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Techniques and Procurements of Audio_Visual Aids for Mathematics

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TECHNIQUES AND PROCUREMENTS OF AUDIO-VISUAL AIDS FOR MATHEMATICS

(TITLE)

BY

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ADVISER

5 August 1964
DATE

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DEPARTMENT HEAD

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CHAPTER I

STATEMENT OF THE PURPOSE

The purpose of this paper is to state some uses of audio-visual materials and equipment in secondary school mathematics. A sampling of some of the techniques will be given. Some good points and several shortcomings will be stated for each different device or procedure used in the paper. Places where materials can be obtained will be pointed out. The materials that are beneficial to the mathematics teacher will also be stated.

It will be necessary to give definitions for several specific terms used in this paper so the reader may have a better understanding of the subject matter.

> Instructional aid: Device which assists an instructor in the teaching-learning process by simply presenting supporting or supplementary material, usually intermittently. They are not self-supporting.
>
> Instructional media: Devices which present a complete body of information, and are largely self-supporting rather than supplementary in the teaching-learning process.
>
> Visual materials: Those instructional materials which communicate primarily through sight: written and printed materials, projected pictures, charts, maps, objects, specimans, and the like.
>
> Audio: Of or pertaining to sound. Specifically, a sound recording. Loosely, any part of or all of the complex of sound equipment, facilities, and personnel.
>
> Audiovisual communications: That branch of educational theory and practice concerned primarily with the design and use of messages which control the learning process.
>
> It undertakes: (a) the study of the unique and relative strengths and weaknesses of both pictorial and nonrepresentational messages which may be employed in the learning process for any purpose; and (b) the structuring and systematizing of messages by men and instruments in an educational environment. These undertakings include the planning, production, selection, management, and utilization of both components and entire instructional systems.

> Its practical goal is the efficient utilization of every method and medium of communication which can contribute to the development of the learner's full potential.[1]

An idea of what is contained in the evergrowing range of audio-visual equipment could be shown by listing a partial list of these aids. The list will be concerned with materials that could be applicable to mathematics. The list is as follows: printed material, globes and maps, filmstrips, photographic slides, television, bulletin boards, chalkboard, flannel board, magnetic board, charts, posters, diagrams, demonstrations, models, manipulations, devices, instruments, field trips, flat pictures, transparencies, and overlays, projectors (film, filmstrip, opaque, overhead, slide). An endless supply of new devices to enhance mathematics teaching could extend the list quite considerably, but a simple name for these would not have meaning.

Is audio-visual instruction worth the effort, complication, and expense involved in their use? On the basis of research, experience, and observation, the answer seems to be yes. More can be learned faster and remembered with more accuracy when appropriate materials have been used. This takes for granted that the materials are skillfully used. Some of these skillful uses will be covered in this paper.

Certain of these materials and equipment are suited for purchase with federal funds now available to elementary and secondary schools under the National Defense Education Act of 1958, Public Law 85-864, Section 303.

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[1]Audio-Visual Communication Review, Vol. 11, No. 1. Jan.-Feb., 1963. pp. 54, 148, 36, 18, 19.

As of September, 1963, 181 million dollars have been spent in the United States on strengthening instruction in mathematics, science and foreign language. This money is made available to the states usually on a 50-50 matching basis.

This bill expired in June of 1964. It is expected that the law will be renewed.

CHAPTER II

WHEN COULD AUDIO-VISUAL MATERIALS BE USED?

When audio-visual aids are intelligently selected and integrated within a specific course of instruction they can greatly enrich both the teaching and learning processes. Poor, or haphazard selection can defeat the purpose of the materials. The best person to make proper selection of materials is the teacher. The teacher should know the aims and objectives of the course, as well as the age, the grade, and the intellectual and scholastic acumen of the students. Therefore, it is the teacher who must decide when to use a particular audio-visual aid. As a guide in the areas when to use supplementary materials, the teacher should follow something closely related to the following set of conditions:

1. When the teaching purpose of the teacher can be supplemented by the use of supplementary materials.

2. When the students will benefit most from the use of additional instructional materials.

3. When appropriate materials are available.

4. When the teacher has a knowledge of the use of the materials that will enhance the use of the media.

The teacher should be careful to avoid using the aids for purposes such as the following:

1. Just to be up-to-date with the modern educational approach.

2. For entertainment unless this is the desired outcome.

3. For the sole purpose of keeping the class busy.

4. Because it has an attractive title and looks like it might be of interest to the class.

5. For the sole reason that the topic is of interest to the teacher.

6. Because the teacher is unprepared for the day's lesson.

7. Because a substitute teacher is in the class.

8. Because it is available.

The fact that instructional aids are used does not insure that the teacher's intentions will be accomplished. The correct material, correct time, and correct way are all necessary if the use of audio-visual materials is to be effective in promoting a good learning situation. Failure to select appropriate material or excessive use of a media could quite easily destroy the intentions behind the use of the aids. Materials used indiscriminately or improperly could create confusion and misunderstanding. Teachers using audio-visual materials should ask: "what unique contribution will this material make toward better learning that cannot be attained without it? Teaching aids should not be used merely because they are desirable and popular, or because the material is available, or because the class period has to be filled in. We must recognize that materials are only part of the learning situation and are always subordinate to aims and methods."[2]

It is the teacher who must evaluate the worth of any specific aid in the attainment of the objectives the teacher wishes to achieve. To make this evaluation, the teacher must have a sound knowledge of the different types of audio-visual equipment, and how they are used.

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[2]John W. Bachman, How to Use Audio-Visual Material, (Associated Press, New York, 1956), p. 1.

In line with this same idea of "how", the N.C.T.M. (National Council of Teachers of Mathematics), has a publication which lists seven steps that suggest in a general way how to make effective use of most types of instructional aids. These seven steps are:

1. Select the proper aid to use in order to attain the desired objective. Select wisely from the wealth of materials that is available, or develop precisely the type of item that will suit your purposes best.
2. Prepare yourself to use the teaching aid that has been selected. Become familiar with techniques for making effective use of the item that you plan to use. If it is a film, preview it; if it is a dynamic device or model, familiarize yourself with the manipulations upon which its use as a learning aid depends. If preparatory materials such as study guides or descriptive literature are available, consult them.
3. Prepare the classroom. Check to insure that all needed materials are on hand and in "running order" before the class meets.
4. Prepare the class. Acquaint students with the purpose of the learning aid, what may be expected of it, why it is being used, what they can hope to learn from the experience, and how they should apply the information which they may gain. Often a written learning guide placed in the hands of students increases the effectiveness of the learning experience.
5. Use the teaching aid selected in the most effective manner possible. Relate the aid to the lesson in progress and to the symbolic representation that will be used later.
6. Provide follow-up activities. Discussions, readings, reports, projects, tests, and repeat performances are needed if maximum learning is to result. Whenever possible, provide opportunity for application of the information learned.
7. Evaluate the effectiveness of the aid. On the basis of your experience with the learning aid, it is wise to make a notation regarding its usefulness. A card file system should be employed so that over a period of time you will know what the best aids for each lesson are.[3]

If the teacher knows "how" to use the different audio-visual materials and equipment most effectively, he will know "when" to use them.

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[3]Berger and Johnson, A Guide to the Use and Procurement of Teaching Aids for Mathematics, (National Council of the Teachers of Mathematics, Washington 6, D.C., 1959), p. 5.

CHAPTER III

USING COMMUNITY RESOURCES

A simple survey of the community may reveal many things that could enrich a secondary mathematics program. If a proper understanding is reached between the school and the community sources, then the area of a field trip is given meaning.

The field trip can be a very effective teaching method for a mathematics teacher. If there are adequate facilities for a trip and there are appropriate places to visit, then the field trip becomes a valuable tool.

A field trip can stimulate interest, present realism, and also show the students the "why" for the things they do in the mathematics classroom. When a civil engineer shows a trigonometry class how he uses trigonometric functions in the planning of a highway or the carpenter shows a geometry class how a knowledge of planes and solids is necessary for his work, then there is much added meaning to the students' work in the classroom.

The N.C.T.M. has published a booklet on field trips that contains a list of goals attainable through the use of the field trip.

> A field trip is always appropriate where it will result in better and more permanent learning than some other form of experience. Carefully planned field trips are well suited to attain goals such as the following:
>
> Field trips <u>provide motivation</u> for the study of a unit. A visit to a computing laboratory with its digital and analog computers may excite interest in the study of various number bases or even in mathematics as a career.

> Field trips provide specific materials for use in the classrooms such as business forms, data, pamphlets.
>
> Field trips encourage maximum active participation in a learning activity. When a class goes outside the classroom to do triangulation and height measurement, the students are using mathematical tools in a real situation.
>
> Field trips generate realistic situations for group planning. The class may participate in choosing a date, gathering materials for field work, making contacts to arrange the trip, scheduling a bus, and sending a courtesy note.
>
> Field trips present material in its natural setting. In an airport the mapping of lights, the measurement of ceiling height, the interpretation of weather reports, and the locus of radio beams are matters of life or death. In a laboratory precision measurement, analysis of data, and the use of formulas are constant activities.
>
> Field trips integrate subject matter of different courses. At the highway office, planning the new toll road involves weather conditions, rock and soil formation, accident causes, transportation surveys, as well as indirect measurement. At the airport, aerodynamics, engine performance, weather conditions, and radar patrol are related to work in science courses.
>
> Field trips provide a means for many community citizens to participate in the school program. At the place visited or in follow-up activities several persons will participate as guides, demonstrators, or speakers.[4]

There are many problems that will arise in planning the trip. The selection of an appropriate place to visit is probably the first. Past experiences, tips from other teachers, and class discussion are some of the ways of deciding where to go. Student interest can be aroused by letting them do much of the destination planning.

Once a site has been selected, the next problem comes in preparing the students so they will benefit most from the trip. They must be made aware of what mathematical aspects are to be seen. The teacher must be sure the students know enough about the topic that they will have a sound basis for understanding what they see, and so that they can relate what they see to their in-class activities.

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[4]Johnson-Brinke-Woodby, How to Use the Field Trip in Mathematics, (N.C.T.M., Washington 6, D.C., 1956), #3, p. 5.

The students must realize that they are expected to bring back a certain amount of knowledge and that the field trip is not just a day of vacation from classes. A method of assuring the teacher that the students are attentive is to include the materials from a trip in the unit examination or have individual reports on what was seen. The students should know what is expected of them before they take the trip. They should also be reminded about proper behavior.

Another problem arises with the making of arrangements. Any trip that a mathematics class will be taking will have to be cleared through the office. Parents must be notified, transportation facilities made available, teachers of the classes that the mathematics students will miss must be advised as to the absences. Sometimes money must be made available. In addition, arrangements must be made with the persons who will be the host to your group. A time, a place, and how many students, must all be decided in advance.

After all of the above and perhaps other necessities are finished, comes the trip itself. As long as the students realize what is expected of them there will be little need for discipline.

A necessary process after the trip is over is the follow-up. The interest that was aroused by the trip should be capitalized upon to stimulate further interest in the mathematics at hand and in general. The teacher must make sure that the class has not developed any wrong impressions. A follow-up activity that is important to the teacher and later classes is recording in a file the details of each field trip taken for future references. In the pamphlet on field trips published by N.C.T.M., these headings are used.

Field Trip Record

Place:
Address:
Telephone:
Person to ask for:
What is to be seen?
When is the best time to go?
How many can go?
How do you get there?
How long does the trip take? [5]

They also list places in a community where applications of mathematics may be seen.

Government agencies. Tax collector's office, social security office, highway department, county surveyor's office, weather bureau, post office, military recruiting office or base, civil air patrol, forestry service, civil service office, water or crop control, land reclamation office.

These government agencies use mathematics in a variety of ways such as determining tax rates, completing tax forms, checking records, determining benefit payments, computing refunds, making maps, surveying land, estimating, computing areas and volumes, determining operating costs, collecting and analyzing data, and determining the competence of applicants for positions.

Community institutions. Museums, art centers, churches, laboratories, observatory, planetarium, college or university mathematics or science departments. Museums may have a variety of exhibits in which the role of mathematics is important (e.g., map projections and ancient measuring devices). Art centers will often have displays of abstract and modern art which emphasize symmetry, perspective, and proportion. Churches may have stained-glass windows and architectural forms that show applied symmetry. Laboratories may have precise measuring devices and experimental techniques that illustrate applied mathematics. Observatories and planetariums utilize a variety of direct astronomical measurements, temperature determinations, space navigation, large units of length, and models illustrating gravitational attraction.

Business enterprises. Insurance companies, banks, stock exchange, grain exchanges, brokers' offices, factories, mechanical computing and recording firms, engineering offices, supermarkets, farms, architects' offices, computing laboratories, scientific laboratories, manufacturers of scientific instruments, aerial survey companies.

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[5] Ibid., p. 3.

> Business firms are highly dependent on mathematics. Arithmetical computations of wages, costs, depreciations, overhead, brokerage fees, market quotations, interest rates, dividends, and rents are involved. Most of these enterprises collect data, prepare tables, draw graphs, predict markets, determine probability, and use a variety of statistical tools. Planning buildings, laying out machine work, measuring stresses, and estimating reconstruction costs require many indirect measurements. Most of these concerns use a variety of business forms and computing machines for efficient operation.
>
> Transportation centers. Airports, train stations, bus depots, freight offices, railway express.
>
> Travel by land, sea, and air involves the use of maps, schedules, rates, traffic control, and time zones. Air and water travel are highly dependent on principles of navigation, aerodynamics, vectors. Measurements are made with compasses, sextants, astrolabes, and driftmeters. Radar and sonar are used everyday. Research is constantly being carried on in accident control, operating efficiency, and new designs.
>
> Public utilities. Telephone center, electric company, gas company, water plant.
>
> Measurement of the amount of service used is the daily problem of public utilities. Their measurements may involve varied types of meters and units. Computing charges, bookkeeping, planning extended services, designing new equipment, and planning new rates are daily activities involving mathematics which are carried on at the public utility rate.
>
> Thus, it is apparent that mathematics is a part of many everyday activities of establishments in one's own community. The listing above involves mathematics from many fields--from elementary arithmetric through trigonometry. Visit potential places for field trips in your city to find the one best suited for your class.[6]

Many places that will be of interest to the mathematics class would also be of interest to classes related to the field of mathematics. Mathematics, physics, and chemistry classes could go together to visit a missile center. Mathematics students should be made aware of the applied mathematics that is used in these related field.

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[6]Ibid., p. 7.

It is very important to mathematics teachers, that the potential worth of the field trip is realized. The learning situation must outweigh such things as cost, time, and trouble, in order to get the most benefits from the field trip. A school-community program should have the support of the administration, the staff, and the community. Community resources can provide students with rich and stimulating experiences that might be impossible to duplicate with any other instructional materials.

CHAPTER IV

USING THE LIBRARY IN MATHEMATICS

The library can be of great use to the mathematics class, especially to the more able students. Generally the student with above average ability will use the library more than other members of the class. Students may become bored with the slow pace of the class, and additional assignments for them in the way of outside reading may be a suitable change of pace.

The use of the library can make a program much more interesting and flexible. If topics are selected for bright, as well as slower students, outside reading can be a good change of pace for the entire class. Students could build up resentment to the constant problem solving method. Those who are not as capable at solving problems may be able to give a report on a mathematics topic which may enrich the learning situation for the remainder of the students.

Brighter students can use this outside learning to whet their appetite for more mathematics. Care should be taken in preparation of this mathematics library. Outstanding books in the field should be kept on reserve or perhaps kept in the mathematics departmental library. Magazines and reports should be easily obtained by students for browsing or for using as sources for reports or reviews.

Students should be made aware of the many books on mathematics topics that are available for recreational reading and of the many books that can be used for research. Ideas to stimulate thought and inquiry, add reality and drama, and provide a different type experience can be built by using these printed materials.

The assignment of projects outside the classroom can be great interest builders. No matter if these are assigned to individuals or groups, the library is a chief source of information for them.

The N.C.T.M. have published a pamphlet concerning uses of the library (see Appendix "A"). Several topics for projects that require the students to use the library are also available.

A list of books and magazines that should be included in a mathematics library is presented in this pamphlet. Bibliographies may be secured from the National Council of Teachers of Mathematics. These are: The Elementary School Mathematics Library, by Hutcheson, Mantor, and Holmbert, 20¢; The High School Mathematics Library, by Schaaf, 20¢; Recreational Mathematics, by Schaaf, $1.20. A Bibliography of Mathematics for Secondary School Libraries, by Robert A. Rosenbaum and Louis J. Rosenbaum, may be obtained from the Department of School Services and Publications, Wesleyan University, Middletown, Connecticut.

Many students read magazines that would very seldom read a book. There are magazines in the field of mathematics that could be of interest to both teacher and students. These magazines could be included in a library: Mathematics Teacher; The Arithmetic Teacher; School Science and Mathematics; The Pentagon; and Scientific American.

There are several newspapers and bulletins concerning mathematics that should be present in the mathematics departmental library or the school library. The Mathematical Pie (97 Chiquer Road, Don-Caster, England), The Mathematic's Student Journal, and the O.U. Mathematics Letter (University of Oklahoma) are examples.

Often the mathematics teacher can obtain information from companies publishing mathematical textbooks. Your Mathematic's Notebook (Scott, Foresman and Co.), and Keeping Posted in Mathematics,

(Ginn and Co.), are examples of materials obtainable free from these respective companies.

Several organizations have publications that could be used as a guide for students selecting mathematics courses. The N.C.T.M., has the following: How to Study Mathematics, by Henry Swain; The Mathematics Teacher's Opportunity for Guidance, by Kenneth E. Brown; Guidance Pamphlet in Mathematics; and Mathematical Preparation for College. Why Study Mathematics?, by General Electric Co., Schenectady, New York; Professional Opportunities in Mathematics, from the Mathematical Association of America; and Mathematical Needs of Prospective Students in the College of Engineering, from the University of Illinois, Urbana, Illinois, are other pamphlets that should be included in the mathematics library.

It is the responsibility of the teacher to see that adequate printed materials are made available in the library for use by the student of mathematics.

In the N.C.T.M. publication, A Guide to the Use and Procurement of Teaching Aids for Mathematics, there is presented a basic list of books, periodicals, pamphlets and charts that should be present in the mathematics library. This list is presented in Appendix "B".

CHAPTER V

MODELS, INSTRUMENTS AND MANIPULATING DEVICES

Models, instruments, and manipulating devices are all materials that bring into use the third dimension in the mathematics class. There are those who claim that since mathematics is a discipline which is in the main, concerned with concepts and principles in the realm of abstract thinking, the use of referents, particularly concrete ones, should be used sparingly. Such a point of view is not a criticism of teaching aids, but rather an indication of a lack of acquaintance and understanding of their possibilities.[7]

The student of mathematics can much more easily understand the workings of a pyramid if a model of the pyramid can actually be handled rather than seeing it illustrated in 2-dimensions on the chalkboard. Actual handling of a sextent to measure the height of an object is often more effective than the pretended operation in class work. Models that can be taken apart and viewed piece by piece present more realism than usually possible through imagining the shape of the components. There are many of these visual aids that can greatly enhance the teaching of mathematics.

The Viking Company, 113 South Edgement, Los Angles 4, California, produce many visual aids for demonstrating geometrical concepts. External shapes, internal dimensions, concealed edges, and enclosed intersecting planes can be seen clearly. Hidden edges, and intersecting surfaces, and penetrating solids are made with differentiated brilliant colors to promote better understanding and faster learning. A model is available to demonstrate almost all

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[7]Berger and Johnson, p. 3.

the figures with which a teacher could work.

A teacher can stimulate many ideas by simply looking through the catalogs by Viking and Company, or a catalog from another similar company. A mathematics teacher could construct some of these models, on his own. If funds are available, the teacher can select the particular models for purchase from various companies.

The Edmund Scientific Co., Barrington, New Jersey, has a catalog on science, mathematics and optics (catalog, 641).

Another such company useful in the selection of visual apparatus for mathematic's teaching is the Welch Scientific Company, 1515 Sedgweck Street, Chicago 10, Illinois. Welch is more concerned with manipulating devices and charts than with models.

The La Pine Scientific Company, 6001 South Knox Avenue, Chicago 29, Illinois, state as advertisement in their catalog the following statement.

> Certainly every teacher should be able to draw three-dimensional objects on paper and on the board. And students, too, must be taught to express three-dimensional ideas by drawing. But models are absolutely essential to the mathematics classroom. They provide an accurate concrete representation of the object being presented.[8]

Geometry teachers can encourage students to make models of figures for their own personal use. Not only do the students gain insight into these geometric figures, but many times the teacher will end up with some home-made models for use in later classes. A display of these models will sometimes encourage other students to make better ones.

Class time spent in the use of such instruments as the sextent, level, transit, etc., can give much added meaning to the student in such areas as measurement. A field trip could be taken to a location

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[8]La Pine Catalog, Models and Aids for Teaching Mathematics, (La Pine Scientific Company, Chicago 29, Illinois).

where these instruments are being used. Then the students could go out and actually use the instruments themselves.

Included in the area of models, instruments and devices, are charts. A Venn diagram chart can be used to show more about sets. A set of tables is needed in working with logarithms and trigonometric functions. Certain graphs can easily be illustrated on a chart. There are other unique uses of the chart such as for graphs and general information. Problems could be worked in advance on a series of charts and presented for study in class. Figures could be placed on a chart and then used over and over.

Figures one through eight (1-8) show some commercially made models, instruments, and manipulating devices. These could suggest to a teacher some materials that could be made for the mathematics classes.

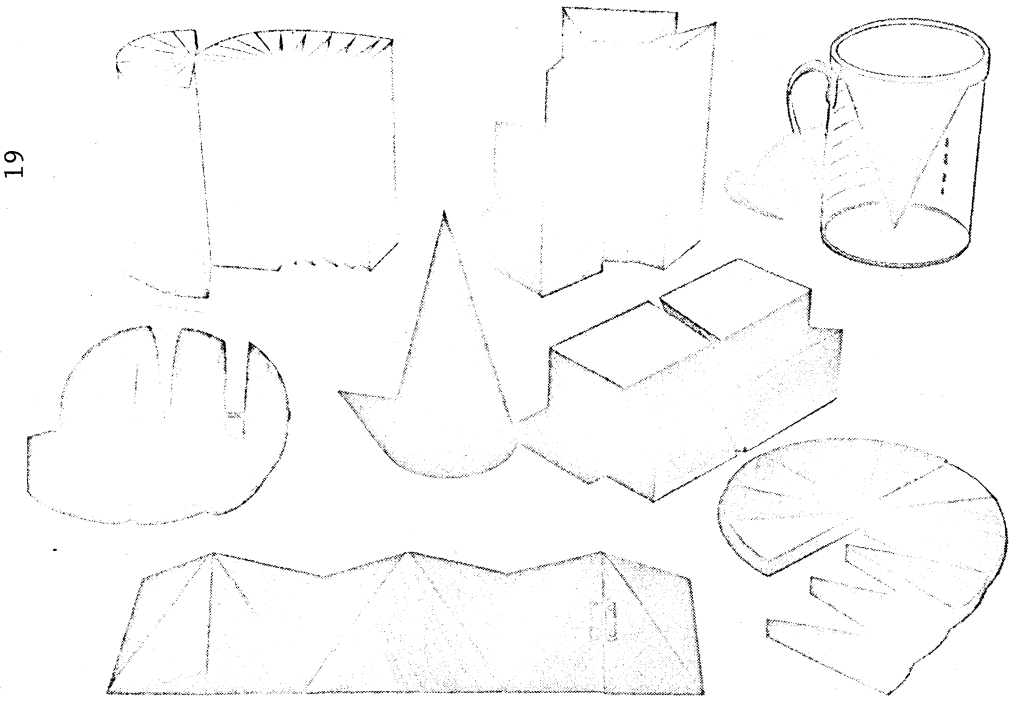
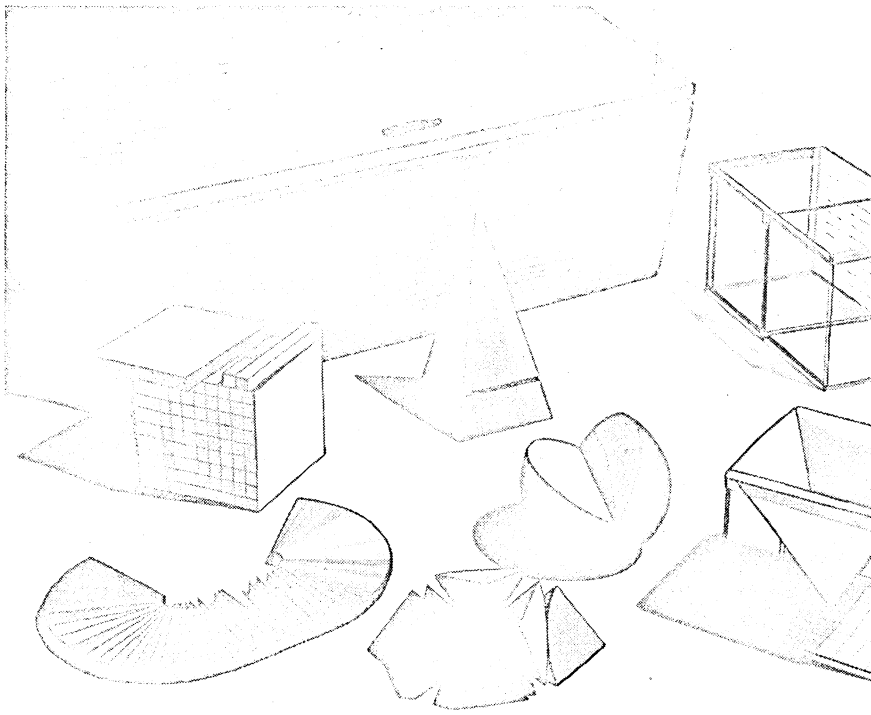


FIGURE I

SET #217 VIKING DIMENSION & CAPACITY MODELS

1. Demonstration apparatus for dimensions of trapezoid.
2. Demonstration apparatus for volume of cube.
3. Demonstration apparatus for the ratio of the circumference of a circle to its diameter.
4. Demonstration apparatus for dimensions of circle, and fan shape.
5. Demonstration apparatus for volume of triangle pillar.
6. Demonstration apparatus for volume of column.
7. Demonstration apparatus for volume of parallel hexahedron.
8. Model of right square cone.
9. Demonstration apparatus for the composition of square cone.
10. Demonstration apparatus for volume of square cone.
11. Demonstration apparatus for volume of square cone.



DIMENSION & CAPACITY MODELS (Continued)

12. Model of cone.
13. Demonstration apparatus for volume of cone.
14. Experimental apparatus for volume of sphere.
15. Demonstration apparatus for volume of sphere.
16. Demonstration apparatus for volume of sphere.

This **QUALITY SET** comes in a beautifully finished box for extra safe and storage. When filled with the complete set it weighs 15 pounds, (e weighs 7 pounds) and measures 20 inches by 13 inches and is 7 inc This set consists of 13 geometrical models made of attractive seaso wood. The average height of each model is approximately 5", though much larger. All models can be dismantled (as shown in photographs clearly show the construction of each solid. Three of the models are sturdy, unbreakable plastic and metal, and may be filled with sand o show capacity. Each model individually boxed and labeled for quick iden

FIGURE II

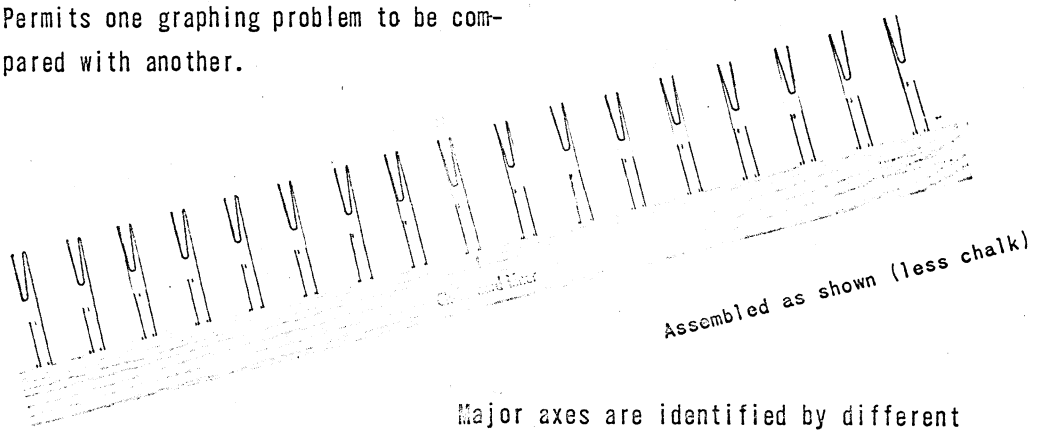
CHALKBOARD LINER

Excellent for: 1. COORDINATE GRAPHS
2. NUMBER LINES

One graph on the board in seconds, many graphs in minutes.

The easy way to make graphs on the chalkboard.

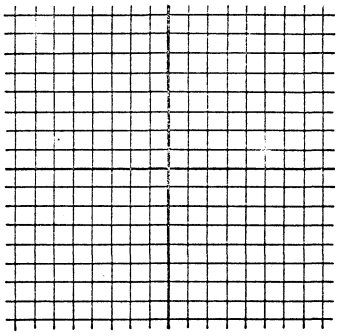
Permits one graphing problem to be compared with another.



Assembled as shown (less chalk)

Major axes are identified by different colored chalk.

GRAPHS



Permits a step by step development of a mathematical concept.

Two simple movements of the hands and the graph is completed.

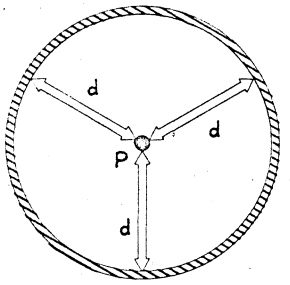
Normal spacing of lines: 2-inches, for easy classroom use. This may be increased in multiples of 2-inches.

Permits several students to graph simultaneously.

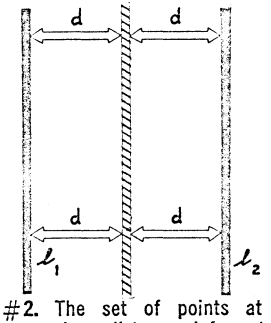
An essential tool in the teaching of mathematics from grade 1 through college.

FIGURE III

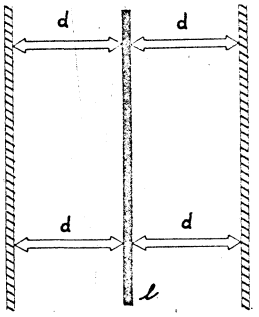
9 fundamental LOCUS problems can be presented and demonstrated.



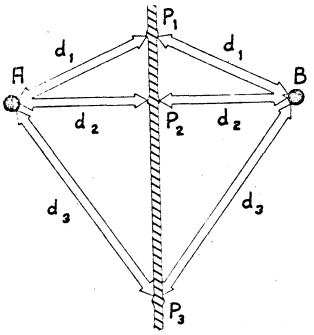
#1. The set of points at a given distance **d** from a given point **P**.



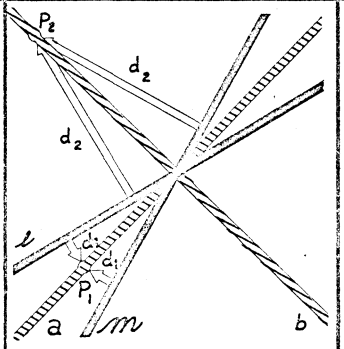
#2. The set of points at a given distance **d** from two given parallel lines l_1 and l_2.



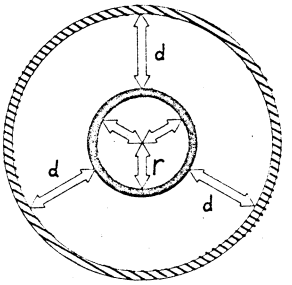
#3. The set of points at a given distance **d** from a given line **l**.



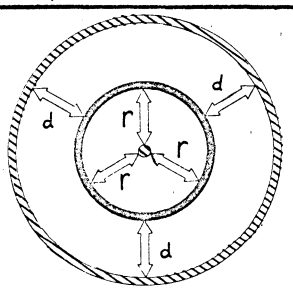
#4. The set of points equidistant from two given points **A** and **B**.



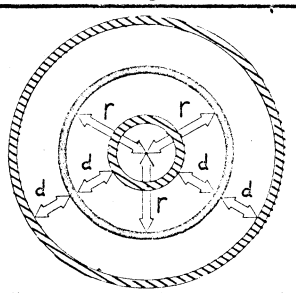
#5. The set of points equidistant from two given intersecting lines **l** and **m**.



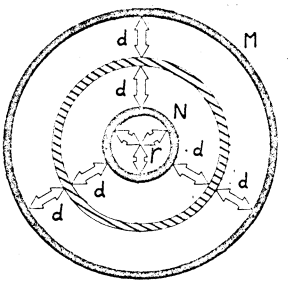
#6. The set of points at a given distance **d** from a given circle, when the given distance **d** is greater than the radius **r** of the circle. $d > r$



#7. The set of points equidistant from a given circle, when the given distance **d** is equal to the radius **r** of the given circle. $d = r$



#8. The set of points equidistant from a given circle, when the given distance **d** is less than the radius **r** of the circle. $d < r$



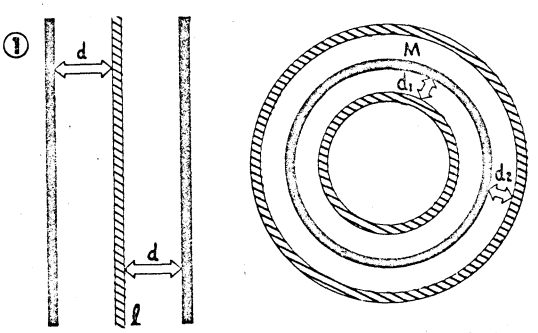
#9. The set of points equidistant from two given concentric circles, **M** and **N**.

This **LOCUS** kit was developed by Don C. Cherrington, head of the Math Dept. at Castlemont High School, Oakland, Calif.

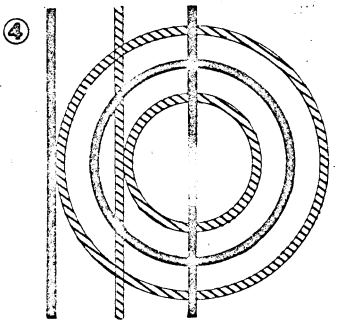
FIGURE IV

Typical example of a compound LOCUS problem.

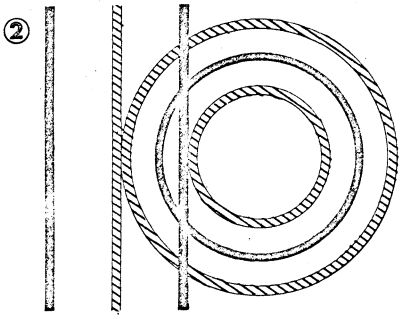
To determine the set of points which are equidistant from two given parallel lines, and a given distance, d_2 is less than the radius of the given circle. Possible solutions:



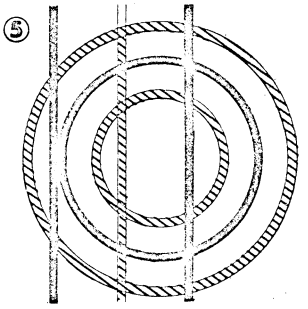
Null or empty set (no solution)



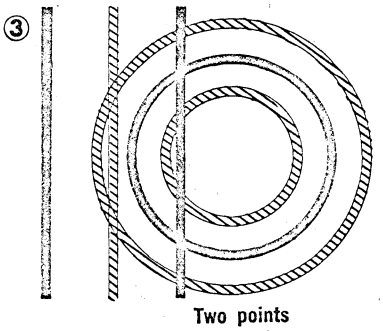
Three points



One point



Four points



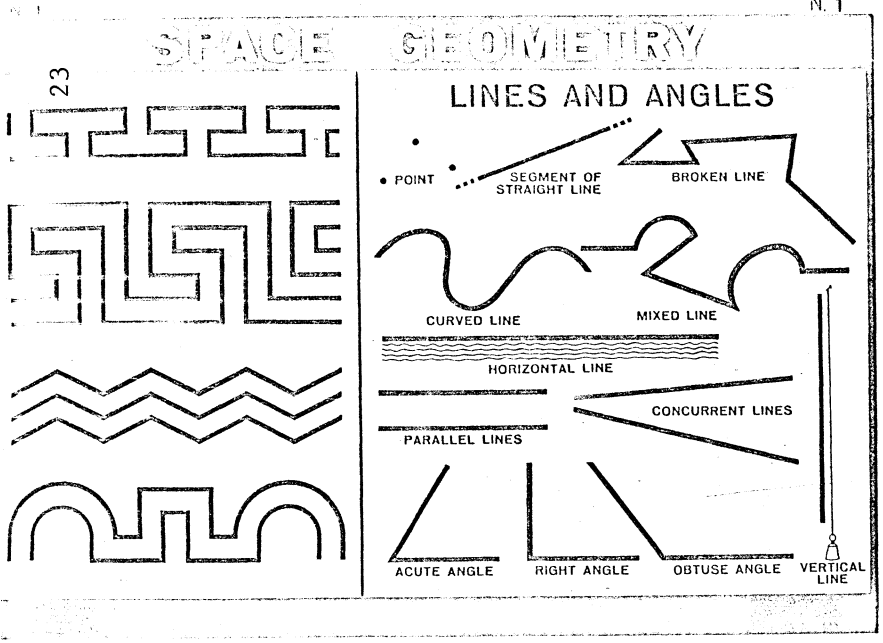
Two points

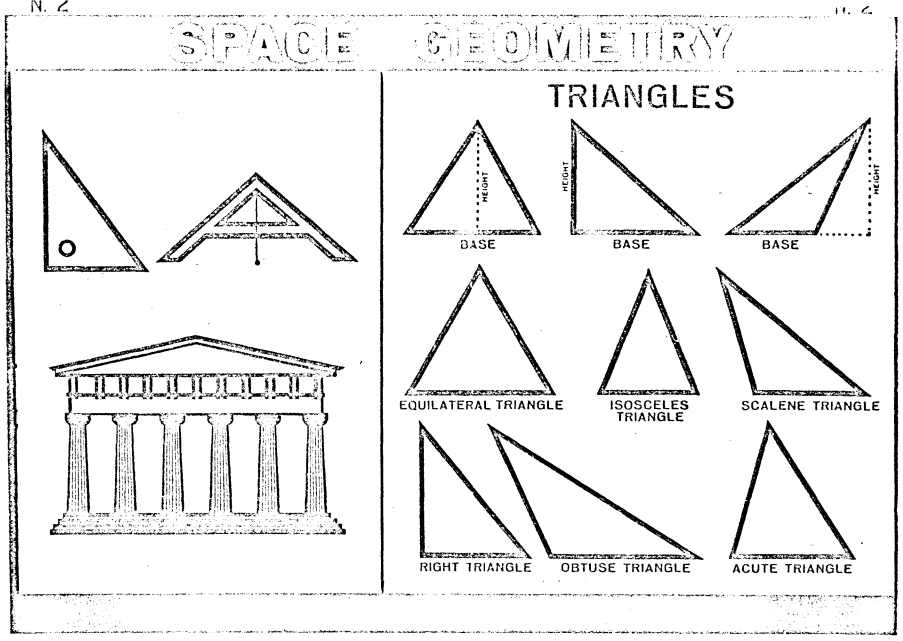
COLOR CODE:

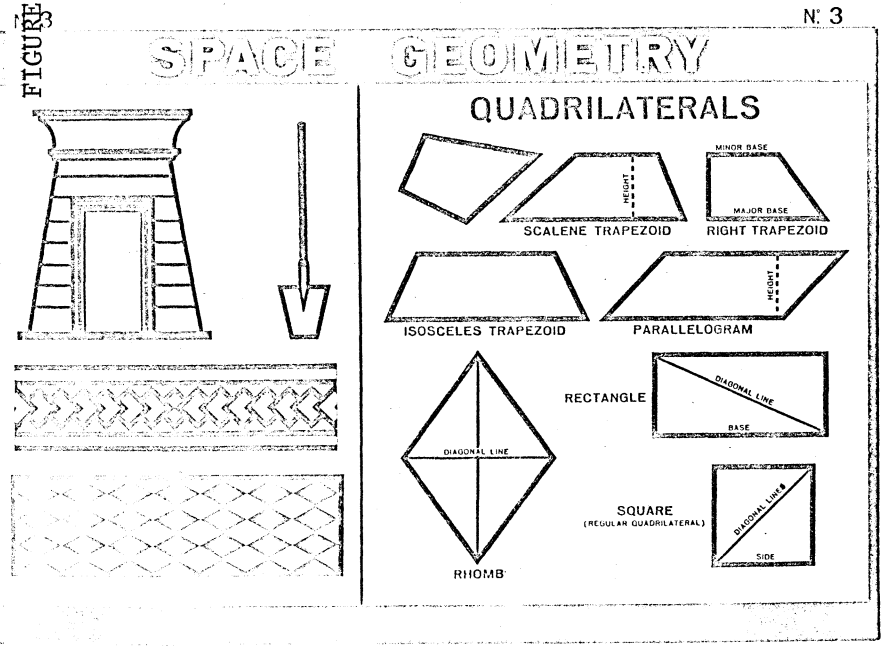
Black

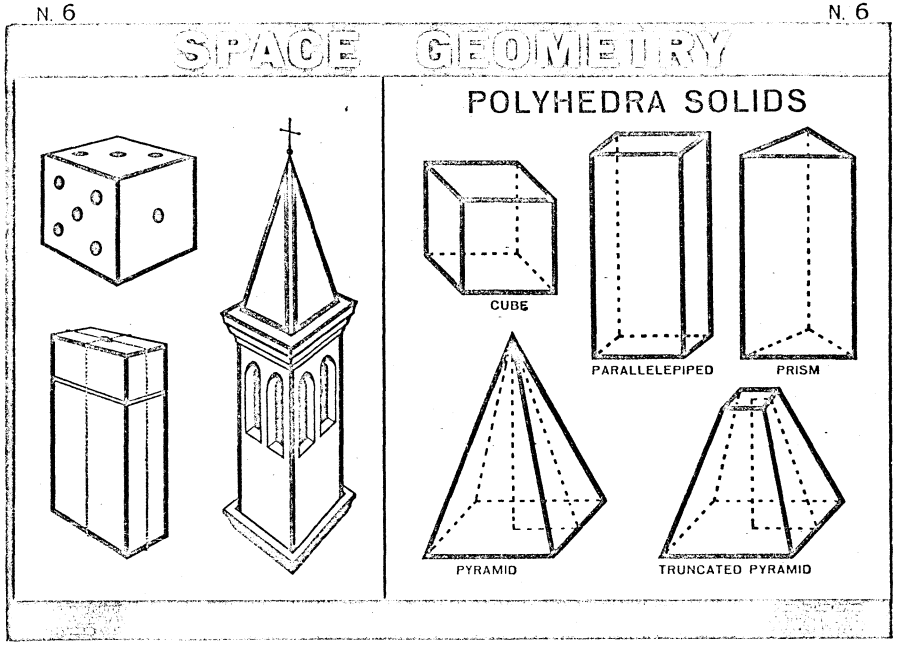
Red

Green









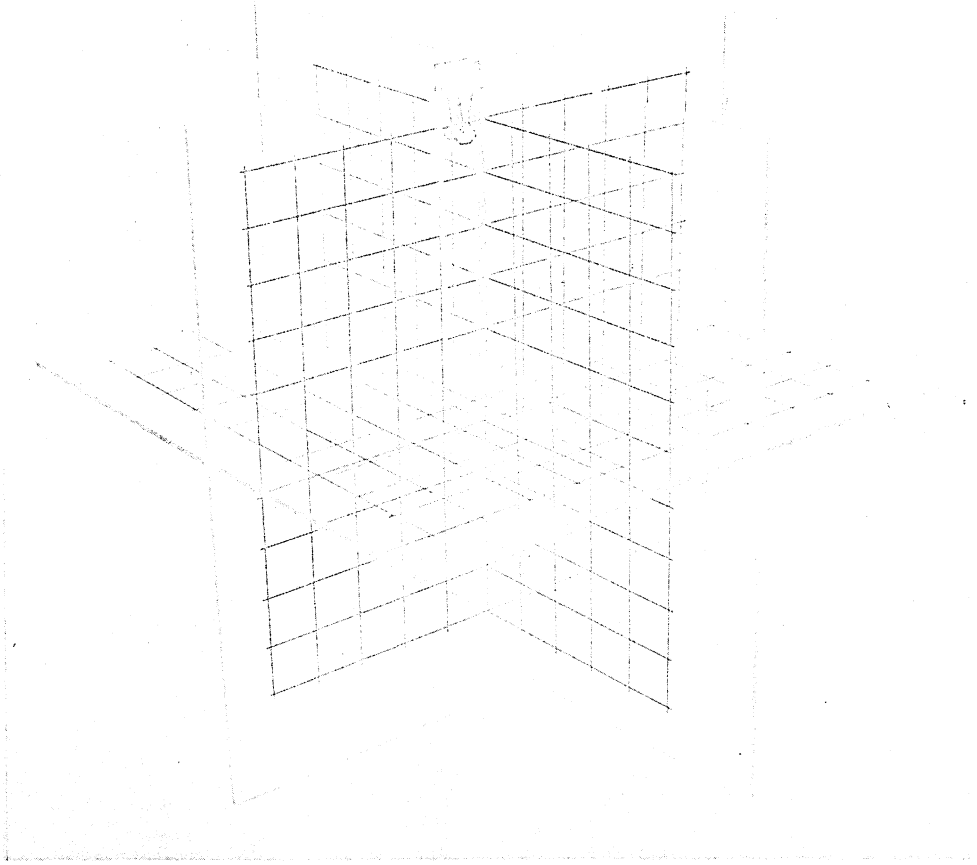
10 CHARTS COMPRISE A SET, AND ILLUSTRATE THE FOLLOWING

1. LINES AND ANGLES
2. TRIANGLES
3. QUADRILATERALS
4. POLYGONS
5. PLANE FIGURES
6. POLYHEDRA SOLIDS
7. ROUND SOLIDS
8. AREAS & VOLUMES OF POLYHEDRA SOLIDS
9. AREAS & VOLUMES OF ROUND SOLIDS
10. REGULAR POLYHEDRA

FIGURE VI

3-DIMENSION GRAPH

FOR HIGH SCHOOL AND COLLEGE -MATHEMATICS TEACHERS

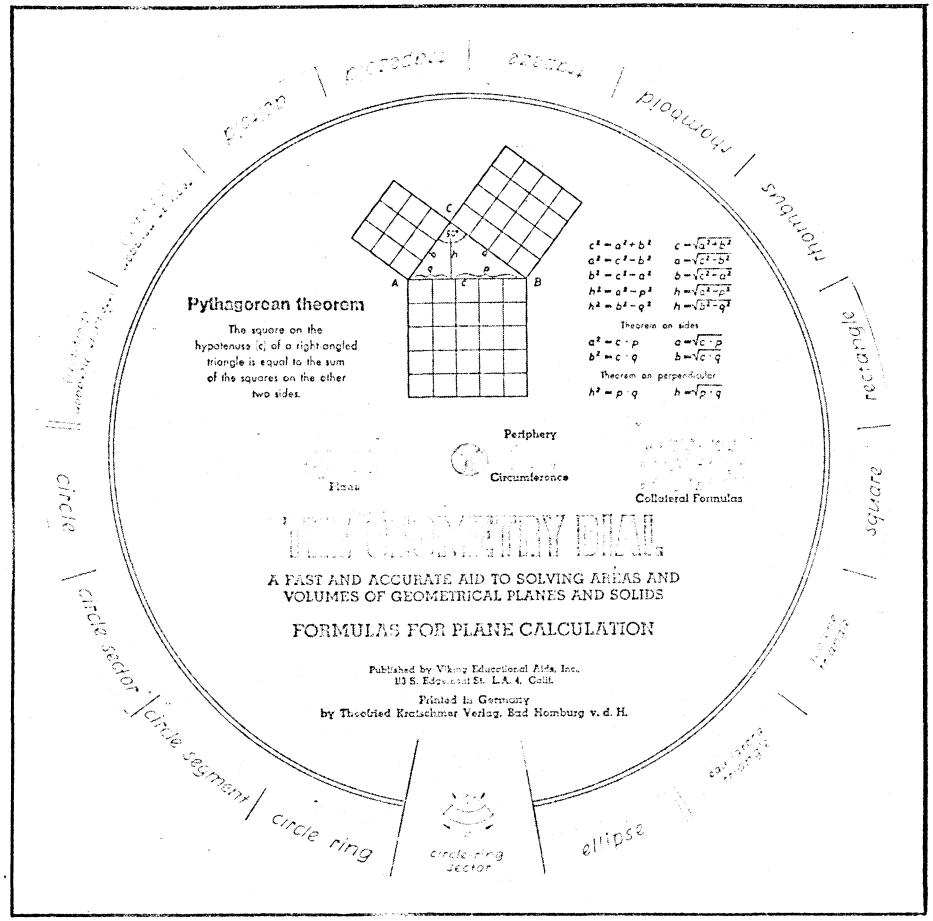


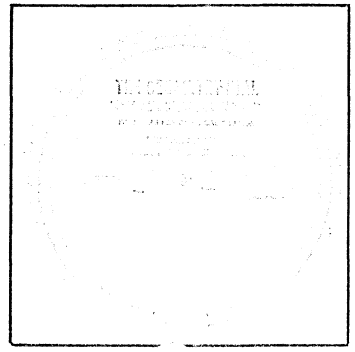
- Graph REAL and IMAGINARY values on the same model
- Graph in THREE dimensions -- x, y and z

©GEOMETRY DIAL

Side One: Gives Formulas for Plane Calculations.
Side Two: Gives Formulas for Cubic Calculations.

FIGURE VII





This new exclusive Geometry Dial is a fast accurate aid in solving areas and volumes of geometrical planes and solids.

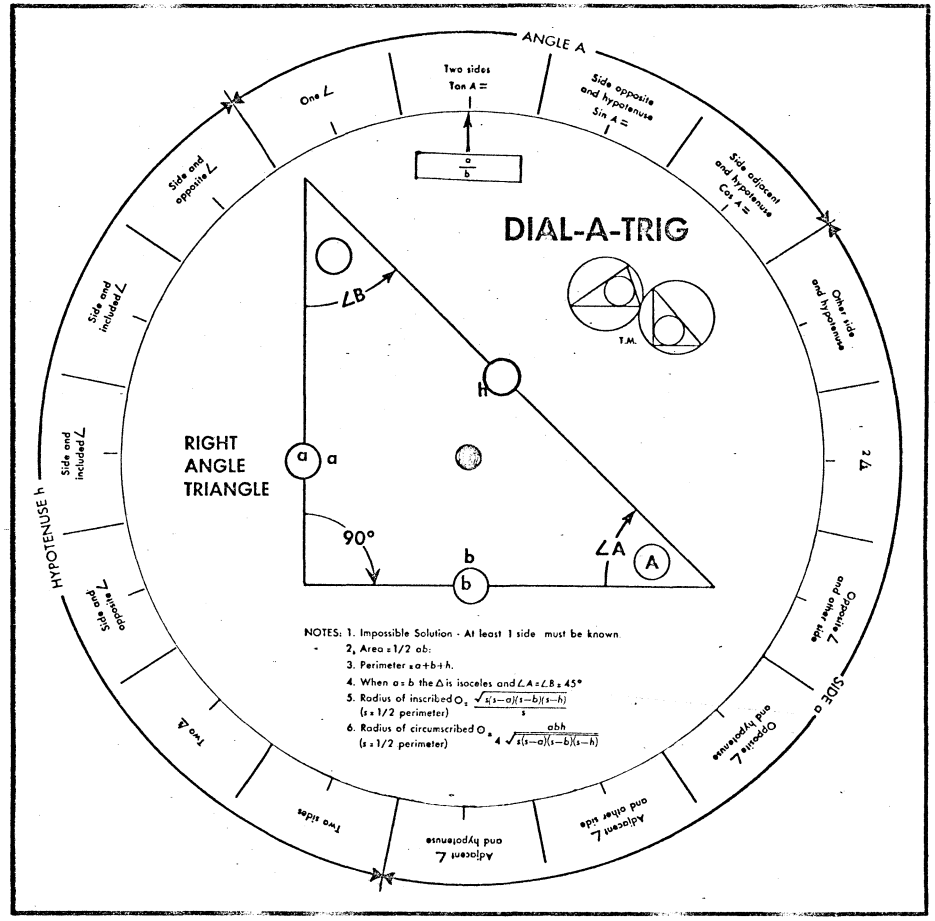
Easy to use, the information contained on this dial represents the consolidation of 100 pages of Geometry text.

Only $1.00 ppd.

(Back)

*DIAL-A-TRIG

Side One: Has all the solutions of the Right Angle Triangle.
Side Two: Has the solution to the oblique Triangle.



Dial-A-Trig is a condensation of all trigonometric formulae in a form that permits the user to select the verbal descriptions of the problem, dial to it, and the visual depiction of the problem is presented in 2 colors and the answer appears on the window. Nothing else like it on the market today.

Only $1.00 ppd.

FIGURE VIII

Instructions and Suggestions
for using your

TRIGTRACKER

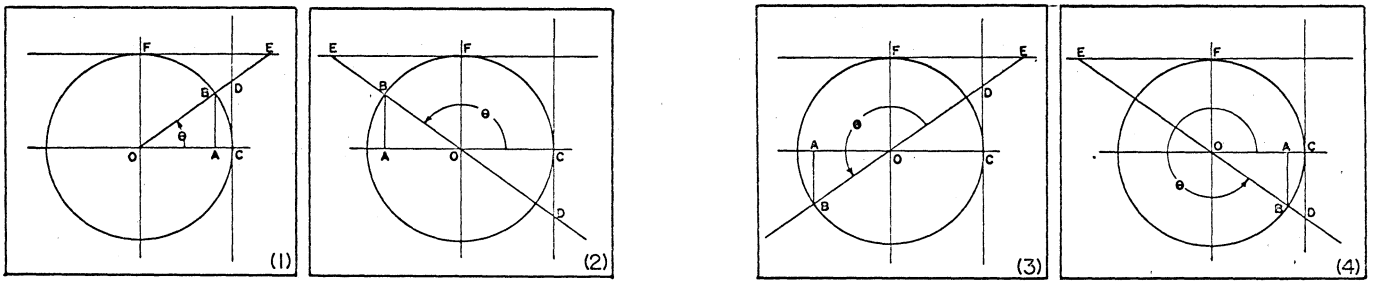
For Easier Learning

TRIGTRACKER is designed to give the student of trigonometry a clear picture of the continuous change in the functions of an angle as the angle changes. Throughout the trigonometry course, TRIGRACKER may be used to assist in the understanding of:

COORDINATES. Rectangular coordinates of a number of points are given. In a unit circle the radius vector is always unity, hence the polar coordinates of any point on this circle are $(1,\theta)$ where θ is the angle formed by the radius vector and the positive part of the x-axis.

TRIGONOMETRIC FUNCTIONS. The functions of the commonly used angles can be determined by using the proper rectangular coordinates in conjunction with the definitions of the functions of an angle in standard position.

FUNCTION VARIATION. Figures below illustrate four angles in standard position, terminating in each of the four quadrants. It can be shown by using similar triangles that the value of each function in the unit circle is represented by the length of the line segment as indicated below:



sin θ = AB *cos θ = OA* *tan θ = CD* *cot θ = EF* *sec θ = OD* *csc θ = OE*

The terminal side of the angle is represented on the TRIGTRACKER by a movable radius strip. The ordinate strip is kept vertical at all times when studying variations of sin θ and cos θ. Align it with the radius strip when studying the variations of the other functions.

Direction is defined as: a. positive when upward from the x-axis, to the right of the y-axis, or on the terminal side of the angle; b. negative when downward from the x-axis, to the left of the y-axis, or on the extension of the terminal side back through the origin.

Revolve the terminal side of θ counterclockwise from 0° to 360°, and note how the length of the line segment representing each function varies as the angle increases. Results can be tabulated and used to sketch the graphs of the functions.

REDUCTION FORMULAS. TRIGTRACKER can be used to generalize the formulas for expressing functions of any angle in terms of an acute angle.

FUNDAMENTAL IDENTITIES. The fundamental identities can be quickly illustrated and recalled by referring to the similar right triangles in any of the above figures.

CHAPTER VI

FILMS AND FILMSTRIPS

The film could be a useful media for the mathematics teacher. Dale states seven reasons justifying the use of films in class.

1. Supply concrete basis for conceptual thinking.
2. Have a high degree of interest for students.
3. Make learning more permanent.
4. Offer experiences which stimulate self-activity.
5. Develops a continuity of thought.
6. Contributes to the growth and development of the desired concepts.
7. Provide experiences not easily acquired in the classroom.[9]

Most schools can provide adequate facilities necessary for good film showing. Projectors and screens should be available so they can be obtained with only several days notice. Rooms should be equipped with shades, drapes, or blinds that will sufficiently darken the room.

Schools should try to develop a student "Projectionist Club". Interested students could be responsible for moving and setting up equipment, checking to see if equipment is in working order, and removing equipment when the teacher is finished with it. This does not mean that the teacher should not check the operation of the equipment. Most troubles can be caught by the students.

There should be a place provided for previewing the films. The teacher should not show a film that has not been previewed. The teacher should be able to point out beforehand any unfamiliar terms

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[9]Edgar Dale, Audio Visual Methods in Teaching, Dryden Press, New York, 1954, p. 65.

and new vocabulary used in the narration of the film.

Filmstrips can also be a very important tool for mathematics teachers. The fact that there is not a set running time for filmstrips, as compared to the predetermined speed of films can make filmstrips very useful in that as much time can be spent on each frame as the teacher sees fit.

Both films and filmstrips are valuable in that they may show a great deal of material in a short period of time. Many things can be included in films and filmstrips that the teacher of mathematics could not possibly have time or skill to prepare. Some of the practical applications of mathematics can easily be shown and thus increase the interest that can be stimulated towards mathematics.

The teacher should use care in the selection of films. The practice of showing a film just because it is available is often without justification. Showing a class a film for the sole reason that the teacher is not going to be in class is another common but unjustifiable practice. Films should be carefully selected as to content, presentation, age group preference, technical quality, and authoritativeness. A careful preview is the first step the mathematics teacher should take in this selection process.

After looking through the summaries presented in the many film catalogs or records of other mathematics teachers, the teacher picks out the films that warrant a preview. These should be previewed to see if they are applicable for use. Only the films that are applicable should be used in class. After the films have been shown to a class these questions should be asked. Did the students understand the film? Did it motivate them for further study? Did

it add something to the students' store of knowledge about mathematics? Did it clarify some hard-to-see idea about mathematics? Did they enjoy the film from a mathematical standpoint and not simply for mere entertainment? The answers to these questions will tell if a film accomplished the purpose the teacher intended it to accomplish.

Class discussion on the film should be part of the follow-up activity. This discussion can also determine the extent that the above questions have been answered. Let the students criticize the film to find its strong and weak points. Observation of the students is another aid in answering these questions. See if the motivating factors of the film has led them to further interest or study.

A record should be kept on file by each teacher of the films and filmstrips previewed and the ones used. An effective method of keeping these records could be one similar to the following methods used in audio-visual classes at Eastern Illinois University. The following are on a 3 x 5 card.

Front side

Title:

Producer:

Sound or Silent: Black & White or Color

Photography: poor, fair, good, excellent

Sound: poor, fair, good, excellent

Grade level:

Purpose:

Reverse side

Summary:

Comments:

In addition to these headings, there will be others that should be recorded for future reference. These will be questions answerable only after a class has viewed the film or filmstrip. The questions as to understanding, motivation, etc., are the headings answerable now.

In a period of years the teacher can have a file that will help very much in the selection of films and filmstrips for the incoming classes. There will be new material to preview, tryout, and record the results.

Vollmar and Peak have devised a check list for the effective use of film. They state that probably no teacher will find that he can answer yes to all questions in the list, but it should give him suggestions on which to base his use of films. It is best used by the teacher as he plans his work; only at that time can he make provision for obtaining the desired results as listed. Also, he should review the list after using the film in order to compare the results with his original plans.

1. Do I have a purpose for using a film?
2. Have I used the available sources for obtaining pre-preview information about films?
3. Have I carefully selected from available information the film to preview?
4. Will the arranged time for previewing allow for planning?
5. During the preview I must consider the following:
   a. Does the vocabulary fit with that used in my class?
   b. Is the level of the film in other aspects appropriate for my class?
   c. Is the film environment sufficiently familiar to my class so that it will not interfere with the objectives?
   d. Is the desired objective presented clearly and prominently in the film?
   e. Have I carefully checked the summary to see if it will aid the pupil in grouping and clinching his ideas?

f. Have I selected interesting and valuable information to present to the class prior to the showing so that the value derived from the film will be increased?
g. Have I noted the technical aspects of the film that may require special attention during the screening?

6. Does my plan of use provide an alternate plan in case of poor timing or an unavailable film?
7. Does my plan provide for directed instruction before as well as after showing the film?
8. Does my plan provide for adjusting the objectives to fit changing needs?
9. Does my plan for the showing include provision for student questions, for pointing out important items in the film, for reshowing of difficult scenes, and for other desired activities?
10. Do I have in mind some suggestions and available materials for the student who wishes to continue his study beyond that given in the film?
11. Does my plan provide for obtaining student reaction and evaluation of the film?
12. Does my plan include a method for noting students' activities and attitudes which result from the film?
13. Have I made provision for keeping a permanent record of pertinent information in relation to this film as a teaching aid?[10]

Another necessity in the use of films and filmstrips would be preparing the students for what they will see. They should be familiar with the mathematical terms used in the showing. Many times a slightly different word or method would have confused a student if the teacher had not made them aware of the differences before showing the film or filmstrip.

Students should be familiar with the subject matter presented. It is the responsibility of the teacher to determine this before the film or filmstrip is shown.

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[10]Robert Vollmar, Philip Peak. "How to Use Films and Filmstrips in Mathematics Classes." (National Council of Teachers of Mathematics, Washington 6, D.C., 1956).

It is best to have a little freedom as to the exact day the material is used. This might rule out films that are so specific that they can only be used at one exact spot in a unit. The film selected should be versatile enough that they could be shown at various points in the unit. Some good mathematical films are versatile enough that they could be used at the beginning of a unit for an introduction and interest builder; at the middle to clarify certain points, and/or at the end of the unit for summarization purposes.

Preparation would also be necessary in securing the equipment. The teacher must be assured that the projector, screen and speaker will be available and in working order at the specified time. The film must be there and in good condition.

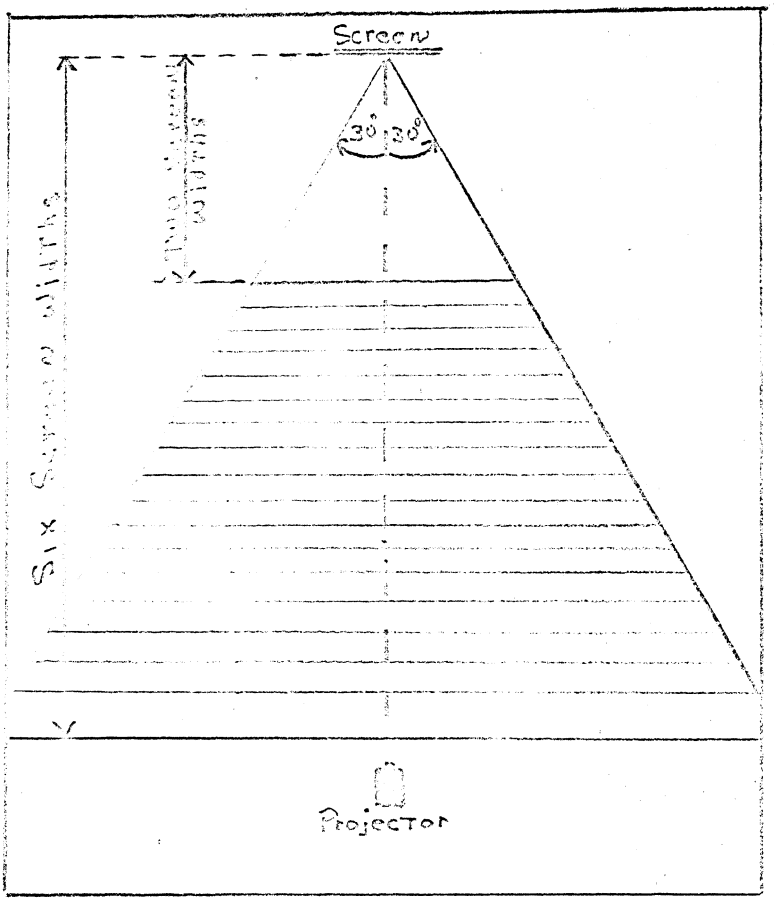
The seating arrangement should be such that would give optimum visual advantages to the class. The screen should be in the darkest portion of the room.

Two typical plans are presented in figure nine (9).

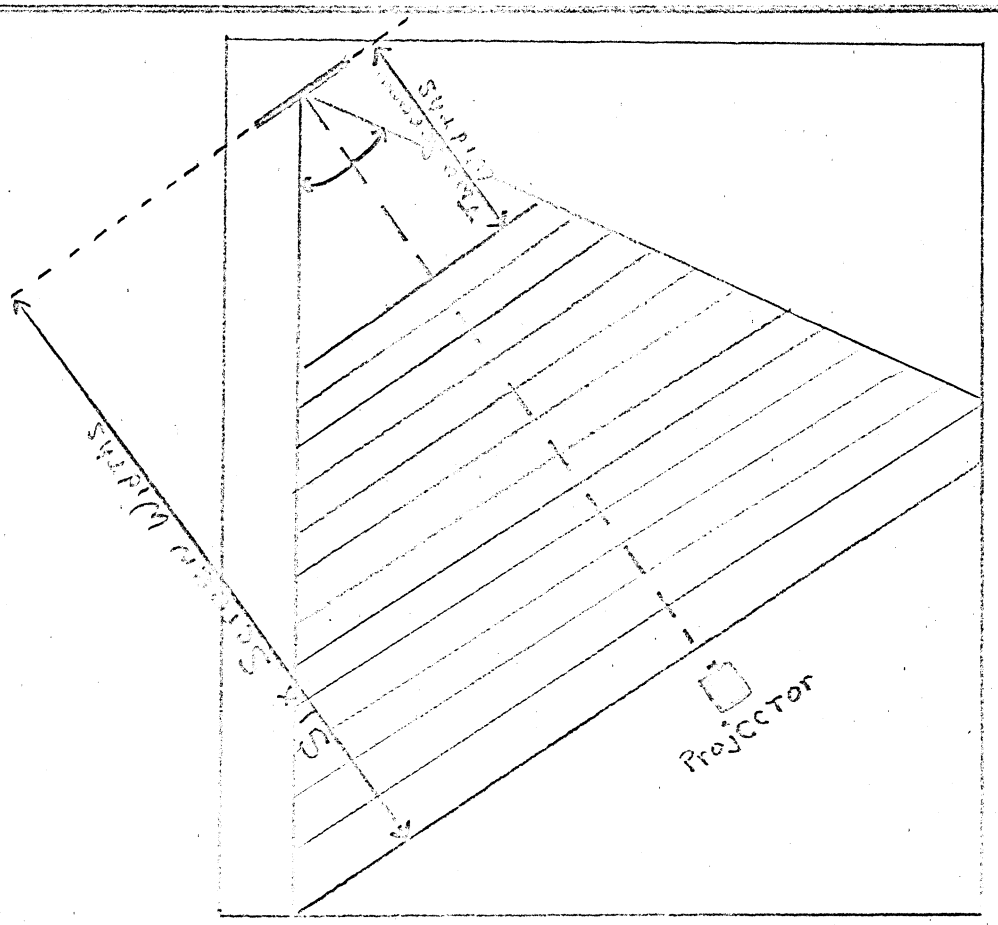
It is necessary in the showing of films and filmstrips that the room be provided with the necessary equipment to properly darken the room. A room that is too bright takes much from the effectiveness. The room need not be completely dark, but there should be no light shinning directly on the screen. This might detract students' attention from contents of the film and especially reduces the effectiveness of color films.

It is also necessary that the room be comfortably heated or cooled. Too warm a room may produce sleepiness. A good film that will enhance the teaching-learning situation is worthwhile. If there is another media that could have done the job better, that media should be used.

FIGURE IX



Two typical plans for assuring satisfactory seating arrangements for showing films, filmstrips, or other visual projections. A different location of the projector will be necessary for the other visual projections.



The follow-up activities are important. Discussion of the film can clear up foggy or misinterpreted ideas the students gained from the film or filmstrip. Re-showing of parts or all of the film can be an effective means of clarification and also give the student a chance to concentrate on the more specific details of the film. Quizzing the class on the main ideas will sometimes promote interest in some of the often unanswered questions in the film.

Sources of Films and Filmstrips.

Many of the schools now are establishing audio-visual centers. If a teacher is in a system that has an active audio-visual center, this is the place to find films on mathematics. If the audio-visual center is not active, it is the responsibility of the teacher to seek out and obtain good films and filmstrips suitable for use in the mathematics class.

An older subject guide of films was published by H. W. Wilson Company, New York:

Educational Film Guide (1953 and annual supplements)

Filmstrip Guide (1954 and annual supplements)

These indexes may still be found in the public library and school systems. They are often available in the audio-visual centers of colleges and universities. The newer guide is the Educational Media Index, published by McGraw-Hill. This Index replaced the Educational Film Guide after 1962.

Educators working in school systems that use commercially sponsored educational aids will find the following guides useful. The following are published by the Educators Progress Service, Dept. St., Randolph, Wisconsin: Educator's Guide to Free Film and Educator's Guide to Free Slide Films.

Mathematical films are often available from the United States Government. For further information on how to secure these listings, write to the Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C.

In Canada, the central government source of films and filmstrips is the National Film Board, which not only produces, but distributes through public libraries and other film depots, motion-pictures. The subject matter includes films in specialized areas such as mathematics. Canadian films available in the United States are listed in the catalog obtainable from National Film Board of Canada, Canada House, 680 Fifth Avenue, New York 19, New York.

The state universities often have film rental libraries available to state residents and often available to residents of other states. For further information and/or a catalog, address an inquiry directly to the film libraries of your state universities.

The Audio-Visual Aids Service at the University of Illinois has one of the largest and best collections of educational films in the United States. They are available on a rental basis. A 1962-1964 catalog may be secured for the areas of science and mathematics. In this catalog there are seventy-seven different films concerned with mathematics. Many other films are listed in related areas of science that could be applied to mathematics. Each film listed has a short summary in the catalog along with the suggested age level, time, black-white or color, and rental cost. Southern Illinois University also has a film rental service.

In Appendix "C" are listed the seventy-seven titles of the films obtainable from the University of Illinois. The purpose of this list

is to show the wide range of topics they cover. Several samples of summaries given next show how much information is available without previewing the film.

> Donald in Mathmagic Land (wdp)
> i-j-h- 26 minutes color; $8.25
> The film is designed to create a new appreciation of the value and importance of mathematics in our daily lives. Donald learns the importance of mathematics from the early Greeks who first discovered some of its basic principles. In successive sequences these principles are related to music, art, architecture, mechanics, sports, and other phases of our daily lives.
> Meaning of Pi (c)
> i-j 10 minutes bw; $2.15
> Develops an understanding of this important mathematical ratio and outlines a study procedure by which the numerical value of pi can be checked and reviewed. In careful step-by-step procedure, the numerical value of pi is arrived at. Sequences showing the use of circles in art, industry, and commerce help to fix the value firmly in the student's mind.
> Understanding Solids in Geometry
> h-e-a- 17 minutes bw; $4.15
> Solid geometry is reviewed in the opening part; however, a new device for construction of geometric figures is introduced. The importance of geometry to industry and society in general completes the treatment of the subject.[11]

The Wilson Educational Film Guide lists several mathematics films that are not available from the University of Illinois. They are: Periodic Functions, (illustrates the graphing of sine angles; relates sine wave to the amount of voltage produced by a generator.)
Introduction to Vectors: Coplanar Concurrent Forces, (Explains the meaning of scalar and vector quantities and how to add them; explains the various methods of vector composition and vector resolution; and shows how vectors may be employed to solve engineering problems.)
Vectors, (Vectors, changes in angle or magnitude, how vectors are ploted, and how the resultant is found are demonstrated.)

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[11]University of Illinois Film Library, Science and Mathematics Catalog, 1962-1964; pp. 22-23.

The filmstrip guides has mathematical filmstrips that are available. They are:

Thinking in Symbols: (Groupings Symbols and Order of Operations; Geometric Figures; Measurement; Variables and Coordinates; Mathematics in Daily Living).
Mathematics in Aviation: (The Compass; Mathematics in Aviation; WIND Drift; Indirect Measurement; Systems of Equations; Slide Rule (Parts 1-2.)
Basic Algebra Filmstrips: (Introduction to Algebra; Graphs; Introduction to Signed Numbers; Formulas; Equations; Addition and Subtraction of Signed Numbers; Multiplication and Division of Signed Numbers).
Living Geometry Filmstrips: (Geometry in Nature; Geometry in the Home; Geometric Solids in Nature and in Architecture).
Plane Geometry Filmstrips: (Areas; Basic Angles and experimental Geometry; Basic Triangles; Common Tangents and Tangent Circles; Congruent and Overlapping Triangles; Introduction to Circles; Introduction to Demonstrative Geometry--Axioms, Theorems, Postulates; Introduction to Plane Geometry; Loci; Parallel Lines and Transversals; Quadrilaterals; Similar Polygons).
Using Geometry Filmstrips: (Angles; Geometric Form; Solids; Geometric Form: Points, Lines, Planes; Using a Protractor).
Integral Calculus Filmstrips: (Areas by Integration; Double Integrals; Triple Integrals; Areas by Integration using Polar Coordinates.[12]

Education Audio-Visual has a catalog of filmstrips available in the fields of science and mathematics. This catalog may be obtained by writing directly to Pleasantville, New York. A series of filmstrips covering basic algebra and another series on basic geometry is available.

Society for Visual Education, Inc., (SVE) 1345 Diversey Parkway, Chicago 14, Illinois, is another source of mathematics filmstrips. A catalog is available containing many new and valuable strips. S.V.E. is probably the largest producer of filmstrips in the United States.

The Film Strip of the Month Club, 355 Lexington Avenue, New York 17, New York, is still another source of mathematics filmstrips. The

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[12]H.W. Wilson Company, Educational Film and Filmstrip Guide. (H. W. Wilson Company, New York 52, New York, 1954-1962.)

cost for each school is $33.00 per year and each subscriber is entitled to nine filmstrips and a teaching guide for each. The filmstrips for the months September 6, 1963, through May, 1964, in the High School Mathematics Club were:

1. The Why of Simplifying Expressions
2. Subtraction of Signed Numbers
3. Equations with Fractions
4. Introduction to Irrational Numbers
5. Perpendicular Lines in Coordinate Geometry
6. The Parabola
7. Area in Coordinate Geometry
8. Graphing Inequalities in Two Variables
9. Arithmetic Series[13]

Since these were published for months of 1963-64, all nine will be sent in one package. The filmstrips for September 1964, through May 1965, will be sent one per month and they are:

1. Solving Inequalities
2. Absolute Value
3. The Parallel Postulate
4. Angle Sums for Polygons
5. Solving Two Linear Equations Algebraically
6. Direct Variation
7. The Law of Sines
8. Locus Problems: The Circle
9. Geometric Series[14]

In the N.C.T.M. publication on films and filmstrips, there is a list of publications which could aid in the teacher's search for good mathematics films. (See Appendix "D" for the above publications list.)

During 1962, several committees of high school and college mathematics teachers met on several occasions to review films for junior and

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[13]Film Strip of-the-Month Club. 355 Lexington Avenue, New York 17, New York.

[14]Ibid.

senior high school use. Meetings were financed by the National Council of Teachers of Mathematics. These mathematics teachers viewed approximately 160 of the then available mathematics films. For each film a critique was submitted. They have been altered only slightly in the Mathematics Teacher publication to save space. These reviews in Mathematics Teacher have been put in booklet form by N.C.T.M. and are available for 40¢. This booklet review of films could very easily be the proper starting place in the narrowing down process that eventually ends with the films you show in class. The reviews given in this booklet are accurate. They are not written with the idea of selling films. The teacher using these sources should still preview the films before using them. No matter how good the reviews by these mathematics teachers are, special words and situations that arise in the film must be picked out by the teacher in the preview so that the class can be prepared before they see the film.

CHAPTER VI

THE CHALKBOARD

The chalkboard is probably the most widely used visual aid in the mathematics classes. Almost every mathematics teacher uses the chalkboard. The degree of effectiveness depends upon how one uses it. It is not the intention of this paper to evaluate the mathematics teacher's methods of board work, nor is it to establish a specific pattern that should be followed. The writer feels there are many different individual techniques that are effective. It is intended to give several rules and techniques that apply to the use of the chalkboard. Some of its shortcomings will be noted.

Dale has a list of suggestions that might prove quite worthwhile to note.

1. Keep the chalkboard clean. This means periodic sponging. It means especially that a teacher should almost automatically check to see that the chalkboard has been erased before beginning a class. A clean board eliminates needless distractions and makes your own writing on it easily read from all parts of the room--a necessary standard for all chalkboard writing.
2. Make your letters and drawings large enough to be seen from all parts of the room. (Five minutes of practice will show you how to make thinner or thicker lines.)
3. Do not cover up the material on the chalkboard by standing in front of it.
4. If a drawing is complex, put it on the board before your class starts. Do not use class time wastefully. You may wish to put in a faint outline of the drawing ahead of time and build the finished drawing before the eyes of your students. Most drawings--even complex ones--should be done with a few bold strokes.
5. Do not put unnecessary and time-consuming accuracy into a drawing when accuracy is not called for. When it is desirable, use ruler, T-square, compass, protractor, stencils. Drawings that cannot be made with these tools

can be produced by means of proportional squares or by using the opaque projector and tracing in the outlines.

6. Forms that are regularly used can be painted or scored on the board. This procedure is particularly efficient for music staffs, map outlines, bookkeeping forms, cross sections, and the like.
7. Occasionally a dramatic visual presentation can be made by preparing the entire chalkboard in advance and covering it with strips of paper which will be removed, one by one, as the demonstration proceeds.
8. Do not put too much material on the chalkboard at one time unless your specific purpose makes this necessary. Remember that the board is a display, a showcase; clutter must be avoided.[15]

One cannot list just exactly which materials will work best in a given situation. It is the responsibility of the teacher to select that material which can be used most effectively under the given circumstances. The reaction of the class towards the different materials should be noted. Which one held their attention best? Which one seemed to give them the most retention? Which one seemed to stimulate them the most? After the teacher has observed the students, with these types of questions, then a judgement can be made about when to use the different media and under what situation.

To use the chalkboard more effectively, drawing aids should be used. For the mathematics teacher this means an enlarged compass, protractor, straight edge, T-square, colored chalks, and several appropriate templates. These are usually figures cut in cardboard or plywood that can be used over and over again for tracing these figures. All of these items except the templates might be purchased. The templates could easily be made by the individual teachers. They usually take the form of certain triangles, square, circles, etc. They will reduce drawing time and give more accuracy.

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[15]Edgar Dale, pp. 312-314.

Many times the mathematics teacher is faced with the problem of difficult drawings, such as trying to represent three dimensional figures on the two dimensional chalkboard. Rather than to take up valuable class time with these detailed drawings, it is a good practice to construct them before the class meets.

Time can be taken for more accuracy and better emphasis. They may be left on the board for several days or until you are finished with them. Thus they save the teacher from drawing several hasty figures.

A process called "pounce drawing" is quite effective in the outlining of complex figures. The original figure can be drawn on a tough but light weight paper material, or old window shade material. Holes would then be punched out on the outline of the figure to form a dotted line using a quarter-inch metal punch. The material is now ready for repeated use as a drawing aid. It could be held up against the board and an eraser full of chalk dust would then be "pounced" on the pattern of holes leaving on the board a dotted line to follow when drawing the figure.

The magnetic board is a sheet of metal to which objects will adhere by means of a magnet. Some chalkboards are magnetic boards that have been painted with chalkboard paint. Many new chalkboards will attract magnets if they have a metal base. Strips with different colors representing different sizes could have small magnets attached to the back for use with this magnetic board. These strips could be arranged in desired figures and then could be moved or adjusted at will. There are many other techniques usable with magnetic boards by mathematics teachers.

In general the chalkboard is best suited for situations where spontaneity, speed, and change are important. Class work at the board is another of the uses of the chalkboard.

CHAPTER VII

THE OVERHEAD PROJECTOR

Many teachers feel that there is more potential use in the overhead projector than there is in any other instructional device now being used. The overhead projector throws a highly illuminated image on the screen by reflection from a mirror.

The first advantage of the overhead projector is that the room need not be dark. Taking notes and asking questions thus becomes easier for the students. While the teacher uses the overhead, he faces the class, thus placing him in a better position, so that he can note facial expression and reaction of the students. The same material written on the chalkboard would require the teacher to turn his back on the class. The teacher and the screen can become the center of attention for the students, since usually, the screen is located above and behind him. The teacher need not move around to new board space, often at the side of the room, or erase some previous work distracting class attention and using valuable time. Simply replace the clear plastic material with new material and the overhead is ready to use. A role of acetate material is often part of the projector. A simple turn of a crank will bring new material to the screen. When the roll is not present, extra sheets of the acetate will provide additional writing space.

The marking device used in connection with the transparent material is a grease or wax pencil. The marks are placed directly on

the acetate and their shadow appears on the screen as a line. These marks may be wiped off with a cloth, leaving the material ready to be used again.

The overhead projector is usable under many adverse conditions. It is best used when the room is slightly darkened. The teacher should make sure that the light rays of the overhead projector strike the screen in a perpendicular fashion, or some distortion will enter into your figures. This is known as the "keystone effect."

Transparencies can now be made right in the school and often in a short period of time. The commonly used office copying machine can usually make transparencies if the proper supplies are available.

Technifax Corporation, 195 Appleton Street, Holyoke, Massachusetts; A-B Dick, 5700 West Touhy Avenue, Chicago 48, Illinois; and Thermofax Visual Products, St. Paul 19, Minnesota, are three companies whose copying machines are usable for the making of transparencies.

The production of transparencies requires that three important factors are considered: conception, production, and presentation.

The mathematics teachers should conceive their needs and prepare their own transparencies. One of the commercial companies, the Technifax Corporation, will provide free training in projectual design, production, and use at its home office in Holyoke, and in 25 training and service centers in principal cities. More information on this can be obtained by writing directly to Technifax in Holyoke or to the regional offices in St. Louis or Chicago.

Teachers can easily make transparencies. Many high schools now have an audio-visual department with skilled personnel and the time to

help the teachers prepare materials. There are also many commercially made transparencies and overlays for the field of mathematics available from an increasing number of sources. With the aid of the commercially-made transparencies, the copying machines, and the services of an audio-visual department, many effective transparencies and overlays have been developed and used quite efficiently in the field of mathematics. Even color and moving parts can also be incorporated in the transparencies.

Transparencies may be used for several years. Rather than marking directly on them, it is best to lay a clear sheet of acetate over them and mark on it, thus protecting the produced transparency from possible damage. The markings can then be wiped off the clear sheet with a cloth.

Charles E. Merrill Books, Inc., 1300 Alum Creek Drive, Columbus, Ohio, has a booklet containing visual masters for the courses of Algebra I and Algebra II. These masters are for the use of preparing transparencies using the Diazo process.

Kueffel and Esser have a loose leaf book of visual masters for supplementing a course in geometry. It is quite elaborate. It costs approximately $60.00. With each series there is a guide and some comments as to how it is best used.

Robert J. Brady Company, 3227 N. Street, N. W. Washington 7, D.C., has a set of visual masters to supplement a course in trigonometry. Two titles from the series of transparencies and overlays, along with the description from the Instructor's Guide will be shown. The series Reduction Formulas and Graphical Addition of Curves, are two typical sets of transparencies concerning trigonometry.

Reduction Formulas

Functions of 90, 180 and 270 degree angles

A student's concept of angle size changes with experience and education. From special triangles--such as equilateral, isosceles, and right--he meets 30, 45, 60 and 90 degree angles. In Geometry, he learns that the sum of the angles of a triangle is 180 degrees and becomes familiar with obtuse angles. The concept of reflex angles and angles of any magnitude may not be appreciated until he studies trigonometry. Wheels, machines, missiles, and the earth itself rotate through angles of tremendous magnitude, but most trigonometric tables stop at 90 degrees. This transparency clearly demonstrated how large angles may be related to acute angles in the first quadrant. Fundamentally, they are the same angles which occur in right triangles.

The basic diagram--which relates angles of the first to the second quadrants--shows that the rotation of the triangle does not change the lengths of the sides, but only the algebraic signs of the sides. Therefore numerical values of trigonometric functions are common to those of acute angles, but their signs may change. When the terminal side of the angle is in the second quadrant, the sine remains positive, but the cosine, tangent and cotangent become negative. Understanding replaces rote learning, when the student sees that sides to the right and up are always positive, and to the left of the origin is negative. Later overlays show that the down direction is consistently negative. The terminal side of the angle (the hypotenuse of the right triangles formed) are always positive.

For example, the cosine of 120^{o} = cos $(90^{o} + 30^{o})$ = - sin 30^{o}.

Overlay 1. When the terminal side is in the third quadrant, the sine and cosine changes signs, but the tangent and cotangent remain the same. The sides of the angles are drawn the same size as in the first quadrant. The corresponding sides are labeled with the same letters and the same consistency occurs in color to emphasize these basic relationships.

Overlay 2. Here the terminal side is in the fourth quadrant, and the cosine is positive; the sine, tangent and cotangent are negative.

This transparency follows one which emphasizes the algebraic signs of trigonometric functions and how they occur in pairs. Each trigonometric function is positive in two quadrants and negative in the other two.

With the complete set of transparencies in trigonometry, the teacher may readily change from this analytic approach, to the graphical one which shows the periodic nature of trigonometric functions. From specific illustrations--graphically and analytically--the student visualizes that since the sine 30^{o} is .5, the sine 150^{o} is also .5, the sine 210^{o} is - .5, and the sine 330^{o} is also - .5. In the graphical situation, again, when the ordinate of the curve is below, the sine has a negative sign.

Graphical Addition of Curves.

The basic transparency is $y_1 = 10 \sin t$. The period is known to be 2 pi. The amplitude is 10. This representation affords the teacher the opportunity to discuss "time histories" or transient responses of functions where the abscissa is not an angle in general but rather an angle which varies with time. As time increases the angle increases.

The first overlay, is $y_2 = 5 \sin 2t$. The period is pi radians and the amplitude is 5. When the basic and the first overlay are super-imposed, there are some points in common--such as 0^o, 180^o and 360^o. The super position emphasizes the period and amplitude relations of the two functions.

The very important concept of graphical addition is attained by adding the signed values of the ordinate of the second to the ordinate of the first. Directed arrows facilitate the understanding of this operation.

This leads to an important advanced concept of adding trigonometric curves as done in the Fourier Series. Some of the many physical applications are found in oscilloscope traces, in the addition of electrical and sound waves. This concept may be reduced to students prior experience by plotting $y_1 = 2$, $y_2 = 2x$ and $y_3 = -3x^2$ and then drawing $y_4 = 2x + 2$, $y_5 = 3x^2 + 2$ and $y_6 = -3x^2 + 2x$.[16]

The Technifax Corporation has sets of transparencies for headings such as: mathematics, geometric construction, plane geometry and trigonometry.

The Encyclopedia Britannica offers transparencies covering some related phases of mathematics. At this time they do not cover any specific part of mathematics.

The Robert J. Brady Company, has a set for plane geometry, one for modern mathematics, and another for clarification of the numeration systems in modern mathematics.

There are other ways besides using the prepared transparencies that make the overhead projector so versatile in the hands of the mathematics teacher. By using it to give examples and solve problems,

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[16]Robert J. Brady Company. Instructor's Guide for Use with the Trigonometry Series, (Washington 7, D.C., 1962) pp. 5, 14.

messy board work can be eliminated. Tests can be written on the acetate with a grease pencil or made into a permanent transparency using an office copying machine. This saves writing tests on the board or having it mimeographed. Problems worked in class on the overhead can be used for review or can also help students who have missed the class. Simply place an opaque sheet over parts you don't want projected, then slide the opaque sheet down at the appropriate time and the next step can be shown.

A transparent slide rule is available that could take the place of the cumbersome slide rule that often hangs over the chalkboard in the mathematics classroom. This slide rule is a small hand rule made of transparent material with markings on one side only. These marks are visible on the screen. Easy manipulation of this small rule can be viewed by the entire class better and more conveniently than can the manipulation of the large slide rule.

Since the overhead projector is so relatively new in the educational classroom, most schools do not have enough projectors for each room. The writer feels that the overhead should be standard equipment in each mathematics classroom. It has so many possible uses, that it could be used everyday.

CHAPTER VIII

THE OPAQUE PROJECTOR

The opaque projector is a device which can project small non-transparent images, such as figures, pictures, or printed pages onto a screen as enlargements. It is probably the easiest to use of all the visual projectors. There are simple steps involved in its use. Place the material on the tray or "platen"; close the projector; and focus the image on the screen.

There is no preparation of the material other than arranging it in the desired order. The opaque projector reflects the images of objects from books, pamphlets, cards, or hand-written pages. The visual masters originally purchased for the making of transparencies may be used in the opaque projector.

The opaque has some limitations. Inability to write on material while it is being projected. Another is that the room must be completely dark. The size of the opening is only 10 x 10. The machine is noisy. The opaque reflects light rather than projecting it through a transparent material. This reduces the amount of light available through the lens, even though the lens is unusually large. This may necessitate placing the projector near the screen. Thus, a rearranging of desks is sometimes necessary when using the opaque projector.

Some of the chief uses of the opaque lie in areas that can also be done as well with medias such as the chalkboard, the mimeograph

machine or the overhead projector. Materials used in the opaque can be used over again as long as the material remains undamaged, thus giving the opaque a slight advantage over the above three medias.

Another characteristic that makes the opaque very useful is that it can reflect in true color an image from any non-transparent object that can fit in the 10 x 10 opening. In this manner, intricately drawn figures printed in textbooks can be projected to the chalkboard and traced thereon leaving an accurate figure large enough to be viewed by the entire class. Simply place the book in proper position on the platten and focus the figure on the screen. The image can be shown on the screen for viewing a problem, proof, or figures not included in the class textbook. The image can be shown on the board. Chalk can be used to draw in details or to trace a more permanent figure. The image can be shown on a large sheet of paper for the purpose of copying a permanent figure and usint it as a chart.

The opaque projector can utilize many free or inexpensive mathematical materials available to the teacher.

SUMMARY

A mathematics teacher should consider which materials make certain types of information more easily understood by the student. The media which the teacher can use most efficiently should be considered. This may vary with different teachers. The availability of instructional materials sometimes restricts the flexibility of the use of materials. In many schools today, the facilities, materials and equipment are available. It is the responsibility of the teacher to see that course material is properly presented by using the appropriate audio-visual materials.

Different media of instruction can be linked together to stimulate even more interest in the students. Perhaps a film or filmstrip would motivate a student to do outside reading in the mathematics library. A field trip may cause a student to desire to use certain instruments and manipulating devices. A student doing research or recreational reading may find something in the library which may be of interest to the entire class. These materials could be presented to the entire class by using the chalkboard, the opaque or the overhead projectors. There are many different ways the overhead and the chalkboard or the opaque and the chalkboard can be used together to the best advantage of everyone concerned.

With the help of audio-visual materials and equipment, the mathematics teacher has a new store of materials that could make the teacher-learning situation a better one. The students' interest may be deepened in mathematics. The teacher may thus be more effective.

APPENDIX A

A. Topics for Historical Reports

B. Things to Make

C. Great Mathematics

APPENDIX A

A. TOPICS FOR HISTORICAL REPORTS

1. Achilles Paradox
2. Ahmes Papyrus
3. Bernoulli's Theorem
4. Boolean Algebra
5. Cartesian Geometry
6. Calculation of pi or epsilon
7. Copernican Theory
8. Curves--Catenary, Cycloid Ellipse, Hyperbola, Parabola
9. De Moivre's Theorem
10. Egyptian Level
11. Fourth Dimension
12. Golden Section
13. History of Algebra, Arithmetic, and Geometry
14. History of Numbers--Arabic, Babylonian, Chinese, Egyptian, Greek, Hebrew, Hindu, Roman
15. Imaginary Numbers
16. Magic Squares
17. Metric System
18. Natural Logarithms
19. Non-Euclidean Geometry
20. Spirals
21. Squaring the Circle
22. Trisection of an Angle
23. Angular Measurement (October 1953, pp. 419-426)
24. Binary System (December 1953, pp. 575-577)
25. Calendar (May 1952, pp. 336-339)
26. Calculation of Logarithms (February 1954, pp. 115-116)
27. Complex Numbers (February 1954, pp. 106-114)
28. Duodecimal System (May 1955, pp. 332-333)
29. Finger Reckoning (March 1955, pp. 153-157)
30. Irrational Numbers (February 1956, pp. 123-127; March 1956, pp. 187-191; April 1956, pp. 282-285; October 1956, pp. 469-472; November 1956, pp. 541-543)
31. Nine-Point Circle (January 1957, pp. 53-54)
32. Recent Discoveries in Babylonian Mathematics (February 1957, pp. 162-163)

B. THINGS TO MAKE

1. Linkages--(18th Yearbook, pp. 117-129)
2. Models for Illustrating Locus Problems--(18th Yearbook, pp. 106-119)
3. Church Windows--(18th Yearbook, pp. 86-87; The Mathematics Teacher, November 1952, pp. 518-521)
4. Curve Stitching--(18th Yearbook, pp. 82-85)
5. Hypsometer, Angle Mirror and Clinometer--(18th Yearbook, 182-193; Bakst, Aaron, Mathematics, Its Magic and Mastery, D. Van Nostrand Co., pp. 434-439 and pp. 450-452)
6. Models for Illustrating Mathematical Concepts--(18th Yearbook, pp. 369-406)

7. Models to Show Conic Sections--(18th Yearbook, pp. 212-223, 273-279; The Mathematics Teacher, October, 1953, pp. 428-429)
8. Magic Squares--(Bragdon, Claude, The Frozen Fountain)
9. Napier's Rods--(The Mathematics Teacher, November 1954, pp. 482-487)
10. Nomographys--(18th Yearbook, pp. 164-181; The Mathematics Teacher, May 1956, pp. 391-392)
11. Paper Folding--(18th Yearbook, pp. 154-159; Yates, Robert C., Geometrical Tools, Educational Publishers, pp. 84-90)
12. Peaucellier's Cell--(Yates, Robert C., Geometrical Tools; Educational Publishers, pp. 54-65)
13. Tesseract--(18th Yearbook, pp. 246-250)
14. An A-Shaped Level--(The Mathematics Teacher, January 1953, p. 41)
15. Angle Mirror--(The Mathematics Teacher, February 1954, pp. 71-72)
16. Curve Stitching in Space--(The Mathematics Teacher, November 1956, pp. 560-561)
17. Golden Section Compasses--(The Mathematics Teacher, May 1954, pp. 338-339)
18. Probability Board--(The Mathematics Teacher, April 1953, pp. 274-277)
19. Telemeter--(The Mathematics Teacher, November 1955, pp. 473-475)
20. Three Dimensional Graphing--(the Mathematics Teacher, May 1953, pp. 339-340)
21. Trigtractor--(The Mathematics Teacher, January 1956, pp. 28-29)

C. GREAT MATHEMATICS

1. Appolonius--conics
2. Archimedes--area of a circle, mechanics, infinite series
3. Bolyai--non-Euclidean geometry
4. Brahe, Tycho--astronomy
5. Briggs, Henry--logarithms with base ten
6. Cardan--cubic equations
7. Copernicus--trigonometry, astronomy
8. Descartes, Rene--exponents, analytic geometry
9. Diophantus--algebrais symbolism
10. Einstein, Albert--relativity
11. Euclid--geometry
12. Euler--symbolism for algebra and trigonometry
13. Fermat, Pieere--probability
14. Ferrari, Lodovico--biquadratic equation (4th degree)
15. Galileo--proportional compasses, physics, astronomy
16. Gauss, Karl--positive and negative square roots
17. Hipparchus--trigonometry
18. Jacobi, Carl--elliptical functions
19. Kepler, Johann--application of conics to astronomy
20. Lagrange, Joseph--Theory of numbers, elliptical functions

21. Laplace, Pierre, --theory of least squares
22. Legrendre, Adrian Marie--geometry
23. Liebniz=-calculus
24. Leonardo da Vinci--geometry, mechanics
25. Lobachevsky--non-Euclidean geometry
26. Napier, John--logarithms, Napier's rod
27. Newton, Isaac--binominal theorem, physics, calculus
28. Omar Khayyam--algebra
29. Pascal, Blaise--binominal theorem, probability
30. Plato--geometry
31. Ptolemy--symbol for zero
32. Pythagoras--geometry, right triangle
33. Rabbi ben Ezra--magic squares, calendar, theory of numbers
34. Riemann--non-Euclidean geometry
35. Stevin--decimal fractions, quadratic equation
36. Thales--geometry
37. Wallis--graphs, negative and fractional exponents, algebraic symbolism, complex numbers[17]

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[17]Allene Archer, How to Use Your Library in Mathematics, (N.C.T.M., Washington 6, D.C., 1958), #5, pp. 4, 5, 6.

APPENDIX B

Senior High School Library List

A. Books

B. Periodicals

C. Pamphlets

APPENDIX B

Senior High School Library List

A. BOOKS

1. Adler, Irving. The New Mathematics. John Day Co., 1958. $3.50.
2. Allendoerfer, C. B. and C. O. Oakley. Principles of Mathematics. McGraw-Hill Book Co., Inc., 1955. $5.75.
3. Bakst, Aaron. Mathematical Puzzles and Pastimes. D. Van Nostrand Co., Inc., 1954, $4.00.
4. Banks, J. Houston, Elements of Mathematics. Allyn and Bacon, 1956. $5.75.
5. Bross, Irwin D. Design for Decision. Macmillan Co., 1953. $4.25.
6. Comrie, L. J. (ed.) Barlow's Tables. Chemical Publishing Company, Inc., Tudor Publishing Co., 1957. $3.95.
7. Courant, R., and H. Robbins. What is Mathematics? Oxford University Press, 1941. $7.50.
8. Dantzig, Tobias. The Bequest of the Greeks. Charles Scribner's Sons, 1955. $3.95.
9. Dubisch, Roy. The Nature of Numbers. Ronald Press Co., 1952. $4.00.
10. Eves, Howard. An Introduction to the History of Mathematics. Rinehart & Co., Inc., 1953. $7.00.
11. Freund, John E. A Modern Introduction to Mathematics. Prentice-Hall, Inc., 1956. $9.00.
12. Gamow, George. One, Two, Three--Infinity. Viking Press, 1947. $5.00.
13. Harris, Charles O. Slide Rule Simplified. American Technical Society, 1953. $3.00.
14. Hart, William L. Calculus. D. C. Heath and Co., 1955. $5.50.
15. Hilbert D. and S. Cohn-Vossen. Geometry and the Imagination. Chelsea Publishing Co., 1952. $6.00.
16. Introduction to Probability and Statistical Inference. Commission on Mathematics, 1958. $1.00.
17. James, Glenn and R. C. James. Mathematics Dictionary. D. Van Nostrand Co., Inc., 1959. $10.00.
18. Johnson, Richard E. et al. Fundamentals of College Mathematics. Rinehart & Co., Inc., 1953. $5.00.
19. Jones, Burton W., Elementary Concepts of Mathematics. Macmillan Co., 1947. $4.85.
20. Kasner, E. and J. Newman. Mathematics and the Imagination. Simon and Schuster, Inc., 1940. $4.50.
21. Kemeny, J. G., J. L. Snell and G. L. Thompson. Introduction to Finite Mathematics. Prentice-Hall, Inc., 1957. $7.65.

22. Kline, Morris. Mathematics in Western Culture. Oxford University Press, 1952. $7.50.
23. Kramer, Edna. The Mainstream of Mathematics. Oxford University Press, 1952. $7.50.
24. Lieber, L. R. and H. G. Lieber. Mits, Wits, and Logic. Galois Institute of Mathematics and Art, 1947. $3.95.
25. Lieber, L.R. and H. G. Lieber, Education of T. C. Mits. Galois Institute of Mathematics and Art, 1947. $3.95.
26. Lieber, L. R. and H. G. Lieber, Non-Euclidean Geometry. Galois Institute of Mathematics and Art. $1.95.
27. Newman. J. R. The World of Mathematics. Simon and Schuster, Inc., 1956. $20.00.
28. Northrup, Eugene. Riddles in Mathematics. D. Van Nostrand Co., Inc., 1944. $4.50.
29. Polya, G. How to Solve It. Princeton University Press. $4.00.
30. Ransom, William. One Hundred Mathematical Curiosities. J. Weston Walch, Publisher, 1954. $3.00.
31. Reid, Constance. From Zero to Infinity. Thomas Y. Crowell Co., 1955. $3.00.
32. Richardson, Moses. Fundamentals of Mathematics. Macmillan Co., 1958. $6.50.
33. Rider, Paul R. Analytic Geometry, Macmillan Co., 1947. $4.25.
34. Sawyer, W. W. Prelude to Mathematics. Penguin Books, Inc., 1955. 85¢.
35. Steinhaus, Hugo. Mathematical Snapshots. Oxford University Press, 1950. $7.00.
36. Williams, John D. The Compleat Strategyst. McGraw-Hill Book Co., Inc., 1954. $4.75.

B. PERIODICALS

1. Current Science and Aviation. American Education Publication, 1260 Fairwood Avenue, Columbus 16, Ohio. $1.40 per year.
2. Mathematical Pie. 97 Chequer Road, Doncaster, England.
3. Mathematics Student Journal. National Council of Teachers of Mathematics, 1201 Sixteenth Street, N. W., Washington 6, D. C. $1.50 per year.
4. O. U. Mathematics Letter. Mathematics and Astronomy Department, University of Oklahoma, Norman, Oklahoma. $1.00 per year.
5. Science and Mechanics. Science and Mechanics Publishing Co., 450 East Ohio Street, Chicago 11, Illinois. $2.00 per year.
6. Science News Letter. Science Service, Inc., 1719 N. Street, N. W., Washington 6, D. C. $5.50 per year.
7. Scientific American. 415 Madison Avenue, New York 17, New York. $5.00 per year.

C. PAMPHLETS

1. Amazing Story of Measurement, The. Lufkin Rule Co., 1730 Hess Street, Saginaw, Michigan. 10¢.
2. Archer, Allene. How to Use Your Library in Mathematics. National Council of Teachers of Mathematics, 1201 Sixteenth Street, N. W., Washington 6, D. C. $1.00.

3. Bartnick, Lawrence P. Designing the Mathematics Classroom. National Council of Teachers of Mathematics, 1201 Sixteenth Street, N. W., Washington, D. C. $1.00.
4. Carnahan, Walter H. Mathematics Clubs in High School. National Council of Teachers of Mathematics, 1201 Sixteenth Street, N. W., Washington, D. C. 75¢
5. Christian, Robert R. Introduction to Logic and Sets. Ginn and Co., Statler Building, Boston 17, Massachusetts. 75¢.
6. Christman, Leon E. A Rhythmic Approach to Mathematics. The author, Yorkville, Illinois. 50¢
7. Concepts of Equation and Inequality. Commission on Mathematics, College Entrance Examination Board, 425 West 117th Street, New York 27, N. Y. 15¢.
8. Dadourian, H. M. How to Study; How to Solve. Addison-Wesley Publishing Co., Inc., Reading, Massachusetts. 50¢.
9. Dent, C. H. and E. F. Tiemann. Bulletin Boards for Teaching. Visual Instruction Bureau, University of Texas, Dallas, Texas. $1.00.
10. Dent, C. H. and E. F. Tiemann. Felt Boards for Teaching. Visual Instruction Bureau, University of Texas, Dallas, Texas. $1.00.
11. Education for the Talented in Math and Science. U. S. Government Printing Office. Washington, D. C. 15¢.
12. Fehr, Howard F. The Role of Insight in the Learning of Mathematics. National Council of Teachers of Mathematics. 1201 Sixteenth Street, N. W. Washington, D. C. 20¢.
13. Fehr, Howard F. Teaching High School Mathematics. National Council of Teachers of Mathematics. 1201 Sixteenth Street, N. W., Washington D.C. 25¢.
14. Glennon, V. S. and C. W. Hunnicutt. What Does Research Say About Arithmetic? National Education Association. 1201 Sixteenth Street, N. W. Washington, D. C. 25¢.
15. Gorn, S. and Manheimer, W. The Electronic Brain and What It Can Do. Science Research Associates, 57 West Grand Avenue,, Chicago 10, Illinois. 50¢
16. Guidance Pamphlet in Mathematics. National Council of Teachers of Mathematics, 1201 Sixteenth Street, N. W. Washington, D.C. 25¢.
17. Johnson, D. A. and C. E. Olander. How to Use Your Bulletin Board. National Council of Teachers of Mathematics, 1201 Sixteenth Street, N. W. Washington, D. C. 50¢.
18. Johnson, D. A. Paper Folding for the Mathematics Class. National Council of Teachers of Mathematics, 1201 Sixteenth Street, N. W. Washington, D. C. 75¢.
19. Kidd, K. P. Instructional Materials for Mathematics Classes. College of Education. University of Florida. Gainesville, Florida. 40¢.
20. Keiffer, Mildred and Anne Marie Evans. How to Develop a Teaching Guide in Mathematics. National Council of Teachers of Mathematics, 1201 Sixteenth Street, N. W. Washington, D.C. 40¢.
21. Krickenberger, W. R. and Helen R. Pearson. An Introduction to Sets and the Structure of Algebra. Ginn and Co., 205 West Wacker Drive, Chicago 6, Illinois. 60¢.

22. Larsen, H. D. Enrichment Program for Arithmetic. A series of pamphlets for grades 4, 5 and 6. Row, Peterson & Co., 1911 Ridge Avenue, Evanston, Illinois.
23. Mathematics Education Research Studies. U. S. Government Printing Office, Washington, D. C.
24. Menger, Karl. You Will Like Geometry. Museum of Science and Industry, Jackson Park, Chicago 37, Illinois. 15¢.
25. Money Management Booklet Service. Household Finance Corporation. 919 North Michigan Avenue, Chicago, Illinois. 11 booklets at 10¢ each.
26. Norris, Willa and Wallace Manheimer. What Good Is Math? Science Research Associates, Inc., 57 West Grand Avenue, Chicago 10, Illinois. 50¢.
27. Potter, Mary and Virgil Mallory. Education in Mathematics for the Slow Learner. National Council of Teachers of Mathematics. 1201 Sixteenth Street, N. W. Washington, D. C. 75¢.
28. Professional Opportunities in Mathematics. Mathematical Association of America, University of Buffalo, Buffalo 14, New York. 25¢.
29. Ransom, William R. Geometry Growing. National Council of Teachers of Mathematics, 1201 Sixteenth Street, N. W. Washington, D. C. 75¢.
30. Ringenberg, Lawrence A. A Portrait of 2. National Council of Teachers of Mathematics, 1201 Sixteenth Street, N. W. Washington, D. C. 75¢.
31. Rourke, Robert E. K. Some Implications of Twentieth-Century Mathematics for High Schools. National Council of Teachers of Mathematics, 1201 Sixteenth Street, N. W., Washington, D. C. 25¢.
32. Schaaf, William L. Recreational Mathematics. N.C.T.M., 1201 Sixteenth Street, N. W. Washington, D. C. $1.20.
33. Sets, Relations, and Functions. Commission on Mathematics, College Entrance Examination Board, 425 West 117th Street, New York 27, N. W.
34. Shuster, Carl N. Computation with Approximate Data. Yoder Instruments, East Palestine, Ohio. 25¢.
35. Surveying. Boy Scouts of America. New Brunswick, New Jersey.
36. Swain, Henry. How to Study Mathematics. N.C.T.M., 1201 Sixteenth Street, N. W. Washington, D. C. 50¢.
37. Vance, E. P., et al. Program Provisions for the Mathematically Gifted Student in the Secondary School. N.C.T.M., N. W. Washington, D. C. 75¢
38. Vollmar, Robert and Philip Peak. How to Use Films and Filmstrips in Mathematics Classes. N.C.T.M., 1201 Sixteenth Street, N. W. Washington D. C. 50¢.
39. Why Study Mathematics? Canadian Mathematical Congress, Engineering Building, McGill University, Montreal, Canada. 50¢.

D. CHARTS

1. Algebra, Basic (Solid Plastic Reference Chart). Data Guide, Inc., 40-05 149th Place, Flushing 54, New York. 79¢.
2. Curveasy Charts: (1) Spherical Area; (2) Plane Parabola. Eugene Dietzgen Co., 2425 North Sheffield Avenue, Chicago 14, Illinois. $2.50.

3. Logarithm and Trigonometric Functions Chart. Welch Scientific Co., 1515 Sedgwick Street, Chicago 10, Illinois. $15.00.
4. Slide Rule Guide (Solid Plastic Reference Chart). Data Guide, Inc., 40-05 149th Place, Flushing 54, New York. 79¢.
5. Stereoscopic Drawings of Figures of Solid Geometry. Keystone View Co., Meadville, Pennsylvania. $41.50.
6. Trig-Easy Charts: (1) Plane Angles; (2) Spherical Angles; (3) Compound Angles. Eugene Dietzgen Co., 2425 North Sheffield Avenue, Chicago 14, Illinois. $3.00.
7. Trigonometry (Solid Plastic Reference Chart) Data Guide, Inc., 40-05 149th Place, Flushing 54, New York. 79¢ [18]

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[18] Berger and Johnson, A Guide to the Use and Procurement of Teaching Aids for Mathematics, (National Council of the Teachers of Mathematics, Washington 6, D. C., 1959), p. 5.

APPENDIX C

Mathematical Film Titles

From the University of Illinois

APPENDIX C

Mathematical Film Titles from the University of Illinois.

1. A Plus B Squared
2. Algebra in Everday Life
3. Angles
4. Angles and Arcs in Circles
5. Areas
6. Asioms in Algebra
7. Chords and Tangents of Circles
8. Circle, The
9. Congruent Figures
10. Decimal Fractions
11. Descriptive Geometry: Finding Lines of Intersection Between Two Solids
12. Donald in Mathmagic Land
13. Formulas in Mathematics
14. Geometry and You
15. Geometry in Action
16. Graphing Linear Equations
17. How to Add Fractions
18. How to Change Fractions
19. How to Divide Fractions
20. How to Find the Answer
21. How to Multiply Fractions
22. How to Subtract Fractions
23. Indirect Measurement
24. Introduction to Fractions
25. Language of Algebra
26. Language of Graphs
27. Language of Mathematics
28. Latitude and Longitude
29. Lines and Angles
30. Locus
31. Meaning of Long Division
32. Meaning of Percentage
33. Meaning of Pi
34. Measuring Areas: Squares, Rectangles
35. Measuring Simple Areas
36. Metric System
37. Number Systems and Its Structure
38. Oblique Cones and Transitional Developments
39. Origin of Mathematics
40. Parallel Lines
41. Percent in Everyday Life
42. Percentage
43. Polygons
44. Precisely So
45. Principles of Scale Drawing

46. Properties of Triangles
47. Proportion at Work
48. Pythagorean Theorem
49. Pythagorean Theorem: Proof by Area
50. Pythagorean Theorem: The Cosine Formula
51. Rectangular Coordinates
52. Quadrilaterals
53. Ratio and Proportion
54. Ratio and Proportion in Mathematics
55. Rectilinear Coordinates
56. Similar Triangles
57. Size of Things, The (Adventure in Science)
58. Slide Rule (The C and D Scales)
59. Slide Rule (Proportion, Percentage, and Square Roots)
60. Solids in the World Around Us (Discovering Solids Series)
61. Story of Our Number System
62. Surface Areas of Solids (Parts I and II)
63. Symbols in Algebra
64. Understanding Numbers #1; Earliest Numbers
65. Understanding Numbers #2; Base and Place
66. Understanding Numbers #3; Big Numbers
67. Understanding Numbers #4; Fundamental Operations
68. Understanding Numbers #5; Short Cuts
69. Understanding Numbers #6; Fractions
70. Understanding Numbers #7; New Numbers
71. Understanding Solids in Geometry
72. Vernier Scale
73. Volume and Its Measurement
74. Volumes of Cubes, Prisms and Cylinders (Discovering Solids Series)
75. Volumes of Pyramids, Cones and Spheres (Discovering Solids Series)
76. What are Decimals?
77. What are Fractions? [19]

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[19] Science and Mathematics Catalog, University of Illinois Film Library, 1962-1964, p. 22-23.

APPENDIX D

List of Sources for

Mathematical Films

APPENDIX D

List of Sources for Mathematical Films

1. American Council on Education. Selected Educational Motion Pictures: A Descriptive Encyclopedia. American Council on Education, 1785 Massachusetts Avenue, N. W. Washington 6, D. C.
2. Audio-Visual Publications, Inc. See and Hear. (Audio-Visual news and discussions of materials for classroom use. Publication discontinued after December 1953. Back issues when available may be useful.) Audio-Visual Publications, Inc., 7064 Sheridan Road, Chicago 26, Illinois.
3. Department of Audio-Visual Instruction. Instructional Materials. Monthly, October through June. Department of Audio-Visual Instruction of the National Education Association, 1201 Sixteenth Street, N. W. Washington 6, D. C. $4.00.
4. Educational and Recreational Guides, Inc. Audio-Visual Guide. Monthly, September through June. (Study guides for commercial films; also lists of new audio-visual materials and a variety of articles.) Educational and Recreational Guides, Inc., 1630 Springfield Avenue, Maplewood, New Jersey. $3.50.
5. Educational Screen. Blue Book of Audio-Visual Materials. Twenty-ninth edition. Educational Screen, 64 East Lake Street, Chicago 1, Illinois. $2.00.
6. Education Screen. Blue Book of 16mm Films. Twenty-eighth edition. Educational Screen, 64 East Lake Street, Chicago 1, Illinois. $2.00.
7. Educational Screen. Educational Screen. Monthly, September through June. (Most widely read audio-visual magazine. Contains evaluations of 16 mm educational and religious films and a variety of articles and regular features.) Educational Screen, 64 East Lake Street, Chicago 1, Illinois. $4.00.
8. Educators Progress Service. Educators Guide to Free Films. Fourteenth edition. Annual Educators Program Service, Randolph, Wisconsin. $6.00.
9. Educators Progress Service. Educators Guide to Free Slide-films. Sixth edition. Annual. Educators Program Service. Randolph, Wisconsin. $5.00.
10. Falconer, Vera M. Filmstrips--A Descriptive Index and Users' Guide. (Discusses the selection and utilization of filmstrips; then gives detailed descriptions and evaluations of about 3000 of the better filmstrips available on March 15, 1947. Illustrated with frames from the filmstrips. Should be seen in the Filmstrip Guide for a complete listing of usable filmstrips.) McGraw-Hill Book Company, Inc., New York, New York. 1948. 572 p.

11. Film News. Film News. (Well-written film evaluations, news stories, and discussions.) Nine issues a year. Film News, 444 Central Park West, New York 25, New York. $3.00.
12. H. W. Wilson Company. Educational Film Guide. Eleventh edition. (Completely revised, with or without 7 semi-annual supplements.) H. W. Wilson Company, 950-72 University Ave., New York 52, New York. 1953. $7.50 or $12.50.
13. H. W. Wilson Company. Filmstrip Guide. Third edition. (Completely revised, including 7 semi-annual supplements.) H. W. Wilson Company, 950-72 University Avenue, New York 52, New York. 1954. $8.50.
14. Indiana University. Educational Motion Pictures. With semi-annual and cumulative supplements, plus monthly mimeographed lists. (An alphabetical list of 3354 films available on a rental basis including film data, description of contents, and recommended grade levels. Preceded by a subject description and grade level index. Cumulative supplement issued on April 1, 1954, contains information on approximately 900 additional films.) Audio-Visual Center, Bloomington, Indiana. 1952. 530 p. $ .85.
15. National Council of Teachers of Mathematics. The Mathematics Teacher. (Each issue contains a section entitled "Reviews and Evaluations.") National Council of Teachers of Mathematics, 1201 Sixteenth Street, N. W., Washington 6, D. C. $3.00.
16. National Council of Teachers of Mathematics. Multi-Sensory Aids in the Teaching of Mathematics. (Discusses the production and use of all types of visual materials for the teaching of mathematics. Many photographs.) National Council of Teachers of Mathematics, 1201 Sixteenth Street, N. W. Washington 6, D. C. 445 p. $3.00.
17. Ohio State University. Sources of Teaching Materials. (A classified listing of sources of teaching materials. Includes sections on (a) references for utilization, (b) sources of films, filmstrips, and slides, (c) radio and television, (d) educational recordings, (e) free and inexpensive teaching aids, (f) keeping currently informed, and (g) sources of materials for the study of other countries. Good address list for free travel materials.) Bureau of Educational Research, The Ohio State University, Columbus 10, Ohio. 1952. 19 p. $ .70.
18. United States Office of Education. 102 Motion Pictures on Democracy. (Describes all motion pictures, filmstrips, and slide sets produced by a group of branches of the U. S. Government and available for public use.) U. S. Government Printing Office, Washington 25, D. C. 1951. 329 p. $ .70.
19. Ver Halen Publishing Company. Film World and A-V World Magazine. Monthly. (Many news stories and film descriptions and evaluations.) Ver Halen Publishing Company, 6327 Santa Monica Blvd, Hollywood 38, California. $4.00.

20. Ver Halen Publishing Company. Teaching Tools. Published eight times yearly. (Case histories of how instructional materials are used in teaching.) Ver Halen Publishing Company, 6327 Santa Monica Blvd., Hollywood 38, California. $4.00. [20]

Educational Film Guide and Filmstrip Guide have been discontinued and Educational Media Index has replaced them. Teaching Tools by Ver Halen Publishing Company has also been discontinued.

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[20]Robert Vollmar and Philip Peak, How to Use Films and Filmstrips in Mathematics Classes, (N.C.T.M., Washington D. C. 1956), p. 11.

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A-V Communication Review, Vol. 11, No. 1. Jan. - Feb., 1963. pp. 54, 148, 36, 18, 19.

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Bachman, John W., How to Use Audio-Visual Material. Associated Press, New York, 1956. p. 1.

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Brady, Robert J. Company, Teacheze Catalog, Robert J. Brady Company, Washington 7, D. C., 1963.

Brinke-Johnson-Woodby, How to Use the Field Trip in Mathematics, N.C.T.M., Washington 6, D. C., 1956. #3, p. 5.

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Dale, Edgar, Audio-Visual Methods in Teaching, Dryden Press, New York 1954. p. 65.

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Educational Filmstrip Catalog 1964, Society for Visual Education, Inc., Chicago, Illinois, 1964.

Haas and Packer, Preparation and Use of Audio-Visual Aids, Prentice Hall, Inc., New York, 1955, p. XI.

Instructional Materials, Illinois Curriculum Program, Administration and Supervision Series, Bulletin A-3, Springfield, Illinois, 1961.

LaPine Catalog, Models and Aids for Teaching Mathematics, LaPine Scientific Company, Chicago 29, Illinois, 1964.

Peak, Philip, and Vollmar, Robert, How to Use Films and Filmstrips in Mathematics Classes, National Council of Teachers of Mathematics, Washington 6, D. C., 1956.

Reviews of Films, Reprinted from the Mathematics Teacher, Dec., 1963, N.C.T.M. Washington 6, D.C.

Science and Mathematics Catalog, University of Illinois Film Library, 1962-1964. p. 22-23.

Selected Welch Apparatus for Teaching Mathematics, The Welch Scientific Company, Chicago, Illinois, 1962.

VOGT Catalog, Transparent Geometric Models for Teaching Mathematics, Viking Company, Los Angles 4, California, Cover.

Wilson, H. W. Company, Educational Film. H. W. Wilson Company, New York 52, New York, 1954.

Wilson, H. W. Company, Educational Filmstrips. H. W. Wilson Company, New York 52, New York, 1954.