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ADAPTIVE SUBOPTIMAL CONTROL OF INPUT CONSTRAINED PLANTS

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Abstract. *This paper deals with adaptive regulation of a discrete-time linear time-invariant plant with arbitrary bounded disturbances whose control input is constrained to lie within certain limits. The adaptive control algorithm exploits the one-step-ahead control strategy and the gradient projection type estimation procedure using the modified dead zone. The convergence property of the estimation algorithm is shown to be ensured. The sufficient conditions guaranteeing the global asymptotical stability and simultaneously the suboptimality of the closed-loop systems are derived. Numerical examples and simulations are presented to support the theoretical results.*

Keywords: adaptive control, asymptotical stability, bounded disturbance, constraint, convergence, estimation algorithm, suboptimality.

Introduction

All real control systems usually have in face some nonlinearity, such as input actuator saturation constraints. Therefore standard control objectives including, in particular, regulation have to be met in the presence of these constraints. Unfortunately, in many situations, dynamic systems with hard limits on the magnitude of the control input may exhibit unexpected performance and can even become unstable if the saturation is not taken into account in the system design. Hence, the achievement of desirable control objectives in the closed-loop systems containing control saturation constraints is a very important problem from both theoretical and practical point of view.

In the case of parametric uncertainties requiring an adaptive approach, stability and good control performance of the amplitude constrained closed-loop systems are a difficult problem that needs more attention.

This paper sheds light on such a difficult problem. It deals with adaptive regulation of a discrete-time linear time-invariant plant with arbitrary bounded disturbances whose control input is constrained to lie within certain limits.

The main effort is focused on establishing the sufficient conditions of the global asymptotical stability and suboptimality.

Analysis of previous researches

Adaptive control methods have been an active research area during the past decades. Stability as well as optimality (suboptimality) and robustness of adaptively controlling linear time-invariant plants with no restrictions on the magnitude of the control input has studied and presented in several textbooks and papers; see, for example, [1–6], etc.

Stability results in the sense of the ultimate boundedness concerning the adaptive discrete-time control systems that contain the input saturated plants of certain type classes and use a direct control approach are reported in [7–9], and an indirect control approach in [10–13].

In all these works, however, it has not been proved that the output error and the control input sequences converge. Nevertheless, it turns out that although these signals remain bounded, such control systems may not be asymptotically stable even when a plant whose parameters are known is strictly stable and stably invertible (minimum phase). Again, in contrast with their unconstrained counterparts, neither the optimality as in [6] nor the suboptimality as in [1; 5] can be achieved in the presence of arbitrary bounded disturbances if their "size" is large enough while it will be asymptotically stable when they are absent.

Significant progress in ensuring a desirable ultimate behavior of adaptive control systems with a saturation input constraint was achieved by

M. M'Saad and his colleagues who presented their first results at the ECC'95 and extended in the paper [14].

They have derived the condition under which, in the disturbance-free case, the output error converges to zero and the control input stops saturating after a finite time instant. A novel breakthrough was later made by these authors to deal with bounded disturbances of some class [15].

Unfortunately, their restriction on these disturbances seems to be hardly verifiable.

Meanwhile, there are no another strong results available in the literature regarding the adaptive control of discrete-time plants with arbitrary bounded disturbances in the presence of input constraints.

More certainty, the question of how the desirable asymptotical properties, in particular, the suboptimality, might be achieved in these cases has not been resolved as yet.

Formulation of the problem

The plant to be controlled is a single input – single output (SISO) discrete-time system whose output can be described by the linear difference equation

$$A(q^{-1})y_t = B(q^{-1})u_t + v_t, \tag{1}$$

where $\{y_t\}$, $\{u_t\}$ and $\{v_t\}$ denote the output, control input and external disturbance sequences, respectively;

$$A(q^{-1}) = 1 + a_1 q^{-1} + \cdots + a_n q^{-n}, \tag{2}$$

$$B(q^{-1}) = b_1 q^{-1} + \cdots + b_n q^{-n} \tag{3}$$

represent the polynomials in the backward shift operator q^{-1} with $b_1 \neq 0$, $|a_n| + |b_n| > 0$.

Suppose the coefficients of A and B in (2), (3) are unknown and the disturbance v_t is unmeasured.

The following basic assumptions are made.

A1) The plant order n is known.

A2) $W_0(z^{-1}) := B(z^{-1}) / A(z^{-1})$ has no unstable poles and zeros, i.e., the plant is asymptotically stable and strictly minimum phase.

A3) One knows a convex compact region $\Omega \subset \mathbf{R}^{2n}$ to which the $2n$-dimensional coefficient vector

$$\theta = [a_1, \ldots, a_n, \ b_1, \ldots, b_n]^T$$

belongs.

A4) The disturbance sequence $\{v_t\}$ is bounded in modulus by an ε to be known:

$$|v_t| \leq \varepsilon \qquad \forall t. \tag{4}$$

As in [2, Remark 6.3.3], the control sequence $\{u_t\}$ is constrained in amplitude so that

$$-\infty < u_{\min} \leq u_t \leq u_{\max} < +\infty, \tag{5}$$

where $u_{\min}$ and $u_{\max}$ are specified minimum and maximum input levels to be known.

Let y^* ($y^* \equiv \text{const}$) denote a desired output y_t. The output error will then be defined as

$$e_t := y^* - y_t. \tag{6}$$

Now, one needs the following definitions introduced in [1, Definition 4.1.1].

Definition 1. $\{u_t\}$ is said to be optimal if the control objective in the form

$$\limsup_{t \to \infty} |e_t| \leq \varepsilon \tag{7}$$

is achieved with e_t given by (6).

Definition 2. $\{u_t\}$ is said to be suboptimal if

$$\limsup_{t \to \infty} |e_t| \leq \varepsilon + \delta, \tag{8}$$

where δ is an arbitrary sufficiently small positive number chosen by the designer.

The aim is to derive conditions under which a simple direct adaptive control algorithm similar to that in [6] and subject to the constraints (5) can ensure the objective (8) for any $\delta > 0$.

Non-adaptive case

Before going to design an adaptive controller that can be able to achieve the goal (8), it makes sense to evaluate whether the regulation problem has a solution in the absence of plant parameter uncertainties. To this end, define the variable

$$u_t' = \frac{1}{b_1}[y^* + a_1 y_t + \ldots + \tag{9}$$

$$+ a_n y_{t-n} - b_2 u_{t-1} - \ldots - b_n u_{t-n+1}]$$

that is the signal formed by the usual one-step-ahead linear controller employed in [1, sect. 3.2.2]; [2, sect. 8.2.1]. Then, taking (5) into account, the amplitude constrained control input u_t is determined by

$$u_t = \text{sat}\{u_t'\}, \tag{10}$$

where $\text{sat}\{\cdot\}$ is the saturation nonlinearity defined as

$$\text{sat}\{x\} = \begin{cases} x_{\max} & \text{for} \quad x > x_{\max}, \\ x & \text{for} \quad x_{\min} \leq x \leq x_{\max}, \\ x_{\min} & \text{for} \quad x < x_{\min}. \end{cases} \quad (11)$$

Notice that the past constrained control signals $u_{t-1}, \ldots, u_{t-n+1}$ are used to calculate u_t' but not the past signals $u_{t-1}', \ldots, u_{t-n+1}'$ (as in the linear case).

It is obvious that if $A(z^{-1})$ has no unstable roots, then the closed-loop system (1), (9)–(11) will always be BIBO stable. Furthermore, noting that (5) causes

$$\left| \Delta u_t' \right| \leq u^+, \quad (12)$$

where

$$\Delta u_t' = u_t' - u^0,$$

$$u^+ = \frac{u_{\max} - u_{\min}}{2} \quad (13)$$

with

$$u^0 = \frac{u_{\min} + u_{\max}}{2}, \quad (14)$$

and using (1) together with (4), within the framework of the modern control theory, one can write

$$\|u\|_\infty := \sup_{0 \leq t < \infty} |u_t| \leq \max\{|u_{\min}|, |u_{\max}|\},$$

$$\|y\|_{\text{ss}} := \limsup_{t \to \infty} |y_t| \leq W_0(1)u^0 + \|W_0\|_1 u^+ +$$

$$+ \|A^{-1}\|_1 \varepsilon < \infty,$$

where $\|\cdot\|_\infty$ and $\|\cdot\|_1$ are the corresponding ℓ_∞- and ℓ_1-norms;

$\|\cdot\|_{\text{ss}}$ denotes the semi-norm.

However, the boundedness of $\{u_t\}$ and $\{y_t\}$ does not imply the asymptotical global stability of this system in the sense of the existence of their limits as t tends to ∞ (for arbitrary initial conditions and $v_t \equiv 0$). An illustrative example demonstrates this fact.

Example 1. Let

$n = 2;$

$u_{\min} = 4,0;$

$u_{\max} = 10,0;$

$A(q^{-1}) = 1 + 1,5q^{-1} + 0,95q^{-2};$

$B(q^{-1}) = = 0,1q^{-1} - 0,05q^{-2}$

be chosen to meet A2). The following conditions were used:

$y^* = 0,5;$

$y_{-1} = -10,0;$

$y_{-2} = 2,0;$

$u_{-1} = 10,0;$

$u_{-2} = 10,0.$

For this case, the system behavior, showing that $\{u_t\}$ and $\{y_t\}$ may not converge even when $A(z^{-1})$ and $B(z^{-1})$ are strictly stable and $v_t \equiv 0$ is presented in fig. 1.

Definition 3. A nonlinear closed-loop system with any finite y^* and $v_t \equiv 0$ is said to be asymptotically stable in the large (globally) if the limits

$$u_\infty = \lim_{t \to \infty} u_t, \quad y_\infty = \lim_{t \to \infty} y_t$$

exist for all initial conditions within a compact set.

To establish the global stability conditions for the regulation system considered in this section, (9) is rewritten as follows:

$$y^* + [A(q^{-1}) - 1]y_t = b_1 q^{-1} u_t' + [B(q^{-1}) - \\ - b_1 q^{-1}] u_t. \quad (15)$$

Multiplying both sides of (15) by $A(q^{-1})$ and utilizing (1) for $v_t \equiv 0$ one obtains

$$b_1 A(q^{-1}) q^{-1} u_t' = A(q^{-1}) y^* - [B(q^{-1}) - \\ - b_1 A(q^{-1}) q^{-1}] u_t. \quad (16)$$

Further, (10)–(14) yield

$$u_t = \text{sat}\{\Delta u_t'\} + u^0, \quad (17)$$

where $\text{sat}\{\Delta u_t'\}$ is now the saturation nonlinearity of the form (11) having the symmetrical bounds $x_{\min} = -u^+$ and $x_{\max} = u^+$. With these equations and also with (17), a resulting regulation system equivalent to the closed-loop system (1), (9)–(11) becomes similar to the nonlinear one studied by Ya. Z. Tsypkin [17] and depicted in fig. 2.

Its open-loop circuit comprises the nonlinearity $\text{sat}\{\Delta u_t'\}$ and the linear dynamic part whose transfer function $H(z^{-1})$ is determined from (16) as

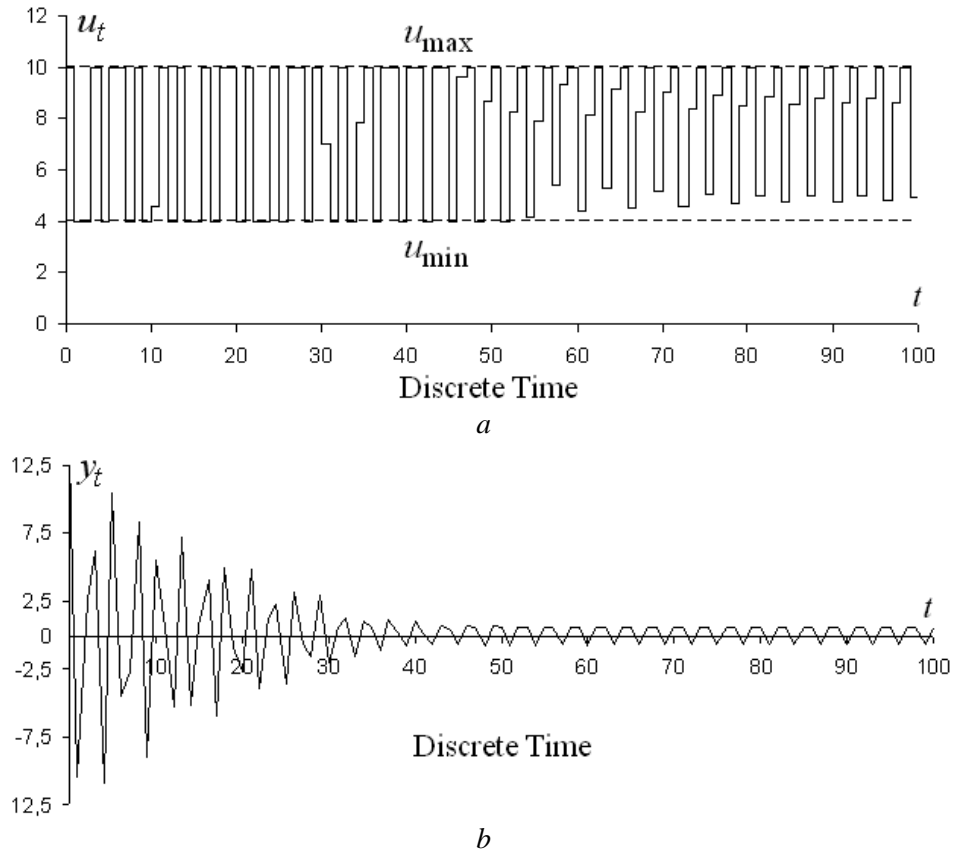$$H(z^{-1}) = \frac{b_1^{-1} B(z^{-1}) - A(z^{-1}) z^{-1}}{A(z^{-1}) z^{-1}}. \quad (18)$$

Fig. 1. Performance of non-adaptive controller in Example 1:
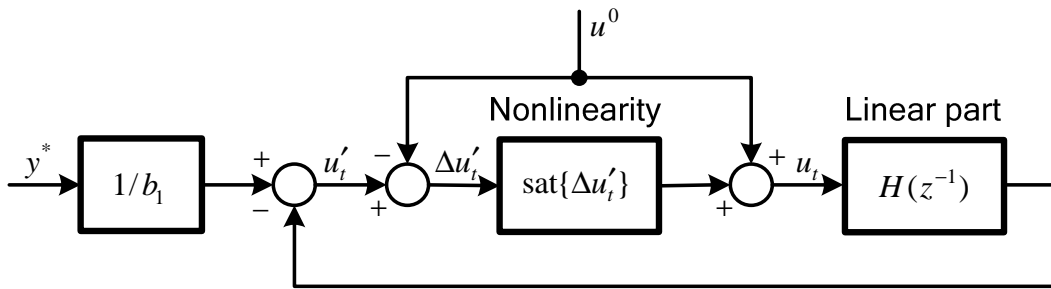*a* – control input;
b – output

Fig. 2. Feedback configuration for the stability investigation

Since the denominator of $H(z^{-1})$ is stable and sat$\{\Delta u'_t\}$ is the sector nonlinearity, the classical Tsypkin's frequency stability criterion [16] is applicable. This criterion allows to establish the final result summarized in the theorem below.

Theorem 1. Let $A(z^{-1})$ be stable. Then, the sufficient condition for global asymptotical stability of the system (1), (9)–(11) is

$$1 + \min_{0 \leq \omega \leq \pi} \text{Re}\, H(e^{-j\omega}) > 0, \tag{19}$$

where $H(e^{-j\omega})$ represents the frequency response obtained by putting $z = \exp(j\omega)$ in (18).

Corollary. The system (1), (9)–(11) is stable in the sense of Definition 3 if

$$\|H\|_\infty < 1. \tag{20}$$

The proof follows immediately from (20) together with the definition

$$\|H\|_\infty := \sup_{0\leq\omega\leq\pi} |H(e^{-j\omega})|.$$

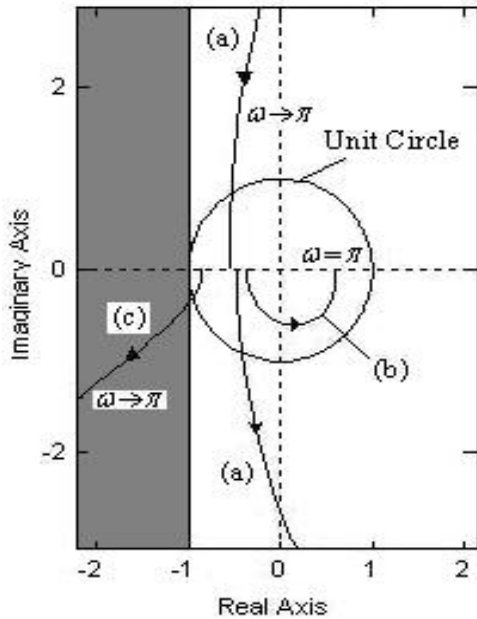The geometrical interpretation of the condition (19) is given in fig. 3.

Fig. 3. Loci of $H(z^{-1})$ for several θs:

(a) $\theta = [1{,}5;\ 0{,}95;\ 0{,}1;\ 0{,}08]^T$;

(b) $\theta = [0{,}7;\ 0{,}2;\ 0{,}1;\ 0{,}02]^T$;

(c) $\theta = [1{,}5;\ 0{,}95;\ 0{,}1;\ -0{,}05]^T$

It is seen that in the cases (a) and (b), the global asymptotical stability is guaranteed, whereas in the case (c), where the vector θ induces the polynomials given in Example 1, the one may not take place (fig. 1). Note that the case (b) obeys the condition (20).

It can be established that if $A(z^{-1}) \equiv 1$ and $B(z^{-1})$ represents the so-called hyperstable polynomial [17] satisfying

$$|b_1| > |b_2| + \cdots + |b_n|$$

then (20) always holds.

It can also be proved that if the plant (1) is free from the disturbance then, under the conditions of Theorem 1, the regulation goal (7) with $\varepsilon = 0$ is achieved and

$$\lim_{t\to\infty} u_t = y^* W_0(1)^{-1},$$

if

$$u_{\min} \leq y^* W_0(1)^{-1} \leq u_{\max},$$

where $W_0(1) = B(1)/A(1)$ represents the static plant gain.

However, in the presence of the disturbance whose amplitude ε is large enough, this goal may not be achieved, in general. This fact is confirmed by

Example 2. Let

$n = 2$;

$u_{\min} = 0$;

$u_{\max} = 10{,}0$;

$y^* = 0{,}5$; $A(z^{-1})$ and $B(z^{-1})$

be induced by the vector θ corresponding to the case (*a*) in fig. 3.

The control input and plant output in the closed-loop system (1), (9)–(11) with v_t representing a pseudo-random variable within the set $[-0{,}2,\ 0{,}2]$ are shown in fig. 4.

It can be observed that the saturation occurs from time to time during which the output error e_t may exceed the admissible bounds equal to 0,2. In this case, the goal (7) is not achieved.

Let $u_{\max} = -u_{\min} = u^+$.

With this additional condition, the following result can be shown to be valid.

Theorem 2. Provided that $y^* = 0$, $u_t \in [-u^+,\ u^+]$ is satisfied in addition to the conditions of Theorem 1, there is an $\varepsilon^* > 0$ which is small enough to satisfy

$$\varepsilon^* \|W_{v/u}\|_1 \leq u^+, \qquad (21)$$

and such that if $0 \leq \varepsilon \leq \varepsilon^*$ then the goal (7) will be achieved, where $\|W_{v/u}\|_1$ denotes the ℓ_i-norms of the transfer function

$$W_{v/u}(z^{-1}) = \overline{A}(z^{-1})/B(z^{-1}),$$

when

$$\overline{A}(z^{-1}) = A(z^{-1}) - 1.$$

Due to space limitation, the proof is omitted.

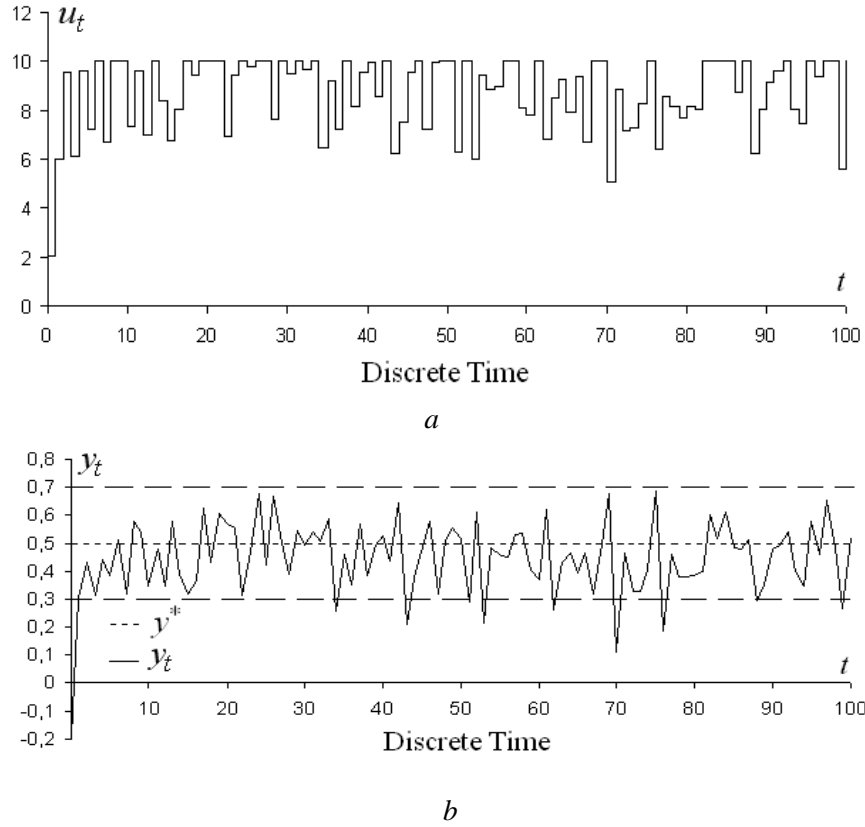Note that condition (21) can always be verified.

Fig. 4. Performance of non-adaptive controller in Example 2:
a – control input;
b – output

Adaptive controller design

The adaptive control law is chosen as

$$u'_t = \frac{1}{b_1(t)}[y^* + a_1(t)y_t + \ldots + a_n(t)y_{t-n} \\ - b_2(t)u_{t-1} - \ldots - b_n(t)u_{t-n+1}]$$ (22)

via replacing the unknown coefficients a_i and b_i in (9) by their estimates $a_i(t)$ and $b_i(t)$, respectively.

The estimation algorithm for updating the vector

$$\theta_t = [a_1(t),\ldots, a_n(t), \; b_1(t),\ldots, b_n(t)]^T$$

is described by

$$\theta_t = \mathrm{Proj}\left\{\theta_{t-1} + \gamma_t \frac{\varphi_{t-1}}{\|\varphi_{t-1}\|^2} f(\tilde{e}_t, \varepsilon, \varepsilon^0)\right\},$$ (23)

where Proj denotes the projection operator necessary to ensure $\theta_t \in \Omega \;\; \forall t$;

$$\tilde{e}_t = y_t - \theta_{t-1}^T \varphi_{t-1}$$ (24)

is the prediction error depending on the past estimate vector θ_{t-1} and on the regresion vector

$$\varphi_{t-1} = [-y_{t-1},\ldots,- y_{t-n}, \; u_{t-1},\ldots,u_{t-n}]^T;$$ (25)

$f(\cdot,\cdot,\cdot)$ represents the modified dead-zone function depicted in fig. 5 and defined as follows:

$$f(\tilde{e}_t,\varepsilon,\varepsilon^0)=\begin{cases}\tilde{e}-\varepsilon & \text{if} \quad \tilde{e}>\varepsilon^0, \\ 0 & \text{if} \quad |\tilde{e}|\leq\varepsilon^0 \quad (\varepsilon^0 >\varepsilon), \\ \tilde{e}+\varepsilon & \text{if} \quad \tilde{e}<-\varepsilon^0;\end{cases}$$ (26)

γ_t is the coefficient chosen from the range

$$0 < \gamma' \leq \gamma_t \leq \gamma'' < 2$$ (27)

with some fixed γ' and γ'' so that $b_1(t)$ in (22) is nonzero;
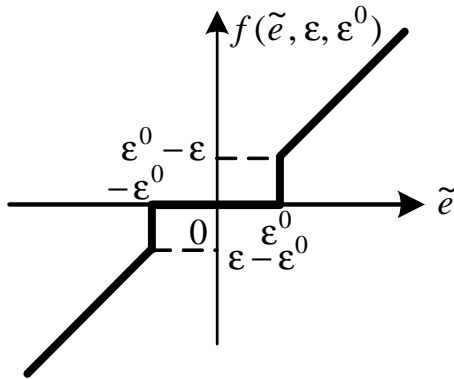
$\|\cdot\|$ denotes the Euclidean vector norm.

Fig. 5. Modified dead-zone function

A distinctive feature of the constrained adaptive control is that $e_t \not\equiv -\tilde{e}_t$, whereas in the absence of the control saturation constraints one gets $e_t \equiv -\tilde{e}_t$.

The convergence properties are given in the following lemma.

Lemma. If $A(z^{-1})$ is strictly stable and $-u^+ \leq u_t \leq u^+$ then the estimation algorithm (23)–(27) with any $\varepsilon^0 > \varepsilon$ converges at a finite time t^* so that: (i) $\theta_t \equiv \theta_{t^*} = \theta^*$ for all $t \geq t^*$, where $\theta^* \in \Omega.$; (ii) $\limsup_{t \to \infty} |\tilde{e}_t| \leq \varepsilon^0$ (independently of how $\{u_t\}$ is generated).

Proof. See [1, Theorem 2.1.1a].

Remark. As in the unconstrained case, the estimates $a_i(t)$ and $b_i(t)$ which are frozen for $t \in [t^*, \infty)$ may not be close to their true values a_i and b_i, respectively. However, the desired control performance in the form (8) is not guaranteed as yet. To show this property, a simulation example will be presented. For the purpose of comparison, simulations are also conducted for an unconstrained adaptive control system.

Example 3. Let

$$n = 2;$$

$$u_{\min} = 0;$$

$$u_{\max} = 10,0;$$

$$y^* = 1,0;$$

$$A(z^{-1}) \equiv 1;$$

$$B(z^{-1}) = 0,1z^{-1} + 0,08z^{-2}.$$

With the initial

$$\theta_0 = [0, \ 0, \ 0,09, \ 0,192]^T$$

and with $\{v_t\}$ as in Example 2, and $\varepsilon^0 = 0,35$, the simulation results are presented in fig. 6.

It can be observed that the suboptimal behavior in the input constrained adaptive case may not be achieved (contrary to the unconstrained one).

Main result

Let A', B' and A'', B'' be the polynomials induced by some θ' and θ'' from Ω. To establish the sufficient condition under which the suboptimality performance can be ensured in the input constrained case, the following additional assumptions on Ω will be required.

A5) The region Ω is such that: (i) A', B' and A'', B'' are all stable for any $\theta', \theta'' \in \Omega$; (ii) $\tilde{B} = B' + A'B'' - A''B'$ is stable; (iii) the condition

$$1 + \min_{\theta_1, \theta_2 \in \Omega} \ \min_{0 \leq \omega \leq \pi} \mathrm{Re}\, H(\theta_1, \theta_2, e^{-j\omega}) > 0$$

is satisfied with $H(\theta_1, \theta_2, z^{-1}) =$
$= [(b_1'')^{-1} \tilde{B}(z^{-1}) - A'(z^{-1})] / A'(z^{-1}) z^{-1}.$ (

Now, the main result is formulated as follows.

Theorem 3. Subject to Assumptions A1)–A5) with $\varepsilon^0 = \varepsilon + \delta$, $y^* = 0$, the adaptive controller described in (22)–(27) and applied to the plant (1) whose control input is constrained to lie within $[-u^+, \ u^+]$ has the properties: (1) the goal (8) is achieved; (2) the control input stops saturating after a finite transient period.

The proof proceeds similarly to the proof of Theorem 2 by using the results of Lemma.

Simulation

A simulated example, showing the successful performance of the adaptive controller is presented in fig. 7.
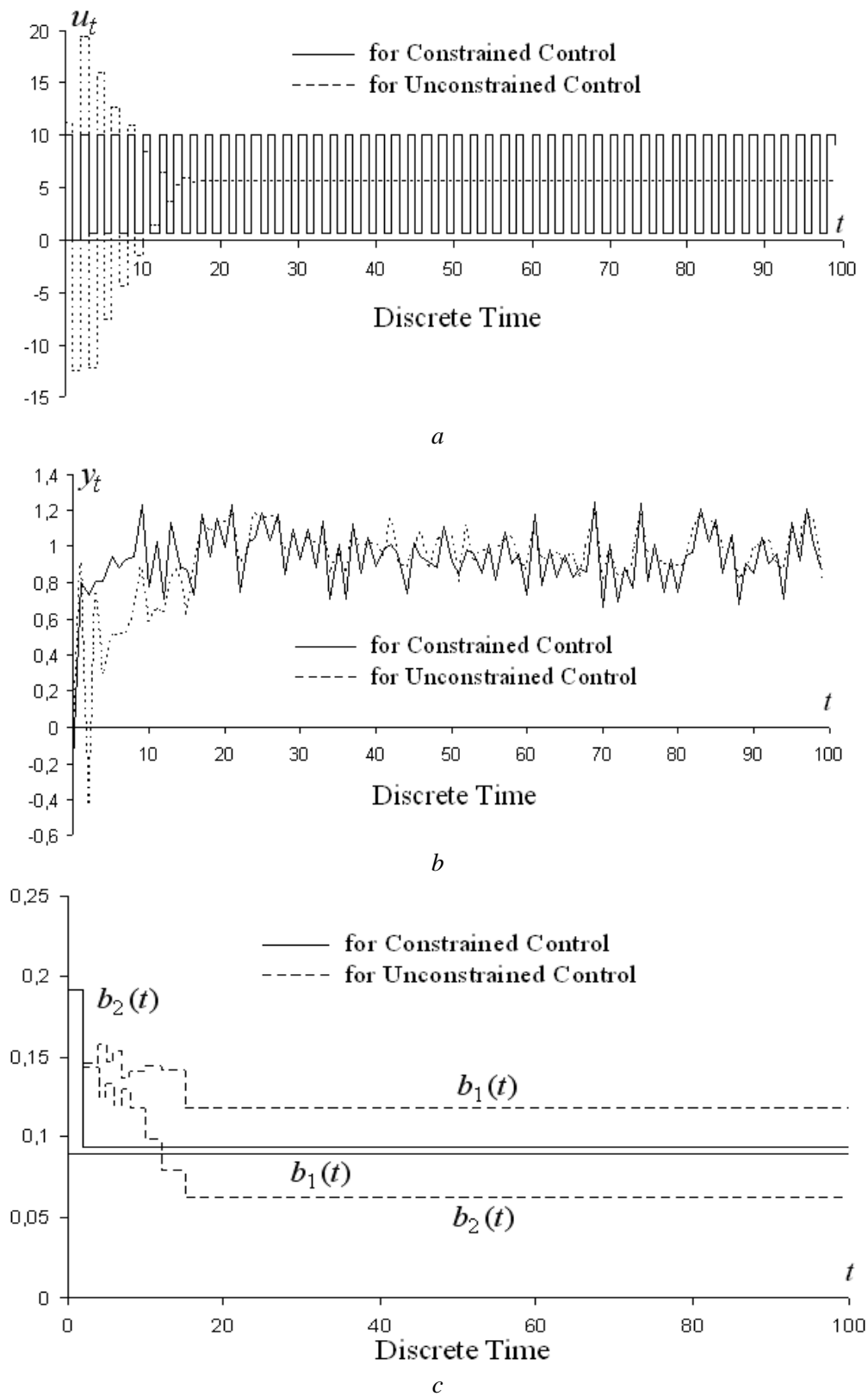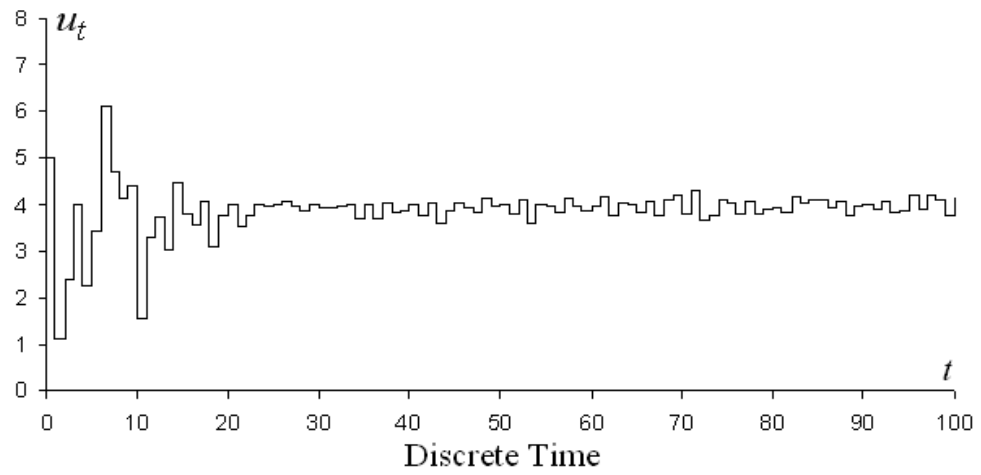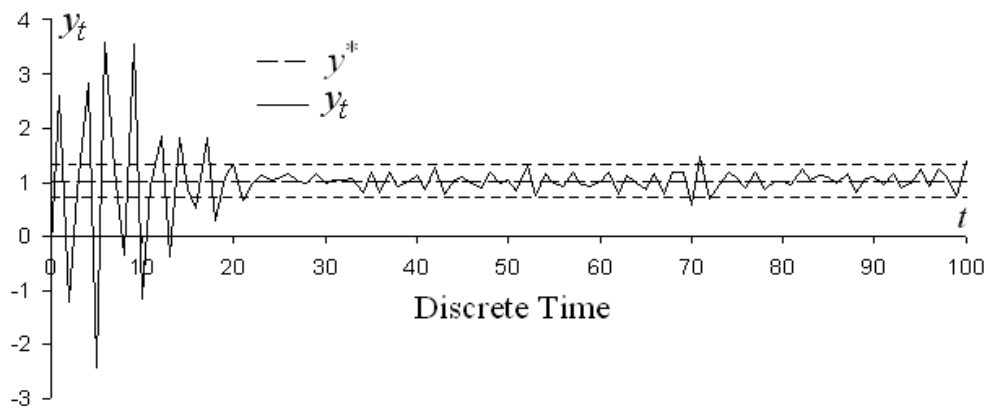
a

b

c

Fig. 6. Performance of adaptive controllers in Example 3:

a – control inputs;

b – outputs;

c – estimate parameters
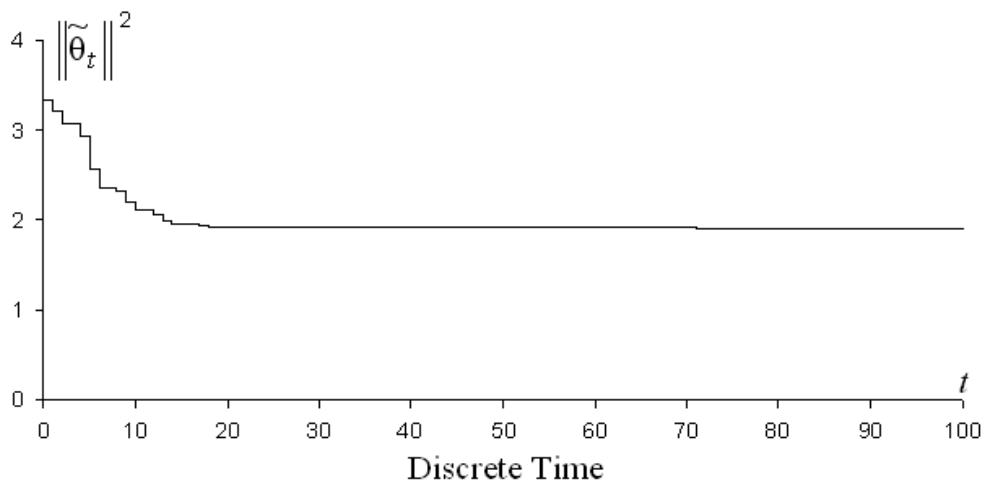
Fig. 7. Adaptive suboptimal control:
a – control input;
b – output (y_t) and desired (y*)
c – Euclidean norm $\tilde{\theta}_t = \theta - \theta_t$

The simulation conditions were chosen as $\varepsilon^0 = 0{,}3$,

$$A(z^{-1}) = 1 + 1{,}5z^{-1} + 0{,}95z^{-2},$$

$$B(z^{-1}) = 0{,}5z^{-1} + 0{,}4z^{-2},$$

$$\theta_0 = [0, 0, 0{,}2, 0{,}1]^T \text{ and } y^* = 1{,}0.$$

Conclusions

The stability and the ultimate regulation performance analysis of the discrete-time adaptive control system with an input amplitude constraint and bounded disturbance are addressed in this paper. Its main contributions are: 1) the sufficient conditions which guarantee the global asymptotical stability of this system; 2) the sufficient conditions under which it will be suboptimal with a given suboptimality index.

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