Average rates of return, working capital, and NPV-consistency in project appraisal: A sensitivity analysis approach

Magni, Carlo Alberto and Marchioni, Andrea

University of Modena and Reggio Emilia, University of Modena and Reggio Emilia

27 April 2020

Average rates of return, working capital, and NPV-consistency in project appraisal: A sensitivity analysis approach

Carlo Alberto Magni
Department of Economics "Marco Biagi" and School of Doctorate E4E
(Engineering for Economics - Economics for Engineering)
University of Modena and Reggio Emilia*

Andrea Marchioni
Department of Economics "Marco Biagi"
University of Modena and Reggio Emilia

Abstract

In project appraisal under uncertainty, the economic reliability of a measure of financial efficiency depends on its strong NPV-consistency, meaning that the performance metric (i) supplies the same recommendation in accept-reject decisions as the NPV, (ii) ranks competing projects in the same way as the NPV, (iii) has the same sensitivity to perturbations in the input data as the NPV. In real-life projects, financial efficiency is greatly affected by the management of the working capital. Using a sensitivity analysis approach and taking into explicit account the role of working capital, we show that the *average return on investment* (ROI) is not strongly NPV-consistent in accept-reject decisions if the working capital is uncertain and changes under changes in revenues and costs. Also, it is not strongly NPV-consistent in project ranking. We also show that the *internal rate of return* (IRR) is not strongly NPV-consistent and economic analysis may even turn out to be impossible, owing to possible nonexistence and multiplicity caused by perturbations in the input data, as well as to possible shifts in the financial meaning of IRR under changes in the project's value drivers. We introduce the *straight-line rate of return* (SLRR), based on the notion of average rate of change, which overcomes all the problems encountered by average ROI and IRR: It always exists, is unique, strongly NPV-consistent for both accept-reject decisions and project ranking, and has an unambiguous financial nature.

Keywords. Finance, project evaluation, working capital, ROI, IRR, sensitivity analysis, net present value, straight-line, project ranking.

JEL codes. C4, C44, D81, M41, G31, G12, D92.

*Corresponding author: tel. +39-059-2056777, e-mail: magni@unimo.it

1 Introduction

In capital asset projects, economic profitability may be measured with absolute metrics, such as the net present value (NPV), expressing value increase in monetary units, or relative metrics, expressing rates of return or profitability indices which aim at identifying a project's financial efficiency.

The preference for absolute metrics or relative metrics in practice may depend on several factors. Capital rationing is one such factor. It may occur in several different forms; for example, the firm may face an upper limit to borrow from banks; headquarters may impose budget limits on expenditures of a division; the firm may have more positive NPVs that it can finance; the firm's owners may exclude issuance of new shares to avoid loss of the firm's control; a given amount of monetary resources may be freed out of current operations and be available for new investments. Other kinds of constraints (limits in management time, skilled labor, equipment, know-how, etc.) and agency conflicts are also frequent in capital investment decisions. These (soft or hard) constraints often induce managers to focus on relative metrics measuring the marginal efficiency of capital (see Pike and Ooi 1988, Berkovitch and Israel 2004, Ross, Westerfield and Jordan 2011, Brealey, Myers and Allen 2011).

Functional areas and educational background of decision makers play also a role. For instance, practitioners seem to be at ease with the intuitive appeal of a rate of return (Evans and Forbes 1993, Graham and Harvey 2001, Sandahl and Sjögren 2003, Lindblom and Sjögren 2009). Managers with a strong financial background generally do not encounter difficulties in using absolute metrics, whereas managers with a traditional accounting or engineering imprinting may be more confident in using rates of return instead of monetary values.

Therefore, the coherence or incoherence between absolute and relative metrics is, comprehensibly, an important theoretical and applicative issue. Net-present-value consistency of a performance metric means that the decisions recommended by the metric are the same as the ones recommended by the NPV criterion. The literature on NPV-consistent (or NPV-compatible) measures is enormous and spans over several decades (e.g., see Hajdasinski 1995, 1997, Hartman 2000, Hartman and Schafrick 2004, Pfeiffer 2004, Gow and Reichelstein 2007, Lindblom and Sjögren 2009, Chiang, Cheng and Lam 2010, Pasqual, Padilla and Jadotte 2013).

Recent studies take a different view on NPV-consistency. Percoco and Borgonovo (2012) and Borgonovo and Peccati (2004, 2006) analyze the influence on the NPV and the internal rate of return (IRR) of the value drivers (also called key parameters or input data, which are the sources of investment risk) via the application of Sensitivity Analysis (SA). They show that the parameters whose uncertainty is most influential on NPV are not the same as the IRR's. More recently, using the average-internal-rate-of-return (AIRR) approach (Magni 2010), Marchioni and Magni (2018) (henceforth, MM 2018) proposed a relative metric, the average Return On Investment (ROI), which enjoys strong NPV-consistency, in the sense that changes in the key parameters have the same effects on NPV and on average ROI, overcoming the deficiency of IRR described in Percoco and

Borgonovo (2012) and Borgonovo and Peccati (2004, 2006). However, all these authors implicitly assumed a working capital equal to zero throughout the project's life. Also, they they did not cope with project ranking.

The influence of working capital (WC) management on financial performance is suggested by several recent works. Among others, Caballero, García-Teruel and Martínez-Solan (2014) find a significant link between working capital management and corporate performance. Chauhan (2019) highlights the long-term role of working capital management, as opposed to the traditional short-term view of working capital. Bian et al (2018) study the effect of working capital requirements on the company's financial situation via a discounted cash flow model over the planning horizon, and Luciano and Peccati (1999) present the application of adjusted present value techniques to an inventory management problem. Huang et al (2019) analyze the role of the supply chain finance to alleviate financing problems of small and medium enterprises and the beneficial effect of efficient working capital management on the selection of reasonable financing modes. Song et al (2019) analyze the role of supply chain finance in reducing information asymmetry and increasing the possibility to raise WC. Pirtillä et al (2019) underline the importance of the supply chain finance on the competitive advantage in the Russian automotive industry. Furthermore, Peng and Zhou (2019) propose three different models describing different level of cooperations into the supply chain and suggest to manage WC according to a supply chain-oriented solution. Moreover, Protopappa-Sieke and Seifert (2010) investigate the advantages of interrelating operational and financial aspects in decision-making about supply chain and working capital. In addition, Wetzel and Hofmann (2019) realize an exploratory network analysis about supply chain finance, financial constraints and corporate performance. Wu et al (2019) consider the role of the payment term and of the payment approach on the financial performance of the supplier and retailer through cash flow optimization.

We build upon the SA literature as a tool for managing risk and we specifically focus on the recent subset of papers which study the reciprocal consistency of different performance metrics. At the same time, we take into explicit account the role of working capital management in selecting an economically significant and reliable measure of efficiency for making financial analyses and capital investment decisions. In particular, we

- show that the average ROI is not strongly NPV-consistent in presence of WC
- introduce a new performance metric, the Straight-Line of Return (SLRR), which allows for nonzero (uncertain) WC while retaining strong NPV-consistency
- extend the notion of strong NPV-consistency to project ranking, showing that the SLRR's ranking is strongly NPV-consistent, if the initial outlays are equal
- measure the degree of inconsistency of the average ROI and the IRR and show that the SLRR outperforms these indices
- introduce some previously unknown pitfalls of the IRR.

Specifically, we show that, if one relaxes the assumption of zero WC, the average ROI is strongly NPV-consistent in accept-reject decisions only if the WC is exogenous, that is, it does not change under changes in the value drivers. However, this case is not frequent, given the strong link which usually occurs between accounts receivable and revenues, between accounts payable and operating costs, and between inventory and production and sales. Also, the average ROI is not strongly NPV-consistent in project ranking. Moreover, albeit a rare case, the average ROI might not exist.

We use the notion of Chisini mean (Chisini 1929) to find possible substitutes for the average ROI: The internal rate of return and the straight-line rate of return. We prove, via several counterexamples, that the IRR is not strongly NPV-consistent (see also Battaglio, Longo and Peccati 1996, Borgonovo and Peccati 2004, 2006, Percoco and Borgonovo 2012 on divergence between IRR and NPV) with non-negligible degrees of inconsistency, as measured via Spearman's (1904) correlation coefficient and Iman and Conover's (1987) top-down coefficient. We discover new, previously unknown deficiencies of IRR in project appraisal under uncertainty: Even in those cases where it exists and is unique, a simple perturbation of the key parameters may cause the IRR to disappear or generate multiple IRRs, with the unpleasant implication of making it impossible to assess the impact of a change in value drivers on the IRR; furthermore, the IRR may change its financial nature (investment rate versus financing rate) under changes in the key parameters, which makes IRR unhelpful.

In contrast, we find that the SLRR is strongly NPV-consistent, even in a strict sense (the relevances of the value drivers are the same as the NPV's) in accept-reject decisions and, if the competing projects share the same initial investment, in project raking. Also, it always exists, is unique, and has an unambiguous meaning.

The remaining part of the paper is structured as follows. Section 2 recalls the definition of strong NPV-consistency proposed in MM (2018) for accept-reject decisions, based on sensitivity analysis, and shows that the strong NPV-consistency of the average ROI rests on the assumption of zero WC or, alternatively, the assumption that WC is exogenously determined (i.e., it does not depend on revenues and costs); without either assumption, strong NPV-consistency of average ROI is not guaranteed. Section 3 uses the notion of Chisini mean to find alternative candidates enjoying strong NPV-consistency. Chisini's invariance requirement supplies the internal rate of return and the straight-line rate of return. The SLRR is shown to exist, be unique, and be strongly NPV-consistent in a strict form for accept-reject decisions, whereas the IRR is not. In sections 4-5 we introduce new types of difficulties suffered by IRR under uncertainty. Section 6 proves, via counterexamples, that, in general, the average ROI is not strongly NPV-consistent under uncertain WC, and it further measures its level of inconsistency. Section 7 extends the notion of strong NPV-consistency to project ranking and shows that, unlike average ROI and IRR, the SLRR fulfills it if the projects' initial investment is the same. Some concluding remarks end the paper and summarize the difference among the three performance metrics.

2 Accept-reject decisions and NPV-consistency of average ROI

2.1 Economic setting of investment decisions

Consider a capital asset project, P, and let $F = (F_0, F_1, \dots, F_p)$, $F_0 \neq 0$, be its estimated stream of free cash flows (FCFs), where p is the number of periods in which the firm operates the project. A positive cash flow means that the capital providers (i.e., shareholders and debtholders) receive money from the firm (i.e., money *flows out* of the firm), a negative cash flow means that the capital providers contribute money to the firm (i.e., money *flows in* the firm). The project's net present value (NPV) is the algebraic sum of the discounted cash flows, and represents the *economic value created*: $\text{NPV} = \sum_{t=0}^{p} F_t(1+k)^{-t}$. The discount rate k is the so-called *cost of capital* (COC) (or minimum attractive rate of return).[1]

Definition 1. (*NPV criterion for accept/reject decisions*) A project creates value (i.e., it is worth undertaking) if and only if NPV > 0.

Following we define the classical notion of NPV-consistency for a rate of return. It provides a notion of weak NPV-consistency based on the decision recommended by a given metric.

Definition 2. (*Weak NPV-consistency for accept/reject decisions*) A rate of return φ is weakly NPV-consistent if and only if a decision maker adopting φ makes the same decision suggested by the NPV criterion. In formal terms, φ is NPV-consistent if, given a cutoff rate k, the following statements are true:

- an investment project creates value if and only if $\varphi > k$
- a financing project creates value if and only if $\varphi < k$.

In real-life applications, to evaluate a project and make a decision on project acceptability, the analyst draws, for each period, the project's pro forma financial statements (balance sheets and income statements) where prospective incomes and book values are determined. More precisely, the analyst estimates, for every $t = 0, 1, \dots, p$, the incomes, I_t, and the book values, b_t, which represents the amount of invested capital at the beginning of period $[t, t+1]$. The initial book value coincides with the initial investment (i.e., $b_0 = -F_0$) and the terminal book value (after liquidation) is equal to zero (i.e., $b_p = 0$). After estimating incomes and book values, the analyst derives the cash flows, often called *free cash flows* (FCF), by subtracting the changes in book value from the incomes:

$$F_t = I_t - \Delta b_t, \tag{1}$$

[1]The COC can be determined in various way, using some asset pricing models, which may be integrated by (or even replaced by) subjectively determined thresholds (see Magni 2009, 2019). In finance, the recommended COC is the *weighted average cost of capital* (WACC). Its significance, estimation and relation with the cost of equity and the cost of debt have been extensively investigated in the literature (see, for example, Arditti and Levy 1977, Miles and Ezzel 1980, Cigola and Peccati 2005, Block 2011, Massari, Roncaglio and Zanetti 2008, Dempsey 2013. See also Magni 2019 and references therein). Consistently with MM (2018), we assume k is exogenously given and time-invariant (a usual assumption in finance).

where $\Delta b_t = b_t - b_{t-1}$. The pro forma financial statements along with Equation (1) represent a standard tool in finance and in industry and are the basis for the financial modelling of capital asset projects.[2] Hence, the NPV may be framed in terms of incomes and changes in book value: $\text{NPV} = -b_0 + \sum_{t=1}^{p}(I_t - \Delta b_t)/(1+k)^t$.

Magni (2010) proved that, for *any* stream $\boldsymbol{C} = (C_0, C_1, C_2, \ldots, C_{n-1})$ of capital amounts such that $C_0 = -F_0$ and any stream $\boldsymbol{J} = (0, J_1, J_2, \ldots, J_n)$ of profits such that

$$F_t = J_t - \Delta C_t, \tag{2}$$

the following equality holds:

$$\text{NPV}(1+k) = C(\bar{\imath} - k) \tag{3}$$

where

$$\bar{\imath} = \frac{J}{C} \tag{4}$$

is an Average Internal Rate of Return (AIRR) and $C = \sum_{t=1}^{p} C_{t-1}(1+k)^{-(t-1)}$ and $J = \sum_{t=1}^{p} J_t(1+k)^{-(t-1)}$ (see also Magni 2013).

If $C > 0$ the project is defined a net investment, whereas if $C < 0$ the project is defined a net financing (Magni 2010, 2013). Therefore, the financial nature of any project (and its associated average ROI) can be identified as an investment project or a financing project (respectively, an investment rate or a financing rate).

Equation (1) is a special case of (2). MM (2018) precisely used eq. (4) picking up the book value capitals invested in the project (i.e., $C_t = b_t$) and the vector of pro forma accounting incomes (i.e., $J_t = I_t$).

With this choice, (4) becomes the so-called *average Return On Investment* (ROI), here denoted as $\bar{\imath}(b)$:

$$\bar{\imath}(b) = \frac{I}{b} = \frac{\text{Total profit}}{\text{Total invested capital}} \tag{5}$$

where $I = \sum_{t=1}^{p} I_t(1+k)^{-(t-1)}$ represents the overall profit which the project is expected to generate and $b = \sum_{t=1}^{p} b_{t-1}(1+k)^{-(t-1)}$ represents the total invested capital (pro forma book values).

It is important to stress that, in an industrial project, the invested capital, quantified by b_t, may consist of net fixed assets or working capital (or both):

- net fixed assets (NFA) are depreciable assets (property, plant and equipment)
- working capital (WC) is made up of inventories and accounts receivables, net of accounts payable.

Therefore, $b_t = \text{NFA}_t + \text{WC}_t$.

[2] "The first thing we need when we begin evaluating a proposed investment is a set of pro forma, or projected, financial statements. Given these, we can develop the projected cash flows from the project. Once we have the cash flows, we can estimate the value of the project" (Ross, Westerfield, and Jordan 2011, p. 271); "free cash flow is the total amount of cash available for distribution to the creditors who have loaned money to finance the project and to the owners who have invested in the equity of the project. In practice this cash flow information is compiled from pro forma financial statements" (Titman, Keown, and Martin 2011, p. 383). Equation (1) is also known as *clean surplus relation* (Brief and Peasnell 1996).

Let R_t and OpC_t be the revenues and operating costs, respectively (excluding depreciation and taxes); let $\mathrm{Dep}_t = -\Delta\mathrm{NFA}_t$ be the depreciation charge for the fixed assets with $\Delta\mathrm{NFA}_t = \mathrm{NFA}_t - \mathrm{NFA}_{t-1}$, and let τ be the company tax rate.[3] Therefore, the project's (operating) income, I_t, is equal to

$$I_t = (\mathrm{Rev}_t - \mathrm{OpC}_t - \mathrm{Dep}_t)(1-\tau).$$

This income is often called in finance *net operating profit after taxes* (NOPAT). Using (1), the FCF is

$$F_t = \overbrace{(\mathrm{Rev}_t - \mathrm{OpC}_t - \mathrm{Dep}_t)(1-\tau)}^{I_t} - \overbrace{(\Delta\mathrm{NFA}_t + \Delta\mathrm{WC}_t)}^{\Delta b_t} \tag{6}$$

where $\Delta\mathrm{WC}_t = \mathrm{WC}_t - \mathrm{WC}_{t-1}$, $\Delta\mathrm{WC}_0 = \mathrm{WC}_0$. According to eq. (6), the NPV depends on several key parameters, including the working capital (via $\Delta\mathrm{WC}_t$). However, in their formulation of the book value capital, MM (2018) implicitly assumed that the working capital is zero, implying that $b_t = \mathrm{NFA}_t$ and

$$\begin{aligned} F_t &= (\mathrm{Rev}_t - \mathrm{OpC}_t - \mathrm{Dep}_t)(1-\tau) - \overbrace{\Delta\mathrm{NFA}_t}^{\Delta b_t} \\ &= (\mathrm{Rev}_t - \mathrm{OpC}_t - \mathrm{Dep}_t)(1-\tau) + \mathrm{Dep}_t \end{aligned} \tag{7}$$

which is eq. (6) with $\Delta\mathrm{WC}_t = 0$ (see MM 2018, eq. (1)).[4]

2.2 Strong NPV-consistency of rates of return

MM (2018) introduced a stronger definition of NPV-consistency presented by taking into account the sources of investment risk. Their definition is based upon the project's value drivers and sensitivity analysis (SA). Specifically, let f be a valuation metric defined on the *parameter space* A, which maps the vector of inputs (or parameters or value drivers) $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n) \in A \subset \mathbb{R}^n$ onto the model output $y(\alpha)$:

$$f : A \subset \mathbb{R}^n \to \mathbb{R}, \quad y = f(\alpha), \quad \alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n). \tag{8}$$

The vector of value drivers, α, collects the key assumptions on sales revenues and costs, including labor costs, energy costs, materials, selling, general, and administrative expenses, etc. Let $\alpha^0 = \left(\alpha_1^0, \alpha_2^0, \ldots, \alpha_n^0\right) \in A$ be the *base-case* value, a representative value for the parameters. The relevance of a parameter α_i, also known as *importance measure*, quantifies the effect on y of a change in α_i. Let R_i^f be the relevance of parameter α_i and let $R^f = \left(R_1^f, R_2^f, \ldots, R_n^f\right)$ be the vector of the relevances: If $|R_i^f| > |R_j^f|$, then parameter

[3]The rate τ is the company's *marginal* tax rate, which is applied to the incremental gross operating profit generated by the project. If it is positive, it means that the project-with-the-firm will pay additional taxes as opposed to the firm-without-the-project; if it is negative, it means that, the firm-with-the-project will pay less taxes than the firm-without-the-project.

[4]As opposed to the zero-WC case, and assuming other tings unvaried, nonzero WC affects cash flows (an, therefore, NPV) in the following way. If, in a given period $[t-1, t]$, WC increases (i.e., $\Delta WC_t > 0$, the FCF is smaller than in the zero-WC case. In contrast, if WC decreases (i.e., $\Delta\mathrm{WC}_t < 0$), the FCF is greater than in the zero-WC case. Overall, the role of working capital on NPV depends on the timeline of signs and magnitudes of changes, $(\Delta\mathrm{WC}_0, \Delta\mathrm{WC}_1, \ldots, \Delta\mathrm{WC}_n)$ with $\Delta\mathrm{WC}_0 = \mathrm{WC}_0$.

α_i has a rank higher than α_j. We denote as r_i^f the rank of parameter α_i and denote as $r^f = \left(r_1^f, r_2^f, \ldots, r_n^f\right)$ the rank vector.

Example 1. Consider the NPV of a project and let φ be a different valuation metric. Assume the vector of relevances are

$$R^{\text{npv}} = (0.1,\ -0.3,\ 0.2,\ 0.05,\ 0.35)$$

for NPV and

$$R^{\varphi} = (0.07,\ 0.35,\ 0.15,\ 0.03,\ 0.40)$$

for φ. Since the rank is determined by the absolute value of the importance measure, NPV and φ determine the same ranking: $r^{\text{npv}} = r^{\varphi} = (4,\ 2,\ 3,\ 5,\ 1)$, which means that parameter 5 has the highest rank, followed by parameter 2, then parameter 3, parameter 1, and, finally, parameter 4, which has the smallest impact. $\Diamond$

MM (2018) supplied the following definition of strong NPV-consistency.

Definition 3. (*Strong NPV-consistency for accept-reject decisions*) Given an SA technique, a metric φ (and its associated decision criterion) is strongly NPV-consistent if

– φ is weakly NPV-consistent (Definition 2)

– the rank vector of φ is equal to the rank vector of NPV: $r^{\text{npv}} = r^{\varphi}$.

If φ is strongly NPV-consistent and, in addition, the vectors of the relevances coincide, $R^{\text{npv}} = R^{\varphi}$, then φ is strictly NPV-consistent.

In Example 1, φ is strongly NPV-consistent, since $r^{\text{npv}} = r^{\varphi}$. However, it is not strictly NPV-consistent, for the relevances are different. For instance, focusing on parameter 1, the relevance is $R_1^{\text{npv}} = 0.1$ for NPV and $R_1^{\varphi} = 0.07$ for φ.

There are many ways of defining a vector of relevances, each one associated with a specific SA technique (see Borgonovo and Plischke 2016, and Pianosi et al. 2016, for review of SA methods). MM (2018) coped with several different techniques. The authors showed that, if φ is an affine transformation of NPV, that is, $\varphi(\alpha) = m \cdot \text{NPV}(\alpha) + q$ for all $\alpha \in A$ with $m, q \in \mathbb{R}$, then φ is strictly NPV-consistent under the following techniques: (i) Standardized regression coefficient (ii) Sensitivity Indices in variance-based decomposition methods (iii) Finite Change Sensitivity Indices (iv) Helton's index (v) Normalized Partial Derivative (NP2) (vi) Differential Importance Measure.
Finally, the authors showed that the average ROI, $\bar{\imath}(b)$, is an affine transformation of NPV. Precisely, they showed that

$$\bar{\imath}(b) = k + \frac{\text{NPV}(\alpha)(1+k)}{b} \tag{9}$$

where $\text{NPV}(\alpha)$ highlights the dependence of NPV on α, the vector of value drivers. Therefore, they concluded that the average ROI is strictly NPV-consistent.

However, note that the typical stream of value drivers α in a capital asset project may be partitioned into three groups:

- sales revenues (prices, quantity, growth rates)
- cost of goods sold (labor costs, material, energy, overhead, etc.)
- selling, general and administrative costs.

All these items affect cash flows. In many cases, working capital is present, either because inventory is needed (e.g., manufacturing firms) and/or because purchases of material is made on credit (so that accounts payable are nonzero) and/or because sales are made on credit (so that accounts receivable are nonzero). If WC is present, it may or may not be affected by the above mentioned value drivers. Overall, there are three possibilities:

1. WC is zero for all t
2. WC is nonzero for some t and is unaffected by revenues and costs (i.e., it is, so to say, *exogenous*)
3. WC is nonzero for some t and is affected by revenues and/or costs (i.e., it is, so to say, *endogenous*).

As mentioned above, MM (2018) assumed zero WC (case 1), which implies that $b = \sum_{t=1}^{n} \text{NFA}_t(1+k)^{-(t-1)}$ does not depend on α. Case 2 might occur, for example, when WC is estimated to be a given percentage of NFA. Or, alternatively, when WC is managed so as to remain constant until the liquidation date (e.g., Hartman 2007. In the latter case, $\Delta\text{WC}_t = 0$ for all t except $t = 0$ and $t = p$). Case 3 may occur, for example, whenever inventory and accounts payable are estimated to be a percentage of operating costs, while accounts receivable are a percentage of the sales revenues (e.g., see Titman and Martin 2011). In this case, FCF is obtained as

$$F_t = (\text{Rev}_t - \text{OpC}_t - \text{Dep}_t)(1-\tau) - \overbrace{(\Delta\text{NFA}_t + \Delta\text{WC}_t(\alpha))}^{\Delta b_t(\alpha)}. \tag{10}$$

Note that in this case the book value depends on α: $b_t = b_t(\alpha) = \text{NFA}_t + \text{WC}_t(\alpha)$. This means that the average ROI,

$$\bar{\imath}(b) = \bar{\imath}\big(b(\alpha)\big) = k + \frac{\text{NPV}(\alpha)(1+k)}{b(\alpha)} \tag{11}$$

ceases to be an affine transformation of NPV, since $(1+k)/b(\alpha)$ is not constant under changes in α. Therefore, strong NPV-consistency of average ROI is not guaranteed. Also, note that, regardless of dependence on α, the overall book value may be equal to zero. In this case, the average ROI does not exist.

Contrary to MM (2018), we allow for the more general case of nonzero working capital ($\text{WC}_t \neq 0$) and, in the next section, we investigate a performance metric which is strongly NPV-consistent.

3 Searching for strongly NPV-consistent measures: IRR and SLRR

The strong NPV-consistency of a rate of return, φ, introduced in MM (2018), enables the analyst to enrich the economic analysis or even replace NPV with a measure which precisely quantifies the economic efficiency of the project, something which the NPV is not capable to convey.[5] Therefore, the use of rates of return and, in general, relative measures, is especially suitable for project valuation and selection under budget constraints, where capital amounts are managed as scarce resources (see also the Introduction). However, contrary to MM (2018), we now allow for nonzero WC and, in particular, for the case where WC is endogenous, meaning that it depends on revenues and costs, which is a most usual case in industrial applications.

Since the average ROI does not guarantee strong NPV-consistency in the presence of uncertain WC, in this work we search for alternative valuation metrics. To this end, we consider the possibility of using the average rate of change of the book value to build an economically significant capital base and a related rate of return which may be strongly NPV-consistent, as opposed to the average ROI, whenever WC is nonzero and is not exogenously determined. In this respect, we stress that the rate of change in pro forma book values is time-varying.

To this end, we make use of Chisini's (1929) invariance requirement: Given a function $g(y_1, y_2, \ldots, y_p)$ of p data, one replaces the p data with a unique value $\bar{y}$ such that the value of the function remains unvaried: $g(y_1, y_2, \ldots, y_p) = g(\bar{y}, \bar{y}, \ldots, \bar{y})$. The number $\bar{y}$ is called the *Chisini mean* of $y_1, y_2, \ldots, y_p$.[6]

We consider the rate of change of the book value between $t-1$ and t. Now, the initial invested capital is $C_0 = b_0$ and there are (at least) two ways to formalize the rate of change of the invested capital, in geometric or linear shape. In the former case, the rate of change, denoted as x_t, is such that $E_t = C_{t-1}(1 + x_t)$, where $E_t = C_t + \text{FCF}_t$ is the end-of-period capital value; in the latter case, the rate of change, denoted as λ_t, is such that $C_t = C_{t-1} - \lambda_t C_0 = C_{t-1} - \lambda_t b_0$. These two mutually exclusive framings imply, respectively,

1. $C_p = -\sum_{t=0}^{p} F_t(1 + x_{t+1}) \cdot (1 + x_{t+2}) \cdot \ldots \cdot (1 + x_p)$
2. $C_p = b_0(1 - \lambda_1 - \lambda_2 - \ldots - \lambda_p)$.

[5]NPV is affected by the project scale and correctly provides the shareholders wealth increase, but it does not tell how efficiently money is managed. For this, one needs a rate of return.

[6]For example, in financial mathematics the compounding factor for a three-period investment is $g(y_1, y_2, y_3) = (1 + y_1)(1 + y_2)(1 + y_3)$, where y_i is the capital growth rate in period i. The Chisini mean of y_1, y_2, y_3 with respect to g is that unique value $\bar{y}$, named average growth rate, such that $(1 + y_1)(1 + y_2)(1 + y_3) = (1 + \bar{y})^3$ that is, $\bar{y} = \sqrt[3]{(1 + y_1)(1 + y_2)(1 + y_3)} - 1$.

Applying Chisini invariance requirement upon both, one gets the equations

$$\sum_{t=0}^{p} F_t(1+x_{t+1})\cdot(1+x_{t+2})\cdot\ldots\cdot(1+x_p) = \sum_{t=0}^{p} F_t(1+x)^{p-t}$$
$$b_0(1-\lambda_1-\lambda_2-\ldots-\lambda_p) = b_0\underbrace{(1-\lambda-\lambda-\ldots-\lambda)}_{1-p\lambda}.$$

The first equation is not solvable analytically. However, recalling that $C_p = 0$, it may be rewritten as

$$\sum_{t=0}^{p} F_t(1+x)^{-t} = 0. \tag{12}$$

The solution of this equation, x, is the well-known internal rate of return (IRR). As a result, the first candidate for replacing the average ROI is the IRR. We denote the associated overall average capital as $C^x = \sum_{t=1}^{p}\sum_{j=t}^{p} F_j(1+x)^{t-1-j}\cdot(1+k)^{-(t-1)}$.

As for the second equation, it has a (unique) solution, λ, such that

$$\lambda = \frac{\sum_{t=1}^{p}\lambda_t}{p} = \frac{1}{p}.$$

This means that the average capital, denoted as C_t^{sl}, is $C_t^{sl} = C_{t-1}^{sl} - b_0/p = b_0(1-t/p)$. Hence, the overall average capital is $C^{sl} = \sum_{t=1}^{p} b_0(1-(t-1)/p)(1+k)^{-(t-1)}$. Picking $C_t = C_t^{sl}$ in (2), and denoting as I_t^{sl} the corresponding "average" profit J_t[7], one gets

$$I_t^{sl} = C_t^{sl} + F_t - C_{t-1}^{sl} = F_t - \lambda b_0 = F_t - \frac{b_0}{p}.$$

Following eq. (4), one divides the overall profit I^{sl} by the total average capital C^{sl}. The result is the second candidate for substituting the average ROI:

$$\bar{\imath}(C^{sl}) = \frac{I^{sl}}{C^{sl}} = \frac{\sum_{t=1}^{p}(F_t - \frac{b_0}{p})\cdot(1+k)^{-(t-1)}}{\sum_{t=1}^{p} b_0\cdot\left(1-\frac{t-1}{p}\right)(1+k)^{-(t-1)}}. \tag{13}$$

We call $\bar{\imath}(C^{sl})$ the *average, straight-line rate of return* (SLRR). For simplicity, we henceforth denote it with the symbol $\bar{\imath}^{sl}$.

Example 2. A 4-period investment project has book value capitals represented by the vector $\mathbf{b} = (100, 60, 70, 15, 0)$. Therefore, in linear shape the period depreciation rates are $\lambda_1 = 40\%$, $\lambda_2 = -10\%$, $\lambda_3 = 55\%$, $\lambda_4 = 15\%$. The invested capital at time 0 is $b_0 = -F_0 = 100$ and the average rate of change is the Chisini mean of period depreciation rates: $\lambda = 25\% = (40\% - 10\% + 55\% + 15\%)/4 = 1/4$; the average capital is then $\boldsymbol{C}^{sl} = (100, 75, 50, 25, 0)$. Figure 1 represents the dynamics of the book value and the average capital. $\Diamond$

As (3) holds for any $\boldsymbol{C}$ and associated $\boldsymbol{J}$, both IRR and SLRR are weakly NPV-consistent (see Hazen 2003 and Magni 2010).

This means that both are good candidates as substitutes for the average ROI whenever WC depends on the value drivers.

[7] More precisely, this is the profit which is associated with the average capital.

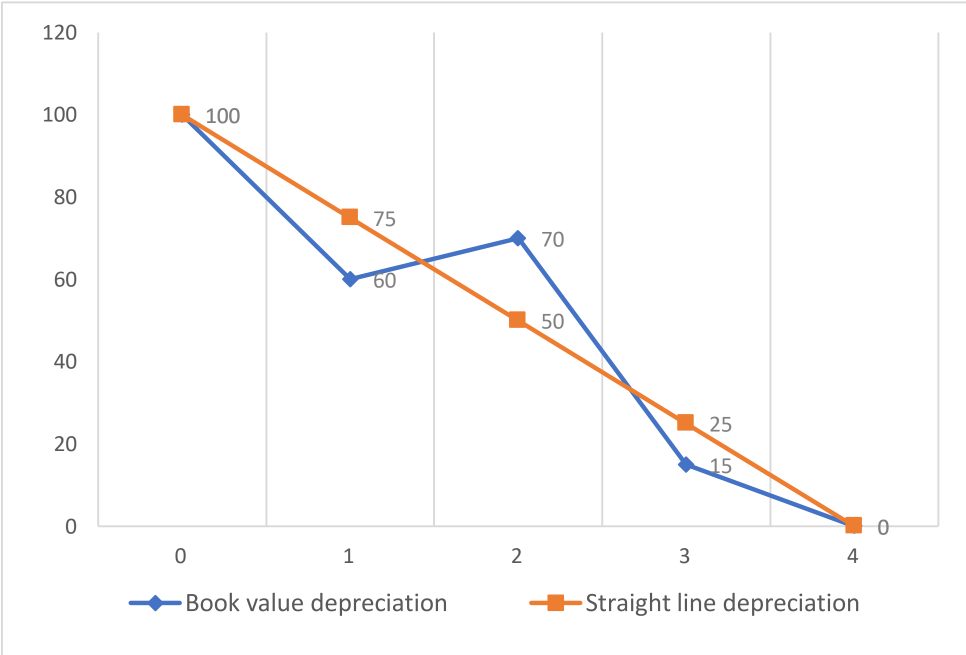

Figure 1: Average depreciation

We now need analyze whether they are strongly NPV-consistent or not and, if not, we aim at measuring their degree of inconsistency, which is a signal of their reliability.

However, we anticipate that, regardless of strong NPV-consistency, IRR is known to be subject to some difficulties. Among others, owing to the way it is derived, it may not exist or multiple IRRs may arise: For instance, engineering projects with considerable length and numerous changes in sign of cash flows, possibly due to disposal and remediation costs, may have no IRR or multiple IRRs (Magni 2013, Hartman 2007). More simply, any project which does not require investment in equity (i.e., outflows are financed with either debt or liquid assets or both) has no IRR for shareholders.[8]

Also, the financial nature of the IRR depends upon the COC, k, as C^x is not necessarily invariant under changes in k (see Magni 2013 for a compendium).

Contrary to IRR and average ROI, the SLRR has the nice property of existence. It always exists, because $b_0 = -F_0 \neq 0$.[9] Also, contrary to IRR, it is unique, since it is derived from a linear equation. Furthermore, its financial nature is not affected by the revenues and costs, being unambiguously determined by the sign of b_0, which coincides

[8]For example, suppose a firm purchases a piece of equipment for an amount of \$10 in order to increase production and sales. Suppose it is financed by withdrawing cash from the firm's bank account (or by selling some marketable securities). Incremental cash flows are expected to be equal to \$3, \$6, \$12 at times 1, 2, and 3, respectively. Suppose the firm's liquid assets are currently invested at 1%. Therefore, there is no incremental outflow for the firm's shareholders ($\$10 - \$10 = 0$) and the prospective incremental inflows for shareholders will be \$3, \$6, and \$1.7 ($= 12 - 10(1.01)^3$). The resulting cash-flow stream is $(0, 3, 6, 1.7)$, which possesses no real-valued IRR.

[9]Even if $F_0 = 0$, one may redefine b_0 as the first nonzero book value and neglect the previous zero cash flows. For example, if $\boldsymbol{F} = (0, 0, 0, -200, 100, 140)$, one may reframe the cash-flow stream as $\boldsymbol{F} = (-200, 100, 140)$ and set $b_0 = 200$.

with the sign of C^{sl} for any given k: $C^{sl} > 0$ if and only if $b_0 > 0$.

Example 3. Consider a project P such that $F = (-10,\ 23,\ -17,\ 24,\ -22)$ and a COC equal to $k = 32\%$. The NPV is $0.86 = -10+23\cdot 1.32^{-1}-17\cdot 1.32^{-2}+24\cdot 1.32^{-3}-22\cdot 1.32^{-4}$; therefore the project is worth undertaking. Two IRRs exist: $x_1 = 11.2\%$ and $x_2 = 67\%$. The former is associated with the stream $\boldsymbol{C}^{x_1} = (10, -6.3, 6.5, -13.2, 0)$, the latter is associated with the stream $\boldsymbol{C}^{x_2} = (10, -11.9, 3.8, -19.8, 0)$. The overall capital associated with x_1 is $C^{x_1} = 2.4 > 0$, the overall capital associated with x_2 is $C^{x_2} = -4.1 < 0$. Therefore, IRR does not unambiguously determine the financial nature of the project: According to the first IRR, the project is an investment, according to the second IRR the project is a financing. The first IRR is a rate of return, the second IRR is a rate of cost. Conversely, the SLRR exists and is unique in any case, and unambiguously identifies the project as an investment, since the associated capital stream is $\boldsymbol{C}^{sl} = (10,\ 7.5,\ 5,\ 2.5,\ 0)$ so that the total average capital is $C^{sl} = 14.9 > 0$. The SLRR is then $\bar{\imath}^{sl} = 0.32+0.86(1+0.32)/14.9 = 37.8\%$. $\Diamond$

We now show that SLRR is strongly NPV-consistent, in a strict sense.

Proposition 1. *For any fixed k, C_0, and p, SLRR is strictly NPV-consistent for accept-reject decisions.*

Proof. Recalling that (3) holds irrespective of the capital stream $\boldsymbol{C}$ and picking $C = C^{sl}$, one gets $\text{NPV}(\alpha)(1+k) = C^{sl}(\bar{\imath}^{sl} - k)$ where $C^{sl} = \sum_{t=1}^{p}\big(b_0(1-(t-1)/p)(1+k)^{-(t-1)}\big)$ does not depend on α. This implies

$$\bar{\imath}^{sl} = k + \frac{\text{NPV}(\alpha)(1+k)}{C^{sl}}. \tag{14}$$

This means $\varphi = q + m\cdot\text{NPV}(\alpha)$ where $\varphi = \bar{\imath}^{sl}$, $q = k$ and $m = (1+k)/C^{sl}$. Therefore, the SLRR is an affine transformation of NPV. The thesis follows from MM (2018, Proposition 1). $\square$

The proposition above shows that SLRR and NPV are identically influenced by the variation of the project's value drivers, not only in terms of ranks ($r^{\text{npv}} = r^{\text{slrr}}$) but also in terms of relevances ($R^{\text{npv}} = R^{\text{slrr}}$). This ensures the equivalence of NPV and SLRR criteria for investment decisions even when working capital is nonzero and is estimated on the basis of revenues and costs.

As for IRR, note that it is an implicit function of the value drivers, since it depends on revenues and costs, both directly (via Rev_t and OpC_t) irrespective of whether WC is zero or not and irrespective of how it is estimated:

$$\sum_{t=0}^{p}\big((\text{Rev}_t - \text{OpC}_t - \text{Dep}_t)(1-\tau) - (\text{NFA}_t - \text{NFA}_{t-1}) - (\text{WC}_t - \text{WC}_{t-1})\big)(1+x)^{-t} = 0.$$

Therefore, in general, it is not possible to determine an analytical relationship between NPV and IRR (see also Borgonovo and Peccati 2004, 2006, Percoco and Borgonovo 2012). Indeed, let $\alpha^* \in A$ be a given value of parameters and x^* be the associated IRR, such that $\text{NPV}(\alpha^*, x^*) = 0$.[10] If there exists a neighbourhood of α^* where function $\text{NPV}(\alpha, k)$ is a continuously differentiable function and $\frac{\partial \text{NPV}}{\partial k}(\alpha^*, x^*) \neq 0$, then there exists a neighbourhood $V(\alpha^*) \subset A$ and a neighbourhood $W(x^*) \subset \mathbb{R}$ such that $x(\alpha) : V \to W$ is the implicitly-defined function from the equation $\text{NPV}(\alpha, k) = 0$ and

$$x(\alpha^*) = x^*,$$
$$\text{NPV}(\alpha, x(\alpha)) = 0, \ \forall \alpha \in V,$$
$$\frac{\partial x}{\partial \alpha_i}(\alpha) = -\frac{\frac{\partial \text{NPV}}{\partial \alpha_i}(\alpha, x(\alpha))}{\frac{\partial \text{NPV}}{\partial k}(\alpha, x(\alpha))}, \ \forall \alpha \in V.$$

In particular,

$$\frac{\partial x}{\partial \alpha_i}(\alpha^*) = -\frac{\frac{\partial \text{NPV}}{\partial \alpha_i}(\alpha^*, x^*)}{\frac{\partial \text{NPV}}{\partial k}(\alpha^*, x^*)}. \tag{15}$$

Therefore, IRR is not an affine transformation of NPV. In the next section, we demonstrate, via some counterexamples, that IRR may not be used for accomplishing *ex ante* risk analysis or *ex post* performance measurement for several different reasons:

- it is not strongly NPV-consistent
- it may not exist in some scenario
- multiple IRRs may arise
- the financial nature of IRR may change under changes in the value drivers.

In contrast, the SLRR always exists, is unique, possesses an unambiguous financial nature, and enjoys strong NPV-consistency.

For reasons of space, we limit the analysis to two SA techniques: The Finite Change Sensitivity Index (FCSI) (Borgonovo 2010a) and Differential Importance Measure (DIM) (Borgonovo and Apostolakis 2001, Borgonovo and Peccati 2004). The FCSI index is particularly useful when two different scenarios for the value drivers are compared, namely, α^0 (base value or base case) and α^1 (perturbed value). It may be used for *ex ante* analysis, when the analyst aims to compare a base case and a possible different scenario or, more compellingly, for *ex post* auditing, when the analyst wants to investigate the source of variation of the actual performance (α^1) with respect to the expected one (α^0). The DIM is useful when not-so-large deviations around the base value are assumed; therefore, it is most useful in ex ante decision-making to measure the major sources of risk in terms of key parameters.

Furthermore, we need avail ourselves of a measure for quantifying the degree of NPV-inconsistency of average ROI or IRR: The higher the degree of inconsistency, the smaller the reliability of average ROI or IRR. We comply with MM's (2018) choice of the Spearman's rank correlation coefficient (Spearman 1904) and top-down correlation coefficient (Iman and Conover 1987). Spearman's coefficient is the correlation coefficient of the rank vectors r^{npv} and r^{φ}: $\rho_{\text{npv},\varphi} = \frac{\text{Cov}(r^{\text{npv}}, r^{\varphi})}{\sigma(r^{\text{npv}}) \cdot \sigma(r^{\varphi})}$. The top-down correlation coefficient, introduced by Iman and Conover (1987), attributes a higher weight to top parameters than to

[10]Let α be a generic value belonging to a neighbourhood of α^*. $\text{NPV}(\alpha, k)$ is the NPV calculated with discount rate k.

low parameters, based on *Savage Score* (Savage 1956). The Savage score of parameter α_i is $S_i^{\text{npv}} = \sum_{h=r_i^{\text{npv}}}^{n} \frac{1}{h}$. For example, considering a vector of $n = 8$ value drivers, such that $\alpha = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8)$ and assuming α_2 has rank $r_2^{\text{npv}} = 3$, then its Savage score will be

$$S_2^{\text{npv}} = \sum_{h=3}^{8} \frac{1}{h} = \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} = 1.218.$$

In general, the Savage scores' vector of f is $S^f = \left(S_1^f, S_2^f, \ldots, S_n^f\right)$. The top-down correlation coefficient between NPV and φ is the correlation coefficient between the Savage scores' vectors S^{npv} and S^{φ} (Iman and Conover 1987): $\rho_{S^{\text{npv}},S^{\varphi}} = \frac{\text{Cov}(S^{\text{npv}},S^{\varphi})}{\sigma(S^{\text{npv}})\cdot\sigma(S^{\varphi})}$.

The coefficients $\rho_{\text{npv},\varphi}$ and $\rho_{S^{\text{npv}},S^{\varphi}}$ are equal to 1 if and only if φ is strongly NPV-consistent. The smaller the value of $\rho_{\text{npv},\varphi}$ and $\rho_{S^{\text{npv}},S^{\varphi}}$, the higher the degree of NPV-inconsistency. The differences $1 - \rho_{\text{npv},\varphi}$ and $1 - \rho_{S^{\text{npv}},S^{\varphi}}$ can be taken as representative of the degree of inconsistency.

4 Comparison of SLRR and IRR using FCSI

In this section, as well as in Section 5, we assume that working capital is equal to zero (e.g., customers pay in cash, suppliers are paid in cash, and no inventory exists) (in section 6 we will remove this assumption). We also assume $\tau = 0$. Therefore, $\text{FCF}_t = \text{Rev}_t - \text{OpC}_t$, $\forall t > 0$. We focus on the FCSI technique (see eqs. (17)-(18) in the Appendix of this paper) and illustrate four numerical applications, aimed at presenting the problems of the IRR:

1. in the first application, IRR exists and is unique but is not strongly NPV-consistent[11]
2. in the second application, despite IRR exists and is unique in the base case α^0 , it does not exist in α^1 (or vice versa), making it impossible to perform the SA
3. in the third application, multiple IRRs arise for $\alpha = \alpha^1$
4. in the fourth application, IRR changes its financial nature from investment rate (in α^0) to financing rate (in α^1).

No such problems will arise with (average ROI and) SLRR, which is strictly NPV-consistent.[12]

We will consider the simple model described in MM (2018), consisting of a firm facing the opportunity of investing in a 4-period project whose estimated revenues and costs are denoted as Rev_t and OpC_t. As anticipated, the FCF is $\text{FCF}_t = \text{Rev}_t - \text{OpC}_t$. The project's value drivers are then $\alpha_i = \text{Rev}_i$ for $i = 1, 2, 3, 4$ and $\alpha_i = \text{OpC}_{i-4}$ for $i = 5, 6, 7, 8$. Hence, the value drivers' vector for the base case is

$$\alpha^0 = (\text{Rev}_1^0, \text{Rev}_2^0, \text{Rev}_3^0, \text{Rev}_4^0, \text{OpC}_1^0, \text{OpC}_2^0, \text{OpC}_3^0, \text{OpC}_4^0)$$

[11] Examples of this kind of shortcoming for IRR are also described in Borgonovo and Peccati (2004, 2006), Percoco and Borgonovo (2012), which show that parameter rankings for NPV and IRR are different.

[12] To compute SLRR, one may either use the definition in (13) or the shortcut in (14).

while the value drivers' vector for the alternative (perturbed) case is

$$\alpha^1 = (\mathrm{Rev}_1^1, \mathrm{Rev}_2^1, \mathrm{Rev}_3^1, \mathrm{Rev}_4^1, \mathrm{OpC}_1^1, \mathrm{OpC}_2^1, \mathrm{OpC}_3^1, \mathrm{OpC}_4^1).$$

NPV is computed as:

$$\mathrm{NPV}(\alpha) = -C_0 + \frac{\mathrm{Rev}_1 - \mathrm{OpC}_1}{1+k} + \frac{\mathrm{Rev}_2 - \mathrm{OpC}_2}{(1+k)^2} + \frac{\mathrm{Rev}_3 - \mathrm{OpC}_3}{(1+k)^3} + \frac{\mathrm{Rev}_4 - \mathrm{OpC}_4}{(1+k)^4}.$$

Example 4. (*NPV inconsistency*) Assume $C_0 = 750$ and $k = 10\%$. Table 1 describes the base value α^0 and reports the corresponding FCFs and valuation metrics. The NPV is $157.37 = -750 + 380/1.1 + 270/(1.1)^2 + 360/(1.1)^3 + 100/(1.1)^4$, the vector of average capitals is $\boldsymbol{C}^{sl} = (750, 562.5, 375, 187.5, 0)$ and the overall average capital is $C^{sl} = 1{,}712.15 = 750 + 562.5/1.1 + 375/(1.1)^2 + 187.5/(1.1)^3$. Therefore, SLRR is equal to $\bar{\imath}^{sl} = 10\% + 157.37/1,712.15 \cdot 1.1 = 20.11\%$. The IRR exists and is unique, $x = 20.86\%$.

Table 1: Investment evaluated in α^0

	0	1	2	3	4
Rev_t^0		580	570	560	400
OpC_t^0		200	300	200	300
F_t	-750	380	270	360	100
Valuation					
NPV	157.37				
$\bar{\imath}^{sl}$	20.11%				
x	20.86%				

Table 2 reports the alternative scenario α^1 and the corresponding new values of F_t, NPV, SLRR, and IRR. In α^1, NPV is 442.92, SLRR is 38.46%, IRR is 41.12% (it exists and is unique). The observed variations are: $\Delta\mathrm{NPV} = 285.55 = 442.92 - 157.37$; $\Delta\bar{\imath}^{sl} = 18.35\% = 38.46\% - 20.11\%$; $\Delta x = 20.25\% = 41.12\% - 20.86\%$.

Table 3 shows the First Order FCSIs ($\Phi_i^{1,f}$), the ranks (r_i^f), and the Savage Scores (S_i^f) for NPV, SLRR and IRR. The (ranks and) importance measures of NPV and SLRR are equal, $\Phi_i^{1,\mathrm{npv}} = \Phi_i^{1,\mathrm{slrr}}$, meaning that SLRR is strictly NPV-consistent. The relevances of NPV and IRR are different, $\Phi_i^{1,\mathrm{npv}} \neq \Phi_i^{1,\mathrm{irr}}$, as well the ranks, $r^{\mathrm{npv}} \neq r^{\mathrm{irr}}$, implying that the IRR is not strongly NPV-consistent according to Definition 3. The degree of NPV-inconsistency, measured via (one minus) Spearman's coefficient or top-down coefficient, is $1 - \rho_{\mathrm{irr,npv}} = 1 - 0.857 = 0.143$ and $1 - \rho_{S^{\mathrm{irr}},S^{\mathrm{npv}}} = 1 - 0.77 = 0.23$.

Table 4 shows Total Order FCSIs ($\Phi_i^{T,f}$), ranks (r_i^f), and Savage scores (S_i^f) for the three metrics. The (ranks and) Total Order FCSIs of NPV and SLRR are equal, $\Phi_i^{T,\mathrm{npv}} = \Phi_i^{T,\mathrm{slrr}}$, therefore SLRR is strictly NPV-consistent, whereas the ranks (and relevances) of NPV and IRR are different, implying that the IRR is not strongly NPV-consistent with degree of incoherence equal to $1 - \rho_{\mathrm{irr,npv}} = 1 - 0.667 = 0.333$ and $1 - \rho_{S^{\mathrm{irr}},S^{\mathrm{npv}}} = 1 - 0.409 = 0.591$. This is especially due to the ranking distortion of OpC_4, with rank 1 according to NPV and SLRR, and rank 5 in terms of IRR. $\diamond$

Example 5. (*Nonexistence of IRR in* α^1) Consider a project P such that $C_0 = 750$ and $k = 10\%$. Hence $C^{sl} = 1,712.15$. The base value is described in the revenue-cost vector

Table 2: Investment evaluated in α^1

	0	1	2	3	4
Rev_t^1		800	810	780	630
OpC_t^1		350	250	380	600
F_t	-750	450	560	400	30
Valuation					
NPV	442.92				
$\bar{\imath}^{sl}$	38.46%				
x	41.12%				

Table 3: First Order FCSI

	NPV			SLRR			IRR		
Parameter	$\Phi_i^{1,\mathrm{npv}}$	r_i^{npv}	S_i^{npv}	$\Phi_i^{1,\mathrm{slrr}}$	r_i^{slrr}	S_i^{slrr}	$\Phi_i^{1,\mathrm{irr}}$	r_i^{irr}	S_i^{irr}
Rev_1	70.04%	2	1.718	70.04%	2	1.718	79.78%	1	2.718
Rev_2	69.46%	3	1.218	69.46%	3	1.218	64.05%	3	1.218
Rev_3	57.89%	4	0.885	57.89%	4	0.885	45.56%	5	0.635
Rev_4	55.01%	5	0.635	55.01%	5	0.635	37.68%	7	0.268
OpC_1	-47.76%	6	0.435	-47.76%	6	0.435	-46.93%	4	0.885
OpC_2	14.47%	8	0.125	14.47%	8	0.125	13.68%	8	0.125
OpC_3	-47.36%	7	0.268	-47.36%	7	0.268	-45.25%	6	0.435
OpC_4	-71.76%	1	2.718	-71.76%	1	2.718	-76.83%	2	1.718
Correlations									
$\rho_{\mathrm{slrr},\mathrm{npv}}$	1								
$\rho_{S^{\mathrm{slrr}},S^{\mathrm{npv}}}$	1								
$\rho_{\mathrm{irr},\mathrm{npv}}$	0.857								
$\rho_{S^{\mathrm{irr}},S^{\mathrm{npv}}}$	0.770								

Table 4: Total Order FCSI

	NPV			SLRR			IRR		
Parameter	$\Phi_i^{T,\mathrm{npv}}$	r_i^{npv}	S_i^{npv}	$\Phi_i^{T,\mathrm{slrr}}$	r_i^{slrr}	S_i^{slrr}	$\Phi_i^{T,\mathrm{irr}}$	r_i^{irr}	S_i^{irr}
Rev_1	70.04%	2	1.718	70.04%	2	1.718	75.79%	1	2.718
Rev_2	69.46%	3	1.218	69.46%	3	1.218	65.33%	2	1.718
Rev_3	57.89%	4	0.885	57.89%	4	0.885	44.78%	4	0.885
Rev_4	55.01%	5	0.635	55.01%	5	0.635	34.09%	6	0.435
OpC_1	-47.76%	6	0.435	-47.76%	6	0.435	-57.78%	3	1.218
OpC_2	14.47%	8	0.125	14.47%	8	0.125	13.18%	8	0.125
OpC_3	-47.36%	7	0.268	-47.36%	7	0.268	-31.29%	7	0.268
OpC_4	-71.76%	1	2.718	-71.76%	1	2.718	-34.93%	5	0.635
Correlations									
$\rho_{\mathrm{slrr},\mathrm{npv}}$	1								
$\rho_{S^{\mathrm{slrr}},S^{\mathrm{npv}}}$	1								
$\rho_{\mathrm{irr},\mathrm{npv}}$	0.667								
$\rho_{S^{\mathrm{irr}},S^{\mathrm{npv}}}$	0.409								

$\alpha^0 = (630, 740, 850, 600, 180, 390, 490, 550)$; the revenue-cost vector for the perturbed scenario is $\alpha^1 = (600, 700, 800, 500, 200, 400, 500, 850)$, a worse situation in terms of both revenues and costs. Table 5 reports cash flows, NPV, SLRR, and IRR. In α^0 IRR exists, is unique, and is equal to 28.52%. In α^1 IRR does not exist. This implies that the sensitivity analysis cannot be applied for IRR: Δx is not defined, hence the First Order and Total Order FCSIs of IRR do not exist.

SLRR does not suffer from this problem because it always exists and is unique. Table 6 shows the First Order and Total Order FCSIs of NPV and SLRR: As expected, SLRR is strictly NPV-consistent.

The opposite case may also occur, whereby the IRR does not exist in α^0 while it exists in α^1, resulting in the same kind of pitfall (e.g., just reverse the base-case value and the perturbed value of this example). $\Diamond$

Table 5: IRR not existing in α^1

	α^0	α^1
Cash flows		
F_0	-750	-750
F_1	450	400
F_2	350	300
F_3	360	300
F_4	50	-350
Valuation		
NPV	252.97	-152.09
$\bar{\imath}^{sl}$	26.25%	0.23%
x	28.52%	–

Table 6: IRR not existing in α^1:
First Order and Total Order FCSIs

	NPV		SLRR		IRR	
Parameter	$\Phi_i^{T,\text{npv}} = \Phi_i^{1,\text{npv}}$	r_i^{npv}	$\Phi_i^{T,\text{slrr}} = \Phi_i^{1,\text{slrr}}$	r_i^{slrr}	$\Phi_i^{T,\text{irr}}$	r_i^{irr}
Rev_1	6.73%	5	6.73%	5	–	–
Rev_2	8.16%	4	8.16%	4	–	–
Rev_3	9.27%	3	9.27%	3	–	–
Rev_4	16.86%	2	16.86%	2	–	–
OpC_1	4.49%	6	4.49%	6	–	–
OpC_2	2.04%	7	2.04%	7	–	–
OpC_3	1.85%	8	1.85%	8	–	–
OpC_4	50.59%	1	50.59%	1	–	–

Example 6. (*Nonuniqueness of IRR*) Consider a project P, with $C_0 = 800$ and $k = 15\%$. Therefore, $C^{sl} = 1,755.70$. The base value is described in the input vector $\alpha^0 = (2,300, 1,100, 1,400, 2,000, 1,300, 1,200, 1,600, 1,300)$; the input vector in the perturbed state is $\alpha^1 = (2,960, 500, 400, 2,300, 600, 1,440, 2,750, 550)$. Table 7 shows the cash flows and the valuation metrics in α^0 and α^1. In α^0, the IRR function supplies a unique value and is equal to 36.72%. For α^1, there exist three different IRRs: $x_1(\alpha^1) = 8.07\%$, $x_2(\alpha^1) = 25.0\%$, $x_3(\alpha^1) = 61.93\%$ so the sensitivity analysis is problematic: It is not clear which one IRR should be the relevant one, if any.

Table 8 shows the First Order and Total Order FCSIs of NPV and SLRR: As obvious, SLRR is strictly NPV-consistent. ♢

Table 7: Multiple IRR in α^1

	α^0	α^1
Cash flows		
F_0	-800	-800
F_1	$1,000$	$2,360$
F_2	-100	-940
F_3	-200	$-2,350$
F_4	700	$1,750$
Valuation		
NPV	262.67	-3.20
$\bar{\imath}^{sl}$	32.21%	14.79%
x	36.72%	8.07%; 25.0%; 61.93%

Table 8: Multiple IRR in α^1:
First Order and Total Order FCSIs

	NPV		SLRR		IRR	
Parameter	$\Phi_i^{T,\text{npv}} = \Phi_i^{1,\text{npv}}$	r_i^{npv}	$\Phi_i^{T,\text{slrr}} = \Phi_i^{1,\text{slrr}}$	r_i^{slrr}	$\Phi_i^{T,\text{irr}}$	r_i^{irr}
Rev_1	-215.86%	4	-215.86%	4	–	–
Rev_2	170.64%	5	170.64%	5	–	–
Rev_3	247.31%	2	247.31%	2	–	–
Rev_4	-64.51%	8	-64.51%	8	–	–
OpC_1	-228.94%	3	-228.94%	3	–	–
OpC_2	68.26%	7	68.26%	7	–	–
OpC_3	284.40%	1	284.40%	1	–	–
OpC_4	-161.29%	6	-161.29%	6	–	–

Table 9: IRR changes its financial nature

	α^0	α^1
Cash flows		
F_0	-500	-500
F_1	-700	-400
F_2	$1,345$	$1,695$
F_3	35	385
F_4	340	$-1,210$
Valuation		
NPV	363.24	-6.43
$\bar{\imath}^{sl}$	37.00%	4.43%
x	22.17%	10.00%
x	(investment rate)	(financing rate)

Example 7. (*Financial nature of IRR*) Consider a project P such that $C_0 = 500$ and $k = 5\%$. Therefore $C^{sl} = 1,191.88$. The base case is described in the input vector $\alpha^0 = $ (800, 2,150, 950, 850, 1,500, 805, 915, 510). The perturbed vector is $\alpha^1 =$ (600, 2,000, 800, 800, 1,000, 305, 415, 2,010). The difference between α^0 and α^1 lies in lower revenues for α^1 and in intertemporal cost allocation: The total amount of costs is the same in the two cases, but in α^1 costs are highly concentrated in period 4 (one

may assume remedial costs at the end of the project have been paid). Table 9 shows the project's cash flows and the corresponding NPV, SLRR, and IRR in α^0 and α^1. In the base case IRR exists, is unique, and is equal to 22.17% and the IRR-implied capital vector is $\boldsymbol{C^x} = (500, 1{,}310.85, 256.45, 278.30, 0)$ whence $C^x(\alpha^0) = 2{,}221.44$; therefore, IRR is an investment rate in α^0. In α^1, IRR exists, is unique, and is equal to 10%, associated with the vector $\boldsymbol{C^x} = (500, 950, -650, -1{,}100, 0)$, implying $C^x(\alpha^1) = -135.03 < 0$ which means that the IRR is a financing rate in α^1. This proves that a change in the value drivers' vector may cause IRR to change financial nature (from investment rate to financing rate or viceversa). The decomposition of the output variation with FCSIs is economically dubious, as the model output does not merely change in quantitative terms, but it changes in meaning: No more a rate of return but a financing rate.

SLRR does not suffer from this problem, because its financial nature only depends on the sign of C_0. In this case, SLRR is an investment rate, regardless of changes in the value drivers.

It is worth noting that two or more of the above mentioned problems may occur simultaneously. For instance, IRR changes financial nature from α^0 to α^1 and, at the same time, the importance measure of one of the value drivers, namely the costs in period 4, suffers from a problem of nonexistence: $x(\alpha^1_8, \alpha^0_{(-8)})$ is not defined because the associated cash flows vector $(-500, -700, 1{,}345, 35, -1{,}160)$ does not admit any real IRR > -1, therefore $\Phi^{1,\text{irr}}_8$ does not exist. Consequently, the parameters ranking for IRR is not possible and correlation coefficients are not computable (see Table 10). $\Diamond$

Table 10: First Order FCSI: IRR changes its financial nature

	NPV			SLRR			IRR		
Parameter	$\Phi^{1,\text{npv}}_i$	r^{npv}_i	S^{npv}_i	$\Phi^{1,\text{slrr}}_i$	r^{slrr}_i	S^{slrr}_i	$\Phi^{1,\text{irr}}_i$	r^{irr}_i	S^{irr}_i
Rev_1	51.53%	5	0.635	51.53%	5	0.635	79.64%	–	–
Rev_2	36.80%	6	0.435	36.80%	6	0.435	53.16%	–	–
Rev_3	35.05%	7	0.268	35.05%	7	0.268	45.59%	–	–
Rev_4	11.13%	8	0.125	11.13%	8	0.125	12.17%	–	–
OpC_1	−128.81%	2	1.718	−128.81%	2	1.718	−268.88%	–	–
OpC_2	−122.68%	3	1.218	−122.68%	3	1.218	−175.42%	–	–
OpC_3	−116.84%	4	0.885	−116.84%	4	0.885	−127.90%	–	–
OpC_4	333.82%	1	2.718	333.82%	1	2.718	–	–	–
Correlations									
$\rho_{\text{slrr},\text{npv}}$	1								
$\rho_{S^{\text{slrr}},S^{\text{npv}}}$	1								
$\rho_{\text{irr},\text{npv}}$	–								
$\rho_{S^{\text{irr}},S^{\text{npv}}}$	–								

5 Comparison of IRR and SLRR using DIMs

In this section, we analyze the behavior of IRR and SLRR under the DIM technique, which presupposes small perturbations in the input data and makes use of derivatives (see eq. (21)). In this model, the first partial derivatives of $\text{NPV}(\alpha)$, evaluated in α^0, are

$$\frac{\partial \mathrm{NPV}}{\partial \alpha_i}(\alpha^0) = \begin{cases} (1+k)^{-i}, & i=1,2,3,4; \\ -(1+k)^{-(i-4)}, & i=5,6,7,8 \end{cases} \tag{16}$$

(see also MM 2018). Using (14), the first partial derivatives of SLRR, evaluated in α^0, are

$$\frac{\partial \bar{\imath}^{sl}}{\partial \alpha_i}(\alpha^0) = \mathrm{NPV}'_{\alpha_i}(\alpha^0) \cdot \frac{(1+k)}{C^{sl}}.$$

This implies that SLRR and NPV share the same DIMs and, therefore, SLRR is strictly NPV-consistent, as already stated in Proposition 1.

The case with IRR is more problematic. From (15) and (16),

$$\frac{\partial x}{\partial \alpha_i}(\alpha^0) = \begin{cases} -(1+x^0)^{-i} \cdot (\mathrm{NPV}'_k(\alpha^0, x^0))^{-1}, & i=1,2,3,4; \\ (1+x^0)^{-(i-4)} \cdot (\mathrm{NPV}'_k(\alpha^0, x^0))^{-1}, & i=5,6,7,8 \end{cases}$$

where

$$\frac{\partial \mathrm{NPV}}{\partial k}(\alpha^0, x^0) = -\frac{\mathrm{Rev}_1^0 - \mathrm{OpC}_1^0}{(1+x^0)^2} - 2\cdot\frac{\mathrm{Rev}_2^0 - \mathrm{OpC}_2^0}{(1+x^0)^3} - 3\cdot\frac{\mathrm{Rev}_3^0 - \mathrm{OpC}_3^0}{(1+x^0)^4} - 4\cdot\frac{\mathrm{Rev}_4^0 - \mathrm{OpC}_4^0}{(1+x^0)^5}.$$

This suggests that IRR is not strongly NPV-consistent.

We now illustrate a numerical application of DIM technique which, being a counterexample, shows that the IRR is indeed NPV-inconsistent under DIM according to Definition 3.

Example 8. We consider an investment P, with $C_0 = 900$ and $k = 8\%$. Therefore $C^{sl} = 2{,}089.41$. The base value is $\alpha^0 = (900,\ 1{,}000,\ 1{,}100,\ 1{,}200,\ 600,\ 700,\ 800,\ 900)$. The corresponding cash-flow vector is $F = (-900, 300, 300, 300, 300)$ and $\mathrm{NPV}(\alpha^0) = 93.64$, $\bar{\imath}(\alpha^0) = 12.84\%$, $x(\alpha^0) = 12.59\%$. Table 11 shows the DIMs, the ranks, and the Savage scores. The DIMs for NPV and IRR are different: $DIM_i^{\mathrm{npv}}(\alpha^0) \neq DIM_i^{\mathrm{irr}}(\alpha^0)$. Not even the ranking is equal, therefore IRR is NPV-inconsistent according to Definition 3 and, since $1 - \rho_{\mathrm{irr,npv}} = 0.262$ and $1 - \rho_{S^{\mathrm{irr}},S^{\mathrm{npv}}} = 0.691$, the degree of NPV-inconsistency is remarkable when using top-down coefficient. $\diamondsuit$

Table 11: Coherence under DIM technique

		NPV			SLRR			IRR		
Parameter	α^0	$DIM_i^{\mathrm{npv}}(\alpha^0)$	r_i^{npv}	S_i^{npv}	$DIM_i^{\mathrm{slrr}}(\alpha^0)$	r_i^{slrr}	S_i^{slrr}	$DIM_i^{\mathrm{irr}}(\alpha^0)$	r_i^{irr}	S_i^{irr}
Rev_1	900	83.87%	4	0.885	83.87%	4	0.885	88.82%	1	2.718
Rev_2	1000	86.28%	3	1.218	86.28%	3	1.218	87.65%	2	1.718
Rev_3	1100	87.88%	2	1.718	87.88%	2	1.718	85.64%	3	1.218
Rev_4	1200	88.77%	1	2.718	88.77%	1	2.718	82.97%	4	0.885
OpC_1	600	−55.91%	8	0.125	−55.91%	8	0.125	−59.21%	8	0.125
OpC_2	700	−60.40%	7	0.268	−60.40%	7	0.268	−61.36%	7	0.268
OpC_3	800	−63.91%	6	0.435	−63.91%	6	0.435	−62.28%	5	0.635
OpC_4	900	−66.58%	5	0.635	−66.58%	5	0.635	−62.23%	6	0.435
Correlations										
$\rho_{\mathrm{slrr,npv}}$	1									
$\rho_{S^{\mathrm{slrr}},S^{\mathrm{npv}}}$	1									
$\rho_{\mathrm{irr,npv}}$	0.738									
$\rho_{S^{\mathrm{irr}},S^{\mathrm{npv}}}$	0.309									

6 Non-strong NPV-consistency of average ROI

In the previous sections, we have shown, by means of counterexamples, that the IRR is not strongly NPV-consistent, even though the WC is not present. With this assumption, the average ROI is strictly NPV-consistent, as shown in MM (2018).

In this section, we deal with nonzero WC and assume it depends on value drivers. This implies that the average ROI is not an affine transformation of NPV. It is then natural to make the conjecture that the average ROI is not strongly NPV-consistent. To prove the conjecture, it suffices to provide one counterexample. For illustrative purposes, we will deal with the FCSI technique and will illustrate two simple applications, where we compare average ROI, IRR, and SLRR:

1. in the first application, working capital is exogenous. Average ROI and SL rate of return are both strictly NPV-consistent; IRR is not strongly NPV-consistent
2. in the second application, working capital is endogenous (it changes under change in α). Average ROI and IRR are not strongly consistent with NPV, whereas SLRR is strictly NPV-consistent.

(Importance measures, ranks, and correlation coefficients inherent to average ROI are denoted with the superscript "roi".)

Example 9. (*Exogenous WC*) Consider a project P with initial investment in fixed assets equal to $\text{NFA}_0 = 500$. Depreciation is equal to $\text{Dep}_1 = 250$, $\text{Dep}_2 = 100$, $\text{Dep}_3 = 50$, and $\text{Dep}_4 = 100$ so that $\text{NFA}_1 = 250$, $\text{NFA}_2 = 150$, $\text{NFA}_3 = 100$. The working capital is assumed to be 50% of the net fixed assets in each period, $\text{WC}_t = 50\% \cdot \text{NFA}_t$. Therefore $\text{WC}_0 = 250$, $\text{WC}_1 = 125$, $\text{WC}_2 = 75$, $\text{WC}_3 = 50$. Hence, the vector of book value capitals is $\mathbf{b} = (750, 375, 225, 150, 0)$, while the vector of average capital is $\boldsymbol{C}^{sl} = (750, 562.5, 375, 187.5, 0)$. Assuming that cost of capital is $k = 6\%$, the overall book value capital is $b = 1{,}429.97$ and the overall SL capital is $\boldsymbol{C}^{sl} = 1{,}771.84$. Revenues and costs in the base case and in the perturbed case are $\alpha^0 = (420, 460, 480, 520, 300, 290, 280, 260)$ and $\alpha^1 = (450, 428, 512, 487, 329, 321, 249, 292)$, respectively. From the estimates of book value capitals and incomes, the cash flow streams in α^0 and α^1 are calculated via (6) and reported in Table 12. Average ROI, SL rate of return, and IRR are calculated from (5), (14), and (12) respectively. The book value of working capital (and, hence, the book value of invested capital) does not depend on revenues and costs, which implies, from (9), that the average ROI is an affine transformation of NPV and, therefore, from MM (2018, Proposition 1), is strictly NPV-consistent under FCSI and DIM. The same applies to SL rate of return, since C^{sl} does not depend on the value drivers. Results of the analysis via Total Order FCSI are shown in Table 13. Since average ROI and SLRR are strictly NPV-consistent, their correlation with NPV is equal to 1 (with Spearman's and top-down coefficients): $\rho_{\text{roi,npv}} = \rho_{S^{\text{roi}},S^{\text{npv}}} = \rho_{\text{slrr,npv}} = \rho_{S^{\text{slrr}},S^{\text{npv}}} = 1$. As expected, IRR is not strongly NPV-consistent, with $\rho_{\text{irr,npv}} = 0.857$ and $\rho_{S^{\text{irr}},S^{\text{npv}}} = 0.611$. $\Diamond$

Example 10. (*Endogenous WC*) We consider an investment project P with initial investment in fixed assets equal to $\text{NFA}_0 = 500$. Revenues and costs in the base case and

Table 12: Exogenous WC:
Average ROI, SLRR, and IRR

	α^0	α^1
Cash flows		
F_0	−750	−750
F_1	245	246
F_2	220	157
F_3	225	288
F_4	310	245
Valuation		
NPV	111.39	57.68
$\bar{\imath}(b)$	14.26%	10.28%
$\bar{\imath}^{sl}$	12.66%	9.45%
x	12.08%	9.22%

Table 13: Exogenous WC:
Total Order FCSIs of average ROI, SLRR, and IRR

	NPV		Average ROI		SLRR		IRR	
Parameter	$\Phi_i^{T,\text{npv}}$	r_i^{npv}	$\Phi_i^{T,\text{roi}}$	r_i^{roi}	$\Phi_i^{T,\text{slrr}}$	r_i^{slrr}	$\Phi_i^{T,\text{irr}}$	r_i^{irr}
Rev_1	−52.69%	2	−52.69%	2	−52.69%	2	−56.26%	1
Rev_2	53.02%	1	53.02%	1	53.02%	1	55.75%	3
Rev_3	−50.02%	5	−50.02%	5	−50.02%	5	−51.76%	5
Rev_4	48.66%	6	48.66%	6	48.66%	6	47.03%	7
OpC_1	50.93%	4	50.93%	4	50.93%	4	55.99%	2
OpC_2	51.36%	3	51.36%	3	51.36%	3	54.01%	4
OpC_3	−48.45%	7	−48.45%	7	−48.45%	7	−50.12%	6
OpC_4	47.19%	8	47.19%	8	47.19%	8	45.64%	8
Correlations								
$\rho_{\text{roi,npv}}$	1							
$\rho_{S^{\text{aroi}},S^{\text{npv}}}$	1							
$\rho_{\text{slrr,npv}}$	1							
$\rho_{S^{\text{slrr}},S^{\text{npv}}}$	1							
$\rho_{\text{irr,npv}}$	0.857							
$\rho_{S^{\text{irr}},S^{\text{npv}}}$	0.611							

perturbed case are, respectively,

$$\alpha^0 = (420, 460, 480, 520, 300, 290, 280, 260)$$

and

$$\alpha^1 = (450, 428, 513, 487, 329, 321, 249, 292).$$

The NFA is assumed to depreciate uniformly, that is, $\text{Dep}_t = 500/7 = 62.5$. The initial investment in working capital is $\text{WC}_0 = 250$. In the following periods, the working capital is equal to 20% of revenues: $\text{WC}_t = 20\% \cdot \text{Rev}_t$, with $0 < t < p$. With such an assumption, the working capital (and, hence the book value of assets) changes under changes in the value drivers: $b_t = b_t(\alpha)$. Cost of capital is assumed to be $k = 10\%$. Table 14 and Table 15 report the book values, b_t (sum of fixed assets and working capital), the average capitals, C_t^{sl}, the FCFs, F_t, and the valuation metrics in the base case and perturbed case, respectively. The FCF streams in α^0 and α^1 are derived from the estimates of incomes and book value capitals. Results of the analysis via Total Order FCSI are collected in Table 16, which shows that average ROI and IRR are not strongly NPV-consistent. The degree of NPV-inconsistency of IRR is higher than the inconsistency of average ROI: $1 - \rho_{\text{irr,npv}} = 0.286$, $1 - \rho_{S^{\text{irr}},S^{\text{npv}}} = 0.646$, $1 - \rho_{\text{aroi,npv}} = 0.048$, and $1 - \rho_{S^{\text{aroi}},S^{\text{npv}}}$=0.201. As expected, the SLRR is strictly NPV-consistent. $\diamondsuit$

Table 14: Endogenous WC:
Average ROI, SLRR, and IRR in α^0

	0	1	2	3	4
Capital amounts					
b_t	750	334	242	146	0
NFA_t	500	250	150	50	0
WC_t	250	84	92	96	0
C_t^{sl}	750	562.5	375	187.5	0
Overall capital					
b	$1,403.06$				
C^{sl}	$1,771.84$				
Cash flows					
F_t	-750	286	162	196	356
Valuation					
NPV	110.54				
$\bar{\imath}(b)$	14.35%				
$\bar{\imath}^{sl}$	12.61%				
x	12.02%				

7 Strong NPV-consistency for project ranking

In this section we deal with the ranking of independent projects available to the firm. We first recall the NPV criterion.

Definition 4. (*NPV criterion for project ranking*) Consider a bundle of N projects which share the same risk. Project j is preferable to project h if and only if the NPV of j is greater than the NPV of h: $\text{NPV}^j > \text{NPV}^h$, $j, h \in \{1, 2, \ldots, N\}$.

Table 15: Endogenous WC:
Average ROI, SLRR, and IRR in α^1

	0	1	2	3	4
Capital amounts					
b_t	750	340	235.6	152.6	0
NFA_t	500	250	150	50	0
WC_t	250	90	85.6	102.6	0
C_t^{sl}	750	562.5	375	187.5	0
Overall capital					
b	$1,408.56$				
C^{sl}	$1,771.84$				
Cash flows					
F_t	-750	281	111.4	247	297.6
Valuation					
NPV	57.35				
$\bar{\imath}(b)$	10.32%				
$\bar{\imath}^{sl}$	9.43%				
x	9.18%				

Table 16: Endogenous WC: Total Order FCSIs
(Average ROI, SLRR, IRR)

	NPV		Average ROI		SLRR		IRR	
Parameter	$\Phi_i^{T,\text{npv}}$	r_i^{npv}	$\Phi_i^{T,\text{roi}}$	r_i^{roi}	$\Phi_i^{T,\text{slrr}}$	r_i^{slrr}	$\Phi_i^{T,\text{irr}}$	r_i^{irr}
Rev_1	-52.61%	2	-51.96%	1	-52.61%	2	-55.56%	2
Rev_2	52.94%	1	51.87%	2	52.94%	1	54.84%	3
Rev_3	-51.50%	4	-50.87%	5	-51.50%	4	-52.73%	5
Rev_4	49.14%	6	48.75%	6	49.14%	6	47.21%	7
OpC_1	51.44%	5	51.02%	4	51.44%	5	56.14%	1
OpC_2	51.87%	3	51.45%	3	51.87%	3	54.17%	4
OpC_3	-48.94%	7	-48.54%	7	-48.94%	7	-50.22%	6
OpC_4	47.66%	8	47.27%	8	47.66%	8	45.81%	8
Correlations								
$\rho_{\text{roi},\text{npv}}$	0.952							
$\rho_{S^{\text{roi}},S^{\text{npv}}}$	0.799							
$\rho_{\text{slrr},\text{npv}}$	1							
$\rho_{S^{\text{slrr}},S^{\text{npv}}}$	1							
$\rho_{\text{irr},\text{npv}}$	0.714							
$\rho_{S^{\text{irr}},S^{\text{npv}}}$	0.354							

The notion of weak NPV-consistency for project ranking may be stated as follows.

Definition 5. (*Weak NPV-consistency for project ranking*) A rate of return φ is weakly NPV-consistent for project ranking if and only if the ranks of projects derived from φ is the same as the ranks of projects derived from NPV. Formally, φ is NPV-consistent for project ranking if the following statements are true:

– for every pair of investment projects j and h, $\text{NPV}^j > \text{NPV}^h$ if and only if $\varphi^j > \varphi^h$

– for every pair of financing projects j and h, $\text{NPV}^j > \text{NPV}^h$ if and only if $\varphi^j < \varphi^h$.

We now define strong NPV-consistency for project ranking and then show that, contrary to IRR and average ROI, the SLRR fulfills it under suitable assumptions.

Definition 6. (*Strong NPV-consistency for project ranking*) Given an SA technique, a metric φ (and its associated decision criterion) is strongly NPV-consistent for project ranking if

– φ is weakly NPV-consistent for project ranking (Definition 5)

– the parameters' rank vector of φ is equal to the parameters' rank vector of NPV for every project: $r^{\text{npv}^j} = r^{\varphi^j}, j \in \{1, 2, \ldots, N\}$.

If φ is strongly NPV-consistent for project ranking and, in addition, the vectors of the relevances coincide, $R^{\text{npv}^j} = R^{\varphi^j}, j \in \{1, 2, \ldots, N\}$, then φ is strictly NPV-consistent for project ranking.

It is worth noting that, if the metric φ is not weakly NPV-consistent, the degree of NPV-(in)consistency is irrelevant. That is, even if the degree of NPV-consistency is 1, the fact that the impact of input changes on φ is the same as the impact of input changes on NPV does not heal the project ranking error, and, therefore, a high degree of correlation in the parameter ranking is useless.[13] Conversely, if the metric φ is weakly NPV-consistent but not strongly NPV-consistent, then it is important to assess its degree of (in)consistency with NPV.

In general, none of the three performance metrics (SLRR, average ROI, and IRR) is weakly NPV-consistent for project ranking (let alone strongly NPV-consistent). However, SLRR is strongly (even strictly) NPV-consistent if the competing projects have the same initial cash flows.

Proposition 2. *Suppose $F_0^j = F_0$ for every $j \in \{1, 2, \ldots, N\}$. Then, the SLRR is strictly NPV-consistent for project ranking.*

Proof. Owing to Proposition 1, given a project j, the rank vector of φ^j is equal to the rank vector of NPV^j and the vectors of relevances coincide.

We then only have to show that $\varphi = \bar{\imath}^{sl}$ is NPV-consistent according to Definition 5. The overall average capital of project j is $C^{sl^j} = \sum_{t=1}^{p} b_0^j(1-(t-1)/p)(1+k)^{-(t-1)} = \sum_{t=1}^{p} -F_0(1-(t-1)/p)(1+k)^{-(t-1)} = C^{sl}$ and is constant for every $j \in \{1, 2, \ldots, N\}$. If $F_0 < 0$, it results that $C^{sl^j} > 0$ for every $j \in \{1, 2, \ldots, N\}$; therefore, every project is an

[13]Indeed, the degree of NPV-(in)consistency if the metric is not weakly consistent is hardly interpretable in one sense or another.

investment project. If $F_0 > 0$, then $C^{sl^j} < 0$ for every $j \in \{1, 2, \ldots, N\}$ and every project is a financing project. According to eq. (14), $\bar{\imath}^{sl^j} = k + \frac{\text{NPV}^j(\alpha)(1+k)}{C^{sl}}$ $\forall j \in \{1, 2, \ldots, N\}$. This implies that the coefficients of the affine transformation $q = k$ and $m = (1+k)/C^{sl}$ are equal for all projects. If $F_0 < 0$, it results that $m > 0$ and, therefore, $\text{NPV}^j > \text{NPV}^h$ if and only if $\bar{\imath}^{sl^j} > \bar{\imath}^{sl^h}$; if $F_0 > 0$, it derives that $m < 0$ and, therefore, $\text{NPV}^j > \text{NPV}^h$ if and only if $\bar{\imath}^{sl^j} < \bar{\imath}^{sl^h}$. □

The proposition says that, whenever the firm has a given amount of capital $b_0 = -F_0$ to be invested, then the SLRR may be employed as a substitute for NPV (or be used in conjunction with it) for selecting the preferred alternative.

In contrast, if initial outlays F_0^j differ across the investments, SLRR and NPV are not consistent for project ranking and the selection of the adequate valuation metric may depend on the presence of capital budget constraints: In case of capital rationing, decision makers may choose the SLRR in place of the NPV, whereas NPV is appropriate if no budget constraints exist and if absolute increase in wealth is set as the objective function instead of financial efficiency.

We now illustrate two simple numerical applications with $N = 2$. They serve as counter-examples for proving that the average ROI and the IRR are not strongly NPV-consistent for project ranking. We use Total Order FCSI to assess degrees of NPV-inconsistency. In the first example, both average ROI and IRR are weakly NPV-consistent for project ranking but not strongly NPV-consistent. In the second example, both the average ROI and the IRR are not even weakly NPV-consistent for project ranking.[14]

Example 11. (*Weak NPV-consistency for project ranking*) Consider projects A and B with equal initial fixed assets, $\text{NFA}_0 = 500$, and equal initial working capital, $\text{WC}_0 = 250$. We assume that the book values of fixed assets are different, such that $\text{NFA}_1^A = 300$, $\text{NFA}_2^A = 100$, $\text{NFA}_3^A = 50$ and $\text{NFA}_1^B = 450$, $\text{NFA}_2^B = 350$, $\text{NFA}_3^B = 150$. The working capital of the two projects is assumed to amount to 20% of revenues, $\text{WC}_t^j = 20\% \cdot \text{Rev}_t^j$, $j = A, B$, for $t = 1, 2, 3$ (and $\text{WC}_4 = 0$ for working capital is recovered at the end of the project).

On the basis of the input data, reported in Tables 17-18, the book values are calculated in the two scenarios: $\mathbf{b}^A(\alpha^0) = (750,\ 380,\ 184,\ 140,\ 0) \neq \mathbf{b}^B(\alpha^0) = (750,\ 520,\ 426,\ 220,\ 0)$ and $\mathbf{b}^A(\alpha^1) = (750,\ 382,\ 186.122,\ 142.246,\ 0) \neq \mathbf{b}^B(\alpha^1) = (750,\ 522,\ 428.122,\ 222.248,\ 0)$. Assuming $k = 6\%$, the overall book value capitals of A are $b^A(\alpha^0) = 1,389.80$ and $b^A(\alpha^1) = 1,395.46$ and the overall book value capitals of B are $b^B(\alpha^0) = 1,804.42$ and $b^B(\alpha^1) = 1,810.08$. The initial invested capital is $C_0 = \text{NFA}_0 + \text{WC}_0 = 500 + 250 = 750$, implying that the average capital vectors of A and B coincide: $\boldsymbol{C}^{sl} = (750,\ 562.5,\ 375,\ 187.5,\ 0)$ such that $C^{sl} = 1,771.84$. The performance metrics are collected in Tables 17-18 (the bold typeface represents the higher value of each performance metric). Project A is preferred to B, since $\text{NPV}^A > \text{NPV}^B$. All the three relative criteria average ROI, IRR, and SLRR satisfy the weak NPV-consistency for project ranking, since

[14] It is worthy of attention that, if working capital is zero or exogenous *and* if every project shares the same capital depreciation schedule, $\mathbf{b}^j = \mathbf{b}$, $\forall j \in \{1, 2, \ldots, N\}$, then average ROI is indeed an affine transformation of NPV, $\bar{\imath}^j(b) = k + \text{NPV}^j(\alpha)(1+k)/b$ with coefficients $q = k$ and $m = (1+k)/b$ equal for every project $j \in \{1, 2, \ldots, N\}$; therefore, under these assumptions, average ROI is strictly NPV-consistent for project ranking.

$\bar{\imath}^A(b) > \bar{\imath}^B(b)$, $\bar{\imath}^{sl^A} > \bar{\imath}^{sl^B}$, and $x^A > x^B$. However, the parameter ranking of average ROI and IRR is different from the NPV's parameter ranking. In particular, the degrees of NPV-consistency of average ROI for project A are $\rho^A_{\text{roi,npv}} = 0.857$ and $\rho^A_{S^{\text{roi}},S^{\text{npv}}} = 0.553$ and, for project B, are $\rho^B_{\text{roi,npv}} = 0.857$ and $\rho^B_{S^{\text{roi}},S^{\text{npv}}} = 0.553$. The degrees of NPV-inconsistency for IRR are very high. Specifically, $\rho^A_{\text{irr,npv}} = 0.571$ and $\rho^A_{S^{\text{irr}},S^{\text{npv}}} = 0.483$ for project A ; $\rho^B_{\text{irr,npv}} = 0.048$ and $\rho^B_{S^{\text{irr}},S^{\text{npv}}} = 0.126$ for B. Therefore, while weakly NPV-consistent, average ROI and IRR are not strongly NPV-consistent for project ranking and their degree of NPV-inconsistency (especially, the IRR's) is remarkable. $\diamondsuit$

Table 17: Weak NPV-consistency
for project ranking in α^0

	A	B
Rev_1	400	350
Rev_2	420	380
Rev_3	450	350
Rev_4	500	350
OpC_1	300	220
OpC_2	290	210
OpC_3	280	195
OpC_4	260	190
Valuation	A	B
NPV	**15.95**	5.77
$\bar{\imath}(b)$	**7.22%**	6.34%
$\bar{\imath}^{sl}$	**6.95%**	6.35%
x	**6.89%**	6.36%

Table 18: Weak NPV-consistency
for project ranking in α^1

	A	B
Rev_1	410	360
Rev_2	430.61	390.61
Rev_3	461.23	361.24
Rev_4	511.67	361.76
OpC_1	290.24	210.26
OpC_2	279.66	199.67
OpC_3	269.05	184.05
OpC_4	248.39	178.39
Valuation	A	B
NPV	**89.97**	79.85
$\bar{\imath}(b)$	**12.83%**	10.68%
$\bar{\imath}^{sl}$	**11.38%**	10.78%
x	**10.92%**	10.83%

Example 12. (*NPV-inconsistency for project ranking*) Suppose, again, that projects A and B have the same initial fixed assets and same initial working capital: $\text{NFA}_0 = 500$ and $\text{WC}_0 = 250$. We assume that the two projects have different book values of fixed assets: $\text{NFA}^A_1 = 250$, $\text{NFA}^A_2 = 150$, $\text{NFA}^A_3 = 50$ and $\text{NFA}^B_1 = 40$, $\text{NFA}^B_2 = 20$, $\text{NFA}^B_3 = 10$. Tables 19-20 describe the input values in base case and perturbed case, respectively. We assume that the working capital of the two projects is endogenously determined: Specifically, it is equal to 20% of revenues in every period, $\text{WC}^j_t = 20\% \cdot \text{Rev}^j_t$, where $j = A, B$ and $t =$

$1, 2, 3$ ($\mathrm{WC}_4^j = 0$). The vectors of book value capitals are different for both cases: In the base case, $\mathbf{b}^A(\alpha^0) = (750,\ 334,\ 242,\ 146,\ 0) \neq \mathbf{b}^B(\alpha^0) = (750,\ 120,\ 88,\ 90,\ 0)$ and in the perturbed case $\mathbf{b}^A(\alpha^1) = (750,\ 340,\ 235.6,\ 152.6,\ 0) \neq \mathbf{b}^B(\alpha^1) = (750,\ 114,\ 94.34,\ 84,\ 0)$. Assuming $k = 6\%$, the overall book value capitals of A are $b^A(\alpha^0) = 1{,}403.06$ and $b^A(\alpha^1) = 1{,}408.56$; the overall book value capitals of B are $b^B(\alpha^0) = 1{,}017.09$ and $b^B(\alpha^1) = 1{,}012.04$. Given the input data, the initial invested capital is the same for A and B, $C_0 = \mathrm{NFA}_0 + \mathrm{WC}_0 = 500 + 250 = 750$; therefore, the vectors of average capital are the same, $\boldsymbol{C}^{sl} = (750,\ 562.5,\ 375,\ 187.5,\ 0)$. The overall SL capital is the same for the two projects and does not depend on the state: $C^{sl} = 1{,}771.84$, regardless of the scenario considered. The valuation metrics in the two cases are reported in Tables 19-20, respectively. Project A creates more value than B, since $\mathrm{NPV}^A > \mathrm{NPV}^B$. The SLRR provides the same answer as the NPV, since $\bar{\imath}^{sl^A} > \bar{\imath}^{sl^B}$. Also, considering Total Order FCSI, the parameters' relevances of NPV and SLRR are equal, implying that the SLRR is strictly NPV-consistent for project ranking. Conversely, the average ROI and the IRR provide an error in ranking projects, since $\bar{\imath}^A(b) < \bar{\imath}^B(b)$ and $x^A < x^B$, so they are not even weakly NPV-consistent.[15] $\Diamond$

Table 19: NPV-inconsistency
for project ranking in α^0

	A	B
Rev_1	420	400
Rev_2	460	340
Rev_3	480	400
Rev_4	520	450
OpC_1	300	200
OpC_2	290	200
OpC_3	280	352
OpC_4	260	100
Valuation	A	B
NPV	**110.54**	105.16
$\bar{\imath}(b)$	14.35%	**16.96%**
$\bar{\imath}^{sl}$	**12.61%**	12.29%
x	12.02%	**12.03%**

8 Concluding remarks

This paper builds upon three strands of literature, namely, i) a methodological one, dealing with the NPV-consistency of measures of financial efficiency, (ii) a managerial one, dealing with management of uncertainty and sensitivity-analysis application to project appraisal, and (iii) an accounting one, dealing with the impact of working capital on financial performance. We introduce a new performance metric for project appraisal, the straight-line rate of return (SLRR), which takes into explicit consideration the presence

[15]The correlation coefficients of average ROI for project A are $\rho^A_{\mathrm{roi,npv}} = 0.952$ and $\rho^A_{S^{\mathrm{roi}},S^{\mathrm{npv}}} = 0.799$ and, for project B, are $\rho^B_{\mathrm{roi,npv}} = 0.976$ and $\rho^B_{S^{\mathrm{roi}},S^{\mathrm{npv}}} = 0.953$. IRR's correlation coefficients are, for project A, $\rho^A_{\mathrm{irr,npv}} = 0.714$ and $\rho^A_{S^{\mathrm{irr}},S^{\mathrm{npv}}} = 0.354$ and, for project B, $\rho^B_{\mathrm{irr,npv}} = 0.976$ and $\rho^B_{S^{\mathrm{irr}},S^{\mathrm{npv}}} = 0.995$. However, these degrees are not relevant, given the error in project ranking.

Table 20: NPV-inconsistency for project ranking in α^1

	A	B
Rev_1	450	370
Rev_2	428	371.7
Rev_3	513	370
Rev_4	487	370
OpC_1	329	190
OpC_2	321	189.3
OpC_3	249	347
OpC_4	292	80
Valuation	A	B
NPV	**57.35**	55.80
$\bar{\imath}(b)$	10.32%	**11.84%**
$\bar{\imath}^{sl}$	**9.43%**	9.34%
x	9.18%	**9.37%**

of (uncertain) working capital. We measure its NPV-consistency in both accept-reject decisions and project ranking and compare it with the average ROI introduced in Marchioni and Magni (2018) and the traditional Internal rate of Return (IRR). To this end, we analyze the impact on them of changes (perturbations) in the input data, also known as *value drivers* or *key parameters* (i.e., project's revenues and costs).

We find that the average ROI is not strongly NPV-consistent whenever working capital (WC) is present, uncertain, and endogenously dependent on the value drivers. We use the notion of Chisini mean to search for a measure which possesses strong NPV-consistency, thereby improving upon the average ROI. Two candidates arise: The well-known IRR and the newly-introduced SLRR, based on the (linear) average rate of change of the invested capital.

We find that the IRR is problematic, for its existence and uniqueness may depend on the project's key assumptions, and its financial nature may turn out to be ambiguous. In other words, a change in the value drivers may turn an investment IRR to a financing IRR (or vice versa) or generate multiple IRRs or make the IRR nonexistent. Further, even in favorable cases (as already displayed in Borgonovo and Peccati 2004, 2006, Percoco and Borgonovo 2012) the IRR is not strongly NPV-consistent for accept-reject decisions. For project ranking, we show that it is not NPV-consistent, not even in a weak sense.

In contrast, the SLRR is strongly NPV-consistent in a strict form for accept-reject decisions, regardless of whether the working capital is zero or not and regardless of whether it is endogenous or exogenous. Furthermore, its existence and uniqueness is guaranteed in every case. Moreover, the SLRR also enjoys strict NPV-consistency in project ranking if the initial cash flows of the competing projects are equal.

To wrap things up, as compared to the strand of literature about sensitivity analysis and project valuation, we make different and incremental findings:

- we show that a necessary condition for the average ROI to be strongly NPV-consistent in accept-reject decisions is that no use of WC is made in the operations (e.g., no inventory, and sales and purchases are made on a cash-only basis) or that the nonzero WC is managed by the firm's managers in such a way that it is unaffected by the value drivers (sales revenues and costs). In all other cases, the average ROI is not strongly consistent

- we introduce the SLRR (associated with the *average* invested capital) and show that it is strongly NPV-consistent, regardless of whether WC is present or not
- we compare the SLRR, the IRR, and the average ROI and measure the degree of NPV-inconsistency of IRR and average ROI
- we extend the study to project ranking and show, that, contrary to average ROI and IRR, the SLRR is (not only strongly but also) strictly NPV-consistent if the competing projects have the same initial outflow.

We illustrate these results by taking into account two sensitivity analysis techniques: FCSI (Borgonovo 2010a) and DIM (Borgonovo and Apostolakis 2001, Borgonovo and Peccati 2004), and assess the degree of NPV-inconsistency of average ROI and IRR via Spearman's (1904) correlation coefficient and Iman and Conover's (1987) top-down coefficient and find that the degree of inconsistency of IRR and average ROI may vary case by case and may be very high.

The properties of average ROI, SLRR, and IRR are summarized in the following table.

Property	**Average ROI**	**SLRR**	**IRR**
Existence guaranteed	no	yes	no
Uniqueness guaranteed	yes	yes	no
Unambiguous financial nature	yes	yes	no
Accept-reject decisions			
Weak NPV-consistency	yes	yes	yes
Strong NPV-consistency			
with exogenous WC	yes	yes	no
with endogenous WC	no	yes	no
Project ranking			
Weak NPV-consistency (if $F_0^j = F_0 \quad \forall j$)	no	yes	no
Strong NPV-consistency (if $F_0^j = F_0 \quad \forall j$)	no	yes	no

These findings show that

- the IRR meets new, previously unknown difficulties in several respects
- the average ROI is more reliable than IRR, but it may incur NPV-inconsistency for both accept-reject decisions and project ranking as well as possible nonexistence
- the SLRR, based on the average rate of change, is reliable and robust and is an appropriate candidate for economic analysis in accept-reject decisions. It is also sound for project ranking if the initial cash flows of the competing projects are equal.

References

Arditti, F.D., Levy, H. (1977). The Weighted Average Cost of Capital as a cutoff rate: A critical analysis of the classical textbook weighted average. *Financial Management*, 6(3) (Autumn), 24–34.

Battaglio, C., Longo, G., Peccati, L. (1996). Restyling of fees in consumers credit and their optimization. *European Journal of Operational Research*, 91, 330-337.

Berkovitch, E., Israel, R. (2004). Why the NPV criterion does not maximize NPV. *The Review of Financial Studies*, 17(1) (January), 239-255.

Bian, Y., Lemoine, D., Yeung, T. G., Bostel, N., Hovelaque, V., Viviani, J. L., Gayraud, F. (2018). A dynamic lot-sizing-based profit maximization discounted cash flow model considering working capital requirement financing cost with infinite production capacity. *International Journal of Production Economics*, 196, 319-332.

Borgonovo, E. (2010a). Sensitivity analysis with finite changes: An application to modified EOQ models. *European Journal of Operational Research*, 200, 127-138.

Borgonovo, E. (2010b). A Methodology for Determining Interactions in Probabilistic Safety Assessment Models by Varying One Parameter at a Time. *Risk Analysis*, 30 (3), 385-399.

Borgonovo, E., Apostolakis, G.E. (2001). A new importance measure for risk-informed decision-making. *Reliability Engineering and System Safety*, 72 (2), 193-212.

Borgonovo, E., Peccati, L. (2004). Sensitivity analysis in investment project evaluation. *International Journal of Production Economics*, 90, 17-25.

Borgonovo, E., Peccati, L. (2006). Uncertainty and global sensitivity analysis in the evaluation of investment projects. *International Journal of Production Economics*, 104, 62-73.

Borgonovo, E., Plischke, E. (2016). Sensitivity analysis: A review of recent advances. *European Journal of Operational Research*, 248(3), 869-887.

Brealey, R.A., Myers, S., Allen, F. (2011). *Principles of Corporate Finance.* Tenth Edition. McGraw-Hill Irwin.

Brief, R.P., Peasnell, K.V. (Eds.) 1996. *Clean surplus: a link between accounting and finance.* Garland Publishing, New York and London.

Caballero, S.B., García-Teruel, P.J., Martínez-Solan, P. (2014). Working capital management, corporate performance, and financial constraints. *Journal of Business Research*, 67(3) (March), 332-338.

Chauhan G.S. (2019). Are working capital decisions truly short-term in nature? *Journal of Business Research*, 99 (June), 238-253.

Chiang, Y., Cheng, E., and Lam, P. (2010). Employing the Net Present Value-Consistent IRR Methods for PFI Contracts. *Journal of Construction Engineering & Management*, 136(7), 811-814.

Chisini, O. (1929) Sul concetto di media. *Periodico di Matematiche*, 4, 106-116.

Cigola, M., Peccati, L. (2005). On the comparison between the APV and the NPV computer via the WACC. *European Journal of Operational Research*, 161, 377-385.

Dempsey, M. (2013). Consistent cash flow valuation with tax-deductible debt: A clarification. *European Financial Management* 19(4) (September), 830-836.

Evans, D.A., Forbes, S.M. (1993). Decision making and display methods: the case of prescription and practice in capital budgeting. *The Engineering Economist*, 39(1), 87-92.

Gow, I., Reichelstein, S. (2007). Capital budgeting: The role of cost allocations. In KH Waldmann and UM Stocker (Eds.), *Operations Research Proceedings 2006*. Springer, Berlin, Heidelberg, pp. 115-121.

Graham, J.R., Harvey, C.R. (2001). The theory and practice of corporate finance: Evidence from the field. *Journal of financial economics*, 60, 187-243.

Hajdasinski, M.M. (1995). Meaningful rates of return for project evaluation. In Singhal RJ et al (Eds.), *Proceedings of the 4th international symposium on Mine planning and equipment selection*, Calgary, Canada, 31 October-3 November. pp. 81-88. AA Balkema, Rotterdam.

Hajdasinski, M.M. (1997). NPV-compatibility, project ranking, and related issues. *The Engineering Economist*, 42(4), 325-339.

Hartman J.C. (2000). On the equivalence of net present value and market value added as measures of a project's economic worth. *The Engineering Economist*, 45(2), 158-165.

Hartman, J.C. (2007). *Engineering Economy and the Decision-making Process*. Pearson, Upper Saddle River NJ.

Hartman, J.C., Schafrick, I.C. (2004). The relevant internal rate of return. *The Engineering Economist*, 49(2), 139-158.

Hazen, G.B. (2003). A new perspective on multiple internal rates of return. *The Engi-*

neering Economist, 48(1), 31-51.

Helton, J.C. (1993). Uncertainty and sensitivity analysis techniques for use in performance assessment for radioactive waste disposal. *Reliability Engineering and System Safety*, 42, 327-367.

Huang, J., Yang, W., Tu, Y. (in press). Financing mode decision in a supply chain with financial constraint. *International Journal of Production Economics.* <https://doi.org/10.1016/j.ijpe.2019.07.014>.

Iman, R.L., Conover, W.J. (1987). A measure of top-down correlation. *Technometrics*, 29 (3), 351-357.

Lindblom, T., Sjögren, S. (2009). Increasing goal congruence in project evaluation by introducing a strict market depreciation schedule. *International Journal of Production Economics*, 121, 519–532.

Luciano, E., Peccati, L. (1999). Capital structure and inventory management: The temporary sale price problem. *International Journal of Production Economics*, 59, 169-178.

Magni, C.A. (2010). Average Internal Rate of Return and investment decisions: A new perspective. *The Engineering Economist*, 55(2), 150-180.

Magni, C.A. (2013). The Internal Rate of Return approach and the AIRR Paradigm: A refutation and a corroboration. *The Engineering Economist*, 58(2), 73-111.

Magni, C.A. (2019). *Investment Decisions and the Logic of Valuation. Linking Finance, Accounting, and Engineering.* Springer Nature Switzerland AG.

Marchioni, A., Magni, C.A. (2018) Investment decisions and sensitivity analysis: NPV-consistency of rates of return. *European Journal of Operational Research*, 268, 361-372.

Massari, M., Roncaglio, F., Zanetti, L. (2008). On the equivalence between the APV and the WACC approach in a growing leveraged firm. *European Financial Management*, 14(1) (January), 152-162.

Miles, J.A., Ezzell, J.R. (1980). The Weighted Average Cost of Capital, perfect capital markets, and project life: A clarification. *Journal of Financial and Quantitative Analysis*, 15(3) (September), 719-730.

Pasqual, J., Padilla, E., Jadotte, E. (2013). Technical note: Equivalence of different profitability criteria with the net present value. *International Journal of Production Economics*, 142(1) (March), 205-210.

Peng, J., Zhou, Z. (2019). Working capital optimization in a supply chain perspective.

European Journal of Operational Research, 277(3), 846-856.

Percoco, M., Borgonovo, E. (2012). A note on the sensitivity analysis of the internal rate of return. *International Journal of Production Economics*, 135, 526-529.

Pfeiffer T (2004). Net Present Value-consistent investment criteria based on accruals: A generalisation of the residual income-identity. *Journal of Business Finance & Accounting*, 31(7), 905-926.

Pianosi, F., Beven, K., Freer, J., Hall, J.W., Rougier, J., Stephenson, D.B., Wagener, T. (2016). Sensitivity analysis of environmental models: A systematic review with practical workflow. *Environmental Modelling & Software*, 79, 214-232.

Pike, R.H., Ooi, T.S. (1988). The impact of corporate investment objectives and constraints on capital budgeting practices. *The British Accounting Review*, 20(2) (August), 159-173.

Pirttilä, M., Virolainen, V. M., Lind, L., Kärri, T. (in press). Working capital management in the Russian automotive industry supply chain. *International Journal of Production Economics*. <https://doi.org/10.1016/j.ijpe.2019.08.009>.

Protopappa-Sieke, M., Seifert, R. W. (2010). Interrelating operational and financial performance measurements in inventory control. *European Journal of Operational Research*, 204(3), 439-448.

Ross, S.A., Westerfield, R.W., Jordan, B.D. (2011). *Essentials of corporate finance*, 7th ed. McGraw-Hill/Irwin, New York.

Sandahl, G., Sjögren, S. (2003). Capital budgeting methods among Sweden's largest groups of companies. The state of the art and a comparison with earlier studies. *International Journal of Production Economics*, 84, 51-69.

Savage, I.R. (1956). Contributions to the theory of rank order statistics − The two-sample case. *The Annals of Mathematical Statistics*, 27(3), 590-615.

Song, H., Yang, X., Yu, K. (in press). How do supply chain network and SMEs' operational capabilities enhance working capital financing? An integrative signaling view. *International Journal of Production Economics*. <https://doi.org/10.1016/j.ijpe.2019.07.020>.

Spearman, C. (1904). The proof and measurement of association between two things. *The American Journal of Psychology*, 15(1), 72-101.

Titman, S., Keown, A.J., Martin, J.D. (2011). *Financial Management. Principles and Applications*. 11th edition, Pearson, Boston.

Titman, S., Martin, J. (2011). *Valuation. The Art and Science of Corporate Investment Decisions*, second edition. Pearson, Boston.

Wetzel, P., Hofmann, E. (2019). Supply chain finance, financial constraints and corporate performance: An explorative network analysis and future research agenda. *International Journal of Production Economics*, 216, 364-383.

Wu, Y., Wang, Y., Xu, X., Chen, X. (in press). Collect Payment Early, Late, or Through a Third Party's Reverse Factoring in a Supply Chain. *International Journal of Production Economics*. <https://doi.org/10.1016/j.ijpe.2019.04.040>.

Appendix. Finite Change Sensitivity Index and Differential Important Measure

Finite Change Sensitivity Indices. The Finite Change Sensitivity Indices (FCSIs) study the effect of a finite change in the inputs on the model output (Borgonovo 2010a, 2010b). Two versions of FCSIs are defined: First Order FCSI and Total Order FCSI. The First Order FCSIs measure the individual effects of the parameters on f, whereas the Total Order FCSIs consider both the individual contributions and the interactions between parameters. The parameters change from the base value α^0 to $\alpha^1 = \left(\alpha_1^1, \alpha_2^1, \ldots, \alpha_n^1\right) \in A$. The corresponding output variation is $\Delta f = f(\alpha^1) - f(\alpha^0)$. The individual effect of α_i on Δf is

$$\Delta_i f = f(\alpha_i^1, \alpha_{(-i)}^0) - f(\alpha^0)$$

where $(\alpha_i^1, \alpha_{(-i)}^0) = (\alpha_1^0, \alpha_2^0, \ldots, \alpha_{i-1}^0, \alpha_i^1, \alpha_{i+1}^0, \ldots, \alpha_n^0)$ is obtained by varying the parameter α_i to the new value α_i^1, while the remaining $n-1$ parameters are fixed at α^0. The First Order FCSI of α_i, denoted as $\Phi_i^{1,f}$, is

$$\Phi_i^{1,f} = \frac{\Delta_i f}{\Delta f} \tag{17}$$

(Borgonovo 2010a). The total effect of the parameter α_i, denoted as $\Delta_i^T f$, is

$$\Delta_i^T f = f(\alpha^1) - f(\alpha_i^0, \alpha_{(-i)}^1),\ \forall i = 1, 2, \ldots, n,$$

(Borgonovo 2010a, Proposition 1) where $(\alpha_i^0, \alpha_{(-i)}^1)$ is the point with all the parameters equal to the new value α^1, except the parameter α_i, which is equal to α_i^0. The Total Order FCSI of the parameter α_i, denoted as $\Phi_i^{T,f}$, is (Borgonovo 2010a):

$$\Phi_i^{T,f} = \frac{\Delta_i^T f}{\Delta f} = \frac{f(\alpha^1) - f(\alpha_i^0, \alpha_{(-i)}^1)}{\Delta f}. \tag{18}$$

Differential Importance Measure. The Differential Importance Measure (DIM) of parameter α_i is the ratio of the partial differential of f with respect to α_i to the total differential of f (Borgonovo and Apostolakis 2001, Borgonovo and Peccati 2004):

$$DIM_i^f(\alpha^0, \mathrm{d}\alpha) = \frac{\mathrm{d}f_{a_i}}{\mathrm{d}f} = \frac{\frac{\partial f}{\partial \alpha_i}(\alpha^0) \cdot \mathrm{d}\alpha_i}{\sum_{j=1}^n \frac{\partial f}{\partial \alpha_j}(\alpha^0) \cdot \mathrm{d}\alpha_j}. \tag{19}$$

Two versions of DIM are defined, according to the assumption made upon the variation structure of parameters: Uniform variation assumption (H1) or proportional variation assumption (H2).

H1 implies $\mathrm{d}\alpha_i = \mathrm{d}\alpha_j,\ \forall \alpha_i, \alpha_j$; the resulting DIM is

$$DIM1_i^f(\alpha^0) = \frac{\frac{\partial f}{\partial \alpha_i}(\alpha^0) \cdot \mathrm{d}\alpha_i}{\sum_{j=1}^n \frac{\partial f}{\partial \alpha_j}(\alpha^0) \cdot \mathrm{d}\alpha_j} = \frac{\frac{\partial f}{\partial \alpha_i}(\alpha^0)}{\sum_{j=1}^n \frac{\partial f}{\partial \alpha_j}(\alpha^0)}. \tag{20}$$

H2 implies $\mathrm{d}\alpha_i = \xi \cdot \alpha_i^0$ for some $\xi \neq 0$; the resulting DIM is

$$DIM2_i^f(\alpha^0) = \frac{\frac{\partial f}{\partial \alpha_i}(\alpha^0) \cdot \xi \cdot \alpha_i^0}{\sum_{j=1}^n \frac{\partial f}{\partial \alpha_j}(\alpha^0) \cdot \xi \cdot \alpha_j^0} = \frac{\frac{\partial f}{\partial \alpha_i}(\alpha^0) \cdot \alpha_i^0}{\sum_{j=1}^n \frac{\partial f}{\partial \alpha_j}(\alpha^0) \cdot \alpha_j^0}. \tag{21}$$