R 95 of the fix is
90                                                                   1.2
91  The R95 DR max is
92
93
94
95
22-Sep-90  07:24 PM
```

```
76
77  TIME ALLOWED FOR DEAD RECKONING
78
79  INPUTS
80
81  The speed of the ship is                    18
82  The angle a is                            -135
83  The product w*sina is                -14.14213
84  The surface Au is                          1.4
85  The sqrt of Au/A1 is                         1
86  The surface A1 is                          1.4
87  The current is                             1.5
88
89  The R 95 of the fix is                     0.5
90
91  The R95 DR max is                          1.2
92
93
94
95
22-Sep-90  07:31 PM
```

```
93  CALCULATIONS
94
95                                         0.2025
96                                      0.0555555
97                                           0.25
98                                      2.0322222
99                                           1.19
100                                     0.5855658
101                                    -0.393512
102
103               OUTPUTS                RESULTS  PRACTICAL RESULTS
104
105 Time allowed for DR in hours        0.6746830               0.7
106
107 Time allowed for DR in minutes      40.480985              40.5
108
109
110
111
112
22-Sep-90  07:33 PM
```

```
119 SAFE DISTANCE FROM DANGER
120 The angle a is                                                        70
121 The speed of the ship is                                              18
122 The rate of the current is                                           1.1
123 The product w*sqrt(Au/A1)*sina is                           1.3937893985
124 The maximum time tmax in hours is                                    1.5
125 The wind rate w is                                                    20
126 The surface Au is                                                 0.0055
127 The surface Al is                                                      1
128                                       The R 95 (transfer) is
129                                       The R 95 of the fix is
130 The given w*sqrt(Au/Al)*sina is                             1.3937893985
131
132
133
134
135
136
137
138
22-Sep-90  07:36 PM
```

```
119
120                                    70 CALCULATIONS
121                                    18    0.2025
122                                   1.1 0.1344444
123                          1.3937893985 0.0005396
124                                   1.5
125                                    20
126                                0.0055 0.3374840
127                                     1
128 The R 95 (transfer) is                1.5307303
129 The R 95 of the fix is                      0.5
130                          1.3937893985 2.0307303
131
132                                         RESULT   PRACTICAL RESULT
133
134      SAFE DISTANCE FROM DANGER        50.768258                50.8
135
136
137
138
22-Sep-90  07:38 PM
```

```
138 GEOMETRIC DILUTION OF ACCURACY
139
140              CONDITIONS
141
142 Number of observations is n
143 The n LOPs satisfy the following
144 conditions :
145
146 1. All LOPs have equal M 95
147 2. All LOPs have indep. errors
148 3. Each pair of neighbouring LOPs
149 intersects at an angle a=360/n
150 (n>=2).
151
152
153
154
155
156
157
22-Sep-90  07:41 PM
```

```
145
146
147
148
149
150                                         INPUTS
151
152 The number of observations is                6
153 The M 95 (1 LOP ) is                         2
154                                       RESULTS  PRACTICAL RESULTS
155 The Geometric Dilution of Accuracy 0.8164965                0.8
156
157 The Position Accuracy R95 is       1.6329931                1.6
158
159
160
161
162
163
164
22-Sep-90  07:42 PM
```

```
194
195            CONDITIONS
196
197 Standard deviation winddrift = 1.5
198 Speed v > 10 knots
199 Log obeys the IMO standards
200 Plot Interval = 3 min
201 Scanner Period = 3 seconds
202 TCPA in minutes
203 Range more than 2.5 Miles
204
205           CALCULATIONS
206
207                                          15.7
208                                        2649.6
209                                          4600
210                                        7265.3 85.236729
211                                             7
212                                     596.65710
213
22-Sep-90  08:01 PM
```

```
213                                                PRACTICAL
214                                       RESULTS  RESULTS
215
216 The R 95 of the Possible Point of
217 Collision PPC in meters is          994.42850     994.4
218        in nautical miles is         0.5369484       0.5
219
220
221
222
223
224
225
226
227
228
229
230
231
232
22-Sep-90  08:04 PM
```

4.7. Results of the references.

Variances and accuracies.

Var GRC = 7.48 , M 95 GRC = 5.5 M
Var Vgr = 0.0576 , M 95 Vgr = 0.5 M

Time allowed for Dead Reckoning = 0.67 hours = 40 min

Safe distance from danger = 50 M

R 95 Point of Possible Collision = 0.6 M

4.8. Comparisons and critical remarks.

The LOTUS results are similar to those mentionned in the listed reference.

4.9. Listing of the LOTUS program.

The listing may be found in Appendix F .

and log

TABLES FOR THE ACCURACIES IN DEAD RECKONING. IX

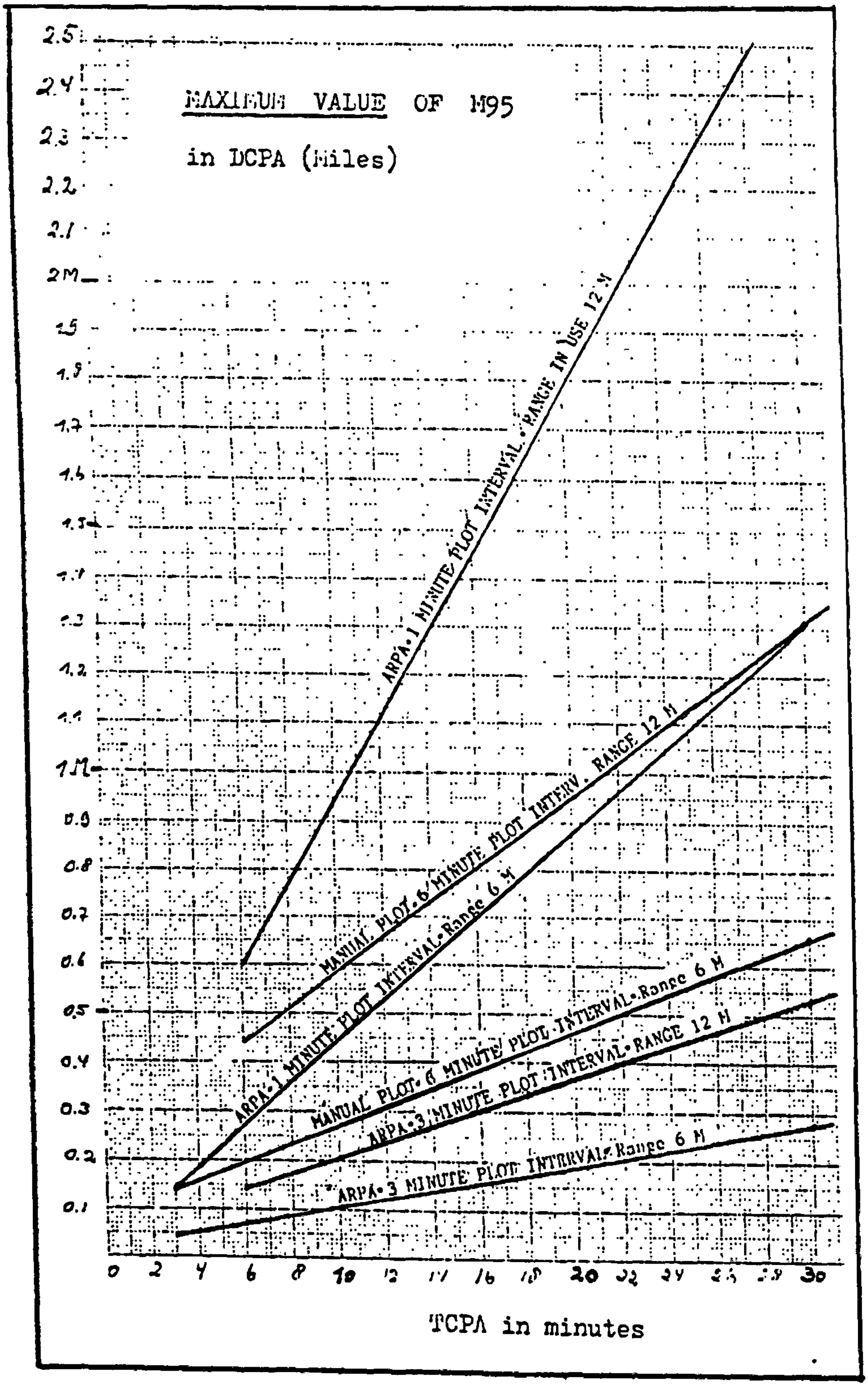
Navigational Quantity	ACCURACIES M95 or R95	Conditions
	Dimensions : w , v and current in knots ; t in hours	
GROUNDCOURSE	$M95^{\circ}_{GrC} = 2 \cdot \sqrt{0.68 + (\frac{w}{v} \cdot \sqrt{\frac{\Lambda u}{\Lambda l}} \cdot \sin\alpha)^2 + (\frac{20 \cdot CT\text{-current}}{v'})^2}$	Gyro obey IMO Performance Standards Latitude ≤ 60° tc is observed regularly $c_w \approx 3$ v ≥ 4·CT-current v ≥ 10 kn v ≤ 30 kn Disturbances fluctuate not very fast nor are they constant
GROUNDSPEED	$M95\text{-}v_{gr} = 2 \cdot \sqrt{(\frac{v}{50})^2 + (\frac{AT\text{-current}}{3})^2}$ kn	
DR-CROSS TRACK ERROR	$M95_{CTE} = \left[\frac{1}{60} \cdot t^{0.68} \cdot v \cdot M95^{\circ}_{GrC}\right]$ M	
DR-ALONG TRACK ERROR	$M95_{ATE} = 2 \cdot t^{0.68} \cdot \sqrt{(\frac{v}{50})^2 + (\frac{AT\text{-current}}{3})^2}$ M	In an environment with constant disturbances the factor $t^{0.68}$ should be replaced by t
DR-POSITION	$\Delta R95(transfer) = 2 \cdot t^{0.68} \sqrt{(\frac{v}{40})^2 + (\frac{w}{60} \sqrt{\frac{\Lambda u}{\Lambda l}} \cdot \sin\alpha)^2 + (\frac{Current}{3})^2}$ M $R95_{DR} = \sqrt{R95^2_{MPP} + \Delta R95^2(transfer)}$ M	See also TABLE 3.18
TRANSFERRED LOP	$M95(transferred\ LOP) = \sqrt{M95^2 + (0.7\, \Delta R95(transfer))^2}$ M	

From the foregoing it will be clear that the number of variables which govern the value of M95 in the plotted Distance to CPA is large. In order to attain a simplified but justified comparison between the accuracies of the various plotting methods on the 12- and 6 Mile ranges the maximum values of M95 with respect to the Distance of Target are pictured in Table XI.

From this table it is concluded that :

- Accuracy from a 1-minute ARPA plot at the 12 Mile-range is the worst
- Accuracies improve with at least a factor two when the 12 Mile-range is replaced by the 6 Mile-range.
- The best accuracy is obtained from a 3-minute ARPA Plot at the 6 Mile-range.

TABLE XI



In the IMO "Performance Standards for Navigational Equipment" accuracy requirements for ARPA are given in Chapter 1.4 paragraph 3.8.
For the accuracy of DCPA, values are presented for 4 scenarios, which are specified in Annex 2 of that chapter. See paragraph 3.1, Table V.

Comparison of these four values with the previous table leads to the conclusion that the graph presents slightly pessimistic accuracies, which it actually intends to do.

The aim of the accuracies given in the previous graph is to aid navigators in assessing a safe DCPA in Collision Avoidance navigation.

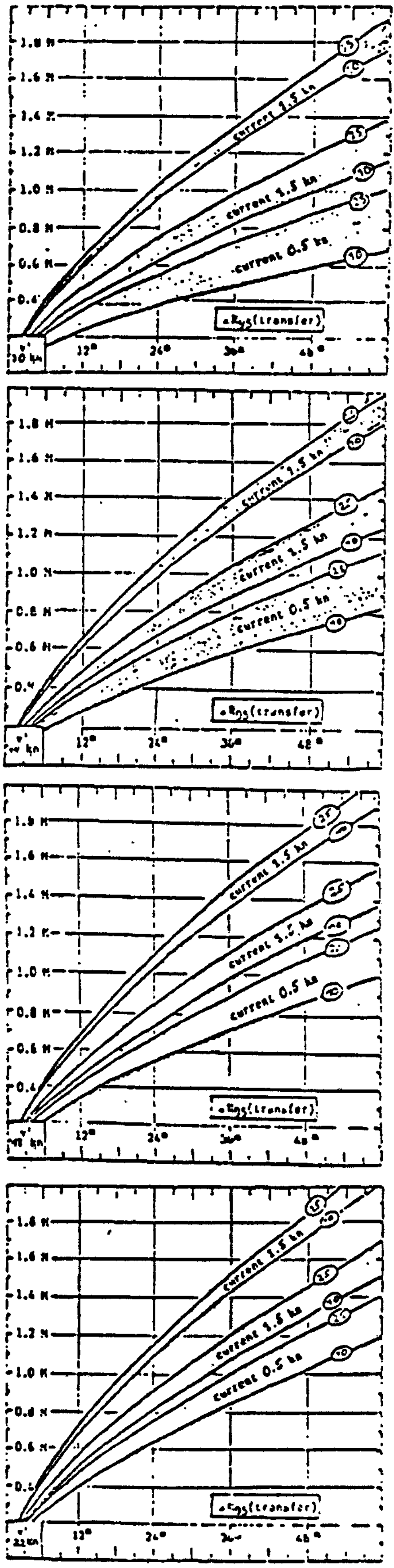
In using the graph however it should be born in mind that:

- there is always a probability that the error in DCPA exceeds the M95 value.

- the accuracy of a 3-minute-plotinterval-ARPA is the better one in "steady course and steady speed" conditions.
  In cases with changes in the relative speed-vector the "long memory" of the 3-minute-interval is a serious drawback, which can lead to critical situations especially when the distance between own ship and target is small (Ref. 10.11)

$$dR95(transfer) = 2\,t^{0.68}\sqrt{\left(\left[\frac{v}{40}\right]^2 + \left[\frac{w}{60}\frac{\sqrt{Au}}{\sqrt{A1}}\sin\alpha\right]^2 + \left[\frac{1}{3}\,current\right]^2\right)}\ M$$

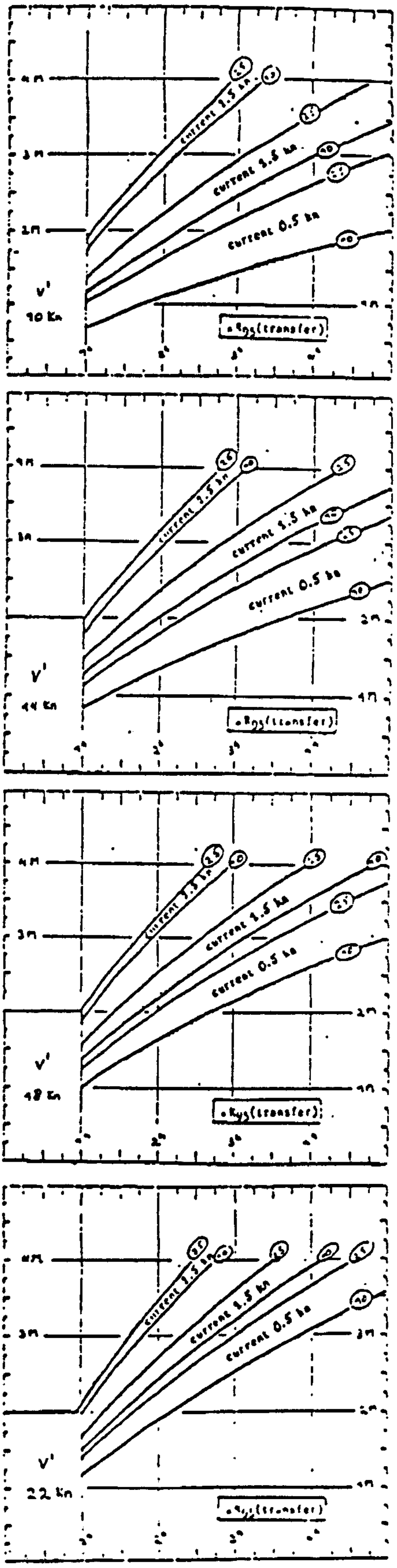
CONDITIONS ARE SIMILAR WITH THOSE OF TABLE 9.1.1



Encircled values are : $w' \cdot \sqrt{\frac{Au}{A1}}\sin\alpha$

Conditions
Lat < 60°
10 kn ≤ v' ≤ 30 kn
lc observed regularly
$c_w \approx 3$
v' ≥ 4·CT-current
Gyrocompass , Log and Autopilot within the IMO Performance Standards for Navigational Equipment.

DISTURBANCES fluctuate not very fast nor are they constant



APPENDIX F :

PROGRAM : ERROR 1 A.

```
A1: [W35] 'NAVIGATIONAL ERRORS
B1: [W35] 'PROGRAM ERRORIA. TATY-BOUSSIANA J.L.
C1: [W35] 'WORLD MARITIME UNIVERSITY
D1: [W35] ^MALMO, SWEDEN.
B2: [W35] ^MET ( N ) 90
A3: [W35] 'INPUTS
A5: [W35] 'CONDITIONS
A7: [W35] 'NORMAL ( GAUSSIAN ) DISTRIBUTION
A9: [W35] 'Gyrocompass and log obey to
A10: [W35] 'IMO Performance Standards
A12: [W35] 'Latitude is less than 60 degrees
A13: [W35] 'Total corr. tc of the Gyrocompass
A14: [W35] 'is observed regularly
A15: [W35] 'Coefficient cw = 3
A16: [W35] 'Speed v of the ship through the
A17: [W35] 'water is >=4*CTcurrent
A18: [W35] 'v >= 10 knots
A19: [W35] 'v <= 30 knots
A22: [W35] 'Disturbances fluctuate not
A23: [W35] 'very fast nor are they constant
A24: [W35] 'In an environment with constant
A25: [W35] 'disturbances the factor t^0.68
A26: [W35] 'should be replaced by t
C26: [W35] ^CALCULATIONS
B28: [W35] 'The variance in the Gc is
C28: [W35] 0.08
B29: [W35] 'The variance in the tc is
C29: [W35] 0.6
A31: [W35] 'The speed v of the ship is
B31: [W35] @N(A31..A31)
A32: [W35] 'The wind force w is
B32: [W35] @N(A32..A32)
A33: [W35] 'The lateral surface Au is
B33: [W35] @N(A33..A33)
A34: [W35] 'The lateral surface Al is
B34: [W35] @N(A34..A34)
D34: [W35] +B33/B34
A35: [W35] 'The calculated sqrt(Au/Al) is
B35: [W35] @SQRT(D34)
A36: [W35] 'The angle a is
B36: [W35] @N(A36..A36)
B37: [W35] @SIN(B36*@PI/180)
A38: [W35] 'The given w*sina is
B38: [W35] +B39
A39: [W35] 'The calculated w*sina is
B39: [W35] +B32*@SIN(B36*@PI/180)
A40: [W35] 'The given sqrt(Au/Al) is
B40: [W35] +B35
A41: [W35] 'The CTcurrent is
B41: [W35] @N(A41..A41)
D41: [W35] ^RESULTS
E41: [W35] ^RESULTS
```

```
B42: [W35] 'The variance in the dr is
C42: [W35] (B39*B40/B31)^2
B43: [W35] 'The variance in the CTcurrent is
C43: [W35] (20*B41/B31)^2
D43: [W35] ^Standard Deviation
E43: [W35] ^ACCURACIES IN DEAD RECKONING
B44: [W35] ^Variance
C44: [W35] ^RESULTS
D44: [W35] ^sd
E44: [W35] ^M95
B46: [W35] 'The variance in the GRC is
C46: [W35] @SUM(C28..C43)
D46: [W35] @SQRT(C46)
E46: [W35] 2*D46
F46: (F1) [W35] +E46
C47: (F1) [W35] +C46
D47: (F1) [W35] +D46
A48: [W35] 'The time t in hours is
B48: [W35] @N(A48..A48)
B49: [W35] (B48)^0.68
B50: [W35] (B48)^1.36
B51: [W35] 'The variance  of the CTE is
C51: [W35] (B31*B49*D46/57.3)^2
D51: [W35] @SQRT(C51)
E51: [W35] 2*D51
A53: [W35] 'The ATcurrent is
B53: [W35] @N(A53..A53)
B54: [W35] 'The variance of the ATE is
C54: [W35] ((0.02*B31)^2+1/9*(B53)^2)*B50
D54: [W35] @SQRT(C54)
E54: [W35] 2*D54
B56: [W35] 'The variance in the Vgr is
C56: [W35] (0.02*B31)^2+1/9*(B53)^2
D56: [W35] @SQRT(C56)
E56: [W35] 2*D56
F56: (F1) [W35] +E56
A58: [W35] 'DR POSITION
A60: [W35] 'The R95 MPP is
B60: [W35] @N(A60..A60)
C60: [W35] (B60)^2
A62: [W35] 'The dR95(transfer) is
B62: [W35] 2*B49*@SQRT((B31/40)^2+(B39*B40/60)^2+1/9*(B41)^2)
C62: [W35] (B62)^2
C64: [W35] @SUM(C60..C62)
C65: [W35] 'RESULTS
B67: [W35] 'The R95 Dead Reckoning Position is
C67: [W35] @SQRT(C64)
D67: (F1) [W35] +C67
A69: [W35] 'TRANSFERRED LOP
A71: [W35] 'The  M95 MPP is
B71: [W35] +B60/1.25
B73: [W35] 'The M95 (transferred LOP ) is
```

```
C73: [W35] @SQRT((B71)^2+(0.7*B62)^2)
D73: (F1) [W35] +C73
A77: [W35] 'TIME ALLOWED FOR DEAD RECKONING
A79: [W35] 'INPUTS
A80: [W35] 'The wind rate is
B80: [W35] @N(A80..A80)
A81: [W35] 'The speed of the ship is
B81: [W35] @N(A81..A81)
A82: [W35] 'The angle a is
B82: [W35] @N(A82..A82)
A83: [W35] 'The product w*sina is
B83: [W35] +B80*@SIN(B82*@PI/180)
A84: [W35] 'The surface Au is
B84: [W35] @N(A84..A84)
A85: [W35] 'The sqrt of Au/Al is
B85: [W35] @SQRT(B84/B86)
A86: [W35] 'The surface Al is
B86: [W35] @N(A86..A86)
A87: [W35] 'The current is
B87: [W35] @N(A87..A87)
A89: [W35] 'The R 95 of the fix is
B89: [W35] @N(A89..A89)
A91: [W35] 'The R95 DR max is
B91: [W35] @N(A91..A91)
A93: [W35] 'CALCULATIONS
B95: [W35] (B81/40)^2
B96: [W35] (B83*B85/60)^2
B97: [W35] 1/9*(B87)^2
B98: [W35] 4*@SUM(B95..B97)
B99: [W35] (B91)^2-(B89)^2
B100: [W35] +B99/B98
B101: [W35] @LN(B100)/1.36
B103: [W35] ^RESULTS
A105: [W35] 'Time allowed for DR in hours
B105: [W35] @EXP(B101)
C105: (F1) [W35] +B105
A107: [W35] 'Time allowed for DR in minutes
B107: [W35] +B105*60
C107: (F1) [W35] +B107
A119: [W35] 'SAFE DISTANCE FROM DANGER
A120: [W35] 'The angle a is
B120: [W35] @N(A120..A120)
C120: [W35] ^CALCULATIONS
A121: [W35] 'The speed of the ship is
B121: [W35] @N(A121..A121)
C121: [W35] (B121/40)^2
A122: [W35] 'The rate of the current is
B122: [W35] +B41
C122: [W35] (B122/3)^2
A123: [W35] 'The product w*sqrt(Au/Al)*sina is
B123: [W35] +B125*@SQRT(B126/B127)*@SIN(B120*@PI/180)
C123: [W35] (B123/60)^2
```

```
A124: [W35] 'The maximum time tmax in hours is
B124: [W35] @N(A124..A124)
A125: [W35] 'The wind rate w is
B125: [W35] @N(A125..A125)
A126: [W35] 'The surface Au is
B126: [W35] @N(A126..A126)
C126: [W35] @SUM(C121..C123)
A127: [W35] 'The surface Al is
B127: [W35] @N(A127..A127)
B128: [W35] 'The R 95 (transfer) is
C128: [W35] 2*(B124)^0.68*@SQRT(C126)
B129: [W35] 'The R 95 of the fix is
C129: [W35] @N(B129..B129)
A130: [W35] 'The given w*sqrt(Au/Al)*sina is
B130: [W35] +B123
C130: [W35] @SUM(C128..C129)
C132: [W35] ^RESULT
B134: [W35] 'Safe Distance from danger in Miles
C134: [W35] 100/4*C130
D134: (F1) [W35] +C134
A138: [W35] ^GEOMETRIC DILUTION OF ACCURACY
A140: [W35] ^CONDITIONS
A142: [W35] 'Number of observations is n
A143: [W35] 'The n LOPs satisfy the following
A144: [W35] 'conditions :
A146: [W35] '1. All LOPs have equal M 95
A147: [W35] '2. All LOPs have indep. errors
A148: [W35] '3. Each pair of neighbouring LOPs
A149: [W35] 'intersects at an angle a=360/n
A150: [W35] '(n>=2).
C150: [W35] ^INPUTS
B152: [W35] 'The number of observations is
C152: [W35] @N(B152..B152)
B153: [W35] 'The M 95 (1 LOP ) is
C153: [W35] @N(C153..C153)
C154: [W35] ^RESULTS
B155: [W35] 'The Geometric Dilution of Accuracy is
C155: [W35] 2/@SQRT(C152)
D155: (F1) [W35] +C155
B157: [W35] 'The Position Accuracy R95 is
C157: [W35] +C155*C153
D157: (F1) [W35] +C157
A161: [W35] ^AUTOMATIC RADAR PLOTTING AIDS
A163: [W35] ^INPUTS
A165: [W35] 'The speed of the own ship is
B165: [W35] @N(A165..A165)
A167: [W35] 'The Plot Interval in min is
B167: [W35] @N(A167..A167)
A168: [W35] 'The number of plots is
B168: [W35] @N(A168..A168)
A169: [W35] 'The Distance of the Target is
B169: [W35] +B165*B167/60
```

```
A170: [W35] 'The TCPA' in minutes is
B170: [W35] @N(A170..A170)
A171: [W35] 'The Range in use is
B171: [W35] @N(A171..A171)
A173: [W35] ^CALCULATIONS
C175: [W35] @SQRT(B175)
C177: [W35] @SQRT(B177)
B181: [W35] ^RESULTS
A183: [W35] 'The R 68 = Rown in naut. miles is
B183: [W35] @SQRT(1.28*B31^2*B167^2)/1852
A185: [W35] 'The R68 n plots in miles is
B185: [W35] @SQRT(941/60*B169^2+1104/60*B171^2)/1852
C185: [W35] +B185^2
B187: [W35] +B183^2
C187: [W35] +B187^2
B188: [W35] +B185^2
A190: [W35] 'The R 68 PPC in nautical miles is
B190: [W35] @SQRT(@SUM(B187..B188))
C191: [W35] @SUM(C185..C187)
A195: [W35] ^CONDITIONS
A197: [W35] 'Standard deviation winddrift = 1.5
A198: [W35] 'Speed v > 10 knots
A199: [W35] 'Log obeys the IMO standards
A200: [W35] 'Plot Interval = 3 min
A201: [W35] 'Scanner Period = 3 seconds
A202: [W35] 'TCPA' in minutes
A203: [W35] 'Range more than 2.5 Miles
A205: [W35] ^CALCULATIONS
B207: [W35] 15.7*B169^2
B208: [W35] 18.4*B171^2
B209: [W35] 11.5*B165^2
B210: [W35] @SUM(B207..B209)
C210: [W35] @SQRT(B210)
B211: [W35] 1/3*B170+1
B212: [W35] +C210*B211
B214: [W35] ^RESULTS
A216: [W35] 'The R 95 of the Potential Point of
A217: [W35] 'Collision PPC in meters is
B217: [W35] 5/3*B212
C217: (F1) [W35] +B217
A218: [W35] ^in nautical miles is
B218: [W35] +B217/1852
C218: (F1) [W35] +B218
```

CHAPTER 5 :

STABILITY AT LARGE ANGLES.

5.1. Introduction.

In the present chapter , calculations are related to the curves of stability of merchant ships.

The curve of stability is used to determine several important characteristics for each displacement, among which are

- the righting arm at any inclination and consequently the rigthing moment which equals the righting arm times the displacement of the ship.

- the metacentric heigth GM

- the angle of maximum righting arm

- the range of stability

- the dynamic stability related to the area under the righting arm curve.

An overview of the theory and an overall description of the LOTUS 1-2-3 program are included.

5.2. Definition of the tasks.
The Mathematical Model.

5.2.1. Task 1: Vertical correction for the position of the center of gravity.

The righting arm is

(1) $GZ = AZo - AGv*\sin\theta$

where AZo = the original or uncorrected righting arm based on the original center of gravity

AGv = the distance that the center of gravity of the ship has moved vertically from the center line.

5.2.2. Task 2 : Transverse correction for the position of the center of gravity.

The righting arm is

(2) $GZ = AZo - AGt*\cos\theta$

where AGt = the distance that the center of gravity of the ship has moved transversely from the center line.

The final stability curve is expressed as:

(3) $GZ(\theta) = AZo - AGv*\sin\theta - AGt*\cos\theta.$

5.2.3. Task 3 : Metacentric height and stability curves.

For small angles$(0 < \theta < 10$ degrees$)$

$GM = GZ / \theta$

For large angles, an approximation of GM is:

(4) $GM = AZ(10^{\circ})/ 10*\pi/180 - AGv.$

5.2.4. Task 4 : Free surface correction at large angles of heel.

For moderate angles of heel,

(5) $G1Z1 = (GM - \frac{d_1}{d_2}*i/Vs*(1 + (\tan\theta)^{2})/2)*\sin\theta$

The horizontal and vertical shifts in the center of buoyancy are:

(6) $BB1t = BM*\tan\theta = I/V*\tan\theta$

(7) $BB1v = BB1t*\tan\theta/2 = I/V*(\tan\theta)^{2}/2$

For large angles,

(8) $G1Z1 = GZ + BB1v*\sin\theta - GG1t*\cos\theta - GG1t*\sin\theta$ or

(9) $G1Z1 = [GM + BM*(\tan\theta)^{2}/2 - FSC*(1+(\tan\theta)^{2}/2)]*\sin\theta$

5.2.5. Task 5 : Righting Arm by using the KN curves.

The corrected righting arm GZ is

$$GZ = KN - KG * \sin\theta$$

KN is obtained from the KN curves for any displacement and any angle of inclination.

5.2.6. Task 6 : Area under the curve of stability.The IMO requirements.

The area under the curve of stability is expressed as :

$$\text{Area } A = \int_{0}^{\theta_f} GZ \, d\theta$$

The IMO stability requirements are the following :

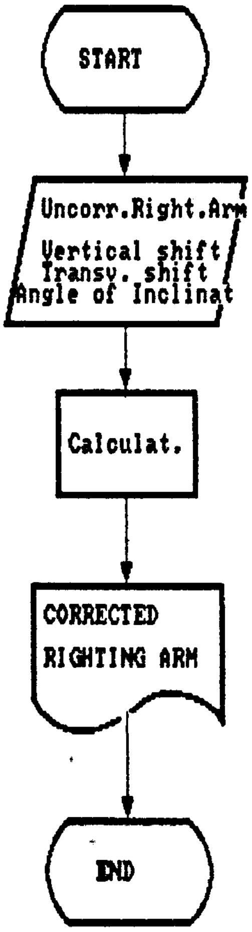
1. Area (0 , 30°) >= 0.055 mrd
2. Area (30 , 40° or θf) >= 0.03 mrd
3. Area (0 , 40° or θf) >= 0.09 mrd
4. GZ >= 0.20 m for θ >= 30 degrees
5. GZ max is obtained for θ >= 30 degrees
6. GM (initial stability) >= 0.15 m

5.3. The flow chart.

The diagrams 1 ,2 and 3 from page 125 to page 127 give details for the computer program.

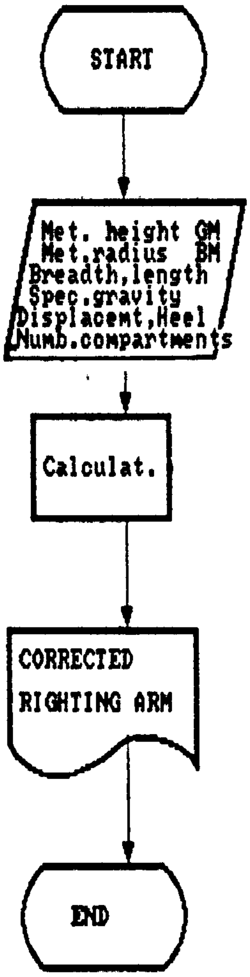
STABILITY AT LARGE ANGLES.

TASK : VERTICAL AND TRANSVERSE CORRECTIONS. RIGHTING ARM.



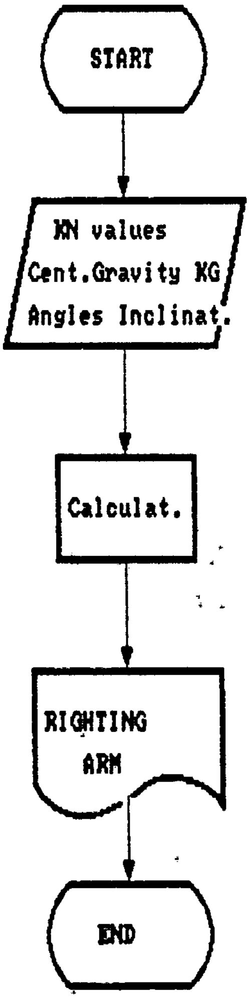
STABILITY AT LARGE ANGLES.

TASK : FREE SURFACE CORRECTION. RIGHTING ARM.



STABILITY AT LARGE ANGLES.

TASK : KN CURVES. RIGHTING ARM.



STABILITY AT LARGE ANGLES.

5.4. ORGANIZATION AND STRUCTURE OF THE LOTUS PROGRAM.

STAB 1 A.

INPUTS	CELLS LOCATION
Uncorrected righting arm	B 13 .. B 19
Vertical shift	C 13 .. C 19
Transverse shift	E 13 .. E 19
Angles of inclination	A 13 .. A 19
	A 45 .. A 50
	A 79 .. A 87
	A 101 .. A 107
	A 126 .. A 134
	A 142 .. A 148
	A 159 .. A 161
	A 173 .. A 177
Righting arms	B 33
	B 45 .. B 50
	B 142 .. B 148
	B 101 .. B 101
	B 159 .. B 161
	B 173 .. B 177
Center of gravity above keel KG	H 79 .. H 87
Transverse Metacenter above keel	I 79 .. I 87
Center of buoyancy above keel	K 79 .. K 87
Breadth of the tank	B 68 .. B 74
Length of the tank	C 68 .. C 74
Number of compartiments	D 68 .. D 74
Specific gravity liquid in tank	A 68 .. A 74
Specific gravity ship flotation liquid	E 79 .. E 87
Displacement	D 79 .. D 87
Simpson's Multipliers	C 101 .. C 107
Common interval	C 110
Displacement	C 114
K N values	B 126 .. B 134
K G values	C 126 .. C 134

OUTPUTS	CELLS LOCATION
CORRECTED RIGHTING ARMS	G 13 .. G 19 P 79 .. P 87 E 126 ..E 134
APPROXIMATED GM (10 degrees)	B 36
AREA UNDER THE STABILITY CURVE	C 112, D 167, D 183
RIGHTING MOMENTS	D 142 .. D 148 G 142 .. G 148 J 142 .. J 148

5.5. Examples.

Example 1 : Ref 12, page 164.

Heel	Tabulated GZ (KG = 9 m)
0	0
15	0.90
30	2.15
45	2.55
60	1.91
75	0.80
90	-0.50

Example 2 : Ref 12, page 165.

KG = 9 m

Heel	KN
5	0.9
10	2.0
15	3.2
20	4.4
30	6.5
45	8.75
60	9.7
75	9.4
90	8.4

5.6 LOTUS results.

The LOTUS results are given from page 131 to page 147.

```
A1: [W35] 'STABILITY AT LARGE ANGLES                                      READ

                    A                                       B
1   STABILITY AT LARGE ANGLES          PROGRAM STAB1A. TATY-BOUSSIANA J.L.
2                                                MET ( N ) 90
3   TASK 1 :
4
5   RIGTHING ARM DUE TO VERTICAL AND
6   TRANSVERSE SHIFT OF THE CENTER OF GRAVITY.
7   STABILITY CURVE.
8
9   INPUTS
10
11       Angle of inclination 0        Uncorrected righting arm AZo
12             in degrees                        in meters
13                                    0                                 0
1                                    15                               0.9
15                                   30                              2.15
16                                   45                              2.55
17                                   60                              1.91
18                                   75                               0.8
1                                    90                              -0.5
20
23-Sep-90  10:37 AM                            CIRC
```

```
1   [illegible]
2
3
4
5
6
7
8
9                                                  CALCULATIONS
1
11  Vertical shift AGv                 Vertical correction AGv*sin0
12              in meters                         in meters
13                                    0                              0.00
14                                  0.3                              0.08
15                                  0.3                              0.15
16                                  0.3                              0.21
17                                  0.3                              0.26
18                                  0.3                              0.29
19                                  0.3                              0.30
20
23-Sep-90  10:38 AM                            CIRC
```

```
1
2
3
4
5
6
7
8
9                                                    CALCULATIONS
10
11  Transverse shift AGt                 Transverse correction AGt*cos0
12               in meters                           in meters
13                                     0                                 0.00
14                                     0                                 0.00
15                                     0                                 0.00
16                                     0                                 0.00
17                                     0                                 0.00
18                                     0                                 0.00
19                                     0                                 0.00
20
23-Sep-90  10:39 AM                               CIRC
```

```
1
2
3
4
5
6
7
8
9              CALCULATIONS                                  RESULTS
10
11  Transverse correction AGt*cos0     Corrected Righting arm GZ(0)
12               in meters                           in meters
13                                 0.00                                  0.00
14                                 0.00                                  0.82
15                                 0.00                                  2.00
16                                 0.00                                  2.34
17                                 0.00                                  1.65
18                                 0.00                                  0.51
19                                 0.00                                 -0.80
20
23-Sep-90  10:40 AM                               CIRC
```

```
21
22
23
24
25   TASK 2 :
26
27   METACENTRIC HEIGTH GM ( in meters )
28   Large angles of inclination (0>10 degrees)
29
30   INPUTS
31
32   The vertical shift AGv is                                        0.3
33   The Righting Arm ( 0 = 10 deg. ) is                              0.82
34
35                                                   RESULT
36   The approximated GM  in meters is                        4.3982539201
37                                                                    4.40
38
39
40
23-Sep-90  10:44 AM                         CIRC
```

```
38   TASK 3 :
39
40   METACENTRIC HEIGTH GM
41   Small angles of inclination (0<10 degrees)
42
43   Angles of inclination              Righting arm GZ
44
45                                 1                                  0.05
46                                 6                                   0.1
47                                 7                                   0.2
48                                 8                                   0.3
49                                 9                                   0.4
50                                10                                   0.5
51
52
53
54
55
56
57
23-Sep-90  10:45 AM                         CIRC
```

```
38
39
40
41
42                                                       RESULTS
43  Righting arm GZ                                Metacentric heigth GM
44
45                                  0.05
46                                                                        2.865
47                                   0.1                                  0.955
48                                   0.2                                  1.637
49                                   0.3                                  2.149
50                                   0.4                                  2.546
51                                   0.5                                  2.865
52
53
54
55
56
57
23-Sep-90  10:46 AM                                CIRC
```

```
53  TASK 4 :
54
55  RIGTHING ARM  GZ
56  FREE SURFACE CORRECTION AT LARGE ANGLES
57
58  INPUTS
59
60  The breadth b of the tank
61  The length of the tank l
62  Specific gravity of the liquid in the tank
63  Specific gravity of the flotation liquid
64  The number of the tank compartments is
65
66  Specific gravity d1                                Breadth b
67
68                                1.025
69                                1.025                                   2
70                                1.025                                   2
71                                1.025                                   2
72                                1.025                                   2
23-Sep-90  10:48 AM                               CIRC                  2
```

```
53
54
55
56
57
58
59
60
61
62
63
64
65
66     Length l    Number of compartments n Moment of inertia i
67
68              5                         3        0.3703703704
69              5                         3        0.3703703704
70              5                         3        0.3703703704
71              5                         3        0.3703703704
72              5                         3        0.3703703704
23-Sep-90  10:50 AM                          CIRC
```

```
53
54
55
56
57
58
59
60
61
62
63
64
65
66          Moment of inertia i                      Product d*i
67
68                        0.3703703704                         0.3796296296
69                        0.3703703704                         0.3796296296
70                        0.3703703704                         0.3796296296
71                        0.3703703704                         0.3796296296
72                        0.3703703704                         0.3796296296
23-Sep-90  10:53 AM                           CIRC
```

```
75
76                                                                      Displacemt
77        Angles of inclination 0          tan0    1 + ( tan0 )^2/2          W
78
79                                     5 0.0874886      1.0038271331       20000
80                                    10 0.1763269      1.0155456021       20000
81                                    15 0.2679491      1.0358983849       20000
82                                    20 0.3639702      1.0662371657       20000
83                                    30 0.5773502      1.1666666667       20000
84                                    45         1               1.5       20000
85                                    60 1.7320508               2.5       20000
86                                    75 3.7320508      7.9641016151       20000
87                                    90  2.94E+18  4.309726524E+36       20000
88
89
90
91
92
93
94
23-Sep-90  10:57 AM                                  CIRC
```

```
                                                                            READ

       F         G                    H
75
76
77  Volumes V Mean draft    Location of G above keel KG
78
79   19512.195         11                                  9
80   19512.195         11                                  9
81   19512.195         11                                  9
82   19512.195         11                                  9
83   19512.195         11                                  9
84   19512.195         11                                  9
85   19512.195         11                                  9
86   19512.195         11                                  9
87   19512.195         11                                  9
88
89
90
91
92
93
94
23-Sep-90  11:01 AM                                  CIRC
```

```
75
76   Location of M                         Location of B
77  above keel KM   Values of GM  above keel KB   Values of BM
78
79              11              2              7              4
80              11              2              7              4
81              11              2              7              4
82              11              2              7              4
83              11              2              7              4
84              11              2              7              4
85              11              2              7              4
86              11              2              7              4
87              11              2              7              4
88
89
90
91
92
93
94
23-Sep-90  11:05 AM                           CIRC
```

```
75
76
77      Free Surface Correction FSC        Product FSC*(1+(tan0)^2/2)
78
79                       0.0000189815                        0.0000190541
80                       0.0000189815                        0.0000192766
81                       0.0000189815                        0.0000196629
82                       0.0000189815                        0.0000202388
83                       0.0000189815                        0.0000221451
84                       0.0000189815                        0.0000284722
85                       0.0000189815                        0.0000474537
86                                  0                                   0
87                                  0                                   0
88
89
90
91
92
93
94
23-Sep-90  11:06 AM                           CIRC
```

```
75
76
77      Product FSC*(1+(tan0)^2/2)        Corrected Righting Arm GZ
78
79                       0.0000190541                                  0.18
80                       0.0000192766                                  0.36
81                       0.0000196629                                  0.55
82                       0.0000202388                                  0.77
83                       0.0000221451                                  1.33
84                       0.0000284722                                  2.83
85                       0.0000474537                                  6.93
86                                  0                                 28.84
87                                  0 ***********************************
88
89
90
91
92
93
94
23-Sep-90  11:08 AM                         CIRC
```

```
92   AREA UNDER THE STABILITY CURVE
93   SIMPSON'S RULES
94   DYNAMIC STABILITY
95
96
97   INPUTS
98
99         Angles of inclination 0      Righting Arm GZ SIMPSON's Multiplier
100
101                                 0               0                    1
102                                15            0.82                    4
103                                30               2                    2
104                                45            2.34                    4
105                                60            1.65                    2
106                                75            0.51                    4
107                                90            -0.8                    1
108
109                                                                     15
110                                   Common interval         0.2617993878
111
23-Sep-90  11:11 AM                         CIRC
```

```
2
3
4
5
6
7
8
9  Righting Arm GZ SIMPSON's Multiplier Product for Areas
00
01              0                   1                   0
02           0.82                   4                3.28
03              2                   2                   4
             2.34                   4                9.36
05           1.65                   2                 3.3
06           0.51                   4                2.04
07           -0.8                   1                -0.8
08
 )                                 15
10 Common interval       0.2617993878
11
3-Sep-90  11:12 AM                           CIRC
```

```
03                                  2                   2
04                               2.34                   4
 5                               1.65                   2
 6                               0.51                   4
07                               -0.8                   1
08
09                                                     15
   Common interval h                         0.2617993878
                                             RESULT
12 The Area under the stab curve is          1.8483036779
13
14 The displacement W is                            20000
15                                           RESULT
16 The dynamic stability is                  36966.073557
17                                               36966.07
18
19
20
21
22
23-Sep-90  11:15 AM                          CIRC
```

```
119 TASK 6 :
120
121 RIGHTING ARM. KN CURVES.
122
123 Angles of inclination 0  Values of KN Values of KG   Product KG*sin0
124
125
126                          5          0.9            9        0.7844016847
127                         10            2            9         1.562833599
128                         15          3.2            9        2.3293714059
129                         20          4.4            9        3.0781812899
130                         30          6.5            9                 4.5
131                         45         8.75            9        6.3639610307
132                         60          9.7            9        7.7942286341
133                         75          9.4            9        8.6933324366
134                         90          8.4            9                   9
135
136
137
138
23-Sep-90  11:18 AM                          CIRC
```

```
119
120
121
122
123 Values of KG   Product KG*sin0   Righting Arm GZ
124
125
126             9        0.7844016847       0.12
127             9         1.562833599       0.44
128             9        2.3293714059       0.87
129             9        3.0781812899       1.32
130             9                 4.5       2.00
131             9        6.3639610307       2.39
132             9        7.7942286341       1.91
133             9        8.6933324366       0.71
134             9                   9      -0.60
135
136
137
138
23-Sep-90  11:19 AM                          CIRC
```

```
135
136 TASK 7 :
137
138 STABILITY CURVE  GZ = f ( 0 )
139
140 Angles of inclination Righting Arm GZ Displacement Righting Moment M
141                                                            W*GZ
142                     0               0       20000                0
143                    15             0.9       20000            18000
144                    30            2.15       20000            43000
145                    45            2.55       20000            51000
146                    60            1.91       20000            38200
147                    75             0.8       20000            16000
148                    90            -0.5       20000           -10000
149
150
151
152
153
154
23-Sep-90  11:28 AM                          CIRC
```

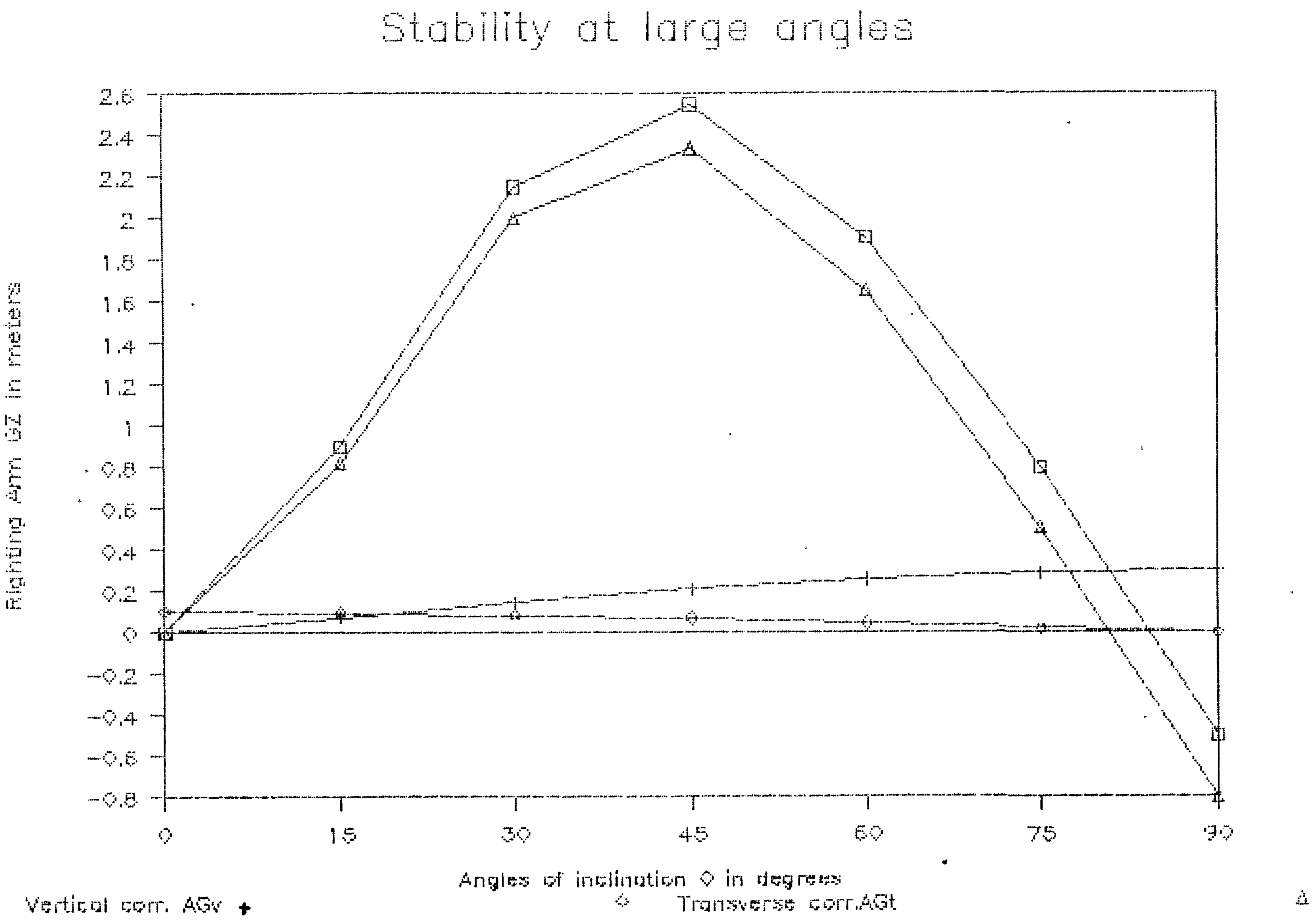
```
135
136
137
138
139 Transverse                   MetacenterCent.Gravity
140   shift d Inclining Moment above keel above keel KG Righting Moment MTi
141                     w*d
142         5        25000.00          11             9                0
143         5        24148.15          11             9     10471.975512
144         5        21650.64          11             9     20943.951024
145         5        17677.67          11             9     31415.926536
146         5        12500.00          11             9     41887.902048
147         5         6470.48          11             9      52359.87756
148         5            0.00          11             9     62831.853072
149
150
151
152
153
154
23-Sep-90  11:36 AM                          CIRC
```

```
151 AREA UNDER THE STABILITY CURVE
152
153 THE IMO REQUIREMENTS
154
155 AREA ( 0 , 30 degrees )
156
157 Angles of inclination Righting Arm GZ
158
159                     0                0
160                    15              0.9
161                    30             2.15
162
163
164
165
166
167
168
169
170
23-Sep-90  11:38 AM                              CIRC
```

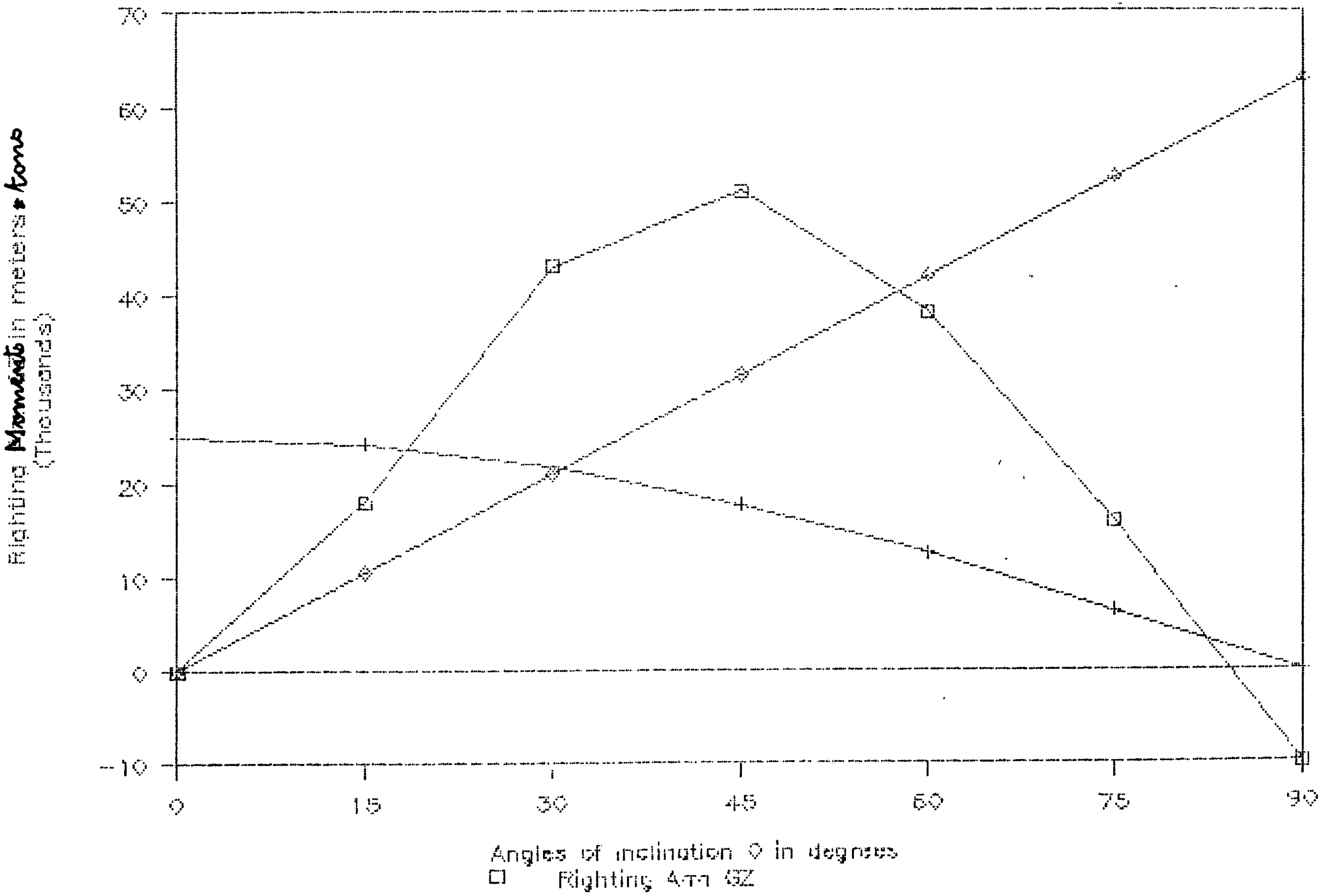
```
151
152
153
154
155
156
157        Simpson's Multipliers       Product for Areas
158
159                                    1                0
160                                    4              3.6
161                                    1             2.15
162
163 The common interval h in degrees is                15
164
165                                          RESULT
166
167      The Area under the curve is         0.5017821599
168                                                  0.50
169
170
23-Sep-90  11:39 AM                              CIRC
```

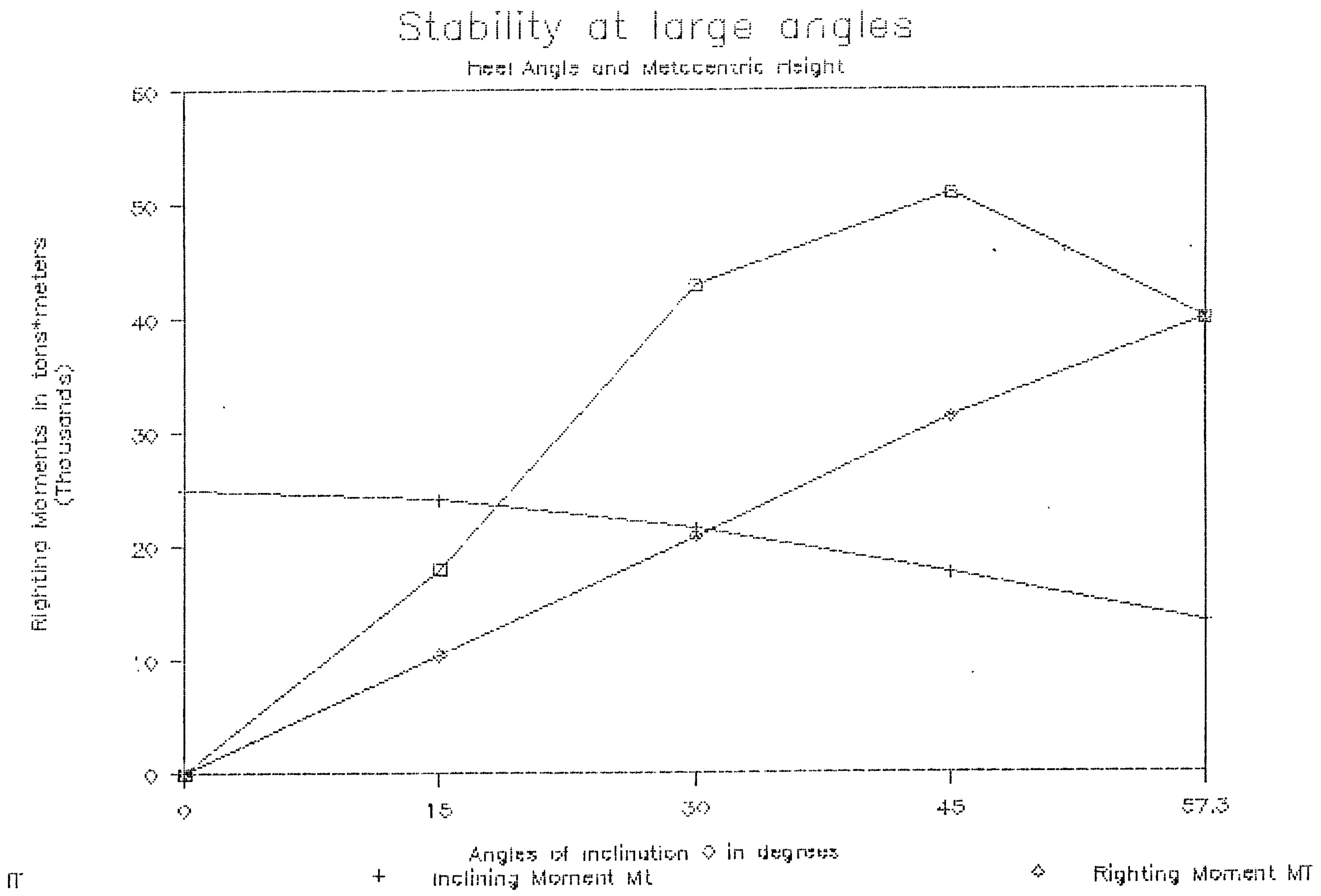
```
163
164
165
166
167
168
169 AREA ( 30 , 90 degrees )
170
171 Angles of inclination Righting Arm GZ
172
173                  30             2.15
174                  45             2.55
175                  60             1.91
176                  75              0.8
177                  90             -0.5
178
179
180
181
182
23-Sep-90  11:40 AM                        CIRC
```

```
169
170
171        Simpson's Multipliers       Product for Areas
172
173                                    1             2.15
174                                    4             10.2
175                                    2             3.82
176                                    4              3.2
177                                    1             -0.5
178
179 The common interval h is                           15
180
181                                           RESULT
182
183 The Area under the curve is          1.6467181493
184                                                  1.65
185
186
187
188
23-Sep-90  11:42 AM                        CIRC
```

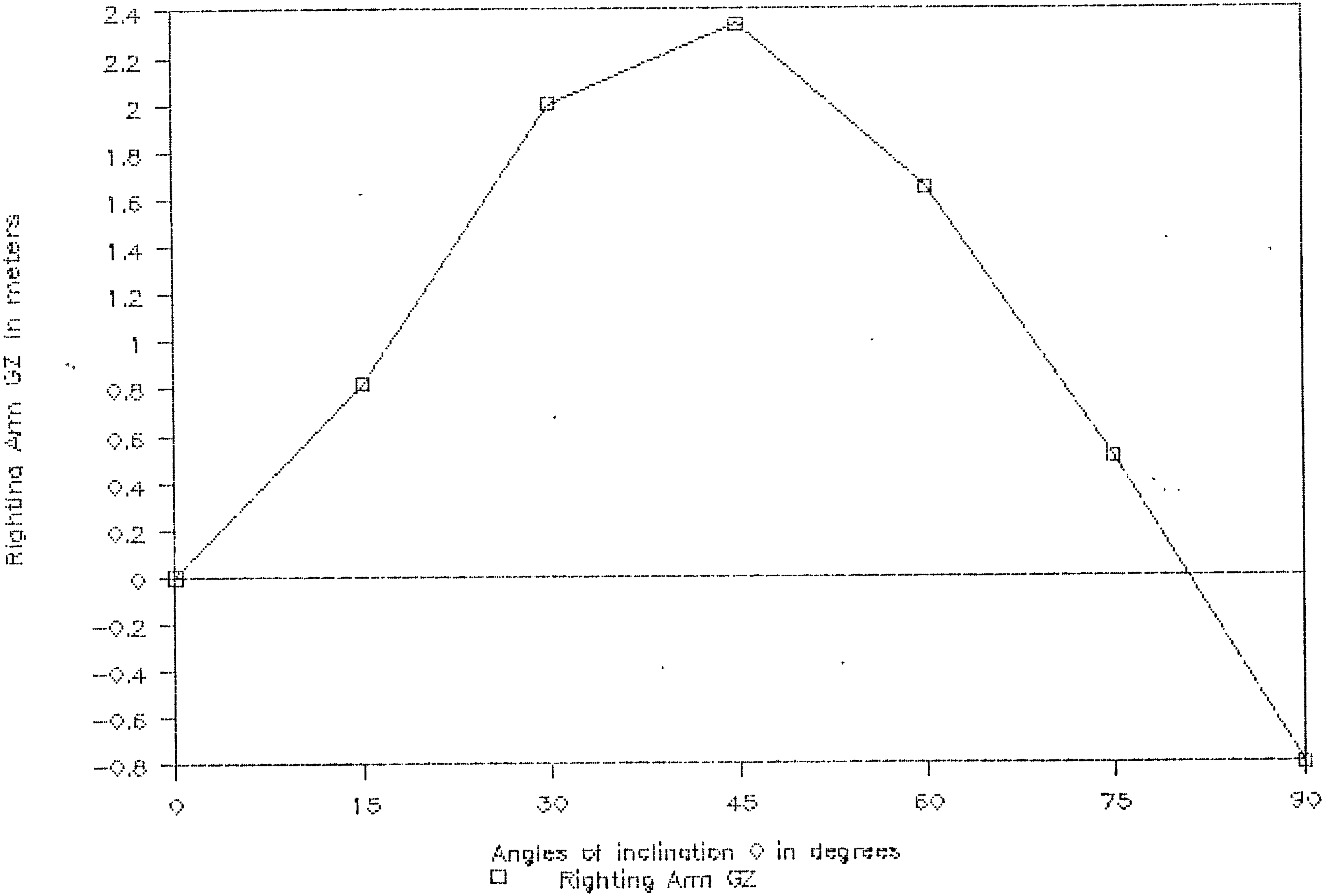

Stability at large angles
Righting Arm GZ in meters
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
15
30
45
60
75
90
Angles of inclination ◊ in degrees
Vertical corr. AGv +
◊ Transverse corr.AGt
△

Stability at large angles




Stability at large angles
Heel Angle and Metacentric Height
Righting Moments in tons*meters
(Thousands)
0
10
20
30
40
50
60
0
15
30
45
57.3
Angles of inclination ◇ in degrees
+ Inclining Moment Mt
◇ Righting Moment MT

Stability at large angles



5.7. Results of the reference.

Example 1 : Required GZ in meters

0
0.82
2.00
2.34
1.65
0.51
\- 0.80

Example 2 : GZ = KN - KG * sin 0

0.12
0.43
0.87
1.32
2.00
2.39
1.91
0.71
0.60

5.8. Comparisons and critical remarks.

The last value of GZ from the reference is numerically correct but the sign minus is missing. This omission may be occured during the printing of the book.

Graph 1 on page 140 shows the effect of transverse and vertical shifts of the center of gravity on the righting arm .

Graph 2 on page 141 gives the curves righting moments MT = W*GZ, MTi (initial stability) = W*GM*0 and Mt = w*d*cos0 (inclining moment).

The intersection between the curves MT=f(0) and Mt=g(0) gives the value of the angle of heel created when the transverse shift is d = 5 meters and the shifted weight w is 5000 tons.
The angle of heel is 18 degrees.

Graph 3 in page 142 gives the value of the metacentric height GM which is 2 meters at angle 0 = 57.3 degrees.

Graph 4 in page 143 is the stability curve obtained from the KN curves.

The GZ = 1.8 m for 0 = 30 degrees

The maximum GZ is obtained at an angle of 45 degrees.

The area (0, 30 deg) is 0.50 m*rd

The area (30, 90 deg) is 1.6 m*rd.

The range of stability is from 0 to 90 degrees.

Conclusion : the given ship complies with the IMO requirements.

5.9. Listing of the LOTUS program.

The listing of the program is given in Appendix G.

Ref 12.

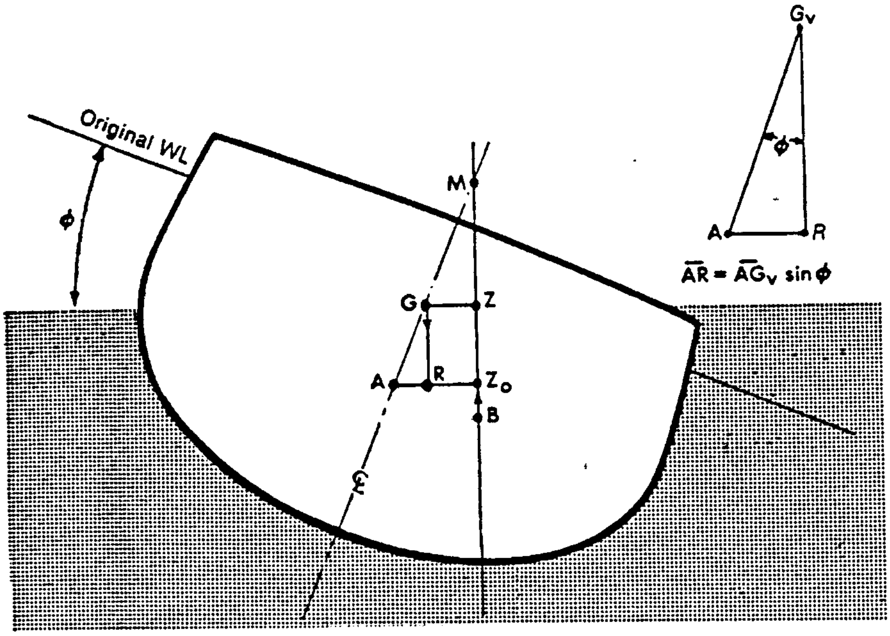


Figure 8–6. Loss of righting arm due to a rise in the center of gravity

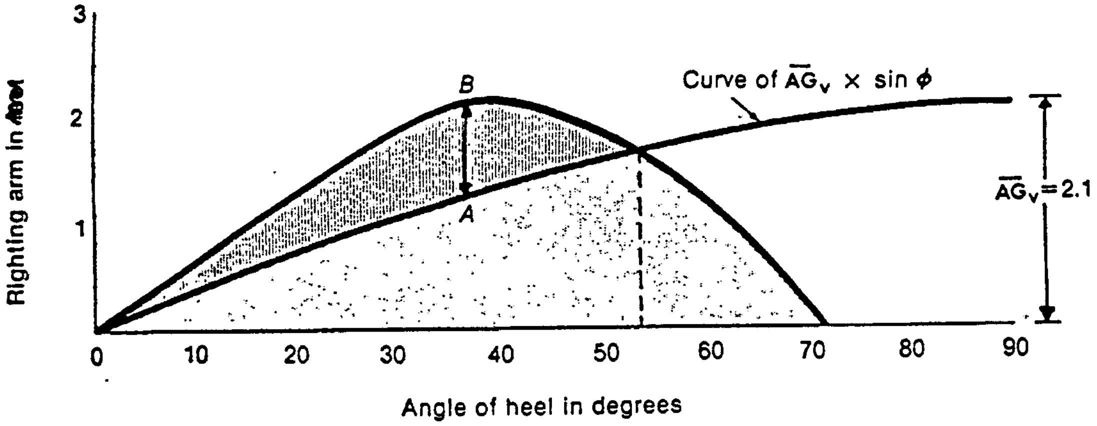


Figure 8–7. Sine curve superimposed on the original stability curve

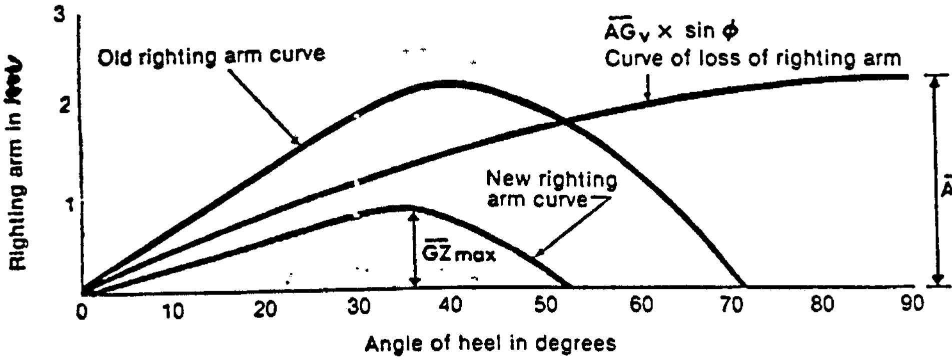


Figure 8–8. Curve of static stability as corrected for a loss of stability due to a vertical weight shift

Ref 12.

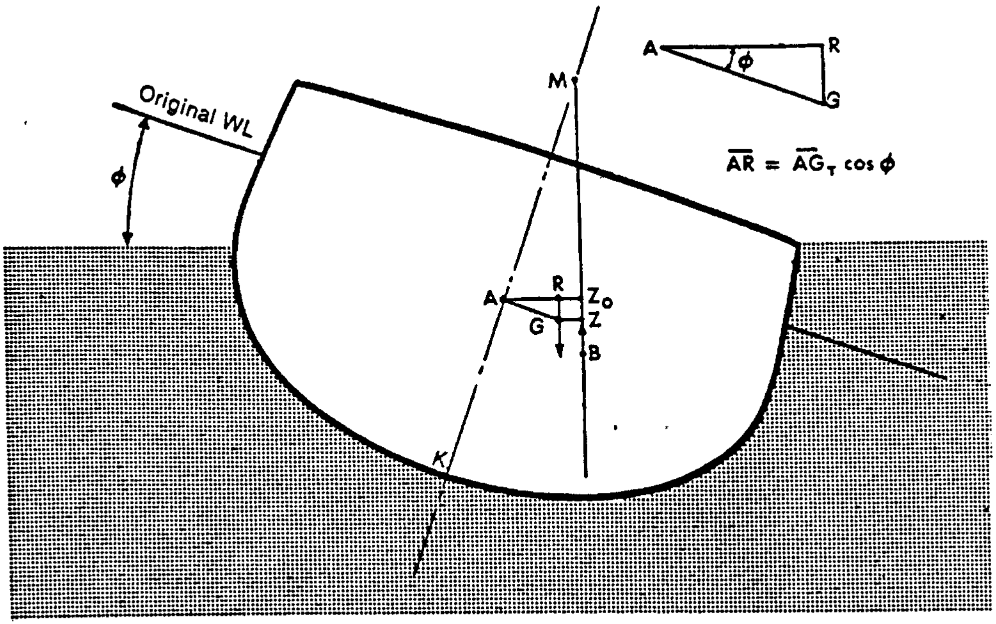


Figure 8-9. Loss of righting arm when the center of gravity is moved off the centerline

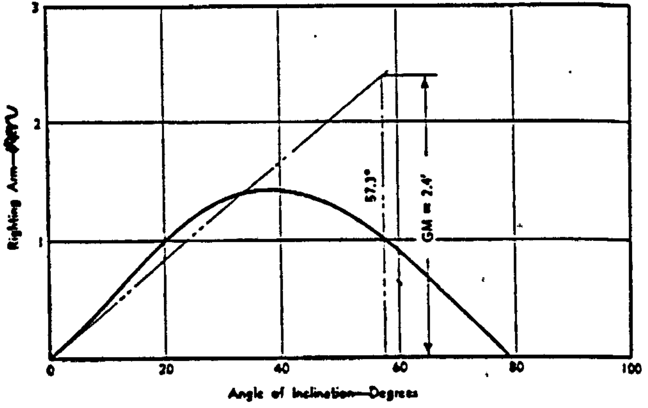


Figure 8-12. Metacentric height

Ref 12.

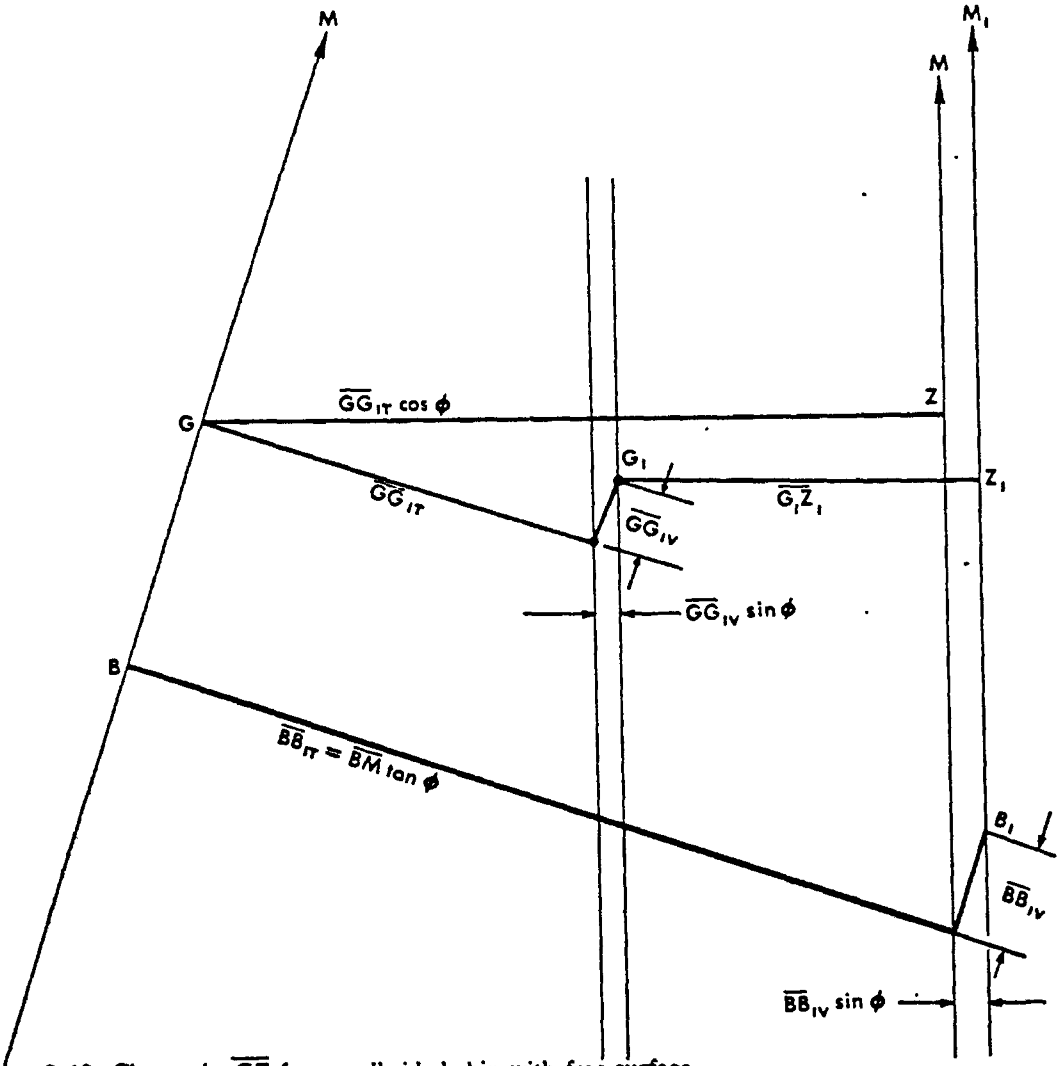


Figure 8–18. Change in $\overline{GZ}$ for a wall-sided ship with free surface

APPENDIX G :

PROGRAM : STAB 1 A.

```
A1: [W35] 'STABILITY AT LARGE ANGLES
B1: [W35] 'PROGRAM STAB1A. TATY-BOUSSIANA J.L.
C1: [W35] 'WORLD MARITIME UNIVERSITY
D1: [W35] ^MALMO, SWEDEN.
B2: [W35] ^MET ( N ) 90
A3: [W35] 'TASK 1 :
A5: [W35] 'RIGTHING ARM DUE TO VERTICAL AND
A6: [W35] 'TRANSVERSE SHIFT OF THE CENTER OF GRAVITY.
A7: [W35] 'STABILITY CURVE.
A9: [W35] 'INPUTS
D9: [W35] ^CALCULATIONS
F9: [W35] ^CALCULATIONS
G9: [W35] ^RESULTS
A11: [W35] ^Angle of inclination 0
B11: [W35] 'Uncorrected righting arm AZo
C11: [W35] 'Vertical shift AGv
D11: [W35] 'Vertical correction AGv*sin0
E11: [W35] 'Transverse shift AGt
F11: [W35] 'Transverse correction AGt*cos0
G11: [W35] 'Corrected Righting arm GZ(0)
A12: [W35] ^in degrees
B12: [W35] ^in meters
C12: [W35] ^in meters
D12: [W35] ^in meters
E12: [W35] ^in meters
F12: [W35] ^in meters
G12: [W35] ^in meters
A13: [W35] @N(A13..A13)
B13: [W35] @N(B13..B13)
C13: [W35] @N(C13..C13)
D13: (F2) [W35] +C13*@SIN(A13*@PI/180)
E13: [W35] @N(E13..E13)
F13: (F2) [W35] +E13*@COS(A13*@PI/180)
G13: (F2) [W35] +B13-D13-F13
A14: [W35] @N(A14..A14)
B14: [W35] @N(B14..B14)
C14: [W35] @N(C14..C14)
D14: (F2) [W35] +C14*@SIN(A14*@PI/180)
E14: [W35] @N(E14..E14)
F14: (F2) [W35] +E14*@COS(A14*@PI/180)
G14: (F2) [W35] +B14-D14-F14
A15: [W35] @N(A15..A15)
B15: [W35] @N(B15..B15)
C15: [W35] @N(C15..C15)
D15: (F2) [W35] +C15*@SIN(A15*@PI/180)
E15: [W35] @N(E15..E15)
F15: (F2) [W35] +E15*@COS(A15*@PI/180)
G15: (F2) [W35] +B15-D15-F15
A16: [W35] @N(A16..A16)
B16: [W35] @N(B16..B16)
C16: [W35] @N(C16..C16)
D16: (F2) [W35] +C16*@SIN(A16*@PI/180)
```

```
E16: [W35] @N(E16..E16)
F16: (F2) [W35] +E16*@COS(A16*@PI/180)
G16: (F2) [W35] +B16-D16-F16
A17: [W35] @N(A17..A17)
B17: [W35] @N(B17..B17)
C17: [W35] @N(C17..C17)
D17: (F2) [W35] +C17*@SIN(A17*@PI/180)
E17: [W35] @N(E17..E17)
F17: (F2) [W35] +E17*@COS(A17*@PI/180)
G17: (F2) [W35] +B17-D17-F17
A18: [W35] @N(A18..A18)
B18: [W35] @N(B18..B18)
C18: [W35] @N(C18..C18)
D18: (F2) [W35] +C18*@SIN(A18*@PI/180)
E18: [W35] @N(E18..E18)
F18: (F2) [W35] +E18*@COS(A18*@PI/180)
G18: (F2) [W35] +B18-D18-F18
A19: [W35] @N(A19..A19)
B19: [W35] @N(B19..B19)
C19: [W35] @N(C19..C19)
D19: (F2) [W35] +C19*@SIN(A19*@PI/180)
E19: [W35] @N(E19..E19)
F19: (F2) [W35] +E19*@COS(A19*@PI/180)
G19: (F2) [W35] +B19-D19-F19
A25: [W35] 'TASK 2 :
A27: [W35] 'METACENTRIC HEIGTH GM ( in meters )
A28: [W35] 'Large angles of inclination (0>10 degrees)
A30: [W35] 'INPUTS
A32: [W35] 'The vertical shift AGv is
B32: [W35] @N(A32..A32)
A33: [W35] 'The Righting Arm ( 0 = 10 deg. ) is
B33: [W35] @N(A33..A33)
B35: [W35] ^RESULT
A36: [W35] 'The approximated GM  in meters is
B36: [W35] +B33/(10*@PI/180)-B32
A38: [W35] 'TASK 3 :
A40: [W35] 'METACENTRIC HEIGTH GM
A41: [W35] 'Small angles of inclination (0<10 degrees)
C41: [W35] ^RESULTS
A43: [W35] 'Angles of inclination
B43: [W35] 'Righting arm GZ
C43: [W35] ^Metacentric heigth GM
A45: [W35] @N(A45..A45)
B45: [W35] @N(B45..B45)
C45: (F3) [W35] +B45/(A45*@PI/180)
A46: [W35] @N(A46..A46)
B46: [W35] @N(B46..B46)
C46: (F3) [W35] +B46/(A46*@PI/180)
A47: [W35] @N(A47..A47)
B47: [W35] @N(B47..B47)
C47: (F3) [W35] +B47/(A47*@PI/180)
A48: [W35] @N(A48..A48)
```

```
B48: [W35] @N(B48..B48)
C48: (F3) [W35] +B48/(A48*@PI/180)
A49: [W35] @N(A49..A49)
B49: [W35] @N(B49..B49)
C49: (F3) [W35] +B49/(A49*@PI/180)
A50: [W35] @N(A50..A50)
B50: [W35] @N(B50..B50)
C50: (F3) [W35] +B50/(A50*@PI/180)
A53: [W35] 'TASK 4 :
A55: [W35] 'RIGTHING ARM  GZ
A56: [W35] 'FREE SURFACE CORRECTION AT LARGE ANGLES
A58: [W35] 'INPUTS
A60: [W35] 'The breadth b of the tank
A61: [W35] 'The length of the tank l
A62: [W35] 'Specific gravity of the liquid in the tank
A63: [W35] 'Specific gravity of the flotation liquid
A64: [W35] 'The number of the tank compartments is
A66: [W35] 'Specific gravity d1
B66: [W35] ^Breadth b
C66: [W35] ^Length l
D66: [W35] ^Number of compartments n
E66: [W35] ^Moment of inertia i
F66: [W35] ^Product d*i
A68: [W35] @N(A68..A68)
B68: [W35] @N(B68..B68)
C68: [W35] @N(C68..C68)
D68: [W35] @N(D68..D68)
E68: [W35] +B68^3*C68/(D68^2*12)
F68: [W35] +A68*E68
A69: [W35] @N(A69..A69)
B69: [W35] @N(B69..B69)
C69: [W35] @N(C69..C69)
D69: [W35] @N(D69..D69)
E69: [W35] +B69^3*C69/(D69^2*12)
F69: [W35] +A69*E69
A70: [W35] @N(A70..A70)
B70: [W35] @N(B70..B70)
C70: [W35] @N(C70..C70)
D70: [W35] @N(D70..D70)
E70: [W35] +B70^3*C70/(D70^2*12)
F70: [W35] +A70*E70
A71: [W35] @N(A71..A71)
B71: [W35] @N(B71..B71)
C71: [W35] @N(C71..C71)
D71: [W35] @N(D71..D71)
E71: [W35] +B71^3*C71/(D71^2*12)
F71: [W35] +A71*E71
A72: [W35] @N(A72..A72)
B72: [W35] @N(B72..B72)
C72: [W35] @N(C72..C72)
D72: [W35] @N(D72..D72)
E72: [W35] +B72^3*C72/(D72^2*12)
```

```
F72: [W35] +A72*E72
A73: [W35] @N(A73..A73)
B73: [W35] @N(B73..B73)
C73: [W35] @N(C73..C73)
D73: [W35] @N(D73..D73)
E73: [W35] +B73^3*C73/(D73^2*12)
F73: [W35] +A73*E73
A74: [W35] @N(A74..A74)
B74: [W35] @N(B74..B74)
C74: [W35] @N(C74..C74)
D74: [W35] @N(D74..D74)
E74: [W35] +B74^3*C74/(D74^2*12)
F74: [W35] +A74*E74
A77: [W35] ^Angles of inclination 0
B77: [W35] ^tan0
C77: [W35] ^1 + ( tan0 )^2/2
D77: [W35] ^Displacement W
E77: [W35] 'Specific gravity d2
F77: [W35] ^Volumes V
G77: [W35] ^Mean draft Tm
H77: [W35] ^Location of G above keel KG
I77: [W35] ^Location of M above keel KM
J77: [W35] ^Values of GM
K77: [W35] ^Location of B above keel KB
L77: [W35] ^Values of BM
M77: [W35] ^Product BM*(tan0)^2/2
N77: [W35] ^Free Surface Correction FSC
O77: [W35] ^Product FSC*(1+(tan0)^2/2)
P77: [W35] ^Corrected Righting Arm GZ
A79: [W35] @N(A79..A79)
B79: [W35] @TAN(A79*@PI/180)
C79: [W35] 1+(B79^2)/2
D79: [W35] @N(D79..D79)
E79: [W35] @N(E79..E79)
F79: [W35] +D79/E79
G79: [W35] @N(G79..G79)
H79: [W35] @N(H79..H79)
I79: [W35] @N(I79..I79)
J79: [W35] +I79-G79
K79: [W35] @N(K79..K79)
L79: [W35] +I79-K79
M79: [W35] +L79*B79^2/2
N79: [W35] +F68/D79
O79: [W35] +N79*C79
P79: (F2) [W35] (J79+M79-O79)*@SIN(A79*@PI/180)
A80: [W35] @N(A80..A80)
B80: [W35] @TAN(A80*@PI/180)
C80: [W35] 1+(B80^2)/2
D80: [W35] @N(D80..D80)
E80: [W35] @N(E80..E80)
F80: [W35] +D80/E80
G80: [W35] @N(G80..G80)
```

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H80: [W35] @N(H80..H80)
I80: [W35] @N(I80..I80)
J80: [W35] +I80-G80
K80: [W35] @N(K80..K80)
L80: [W35] +I80-K80
M80: [W35] +L80*B80^2/2
N80: [W35] +F69/D80
O80: [W35] +N80*C80
P80: (F2) [W35] (J80+M80-O80)*@SIN(A80*@PI/180)
A81: [W35] @N(A81..A81)
B81: [W35] @TAN(A81*@PI/180)
C81: [W35] 1+(B81^2)/2
D81: [W35] @N(D81..D81)
E81: [W35] @N(E81..E81)
F81: [W35] +D81/E81
G81: [W35] @N(G81..G81)
H81: [W35] @N(H81..H81)
I81: [W35] @N(I81..I81)
J81: [W35] +I81-G81
K81: [W35] @N(K81..K81)
L81: [W35] +I81-K81
M81: [W35] +L81*B81^2/2
N81: [W35] +F70/D81
O81: [W35] +N81*C81
P81: (F2) [W35] (J81+M81-O81)*@SIN(A81*@PI/180)
A82: [W35] @N(A82..A82)
B82: [W35] @TAN(A82*@PI/180)
C82: [W35] 1+(B82^2)/2
D82: [W35] @N(D82..D82)
E82: [W35] @N(E82..E82)
F82: [W35] +D82/E82
G82: [W35] @N(G82..G82)
H82: [W35] @N(H82..H82)
I82: [W35] @N(I82..I82)
J82: [W35] +I82-G82
K82: [W35] @N(K82..K82)
L82: [W35] +I82-K82
M82: [W35] +L82*B82^2/2
N82: [W35] +F71/D82
O82: [W35] +N82*C82
P82: (F2) [W35] (J82+M82-O82)*@SIN(A82*@PI/180)
A83: [W35] @N(A83..A83)
B83: [W35] @TAN(A83*@PI/180)
C83: [W35] 1+(B83^2)/2
D83: [W35] @N(D83..D83)
E83: [W35] @N(E83..E83)
F83: [W35] +D83/E83
G83: [W35] @N(G83..G83)
H83: [W35] @N(H83..H83)
I83: [W35] @N(I83..I83)
J83: [W35] +I83-G83
K83: [W35] @N(K83..K83)
```

```
L83: [W35] +I83-K83
M83: [W35] +L83*B83^2/2
N83: [W35] +F72/D83
O83: [W35] +N83*C83
P83: (F2) [W35] (J83+M83-O83)*@SIN(A83*@PI/180)
A84: [W35] @N(A84..A84)
B84: [W35] @TAN(A84*@PI/180)
C84: [W35] 1+(B84^2)/2
D84: [W35] @N(D84..D84)
E84: [W35] @N(E84..E84)
F84: [W35] +D84/E84
G84: [W35] @N(G84..G84)
H84: [W35] @N(H84..H84)
I84: [W35] @N(I84..I84)
J84: [W35] +I84-G84
K84: [W35] @N(K84..K84)
L84: [W35] +I84-K84
M84: [W35] +L84*B84^2/2
N84: [W35] +F73/D84
O84: [W35] +N84*C84
P84: (F2) [W35] (J84+M84-O84)*@SIN(A84*@PI/180)
A85: [W35] @N(A85..A85)
B85: [W35] @TAN(A85*@PI/180)
C85: [W35] 1+(B85^2)/2
D85: [W35] @N(D85..D85)
E85: [W35] @N(E85..E85)
F85: [W35] +D85/E85
G85: [W35] @N(G85..G85)
H85: [W35] @N(H85..H85)
I85: [W35] @N(I85..I85)
J85: [W35] +I85-G85
K85: [W35] @N(K85..K85)
L85: [W35] +I85-K85
M85: [W35] +L85*B85^2/2
N85: [W35] +F74/D85
O85: [W35] +N85*C85
P85: (F2) [W35] (J85+M85-O85)*@SIN(A85*@PI/180)
A86: [W35] @N(A86..A86)
B86: [W35] @TAN(A86*@PI/180)
C86: [W35] 1+(B86^2)/2
D86: [W35] @N(D86..D86)
E86: [W35] @N(E86..E86)
F86: [W35] +D86/E86
G86: [W35] @N(G86..G86)
H86: [W35] @N(H86..H86)
I86: [W35] @N(I86..I86)
J86: [W35] +I86-G86
K86: [W35] @N(K86..K86)
L86: [W35] +I86-K86
M86: [W35] +L86*B86^2/2
N86: [W35] +F75/D86
O86: [W35] +N86*C86
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P86: (F2) [W35] (J86+M86-O86)*@SIN(A86*@PI/180)
A87: [W35] @N(A87..A87)
B87: [W35] @TAN(A87*@PI/180)
C87: [W35] 1+(B87^2)/2
D87: [W35] @N(D87..D87)
E87: [W35] @N(E87..E87)
F87: [W35] +D87/E87
G87: [W35] @N(G87..G87)
H87: [W35] @N(H87..H87)
I87: [W35] @N(I87..I87)
J87: [W35] +I87-G87
K87: [W35] @N(K87..K87)
L87: [W35] +I87-K87
M87: [W35] +L87*B87^2/2
N87: [W35] +F76/D87
O87: [W35] +N87*C87
P87: (F2) [W35] (J87+M87-O87)*@SIN(A87*@PI/180)
A90: [W35] 'TASK 5 :
A92: [W35] 'AREA UNDER THE STABILITY CURVE
A93: [W35] 'SIMPSON'S RULES
A94: [W35] 'DYNAMIC STABILITY
A97: [W35] 'INPUTS
A99: [W35] ^Angles of inclination 0
B99: [W35] ^Righting Arm GZ
C99: [W35] ^SIMPSON's Multipliers
D99: [W35] ^Product for Areas
A101: [W35] @N(A101..A101)
B101: [W35] @N(B101..B101)
C101: [W35] @N(C101..C101)
D101: [W35] +B101*C101
A102: [W35] @N(A102..A102)
B102: [W35] @N(B102..B102)
C102: [W35] @N(C102..C102)
D102: [W35] +B102*C102
A103: [W35] @N(A103..A103)
B103: [W35] @N(B103..B103)
C103: [W35] @N(C103..C103)
D103: [W35] +B103*C103
A104: [W35] @N(A104..A104)
B104: [W35] @N(B104..B104)
C104: [W35] @N(C104..C104)
D104: [W35] +B104*C104
A105: [W35] @N(A105..A105)
B105: [W35] @N(B105..B105)
C105: [W35] @N(C105..C105)
D105: [W35] +B105*C105
A106: [W35] @N(A106..A106)
B106: [W35] @N(B106..B106)
C106: [W35] @N(C106..C106)
D106: [W35] +B106*C106
A107: [W35] @N(A107..A107)
B107: [W35] @N(B107..B107)
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C107: [W35] @N(C107..C107)
D107: [W35] +B107*C107
C109: [W35] @N(C109..C109)
B110: [W35] 'The common interval h is
C110: [W35] +C109*@PI/180
C111: [W35] ^RESULT
B112: [W35] 'The Area under the stab curve is
C112: [W35] +C110/3*@SUM(D101..D107)
B114: [W35] 'The displacement W is
C114: [W35] @N(C114..C114)
C115: [W35] ^RESULT
B116: [W35] 'The dynamic stability is
C116: [W35] +C114*C112
A119: [W35] 'TASK 6 :
A121: [W35] 'RIGHTING ARM. KN CURVES.
A123: [W35] 'Angles of inclination 0
B123: [W35] ^Values of KN
C123: [W35] ^Values of KG
D123: [W35] ^Product KG*sin0
E123: [W35] ^Righting Arm GZ
A126: [W35] @N(A126..A126)
B126: [W35] @N(B126..B126)
C126: [W35] @N(C126..C126)
D126: [W35] +C126*@SIN(A126*@PI/180)
E126: (F2) [W35] +B126-D126
A127: [W35] @N(A127..A127)
B127: [W35] @N(B127..B127)
C127: [W35] @N(C127..C127)
D127: [W35] +C127*@SIN(A127*@PI/180)
E127: (F2) [W35] +B127-D127
A128: [W35] @N(A128..A128)
B128: [W35] @N(B128..B128)
C128: [W35] @N(C128..C128)
D128: [W35] +C128*@SIN(A128*@PI/180)
E128: (F2) [W35] +B128-D128
A129: [W35] @N(A129..A129)
B129: [W35] @N(B129..B129)
C129: [W35] @N(C129..C129)
D129: [W35] +C129*@SIN(A129*@PI/180)
E129: (F2) [W35] +B129-D129
A130: [W35] @N(A130..A130)
B130: [W35] @N(B130..B130)
C130: [W35] @N(C130..C130)
D130: [W35] +C130*@SIN(A130*@PI/180)
E130: (F2) [W35] +B130-D130
A131: [W35] @N(A131..A131)
B131: [W35] @N(B131..B131)
C131: [W35] @N(C131..C131)
D131: [W35] +C131*@SIN(A131*@PI/180)
E131: (F2) [W35] +B131-D131
A132: [W35] @N(A132..A132)
B132: [W35] @N(B132..B132)
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C132: [W35] @N(C132..C132)
D132: [W35] +C132*@SIN(A132*@PI/180)
E132: (F2) [W35] +B132-D132
A133: [W35] @N(A133..A133)
B133: [W35] @N(B133..B133)
C133: [W35] @N(C133..C133)
D133: [W35] +C133*@SIN(A133*@PI/180)
E133: (F2) [W35] +B133-D133
A134: [W35] @N(A134..A134)
B134: [W35] @N(B134..B134)
C134: [W35] @N(C134..C134)
D134: [W35] +C134*@SIN(A134*@PI/180)
E134: (F2) [W35] +B134-D134
A136: [W35] 'TASK 7 :
A138: [W35] 'STABILITY CURVE  GZ = f ( 0 )
A140: [W35] 'Angles of inclination 0
B140: [W35] ^Righting Arm GZ
C140: [W35] ^Displacement W
D140: [W35] ^Righting Moment MT
E140: [W35] ^Weight w
F140: [W35] ^Transverse shift d
G140: [W35] ^Inclining Moment Mt
H140: [W35] ^Metacenter above keel KM
I140: [W35] 'Center of gravity above keel KG
J140: [W35] ^Righting Moment MTi
D141: [W35] ^W*GZ
G141: [W35] ^w*d
A142: [W35] @N(A142..A142)
B142: [W35] @N(B142..B142)
C142: [W35] @N(C142..C142)
D142: [W35] +B142*C142
E142: [W35] @N(E142..E142)
F142: [W35] @N(F142..F142)
G142: (F2) [W35] +E142*F142*@COS(A142*@PI/180)
H142: [W35] @N(H142..H142)
I142: [W35] @N(I142..I142)
J142: [W35] ($H$142-$I$142)*$C$142*A142*@PI/180
A143: [W35] @N(A143..A143)
B143: [W35] @N(B143..B143)
C143: [W35] +$C$142
D143: [W35] +B143*C143
E143: [W35] +$E$142
F143: [W35] +$F$142
G143: (F2) [W35] +E143*F143*@COS(A143*@PI/180)
H143: [W35] +$H$142
I143: [W35] +$I$142
J143: [W35] ($H$142-$I$142)*$C$142*A143*@PI/180
A144: [W35] @N(A144..A144)
B144: [W35] @N(B144..B144)
C144: [W35] +$C$142
D144: [W35] +B144*C144
E144: [W35] +$E$142
```

```
F144: [W35] +$F$142
G144: (F2) [W35] +E144*F144*@COS(A144*@PI/180)
H144: [W35] +$H$142
I144: [W35] +$I$142
J144: [W35] ($H$142-$I$142)*$C$142*A144*@PI/180
A145: [W35] @N(A145..A145)
B145: [W35] @N(B145..B145)
C145: [W35] +$C$142
D145: [W35] +B145*C145
E145: [W35] +$E$142
F145: [W35] +$F$142
G145: (F2) [W35] +E145*F145*@COS(A145*@PI/180)
H145: [W35] +$H$142
I145: [W35] +$I$142
J145: [W35] ($H$142-$I$142)*$C$142*A145*@PI/180
A146: [W35] @N(A146..A146)
B146: [W35] @N(B146..B146)
C146: [W35] +$C$142
D146: [W35] +B146*C146
E146: [W35] +$E$142
F146: [W35] +$F$142
G146: (F2) [W35] +E146*F146*@COS(A146*@PI/180)
H146: [W35] +$H$142
I146: [W35] +$I$142
J146: [W35] ($H$142-$I$142)*$C$142*A146*@PI/180
A147: [W35] @N(A147..A147)
B147: [W35] @N(B147..B147)
C147: [W35] +$C$142
D147: [W35] +B147*C147
E147: [W35] +$E$142
F147: [W35] +$F$142
G147: (F2) [W35] +E147*F147*@COS(A147*@PI/180)
H147: [W35] +$H$142
I147: [W35] +$I$142
J147: [W35] ($H$142-$I$142)*$C$142*A147*@PI/180
A148: [W35] @N(A148..A148)
B148: [W35] @N(B148..B148)
C148: [W35] +$C$142
D148: [W35] +B148*C148
E148: [W35] +$E$142
F148: [W35] +$F$142
G148: (F2) [W35] +E148*F148*@COS(A148*@PI/180)
H148: [W35] +$H$142
I148: [W35] +$I$142
J148: [W35] ($H$142-$I$142)*$C$142*A148*@PI/180
A151: [W35] 'AREA UNDER THE STABILITY CURVE
A153: [W35] 'THE IMO REQUIREMENTS
A155: [W35] 'AREA ( 0 , 30 degrees )
A157: [W35] ^Angles of inclination 0
B157: [W35] ^Righting Arm GZ
C157: [W35] ^Simpson's Multipliers
D157: [W35] ^Product for Areas
```

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A159: [W35] @N(A159..A159)
B159: [W35] @N(B159..B159)
C159: [W35] @N(C159..C159)
D159: [W35] +B159*C159
A160: [W35] @N(A160..A160)
B160: [W35] @N(B160..B160)
C160: [W35] @N(C160..C160)
D160: [W35] +B160*C160
A161: [W35] @N(A161..A161)
B161: [W35] @N(B161..B161)
C161: [W35] @N(C161..C161)
D161: [W35] +B161*C161
C163: [W35] ^The common interval h in degrees is
D163: [W35] @N(D163..D163)
D165: [W35] ^RESULT
C167: [W35] ^The Area under the curve is
D167: [W35] +D163/3*@PI/180*@SUM(D159..D161)
A169: [W35] 'AREA ( 30 , 90 degrees )
A171: [W35] ^Angles of inclination 0
B171: [W35] ^Righting Arm GZ
C171: [W35] ^Simpson's Multipliers
D171: [W35] ^Product for Areas
A173: [W35] @N(A173..A173)
B173: [W35] @N(B173..B173)
C173: [W35] @N(C173..C173)
D173: [W35] +B173*C173
A174: [W35] @N(A174..A174)
B174: [W35] @N(B174..B174)
C174: [W35] @N(C174..C174)
D174: [W35] +B174*C174
A175: [W35] @N(A175..A175)
B175: [W35] @N(B175..B175)
C175: [W35] @N(C175..C175)
D175: [W35] +B175*C175
A176: [W35] @N(A176..A176)
B176: [W35] @N(B176..B176)
C176: [W35] @N(C176..C176)
D176: [W35] +B176*C176
A177: [W35] @N(A177..A177)
B177: [W35] @N(B177..B177)
C177: [W35] @N(C177..C177)
D177: [W35] +B177*C177
C179: [W35] 'The common interval h is
D179: [W35] +D163
D181: [W35] ^RESULT
C183: [W35] 'The Area under the curve is
D183: [W35] +D179/3*@PI/180*@SUM(D173..D177)
```

CHAPTER 6 :

ROLL MOTION.

6.1. Introduction.

This chapter deals with the differential equation of the roll motion which is defined as the oscillatory motion of the ship about its longitudinal axis.
The angle of roll motion is expressed , then the natural roll period of the ship.

This natural period of the ship is compared to the encounter period of the waves .

The results of the comparison lead to the so-called Mathieu resonance criteria.

This chapter aims to designe a LOTUS 1-2-3 program for the computation of the parameters of the roll motion

6.2. Definition of the problem.
The Mathematical Model.

6.2.1. The parameters of the Rolling motion equation.

The differential equation governing the uncoupled linear roll motion (coupling moments coming from other motion directions rather than roll neglected) may be written as follows :

$$a*\ddot{\theta} \quad + \quad b*\dot{\theta} \quad + c*\theta \quad = \quad d$$

where

$a = I = W * r$ = the inertia mass moment of the rolling ship including the hydrodynamic mass moment effect of the surrounding water with W the ship displacement

and r = the radius of roll gyration.

b = the damping coefficient = $2 * d * a$.

c = restoring moment / roll angle = $g * W * GM$

For small angles of roll, the ratio c/a is equal to the natural circular roll frequency squared :

$$c \,/\, a = g * GM \,/\, r \quad = \quad wo$$

The natural roll period of the ship is

$$To = 2 * \pi \,/\, wo$$

6.2.2. The WEISS formula.

If it is assumed that the radius of gyration r is proportional to the half beam 0.5*B , with f being the proportional constant,

$r = f*B/2$

the Weiss formula is

$$GM = (f * B / To)^2$$

6.2.3. The Roll angle versus time t in beam sea.
The wave contour versus time.
The wave slope versus time.

The wave contour is written as :

$$\varphi(t) = 0.5 * Hw * \cos (2 * \pi * t / Tw)$$

The wave slope is written as :

$$v (t) = \pi * Hw / Lw * \sin (2 * \pi * t / Tw)$$

where

Hw = the wave height
Lw = the wave length
Tw = the wave period

The roll angle versus time in beam sea may be written as:

$$\theta (t) = \theta max * \sin (2 * \pi * t / Tw + x)$$

where

$$\theta max = \pi *Hw/Lw/[1-(To/Tw)^2 + 4*D^2*(To/Tw)^2]$$

$$x = - \arctan [2*D*(To/Tw)/1-(To/Tw)^{2}]$$

6.2.4. The encounter period of ship and waves.

The motion excitation of a ship from waves is governed by the wave encounter period, which is the time elapsing from wave crest to the next wave crest passing the ship.

The encounter period is given by :

$$Te = Tw / [Tw - (2*\pi*V*\cos X /g)]$$

or

$$Te = Lw / (g*Tw/2*\pi - V*\cos X)$$

6.2.5. Mathieu resonance in following and quartering sea.

6.2.5.1. External resonance in beam sea.

Beam sea resonance occurs at a ratio of wave period Tw to the ship's natural roll period To equal or close to 1.

$$Tw = To$$

6.2.5.2. Parametric roll resonance in following and quartering sea.

Mathieu resonance for roll motion due to time variations of the uprighting moment of the vessel occurs mainly in the following and aft quartering seas, when the wave period of encounter Te is either

$$Te = 0.5 * To$$

$$Te = To$$

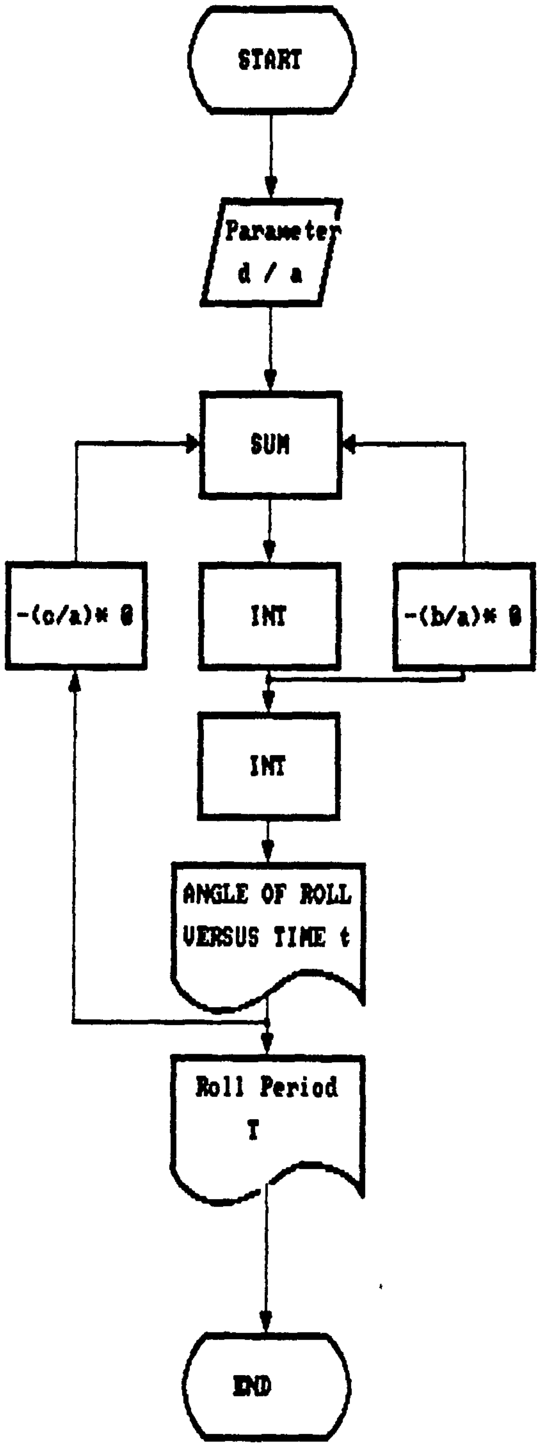
6.3. The flow chart.

The diagrams 1 and 2 from page 156 to page 157 give the steps for the computation of the roll angle versus time and the encounter period versus ship's speed.

The natural period is obtained from the expression of the roll angle.

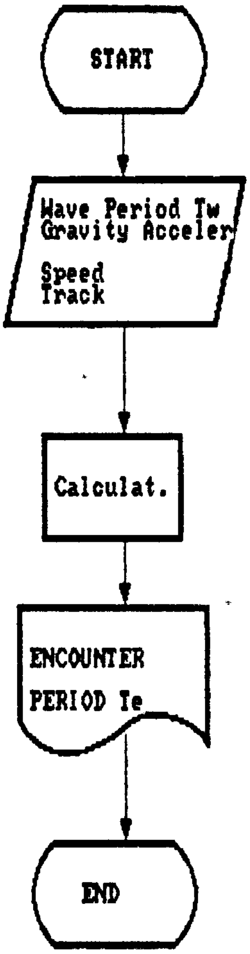
In practice, the natural period is obtained from the Weiss formula.

TASK : ANGLE OF ROLL VERSUS TIME t. ROLL PERIOD T.



ROLL MOTION.

TASK : ENCOUNTER PERIOD OF SHIP AND WAVES.



ROLL MOTION.

6.4. ORGANIZATION AND STRUCTURE OF THE LOTUS PROGRAM.

ROLL 1 A.

INPUTS	CELLS LOCATION
Coefficient f	B 7
Ship's breadth	B 9
Metacentric height	B 8
Wave height	B 22
Wave length	B 23
Wave period	B 24
Damping coefficient	B 25
Gravity acceleration	A 46 .. A 58
Wave period	B 46 .. B 58
Wave length	C 46 .. C 58
Speed	D 46 .. D 58
Track	E 46 .. E 58

CALCULATIONS (See listing)

OUTPUTS	CELLS LOCATION
ROLL PERIOD	C 12
MAXIMUM ANGLE OF ROLL	C 34
ENCOUNTER PERIOD OF SHIP AND WAVES	G 46 .. G 58

6.5. Examples.

Example 1 :

Coefficient f = 0.8

Metacentric Height GM = 0.36

Ship breadth B = 9 m

Example 2 :

Roll period To = 20 sec
Wave Height Hw = 13 m
Wave Period Tw = 19 sec
Damping coefficient D = 0.1

Example 3 :

Curve Encounter Period Te = f (V)
Speed V

6.6. LOTUS results.

The results of the LOTUS 1-2-3 program are given from page 160 to page 163.

```
                          A                                          B
1    ROLL MOTION                                   PROGRAM ROLL. TATY-BOUSSIANA J.L.
2    TASK 1 :                                                  MET ( N ) 90
3     ROLL PERIOD To . Weiss formula
4
5    INPUTS
6
7    The coefficient f is                                                        0.8
8    The metacentric height GM is                                               0.36
9    The breadth of the ship B is                                                  9
10
11
12                                                 The Roll Period To in seconds is
13
14
15
16
17
18
19
20
23-Sep-90   11:43 AM
```

```
1
2
3
4
5
6
7                                        0.8
8                                       0.36
9                                          9
10                                                         RESULT
11
12   The Roll Period To in seconds is                                          12.00
13
14
15
16
17
18
19
20
23-Sep-90   11:44 AM
```

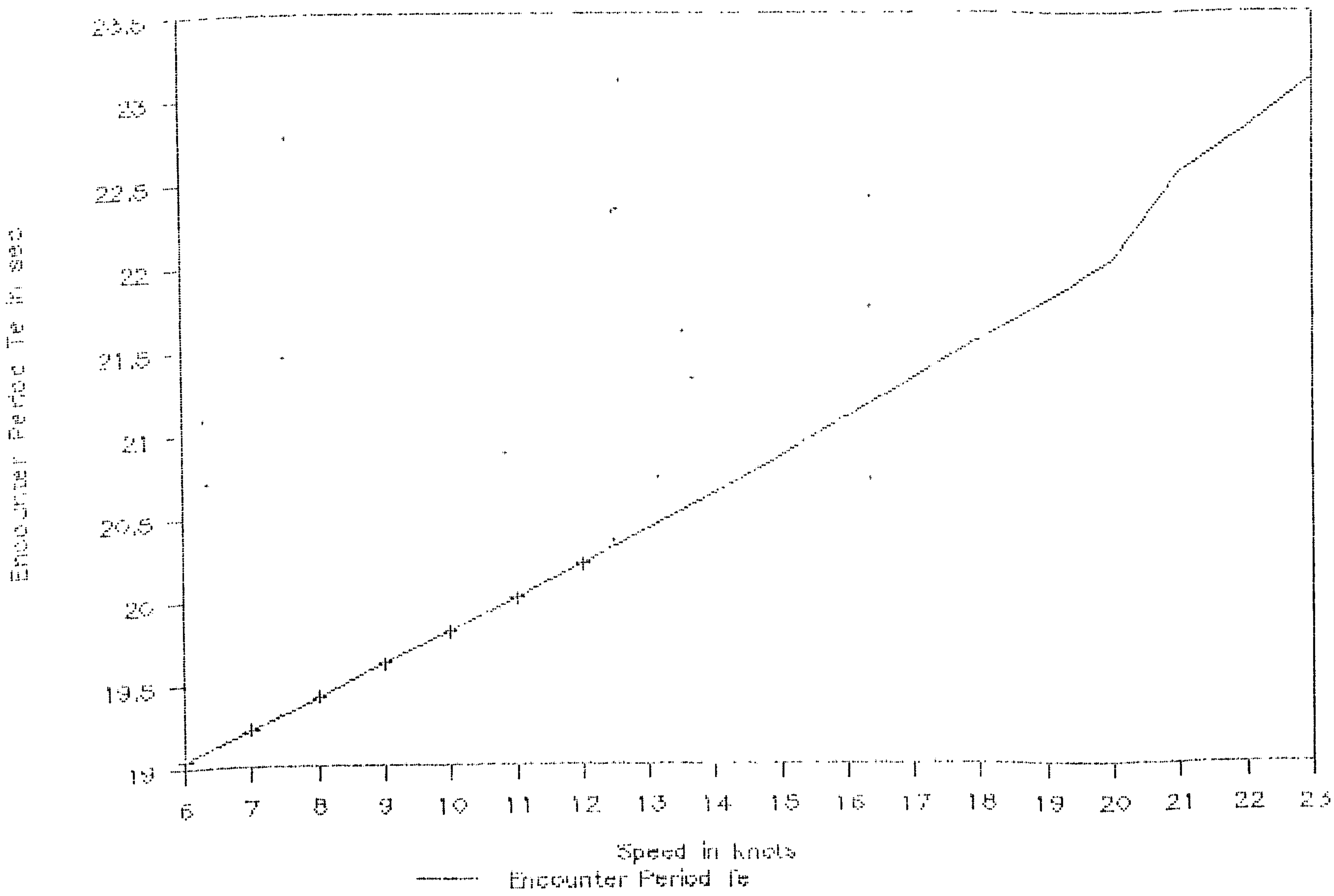
```
15  TASK 2 :
16
17  THE ROLL ANGLE VERSUS TIME t
18  IN BEAM SEA
19
20  INPUTS
21  The roll period is                                               20
22  The wave height Hw in meters is                                  13
23  The wave length Lw in meters is                              624.52
24  The wave period Tw in seconds is                                 19
25  The damping coefficient D is                                    0.1
2
27  CALCULATIONS
28
29                                                         0.0653953508
30                                                         0.0116711812
31                                                         0.0443213296
32                                                          0.236627367
33
34
23-Sep-90  11:45 AM
```

```
22                             13
23                         624.52
24                             19
25                            0.1
2
27
28
29                   0.0653953508
30                   0.0116711812                     0.108033241
31                   0.0443213296                    0.2105263158
32                    0.236627367
33                                           RESULT
34  The Maximum Angle of roll is                     0.2763642754
35
36  The phase angle x is                             0.4741184305
37
38
39
40
41
23-Sep-90  11:47 AM
```

```
TASK 3 :

THE ENCOUNTER PERIOD OF SHIP
AND WAVES

Grav.acceler. g Wave Period TwWave length Lw       Speed V in knots

           9.81                6   56.207159702                             6
           9.81                7   76.504189595                             7
           9.81                8   99.923839471                             8
           9.81                9   126.46610933                             9
           9.81               10   156.13099917                            10
           9.81               11     188.918509                            11
           9.81               12   224.82863881                            12
           9.81               13    263.8613886                            13
           9.81               14   306.01675838                            14
           9.81               15   351.29474814                            15
           9.81               16   399.69535788                            16
           9.81               17   451.21858761                            17
           9.81               18   505.86443732                            18
-Sep-90   11:51 AM
```

```
                                                   RESULTS

 Track X in deg.Product  2*PI/g*V*cosX       Encounter Period Te

             60          1.0375072336                                 7.3
             60          1.2104251059                                 8.5
             60          1.3833429782                                 9.7
             60          1.5562608505                                10.9
             60          1.7291787227                                12.1
             60           1.902096595                                13.3
             60          2.0750144673                                14.5
             60          2.2479323395                                15.7
             60          2.4208502118                                16.9
             60          2.5937680841                                18.1
             60          2.7666859564                                19.3
             60          2.9396038286                                20.6
             60          3.1125217009                                21.8
-Sep-90   11:52 AM
```

Roll Motion


Encounter Period Te in sec
23.5
23
22.5
22
21.5
21
20.5
20
19.5
19
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
Speed in knots
Encounter Period Te

6.7. Results of the references.

The examples included in this chapter have been designed by the author.

6.8. Comparisons and critical remarks.

The graph in page 160 is obtained from the following data:

Ship track (course) X = 60 degrees
Ship speed V varies from 6 to 23 knots, step = 1 knot
Wave period Tw = 18 seconds
Gravity acceleration g = 9.81 m / s^2.

6.9. Listing of the LOTUS program.

The listing of the program is given in Appendix H.

APPENDIX H :

PROGRAM : ROLL 1 A.

```
A1: [W35] 'ROLL MOTION
B1: [W35] 'PROGRAM ROLL1A. TATY-BOUSSIANA J.L.
C1: [W35] 'WORLD MARITIME UNIVERSITY
D1: [W35] ^MALMO, SWEDEN.
A2: [W35] 'TASK 1 :
B2: [W35] ^MET ( N ) 90
A3: [W35] ' ROLL PERIOD To . Weiss formula
A5: [W35] 'INPUTS
A7: [W35] 'The coefficient f is
B7: [W35] @N(A7..A7)
A8: [W35] 'The metacentric height GM is
B8: [W35] @N(A8..A8)
A9: [W35] 'The breadth of the ship B is
B9: [W35] @N(A9..A9)
C10: [W35] ^RESULT
B12: [W35] 'The Roll Period To in seconds is
C12: (F2) [W35] +B7*B9/@SQRT(B8)
A15: [W35] 'TASK 2 :
A17: [W35] 'THE ROLL ANGLE VERSUS TIME t
A18: [W35] 'IN BEAM SEA
A20: [W35] 'INPUTS
A21: [W35] 'The roll period is
B21: [W35] @N(A21..A21)
A22: [W35] 'The wave height Hw in meters is
B22: [W35] @N(A22..A22)
A23: [W35] 'The wave length Lw in meters is
B23: [W35] @N(A23..A23)
A24: [W35] 'The wave period Tw in seconds is
B24: [W35] @N(A24..A24)
A25: [W35] 'The damping coefficient D is
B25: [W35] @N(A25..A25)
A27: [W35] 'CALCULATIONS
B29: [W35] @PI*B22/B23
B30: [W35] (1-(B21/B24)^2)^2
C30: [W35] @SQRT(B30)
B31: [W35] 4*B25^2*(B21/B24)^2
C31: [W35] @SQRT(B31)
B32: [W35] @SQRT(B30+B31)
C33: [W35] ^RESULT
B34: [W35] 'The Maximum Angle of roll is
C34: [W35] +B29/B32
D34: [W35] +C34*@PI/180
B36: [W35] 'The phase angle x is
C36: [W35] @ATAN2(C31,C30)
D36: [W35] +C36*180/@PI
A39: [W35] 'TASK 3 :
A41: [W35] 'THE ENCOUNTER PERIOD OF SHIP
A42: [W35] 'AND WAVES
B42: [W35] ^RESULTS
A44: [W35] 'Gravity acceleration g
B44: [W35] ^ Wave Period Tw
C44: [W35] ^Wave length Lw
```

```
D44: [W35] ^Speed V in knots
E44: [W35] ^Track X in degrees
F44: [W35] ^Product 2*PI/g*V*cosX
G44: [W35] ^Encounter Period Te
A46: [W35] @N(A46..A46)
B46: [W35] @N(B46..B46)
C46: [W35] +A46*B46^2/(2*@PI)
D46: [W35] @N(D46..D46)
E46: [W35] @N(E46..E46)
F46: [W35] 2*@PI*(D46/1.94384)*@COS(E46*@PI/180)/A46
G46: (F1) [W35] +B46^2/(B46-F46)
A47: [W35] +$A$46
B47: [W35] +$B$46
C47: [W35] +$C$46
D47: [W35] @N(D47..D47)
E47: [W35] +$E$46
F47: [W35] 2*@PI*(D47/1.94384)*@COS(E47*@PI/180)/A47
G47: (F1) [W35] +B47^2/(B47-F47)
A48: [W35] +$A$46
B48: [W35] +$B$46
C48: [W35] +$C$46
D48: [W35] @N(D48..D48)
E48: [W35] +$E$46
F48: [W35] 2*@PI*(D48/1.94384)*@COS(E48*@PI/180)/A48
G48: (F1) [W35] +B48^2/(B48-F48)
A49: [W35] +$A$46
B49: [W35] +$B$46
C49: [W35] +$C$46
D49: [W35] @N(D49..D49)
E49: [W35] +$E$46
F49: [W35] 2*@PI*(D49/1.94384)*@COS(E49*@PI/180)/A49
G49: (F1) [W35] +B49^2/(B49-F49)
A50: [W35] +$A$46
B50: [W35] +$B$46
C50: [W35] +$C$46
D50: [W35] @N(D50..D50)
E50: [W35] +$E$46
F50: [W35] 2*@PI*(D50/1.94384)*@COS(E50*@PI/180)/A50
G50: (F1) [W35] +B50^2/(B50-F50)
A51: [W35] +$A$46
B51: [W35] +$B$46
C51: [W35] +$C$46
D51: [W35] @N(D51..D51)
E51: [W35] +$E$46
F51: [W35] 2*@PI*(D51/1.94384)*@COS(E51*@PI/180)/A51
G51: (F1) [W35] +B51^2/(B51-F51)
A52: [W35] +$A$46
B52: [W35] +$B$46
C52: [W35] +$C$46
D52: [W35] @N(D52..D52)
E52: [W35] +$E$46
F52: [W35] 2*@PI*(D52/1.94384)*@COS(E52*@PI/180)/A52
```

```
G52: (F1) [W35] +B52^2/(B52-F52)
A53: [W35] +$A$46
B53: [W35] +$B$46
C53: [W35] +$C$46
D53: [W35] @N(D53..D53)
E53: [W35] +$E$46
F53: [W35] 2*@PI*(D53/1.94384)*@COS(E53*@PI/180)/A53
G53: (F1) [W35] +B53^2/(B53-F53)
A54: [W35] +$A$46
B54: [W35] +$B$46
C54: [W35] +$C$46
D54: [W35] @N(D54..D54)
E54: [W35] +$E$46
F54: [W35] 2*@PI*(D54/1.94384)*@COS(E54*@PI/180)/A54
G54: (F1) [W35] +B54^2/(B54-F54)
A55: [W35] +$A$46
B55: [W35] +$B$46
C55: [W35] +$C$46
D55: [W35] @N(D55..D55)
E55: [W35] +$E$46
F55: [W35] 2*@PI*(D55/1.94384)*@COS(E55*@PI/180)/A55
G55: (F1) [W35] +B55^2/(B55-F55)
A56: [W35] +$A$46
B56: [W35] +$B$46
C56: [W35] +$C$46
D56: [W35] @N(D56..D56)
E56: [W35] +$E$46
F56: [W35] 2*@PI*(D56/1.94384)*@COS(E56*@PI/180)/A56
G56: (F1) [W35] +B56^2/(B56-F56)
A57: [W35] +$A$46
B57: [W35] +$B$46
C57: [W35] +$C$46
D57: [W35] @N(D57..D57)
E57: [W35] +$E$46
F57: [W35] 2*@PI*(D57/1.94384)*@COS(E57*@PI/180)/A57
G57: (F1) [W35] +B57^2/(B57-F57)
A58: [W35] +$A$46
B58: [W35] +$B$46
C58: [W35] +$C$46
D58: [W35] @N(D58..D58)
E58: [W35] +$E$46
F58: [W35] 2*@PI*(D58/1.94384)*@COS(E58*@PI/180)/A58
G58: (F1) [W35] +B58^2/(B58-F58)
A59: [W35] +$A$46
B59: [W35] +$B$46
C59: [W35] +$C$46
D59: [W35] @N(D59..D59)
E59: [W35] +$E$46
F59: [W35] 2*@PI*(D59/1.94384)*@COS(E59*@PI/180)/A59
G59: (F1) [W35] +B59^2/(B59-F59)
A60: [W35] +$A$46
B60: [W35] +$B$46
```

```
C60: [W35] +$C$46
D60: [W35] @N(D60..D60)
E60: [W35] +$E$46
F60: [W35] 2*@PI*(D60/1.94384)*@COS(E60*@PI/180)/A60
G60: (F1) [W35] +B60^2/(B60-F60)
A61: [W35] +$A$46
B61: [W35] +$B$46
C61: [W35] +A61*B61^2/(2*@PI)
D61: [W35] @N(D61..D61)
E61: [W35] +$E$46
F61: [W35] 2*@PI*(D61/1.94384)*@COS(E61*@PI/180)/A61
G61: (F1) [W35] +B61^2/(B61-F61)
A62: [W35] +$A$46
B62: [W35] +$B$46
C62: [W35] +A62*B62^2/(2*@PI)
D62: [W35] @N(D62..D62)
E62: [W35] +$E$46
F62: [W35] 2*@PI*(D62/1.94384)*@COS(E62*@PI/180)/A62
G62: (F1) [W35] +B62^2/(B62-F62)
A63: [W35] +$A$46
B63: [W35] +$B$46
C63: [W35] +A63*B63^2/(2*@PI)
D63: [W35] @N(D63..D63)
E63: [W35] +$E$46
F63: [W35] 2*@PI*(D63/1.94384)*@COS(E63*@PI/180)/A63
G63: (F1) [W35] +B63^2/(B63-F63)
```

CHAPTER 7 :

VOYAGE PLANNING.

7.1. Introduction.

In the previous chapters, LOTUS 1-2-3 programs have been designed in order to compute navigational and ship's stability and dynamics quantities and to enable the student to be acquainted with the use of computer in maritime education and training.

In other words, the student has been given the opportunity to take advantage of the computer capabilities

For the efficient achievement of this goal, this final chapter has been designed, gathering information from the previous chapters.

The student is requested to solve a day to day practical problem faced by deck officers on board merchant ships.

7.2. Definition of the tasks.

For the voyage, the following data for the geographical area and the ship's characteristics are given:

7.2.1. Point of departure :

Latitude L = 45 N

Longitude G = 140 E

Point of destination:

Latitude L' = 65 N

Longitude G' = 110 W

Estimated distance d = 3444.5 miles.

Ship's speed v = 18 knots.

Estimated track C = N 028 E degrees.

7.2.2. Ship environment.

Wind force w = 20 knots.

Angle of attack a = 60 degrees.

Current rate c = 1,1 knots.

Wave period Tw = 20 seconds.

Specific gravity of the sea water d1 = 1.025

The disturbances fluctuate not very fast nor are constant (see Annex 1).

7.2.3. Ship's characteristics. Bulk Carrier.

Length between perpendiculars Lpp = 268 meters.

Breadth B = 30 meters.

Surface ratio Au/Al = 1.4

Light vessel :

Displacement Wo = 19091 tons

Location of center of gravity to aft Xgo = 118.77 meters.

Location of center of gravity above keel = 12.46 meters.

7.2.3.1. Hydrostatic curves :

T	W	LCB	LCF	KB	KMl	KMt	MTC
7.60	66365	144.55	142.40	3.91	21.78	602.19	1482
7.70	67303	144.52	142.31	3.96	21.61	595.11	1485
7.80	68243	144.49	142.23	4.02	21.45	588.21	1488
7.90	69183	144.46	142.15	4.07	21.29	581.50	1491

T = mean draft in meters
W = displacement in tons
LCB=Location of Center of Buoyancy from aft perpendicular
LCF=Location of Center of Flotation from aft perpendicular
KB =Location of Center of Buoyancy above keel.
KMl=Location of Longitudinal Metacenter above keel
KMt=Location of Transverse Metacenter above keel.
MTC = Moment To change the trim 1 cm.

7.2.3.2. KN curves.

Angles of inclination (in degrees)	KN (in meters)
10	3.78
20	7.63
30	10.90
40	13.30
50	14.78
60	15.23
70	14.90
80	13.95
90	12.52

7.2.3.3. Cargo Plan.

	Weights (t)	Moments wixi	Moments wizi	Free Surface Correction . (t*m)
Dead weight	284	5254	4518	207
Fresh water	242	1418	4840	277
Fuel	5159	317271	54681	29023
Sea water	43330	6666908	476084	4152

7.2.3.4. Coefficient f = 1.6 (Roll Motion)

Breadth B = 30 m

Wave period Tw = 12 seconds

Track X = 30 degrees

Gravity acceleration g = 9.81 m/s^2

Speed V in knots

6 , 8 , 10 , 12 , 14 , 16 , 18 , 20 , 22.

7.2.4. Navigational Equipment.

The navigational equipment complies with the International Maritime Organization (IMO) requirements. See Annex 2.

7.2.4.1. TRANSIT Satellite Receiver .

Angles of elevation (in degrees)	Angles ai C.A. (in degrees)	Accuracy R68 (in meters)
0	31.5	
5		1105.3
10	22.75	515.8
20	16.75	336.8
30	12.5	284.2
40	9.25	252.6
50	6.75	273.7
60	4.75	389.5
70	3.0	600
80		1105.3

7.2.4.2. Automatic Radar Plotting Aids ARPA .

Number of plots n = 60

Plot Interval t = 3 minutes

TCPA′ = 15 minutes

Range R = 12 Miles

Ship speed = 18 knots

7.2.4.3. Star observations.

True Bearings (Azimuths)	Intercepts
309.5	-2.2
67.0	0.1
153.0	1.8
345.5	-2.3
123.9	3.2
0.8	-1.6

M95 (1 LOP) = 2 Miles.

7.3. The following quantities to be obtained from a LOTUS 1-2-3 program are required for the voyage.

Regarding the results, the attention of the student is drawn on the fact that the final answer is usually rounded off to :

- the nearest degree for track, angle of inclination
- two decimals for the latitude and longitude, then the number of decimals is multiplied by 60 to obtain the number of minutes of degrees.
- 0.1 mile for distances, accuracies
- 0.1 meter for righting arms
- 1 centimeter for drafts
- 1 ton for displacements.

7.3.1. Rhumb line and Great circle sailings.

Distances and Tracks
Gain in nautical miles
Initial track (composite sailing)
Total distance , coordinates of the Vertex.
Longitudes Gb and Gc (limiting parallel)
Limiting parallel Lmax = 67 N

7.3.2. Navigational Errors. Accuracies.

R 95 DR Position (R 95 MPP = 0.5 M , t = 2 h)
Time allowed for Dead Reckoning for dR 95 = 2 M
R95 DR = 4 M
Safe Distance from the danger (t = 2 h).
R 95 PPC (Possible Point of Collision)

7.3.3. Satellite Navigation.

Graph R 95 = f (EL)
Graph tobs = f (EL)

7.3.4. **Celestial Navigation.**

Location of the MPP (using matrix)
Geometric Dilution of Accuracy GDOA
R95 MPP.
Fix from the DR.

7.3.5. Stability at large angles.

Metacentric height GM
Center of gravity of the ship (Xg,Zg)
Aft and fore drafts
Graph Righting Arm GZ = f (0)

From the GZ curve obtained, specify the following:

Maximum GZ
Metacentric height GM
Angle of heel (weight w = 1000t is shifted from port to starboard d = 10 m)
Angle of maximum GZ
Vanishing angle Of (GZ = 0)
Range of stability
Dynamic stabitlity
Area under the curve of stability.

Discuss about the stability of the ship for loaded condition referring to the IMO requirements. Is the ship in compliance with the safety regulations ? See Annex 2.

7.3.5. Roll Motion.

Roll period of the ship To

Graph Encounter period versus speed.

Actions to be undertaken in case of Mathieu resonance.

CONCLUSION.

The results of the method presented in this paper demonstrate its appropriateness for calculations related to navigation, ship's stability and dynamics.

Image generation and decision making are not features of the proposed software package.

Research and development continue towards improving the capabilities of the existing computer systems with most attention being directed to the development of the new generation of " intelligent " computers.

Knowledge based systems (expert systems) on board ships are a new challenge for those who, by the nature of their vocation , have to transfer knowledge to the new generation of highly trained deck officers and marine engineers.

In a didactical point of view, the best way of knowledge transfer is to start with a very simple and comprehensive method.

The author hopes that this goal has been efficiently achieved for the training of the future decision makers in the maritime field in general, and the students of the emergent Académie Régionale des Sciences et Techniques de la Mer d'Abidjan (Côte d'Ivoire), in particular.

ANNEX 1.

1.2 IMO "ACCURACY STANDARDS FOR NAVIGATION".

Annex 1.

In this paragraph the IMO Resolution A.529 (13) is presented in its original form. (Ref.1)
The importance of these Standards is emphasized in the previous chapter 1.1. and throughout this book these standards principaly will be used and referred to.
An explanation of the theoretical backgrounds of these practical standards is given in paragraph 3.11 and there also will be presented the possibilities to estimate the DR-accuracies under conditions which differ from the average values for ship and environment, which are used in the IMO Accuracy Tables.

In paragraph 12.2 the proof of the validity of the formula for the accuracy in DR will be given. A comparison between this formula and other empirical formalae for the same purpose, which were produced for statistical analysis, will also be given. The comparison will show that the differences between various formulae sprout from differences in ship's and environmental parameters.

INTERNATIONAL MARITIME
ORGANIZATION
ASSEMBLY - 13TH session
Agenda item 10(b)

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RESOLUTION A.529(13)
adopted on 17 November 1983

ACCURACY STANDARDS FOR NAVIGATION

THE ASSEMBLY,

RECALLING Article 16(j) of the Convention on the International Maritime Organization concerning the functions of the Assembly in relation to regulations concerning maritime safety,
RECOGNIZING the need to provide guidance to Administrations on the standards of navigation accuracy for assessing position-fixing system,
HAVING CONSIDERED the recommendation made by the Maritime Safety Committee at its forty-eight session,

1. ADOPTS the navigation accuracy standards set out in the Annex to the present resolution;
2. RECOMMENDS Member Governments to use these accuracy standards as guidance when assessing the performance of position-fixing system.

ANNEX
ACCURACY STANDARDS FOR NAVIGATION

1 Introduction

1.1 The objective is to provide guidance to Administrations on the standards of navigation accuracy for assessing position-fixing systems, in particular radio-navigation systems, including satellite systems. These standards do not apply to specialized activities such as offshore exploration or those performed by the hydrographic services.

1.2 The navigator needs to be able to determine his position at all times. This requires accurate position fixes and, where position fixed are not available continuously, a method of estimating the position between fixes which may be dead reckoning.

2. Factors affecting accuracy requirements

2.1 Accuracy requirements depend upn various factors which include:

.1 ship speed; and

.2 distance from nearest navigational danger. A navigational danger is considered to be any recognized feature or charted feature or boundary which might present or encompass a hazard to the ship or prescribe a limit to navigation.

3. Phases of the voyage

3.1 The phases can be divided into:

.1 harbour entrances and approaches and waters in which the freedom to manoeuvre is limited; and

.2 other waters

3.2. In the first phase navigation will be generally by visual observations, radar, echo sounder, etc. or by specialized electronic positioin-fixing system. Accuracy requirements will depend upon local circumstances. The division between the two phases is not precise and will depend upon local circumstances.

4. Definition of accuracy standards

4.1 The following table sets out the accuracy standards against which position-fixing systems can be assessed in respect of a ship proceeding at a speed of not more than 30 knots.

TABLE 1

Phase of the voyage	Order of accuracy
Harbour entrances, etc.	Depends on local circumstances
Other waters	4% of distance from danger with a maximum of 4 nautical miles

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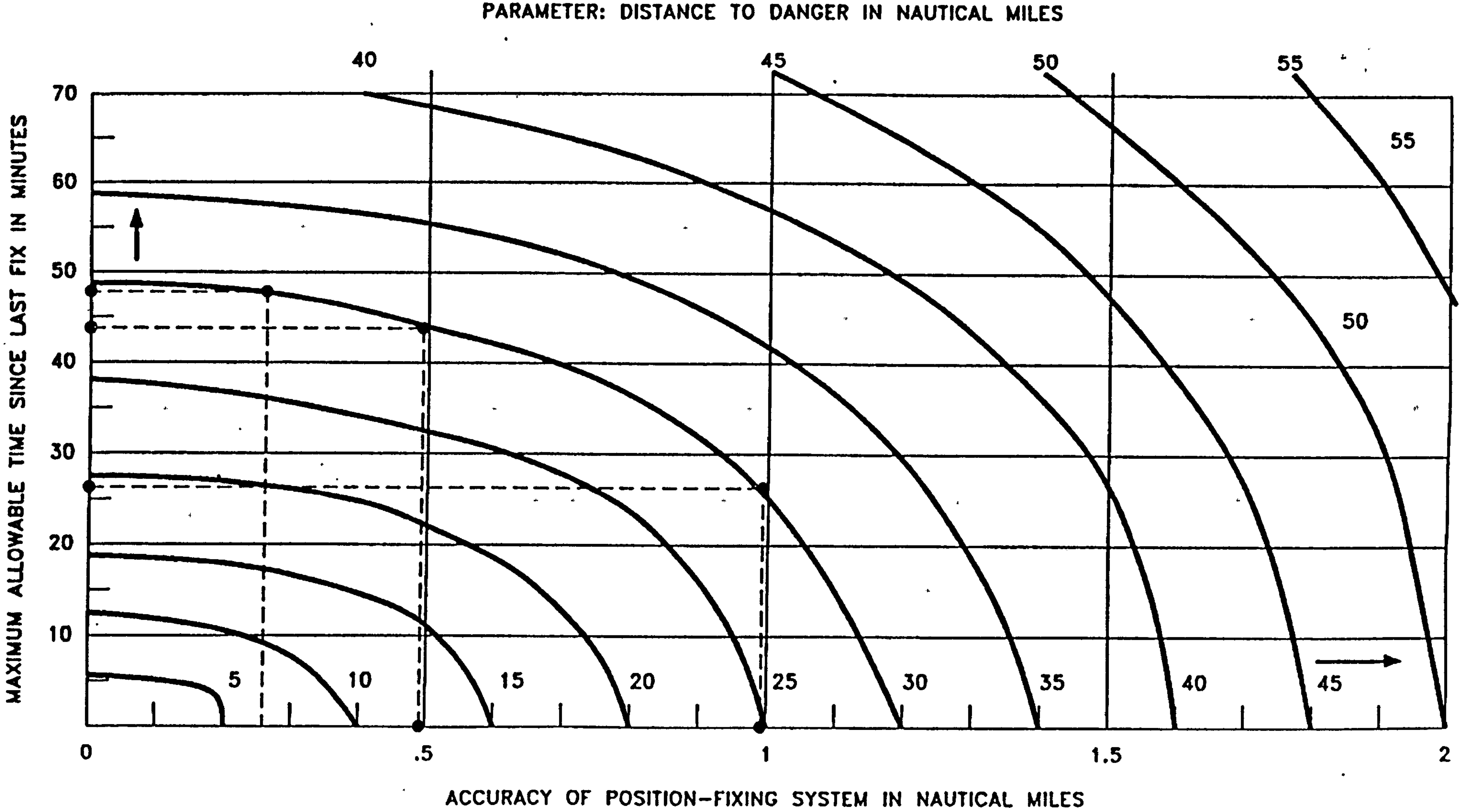
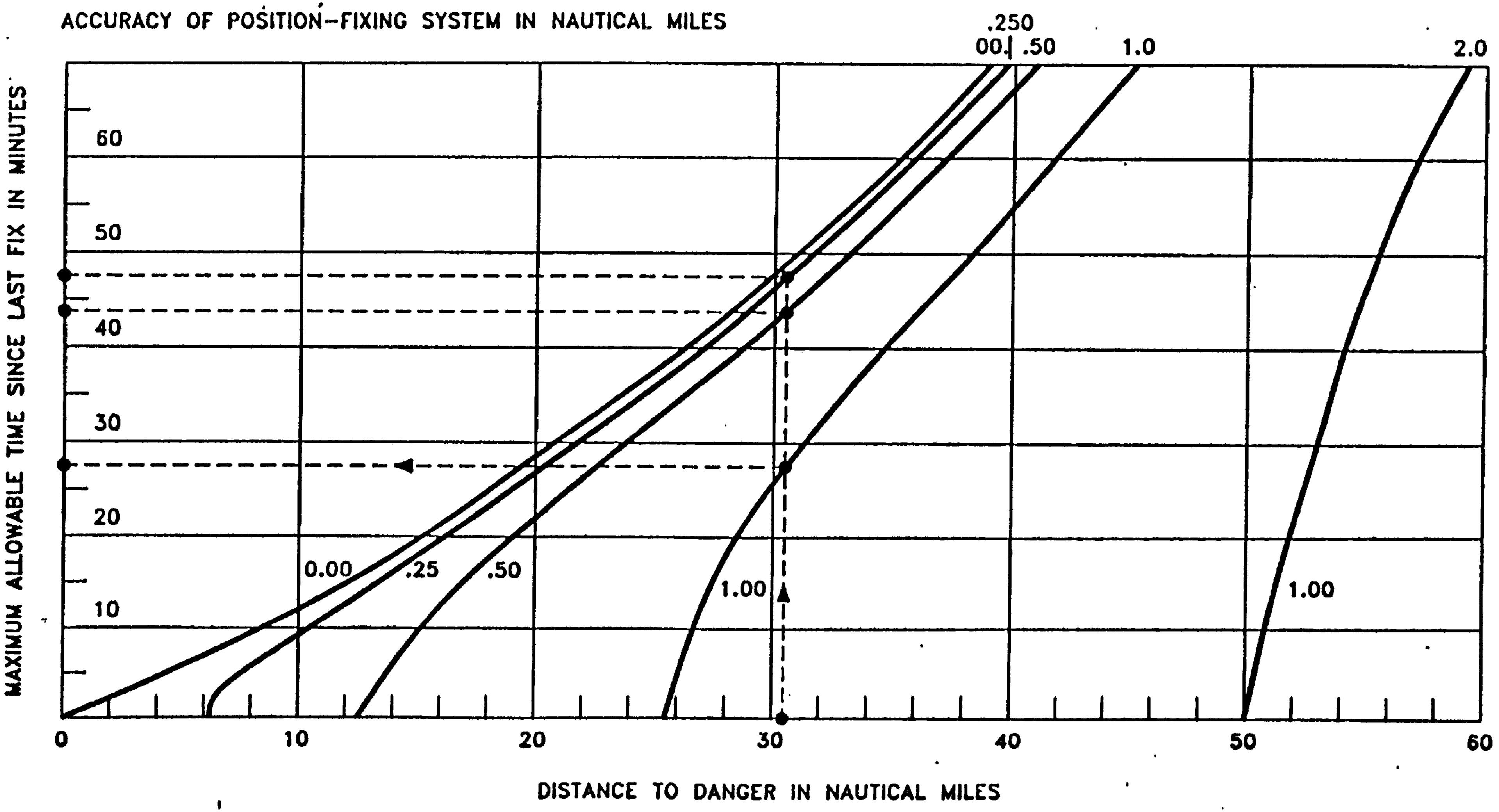
MINIMUM DISTANCE FROM DANGER (n.m.)	ACCURACY REQUIRED (n.m.)	ACCURACY OF POSITION-FIXING SYSTEM (n.m.)					
		0	0.1	0.25	0.5	1	2
		MAXIMUM ALLOWABLE TIME *SINCE* LAST FIX (MIN)					
10	0.4	12	12	9	-	-	-
20	0.8	28	28	27	22	-	-
30	1.2	48	48	47	44	27	-
40	1.6	72	72	71	68	56	-
50	2.0	100	100	99	97	87	0
60	2.4	132	132	131	129	120	73
70	2.8	168	168	167	165	157	118
80	3.2	208	208	207	206	198	162
90	3.6	252	252	251	250	242	210
100 or more	4.0	300	300	300	298	291	260


PARAMETER: DISTANCE TO DANGER IN NAUTICAL MILES
40
45
50
55
55
50
MAXIMUM ALLOWABLE TIME SINCE LAST FIX IN MINUTES
70
60
50
40
30
20
10
5
10
15
20
25
30
35
40
45
0
.5
1
1.5
2
ACCURACY OF POSITION-FIXING SYSTEM IN NAUTICAL MILES



ACCURACY OF POSITION-FIXING SYSTEM IN NAUTICAL MILES
.250
00.
.50
1.0
2.0
MAXIMUM ALLOWABLE TIME SINCE LAST FIX IN MINUTES
60
50
40
30
20
10
0.00
.25
.50
1.00
1.00
0
10
20
30
40
50
60
DISTANCE TO DANGER IN NAUTICAL MILES

ANNEX 2.

SHIPS IN GENERAL

Structural Strength and Stability

2.—(1) The construction of the ship shall be such that her general structural strength will be sufficient for the freeboards to be assigned to her.

(2) The design and construction of the ship shall be such as to ensure that her stability in all probable loading conditions will be sufficient for the freeboards to be assigned to her, and for this purpose regard shall be had, in addition to the intended service of the ship and to any relevant requirements of Rules made under the Merchant Shipping (Safety Convention) Act 1949 (a) and the Merchant Shipping Act 1964 (b), to the following criteria:—

(a) The Area under the curve of Righting Levers (GZcurve) shall not be less than-
0·055 metre-radians up to an angle of 30 degrees:

(b) The Righting Lever (GZ) shall be at least 0·20 metres at an angle of heel equal to or greater than 30 degrees.

(c) The maximum Righting Lever (GZ) shall occur at an angle of heel not less than 30 degrees.

(d) The initial transverse metacentric height shall not be less than 0·15 metres. In the case of a ship carrying a timber deck cargo which complies with sub-paragraph (a) by taking into account the volume of timber deck cargo the initial transverse metacentric height shall not be less than 0·05 metres.

(3) To determine whether the ship complies with the requirements of sub-paragraph (2) the ship shall, unless the Board otherwise permit, be subjected to an inclining test carried out in the presence of a surveyor appointed by the Board, and the Board shall notify the Assigning Authority whether or not they are satisfied that the ship complies with those requirements.

Extract of safety regulations on the intact stability

ANNEX 3.

Annex 3.

MEMORANDUM TO ADMINISTRATIONS ON AN APPROXIMATE DETERMINATION OF SHIP'S STABILITY BY MEANS OF THE ROLLING PERIOD TESTS

(for ships up to 70 m in length)

1. Recognizing the desirability of supplying to masters of small ships instructions for a simplified determination of initial stability, attention was given to the rolling period tests. Studies on this matter have now been completed with the result that the rolling period test may be recommended as a useful means of approximately determining the initial stability of small ships when it is not practicable to give approved loading conditions or other stability information, or as a supplement to such information.

2. Investigations comprising the evaluation of a number of inclining and rolling tests according to various formulae showed that the following formula gave the best results and it has the advantage of being the simplest:

$$GM_o = (\frac{fB}{T_r})^2,$$

Where:

f = factor for the rolling period/rolling coefficient (different for feet and metric system)

B = breadth of the ship in feet or metric units

T_r = time for a full rolling period in seconds (i.e. for one oscillation "to and fro" port – starboard – port, or vice versa).

3. The factor "f" is of the greatest importance and the data from the above tests were used for assessing the influence of the distribution of the various masses in the whole body of the loaded ship.

4. For coasters of normal size (excluding tankers), the following average values were observed:

	metric system	feet system
(a) Empty ship or ship carrying ballast	f ~ 0.88	f ~ 0.49
(b) Ship fully loaded and with liquids in tanks comprising the following percentage of the total load on board (i.e. cargo, liquids, stores, etc.)		
1. 20 per cent of total load	f ~ 0.78	f ~ 0.435
2. 10 per cent of total load	f ~ 0.75	f ~ 0.415
3. 5 per cent of total load	f ~ 0.73	f ~ 0.405

The stated values are mean values. Generally, observed f-values were within ±0.05 of those given above.

5. These f-values were based upon a series of limited tests and, therefore, Administrations should [illegible] these in the light of any different circumstances applying to their own ships.

6. It must be noted that the greater the distance of masses from the rolling axis, the greater the rolling coefficient will be.

Therefore it can be expected that:

The rolling coefficient for an unloaded ship, i.e. for a hollow body, will be higher than that for a loaded ship.

The rolling coefficient for a ship carrying a great amount of bunkers and ballast – both groups are usually located in the double bottom, i.e. far away from the rolling axis – will be higher than that of the same ship having an empty double bottom.

7. The above recommended rolling coefficients were determined by tests with vessels in port and with their consumable liquids at normal working levels; thus, the influences exerted by the vicinity of the quay, the limited depth of water and the free surfaces of liquids in service tanks are covered.

8. Experiments have shown that the results of the rolling test method get increasingly less reliable the nearer they approach GM-values of 0.20 m and below.

9. For the following reasons, it is not generally recommended that results be obtained from rolling oscillations taken in a seaway:

(a) Exact coefficients for tests in open waters are not available.

(b) The rolling periods observed may not be free oscillations but forced oscillations due to seaway.

(c) Frequently, oscillations are either irregular or only regular for too short an interval of time to allow accurate measurements to be observed.

(d) Specialized recording equipment is necessary.

10. However, sometimes it may be desirable to use the vessel's period of roll as a means of approximately judging the stability at sea. If this is done, care should be taken to discard readings which depart appreciably from the majority of other observations. Forced oscillations corresponding to the sea period and differing from the natural period at which the vessel seems to move should be disregarded. In order to obtain satisfactory results, it may be necessary to select intervals when the sea action is least violent, and it may be necessary to discard a considerable number of observations.

11. In view of the foregoing circumstances, it needs to be recognized that the determination of the stability by means of the rolling test in disturbed waters should only be regarded as a very approximate estimation.

12. The formula given in paragraph 2 above can [illegible]educed to:

$$GM_o = \frac{F}{T_r^2}$$

and the Administration should determine the F-value(s) for each vessel.

13. The determination of the stability can be simplified by giving the master permissible rolling periods, in relation to the draughts, for the appropriate value(s) of F considered necessary.

14. The initial stability may also be more easily determined graphically by using one of the attached sample nomograms for feet and/or metric units as described below:

(a) The values for B and f are marked in the relevant scales and connected by a straight line (1). This straight line intersects the vertical line (mm) in the point (M).

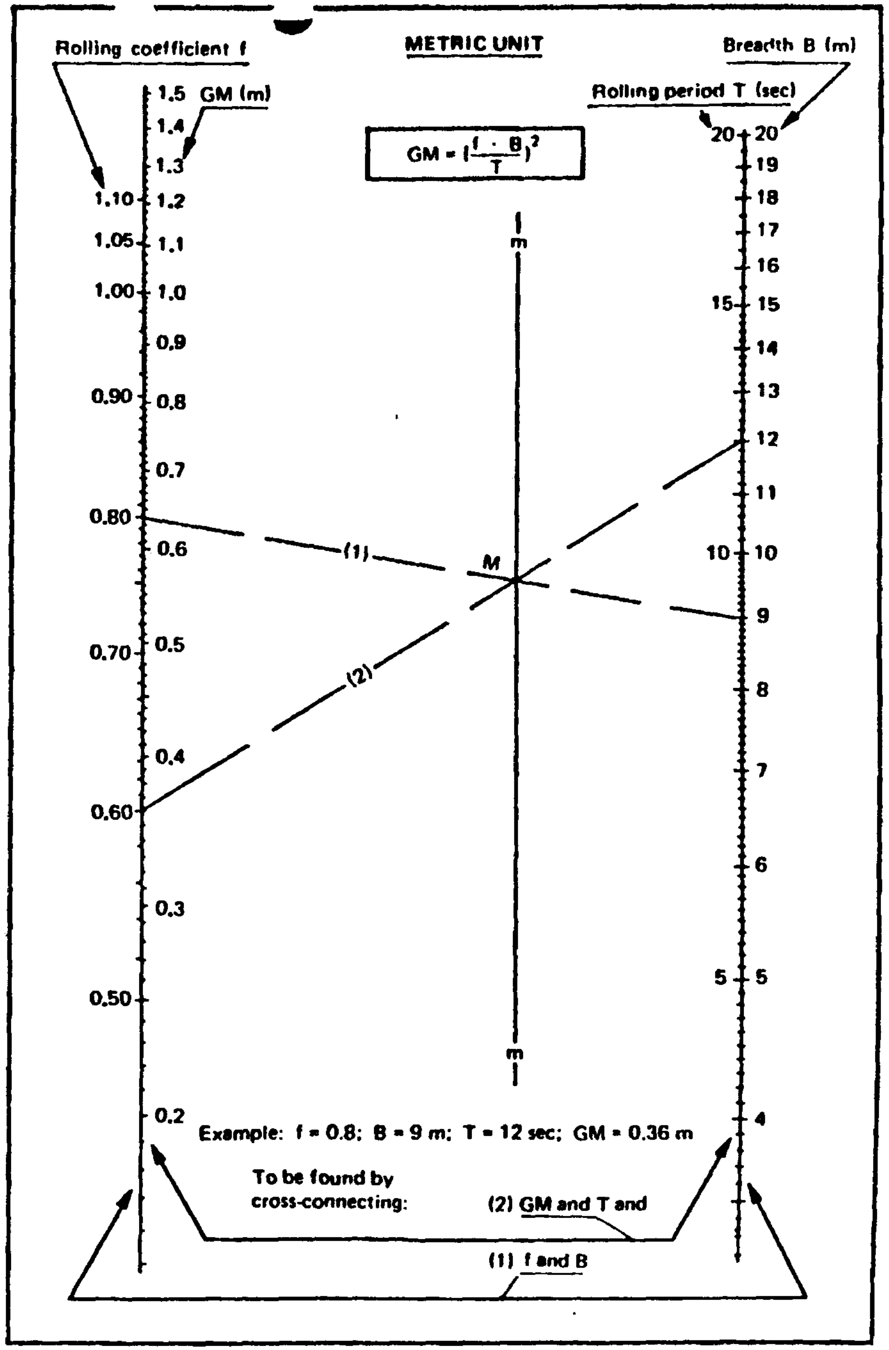
(b) A second straight line (2) which connects this point (M) and the point on the T_r scale corresponding with the determined rolling period, intersects the GM scale at the requested value.

15. The Annex to Appendix III shows an example of a recommended form in which these instructions might be presented by each Administration to the masters. It is considered that each Administration should recommend the F-value or values to be used.

RECOMMENDATION ON INTACT STABILITY FOR PASSENGER AND CARGO SHIPS UNDER 100 METRES IN LENGTH

as amended with respect to ships carrying deck cargoes

LONDON 1981
Reprinted 1983
(First edition 1975)



ANNEX 4.

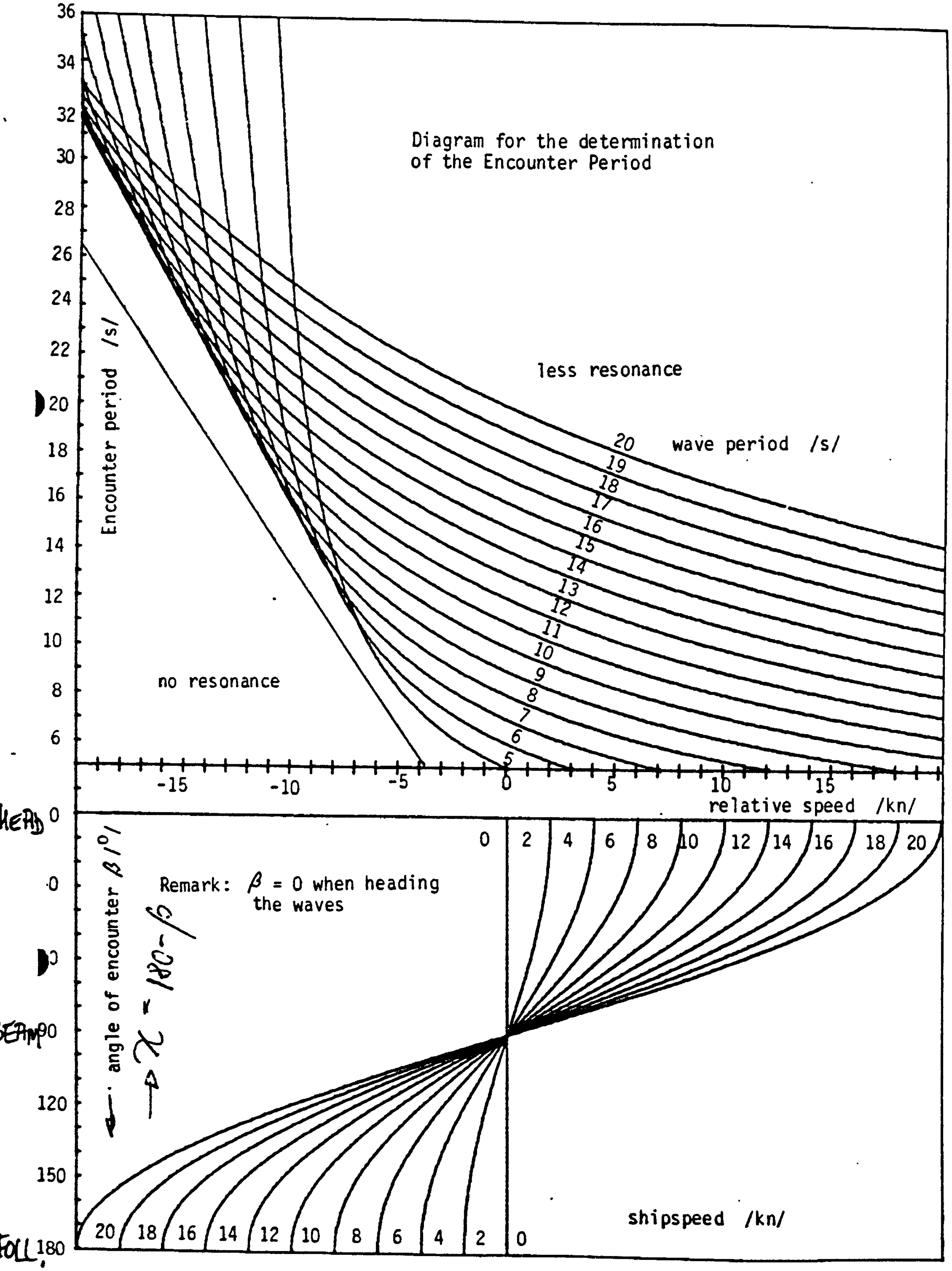


Diagram for the determination of the Encounter Period

ANNEX 5.

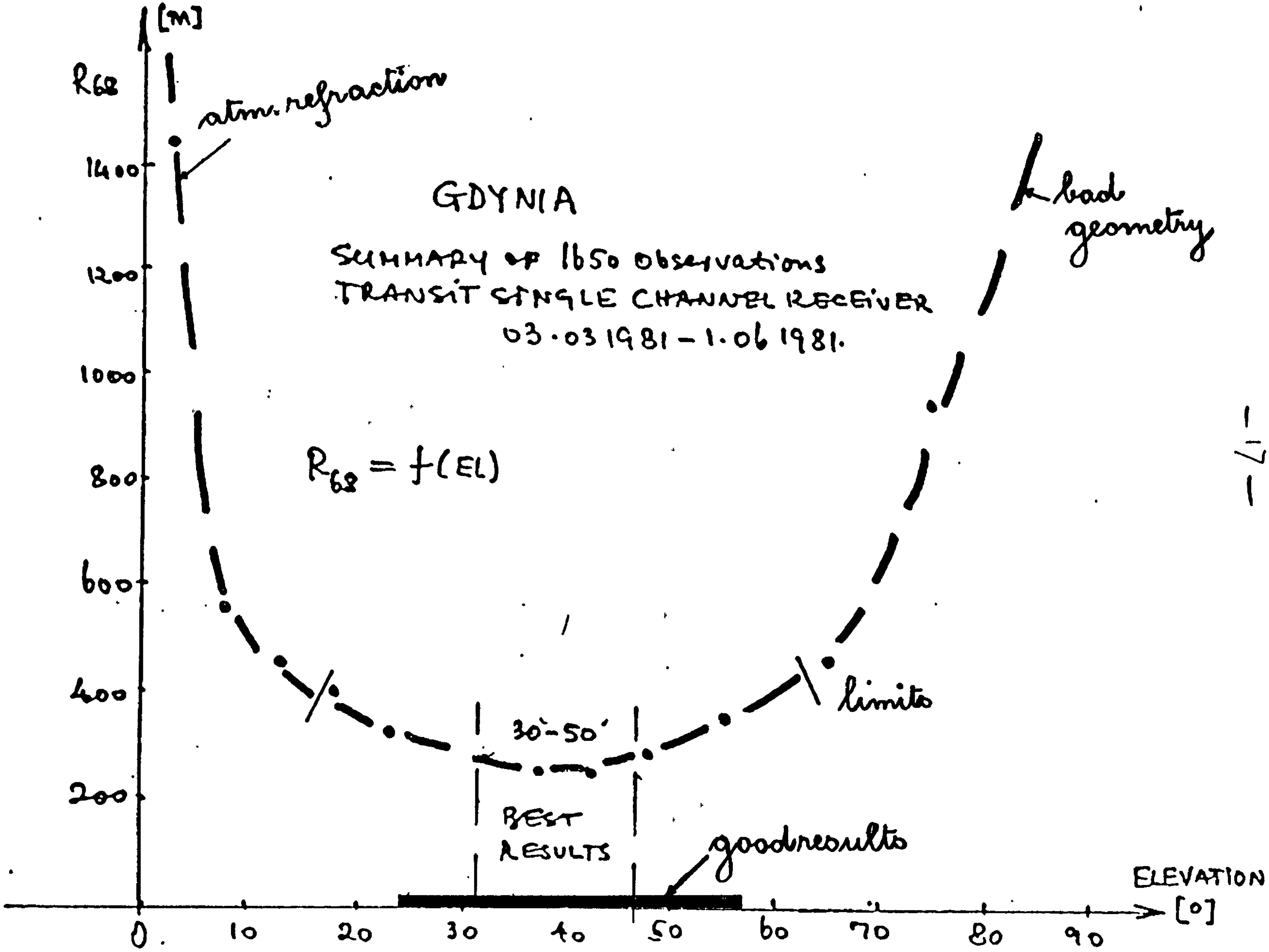


FIG 12. TRANSIT POSITION R_{68}-ERROR AGAINST ELEVATION OF SATELLITES.

— REFERENCES —

AND

BIBLIOGRAPHY.

I. NAVIGATION.

1. Admiralty Manual of Navigation, Vol.1 : General Navigation, Coastal Navigation and Pilotage, Her Majesty's Stationary Office, London, 1987.

2. Appleyard S.F., Linford R.S. and Yarwood P.J.: Marine Electronic Navigation, Routledge and Kegan Paul Inc., New York, 1988.

3. Bowditch N., American Practical Navigator, Vol. 1 and 2, Defence Mapping Agency Hydrographic Topographic Center, Washington, 1984.

4. Cours de Navigation, Ecole Nationale de la Marine Marchande, Le Havre, 1986.

5. Jurdzinski M., Satellites for Navigation, World Maritime University, Malmo, 1990.

6. Mulders J.H., Introduction in Satellites Systems, World Maritime University, Malmo, 1990.

7. Mulders J.H., Error Analysis of Position Fixing . Methods and Systems, World Maritime University, Malmo,1990.

8. Sonnenberg G.J., Radar and Electronic Navigation, Butterworths and Co. Ltd., University Press Cambridge, U.K., 1988.

9. Tetley T. and Calcutt D., Electronic Aids to Navigation, Edward Arnold Ltd., London, 1986.

II. SHIP'S STABILITY AND DYNAMICS.

10. Bhattacharyya R., Dynamics of Marine Vehicles, John Wiley and Sons Ltd., New York, 1978.

11. Cours d'Assiette, Stabilité et Fatigues de Coque, Ecole Nationale de la Marine Marchande, Le Havre, 1986.

12. Derett D.R., Ship's Stability for Master and Mates, 4th edition, London, 1987.

13. Gillmer T.C. and Johnon B., Introduction to Naval Architecture, E.and F.N. Spon.,London, 1982.

14. Kastner S., Ship Dynamics and Safe Transport of Cargo, World Maritime University, Malmo, 1989.

15. The Society of Naval Architects and Marine Engineers, Principles of Naval Architecture, Vol.1, 2 and 3., 2nd revision, Edward V. Lewis, 1988.

III. LOTUS 1-2-3

16. Bookbinder D.J., The LOTUS Guide to 1-2-3- Release 3, Addison Wesley Publishing Company, Inc., New York, 1989.

17. Campbell M., 1-2-3 Release 3, The complete reference, Osborne McGraw Hill, Berkeley, California USA, 1989.

18. Hans van Walen, Computers in Maritime Education and Training, World Maritime University, Malmo, 1990.

19. Metzelaar L.C. and Fox M.B., Hands-on MS DOS, Word Perfect, dBase III Plus, LOTUS 1-2-3, The Benjamin Cummings Publishing Company, Menlo Park, California, USA, 1988.

20. Napier H.A. and Judd P.J., Mastering and Using LOTUS 1-2-3 Release 3.,Boyd and Fraser Publishing Company, 1990.

IV. ENGINEERING MATHEMATICS.

21. Angot A., Compléments de Mathématiques à l'usage des Ingénieurs de l'Electrotechnique et des Télécommunications, Masson et Cie, Paris, 1972.

22. Chossat M., Mathématiques de l'Ingénieur, Dunod, Paris, 1977.

23. Kreyzig E., Advanced Engineering Mathematics, John Wiley and Son's, New York, 1988.

24. Quinet E., Cours Elémentaire de Mathématiques Supérieures , Vol. 3 et 5, Dunod, Paris, 1980.

25. Tuma J.J., Engineering Mathematics Handbook, 3rd edition McGraw Hill Book Company, New York, 1987.