Demographic transition and Economic development : the role of child costs

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: the role of child costs

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Abstract

This paper analyzes the interactions between demographic transition and economic development by focusing on two child costs: time child-rearing cost and physical child-rearing cost. To analyze the interactions, we construct two overlapping generations model: human capital accumulation model and physical capital accumulation model. The two child costs, in particular, physical child cost plays crucial role in appearing non-monotonous fertility dynamics since it generates income effect. In both growth models, increase in physical child cost decreases the fertility, while it promotes economic development by dilution effect. Since increase in physical child cost encourages to start investing in human capital, it facilitates more rapid the timing of demographic transition in human capital accumulation model and therefore it gets the economy out of development trap. In contrast, it slows down the timing in physical capital accumulation model due to increase in income effect.

JEL classifications: I25, J11, J13, O11

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1. Introduction

This paper analyzes the interactions between demographic transition and economic development focusing on two child costs: time child-rearing cost and physical child-rearing cost. As has been indicated by many studies and historical data, the shift from positive relationship between income and fertility to negative relationship with economic development, i.e., demographic transition has been observed in developed countries. As has been indicated by Liao (2011), demographic transition is important issue for economic development since it influences the growth path through increase or decrease in population growth. The analysis is important and useful not only for clarifying the growth path in developed countries, but also for considering economic development of developing countries.

Many previous studies attempt to explain the relationship between demographic transition and economic development. Early works on Becker (1960), Becker and Lewis (1973), Willis (1973), and Barro and Becker (1989) show the trade-off between child-quality and child-quantity. As the economy develops, individuals chooses high-quality, i.e., more educational investment and the low-quantity, i.e., fewer children and therefore the fertility decreases with economic development. In fact, Cáceres-Delpiano (2006) and Fernihough (2017) provide the empirical evidence of Quality-Quantity trade-off. In line with quality-quantity theory, de la Croix (2003) explains the relationship between inequality in human capital among individuals and economic growth with differential fertility model. [1] However, the trade-off appears only at a mature stage of economic development and therefore they do not consider the early stage of economic development. Along with lines of unified growth theory, the seminal work by Galor and Weil (1999, 2000), Galor (2005a, b) and Becker et al. (2010) show the transition from Malthusian stagnation to modern growth focusing on investment in human capital.

Using Stone-Geary utility function, Jones (2001) and Nakamura (2018) analyze the interactions between growth and demographic transition. Supposing that income effect by time rearing-cost and substitution effect by the elasticity of substitution between consumption and the number of children, Nakamura (2018) show non-monotonous fertility dynamics in simple physical capital accumulation model. Also, there are many previous studies which focuses on mortality or life expectancy. The analysis which the reduction of mortality decreases the fertility and therefore it leads to demographic transition exists such as Soares (2005), Cervelani and Sunde (2011, 2015), Strulik and Weisdorf (2014), among others. However, this explanation is not

[1] In fact, demographic variables, such as fertility and life expectancy, have a significant impact on economic growth. Yakita (2010) demonstrate sustainable development with endogenous fertility. Yakita (2001), Chacraborty (2004) and Fanti and Gori (2014) analyze the effects of adult life expectancy on economic growth. In addition, Using human capital accumulation model, the effects of child mortality on economic development is analyzed by Azazert (2006) and Fioroni (2010). Focusing on endogenous labor supply, Manfredi and Fanti (2006) study Malthusian cycle.

supported by some empirical studies such as Mateos-Planas (2002), Doepke (2005) and Murphy (2009).

This paper analyzes the effects of child cost, in particular, physical child-rearing cost on demographic transition and economic development. To analyze the effect, we construct two overlapping generations model: human capital accumulation model and physical capital accumulation model. In this paper, the existence of physical child cost plays crucial role in appearing the non-monotonous fertility dynamics, i.e., demographic transition since it generates income effect. In other words, if there is no physical child cost, the non-monotonous dynamics does not appear in the economy.

In human capital accumulation model, demographic transition is derived from income effect by physical child cost and substitution effect by the substitution between educational investment for children and the number of children. At an early stage of economic development, individuals chooses no investment in human capital and high fertility due to low income and hence the economy has no substitution effect. Since there is no substitution effect, the fertility increases with income due to income effect by physical child cost at the early stage of the economy. As the economy develops, individual's income also increases. At a mature stage of economic development, individual stars to invest in human capital for children and therefore the economy has both substitution effect and income effect. Since substitution effect is dominant, the fertility decreases with income. The demographic transition is a crucial factor for economic development. If the timing of demographic transition slows, that is, individuals needs larger income to start educational investment, then the economy falls into development trap due to high fertility and no education.

Nakamura (2018) consider income effect by the existence of minimum quality of consumption using Stone-Geary preferences. Incorporating physical child cost into Nakamura (2018) instead of minimum consumption, we analyze the effects of physical child cost on the relationship between demographic transition and economic development in physical capital accumulatio model. At an early stage of the economy, in contrast to human capital accumulation model, the economy has both income effect by physical child cost and substitution effect by the substitution between consumption and fertility in physical accumulation model. Since income effect dominates substitution effect, the fertility decreases with income, while the fertility eventually decreases with income due to larger substitution effect.

In addition, we show the effects of increase in physical child cost on demographic transition and economic development. It always the fertility by additional child-rearing cost, and increases economic development by dilution effect in both growth models. However, the effects on the fertility in equilibrium and the timing of demographic transition since source of substitution effect is different in each models.

Physical child cost is crucial factor for appearing demographic transition in this paper. In fact, Mateos-Planas (2002) find that child costs and technological progress rather than mortality decline has been main factors for demographic transition in Europe using numerical simulation. Similarly,

Hence, focusing on the role of child costs, this paper presents the relationships between demographic transition and economic development in simple but useful overlapping generations framework.

The rest of this paper is organized as follows. Section 2 sets up the human capital accumulation model. Section 3 analyzes the dynamics of demographic transition and economic development, and Section 4 demonstrates the effects of increase in physical child cost them in human capital accumulation model. Section 5 sets up the physical capital accumulation model. Section 6 analyzes the dynamics of demographic transition and economic development, and Section 7 shows the effects of physical child cost on them in physical capital accumulation model. Section 8 concludes the paper.

2. Human capital accumulation model

In this section, we analyze the interactions between demographic transition and economic development in human capital accumulation model. Consider the competitive equilibrium of an overlapping generations economy. Each individual lives for two periods: childhood and adulthood. In the first period, she receives education, while she has children, works and divides her income between consumption, child rearing costs and educational investment for children.

2.1 Production and technology

For simplicity, we assume that production function is linear in labor.

$$Y_t = L_t \,, \tag{1}$$

where L_t is the total population in period t. We assume that the wage is always equal to one in the equilibrium in the labor market, i.e., $w_t = 1$ for all t in human capital accumulation model.

2.2 Individuals

The human capital of an individual in adulthood in period $t + 1$ is assumed as follows.

$$h_{t+1} = \varepsilon(\theta + e_t)^{\eta} h_t^{\delta} \,, \qquad \varepsilon, \theta, \eta, \delta > 0, \quad \eta + \delta < 1 \,, \tag{2}$$

where e_t is educational investment per child and h_t is the stock of human capital in period t. Since $\theta > 0$, human capital is positive even if parent do not invest in education, i.e., $e_t = 0$.

People gains utility from consumption c_t, the number of children n_t and human capital of their children h_{t+1}. Hence, the preference of individual of generation t is expressed by the following utility function.

$$(1-\gamma)log c_t + \gamma log n_t h_{t+1}, \qquad \gamma > 0. \tag{3}$$

Individual allocates her income between consumption, child rearing costs and educational expenditure for children. In particular, we assume two child costs, $m + \phi h_t$: time child-rearing cost ϕh_t and physical child-rearing cost m needed to care for children (Boldrin and Jones; 2002, and Fanti and Gori; 2012). Thus, her budget constraint become

$$h_t = c_t + \phi n_t h_t + e_t n_t + m n_t, \qquad 0 < \phi < 1, \quad m > 0. \tag{4}$$

Individual of generation t choose own consumption, the number of children and educational expenditure for children. Substituting (2) into (3), it solves the following utility maximization problem:

$$\max_{c_t, n_t, e_t} \ (1-\gamma)log c_t + \gamma log n_t + \gamma log\left[\varepsilon(\theta + e_t)^{\eta} h_t^{\delta}\right],$$

$$subject\ to \quad h_t = c_t + \phi n_t h_t + e_t n_t + m n_t.$$

From first-order condition for maximization, we have optimal educational expenditure and the number of children.

$$n_t = \begin{cases} \dfrac{\gamma h_t}{(\phi h_t + m)} & if \quad h_t \le \hat{h} \\ \dfrac{(1-\eta)\gamma h_t}{(\phi h_t + m - \theta)} & if \quad h_t > \hat{h}, \end{cases} \tag{5}$$

$$e_t = \begin{cases} 0 & if \quad h_t \le \hat{h} \\ \dfrac{\eta(\phi h_t + m) - \theta}{1-\eta} & if \quad h_t > \hat{h}, \end{cases} \tag{6}$$

where $\hat{h} \equiv \theta/\eta\phi - m/\phi$, which represents the threshold or turning point for demographic transition and education investment. When $h_t > \hat{h}$, whether an increase in income increases or decreases the number of children depends on $m - \theta$. Since this paper focuses on demographic transition and economic development, we assume that $\theta > m$. It implies the number of children decreases with income since substitution effect is larger than income effect when $h_t > \hat{h}$.[2] As can be seen from (5), if $m = 0$, fertility is constant when $h_t \le \hat{h}$, while decreases with income when $h_t > \hat{h}$ since income effect disappear. Hence, the physical child rearing costs m plays

[2] If $\theta < m$, an increase in income always increase the number of children since income effect is larger than substitute effect.

crucial role in demographic transition.

Assumption 1

$$\theta > m$$

Proposition 1 In human accumulation model, under Assumption 1, the fertility increases with an income at an early stage of economy, while the fertility decreases with income at a mature stage of economy since individual starts to invest in education for children. If $m = 0$, then this non-monotonous motion of the fertility does not appear in the economy.

3. The dynamical system in human capital accumulation model

In human accumulation model, the dynamical system is expressed as follows:

$$h_{t+1} = \begin{cases} \varepsilon \theta^{\eta} h_t^{\delta} & if \quad h_t \leq \hat{h} \\ \varepsilon \left[\frac{\eta(\phi h_t + m) - \theta}{1-\eta} \right]^{\eta} h_t^{\delta} & if \quad h_t > \hat{h}\,. \end{cases} \tag{7}$$

Fig.1 demonstrates the relationship between economic development and demographic transition in human capital accumulation model. When an economy is in the early stage of development, i.e., $h_t \leq \hat{h}$, individual does not invest in education for children due to low income. Hence, the fertility increases as income grows due to income effect by physical child cost. However, when the economy is sufficiently developed, i.e., $h_t > \hat{h}$, since individual starts to invest in education for children, the fertility decreases as income grows due to substitution effect. As a result, the relationship between fertility and income shifts from positive to negative, that is, demographic transition occurs with economic development.[3]

[Insert Fig.1 about here]

When initial value of human capital $h_0 = 0$, the economy has trivial steady state $h_{t+1}(0) = 0$. In addition, when $h_{t+1}(\hat{h}) \leq \hat{h}$, we can show the steady state $h_1^* = (\varepsilon\theta^{\eta})^{1/1-\delta}$. Hence, when $h_1^* \leq \hat{h}_t$, $h_{t+1}(\hat{h}) \leq \hat{h}$. In contrast, when $h_1^* > \hat{h}_t$, $h_{t+1}(\hat{h}) > \hat{h}$ and therefore the economy never converges to h_1^*. From (7), h_1^* and $\hat{h}$, we obtain following proposition.

[3] Similarly, Galor (2012) indicate that demand for human capital is main trigger for economic development and decreasing fertility.

Proposition 2 When $h_0 > 0$ and $h_1^* > \hat{h}$, the dynamical system described by (7) has unique steady state. In contrast, when $h_0 > 0$ and $h_1^* \leq \hat{h}$, the economy has multiple steady states and therefore the economy falls into development trap.

Proof. First, suppose that $h_0 > 0$ and $h_1^* > \hat{h}$. Since $h_{t+1}(\hat{h}) > \hat{h}$, the economy never converges to h_1^*. $\partial h_{t+1}/\partial h_t = \varepsilon\eta^2\phi D_t^{\eta-1}h_t^{\delta} + \delta\varepsilon D_t^{\eta}h_t^{\delta-1} > 0$, where $D_t \equiv (\eta\phi h_t + \eta m - \theta)/1 - \eta$. Also, $\lim_{h_t \to \infty} h_{t+1}/h_t = \varepsilon(\eta\phi/1-\eta)^{\eta}\infty^{\eta+\delta-1}$. Since we assume that $\eta + \delta < 1$, $\lim_{h_t \to \infty} h_{t+1}/h_t = 0$. It implies that $\partial^2 h_{t+1}/\partial h_t^2 < 0$. Hence, h_{t+1} is concave function for h_t and therefore unique and locally asymptotically stable steady state exists in the economy. Next, suppose that $h_0 > 0$ and $h_1^* \leq \hat{h}$. Since $h_{t+1}(\hat{h}) \leq \hat{h}$, the economy shows the low steady state h_1^*. If $\forall h_{t+1} < h_t$ for $h_t \in (\hat{h}, \infty)$, then unique and local asymptotically stable steady state h_1^* exist in the economy. If $\exists h_{t+1} \geq h_t$ for $h_t \in (\hat{h}, \infty)$, two steady states $\{h_1^*, h_2^*\}$, where h_2^* is local asymptotically unstable, or three steady states $\{h_1^*, h_2^*, h_3^*\}$, where h_3^* is locally asymptotically stable, exist in the economy. Hence, the economy falls into development trap when $h_{t+1}(\hat{h}) \leq \hat{h}$.

Proposition 2 implies that the economy converges higher steady state when the turning point of demographic transition $\hat{h}$ is sufficiently small. In contrast, when the turning point of demographic transition $\hat{h}$ is sufficiently large, the economy falls into development trap and converges lower steady state h_1^* with high fertility and low income, i.e., Malthusian stagnation, or higher steady state h_3^* with low fertility and high income as shown in Fig.1. Fig.1 illustrates that the economy with multiple steady states: locally asymptotically stable low steady state h_1^* and locally asymptotically stable high steady state h_3^*. At an early stage of economy, individual does not invest in human capital for children due to low income. Increase in income has income effect by child costs on fertility, while there is no substitution effect by educational investment. Hence, the fertility increases with economic development. When individuals dose not invest in education and has high fertility due to low income, the economy converges to h_1^*, This situation is considered as Malthusian stagnation since the economy is characterized by the high population growth and low income. At a sufficiently mature stage of economy, individual starts to invest in human capital for children due to higher income. Since the substitution effect by investing in human capital is dominant, the fertility decreases with income. As the economy develops, educational investment increases, while the fertility decreases and therefore the economy converges to high steady state h_3^*, which is characterized by low fertility and high education. Whether the economy falls into development or not depends on the threshold value of demographic transition. When the threshold is sufficiently small, the economy develops with accumulating human capital by more quick shifting from increasing population growth to decreasing population growth. However, when the threshold is sufficiently large, the economy falls into development trap since individual does not invest in human capital due to high fertility and low income. As a result, economic development and demographic transition have a high degree of interdependence.

4. The effects of child costs in human capital accumulation model

In this section, we analyze the effects of increase in physical child cost m on fertility, demographic transition and economic development in human capital accumulation model. First, we show the effects of increase in physical child cost on economic development. When $h_t \leq \hat{h}$, increase in physical child cost has no effects on h_{t+1} since $\partial e_t / \partial m = 0$. In contrast, when $h_t > \hat{h}$, increase in physical child cost increases h_{t+1} since $\partial e_t / \partial m > 0$.

$$\frac{dh_{t+1}}{dm} = \begin{cases} 0 & if \quad h_t \leq \hat{h} \\ \varepsilon \eta^2 \left[\frac{\eta(\phi h_t + m - \theta)}{1-\eta} \right]^{\eta-1} h_t^{\delta} > 0 & if \quad h_t > \hat{h}, \end{cases} \tag{8}$$

When $h_t > \hat{h}$, $dh^*/dm > 0$ since $dh_{t+1}/dm > 0$. Hence, the increase in physical child cost promotes economic development since it encourages educational investment for children.

Next, we show the effects on demographic transition and fertility in equilibrium. From (5) and $\hat{h} \equiv \theta/\eta\phi - m/\phi$, we get the result as follows:

$$\frac{\partial \hat{h}}{\partial m} = -\frac{1}{\phi} < 0\,, \qquad \text{: and} \tag{9}$$

$$\frac{dn^*}{dm} = \begin{cases} \underbrace{\frac{\partial n^*}{\partial m}}_{-} < 0 & if \quad h_t \leq \hat{h} \\ \underbrace{\frac{\partial n^*}{\partial m}}_{-} + \underbrace{\underbrace{\frac{\partial n^*}{\partial h^*}}_{-} \underbrace{\frac{\partial h^*}{\partial m}}_{+}}_{-} < 0 & if \quad h_t > \hat{h}, \end{cases} \tag{10}$$

From (9) and (10), increase in physical child cost always decreases the threshold of demographic transition and the fertility in equilibrium. Increase in physical child cost has two effects on fertility in equilibrium, which is direct effects through increasing rearing children cost and indirect effects through substitution effect with increasing income. Hence, we have the following proposition.

Proposition 3 Increase in physical child cost always decreases fertility in equilibrium and facilitates more rapid the timing of demographic transition. Since they encourage invest in human capital, the economy develops with human capital accumulation. Hence, the sufficiently larger increase in physical child cost gets the economy out of development trap.

Fig.2 clearly illustrates the effects of increase in physical child cost on economic development, fertility and demographic transition. Increase in physical fertility increases human capital stock and greatly decreases fertility by two effects: direct effect and indirect effect. Hence, the fertility greatly decreases as shown in Fig.2(b) In addition, increase in physical child cost promotes the shift from increasing population growth to decreasing population growth. In other words. it shifts of threshold from $\hat{h}_1$ to $\hat{h}_2$. This intuition can be explained as follows. Physical child cost itself generates income effect. However, increase in physical child cost increases the marginal cost of additional child and therefore individual greatly decreases the number of children. Due to this great decrease, individual with low income can starts to invest in human capital for children and therefore the economy develops without falling into development trap and eventually converges to h'^{*} with high income and low fertility. [4] Hence, demographic transition and economic development depend crucially on physical child cost in human capital accumulation model.

[Insert Fig.2 about here]

5. Physical capital accumulation model

In this section, we analyze the interactions between demographic transition and economic development in physical capital accumulation model. The model is based on incorporating physical child cost into Nakamura (2018) instead of minimum quality of consumption. In doing so, we can show the role of child costs in demographic transition and economic development. Consider the competitive equilibrium of an overlapping generations economy. Each individuals lives for two period: childhood and adulthood. All economic decisions are made in the adulthood. Each individuals determines consumption, bequest for children and the number of children as in order to maximize utility. In addition, the bequest is used in capital market in the economy.

5.1 Production and technology

We assume that the production function is characterized by constant return to scale production

[4] Similar main results can be obtained with an increase in rearing child cost ϕ.

function.

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha}, \qquad A > 0,\ 0 < \alpha < 1, \tag{11}$$

where K_t is the physical capital and L_t is labor. Therefore, per worker output becomes $y_t = Ak_t^{\alpha}$, where $k_t = K_t/L_t$ represents per worker stock of capital. We have the following in equilibrium under assumption with full depreciation of capital.

$$1 + r_t = \alpha A k_t^{\alpha-1}, \qquad w_t = (1-\alpha)Ak_t^{\alpha} \tag{12}$$

5.2 Individuals

Individuals gains from consumption c_t, bequest b_t and the number of children n_t. Hence, we assume CES utility function as follows:

$$u_t = \frac{\left(c_t^{\beta} b_t^{1-\beta}\right)^{1-(1/\sigma)}}{1-(1/\sigma)} + \frac{n_t^{1-(1/\sigma)}}{1-(1/\sigma)} \qquad 0 < \beta < 1, \quad \sigma > 0, \tag{13}$$

where σ represents the elasticity of substitution between $c_t^{\beta} b_t^{1-\beta}$ and n_t. Individual allocates her income between consumption, child rearing costs and bequest for children. Similar to human capital accumulation model in section 2, we assume two child costs: physical child-rearing cost and time child-rearing cost needed to care for children. Thus, her budget constraint is given by

$$w_t = c_t + \phi n_t w_t + b_t + m n_t \tag{14}$$

In adulthood, each individuals decides consumption c_t, bequest b_t and the number of children n_t. Similar to Nakamura (2018), we solve utility maximization problem in two steps. At the first step, we solve the following utility maximization with optimal the number children as given.

$$\max_{c_{t+1}^i, b_{t+1}^i} c_t^{\beta} b_t^{1-\beta} \qquad \textit{subject to } w_t = c_t + \phi n_t w_t + b_t + m n_t\,.$$

We obtain optimal consumption and bequest.

$$c_t = \beta(w_t - \phi n_t w_t - m n_t), \tag{15}$$

$$b_t = (1-\beta)(w_t - \phi n_t w_t - m n_t). \tag{16}$$

Substituting (15) and (16) into utility function, we get the following utility function as a function of only n_t.

$$v(n_{t+1}) = \frac{\tilde{\beta}(w_t - \phi n_t w_t - m n_t)^{1-(1/\sigma)}}{1-(1/\sigma)} + \frac{{n_t}^{1-(1/\sigma)}}{1-(1/\sigma)}, \tag{17}$$

where $\tilde{\beta} = \left[\beta^{\beta}(1-\beta)^{1-\beta}\right]^{\frac{\sigma-1}{\sigma}}$. At the second step, we obtain optimal the number of children to maximize utility function (17).

$$\max_{n_t} \left\{\frac{\tilde{\beta}(w_t - \phi n_t w_t - m n_t)^{1-(1/\sigma)}}{1-(1/\sigma)} + \frac{{n_t}^{1-(1/\sigma)}}{1-(1/\sigma)}\right\}.$$

The first-order condition is

$$n_t = \left(\tilde{\beta}\phi w_t + \beta m\right)^{-\sigma}[w_t - \phi n_t w_t - m n_t]\,. \tag{18}$$

From (18), we obtain optimal fertility:

$$n_t = \frac{w_t}{(\phi w_t + m) + \left(\tilde{\beta}\phi w_t + \tilde{\beta} m\right)^{\sigma}}. \tag{19}$$

If there is no physical child cost $m = 0$, we can rewrite (19) as follows.

$$n_t = \frac{w_t}{\phi w_t + \left(\tilde{\beta}\phi w_t\right)^{\sigma}}. \tag{20}$$

When $m = 0$, differentiating (20) with respect to w_t, we obtain

$$\frac{\partial n_t}{\partial w_t} = \frac{\left(\tilde{\beta}\phi w_t\right)^{\sigma}(1-\sigma)}{\left[\phi w_t + \left(\tilde{\beta}\phi w_t\right)^{\sigma}\right]^2}\,. \tag{21}$$

and hence,

$$\frac{\partial n_t}{\partial w_t} \lesseqgtr 0 \;\Leftrightarrow\; \sigma \lesseqgtr 1\,. \tag{22}$$

When $m = 0$, the behavior of fertility depends on the elasticity of substitution. If $\sigma < 1$, the fertility increases with wage since income effect is dominant. In contrast, if $\sigma > 1$, the fertility decreases with wage since substitution effect is dominant. Hence, if there is no physical child cost, the monotonous behavior of fertility exhibits. In other words, non-monotonous behavior of fertility, i.e., demographic transition does not appear without physical child cost.

Next, suppose that $m > 0$. Differentiating (19) with respect to w_t, the behavior of fertility is given.

$$\frac{\partial n_t}{\partial w_t} = \frac{\left(\tilde{\beta}\phi w_t + \tilde{\beta} m\right)^{\sigma-1} \left\{ m \left[\left(\tilde{\beta}\phi w_t + \tilde{\beta} m\right)^{1-\sigma} + \tilde{\beta} \right] - (\sigma - 1)\tilde{\beta}\phi w_t \right\}}{\left[\phi w_t + m + \left(\tilde{\beta}\phi w_t + \tilde{\beta} m\right)^{\sigma}\right]^2}. \quad (23)$$

As can be shown in Fig.3, the threshold or the turning point of demographic transition $\widehat{w}$ exists when $\sigma > 1$. The relationships between n_t and w_t is expressed as follows:

$$\frac{\partial n_t}{\partial w_t} \gtreqless 0 \Leftrightarrow w_t \lesseqgtr \widehat{w}\,. \quad (24)$$

As can be illustrated in Fig.3, when $w_t < \widehat{w}$, i.e., the economy is in the early stage of economic development such as low income, the fertility increases with wage, $\partial n_t/\partial w_t > 0$. In contrast, when $w_t > \widehat{w}$, i.e., the economy is in mature stage of economic development such as high wage, the fertility decreases with wage, $\partial n_t/\partial w_t < 0$. With physical child cost, hence, the behavior of fertility is non-monotonous i.e. demographic transition appears in the economy as shown in Fig.3. The existence of physical child cost is the hurdles to rearing children and therefore generates income effect. When income is very low, the marginal benefit of additional child is very high and this prevents the substitution of consumption, bequest and fertility. Even if $\sigma > 1$, when the economy is in the early stage i.e., wage is very low, the elasticity of substitution between consumption, bequest and fertility is very low due to the existence of physical child cost. Hence, when $w_t < \widehat{w}$, i.e., wage is sufficiently low and therefore fertility increases with wage, income effect by physical rearing child cost dominates the substitution effect. As the economy develops and wage increases, income effect is smaller since the hurdles by physical child cost is relatively lower. Thus, when wage is sufficiently high, i.e., $w_t > \widehat{w}$, the fertility decreases with income since the substitution effect is larger than income effect. As a result, the physical child cost generates non-monotonous behavior of fertility with wage. These results are summarized in the following proposition.

Proposition 4 Without physical child cost, the fertility dynamics depends on only the elasticity of substitution and therefore the monotonous motion of fertility appears in the economy. With physical child cost, the fertility dynamics changes dramatically with wage. When $\sigma > 1$ and wage is sufficiently low, the fertility increases with wage since income effect by physical child cost dominates substitution effect. In contrast, when $\sigma > 1$ and wage is sufficiently high, the fertility decreases with wage since substitution effect is larger than income effect. As a result, with physical capital cost and $\sigma > 1$, the non-monotonous behavior of fertility, i.e., demographic transition appears in the economy.

[Insert Fig.3 about here]

6. The dynamical system in physical capital accumulation model

In this section, we analyze the interactions between demographic transition and economic development in physical accumulation model. Inherited bequest is capital stock in current period and bequest to leave the children is capital stock in next period. Aggregate capital stock in period $t+1$ becomes $K_{t+1} = L_t b_t$. Hence, per worker capital stock in period $t+1$ is $k_{t+1} = b_t/n_t$. Substituting (16) and (19) into $k_{t+1} = b_t/n_t$, the dynamical system is given by

$$k_{t+1} = (1-\beta)\left(\tilde{\beta}\phi A k_t^{\alpha} + \tilde{\beta} m\right)^{\sigma}, \tag{25}$$

where $\partial k_{t+1}/\partial k_t = (1-\beta)\alpha\sigma\tilde{\beta}\phi A k_t^{\alpha-1}\left(\tilde{\beta}\phi A k_t^{\alpha} + \tilde{\beta} m\right)^{\sigma-1} > 0$, In addition, $\lim_{k_t\to\infty} k_{t+1}/k_t = (1-\beta)\left(\tilde{\beta}\phi A\right)^{\sigma}\infty^{\alpha\sigma-1}$. When $\sigma < 1/\alpha$, $\lim_{k_t\to\infty} k_{t+1}/k_t = 0$, i.e., $\partial^2 k_{t+1}/\partial k_t^2 < 0$ and k_{t+1} is concave function for k_t and therefore unique locally asymptotically stable steady state exists in the economy. To ensure stable steady state, and to analyze demographic transition, we assume that $1 < \sigma < 1/\alpha$. This assumption implies that the elasticity of substitution is not too large.

In addition, since $w_t = w(k_t)$, the following relationship holds for k_t and n_t.

$$\frac{\partial n_t}{\partial k_t} \gtreqless 0 \Leftrightarrow k_t \lesseqgtr \hat{k}\,. \tag{26}$$

Fig.4 demonstrates the relationship between economic development and demographic transition. The relationship between the fertility and per worker capital changes positive from negative. When the technology is low, i.e., $A = A_L$, the economy converges to k_L^*. Since stable steady state k_L^* is characterized by high fertility and low income. This situation is considered as the Malthusian economy with low income and high fertility. When the technology is high, i.e., $A = A_H$, the economy converges to k_H^* with low fertility and high income. As the economy develops, the fertility increases due to income effect when income is low, i.e., $k_t < \hat{k}$, and it eventually decreases when k_t exceeds a certain level, i.e., $k_t > \hat{k}$. Hence, the fertility dynamics become non-monotonous motion as the economy develops. Hence, we have the following proposition.

Proposition 5 Suppose that $1 < \sigma < 1/\alpha$. Then, the economy eventually converges to the locally asymptotically stable steady state. When the technology is low, the economy has steady state with low income and high fertility. In contrast, when the technology is high, the fertility increases with income at an early stage of economic development, and eventually decreases with income at a mature stage of economic development. Hence, the economy has steady state with high income and low fertility.

7. The effects of child costs in physical capital accumulation model

In this section, we show the effects of increase in physical child cost m on fertility, demographic transition and economic development in physical capital accumulation model. First, we show the effects on k_{t+1}.

$$\frac{\partial k_{t+1}}{\partial m} = (1-\beta)\tilde{\beta}\sigma\left(\tilde{\beta}\phi A k_t^{\alpha} + \tilde{\beta}m\right)^{\sigma-1} > 0 \tag{27}$$

Hence, increase in physical child cost encourages economic development. Next, we analyze the effects on the fertility.

$$\frac{\partial n_t}{\partial m} = -\frac{1+\tilde{\beta}\sigma\left(\tilde{\beta}\phi k_t + \tilde{\beta}m\right)^{\sigma-1}}{\left[\phi k_t + m + \left(\tilde{\beta}\phi k_t + \tilde{\beta}m\right)^{\sigma}\right]^2} < 0, \tag{28}$$

and hence

$$\frac{dn^*}{dm} = \begin{cases} \underbrace{\frac{\partial n^*}{\partial m}}_{-} + \underbrace{\underbrace{\frac{\partial n^*}{\partial w^*}}_{+}\underbrace{\frac{\partial w^*}{\partial k^*}}_{+}\underbrace{\frac{\partial k^*}{\partial m}}_{+}}_{+} \gtreqless 0 & if \quad k_t \leq \hat{k} \\ \underbrace{\frac{\partial n^*}{\partial m}}_{-} + \underbrace{\underbrace{\frac{\partial n^*}{\partial w^*}}_{-}\underbrace{\frac{\partial w^*}{\partial k^*}}_{+}\underbrace{\frac{\partial k^*}{\partial m}}_{+}}_{-} < 0 & if \quad k_t > \hat{k}, \end{cases} \tag{29}$$

From (28), increase in physical child cost always decreases n_t. This decrease of the fertility encourages per worker physical capital accumulation due to dilution effect. In contrast, the effects on the fertility in equilibrium n^* is ambiguous. Increase in physical child cost on the fertility in equilibrium have two effects: direct effect and indirect effect. The direct effect is the effect by increasing the hurdle to rearing and therefore it decreases fertility. On the other hand, indirect effect is the effect on the fertility through increasing income. At an early stage of economy, i.e., $k_t \leq \hat{k}$, increase in income increases the fertility since income effect is dominant, i.e., indirect effect is positive, while, at the mature stage of economy, i.e., $k_t > \hat{k}$, it decreases since substitution effect is dominant, i.e., indirect effect is negative. As the result, the effect of increase in physical child cost on the fertility in equilibrium is ambiguous when $k_t \leq \hat{k}$ due to negative direct effect and positive indirect effect although it always decreases the fertility in equilibrium when $k_t > \hat{k}$.

Finally, we show the effects on demographic transition. As can be seen from (23), the threshold of demographic transition $\hat{k}$ depend on $g(w_t, m) = m\left[(\tilde{\beta}\phi w_t + \tilde{\beta}m)^{1-\sigma} + \tilde{\beta}\right]$ and $(\sigma - 1)\tilde{\beta}\phi w_t$. Hence, increase in m shifts only $g(w_t, m)$. Differentiating $g(w_t, m)$ with respect to m, we can analyze the effects on the threshold $\hat{k}$.

$$\frac{\partial g(w_t, m)}{\partial m} = \tilde{\beta} + \left(\tilde{\beta}\phi w_t + \tilde{\beta}m\right)^{-\sigma}\left\{\tilde{\beta}m(2-\sigma) + \tilde{\beta}\phi w_t\right\}. \tag{30}$$

From (30), the effects of increase in physical child cost on the threshold depend on the sign of $\tilde{\beta}m(2-\sigma) + \tilde{\beta}\phi w_t$. If the elasticity of substitution is not too large, i.e., $\sigma < 2 + \phi w_t/m$, then increase in m increases the threshold $\hat{k}$. We already assume that the elasticity is not too large, i.e., $\sigma < 1/\alpha$. Hence, it is plausible to assume that $\sigma < 2 + \phi w_t/m$.

Assumption 2

$$1 < \sigma < 2 + \frac{\phi w_t}{m}$$

Under Assumption 2, increase in physical child cost increases $g(w_t, m)$ and therefore it increases the threshold $\hat{k}$. In other words, increase in physical child cost slows down the timing of demographic transition.

$$\frac{\partial g(w_t, m)}{\partial m} = \tilde{\beta} + \left(\tilde{\beta}\phi w_t + \tilde{\beta}m\right)^{-\sigma}\left\{\tilde{\beta}m(2-\sigma) + \tilde{\beta}\phi w_t\right\} > 0\,. \tag{32}$$

and hence

$$\frac{\partial \hat{k}}{\partial m} > 0\,. \tag{33}$$

Unlike human capital accumulation model, hence, increase in physical child cost slows down the timing of demographic transition in physical accumulation model. This intuition can be explained as follows. In human capital accumulation model, the economy greatly develops by investing in human capital. However, individuals does not invest in human capital for children while income is low and the fertility is high, and therefore the fertility increases with income due to income effect by physical child cost since there is no substitution effect at the early stage of economic development. When income is sufficiently large, i.e, the economy is in the mature stage, individuals starts to invest in human capital, and the fertility decreases with income since the substitution effect emerges. Hence, starting educational investment generates great substitution effect in human capital accumulation model. Increase in physical child cost promotes the timing of demographic transition since it encourages starting to invest in human capital, which generates great substitution effect. In contrast to human capital accumulation model, both income effect and substitution effect always exist from the early stage of the economy to the mature stage of economy in physical capital accumulation model. Since increase in physical child cost increases the hurdle of having children, it decreases the fertility, while it facilitates larger income effect. Assuming that the elasticity of substitution is not too large, individuals have more children by increasing in income effect. This observation explains the slow-down of the timing of demographic transition. The effects of increase in physical child cost in physical capital accumulation model are summarized in the following proposition.

Proposition 6 Suppose that $1 < \sigma < 2 + \phi w_t/m$. Increase in physical child cost always decreases the fertility, while it encourages per worker capital stock by dilution effect. The effects of increase in physical child cost on the fertility in equilibrium depend on the stage of economic development. At the early stage of economic development, the effects are ambiguous since direct effect through increasing marginal cost of additional child is negasitive and indirect effect through increasing income is positive. In contrast, at the mature stage of economic development, the effects always decrease since both direct effect and indirect is negative. Increase in physical child cost slows down the timing of demographic transition since it increases income effect.

Fig. 5 illustrates the effects of increase in physical child cost on economic development, fertility and demographic transition. First, suppose that $A = A_L$, i.e., $k_{t+1} = k_{t+1}^L$. Increase in physical child cost decreases the fertility from n_t to n_t'. If the effects of it on k_{t+1}^L is smaller, then per worker capital stock shifts from k_{t+1}^L to $k_{t+1}^{\prime 1L}$ due to (smaller) dilution effect, while it decrease the fertility in the equilibrium from n_L^* to $n_L^{\prime 1*}$ since the (negative) direct effect through increasing physical child cost is larger than (positive) indirect effect through increasing income. Hence the steady state $k_L^{\prime 1*}$ or $k_L^{\prime 2*}$ is characterized by Malthusian stagnation with low income and high fertility. In contrast, if the effects of additional physical child cost on k_{t+1}^L is larger, then per worker capital stock shifts from k_{t+1}^L to $k_{t+1}^{\prime 2L}$ due to (larger) dilution effect, while it increase the fertility in the equilibrium from n_L^* to $n_L^{\prime 2*}$ since the (positive) indirect effect is larger than the (negative) direct effect. Next, suppose that $A = A_H$. Since the technology is high, i.e., $k_{t+1} = k_{t+1}^H$, the economy eventually converges to k_H^*. Increase in physical child cost increases per worker capital stock from k_{t+1}^H to $k_{t+1}^{\prime H}$ and therefore the equilibrium shifts from k_H^* to $k_H^{\prime *}$. In addition, the effects of it on the fertility in equilibrium greatly decreases from

n_H^* to $n_H'^*$ since both direct effect and indirect direct has negative effects on the fertility. Hence, the economy converges to the steady state $k_H'^*$ with high income and low fertility. Finally, we show the effects of increase in physical child cost on demographic transition. As can be illustrated in Fig.5, it increases the threshold from $\hat{k}_1$ to $\hat{k}_2$ since it encourages income effect. Hence, it slows down the shift from increasing population to decreasing population growth, i.e., the timing of demographic transition. As a result, similar to human capital accumulation model, physical child cost plays crucial role in economic development and demographic transition in physical capital accumulation model.

8. Concluding remarks

This paper analyzes interactions between demographic transition and economic development in both human capital accumulation model and physical capital accumulatio model, focusing on the child costs. We assume that two child costs: physical child-rearing cost and time child rearing-cost. In particular, the existence of physical child cost generates the non-monotonous fertility dynamics since it makes income effect. Without physical child cost, in other words, the non-monotonous fertility behavior does not appear in both growth model.

In human capital accumulation model, the non-monotonic relationships between the fertility and income is derived from income effect by physical child cost and substitution effect by investing in human capital. At an early stage of the economy, individuals does not invest in education for children due to low income and high fertility. Hence, the economy has only income effect by physical child cost and therefore the fertility increases with income. When the economy is sufficiently developed, individuals become to start investing in human capital due to high income, and therefore the fertility decreases with income since the substitution effect by educational investment is dominant. When the timing of demographic transition is sufficiently slow, the economy falls into development trap since individuals chooses high fertility and no investment for education.

In physical capital accumulation model, both income effect by physical child cost and substitution effect by the elasticity od substitution between consumption, bequest and fertility exist at an early stage of economic development in contrast to human capital accumulation model. Individuals gains the larger utility from the number of children while income is low, and therefore the fertility increases with income since income effect is larger than substitution effect. At a mature stage of the economy, the utility from the additional number of children is smaller since the hurdle by physical child cost is relatively lower due to high income. Hence, when income is sufficiently is high, the fertility decreases with income since substitution effect is dominant.

In both growth models, increase in physical child cost decreases the fertility due to increase in rearing-child cost, while it promotes economic development by dilution effect. However, the effects of it on the fertility in equilibrium and demographic transition is different for human capital accumulation model and physical capital accumulation model. The effects on the fertility in equilibrium depends on direct effect through increasing child-rearing cost, which decreases the fertility, and indirect effect through increasing income, which depends on the stages of economic development. At an early stage of the economy, the effects are ambiguous in physical capital accumulation model since direct effect is positive and indirect effect is positive. At a mature stage of the economy, it always decreases the fertility in equilibrium since both direct effect and indirect is negative. In contrast, regardless of stages of the economic development, it always decreases the fertility in equilibrium since direct effect is always dominant. As a result, increase in physical child cost always decreases the fertility in equilibrium in human capital accumulation, while it is ambiguous in physical capital accumulation model. Since increase in physical child cost encourages to starting investment for education, it facilitates more rapid the timing of demographic transition in human capital accumulation model and therefore the economy gets out of development trap. In contrast, increase in physical child cost slows down the timing of demographic transition in physical capital accumulation model since it increases income effect.

We assume that exogenous technology in this paper. However, the technological progress is impossible factor for long-run growth and it changes population composition as indicated by Nakamura (2018). It deserves future research to endogenize technological progress to derive the implications of demographic transition and economic development.

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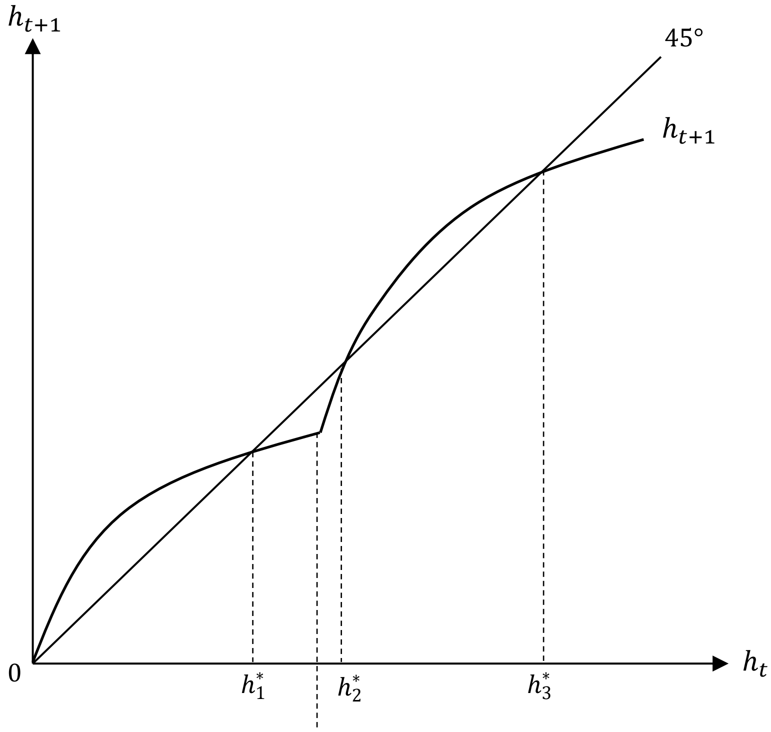
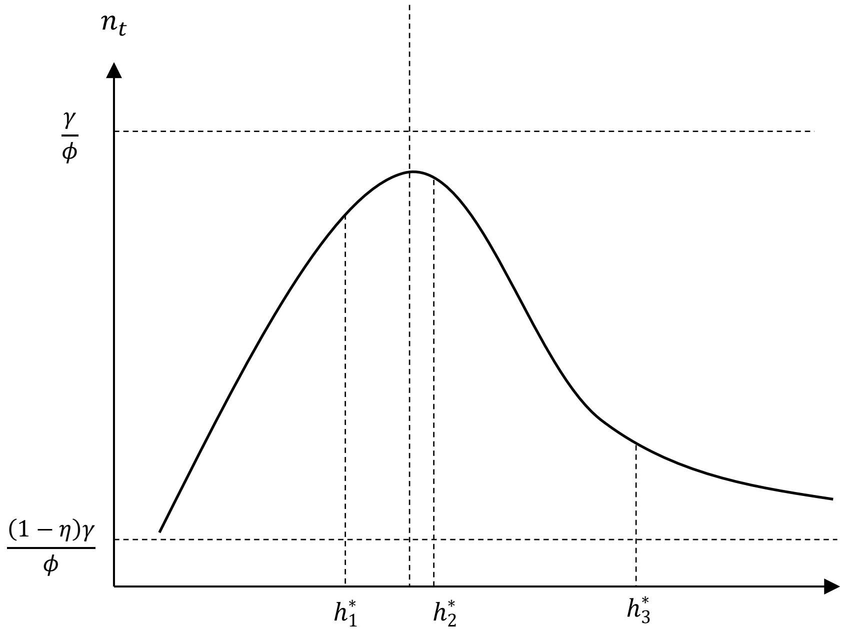
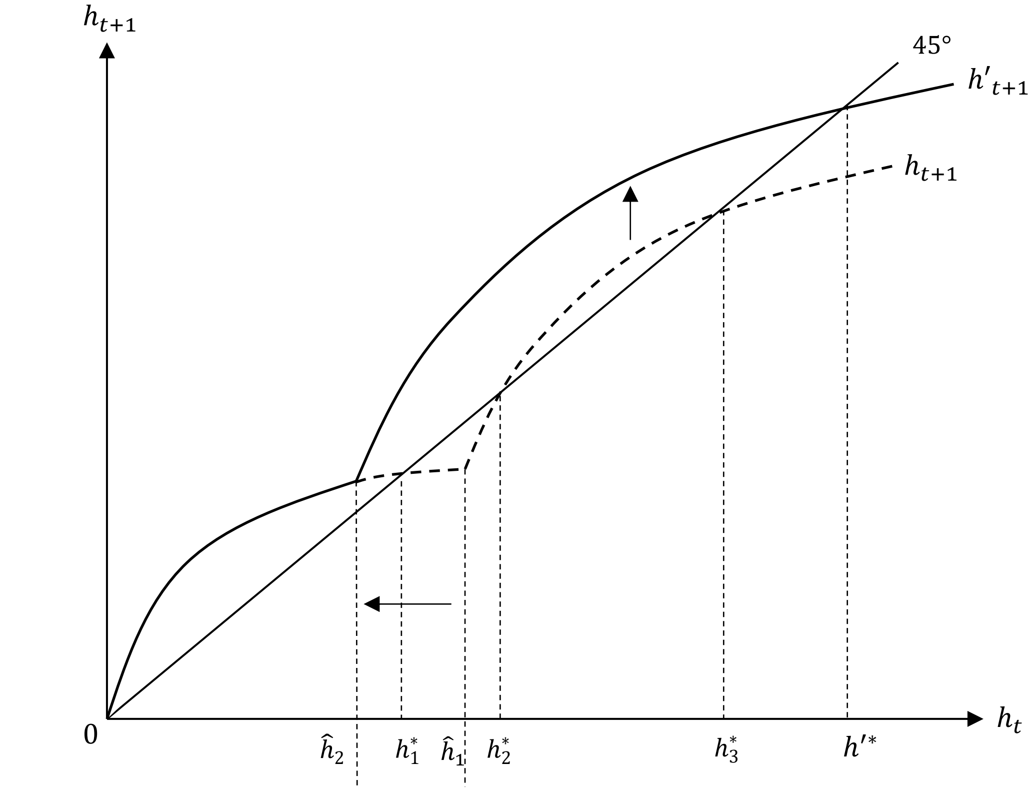


(a) The dynamical system of h_t

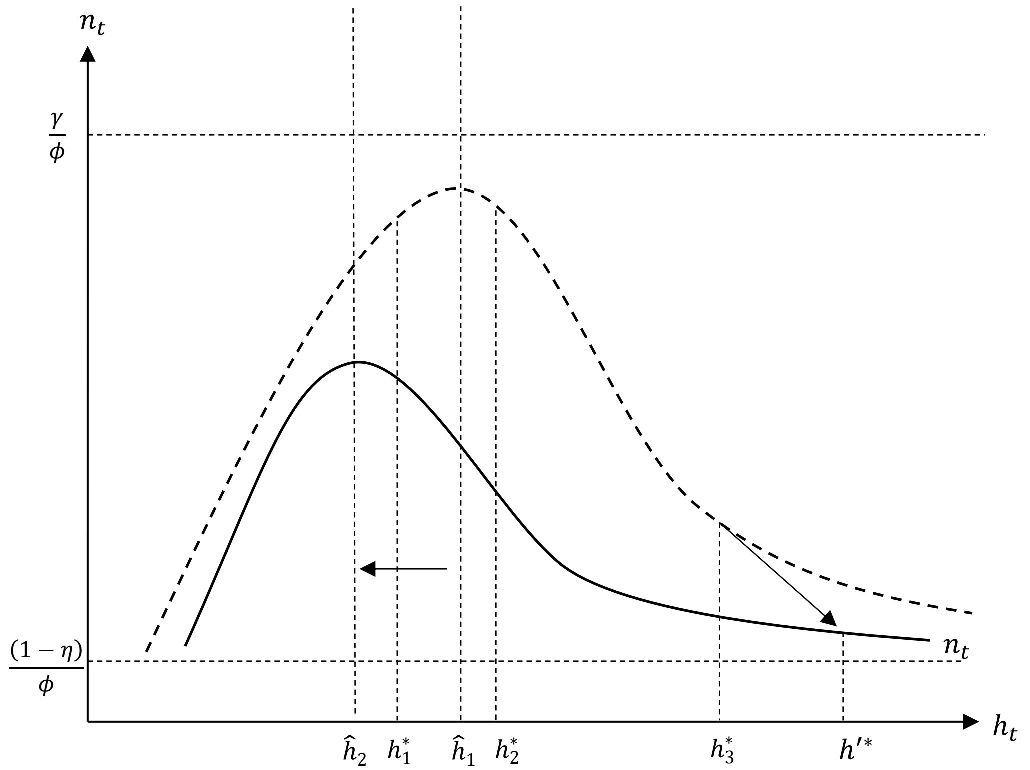


(b) The fertility dynamics of n_t

Fig. 1 Relationship between income and fertility



(a) Increase in the dynamical system of h_t



(b) Decrease in the fertility dynamics n_t

Fig. 2 The effects of increase in physical child cost

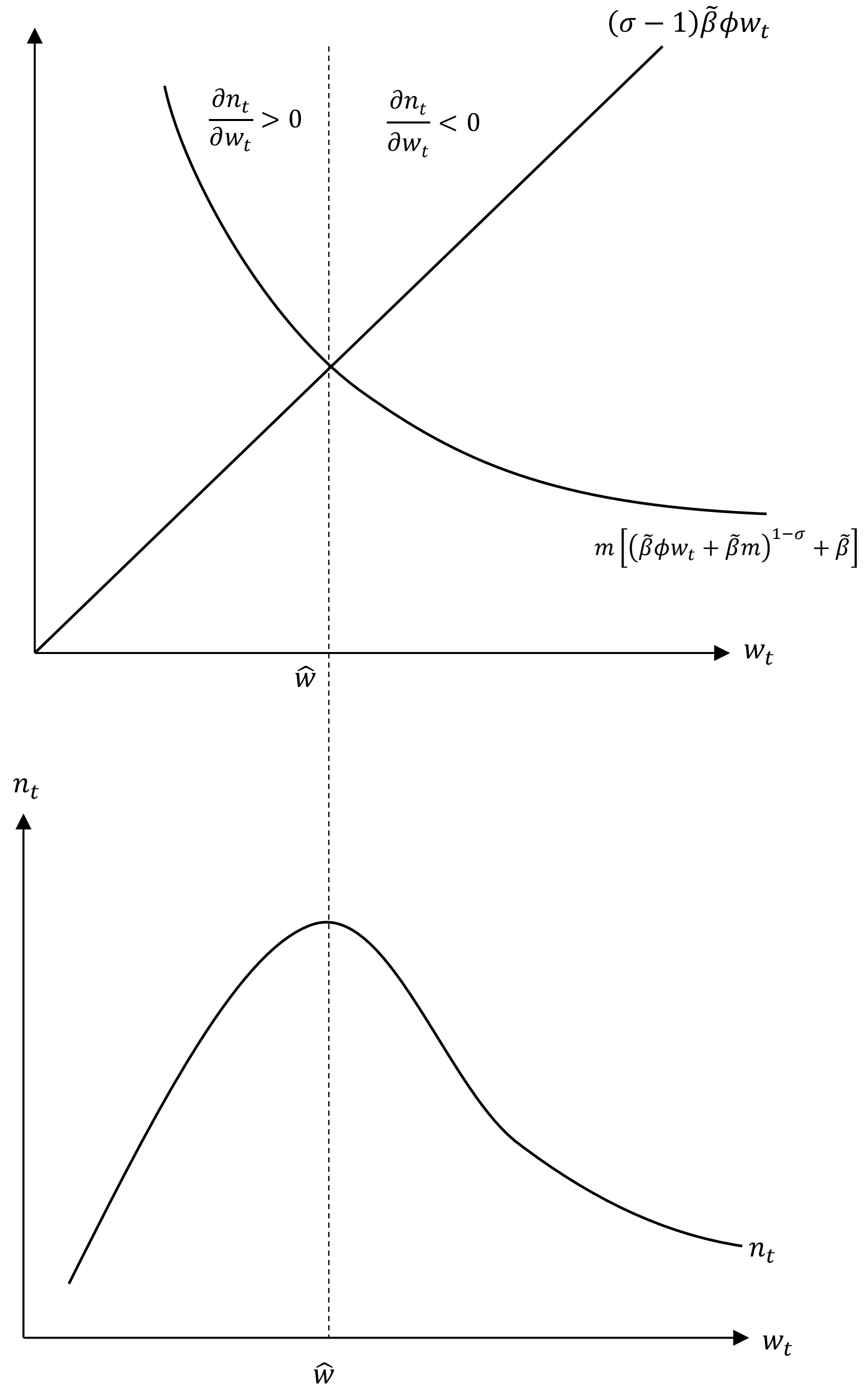
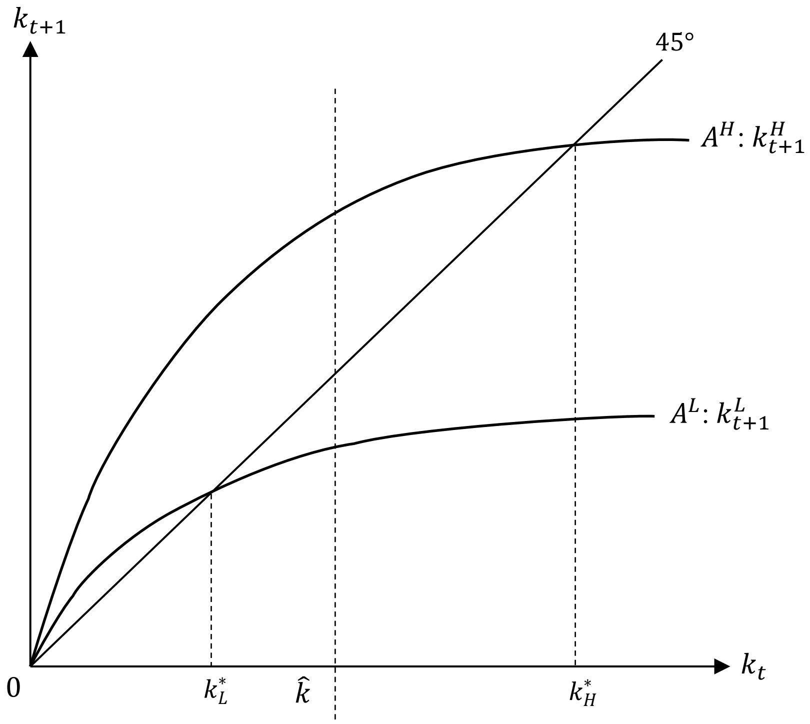
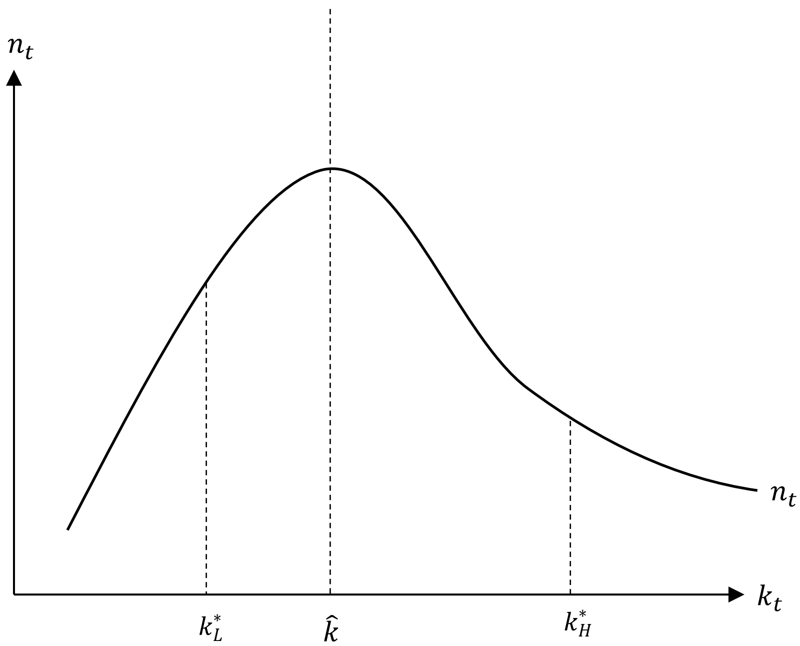


Fig. 3 Relationship between income and fertility

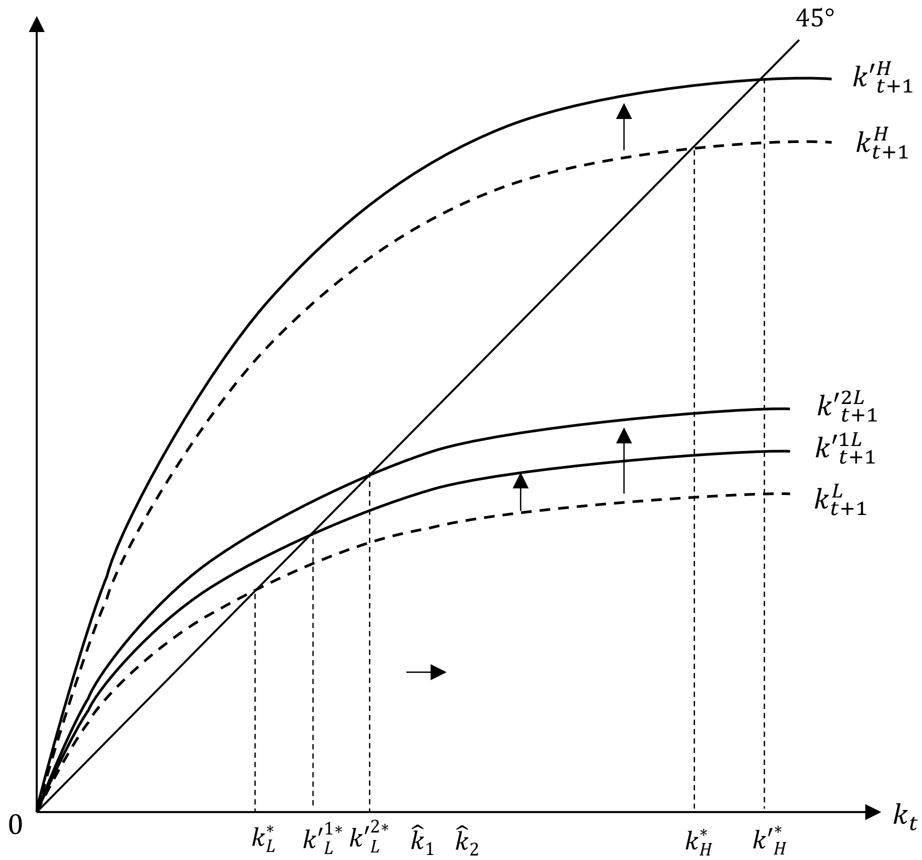


(a) The dynamical system of k_t for different technology

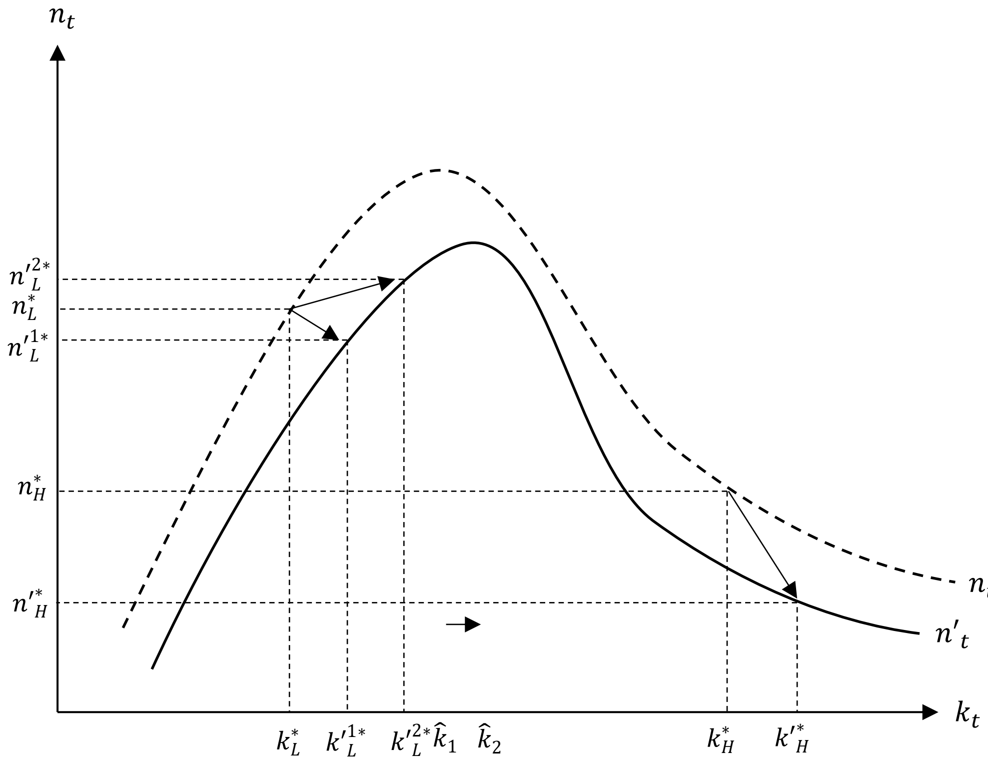


(b) The fertility dynamics of n_t for different technology

Fig. 4 Relationship between income and fertility



(a) Increase in k_t for different technology



(b) Changes in n_t for different technology

Fig. 5 The effects of increase in physical child cost