Current account sustainability for 21 African economies: Evidence based on nonlinear flexible Fourier stationarity and unit-root tests

Husein, Jamal

Angelo State University

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Jamal Husein[a]

[a] Department of Accounting, Economics and Finance
Angelo State University
ASU Station # 10908, San Angelo, TX 76909, USA
Telephone number: (325) 486-6457

* Corresponding author
E-mail: jamal.husein@angelo.edu

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ABSTRACT

We examine the mean reversion properties in the current account balance as a percentage of GDP under assumptions of smooth breaks and nonlinearity for twenty one African economies. Since there are reasons to indicate that the dynamic adjustment in the current account may follow a nonlinear process, we utilize a range of nonlinear stationarity and unit-root tests and compare across them to obtain a comprehensive picture of the pattern characterizing imbalances in part of the region. In particular, we apply the newly introduced Fourier stationarity test of Tsong, Lee, and Tsai (2019) in conjunction with Enders and Lee (2012a and 2012b) and Rodrigues and Taylor (2012) Fourier unit-root tests. However, rather than assuming a nonlinear current account adjustment, we test for "nonlinearity" using an approach that is robust to whether the current account series are stationary or integrated. We find strong evidence in favor of nonlinearity in eighteen current accounts out of the twenty one examined. Our empirical results show that the traditional linear and the widely used nonlinear unit-root tests of Kapetanios et al. (2003), Sollis (2009), and Kruse (2011) confirm sustainability of the current account balance for a small number of countries. Meanwhile, the Fourier based stationarity and unit-root tests confirm sustainability in a much larger group of countries.

Keywords: Unit-root, Stationarity, Fourier approximation, Current account
JEL Classification: C22, F32

1. Introduction

Current account (CA) sustainability has been the focus of research and debate among economists and policy makers for a long time. CA sustainability refers to whether an economy is able to satisfy its intertemporal long-run budget constraint (LRBC) without experiencing drastic changes in public policy and/or private sector behavior. Temporary or short-run CA shocks cause fewer problems, if any, as they may represent a natural outcome of reallocating capital to the country where the factor of production tends to receive the highest rate of return (Hakkio, 1995). In this situation, a stationary or mean reverting CA to GDP ratio is a sufficient condition for the LRBC to be satisfied (Trehan and Walsh, 1991 and Taylor, 2002). Hence, the growth rate of the CA deficit is not larger than that of the expected growth rate of output, indicating a finite debt to output ratio. Under stationary current account to GDP ratio, deviations from the mean level of the CA would eventually be corrected and perpetual CA deficits could be run without inferring a costly adjustment process on the economy (Akdogan, 2014).

On the other hand, a nonstationary CA during the period of study, implies that the economy has violated its LRBC and that some form of "unexpected" adjustment will have to take place in the future. Large and persistent CA deficits present an economic problem that requires a policy response. Rising long-run deficits can lead to unsustainable levels of external indebtedness and thus to either a default or a costly adjustment process. Prolonged periods of CA deficits may end either abruptly by generating debt, exchange rate crises and an output collapse or by achieving a soft landing that will ultimately cause consumption, investment and growth slowdowns (Christopoulos and Leon-Ledesma, 2010, and Chen, 2014).[1]

Despite Bohn's (2007) critique that stationarity is not necessary for the LRBC to hold, and that external deficit may satisfy the transversality condition for orders of integration higher than zero, the time-series

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[1] Large external imbalances are often assumed to play a role in the propagation of currency crises, i.e., Chile and Mexico in the early 1980s, UK and Nordic countries in the late 1980s, Mexico and Argentina in the mid-1990s, and East-Asian countries in the late 1990s (Baharumshah et al. (2003)).

properties of the CA are quite informative. Within the literature, stationarity of the CA balance to GDP ratio is seen as a strong form of sustainability, and Policy makers and investors may want to learn when and how CA mean reversion happens (Cuestas, 2013).

Many first generation studies, based on Trehan and Walsh (1991), investigated the mean-reversion behavior of the CA employing linear unit-root and cointegration tests, however, in recent studies researchers began to emphasize that the dynamic adjustment of the CA may be a nonlinear process.[2] Christopoulos and Leon-Ledesma (2010) argue that changes in agents' perceptions about risk, portfolio allocation decisions, future policy changes, transaction costs in international markets, etc. can lead to changes in the dynamics of CA mean reversion and hence equilibrium values of the CA. That is, changes in the CA affecting agents' perceptions can trigger adjustment dynamics that are not necessarily linear as commonly assumed. Moreover, government policy and market forces can induce faster CA corrections when deficits reach a certain "danger zone", leading to a nonlinear adjustment process in the CA (Clarida et al 2006).

As a result, a growing body of literature turned their attention to utilizing advanced nonlinear models to assess CA sustainability (e.g., Chen, 2014; Cuestas, 2013; Kim et al., 2009; Holmes et al., 2011; Holmes, M. J., Otero, J., & Panagiotidis, T., 2010; Chortareas et al., 2004; Christopoulos and Leon-Ledesma, 2010; Cecen and Xiao, 2014) to name a few. However, as emphasized by Choi and Moh (2007), existing nonlinear unit-root tests, when introduced, tend to highlight their usefulness by showing power improvement over conventional unit-root tests under a specific nonlinear functional form maintained in the alternative hypothesis.[3] This outperformance could be a natural consequence from the design of the tests themselves and there are no guarantees that such performance will be witnessed in other nonlinear models.[4] Moreover,

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[2] See Chen (2014) for an extensive list of studies utilizing both approaches linear and nonlinear.

[3] Available nonlinear models include exponential smooth transition autoregressive (ESTAR), logistic smooth transition autoregressive (LSTAR), threshold, artificial neural network (ANN), Markov switching, generalized autoregressive (GAR), and multiple adaptive regressive splines (MARS), to name a few.

[4] As detailed in Terasvirta et al. (2010), several Lagrange multiplier (LM) and other general specification tests fail to pinpoint the precise form of nonlinearity .

since the true underlying model is usually unknown in practice, it is imperative to evaluate the performance of such nonlinear tests under a wide class of models.

Given that the class of nonlinear models includes virtually an infinite number of models and specifications, in this study, we utilize a recently developed stationarity and unit-root tests that mitigate the problem of selecting the most appropriate functional form in unit-root testing.[5] Furthermore, since it is natural to test the validity of a theory, i.e., current account sustainability, such that the theory itself forms the basis of the null hypothesis and its refutation forms the alternative, we consider the newly introduced Fourier stationarity test of Tsong, Lee, and Tsai (2019). Such test extends Leybourne and McCabe (1994) stationarity test, LMC, by approximating the unknown nonlinearities present in the data generating process, DGP, using a selected frequency component from a Fourier approximation. An important feature of the flexible Fourier form is that it can mimic various types of smooth and sharp breaks including LSTAR and ESTAR type of nonlinearity.

In addition, some authors have advocated the joint application of stationarity and unit-root tests as a way to obtain robust conclusions about the stochastic properties of univariate time-series (e.g., Maddala and Kim 1998; Carrion-i-Silvestre et al. 2001; Carrion-i-Silvestre and Sansó 2006), and since there exists Fourier unit-root tests, for completeness, we also report unit-root test results based on Enders and Lee (2012a), Enders and Lee (2012b)) and Rodrigues and Taylor (2012). Enders and Lee (2012a)-hereafter EL^A- is a Lagrange Multiplier (LM) type unit-root test based on Schmidt and Phillips (1992) and Amsler and Lee (1995) that employs first difference (FD) detrending. Enders and Lee (2012b)) -hereafter EL^B- is a unit-root test with a Fourier function in the deterministic term in an OLS detrending Dickey-Fuller (DF) type regression. Rodrigues and Taylor (2012)-hereafter RT- apply the flexible Fourier form to the local GLS unit-root testing procedure of Elliott, Rothenberg and Stock (1996).

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[5] In this paper, the term "unit-root" refers to tests with the null hypothesis of a unit root and stationarity as the alternative. The term "stationarity" test describes a test with the null hypothesis of stationarity and an alternative of a unit-root.

Since structural breaks shift the spectral density function towards zero, it seems reasonable to control for breaks using the low frequency component of a Fourier approximation. Gallant (1981), Davies (1987), Becker, Enders and Hurn (2004), Becker, Enders, and Lee (2006)-hereafter BEL-, show that a Fourier approximation can often capture the behavior of an unknown function even if the function itself is not periodic. Although this methodology works best when breaks are gradual, BEL show that the Fourier approximation has good power to detect U-shaped breaks and smooth breaks located near the end of a series. Hence, instead of selecting (or estimating) specific break dates, the number of breaks, and the functional form of the break, the specification problem is transformed into choosing the proper frequency component to incorporate in the testing regression.

Most of the African economies included in this study face high levels of external debt, limited sources of revenue and can also generally be characterized by a lack of credibility that renders external borrowing more difficult and/or more costly. Moreover, most of these economies have experienced and are still vulnerable to internal and external shocks that may potentially prevent them from honoring their external commitments (Marius et al. (2017). As such, one important contribution of this paper is to assess whether the CA imbalances for these African economies are sustainable. Table 1, column two, lists the average CA balance to GDP ratios. It can be seen that the majority of the countries recorded CA deficits for the most, if not, the entire period of study. The time path of the CA to GDP ratios are depicted in Figure 1. For most countries, it is difficult to observe a clear pattern of strong mean reversion as current accounts deviate from their mean for a relatively large number of years. This illustrates the empirical difficulty to find a strong evidence supporting CA sustainability.

Even though there exists a large volume of empirical literature on the CA sustainability, most of the studies focus on advanced economies while developing economies, especially those in Africa, attracted much less attention. Similar to the theme of this paper, what follows is a very brief review of recent studies that utilized a nonlinear approach to assessing CA sustainability. Christopoulos and Leo-Ledesma (2010) examine the CA sustainability for the USA during the 1960:4 to 2008:4 period. They utilize a nonlinear

ESTAR unit-root test based on Kapetanios et al. (2003) and Kilic (2003) and conclude that the USA's CA is sustainable. Cuesta (2013) tests the sustainability of the CA for a group of central and eastern European countries. By means of KSS nonlinear unit-root test and fractional integration, he shows that the GDP to CA ratio is stationary and mean reverting process in most cases.

From the empirical literature surveyed, we found one recent study testing the sustainability of the CA for few African economies, although the methodology is different from the one utilized here. Marius, Mbratana, and Quentin (2017) examine the sustainability of CA deficits for Cameroon, Chad, Congo DR., Congo Rep., Gabon, and Rwanda for the 1970-2015 period. Their country analysis finds no evidence of cointegration between exports and imports based on Johansen cointegration technique. However, the Enders & Siklos (2000) threshold cointegration technique reveals the existence of weak-sustainability for all six countries.

Based on available data, this paper investigates the mean reversion properties of the CA for twenty one African economies. We rely on stationarity and unit-root tests that use trigonometric terms to capture unknown nonlinearities. These economies are: Benin, Botswana, Cabo Verde, Cameroon, Congo, DR., Eswatini, Ethiopia, Gabon, Ghana, Kenya, Lesotho, Madagascar, Malawi, Mali, Mauritius, Niger, Nigeria, Senegal, South Africa, Togo, and Uganda. This study and its results contribute to the existing relevant literature in several ways. First, to the best of our knowledge, this is the first study examining the sustainability of the CA in Africa utilizing "nonlinear" univariate stationarity and unit-root tests (to perform the so-called confirmatory analysis).

Second, with few exceptions, many previous empirical studies assert that the dynamic adjustment of the CA follows a nonlinear process but do not formally test for the presence of nonlinearity in the DGP. In the absence of nonlinearity in the CA series, linear stationarity and unit-root tests tend to be more powerful than nonlinear tests. As such, this study, to date, is the first to utilize the newly developed test by Perron, Shintani, and Yabu (2017) to test all CA series for the presence of a nonlinear deterministic component.

This linearity test can be implemented without any "prior" knowledge as to whether the noise component in the univariate time-series is stationary or contains a unit-root.

Third, we test for nonstationarity in the presence of an unknown number and form of smooth breaks based on the newly developed nonlinear stationarity and unit-root tests, i.e., Tsong, Lee and Tsai (2019)-hereafter TLT-, Enders and Lee (2012a)-Hereafter EL^{DF}-, Enders and Lee (2012b)-Hereafter EL^{LM}- and Rodrigues and Taylor (2012)-Hereafter RT^{GLS}-. The results of these tests are then compared to a battery of linear unit-root tests and to a number of competing nonlinear unit-root tests such as Kapetanios, Shin and Snell (2003), Sollis (2009), and Kruse (2011).

The remainder of this paper is organized as follows. Section 2 briefly discusses the theoretical model of the current account. Section 3 introduces the econometric methodologies we employ. Section 4 describes the data and the empirical results. Section 5 presents the conclusions from this research.

2. Theoretical background

We follow Taylor (2002) and Christopoulos and Leon-Ledesma (2010) and use the concept of sustainability as the economy's ability to meet its LRBC. Trehan and Walsh (1991) show that CA stationarity is a sufficient condition for the LRBC to hold. The open economy's two period budget constraint is:

$$C_t + I_t + G_t + B_t = Y_t + (1 + r_t)B_{t-1} \tag{1}$$

here C_t is consumption; I_t is investment; G_t is government spending; B_t is net stock of debt; Y_t is income; and r_t is (non-constant) one period world interest rate. Rearranging equation 1 and from national accounts identities we have

$$B_t = (1 + r_t)B_{t-1} + Y_t - C_t - I_t - G_t = (1 + r_t)B_{t-1} + NX_t \tag{2}$$

where $NX_t \equiv Y_t - (C_t + I_t + G_t)$ is the economy's net exports. Following Trehan and Walsh (1991, p. 209), iterating equation 2 forward in time assuming φ_{t-1} is the information set available at $t - 1$, we obtain the result that current debt (credit) position must be offset, in expected value terms, by future surpluses (deficits):

$$B_{t-1} = -\sum_{j=0}^{\infty} E\left(\prod_{i=0}^{j}\left(\frac{1}{1+r_{t+i}}\right) NX_{t+j} \middle| \varphi_{t-1}\right) + \lim_{j\to\infty} E\left(\prod_{i=0}^{j}\left(\frac{1}{1+r_{t+i}}\right) B_{t+j} \middle| \varphi_{t-1}\right) \tag{3}$$

the above equation implies that international investors are willing to lend an economy if they expect that the present value of the future stream of *NX* surpluses to equal the current stock of external debt. Hence, the sustainability hypothesis, or LRBC implies that the last term of equation 3 must equal zero:

$$\lim_{j\to\infty} E\left(\prod_{i=0}^{j}\left(\frac{1}{1+r_{t+i}}\right) B_{t+j} \middle| \varphi_{t-1}\right) = 0 \tag{4}$$

This transversality condition (i.e., non-Ponzi scheme condition) means that the present value of the expected stock of debt must converge to zero as t tends to infinity. Trehan and Walsh (1991) show that given that the $CA_t = \Delta B_t = B_t - B_{t-1}$, a sufficient condition for equation (4) to hold is that the CA is stationary. That is if ΔB_t is stationary, then the exponential growth of the realized gross rate of return $(1 + r_t)$ dominates the linear growth in B_t, ensuring that Eq. (4) converges to zero (Christopoulos and Leon-Ledesma , 2010). In an economy with a positive rate of economic growth, CA sustainability will hold if the current account to GDP ratio, $CAY_t, = \frac{CA_t}{Y_t}$, is stationary. This implies that sustainability is possible with perpetual CA deficits as long as the deficits do not grow faster that the economy's output in expected value. In this case, sustainability implies that long-run debt to GDP ratio is constant.

CA nonstationarity, if observed, should be interpreted such that the behavior of the CA is not mean reverting and hence incompatible with the LRBC during the sample period observed. Such behavior, if perpetuated in the future, may result in a default unless some form of an unexpected shock or shocks brings it back to equilibrium (Christopoulos and Leon-Ledesma , 2010). Hence, an obvious mechanism to test the sustainability hypothesis is a unit-root and/or stationarity test on CAY_t series.

3. Econometric methodology

Perron (1989) show that there is a bias against rejecting a false unit-root if an extant structural break is ignored in the standard DF type tests.[6] Two distinct problems can arise if a univariate time-series contains an unknown number of breaks. First is that even if the breaks are sharp, as extant structural break unit-root tests assume, the number of breaks and the break dates themselves are unknown and in most instances need to be estimated along with other parameters in the model. Second, even if the number of breaks are known, the possibility of a smooth break means that the functional form of the break is unknown.[7] Finally, Prodan (2008) show that it can be difficult to properly estimate the number and the magnitudes of multiple breaks, particularly when the breaks are of opposite signs.

Since gradual structural breaks and other nonlinearities in the DGP can be approximated using a Fourier function, EL^{DF}, EL^{LM}, RT^{GLS} and Tsong et al. (2013) developed unit-root tests without estimating break location, number of breaks or the functional form of structural breaks. However, the above tests consider nonstationarity as the null hypothesis. Such set up may bias test results in favor of the null hypothesis, which can't be rejected unless there is a strong evidence against it (Tsong et al. 2019). Additionally, such problem of low power associated with unit-root tests could be exacerbated when a theory, such as current account sustainability, can be more naturally tested under the null of stationarity (Becker, Enders and Lee, 2006-hereafter BEL-). As a complement to Fourier unit-root tests, BEL introduced a Fourier stationarity

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[6] In the DF type unit-root tests, the issue is a loss of power when structural breaks are ignored. Since the null and the alternative are reversed in stationarity tests, standard stationarity tests, i.e., KPSS, exhibit size distortions under the null rather than a loss of power. BEL observe a similar problem when a nonlinear trend present in the DGP is ignored.

[7] In applied research, both problems can coexist.

test based on Kwiatkowski et al. (1992) KPSS test (FKPSS) and TLT introduced a similar stationarity test based on Leybourne and McCabe (1994) LMC test.

3.1 The flexible Fourier stationarity test

Linear and nonlinear unit-root tests have the unit-root as the null hypothesis and stationarity as the alternative. However, if the presumption is that a univariate time-series is stationary, it seems natural to operate with the null hypothesis of stationarity and the alternative of unit-root. Moreover, some authors have advocated the joint application of the two kinds of tests as a way to obtain robust conclusions about the stochastic properties of such univariate time-series (e.g., Maddala and Kim 1998; Carrion-i-Silvestre et al. 2001). To mitigate the problem of controlling for breaks of an unknown form and number in stationarity tests, TLT and BEL propose tests to investigate the stationary null against the unit-root alternative employing a Fourier component to accommodate possible nonlinear trend.

As noted in Leybourne and McCabe (1994), the LMC test is superior to the KPSS in several ways. First, the value of the KPSS test statistic can be very sensitive to the value of the autocorrelation lag truncation number, l, and hence also the test inference. Although the LMC test involves choosing a number, $\mathbf{P}$, of AR components to include, the value of the LMC test is not heavily influenced by choosing $\mathbf{P} > p$, hence LMC's inferences tend to be more robust. Second,, the power of the KPSS would be reduced with increasing l, and in general, the LMC test is more powerful than the KPSS test. Consequently, TLT develop a Fourier LMC (FLMC) stationarity test based on Leybourne and McCabe (1994) and show that the aforementioned properties of the LMC test carry over to the newly developed FLMC test.

TLT derive the asymptotic distribution of the FLMC test under the null and find that both FLMC and FKPSS to have the same large-sample properties. In addition, they show that FLMC is consistent to the order $O_p(\mathrm{T})$ and prove that when a series is stationary and exhibits nonlinear trend, the LMC test, ignoring nonlinearity, diverges at a rate of $O_p(\mathrm{T})$ under the null causing severe size distortions. Finally, they show that the FLMC test has better size and power properties than FKPSS even in small samples. To illustrate the FLMC stationarity test, consider the following DGP:

$$y_t = \sum_{i=0}^{m} \delta_i \, t^i + d(t) + v_t, \qquad t = 1,2,3 \ldots, T \tag{5}$$

where $d(t)$ is defined as

$$d(t) = \delta_0 + \delta_{si} \sin\left(\frac{2\pi kt}{T}\right) + \delta_{ci} \cos\left(\frac{2\pi kt}{T}\right) \tag{6}$$

where n represents the number of frequencies contained in the approximation, k represents a particular frequency, δ_{si} and δ_{ci} measure the amplitude and displacement of the frequency component, T is the number of observations and v_t follows the stochastic process:

$$\Phi(L)v_t = \gamma_t + \varepsilon_t, \tag{7}$$

$$\gamma_t = \gamma_{t-1} + \eta_t, \, \gamma_0 = 0, \tag{8}$$

where $\Phi(L) = 1 - \varphi_1 L - \cdots - \varphi_p L^p$ being a pth order autoregressive polynomial in then lag operator with roots outside the unit circle. ε_t are i.i.d. (0, σ_{ε}^2), and η_t is i.i.d. (0, σ_{η}^2). As in Leybourne and McCabe (1994), ε_t and η_t are assumed to be independent of each other. If σ_{η}^2 = 0, v_t is an I(0) stationary AR(p) process, and so is y_t; otherwise, γ_t is an I(1) unit root process, and so are v_t and y_t. Accordingly, the stationary null and the unit root alternative can be tested as:

$$H_0: \sigma_\eta^2 = 0 \; vs. H_1: \sigma_\eta^2 > 0. \tag{9}$$

As suggested by Gallant (1981), BEL, and EL^{DF} and EL^{LM}, a Fourier term with a single frequency, k, is incorporated in Eq.(6) to accommodate possible nonlinearities in the deterministic component of y_t.[8]

The FLMC test statistic can be computed by first estimating Eq. (5) with least squares (LS) and obtain the LS residuals $\hat{v}_t$ by

$$\hat{v}_t = y_t - \sum_{i=0}^{m} \hat{\delta}_i \, t^i - \hat{\delta}_{si} \sin\left(\frac{2k\pi t}{T}\right) - \hat{\delta}_{ci} \cos\left(\frac{2k\pi t}{T}\right), \tag{10}$$

where $\hat{\delta}_i$, $\hat{\delta}_{si}$ and $\hat{\delta}_{ci}$ are the LS estimates for δ_i, δ_{si} and δ_{ci}, respectively. In the second step, compute $\hat{\varepsilon}_t = \hat{v}_t - \sum_{i=1}^{p} \hat{\varphi}_i \, \hat{v}_{t-i}$, where $\hat{\varphi}_i$ are the maximum likelihood (ML) estimates of the AR coefficients when the residuals $\hat{v}_t$ is fitted with an ARIMA (p, 1,1) model. Finally, the FLMC^m stationarity is computed as follows:

$$FLMC^m(p) = \frac{\sum_{t=1}^{T} S_t^2}{T^2 \hat{\sigma}_\varepsilon^2}, \tag{11}$$

where $S_t = \sum_{i=1}^{t} \hat{\varepsilon}_i$ is the partial sum of $\hat{\varepsilon}_t$, and $\hat{\sigma}_\varepsilon^2 = 1/T(\sum_{t=1}^{T} \hat{\varepsilon}_t^2)$ is an estimator for σ_ε^2. Note that the AR estimates in the second step are obtained by maximum likelihood, ML, after fitting the residuals $\hat{v}_t$ to an ARIMA(p,1,1) model, rather than estimate the parameters with LS in an AR(p) specification for the residuals. Doing so ensures that $\hat{\varphi}_i$ is consistent for φ_i for $i = 1, 2, 3, \ldots p$, no matter the series is generated from the stationary null or from the unit root alternative as argued in Leybourne and McCabe (1994). As such, the FLMC test constructed with the residuals $\hat{\varepsilon}_t$ will not suffer from power losses under the unit root alternative.

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[8] We do not consider *d(t)* containing multiple frequencies to avoid the overfitting problem. Moreover, EL^{DF}, EL^{LM} and RT^{GLS} note that using a single frequency, k, can effectively capture several types of structural breaks commonly seen in economic analysis.

The asymptotic distribution of the FLMC stationarity test depends on the Fourier frequency, k, but invariant to other parameters in the model. Similar to LMC and KPSS tests, FLMC and FKPSS also share the same asymptotic distribution. Table 1 of TLT reports the critical values of the FLMC test, which are very similar to the FKPSS test, for k = 1, 2 and 3 and for m = 0 (level stationary process) and for m = 1 (trend stationary process).

Calculating the FLMC test involves selecting the appropriate frequency, k. BEL, EL$^{\mathrm{LM}}$ and TLT suggest that choosing $k = 1$ is sometimes sufficient to capture various nonlinearities present in the DGP. An alternative approach is to select the frequency component, k, using a statistical mean. Following Davis (1987), BEL use a grid-search method such that the value $k = \hat{k}$ minimizes SSR from Eq. (10). As such, for each integer value of k in the interval $1 \leq k \leq 3$, we estimate equation 10 and select the value yielding the best fit.

3.2 linearity test

Without a nonlinear trend, i.e., if δ_{si}= δ_{ci}= 0, the process is linear and traditional unit-root testing methodologies are appropriate. However, if there is a break or a nonlinear trend, at least one frequency component, typically with a small k value, must be significant. BEL and TLT propose a pretesting for nonlinearities by performing the usual F-test for the null hypothesis δ_{si} = δ_{ci}= 0.

The practical implication of such tests could be problematic since they assume that the order of integration of the data to be known a priori.[9] This creates a circular testing problem as one needs to know the order of integration of the time-series before performing any stationarity tests; however, prior to performing such tests, one would also need to specify the form of the deterministic component, $d(t)$.

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[9] Specifically, in deriving the critical values for the $F^m(\hat{k})$ linearity test, the null of stationarity is imposed on the DGP. Hence, the critical values are valid only when the null hypothesis of stationarity is not rejected.

We address this circularity in the testing problem by using a robust test for a nonlinear deterministic component that does not depend on the stationarity properties of the data. We apply the newly developed test of Perron, Shintani, and Yabu (2017)- hereafter PSY- to test all CAY_t series for the presence of a nonlinear deterministic component. This pretesting methodology is robust to the presence of I(0) or I(1) errors by considering a Fourier expansion with a number of frequencies that can capture the main characteristics of a very general class of nonlinear functions.

PSY testing strategy builds on the work of Perron and Yabu (2009) and is based on a Feasible Generalized Least Squares (FGLS) procedure that uses a super-efficient estimator of the sum of the autoregressive coefficients, α, when $\alpha = 1$. To improve small sample properties of the test, PSY use a bias-corrected version of the OLS estimator of α proposed by Roy and Fuller (2001).[10]

PSY derive the Wald statistics, $W_{\hat{\gamma}}$, for testing the null hypothesis for the absence of nonlinear components, H_0: $\delta_{si} = \delta_{ci} = 0$, against the alternative of the presence of a nonlinear component approximated by the Fourier series expansion, H_1: $\delta_{si} \neq \delta_{ci} \neq 0$. PSY show that the $W_{\hat{\gamma}}$ test has a χ^2 (2) distribution in both the I(0) and the I(1) cases. To choose the number of frequencies, k, in implementing the $W_{\hat{\gamma}}$ test statistic, we follow PSY and use a general-to-specific procedure based on the sequential application of the variant of $W_{\hat{\gamma}}$ for subset of coefficients. We set the maximum number of frequencies, k, in the following testing regression to 3:

$$y_t = \sum_{i=0}^{p_d} \beta_t \, t^i + \sum_{k=1}^{3} \left(\delta_{si} \sin(\frac{2\pi kt}{T}) + \delta_{ci} \cos(\frac{2\pi kt}{T}) \right) + \mu_t \tag{14}$$

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[10] Perron et al. (2017) show that the FGLS procedure has many econometric advantages and is substantially more powerful than currently available alternatives (i.e., Harvey et al. (2010) and Astill et al. (2015)). See Perron et al. (2017) for a detailed analysis of testing for flexible nonlinear trends procedure.

$$u_t = \alpha u_{t-1} + e_t \tag{15}$$

and test the null hypothesis that the coefficients related to the maximum frequency, $k = 3$, are zero when $p_d = 0$ and when $p_d = 1$. If the linearity null hypothesis is rejected, we set $k = 3$. If not, we set $k = 2$ and test whether the coefficients related to $k = 2$ are zero. We continue until we reject the null or reach $k = 0$. Note that all the tests share the same critical value from the χ^2 distribution as the number of restrictions in each step is 2 (=2m).

4. Data and empirical results

The data include annual observations of the current account balance as a percentage of GDP, CAY_t. We conduct linear and nonlinear stationarity and unit-root tests for 21 African economies (see Table 1 for each country's sample period. Table 1, columns 1 and 2, show sample period and the number of annual observations. Column 3 ranks the countries according to their average current account to GDP ratio with Malawi being historically the highest deficit economy and Gabon the highest surplus country. We also show the time path of the current account to GDP ratios as the solid lines in Figure 1. The Figures show that most of the countries recorded large and persistent deficits for extended periods of time. All data are obtained from the World Bank-World Development Indicators (WDI)- database (2019).

4.1 linear unit-root test results

As a preliminary and a benchmark for subsequent analysis, we first utilize a battery of linear unit-root tests that ignore possible nonlinearity and structural breaks in the CAY_t. In particular, we employ the augmented Dickey-Fuller test (ADF), the DF-GLS of Elliott et al. (1996) and the Ng and Perron (2001) modified unit-root tests (M-type tests) with GLS detrending, i.e., MZ_α^{GLS}, MZ_t^{GLS}, MSB and MP_t^{GLS}.

The results of applying these linear tests are reported in Table 2. Even at 10% level of significance, the ADF fails to reject the null hypothesis of nonstationary CAY_t series for 14 of 21 countries, while the DF-GLS and the 3 *M*-type unit-root tests fail to reject the same null, also at 10% level, for 13 and 12 countries, respectively. Note that, all 5 linear unit-root tests (i.e., ADF, DF-GLS and 3 *M*-type tests), simultaneously fail to reject the null hypothesis of nonstationary CAY_t , at 5 % level of significance, for the following 11 countries: Benin, Botswana, Cabo Verde, Cameroon, Ethiopia, Ghana, Lesotho, Madagascar, Niger, Senegal, and South Africa, which would indicate that their CAY_t is a nonstationary process and sustainability is not supported for more than half of the countries in our study. As stated earlier, standard unit-root tests may have low power distinguishing a unit-root process from a structural break or any other form of nonlinearity present in the DGP.

4.2 structural break unit-root test results

In the presence of a structural break and/or nonlinearities, the power to reject a unit-root decreases if the stationary alternative is true and the structural break is ignored (Perron, 1989). To address the structural break issue, we employ Cavaliere, Harvey, Leybourne and Taylor (2011)-hereafter CHLT- and Harvey, Leybourne and Taylor (2013)-hereafter HLT- unit-root tests and to address any nonlinearity, we apply three widely used nonlinear unit-root tests, i.e., the nonlinear ESTAR unit root test of Kapetanios, Shin, and Snell (2003)-KSS-, the unit-root test against asymmetric ESTAR nonlinearity of Sollis (2009)-Sollis-, and the unit-root test against ESTAR alternative of Kruse (2011)-Kruse-.[11]

CHLT consider testing a time series for a unit-root in the possible presence of a single break in a linear deterministic trend at an unknown point of time. They consider the impact of nonstationary volatility on the ADF-type test of Harris et al. (2009) together with analogous *M*-type tests studied in Perron and Rodriguez (2003). They show that the limiting null distributions of unit-root statistics, based around the

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[11] Structural break unit-root tests depend on some past shock that permanently changes the mean and/or the slope of the time-series investigated; while nonlinear unit-root tests incorporate the possibility of changing autoregressive parameters, depending on the size of shocks (Cuestas, 2013).

break fraction estimator of Harris et al. (2009), are not pivotal under nonstationary volatility and propose a solution to such inference problem using a wild bootstrap method that is shown to perform well in practice. The four structural break unit-root tests considered by CHLT are the $ADF\text{-}GLS^{tb}$, MZ_{α}^{tb}, MZ_{t}^{tb}, and MSB^{tb}.

HLT unit-root tests allow for the presence of one and two structural breaks obtained by taking the infimum of the sequence (across all candidate break points in a trimmed range) of local GLS detrended augmented Dickey-Fuller type statistics. HLT show that their procedure has power that is robust to the magnitude of any trend breaks, thereby retaining power in the presence of plausibly sized breaks. HLT demonstrate that, unlike the OLS detrended infimum tests of Zivot and Andrews (1992), their tests display no tendency to spuriously reject in the limit when fixed magnitude trend breaks occur under the null of unit-root.

Allowing for a structural break did not result in more rejections of the unit-root null hypothesis compared to linear tests. For example, using a 5% wild bootstrap critical values, the CHLT four structural break unit-root tests fail to reject the null hypothesis of nonstationary CAY_t series for twelve countries. The HLT one and two structural break tests, MDF_1 and MDF_2, fail to reject the same null, at 5% level, for 12 and 10 countries, respectively. These findings echo the majority of results obtained from conventional unit-root tests.[12]

4.3 nonlinear unit-root test results

Next, we consider nonlinear unit-root tests that ignore possible structural breaks in the current account to GDP ratio. Table 4 reports results of KSS, Sollis and Kruse nonlinear unit-root tests applied to the demean CAY_t data. As seen in Table 4, the three nonlinear unit-root tests produce very similar results.

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[12] We also applied the GLS-based unit-root tests with multiple structural breaks of Carrion, Kim, and Perron (2009). The stationarity of the current account to GDP ratio was rejected for the vast majority of countries based on their test. The results of these tests can be supplied upon request.

Table 4 shows that the KSS, Sollis, and Kruse tests, even at the 10% level of significance, fail to reject the null hypothesis of nonstationary CAY_t series for 15, 13 and 14 countries, respectively. With the exception of Benin and Nigeria, the 3 nonlinear unit-root tests simultaneously fails to reject the null hypothesis of nonstationary current account to GDP ratios for the following 13 countries: Botswana, Cabo Verde, Cameroon, Congo, Eswatini, Ethiopia, Gabon, Lesotho, Madagascar, Malawi, Niger, Togo, and Uganda, leading one to conclude that the current accounts, for these countries, are not on a unsustainable path.

4.4 Linearity pretest results

Since a standard unit-root test is more powerful than tests including Fourier terms if the true DGP precludes a nonlinear trend, we apply PSY linearity test to ascertain whether CAY_t series contain nonlinear components that can be addressed using a Fourier function. Table 1 reports the $W_{\hat{\gamma}}$ test of whether there exists significant nonlinear shifts in the current account to GDP ratios. The computed $W_{\hat{\gamma}}$ test statistics are compared to the relevant χ^2 critical values as reported in Table 1. The results show a strong evidence against the null hypothesis of linearity for 17 CAY_t . Hence, it is necessary to include a trigonometric function when testing for stationarity and unit-root for all CAY_t except for the 4 current account-GDP ratios of Congo, Kenya, Niger and South Africa. To assess the mean reversion properties of those 4 current accounts, we rely on standard and structural break unit-root and stationarity tests.

4.5 The flexible Fourier stationarity test results

The ambiguity of results regarding the current account sustainability issue reported previously in this and possibly in many published papers may be attributed to neglecting the presence of smooth structural breaks present in the current account to GDP ratios. Moreover, problem of low power associated with unit-root tests could be exacerbated when a theory, such as current account sustainability, can be more naturally tested under the null of stationarity (Becker, Enders and Lee, 2006).

In this section, we first report the results of the linear stationarity test of Leybourne and McCabe (1994), LMC, for comparison. As Table 5 shows, the LMC level stationarity test rejects the null of stationary current account to GDP ratio, at the 5% level of significance, for 13 out of 21 countries. If accurate, a result that would indicate that the current accounts for those countries are not mean reverting and the LRBC does not hold. However, the results are quite different when we apply the nonlinear version of the LMC test to the current account to GDP ratios.[13]

Note the reported high rejection of the stationarity null hypothesis, by the LMC test, could be attributed to a genuine nonlinear deterministic component present in the current account to GDP ratios. Such nonlinearity, when ignored, tends to bias standard stationarity and unit-root tests. This demonstrates the importance of linearity testing and for allowing for both $I(0)$ and $I(1)$ noise when testing for the presence of nonlinear components in the trend.

The nonlinear LMC test, i.e., FLMC, is computed based on equation 11 with the optimal frequency, $\hat{k}$, determined by using a grid-search method. For each integer value of k in the interval $1 \leq k \leq 3$, we estimate equation 10 and select the value yielding the best fit. The optimal AR lag, p^*, in the ARIMA(p, 1, 1) model is selected by the general-to-specific ($GTOS$) procedure given a maximum lag of 6. Table 5 shows that the FLMC test fails to reject the stationarity null hypothesis, at the 5% significance level, for 12 out of 17 countries where including a Fourier component deemed necessary to model the smooth breaks based on PSY linearity test. These countries are: Malawi, Cabo Verde, Togo, Mali, Madagascar, Benin, Senegal, Mauritius, Cameroon, Eswatini, Nigeria and Gabon. The five countries, where the FLMC test rejects the null of stationarity, are Botswana, Ethiopia Ghana, Lesotho and Uganda.

The sharp contrast between LMC and FLMC test results are simply due to neglecting the presence of smooth breaks corroborated by the PSY test. Taking into account smooth breaks with a Fourier component,

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[13] We consider level stationary tests for the CAY_t series and exclude “time” as a regressor in equation 5. Any type of trend component is inconsistent with mean reversion properties of the current account to GDP ratios.

the FLMC test can uncover robust evidence in favor of sustainable CAY_t series for a larger number of countries. Note that for the four countries, Congo, Kenya, Niger, and South Africa, where the linearity null hypothesis was not rejected, only Congo's and Kenya's CAY_t is stationary based on the LMC test.[14]

4.6 The flexible Fourier unit-root test results

Since some authors advocate the joint application of stationarity and unit-root tests as a way to obtain robust conclusions about the stochastic properties of univariate time-series data such as the CAY_t, we compute and report the results of three available Fourier unit-root tests. The LM and DF tests of Enders and Lee (2012a) and (2012b) and the DF-GLS test of Rodrigues and Taylor (2012). The three test statistics, τ_{LM}, τ_{DF_C}, and $t_{\emptyset}^{ERS,\mu}$, for the 17 countries are reported in Table 6.

For all the 12 out of the 17 countries, where FLMC finds strong empirical evidence of mean reverting CAY_t, at least one Fourier unit-root test supports such finding. Specifically, For Malawi, Mali, Benin and Nigeria, we find compelling evidence of mean reversion current account to GDP ratios as the three Fourier unit-root tests rejects the null of nonstationarity at 5% or better significance levels. The same null hypothesis is rejected for Madagascar, Senegal, Cameroon, Eswatini, and Gabon by two Fourier unit-root tests at least at 5% significance level, while the null of nonstationary CAY_t is rejected for Cabo Verde, Togo and Mauritius by one Fourier unit-root test at least at 5% significance level. The above results unequivocally show that the current account to GDP ratios for these 12 countries are mean reverting and their LRBCs hold.

Additionally, the three Fourier unit-root tests uncover strong evidence in favor of current account sustainability for Ghana and Ethiopia as the null hypothesis of nonstationary current account to GDP ratio for both countries is rejected, at 5% or better level of significance, by the three Fourier unit-root tests. We

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[14] Table 5 reports the linearity test, $F^m(\hat{k})$ as suggested by Becker et al. (2006) and Tsong et al. (2019). The results confirm those reached by Perron et al. (2017) linearity tests.

also find evidence supporting the stationarity of Uganda's current account to GDP ratio as the τ_{LM} and $t_{\emptyset}^{ERS,\mu}$ test statistics reject the null of unit-root at 10% and 1%, respectively, in conjunction with the strong evidence provided by ADF, DF-GLS and Ng and Perron (2001) M-type unit-root tests.[15] Furthermore, the results based on Fourier tests, and a battery of linear and nonlinear unit-root tests indicate that the current account to GDP ratios for Botswana and Lesotho are not on a sustainable path.

Figure 1 shows the time path of CAY_t series (solid black lines) and the estimated time paths of the time varying intercept (smooth solid blue lines), based on the preferred frequency, for the current account to GDP ratio for the 17 countries. These figures show that the breaks in the CAY_t series follow a similar pattern to the Fourier approximation and it is very plausible to argue that each current account to GDP ratio is subject to one or more structural breaks.

Finally, since the null of linearity, based on PSY test, is not rejected For Congo, Niger, Kenya and South Africa, we resort to standard stationarity and unit-root tests to ascertain their current account sustainability. The LMC, ADF, DF-GLS and Ng and Perron (2001) *M*-type tests show strong evidence in favor of a stationary current account to GDP ratio for Congo and Kenya but not for Niger and South Africa. Similar conclusions for the 4 countries are reached if we are to entertain CHLT structural break unit-root tests. However, nonlinear unit-root tests of KSS, Sollis and Kruse confirm the same conclusions for Kenya and Niger but not for South Africa. The three nonlinear unit-root tests strongly reject the unit-root null hypothesis for South Africa's current account to GDP ratio, at 1% level of significance, an indication that the adjustment in South Africa's current account to GDP ratio may follow a STAR-type nonlinearity that is not captured by standard and Fourier type stationarity tests.

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[15] Cavaliere et al (2011) structural break unit root *M*-type tests also reject the null of unit-root for Ethiopia, Ghana and Uganda at 5% level of significance.

On the whole, the above empirical results indicate that taking into account possible smooth breaks in the current account to GDP ratios, with a Fourier component, the stationarity and Fourier unit-root tests can reveal significant evidence in favor of a sustainable CAY_t for the majority of African economies.

Finally, Given that the CAY_t series for all countries, except Botswana, Lesotho and Niger are a stationary process based on the results of a battery of stationarity and unit-root tests, we examine the breaks that are identified by the Bai and Perron (2003) procedure for the remaining 18 countries. Allowing for a maximum of three breaks, with a minimum break-size of about 8 years, the Bayesian Information Criterion (BIC) identified one or two breaks for each country with the exception of Benin, Mali, Senegal and Togo. The dashed lines in Figures 1 show the break points and the sub-period means of the CAY_t series (i.e., two and three sub-periods for one and two breaks, respectively). The Figures show that the sharp breaks are similar to the Fourier intercept except that the Fourier intercept is of a smooth nature and the Bai and Perron (2003) procedure does not embody a unit-root test. For Benin, Mali, Senegal and Togo, the graphs show the time path of CAY_t series (solid lines) and the estimated time paths of the time varying intercept (smooth lines) based on the preferred frequency.

5. CONCLUSIONS

The aim of this paper is to investigate the mean reversion properties in the CA balances as a percentage of GDP for twenty one African economies. First, we apply a battery of linear and structural break unit-root tests that provide negligible evidence in favor of CA sustainability. Moreover, we apply the widely used ESTAR based nonlinear unit-root tests of Kapetanios et al. (2003), Sollis (2009), and Kruse (2011). The empirical results of the three nonlinear unit-root tests are similar to the standard and structural break unit-root tests in that they confirm sustainability of the CA balance for a small number of countries.

To investigate further, we apply the Fourier stationarity test of Tsong et al. (2019) and three Fourier unit-root tests of Enders and Lee (2012a) and (2012b) and Rodrigues and Taylor (2012). However, rather than assuming a nonlinear CA adjustment, we formally test for “nonlinearity” using Perron et al. (2017)

new test for the presence a nonlinear deterministic trend using a flexible Fourier approximation. The Perron et al. (2017) test confirms nonlinearity of the CAY_t for 17 of 21 countries. Employing a Fourier function to approximate smooth structural breaks and other forms of nonlinearities in the CAY_t for the 17 countries, The Fourier stationarity and unit-root test results confirm that the current accounts of 15 out of 17 countries (i.e., Malawi, Cabo Verde, Togo, Mali, Madagascar, Benin, Senegal, Mauritius, Cameroon, Eswatini, Nigeria, Gabon, Ghana, Ethiopia, and Uganda) were on a sustainable path. Moreover, relying on non-Fourier standard unit-root tests confirm that the current account to GDP ratios for Congo, Kenya and South Africa were also on a sustainable path.

All in all, our extensive empirical analysis reveal that employing the Fourier tests yield a lot more rejections of the nostationarity hypothesis of the current account to GDP ratio compared to standard, structural break, and ESTAR based nonlinear unit-root tests of Kapetanios et al (2003), Sollis (2009), and Kruse (2011). This conclusion is not meant to imply that Fourier unit-root tests have absolute power over other procedures in all cases. Moreover, the empirical results indicate that the nonlinear adjustment in the current account is not exclusively present for countries with high historical average current account deficits, but rather for the vast majority of countries in our sample.

Finally, the conclusion that every country, in our study, appears to be obeying it's LRBC does not necessarily mean that, during some periods, some countries were unable to run "unsustainable" current account deficits. All this means that deficits and/or shocks, during the study period, were temporary or too limited in duration to cause nonstationarity of debt in the Ponzi scheme sense.

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Table 1

Perron, Shintani, and Yabu (2017) test for flexible nonlinear trends

				$p_{d=0}$			$p_{d=1}$		
Country	avg. cay_t	date	T	$W_{\hat{\gamma}}$		$\hat{k}$	$W_{\hat{\gamma}}$		$\hat{k}$
Malawi	-10.7	1977-2017	41	5.656	***	1	2.885		0
Niger	-9.41	1974-2018	44	0.852		0	0.729		0
Congo, Rep. of	-9.27	1978-2016	39	2.318		0	2.301		0
Cabo Verde	-7.86	1980-2018	39	12.69	*	1	11.99	*	1
Togo	-7.55	1974-2017	44	10.29	*	2	16.72	*	2
Mali	-7.53	1975-2017	43	6.604	**	1	4.467		0
Madagascar	-7.19	1974-2017	44	5.935	***	3	5.592	***	3
Benin	-6.61	1974-2017	44	4.849	***	1	4.463	***	0
Kenya	-6.03	1975-2017	43	0.026		0	0.009		0
Senegal	-5.96	1974-2017	44	9.300	*	2	7.181	**	2
Ghana	-5.01	1975-2017	43	8.617	*	1	4.892	***	3
Uganda	-4.31	1980-2018	39	11.97	*	2	10.69	*	2
Mauritius	-4.11	1976-2018	43	6.345	**	3	6.356	**	3
Ethiopia	-3.82	1981-2017	37	6.828	**	1	2.29		0
Cameroon	-3.31	1977-2017	41	7.792	*	1	3.681		0
South Africa	-1.30	1960-2018	59	0.829		0	0.827		0
Eswatini	-0.43	1974-2017	44	14.89	*	3	14.91	*	3
Nigeria	2.48	1977-2017	41	1.501		0	5.045	***	2
Botswana	3.66	1975-2017	43	0.669		0	4.730	***	3
Lesotho	4.20	1975-2018	44	9.946	*	2	9.951	*	2
Gabon	5.58	1978-2015	38	11.51	*	1	7.963	**	1

Notes: (1) *, **, *** denote significance at the 1%, 5%, and 10% respectively. (2) The $W_{\hat{\gamma}}$ is the Wald test statistic for testing the null hypothesis of the absence of nonlinear deterministic component based on the FGLS proposed by Perron et al. (2017); (3) the 1%, 5% and 10% critical values for the Wald test are 9.2103, 5.9915 and 4.6052, respectively; (3) $\hat{k}$ is the single frequency. (4) The countries are ranked according to their current account to GDP ratio, avg. cay_t, with Malawi being historically the highest deficit country and Gabon is the highest surplus country.

Table 2
Linear unit root tests

	ADF		*DF-GLS*		*Ng & Perron (2001) M-type unit root tests*					
Country	τ		η_{μ}^{GLS}		MZ_{α}^{GLS}		MZ_{t}^{GLS}		MSB	
Benin	-1.66		-1.68		-5.63		-1.63		0.28	
Botswana	-2.00		-1.58		-5.23		-1.53		0.292	
Cabo Verde	-2.04		-1.59		-3.67		-1.35		0.368	
Cameroon	-1.96		-2.04		-5.83	***	-1.71	***	0.292	
Congo, Rep. of	-3.38	**	-2.36	**	-11.4	**	-2.30	**	0.202	**
Eswatini	-1.93		-2.55	**	-12.5	**	-2.49	**	0.199	**
Ethiopia	-0.27		-0.53		-1.15		-0.68		0.589	
Gabon	-2.69	***	-2.71	*	-10.4	**	-2.28	**	0.218	**
Ghana	-1.15		-0.88		-1.92		-0.976		0.508	
Kenya	-3.54	**	-3.58	**	-15.3	*	-2.76	*	0.181	**
Lesotho	-1.96		-1.97		-6.69	***	-1.82	***	0.273	***
Madagascar	-2.24		-1.74		-6.62	***	-1.77		0.268	***
Malawi	-2.37		-2.43	**	-5.42		-1.56		0.287	
Mali	-4.44	*	-4.49	*	-18.6	*	-3.05	*	0.163	*
Mauritius	-2.27		-2.24	***	-8.59	**	-2.07	**	0.241	**
Niger	-1.29		-0.83		-2.52		-0.94		0.373	
Nigeria	-3.15	**	-1.58		-4.85		-1.55		0.318	
Senegal	-1.89		-1.27		-3.95		-1.33		0.336	
South Africa	-1.82		-1.48		-4.55		-1.44		0.315	
Togo	-3.96	*	-0.49		-0.11		-0.16		1.455	
Uganda	-2.82	***	-2.63	*	-10.5	**	-2.21	**	0.211	**
cv (1%)	-3.59		-2.63		-13.8		-2.58		0.174	
cv (5%)	-2.94		-2.32		-8.10		-1.98		0.233	
cv (10%)	-2.60		-2.08		-5.70		-1.62		0.275	

(1) *, ** and *** denote significance at the 1%, 5%, and 10% respectively. (2) Tests are computed with a constant. (3) MZ_{α}^{GLS}, MZ_{t}^{GLS} and MSB are Ng and Perron (2001) modified unit root tests with GLS de-trending. MZ_{α}^{GLS} and $MZ_{t}^{GLS}$are modified versions of Phillips (1987) and Phillips-Perron (1988) and MSB is Modified Sargan-Bhargava (Bhargava 1986 and Sargan and Bhargava (1983). (4) The lag truncation parameter is based on modified AIC (MAIC) as proposed by Ng and Perron (2001). The maximum autoregressive order, *p*, is set equal to 6 (p_{max} = $\left\lfloor int(8\,(\frac{T}{100})^{1/4}\right\rfloor$).

Table 3
Structural break unit root tests

	Cavaliere, Harvey, Leybourne, and Taylor (2011)-CHLT- structural break unit-root test				*Harvey, Leybourne and Taylor (2013)-HLT-*	
Country	*ADF-GLS*^{tb}	MZ_{α}^{tb}	MZ_{t}^{tb}	*MSB*^{tb}	MDF_1	MDF_2
Benin	-1.73	-6.66	-1.80	0.271	-5.51 **	-6.32 **
Botswana	-2.87	-13.8	-2.61	0.189	-3.08	-3.58
Cabo Verde	-2.81	-9.41	-2.13	0.227	-4.50 **	-5.31 **
Cameroon	-3.32	-16.8 **	-2.90 **	0.172	-5.47 **	-5.98 **
Congo, Rep. of	-3.45 **	-16.1 **	-2.68 **	0.167 **	-4.17 **	-4.78 **
Eswatini	-1.94	-5.76	-1.66	0.288	-2.21	-2.61
Ethiopia	-3.54 **	-14.7 **	-2.71 **	0.184	-4.18 **	-4.42
Gabon	-3.03	-14.8 **	-2.62	0.178	-3.44	-5.04 **
Ghana	-4.51 **	-20.3 **	-3.09 **	0.152 **	-5.15 **	-5.35 **
Kenya	-3.59 **	-16.4 **	-2.86 **	0.175 **	-3.60	-4.24
Lesotho	-2.11	-7.79	-1.97	0.253	-2.30	-3.56
Madagascar	-1.98	-11.4	-2.29	0.200	-2.77	-3.39
Malawi	-3.52	-13.2	-2.57	0.194	-3.77	-3.79
Mali	-5.66 **	-22.2 **	-3.31 **	0.149 **	-5.81 **	-6.35 **
Mauritius	-3.17	-14.3	-2.68	0.187	-3.38	-4.15
Niger	-3.34	-16.9	-2.89	0.171	-3.81	-4.57
Nigeria	-3.37 **	-15.3	-2.76	0.180	-4.04 **	-4.68 **
Senegal	-1.61	-7.17	-1.89	0.264	-3.33	-5.01 **
South Africa	-1.88	-7.29	-1.91	0.262	-3.81	-4.79 **
Togo	-3.32	-11.7	-2.34	0.200	-4.33 **	-5.06 **
Uganda	-3.60 **	-14.8 **	-2.67 **	0.181	-3.87	-4.52

(1) For CHLT test, ** denote significance at the 5%, level based on wild bootstrapped critical values. (2) For MDF1 and MDF2, ** denotes significance at the 5% level based on asymptotic critical values reported in Harvey et al. (2013) Table 1. (3) The unit-root null hypothesis is rejected for large negative values for *ADF-GLS*^{tb}, MZ_{α}^{tb}, and MZ_{t}^{tb} statistics, whereas a test based on *MSB*^{tb} rejects for small values. (4) The lag truncation parameter, *p*, is based on modified MAIC with $p_{max} = 6$.

Table 4
Nonlinear unit root test results

Country	*Kapetanios, Shin, and Snell (2003)*		*Sollis (2009)*		*Kruse (2011)*	
	$t_{NL,\mu}$		$F_{AE,\mu}$		τ_{μ}	
Benin	-2.99		10.0	*	12.9	**
Botswana	-2.55		4.24		9.73	
Cabo Verde	-1.89		2.04		4.17	
Cameroon	-1.79		1.81		3.72	
Congo, Rep. of	-0.77		0.78		0.96	
Eswatini	-2.72		3.76		7.43	
Ethiopia	-1.92		2.51		4.94	
Gabon	-2.28		3.83		5.67	
Ghana	-3.53	**	6.13	**	12.2	***
Kenya	-3.40	**	7.89	**	14.1	**
Lesotho	-2.37		2.82		7.27	
Madagascar	-2.44		2.91		6.95	
Malawi	-1.60		2.65		5.08	
Mali	-3.59	**	9.21	*	14.4	**
Mauritius	-3.16	***	5.79	***	10.3	***
Niger	-2.67		3.51		8.28	
Nigeria	-2.51		7.60	**	10.8	
Senegal	-5.11	*	13.6	*	29.7	*
South Africa	-4.19	*	9.41	*	18.9	*
Togo	-2.01		4.55		9.23	
Uganda	-2.74		3.82		7.49	
critical values						
cv (1%)	-3.93		8.799		17.1	
cv (5%)	-3.4		6.546		12.82	
cv (10%)	-3.13		5.415		11.1	

(1) *, **, *** denote significance at the 1%, 5%, and 10% respectively . (2) Nonlinear tests are applied to the demeaned current account to GDP ratios. (3) The lag truncation parameter, p, in the testing regressions is based on SBC. The maximum autoregressive order, p, is set equal to 6.

Table 5
Stationarity tests results

	Nonlinear tests					*Linear tests*	
Country	*FLMC*		P^*	$\hat{k}$	$F^m(\hat{k})$	*LMC* (P^*)	
Benin	0.038		1	1	3.691^{c}	0.146	
Botswana	0.467^{**}	**	1	1	=	1.287^{**}	**
Cabo Verde	0.109		1	1	10.22^{a}	0.724^{**}	**
Cameroon	0.052		1	1	5.733^{a}	0.118	
Congo, Rep. of	-		1	-	-	0.162	
Eswatini	0.127		2	3	13.55^{a}	0.567^{**}	**
Ethiopia	0.186^{**}	**	1	1	=	0.910^{**}	**
Gabon	0.115		1	1	17.06^{a}	0.692^{**}	**
Ghana	0.177^{**}	**	1	1	=	0.782^{**}	**
Kenya	-		1	-	-	0.062	
Lesotho	1.048^{**}	**	1	2	=	0.831^{**}	**
Madagascar	0.084		1	2	9.060^{a}	0.081	
Malawi	0.074		1	1	12.09^{a}	0.547^{**}	**
Mali	0.067		1	1	5.492^{b}	0.478^{**}	**
Mauritius	0.034		1	1	11.25^{a}	0.299	
Niger	-		1	-	-	2.141^{**}	**
Nigeria	0.095		1	1	6.820^{a}	0.892^{**}	**
Senegal	0.167		1	2	12.83^{a}	0.076	
South Africa	-		1	-	-	0.706^{**}	**
Togo	0.122		1	3	1.61	0.155	
Uganda	0.649^{**}	**	1	2	=	1.451^{**}	**

(1) ** indicate rejection of the null hypothesis of stationarity at 5% significance level (2) *a*, *b* and *c* denote significance at 1%, 5% and 10% levels, respectively. (3) p^* denotes the optimum AR lag p in the ARIMA (p, 1, 1) model selected with the top down procedure, with a 5% significance level, given a maximum lag set to 6. (4) LMC 5% critical value is 0.463. (5) The FLMC 5% critical values for $k = 1$, $k = 2$ and $k = 3$ are 0.173, 0.416 and 0.435, respectively. (6) $F^m(\hat{k})$ test the null of intercept constancy against the alternative nonlinear intercept based on Becker et al. (2006). Critical values for F^m test are reported in the lower part of Table 1 of Tsong et al. (2019). (7) "=" implies that no test is performed since the null of stationarity is rejected.

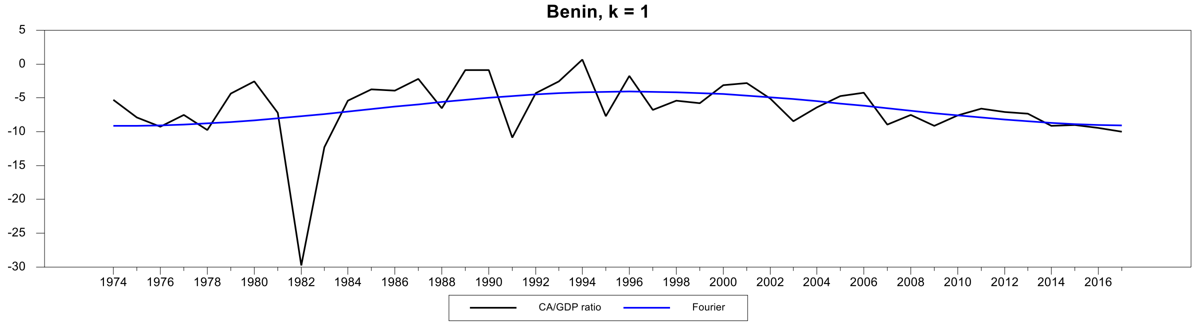
Table 6

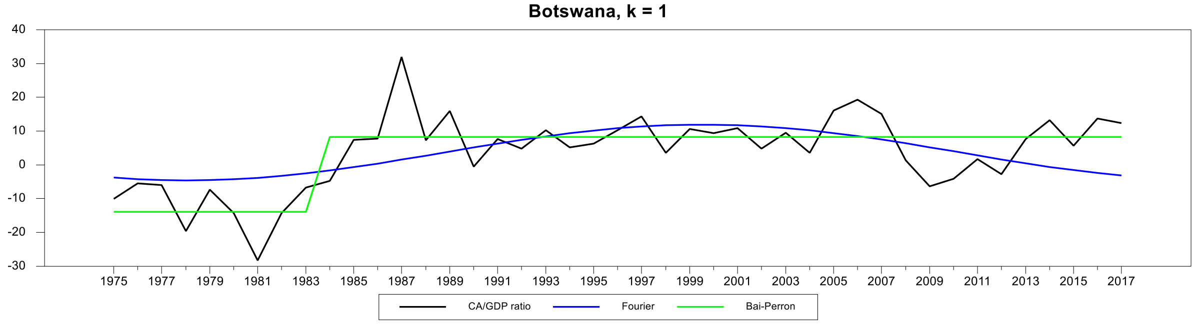
Fourier unit root tests

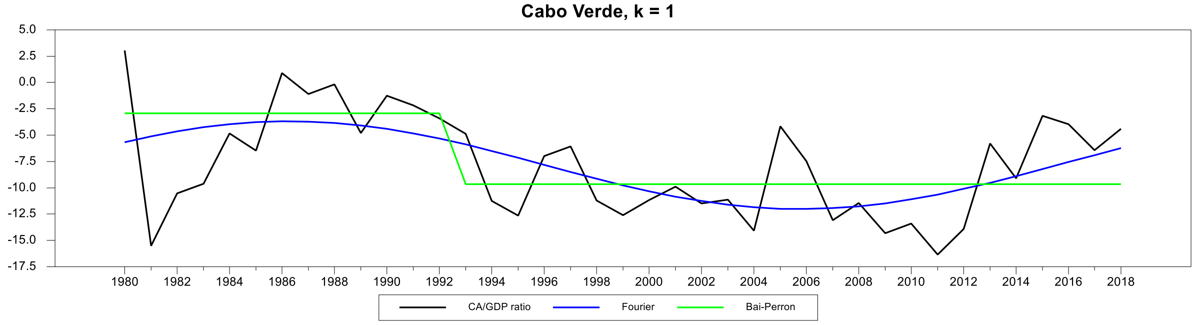
Country	Enders & Lee (2012a) τ_{DF_c}		Enders & Lee (2012a) τ_{LM}		$\hat{k}$	*Rodrigues & Taylor (2012)* $t_{\emptyset}^{ERS,\mu}$		$\hat{k}$
Malawi	-4.08	**	-5.20	**	1	-4.32	*	1
Niger	-	-	-	-	-	-	-	-
Congo, Rep. of	-	-	-	-	-	-	-	-
Cabo Verde	-3.50		-2.99		1	-3.58	**	1
Togo	-4.51*	*	-1.95		3	-1.13		1
Mali	-4.18	**	-4.21	**	1	-4.37	*	1
Madagascar	-2.94	***	-2.85		2	-3.29	**	2
Benin	-5.29		-5.01	*	1	-5.53*	*	1
Kenya	-	-	-	-	-	-	-	-
Senegal	-3.701	***	-2.83		2	-4.08	*	2
Ghana	-4.28	**	-4.52	**	1	-4.41	*	1
Uganda	-2.87		-3.44	***	2	-3.57	*	2
Mauritius	-3.51		-3.47		1	-3.62	**	1
Ethiopia	-5.22	*	-5.06	*	1	-5.69	*	1
Cameroon	-3.87	***	-3.85		1	-5.41	*	1
South Africa	-		-		-	-		-
Eswatini	-2.94	***	-2.60		3	-2.73	**	3
Nigeria	-4.82	*	-4.29	**	1	-4.75	*	1
Botswana	-2.29		-3.61		1	-2.44		1
Lesotho	-2.12		-2.85		2	-2.22		2
Gabon	-3.75	***	-3.91		1	-3.92	*	1

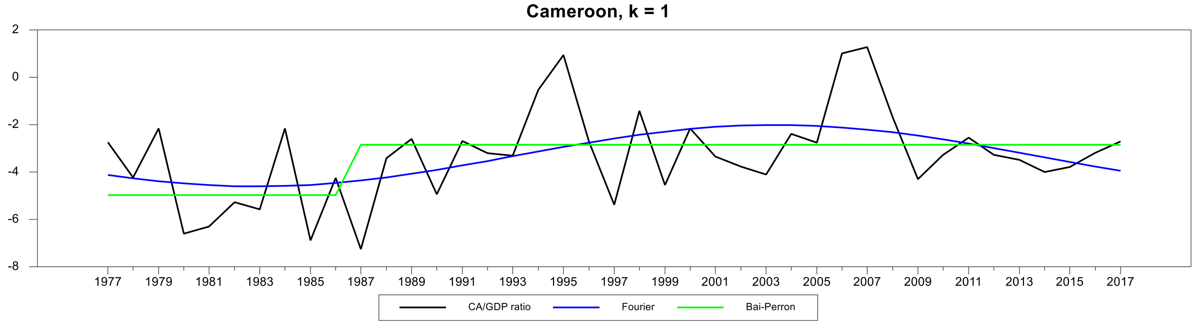
Notes: (1) *, **and *** indicate rejection of the null hypothesis at 1%, 5% and 10% levels, respectively; (2) See Enders and Lee (2012a), Tables 1a and 1b, for theτ_{DF_t} critical values. (3) See Rodrigues and Taylor (2012), Table 2, for the $t_{\emptyset}^{ERS,\mu}$ and $t_{\emptyset}^{ERS,\tau}$ critical values. (4) See Enders and Lee (2012b), Table 1, for τ_{LM} critical values. (5) SBC is used in selecting the optimal lag.

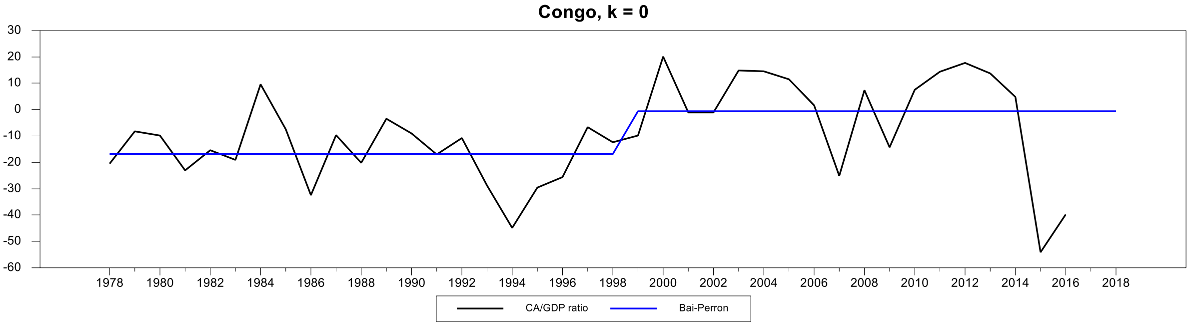
Figure 1: current account balance (as a percentage of GDP), fitted nonlinearities and the intercepts







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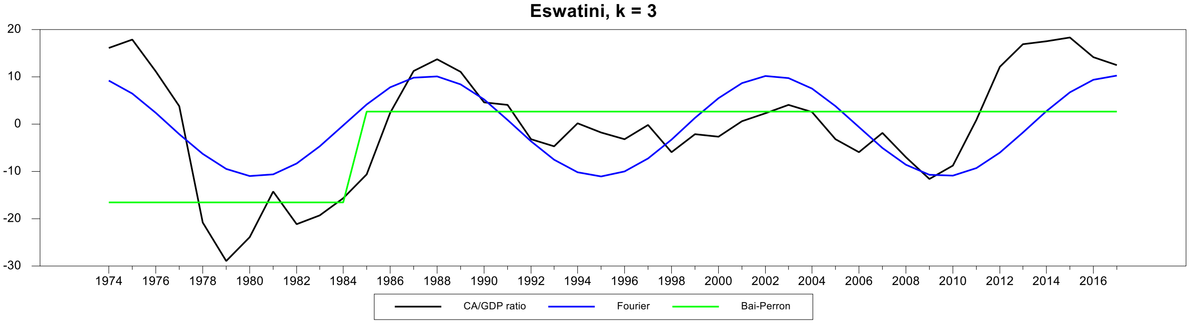
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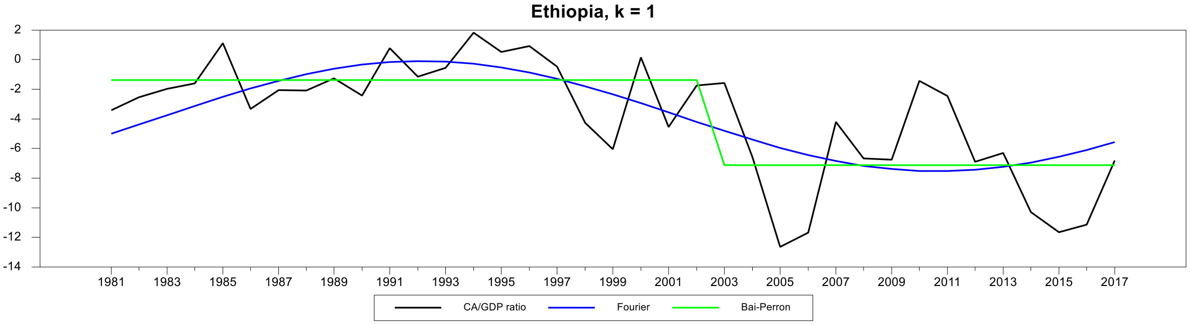
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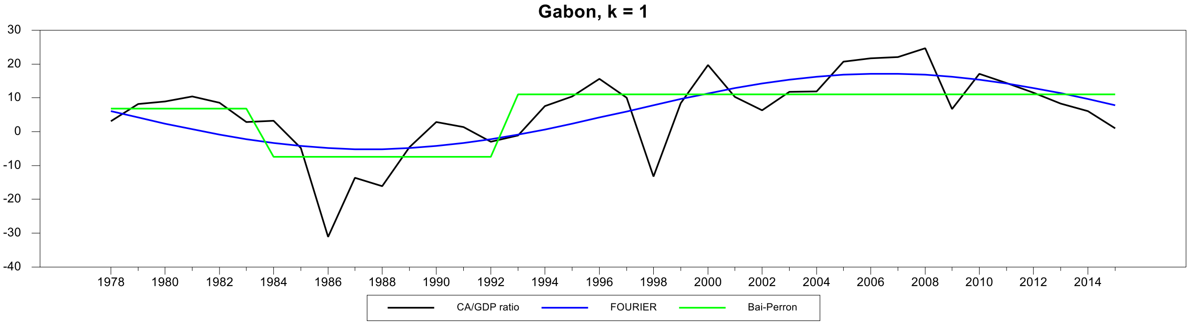
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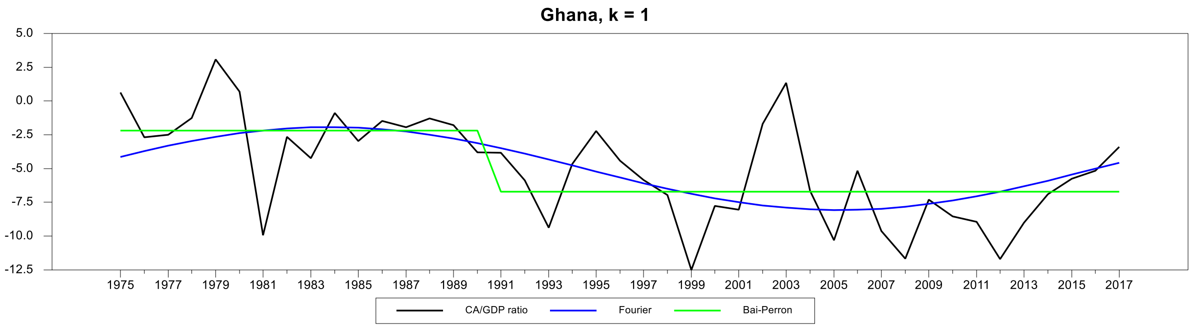
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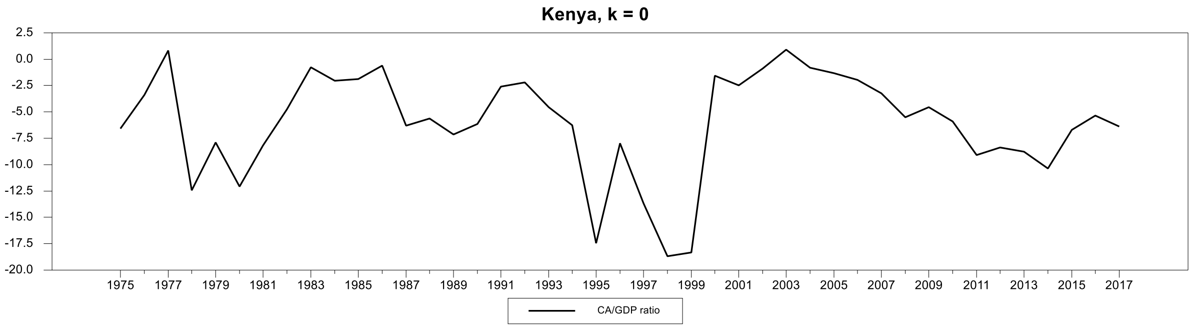
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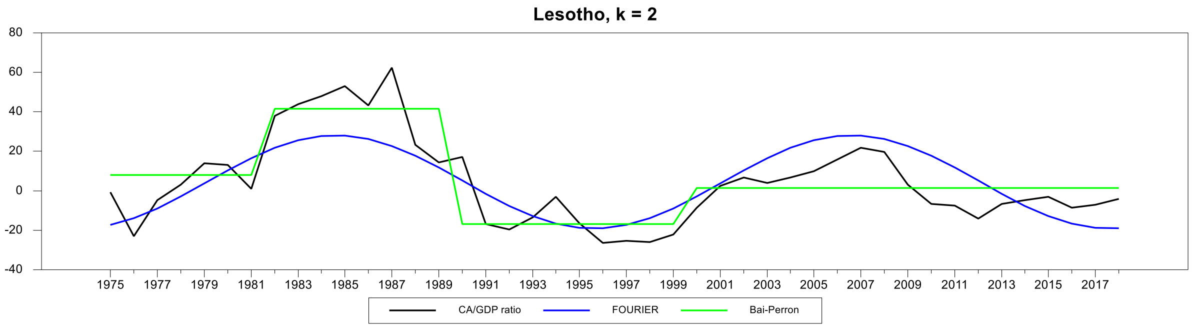
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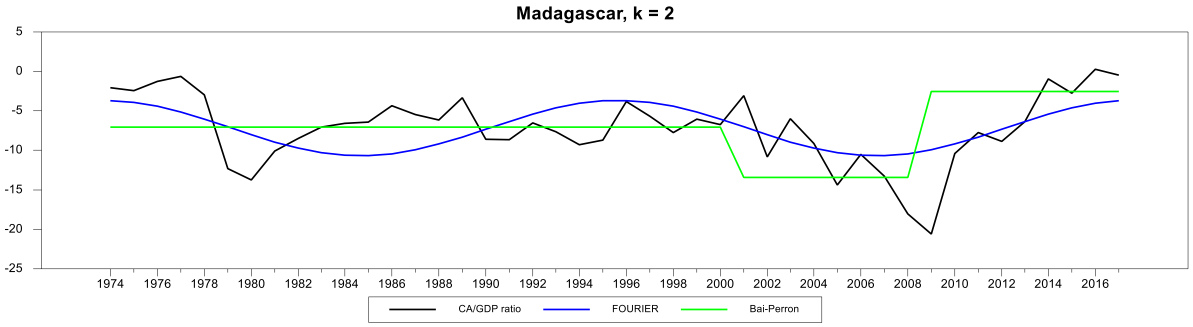
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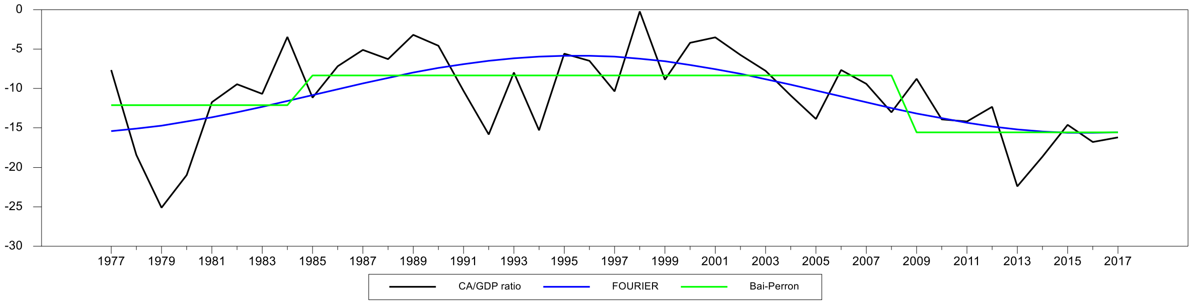

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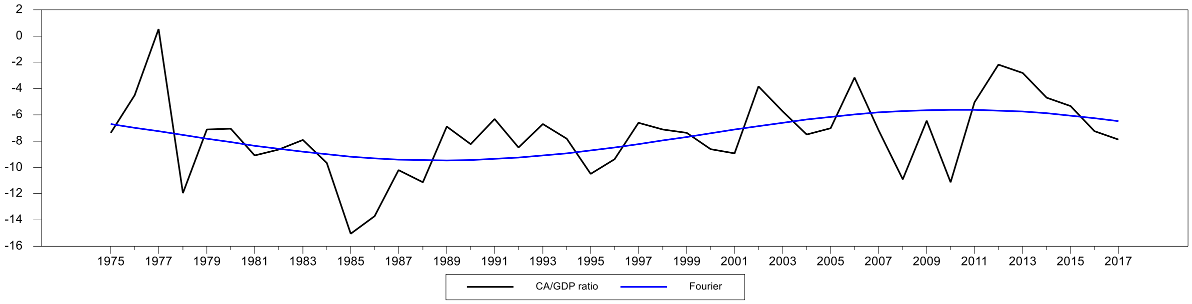

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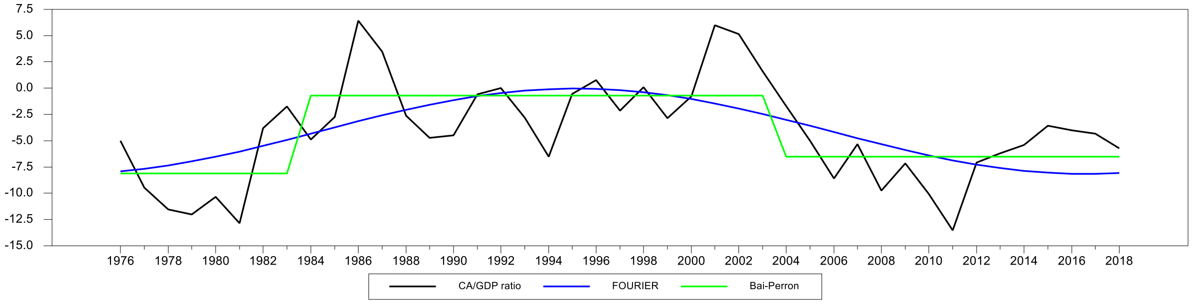

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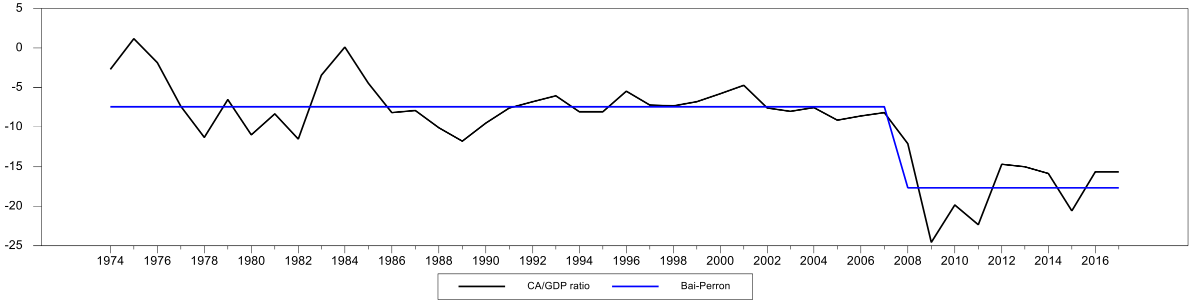

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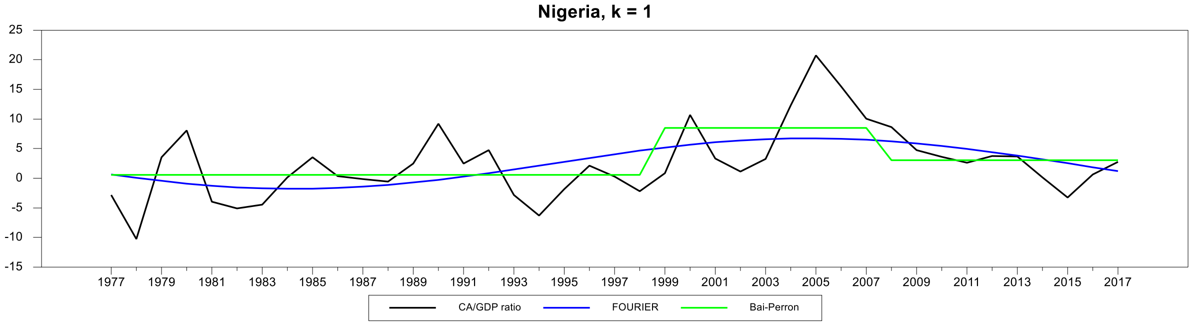
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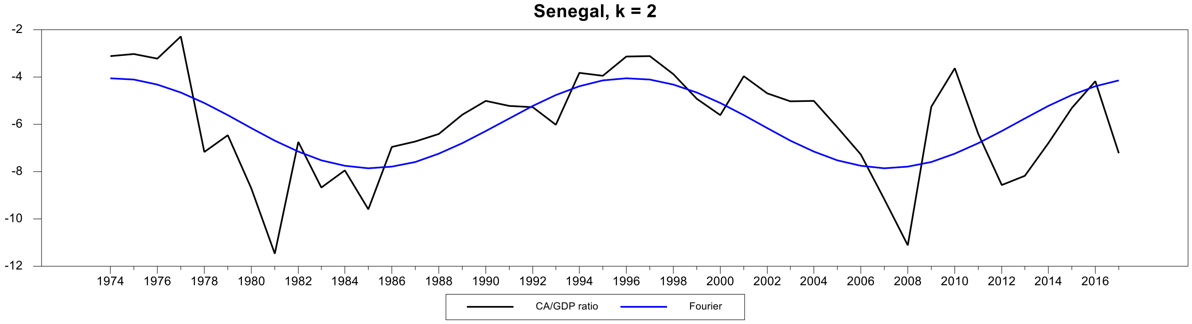
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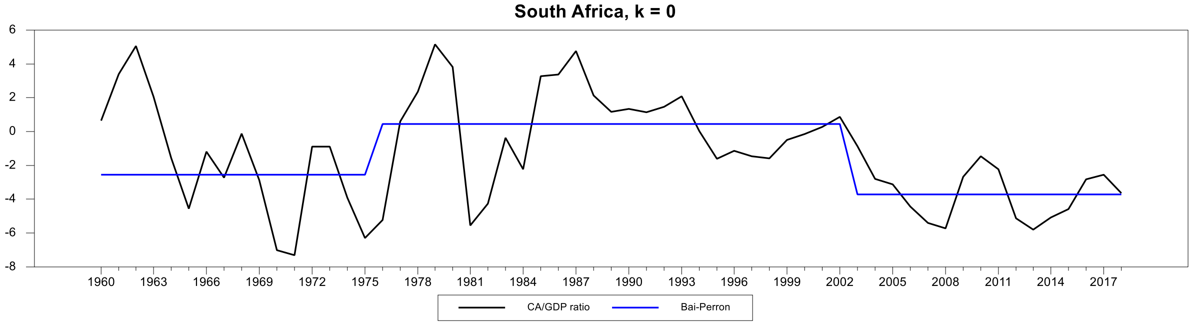
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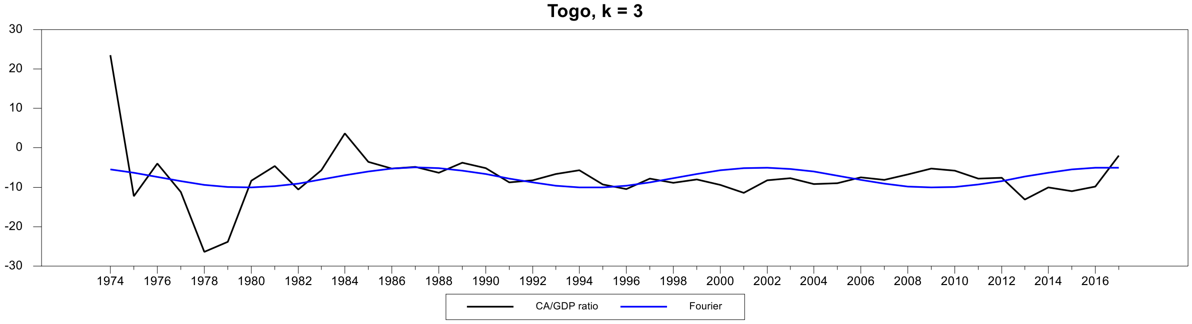
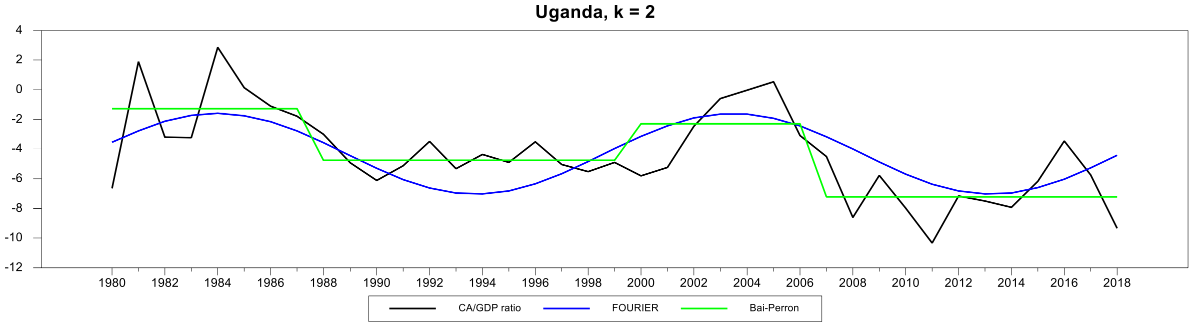

Togo, k = 3


Uganda, k = 2
4
2
0
-2
-4
-6
-8
-10
-12
1980
1982
1984
1986
1988
1990
1992
1994
1996
1998
2000
2002
2004
2006
2008
2010
2012
2014
2016
2018
CA/GDP ratio
FOURIER
Bai-Perron