Proof of the invisible hand: the optimal consumer allocation of time under price dispersion

Malakhov, Sergey

22 April 2020

Sergey MALAKHOV
Ph.D., Applied Economics,1998,
Pierre-Mendès-France University
(Université Grenoble-Alpes), France

Proof of the Invisible Hand:
the optimal consumer allocation of time under price dispersion.

"…most physicists ignored the weird philosophical implications
of the Copenhagen interpretation,
and just used the Schrödinger equation as a tool to do a job,
working out how things like electrons behaved in the quantum world.
Just as a car driver doesn't need to understand
what goes on beneath the bonnet of the car in order to get from A to B,
as long as quantum mechanics worked, you didn't have to understand it…"
John Gribbin,
Introduction to "Q is For Quantum: an encyclopedia of particle physics" ,p.7,
Touchstone Book 2000.

Abstract

The analysis of the static labor-search-leisure model discovers some special optimality properties of the consumer choice under price dispersion. If the consumer is realistic about what he can buy with his efforts, the unit elasticity of his labor and search efforts with respect to consumption reproduces his initial consumption-leisure trade-off; it results in the equality of the marginal loss on search with its marginal benefit and the optimal consumption-leisure choice for any quantity purchased. The inequality of the marginal values of search, described by the satisficing approach, doesn't take place, because this inequality represents the reproduction of the corner prior expectations by the same unit elasticity rule. If the consumer challenges the initial corner solution and start to work and to search, the unit elasticity rule reproduces high prior expectations and the following outcome produces the disappointment. As a result, the consumer either buys satisfactorily as well as optimally under the equality of the marginal values of search, or he quits the market without purchase. In this way the unit elasticity rule provides a powerful illustration of the consistency of consumer's preferences.

The consumer avoids the computational complexity of the marginal analysis because the unit elasticity rule automatically reproduces his realistic prior expectations of how much leisure should be given up for consumption. However, the unit elasticity rule provides the optimality of the total efforts and tells nothing about their distribution between labor and search. But this rule cannot work without the optimal consumer labor-search trade-off. It means that there is some inner market mechanism, which leads the producer along the production possibility frontier to make an offer with respect to the consumer optimal allocation of time. So to the consumer it feels under the unit elasticity rule like the producer is more about being at the right place and the right time with a price, which optimizes the consumer labor-search trade-off and in this way maximizes his consumption-leisure utility function.

Key words: invisible hand, optimal consumption-leisure choice, search, price dispersion, unit elasticity rule

JEL classification: D11, D83.

Introduction

When Kenneth Arrow presented his outstanding analysis of the potentials and limits to the market, he paid attention to the fact, that "…*the view that competitive equilibria have some special optimality properties is at least as old as Adam Smith's invisible hand*…" (Arrow 1985, p.110). The transformation of the classical labor-leisure choice into the labor-search-leisure choice under price dispersion discovers some particular attributes of imperfect markets that provide the optimality of consumer behavior, which is not charged by the computational complexity of the marginal analysis and is based on the common sense.

While this paper represents the result of the long-term study of the optimal consumption-leisure choice under price dispersion (Malakhov 2003-2020), the importance of the topic needs the brief presentation of the labor-search-leisure model, enriched here by the analysis of the corner solution. This analysis discovers the rather common misconception of the potential inequality of the marginal values of search, usually presented as the key attribute of the satisficing approach. In 1978 Herbert Simon wrote:

"*In an optimizing model, the correct point of termination is found by equating the marginal cost of search with the (expected) marginal improvement in the set of alternatives. In a satisficing model, search terminates when the best offer exceeds an aspiration level that itself adjusts gradually to the value of the offers received so far*." (Simon 1978, p.10).

The paper challenges this thesis and argues that from the point of view of the consumption-leisure choice the inequality of the marginal values of search represents the reproduction of the corner solution, when the market cannot provide the consumer with a satisficing purchase, because his aspiration level is based on the unrealistic trade-off between leisure and consumption. However, if prices are rigid or unfair, the former corner solution disappears; the arbitrage starts and the consumer again optimizes his choice, now on the new price level. In this way the equality of the marginal values of search always follows any consumer decision for the given time horizon that confirms the other Simon's assumption that satisficing procedure can also be optimal (Simon 1972). The commonsense idea how much leisure time should be given up to buy the quantity demanded results in the reasoning "it's enough to search", but this reasoning makes the purchase satisficing as well as optimal.

The general presentation of the labor-search-leisure model

The key variable of the labor-search-leisure model is the individual willingness to substitute the labor income by the search income, derived from the price dispersion. This individual behavioral attribute can be presented as the ***propensity to search*** *∂L/∂S<0* and described with the help of the Archimedes' principle. Indeed, the search *S* displaces both labor *L* and leisure time *H* in the given time horizon *T* like the ice squeezes both whiskey and soda from the glass:

$$L + S + H = T; \quad (1.1)$$

$$(-\partial L / \partial S) + (-\partial H / \partial S) = 1; \quad (1.2)$$

$$dH(S) = dS\frac{\partial H}{\partial S} = -dS\frac{H}{T}; \rightarrow \frac{\partial H}{\partial S} = -\frac{H}{T}; \quad (1.3)$$

$$\frac{\partial L}{\partial S} = \frac{H-T}{T} = -\frac{L+S}{T} \quad (1.4)$$

$$\frac{L+S}{T} + \frac{H}{T} = 1 \quad (1.5)$$

If we multiply the propensity to search *∂L/∂S* by the wage rate *w*, we get the value of the marginal loss of monetary labor income during the search *w∂L/∂S<0*. According to the famous George Stigler's rule (Stigler 1961), we can equalize it with the marginal benefit of the search *Q∂P/∂S<0*, where quantity demanded *Q* is given and the price of purchase depends on search *P(S)*.[1] This behavioral explicit rule can be used as the constraint to some utility function *U(Q,H)*, where the quantity to be purchased *Q* becomes the variable value and the value of the marginal benefit per unit of purchase *∂P/∂S<0* becomes constant value because it is given by the place of purchase:

[1] G.Stigler in his paper paid attention to the unimportance of interest rate for the marginal search (p.219) (S.M)

$$\max U(Q,H)\ subject\ to\ w\frac{\partial L}{\partial S}=Q\frac{\partial P}{\partial S} \quad (2.1)$$

$$\Lambda = U(Q,H)+\lambda(w-\partial P/\partial S\frac{Q}{\partial L/\partial S}) \quad (2.2)$$

$$\frac{\partial U}{\partial Q}=\lambda\frac{\partial P/\partial S}{\partial L/\partial S} \quad (2.3)$$

$$\frac{\partial U}{\partial H}=-Q\frac{\partial P/\partial S}{(\partial L/\partial S)^2}\partial^2 L/\partial S\partial H=-\frac{w}{\partial L/\partial S}\partial^2 L/\partial S\partial H \quad (2.4)$$

$$MRS(H\ for Q)=-\frac{w}{\partial P/\partial S}\partial^2 L/\partial S\partial H \quad (2.5)$$

$$\partial^2 L/\partial S\partial H=\frac{\partial(H-T/T)}{\partial H}=1/T \quad (2.6)$$

$$MRS(H\ for Q)=-\frac{w}{T\partial P/\partial S}=-\frac{Q}{T\partial L/\partial S}=\frac{QT}{T(L+S)}=\frac{Q}{L+S} \quad (2.7)$$

$$MRS(H\ for Q)=\frac{Q}{L+S}\frac{H/T}{H/T}=\frac{Q}{H}\frac{(-\partial H/\partial S)}{(-\partial L/\partial S)} \quad (2.8)$$

$$U(Q,H)=Q^{-\partial L/\partial S}H^{-\partial H/\partial S} \quad (2.9)$$

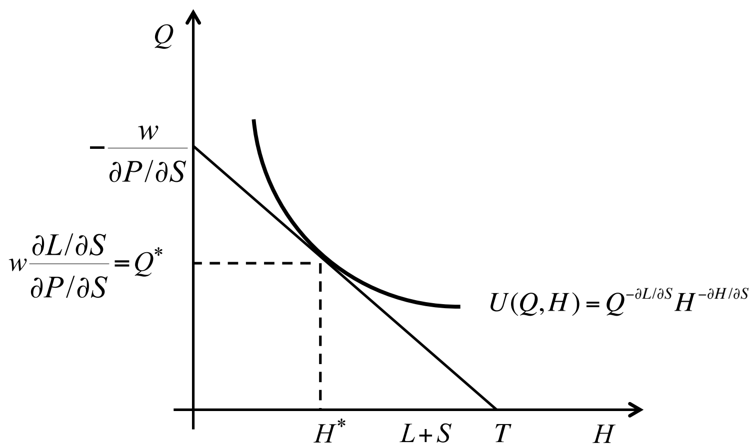

Fig.1. Implicit consumption-leisure choice under the search

While at the optimum level this implicit solution should match the explicit behavioral constraint, we can present this explicit choice of the fixed quantity demanded, now with the help of the sets of equations (1) and (2) (Figure 2):

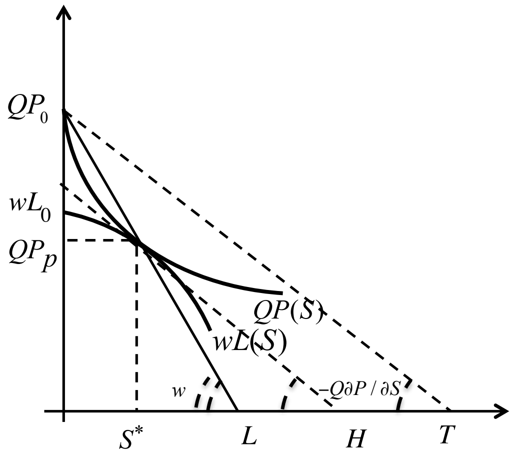

Fig.2.Explicit choice of the pre-determined quantity to be purchased

The individual reserves the labor income wL_0 for the given time horizon T until the next purchase; he starts to search and finds the price $QP_P<wL_0$. It is easy to derive the shape of the labor income $wL(S)$ from the equation (1.4) for the value of the propensity to search $\partial L/\partial S>-1$ as $\partial^2L/\partial S^2<0$[2]. The $Q\partial^2P/\partial S^2>0$ shape of the search income can be confirmed by the refutation of the so-called paradox of the little pre-purchase search for big ticket items (Kapteyn et al. 1979, Thaler 1980, 1987, Grewal and Marmorstein 1994), but here it is enough simply to assume the diminishing efficiency of search. The fact of the purchase means the intersection of the labor income curve $wL(S)$ with some $QP(S)$ curve. Indeed, the individual doesn't know the actual $QP(S)$ curve. He simply reproduces it by the trial and error method.

However, at the moment of purchase some $Q\partial P/\partial S$ value exists as well as the $w\partial L/\partial S$ value. And we can help to this individual, who is not worried about the mathematical efficiency of his purchase. We can take the total of his efforts $w(L+S)$ and get some QP_0 value at the zero search level. And if we divide it by the value of the time horizon T, we will get the same value of the marginal savings on purchase $Q\partial P/\partial S$. It means, that the moment of purchase doesn't represent the intersection of $wL(S)$ and $QP(S)$ curves, but their touching. The $wL(S)$ and $QP(S)$ curves become tangent:

$$Q\frac{\partial P}{\partial S}=w\frac{\partial L}{\partial S}=-w\frac{L+S}{T} \quad (3.1)$$

$$w(L+S)=-QT\frac{\partial P}{\partial S}=QP_0 \quad (3.2)$$

$$MRS(H\,for\,Q)=-\frac{w}{\partial P/\partial S}\partial^2L/\partial S\partial H=-\frac{w}{T\partial P/\partial S}=\frac{w}{P_0}=\frac{Q}{L+S} \quad (3.3)$$

[2] The value $\partial^2L/\partial S^2=-\partial(L+S)/T/\partial S=-(\partial L/\partial S+1)/T>-1$. The value $\partial L/\partial S<-1$ goes beyond the time horizon and produces «the leisure model» of behavior $(\partial Q/\partial H>0)$, that has been presented by the analysis of the service augmenting technical progress in Malakhov (2020).

We see that for any purchase QP_P there is some P_0 value at the zero search level that equalizes the marginal savings on purchase $w\partial L/dS$ with its marginal benefit $Q\partial P/\partial S$ for the given time horizon T. And this value is not virtual. If the search discovers some offer QP_P, there should be a corresponding offer QP_0 "at the door", i.e., at the zero search level. This value is not mute because it describes the market optics of the individual trade-of between leisure and consumption $w/P_0=Q(L+S)$.

The equilibrium price dispersion

The concept of the zero search level significantly reduces the price dispersion. If there is no internal hidden side effect in quality and terms of delivery, and if there is no external side effect, like consumers' brand loyalty, the zero search level should exhibit the lowest willingness to pay of *shoppers*, consumers with zero search costs.

The price dispersion exhibits the consumer's heterogeneity (Stahl 1989). Indeed, the P_0 value represents the price paid by *shoppers*, consumers with zero search costs with respect to the purchase prices P_P, paid by *searchers*, consumers with positive search costs. It means that if the *searcher* wants to resell the bought item to the *shopper*, he will do it at the QP_0 level that collects all his costs and is equal to his willingness to sell or to accept *WTA*. This level represents the equilibrium price because when it matches the lowest *WTP* of *shoppers* with *WTA* of *searchers*, it also equalizes the marginal costs on purchase with its average costs:

$$MRS(H\ for Q) = \frac{Q}{L+S} = \frac{w}{P_0} \Rightarrow P_0 = \frac{w(L+S)}{Q} = AC \quad (4.1)$$

$$MC = \frac{\partial w(L+S)}{\partial Q} = \frac{\partial QP_0}{\partial Q} = P_0 \quad (4.2)$$

$$P_0 = AC = MC = P_e \quad (4.3)$$

The equilibrium price produces the equilibrium price dispersion, where different purchase prices $wL=QP_P$ correspond to different wages rates and time horizons with its particular allocation between labor, search, and leisure. But the equilibrium price holds for any allocation of time (Figure 3):

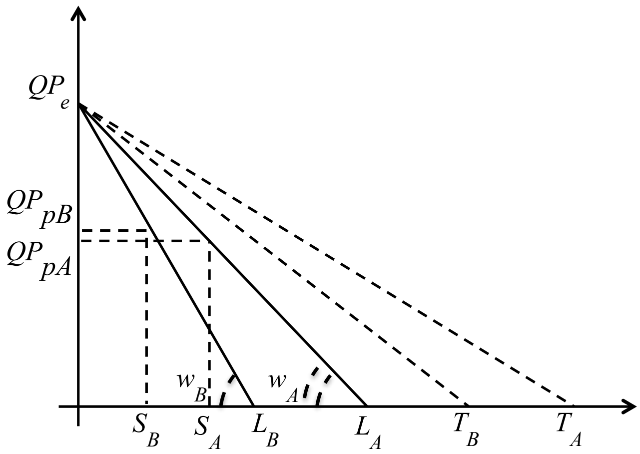

Fig.3. The equilibrium price dispersion

Moreover, it gives an opportunity to sellers to discriminate consumers with respect to their labor income *w* along the production possibility frontier (Figure 4):

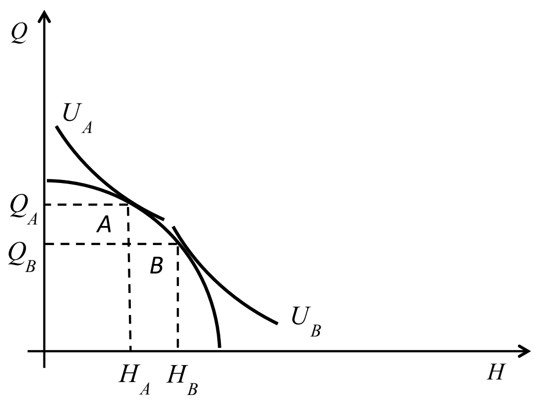

Fig.4. The production possibility frontier under equilibrium price dispersion

The production possibility frontier *PPF* represents the production itself Q with complementary services, which increase consumers' leisure time H. The producer can make more but cheap consumption units Q_A with minimum services for consumers with low wage rate w_A, who pay less but search more, or less but expensive consumption units Q_B with maximum services for consumers with high wage rate w_B, who pay more but search less.

The idea of $(Q;H)$ production possibility frontier is very important. It describes the level of technology for any quantity to be purchased. The labor augmenting technological progress can shift the *PPF*; it reduces the marginal costs of production and makes the quantity demanded Q^* cheaper. The decrease in the consumer's labor time L raises his leisure time from H_0 to H_1 , where the $|\partial P/\partial S|$ value follows the decrease in the purchase price QP_P and exhibits the growth of the consumer's utility (Figure 5):

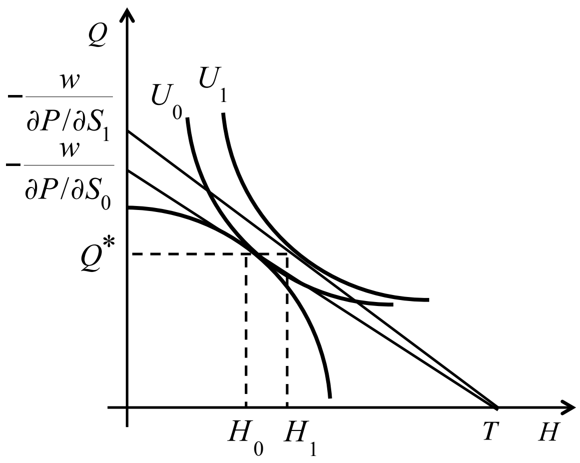

Fig.5. The labor augmenting technological progress

Marginal utilities and corner solutions

If we follow the set of equations (3) in the reverse order, now with respect to the equilibrium price $P_0=P_e$, we can see that the equality of the marginal values of search (3.1) is automatically generated by the equality of the rate w/P_e, at which leisure H can be traded for consumption Q in the market, to the consumer's individual psychic trade-off $Q/(L+S)$ (3.3).

This consideration is very important. It shows that the equality of marginal values of search appears regardless the utility function. From the beginning it emerges from some individual consumption preferences with regard to leisure time to be given up with the purchase and the market real wage rate, or the individual purchasing power. Both these values don't need specific mathematical ability. Moreover, the following utility maximization also looks like an automatic process, which is not charged by the computational complexity.

However, the equality of the marginal values of search itself is not the indisputable concept. If we try to describe the satisficing approach by means of the labor-search-leisure model, we get the following conclusion:

$$w\frac{L+S}{T} < -Q\frac{\partial P}{\partial S}; -w\frac{L+S}{T} > Q\frac{\partial P}{\partial S}; \left|-w\frac{L+S}{T}\right| < \left|Q\frac{\partial P}{\partial S}\right| \quad (5)$$

or the marginal loss of the satisficing decision is less that its marginal benefit because the consumer cuts search efforts.

However, the set of equations (3) gives an idea that if the marginal values of search are not equal, the market and individual trade-off of leisure for consumption also are not equal. This inequality is known as a corner solution. Thus, we need to describe the corner solution for the labor-search-leisure model. But here we cannot challenge the basic principles of the classical labor-leisure choice.

First, we need to prove the identity of marginal utility of both consumption and leisure under the classical labor-leisure choice and the choice on imperfect market under the search that can be done with the help of the methodology for the analysis of the Lagrangian multiplier, proposed once by American mathematicians J.V.Baxley and J.C.Moorhouse (Baxley and Moorhouse 1984, Malakhov 2015):

$$labor-leisure\ choice:$$

$$\lambda = \frac{MU_w}{T-H} \quad (6.1);$$

$$MU_Q = \lambda P = MU_w \frac{P}{T-H} \quad (6.2);$$

$$MU_H = \lambda w = MU_w \frac{w}{T-H} \quad (6.3);$$

$$labor-search-leisure\ choice:$$

$$\lambda = MU_w \quad (6.4);$$

$$MU_Q = \lambda \frac{\partial P / \partial S}{\partial L / \partial S} = -MU_w \frac{T \partial P / \partial S}{L+S} = MU_w \frac{P_e}{T-H} \quad (6.5);$$

$$MU_H = \lambda w = -MU_w \frac{w}{\partial L / \partial S} \partial^2 L / \partial S \partial H = MU_w \frac{wT}{L+S} \frac{1}{T} = MU_w \frac{w}{T-H} \quad (6.6).$$

After the identity of marginal utilities is confirmed, we can proceed to the comparative analysis of the corner solution:

$$labor-leisure\ choice: \lambda = \frac{MU_w}{T-H}; MU_Q = \lambda P; MU_H = \lambda w \ (7.1)$$

$$corner\ solution: P > \frac{MU_Q}{\lambda}; \frac{P}{\lambda w} > \frac{MU_Q}{\lambda MU_H}; \frac{MU_H}{MU_Q} > \frac{w}{P} (7.2);$$

$$labor-search-leisure\ choice: \lambda = w; MU_Q = \lambda \frac{P_e}{T-H}; MU_H = \lambda \frac{w}{T-H} (7.3);$$

$$corner\ solution: \frac{MU_Q}{\lambda} < \frac{P_e}{T-H} (7.4);$$

$$\frac{MU_Q}{\lambda} < \frac{P_e}{T-H}; \frac{MU_Q}{MU_H} < \frac{\lambda}{MU_H} \frac{P_e}{T-H} = \frac{\lambda (T-H)}{\lambda w} \frac{P_e}{T-H} = \frac{P_e}{w} (7.5);$$

$$\frac{MU_Q}{MU_H} < \frac{P_e}{w} \Rightarrow \frac{MU_H}{MU_Q} = \frac{Q}{L+S} > \frac{w}{P_e} (7.6)$$

We see that like in the labor-leisure model, the corner solution under the search occurs when the rate w/P_e, at which leisure H can be traded for consumption Q in the market is lower than the consumer's psychic trade-off, or $w/P_e < MRS\ (H\ for\ Q) = Q/(L+S)$.

The corner solution reduces the options of both labor and search. It is clear that the consumer will not purchase an item that he believes is not worth the efforts on labor and search and the level of consumption stays equal to zero.

While the corner solution works as the attribute of the utility theory, it represents the psychological phenomenon, because the equation (7.6) is easily transformed into the consideration that the expected consumption level is greater than the actual one or $Q_{expected}>Q_{actual}$. If a consumer is unaware of the implicit corner solution or he challenges it and decides to search an item, he simply forms a prior optimistic expectation on the purchase but the outcome is worse than expected. And the consumer experiences an emotion, which is called as the disappointment. Coming back to the utility theory, we can say that the corner solution $Q/(L+S)>w/P_e$ means that the disappointment appears because the consumer has simply overestimated the efficiency of his efforts or his ***purchasing power***.

The last consideration directs us to the more profound analysis of the prior expectations at the moment when the intention to buy is formed.

The moment of the intention to buy

At the moment of the intention to buy, when the consumer's wallet is almost empty and his supplies have also run desperately low $(L\to 0;Q\to 0)$, as well as he has no actual information about the price dispersion $(S\to 0)$, he needs to work and to search the quantity demanded if he doesn't want to stay in the following time period with plenty of leisure time ($H\to T$) and empty fridge. At this moment his psychic trade-off of leisure for consumption $MRS\ (H\ for\ Q)=Q/(L+S)$ takes the indeterminate form of $0/0$. However, this is not the corner solution because the consumer doesn't prefer to get T hours of leisure and zero consumption. He really wants to reduce leisure in favor of labor and search in order to buy. Both the consumption Q and the total efforts $(L+S)$ represent the functions of leisure time, or $Q(H)$ and $(L+S)(H)$, that justifies the use of the l'Hôpital's rule for the given time horizon $T=L+S+H$ where $H\to T$:

$$\lim_{H\to T} Q(H) = \lim_{H\to T}(L+S)(H) = 0; \partial(L+S)|_{Tconst} / \partial H = -1 \quad (8.1);$$

$$\lim_{H\to T} \frac{\partial Q/\partial H}{\partial(L+S)/\partial H} = -\frac{\partial Q}{\partial H} = \lim_{H\to T}\frac{Q}{L+S} \quad (8.2)$$

We see that the prior psychic trade-off of leisure for consumption or $MRS\ (H\ for\ Q)=Q/(L+S)=(-\partial Q/\partial H)$ really exists before the consumer start to work and to search the quantity demanded. However, this prior psychic trade-off can be different (Figure 6):

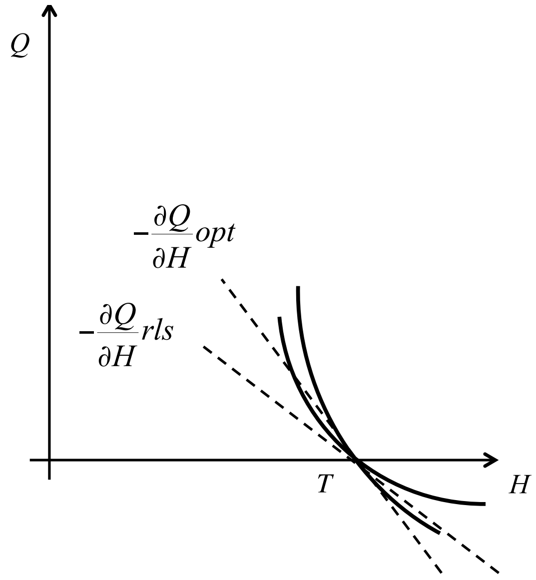

Fig.6. Initial psychic trade-offs of leisure for consumption

From the beginning (*H=T;Q;L;S=0*) the prior preferences are not monetary. They simply exhibit the feelings how much leisure *(T-H)=L+S* should be given up for the purchase *Q*, or at the margin *Q/(L+S) = -dQ/dH*. There are two scenarios – the realistic and the optimistic one. According to the equation (8.2) the realistic expectations result in the realistic trade-off of leisure for consumption $(-\partial Q/\partial H)_{rls}=Q/(L+S)_{rls}$. The optimistic expectations are based on the reasoning that the quantity demanded *Q* is not worth such efforts, but only $(L+S)_{opt}<(L+S)_{rls}$ efforts with respect to the initial psychic trade-off of leisure for consumption $(-\partial Q/\partial H)_{opt}=Q/(L+S)_{opt}$. We can ask the question whether these initial expectations stay constant, when the consumer starts to work and to search. Let's assume that the time horizon until next purchase doesn't depend on the quantity demanded for this current time period, or *T≠T(Q)*. This assumption looks rather strong but it can be accepted for some relevant range of consumption, for example, when we buy one or three bears for today and don't leave the stock in the fridge for tomorrow.

This assumption gives us the proof that the consumer's preferences are consistent:

$$e_{(L+S),Q}=\frac{\partial(L+S)}{\partial Q}\frac{Q}{L+S}\bigg|_{Tconst;Q_0;L_0;S_0=0}=\frac{\partial(T-H)}{\partial Q}\frac{0}{0}=\left(-\frac{\partial H}{\partial Q}\right)\left(-\frac{\partial Q}{\partial H}\right)=1 \quad (9)$$

The total efforts (*L+S*) on purchase are unit elastic with respect to any quantity demanded *Q* for the given time horizon *T*. It means that the value *Q/(L+S)* stays constant for any purchase.

We see that the equation (9) holds regardless the monetary values of the market. And it doesn't need complex computations. The equation (9), let's call it ***the unit elasticity rule***, works automatically with respect to some commonsense reasoning how much leisure the consumption is worth.

The next question is very simple – does the market support this automatic process or not. And it does, if it has the price, independent on preferences of *searchers*, consumers with positive search costs, because it is given to them by the lowest willingness to pay of *shoppers*, consumers with zero search costs, for the unit of consumption "at the door". This is the equilibrium price:

$$MC = \frac{\partial w(L+S)}{\partial Q} = AC = \frac{w(L+S)}{Q} \Rightarrow e_{(L+S),Q} = \frac{\partial w(L+S)}{\partial Q}\frac{Q}{w(L+S)} = \frac{MC}{AC} = 1 \quad (10.1);$$

$$e_{w(L+S),Q} = 1; w(L+S) = QP_e \rightarrow e_{w(L+S),Q} = e_{QP_e,Q} = 1 + e_{Pe,Q} \Rightarrow e_{P_e,Q} = 0 \quad (10.2)$$

However, the sets of equations (3), (4), and (10) tell us that the simply unit elasticity rule states the fact that ***any purchase for the given time horizon is optimal***, because it equalizes for any quantity demanded the marginal values of search and maximizes the consumption-leisure utility with respect to the equality of these marginal values.[3]

Now we can come back to the production possibility frontier (Figure 5) and the optimistic consumption-leisure trade-off (Figure 6). If there is no producer with advanced technology and the production possibility frontier is stable, any optimistic consumption-leisure trade-off results in the consumer's disappointment. He quits the market without purchase because his prior expectations have represented the corner solution, and the purchase will mean the loss in the utility with respect to these prior expectations (Figure 7):

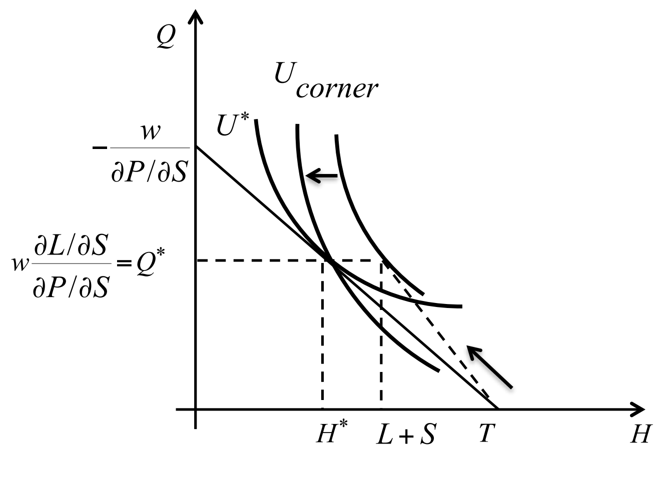

Fig.7. The *ex post* corner solution

If a man is realistic about what he can buy with his efforts, his prior expectations or the initial trade-off of leisure for consumption will be equal to the real wage rate w/P_e. Once it is

[3] This conclusion illustrates the idea why the consumption-leisure utility function doesn't need money. The unit elasticity rule tells us that the purchase of any quantity under another constraint, either of money in utility function, or interest rate for future consumption, also stays optimal. Any additional constraint simply results in some particular choice within the set of optimal consumption-leisure solutions (S.M.)

determined, the unit elasticity rule moves the consumer to the optimal choice for any level of consumption.

The equation (9) tells us that the efforts' spending doesn't depend on the consumption-leisure trade-off itself. If the consumer is unaware of the corner solution or he challenges it, the unit elasticity rule doesn't mind. The consumer simply evaluates his real wage rate or his purchasing power and gives himself up to the unit elasticity rule, which mechanically reproduces his feelings about his purchasing power for any level of consumption. If he makes a mistake, this is a sad thing, but it is his choice.

However, Figure 5 also describes the other scenario. If the optimistic consumption-leisure trade-off results in the purchase, it means that the consumer has got a chance to meet a producer with some advanced technology, which moves his offer outside the production possibility frontier. But it simply means that the equilibrium price level is ether rigid or unfair. And once the low price P_P appears under the search, it should generate some new P_0 offer at the zero search level for *shoppers*. The *searchers-shoppers* relationship is mutually advantageous. The *shoppers* set the equilibrium price level, but the *searchers* check its reliability. And if the equilibrium price level is not correct, the arbitrage process starts, and it shifts the production possibility frontier.

The last consideration is very important because it describes the advantageous purchase not only with regard to the price level, but also with regard to the process of search. If the consumer easily finds the interesting offer, it means that the seller cures the customer's headache and enables his particular manner the search. But if the seller can make his goods more accessible to one consumer, why he cannot make them absolutely accessible for many consumers with zero search costs? And he starts to sell "at the door" on the new price level P_0.

There is another case that could fuel the skepticism of proponents of the satisficing approach. What happens if the consumer quickly finds the unexpectedly low price? In this way the absolute value of the price reduction $|\partial P/\partial S|$ is great, and it should establish the inequality of the marginal values of search (5). The labor-search-leisure model challenges this view. There are should be reasons for the seller to cut the price – huge inventories, short shelf life, etc. There are also might be hidden defects like it happens with "lemons". The labor-search-leisure model summarizes all evident and hidden defects by the assumption of the short time horizon of the use of the bought item. And, if we come back to the inequality of the marginal values (5) with this assumption, we can see that the short time horizon re-establish their equality.

All these considerations tell us that ***any purchase results in the equality of the marginal values of search***. The inequality corresponds to the corner solution. The consumer either doesn't start to work and to search at the beginning, or he challenges the prior corner solution and tries to make the purchase, but he quits the market without it. However, coming back to our lucky consumer, we can see that the former corner solution can disappear, and his purchase again becomes optimal, now on the new price level:

:

$$w\frac{\partial L}{\partial S} = -w\frac{L+S}{T} > Q\frac{\partial P}{\partial S}; \left|-w\frac{L+S}{T}\right| < \left|Q\frac{\partial P}{\partial S}\right| \quad (11.1)$$

$$w(L+S) < -TQ\frac{\partial P}{\partial S} = QP_e \quad (11.2)$$

$$\frac{w}{P_e} < \frac{Q}{L+S} = -\frac{dQ}{dH} = \frac{MU_H}{MU_Q} \quad (11.3)$$

$$P_0 < P_e; \frac{w}{P_0} = \frac{Q}{L+S}; w(L+S) = -TQ\frac{\partial P}{\partial S} = QP_0 \quad (11.4)$$

$$w\frac{\partial L}{\partial S} = -w\frac{L+S}{T} = Q\frac{\partial P}{\partial S} \quad (11.5)$$

It looks like the commonsense reasoning "it's enough to search" exhibits not only the explicit satisficing procedure, but also the implicit optimal one. The marginal values of search as well as the maximum of the utility function are not calculated. The decision-making is limited by the allocation of time with respect to quantity demanded that consumers make every time when they are making purchases for some time horizon.

The unit elasticity rule holds itself regardless the problem of the optimization of utility. It seems that this rule works like a law of nature. However, the unit elasticity rule tells nothing about the allocation of time between the labor *L* and the search *S*. But while it equalizes marginal loss on the search with its marginal benefit at any level of consumption, it means that consumer's efforts are divided between labor and search optimally for any level of consumption. It looks like the consumer makes intuitive decisions or he is led by some invisible clues $\partial L/\partial Q$ and $\partial S/\partial Q$ how much to spend and to search.[4] The only thing he definitely knows about the imperfect market that the increase in quantity demanded should cut the purchase price, but the following implicit relationships $\partial L/\partial Q>0;\partial^2 L/\partial Q^2<0$ and $\partial S/\partial Q>0;\partial^2 S/\partial Q^2>0$ don't change the logic of the unit elasticity rule:

$$T_{const} = L+S+H; -\partial L/\partial H - \partial S/\partial H = 1 \quad (12.1)$$

$$e_{(L+S),Q} = \frac{\partial(L+S)}{\partial Q}\frac{Q}{L+S} = \left(\frac{\partial L}{\partial Q}+\frac{\partial S}{\partial Q}\right)\left(-\frac{\partial Q}{\partial H}\right) = -\frac{\partial L}{\partial Q}\frac{\partial Q}{\partial H} - \frac{\partial S}{\partial Q}\frac{\partial Q}{\partial H} = 1 \quad (12.2)$$

It means that the market really has some hidden mechanism that provides the optimality of exchange under the two-sided heterogeneity of its actors. This optimality is confirmed by

[4] While $Q \neq Q(S)$, the inverse relationship $S=S(Q)$ exists (S.M.)

successful purchases under the unit elasticity rule. The consumer allocation of time also becomes optimal but it happens only *ex post*, i.e., after the purchase, while *ex ante* the consumer doesn't definitely know in what manner the leisure he is ready to give up, will be allocated between labor and search. But the market helps him, and the consumer certainly finds the price, which allocates his time optimally.

When Adam Smith described the economic behavior of a man, he used the metaphor of the invisible hand, which led self-interested producers to meet the wishes of consumers for their common social benefit (Smith, The Wealth of Nations, Book IV, Chapter II, p.456, para.9). So to the consumer it feels under the unit elasticity rule like the producer is more about being at the right place and the right time.

Conclusion

The static nature of the labor-search-leisure model provides the powerful illustration of the basic principles of exchange on imperfect markets under price dispersion. It presents the problem of the optimization like the automatic process, which subordinates the individual buying behavior and pre-purchase intentions. If a man doesn't loose control of his appetites and if he recognizes well his own capacities, his purchasing decisions should be optimal without any computational complexity. The unit elasticity rule easily paves his way under the uncertainty of price dispersion, where the knowledge of both sides of the market is limited. It confirms the assumption of the market self-organization, brilliantly illustrated once by Kenneth Arrow:

"*The notion of the inner coherence of the economy – the way markets and the pursuit of self-interest could in principle achieve a major degree of coordination without any explicit exchange of information, but where the results may diverge significantly from those intended by the individual actors – is surely the most important intellectual contribution that economic thought has made to the general understanding of social processes*." (Arrow, op.cit., p.108)

References

Arrow, K. (1985) 'The Potentials and Limits to the Market in Resource Allocation', 107-124, *'Issues in Contemporary Economics'*, George R.Feiwel (eds.), Springer, 510 p.

Baxley J.V, Moorhouse J.C. (1984) 'Lagrange Multiplier Problems in Economics' *American Mathematical Monthly*, 91, (7), 404–412.

Grewal D., Marmorstein, H. (1994) 'Market Price Variation, Perceived Price Variation, and Consumers' Price Search Decisions for Durable Goods.' *Journal of Consumer Research*, 21, (3), 453-460

Kapteyn, A., Wansbeek, T., Buyze, J. (1979) 'Maximizing or Satisficing', *The Review of Economics and Statistics*, 61, (4), 549-563.

Malakhov, S. (2003) 'Transaction Costs, Economic Growth and Labor Supply',*VOPROSY ECONOMIKI,* 9,49-61.

Malakhov, S. (2011). 'Towards a New Synthesis of Neoclassical Paradigm and Search – Satisficing Concept.' *IAREP/SABE Annual Conference*, Exeter, UK.

Malakhov, S. (2015) 'Propensity to Search: common, leisure, and labor models of behavior', *Expert Journal of Economics,* 3(1),63-76 (http://economics.expertjournals.com/wp-content/uploads/EJE_308malakhov63-76.pdf)

Malakhov, S. (2016a) 'Search and Home Production: do empirical studies of happiness overlook relative overconsumption?' *Paper presented at the HEIRS International Conference*, Rome 2016, March 15-16.

Malakhov, S. (2016b) 'Law of One Price and Optimal Consumption-Leisure Choice Under Price Dispersion', *Expert Journal of Economics*,4(1),1-8.

Malakhov, S. (2019) 'Willingness to Take Care of Good Cars: from the theorem of lemons to the Coase theorem', *Paper presented at the 68th AFSE Congress*, Orléans 2019, June 17-19.

Malakhov, S. (2020a) 'Consumers' Diligence as a Natural Way to Copy with Externalities: from the Coase theorem to the invisible hand' *Journal of Institutional Studies*, 12(1), 38-65

Malakhov, S. (2020b) 'Deriving Utility: consumers' diligence under externalities and technical progress,' *Paper presented at the 69th AFSE Congress*, Lille 2020, June 10-12. (https://works.bepress.com/sergey_malakhov/21/)

Simon, H. (1972). 'Theories of Bounded Rationality. In: Decision and Organization.' C.B.Mcguire and Roy Radner (eds.), Nort-Holland Publishing Company, 161 – 176.

Simon, H. (1978). 'Rationality as Process and as Product of Thought.' *American Economic Review*, 68(2), 1-16.

Smith, A. (1976) 'An Inquiry into the Nature and Causes of the Wealth of Nations', R.H. Campbell, A.S. Skinner, W. B. Todd, eds. Oxford: Oxford University Press.

Stahl, D. O. (1989) 'Oligopolistic Pricing with Sequential Consumer Search' *American Economic Review*, 79(4),700–712.

Stigler, G. (1961) 'The Economics of Information', *Journal of Political Economy*, 69(3), 213-225.

Thaler, R. (1980) 'Toward a Positive Theory of Consumer Choice', *Journal of Economic Behavior and Organization*, 1,39-60.

Thaler, R. (1987) 'The Psychology of Choice and the Assumptions of Economics', *Laboratory experimentation in economics: six points of view*. Roth, A.E. (eds.) Cambridge University Press, 99-130.