Trade in value-added and the welfare gains of international fragmentation

NJIKE, ARNOLD

Université Paris Dauphine

16 May 2020

Trade in value-added and the welfare gains of international fragmentation

Arnold NJIKE

arnold.njike-oya@dauphine.eu

Université Paris Dauphine, PSL Research University, LEDA, DIAL,75016 Paris, France

May 16, 2020

Abstract: In order to take profit from the differences in factor endowments and technology that exist between countries, firms delocalize or externalize a share of their goods' production process to other countries. This phenomenon is so widespread today that very few manufactured goods are produced entirely within the borders of a single country. We examine in this paper the macroeconomic gains related to this phenomenon by calculating the net share of international fragmentation in the welfare gains of trade. To do so, we propose a model that allows us to identify all the components related to international fragmentation in these welfare gains, something that most of the classical trade models fail to do. We show that the net share of international fragmentation in the welfare gains of trade represents on average 22% of the gains of trade, a way lower figure than the share that could be inferred from standard trade models. The shutdown of international fragmentation would, therefore, only reduce the average real wage by 3%.

Keywords: Global supply chains,Welfare effects of trade, Trade in value-added,Computable general equilibrium

JEL classification code : F100

1 Introduction

In 2018, the WTO reported that trade in manufactured goods represented 68 % of world trade, far ahead primary goods like agricultural and fuel and mining products. In the current era of global supply chains, few countries carry out the production process of these goods from the upstream to the downstream. Rather, the process is fragmented between a lot of countries so that each one is specialized on particular tasks that are realized with the highest degree of efficacy, provided that the gains from fragmentation exceed the costs. It means that before a good reaches its final destination, it could cross the border of a country as many times as required for the completion of the production process.

As international fragmentation is a widespread phenomenon today, there is no doubt that its gains exceed its costs whenever it occurs because otherwise, companies would be losing money. These gains, however, could be under threat in the current context marked by a protectionist temptation, notably in the United states since the election of Donald Trump as President. The ongoing trade war launched by this president with China have seen tariffs hikes from both sides. This could have detrimental consequences on vertical specialization between the two countries because as Yi (2003) for instance have demonstrated, a small variation in tariffs can have magnified and non-linear effects on the growth of trade and especially on vertically specialized goods.

Since, vertically specialized goods cross the borders of many countries or could cross the border of a single country several times, the impact of a one-percentage-point tariff reduction on their trade is logically amplified. A tariff increase, however, as it is currently done by both countries could possibly render vertical specialization economically unsustainable between them. It is therefore interesting to determine the welfare reduction that would imply such a situation.

From a microeconomic standpoint, it is straightforward to determine the net gains of international fragmentation because companies are able to identify what they earn by delocalizing or externalizing a share of their production process to other countries. From a macroeconomic standpoint however, the answer to this question is a bit more complicated.

A solution to provide an answer to this question is to calculate the welfare gains of trade with a model that takes into account the fragmented organization of the world production process and make the difference with the gains predicted by a model that does not. This difference would represent the share of international fragmentation in the welfare gains of trade. It should be understood that the model that assumes no production linkages between countries should be a model where each unit of final good is produced using only value-added of the producing country and should therefore be calibrated on data that reflect this reality for the calculation to be reliable.

In principle, any trade model that allows for tradable intermediate inputs takes into account in a certain way the fragmented organization of the world production network. Costinot and Rodríguez-Clare (2014) using trade models featuring two different market structures, notably perfect and monopolistic competition, calculate the welfare gains of trade in two cases where trade in intermediate goods is allowed and not. They find that the welfare gains of trade are almost twice as high when trade in intermediate goods is allowed than without, suggesting that the share of fragmentation in the welfare gains of trade is 50% of the total gains.

However, as explained earlier, for this calculation to be reliable, the models without trade in intermediate goods should have been calibrated on final goods trade data where each unit of final good is obtained only with value-added of the producing country. Instead Costinot and Rodríguez-Clare (2014) calibrate their model using actual trade data that do not satisfy this requirement. We label the share obtained by performing this calculation the gross share of international fragmentation in the welfare gains of trade, which is the gross share of the gains related to trade in intermediate goods. We use the term gross because it is obvious that the model which is supposed to be without intermediate goods in fact hides a component of the impact of international fragmentation in the welfare gains of trade.

To provide a reliable estimation of the net share of international fragmentation in the welfare gains of trade, we propose a model that allows us to identify in these gains all the components related to international fragmentation such that this net share be identified. It is a model that is based upon value-added trade flows rather than gross trade flows.

Gross trade flows are trade statistics that are obtained by recording the value of goods crossing a country's borders. This strategy renders difficult the identification of the true country of origin and the destination of final consumption of a good, especially in presence of international fragmentation. Value-added trade flows however, which are obtained through a specific statistical transformation[1] identify the value-added of each country incorporated in the goods and services that are produced and exported worldwide. Unlike bilateral gross exports that depend only on direct bilateral trade costs, a given country value-added exports to a particular destination depend on intermediate countries final goods exports to this destination, and therefore, on intermediate countries trade costs with it Koopman et al. (2014).

[1] see Daudin et al. (2011), Johnson and Noguera (2012), or Koopman et al. (2014)

Standard trade models[2] do not take into account this more complex structure of value-added exports and thus, are not suitable to explain this kind of trade flows. Authors such as Noguera (2012) or Aichele and Heiland (2018) have already proposed a structural model for value-added exports, but none of them derive the welfare formula for the gains of trade with their models. To our knowledge, no other papers rely explicitly upon value-added exports to do so.

Theoretically, it should be noted that the welfare gains of trade are not supposed to be different with a value-added exports model in comparison to a gross exports model, as the economy's total expenditures remain the same in the two approaches. In fact as value-added trade flows are obtained by a statistical transformation of gross trade flows, we need to specify a full gross trade flows model to get our value-added trade flows model. The gross trade flows model that we specify is close to Eaton and Kortum (2002) and Caliendo and Parro (2015).

However, these two models assume implicitly that the share of a given origin country in the total demand of inputs by firms in a destination country is also the share of this origin country in the destination country total demand of final goods. This assumption is not confirmed by the data. To solve this problem, we specify a different model for the two kinds of trade as Alexander (2017). Unlike the latter though who assumes that the market structure for both trade in intermediate and final goods is perfect competition, we assume that only trade in intermediate goods is based upon perfect competition while for trade in final goods, we assume that consumers have a "love of variety-like" utility function and consume all the varieties produced and exported by each country in the world.

Using a value-added exports model rather than a gross exports model to infer the welfare gains of trade allows us to identify what we labelled earlier the gross share of international fragmentation in the welfare gains, but also allows us to identify what we label the macroeconomic cost of fragmentation. This cost is the accumulated cost that appears when intermediate goods go back and forth between countries before reaching the country of final transformation. It is the hidden component of the impact of international fragmentation in the welfare gains of trade that we mentioned earlier, which is critical to calculate the net share of fragmentation in these gains.

Besides, this model also allows us to determine the real implications of a trade costs reduction on a given country's participation in the global supply chain. This is also

[2] By standard trade models we refer to models with Armington utility functions such as the one of Anderson and Van Wincoop (2003) or models with perfect and monopolistic competition as market structures such as the one of Eaton and Kortum (2002) or Chaney (2008) respectively.

worthy of interest because, as many countries anticipate that participating more in the global value chain will foster their exports and GDP growth, a lot of them are devising policies in order to stimulate their integration into the world production process. Among these policies, reducing the level of trade costs is one of the top priorities.

Our results show that a reduction in the bilateral trade costs of a given country with each of its trading partners unambiguously increase in absolute terms its participation in the global supply chain whether backward or forward but, could imply relatively less forward participation. In fact, the origin country could become more efficient at exporting final goods to the country of final consumption than exporting intermediate goods to intermediate countries which are then transformed before being exported to the destination of final consumption. This results in more final goods directly exported to a particular destination of final consumption by the origin country than intermediate goods indirectly exported to this particular destination embedded in intermediate countries final goods exports, which means a relatively lower forward participation.

Moreover, we show that the change in welfare that would imply a move to autarky is different when estimated using our model rather than a model that does not distinguish trade in intermediate and final goods. Specifically, it appears similarly to the findings of Fally and Hillberry (2018) or Alexander (2017) that downstream countries feature higher welfare gains than upstream countries compared to what predicts a classical model.

We finally show that the net share of international fragmentation in the welfare gains from trade represents 22% of the gains of trade, a way lower figure than the gross share that we inferred from Costinot and Rodríguez-Clare (2014). The structure of the paper is as follows. The second section describes the model, the third and fourth sections present respectively the data with their different sources and the results of our estimations, and the last section concludes.

2 The model

The presentation of the model is organized in three sub-sections. In the first, we describe how goods and value-added are produced and traded between countries. In the following, we derive the welfare formula that is used to infer the gains from trade against autarky and in the third we present the method used to infer the change in the welfare gains related to any trade costs shock other than a move to autarky.

While trade in goods implies a bilateral relationship between the origin country of the goods and the country of destination, trade in value-added involves a set of other actors that we label intermediate countries which is the set of countries through which the value-added of the origin country passes to reach its final destination. Let “i”, “s” and “j” be any three countries in the set of countries N. Throughout this model, we use indices “i” and “j” alternatively for the origin country and the destination country of the trade flows. When it comes to trade in value-added, we use index $s \in$ S for the intermediate countries. It is worth to note that the set of origin countries, the set of destination countries and the set of intermediate countries are composed of the same countries which means that a country can be simultaneously origin, destination and intermediate.

2.1 Production

To produce a unit of good either intermediate or final, a given country combines labor with intermediate inputs coming from itself and other countries. We assume that the production technology takes the form of the following Cobb-Douglas function:

$$q_j(\omega) = z_j(\omega) \quad l_j(\omega)^{\alpha_j} \; m_j(\omega)^{1-\alpha_j} \tag{1}$$

Where $z_j(\omega)$ represents country “j” efficiency at producing good ω, $l_j(\omega)$ is labor, $m_j(\omega)$ represents the composite intermediate inputs used in order to produce good ω and where α_j is the share of labor required to produce a unit of good in country “j”. We assume that countries do not have the same access to technology but also that producing a given good implies a specific technology requirement. $z_j(\omega)$, therefore, vary by country and by good.

Following (1), the total quantity of output produced in the economy is given by:

$$Q_j = \int_0^1 q_j(\omega) d\omega \tag{2}$$

2.2 Trade in intermediate goods

In order to get the composite intermediate inputs, producers purchase intermediate goods from suppliers across all countries at the lowest price possible and aggregate them according to the following production technology:

$$I_j = \left[\int_0^1 k_j(\omega)^{\frac{\varepsilon-1}{\varepsilon}} d\omega \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{3}$$

Where I_j is the total quantity of composite intermediate inputs produced in country “j” and used to produce either intermediate or final goods[3], $k_j(\omega)$ is country "j" demand of input ω and ε the elasticity of substitution across inputs. As in Caliendo and Parro (2015), the solution to the intermediate input producer problem is thus given by:

$$k_j\left(\omega\right) = \left(\frac{p_j(\omega)}{P_j}\right)^{-\varepsilon} I_j \tag{4}$$

$$\text{With } P_j = \left[\int_0^1 p_j(\omega)^{1-\varepsilon} d\omega\right]^{\frac{1}{1-\varepsilon}} \tag{5}$$

P_j is the unit price of the composite intermediate input in country "j" and $p_j(\omega)$ the price at which is bought intermediate input ω by country "j".

Let $p_{ij}\left(\omega\right)$ be the price of producing and exporting input ω from country "i" to country "j". Following Eaton and Kortum (2002), this price is given by:

$$p_{ij}\left(\omega\right) = \left(\frac{c_i}{z_i(\omega)}\right) t_{ij} \tag{6}$$

Where $\frac{c_i}{z_i(\omega)}$ represents the unitary cost for producing input ω in country "i" with c_i the cost of a bundle of production factors which is the same for each input as we assume the production factors to be mobile across activities within a country and $z_i(\omega)$ country "i" efficiency at producing input ω as in equation (1). t_{ij} represents the bilateral trade cost factor between country "i" and country "j". This trade cost factor is composed of iceberg costs and ad-valorem flat rate tariffs[4]. It is such that the internal trade cost of a country be equal to unity ($t_{ii} = 1$). Assuming that bilateral barriers obey the triangle inequality because of cross-border arbitrage, we have for any three countries "i", "j", "s", $t_{ij} \leq \ t_{is}t_{sj}$.

The price of a given input is therefore:

$p_j\left(\omega\right) = min\left\{p_{ij}\left(\omega\right); i = 1, \ldots,\ N\right\}$

With N being the number of countries. We use the same probabilistic representation of technologies as proposed by Eaton and Kortum (2002). More precisely, we assume that country "i" efficiency in producing input ω, $z_i\left(\omega\right)$ is the realization of a random variable

[3] We have $I_j = \int_0^1 m_j\left(\omega\right) d\omega = \frac{\int_0^1 p_j(\omega) k_j(\omega) d\omega}{P_j}$ with $\int_0^1 p_j\left(\omega\right) k_j\left(\omega\right) d\omega = I_j P_j$ the budget constraint of the intermediate good producer.

[4] It is worth to mention that our dataset does not provide data on ad-valorem tariffs. To perform our counterfactual analysis, we will calibrate them using actual data on tariff revenues and bilateral trade flows.

Z_{i} drawn for each input independently from its country-specific probability distribution. This probability distribution is $F_i(z) = Pr[Z_i \leq z]$ which is also by the law of large numbers the fraction of inputs for which country i's efficiency is below z. Assuming a Fréchet distribution, we have:

$$F_i(z) = e^{\Upsilon_i z^{-\theta}} \tag{7}$$

Where $\Upsilon_i > 0$ is a country-specific state of technology parameter whose value indicates the likeliness of a good efficiency draw. The bigger its value, the higher the likeliness of a good efficiency draw for any input ω. $\theta > 1$, the shape parameter of the Fréchet distribution is not country specific. As explained by Eaton and Kortum (2002), the higher its value the lesser is the variability of efficiency draws within the countries. This parameter therefore regulates the heterogeneity of efficiencies across inputs in the countries. It follows that the probability π_{ij} that country "i" provides an input at the lowest price in country "j" is[5]:

$$\pi_{ij} = \frac{\Upsilon_i (c_i t_{ij})^{-\theta}}{\Phi_j} \tag{8}$$

This probability is the same regardless of the type input. It also represents the share of country "i" in the total demand of inputs by firms in country "j". Let H_j be this demand exclusive of intermediate goods imports tariff revenues such that $H_j = (I_j * P_j) - R_j^I$ where R_j^I represents tariff revenues on intermediate goods.

The value of country "j" bilateral demand of inputs or intermediate goods from country "i" exclusive of intermediate goods imports tariff revenues[6] is therefore:

$$h_{ij} = \frac{\Upsilon_i (c_i t_{ij})^{-\theta}}{\Phi_j} H_j \tag{9}$$

$$\text{With } \Phi_j = \sum_{i=1}^{n} \Upsilon_i (c_i t_{ij})^{-\theta} \tag{10}$$

Φ_j is a parameter of the composite intermediate input price in country "j". Assuming as Eaton and Kortum (2002) that $\varepsilon < 1+ \theta$ for the price index to be well defined, we

[5] For more details, see Eaton and Kortum (2002)

[6] We need intermediate goods imports exclusive of tariff revenues because it is what is required to obtain the input requirements matrix necessary for the calculation of value-added exports.

get the exact price index from equation (5) and the distribution of $p_j(\omega)$ implied by the assumptions made earlier, which gives[7]:

$$P_j = \gamma \left(\sum_{i=j}^{N} \Upsilon_i \left(c_i t_{ij} \right)^{-\theta} \right)^{\frac{-1}{\theta}} \tag{11}$$

Where $\gamma = \left[\Gamma \left(\frac{1-\varepsilon+\ \theta}{\theta} \right) \right]^{\frac{1}{1-\varepsilon}}$ with Γ the gamma function.

The cost of a bundle of production factors c_i net of export trade costs is then given by:

$$c_i = \zeta_i \ {w_i}^{\alpha_i} \ P_i^{1-\alpha_i} \tag{12}$$

Where w_i is the nominal wage in country "i" and $\zeta_i = {\alpha_i}^{-\alpha_i} \ 1{-\alpha_i}^{\alpha_i - 1}$ a constant.

Eaton and Kortum (2002) or Caliendo and Parro (2015) assume implicitly that π_{ij}, the share of country "i" in the total demand of inputs by firms in country "j" (8) is also the share of country "i" in country "j" total demand of final goods. This is because π_{ij} in their framework is the share of goods (not only intermediate goods as in our model but also final goods) imported from country "i" by country "j" in its total demand but also the probability that country "i" provides a good at the lowest price in country "j". Once again, this probability is the same regardless of the type of good.

As Antràs and De Gortari (2017) suggested, the implicit assumption of these authors is not confirmed by the data. To solve this problem, Alexander (2017) assumed that for a given country, the average technology parameter Υ_i for producing intermediate and final goods is different. It allows him to stay in this Ricardian framework for modelling trade in final goods. We do not follow this approach.

Instead, we assume that consumers have a "love of variety-like" utility function which has different implications in terms of final goods price indexes, trade shares and the trade elasticity. More precisely, consumers do not necessarily search for the lowest cost supplier but want to consume all the varieties of goods supplied by each country. This assumption leads to a different model as regards trade in final goods, a model that is similar to the standard Anderson and Van Wincoop (2003) gravity equation.

[7] See Appendix A in Caliendo and Parro (2015) for more details

2.3 Trade in final goods

Let us define v as a variety of final good produced by country "i". Country "i" supply of final goods follows from the production function defined in equation (1):

$$f_i\left(v\right) = z_i \; l_i\left(v\right)^{\alpha_i} \; m_i\left(v\right)^{1-\alpha_i} \tag{13}$$

Where $f_i\left(v\right)$ is the quantity of final goods of variety v produced by country "i", z_i represents country "i" efficiency at producing a final good which we assume for simplicity to be the same across final goods, $l_i\left(v\right)$ is labor and $m_i\left(v\right)$ the composite intermediate inputs used in order to produce variety v' final goods. The cost of producing a unit of good v is such that :

$$\frac{c_i}{z_i} = \frac{\zeta_i \; w_i^{\alpha_i} P_i^{1-\alpha_i}}{z_i} \tag{14}$$

Where c_i is the cost of a bundle of production factors defined in equation (12). Country "i" nominal total supply of final goods from is thus given by :

$$F_i = c_i \int_0^1 \frac{z_i \; l_i\left(v\right)^{\alpha_i} \; m_i\left(v\right)^{1-\alpha_i}}{z_i} dv \tag{15}$$

Country "j" consumers maximize the following utility function:

$$\left(\sum_i \beta_i^{\frac{1-\sigma}{\sigma}} f_{ij}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{16}$$

Subject to the budget constraint:

$$\sum_i p_{ij} f_{ij} = X_j \tag{17}$$

Where β_i is a positive distribution parameter, f_{ij} the consumption of country "i" final good by country "j" consumers, p_{ij} the price of country "i" final good for country "j" consumers and X_j represents country "j" total demand of final goods inclusive of final goods tariff revenues (the economy's total expenditures). We have $p_{ij} = \frac{c_i}{z_i} t_{ij}$ where the exporter's supply price net of trade costs is $\frac{c_i}{z_i}$ as in equation (14) and t_{ij} the trade cost factor between "i" and "j". The nominal value of country "i" final goods imports from "j" inclusive of tariff revenues is therefore $p_{ij} f_{ij}$. A simple maximization of the utility function under the budget constraint yields:

$p_{ij}f_{ij} = \frac{\left(\ddot{\beta}_i c_i t_{ij}\right)^{1-\sigma}}{\sum_i \left(\ddot{\beta}_i c_i t_{ij}\right)^{1-\sigma}} \sum_i p_{ij}f_{ij}$ with $\ddot{\beta}_i = \frac{\beta_i}{z_i}$

In order to determine value-added exports flows as we do in the following section, we need final goods imports exclusive of tariff revenues. Let us define country "i" bilateral imports of final goods from "j" exclusive of tariff revenues as $x_{ij} = \frac{p_{ij}f_{ij}}{1+\tau_{ij}^F}$ with τ_{ij}^F representing the bilateral ad-valorem flat rate tariff for final goods imports. We will get:

$$x_{ij} = \frac{\left(\ddot{\beta}_i c_i t_{ij}\right)^{1-\sigma} E_j}{P_j^{F\,1-\sigma}} \tag{18}$$

Where $E_j = \sum_s \frac{p_{ij}f_{ij}}{1+\tau_{ij}^F}$ is country "j" demand of final goods exclusive of final goods tariff revenues and :

$$P_j^F = \left(\sum_i \left(\ddot{\beta}_i c_i t_{ij}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{19}$$

P_j^F is, therefore, the price index of final goods in country "j". The market clearing condition implies that country "i" total supply of final goods is also equal to follows from equation (18) and is given by $F_i = \sum_j x_{ij}$. As it should be clear now, this supply of final goods is as total output composed of value-added from different origins, be it local or foreign, so as bilateral exports of final goods. If we are interested in bilateral value-added exports which are exports that embed only value-added from local origin, a different model should be used.

2.4 Trade in value-added

We can determine the amount of value-added that a given country exports to its trading partners including itself as a function of the total supply of final goods. This amount is equivalent to its GDP. Let us define α_{is} as the fraction of country "i" GDP required by country "s" in order to produce a unit of final good. The GDP of country "i" is equal to the sum of the value-added that it provides to each country "s" including itself. We have:

$$w_i L_i = \left(\sum_{s=i}^{S} \alpha_{is} F_s\right) \tag{20}$$

As each country "s" exports its final goods to the countries of final consumption including itself, we can also determine the value-added exported by a given origin country "i" to a given destination of final consumption "j". As shown in equation (21), it is the sum of the value-added originated in "i" that is firstly sent to intermediate countries "$s \in$ S" for transformation into final goods before being sent to the country of final consumption "j".

$$v_{ij} = (\sum_{s=i}^{S} \alpha_{is} X_{sj}) \tag{21}$$

Where v_{ij} represents bilateral value-added exports from country "i" to country "j"[8], X_{sj} defined as in equation (18) represents final goods exports from country "s" to country "j" and α_{is} as said earlier is the fraction of country "i" GDP required by country "s" in order to produce a unit of final good.

Note that the set S includes the origin country "i" and the destination country "j". Hence, when $i = s$ country "i" exports directly its value-added to country "j". When $i \neq s$, country "i" exports indirectly its value-added to country "j" via the other intermediate countries' final goods exports to "j". When $s = j$, country "i" exports its value added to the destination of final consumption "j", but this value-added is transformed in final good in "j" before consumption.

By combining equation (18) and equation (21), it follows that:

$$v_{ij} = \sum_{s=i}^{S} \frac{\left(\ddot{\beta}_s c_s t_{sj}\right)^{1-\sigma} E_j}{P_j^{F\,1-\sigma}} \alpha_{is}$$

$$= \left(\frac{\left(\ddot{\beta}_i c_i t_{ij}\right)^{1-\sigma} E_j}{P_j^{F\,1-\sigma}} \alpha_{ii}\right) + \left(\sum_{s\neq i}^{S} \frac{\left(\ddot{\beta}_s c_s t_{sj}\right)^{1-\sigma} E_j}{P_j^{F\,1-\sigma}} \alpha_{is}\right)$$

$$\Rightarrow v_{ij} = \left(\frac{\left(\ddot{\beta}_i c_i t_{ij}\right)^{1-\sigma} E_j}{P_j^{F\,1-\sigma}} \alpha_{ii}\right) \left(\frac{\frac{\left(\ddot{\beta}_i c_i t_{ij}\right)^{1-\sigma} E_j}{P_j^{F\,1-\sigma}} \alpha_{ii} + \sum_{s\neq i}^{S} \frac{\left(\ddot{\beta}_s c_s t_{sj}\right)^{1-\sigma} E_j}{P_j^{F\,1-\sigma}} \alpha_{is}}{\frac{\left(\ddot{\beta}_i c_i t_{ij}\right)^{1-\sigma} E_j}{P_j^{F\,1-\sigma}} \alpha_{ii}}\right)$$

$$= \left(\frac{\left(\ddot{\beta}_i c_i t_{ij}\right)^{1-\sigma} E_j}{P_j^{F\,1-\sigma}}\right) \left(\frac{\sum_{s=i}^{S} \alpha_{is} \left(\ddot{\beta}_s c_s t_{sj}\right)^{1-\sigma}}{\left(\beta_i c_i t_{ij}\right)^{1-\sigma}}\right)$$

$$\Rightarrow v_{ij} = \left(\frac{\left(\ddot{\beta}_i c_i t_{ij} t_{\mathrm{iSj}}\right)^{1-\sigma} E_j}{P_j^{F\,1-\sigma}}\right) \tag{22}$$

[8] It is straightforward to see that $w_i L_i = \sum_{s=i}^{S} \sum_j \alpha_{is} X_{sj}$

$$\text{Where } t_{iSj} = \left(\frac{\sum_{s=i}^{S} \alpha_{is} \left(\ddot{\beta}_s c_s t_{sj} \right)^{1-\sigma}}{\left(\ddot{\beta}_i c_i t_{ij} \right)^{1-\sigma}} \right)^{\frac{1}{1-\sigma}} \tag{23}$$

This term t_{iSj} is a function of the weighted relative price between the indirectly exported flows by the origin country "i" to the destination country "j" through intermediate countries "$s \in S$" over the directly exported flows by the origin country "i" to the destination country "j". Besides, we can see that equation (22) is nothing more than the Anderson and Van Wincoop's gravity equation scaled by this term t_{iSj} that we label "the cost of fragmentation".

As bilateral trade costs, this term exerts a negative effect on bilateral value-added exports. However, it decreases with the amount of value added exported as input by the origin country to intermediate countries, that is to say $\frac{\partial t_{iSj}}{\partial \alpha_{is}} < 0$. It means that the more a country exports its intermediate inputs to a given destination's main providers of final goods, the lesser its cost of fragmentation will be and the higher will be its indirectly exported flows to this destination, comparatively to a country that exports less intermediate inputs to the said providers of final goods. Consequently, upstream countries, that are countries with a high forward participation in a given chain of production undergo a low cost of fragmentation, whereas the most downstream countries that have a low forward participation in comparison to the previous but a higher backward participation undergo a higher cost of fragmentation and, therefore, export more directly their goods to final consumers[9].

t_{iSj} therefore, measures the proximity of country "i" to the final consumers in country "j".[10] The lower it is, the further away is the origin country from the final consumer. It implies higher indirectly exported flows to the country of final consumption. On the contrary, the higher it is, the closer is the origin country from the final consumer. The indirectly exported flows are, therefore, lesser and exports of final goods are higher. As equation (23) shows, t_{iSj} depends critically on α_{is} which is the fraction of country "i" value-added required by country "s" in order to produce a unit of final good. The latter is obtained using input-output analysis. More precisely, we have:

$$\alpha_{is} = \alpha_i * B_{is} \tag{24}$$

[9] Direct exports of goods to final consumers refer to final goods exports.

[10] Fally (2012) and Antràs et al. (2012) also proposed indexes to measure the distance of industries to final demand or the average position of countries in global supply chains.

Where $\alpha_i = \frac{w_i L_i}{G_i}$ is the share of GDP (total value-added) in total output, and where B_{is} is the quantity of output sourced by country "s" from country "i" in order to produce a unit of final good. It is thus an element of the input requirements matrix also known as the Leontief inverse matrix. Let A be the input-coefficient matrix obtained from an input-output table with $\frac{h_{ij}}{G_j}$ as elements; h_{ij} being the value of country "i" bilateral supply of intermediate goods to country "j" and G_j the nominal output of country "j" such that :

$$G_j = \int_0^1 \frac{c_j}{z_j(\omega)} q_j(\omega) d\omega \tag{25}$$

$$= \int_0^1 \zeta_j \ (w_j l_j(\omega))^{\alpha_j} \left(P_j m_j(\omega)\right)^{1-\alpha_j} d\omega$$

The Leontief inverse is given by $B = (ID - A)^{-1}$ with ID being an identity matrix. From matrix algebra, we know that $ID = (ID - A)^{-1}(ID - A)$. If we define ID_{ij} as an element of the identity matrix, it follows that the Leontief inverse can be obtained by solving:

$$ID_{ij} = \sum_{s=1}^{S} B_{is} \left(ID_{sj} - \frac{h_{sj}}{G_j} \right) \tag{26}$$

2.5 Total expenditures and trade balance

Let us set country "j" for the sake of presentation as the benchmark country in this section. The economy's total expenditures X_j are given by the following equation:

$$X_j = w_j L_j + R_j \ + \ D_j \tag{27}$$

X_j also represents the final absorption of country "j" which is the sum of labor income $w_j L_j$, tariff revenues R_j and the trade deficit D_j; where $R_j = R_j^I + R_j^F$ is the sum of tariff revenues on intermediate goods R_j^I and final goods R_j^F and labor income is also the sum of value-added exports such that $w_j L_j = \sum_j \ v_{ji} = \sum_{s=j}^{S} \sum_i \alpha_{js} X_{si}$.We have:

$$R_j^I = \sum_i \tau_{ij}^I \ h_{ij} \tag{28}$$

$$R_j^F = \sum_i \tau_{ij}^F \ x_{ij} \tag{29}$$

With τ_{ij}^I and τ_{ij}^F representing the bilateral ad-valorem flat-rate tariffs respectively for intermediate and final goods imports.

$D_j = \sum_i (h_{ij} + x_{ij}) - \sum_i (h_{ji} + x_{ji})$ is the difference between total imports of intermediate and final goods and total exports. As Caliendo and Parro (2015) we assume the country's trade deficit to be exogeneous in this model and the sum of trade deficits across countries to be equal to zero.

The economy's total expenditures X_j are also given by $X_j = E_j + R_j^F$ where E_j as said earlier is country "j" total demand of final goods exclusive of final goods tariff revenues. It follows that:

$$E_j = w_j L_j + R_j^I + D_j \tag{30}$$

In equilibrium, the country total supply of goods G_j which is defined in equation (25) should be equal to the total expenditures excluding tariff revenues of the economy, final goods and intermediate goods included, minus the trade deficit. We thus have:

$$G_j = H_j + E_j - D_j \tag{31}$$

where H_j is the total demand of intermediate goods exclusive of intermediate goods tariff revenues. Writing equation (31) differently, we would get:

$$G_j = H_j + R_j^I + E_j + R_j^F - R_j - D_j \tag{32}$$

We can directly see from equation (30) that $E_j + R_j^F - R_j - D_j = w_j L_j$ is by definition the GDP of country "j". We thus get:

$$G_j = H_j + R_j^I + w_j L_j \tag{33}$$

Moreover, in equilibrium, a given country's total supply of goods should be equal to the total expenditures (excluding tariff payments) of all the countries in the world on goods from this given country. Using equation (33) as the definition of country "j" total supply of goods which is equivalent to equation (25), it follows that:

$$\sum_i h_{ij} \left(1 + \tau_{ij}^I\right) + w_j L_j = \sum_i (h_{ji} + x_{ji}) \tag{34}$$

In appendix 6.C, I represent these equations in an inter-country input-output table.

2.6 Welfare predictions

In trade theory, welfare is generally defined as the real expenditures of the economy. It is represented in this work by $\frac{X_j}{P_j^F}$ where X_j, given by equation (27) is the nominal value of the economy's total expenditures and P_j^F given by equation (19) is the price index of final goods. This variable, thus, depends on tariff revenues and trade imbalances. A lot of static models[11], however, generally assume that there are no trade imbalances and abstract from tariff revenues, which implies that the welfare variable depends only on the real wage. In this model, we allow for trade imbalances and assume that they are lump-sum transfers which remain unchanged between the initial and the counterfactual equilibrium as suggested by Costinot and Rodríguez-Clare (2014) and following Caliendo and Parro (2015). Nevertheless, to ensure comparability with the above-mentioned static models and for simplicity, we will focus on analyzing the real wage and especially the change in real wage following a trade shock as our measure of welfare, given that the real wage should be the same regardless of trade imbalances.

To determine this real wage, we firstly combine the trade equation for a given country's intermediate goods internal flows (equation (8)), with equations (11) and (12) representing respectively the price index for intermediate goods and the unit cost of production. This allows us to obtain the nominal wage. Then, the relevant price index which is the price of final goods is obtained by rearranging equation (18), the final goods trade equation, in order to express it in terms of trade data. We have with $t_{jj} = 1$:

$$\left(\frac{\pi_{jj}\Phi_j}{\Upsilon_j}\right)^{\frac{-1}{\theta}} = c_j = \zeta_j \ {w_j}^{\alpha_j} \ {p_j}^{1-\alpha_j}$$

$$\implies {w_j}^{\alpha_j} = \frac{1}{\zeta_j}\left(\gamma {\Phi_j}^{\frac{-1}{\theta}}\right)^{\alpha_j - 1}\left(\frac{\pi_{jj}\Phi_j}{\Upsilon_j}\right)^{\frac{-1}{\theta}}$$

$$\implies w_j = \left(\frac{1}{\zeta_j}\right)^{\frac{1}{\alpha_j}} (\gamma)^{\frac{\alpha_j - 1}{\alpha_j}} \left(\frac{\pi_{jj}{\Phi_j}^{\alpha_j}}{\Upsilon_j}\right)^{\frac{-1}{\theta\alpha_j}} \tag{35}$$

We also have from equation (18):

$$\implies P_j^F = \frac{\ddot{\beta}_j c_j}{{\lambda_{jj}}^{\frac{1}{1-\sigma}}} = \frac{\ddot{\beta}_j}{{\lambda_{jj}}^{\frac{1}{1-\sigma}}}\left(\frac{\pi_{jj}\Phi_j}{\Upsilon_j}\right)^{\frac{-1}{\theta}} \tag{36}$$

Where $\lambda_{jj} = \frac{x_{jj}}{E_j}$

[11] See for instance Fally and Hillberry (2018), Alexander (2017), Arkolakis et al. (2012), Eaton and Kortum (2002)

Combining equation (35) and (36), we get the following real wage equation:

$$\frac{w_j}{P_j^F} = \left(\frac{1}{\zeta_j}\right)^{\frac{1}{\alpha_j}} (\gamma)^{\frac{\alpha_j - 1}{\alpha_j}} \left(\frac{\pi_{jj}}{\Upsilon_j}\right)^{\frac{-1}{\theta}\left(\frac{1-\alpha_j}{\alpha_j}\right)} \left(\frac{\lambda_{jj}}{\ddot{\beta}_j^{\,1-\sigma}}\right)^{\frac{1}{1-\sigma}} \tag{37}$$

This equation is in many regards similar to the real wage formula that we would get from a standard one sector Eaton and Kortum (2002) or Armington model with tradable intermediate goods. Precisely, it must be assumed for the formulas to be equivalent that the share of intermediate goods sourced locally in the total demand of intermediate goods is the same as the share of final goods sourced locally in the total demand of final goods but also that trade elasticities are the same regardless of the type of trade flows (intermediate or final goods). If it is the case, there would be no need to model trade in final goods differently than trade in intermediate goods, and we could get the real wage by dividing the nominal wage in equation (37) with the price index of intermediate goods in equation (11). This would give:

$$\frac{w_j}{P_j} = (\gamma\zeta_j)^{\frac{-1}{\alpha_j}} \left(\frac{\pi_{jj}}{\Upsilon_j}\right)^{\frac{-1}{\theta\alpha_j}} \tag{38}$$

$$\Longrightarrow \frac{w_j}{P_j} = (\gamma\zeta_j)^{\frac{-1}{\alpha_j}} \left(\frac{\pi_{jj}}{\Upsilon_j}\right)^{\frac{-1}{\theta}\left(\frac{1-\alpha_j}{\alpha_j}\right)} \left(\frac{\pi_{jj}}{\Upsilon_j}\right)^{\frac{-1}{\theta}} \tag{39}$$

It is the same real wage equation determined by Eaton and Kortum (2002) [12]

Thus, assuming that the share of intermediate goods sourced locally in the total demand of intermediate goods is equivalent to the share of final goods sourced locally in the total demand of final goods, an assumption non consistent with trade data, have implications as Alexander (2017) already showed on the welfare gains of trade. The results section will make it clear.

We could also derive equation (37) using value-added trade flows instead of final goods trade flows. Theoretically, this should not modify the real wage value as the price index of final goods is not supposed to change between the two models. Using equation (22), we can express the price of final goods in terms of value-added trade data as following:

$$P_j^F = \frac{\ddot{\beta}_j c_j t_{jSj}}{\lambda_{jj_{va}}^{\frac{1}{1-\sigma}}} = \frac{\ddot{\beta}_j t_{jSj}}{\lambda_{jj_{va}}^{\frac{1}{1-\sigma}}} \left(\frac{\pi_{jj}\Phi_j}{\Upsilon_j}\right)^{\frac{-1}{\theta}} \tag{40}$$

[12] See equation 15 in Eaton and Kortum (2002)

Where $\lambda_{jj_{va}} = \frac{v_{jj}}{D_j}$

$$\Longrightarrow \frac{w_j}{P_j^F} = \left(\frac{1}{\zeta_j}\right)^{\frac{1}{\alpha_j}} (\gamma)^{\frac{\alpha_j - 1}{\alpha_j}} \left(\frac{\pi_{jj}}{\Upsilon_j}\right)^{\frac{-1}{\theta}\left(\frac{1-\alpha_j}{\alpha_j}\right)} \left(\frac{\lambda_{jj_{va}}}{\ddot{\beta}_j^{\,1-\sigma} \quad t_{jSj}{}^{1-\sigma}}\right)^{\frac{1}{1-\sigma}} \tag{41}$$

The condition required for equation (37) and equation (41) to be equal is that $\lambda_{jj} = \frac{\lambda_{jj_{va}}}{t_{jSj}{}^{1-\sigma}}$ i.e. that the ratio of the share of internal trade in value-added over the internal cost of fragmentation be equal to the share of final goods internal trade. As we will see in the data section, this condition is met.

It appears as (37) and (41) show that the real wage decreases with internal trade be it internal trade in intermediate goods π_{jj} , internal trade in final goods λ_{jj} or internal trade in value-added $\lambda_{jj_{va}}$, but increases with technology Υ_j. We can also see that it decreases with the trade cost of fragmentation t_{jSj}. As said earlier t_{jSj} summarizes the production linkages of the origin country with all the indirect exporters "$s \in S$" of its value-added. In this case, the value-added is exported as intermediate inputs in the first step by the origin country to intermediate countries and exported back to the origin country embedded in these intermediate countries' final goods. From this formula, we see which factors can drive a given country's welfare gains from a change in trade costs for example. Expressing (41) in relative change assuming ζ_j, Υ_j and $\ddot{\beta}_j$ to be constant across equilibria gives:

$$\ln \frac{\widehat{w_j}}{\widehat{P_j}} = \frac{-1}{\theta}\left(\frac{1-\alpha_j}{\alpha_j}\right) \ln \widehat{\pi_{jj}} - \frac{1}{\sigma - 1} \ln \widehat{\lambda_{jj_{va}}} - \ln \widehat{t_{jSj}} \tag{42}$$

Where a variable with a hat, for instance $\widehat{X}$ represents the relative change of the variable between an initial and a counterfactual equilibrium such that $\widehat{X}=\frac{X'}{X}$ with X the variable in the initial equilibrium and X' the variable in the counterfactual equilibrium.

Consider for example a reduction in the level of a given country's bilateral trade costs on imports and exports with its trading partners. This shock would reduce the import price of its intermediate inputs, which is the source of the first gain. In this case, the share of internal trade in intermediate goods π_{jj} decreases between the initial and the counterfactual equilibrium because more intermediate inputs are imported from other countries as a result of the decrease in bilateral trade costs. $\frac{-1}{\theta}\left(\frac{1-\alpha_j}{\alpha_j}\right) \ln \widehat{\pi_{jj}}$ which represents the first source of gains is thus positive. The second source of change in the gains, $\frac{1}{1-\sigma} \ln \widehat{\lambda_{jj_{va}}}$, is also affected positively by the decrease in the level of bilateral trade costs. In fact, the share of value-added exported to itself by the given country decreases between the initial and the counterfactual equilibrium, because more value-added is imported from other countries. This implies a positive value of $\frac{1}{\sigma-1}\ln\widehat{\lambda_{jj_{va}}}$ and, therefore, a positive change of the given country real wage.

The last source of change in the gains, $\ln\widehat{t_{jSj}}$, is the change in the trade costs undergone by the given origin country "j" for the inputs exported to its partners or intermediate countries ($s \in S$), and that are exported back by them to "j" embedded in their final goods. In autarky, this term is equal to one, the lower bound trade cost when a country trades with itself. It means that decreasing the level of bilateral trade costs should have a positive impact on this variable and therefore, exert a negative impact on the welfare gains change.

This last source of gains as explained earlier appears in the welfare formula because of the use of the value-added trade equation to determine the price index. Had we used the final goods trade equation that it would have been captured by the gains related to trade in final goods so that : $\frac{1}{\sigma-1}\ln\widehat{\lambda_{jj_{va}}} - \ln\widehat{t_{jSj}} = \frac{1}{\sigma-1}\ln\widehat{\lambda_{jj}}$. It follows that this approach based on value-added exports allows us to determine the net share of the welfare gains that can be attributed to international fragmentation. This is because we identify the impact of being able to import and export cheap intermediate inputs on the change in real wage $\frac{-1}{\theta}\left(\frac{1-\alpha_j}{\alpha_j}\right)\ln\widehat{\pi_{jj}}$ and also the indirect cost implied by this international organization of production $-\ln\widehat{t_{jSj}}$. With a standard trade model à la Eaton and Kortum (2002) or Anderson and Van Wincoop (2003) with tradable intermediate goods, we would not be able to do so because we could only identify $\frac{1}{\sigma-1}\ln\widehat{\lambda_{jj}}$ which embeds $-\ln\widehat{t_{jSj}}$ as explained earlier. We define the net share of international fragmentation in the welfare gains of trade as following:

$$\ln\frac{\widehat{w_j}}{\widehat{P_j}}^F = \frac{\frac{-1}{\theta}\ln\widehat{\pi_{jj}} - \ln\widehat{t_{jSj}}}{\ln\frac{\widehat{w_j}}{\widehat{P_j}}} \tag{43}$$

Consider for example the hypothetical situation of a move to autarky for country "j". $\frac{-1}{\theta}\left(\frac{1-\alpha_j}{\alpha_j}\right)\ln\widehat{\pi_{jj}}$ on one hand that would be negative, represents the log change in real wage related to the fact that country "j" could not anymore source cheap inputs from other countries in order to produce its final goods. On the other hand, $-\ln\widehat{t_{jSj}}$ which would be positive represents the log change in real wage related to the trade costs that country "j" would not have to undergo anymore to send its inputs to intermediate countries before re-importing them embedded in final goods or intermediate inputs used in the production of its final goods. With a one stage production process the log change in real wage would simply be $-\frac{1}{\sigma-1}\ln\widehat{\lambda_{jj_{va}}}$ where $\lambda_{jj_{va}}$, the share of domestic expenditures on value-added would be equal to the share of domestic expenditures on final goods (18).

In this regard, our results share similarities with the model of Fally and Hillberry (2018) who proposed a sequential model of global supply chains. More precisely, they proposed a welfare formula for a two-country case with one country upstream, the other one downstream, and they showed that the welfare gains in presence of fragmentation are lower than without for the upstream country and higher for the downstream country. This is due to the fact that the upstream country re-imports its previously exported inputs to the downstream one embedded in the latter final goods exports. As this amounts to an indirect export to oneself and that welfare decreases with internal trade, this result is totally sensical. The downstream country however does not export inputs whatsoever in their framework, but sources some of its inputs from the upstream one, everything that increases its welfare.

Their welfare formula is, therefore, suitable to analyze the net welfare gains of international fragmentation, but ours is more general because it works also for a "more than two country-case" where both upstream and downstream countries import and export intermediate inputs.

2.7 Counterfactual analysis

Different kind of trade costs shocks are often envisaged to determine the welfare gains of trade. The most commonly used in the literature is a move to autarky.

2.7.1 Autarky

It is straightforward to see that one does not need to solve the full general equilibrium model to get the change in real wage as in autarky, the internal trade shares and the internal cost of fragmentation would be equal to 1.

From equation (42), It follows that the welfare formula (the log change in real wage) after a move to autarky is given by:

$$W = \frac{1}{\theta}\left(\frac{1-\alpha_j}{\alpha_j}\right)\ln \pi_{jj} + \frac{1}{\sigma-1}\ln \lambda_{jj_{va}} + \ln t_{jSj} \tag{44}$$

Something interesting to mention is that we don't need to calculate the internal cost of fragmentation t_{jSj} as it could be straightforwardly approximated through the data. We can see this from equation (23) which defines the cost of fragmentation.

$$t_{jSj}= \left(\frac{\sum_{s=j}^{S}\alpha_{js}\left(\ddot{\beta}_s c_s t_{sj}\right)^{1-\sigma}}{\left(\ddot{\beta}_j c_j t_{jj}\right)^{1-\sigma}}\right)^{\frac{1}{1-\sigma}}$$

$$\implies t_{jSj} = \left(\frac{\alpha_{jj}\left(\ddot{\beta}_j c_j t_{jj}\right)^{1-\sigma} + \sum_{s\neq j}^{S} \alpha_{js}\left(\ddot{\beta}_s c_s t_{sj}\right)^{1-\sigma}}{\left(\ddot{\beta}_j c_j t_{jj}\right)^{1-\sigma}} \right)^{\frac{1}{1-\sigma}}$$

$$\implies t_{jSj} = \left(\alpha_{jj} + \frac{\sum_{s\neq j}^{S} \alpha_{js}\left(\ddot{\beta}_s c_s t_{sj}\right)^{1-\sigma}}{\left(\ddot{\beta}_j c_j t_{jj}\right)^{1-\sigma}} \right)^{\frac{1}{1-\sigma}}$$

Where $\frac{\sum_{s\neq j}^{S} \alpha_{js}\left(\ddot{\beta}_s c_s t_{sj}\right)^{1-\sigma}}{\left(\ddot{\beta}_j c_j t_{jj}\right)^{1-\sigma}}$ represents the ratio of the value-added exported as intermediate good by "j" to intermediate countries $S \neq j$ and that comes back to "j" embedded in its final goods imports from the intermediate countries, over the internal trade in final goods of country "j". To see this, we can rewrite the ratio as following:

$$\frac{\sum_{s\neq j}^{S} \alpha_{js}\left(\ddot{\beta}_s c_s t_{sj}\right)^{1-\sigma}}{\left(\ddot{\beta}_j c_j t_{jj}\right)^{1-\sigma}} = \frac{\sum_{s\neq j}^{S} \frac{\alpha_{js}\left(\ddot{\beta}_s c_s t_{sj}\right)^{1-\sigma} E_j}{P_j^{F\,1-\sigma}}}{\frac{\left(\ddot{\beta}_j c_j t_{jj}\right)^{1-\sigma} E_j}{P_j^{F\,1-\sigma}}}$$

It is straightforward to see that this ratio is negligible as countries tend to trade more with themselves than with others. Besides, the numerator of the ratio is low by definition as it is only a tiny fraction (α_{js}) of the final goods imports from intermediate countries. We show that in the data section. Thus, the internal cost of fragmentation can be approximated by:

$$t_{jSj} \approx (\alpha_{jj})^{\frac{1}{1-\sigma}} \tag{45}$$

Where $\alpha_{jj} = \alpha_j * B_{jj}$ as shown in equation (24) is the fraction of local value-added required to produce a unit of final good in country "j", with α_j the share of GDP in total output and B_{jj} the fraction of local output required to produce a unit of final good in country "j". As shows equation (26), B_{jj} is obtained through the Leontief inverse. These data are generally observable or could be obtained with minimal transformations.

When the shock is not a move to autarky but an infinitesimal change in trade costs for example, one needs to solve the full general equilibrium model to get the counterfactual shares of internal trade and the counterfactual cost of fragmentation. To do so, we follow the approach of Dekle et al. (2008) which is to solve the model in change and, therefore, avoid having to calibrate unobservable parameters such as preferences or technology.

2.7.2 Other trade costs shocks

We assume as Caliendo and Parro (2015) that the share of value-added in total output $\alpha_j = \frac{w_j L_j}{G_j}$ is fixed across equilibria as well as technology and preference parameters. It implies that:

$$\ln \widehat{w_j} = \ln \widehat{G_j} \tag{46}$$

This change in the nominal wage $\ln \widehat{w_j}$ affects the change in the unit cost of a bundle of inputs associated to a trade costs shock $\ln \widehat{c_j}$. Equation (12) states that this cost is $c_j = \zeta_j \ w_j^{\alpha_j} \ P_j^{1-\alpha_j}$. The log change is thus equal to:

$$\ln \widehat{c_j} = \alpha_j \ln \widehat{w_j} + (1 - \alpha_j) \ln \widehat{P_j} \tag{47}$$

With $\ln \widehat{P_j}$ the change in the intermediate inputs price index given by:

$$\ln \left(\widehat{P_j}\right) = \frac{-1}{\theta} \ln \left(\widehat{\Phi_j}\right) \tag{48}$$

The log change in Φ_j, the intermediate goods price index parameter follows from equation (10) which states that $\Phi_j = \sum_{i=1}^{N} \Upsilon_i \left(c_i t_{ij}\right)^{-\theta}$. It follows that:

$$\ln \left(\widehat{\Phi_j}\right) = \sum_{i=1}^{N} \frac{h_{ij}}{H_j} \ln \left(\widehat{c_i t_{ij}}\right)^{-\theta} \tag{49}$$

Where bilateral imports in intermediate goods, $h_{ij} = \frac{\Upsilon_i (c_i t_{ij})^{-\theta} H_j}{\Phi_j}$ come from equation (9). Expressed in log change, it gives:

$$\ln \widehat{h_{ij}} = \ \ln \left(\widehat{c_i}\right)^{-\theta} + \ln \left(\widehat{t_{ij}}\right)^{-\theta} + \ln \widehat{H_j} - \ln \widehat{\Phi_j} \tag{50}$$

The change in country "j" total demand of intermediate inputs exclusive of tax $\ln \widehat{H_j}$ follows from equation (33), which states that $G_j = H_j + R_j^I + \ w_j L_j$. This implies that:

$$\ln \widehat{G_j} = \frac{H_j}{G_j} \ln \widehat{H_j} + \frac{R_j^I}{G_j} \ln \widehat{R_j^I} + \frac{w_j L_j}{G_j} \ln \widehat{w_j} \tag{51}$$

Where $\ln \widehat{R_j^I}$, the log change of tariff revenues on intermediate goods follows from equation (28) with $R_j^I = \sum_{i=1}^{N} \tau_{ij}^I \ h_{ij}$. In log change, we would have:

$$\ln \widehat{R_j^I} = \sum_{i=1}^{N} \frac{\tau_{ij}^I h_{ij}}{R_j^I} \ \ln \widehat{h_{ij}} \tag{52}$$

As regards bilateral exports of final goods, equation (18) states that $x_{sj} = \frac{\left(\ddot{\beta}_s c_s t_{sj}\right)^{1-\sigma} E_j}{P_j^{F\,1-\sigma}}$, which implies in relative change :

$$\ln \widehat{x_{sj}} = \ \ln \left(\widehat{c_s}\right)^{1-\sigma} + \ln \left(\widehat{t_{sj}}\right)^{1-\sigma} + \ln \widehat{E_J} - \ln \left(\widehat{P_j^F}\right)^{1-\sigma} \tag{53}$$

And where the log change of the final goods price index P_j^F is given by:

$$\ln\left(\widehat{P_j^F}\right)^{1-\sigma} = \sum_{s=1}^{n} \frac{x_{sj}}{E_j} \ln\left(\widehat{c_s t_{sj}}\right)^{1-\sigma} \tag{54}$$

The log change of the economy's total expenditures net of final goods' tariff revenues $\ln \widehat{E_J}$ follows from equation (30) where $E_j = w_j L_j + \ R_j^I + \ D_j$. We thus get:

$$\ln \widehat{E_J} = \frac{w_j L_j}{E_j} \ln \widehat{w}_j + \frac{R_j^I}{E_j} \ln \widehat{R_j^I} + \frac{D_j}{E_j} \ln \widehat{D}_j \tag{55}$$

As mentioned earlier, we assume trade deficits (the difference between imports and exports) to be exogeneous in this model. It follows that:

$$\ln \widehat{D}_j = 0 \tag{56}$$

We now turn to the determination of the log change in bilateral value-added exports. From equation (22), we know that $\lambda_{ij\,va} = \left(\frac{\ddot{\beta}_i c_i t_{ij} t_{iSj}}{P_j^F}\right)^{1-\sigma}$ with $\lambda_{ij\,va} = \frac{v_{ij}}{E_j}$. It implies in log change:

$$\ln \widehat{\lambda_{ij\,va}} = (1-\sigma)\left[\ \ln\widehat{(c_i)} + \ln \widehat{t_{ij}} + \ln \widehat{t_{iSj}} - \ln\ \widehat{P_j^F}\right] \tag{57}$$

The change in the cost of fragmentation $\ln \widehat{t_{iSj}}$ comes from equation (22), (23) and (24). Specifically:

$$t_{iSj} = \left(\frac{\sum_{s=i}^{S}\ \alpha_{is}\ \left(\ddot{\beta}_s c_s t_{sj}\right)^{1-\sigma}}{\left(\beta_i c_i t_{ij}\right)^{1-\sigma}}\right)^{\frac{1}{1-\sigma}} \text{ and } \alpha_{is} = \alpha_i * B_{is}$$

$$\Longrightarrow d\ln t_{iSj} = \frac{1}{1-\sigma} \frac{\sum_{s=i}^{S}(1-\sigma)\alpha_i B_{is}\left[\frac{\left(\ddot{\beta}_s c_s t_{sj}\right)^{-\sigma} d\left(\ddot{\beta}_s c_s t_{sj}\right)}{\left(\ddot{\beta}_i c_i t_{ij}\right)^{1-\sigma}} - \frac{\left(\ddot{\beta}_s c_s t_{sj}\right)^{1-\sigma} d\left(\ddot{\beta}_i c_i t_{ij}\right)}{\left(\ddot{\beta}_i c_i t_{ij}\right)^{2-\sigma}} + \frac{\left(\ddot{\beta}_s c_s t_{sj}\right)^{1-\sigma} d(\alpha_i B_{is})}{\left(\ddot{\beta}_i c_i t_{ij}\right)^{1-\sigma}(1-\sigma)\alpha_i B_{is}}\right]}{t_{iSj}{}^{1-\sigma}}$$

We know from equation (22) that $\frac{1}{t_{iSj}{}^{1-\sigma}} = \frac{E_j}{v_{ij}}\left(\frac{\ddot{\beta}_i c_i t_{ij}}{P_j^F}\right)^{1-\sigma}$

$$\Longrightarrow \frac{1}{t_{iSj}{}^{1-\sigma}} = \frac{x_{sj}}{v_{ij}}\left(\frac{\ddot{\beta}_i c_i t_{ij}}{\ddot{\beta}_s c_s\ t_{sj}}\right)^{1-\sigma}$$

where $P_j^{F\,1-\sigma}$ is given by : $P_j^{F\,1-\sigma} = \frac{E_j\left(\ddot{\beta}_s c_s\ t_{sj}\right)^{1-\sigma}}{x_{sj}}$

$$\Longrightarrow\ \ln \widehat{t_{iSj}} = \sum_{s=i}^{S} \frac{\alpha_i B_{is} x_{sj}}{v_{ij}}\left[\ln\widehat{(c_s t_{sj})} - \ln\widehat{(c_i t_{ij})} + \ln \widehat{B_{is}}^{\frac{1}{(1-\sigma)}}\right] \tag{58}$$

As we can see, the change in the cost of fragmentation $\ln \widehat{t_{iSj}}$ depends critically on the change in the input requirements $\ln \widehat{B_{is}}$. From equation (26), we have $ID_{ij} = \sum_{s=1}^{S} B_{is}\left(ID_{sj} - \frac{h_{sj}}{G_j}\right)$ where ID_{ij} is an element of the identity matrix.

Expressing this equation in change gives:

$$d\ ID_{ij} = \sum_{s=1}^{S}\left[\left(ID_{sj} - \tfrac{h_{sj}}{G_j}\right) d\ B_{is} + B_{is}\ d\ ID_{sj} - \tfrac{B_{is}}{G_j}\ d\ h_{sj} + \tfrac{B_{is}h_{sj}}{{G_j}^2}\ d\ G_j\right]$$

$$\Longrightarrow 0 = \sum_{s=1}^{S}\left[\left(ID_{sj} - \tfrac{h_{sj}}{G_j}\right) d\ B_{is} - \tfrac{B_{is}}{G_j}\quad d\ h_{sj} + \tfrac{B_{is}h_{sj}}{{G_j}^2}\ d\ G_j\right]$$

$$= \sum_{s=1}^{S}\left[\left(B_{is}ID_{sj} - \tfrac{B_{is}h_{sj}}{G_j}\right) d\ln B_{is} - \tfrac{B_{is}h_{sj}}{G_j}\ \left(d\ln h_{sj} - d\ln G_j\right)\right]$$

$$\Longrightarrow 0 \ = \sum_{s=1}^{S}\left[(B_{is}ID_{sj})\ln\widehat{B_{is}} - \frac{B_{is}h_{sj}}{G_j}\ \left(\ln\widehat{h_{sj}} - \ln\widehat{G_j} + \ln\widehat{B_{is}}\right)\right] \tag{59}$$

To close the model, we use the equilibrium condition defined in equation (34) which states that $\sum_{i=1}^{N} h_{ij}\left(1+\tau_{ij}^{I}\ \right) + w_jL_j = \sum_{i=1}^{N}\left(h_{ji} +\ x_{ji}\right)$. Writing this condition in change gives the following expression:

$$\sum_{i=1}^{N}\left(\left(1+\tau_{ij}^{I}\ \right)\frac{h_{ij}}{G_j}\ln\widehat{h_{ij}} + \ \frac{w_jL_j}{G_j}\ln\widehat{w_j}\right) = \sum_{i=1}^{N}\left(\frac{h_{ji}}{G_j}\ln\widehat{h_{ji}} + \ \frac{x_{ji}}{G_j}\ln\widehat{x_{ji}}\right) \tag{60}$$

Equations (46) to (60) represent the set of 15 equations and 15 unknowns that describe our model in relative change between an initial and a counterfactual equilibrium. As we can see, solving it requires mostly data that are readily observables with the exception of the trade elasticities $(1-\sigma)$ and $-\theta$. As they play a critical role in determining the results, we provide a discussion on their calibration in the next section.

Before that, it is interesting to analyze the conditions required for a decrease in the bilateral cost of fragmentation following a decrease in the level of trade costs; which would mean for the exporting country a higher forward participation in the production network of the goods bought by the importing country. For this to occur, it is necessary that the impact of a decrease in the level of trade costs regarding the indirect relationship from the origin country "i" to the destination of final consumption "j" through intermediate countries $s \neq i \in S$ which is represented by "$\sum_{s\neq i}^{S}\frac{\alpha_i B_{is}x_{sj}}{v_{ij}}\left[\ln\widehat{(c_s t_{sj})} + \ln\widehat{B_{is}}^{\frac{1}{(1-\sigma)}}\right] < 0$", be higher than the impact of trade costs on the direct relationship from the origin country "i" to the destination country "j" represented by "$\frac{\alpha_i B_{ii}}{{t_{iSj}}^{1-\sigma}}\ln\widehat{B_{ii}}^{\frac{1}{(1-\sigma)}} + \sum_{s\neq i}^{S}\frac{\alpha_i B_{is}x_{sj}}{v_{ij}}\left[-\ln\widehat{(c_i t_{ij})}\right] > 0$".

As the change in the elements of the Leontief inverse $\ln\widehat{B_{is}}$ depends as shown in equation (59) on the change in intermediate goods trade flows, it follows that the change in the cost of fragmentation depends critically on the intermediate goods flows trade elasticity $-\theta$. Ceteris paribus, the higher the absolute value of this elasticity, the more $\ln \widehat{B_{is}}^{\frac{1}{(1-\sigma)}}$ would change up to the point where the cost of fragmentation would decrease. However, the higher $(\sigma - 1)$, the less $\ln \widehat{B_{is}}^{\frac{1}{(1-\sigma)}}$ would change such that the cost of fragmentation would increase. Hence, if the trade costs of intermediate countries remain constant, we can conjecture that a necessary condition for the cost of fragmentation to decrease is that the trade elasticity for intermediate goods be sufficiently high in comparison to the trade elasticity for final goods.

It is also straightforward to see that the change in country "j" internal cost of fragmentation, $\ln \widehat{t_{jSj}}$ would be equal to $\ln \widehat{B_{jj}}^{\frac{1}{(1-\sigma)}}$ because as shown in equation (45), $\frac{\alpha_j B_{jj}}{t_{jSj}{}^{1-\sigma}} \approx 1$ with $\sum_{s \neq j}^{S} \frac{\alpha_j B_{js} x_{sj}}{v_{jj}} \left[-\ln \widehat{(c_j t_{jj})} \right]$ as well as $\sum_{s \neq j}^{S} \frac{\alpha_j B_{js} x_{sj}}{v_{jj}} \left[\ln \widehat{(c_s t_{sj})} + \ln \widehat{B_{js}}^{\frac{1}{(1-\sigma)}} \right]$ being negligible.

We summarize the results of this model as following:

- Classical models implicitly assume that the share of intermediate goods sourced locally in the total demand of intermediate goods is equivalent to the share of final goods sourced locally in the total demand of final goods for a given country. As we relax this assumption, the welfare gains of trade in this model are different.
- Deriving the welfare gains of trade using the value-added exports equation rather than the gross trade flows equation allows the identification of the net share of international fragmentation in these welfare gains.
- Calculating the welfare gains of trade against autarky from the value-added trade equation only requires a supplementary parameter obtained after minimal transformations from observable data on top of the internal trade shares and the trade elasticities. This parameter is the fraction of local value-added required to produce a unit of final good in a given country.
- A decrease of a country direct bilateral trade costs, those of intermediate countries remaining constant implies a move towards downstream stages of the production process, provided that the trade elasticity for intermediate goods be sufficiently low.

2.8 Calibration of the trade elasticities

The elasticity of import with respect to variable trade costs generally referred in the literature as the trade elasticity is a key parameter required to infer the gains from trade. Hillberry and Hummels (2013) even go so far as to say that it is the most important parameter in modern trade theory. Estimating it does not come without difficulties regarding notably the identification assumptions, as well explained by the previous authors. This is why a lot of trade theory practitioners have relied upon off-the-shelf elasticities provided by the literature. We follow the same path; however, the particularity of our model imposes us some restrictions.

First of all, as we distinguish between intermediate and final goods trade flows and assume a specific market structure for the trade in intermediate goods, notably perfect competition, the trade elasticities have different interpretations for these two kinds of trade. In a model with perfect competition, the trade elasticity is the shape parameter of the distribution of productivity. It determines the extensive and the intensive margins of the change in trade flows following a change in trade costs and is a sufficient parameter along with the internal trade shares to derive the welfare gains of trade provided that certain conditions are met. We should therefore use a trade elasticity obtained from a method that allows the identification of this parameter specifically. Simonovska and Waugh (2014) or Caliendo and Parro (2015) provide these estimates with a preferred value for the former equal to 4.14, and an aggregate value for the latter equal to 4.45.

Secondly, as regards trade in final goods, we did not assume perfect competition as the market structure and derived our model using an Armington utility function. In this environment, the trade elasticity depends on the elasticity of substitution across varieties. As explained by Simonovska and Waugh (2014), estimations that are based upon the method of Feenstra (1994) allows the identification of this parameter. Imbs and Mejean (2015) use this method and find estimates between 2.2 and 54 with an average of 5.4. Ossa (2015) also provides estimates of this parameter for 251 industries.

Thirdly, it is important to note that we use one sector models for the two kinds of trade, and as Imbs and Mejean (2015) suggests, for a one sector model to mimic the welfare gains of trade that a multi sector-model could predict, the trade elasticity should be a weighted average of sector level elasticities instead of being obtained using aggregated trade data. Unfortunately, we are not able to perform such a calculation without proper weights and trade elasticities matching our disaggregated data.

Another point worth to mention is that, it is common in the theoretical literature as mentioned after equation (10) to assume that the shape parameter of the distribution of productivity is higher than the elasticity of substitution across goods minus one "$\varepsilon - 1 < \theta$". Crozet and Koenig (2010) verify empirically this assumption for a set of firms' data calibrated upon a model of monopolistic competition. This assumption is critical in our perfect competition model for the price index to be well defined (See Eaton and Kortum (2002)).

Lastly, as Antràs and De Gortari (2017) suggest, the trade elasticity seems to be lower on average for intermediate inputs than for final goods. The findings of Fally and Hillberry (2018) could help understanding this point. They explain that with international fragmentation, the final goods trade elasticity is higher than without fragmentation. To illustrate that, they take a two-country case with an upstream and a downstream country, and explain that a 1% increase in trade costs increases the price of the final goods imported by the upstream country by more than 1% since these goods embeds intermediate goods previously exported as inputs to the downstream country.

This point seems to be confirmed by the data. To show it, using the work of Ossa (2015) who provides a set of substitution elasticities for 251 SITC-Rev3 sectors at the 3 digits level, we calculate the average elasticity for intermediate goods and final goods sectors. To do so, using a table of concordance between SITC-Rev3 and the UN classification of goods by end-use (UN-BEC), we select sectors corresponding exclusively to intermediate goods and final goods taken separately according to the UN BEC-Rev4 classification and compute the average. We are left with 129 industries for intermediate goods and 32 industries for final goods, with averages that are respectively 3.08 and 4.75.

In sum, the trade elasticity that we should set for intermediate goods trade flows should be higher than the elasticity of substitution for intermediate goods minus one, but lower than the elasticity of substitution for final goods minus one such that $\varepsilon - 1 < \theta < \sigma - 1$. We select the aggregate estimate of Caliendo and Parro (2015) which is obtained using gross trade flows (final and intermediate goods included) as our benchmark. We do so because their gravity-based estimation of the trade elasticity can fit with models using different market structures provided that they can generate a gravity equation. As this value is equal to 4.45 for all the trade flows combined, we set $\theta = 4.25$ for the intermediate goods model and $\sigma - 1 = 4.85$ for the final goods model.

3 Data

To calculate the net share of fragmentation in the welfare gains of trade, we need a dataset of value-added trade flows. To obtain these data, we use the GTAP 9 database which is a multi-country input-output table. The table comprises 140 entities which are countries or aggregations of countries and 57 sectors that we aggregate into one to simplify the analysis. Released in 2015, it has 3 base years among which we choose 2011 to carry out our analysis. We obtained our value-added trade flows data using the methodology developed by Koopman et al. (2014). As the table is a multi-country table, imports of intermediate inputs are not broken down by countries of origin just as final demand imports. This poses a problem because we need the complete set of bilateral intermediate and final demand imports in order to calculate each country bilateral value-added exports.

To solve this problem, two solutions are generally used in the literature. Applying a proportionality assumption which amounts to assume that the imports of intermediate and final goods of a given country from a particular source are proportional to its total imports from this source. The second solution is to use the UN BEC classification of products by end-use category along with the UN COMTRADE database which reports bilateral exports and imports between countries at the HS 6 digits level, in order to obtain the share of intermediate and final goods in the exports of a given country to a particular destination. These shares are then applied to the export data from the GTAP database to disentangle bilateral exports between intermediate and final goods and calculate the value-added exports. By disentangling bilateral exports by type, we get a new table which is an inter-country input output table and that should be consistent with the initial multi-country input output table.

We decided to choose the second option as it is done in the seminal work of Koopman et al. (2014). To ensure the consistency between the inter-country and the multi-country input-output tables, we used the quadratic mathematical programming model formulated by Tsigas et al. (2012).[13]

It is worth to note that our value-added exports include both goods and services. We therefore use the comprehensive database on trade in services of Francois and Pindyuk (2013) along with a preliminary draft of the UN BEC revision 5 classification by broad economic categories to perform our calculations. This revision, unlike previous ones, does a better job at distinguishing goods and services and classifying them by end-use categories.

Using our reconstructed inter-country input-output table, we calculate the cost of fragmentation with the method presented in Njike (2019)[14] and the unobservable variables $\ddot{\beta}_s c_s$ are approximated using the fixed-effects estimates following Fally (2015). In what follows, we present:

- The relationship between the inverse internal cost of fragmentation ${t_{jSj}}^{1-\sigma}$ and the

[13]The GAMS code is available upon request

[14]To calculate the cost of fragmentation $t_{iSj} = \left(\frac{\sum_{s=i}^{S} \alpha_{is} \left(\ddot{\beta}_s c_s t_{sj}\right)^{1-\sigma}}{\left(\ddot{\beta}_i c_i t_{ij}\right)^{1-\sigma}} \right)^{\frac{1}{1-\sigma}}$ we need proxies for the bilateral trade costs indexes $(t_{sj})^{1-\sigma}$ and the unobservable variables $\ddot{\beta}_s c_s$. To obtain them, an econometric estimation with importer and exporter fixed effects is performed on final goods trade flows. We then predict $(t_{sj})^{1-\sigma}$ using the specified trade costs function

fraction of local value-added "α_{jj}" required to produce a unit of final good in a given country.

- The relationship between internal trade in final goods x_{jj} and the ratio of internal value-added trade flows over the inverse internal cost of fragmentation $\frac{v_{jj}}{t_{jsj}^{1-\sigma}}$.

Chart 1 suggests that there is a perfect correlation as mentioned earlier in equation (45) between the inverse internal cost of fragmentation $t_{jSj}^{1-\sigma}$ and the fraction of local value-added required to produce a unit of final good in a given country "α_{jj}" that we label internal total requirement in the left panel of the chart.

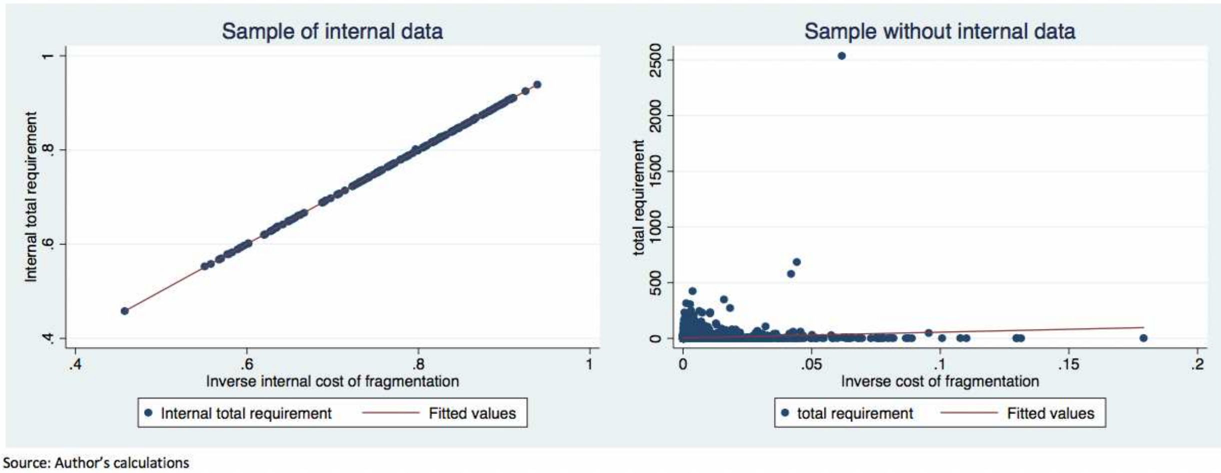


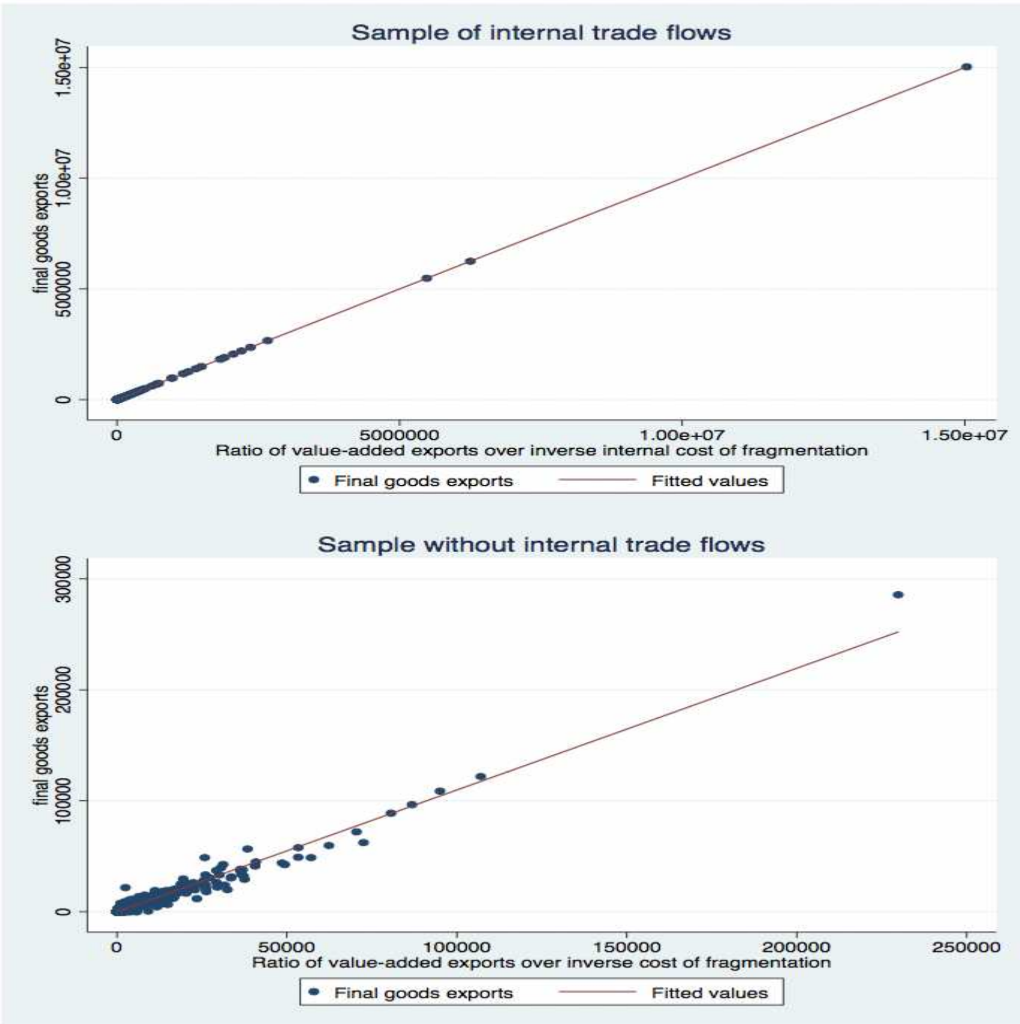
Figure 1: Correlation between total requirements and the cost of fragmentation

It appears as the right panel of the chart shows, that there is no correlation when it comes to non-symmetric relationships i.e. when the exporter is not also the importer. This is perfectly sensical. To see why, let us analyze again the cost of fragmentation formula:

$$t_{iSj} = \left(\frac{\sum_{s=i}^{S} \alpha_{is} \left(\ddot{\beta}_s c_s t_{sj} \right)^{1-\sigma}}{\left(\ddot{\beta}_i c_i t_{ij} \right)^{1-\sigma}} \right)^{\frac{1}{1-\sigma}}$$

When the exporter is also the importer, we have $t_{jSj} = \left(\alpha_{jj} + \frac{\sum_{s \neq j}^{S} \alpha_{js} \left(\ddot{\beta}_s c_s t_{sj} \right)^{1-\sigma}}{\left(\ddot{\beta}_j c_j t_{jj} \right)^{1-\sigma}} \right)^{\frac{1}{1-\sigma}}$ and we explained in page 18 that the term $\frac{\sum_{s \neq j}^{S} \alpha_{js} \left(\ddot{\beta}_s c_s t_{sj} \right)^{1-\sigma}}{\left(\ddot{\beta}_j c_j t_{jj} \right)^{1-\sigma}}$ is negligible because the denominator, internal trade in final goods is very high for all the countries. When the exporter is not the importer, this denominator is not that high anymore which explains why the term is no longer negligible.

Chart 2 shows the correlation between bilateral final goods exports "x_{ij}" and the ratio of value-added trade flows over the inverse cost of fragmentation "$\frac{v_{ij}}{t_{iSj}^{1-\sigma}}$". This ratio can be interpreted as the value-added that would have been directly exported by country "i" to country "j" in the absence of fragmentation. We can see in the upper panel of the chart dedicated to internal trade flows that the correlation is perfect. In the lower panel of the chart related to non-internal trade flows, the correlation is also very high, but not as perfect as for internal trade flows. This is normal since the inverse cost of fragmentation is estimated with error. As shown in Njike (2019) it is obtained via gravity-based estimates of bilateral trade costs.



Source: Author's calculations

Figure 2: Correlation between final goods and value-added trade flows

However, given that $(t_{jSj})^{1-\sigma} \approx \alpha_{jj}$ and that internal value-added trade flows by definition are given by $v_{jj} = X_{jj}\left(\alpha_{jj} + \frac{\sum_{s\neq j}^{S}\alpha_{js}X_{sj}}{X_{jj}}\right)$ with $\frac{\sum_{s\neq j}^{S}\alpha_{js}X_{sj}}{X_{jj}}$ being negligible, the approximation of trade costs that affects only $\frac{\sum_{s\neq j}^{S}\alpha_{js}X_{sj}}{X_{jj}}$ has a little impact on ${t_{jSj}}^{1-\sigma}$ such that $\frac{v_{jj}}{{t_{jSj}}^{1-\sigma}} \approx X_{jj}$. Hence, equations (37) and (39) that represent respectively the real wages obtained with the final goods exports model and the value-added exports model are equivalent as suggested theoretically, so as the welfare gains from trade derived with the two methods. The counterfactual analysis results will render this more explicit.

4 Counterfactual analysis results

In this section we perform two counterfactual exercises featuring two different trade costs shocks. The first trade costs shock is a move from 2011 levels of trade openness to autarky for all the countries in the world, and the second a 20% reduction of the trade costs indexes regarding African trade flows.[15] We first analyse the differences in predictions between our model based upon value-added exports and a classical model with gross exports allowing tradable intermediate goods but not sectoral linkages. The welfare formula[16] regarding the latter is a special case of equation 29 in Costinot and Rodríguez-Clare (2014) or equation 7 in Ossa (2015) and is equal to:

$$\ln \frac{\widehat{w}_i}{\widehat{P}_i} = \frac{-1}{\sigma - 1} \left(\frac{1 - \alpha_i}{\alpha_i} \right) \ln \widehat{\lambda_{iig}} - \frac{1}{\sigma - 1} \ln \widehat{\lambda_{iig}} \tag{61}$$

Where λ_{iig} represents the share of domestic expenditures on gross exports and α_i the value-added to gross output ratio.

4.1 Autarky

Table 1 presents summary statistics on the results regarding the move to autarky. The detailed results are available in appendix 6.A. This table is composed of five parts, the first presenting the results for the entire set and the two following respectively for the countries the less open of the sample and for the most open ones. We define the less open countries as those who present a ratio of internal trade in value-added over GDP $> 77\%$ and the more open ones as those who present a ratio of internal trade in value-added over GDP $< 60\%$.

The last two parts of table 1 present respectively the results for the most downstream countries i.e. with an upstreamness level of less than 1.7, and for the most upstream countries in the production process with an upstreamness level of more than 2.4 where the upstreamness level is calculated following Fally and Hillberry (2018). We discuss how to obtain it in equation (62).

The first two rows in each part of table 1 represent respectively the welfare gains obtained using the model with gross exports as in equation (61) (W_gross exports) and the welfare

[15] Unlike the shock related to autarky, the second shock requires to solve the system of equations presented in section (2.7.2). We do so by using GAMS. The code is available upon request.

[16] Our welfare formula is as said earlier the change in real wage instead of the change in the economy's real expenditures to ensure the comparability with previous studies.

gains using our approach with value-added exports as equation (41) (W_value-added exports).

Table 1: The welfare gains of trade (Autarky)

Variable	Obs	Mean	Min	Max
Entire set (Part 1)				
W_gross exports	139	-11,56%	-43,31%	-1,88%
W_value-added exports	139	-13,59%	-56,70%	-2,76%
Gross share of fragmentation (G)	139	50,80%	21,06%	66,71%
Gross share of fragmentation (VA)	139	65,68%	30,64%	96,14%
Net share of fragmentation (VA)	139	21,81%	6,57%	55,70%
Ratio of internal trade in value-added over GDP >0,77 (Part 2)				
Variable	Obs	Mean	Min	Max
W_gross exports	38	-5,25%	-7,42%	-1,88%
W_value-added exports	38	-8,33%	-28,00%	-2,76%
Gross share of fragmentation (G)	38	47,20%	21,06%	66,59%
Gross share of fragmentation (VA)	38	65,88%	35,13%	96,14%
Net share of fragmentation (VA)	38	17,27%	7,45%	35,38%
Ratio of internal trade in value-added over GDP <0,6 (Part 3)				
Variable	Obs	Mean	Min	Max
W_gross exports	39	-20,08%	-43,31%	-12,78%
W_value-added exports	39	-19,71%	-56,70%	-5,02%
Gross share of fragmentation (G)	39	53,22%	29,07%	66,71%
Gross share of fragmentation (VA)	39	67,37%	30,64%	86,91%
Net share of fragmentation (VA)	39	27,85%	7,35%	55,70%
Upstreamness <1.7 (Part 4)				
Variable	Obs	Mean	Min	Max
W_gross exports	20	-8,46%	-21,05%	-1,88%
W_value-added exports	20	-17,04%	-36,27%	-4,72%
Gross share of fragmentation (G)	20	47,05%	21,06%	58,56%
Gross share of fragmentation (VA)	20	61,34%	35,13%	96,14%
Net share of fragmentation (VA)	20	22,10%	7,45%	37,16%
Upstreamness >2.4 (Part 5)				
Variable	Obs	Mean	Min	Max
W_gross exports	20	-17,94%	-35,22%	-6,41%
W_value-added exports	20	-12,94%	-31,54%	-5,02%
Gross share of fragmentation (G)	20	50,43%	29,07%	66,71%
Gross share of fragmentation (VA)	20	60,55%	30,64%	86,91%
Net share of fragmentation (VA)	20	19,85%	7,35%	44,09%

The last three rows in each part of table 1 represent the gross and net shares of the welfare gains related to international fragmentation. For the model with gross exports (G), the share is obtained using $1 - \frac{-\frac{1}{\sigma-1}\ln\widehat{\lambda_{iig}}}{\ln\frac{\widehat{w_i}}{\widehat{P_i}}}$. For value-added exports (VA), the net share is obtained using $\frac{\frac{-1}{\theta}\left(\frac{1-\alpha_i}{\alpha_i}\right)\ln\widehat{\pi_{ii}} - ln\widehat{t_{\text{isi}}}}{\ln\frac{\widehat{w_i}}{\widehat{P_i}}}$, and the gross share just $\frac{\frac{-1}{\theta}\left(\frac{1-\alpha_i}{\alpha_i}\right)\ln\widehat{\pi_{ii}}}{\ln\frac{\widehat{w_i}}{\widehat{P_i}}}$.

As regards the "upstreamness" indexes we follow Fally and Hillberry (2018). They calculate upstreamness indexes for each sector in a country and then obtain the aggregate country index by computing an export-weighted average of sectoral indexes. To follow their words, these sectoral indexes measure the distance of each industry from final demand where distance is "the number of stages of production an industry's output passes through before reaching final consumers". As said in the data section, we use an inter-regional input-output matrix that does not feature sectoral linkages within and between countries but only aggregate trade linkages, since we aggregated the 57 original sectors of the GTAP database into a unique sector. Our index is therefore not sectoral, and we don't need to apply a weighting scheme to get the aggregate index. More precisely, we have:

$$U_i = 1 + \varphi_{ii} U_i + \sum_{i \neq j} \varphi_{ij} U_j \tag{62}$$

Where U_i is the upstreamness index of country "i" and φ_{ij} denotes the share of output from country "i" that is needed to produce one unit of output in country "j".

As the table shows, on average, a move to autarky would reduce real wage by 11,56 % if we follow the standard model with gross exports, and by 13.59 % if we follow the model with value-added exports (See the first part of table 1 named "entire set"). These results seem quite close, however, the correlation between the two models' results is only 76 %, which means that there are differences. Among these differences, it appears that the welfare loss for the countries that are less open is higher by $(-5.25 - (-8.33)) = 3.08$ percentage points on average with value-added exports than with gross exports, whereas it is just slightly lower, less than 0.5 percentage points on average for the more open countries (See respectively the second and the third part of table 1). It means that the gains from trade are understated for the less open countries when we use the standard gross exports model.

A result that is also worth mentioning is that the welfare gains of trade are higher, $(-8.46 - (-17.04)) = 8.58$ percentage points on average for the most downstream countries with the value-added exports model in comparison to the standard gross exports model (See the fourth part of table 1). On the contrary, the gains for the most upstream countries are $(-12, 94 - (-17.94)) = 5$ percentage points lower (See the fifth part of table 1). This result relates as said earlier to the work of Fally and Hillberry (2018) who built a sequential model of international fragmentation and also found that downstream countries feature higher welfare gains compared to the prediction of a standard model of trade. The difference is that the model that they use as a benchmark for comparison is a model of trade without intermediate goods flows. Unlike them, we compare our model predictions to a standard trade model featuring intermediate goods flows. This benchmark is the relevant one because our model allows back and forth trade in intermediate goods unlike theirs.

Alexander (2017) using a model that distinguishes intermediate and final goods trade flows, also find similar results regarding the difference between the gains on average in comparison to the standard trade model, but also as regards the difference between upstream and downstream countries. This suggests that the difference between the gains comes from the assumption regarding intermediate and final goods trade shares.

As regards the net share of fragmentation in the welfare gains of trade, it appears that this share is not as high as one could expect. To see this, we multiply the net share of fragmentation in the welfare gains from trade (the fifth row of each part of table 1) with the estimated total gains by the value-added exports model for each category of countries be it the entire set or the most upstream countries for example. More precisely, the net share of fragmentation in the total welfare gains from trade represents 21.81 % on average of the welfare reduction for the entire set as it is shown in the fifth row of the first part of the table. On this basis, our model predicts that turning off trade in intermediate goods would only reduce the average real wage by $(21.81 * (13.59)) \approx 3$ percentage points, with $(27.85 * (19.71)) \approx 5.5$ percentage points on average for the more open countries (See the third part of the table) and $(17.27 * (8.33)) \approx 1.5$ percentage point for the less open countries (See the second part of the table).

Paradoxically, the gross share of the welfare gains related to fragmentation is higher (14.88 percentage points more on average) with our approach compared to the gross exports approach as it is shown in the rows 3 and 4 of the first part of the table. It represents 65,68% of the total gains with our model compared to 50,80% with the standard model. There is anyway a striking difference between the gross and net shares of the welfare gains related to fragmentation. This substantiates the necessity to take into account the trade costs associated to international fragmentation that would disappear because of autarky in the calculation of the net share of international fragmentation in the welfare gains from trade, as our model allows us to do.

More precisely, because of autarky, the trade costs that are borne when a given country exports its intermediate goods to intermediate countries and imports them back embedded in the latter final goods exports "t_{jsj}" would not be borne anymore, which attenuates the welfare losses. The evidence that the gross welfare gains from international fragmentation are high but largely compensated by its costs has a trivial implication: reducing significantly the cost of fragmentation ceteris paribus could drastically improve the gains from trade. As shown in section 2.7.2, this seems to be impossible as the change in the cost of fragmentation depends on the change in the cost of a bundle of inputs, the change in the total requirements (the elements of the Leontief inverse) and the change in bilateral trade costs. All these variables also affect trade flows. As amongst them only bilateral trade costs are exogeneous, we will analyse what is the impact of a reduction in trade costs on the cost of fragmentation and on the gains from trade.

4.2 Decrease of African bilateral trade costs

In this section, we simulate the impact of a 20 % decrease on the level of African countries' direct bilateral trade costs[17] which are among the countries with the highest level of trade costs in the world. We analyse the consequences of this shock in term of welfare, and also in term of participation in the global supply chain. We both take into consideration the change in real wage and the change in the economy's real expenditures.

Table 2 presents the results, with the second column representing the change in real wage, the third column the change in real expenditures, the fourth column the average change in the cost of fragmentation, and the last two columns the changes in the inverse internal cost of fragmentation "$\ln \widehat{t_{jSj}^{1-\sigma}}$" and the internal total requirement $\ln \widehat{B_{ii}}$ [18] respectively.

As we can expect, a 20 % reduction in the level of African countries' direct bilateral trade costs would increase real wage by as much as 22% for small open economies like Togo or as much as 5 % for relatively closed and large economies like Nigeria. The results are qualitatively the same when it comes to real expenditures. In term of magnitude, the change in real expenditures is systematically higher than the change in real wage for all the countries. This is due to the fact that the economy's total expenditures is the sum of labor revenues and tariff revenues as shown in equation (27). These revenues increase as the countries imports more goods following the decrease in trade costs.[19]

It is interesting to note that the cost of fragmentation would increase on average. This result is consistent with what we could expect theoretically. The intermediate goods trade elasticity is sufficiently lower than the final goods trade elasticity, a critical condition for this result to occur. Besides, we have only 32 African countries and aggregated regions for which trade costs decrease.

The remaining others are countries for which bilateral trade costs remain constant except with their African partners. As they are considered as intermediate countries for African

[17]The trade costs that are borne when they export or import directly a good from a given country.

[18]The fraction of local output required by a given country to produce a unit of final good.

[19]It should be recalled that we imposed that trade imbalances remain constant between equilibria as well as bilateral tariffs.

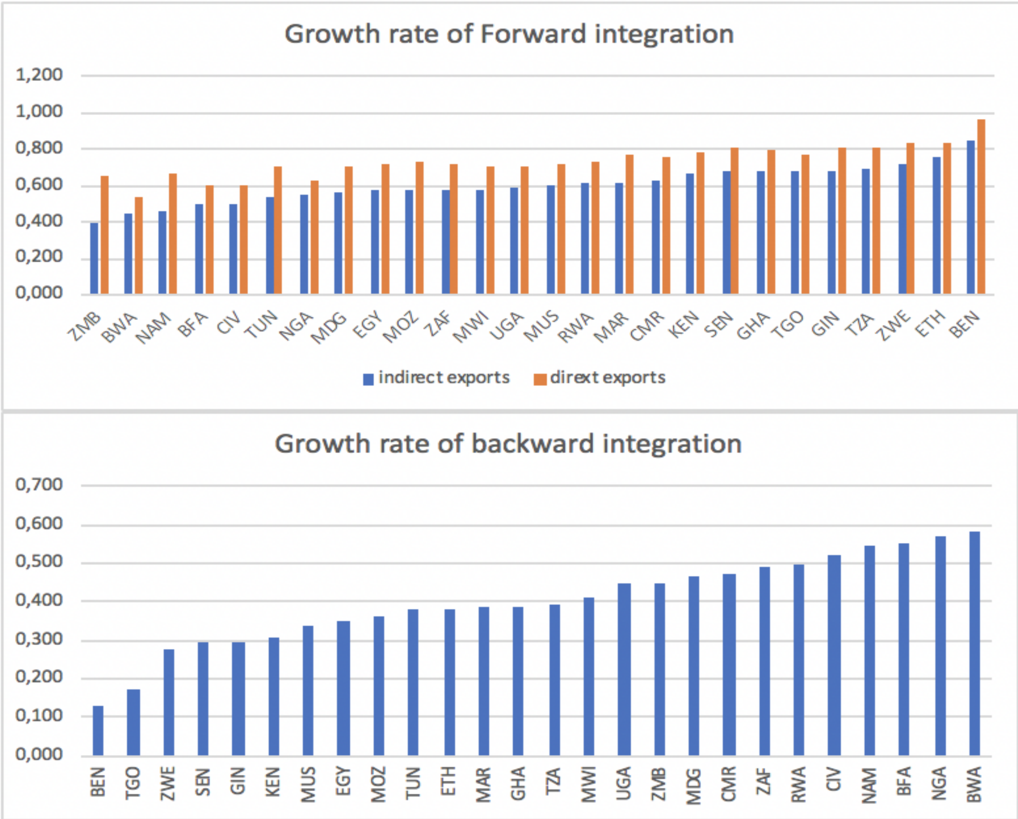
Table 2: The welfare gains of trade (20% decrease of African trade costs)

Countries	Real wages	Real expenditures	Cost of fragmentation	Inverse internal cost of fragmentation	Internal total requirement
TGO	20,29%	21,03%	4,90%	-14,40%	-14,40%
TUN	13,54%	15,61%	5,70%	-17,50%	-17,50%
MUS	13,02%	13,10%	4,70%	-13,10%	-13,10%
GIN	12,49%	15,53%	3,60%	-11,90%	-11,90%
ZMB	12,40%	17,73%	5,30%	-23,70%	-23,80%
MOZ	11,96%	13,71%	2,70%	-14,20%	-14,30%
MAR	10,26%	11,06%	6,00%	-16,70%	-16,70%
BEN	10,07%	16,21%	4,50%	-7,40%	-7,40%
CIV	9,35%	12,75%	4,10%	-14,60%	-14,60%
ZWE	9,35%	13,71%	4,20%	-12,10%	-12,10%
SEN	9,08%	11,15%	4,80%	-12,30%	-12,30%
NAM	8,98%	12,04%	3,90%	-16,10%	-16,10%
KEN	8,89%	10,80%	5,50%	-14,80%	-14,80%
MWI	8,80%	11,42%	4,80%	-15,00%	-15,00%
GHA	8,34%	10,80%	4,30%	-10,80%	-10,80%
EGY	8,25%	9,44%	5,30%	-15,60%	-15,60%
TZA	8,16%	11,15%	4,50%	-9,30%	-9,30%
MDG	7,79%	9,53%	4,40%	-15,00%	-15,00%
BWA	7,33%	10,44%	2,60%	-11,10%	-11,10%
BFA	7,33%	10,53%	2,90%	-10,70%	-10,70%
UGA	7,23%	9,17%	3,60%	-11,60%	-11,60%
ZAF	7,05%	8,34%	5,20%	-14,10%	-14,20%
CMR	6,30%	8,71%	4,20%	-11,50%	-11,50%
RWA	5,64%	7,05%	3,10%	-8,50%	-8,50%
NGA	5,07%	7,23%	1,40%	-5,00%	-5,00%
ETH	4,59%	6,58%	4,20%	-7,70%	-7,70%

value-added exports, the increase in the average cost of fragmentation makes even more sense. There are however instances where the bilateral cost of fragmentation decreases, especially for intra-African trade. This is natural since the trade costs of intermediate countries with African ones do decrease. Appendix 6.B presents detailed results for the change in the bilateral cost of fragmentation regarding Cameroon. We also include a case where the trade elasticity for intermediate goods is higher (7.25) than the trade elasticity for final goods (4.85). In this case, the cost of fragmentation would decrease on average, which confirms our theoretical results.

We presented this cost of fragmentation in section 2.4 as a function of the relative price between the indirectly exported flows over directly exported ones. If this cost increases for a given country, it becomes relatively more expensive for this country to indirectly export goods through intermediate countries than directly exporting final goods to end consumers.

The chart below suggests that this is the case for our set of African countries. This chart presents different measures of integration in the global supply chain. We can for instance



Source: Author's calculations

Figure 3: Participation of selected countries in the global supply chain

see in the upper panel of the chart that indirectly exported flows grow, which means that the countries' forward participation in the global supply chain increase in absolute terms, but these flows grow less than the directly exported ones (final goods exports), which is consistent with our previous result. It means that the countries moved to downstream stages of the production process, at least in relative terms. Another way to see it is to analyze the growth of backward participation that is shown in the lower panel of the chart. As it becomes cheaper to import intermediate inputs, the countries import more of them, everything that increases their backward participation[20] in the global supply chain.

The last result that highlights table 2 is the exact similarity between the variation of the internal cost of fragmentation and the fraction of local output required to produce a unit of final good in country "i" (internal total requirement). This result confirms our previous finding that the internal cost of fragmentation could be approximated by the latter, which makes possible the calculation of the share of international fragmentation in the gains from trade against autarky using only observable data.

[20]In this chart backward integration is represented by the share of foreign output required to produce a unit of final good.

5 Concluding remarks

The goal of this paper was to propose a trade model for the determination of the net share of international fragmentation in the welfare gains of trade. To do so, we relied upon value-added exports as the variable of interest instead of gross exports. It allowed us to highlight the macroeconomic cost of fragmentation, a critical variable for the computation of this net share.

Our model predicts that the net share of international fragmentation in the welfare gains of trade is not as high as one could expect, at least in comparison to the gross share that could be inferred from a classical model. It represents only 22% on average of the gains of trade. As the model allows us to deduct from the gross welfare loss that would imply the shutdown of international fragmentation the cost of fragmentation that would not be supported anymore in its absence, the net welfare loss is thus reduced.

We also show that using our framework to derive the welfare gains of trade in comparison to a standard trade model based upon gross exports give different results. This is due as explained Alexander (2017) to the implicit assumption made by standard trade models that the share of intermediate goods sourced from a given origin country in the total demand of intermediate goods of a given destination country is equivalent to the share of final goods sourced from this origin country in the total demand of final goods of the destination country. Specifically, we show that the reduction in real wage that a move to autarky would provoke is lower using our approach than the traditional one for upstream countries, and higher for downstream countries and countries that are less open in terms of the imports in value-added penetration ratio. The gains from trade are thus understated by the classical model for this last category even if they remain way lower than the gains associated to the more open countries with our model.

Finally, we show that reducing the level of a country's bilateral trade costs with its trading partners does not necessarily imply more forward participation in the global supply chain. In fact, unless the reduction in trade costs affects more the indirectly exported flows than the directly exported ones, the increase in exports would be biased towards the latter, which implies a weaker forward participation in relative terms to the global production network. Backward integration, however, undoubtedly increase, and the countries are closer to the final consumers than before. This result has interesting implications in term of trade policies since increasing the participation in the global supply chain is a key concern for many countries.

References

Aichele, R. and Heiland, I. (2018). Where is the value added? trade liberalization and production networks. *Journal of International Economics*, 115:130–144. 4

Alexander, P. D. (2017). Vertical specialization and gains from trade. Technical report, Bank of Canada Staff Working Paper. 4, 5, 9, 16, 17, 34, 38

Anderson, J. E. and Van Wincoop, E. (2003). Gravity with gravitas: a solution to the border puzzle. *American economic review*, 93(1):170–192. 4, 9, 19

Antràs, P., Chor, D., Fally, T., and Hillberry, R. (2012). Measuring the upstreamness of production and trade flows. *American Economic Review*, 102(3):412–16. 13

Antràs, P. and De Gortari, A. (2017). On the geography of global value chains. Technical report, National Bureau of Economic Research. 9, 27

Arkolakis, C., Costinot, A., and Rodriguez-Clare, A. (2012). New trade models, same old gains? *American Economic Review*, 102(1):94–130. 16

Caliendo, L. and Parro, F. (2015). Estimates of the trade and welfare effects of nafta. *The Review of Economic Studies*, 82(1):1–44. 4, 7, 9, 15, 16, 21, 26, 27

Chaney, T. (2008). Distorted gravity: the intensive and extensive margins of international trade. *American Economic Review*, 98(4):1707–21. 4

Costinot, A. and Rodríguez-Clare, A. (2014). Trade theory with numbers: Quantifying the consequences of globalization. In *Handbook of international economics*, volume 4, pages 197–261. Elsevier. 3, 5, 16, 31

Crozet, M. and Koenig, P. (2010). Structural gravity equations with intensive and extensive margins. *Canadian Journal of Economics/Revue canadienne d'économique*, 43(1):41–62. 26

Daudin, G., Rifflart, C., and Schweisguth, D. (2011). Who produces for whom in the world economy? *Canadian Journal of Economics/Revue canadienne d'économique*, 44(4):1403–1437. 3

Dekle, R., Eaton, J., and Kortum, S. (2008). Global rebalancing with gravity: Measuring the burden of adjustment. *IMF Staff Papers*, 55(3):511–540. 21

Eaton, J. and Kortum, S. (2002). Technology, geography, and trade. *Econometrica*, 70(5):1741–1779. 4, 7, 8, 9, 16, 17, 19, 26

Fally, T. (2012). Production staging: measurement and facts. *Boulder, Colorado, University of Colorado Boulder, May*, pages 155–168. 13

Fally, T. (2015). Structural gravity and fixed effects. *Journal of International Economics*, 97(1):76–85. 28

Fally, T. and Hillberry, R. (2018). A coasian model of international production chains. *Journal of International Economics*, 114:299–315. 5, 16, 20, 27, 33

Feenstra, R. C. (1994). New product varieties and the measurement of international prices. *The American Economic Review*, pages 157–177. 26

Francois, J. and Pindyuk, O. (2013). Consolidated data on international trade in services v8. 9. *IIDE Discussion Paper 20130101*. 28

Hillberry, R. and Hummels, D. (2013). Trade elasticity parameters for a computable general equilibrium model. In *Handbook of computable general equilibrium modeling*, volume 1, pages 1213–1269. Elsevier. 25

Imbs, J. and Mejean, I. (2015). Elasticity optimism. *American Economic Journal: Macroeconomics*, 7(3):43–83. 26

Johnson, R. C. and Noguera, G. (2012). Accounting for intermediates: Production sharing and trade in value added. *Journal of international Economics*, 86(2):224–236. 3

Koopman, R., Wang, Z., and Wei, S.-J. (2014). Tracing value-added and double counting in gross exports. *American Economic Review*, 104(2):459–94. 3, 27, 28

Njike, A. (2019). Are african exports that weak? a trade in value-added approach. 28, 30

Noguera, G. (2012). Trade costs and gravity for gross and value added trade. *Job Market Paper, Columbia University*. 4

Ossa, R. (2015). Why trade matters after all. *Journal of International Economics*, 97(2):266–277. 26, 27, 31

Simonovska, I. and Waugh, M. E. (2014). The elasticity of trade: Estimates and evidence. *Journal of international Economics*, 92(1):34–50. 26

Tsigas, M., Wang, Z., and Gehlhar, M. (2012). How a global inter-country input-output table with a processing trade account is constructed from the gtap database. 28

Yi, K.-M. (2003). Can vertical specialization explain the growth of world trade? *Journal of political Economy*, 111(1):52–102. 2

6 Appendices

6.A Detailed results, trade and welfare

Autarky			20 % decrease African trade costs		
Country	**Real wage benchmark**	**Real wage value-added exports**	**Real wage (VA)**	**(VA) Real expenditures**	**Cost of fragmentation**
TGO	-16,82%	-36,27%	20,28%	21,05%	4,91%
ZMB	-16,37%	-12,65%	12,41%	17,71%	5,33%
TUN	-13,44%	-15,72%	13,57%	15,59%	5,65%
MUS	-12,20%	-16,40%	12,98%	13,12%	4,65%
MOZ	-11,96%	-14,79%	11,97%	13,75%	2,74%
CIV	-11,48%	-9,13%	9,32%	12,74%	4,15%
NAM	-11,40%	-8,47%	9,03%	12,08%	3,90%
BWA	-10,43%	-6,39%	7,35%	10,40%	2,60%
GIN	-10,43%	-18,30%	12,48%	15,54%	3,64%
BFA	-9,42%	-6,74%	7,30%	10,56%	2,87%
MAR	-9,05%	-13,14%	10,29%	11,03%	6,05%
MWI	-8,67%	-10,58%	8,78%	11,41%	4,84%
MDG	-7,89%	-8,61%	7,79%	9,57%	4,44%
ZAF	-7,27%	-7,78%	7,01%	8,37%	5,18%
SEN	-7,20%	-14,47%	9,06%	11,15%	4,82%
GHA	-7,15%	-11,08%	8,34%	10,79%	4,31%
UGA	-7,14%	-8,11%	7,23%	9,19%	3,61%
ZWE	-6,84%	-14,99%	9,30%	13,74%	4,15%
TZA	-6,43%	-11,00%	8,13%	11,14%	4,46%
NGA	-6,37%	-4,72%	5,09%	7,19%	1,44%
KEN	-6,24%	-13,31%	8,85%	10,76%	5,49%
EGY	-6,17%	-10,69%	8,23%	9,42%	5,28%
BEN	-5,73%	-28,00%	10,09%	16,18%	4,55%
RWA	-5,71%	-6,13%	5,66%	7,06%	3,09%
CMR	-5,16%	-7,29%	6,29%	8,73%	4,20%
ETH	-3,28%	-6,98%	4,58%	6,56%	4,19%
LUX	-44,00%	-56,70%	0,37%	0,30%	1,31%
IRL	-35,78%	-25,30%	0,31%	0,64%	1,42%
MLT	-32,59%	-50,98%	0,46%	0,17%	1,23%
SGP	-31,27%	-31,54%	0,31%	0,50%	1,40%

Continued from previous page

	Autarky		20 % decrease African trade costs		
Country	**Real wage benchmark**	**Real wage value-added exports**	**Real wage (VA)**	**(VA) Real expenditures**	**Cost of fragmentation**
XCA	-26,44%	-21,59%	4,23%	8,44%	1,52%
HUN	-25,84%	-24,96%	0,20%	0,25%	1,05%
TTO	-23,75%	-12,26%	1,05%	1,06%	0,84%
KHM	-23,66%	-27,06%	0,00%	-0,04%	1,18%
MYS	-23,59%	-21,58%	0,35%	0,47%	1,36%
BEL	-23,13%	-28,66%	0,63%	0,58%	1,35%
SVK	-22,95%	-24,41%	0,10%	0,09%	1,34%
VNM	-22,34%	-29,64%	0,29%	0,22%	0,96%
TWN	-22,11%	-18,50%	0,22%	0,29%	1,19%
EST	-21,77%	-27,24%	0,39%	0,34%	1,50%
MNG	-21,39%	-23,43%	0,06%	0,04%	1,45%
THA	-21,25%	-21,61%	0,37%	0,41%	0,93%
KWT	-20,96%	-8,40%	0,28%	0,86%	1,06%
CZE	-20,78%	-20,37%	0,14%	0,17%	1,25%
OMN	-18,79%	-11,01%	0,20%	0,33%	0,99%
HKG	-18,11%	-19,55%	0,27%	0,29%	1,44%
SVN	-18,08%	-21,45%	0,23%	0,21%	1,36%
BGR	-17,57%	-21,41%	0,29%	0,27%	1,25%
XWF	-16,83%	-33,37%	20,11%	20,58%	5,33%
SAU	-16,78%	-8,93%	0,34%	0,65%	0,75%
QAT	-16,70%	-5,02%	0,08%	0,38%	1,16%
BHR	-16,65%	-14,19%	0,77%	1,20%	0,22%
BRN	-16,29%	-9,35%	0,06%	0,12%	1,32%
LTU	-16,13%	-21,56%	0,32%	0,24%	1,38%
CRI	-16,08%	-13,66%	0,06%	0,08%	1,30%
NIC	-15,54%	-16,81%	0,36%	0,34%	1,28%
XCF	-15,43%	-9,36%	10,94%	19,58%	3,66%
AZE	-15,41%	-7,57%	0,06%	0,18%	1,33%
CYP	-15,00%	-22,91%	0,43%	0,32%	1,18%
PAN	-14,85%	-30,60%	0,12%	0,01%	1,21%
XEF	-14,39%	-16,88%	0,36%	0,35%	1,25%
CHE	-14,20%	-12,60%	0,38%	0,45%	1,22%

Continued from previous page

Autarky			20 % decrease African trade costs		
Country	**Real wage benchmark**	**Real wage value-added exports**	**Real wage (VA)**	**(VA) Real expenditures**	**Cost of fragmentation**
AUT	-14,07%	-15,05%	0,15%	0,16%	1,26%
HND	-13,94%	-15,46%	0,01%	0,00%	1,33%
XNA	-13,81%	-22,59%	0,60%	0,38%	1,44%
XAC	-13,70%	-8,59%	9,73%	15,27%	2,58%
DNK	-13,66%	-13,71%	0,18%	0,21%	1,27%
UKR	-13,41%	-15,42%	0,28%	0,29%	1,42%
ARE	-13,28%	-15,14%	0,32%	0,31%	1,10%
KOR	-13,04%	-14,26%	0,26%	0,39%	1,33%
NLD	-12,99%	-12,06%	0,34%	0,39%	1,20%
KGZ	-12,78%	-32,67%	0,07%	-0,04%	1,36%
LVA	-12,26%	-18,85%	0,18%	0,11%	1,45%
XSC	-11,98%	-8,36%	9,16%	12,48%	4,40%
JOR	-11,96%	-23,00%	0,64%	0,43%	1,23%
SWE	-11,73%	-11,04%	0,19%	0,23%	1,06%
DEU	-11,66%	-10,96%	0,22%	0,25%	1,08%
BLR	-11,59%	-18,50%	0,24%	0,13%	1,49%
XOC	-11,49%	-18,40%	0,11%	0,03%	1,36%
XEA	-11,28%	-8,97%	0,15%	0,21%	1,30%
XWS	-10,95%	-9,19%	0,27%	0,29%	1,19%
KAZ	-10,80%	-6,40%	0,06%	0,09%	1,43%
XSM	-10,75%	-11,61%	0,28%	0,29%	1,40%
XEE	-10,73%	-25,82%	0,21%	0,06%	1,47%
HRV	-10,55%	-11,67%	0,17%	0,18%	1,09%
LAO	-10,22%	-13,04%	0,03%	0,00%	1,22%
POL	-10,11%	-12,34%	0,11%	0,10%	1,34%
FIN	-10,00%	-10,79%	0,15%	0,15%	1,10%
XSU	-9,97%	-8,03%	0,06%	0,09%	1,30%
ROU	-9,78%	-12,11%	0,16%	0,13%	1,07%
NOR	-9,62%	-7,46%	0,10%	0,21%	1,34%
CHL	-9,50%	-8,78%	0,07%	0,10%	1,23%
JAM	-9,42%	-15,72%	0,14%	0,09%	1,25%
ALB	-9,29%	-15,95%	0,17%	0,14%	1,39%

Continued from previous page

	Autarky		20 % decrease African trade costs		
Country	**Real wage benchmark**	**Real wage value-added exports**	**Real wage (VA)**	**(VA) Real expenditures**	**Cost of fragmentation**
PRT	-9,22%	-11,13%	0,70%	0,67%	1,17%
ISR	-8,71%	-10,28%	0,14%	0,15%	1,37%
PRY	-8,45%	-10,20%	0,16%	0,15%	1,03%
XNF	-8,44%	-6,98%	7,11%	9,42%	2,78%
IRN	-8,35%	-5,12%	0,12%	0,26%	0,93%
XCB	-7,92%	-10,85%	0,88%	0,85%	1,20%
MEX	-7,90%	-7,36%	0,03%	0,03%	1,23%
PHL	-7,77%	-10,63%	0,03%	0,02%	1,20%
SLV	-7,68%	-11,64%	0,04%	0,04%	1,23%
NZL	-7,55%	-7,07%	0,16%	0,19%	1,00%
GRC	-7,54%	-12,06%	0,23%	0,18%	1,34%
BOL	-7,46%	-7,96%	0,07%	0,07%	1,31%
ESP	-7,27%	-8,45%	0,40%	0,39%	1,20%
GBR	-7,23%	-9,11%	0,22%	0,21%	1,07%
ECU	-7,16%	-7,89%	0,02%	0,03%	1,28%
PER	-7,13%	-5,64%	0,10%	0,11%	1,14%
ITA	-7,12%	-7,87%	0,29%	0,27%	1,11%
GTM	-7,11%	-8,91%	0,04%	0,03%	1,25%
CAN	-6,98%	-7,05%	0,08%	0,09%	1,37%
FRA	-6,84%	-7,94%	0,34%	0,33%	1,02%
CHN	-6,51%	-5,98%	0,22%	0,26%	1,02%
BGD	-6,50%	-7,91%	0,09%	0,10%	1,10%
VEN	-6,50%	-4,37%	0,00%	0,00%	1,37%
XEC	-6,40%	-6,92%	6,31%	8,96%	3,50%
DOM	-6,37%	-9,09%	0,09%	0,09%	1,20%
URY	-6,15%	-7,14%	0,18%	0,19%	1,18%
IDN	-6,14%	-5,91%	0,09%	0,10%	1,12%
RUS	-6,11%	-5,18%	0,07%	0,10%	1,31%
XER	-5,94%	-9,24%	0,21%	0,17%	1,22%
GEO	-5,61%	-14,57%	0,19%	0,10%	1,57%
TUR	-5,61%	-8,46%	0,39%	0,34%	1,38%
LKA	-5,18%	-9,18%	0,06%	0,08%	1,07%

Continued from previous page

Country	Autarky: Real wage benchmark	Autarky: Real wage value-added exports	20 % decrease African trade costs: Real wage (VA)	20 % decrease African trade costs: (VA) Real expenditures	20 % decrease African trade costs: Cost of fragmentation
AUS	-5,18%	-4,68%	0,07%	0,08%	1,21%
ARM	-4,73%	-9,96%	0,06%	0,01%	1,41%
IND	-4,62%	-7,47%	0,38%	0,40%	1,19%
ARG	-4,40%	-3,74%	0,13%	0,18%	0,75%
XSA	-4,25%	-12,85%	0,13%	0,15%	1,30%
COL	-4,14%	-4,27%	0,03%	0,03%	1,19%
XSE	-3,89%	-5,61%	0,04%	0,02%	1,14%
JPN	-3,61%	-3,96%	0,06%	0,06%	1,02%
PAK	-3,46%	-6,36%	0,16%	0,16%	0,69%
PRI	-3,05%	-4,45%	0,15%	0,15%	1,33%
USA	-2,84%	-4,05%	0,12%	0,12%	1,21%
BRA	-2,76%	-2,76%	0,11%	0,12%	0,86%
NPL	-1,91%	-6,92%	0,04%	0,06%	1,30%

6.B Change in the bilateral cost of fragmentation (Cameroon)

Countries	$\theta = 4.25$ and $\sigma - 1 = 4.85$	$\theta = 7.25$ and $\sigma - 1 = 4.85$
ALB	5.45%	-5.85%
ARE	4.34%	-1.99%
ARG	3.56%	0.24%
ARM	5.4%	-6.01%
AUS	3.36%	-0.85%
AUT	5.54%	-5.86%
AZE	5.43%	-5.99%
BEL	5.76%	-5.47%
BEN	-0.95%	-1.55%
BFA	4.3%	3,00%
BGD	2.7%	3.15%

Continued from previous page

Countries	$\theta = 4.25$ **and** $\sigma - 1 = 4.85$	$\theta = 7.25$ **and** $\sigma - 1 = 4.85$
BGR	5.59%	-5.8%
BHR	6,00%	-5.64%
BLR	3.51%	0.34%
BOL	5.39%	-6.11%
BRA	5.25%	-6.22%
BRN	3.36%	0.98%
BWA	4.37%	-8.18%
CAN	3.2%	0.58%
CHE	4.06%	-1.62%
CHL	3.45%	0.54%
CHN	5.11%	-6.44%
CIV	1.11%	-2.02%
CMR	2.37%	4.38%
COL	5.35%	-6.07%
CRI	5.34%	-6.05%
CYP	5.69%	-5.63%
CZE	3.96%	-0.75%
DEU	5.16%	-4.86%
DNK	3.4%	1.06%
DOM	2.9%	2.43%
ECU	5.26%	-6.14%
EGY	9.56%	1.47%
ESP	5.74%	-5.43%
EST	5.58%	-5.76%
ETH	-0.54%	-9.23%
FIN	5.48%	-5.99%
FRA	5.65%	-5.67%
GBR	5.51%	-5.86%
GEO	5.36%	-6.05%
GHA	0.59%	0.85%
GIN	-1.36%	-9.11%
GRC	5.42%	-5.89%
GTM	5.3%	-6.1%

Continued from previous page

Countries	$\theta = 4.25$ **and** $\sigma - 1 = 4.85$	$\theta = 7.25$ **and** $\sigma - 1 = 4.85$
HKG	5.37%	-5.91%
HND	5.33%	-6.06%
HRV	5.54%	-5.93%
HUN	5.53%	-5.9%
IDN	5.28%	-6.15%
IND	5.45%	-5.71%
IRL	5.59%	-5.74%
IRN	4.34%	-2.75%
ISR	5.47%	-5.87%
ITA	5.56%	-5.75%
JAM	5.41%	-6,00%
JOR	5.53%	-5.69%
JPN	5.31%	-6.17%
KAZ	3.43%	0.57%
KEN	4.28%	-3.18%
KGZ	3.57%	-0.14%
KHM	3.9%	-0.78%
KOR	5.48%	-5.99%
KWT	5.42%	-6.05%
LAO	3.98%	-0.94%
LKA	5.35%	-6.04%
LTU	5.47%	-5.9%
LUX	3.19%	1.9%
LVA	5.52%	-5.9%
MAR	-0.56%	-2.3%
MDG	-1.5%	-5.05%
MEX	5.3%	-6.07%
MLT	5.62%	-5.75%
MNG	3.5%	0.25%
MOZ	1.8%	-6.8%
MUS	-0.25%	-1.16%
MWI	5.18%	-3.6%
MYS	5.47%	-5.95%

Continued from previous page

Countries	$\theta = 4.25$ **and** $\sigma - 1 = 4.85$	$\theta = 7.25$ **and** $\sigma - 1 = 4.85$
NAM	-2.06%	-7.2%
NGA	-2.18%	-5.96%
NIC	5.47%	-6.04%
NLD	5.66%	-5.66%
NOR	5.45%	-6,00%
NPL	5.34%	-5.97%
NZL	3.36%	0.83%
OMN	3.29%	1.02%
PAK	5.34%	-6.11%
PAN	3.72%	-0.37%
PER	4.57%	-7.05%
PHL	5.35%	-6.09%
POL	5.45%	-5.97%
PRI	5.01%	-6.37%
PRT	6,00%	-5.29%
PRY	3.41%	0.77%
QAT	5.43%	-6.04%
ROU	5.53%	-5.89%
RUS	5.39%	-6.08%
RWA	1.66%	-8.77%
SAU	5.52%	-5.94%
SEN	1.57%	1.23%
SGP	5.47%	-5.94%
SLV	5.3%	-6.09%
SVK	5.46%	-5.96%
SVN	5.54%	-5.81%
SWE	5.55%	-5.9%
TGO	-1.25%	-2.59%
THA	5.55%	-5.89%
TTO	-1.15%	-12.49%
TUN	0.18%	-0.81%
TUR	5.6%	-5.62%
TWN	4.17%	-4.95%

Continued from previous page

Countries	$\theta = 4.25$ **and** $\sigma - 1 = 4.85$	$\theta = 7.25$ **and** $\sigma - 1 = 4.85$
TZA	-1.44%	-10.25%
UGA	5.44%	-3.38%
UKR	5.46%	-6,00%
URY	2.85%	-1.24%
USA	5.16%	-6.3%
VEN	3.47%	0.5%
VNM	5.5%	-5.89%
XAC	12.61%	2.88%
XCA	7.37%	-4.93%
XCB	5.97%	-5.96%
XCF	4.96%	4.09%
XEA	5.45%	-6.03%
XEC	-3.83%	-8.85%
XEE	3.61%	0.22%
XEF	5.62%	-5.72%
XER	5.5%	-5.96%
XNA	3.36%	0.83%
XNF	1.83%	-3.34%
XOC	3.65%	-0.03%
XSA	5.14%	-6.2%
XSE	5.38%	-6.13%
XSC	-2.1%	-5.3%
XSM	5.37%	-5.97%
XSU	5.36%	-6.11%
XWF	8.69%	5.08%
XWS	5.36%	-5.97%
ZAF	-3.64%	-10.2%
ZMB	9.52%	-1.52%
ZWE	2.17%	-4.03%

6.C A two-country inter-country input output table

	Intermediate use		Final Demand		Gross output	Trade Balance
	Country j	Country i	Country j	Country i		
Country j	h_{jj}	h_{ji}	x_{jj}	x_{ji}	$G_j = \sum_{i=j} (h_{ji} + x_{ji})$	$D_j = \sum_i (h_{ij} + x_{ij}) - \sum_i (h_{ji} + x_{ji})$
Country i	h_{ij}	h_{ii}	x_{ij}	x_{ii}	$G_i = \sum_{j=i} (h_{ij} + x_{ij})$	$D_i = \sum_j (h_{ji} + x_{ji}) - \sum_j (h_{ij} + x_{ij})$
Total	$H_j = \sum_{i=j} h_{ij}$	$H_i = \sum_{j=i} h_{ji}$	$E_j = \sum_{i=j} x_{ij}$ $= X_j - R_j^F$ $= w_j L_j + R_j^I + D_j$	$E_i = \sum_{j=i} x_{ji}$ $= X_i - R_i^F$ $= w_i L_i + R_i^I + D_i$		
Custom duties and tax	$R_j^I = \sum_{i=j} \tau_{ij}^I h_{ij}$	$R_i^I = \sum_{j=i} \tau_{ji}^I h_{ji}$	$R_j^F = \sum_{i=j} \tau_{ij}^F x_{ij}$	$R_i^F = \sum_{j=i} \tau_{ji}^f x_{ji}$		
Value-added	$w_j L_j = \sum_{s=j}^S \sum_i \alpha_{js} X_{si}$	$w_i L_i = \sum_{s=i}^S \sum_j \alpha_{is} X_{sj}$	$w_j L_j = X_j - R_j^F - R_j^I - D_j$	$w_i L_i = X_i - R_i^F - R_i^I - D_i$		
Gross output	$G_j = H_j + R_j^I + w_j L_j$ $= \sum_{i=j} h_{ij} \left(1 + \tau_{ij}^I\right) + w_j L_j$	$G_i = H_i + R_i^I + w_i L_i$ $= \sum_{j=i} h_{ji} \left(1 + \tau_{ji}^I\right) + w_i L_i$	$G_j = H_j + E_j - D_j$	$G_i = H_i + E_i - D_i$		

With $\{i, j\} = S$