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The Works of Omar Khayyam in the History of Mathematics

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Abstract: The exact time when the mathematician Omar Khayyam lived is not well-defined, but it is generally agreed upon that he lived from the end of the 11th century to the beginning of the 12th century C.E. in Nishapur, which is in modern-day Iran and Afghanistan (Struik, 1958). Other than mathematics, Omar Khayyam also made considerable contributions to other fields, such as astronomy, philosophy, and poetry (Struik, 1958). He is probably most famous for his poem titled *Rubaiyat of Omar Khayyam*, which was translated by Edward Fitzgerald (Struik, 1958). Although famous for his poetry, he was professionally inclined to astronomy and mathematics. In mathematics, he is well-known for being the first individual to find positive root solutions to multiple cubic equations, and he is also known for furthering understanding of the parallel axiom (Eves, 1958, p. 285; Struik, 1958). In this report, details of Omar Khayyam's life will be mentioned, but the focus will be on his contributions to mathematics and his role in the history of mathematics.

Keywords: Omar Khayyam; cubic equations; geometric solutions

Of many talents, Omar Khayyam was exceptional in mathematics. Omar Khayyam's lifetime is not well-defined, but the general consensus is that he lived sometime between 1040 and 1131 C.E. (Struik, 1958). His full name was Abu'l-Fath Umar ibn Ibrahim Khayyam, and it indicates that either he or his family were tent-makers (Struik, 1958; Joseph, 2011). Moreover, since only Sunni Muslims used the name Omar, his family likely followed that branch of Islam (Richardson, 2016). He lived in Nishapur in the province of Khurasan, which is now part of Afghanistan and Iran (Joseph, 2011). Nishapur was the capital of Khurasan, and Khurasan was northeast of what was then Persia (Joseph, 2011; Richardson, 2016). Omar Khayyam's lifetime overlapped with the high point of the Turkish Seljuq era (Richardson, 2016). The Seljuqs were an Islamic dynasty governed by sultans and are currently noted as being underrecognized (Richardson, 2016). Prior to the Seljuq era, Islamic rule, initiated by the prophet Muhammad, over a vast territory in what is currently considered the middle east began to be established in about 630 C.E. (Joseph, 2011). From this conquest, two dynasties, the Abbasid and Umayyad, arose, but in 750 C.E., the Abbasid dynasty gained power of the territory (Joseph, 2011). Deputies of the prophet Muhammad, known as caliphs, ruled both dynasties, but upon gaining power, the Abbasid caliphs reigned (Joseph, 2011). However, one dynasty having complete power over such a large area proved difficult to handle. In 1038 C.E., near the time of Omar Khayyam's birth, the first Seljuq, Tughril, captured the city of Nishapur (Richardson, 2016). Then, in 1064, Alp Arslan became sultan of the Seljuq dynasty (Richardson, 2016). Alp Arslan was a master of archery, and he was sure to acknowledge it (Richardson, 2016). He was known for being proud of his archery skills, and in 1074, a prisoner threatened the proud master with a knife in front of thousands of

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people (Richardson, 2016). Alp Arslan's guards immediately pursued the prisoner, but the sultan commanded them to withdraw (Richardson, 2016). The sultan instead decided to execute the prisoner himself with his bow and arrow in front of the crowd (Richardson, 2016). While aiming the arrow at the prisoner, his foot slipped, and the arrow did not strike the prisoner (Richardson, 2016). The prisoner caught up with the sultan, and Alp Arslan was executed that day instead (Richardson, 2016). Alp Arslan's son, Malik-Shah, then inherited the dynasty. The Seljuq court at that time was a military court, and much of its resources were directed towards the army (Richardson, 2016). Due to the urgency of the military's needs, intellectual pursuits were likely considered less significant. However, Malik-Shah still supported the arts, science, and learning in general, and he did what he could to support these disciplines with the limited resources he could direct to those areas (Richardson, 2016). In 1074, Malik-Shah established an observatory in his favorite city, Isfahan, and there, he started a project to reform the calendar (Richardson, 2016). Omar Khayyam was invited by Malik-Shah to work at the observatory as a member of a group of leading mathematicians and astronomers (Richardson, 2016; Khayyam, 1896). Khayyam likely became acquainted with Malik-Shah through Malik Shah's vizier, Nizam ul Mulk. Khayyam and Nizam ul Mulk shared the same mentor, Imam Mowaffak (Khayyam, 1935). A universal belief at that time was that anyone who was a pupil of Imam Mowaffak would attain honor and happiness (Khayyam, 1935). Another pupil of Mowaffak was Hasan Ben Sabbah (Khayyam, 1935). One day, Hasan mentioned to Khayyam and Nizam ul Mulk the common belief that Imam Mowaffak's pupils were almost guaranteed good fortune, and he asked the other two pupils to make a pact with him that whomever of the three received this reputed fortune would share it equally with the other two pupils (Khayyam, 1935). Khayyam and Nizam ul Mulk both agreed, and the pact was made (Khayyam, 1935). The rumored fortune did in fact come to one of the three pupils: Nizam ul Mulk (Khayyam, 1935). Nizam ul Mulk became vizier to Alp Arslan and later to Malik-Shah, and he was well compensated (Khayyam, 1935). Hasan discovered Nizam ul Mulk's new position and asked if Nizam ul Mulk would keep his vow (Khayyam, 1935). The vow was kept, and Hasan was granted a position in the government (Khayyam, 1935). However, Hasan was not able to maintain the position, and he then resorted to a darker path (Khayyam, 1935). He became an assassin and was known among the crusaders as 'Old Man of the Mountains' (Khayyam, 1935). Eventually, he was even responsible for the death of Nizam ul Mulk (Khayyam, 1935). While the vizier lived though, he was also approached by Omar Khayyam, but unlike Hasan, Khayyam did not seek power and wealth (Khayyam, 1935). Khayyam asked merely for a living wage, so he could devote his time to science (Khayyam, 1935). The vizier appreciated Khayyam's sincerity, and Khayyam was granted a yearly pension (Khayyam, 1935). Omar was thus able to pursue his intellectual interests when having such an opportunity was rare. His talents were clearly noticed though, and he later accepted the offer from Malik-Shah to work at the new observatory in Isfahan (Khayyam, 1935; Richardson, 2016). Throughout the time he worked at the observatory, he became well-acquainted with Malik-Shah and his court (Richardson, 2016). In fact, he was known for being a companion and 'drinking buddy' of the sultan (Richardson, 2016). At the observatory, he compiled astronomical tables titled *Ziji-Malikshahi* and contributed to the calendar reform (Khayyam, 1935). Also, with great precision, Omar Khayyam measured the length of the solar year to be 365.2424 days (Richardson, 2016). After four years of working on the reform, the new calendar was introduced in 1079, and Malik-Shah announced that time as the Jalalian Era (Richardson, 2016). The calendar composed was more accurate than the calendar that would be proposed centuries later by Pope Gregory XIII (Struik, 1958). Aside from being an astronomer and mathematician, Omar Khayyam was also a philosopher and poet (Struik, 1958). After the calendar was completed, he returned to Nishapur, and in 1083, he wrote a treatise in philosophy about being and necessity (Richardson, 2016; Struik, 1958). As a philosopher, he followed Aristotle, but unlike many philosophers, he did not seem to find contemplating possibilities worthwhile (Struik, 1958). Instead, he focused on what was real, concrete, and could be known, and this was expressed in his poetry. Currently, Omar Khayyam is probably best known for his poem, *The Rubaiyat of Omar Khayyam*, which was translated by Edward Fitzgerald (Struik, 1958). In it, he notably expressed the brevity of life. For example, in one verse he stated,

"Ah, make the most of what we yet may spend,

Before we too into the Dust descend;

Dust into Dust, and under Dust, to lie,

Sans Wine, sans Song, sans Singer and -sans End!" (Khayyam, 1935, p. 14).

He later exclaimed,

"Why, all the Saints and Sages who discuss'd

Of the two Worlds so learnedly, are thrust

Like foolish Prophets forth; their Words to Scorn

Are scatter'd, and their mouths are stopt with Dust.

Oh, come with old Khayyam, and leave the Wise

To talk; one thing is certain, that Life flies;

One thing is certain, and the Rest is Lies;

The Flower that once has blown for ever dies." (Khayyam, 1935, p. 14).

With his insightful reflections in addition to being knowledgeable of mathematics and astronomy, it is no wonder why Malik-Shah wanted Omar Khayyam at the forefront of the calendar reform. During a time when scholarly pursuits were only possible in special circumstances, Omar Khayyam was able to have a career in astronomy, and through philosophy and poetry, he was also able to reflect on what it actually means to be alive and whether it means anything at all. However, his reflections in poetry and philosophy and his leading work in astronomy were only fractions of how he expanded human knowledge and insight. He also made groundbreaking contributions to mathematics. In his placement in the history of mathematics, Omar Khayyam was the first to geometrically solve, in terms of positive roots, multiple cubic equations, and he also furthered understanding of the parallel axiom (Eves, 1958; Struik, 1958).

In 1070, Omar Khayyam travelled to Samarcand to write a treatise on cubic equations (Richardson, 2016). In his book *Al-jabr w'al-muqabala*, which is often instead referred to as *Algebra*, Omar Khayyam addressed how to geometrically find positive root solutions to cubic equations (Joseph, 2011; Struik, 1958). In *Algebra*, Omar Khayyam extended the methods in Euclid's *Elements* of finding second-degree equations (Struik, 1958). Euclid was able to find the positive solutions to second-degree equations using a compass and straight-edge (Struik, 1958).

That is, quadratic equations represented problems of areas, so the equation $x^2 - ax - b = 0$ was interpreted as finding a square with side x that, when added to a given area b, would yield a rectangle with a given side a and another side equal to x (Struik, 1958). For cubic equations, however, Omar could not resort only to the two-dimensional world of compasses and straight-edges. He had to find his solutions through intersections of conics (Struik, 1958). That is, instead of circles, he had to use hyperbolas to find solutions to cubic equations. As an example of how Omar Khayyam found positive root solutions to cubic equations, consider the equation $x^3 + b^2x + a^3 = cx^2$, where a, b, and c = known values of line segments. First, a line segment z such that b:a = a:z must be found. Next, a line segment m such that b:z = a:m needs to be found. We can see that,

$b/a = a/z$

$z(b/a) = a$

$z = a^2/b$.

We also see that with $b/z = a/m$, $z = a^2/b$, so

$b^2/a^2 = a/m$

$m(b^2/a^2) = a$

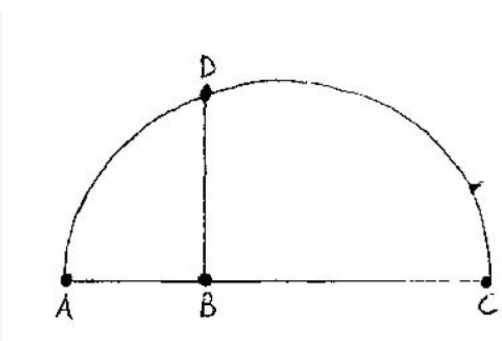
$m = a^3/b^2$.

Then, we construct a line $AB = m = a^3/b^2$ and a line BC = c (**Figure 1**).

Figure 1. Line AC (not drawn to scale)

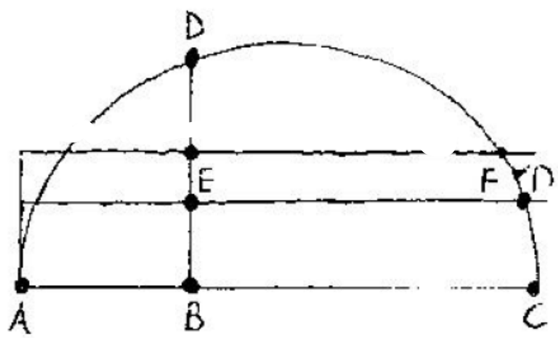
A semicircle with line AC as its diameter is then drawn, and a line perpendicular to line AC at point B is also drawn. This line cuts the semicircle at point D (**Figure 2**).

Figure 2. Semicircle with Line AC as its Diameter (not drawn to scale)



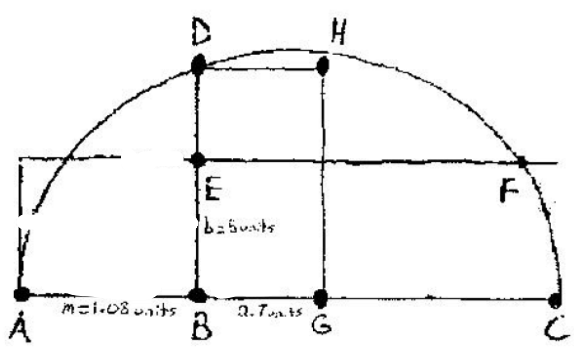
A line parallel to AC through line BD is then drawn such that the point where this line cuts line BD is labelled E, and line BE is equal to b (**Figure 3**) Also, the point where line EF cuts the semicircle is labelled point F.

Figure 3. Semicircle with Line EF (not drawn to scale)



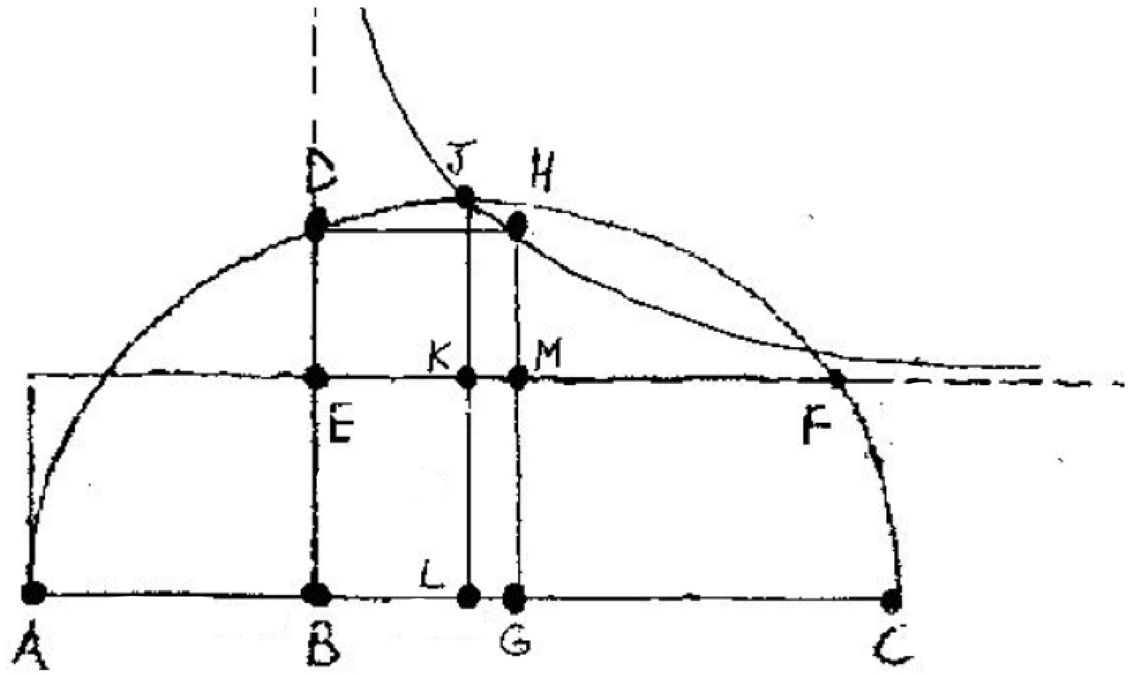
Next, a point, G, on the line AC must be found. This point must be a certain length away from point B, so it satisfies the equation ED:BE = AB: BG. It is known that BE = b, and AB = m, and line ED can be measured. Therefore, BG = mb/ED. Next, another point, point H, is drawn perpendicular to line AC at point G and at a height equal to point D, and a rectangle DBGH is then formed (**Figure 4**).

Figure 4.Semicircle with Rectangle DBGH (not drawn to scale)



Now, at point H, a hyperbola will be drawn that has lines ED and EF as asymptotes. Lastly, the point where the hyperbola cuts the semicircle will be labelled J, and a line perpendicular to line AC will be drawn from point J. The point where this line cuts line EF will be labelled K, and the point where this line cuts line AC will be labelled L. Furthermore, the point where line GH cuts line EF will be labelled point M (**Figure 5**).

Figure 5. Final Product with Hyperbola (not drawn to scale)



From this, we can gather that:

since J and H are on the hyperbola, (EK)(KJ) = (EM)(MH)

and since ED:BE = AB: BG, this can be rearranged to yield (ED)(BG) = (BE)(AB).

Moreover, since ED = MH and BG = EM, it can be written that (ED)(BG) = (EM)(MH) = (EK)(KJ) = (BE)(AB).

Next, as can be observed in **Figure 5**, BL = EK and LJ = BE + KJ, so it can be stated that (BL)(LJ) = (EK)(BE+KJ) = (EK)(BE) + (EK)(KJ),

which can also be stated as (BL)(LJ) = (EK)(BE) + (AB)(BE) = (BE)(EK+AB).

From **Figure 5**, it can be observed that EK + AB = AL, so it can also be written that

(BL)(LJ) = (BE)(AL).

It can also be written that $(BL)^2(LJ)^2 = (BE)^2(AL)^2$.

Since J is located at the highest point of the semicircle and L cuts halfway through line AC, $(LJ)^2$ can be considered equivalent to the radius squared, and it can be written that

$(LJ)^2 = (AL)(LC)$.

Therefore, using $(BL)^2(LJ)^2 = (BE)^2(AL)^2$

$(BL)^2(AL)(LC) = (BE)^2(AL)^2$

$(BL)^2(LC) = (BE)^2(AL)$.

As can be seen in **Figure 5**, $(AL) = BL + AB$, and $LC = BC – BL$. Therefore, $(BL)^2(LC) = (BE)^2(AL)$ can also be written as $(BL)^2(BC – BL) = (BE)^2(BL + AB)$.

Now, it is known that $BE = b$, $AB = a^3/b^2$, and $BC = c$. These values can be substituted into the above equation to give,

$(BL)^2(c – BL) = b^2(BL + a^3/b^2)$.

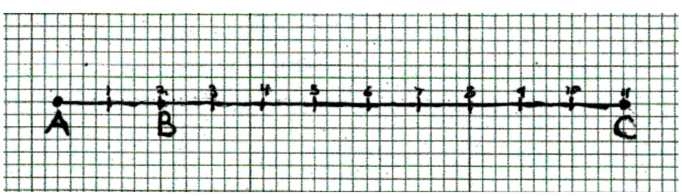
Rearranging, we find:

$(BL)^2(c – BL) = b^2(BL + a^3/b^2)$

$BL^2c – BL^3 = b^2BL + a^3$

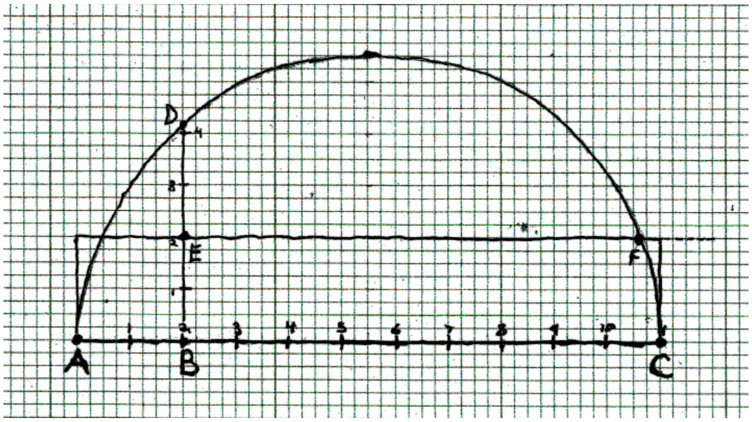
$BL^2c = BL^3 + b^2BL + a^3$, which is the cubic equation we were trying to find a solution to! Using Omar Khayyam's method of finding a positive value to a cubic equation, we find $BL^3 + b^2BL + a^3 = BL^2c$, and since we were trying to find the solution to $x^3 + b^2x + a^3 = cx^2$, we can state that the magnitude of line $BL = x$. The magnitude of line BL would be obtained from direct measurement, and we would find the positive root solution to the equation. This problem and its solution were taken from Eves (1958). As an example of this problem, consider $x^3 + 4x + 8 = 9x^2$, where $a = 2$, $b = 2$, and $c =9$. Following the steps above, we first draw a line AB equal to $m = a^3/b^2 = 8/4 = 2$, and then, a line BC equal to $c = 9$ is drawn (**Figure 6**).

Figure 6. Line AC



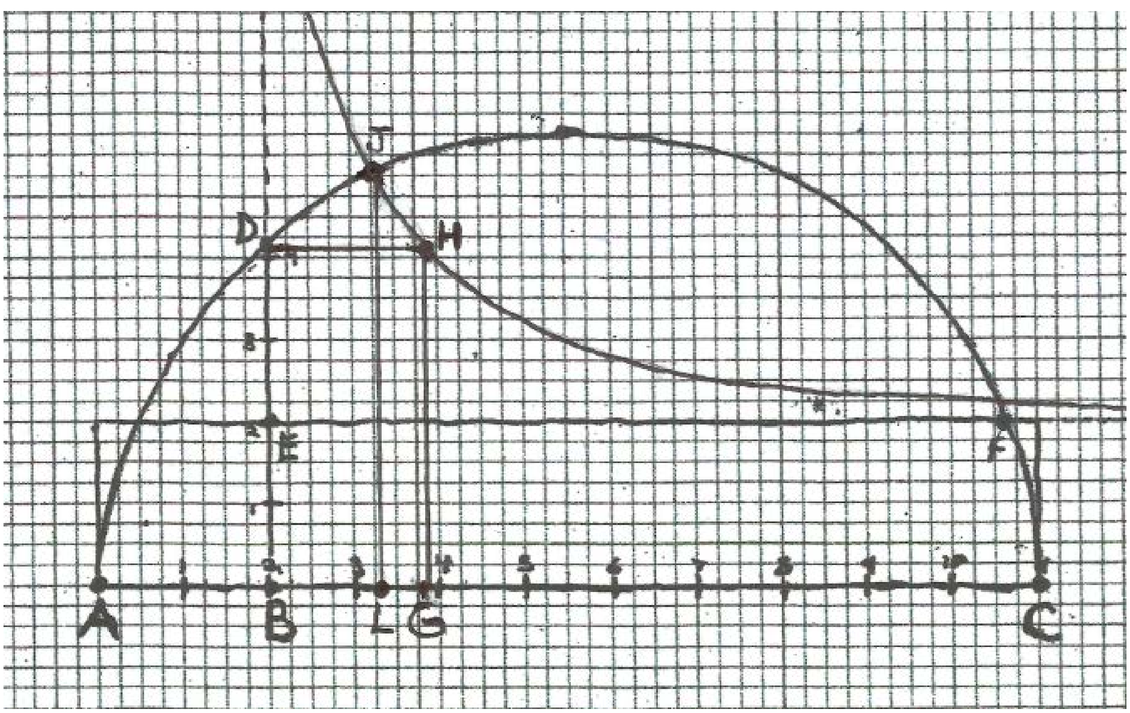
Next, a semicircle with diameter AC is drawn. Line BD is also drawn, and on this line, point E is drawn. Line BE then is equal to $b = 2$ (**Figure 7**).

Figure 7. Semicircle with Line AC as Diameter



We must also construct line BG = mb/ED. m equals 2; b = 2, and from **Figure 7**, it can be observed that line ED is approximately equal to 2.2. Therefore, line BG is about equal to 2(2)/2.2 = 1.8. After constructing rectangle BDGH, a hyperbola with the same asymptotes as those in **Figure 5** is then drawn (**Figure 8**). The point on the semicircle where the hyperbola cuts is labelled point J, and from point J, a line to perpendicular to line AC is drawn. This line cuts line AC at point L (**Figure 8**).

Figure 8. Final Product with Hyperbola



Based on the magnitude of its length in **Figure 8**, we now know the approximate magnitude of line BL. From **Figure 8**, we can see the approximate length of line BL is about 1.3. This is a reasonable approximation of x considering that when x = 1.3:

$x^3 + 4x + 8 = (1.3)^3 + 4(1.3) + 8 = 15.397$

and

$9x^2 = 15.21$

Therefore, the answer provided in **Figure 8** by Omar Khayyam's method is close to the correct positive root solution for x. The preceding form of a cubic equation was only one type of cubic equation Omar Khayyam developed a method of finding a solution to. In fact, he found solutions to 19 types of cubic equations, and of these, 14 were solved by means of conic sections; the remaining 5 were reduced to quadratic equations (Joseph, 2011). Although Omar Khayyam has been noted as being the first to conceive such a variety of solutions to multiple cubic equations, the method for finding complete solutions to cubic equations would later be developed by Cardano in 1545 (Struik, 1958). Regardless of being rudimentary and in its infancy, providing solutions to multiple cubic equations was certainly a groundbreaking novelty. However, this was not Omar Khayyam's only major contribution to mathematics.

Omar Khayyam wrote another mathematical text titled *Commentaries on the Difficulties in the Postulates of Euclid's Elements*, and in it, he addressed the parallel axiom (Struik, 1958). The parallel axiom was the fifth postulate in Euclid's *Elements*, and according to the parallel axiom, when given a straight line and a point that is not on that line, there exists only one line that will pass through the point not on the straight line that will never intersect with the straight line (Struik, 1958). The axiom seems straightforward, but it was difficult to prove (Struik, 1958). In an attempt to prove the axiom, Omar Khayyam first set forth five principles (Struik, 1958). The principles were,

"(I) Magnitudes are infinitely divisible, that is, they do not consist of indivisibles;

(II) A straight line can be produced to infinity;

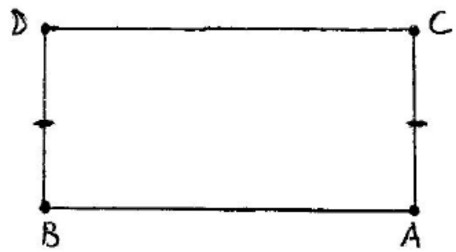
(III) Any two intersecting lines open and diverge to the extent to which they move away from the vertex of the angle of intersection;

(IV) Two converging lines intersect and it is impossible for the converging straight lines to diverge in the direction of convergence;

(V) Of two unequal bounded magnitudes the smaller can be taken with such multiplicity that it exceeds the larger" (Rosenfeld, 1988, p.38).

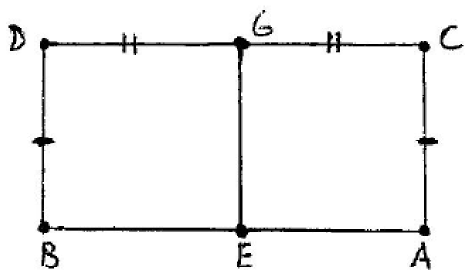
Considering most of the principles were taken from Aristotle, Khayyam's background in philosophy becomes apparent (Struik, 1958). All but Principle IV had been asserted by Aristotle (Rosenfeld, 1988). As a note, although Principle V can be found in the works of Aristotle, it is in fact the axiom of Eudoxus and Archimedes (Rosenfeld, 1988). Moreover, Principle IV can be traced back to Euclid (Rosenfeld, 1988). Omar had enough confidence in these to principles to claim that they should replace proposition 29 in book 1 of Euclid's *Elements*, where Euclid starts the exposition of the theory of parallel lines (Rosenfeld, 1988). After asserting these principles, Khayyam then used them to set forth and prove eight propositions that would establish Euclid's parallel axiom. In the first proposition, Khayyam asks us to consider two equal perpendiculars, AC and BD, that are erected at the ends of the line AB, and the upper ends of these two equal lines are joined (**Figure 9**; Rosenfeld, 1988).

Figure 9. Quadrilateral CABD



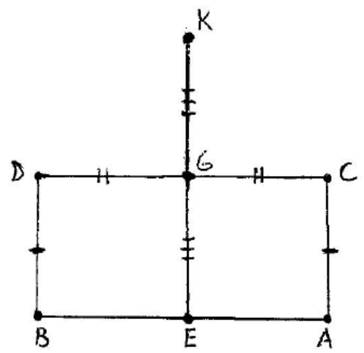
Khayyam then proposes that angle ACD is equal to angle BDC (Rosenfeld, 1988; Struik, 1958). In the second proposition, Khayyam asks that we continue to consider the lines described in the first proposition (Rosenfeld, 1988). As shown in **Figure 9**, these lines form a quadrilateral, and if a perpendicular, line EG, at midpoint E of line AB is erected, Khayyam states in the second proposition that EG is also perpendicular to line DC, given that CG = GD when line DC is cut by the line extended from point E (**Figure 10**; Rosenfeld, 1988; Struik, 1958).

Figure 10. Quadrilateral CABD Cut at Midpoints by Line EG



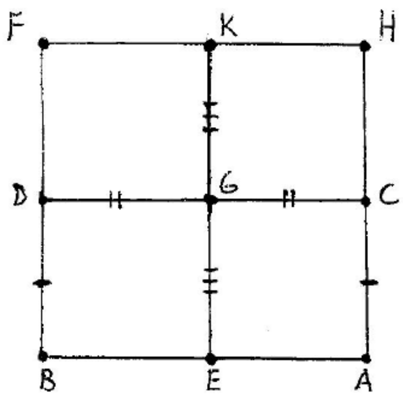
In the third proposition, Khayyam repeats the first proposition, but in this proposition, he provided a proof for Proposition 1 (Rosenfeld, 1988). The third proposition can also be thought of as a proposal that angles ACD and BDC in **Figure 10** are right angles. From the former perspective, according to Khayyam, to prove Proposition 1, we first extend a line from point G in **Figure 10** that is equal to line GE, and this line is then labelled GK (**Figure 11**; Rosenfeld, 1988).

Figure 11. Line GK Extended from Quadrilateral CABD



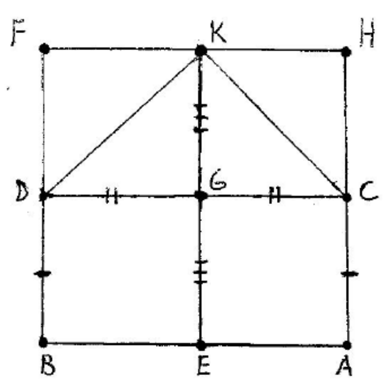
Next, a line, HF, perpendicular to line EK and cutting point K is drawn, and lines AC and BD are extended to points H and F, respectively (**Figure 12**; Rosenfeld 1988).

Figure 12. Quadrilateral HABF



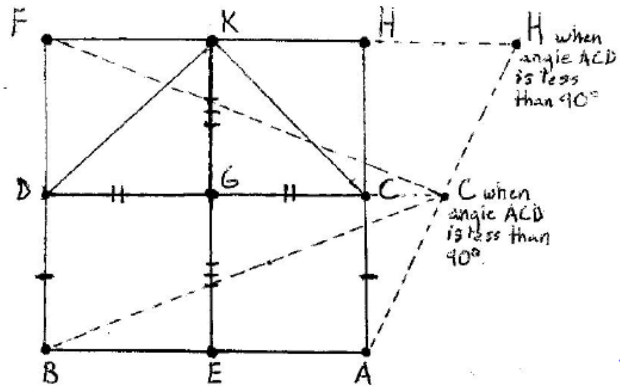
Then, point C is joined to point K, and point D is also joined to point K (**Figure 13**; Rosenfeld 1988).

Figure 13. Quadrilateral HABF with Diagonals



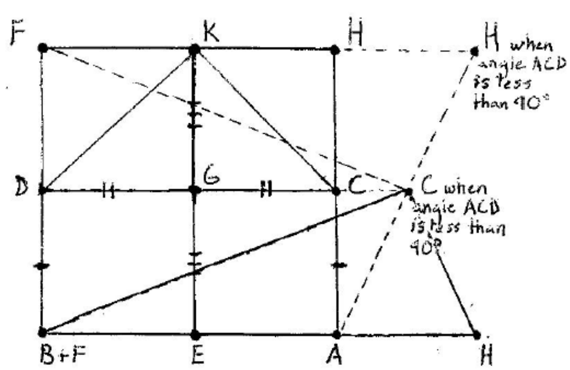
Since lines DG and GC are equal and both share the perpendicular GK, lines DK and KC must be equal. Therefore, angles GCK and GDK must be equal (Rosenfeld, 1988). Since these angles are equal, angles HCK and KDF are equal; angles DKG and CKG are equal, and angles KHC and KFD are equal (Rosenfeld, 1988). Now we see why Khayyam's third proposition can also be viewed as a proposal that angles ACD and BDC are right angles. Since lines EG and GK are equal, and as initially established, lines AC and BD are equal, lines CH and DF must also be equal. Moreover, since lines DG and GC are equal and share the perpendicular GK, since lines DK and KC are equal, and since lines CH and DF are equal, lines HK and KF must be equal. In order for lines HK and KF to be equal and lines CH and DF to also be equal, angles ACD and BDC must be right angles (Rosenfeld, 1988). Khayyam then shows the contradiction of angles ACD and BDC being larger or less than 90°. Khayyam considers the scenario where angle ACD is less than 90°. In this case, he connects point C, when angle ACD is less than 90°, to points F and B (**Figure 14**).

Figure 14. Scenario When Angle ACD is Less Than 90°



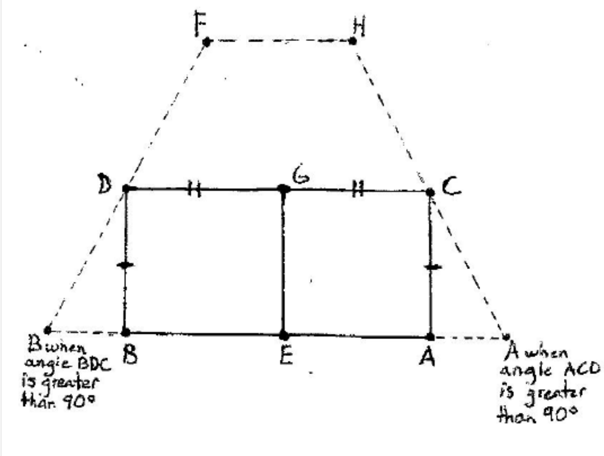
Plane figure CF is then superposed on plane figure CB (**Figure 15**).

Figure 15. Superposition of Plane Figure CF on Plane Figure CB



As shown in **Figure 15**, line HF in this scenario is larger than line AB, which cannot be true. Therefore, angle ACD nor its the angle BDC can be less than 90°. He also demonstrated that if both angles are less than 90°, and using Principle II, if lines CH and DF are extended indefinitely, line HF will continuously increase in length. Due to these conditions and considering Principle III, lines BD and AC are shown to diverge at that end, and therefore, these lines cannot be perpendicular to line AB, which contradicts the initial conditions. Khayyam then considers the situation where angles ACD and BDC are greater than 90° (**Figure 16**).

Figure 16. Scenario When Angles ACD and BDC are Greater Than 90°

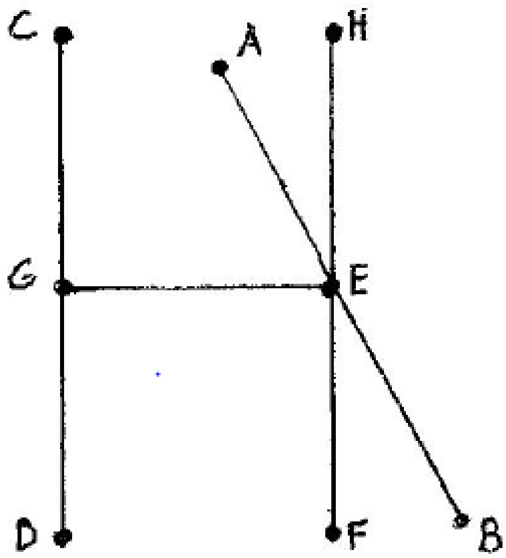


In this case, when lines HF and AB are superposed, line HF will be less than line AB, which cannot be true. Moreover, if lines CH and DF are extended indefinitely, they will eventually converge, indicating lines AC and BD would not be perpendicular to line AB. By demonstrating the contradictions of angles ACD and BDC being less or greater than 90°, Khayyam establishes in his third proposition, through use of some of the principles he set forth, that these angles must be 90°. Khayyam's fourth, fifth, and sixth propositions can be briefly summarized as follows. In the fourth proposition, Khayyam shows that the opposite sides of a rectangle must be equal; in the fifth proposition, Khayyam claims that when two lines are perpendicular to the same line, any line perpendicular to one of these lines will also be perpendicular to the second line, and in Khayyam's sixth proposition, he states that two lines that are parallel and do not intersect upon extension will constitute two perpendicular lines on a straight line (Rosenfeld, 1988). Khayyam's seventh proposition is a reiteration of Euclid's proposition 29 of book 1 in the *Elements*, and in it, Khayyam essentially states that whenever a straight line falls on another straight line and the two angles where the straight line falls on the other are equal, both of the angles then must equal 90° (Rosenfeld, 1988). Although this was already stated by Euclid, Khayyam used his own propositions to prove it (Rosenfeld, 1988). Then in Khayyam's eighth proposition, he completed his proof of Euclid's parallel axiom (Rosenfeld, 188). Khayyam asserts,

"The line EG is a straight line. From it are drawn two lines EA and CG such that the angles AEG and CEG are [together] less than two right angles […]. I claim that they intersect on the side of A" (Rosenfeld, 1988, p. 70).

These lines are shown in **Figure 17**.

Figure 17. Lines in Khayyam's Eighth Proposition



To prove the final proposition, Khayyam instructs us to make angle AEG smaller than angle EGD, and angle HEG should be made equal to angle EGD (Rosenfeld, 1988). Based on these conditions we can see that lines HEF and DGC are parallel, and line AE, which intersects line HF will intersect line CD on the side of A (Rosenfeld, 1988). Angle AEG must only meet the mentioned conditions, and since it only has to be less than 180° when combined with angle CEG, it can have many values. Since it can have many values, there is a variety of lines that can meet these conditions, but none of these lines are parallel to line DGC, and as Khayyam proved, all will intersect line DGC on the side of A. There then remains one and only one line that will remain parallel to line DGC and that will never intersect it. That is the line equivalent to two right angles when facing line DGC: line FEH. Of the individuals who tried prove the parallel axiom, Omar Khayyam stood out in his argument and was a notable figure in furthering understanding of the axiom (Struik, 1958).

Although Omar Khayyam largely impacted mathematics and other fields, the nature of the turmoil surrounding him at that time eventually caught up with him and stunted his progress. After 1095 C.E., Malik-Shah had died, and his 15-year-old son was left to inherit a dwindling empire (Richardson, 2016). This likely weakened Omar Khayyam's relationship with the Seljuq court, and with the political uncertainty, it is clear that intellectual pursuits were still not a societal concern. During that period, Omar's work was significantly fragmented (Richardson, 2016). Furthermore, although Malik-Shah favored Khayyam, he was not very popular among his peers (Khayyam, 1896). Due to being very unpopular in his own country, his work was barely shared abroad (Khayyam, 1896). His ideas were honest but daring, and many of his peers did not appreciate him nor his ideas, regardless of how revolutionary they were (Khayyam, 1896). Particularly, he was hated by Sufi Muslims whose practice he was critical of (Khayyam, 1896). With political, social, and religious influences disfavoring his reputation, intellectual curiosity, and pursuits, Omar's strengths became weaknesses, and his options in that environment were limited. After achieving great feats in mathematics and many other fields, Omar was still an outcast. After awakening from a dream, Omar wrote a poem in response to a question his mother asked him in the dream (Khayyam, 1896). In his dream, Omar's mother asked him what his fate would be, and in response, Omar wrote,

"Oh, Thou who burn'st in Heart for those who burn

In Hell, whose fires thyself shall feed in turn;

How long be crying, 'Mercy on them, God!'

Why, who art Thou to teach, and He to learn?" (Khayyam, 1896, p. xclvii).

Indeed, what Omar had to teach the world was valuable and is still recognized today, but it did not gain him acceptance. Although Omar was shunned by his peers for his boldness in expressing his honest ideas, his work unleashed knowledge to humankind that is still cherished and revered today.

On December 4th, 1131 C.E., Omar Khayyam's son-in-law, Imam Muhammad Baghdadi, recorded Omar Khayyam's death (Richardson, 2016). According to Imam Muhammad Baghdadi,

"[Omar Khayyam] was studying the Shifa, while he was using a golden toothpick, until he reached the section on … "unity and multiplicity." He marked the section with his toothpick, closed the book, and asked his companions to gather round…When his companions gathered, they stood up and prayed and Khayyam refused to eat or drink until he performed his night prayer. He prostrated [himself] by putting his forehead on the ground and said, "O Lord, I know you as much as it is possible for me; forgive me, for my knowledge of you is my way of reaching you" and then died" (Richardson, 2016, p. 62 -63).

Before Omar Khayyam died, he had a conversation with one of his pupils, Khwajah Nizami, and during that conversation, he said to Nizami, "My tomb shall be in a spot where the north wind may scatter roses over it" (Khayyam, 1935, p. 6). Nizami stated that he did not believe those were idle words, and when he returned to Nishapur years after having the conversation with Omar, he searched for the tomb and found it exactly where Omar said it would be (Khayyam, 1935; Richardson, 2016). It was located outside of a garden, and trees from the garden dropped their flowers on the tomb (Khayyam, 1935). Whether this was arranged, coincidental, or predicted by Khayyam is unclear. Whatever foresight or planning he had about the location of his tomb, he constantly acknowledged the one prediction we can all accurately make. Khayyam constantly exposed the limitations of this life, and during his time on Earth, he made an astounding amount of progress in multiple disciplines. Indeed, some individuals have even suggested that the philosopher-mathematician Khayyam could not have been the same person as the poet Khayyam (Richardson, 2016)! Making such outstanding contributions to multiple disciplines does seem to be a bit of a stretch for one person to accomplish in a lifetime, but maybe it was his unshaken knowledge of the limited time we have on this Earth that drove him to accomplish what he did.

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