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DESIGNING COUPLED ENGINEERED SYSTEMS UNDER UNCERTAINTY

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BY

Dr. Janet K.Allen

Dr. Farrokh Mistree

Dr. Harold L.Stalford

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Abstract

The evolving technology and state of art research have provided various platforms for transforming engineering design by merging product and process design with materials. This merger gives us an extended design space and a larger search space with a potential benefit of discovering engineering solutions that include better-quality product without compromising performances. The opportunities also pose serious challenges. The realization and modeling of the extended design space in itself is very complex as result of numerous interacting decisions (coupled decisions) at varying levels of priority. With a plethora of materials and manufacturing processes to choose from, the need for decision support to aid designers to efficiently explore the design space becomes imperative. Furthermore, the uncertainty that lies at each stage of decision making need to be properly addressed to render the effectiveness and accuracy of the undertaken decisions.

The design of engineered systems, in context of this thesis, is viewed from the Decision-Based Design (DBD) perspective. In Decision-Based Design (DBD), the principal role of a human designer is to make decisions and engineering design is recognized as a decision- making process. The implementation of Decision-Based Design can take many forms, one manifestation of the Decision-Based Design (DBD) construct is the Decision Support Problem Technique (DSPT) developed to provide support to human designers in exercising judgment in making design

decisions. All decisions identified in the DSPT are categorized as selection, compromise, or a combination of these. Selection decisions are modeled as selection Decision Support Problems (sDSP) and the compromise decisions are modeled as compromise Decision Support Problems (cDSP).

In this thesis, a framework for modeling design decisions involving multiple interacting decisions, called the Multilevel Decision Scenario Matrix (MDSM) is proposed. The decision pattern pertaining to several interacting decisions is identified for a given engineering design problem using MDSM and a mathematical formulation with robustness metrics is implemented for the identified decision pattern to explore decisions that are relatively insensitive to uncertainties. Then, a generic robust decision method, based on compromise Decision Support Problem Construct is proposed. The integration of coupled decisions with robustness metrics, specifically, Design Capability Index (DCI) and Error Margin Index (EMI) is detailed as a method for designing engineered systems under uncertainty. The proposed method is applied in designing of fender, one-stage reduction gearbox and, composite structures.

Chapter 1: Coupled Decisions In Engineered Systems: Establishing Decision Scenario Matrix with DSPs for Coupled Problems

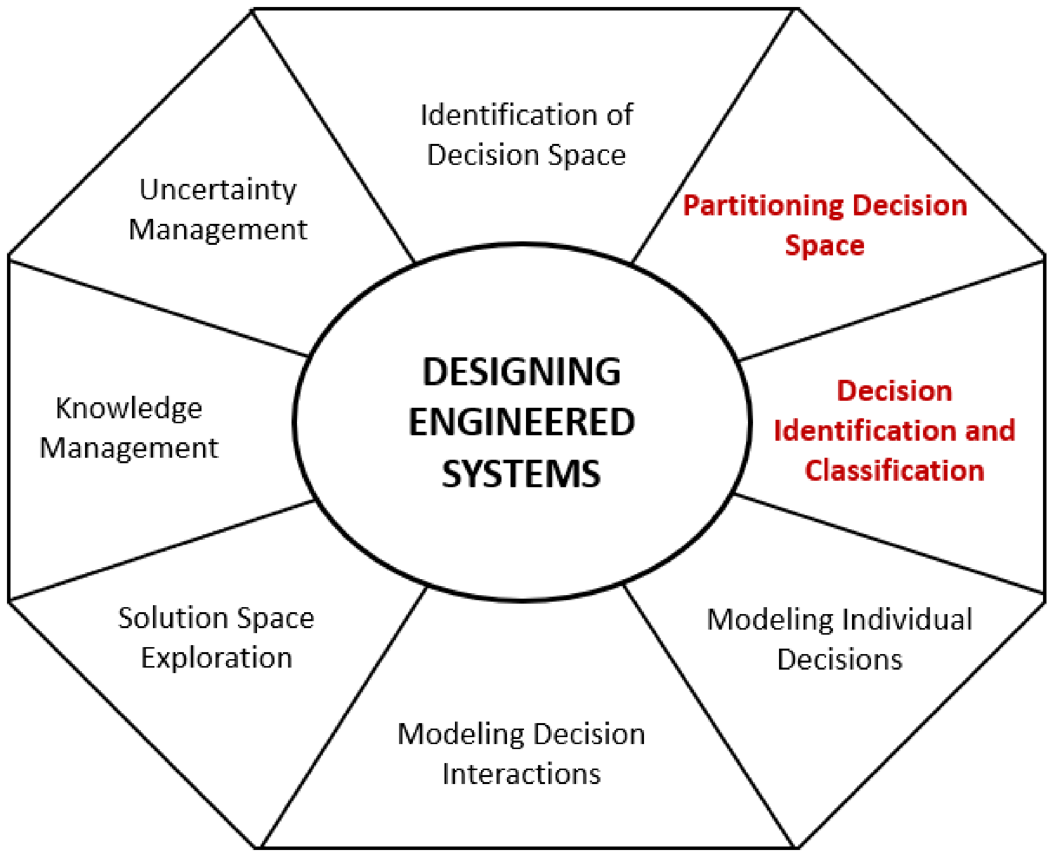


Figure 1.1: Elements in Design of Coupled Engineered Systems Discussed in Chapter 1

The assessment to internal consistency for establishing the logical soundness of the design method is dealt in Chapters 1 and 2. In this context, discussion on two major elements in the design of engineered systems is contained in this chapter as shown in Figure 1.1 (highlighted in red). Particularly, in Chapter 1, the need to address the decision coupling and robust decision making in design of an engineered system is established. Also, the suitability of Decision Support Problem Technique (DSPT) for modelling decisions as DSPs is discussed. The creation and utility of Multileveled Decision Scenario Matrix (MDSM) for classifying decisions is explained. Finally, the scope of the work, including the research questions posed, hypothesis proposed, and the boundary of the present work is detailed.

1.1 Coupled Decisions in the Design of Engineered System

1.1.1 *Introduction to Design of Coupled Engineered System*

"*Engineering Systems combines engineering with perspectives from management, economics, and the social science in order to address the design and development of the complex, large-scale, sociotechnical systems that are so important in all aspects of modern society.*"[1] These systems also involve multiple associated subsystems that interact with one another. Such influence from various knowledge domains and interactions among associated subsystems make the design and development of engineered systems very challenging. It calls for the design process associated with such complex engineered systems to be decomposed into subsystem modules which are coupled through transference of output data (Bloebaum 1992). The assumption in this approach is that the ability to determine subsystems and model interactions among subsystems exist (Bloebaum 1992). What subsystems exist and how they interact are two important

[1] https://mitpress.mit.edu/books/series/engineering-systems

aspects of coupled engineered system. Therefore, design of coupled engineered systems require designer to ascertain subsystems and model their interactions.

Design of an engineered system requires information from several disciplines. Such information forms the basis for design decisions. A decision based on the information from one discipline has an influence on decision based on information from another discipline. This is common in engineering design where decisions are modeled using information from, say, fluid dynamics, thermal science, manufacturing science, economics, material science, etc. My contention is that failure to account for the interaction among decisions leads to poor decisions.

1.1.2 *Design as a Decision-Making Process*

Decision-Based Design (DBD) is a design perspective that emerged to develop design methods to support human designers. In Decision-Based Design (DBD), the principal role of a human designer is to make decisions. The decision-making process converts the information into knowledge. The characteristics of the design decisions are summarized by following sentences (Mistree, Smith et al. 1990):

- Design decisions are invariably multidimensional and multileveled in nature.
- Decisions in design involve information coming from different sources and disciplines.
- Decisions in design are governed by multiple measures of merit and performance.

- All the information needed to make decisions may not be available.
- Some of the information required to make a decision may be hard, that is, based on scientific principles and some of the information may be soft, that is, based on the designer's judgment and experience.
- The problem for which design decisions are being made are invariably loosely defined and open and are characterized by the lack of a singular, unique solution. The decisions are less than optimal which represent satisficing solutions.

Given the characteristics of design decisions, outlining a systematic process involving this decision-making process is vital. Smith and co-authors (Smith, Kamal et al. 1987) suggest that a decision-based design process involves:

- a series of decisions, some being made concurrently and some sequentially.
- multilevel, multidisciplinary and multidimensional decision-making where interactions occur among subsystems on various levels of the decision tree on one or both directions.

One foundational demonstration of the decision-based design construct is the **D**ecision **S**upport **P**roblem **T**echnique (DSPT). In DSPT, "the principal role of an engineer, in the design of an artifact, is to make decisions (Mistree, Smith et al. 1993)." In this sense, DSPT was developed to provide support to human designers in exercising judgement in the process of making design decisions. There are two axioms that are needed to characterize "decisions" as Decision Support Problems (DSPs) that are stated below (Mistree, Smith et al. 1991).

Axiom 1: Existence of Decisions in the DSPT

"The application of the DSPT results in the identification of decisions associated with the system (and subsystems that may be relevant)."

Axiom 2: Type of Decisions in the DSPT

"All decisions identified in the DSPT are categorized as selection, compromise, or a combination of these."

In the DSPT, the selection decision is defined as, 'the process of making a choice between a number of possibilities considering a number of measures of merit or attributes." Similarly, the compromise decision is defined as, "the decision that requires the 'right' values (or combination) of design variables (or parameters) be determined, such that, the system is feasible with respect to constraints and system performance is maximized." In the DSPT, selection decisions are modeled as selection Decision Support Problems (sDSP) and the compromise decisions are modeled as compromise Decision Support Problems (cDSP). Bannerot and coauthors describe three principal components of DSPT: a design philosophy expressed at present in terms of paradigms, an approach for identifying and formulating DSPs and the software necessary for solution (Bannerot and Mistree 1989, Bascaran, Bannerot et al. 1989).

1.2 Identifying Gaps and Research Questions

Having discussed decisions in the design of engineered system, the need is for a framework that can assist designers to design coupled systems for robust performance. In coupled systems, there exist interaction among design decisions which influence one another. Besides, for robust performance we need techniques to manage uncertainties when the design decisions are interacting. In developing a framework for designing coupled engineering system and simultaneously managing the associated uncertainties, some challenges lie ahead. Some of the challenges are, but not limited to

- Representation of the decision interactions in a coupled engineering system.
- Representation of the interactions between decisions made at various priority levels.
- Identifying and establishing interaction among decisions made at same priority level.
- Identifying and establishing interaction among decisions made at various priority level.

- Classifying and identifying decision scenarios in a coupled engineering system.
- Managing uncertainties in the design of coupled engineering systems.
- Capture, storage, reuse and update knowledge in the design of coupled systems.

In the context of these challenges, the focus in this thesis is to establish scientific foundations required for designing coupled engineered systems for an uncertain environment. The key elements are identified and shown in Figure 1.2.

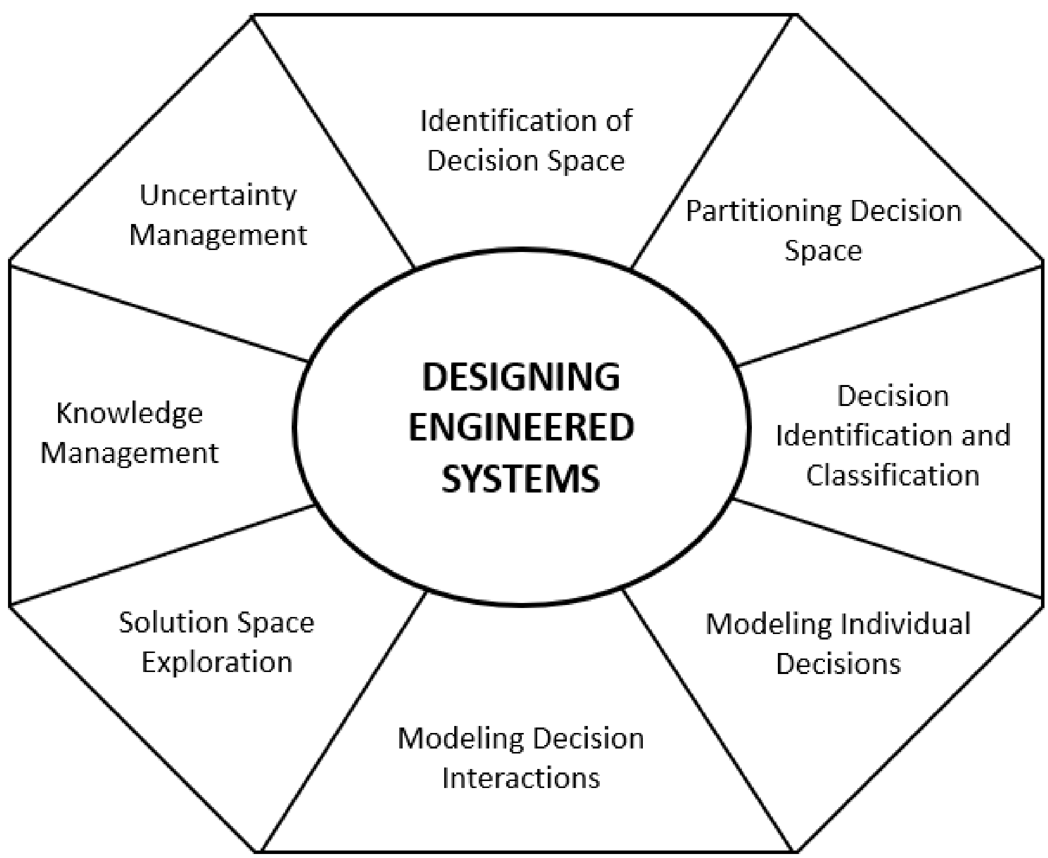


Figure 1.2: Elements in Design of Coupled Engineered Systems

In context of these design elements, key challenges to be addressed in this thesis and the associated research gaps are mentioned below:

Table 1.1: Identified Research Gaps

	Gap
G1	Modeling decision coupling among decisions in the design of coupled engineered system
G2	Framework to identify decision pattern for a given design problem
G3	Mathematical representation to model and analyze coupling in decisions
G4	Mitigating the effects of uncertainty pertaining to coupled decision problems in engineering design

As the principal goal in this thesis is to establish the scientific foundations that are required for the design of coupled engineered system in face of uncertainties. The design of such systems requires information from various domains and incorporation of knowledge and experience in design, materials and, manufacturing. This necessitates the need to have systematic approaches in representing those information and how they interact to influence one another, which gives rise to the following research question for this thesis:

Primary Research Question: What are the necessary scientific foundations necessary for designing and analyzing coupled engineered systems in an uncertain environment?

Before developing a scientific foundations, there is a need for understanding and representing coupling among various design decision. Given that we have two types of decisions, selection and compromise, it is important to establish coupling among these decisions that represent interactions at same and between various priority levels. This leads to a secondary research question associated with the primary research question (RQ1).

Secondary Research Question Associated with Primary Research Question (RQ1): What is the necessary mathematical foundation for modeling coupling among various design decisions required for designing and analyzing coupled engineered systems?

The hypotheses (H1) for answering the research question (RQ1) are as follows:

- By establishing a method to represent coupling among decisions lying at the same level and at different levels.
- Establishing the concept of horizontal and vertical coupling to represent coupling among various design decisions.

Given that a method to represent decision coupling is developed, the need is for a decision framework that can be utilized for modeling coupled design problem. This gives rise to the following research question:

Secondary Research Question Associated with Primary Research Question (RQ2): What is the necessary foundation for integrating the decision coupling to create a generalized decision framework suitable for designing coupled engineered systems?

This research question (RQ2) is supported by the following hypotheses (H2):

- Developing a classification scheme for representing coupled design problems.
- By establishing a multi-leveled decision scenario matrix that gives a generalized decision framework for coupled problems with two primary decisions (selection and compromise), varying strength of interaction and multi-level decision using DSPs.

By answering the above two research questions (RQ1 and RQ2), a decision framework to capture and model decision interactions for designing coupled engineered system is established. Now, the next question is, given any coupled design problem, how do we identify decision scenario/s from the decision framework. This can be done as explained below:

By identifying the nature and type of decision a preliminary selection of decision scenario could be made. Two or more scenarios when suitable may be selected and evaluated for specific problems.

Given that there lies a method to generate decision scenarios from a decision framework for modeling a coupled design problem, the need is also to establish mathematical foundations that

- Enable us to systematically explore the design space for effective decision.
- Mitigate the effect of uncertainty in decision-making.

There are various sources of uncertainty that may preclude a designer from creating a robust design. Uncertainty is pervasive and must be either mitigated or managed. For a coupled design problem, how do we address this issue of uncertainty in design of coupled systems. This gives rise to the following research question (RQ3):

Secondary Research Question Associated with Primary Research Question (RQ3): What is the mathematical foundation for designing and analyzing coupled engineered system under uncertainty?

The hypotheses (H3) for answering the research question (RQ3) are as follows:

- Developing the mathematical representation for defining the couplings identified by answering RQ1 and RQ2.
- By incorporating robustness metrics in the form of system constraints and goals in coupled DSPs. Depending on the kind of robustness required, different metrics may be applied, namely Error Margin Index (EMI) and Design Capability Index (DCI).

For designing a coupled engineered system, the challenges is discussed in this section. Following this, the gaps are identified and the hypotheses to fill this gaps are proposed. Being able to fill these gaps lead us to new knowledge, which are identified and tabulated in Table 1.2.

Table 1.2: Mapping Research Questions and Hypothesis to the New Knowledge

Research Gap	Hypothesis	New Knowledge	Research Questions
Modeling decision coupling among decisions in the design of coupled engineered system	- By establishing a method to represent coupling among decisions lying at the same level and at different levels. - Establishing the concept of horizontal and vertical coupling to represent coupling among various design decisions.	Method to represent coupling among design decisions	What is the necessary mathematical foundation for modeling coupling among various design decisions required for designing and

			analyzing coupled engineered systems?
Framework to identify decision pattern for a given design problem	- Developing a classification scheme for representing coupled design problems. - By establishing a decision scenario matrix that gives a generalized decision framework for coupled problems with two primary decisions (selection and compromise), varying strength of interaction and multi-level decision using DSPs.	DSP based decision scenario matrix for classifying decisions using decision scenario matrix for coupled design problems	What is the necessary foundation for integrating the decision coupling to create a generalized decision framework suitable for designing coupled engineered systems?
- Mathematical representation to model and analyze coupling in decision - Managing the effects of uncertainty pertaining to coupled decision problems in engineering design	- Developing the mathematical representation for defining the couplings identified by answering RQ1 and RQ2. - By incorporating robustness metrics in the form of system constraints and goals in coupled DSPs. Depending on the kind of robustness required, different metrics may be applied, namely Error Margin Index (EMI) and Design Capability Index (DCI).	Foundation to designing and analyzing coupled design problems under uncertainty	What is the mathematical foundations for designing and analyzing coupled engineered system under uncertainty?

In context of the following hypotheses, the next section is devoted to discussing the approach taken for representing decision interaction and the classification scheme for representing coupled design problems.

- By establishing a method to represent interaction among DSPs lying at the same level and at different levels.
- Developing a classification scheme for representing coupled design problems.
- By establishing a decision scenario matrix that gives a generalized decision framework for coupled problems with two primary decisions (selection and compromise), varying strength of interaction and multi-level decision using DSPs.

1.3 DSPs for Coupled Engineered Systems

The complexity in the analysis and synthesis of engineered systems as a single problem necessitates the need to decompose the design problem as dependent subsystems and then after solving subsystems recompose them (Bascaran, Karandikar et al. 1992). As mentioned in Section 1.1.2, the DSPT enables us to classify design decisions as either selection or compromise or combination of these decisions where selection decisions are modeled as selection Decision Support Problems (sDSP) and the compromise decisions are modeled as compromise Decision Support Problems (cDSP). Hence, any engineered system can be modeled as selection, or compromise or combination of these decisions. When there exists an interaction among these decisions in the given engineered system, the engineered system is referred to as coupled engineered system and the decisions (DSPs), either sDSP, or cDSP, or their combination is referred to as coupled DSPs. Coupled decision refers to the decision taken by accounting the interaction between the system/subsystem that are coupled through interacting variables. In essence, decision/s taken by accounting the influence of one decision over the another defines the decision coupling. Based on the strength of interaction, the coupling is shown in Figure 1.3 and Figure 1.4.

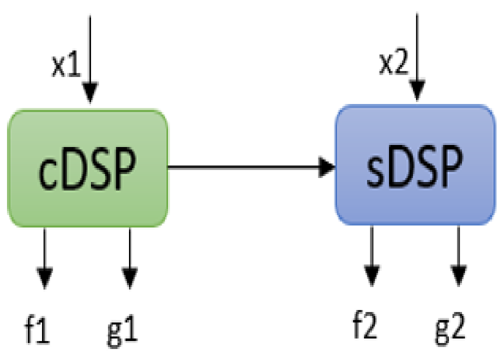


Figure 1.3: Weak Coupling

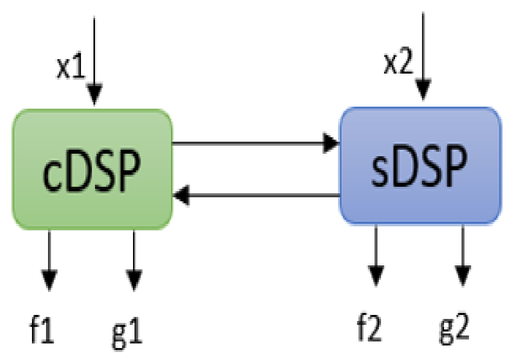


Figure 1.4: Strong Coupling

In Figure 1.3 and Figure 1.4,

x1 and x2 = Set of design variables and/or attributes for cDSP and sDSP respectively

f1 and f2 = Constraint functions for cDSP and sDSP respectively and,

g1 and g2 = Goal functions for cDSP and sDSP respectively

Strong coupling: In strong coupling, there is a two-way flow of information between the systems. For example, in a decision involving selection and compromise, the selection of an alternative affects the attainment of compromise goals whereas the attribute selected depends on the values of the compromise variables.

Weak coupling: In weak coupling, there is one-way flow of information between the system. In weak coupling, either the selection of an alternative affects the attainment of compromise goals or the attribute selected depends on the values of the compromise variables.

1.3.1 *Decision Scenario Matrix Using DSPs*

Decisions in the design of complex engineered system involve interactions. These interactions define the influence of one decision over other. To effect better decisions in the design of complex engineered system, it is imperative to capture these interactions and represent the complex system with numerous interacting decisions. To enable such representation of a complex engineered system, a classification scheme called the Multi-leveled Decision Scenario Matrix (MDSM) is illustrated in Figure 1.5. This is an extension of the Decision Scenario Matrix (DSM) described in (Sharma, Allen et al. 2019). The MDSM is created by identifying and classifying decision scenarios based on three criteria: (i) decision types (selection or compromise), (ii) strength of interaction and, (iii) decision levels. Three axes are used to represent these criteria. The Y-axis represents the type of decisions which may take three forms:

- Both design decisions involve compromise
- Both design decisions involve selection
- Design decisions involve combination of selection and compromise

Similarly, the X-axis represents strength of interaction. The strength of interaction between decisions are coupled through horizontal coupling which may also take three forms:

- There exists no interaction
- There exists a weak or one-way interaction
- There exists a strong or two-way interaction

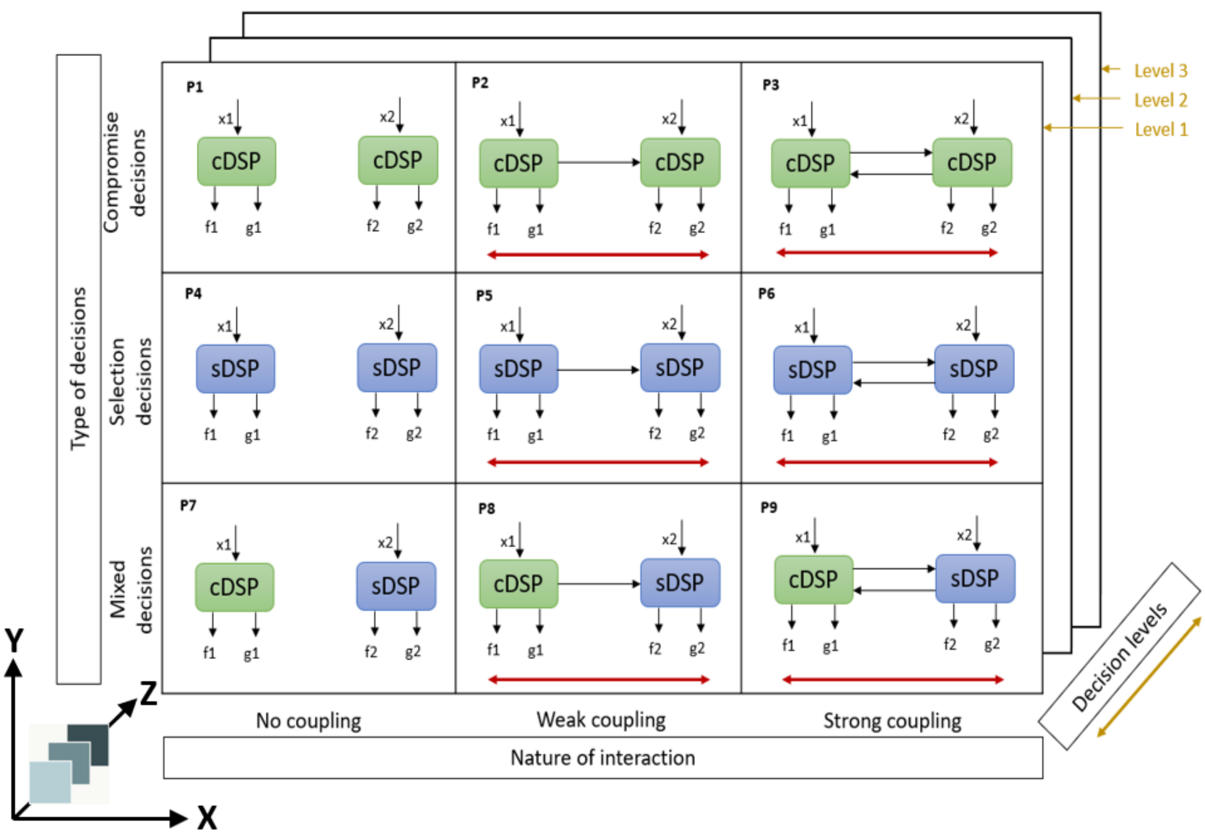


Figure 1.5: Multi-leveled Decision Scenario Matrix (MDSM)

Finally, the Z-axis represents the hierarchy in decisions and are assigned levels which represent the order in which hierarchical decisions are executed. Considering we have multiple decisions at various levels, we have Z-axis to represent such decisions. The leveled decisions are executed in a hierarchical fashion. Level 1 decisions have the highest priority and so on. The decisions at various levels are coupled with adjacent levels defined through vertical coupling.

Horizontal coupling defines the influence of one DSP over other at the same level. For instance: Compromise among variables defining gear geometry and selection of gear material form concurrent decisions which lie at the same level and are coupled through horizontal coupling. Horizontal coupling with two-way arrow indicates strong coupling, which means there is two-way flow of information between the decisions. For example, in a decision involving selection and compromise, the selection of an alternative affects the attainment of compromise goals whereas the attribute selected depends on the values of the compromise variables. Horizontal coupling with one-way arrow indicates weak coupling, which means, there is one-way flow of information between the system. In weak coupling, either the selection of an alternative affects the attainment of compromise goals or the attribute selected depends on the values of the compromise variables. Similarly, vertical coupling defines the influence of decisions among adjacent levels. For instance: Gear design (gear geometry and gear material) influences shaft design.

1.3.2 *Design of a Gearbox – Coupled Decision Scenarios*

To understand the coupling in design decisions, the example of designing a gearbox is taken. In context of a gearbox involving gear, pinion and shafts at input and output, let us consider that we are interested in the following 4 design decisions:

- Gear dimensions
- Gear material
- Shafts dimensions
- Shaft material

Let us also assign the hierarchy in decision with two levels as:

Level 1: Gear decisions (Gear dimensions and gear material)

Level 2: Shaft decisions (Shaft dimensions and Shaft material)

The two levels will be coupled together by the performance requirement Z. For instance: Torque is one of the Z's that binds the two levels together.

Moving further, these decisions are formulated using one of the three decision pattern (P1, P8 and P9) or the combinations of these patterns at two levels. For instance: P1 could be implemented at level 1 while P1, P8 or P9 at level 2 and so on. As such, we could have one of the 3 ways to formulate decision at level 1 and 3 ways to formulate decision at level 2. Hence, we have 9 types of scenarios to implement the 4 decisions in the design of a gearbox.

Table 1.3: Decision Classification for Modeling Decisions as DSPs

SN	Decision	Decision classification as DSP
1	Gear geometry	cDSP
2	Shaft geometry	cDSP
3	Gear material	sDSP - Selection from pool of materials
		cDSP – Design of material
4	Shaft material	sDSP - Selection from pool of materials
		cDSP – Design of material

All the decisions identified in Table 1.3 have been assigned levels, i.e., they can be modeled sequentially. The following table contains the hierarchical information:

Table 1.4: Hierarchy of Decisions in the Design of Gearbox

Hierarchy	Decision	Coupled DSP
Level 1	Coupled gear geometry – gear material	(cDSP – sDSP) or (cDSP)
Level 2	Coupled shaft geometry – shaft material	(cDSP – sDSP) or (cDSP)

Following the information tabulated in Table 1.3 and Table 1.4, the decision patterns that are utilized in modeling the decisions involved in the decision of a gearbox are identified. In both level 1 and 2 , we can execute either (cDSP – sDSP) or (cDSP). If we look for such decision in the Decision Scenario Matrix (DSM), we can identify 3 patterns at level 1, that is, P1, P8 and P9.

1.4 Verification and Validation of Thesis Chapters

Validation square framework introduced by Pederson and co-authors (Pedersen, Emblemsvag et al. 2000, Seepersad, Pedersen et al. 2006) is used in this thesis for implementing verification and validation strategy. Verification deals with the internal consistency in the method proposed while validation deals with the justification of knowledge claims. The validation square construct to validate design methods is shown in Figure 1.6.

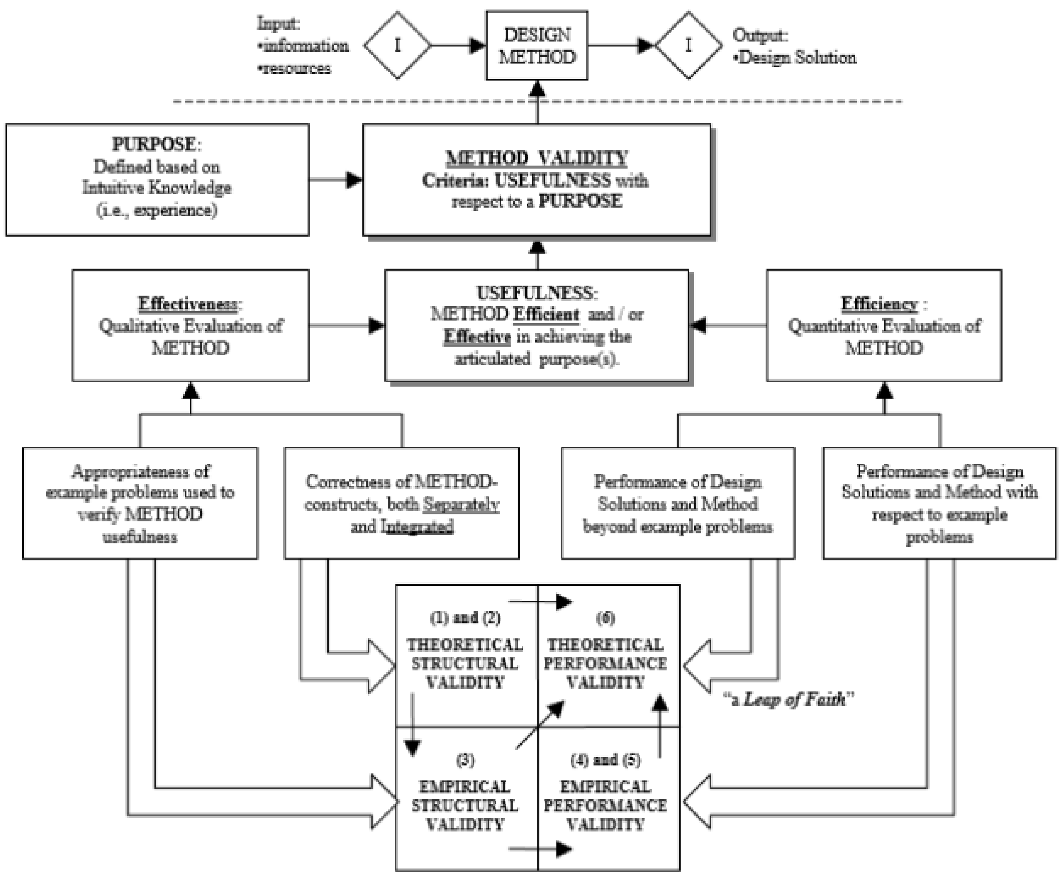


Figure 1.6: Validation Square Framework for Validating Design Methods - Adapted from Seepersad and Co-authors (Seepersad, Pedersen et al. 2006)

The Validation Square shown in Figure 1.6 involves the process of building confidence in the usefulness with respect to a purpose. In philosophical sense, validation refers to internal consistency while verification deals with the justification of knowledge claims. However, from modeling perspective, verification refers to the internal consistency and validation refers to the justification of knowledge claims. Validation Square consists of four quadrants as explained below:

Theoretical Structure Validity (TSV): It involves assessing the internal consistency, i.e., logical soundness of the individual constructs as well as integration of the constructs. The validation of TSV comes from its utility that it can be used for ESV. It requires the following steps:

- Ascertaining the requirements (outcomes as well as process) of the design method.
- Critical evaluation of technical literature in context of design requirements
- Establishing internal consistency of the design method (Individual and Integrated)

Empirical Structural Validity (ESV): It involves examining the appropriateness of the test problems selected to illustrate and verify the design method. The validation of ESV comes from its utility that it can be used for EPV. ESV involves following steps:

- Documenting the appropriateness of the test example with respect to the design method.
- Verifying that the results from the test problem support the use of design method.

Empirical Performance Validity (EPV): It involves examining the appropriateness of the comprehensive test problems selected to illustrate and verify the design method. The validation of EPV comes from its utility that it can be used for TPV. EPV involves following steps:

- Establishing usefulness of the results by applying the design method on the test examples.

Theoretical Performance Validity (TPV): It involves establishing confidence in the generality of the design method. It involves speculation but is anchored in the foundations that are laid on TSV, ESV and EPV. Verification for TPV comes from all the three quadrants (TSV, ESV and EPV). The validation to TPV comes from the idea that the method can be extended, that is, establishing the utility of the presented method in examples not presented in the thesis. It involves establishing confidence in using the design method beyond the examples that have been presented in the thesis. TPV involves following steps:

- Verification anchored in what have been shown in TSV, ESV and EPV.

- Establishing usefulness of the design method to provide useful results beyond the test problems.
- Showcasing the design method as a generic method that can be applied to other design problems.

1.4.1 *Verification and Validation Framework Applied in the Thesis*

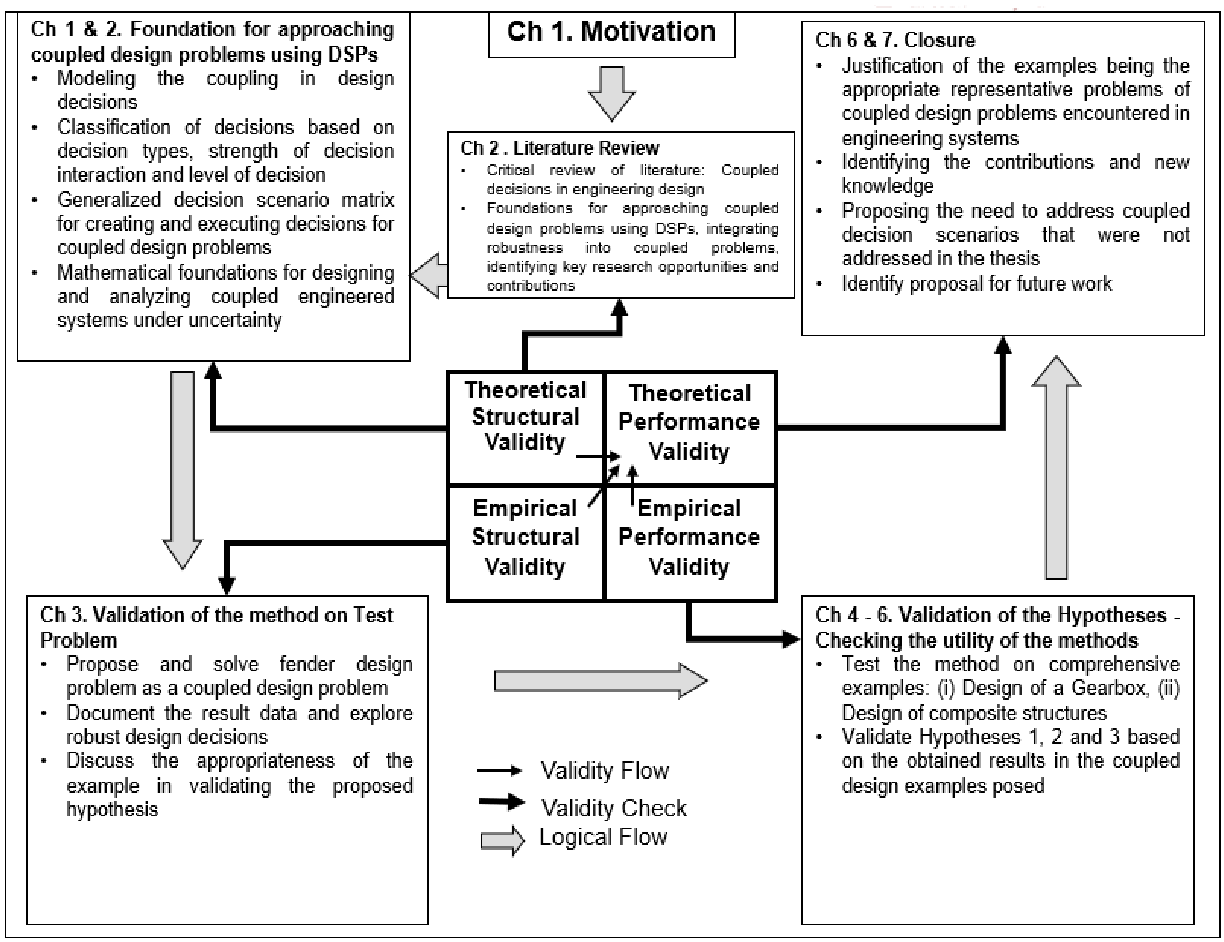


Figure 1.7: Organization of Thesis Chapters with Verification and Validation Square

Table 1.5: Overview of Verification and Validation Strategy used in Thesis Chapters

The foundation for the thesis is established in Chapter 1, along with the motivation for doing this thesis in context of the research gaps. Chapter 1 also includes DSP based classification scheme for coupled design problems.	
Quadrants in Validation Square	**Verification and Validation Strategy Applied to the Thesis Chapters**
1	**Theoretical Structure Validity** The assessment to internal consistency for establishing the logical soundness of the design method is dealt in Chapters 1 and 2. In Chapter 1, the need to address the decision coupling and robust decision making in design of engineered systems is established. Also, the suitability of Decision Support Problem Technique (DSPT) for modeling decisions as DSPs is discussed. The creation and utility of Multi-leveled Decision Scenario Matrix (MDSM) is explained. Finally, the scope of the work, including the research questions posed, hypothesis proposed, and the boundary of the present work is detailed. Chapter 2 contains the detailed discussion about the about all the tools, techniques, formulation and mathematical framework that will

	be applied in this work. In particular, the discussion is on coupled decisions, robustness, compromise Decision Support Problem (cDSP) construct, selection Decision Support Problem (sDSP) construct, Design Capability Index (DCI) and Error Margin Index (EMI).
2	**Empirical Structural Validity** The examination of the appropriateness of the test problem selected to illustrate and verify the design method is dealt in Chapters 3 and 6. In Chapter 3, first demonstrative instance of a coupled design problem is introduced. The coupling in decision in the design of a fender is discussed. The mathematical formulations for solving the fender design problem as (i) a coupled problem approach and, (ii) material design approach is detailed. Following this, mathematical formulations for addressing uncertainties pertaining to the design of fender as a coupled decision problem is presented. In Chapter 6, the results obtained in Chapter 3 is discussed. The results pertaining to each mathematical formulations in Chapter 3 are presented and details regarding the solution exploration approach is discussed. In detail, the discussion about the validity and usefulness of the method is outlined.

3	**Empirical Performance Validity** The examination of the appropriateness of the comprehensive test problems selected to illustrate and verify the design method is dealt in Chapters 4, 5, and 6. In Chapter 4, design decision making in the design of a gearbox is introduced as a multi-level coupled design problem. This followed with the DSP based mathematical formulations for solving a multi-level coupled design problem. In Chapter 5, the overall picture of decision problem in the design of composite structures is presented. First, the DSP based mathematical formulations for the design of composite structures as (i) a coupled problem approach and, (ii) multiscale approach is presented. Following this, the DSP based mathematical formulations for the robust design of composite structures as multiscale problem is presented. In Chapter 6, the results obtained from Chapters 4 and 5 are respectively presented and discussed. The results pertaining to each mathematical formulations in Chapters 4 and 5 are presented and details regarding the solution exploration approach is discussed. In detail, the discussion about the validity and usefulness of the method is outlined.

4	**Theoretical Performance Validity** It involves speculation but is anchored in the foundations that are laid on TSV, ESV and EPV . Verification for TPV comes from all the three quadrants (TSV, ESV and EPV). The validation to TPV comes from the idea that the method can be extended, that is, establishing the utility of the presented method in examples not presented in the thesis. Establishing confidence in the generality of the design method is dealt in Chapter 6 and 7. In Chapter 6, the results pertaining to the test problems are presented and their usefulness is discussed. Following this, the discussion is on the generality of the method. In Chapter 7, a summary of this thesis is given at first. The research questions are then revisited and discussion on the research questions and hypotheses are made. Further, the achievements and contributions made on this thesis are summarized. Finally, the author's vision for opportunities in further research is presented.

Table 1.6: Layout of Thesis Chapters

		Relevance	Hypothesis
Introduction and Problem Identification	**Chapter 1** Frame of reference – Decision matrix with DSPs for coupled problems in engineering design problems, propose RQs and hypotheses	• **Chapter 1: Introduction to the coupled problems in engineered system. Creation of decision matrix using DSPs for coupled problems. Propose RQs and hypotheses**	Establish problem background
Foundations, Theories and Methods	**Chapter 2** Critical review of Literature - Decisions in engineering design, robust DSP constructs	• **Chapter 2: Review literatures, introduce existing mathematical/non-mathematical techniques, methods, tools, etc. to be applied in addressing gaps to be filled**	Propose and Elaborate
Framework Development and introduction to test problem	**Chapter 3** Designing a Fender **Chapter 4** Designing a Gearbox **Chapter 5** Designing a composite	• **Chapter 3, 4 and 5: Develop a computational framework for exploring robust design solutions for coupled design problems. Application of the developed framework in 3 test problems to synthesize robust decisions.**	Step on hypotheses (H1, H2 and H3) to establish framework
Hypothesis Testing	**Chapter 6** Demonstration of the developed framework in achieving the goals and focus of the work by answering the research questions	• **Chapter 6: Demonstrate how the coupled problems has been addressed. Show the results from each coupled DSPs. Discuss on how the decisions are inter-related and how robust decisions can be taken in an integrated fashion. Verify the hypothesis.**	Discuss outcomes and verify hypothesis
Closure	**Chapter 7** Closure	• **Chapter 7: Summarizing, evaluate the extent to which objectives of the work has been achieved, critically review answers to research questions, discuss limitations of the framework and propose future directions.**	Summarize, evaluate and explore possible dimension to future research

Chapter 2: Mathematical Tools and Constructs for Framing and Exploring Robust Decisions in Coupled Problems

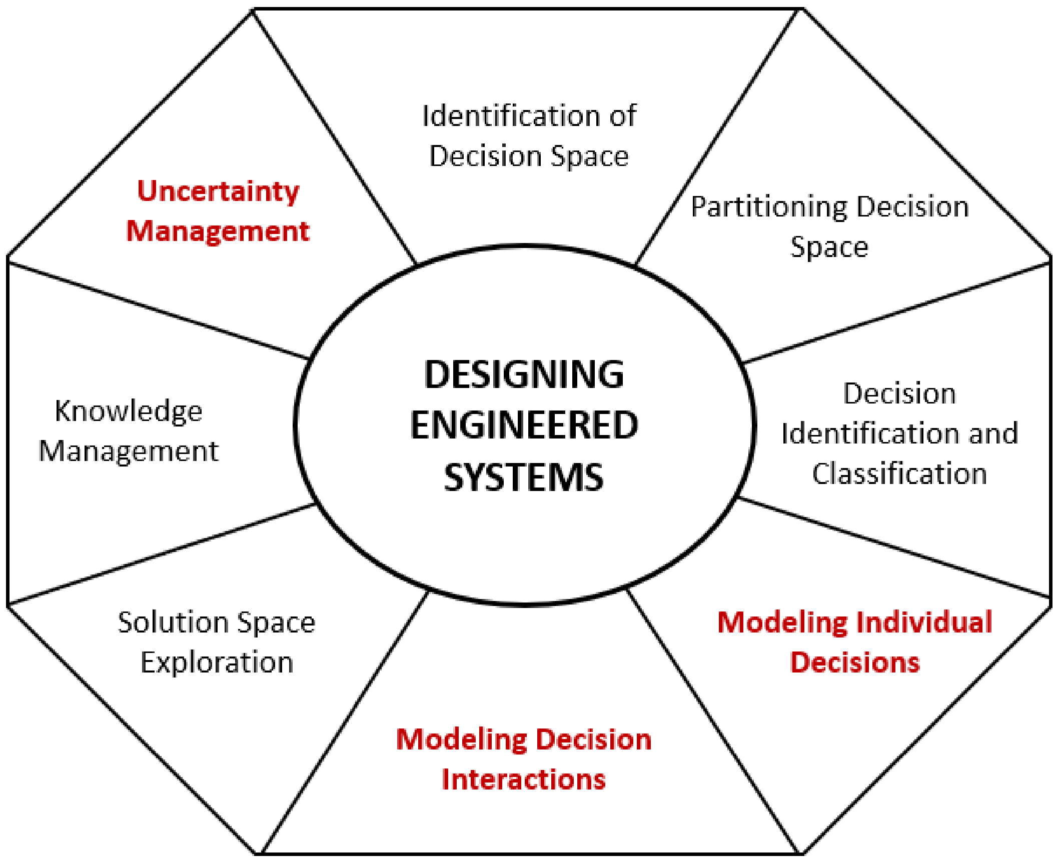


Figure 2.1: Elements in Design of Coupled Engineered Systems Discussed in Chapter 2

In Chapter 2, three elements in the design of engineered systems is discussed as shown in Figure 2.1 (highlighted in red). In this context, Chapter 2 contains the detailed discussion about all the tools, techniques, formulation and mathematical framework that is applied in this thesis. In detail discussion is on coupled decision, robustness, compromise Decision Support Problem (cDSP) construct, Design

Capability Index (DCI) and Error Margin Index (EMI). All discussion includes the mathematics behind each tools, techniques and constructs that will be used in the thesis. Section 2.1, 2.2 and 2.3 will detail the foundational design constructs used in this thesis. In Section 2.4, introduction to robust design methods for managing and mitigating the effect of uncertainty in the design of engineered systems is presented.

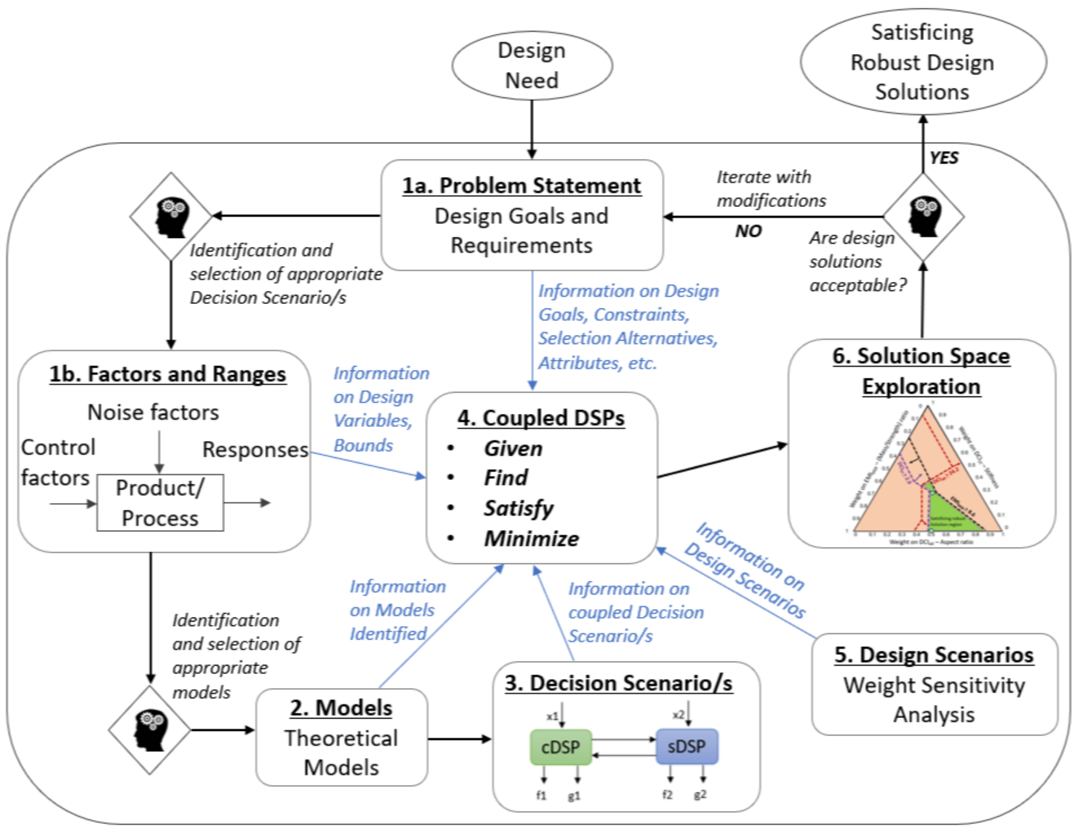


Figure 2.2: Procedure for Exploring Robust Design Solutions for Coupled Problems

2.1 Decision Based Design

In this thesis, the design of engineered systems is viewed from the Decision-Based Design (DBD) perspective. In Decision-Based Design (DBD), engineering design is recognized as a decision- making process. The underlying notions of decision-based design are discussed at greater detail in (Shupe 1988, Mistree, Smith et al. 1990, Hazelrigg 1998). The foundational premise in DBD is that the principal role of an engineer, in the design of an artifact, is to make decisions. There are two important characteristics of a decision (Hazelrigg 1996):

- A decision is made at an instant in time.
- A decision must be made based on the information available at the time it is made.

Several characteristics associated with design decisions are identified and are summarized as descriptive sentences (Mistree and Muster 1990):

- Design decisions are invariably multidimensional and multileveled in nature.
- Decisions in design involve information coming from different sources and disciplines.
- Decisions in design are governed by multiple measures of merit and performance.
- All the information needed to make decisions may not be available.
- Some of the information required to make a decision may be hard, that is, based on scientific principles and some of the information may be soft, that is, based on the designer's judgment and experience.

- The problem for which design decisions are being made are invariably loosely defined and open and are characterized by the lack of a singular, unique solution. The decisions are less than optimal which represent satisficing solutions.

Smith and co-authors (Smith, Kamal et al. 1987) suggest that a decision-based design process involves:

- a series of decisions, some being made concurrently and some sequentially.
- multilevel, multidisciplinary and multidimensional decision-making where interactions occur among subsystems on various levels of the decision tree on one or both directions.

2.2 The Decision Support Problem Technique (DSPT)

Since, its inception DBD has become a topic of discussion among design community that has led to the development of design methods. As such, the implementation of Decision-Based Design can take many forms (Mistree and Muster 1990). One manifestation of the Decision-Based Design (DBD) construct is the Decision Support Problem Technique (DSPT) developed to provide support to human designers in exercising judgment in making design decisions (Mistree, Muster et al. 1989). The three components that consists DSP Technique are: a design philosophy rooted in systems thinking, an approach for identifying and formulating DSPs, and software (Marston and Mistree 1997). In DSP Technique, designers are required to implement two phases, that is, a meta-design and a computer-based design phase (Marston and Mistree 1997). Meta-design phase is achieved by partitioning the problem into constituent DSPs and devising a plan of action required to convert information that characterizes the needs and requirements for a product into knowledge about a prototype of a product that can be manufactured and maintained. In computer-based design phase, computer assistance is sought in making calculations and visualizations to support human designers in making informed decisions. This phase involves a constant interaction between a computer and a human designer. The two phases in DSP Technique is summarized in the table below.

Table 2.1: The Phases of DSP Technique (Mistree and Muster 1990)

Phase I: Meta-Design	Phase II: Design
STEP 1: IDENTIFY/CLARIFY PROBLEM Create Problem Story ↓ Technical brief	STEP 4: STRUCTURE • Organize domain-dependent information and formulate DSP templates • Develop DSP word formulations. • Develop DSP mathematical formulations.
STEP 2: PARTITION AND PLAN Partition each abstract into problem statements and identify decisions associated with each problem statement.	STEP 5: SOLVE • Solve the DSPs using appropriate means to obtain solutions.

STEP 2: PARTITION AND PLAN	STEP 6: POST-SOLUTION ANALYSIS
Identify the Decision Support Problems and Decision Blocks. Create plan for sequence of solutions	• Verify and validate solutions • Sensitivity analysis. • Check for consistency. • Check for need for iteration. • Make design decisions.

For formulating a design problem as DSPs, the following types of decisions are identified:

Selection decisions – It deals with making a choice between a number of alternatives taking into account a number of measures of merit or attributes (Kuppuraju, Ittimakin et al. 1985, Mistree, Marinopoulos et al. 1988, Vadde, Allen et al. 1994).

Compromise decisions – It deals with the determination of the "right" values (or combination) of design variables to describe the best satisficing system design with respect to constraints and multiple goals (Mistree, Hughes et al. 1993).

Derived DSPs (see Figure 2.3) – It deals with decisions that requires a combination of primary DSPs in order to model a complex decision, e.g., selection/selection, compromise/compromise and selection/compromise decisions (Bascaran, Bannerot et al. 1989, Karandikar and Mistree 1991, Mistree, Smith et al. 1991, Vadde, Allen et al. 1994).

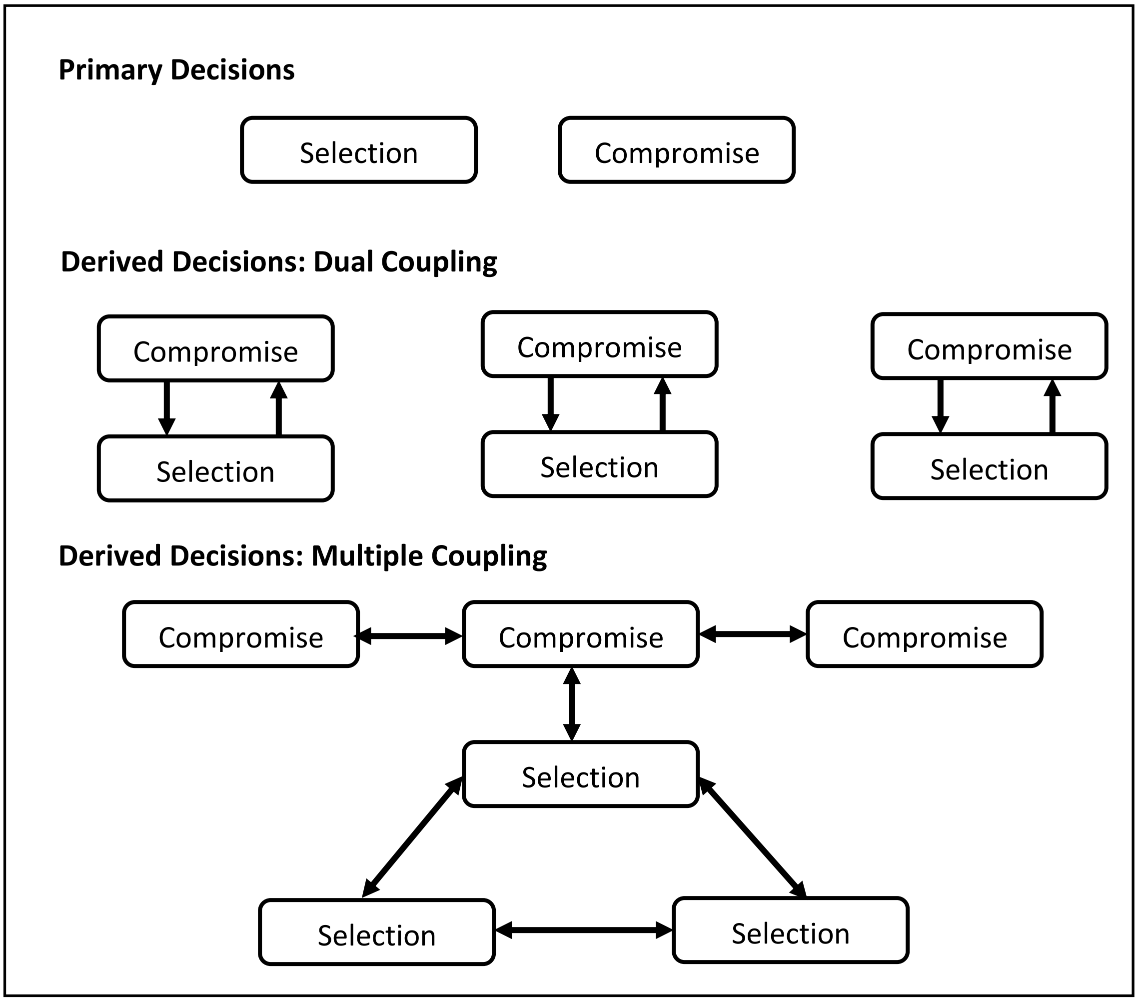


Figure 2.3: Types of Decisions to Model as DSPs

Selection decisions are modeled as selection Decision Support Problems (sDSP) and the compromise decisions are modeled as compromise Decision Support Problems (cDSP). Coupled decisions are modeled by accounting for the interaction between the DSPs as opposed to independent decisions when the individual DSPs do not interact with each other and the decisions can be taken independently. Karandikar and co-authors provide a method for dealing with coupled DSPs (Bascaran, Bannerot et al. 1989, Karandikar and Mistree 1992).

2.3 The Compromise Decision Support Problem Construct

The cDSP is proposed by Mistree and coauthors for modeling engineering decisions involving multiple trade-offs (Mistree, Hughes et al. 1993, Bras and Mistree 1994). By implementing the cDSP construct several design solutions are identified by carrying out trade-offs among multiple conflicting goals. Solutions thus, obtained are evaluated by carrying out solution space exploration for identifying best solutions that satisfy the designers requirements.

The compromise DSP formulation is a multi-objective programming model that incorporates concepts from both traditional mathematical programming and goal programming. The compromise DSP is similar to goal programming in that the multiple objectives are formulated as system goals, involving both system and deviation variables and the deviation function is solely a function of the goal deviation variables (for correspondences between terms used in goal programming and compromise DSP, see Table 2.2). This contrasts from the traditional mathematical programming where multiple objectives are modeled as a weighted function of the systems variables only. From the traditional constrained optimization formulation, it retains the concept of system constraints. In compromise DSP, special emphasis is placed on the bounds of the system variables. For feasibility, the system constraints and bounds must be satisfied. Further, in cDSP, the feasible design space is defined by the set of system constraints and bounds while the set of system goals define the aspiration space, see Figure 2.4. A satisficing solution then is that feasible point which achieves the system goals as far as possible. The solution to this problem represents a tradeoff between that which is desired (as modeled by the aspiration space) and that which can be achieved (as modeled by the design space) (Mistree, Smith et al. 1993).

Table 2.2: Correspondences between Terms Used in Goal Programming and Compromise DSP

GOAL PROGRAMMING	COMPROMISE DSP
Vector of problem variables	Vector of system variables
Rigid or hard goal	System constraint
Flexible or soft goal	System goal
Achievement function	Deviation function

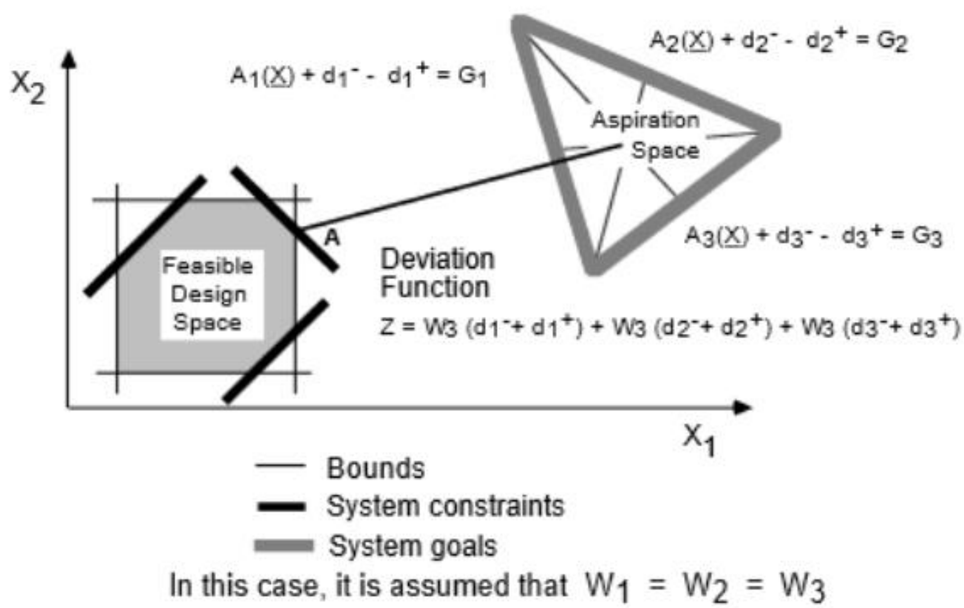


Figure 2.4: Graphical Representation of a Two-Dimensional Compromise DSP, Archimedean Formulation (Mistree, Smith et al. 1993)

There are four keywords used in the formulation of a compromise DSP. The four keywords are GIVEN, FIND, SATISFY and MINIMIZE. Using these keywords, compromise DSPs can been formulated as shown in Table 2.3.

Table 2.3: The cDSP Formulation (Mistree, Hughes et al. 1993)

GIVEN An alternative to be improved, domain dependent assumptions The system parameters: n number of system variables, q inequality constraints, p + q number of system constraints, m number of system goals, $g_i(X)$ system constrain functions $f_k(d_i)$ function of deviation variables to be minimized at priority level k for the preemptive case FIND *System variables:* The values of the independent system variables. Xi ; i = 1, 2, …, n (They describe the physical attributes of an artifact) *Deviation variables:* The values of the deviation variables. di^-, di^+ ; i = 1, 2,…,m (They indicate the extent to which the goals are achieved) SATISFY *System constraints:* These must be satisfied for the solution to be feasible (linear, non-linear)

$g_i(X) = 0$; $i = 1....p$

$g_i(X) \geq 0$; $i = p+1.....p+q$

System goals: These need to achieve a specified target value as far as possible

(linear, non-linear)

$A_i(X) + di^- - di^+ = G_i$; $i = 1...m$

Bounds: Lower and upper limits on the system variables.

$X_i^{min} \leq X_i \leq X_i^{max}$; $i = 1...n$

$di^-, di^+ \geq 0, di^- * di^+ = 0$; $i = 1...m$

MINIMIZE

A deviation function: A function that measures the deviation of the system

performance from that indicated by the set of goals and their associated priority

levels or relative weights.

Case a: Preemptive formulation (lexicographic minimum)

$$Z = [f_1(di^-, di^+), \ldots\ldots, f_k(di^-, di^+))]$$

Case b: Archimedean

$$Z = \sum_{i=1}^{m} w_i \cdot (di^- + di^+) \ , \ \sum_{i=1}^{m} w_i = 1$$

The selection DSP can be reformulated as a compromise DSP, the compromise DSP is considered the principal mathematical DSPT formulation (Bascaran, Bannerot et al. 1989). This transformation of selection to compromise makes it possible to formulate and solve coupled selection-selection DSPs and coupled selection-compromise DSPs (Smith, Kamal et al. 1987, Karandikar, Srinivasan et al. 1989, Bascaran, Karandikar et al. 1992). Similar to compromise DSP, there are also four keywords used in the formulation of a selection DSP. The four keywords are GIVEN, IDENTIFY, RATE and RANK. Using these keywords, compromise DSPs can been formulated as shown in Table 2.4.

Table 2.4: The sDSP Formulation

GIVEN A set of candidate alternatives. IDENTIFY The principal attributes influencing selection. The relative importance of attributes. RATE The alternatives with respect to their attributes. RANK The alternatives in order of preference based on the computed merit function values.

The solution to the DSPs are solved in a software called DSIDES (Decision Support In the Design of Engineering Systems). The compromise DSP is solved using a unique optimization scheme called Adaptive Linear Programming. The ALP algorithm with its multilevel, multigoal feature is incorporated in DSIDES, a tailored computational infrastructure for formulating, solving and analyzing Decision Support Problems (Mistree and Kamal 1985, Reddy, Smith et al. 1996).

Mistree and coauthors believe three important features contribute to the success of the ALP algorithm (Mistree, Hughes et al. 1993), the use of second-order terms in linearization; the normalization of the constraints and goals and their transformation into generally well-behaved convex functions in the region of interest; an “intelligent” constraint suppression and accumulation scheme.

There are templates available in DSIDES for designing thermal energy systems, composite structures, gearbox, pressure vessels, etc. Currently, Platform for Decision Support in the Design of Engineering Systems (PDSIDES), that is a knowledge-based platform is being developed (Ming, Nellippallil et al. 2018). The principal idea in PDSIDES is to allow designers to reuse previous knowledge (which is archived in a knowledge base) to compose decision workflow templates by configuring, reconfiguring, combining different building blocks.

2.3.1 *Modeling Decision Interactions*

The decision interaction is the result of decision influence that exists between decisions. Most of the time when multiple decisions are to be taken for subsystems that represent a system, very rarely can decisions be taken in isolation to one another. Hence, it is imperative to account for the influence that one decision might exert on other. To model such decision interactions, “decisions” are characterized as Decision Support Problems (DSPs) and two major kind of interactions are defined. Horizontal coupling defines and models the interaction between DSPs that lie at the same hierarchical level while vertical coupling defines and models the interaction between DSPs at adjacent hierarchical levels.

Table 2.5: Simplified Mathematical Form for Demonstrating Coupled Selection – Compromise Decision using DSPs

Coupled selection – compromise DSPs	
compromise DSP	selection DSP
Find	**Find**
Compromise System Variables	Selection System Variables
X	**Y**
Deviation Variables	Deviation Variables
d_i^- , d_i^+	e_i^- , e_i^+
Satisfy	**Satisfy**
Design Constraints	Selection Constraint
$g_j (X, \mathbf{Y}) > 0$	$\sum_{i=1}^{n} \mathbf{Y_j} = 1$
Constraints on Deviation Variables	Constraints on Deviation Variables
$d_i^+ \geq 0, d_i^- \geq 0, d_i^+ . d_i^- = 0$	$e_i^+ \geq 0$, $e_i^- \geq 0$, $e_i^+ . e_i^- = 0$
Compromise Goals	**Selection Goal**
$A_i (X, \mathbf{Y}) + d_i^- - d_i^+ = G_i$	$MF_j (\mathbf{X}) Y_i + e_1^- - e_1^+ = 1$
Bounds	**Bounds**
$B: X^{(min)} \leq X \leq X^{(max)}$	$B: 0 \leq Y_j \leq 1$
Minimize $Z = \{e1^-, \sum_{i=1}^{n} w_i \cdot (di^- + di^+) , \sum_{i=1}^{3} w_i = 1$	

Based on the strength of interaction between the DSPs, two formulations are defined. The weak formulation defines an interaction in which there is one-way flow of information between DSPs. The strong formulation defines an interaction in which there is two-way flow of information between DSPs. The concise mathematical form for strong interaction between DSPs (selection and compromise) is shown in Table 2.5. It is worth noting that system variables (X) from compromise DSP influence selection goal (MF) in selection DSP and selection alternatives (Y) from selection DSP influence compromise constraints gi(X,Y) and goals Ai(X,Y) in compromise DSP.

The mathematical formulation in Table 2.5 is utilized in developing a mathematical formulation for modeling interactions among decisions for designing a fender, designing a one-stage reduction gearbox and, designing composite structures.

2.4 Robust Design of Engineered Systems Under Uncertainty

In the thesis, the idea of robust design deals with the identification of design solutions that are relatively insensitive to uncertainties. In the design of an engineered system, one fundamental challenge lies in accounting for the various sources of uncertainties. However, uncertainties and risks are pervasive and must be managed to effect robust solutions. Also, as the computational models are abstractions of reality, we need design solutions that are relatively insensitive to uncertainties. In this section, the review of various sources of uncertainties are made and consequently, the robust design methods that are developed to mitigate the impact of such uncertainties are discussed.

2.4.1 *Classification of Uncertainties*

4th century BC Greeks have the first recorded history to have considered uncertainty in the context of epistemology (Thunnissen 2003). The word epistemology is derived from the Greek episteme, meaning "knowledge", and logos, which has several meanings, including "theory". Research efforts in uncertainty has come from researchers from wide variety of domains, including, social sciences, economics, engineering, medicine and more. There are numerous classification of uncertainties. One fundamental classification comes from management science. In the field of management science, particularly the probabilistic risk analysis community, define uncertainty as "that which disappears when we become certain" (Bedford and Cooke 2001). The uncertainty classification and their definitions are provided in the figure and the table that follow.

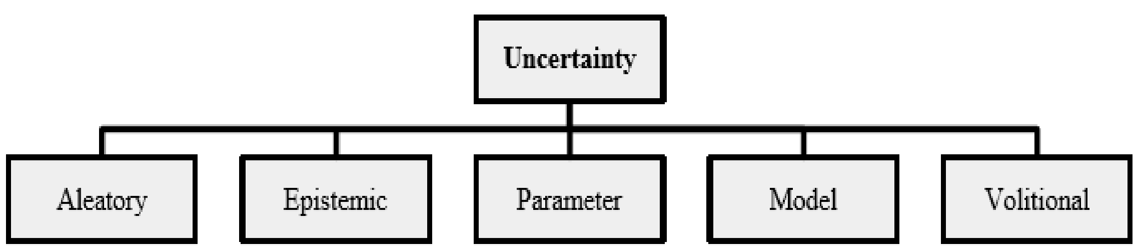


Figure 2.5: Uncertainty Classification for Management Science (Bedford and Cooke 2001)

Table 2.6: Uncertainty Definitions for Management Science (Bedford and Cooke 2001)

Uncertainty	**Definition**
Aleatory	Arises through natural variability in a system
Epistemic	Arises through lack of knowledge of a system
Parameter	Uncertainty about the 'true' value of a parameter in a mathematical model
Model	Uncertainty about the truth of the model
Volitional	Uncertainty that an individual has in whether or not he will do what he agreed to do

Another way to categorize the sources of uncertainty is available in (Kennedy and O'Hagan 2001).

Parameter uncertainty

Parameter uncertainty comes from the model parameters that are inputs to the computer model (mathematical model) but whose exact values are unknown to experimentalists and cannot be controlled in physical experiments, or whose values cannot be exactly inferred by statistical methods. For example, material properties in a finite element analysis for engineering.

Parametric variability

Parametric variability comes from the variability of input variables of the model. For example, the dimensions/surface finish of a work piece in a process of manufacture may not be exactly as designed and instructed, which would cause variability in its performance.

Structural uncertainty

Structural uncertainty comes from the lack of knowledge of the underlying physics in the problem and depends on how accurately a mathematical model describes the true system, considering the fact that models are almost always only approximations to reality.

Algorithmic uncertainty

Algorithmic uncertainty comes from numerical errors and numerical approximations per implementation of the computer model. Most models are too complicated to solve exactly. For example, the finite element method or finite difference method may be used to approximate the solution of a partial differential equation resulting in numerical errors.

Experimental uncertainty

Experimental uncertainty comes from the variability of experimental measurements. The experimental uncertainty is inevitable and can be noticed by repeating a measurement for many times using exactly the same settings for all inputs/variables.

Interpolation uncertainty

Interpolation uncertainty comes from a lack of available data collected from computer model simulations and/or experimental measurements. For inputs other than simulation data or experimental measurements, it is required to interpolate or extrapolate in order to predict the responses.

The understanding of various types of uncertainties is starting point of developing methods to quantify and address them. These methods help us deal with uncertainties by mitigating the effect of uncertainties. Two major types of problems lies in uncertainty quantification[2]. One is the forward propagation of uncertainty, where the various sources of uncertainty are propagated through the model to predict the overall uncertainty in the system response. Other one is the inverse assessment of model uncertainty and parameter uncertainty, where the model parameters are calibrated simultaneously using test data. Engineering design community is increasingly attracted to the inverse uncertainty quantification method since, uncertainty quantification of a model and the subsequent predictions of the true system response(s) are of great interest in designing robust systems.

In this thesis, the foundational concepts of uncertainty classification and robust design methods is based on the works by (Chen, Allen et al. 1996, Choi, Austin et al. 2005, Seepersad, Allen et al. 2005, Allen, Seepersad et al. 2006, McDowell, Panchal et al. 2009, Allen, Panchal et al. 2015). Uncertainty classification by Isukapalli and coauthors (Isukapalli, Roy et al. 1998) is extended and presented by

[2] https://en.wikipedia.org/wiki/Uncertainty_quantification

Choi and co-authors (Choi, Austin et al. 2005), who categorize the types of uncertainty as follows:

Natural uncertainty (NU): Uncertainty due to the inherent randomness or unpredictability of a physical system. Such uncertainty is irreducible and can only be quantified in a statistical sense.

Model parameter uncertainty (MPU): Uncertainty due to the incomplete knowledge of model parameters/inputs due to insufficient or inaccurate data. Such uncertainty is reducible by sufficient data or accurate measurements.

Model structure uncertainty (MSU): Uncertainty due to uncertain model formulation due to approximations and simplifications in a model. Such uncertainty is reducible by improving the model formulation.

Propagated uncertainty (PU): Uncertainty expanded by a combination of the above two types of uncertainty in a chain of models come under this category. As a result, the final performance estimation of the chain of models may have a large degree of uncertainty.

Given the various types of uncertainty prevalent in designing an engineered system, the need is to have robust design methods to address such uncertainties. One way would be to reduce the uncertainty itself and the other would be to manage or mitigate the impact arising due to such uncertainties. The focus in this thesis is to address uncertainties by designing engineered systems to be insensitive to the uncertainties without actually eliminating or reducing the uncertainties itself. There are four types of robust design method (Chen, Allen et al. 1996, Choi, Austin et al. 2005, Seepersad, Allen et al. 2005, Allen, Seepersad et al. 2006, McDowell, Panchal et al. 2009, Allen, Panchal et al. 2015).

2.4.2 *Robust Design Method*

The robust design methods have been identified based on the various sources of uncertainties. In this section, four types of robust design methods are discussed.

Type-I robust design deals with designing a system that is insensitive to the parameters that cannot be controlled (noise factors). This method can be used to identify controllable parameter (design variable) values that satisfy a set of performance requirement despite variations in uncontrollable parameters (noise factors). Type I robust design was first proposed by Genichi Taguchi (Taguchi 1986, Taguchi and Clausing 1990, Taguchi 1993) and has been carried forward by many researchers (Vining and Myers 1990, Welch, Yu et al. 1990, Shoemaker, Tsui et al. 1991, Chen, Allen et al. 1996).

Type-II robust design deals with designing a system that is insensitive to the parameters that can be controlled (design variables). This method can be used to identify controllable parameter (design variable) values that satisfy a set of performance requirement despite variations in controllable parameters (design variables) themselves. In type II robust design, the idea is to search for region wherein there is minimal variation in system performance for the variations in control factors. Type II robust design was first proposed by Chen (Chen, Allen et al. 1996).

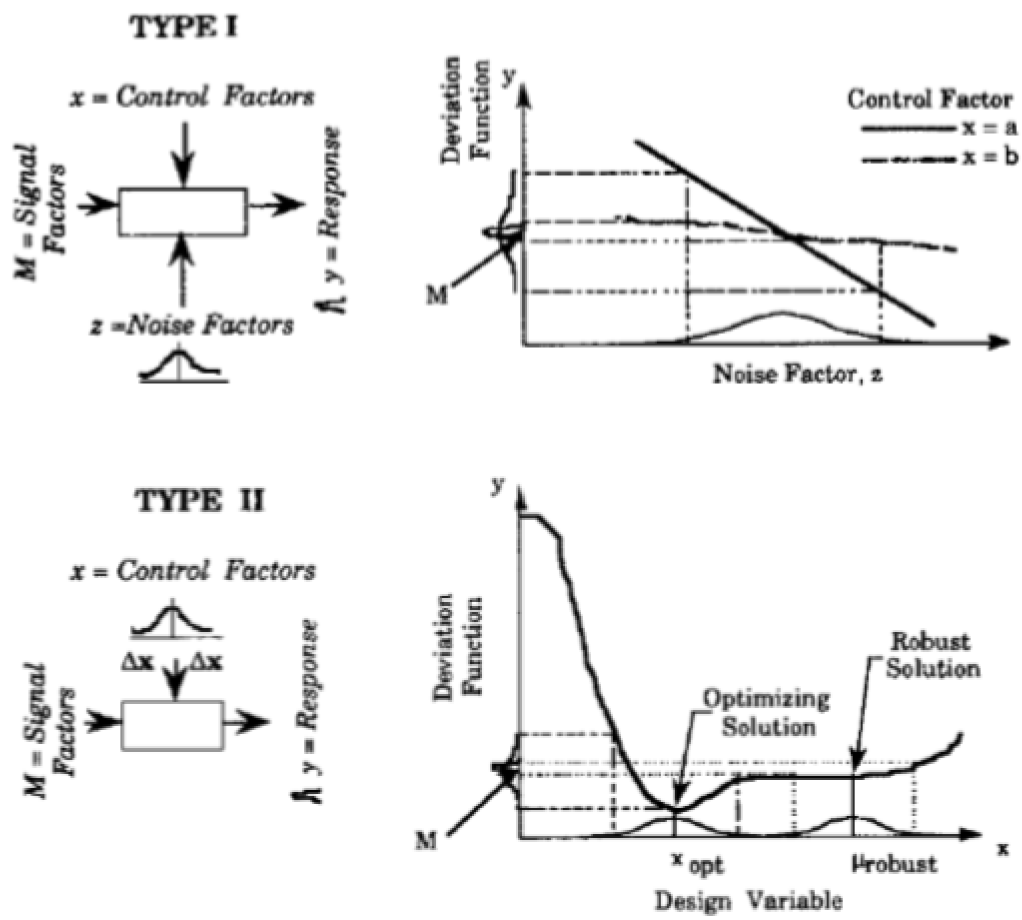


Figure 2.6: Robust Design for Variations in Noise Factors (Type I) and Control Factors (Type II) (Chen, Allen et al. 1996, Seepersad, Allen et al. 2005)

Type-III robust design deals with designing a system that is insensitive to the variability embedded within the model used. This method can be used to identify controllable parameter (design variable) values that satisfy a set of performance requirement despite variations associated with the models being used.

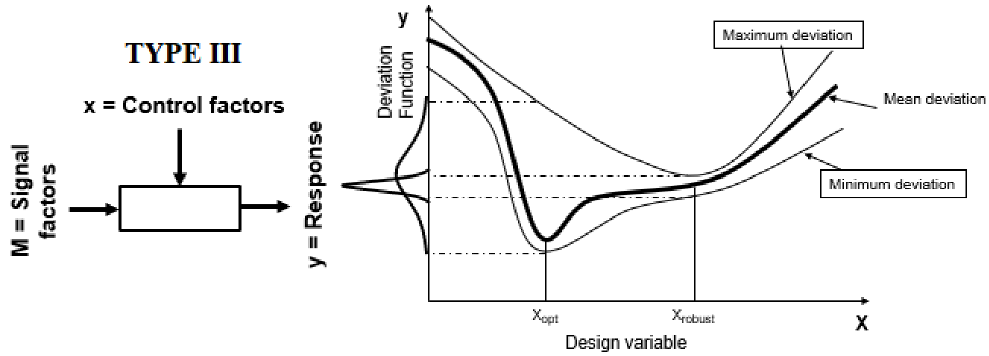


Figure 2.7: Type III Robust Design

Type-IV robust design deals with the integrated multiscale design of material and product. This method can be used to identify controllable parameter (design variable) values that satisfy a set of performance requirement despite the propagation of uncertainty (PU) through the scales (Choi, McDowell et al. 2008).

A domain-independent, systematic, method that integrates statistical experimentation, approximate models (metamodels/response surface models), multi-objective decisions and multidisciplinary analyses, to carry out robust design at early stages of design, called Robust Concept Exploration Method (RCEM) has been proposed by Chen and co-authors. The schematic showing the steps in RCEM is shown in Figure 2.8.

Using RCEM designers can formulate design problems for robust exploration of solution space. The RCEM uses specific goals in cDSP formulation that are meant to bring the mean on performance target and minimize performance variation. The RCEM has been used for variety of applications to design robust systems.

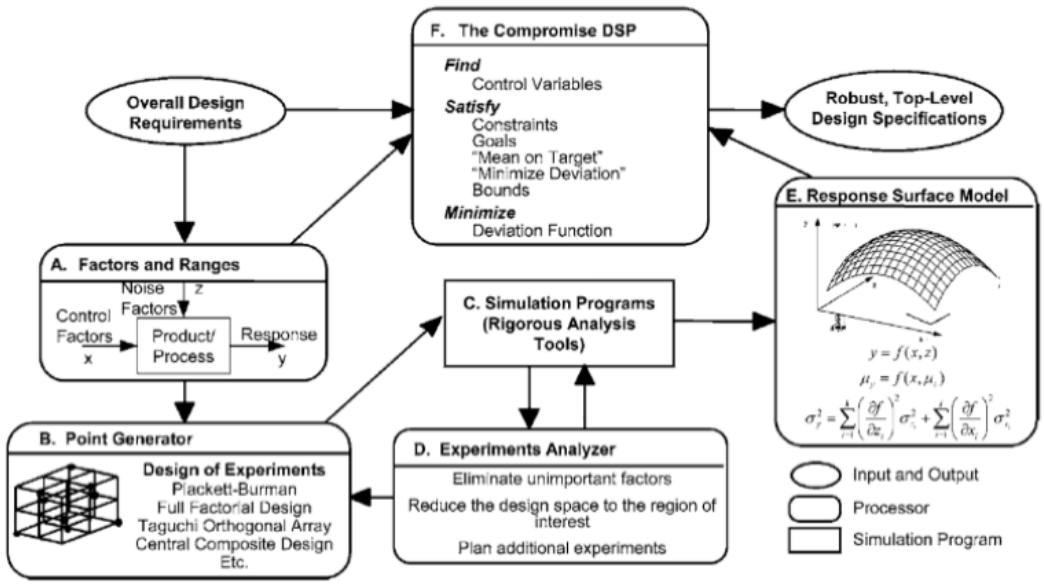


Figure 2.8: Modified version of Computational Infrastructure of RCEM Developed by Chen and Coauthors (Chen, Allen et al. 1996, Seepersad, Allen et al. 2005).

2.4.3 *The Robust Concept Exploration Method with Robustness Metrics*

In this section, the concept of robustness metrics called Design Capability Index (DCI) and Error Margin Index (EMI) to manage and mitigate the effects of uncertainty is presented. In following two figures, we respectively show the uncertainty bounds due to variations in design variable and model, and the development of mathematical constructs to address such uncertainties (Choi 2005, Choi, Austin et al. 2005).

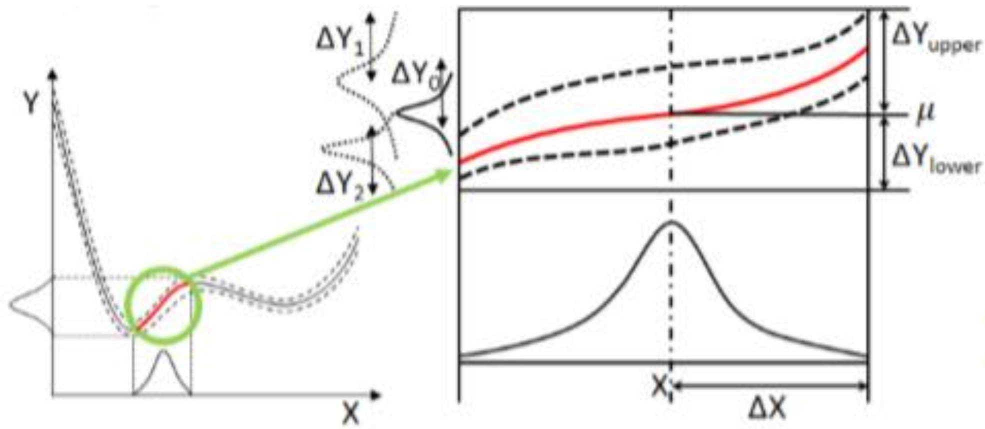


Figure 2.9: Formulation of Uncertainty Bounds Due to Variations in a Design Variable and a Model (Choi, Austin et al. 2005)

In Figure 2.9, the mean response (μ) for the model is illustrated as a solid red curve and two adjacent dotted curves represent the uncertainty bounds associated with the system model. At x, for a variation of + Δx in design variable, the expected variation in response given by the mean response model is ΔY_0. Similarly, for the same change in design variable at x, the expected variation in response for the two uncertainty bounds are ΔY_1 and ΔY_2 respectively as shown in the figure. This will let us calculate the maximum expected deviation in response for any given value of x and Δx.

In Figure 2.10, the mathematical formulations for implementing EMIs or DCIs as a goal in DSPs are shown. “Smaller is better” means that we are looking to minimize the targeted function while “Larger is better” means that we are looking to maximize the targeted function. Further, “Nominal is better” means that we are interested in getting a value as nearer as possible to the target set, that is, we want to avoid underachievement as well as overachievement.

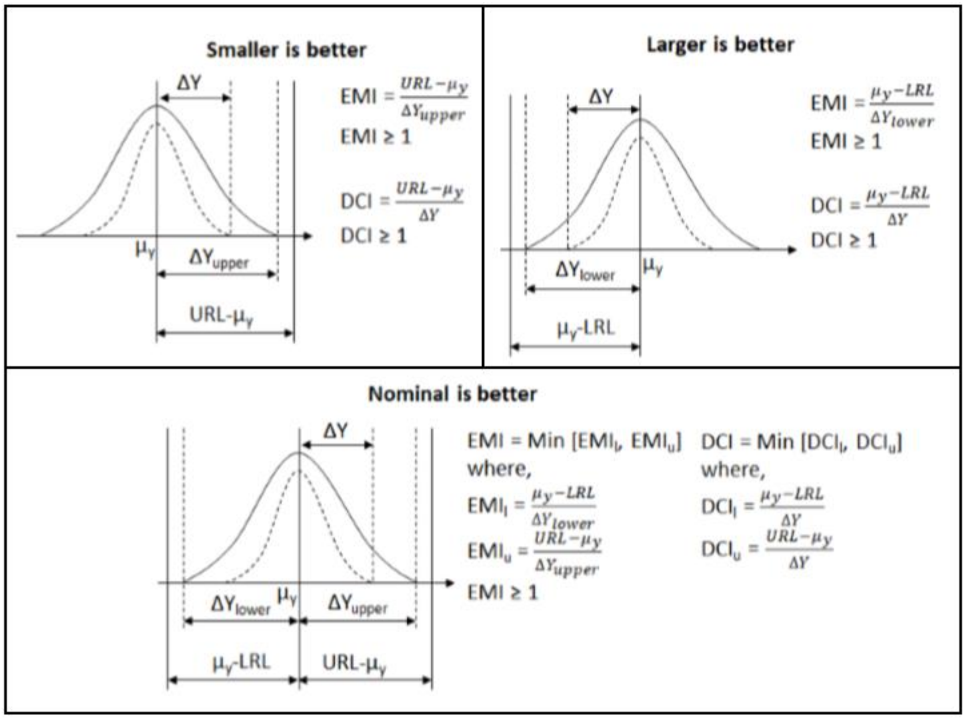


Figure 2.10: Mathematical Constructs of EMIs and DCIs (Choi, Austin et al. 2005)

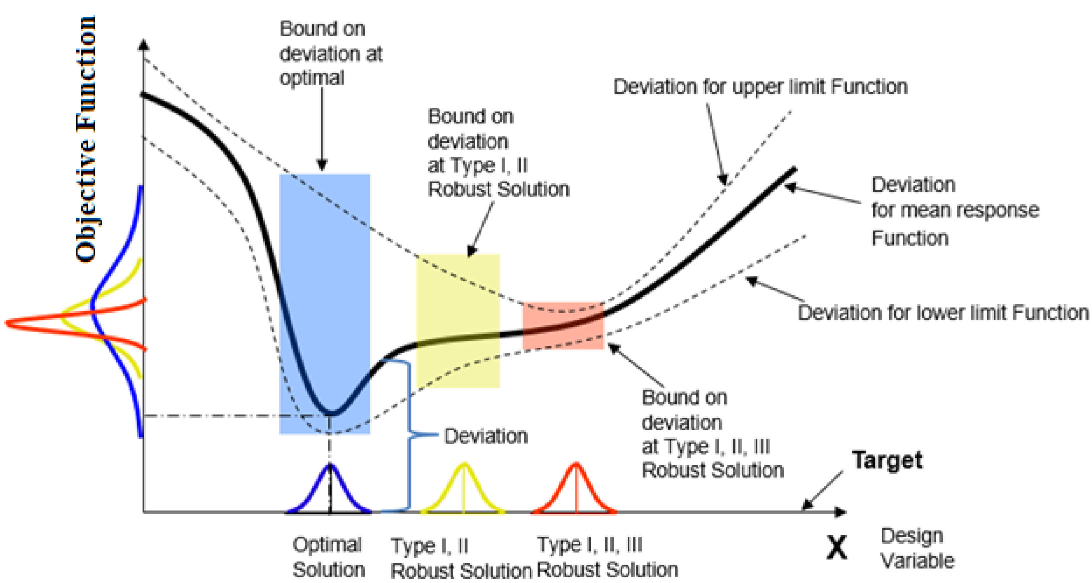


Figure 2.11: Type I, II and III Robust Design (Choi 2005)

Steps for Formulating Goals as DCIs

Step 1: Using a first order Taylor series expansion, the response variation due to variation in the design variable vector x = {x1, x2,……, xn} is estimated. The response variation(Δy) for small variations in design variables is as

$$\Delta y = \sum_{i=1}^{n} \left|\frac{\partial f}{\partial x_i}\right| . \Delta x_i$$

Step 2: Using the mean response (μ_y) obtained from the mean response model ($f_0(x)$) and the response variation due to variation in design variables (Δy), calculate the DCIs. For a 'Larger is Better' case, the DCI is calculated as

$$DCI = \frac{\mu_y - LRL}{\Delta y}$$

where, LRL is the lower requirement limit. A DCI ≥ 1 means that the ranged set of design specifications satisfies a ranged set of design requirements and the system is robust against uncertainty in design variables. Higher the value of DCI, higher is the measure of safety against failure due to uncertainty in design variables.

Steps for Formulating Goals as EMIs

Step 1: Given a system model has k uncertainty bounds, the response variation (ΔY_j) for each of them for small variation in design variables is calculated as

$$\Delta y_j = \sum_{i=1}^{n} \left|\frac{\partial y_j}{\partial x_i}\right| . \Delta x_i$$

where j = 0, 1, 2, …, k (number of uncertainty bounds).

Step 2: After the evaluation of the multiple response variations of mean response function and the k uncertainty bound functions for variations in design variables, the minimum and maximum responses by considering the variability in design variables and uncertainty bounds around the mean response are calculated as shown below.

$$Y_{max} = Max[f_j(x) + \Delta y_j] \text{ and,}$$

$$Y_{\min} = Max[f_j(x) - \Delta y_j]$$

where j = 0, 1, 2, …, k (number of uncertainty bounds), $f_0(x)$ is the mean response function, and $f1(x)$….$fk(x)$ are the uncertainty bound functions.

In Figure 2.9, a mean response function (solid red curve) and two uncertainty bounds (dotted curves in black) is shown. At any value of x, we are able to calculate the value of maximum (Y_{max}), minimum ($Y_{\min}$) and mean response (μ_y) arising due to uncertainty bounds. This calculation will let us also calculate the maximum expected deviation in response for any given value of x.

Step 3: Calculate the upper and lower deviation of response at x as

$$\Delta Y_{upper} = Y_{max} - f_o(x) \text{ and}$$

$$\Delta Y_{lower} = f_o(x) - Y_{min}$$

Step 4: Using the mean response (μ_y) obtained from the mean response model ($f_0(x)$) and the upper and lower deviations (ΔY_{upper} and ΔY_{lower}), the EMIs are calculated as shown below. For a 'Larger is Better' case, the EMI is calculated:

$$\text{EMI} = \frac{\mu_y - \text{LRL}}{\Delta Y_{lower}}$$

Similar, calculations follow for other cases.

By incorporating robustness metrices in representing the original design goals, compromise DSP for robust exploration can be formulated.

Illustrative Calculation for DCI (Transforming Stiffness Goal as DCI)

The stiffness calculation for a fender design example used in Chapter 3 is shown here.

Step 1: Establish the functional relationship of Stiffness (ST) goal in terms of design variables

$$\text{ST = Beam Stiffness} = \frac{48\ \text{EI}}{\text{L}^3} = \frac{3\pi\text{E}\{\text{D}^4 - (\text{D}-2\text{t})^4\}}{4\text{L}^3}$$

Step 2: Evaluate the partial differentiation of ST with respect to the design variables

$$\frac{\partial \text{ST}}{\partial \text{D}} = \frac{3\pi\text{E}\{\text{D}^3 - (\text{D} - 2\text{t})^3\}}{\text{L}^3}$$

$$\frac{\partial \mathrm{ST}}{\partial \mathrm{t}} = -\frac{6\pi \mathrm{E}(\mathrm{D}-2\mathrm{t})^3}{\mathrm{L}^3}$$

$$\frac{\partial \mathrm{ST}}{\partial \mathrm{E}} = \frac{3\pi\{\mathrm{D}^4-(\mathrm{D}-2\mathrm{t})^4\}}{4\mathrm{L}^3}$$

Step 3: Using a first order Taylor series expansion, estimate the response variation due to variation in the design variables. The response variation (Δy) for small variations in design variables is

$$\Delta \mathrm{y} = \left|\frac{\partial \mathrm{ST}}{\partial \mathrm{D}}\right| . \Delta \mathrm{D} + \left|\frac{\partial \mathrm{ST}}{\partial \mathrm{t}}\right| . \Delta \mathrm{t} + \left|\frac{\partial \mathrm{ST}}{\partial \mathrm{E}}\right| . \Delta \mathrm{E}$$

Step 4: Using the mean response obtained from the mean response model (Equation derived in Step 1) and the response variation due to variation in design variables (Δy), calculate the DCI. For a ‘Larger is Better’ case, the DCI is calculated as

$$\mathrm{DCI} = \frac{\frac{3\pi \mathrm{E}\{\mathrm{D}^4-(\mathrm{D}-2\mathrm{t})^4}{4\mathrm{L}^3} - \mathrm{LRL}}{\Delta \mathrm{y}}$$

where, LRL is the lower requirement limit, which can be set based on the design requirement.

2.5 Summary of Chapter 2

In this chapter, the design foundations and the fundamental constructs in decision-based design for designing a robust concept exploration framework in context of coupled engineered system is presented and discussed. The objective in this chapter is also to lay down the mathematical foundations used in this thesis. The outcome of this chapter is a modified robust concept exploration framework for designing coupled engineered systems, shown below in Figure 2.12.

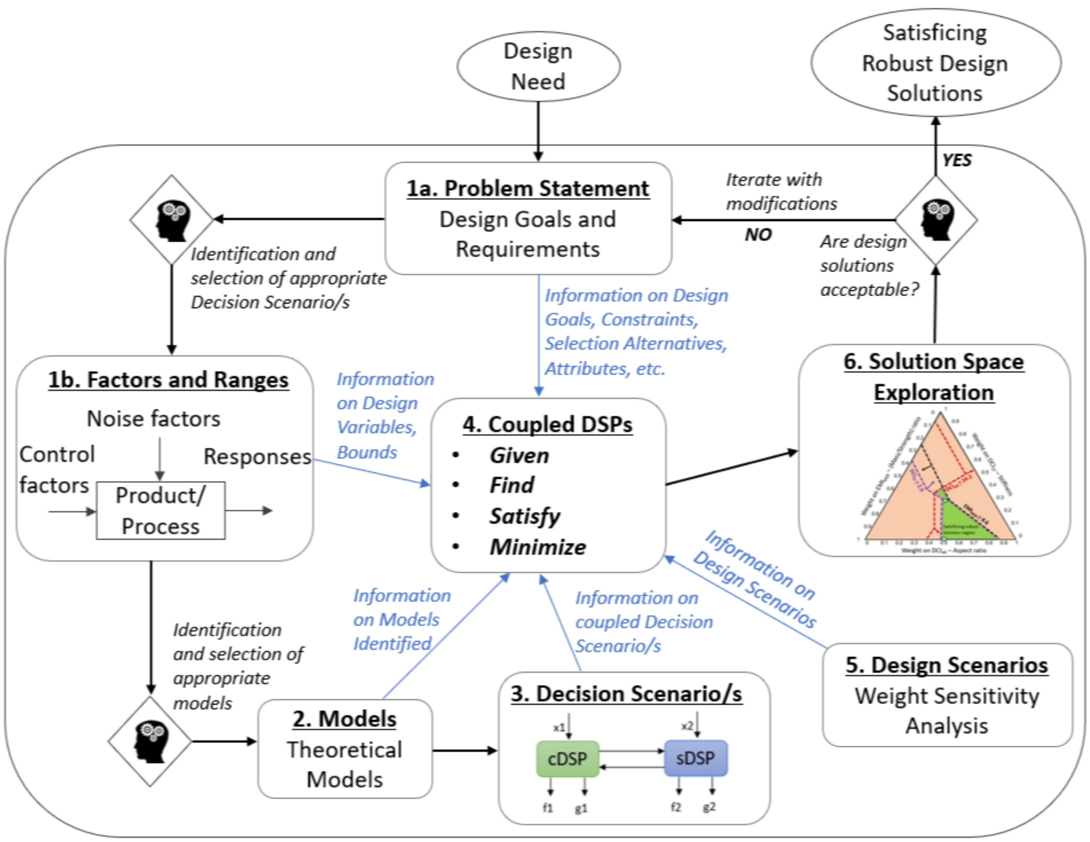


Figure 2.12: Procedure for Exploring Robust Design Solutions for Coupled Problems

Chapter 3: Designing a Fender

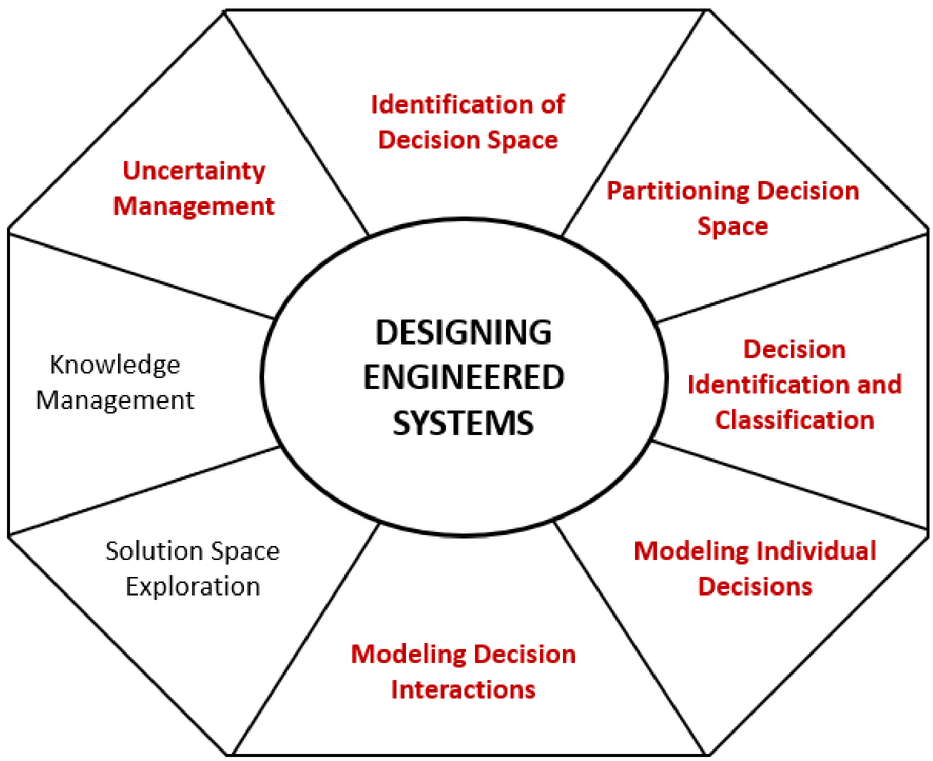


Figure 3.1: Elements in Design of Coupled Engineered Systems Discussed in Chapter 3

In Chapter 3, six elements in the design of engineered systems (as shown in Figure 3.1 - highlighted in red) in context of designing a fender is discussed. In this chapter, a test problem involving the design of a fender is presented. In Section 3.1, after brief introduction to the problem, problem statement and mathematical foundations for designing fender is shown. In Section 3.2, the mathematical foundation for addressing coupled design problem using DSPs for design of fender is presented. In Section 3.3, compromise DSP is presented for designing fender under uncertainty. By an example involving design of fender, a method to manage uncertainties while modeling decision interactions in design of engineered systems is demonstrated.

3.1 Designing a Fender

3.1.1 *Establishing the Mathematical Foundation*

A fender is a tubular beam structure used in marine applications, for example, as a damage mitigator between oil rig and a supply vessel. Hence, fender can be modeled as a simply supported beam.

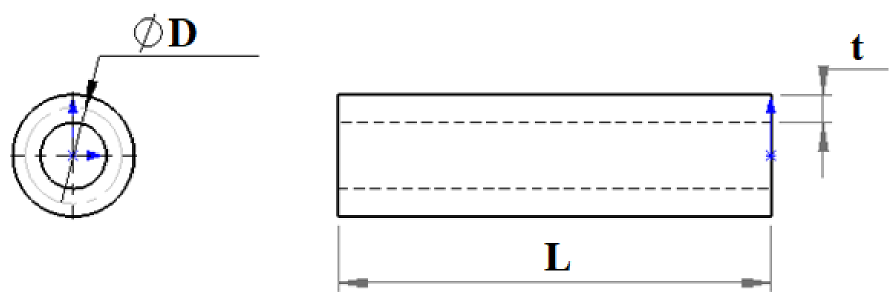


Figure 3.2: Illustration of Fender Geometry

The determination of deflection and stresses in beams as a result of load is critical in designing a beam that is safe. The stresses and deflection in different geometry for various loading conditions can be derived from (Gere and Timoshenko 1997).

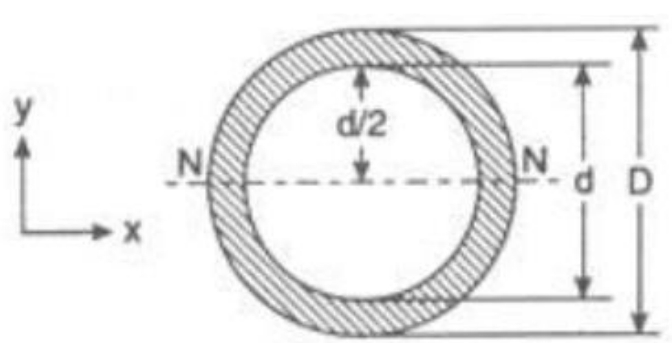


Figure 3.3: Tubular Cross-Section

Considering a hollow tubular structure as shown in Figure 3.3, the moment of inertia (I) can be derived as

$$I = Ixx = Iyy = \frac{\pi}{64}(D^4 - d^4) \qquad \text{Equation 3.1}$$

The bending stress at different parts of the beam can be calculated using the following flexural formula

$$\frac{M}{I} = \frac{\sigma}{y} \qquad \text{Equation 3.2}$$

Where, σ = Bending stress

I = Moment of Inertia

Ixx = Moment of Inertia about X-axis

Iyy = Moment of Inertia about Y-axis

M = Bending moment

y = Distance from neutral axis

The maximum bending stress is seen at the surface of the beam and is calculated using the above equation.

When a point load P is applied at the center of the beam, the formula for deflection in tubular beam is as

$$\text{Deflection } (\delta) = \frac{PL^3}{48EI} \qquad \text{Equation 3.3}$$

Where, E = Young's Modulus of Elasticity

Also, the 3 formulae that are used in the math formulation (Table 3.9 and Table 3.10) are derived as

$$\text{MSR = Mass /Strength} = \frac{(rho)\frac{\Pi}{4}\{D^2-(D-2t)^2\}L}{\sigma y} \qquad \text{Equation 3.4}$$

$$\text{AR = Aspect ratio} = \frac{D}{t} \qquad \text{Equation 3.5}$$

$$\text{ST = Beam Stiffness} = \frac{P}{\delta} = \frac{48\ EI}{L^3} \qquad \text{Equation 3.6}$$

Where,

rho = Density of Material

σy = Yield Strength of Material

3.1.2 *Problem Statement*

The design of a beam, that is to be used as a fender for a floating steel-jacketed platform, is required. This fender must be compatible with the design of floating platform, which specifies a fixed length value L and the specified load P. A tubular cross-section is selected and is characterized by the mean diameter D and the wall thickness t. Restrictions regarding maximum bending stress and deflection on the beam is specified. The quality of the design is measured in terms of design goals which are to be achieved as nearly as possible. Specifically, we need a design that has low weight, stress and aspect ratio while having high beam stiffness. Two important material properties are considered for the design, that is, Young's modulus and yield strength. The design decisions are to be taken to minimize the performance impact from expected variability in design variables and material properties.

3.1.3 *Specific Problem Statements*

For the design problem stated in section 3.1.2, two design approaches are implemented. One design approach considers the selection of suitable material from the pool of available materials while the other approach considers the determination of suitable material properties (Young's modulus and yield strength).

Example 1 - Robust design with material properties as design variables: The task is to recommend the value of material properties and the beam dimensions for best performance with respect to the constraints and design quality specified. The material properties are available for selection within the specified bounds.

Example 2 - Strongly coupled robust design with 3 material alternatives: The task is to recommend the suitable material and the beam dimensions for best performance with respect to the constraints and design quality specified. There are 3 material alternatives, that is, Cast Iron, Titanium and Copper available for selection.

3.1.4 *Selection of Decision Scenarios*

Example 1 is formulated and executed as one compromise Decision Support Problem (cDSP) as there is no selection part to the problem. On the other hand, Example 2 involves selection of suitable material from an available pool while also exploring suitable dimensions with respect to design quality specified. In this example, the influence in selection of material on beam dimensions as well as the influence of beam dimensions on selection of material has been considered. This example fits the pattern P9 proposed in the Decision Scenario Matrix (DSM) as shown in figure below.

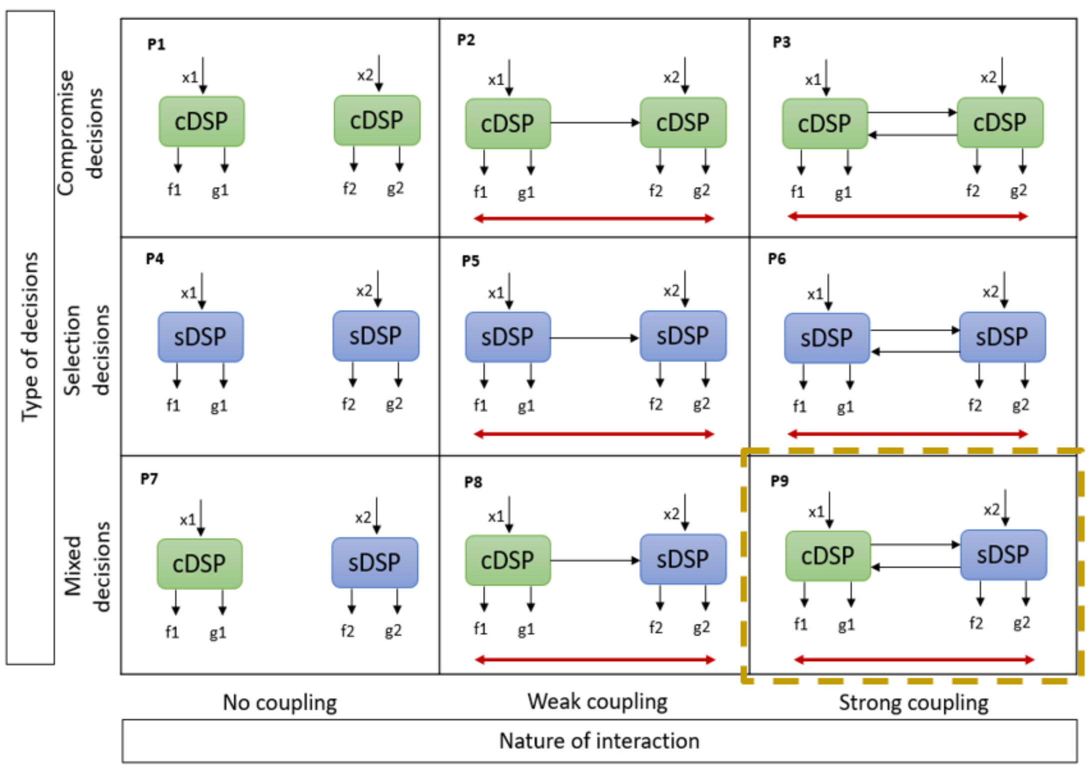


Figure 3.4: Scenario Selection from Decision Scenario Matrix

3.2 Developing a cDSP for Coupled Decision

3.2.1 *General sDSP Template for Design Problems*

As discussed in Chapter 1, selection decisions are modeled through selection Decision Support Problem (sDSP). The selection Decision Support Problem (sDSP) is developed as a tool for solving engineering design problems involving selection among feasible alternatives based on their relative measure of merit (Kuppuraju, Ittimakin et al. 1985). The selection DSP in words can be stated as shown in Table 3.1.

Table 3.1: Word Formulation for Selection DSPs

Given	**A set of candidate alternatives obtained from a preliminary selection process**
Identify	The principal attributes influencing selection and the relative importance of attributes.
Rate	The alternatives with respect to each attribute.
Rank	The alternatives in order of preference based on attributes and their relative importance.
Post-Solution Analysis	Validate the results. Perform sensitivity analysis.

With the word formulation shown in Table 3.1, math formulation for the selection DSPs are developed. The math formulation are solved in a software called DSIDES (Decision Support In the Design of Engineering Systems). Concisely, the math formulation for selection DSP can be stated as shown in Table 3.2.

Table 3.2: Math Formulation for Selection DSPs

selection DSP
Find Selection System Variables Y Deviation Variables ei^- , ei^+ Satisfy Selection Constraint $\sum_{i=1}^{n} \mathbf{Y_j}$ = 1 Constraints on Deviation Variables $ei^+ \geq 0$, $ei^- \geq 0$, $ei^+ . ei^- = 0$ Selection Goal MFj (X) Yj + $e1^-$ - $e1^+$ = 1 Bounds B: 0 ≤ Yj ≤ 1
Minimize $Z = \{e1^-\}$

3.2.2 *General cDSP Template for Design Problems*

As discussed in Chapter 1, compromise decisions are modeled through compromise Decision Support Problem (cDSP). The compromise Decision Support Problem (cDSP) was developed as a tool for solving engineering design problems involving multiple conflicting goals (Mistree, Muster et al. 1989). The compromise DSP in words can be stated as shown in Table 3.3.

Table 3.3: Word Formulation for Compromise DSPs

Given	**The design variables and their respective bounds. The design goals and targets set to those goals.**
Find	The values of design variables and deviation variables.
Satisfy	The system constraints and goal constraints. The bounds on design variables.
Minimize	The deviation of the design's performance modeled by the set of goal constraints.
Post-Solution Analysis	The validity of the solution. Perform sensitivity analysis.

With the word formulation shown in Table 3.3, math formulation for the compromise DSPs are developed. The math formulation are solved in a software called DSIDES (Decision Support In the Design of Engineering Systems). Concisely, the math formulation for compromise DSP can be stated as shown in Table 3.4.

Table 3.4: Math Formulation for Compromise DSPs

compromise DSP
Find Compromise System Variables X Deviation Variables di^- , di^+ Satisfy Design Constraints $gj\ (X, Y) \geq 0$ Constraints on Deviation Variables $di^+ \geq 0, di^- \geq 0,\ di^+ . di^- = 0$ Compromise Goals $Ai\ (X, Y) + di^- - di^+ = Gi$ Bounds B: $X(min) \leq X \leq X(max)$
Minimize $Z = \{\sum_{i=1}^{n} w_i \cdot (di^- + di^+)\}, \ \sum_{i=1}^{3} w_i = 1$

3.2.3 *General Coupled DSP Template for Design Problems*

Coupled DSPs allow designers to model engineering design problems involving interaction among DSPs. Concisely, the math formulation for coupled selection - compromise DSP can be stated as

Table 3.5: Math Formulation for Coupled Selection - Compromise DSP

Coupled selection – compromise DSPs	
compromise DSP	selection DSP
Find	Find
Compromise System Variables	Selection System Variables
X	Y
Deviation Variables	Deviation Variables
di^- , di^+	ei^- , ei^+
Satisfy	Satisfy
Design Constraints	Selection Constraint
gj (X, Y) > 0	$\sum_{i=1}^{n} \mathbf{Y_j} = 1$
Constraints on Deviation Variables	Constraints on Deviation Variables
$di^+ \geq 0, di^- \geq 0, di^+ . di^- = 0$	$ei^+ \geq 0, ei^- \geq 0, ei^+ . ei^- = 0$
Compromise Goals	Selection Goal
Ai (X, Y) + di^- - di^+ = Gi	MFj (X) Yj + $e1^-$ - $e1^+$ = 1
Bounds	Bounds
B: X(min) ≤ X ≤ X(max)	B: 0 ≤ Yj ≤ 1
Minimize	
$Z = \{e1^-, \sum_{i=1}^{n} w_i \cdot (di^- + di^+)\}, \ \sum_{i=1}^{3} w_i = 1$	

The mathematical formulation in Table 3.5 is utilized in developing a mathematical formulation (Table 3.10) for modeling interactions among 2 decisions for designing a fender. The selection decision (G1 in Table 3.10) and compromise decision (G2, G3 and G4 in Table 3.10) is formulated as a strong decision interaction.

3.3 Developing a Robustness Based CDSP For Coupled Decision

Before developing robustness based cDSP for coupled decision, first general coupled DSP template for design of a fender is shown in Table 3.6.

3.3.1 *General Coupled DSP Template for Design of a Fender*

Table 3.6: Coupled DSP Template for Design of a Fender

<table>
<tr><td colspan="2">Design of Fender</td></tr>
<tr><td colspan="2">Find
MSR, AR, ST, X
Satisfy
$MSR + d_M^- - d_M^+ = MSR_{Target}$
$AR + d_A^- - d_A^+ = AR_{Target}$
$ST + d_S^- - d_S^+ = ST_{Target}$
$MF + d_{MF}^- - d_{MF}^+ = 1$</td></tr>
<tr><td>Compromise DSP</td><td>Selection DSP</td></tr>
<tr><td>Given
Length (L)
Find
Compromise Variables
Diameter (D)</td><td>Given
Material Alternatives (X_j)
Find
Selection Variables
Material Alternative (X)</td></tr>
</table>

<table>
<tr><td>

Thickness (t)

Deviation variables

d_i^- , d_i^+

Satisfy

Design Constraints

Deflection Constraint (DC)

DC (D,t,L,X) > 0

Bending constraint (BC)

BC (D,t,L,X) > 0

Constraints on deviation variables

$d_i^+ \geq 0, d_i^- \geq 0, d_i^+ . d_i^- = 0$

for i = 1, 2 & 3

Compromise Goals

$\frac{MSR_{Target}}{MSR(D,t,L,\mathbf{X})} + d_1^- - d_1^+ = 1$

$\frac{AR_{Target}}{AR(D,t,\mathbf{X})} + d_2^- - d_2^+ = 1$

$\frac{ST(D,t,L,\mathbf{X})}{ST_{Target}} + d_3^- - d_3^+ = 1$

Bounds

B1: $D^{(min)} \leq D \leq D^{(max)}$

B2: $t^{(min)} \leq t \leq t^{(max)}$

</td><td>

Deviation variables

e_i^- , e_i^+

Satisfy

Selection Constraint

$\sum_{j=1}^{n} X_j = 1$

Constraints on deviation variables

$e_i^+ \geq 0, e_i^- \geq 0, e_i^+ . e_i^- = 0$

for i = 1

Selection Goal

$MF_j (D, t, L) X_i + e_1^- - e_1^+ = 1$

Bounds

B1: $0 \leq X_j \leq 1$

</td></tr>
<tr><td colspan="2">

Minimize

$Z = \{e1^-, \sum_{i=1}^{3} w_i \cdot (di^- + di^+)\},\ \sum_{i=1}^{3} w_i = 1$

</td></tr>
</table>

3.3.2 *Word Formulation (Robust Exploration): cDSP*

The word formulations for robust design of fender is presented here. Example 1 deals with the robust design of fender with material properties as design variables. Two material properties are considered as design variables, that are, yield strength and young's modulus of the material. Table 3.7 is the word formulation for Example 1.

Table 3.7: Word Formulation for Robust Design of a Fender (Example 1)

Example 1: Word formulation – Robust Design of fender

Given
System parameters
    Load on the beam (P)
    Length of the beam (L)
    Maximum allowable deflection (δ)
    System constants
    Density of the material (rho)
    PI(Π)
Find
System variables
    Wall thickness (t)
    Diameter (D)
    Yield Strength of material (σy)
    Young's modulus of material (E)
Deviation variables

Over- and underachievement of mass/strength ratio goal with robustness

Over- and underachievement of aspect ratio goal with robustness

Over- and underachievement of stiffness goal with robustness

Satisfy

Design Constraints

Maximum allowable deflection constraint

Maximum allowable bending stress constraint

System Constraints

Constraints on deviation variables

Robust Solution Constraint on mass/strength ratio goal

Robust Solution Constraint on aspect ratio goal

Robust Solution Constraint on stiffness goal

System Goals

G1 – Goal for weight/strength ratio

G2 – Goal for aspect ratio

G3 - Goal for stiffness

System Bounds

Upper and lower values for system variables

Minimize

Deviation functions

Distance from target set for mass/strength ratio goal

Distance from target set for aspect ratio goal

Distance from target set for stiffness goal

Example 2 deals with the robust design of fender with material as selection alternatives. Three materials are considered as selection alternatives, that are, Iron, Titanium and Copper. Table 3.8 is a word formulation for Example 2.

Table 3.8: Word Formulation for Robust Design of a Fender (Example 2)

Example 2: Word formulation – Robust Design of fender
Given Selection system parameters Cast Iron yield strength (AS1) Titanium yield strength (AS2) Copper yield strength (AS3) Cast Iron young’s modulus (E1) Titanium young’s modulus (E2) Copper young’s modulus (E3) Cast Iron density (R1) Titanium density (R2) Copper density (R3) Compromise system parameters Load on the beam (P) Length of the beam (L) Maximum allowable deflection (δ) System constants PI(Π) Find Selection system variables Cast Iron yield (X1) Titanium (X2) Copper (X3) Compromise System variables Wall thickness (t) Diameter (D)

Deviation variables

Over- and underachievement of MF goal

Over- and underachievement of EMI_{MSR} goal

Over- and underachievement of DCI_{AR} goal

Over- and underachievement of DCI_{ST} goal

Satisfy

Selection system Constraints

Selection constraint for material alternatives

Compromise Design Constraints

Maximum allowable deflection constraint

Maximum allowable bending stress constraint

Compromise system Constraints

Robust solution constraint on EMI_{MSR} goal

Robust solution constraint on DCI_{AR} goal

Robust solution constraint on DCI_{ST} goal

Constraints on deviation variables

Coupled selection Goal

G1 – Goal for material alternatives

Coupled compromise Goals

G2 – Goal for EMI_{MSR}

G3 – Goal for DCI_{AR}

G4 - Goal for DCI_{ST}

System Bounds

Upper and lower values for system variables

Minimize

Deviation functions (Preemptive form)

Distance from target set for MF goal

Distance from target set for EMI_{MSR} goal

Distance from target set for DCI_{AR} goal
Distance from target set for DCI_{ST} goal

3.3.3 *Math Formulation (Robust Exploration): cDSP*

As explained in Section 2.4.3, for incorporating robustness in the design, we need to convert the original goals into goals that carry robustness metrics such as DCI and EMI. Furthermore, we need to add robustness constraints to ensure that the design solutions are robust. In both the examples, the first goal, that is, Mass to Strength Ratio (MSR) is converted to EMI while other two goals, that are, Aspect Ratio (AR) and Stiffness (ST) are converted to DCI. Table 3.9 is math formulation for Example 1 and Table 3.10 is math formulation for Example 2. The goals derived in Equation 3.4, Equation 3.5 and, Equation 3.6 are converted to respective robustness goals. The conversion of the stiffness goals to a robustness goal is shown through the following equations:

$$\frac{\partial ST}{\partial D} = \frac{3\pi E\{D^3 - (D - 2t)^3\}}{L^3} \qquad \text{Equation 3.7}$$

$$\frac{\partial ST}{\partial t} = \frac{6\pi E(D - 2t)^3}{L^3} \qquad \text{Equation 3.8}$$

$$\frac{\partial ST}{\partial E} = \frac{3\pi\{D^4 - (D - 2t)^4\}}{4L^3} \qquad \text{Equation 3.9}$$

$$\Delta y3 = \left|\frac{\partial ST}{\partial D}\right| . \Delta D + \left|\frac{\partial ST}{\partial t}\right| . \Delta t + \left|\frac{\partial ST}{\partial E}\right| . \Delta E \qquad \text{Equation 3.10}$$

$$DCI_{ST} = \frac{\{(48*E*10^6* I)/L^3\} - 60000}{\Delta y3} \qquad \text{Equation 3.11}$$

Table 3.9: Math Formulation for Robust Design of a Fender (Example 1)

Example 1: Math formulation – Robust Design of fender
Given System Parameters Load on the beam (P) = 10,000 lbf Length of the beam (L) = 100 in Maximum allowable deflection (δ) = 0.025 in System Constants Density of the material (rho) = 0.28 lb/in^3 PI(Π) = 3.142
Find System Variables Wall thickness t (in) Diameter D (in) Yield Strength of material σy (ksi) Young's modulus of material E (Mpsi) Deviation Variables d_1^+ = Overachievement of EMI_{MSR} goal

d_1^- = Underachievement of EMI_{MSR} goal

d_2^+ = Overachievement of DCI_{AR} goal

d_2^- = Underachievement of DCI_{AR} goal

d_3^+ = Overachievement of DCI_{ST} goal

d_3^- = Underachievement of DCI_{ST} goal

Satisfy

Design Constraints (From the Problem Statement)

Maximum allowable deflection constraint

$1-\frac{PL^3}{48EI(\delta)} > 0$ (Normalized)

Maximum allowable bending stress constraint

$1-\frac{PLD}{8I(\sigma y)} > 0$ (Normalized)

System Constraints

Robust solution constraint on EMI_{MSR} goal

$EMI_{MSR} \geq 1$

Robust Solution Constraint on DCI_{AR} goal

$DCI_{AR} \geq 1$

Robust Solution Constraint on DCI_{ST} goal

$DCI_{ST} \geq 1$

Constraints on Deviation Variables

$d_i^+ \geq 0$

$d_i^- \geq 0$

$d_i^+ . d_i^- = 0$ for i = 1, 2 and 3

System Goals

G1 – Maximize EMI_{MSR} for Mass to Strength Ratio goal

$$\frac{EMI_{MSR}}{EMI_{MSR,Target}} + d1^- - d1^+ = 1$$

G2 – Maximize DCI_{AR} for Aspect Ratio goal

$$\frac{DCI_{AR}}{DCI_{AR,Target}} + d2^- - d2^+ = 1$$

G3 - Maximize DCI_{ST} for Stiffness goal

$$\frac{DCI_{ST}}{DCI_{ST,Target}} + d3^- - d3^+ = 1$$

System Bounds

B1: 0.12 in $\leq$ t $\leq$ 0.75 in

B2: 3 in $\leq$ D $\leq$ 24 in

B3: 30 ksi $\leq$ σy $\leq$ 36 ksi

B4: 27.5 Mpsi $\leq$ E $\leq$ 30.5 Mpsi

Minimize

Deviation Functions

$$Z = \sum_{i=1}^{3} w_i \cdot (di^- + di^+) \ , \ \sum_{i=1}^{3} w_i = 1$$

Table 3.10: Math Formulation for Robust Design of a Fender (Example 2)

Example 2: Math formulation – Robust Design of fender
Given Selection System Parameters Cast Iron yield strength (AS1) = 28 ksi Titanium yield strength (AS2) = 34.8 ksi Copper yield strength (AS3) = 27.5 ksi Cast Iron young's modulus (E1) = 26 Mpsi Titanium young's modulus (E2) = 15.2 Mpsi Copper young's modulus (E3) = 19 Mpsi Cast Iron density (R1) = 0.272 lb/in^3 Titanium density (R2) = 163 lb/in^3 Copper density (R3) = 0.298 lb/in^3 Relative importance of attribute j (I_j) Normalized rating of alternative i wrt attribute j (R_{ij}) Compromise System Parameters Load on the beam (P) = 10,000 lbf Length of the beam (L) = 100 in Maximum allowable deflection (δ) = 0.025 in System Constants

PI(Π) = 3.142
Find Selection System Variables Cast Iron yield (X1) Titanium (X2) Copper (X3) Compromise System Variables Wall thickness t (in) Diameter D (in) Deviation Variables e_1^- = Underachievement of MF goal d_1^+ = Overachievement of EMI_{MSR} goal d_1^- = Underachievement of EMI_{MSR} goal d_2^+ = Overachievement of DCI_{AR} goal d_2^- = Underachievement of DCI_{AR} goal d_3^+ = Overachievement of DCI_{ST} goal d_3^- = Underachievement of DCI_{ST} goal
Satisfy Selection System Constraints Selection constraint for material alternatives

$\sum_{i=1}^{3} X_i = 1$

Compromise Design Constraints

Maximum allowable deflection constraint

$1 - \frac{PL^3}{48EI(\delta)} > 0$ (Normalized)

Maximum allowable bending stress constraint

$1 - \frac{PLD}{8I(\sigma y)} > 0$ (Normalized)

Compromise System Constraints

Robust Solution Constraint on EMI_{MSR} goal

$EMI_{MSR} \geq 1$

Robust Solution Constraint on DCI_{AR} goal

$DCI_{AR} \geq 1$

Robust Solution Constraint on DCI_{ST} goal

$DCI_{ST} \geq 1$

Constraints on Deviation Variables

$d_i^+ \geq 0$

$d_i^- \geq 0$

$d_i^+ . d_i^- = 0$ for i = 1, 2 and 3

Coupled selection Goal

G1 – Maximize Merit Function (MF)

$MF_i (D,t) X_i + e_1^- - e_1^+ =1$

Coupled compromise Goals G2 – Maximize EMI_{MSR} for mass to strength ratio goal $$\frac{EMI_{MSR}}{EMI_{MSR,Target}} + d1^- - d1^+ = 1$$ G3 – Maximize DCI_{AR} for aspect ratio goal $$\frac{DCI_{AR}}{DCI_{AR,Target}} + d2^- - d2^+ = 1$$ G4 - Maximize DCI_{ST} for stiffness goal $$\frac{DCI_{ST}}{DCI_{ST, Target}} + d3^- - d3^+ = 1$$ System Bounds B1: 0.12 in $\leq$ t $\leq$ 0.75 in B2: 3 in $\leq$ D $\leq$ 24 in B3: 0 $\leq$ X1 $\leq$ 1 (BOOLEAN) B4: 0 $\leq$ X2 $\leq$ 1 (BOOLEAN) B5: 0 $\leq$ X3 $\leq$ 1 (BOOLEAN)
Minimize $$Z = \{e1^-, \sum_{i=1}^{3} w_i \cdot (di^- + di^+)\},\ \sum_{i=1}^{3} w_i = 1$$

In Table 3.10, there are 4 goals. G1 deals with the selection of material for fender design. G2, G3, and G4 combinedly deal with the compromise decision in the design of fender. The above mentioned four goals form coupled decisions, where selection decision (G1) in the design of fender is horizontally coupled with compromise decision in the design of fender (G2, G3 and G4). The two decisions are formulated with strong interaction between the DSPs.

3.4 Summary of Chapter 3

In this chapter, first test example to validate the method proposed in this thesis for dealing with coupled design problems is formulated. The design example deals with design of a fender. Specifically, two formulations has been presented. One formulation deals the design example as a single DSP, meaning, all design decisions are dealt as compromise decisions. Second formulation approaches the design example as a coupled design decision. In this formulation, design decisions is bifurcated into two such that selection decision and compromise decision are concurrently taken by considering the influence of one decision over the other. Mathematics to manage uncertainty and model decisions interactions is presented. The results to the math formulations in this chapter is presented in Chapter 6 (Section 6.1).

Chapter 4: Designing a Gearbox

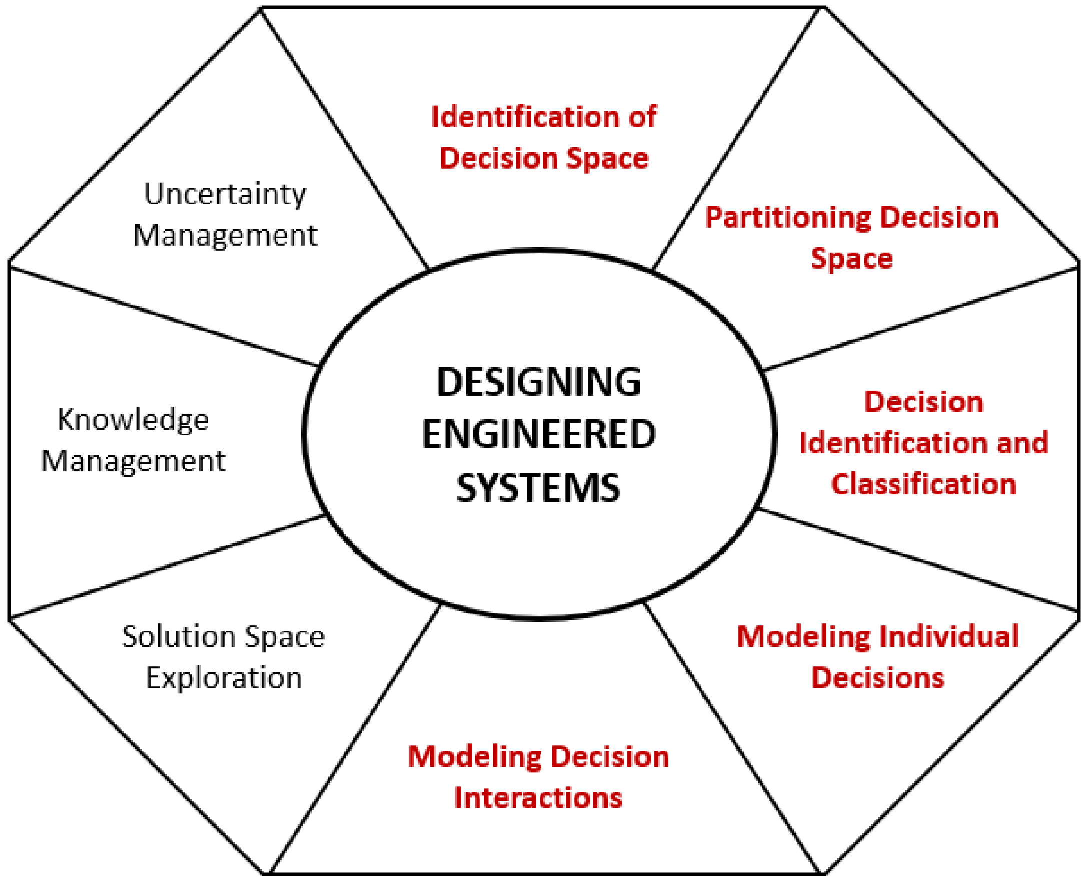


Figure 4.1: Elements in Design of Coupled Engineered Systems Discussed in Chapter 4

In Chapter 4, five elements in the design of engineered systems (as shown in Figure 4.1 - highlighted in red) in context of designing a gearbox is discussed. In this chapter, a test problem involving the design of a one-stage reduction gearbox is presented. In Section 4.1, after brief introduction to the problem, the mathematical foundations for designing gearbox is discussed. In Section 4.2, the problem statement and decision scenarios for designing gearbox is discussed.

Section 4.3 is reserved for the mathematical formulation for addressing coupled design problem using DSPs. By an example involving design of gearbox, a method to model multilevel decision interactions in design of engineered systems is demonstrated.

4.1 Designing a One-Stage Reduction Gearbox

4.1.1 *Establishing the Mathematical Foundations*

Gearbox is a fundamental component used in the transmission of mechanical power. It provides variety of output speed for one input speed. The basic requirements for a gearbox are:

- Provide means of connection and disconnection of power source with rest of the power train without shock and smoothly.
- Provide a varied leverage between the power source and the driven components.
- Provide means to transfer power in opposite direction.
- Enable power transmission at varied angles and varied lengths.
- Enable speed reduction between power source and the driven components.
- Enable diversion of power flow at right angles.
- Bear the effect of torque reaction, driving thrust and braking effort effectively.

In addition to it, a gearbox designer is also expected to fulfil a number of design constraints while also fulfilling the functional requirements to

- Minimize the overall weight
- Come out with compact design
- Reduce the overall cost involved in manufacturing the gearbox
- Reduce noise/vibration
- Improve efficiency
- Avoid heat accumulation

Gearboxes are designed to transfer torque load at rated speed. The major cause of stress on shafts is due to torsion resulting from torque being transmitted. Similarly, the gear teeth are subjected to fatigue due to the bending stress and contact stress on the teeth. The mathematical foundation for designing a gearbox is available in (Shigley 2011).

The American Gear Manufacturers Association (AGMA) is an important authority responsible for the dissemination of knowledge pertaining to the design and analysis of gearing. The methods presented by this organization are in general use in the United States when strength and wear are of primary concerns. AGMA provides relevant equations required for designing gears. The two fundamental stress equations are bending stress and contact stress.

Bending Stress Equation

$$\sigma = W^t K_0 K_v K_s \frac{1}{bm} \frac{K_H K_B}{Y_J} \quad \text{(SI Units)} \qquad \text{Equation 4.1}$$

Where,

σ is the bending stress number

W^t is the tangential transmitted load (N)

K_o is the overload factor

K_v is the dynamic factor

K_s is the size factor

b is the face width of the narrower member (mm)

K_H is the load-distribution factor

K_B is the rim-thickness factor

Y_J is the geometry factor for bending strength (which includes root fillet stress-concentration factor)

m is the transverse metric module

Contact Stress Equation

$$\sigma_c = Z_E \sqrt{W^t K_0 K_v K_s \frac{K_H}{d_{w1} b} \frac{Z_R}{Z_I}} \qquad \text{(SI Units)} \qquad \text{Equation 4.2}$$

Where,

σ_c is the bending stress number

Z_E is an elastic coefficient $(\sqrt{N/mm^2})$

Z_R is the surface condition factor

d_{w1} is the pitch diameter of the pinion (mm)

Z_I is the geometry factor for pitting resistance

AGMA Strength Equations

Instead of using the term strength, AGMA uses data termed allowable stress numbers and designates these by the symbols s_{at} and s_{ac}.

Allowable Bending Stress

$$\sigma_{all} = \frac{S_t}{S_F}\frac{Y_N}{Y_\theta Y_Z} \qquad \text{(SI Units)} \qquad \text{Equation 4.3}$$

Where,

S_t is the allowable bending stress (N/mm^2)

Y_N is the stress cycle factor for bending stress

Y_θ is the temperature factor

Y_Z is the reliability factor

S_F is the AGMA factor of safety, a stress ratio

Allowable Contact Stress

$$\sigma_{c,all} = \frac{S_c}{S_H}\frac{Z_N Z_W}{Y_\theta Y_Z} \qquad \text{(SI Units)} \qquad \text{Equation 4.4}$$

Where,

S_c is the allowable contact stress (N/mm^2)

Z_N is the stress cycle life factor

Z_W is the hardness ratio factor for pitting resistance

Y_θ is the temperature factor

Y_Z is the reliability factor

S_H is the AGMA factor of safety, a stress ratio

The critical locations in shaft are at locations where the bending moment is large, where the torque is present, and where stress concentrations exist. In the present analysis of shafts, shafts are considered to fail due to static shear stress resulting from the torque being transferred. The static shear stress (τ) in shaft due to torsion are given by

$$\tau = \frac{16T}{\pi d^3} \qquad \text{Equation 4.5}$$

Where,

T is the transmitted torque

d is the shaft diameter

4.2 Design Problem - Gearbox

We are required to design one-stage reduction gearbox consisting of a gear-pinion arrangement and shafts, one each at input and output end of the gear-pinion pair. Broadly, our task is to recommend the dimensions and material for the design. The design decisions are to be taken considering the following design requirements:

- Satisficing solutions against multiple conflicting goals
- The influence of gear-pinion design on shaft design and vice-versa
- The influence of selected material on dimensions and vice-versa
- The expected variability in design variables, materials and manufacturing processes

4.2.1 *Problem Statement - Gearbox*

The design of a one-stage reduction gearbox with gear ratio of 4 is required. The torque at input is at least 80 Nm @ 3500 rpm. The gears are required to endure at least 10^7 fatigue cycles. The gears are cut using rack cutter arrangement with pressure angle (α) = 20^0. The reliability for gears is at least 99 %. The gearbox is to be designed for uniform power source and moderate shock in loads. Restrictions regarding the maximum allowable stresses on the gears and shafts are specified. The quality of the design is measured in terms of design goals which are to be achieved as much as possible. Specifically, we need a design that has low weight and smaller height while achieving maximum torque. The task is to select gear material from given pool of materials and dimensions for gears and to recommend shear strength for shaft material and shaft dimensions that give the best performance with respect to the constraints and design quality specified. The material properties for shafts are available for selection within the specified bounds while gear materials are available for selection.

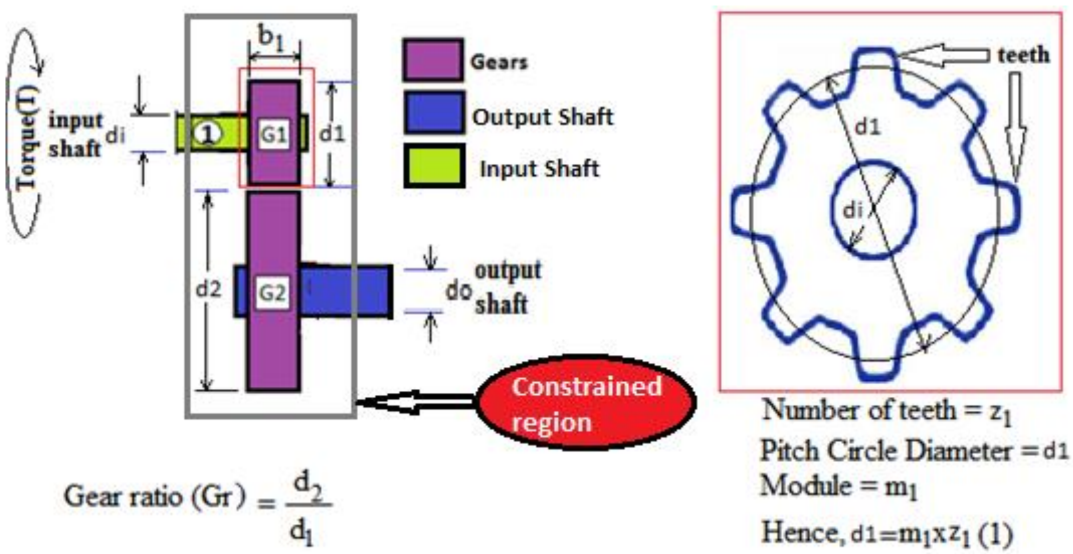


Figure 4.2: Schematic of a One-Stage Reduction Gearbox

We will explore the solution space for gearbox in regards to those solutions which better satisfy following goals

Goal 1. Minimum Weight

Goal 2. Maximum Reliability

Goal 3. Maximum Torque

Table 4.1: Summary of Design Requirements

Attributes	Requirements
Torque	Min 80 N.m.
Gear ratio	4
Input speed	3500 rpm
Case height	620 mm
Gear fatigue life	10^7 cycles
Pressure angle	20 deg
Reliability	95%

Table 4.2: Summary of Design Variables Considered

S.N.	Components	Design variables
(1)	Input Shaft	Diameter (di)
(2)	Gear G1	Module (m1) Number of Teeth (z1) Pitch Circle Diameter (d1) Face width (b1)
(3)	Gear G2	Module (m2) Number of Teeth (z2) Pitch Circle Diameter (d2) Face width (b2)
(4)	Output Shaft	Diameter (do)

In designing gears, it is required for both the gears to have same module and face width and hence m1 = m2 and b1 = b2. Also, the pitch circle diameter is a function of module and number of gear teeth, that is, Pitch Circle Diameter = Module x Number of Teeth. Finally, the two gears are required to have a gear ratio of 4 hence,

$$\text{Gear Ratio} = \frac{\text{Number of Teeth (z2)}}{\text{Number of Teeth (z1)}} = 4 \qquad \text{Equation 4.6}$$

4.2.2 *Selection of Decision Scenarios*

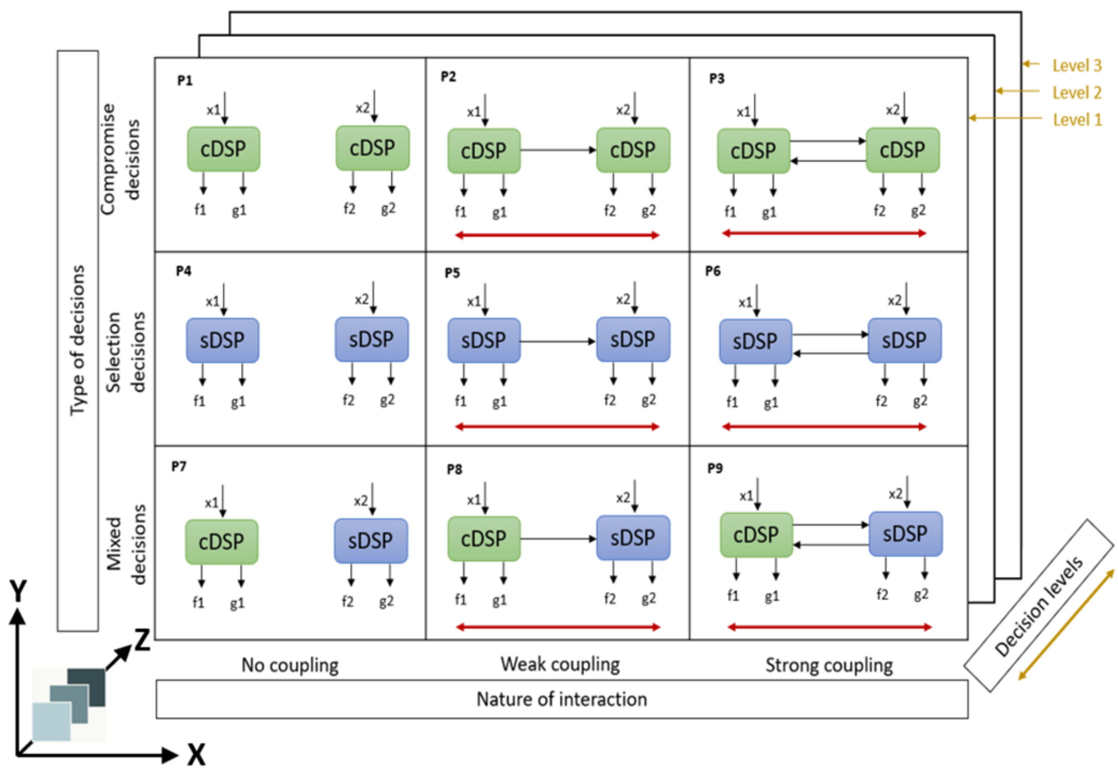


Figure 4.3: Scenario Selected for the Design of One-Stage Reduction Gearbox

The design problem involves selection of suitable material from an available pool while also exploring suitable dimensions with respect to design quality specified.

In this example, the influence in selection of material on gear dimensions as well as the influence of gear dimensions on selection of material has been considered. This example fits the pattern P9 at Level 1 proposed in the Multilevel Decision Scenario Matrix (MDSM) as shown in Figure 4.3. This decision is followed by a compromise decision at Level 2. The decisions in Level 1 and Level 2 are vertically coupled.

4.2.3 *Scenarios for Exploration*

For the exploration of design space, multiple decision scenarios are obtained using Multi-level DSM (Figure 4.3). For each decision scenario, different design scenarios are created by assigning different weights to the design goals. Based on the number of DSPs at each level, order of execution of these DSPs and weight assignment in each DSPs, different decision scenarios are determined. In this thesis, a gearbox design example is partitioned into three individual decisions that form two levels of hierarchy as shown in Figure 4.4. The first level of the hierarchy involve concurrency among two decisions:

1. Compromise decisions in the design of gears, that is, dimensions of the gear.
2. Selection decision in the design of gears, that is, selection of material.

The compromise decision in the design of gears involves determining design parameters (gear design variables shown in Table 4.4) against compromise goals (G2, G3 and G4 in Table 4.4). Similarly, selection decision in the design of gears involve selection of gear material from standard gear material alternatives shown in Table 4.4. The concurrency among the two decisions is modeled through horizontal coupling that accounts for the mutual influence among the two decisions. Following the two concurrent decisions in the design of gears, the compromise decision in the design of shafts forms the second level of the hierarchy. The compromise decision in the design of shafts involve determining design parameters (shaft design variables shown in Table 4.4) against compromise goals (G5 in Table 4.4). The hierarchy between the two levels is modeled through the vertical coupling that accounts for the influence of decisions at Level 1 on decisions at Level 2. All the compromise decisions are modeled as cDSPs and selection decision as sDSPs.

Level 1: Coupled Gear Decisions (cDSP + sDSP)

Level 2: Shaft Decisions (cDSP)

For the exploration of design space, 3 decision scenarios are created from the coupled decision representation shown through Figure 4.4. The 3 decision scenarios are shown in Figure 4.5 . Each DSP is denoted as $cDSP_{ij}$ or $sDSP_{ij}$ where i denotes the order of execution and j denotes the weight assignment in DSPs. The total value of i is equal to the total number of DSPs as each DSP has one order of execution. The value for j is either F or V meaning fixed or variable. F means the weight for goals/attributes are fixed at certain value and V means weights are varied for goals/attributes to obtain multiple design scenarios. For a particular decision scenario, only one DSP will take varying weights for goals/attributes (V) while all other DSPs take fixed weights for goals/attributes (F). Other decision scenarios can be obtained by changing the way in which i and j are assigned to DSPs.

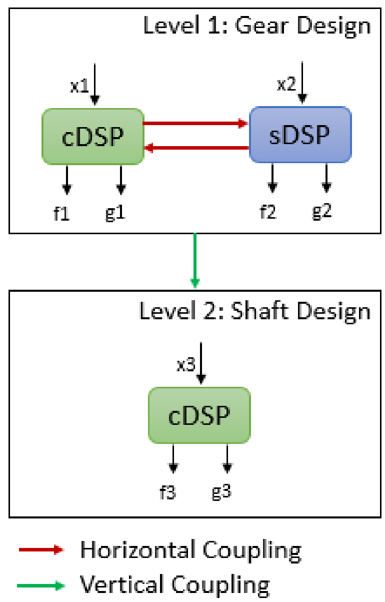


Figure 4.4: Coupled Representation and Modeling of Gearbox Design Problem by 3 Interacting Decisions

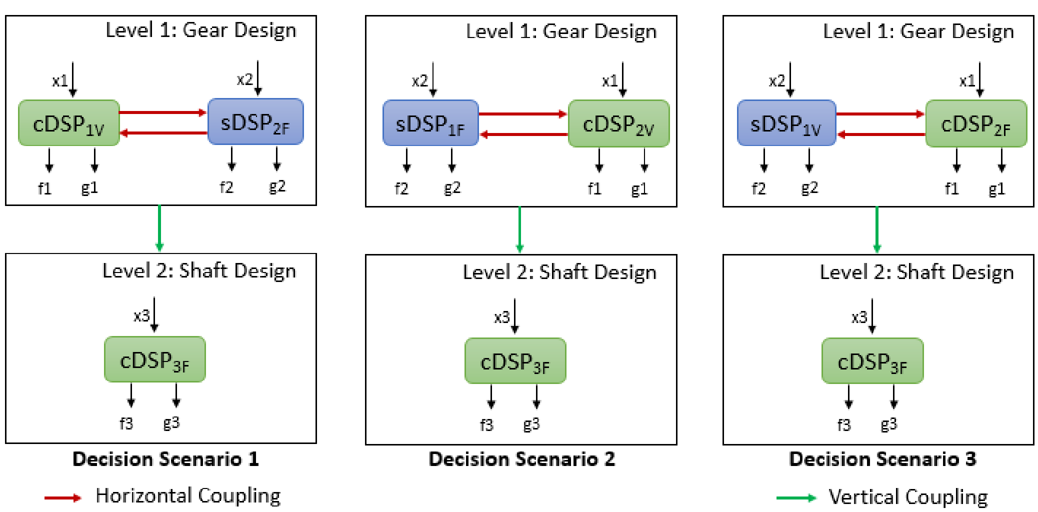


Figure 4.5: Decision Scenarios for Exploration

Decision Scenario 1

Level 1: Compromise of design variables for gear with different weights + Selection of gear material with equal weights to all attributes

Level 2: Compromise of design variables for shaft

Decision Scenario 2

Level 1: Selection of gear material with equal weights to all attributes + Compromise of design variables for gear with different weights

Level 2: Compromise of design variables for shaft

Decision Scenario 3

Level 1: Selection of gear material with varying weights to all attributes + Compromise of design variables for gear with total weight to torque

Level 2: Compromise of design variables for shaft

4.3 Developing a CDSP for Coupled Decisions

In the design of one-stage reduction gearbox, 5 materials are considered as selection alternatives and are shown in Table 4.4. The three compromise goals that are considered for this design are minimization of mass and size while maximizing torque. Table 4.3 is word formulation for the design of one-stage reduction gearbox and Table 4.4 is a math formulation for the design of one-stage reduction gearbox.

Table 4.3: Word Formulation for the Design of One-Stage Reduction Gearbox

Word Formulation for the Gearbox design – Coupled Problem (sDSP and cDSP)

Given
Selection system parameters
    Gear material alternatives
Compromise system parameters
    Torque (T) $\geq$ 80 Nm
    Gear reduction ratio (G) = 4
    Pressure Angle (α) = 20^0
    Density (δ) = 7800 Kg/m^3
System constants
    PI(Π)
    K_o = Overload factor
    K_v = Dynamic factor
    Ks = Size factor
    KH = Load distribution factor
    KB = Rim thickness factor
    YJ = the geometry factor for bending strength (which includes root fillet stress-concentration factor Kf)
    ZE = is an elastic coefficient, (√N/mm2)
    ZR = surface condition factor
    ZI = geometry factor for pitting resistance
    AGMA factor of safety for bending SF = 1
    AGMA factor of safety for contact SH = 1
    Stress cycle factor for bending stress YN = 1
    Temperature factor Y = 1
    Reliability factor YZ = 0.99 – 0.9999= 0.50 – 0.109 ln (1-Reliability)

Stress cycle life factor for contact ZN = 1

Hardness ratio factor for pitting ZW = 1

Find

Selection system variable

Gear Material

Compromise System variables

Module (m)

Number of teeth (z)

Face width (b)

Shear Strength for Shaft Material Hardness (Sy)

Input Shaft Diameter (Di)

Output Shaft Diameter (Do)

Deviation variables

Over- and underachievement of MF goal

Over- and underachievement of DCI_M goal

Over- and underachievement of DCI_H goal

Over- and underachievement of DCI_T goal

Satisfy

Selection system Constraints

Selection constraint for shaft alternatives

Compromise Design Constraints (From the Problem Statement)

Maximum allowable bending stress constraint

Maximum allowable contact stress constraint

Maximum allowable shear stress constraint

Compromise system Constraints

Constraints on deviation variables

Coupled selection Goal

G1 – Goal for gear material alternatives

Coupled compromise Goals
G2 – Goal for Gear Mass
G3 – Goal for Gear Size
G4 - Goal for Gear Torque
Coupled selection Goal
G4 – Goal for Shaft Mass
System Bounds
Upper and lower values for system variables
Minimize
Deviation functions (Preemptive form)
Distance from target set for MF goal
Distance from target set for Gear Mass goal
Distance from target set for Gear Size goal
Distance from target set for Gear Torque goal

By incorporating AGMA design factors in equations 4.1 through 4.4, the following equations are derived for bending stress and contact stress respectively and used in the math formulation shown in Table 4.4.

$$1 - 10.76Y_z \frac{T}{St\, m^2 z1^2 b} \geq 0 \qquad \text{Equation 4.7}$$

$$1 - \frac{186.42Y_z}{Sc} \sqrt{3.88 \frac{T}{mz1} \frac{1}{bmz1}} \geq 0 \qquad \text{Equation 4.8}$$

Table 4.4: Math Formulation for the Design of One-Stage Reduction Gearbox

Math Formulation for the Gearbox design – Coupled problem (sDSP and cDSP)

Given

Selection system parameters

Standard Gear Material alternatives: X1, X2, ………., X5

Design Variables	Materials
X1	AISI 1018
X2	AISI 4140 G1
X3	AISI 4350
X4	AISI 4140 G2
X5	AISI

Compromise system parameters

Torque (T) $\geq$ 80 Nm

Gear reduction ratio (G) = 4

Pressure Angle (α) = 20^0

Density (δ) = 7800 Kg/m^3

System constants

PI(Π)

Ko = Overload factor

Kv = Dynamic factor

Ks = Size factor

KH = Load distribution factor

KB = Rim thickness factor

YJ = the geometry factor for bending strength (which includes root fillet stress-concentration factor Kf)

ZE = is an elastic coefficient, (√N/mm2)

ZR = surface condition factor

ZI = geometry factor for pitting resistance

AGMA factor of safety for bending SF = 1

AGMA factor of safety for contact SH = 1

Stress cycle factor for bending stress YN = 1

Temperature factor Y = 1

Reliability factor YZ = 0.99 – 0.9999= 0.50 – 0.109 ln (1-Reliability)

Stress cycle life factor for contact ZN = 1

Hardness ratio factor for pitting ZW = 1

Find

Selection system variables

Gear Material

Compromise System variables

Module (m)

Number of teeth (z)

Face width (b)

Shear Strength for Shaft Material Hardness (Sy)

Input Shaft Diameter (Di)

Output Shaft Diameter (Do)

Deviation variables

e_1^- = Underachievement of MF goal

d_1^+ = Overachievement of Gear Mass goal

d_1^- = Underachievement of Gear Mass goal goal

d_2^+ = Overachievement of Gear Size goal

d_2^- = Underachievement of Gear Size goal

d_3^+ = Overachievement of Gear Torque goal

d_3^- = Underachievement of Gear Torque goal

d_4^- = Underachievement of Shaft Mass goal

Satisfy

Selection system Constraints

Selection constraint for gear material alternatives

$$\sum_{i=1}^{5} X_i = 1$$

Compromise Design Constraints (From the Problem Statement)

Maximum allowable bending stress constraint

$$1 - 10.76Y_z \frac{T}{St\, m^2 z1^2 b} \geq 0$$

Maximum allowable contact stress constraint

$$1 - \frac{186.42Y_z}{Sc}\sqrt{3.88\frac{T}{mz1}\frac{1}{bmz1}} \geq 0$$

Compromise system Constraints

Constraints on deviation variables

$d_i^+ \geq 0$

$d_i^- \geq 0$

$d_i^+ . d_i^- = 0$ for i = 1, 2, 3 and 4

Coupled selection Goal

G1 – Maximize Merit Function (MF)

$MF_i (m,b,z1) X_i + e_1^- - e_1^+ = 1$

Coupled compromise Goals

G2 – Minimize mass of gear

$$\frac{\text{Mass target}}{\text{Mass}} + d_1^- - d_1^+ = 1$$

G3 – Minimize size of gear

$$\frac{\text{Size target}}{\text{Size}} + d_2^- - d_2^+ = 1$$

G4 - Maximize torque of gear

$$\frac{\text{Torque}}{\text{Torque target}} + d_3^- - d_3^+ = 1$$

Coupled compromise Goals

G5 – Minimize mass of shafts

$$\frac{\text{Mass target}}{\text{Mass}} + d_4^- - d_4^+ = 1$$

Where,

MF_i (m,b,z1) = $\sum_{j=1}^{4} I_j R_{ij}(m, b, z1)$

System Bounds

B1: $24 \leq b \leq 72$ (mm)

B2: $3 \leq m \leq 6$ (mm)

B3: $18 \leq z \leq 30$

B4: $200 \leq Sy \leq 400$

B5: $20 \leq Di \leq 40$

B6: $30 \leq Do \leq 50$

B7: $0 \leq X1 \leq 1$

B8: $0 \leq X2 \leq 1$

B9: $0 \leq X3 \leq 1$

B10: $0 \leq X4 \leq 1$

B11: $0 \leq X5 \leq 1$

Minimize

Deviation functions (Preemptive form)

$$Z = [\, e1^-,\ \sum_{i=1}^{3} w_i \cdot (di^- + di^+),\ d4^-] \ ,\ \sum_{i=1}^{3} w_i = 1$$

In Table 4.4, there are 5 goals. G1 deals with the selection of material for gear design. G2, G3, and G4 combinedly deal with the compromise decision in the design of gears. The above mentioned four goals form the first level of hierarchy where selection decision (G1) in the design of gears is horizontally coupled with compromise decision in the design of gears (G2, G3 and G4). The decisions pertaining to first hierarchical level (G1, G2, G3 and G4) is vertically coupled to second hierarchical level, involving compromise decision (G5) in the design of shafts.

4.4 Summary of Chapter 4

In this chapter, design decision making in the context of designing a one-stage reduction gearbox is introduced as a multi-level coupled design problem. Design decisions pertaining to the design of gears is considered as Level 1 decisions while the design decisions pertaining to shafts is considered as Level 2 decisions. Consequently, 3 decision scenarios for exploring the design space is discussed. This followed with the DSP based mathematical formulations for solving a multi-level coupled design problem for 3 decision scenarios. Mathematical formulation for modeling horizontal and vertical coupling for the design of one-stage reduction gearbox is presented. The results to the math formulations in this chapter is presented in Chapter 6 (Section 6.2).

Chapter 5: Designing Composite Structures

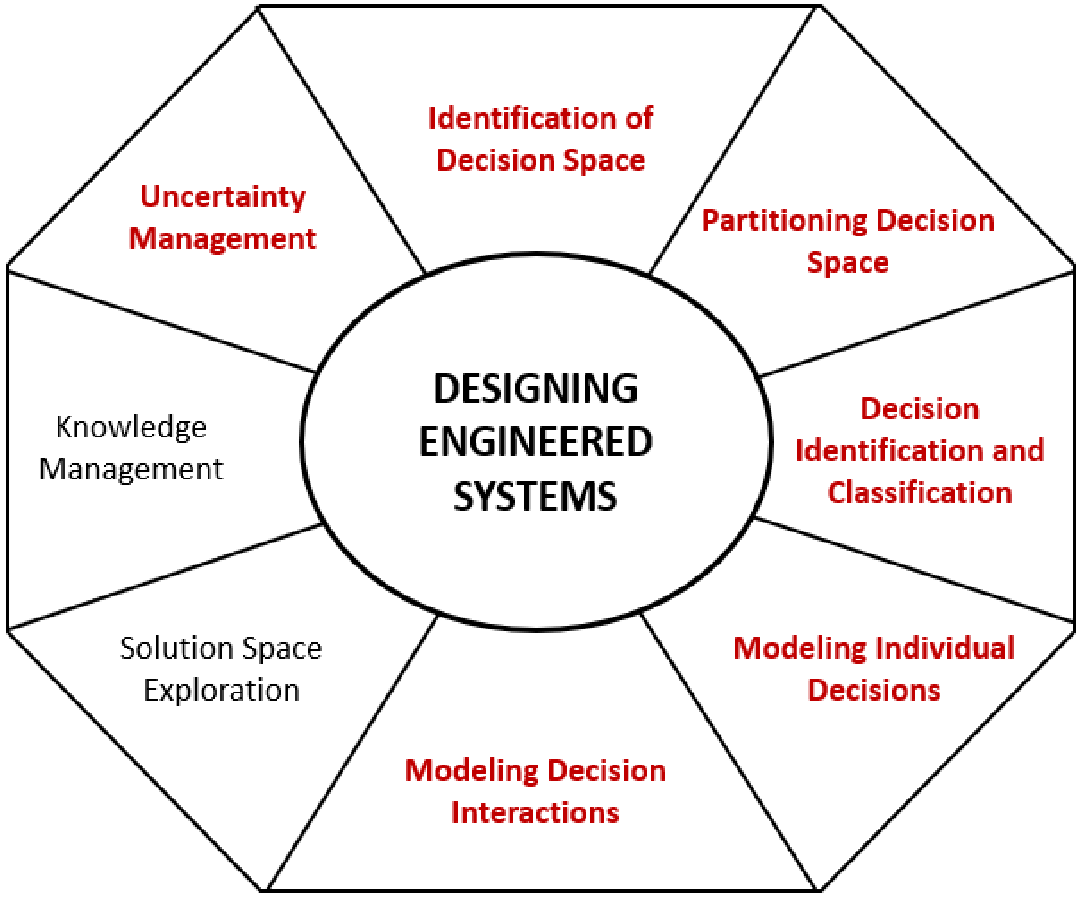


Figure 5.1: Elements in Design of Coupled Engineered Systems Discussed in Chapter 3

In Chapter 5, six elements in the design of engineered systems (as shown in Figure 5.1 - highlighted in red) in context of designing composite structures is discussed. In this chapter, test problem involving the design of composite structures is presented. In Section 5.1, after brief introduction to the problem, the mathematical foundations for designing composite structures is established. In

Section 5.2, the problem statement is stated and math formulation for compromise DSPs are shown. Section 5.3 is reserved for the problem statement and math formulation for compromise DSPs for designing composite structures under uncertainty. By an example involving design of composite structures, a method to design engineered systems under uncertainty is demonstrated.

5.1 Designing a Composite Structure

Designing of composite structures is a complex task as it involves solving multi-level multiple conflicting goals that contains uncertainties at each level of designing and manufacturing. In addition, the unavailability of best performing materials suitable for a given problem adds to the complexity of designing task. The non-availability of best performing materials is due to lack of a design technique in which the composite material is tailored according to the requirements and constraints of a test case.

In this thesis, an approach for design of a composite structure is presented. A test case of designing a sandwich composite cantilever beam is performed. The design problem involves sizing and material selection for skin and core of a sandwich composite beam based on the requirements and constraints. Broadly, the approach is bifurcated into two, that are, Coupled Problem Approach and Multiscale Approach (discussed in Section 5.2).

5.1.1 *Design of Composite Structures*

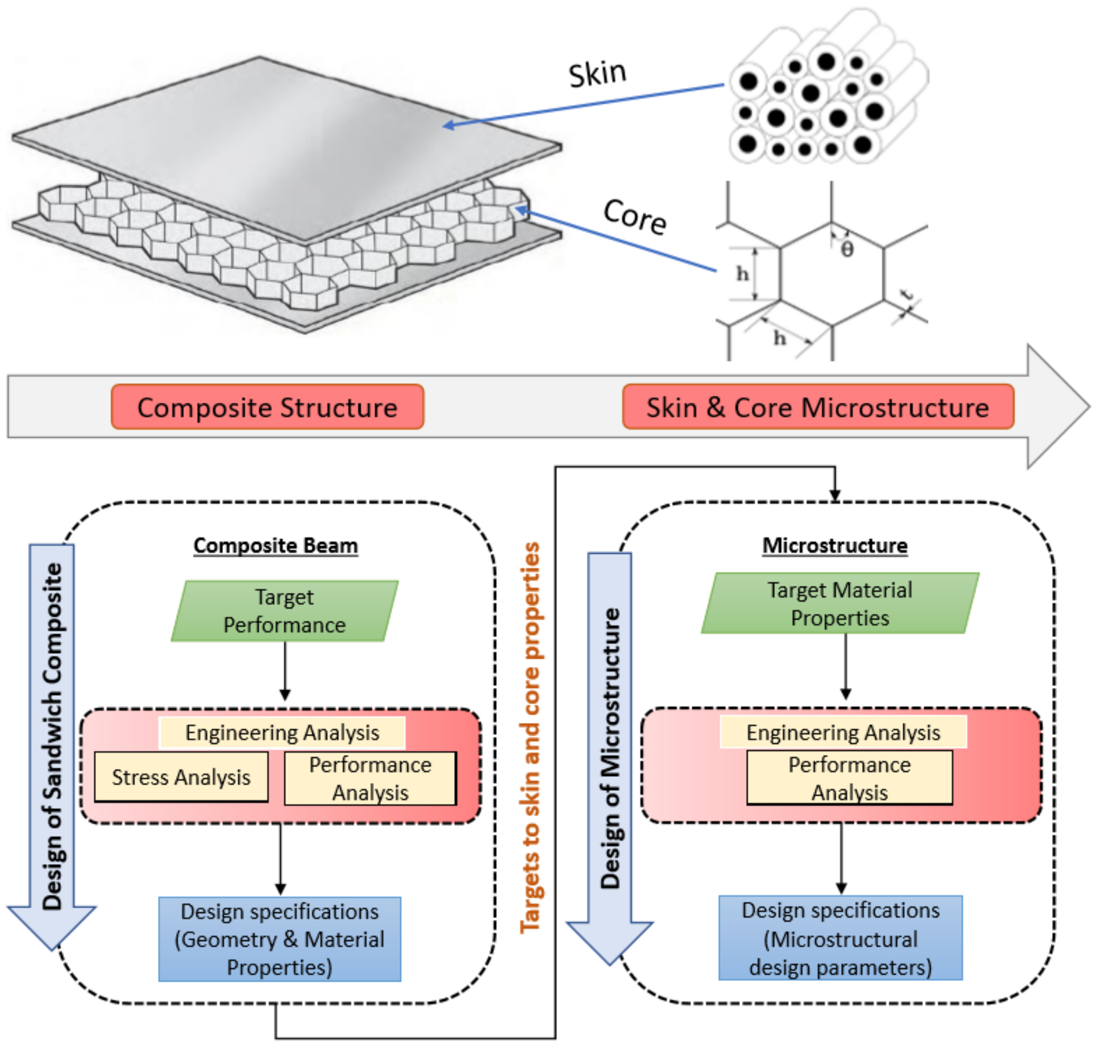


Figure 5.2: Design Exploration Framework for Composite Structures

Sandwich Composite

A sandwich-structured composite is a special class of composite material that consist of two thin but stiff skins and a lightweight but thick core. The core material is a less stiff material, but its higher thickness provides high bending stiffness with overall low density.

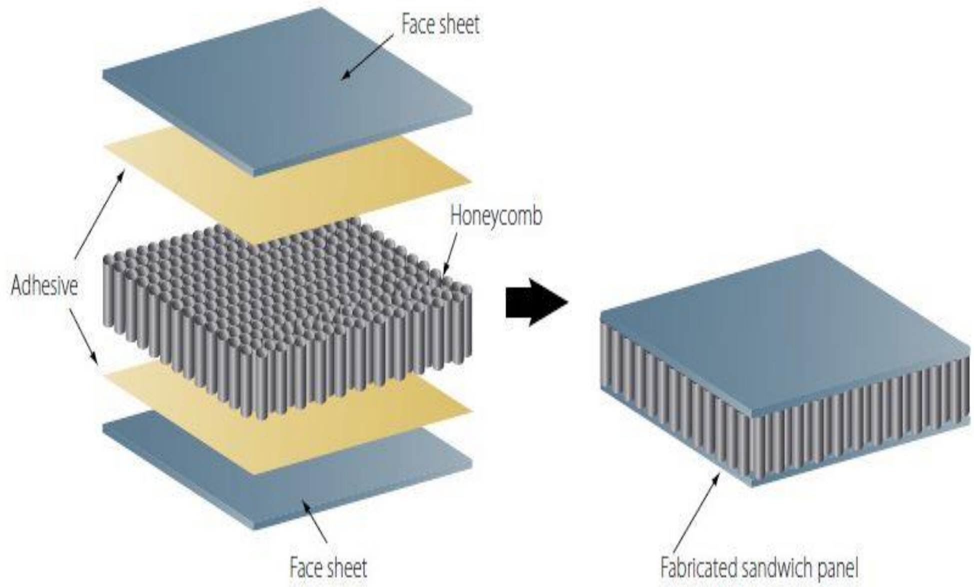


Figure 5.3: Sandwich Composite.

5.1.2 *Description of the design problem*

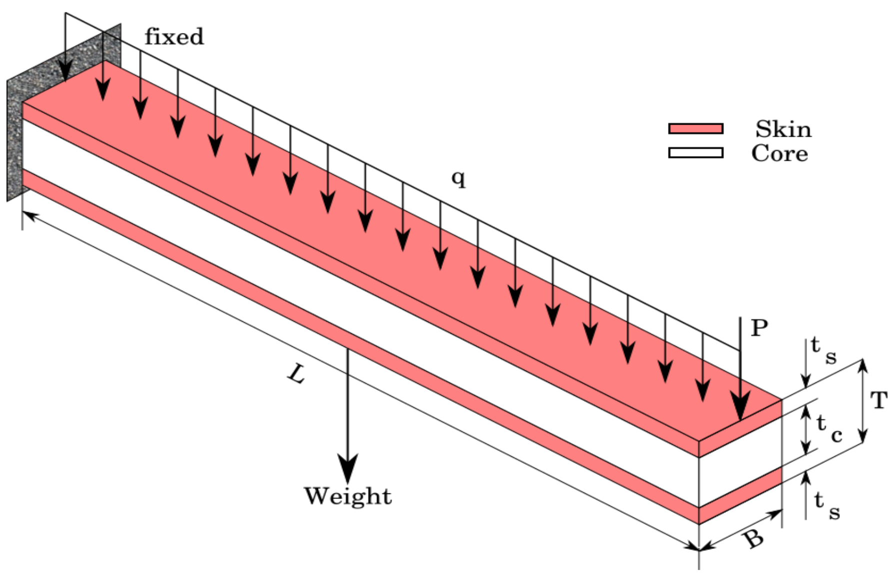


Figure 5.4: Sandwich Composite Cantilever Beam Structure

The sandwich design problem involves determination of

- Material to be used for skin and core.

- Skin thickness (t_s), Core thickness (t_c).

For required target weight and deflection.

The above design problem has been solved for three Load Case Scenarios (LCS) with Uniformly Distributed Load (UDL), concentrated point load and self-weight as shown below:

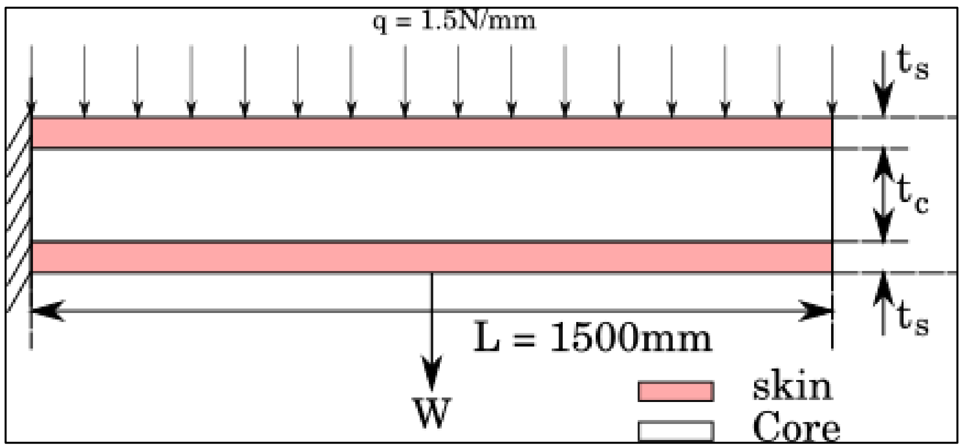


Figure 5.5: UDL with Self-Weight (LCS1)

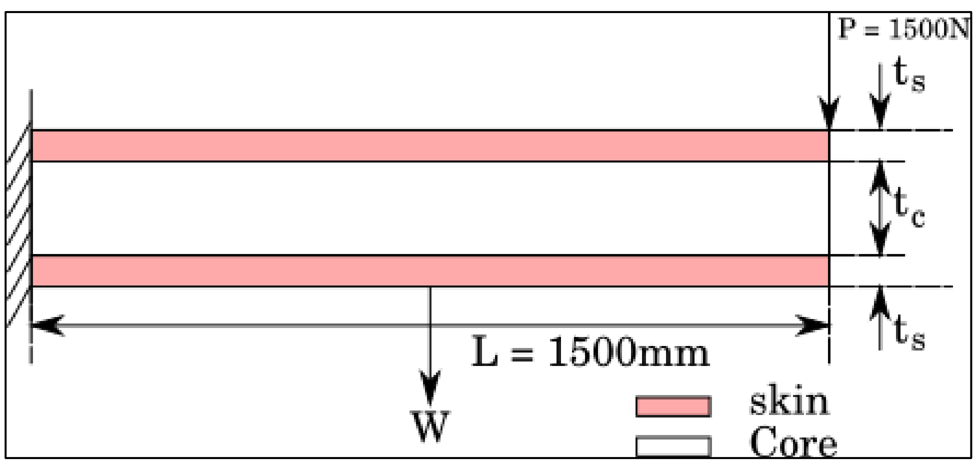


Figure 5.6: End Load with Self-Weight (LCS2)

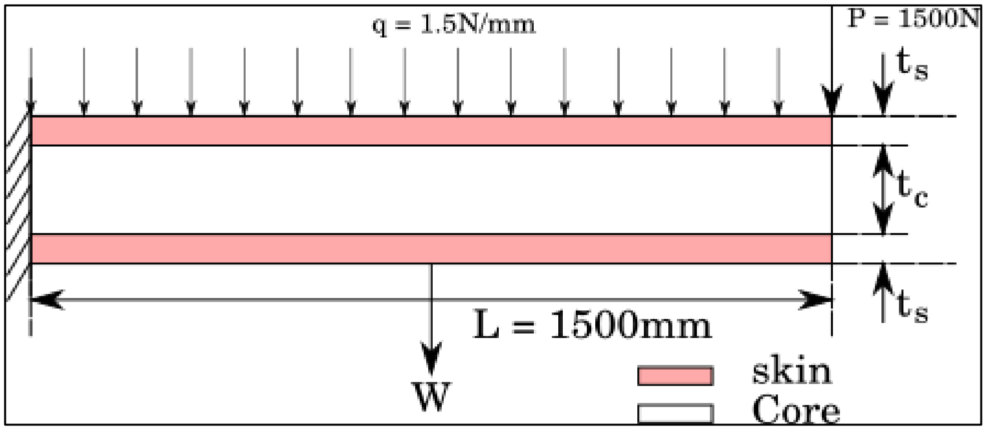


Figure 5.7: UDL, End Load with Self-Weight (LCS3)

In LCS1 sandwich composite beam is subjected to uniformly distributed load (q) of 1.5 N/mm and self-weight W as shown in Figure 5.5, in LCS2 concentrated point load (P) of 1500 N and self-weight as shown in Figure 5.6 and in LCS3 uniformly distributed load, concentrated point load and self-weight as shown in Figure 5.7.

5.1.3 *Establishing the Mathematical Foundation*

The sandwich-structured composite is a special class of composite material that consist of two thin but stiff skins and a lightweight but thick core. The outer skins carry bending stresses while the inner core carries shear stresses. In this design, skins are designed as a fiber-reinforced composites and cores as honeycomb structure to be made out of aluminum. The deflection on sandwich beams depend on the bending and shear rigidity of the beam. The use of skins and core with increased thickness offers high bending rigidity but also adds to an increased beam weight. Thus, the design of a composite beam requires exploring solution against multiple conflicting goals. The mathematical models applied in current design analysis and exploration in explained in (Pathan, Beemaraj et al. 2019).

Deflection of the Beam

The beam deflection due to UDL (δ_q), self-weight (δ_w), and end point load (δ_q) are shown in Equation 5.1, Equation 5.2 and, Equation 5.3 (Allen 2013).

$$\delta_q = \frac{qL^4}{8(EI)_{eff}} + \frac{qL^2}{2(GA)_{eff}} \quad \text{Equation 5.1}$$

$$\delta_w = \frac{WL^3}{8(EI)_{eff}} + \frac{WL}{2(GA)_{eff}} \quad \text{Equation 5.2}$$

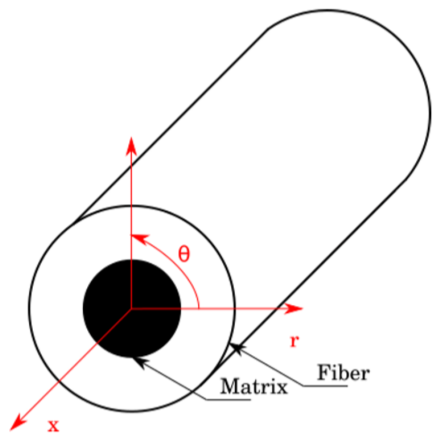
$$\delta_w = \frac{PL^3}{3(EI)_{eff}} + \frac{PL}{(GA)_{eff}} \quad \text{Equation 5.3}$$

Where, $(EI)_{eff}$ and $(GA)_{eff}$ are referred to as effective bending rigidity and shear rigidity respectively and can be calculated as shown in Equation 5.4 and Equation 5.5 (Allen 2013).

$$(EI)_{eff} = \frac{E_s B t_s^3}{6} + \frac{E_s B t_c T^2}{2} \quad \text{Equation 5.4}$$

$$(GA)_{eff} = \frac{G_c B T^2}{t_c} \quad \text{Equation 5.5}$$

Structure-Property Relationships for Skin



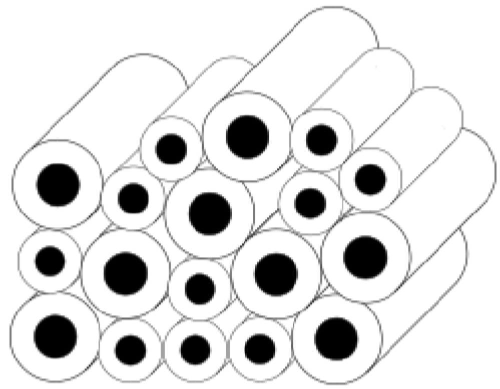


Figure 5.8: CCAM Fibre and Matrix   Figure 5.9: Assemblage of Cylinder

Concentric Cylinder Assemblage Model (CCAM) for the micromechanical modeling of unidirectional laminated composite was proposed by (Hashin and Rosen 1965). The CCAM model assumes unidirectional continuous fiber composite is assemblage of fiber core surrounded by a matrix annulus as shown in Figure 5.8 and each assemblage is having constant fiber volume fraction (see Figure 5.9). The density and stiffness of the assemblage are calculated using fiber properties, matrix properties and fiber volume fraction as

$$\rho_s = \rho_f V_f + \rho_m (1 - V_f) \qquad \text{Equation 5.6}$$

$$E_s = V_f E_f + (1 - V_f) E_m + \frac{4 V_f (1 - V_f)(\nu_f - \nu_m)^2 \mu_m}{\frac{(1 - V_f)\mu_m}{K_f + \frac{\mu_f}{3}} + \frac{V_f \mu_m}{K_m + \frac{\mu_m}{3}} + 1} \qquad \text{Equation 5.7}$$

Where,

E_s is the shear stiffness of the skin

ρ_s is the density of the skin

ν is the Poisson's ratio

K is the bulk modulus

μ is the shear modulus

f and m denote the fiber and matrix, respectively.

Structure-Property Relationships for Skin

Based on unit deflection method, the equation for density and ribbon direction shear modulus of hexagonal honeycomb is obtained (Kelsey, Gellatly et al. 1958). The density as well as the shear modulus is function of cell wall length (h), cell wall thickness (t), cell wall angle(θ), and cell wall material as shown below in Equation 5.8 and Equation 5.9.

$$\rho_c = \frac{2}{(1+\cos\theta)\sin\theta}\frac{t}{h}\rho \qquad \text{Equation 5.8}$$

$$G_c = \frac{1+\cos^2\theta}{(1+\cos\theta)\sin\theta}\frac{t}{h}G \qquad \text{Equation 5.9}$$

Where,

G_c is the shear stiffness of the core

ρ_c is the density of the core

5.2 Developing a cDSP for Design of Composite Structures

5.2.1 *Coupled Problem Approach*

Problem Statement – Design of Structure (Coupled Problem Approach)

Material selection and sizing of a sandwich composite beam needs to be performed concurrently. The material selection involves both for skin and core from materials listed in Table 5.1. Three load cases explained in Figure 5.5, Figure 5.6 and Figure 5.7 are to be considered. The quality of the design is measured in terms of design goals, which are to be achieved as much as possible. Specifically, we need a beam design that achieves target values of weight (Tw) and tip deflection (Tδ) that are 14 N and 10 mm respectively.. The task is to recommend the skin and core thicknesses and material for both skin and core that give the best performance with respect to the design quality specified.

Table 5.1: Skin and Core Materials (Pathan, Beemaraj et al. 2019)

			Functional Attributes			Non-functional Attributes					
						Resistance to					
Material Name	Material family	Type	E(MPa)	G(MPa)	ρ ($^{Kg}/_{m^3}$)	Moisture	Corrosion	Chemical	Fatigue	Cost	Thermal
ELR0908	E Glass-Epoxy UD	Skin	31830	4910	1800	1.0	0.8	0.6	0.8	0.4	0.6
CL0900	Carbon-Epoxy UD	Skin	108450	4090	1530	0.4	0.6	0.6	0.6	0.8	0.4
CLT0900	Carbon-Epoxy BX	Skin	56380	4270	1530	0.4	0.6	0.6	0.6	0.8	0.4
2024T3	Aluminum	Skin	68900	24950	2700	1.0	0.8	0.6	0.6	0.8	0.4
PCGA-XR23003	Aluminum HC	Core	1020	434	83	1.0	1.0	0.8	0.6	0.4	0.2
DivinycellP150	Polymer Foam	Core	152	40	1507	1.0	0.8	0.6	1.0	0.8	0.8
BaltekSB	Balsa	Core	442.8	362	285	0.2	0.8	0.4	0.8	0.6	0.8
AIREXT92130	Polymer Foam	Core	140	34	135	1.0	0.8	0.8	1.0	1.0	0.8

In this approach, the design problem has been bifurcated as decision making process involving two interacting decisions, that is, selection and compromise decision. Selection decision involves the choice of fiber and matrix combination for design of skin. Compromise decision involves the determination of sizing parameters, that is, the thickness of skin and core material. In this problem, core is considered to have honeycomb structure to be made out of aluminum. The goals and constraints used in math formulation are shown in Table 5.2 and follows from the equations discussed in Equation 5.1 through Equation 5.9.

Table 5.2: Math formulation for the Coupled Design of Composite Structure

Math Formulation for the Design of Composite Structure – Coupled problem (sDSP and cDSP)

Given
Selection system parameters
    Standard Material alternatives for skin: X1s, X2s, ………., Xns
    Standard Material alternatives for core: X1c, X2c, ………., Xmc
Compromise system parameters
    Length (L) = 1500 mm
    Width (B) = $\frac{L}{2}$
    Three load cases LCS1, LCS2 and LCS3
Find
Selection system variables
    Skin Material
    Core Material

Compromise System variables

Skin thickness (ts)

Core thickness (tc)

Deviation variables

e_1^- = Underachievement of MF goal for skin material

e_2^- = Underachievement of MF goal for core material

d_1^+ = Overachievement of beam weight goal

d_2^+ = Overachievement of beam deflection goal

Satisfy

Selection system Constraints

Selection constraint for skin material alternatives

$$\sum_{i=1}^{4} X_{i,s} = 1$$

Selection constraint for skin material alternatives

$$\sum_{i=1}^{4} X_{i,c} = 1$$

Compromise Design Constraints (From the Problem Statement)

Maximum strength criteria for skin

Maximum stress in skin < 0.5 x (Skin failure strength)

Maximum strength criteria for core

Maximum stress in core < 0.5 x (Core failure strength)

Compromise system Constraints

Constraints on deviation variables

$d_i^+ \geq 0$

$d_i^- \geq 0$

$d_i^+ . d_i^- = 0$ for i = 1, 2 and 3

Coupled selection Goal

G1 – Maximize Merit Function (MF) for skin material

$MF_i (ts,tc) X_{i,s} + e_1^- - e_1^+ =1$

G2 – Maximize Merit Function (MF) for core material

$MF_i (ts,tc) X_{i,c} + e_2^- - e_2^+ =1$

Coupled compromise Goals

G3 – Minimize beam weight

$$\frac{10}{\text{Weight}} + d_1^- - d_1^+ = 1$$

G4 – Minimize beam deflection

$$\frac{14}{\text{Deflection}} + d_2^- - d_2^+ = 1$$

Where,

$MF_i (m,b,z1) = \sum_{j=1}^{4} I_j R_{ij}(m, b, z1)$

System Bounds

B1: $5 \leq ts \leq 15$ (mm)

B2: $70 \leq tc \leq 90$ (mm)

B3: $0 \leq X1s \leq 1$

B4: $0 \leq X2s \leq 1$

B5: $0 \leq X3s \leq 1$

B6: $0 \leq X4s \leq 1$

B7: $0 \leq X1c \leq 1$

B8: $0 \leq X2c \leq 1$

B9: $0 \leq X3c \leq 1$

B10: $0 \leq X4c \leq 1$

Minimize

Deviation functions

$$Z = [0.25\ e1^- + 0.25\ e2^- + \ 0.25d1^+\ 0.25d2^+]$$

5.2.2 *Multiscale Design Approach*

Problem Statement – Design of Structure (Multiscale Design Approach)

A composite structure is to be designed wherein the material properties of skin, core and their thicknesses are treated as variables and given appropriate ranges. Three load cases explained in Figure 5.5, Figure 5.6 and Figure 5.7 are to be considered. The quality of the design is measured in terms of design goals, which are to be achieved as much as possible. Specifically, we need a beam design that achieves target values of weight (Tw) and tip deflection (Tδ) that are 14N and 10mm respectively.. The task is to recommend the skin and core thicknesses and material properties for both skin and core that give the best performance with respect to the design quality specified. The material properties considered for both skin and core are shown in Table 5.3.

Table 5.3: Range for Material Properties of Skin and Core

Material Properties	Type	Min	Max
Elastic modulus (Mpa)	Skin	94060	204310
Density $\left(\frac{Kg}{m^3}\right)$	Skin	1406	1651
Shear Modulus(Mpa)	Core	21.6	536.6
Density $\left(\frac{Kg}{m^3}\right)$	Core	3.4	86.3

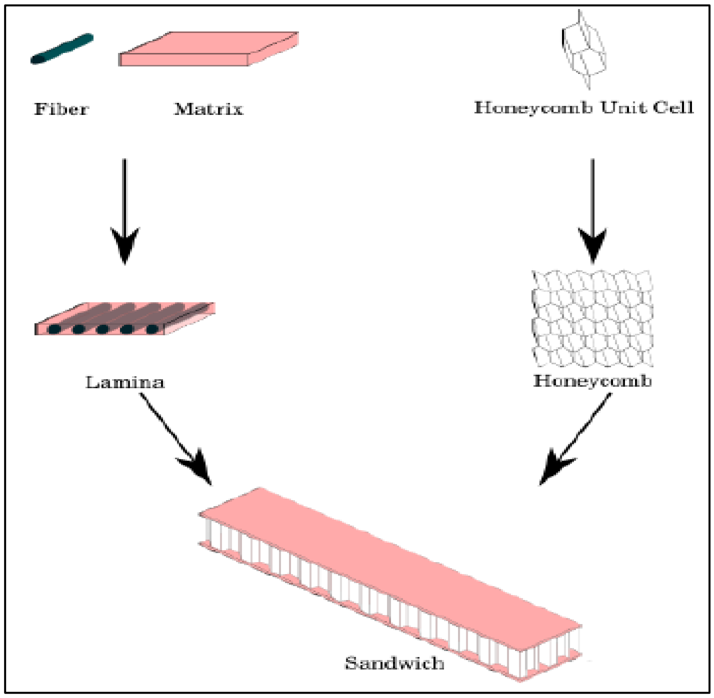


Figure 5.10: Hierarchical Nature of Composite Material

Composite material has hierarchical nature as shown in Figure 5.10, that is, the skin and core microstructure (i.e. fiber and matrix in skin and honeycomb in core) influence the macro properties for the sandwich composite. However, material selection in concurrent design approach is carried out using discrete materials mentioned in manufacturer's datasheets. Thus, the approach does not exploit the tailorable nature of composites entirely. Concurrent design solutions can be further improved upon by including this tailorable nature of composites in the design workflow itself. Hence, in this approach, two steps are involved. First, the design space and material space for skin and core are simultaneously explored against the performance requirements by treating skin and core materials as design variables. The second step involves tailoring the microstructures to achieve skin and core properties value achieved in first step and also required for target performance. The goals and constraints used in math formulation shown in Table 5.4 follows from the equations discussed in Equation 5.1 through Equation 5.9.

Table 5.4: Math Formulation for the Design of Composite Structure – Multiscale Approach

Math Formulation for the Design of Composite Structure – Multiscale Approach

Given
Compromise system parameters
    Length (L) = 1500 mm
    Width (B) = $\frac{L}{2}$
    Three load cases LCS1, LCS2 and LCS3
Find
Compromise System variables
    Skin thickness (ts)
    Core thickness (tc)
    Elastic modulus for skin material (Es)
    Density for skin material (ρs)
    Elastic modulus for core material (Gc)
    Density for skin material (ρc)
Deviation variables
    d_1^+ = Overachievement of beam weight goal
    d_2^+ = Overachievement of beam deflection goal
Satisfy
Compromise Design Constraints (From the Problem Statement)

Maximum strength criteria for skin

Maximum stress in skin < 0.5 x (Skin failure strength)

Maximum strength criteria for core

Maximum stress in core < 0.5 x (Core failure strength)

Compromise system Constraints

Constraints on deviation variables

$d_i^+ \geq 0$

$d_i^- \geq 0$

$d_i^+ . d_i^- = 0$ for i = 1 and 2

Coupled compromise Goals

G1 – Minimize beam weight

$$\frac{10}{\text{Weight}} + d_1^- - d_1^+ = 1$$

G2 – Minimize beam deflection

$$\frac{14}{\text{Deflection}} + d_2^- - d_2^+ = 1$$

System Bounds

B1: $5 \leq ts \leq 15$ (mm)

B2: $70 \leq tc \leq 90$ (mm)

B3: $94060 \leq Es \leq 204310$ (MPa)

B4: $1406 \leq \rho s \leq 1651$ (Kg/m^3)

B5: $21.6 \leq Gc \leq 536.6$ (MPa)

B6: $3.4 \leq \rho c \leq 86.3$ (Kg/m^3)

Minimize

Deviation functions

$Z = [\ 0.5d1^+\ 0.5d2^+]$

5.3 Robust Design of Composite Structures

In this section, an approach for robust design of composite structures is presented. A test case of designing a sandwich composite cantilever beam is performed. The design problem involves sizing and material selection for skin and core of a sandwich composite beam based on the requirements and constraints while accounting for the material and structural uncertainties. The design approach follows two steps. First, the design space and material space for skin and core are simultaneously explored against the performance requirements. In addition to the performance requirement, design exploration is carried out by putting an emphasis on the mitigation of impact on performance due to perturbation in dimensions and properties of skin and core. The second step involves tailoring the microstructures to achieve skin and core properties required for target performance. In this step, the mitigation of impact on skin and core properties due to perturbation in microstructural parameters is also considered.

Description of the Problem

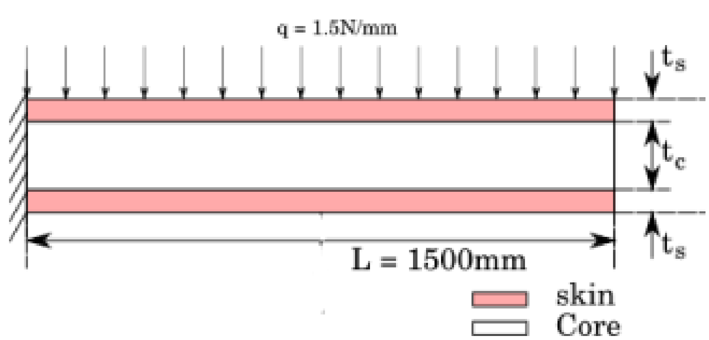


Figure 5.11: Load Case for Robust Design Consideration

The design of a sandwich composite beam with the following load case shown in Figure 5.11 is considered.

There are two problem statements corresponding to the two steps, that is,

- Robust Design of Structure (cDSP1)
- Robust Design of Microstructure (cDSP2)

Problem Statement – Robust Design of Structure (cDSP1)

Material selection and sizing of a sandwich composite beam needs to be performed. A uniformly distributed load is applied on the top of the beam. The quality of the design is measured in terms of design goals, which are to be achieved as much as possible. Specifically, we need a beam design that meets the robustness target of deflection and weight. The task is to recommend the skin and core thicknesses and modulus for both skin and core that give the best performance with respect to the design quality specified.

Problem Statement – Robust Design of Microstructure (cDSP2)

The design of skin and core microstructure of a sandwich composite is required. The target density and modulus for skin are given along with the density and modulus of the core material. The quality of the design is measured in terms of design goals which are to be achieved as much as possible. Specifically, we need to design microstructure for skin and core that meets the robustness target of density and modulus for both skin and core.

5.3.1 *Developing a CDSP for Robust Design of Composite Structures*

As explained in Section 2.4.3, for incorporating robustness in the design, we need to convert the original goals into goals that carry robustness metrics such as DCI and EMI. Furthermore, we need to add robustness constraints to ensure that the design solutions are robust. Table 5.5 is word formulation for robust design of structure and Table 5.6 is a word formulation for robust design of microstructure. All the goals in both formulations are converted to DCI. Table 5.7 is math formulation for robust design of structure and Table 5.8 is a math formulation for robust design of microstructure. All the goals in both formulations is convert to DCI.

5.3.2 *Word Formulation for the Robust Design of Composite Structures*

Table 5.5: Word Formulation for Robust Design of Structure

Word Formulation for Robust Design of Composite Beam – Robust Design of Structure
Given Skin design parameters Length of skin = 1500 mm Breadth of skin = 50 mm Core design parameters Length of skin = 1500 mm Breadth of skin = 50 mm System constants PI(Π) = 3.14

System variables and variability

S.N.	Design variables (X)	Variability (Δx)
(1)	X1, Skin thickness (Tf)	± 0.2 mm
(2)	X2, Density of skin (Rs)	± 4.0 kg/m^3
(3)	X3, Elastic Modulus of skin (Es)	± 5.0 Mpa
(4)	X4, Core thickness (Tc)	± 0.2 mm
(5)	X5, Density of core (Rc)	± 1.0 kg/m^3
(6)	X6, Shear modulus of core (Gc)	± 5.0 Mpa

Find

Skin design variables

Skin thickness (Tf)

Density of skin (Rs)

Elastic Modulus of skin (Es)

Core design variables

Core thickness (Tc)

Density of core (Rc)

Shear Modulus of core (Gc)

Deviation variables

Over- and underachievement of $DCI_{Deflection}$ goal

Over- and underachievement of DCI_{weight} goal

Satisfy

Robust design constraints

$DCI_{Deflection} \geq 1$

$DCI_{weight} \geq 1$

Goals

G1 – Goal for robust deflection

G2 – Goal for robust weight

System Bounds Upper and lower values for system variables Minimize Deviation functions Distance from target set for $DCI_{Deflection}$ goal Distance from target set for DCI_{weight} goal

Table 5.6: Word Formulation for Robust Design of Microstructure

Word Formulation for Robust Design of Composite Beam – Robust Design of Microstructure
Given Skin design parameters Density of fibre = 1760 Kg/m^3 Density of matrix = 1280 Kg/m^3 Modulus of fibre = 230000 Mpa Modulus of matrix = 3700 Mpa Core design parameters Density of core material = 2700 Kg/m^3 Shear Modulus of core material = 26000 Mpa System constants PI(Π) = 3.14 System variables and variability

S.N.	Design variables (X)	Variability (Δx)
(1)	X1, Volumetric fraction (Vf)	$\pm$ 0.05
(2)	X2, Wall angle (Θ)	$\pm$ 0.3 ^{0}
(3)	X3, Wall length (h)	$\pm$ 0.3 mm
(4)	X4, Wall thickness (t)	$\pm$ 0.01 mm

Find

Skin design variables

Volumetric fraction (Vf)

Core design variables

Wall angle (Ɵ)

Wall length (h)

Wall thickness (t)

Deviation variables

Over- and underachievement of DCI_{DS} goal

Over- and underachievement of DCI_{ES} goal

Over- and underachievement of DCI_{DC} goal

Over- and underachievement of DCI_{GC} goal

Satisfy

Skin robust design constraints

$DCI_{DS} \geq 1$

$DCI_{ES} \geq 1$

Core robust design Constraints

$DCI_{DC} \geq 1$

$DCI_{GC} \geq 1$

Skin Properties Goal

G1 – Goal for robust density for skin

G2 – Goal for robust modulus for skin

Core Properties Goal

G3 – Goal for robust density for core

G4 - Goal for robust modulus for skin

System Bounds

Upper and lower values for system variables

Minimize
Deviation functions
    Distance from target set for DCI_{DS} goal
    Distance from target set for DCI_{ES} goal
    Distance from target set for DCI_{DC} goal
    Distance from target set for DCI_{GC} goal

The equations discussed in Equation 5.1 through Equation 5.9 are used in the math formulations shown in Table 5.7 and Table 5.8.

5.3.3 *Math Formulation for the Robust Design of Composite Structures*

Table 5.7: Math Formulation for Robust Design of Structure

Math Formulation for robust design of composite beam – Robust Design of Structure

Given
Skin design parameters
    Length of skin = 1500 mm
    Breadth of skin = 50 mm
Core design parameters
    Length of skin = 1500 mm
    Breadth of skin = 50 mm
System constants
    PI(Π) = 3.14
Find
Skin design variables
    Skin thickness (Tf)
    Density of skin (Rs)
    Elastic Modulus of skin (Es)

Core design variables

Core thickness (Tc)

Density of core (Rc)

Shear Modulus of core (Gc)

Deviation variables

Over- and underachievement of $DCI_{Deflection}$ goal

Over- and underachievement of DCI_{weight} goal

Satisfy

Robust design constraints

$DCI_{Deflection} \geq 1$

$DCI_{weight} \geq 1$

Goals

G1 – Goal for robust deflection

$$\frac{DCI_{Deflection}}{DCI_{Deflection,Target}} + d_1^- - d_1^+ = 1$$

G2 – Goal for robust weight

$$\frac{DCI_{Weight}}{DCI_{Weight,Target}} + d_2^- - d_2^+ = 1$$

System Bounds

B1: $5 \leq Tf \leq 15$ (mm)

B2: $70 \leq Tc \leq 90$ (mm)

B3: $94060 \leq Es \leq 204310$ (Mpa)

B4: $21.6 \leq Gc \leq 536.6$ (Mpa)

B5: $1406 \leq Rs \leq 1651$ (Kg/m^3)

B6: $3.4 \leq Rc \leq 86.3$ (Kg/m^3)

Minimize

Deviation functions

$$Z = \left[\sum_{i=1}^{2} w_i \cdot (di^- + di^+)\right], \; \sum_{i=1}^{2} w_i = 1$$

Table 5.8: Math Formulation for Robust Design of Microstructure

Math Formulation for robust design of composite beam – Robust Design of Microstructure
Given Skin design parameters Density of fibre = 1760 Kg/m^3 Density of matrix = 1280 Kg/m^3 Modulus of fibre = 230000 Mpa Modulus of matrix = 3700 Mpa Core design parameters Density of core material = 2700 Kg/m^3 Shear Modulus of core material = 26000 Mpa System constants PI(Π) = 3.14 Find Skin design variables Volumetric fraction (Vf) Core design variables Wall angle (Θ) Wall length (h) Wall thickness (t) Deviation variables Over- and underachievement of DCI_{DS} goal Over- and underachievement of DCI_{ES} goal Over- and underachievement of DCI_{DC} goal Over- and underachievement of DCI_{GC} goal Satisfy

Skin robust design constraints

$DCI_{DS} \geq 1$

$DCI_{ES} \geq 1$

Core robust design Constraints

$DCI_{DC} \geq 1$

$DCI_{GC} \geq 1$

Skin Properties Goal

G1 – Goal for robust density for skin

$$\frac{DCI_{DS}}{DCI_{DS,Target}} + d_1^- - d_1^+ = 1$$

G2 – Goal for robust modulus for skin

$$\frac{DCI_{ES}}{DCI_{ES,Target}} + d_2^- - d_2^+ = 1$$

Core Properties Goal

G3 – Goal for robust density for core

$$\frac{DCI_{DC}}{DCI_{DC,Target}} + d_3^- - d_3^+ = 1$$

G4 - Goal for robust modulus for skin

$$\frac{DCI_{GC}}{DCI_{GC,Target}} + d_4^- - d_4^+ = 1$$

System Bounds

B1: $0.4 \leq Vf \leq 0.7$

B2: $30 \leq \Theta \leq 60$ (deg.)

B3: $2 \leq h \leq 25$ (mm)

B4: $0.01 \leq t \leq 0.11$ (mm)

Minimize

Deviation functions

$$Z = [\ \sum_{i=1}^{4} w_i \cdot (di^- + di^+)]\ ,\ \sum_{i=1}^{4} w_i = 1$$

5.4 Summary of Chapter 5

In this chapter, design of composite structure beam as a coupled design problem is discussed. Specifically, design of a cantilever beam with 3 loading conditions is presented. Also, the coupling in design decisions in context of the design problem is discussed. Finally, the DSP based mathematical formulations for the design of composite structures as (i) a coupled problem approach and, (ii) multiscale approach is presented. Following this, the DSP based mathematical formulations for the robust design of composite structures with multiscale approach is presented. The results to the math formulations in this chapter is presented in Chapter 6 (Section 6.3 and Section 6.4).

Chapter 6: Results and Discussion

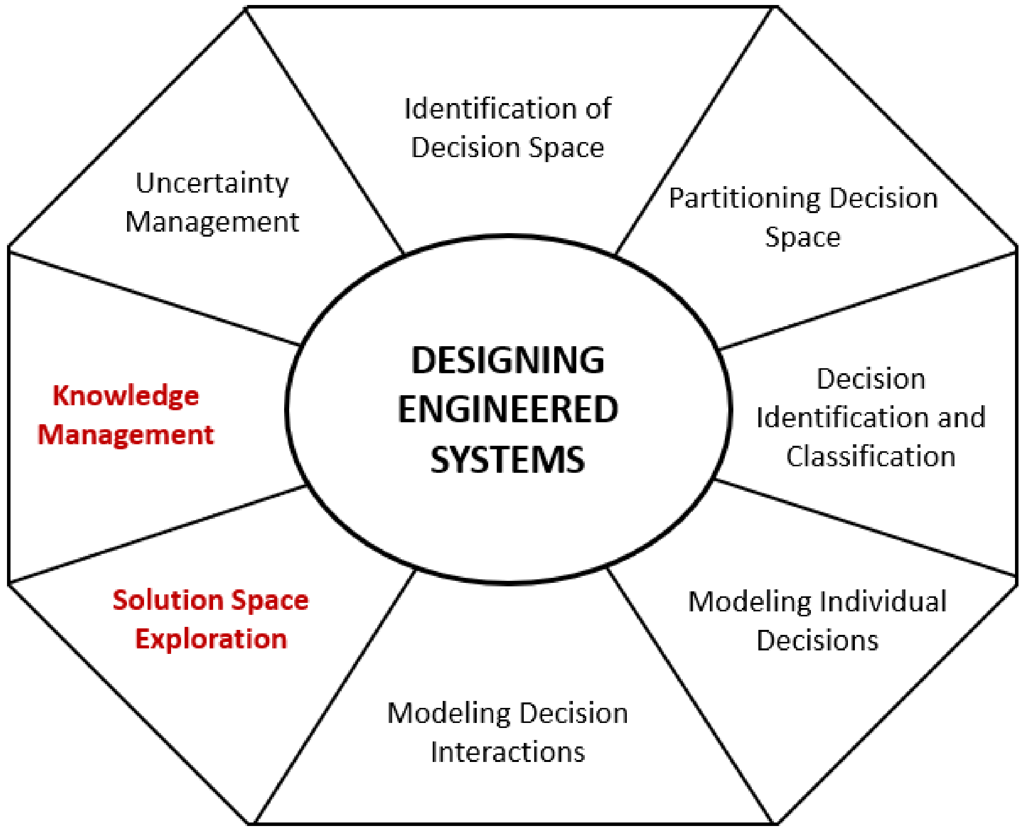


Figure 6.1: Elements in Design of Coupled Engineered Systems Discussed in Chapter 6

In Chapter 6, two elements in the design of engineered systems (as shown in Figure 6.1 - highlighted in red) in context of the test problems (fender, gearbox and composite structures) is discussed. In this chapter, the results pertaining to the math formulations derived in Chapter 3, Chapter 4 and Chapter 5 are presented. In each section, the discuss is on the design scenarios and results from each compromise DSPs. In Section 6.1 and 6.2 the results pertaining to design of fender and gearbox are respectively presented. Section 6.3 and 6.4 is reserved of discussing the results for composite structures. Section 6.5 is reserved for answering the research questions posed in the thesis.

6.1 Exploring Solution Space in the Design of Fender

6.1.1 *Design Scenarios*

In Chapter 3, two design examples for fender has been discussed and formulated. The first example (Example 1) deals with a single DSP, that is, compromise Decision Support Problem (cDSP). The second example (Example 2) deals with a coupled DSP, that is, coupled selection Decision Support Problem (sDSP) - compromise Decision Support Problem (cDSP). In the first example, 7 design scenarios are created by varying the weights on goals. These weights are based on designer's preference on goals. The example 1 is solved for the 7 design scenarios shown below in Figure 6.2.

Figure 6.2: Design Scenarios Explored for the Design of Fender

For the second example (Example 2) involving coupled DSP, 12 design scenarios are solved. These design scenarios are created by assigning different weights to the selection attributes, that is, cost, manufacturability, corrosion resistance and hardness. However, the weight assigned to the 3 compromise goals (EMI_{MSR}, DCI_{AR} and DCI_{ST}) are given equal weights, that is, 0.33. The mathematical formulations, design scenarios and results are also discussed in (Sharma, Allen et al. 2019).

How are ternary plots created for solution space exploration?

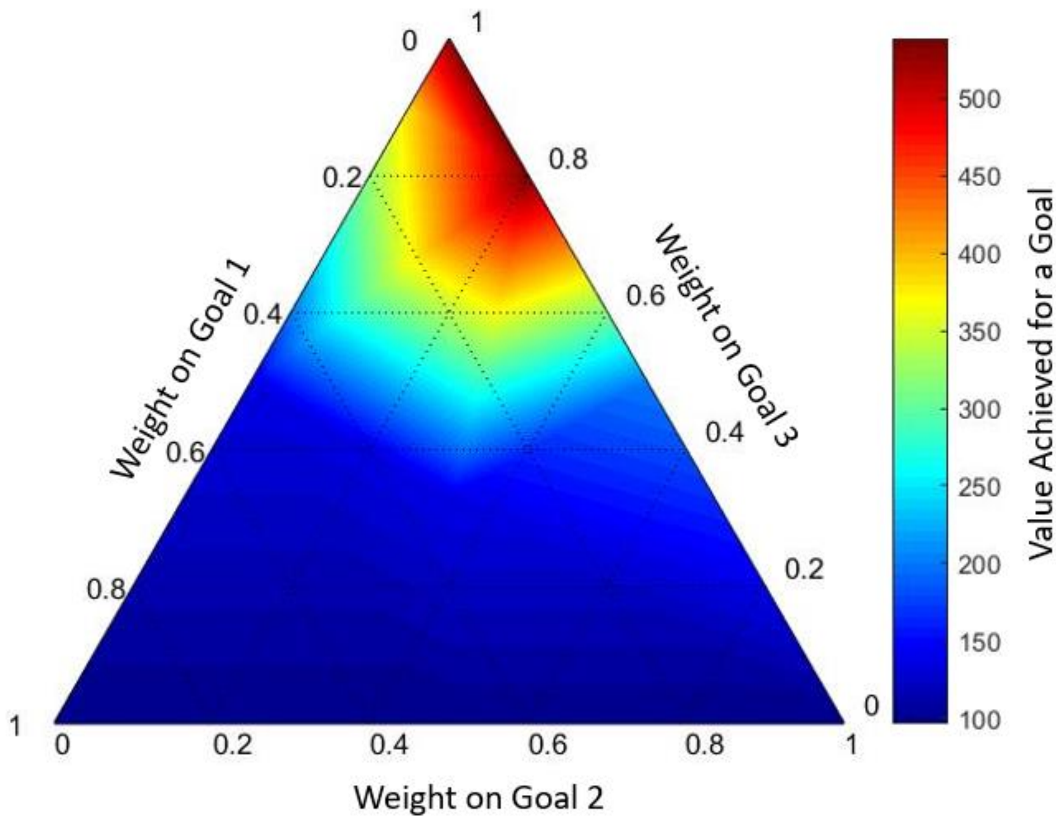


Figure 6.3: Ternary Plot for Solution Space Exploration

A ternary plot is drawn using a triangle as shown in Figure 6.3. Each sides of the triangle represent a variable. In a ternary plot, the values of the three variables a, b, and c must sum to some constant, K. Usually, this constant is represented as 1.0 or 100%. For solution space exploration, the value of K = 1 and each side represent the weights assigned to the goal. Every point on a ternary plot represents a different combination of weights for the goals. The interior color coding indicates the value achieved for a goal when a particular combination of weights is assigned to the three goals. In Figure 6.3, the different colors in the interior of the triangle indicate the values achieved for either one of the goals when different combination of weights to the goals are assigned. Similarly, plots are drawn for the other remaining goals. In each plot, an acceptable region for the particular goal is identified. Finally, a superimposed plot is made to ascertain region of overlap, that is, region where all different goals are met simultaneously.

6.1.2 *Exploration of Solution Space*

Example 1 - Robust Design of fender

In this approach, as discussed in Chapter 3 the material properties has been treated as design variables and the design problem is solved as one compromise DSP (codes available in Appendix). By using different weights on goals, 7 different design scenarios are explored, the results of which are tabulated in Table 6.1.

Table 6.1: Goals and Design Variables Achieved for Different Scenarios

Scenarios	Weights			Goals achieved			Design variables			
	w1	w2	w3	EMI_{MSR}	DCI_{AR}	DCI_{ST}	t (in)	D (in)	σy (ksi)	E (Mpsi)
1	1	0	0	14.309	1.099	1.983	0.1337	18.33	35.99	30.5
2	0	1	0	7.586	33.423	2.998	0.4448	13.08	30	27.5
3	0	0	1	0.638	44.438	5.771	0.6822	23.7	30.98	30.48
4	0.5	0.5	0	10.798	30.020	2.881	0.4158	12.83	35.99	30.48
5	0	0.5	0.5	1.632	45.270	5.750	0.6859	23.48	35.92	30.46
6	0.5	0	0.5	15.733	1.804	1.679	0.1413	16.37	35.99	30.49
7	0.34	0.33	0.3	10.787	30.013	2.884	0.4159	12.84	35.99	30.49

These design scenarios are selected with an intent to effectively cover the design space for the exploration of solution space using different combination of weights on goals. The different weights assigned to the goals indicate designer's interest to achieve target set to the goals. Assigning weight as 1 (Scenarios 1, 2 and 3) to a goal means that the designer's interest is to achieve target set to the goal as closely as possible while ignoring the other goals. For instance, assigning weight w1=1 to EMI_{MSR} (G1) would mean that the designer is interested to achieve the target set to EMI_{MSR} as closely as possible while not considering the other two goals. Similarly, assigning 0.5 (Scenarios 4, 5 and 6) to two goals means that the designer is equally interested in achieving the target set to the two goals while not considering the third goal. At last, Scenario 7 means that designer is equally interested in achieving the target set to all three goals. With the solutions obtained for all the scenarios, the designer is now most interested in exploring the solution space to obtain solutions that are of prime importance to the decision maker, that is, the designer of fender in the present context. With the information tabulated in Table 6.1, ternary plots for each goal are created. The axes in the ternary plots indicate the weights assigned to each goal while the colored ternary space in the interior indicate the value achieved for that specific goal. For instance, ternary plot for EMI_{MSR} goal shows the value achieved for EMI_{MSR} goal within the ternary space, when different weights are assigned to each goal. Once the ternary plots for the goals are drawn, an acceptable region within each ternary plot is identified. Finally, acceptable regions identified from each ternary plot are superimposed into one plot to explore feasible solution region considering all 3 goals.

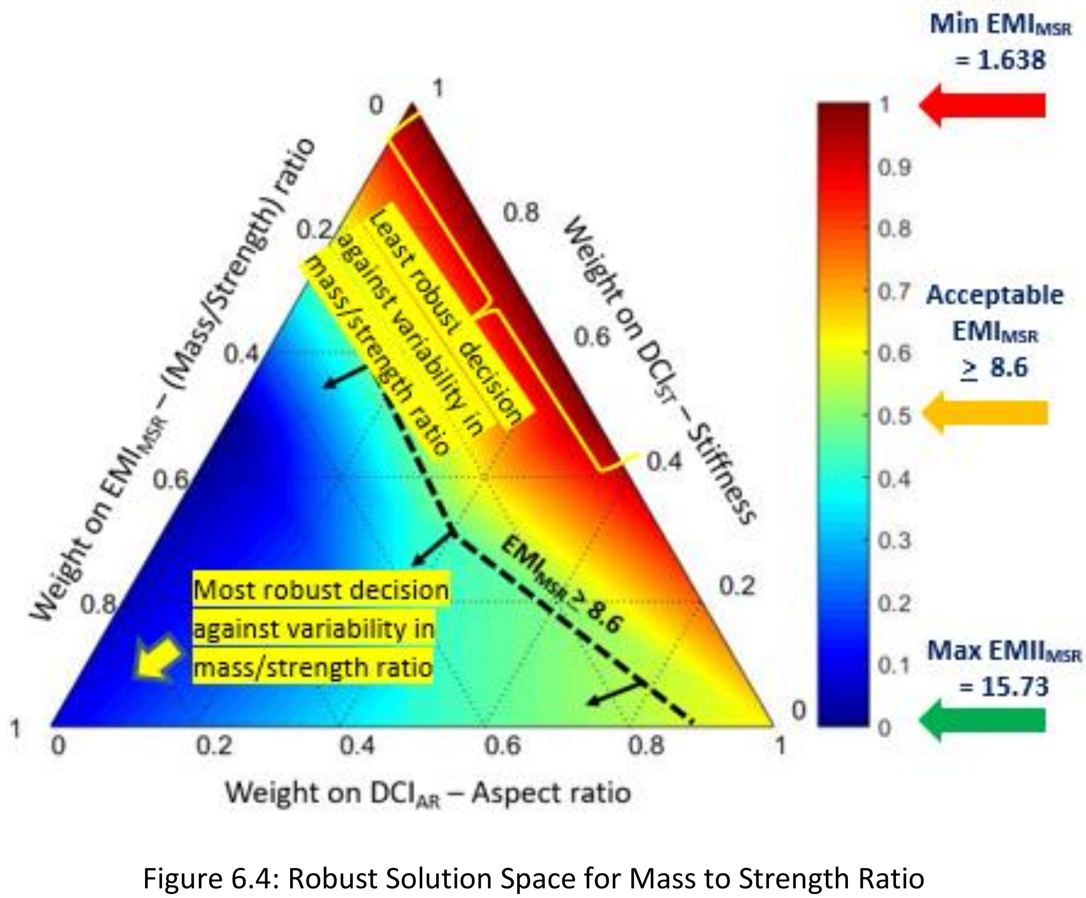


Figure 6.4: Robust Solution Space for Mass to Strength Ratio

The ternary plot for EMI_{MSR} goal (G1) is shown in Figure 6.4. As discussed in Chapter 2, we are interested in achieving a higher value for each robustness goal. For EMI_{MSR} goal (G1), our interest is to identify regions where higher values for EMI_{MSR} have been achieved. The solution space in Figure 6.4 comprises of robust design solutions with $EMI_{MSR} \geq 1$ ensuring robustness against model uncertainty as well as parameter uncertainty. The blue region comprises the robust design solutions that achieve the maximum value for EMI_{MSR} goal whereas the red region comprises the robust design solutions that achieve the minimum value for EMI_{MSR} goal. The maximum value achieved for EMI_{MSR} goal is 15.730 while the minimum achieved value is 1.638. The achieved values for EMI_{MSR} are also represented in terms of deviation from target and normalized. The maximum achieved value is indicated as 0 while the minimum achieved value is indicated as 1. Our interest is now to look for region with least deviation from the normalized minimum deviation or maximum value of EMI_{MSR}. We now define an acceptable robust region within the solution space as $EMI_{MSR} \geq 8.6$ (corresponding to 0.5 deviation) identified by the black dashed lines. Any design solutions contained within this region is acceptable for us as it satisfies the requirement for mass to strength ratio under model and parameter uncertainty.

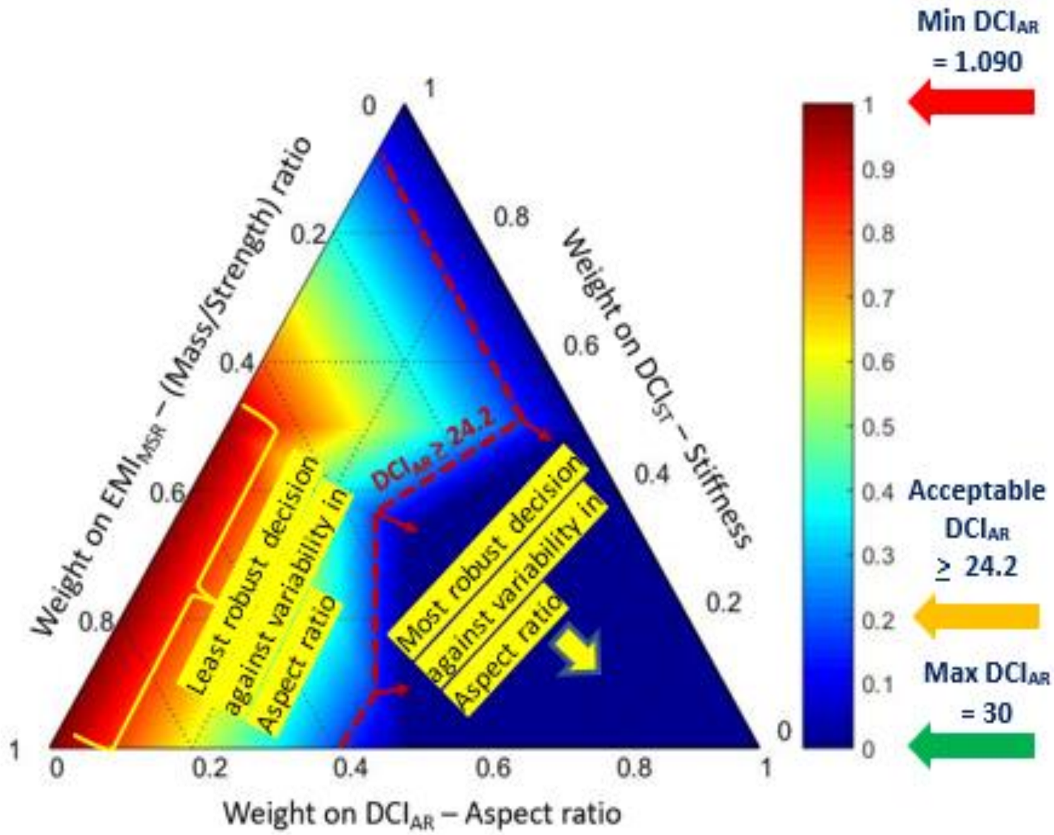


Figure 6.5: Robust Solution Space for Aspect Ratio

As discussed in Chapter 2, we are interested in achieving a higher value for each robustness goal. For DCI_{AR} goal (G2), our interest is to identify regions where higher values for DCI_{AR} have been achieved. The solution space in Figure 6.5 comprises of robust design solutions with $DCI_{AR} \geq 1$ ensuring robustness against model uncertainty as well as parameter uncertainty. The blue region comprises the robust design solutions that achieve the maximum value for DCI_{AR} goal whereas the red region comprises the robust design solutions that achieve the minimum value for DCI_{AR} goal. The maximum value achieved for DCI_{AR} goal is 30 while the minimum achieved value is 1.090. The achieved values for DCI_{AR} are also represented in terms of deviation from target and normalized. The maximum achieved value is indicated as 0 while the minimum achieved value is indicated as 1. Our interest is now to look for region with least deviation from the normalized minimum deviation or maximum value of DCI_{AR}. We now define an acceptable robust region within the solution space as $DCI_{AR} \geq 24.2$ (corresponding to 0.2 deviation) identified by the red dashed lines. Any design solutions contained within this region is acceptable for us as it satisfies the requirement for aspect ratio under parameter uncertainty.

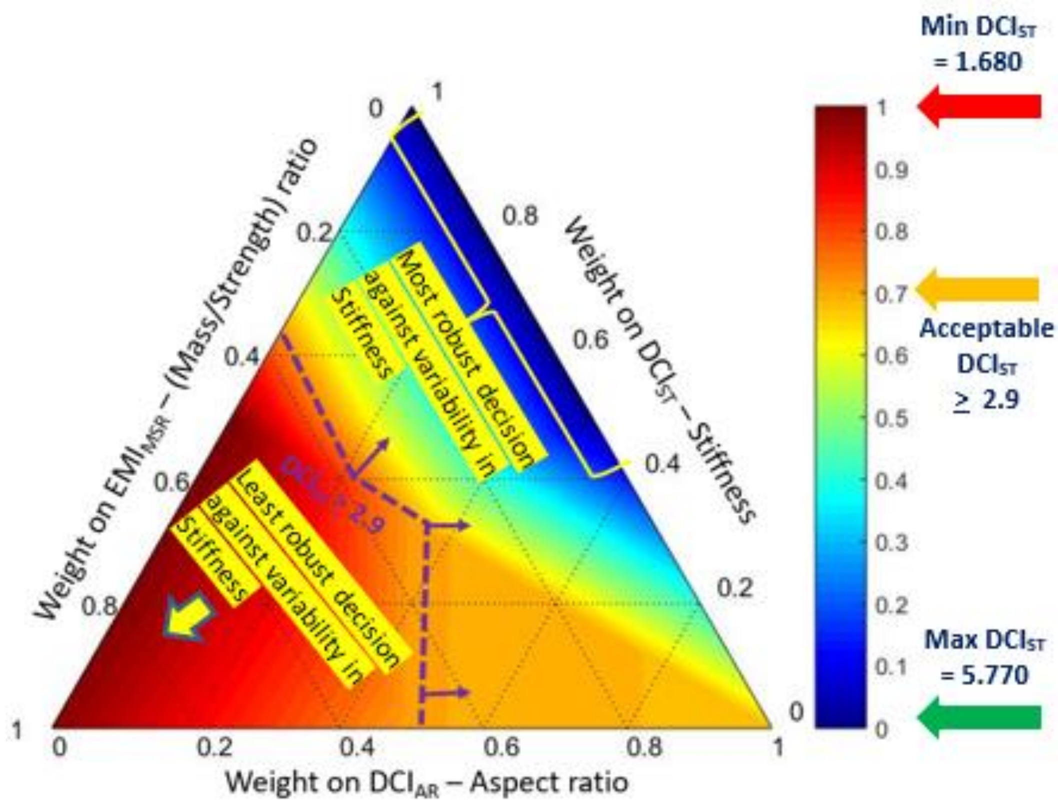


Figure 6.6: Robust Solution Space for Stiffness

As discussed in Chapter 2, we are interested in achieving a higher value for each robustness goal. For DCI_{ST} goal (G3), our interest is to identify regions where higher values for DCI_{ST} have been achieved. The solution space in Figure 6.6 comprises of robust design solutions with $DCI_{ST} \geq 1$ ensuring robustness against model uncertainty as well as parameter uncertainty. The blue region comprises the robust design solutions that achieve the maximum value for DCI_{ST} goal whereas the red region comprises the robust design solutions that achieve the minimum value for DCI_{ST} goal. The maximum value achieved for DCI_{ST} goal is 5.770 while the minimum achieved value is 1.680. The achieved values for DCI_{ST} are also represented in terms of deviation from target and normalized. The maximum achieved value is indicated as 0 while the minimum achieved value is indicated as 1. Our interest is now to look for region with least deviation from the normalized minimum deviation or maximum value of DCI_{ST}. We now define an acceptable robust region within the solution space as $DCI_{ST} \geq 2.9$ (corresponding to 0.7 deviation) identified by the purple dashed lines. Any design solutions contained within this region is acceptable for us as it satisfies the requirement for stiffness under parameter uncertainty.

The acceptable region for all the requirements (mass to strength ratio, aspect ratio and stiffness) with uncertainty consideration is identified. Following this, a superimposed ternary plot will be drawn to identify design solutions that satisfy all requirements.

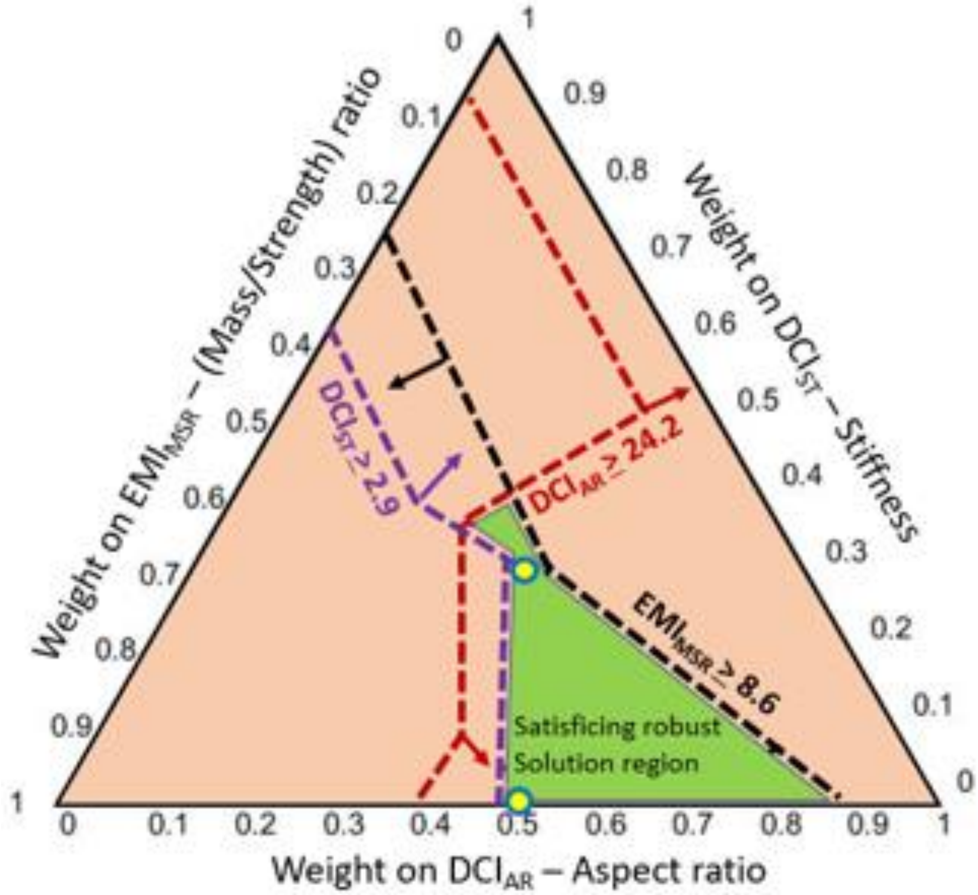


Figure 6.7: Superimposed Satisficing Robust Solution Space

The acceptable solution region identified from all the three individual ternary plots are superimposed in one plot. As our interest lies in identifying a satisficing robust solution region against multiple conflicting goals, we derive a superimposed robust solution space as discussed earlier and shown in Figure 6.7. The green region in Figure 6.7 is our search space for identifying robust solutions that meet our conflicting need of minimizing mass to strength ratio and aspect ratio while maximizing stiffness. We identify two robust design solutions (Scenario 4 and 7) to lie within the green region and are marked by yellow dots with blue edge. The design variables corresponding to these robust solutions are tabulated in Table 6.2.

Table 6.2: Robust Solutions Selected

Scenarios	Design Variables			
	t (in)	D (in)	σy (ksi)	E (Mpsi)
4	0.4158	12.83	35.99	30.48
7	0.4159	12.84	35.99	30.49

Example 2 - Robust Design of fender

Table 6.3: Goals and Design Variables Achieved for Different Scenarios

Scenarios	Weights				Deviations			Design variables		
	Cost	Corrosion resistance	Machinability	Hardness	EMI_{MSR}	DCI_{AR}	DCI_{ST}	D (in)	t (in)	Material
S1	0.5	0.5	0	0	0.139	0	0.474	18.61	0.516	Titanium
S2	0.5	0	0.5	0	0.8	0.002	0.662	12.98	0.417	Copper
S3	0.5	0	0	0.5	0.762	0	0.643	12.97	0.419	Cast Iron
S4	0	0.5	0.5	0	0.8	0	0.666	12.98	0.42	Copper
S5	0	0.5	0	0.5	0	0	0.738	11.05	0.514	Titanium
S6	0.1	0.3	0	0.6	0.765	0	0.643	12.92	0.424	Cast Iron
S7	0.3	0.3	0.2	0.2	0.362	0	0.39	21.51	0.545	Titanium
S8	0.1	0.2	0.3	0.4	0.814	0	0.663	12.71	0.447	Copper
S9	0.2	0.3	0.1	0.4	0	0	0.509	17.68	0.489	Titanium
S10	0.3	0.2	0.1	0.4	0.761	0	0.643	12.98	0.418	Cast Iron
S11	0.2	0.3	0.2	0.3	0	0	0.552	15.93	0.495	Titanium
S12	0.25	0.25	0.25	0.25	0.885	0	0.569	14.48	0.527	Copper

In this approach, as discussed in Chapter 3 the material is selected from the predefined list and hence, material selection and sizing has been considered as a coupled problem. The selection DSP deals with the material selection while compromise DSP deals with dimensional synthesis, that is, determination of design dimensions. Four attributes are considered for the selection of material which are cost, corrosion resistance, manufacturability and hardness. By giving different weights to the selection attributes, 12 different design scenarios are explored, the results of which are tabulated in Table 6.3 (codes available in Appendix).

These scenarios are chosen based on designer's interest to effectively capture the design space for the exploration of solution space using different combination of weights on selection attributes. However, the weights for all compromise goals were assigned equal weights, that is, 0.33 while the weights for attributes in selection DSP are assigned as shown in Table 6.3. Different weights are assigned to different selection attribute which indicate the designer's interest to explore robust design solutions for different preferences. With the solutions obtained for all the scenarios, the designer is now most interested in exploring the solution space to obtain solutions that are of prime importance to the decision maker, that is, the designer of fender in the present context. Various scenarios are generated and presented to the designer for making decision. The designer then chooses designs that most fit the designer's aspiration. In the present context, the designer wishes to meet the compromise goals (G2-EMI_{MSR}, G3-DCI_{AR} and G4-DCI_{ST} mentioned in Table 6.3) as closely as possible and simultaneously select material that can be used to create designs which are corrosion resistant, less expensive and easier to machine.

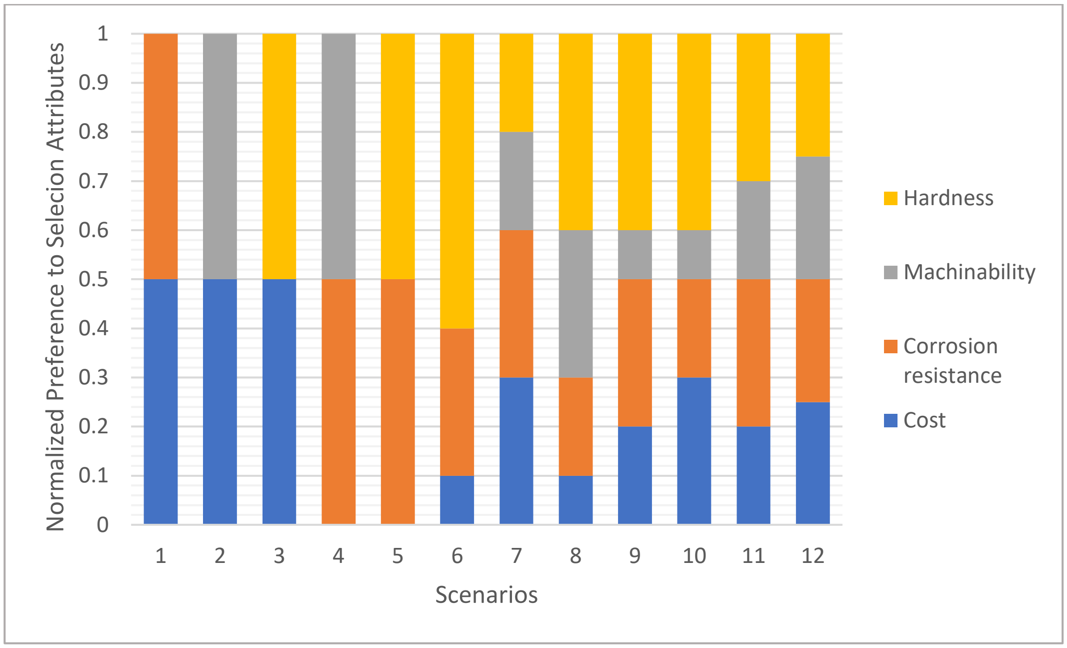


Figure 6.8: Design Scenarios for Selection Attributes

In Figure 6.8, we can see the weights assigned to different selection attributes for all 12 design scenarios. We see that in Scenarios 2, 4, 7, 8, 9, 10, 11 and 12 consideration for materials with easier machinability is made and in Scenarios 1, 4, 5, 6, 7, 8, 9, 10, 11 and 12 consideration for corrosion resistant materials is made. Further from Figure 6.8, we also see that in Scenarios 1, 2, 3, 6, 7, 8, 9, 10, 11 and 12 consideration for cost is made. As the designer is looking for all four attributes (Machinability, Corrosion Resistance and Cost) in selection of material, Scenarios 7, 8, 9, 10, 11 and 12 are the candidate for potential design solutions. These potential scenarios are to be compared to see which of them satisfy the compromise goals more closely.

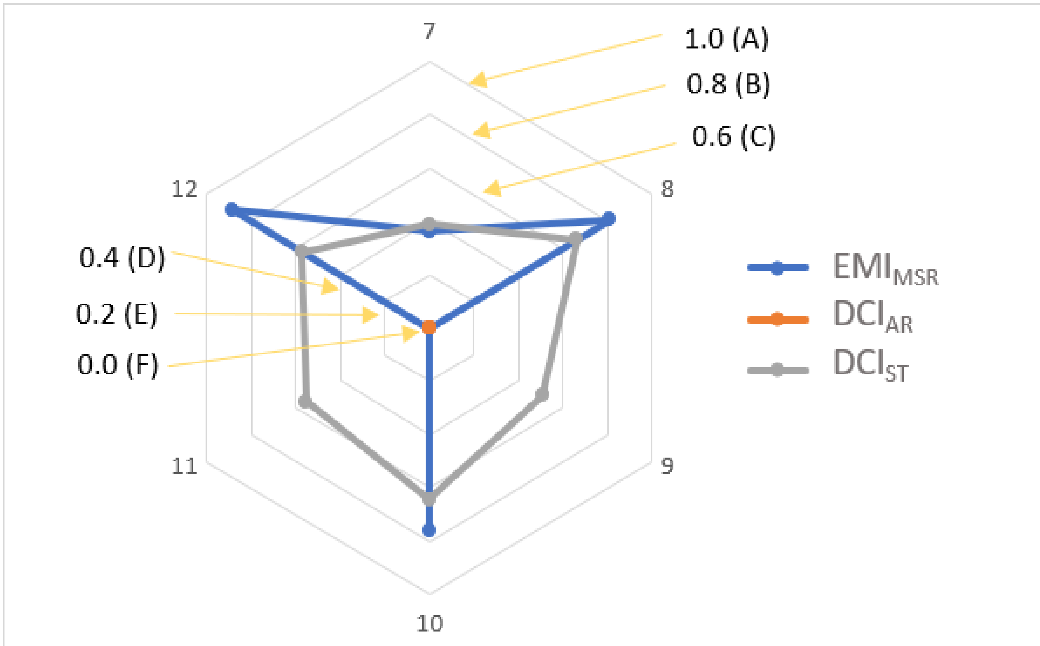


Figure 6.9: Design Scenarios with Deviations from Compromise Goals

The corners of the hexagon in Figure 6.9 represent the six potential design Scenarios 7, 8, 9, 10, 11 and 12. Further, each hexagon represents the normalized deviations from the target set for compromise goals with the outermost hexagon (A) indicating normalized deviation equal to 1. The hexagon second to the outermost hexagon (B) indicates normalized deviation equal to 0.8 and so on with center of the hexagon signifying normalized deviation equal to 0. In Figure 6.9, as we can see that the normalized deviation for DCI_{AR} goal is 0 for all the scenarios, we are now looking for design scenarios that satisfy EMI_{MSR} and DCI_{ST} goals as closely as possible. We do not see any scenarios that have normalized deviation of value equal to 0 for all three compromise goals. There are also no scenarios that have normalized deviation within 0.2 for all three compromise goals. However, we see that Scenario 7 has normalized deviation within 0.4 for all three compromise goals. Hence, Scenario 7 is the design scenario that closely achieves the three compromise goals while also satisfying selection requirements.

Based on the designer's aspiration to meet the compromise goals as closely as possible and select material that that compromises all the selection attributes, the robust solution alternative that most closely satisfies designer's aspiration is shown in Table 6.4.

Table 6.4: Robust Solution Selected

Scenario	Design Variables		
	Compromise Variables		Selection Variable
	t (in)	D (in)	Material
7	0.545	21.51	Titanium

Based on the designer's interest in this specific design problem, the choice of titanium as a material and dimensions as shown in Table 6.4 seem suitable. The intention in this conclusion is not to justify the use of Titanium in the design of fender but to demonstrate the solution approach for coupled design problems. The 3 material alternatives (Cast Iron, Copper and Titanium) are chosen as these materials that stand out from each other in terms of cost, machinability, corrosion resistance and hardness, which allows us to verify if the influence among DSPs are effectively captured. The 12 scenarios tabulated in Table 6.3 are captured by assigning equal weights to the 3 compromise goals while varying the weights for attributes in selection DSP. From this table, we can see that as the solutions for selection DSP (material selection) are changing, the solutions in compromise DSP (thickness and diameter) are also changing and vice-versa. This lets us validate that the mutual influence among DSPs have been successfully captured. Also, varying the designer’s preference allows us to explore other robust solutions. Further, providing a pool of materials that are more suited for a particular application would allow us to effectively explore robust design solutions for practical applications. For instance: Gear design problem can be solved by providing material alternatives that are specifically designed to suit gear applications thus, enabling us to compare and make tradeoff study among the available material alternatives for better decision making in exploring robust gear designs.

6.2 Exploring Solution Space in the Design of a Gearbox

6.2.1 *Decision Scenarios for Design Exploration*

As discussed in Chapter 4, the design will be explored for 3 decision scenarios (codes available in Appendix). The 3 decision scenarios for design exploration are

Decision Scenario 1

Level 1: Compromise of design variables for gear with different weights + Selection of gear material with equal weights to all attributes

Level 2: Compromise of design variables for shaft

Decision Scenario 2

Level 1: Selection of gear material with equal weights to all attributes + Compromise of design variables for gear with different weights

Level 2: Compromise of design variables for shaft

Decision Scenario 3

Level 1: Selection of gear material with varying weights to all attributes + Compromise of design variables for gear with total weight to torque

Level 2: Compromise of design variables for shaft

6.2.2 *Exploration of Solution Space for Decision Scenarios*

Decision Scenario 1

Decision Scenario 1 is solved for 9 different design scenarios. These scenarios are selected based on designer's aspiration to effectively capture the design space for the exploration of solution space using different combination of weights on goals. The design scenarios and the results for Level 1 decisions (coupled cDSP-sDSP) pertaining to gear decisions are summarized in Table 6.5.

These design scenarios are selected with an intent to effectively cover the design space for the exploration of solution space using different combination of weights on goals. The different weights assigned to the goals indicate designer's interest to achieve target set to the goals. Assigning weight as 1 (Scenarios 1, 2 and 3) to a goal means that the designer's interest is to achieve target set to the goal as closely as possible while ignoring the other goals. For instance, assigning weight w1 = 1 to Mass would mean that the designer is interested to achieve the target set to mass as closely as possible while not considering the other two goals. Similarly, assigning 0.5 (Scenarios 4, 5 and 6) to two goals means that the designer is equally interested in achieving the target set to the two goals while not considering the third goal. At last, Scenario 7 means that designer is equally interested in achieving the target set to all three goals and so on.

Table 6.5: Goals and Design Variables Achieved for Different Design Scenarios-Gear

Scenarios	Weights			Design variables-Gear				Goals		
	Mass	Size	Torque	m mm	b mm	z	Material	Mass Kg	Size mm	Torque Nm
S1	1	0	0	3	24	18	X2	7.29	270	96.43
S2	0	1	0	3	24	18	X2	7.29	270	96.43
S3	0	0	1	5.04	40.36	30	X1	96.09	756	526.63
S4	0.5	0.5	0	3	24	18	X2	7.29	270	96.43
S5	0	0.5	0.5	3	35.38	18	X5	10.74	270	202.42
S6	0.5	0	0.5	3	24	18	X5	7.29	270	137.31
S7	0.33	0.33	0.34	3	24	18	X5	7.29	270	137.31
S8	0	0.2	0.8	4.06	44.05	21	X5	33.35	426.3	538.50
S9	0.1	0	0.9	3.79	30.37	25	X5	28.39	473.75	385.15

With the solutions obtained for all the scenarios, the designer is now most interested in exploring the solution space to obtain solutions that are of prime importance to the decision maker, that is, the designer of the gearbox in the present context. With the information tabulated in above table, ternary plots for each goal are created. The axes in the ternary plots indicate the weights assigned to each goal while the colored ternary space in the interior indicate the value achieved for that specific goal. For instance, ternary plot for Mass goal shows the value achieved for Mass goal within the ternary space, when different weights are assigned to each goal. Once the ternary plots for the goals are drawn, an acceptable region within each ternary plot is identified. Finally, acceptable regions identified from each ternary plot are superimposed into one plot to explore feasible solution region considering all 3 goals.

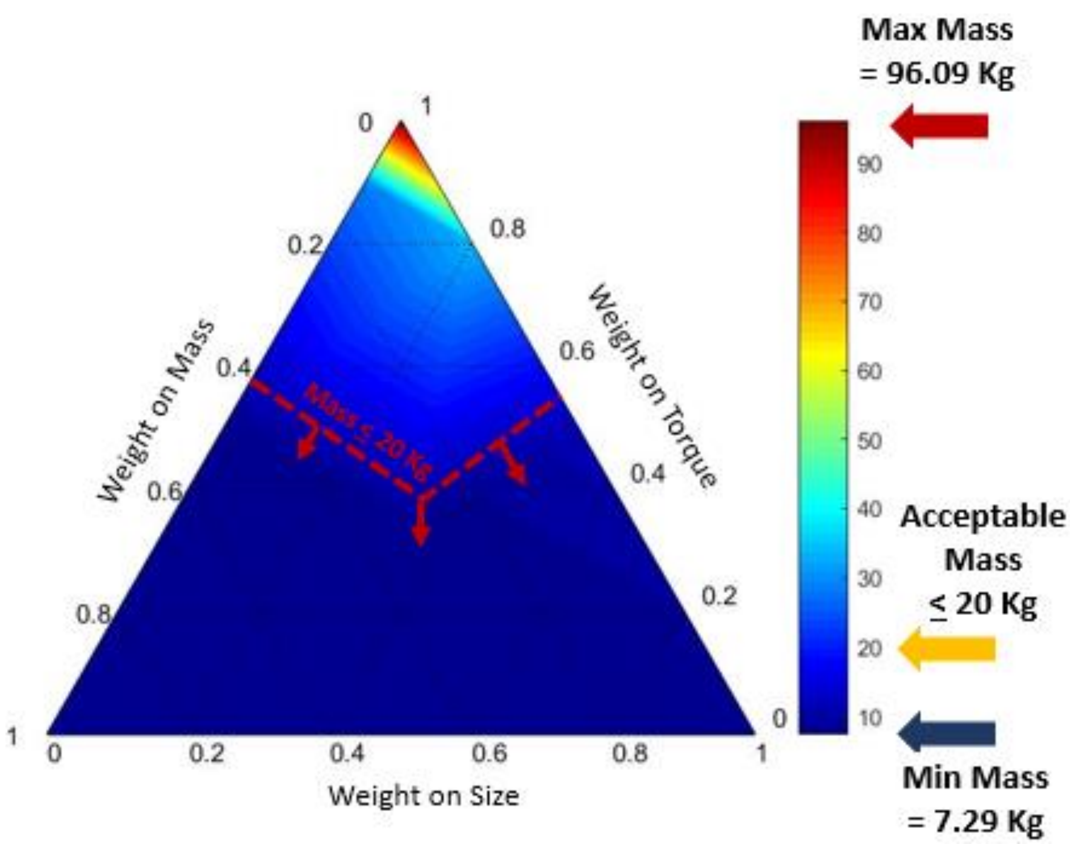


Figure 6.10: Solution Space for Mass

As discussed in Chapter 4, we are interested in achieving a lower value for Mass goal. For Mass goal, our interest is to identify regions where lower values for Mass have been achieved. The blue region comprises the design solutions that achieve the lower value for Mass goal whereas the red region comprises the design solutions that achieve the maximum value for Mass goal. The maximum value achieved for Mass goal is 96.09 Kg while the minimum achieved value is 7.29 Kg. Our interest is now to look for region with lower value of Mass. We now define an acceptable region within the solution space as Mass ≤ 20 Kg identified by the red dashed lines. Any design solutions contained within this region is acceptable for us as it satisfies the requirement for mass.

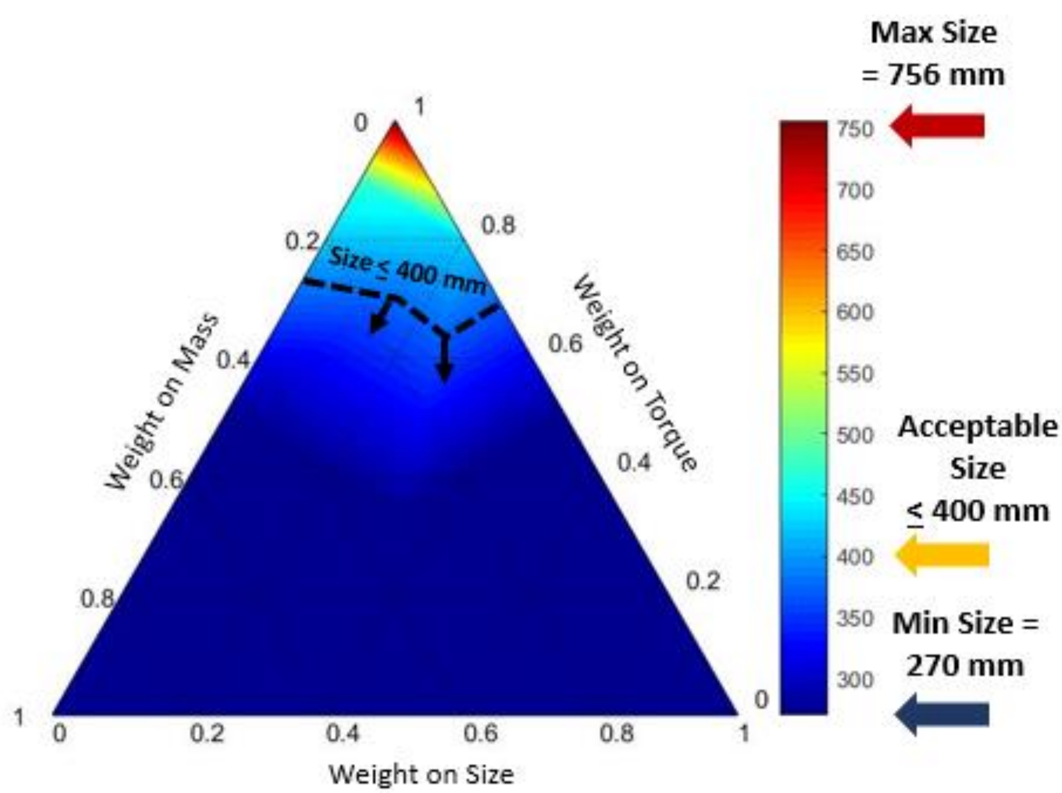


Figure 6.11: Solution Space for Size

As discussed in Chapter 4, we are interested in achieving a lower value for Size goal. For Size goal, our interest is to identify regions where lower values for Size have been achieved. The blue region comprises the design solutions that achieve the lower value for Size goal whereas the red region comprises the design solutions that achieve the maximum value for Size goal. The maximum value achieved for Size goal is 756 mm while the minimum achieved value is 270 mm. Our interest is now to look for region with lower value of Size. We now define an acceptable region within the solution space as Size $\leq$ 400 mm identified by the black dashed lines. Any design solutions contained within this region is acceptable for us as it satisfies the requirement for Size.

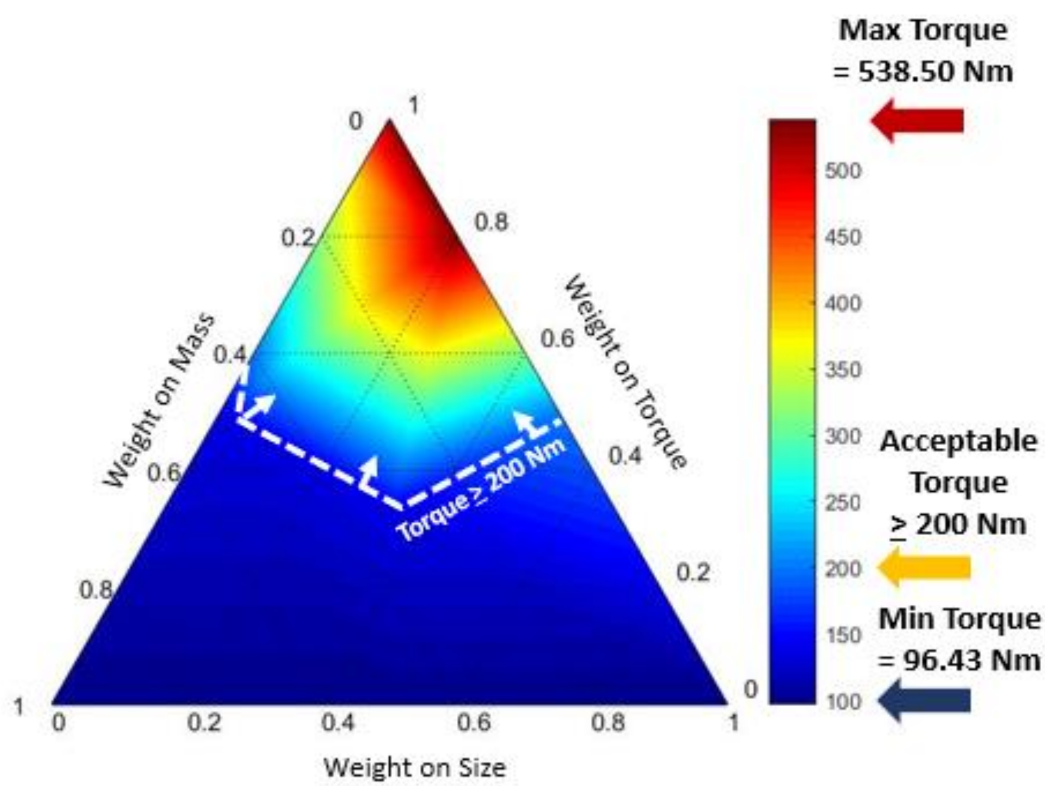


Figure 6.12: Solution Space for Torque

As discussed in Chapter 4, we are interested in achieving a higher value for Torque goal. For Torque goal, our interest is to identify regions where higher values for Torque have been achieved. The blue region comprises the design solutions that achieve the lower value for Torque goal whereas the red region comprises the design solutions that achieve the maximum value for Torque goal. The maximum value achieved for Torque goal is 538.50 Nm while the minimum achieved value is 96.43 Nm. Our interest is now to look for region with higher value of Torque. We now define an acceptable region within the solution space as Torque $\geq$ 200 Nm identified by the white dashed lines. Any design solutions contained within this region is acceptable for us as it satisfies the requirement for Torque.

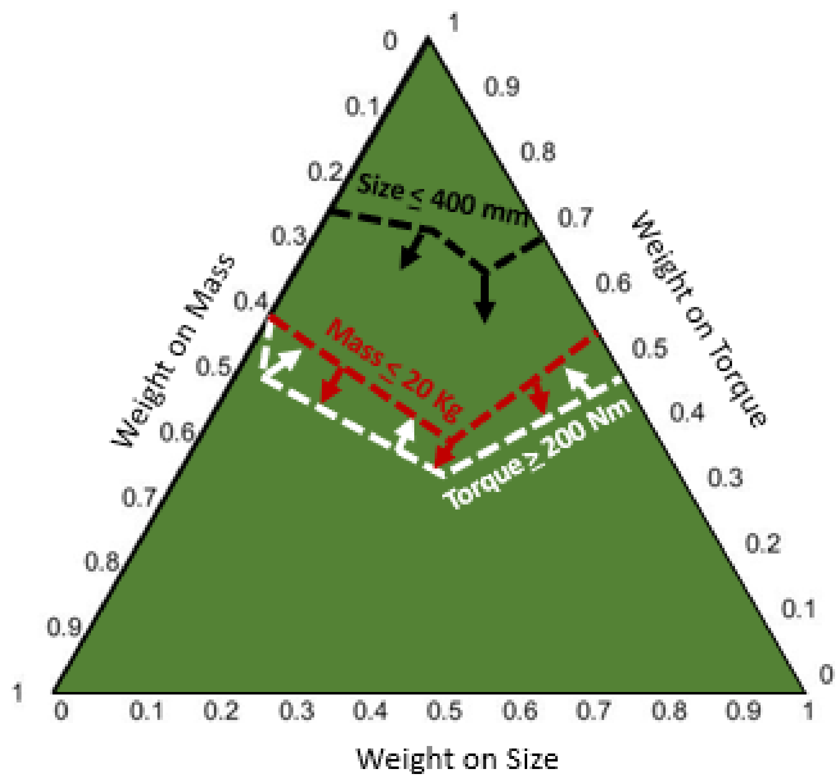


Figure 6.13: Superimposed Satisficing Solution Space

As our interest lies in identifying a satisficing solution region against multiple conflicting goals, we derive a superimposed solution space as discussed earlier and shown in Figure 6.13. The overlap region in Figure 6.13 is our search space for identifying design solutions that meet our conflicting need of minimizing mass and size while maximizing torque. We identify one design solutions (Scenario 5) to lie within the overlap. The design variables corresponding to these robust solutions are tabulated in Table 6.6.

Table 6.6: Design Scenario Selected for Gear

Scenario	Design variables-Gear				Goals		
	m mm	b mm	z	Material	Mass Kg	Size mm	Torque Nm
S5	3	35.38	18	X5	10.74	270	202.42

Note: X5 = AISI 4140 G2

The design solution at Level 1 (coupled cDSP-sDSP shown in Table 6.6) pertaining to gear decisions are coupled to Level 2 (Shaft decisions) functionally. The functional coupling is because of the fact that the torque transmission capability of shafts has to match the torque transmission capability for which the gears have been designed. Following the selection of design scenario for gear, we need to select design variables for shafts that are compatible with the gear thus, designed. The design variables for shaft in Scenario S5 (highlighted in green in Table 6.7) is the shaft design corresponding to the gear designed.

Table 6.7: Goals and Design Variables Achieved for Different Design Scenarios-Shaft

Scenarios	Weights			Design variables-Shaft			Goals	
	Mass	Size	Torque	Di mm	D0 mm	Material MPa	Mass Kg	Torque Nm
S1	1	0	0	20	30	208.212	1.593	96.43
S2	0	1	0	20	30	208.212	1.593	96.43
S3	0	0	1	30.24	49.38	393.750	4.107	526.63
S4	0.5	0.5	0	20	30	208.212	1.593	96.43
S5	0	0.5	0.5	25.83	41.36	214.550	2.913	202.42
S6	0.5	0	0.5	20.55	31.74	303.230	1.751	137.31
S7	0.33	0.33	0.34	20.55	31.74	303.230	1.751	137.31
S8	0	0.2	0.8	31.03	49.25	390.630	4.151	538.50
S9	0.1	0	0.9	29.66	46.81	362.500	3.761	385.15

Decision Scenario 2

Level 1: Selection of gear material with equal weights to all attributes + Compromise of design variables for gear with different weights

Level 2: Compromise of design variables for shaft

Decision Scenario 2 is also solved for 9 different design scenarios. These scenarios are selected based on designer’s aspiration to effectively capture the design space for the exploration of solution space using different combination of weights on goals. The design scenarios and the results for Level 1 decisions (coupled cDSP-sDSP) pertaining to gear decisions are summarized in the Table 6.8.

Table 6.8: Goals and Design Variables Achieved for Different Design Scenarios-Gear

Scenarios	Weights			Design variables-Gear				Goals		
	Mass	Size	Torque	m mm	b mm	z	Material MPa	Mass Kg	Size mm	Torque Nm
S1	1	0	0	3	24	18	X2	7.288	270	96.43
S2	0	1	0	3	24	18	X2	7.288	270	96.43
S3	0	0	1	3.8	30.33	26	X2	30.833	494	282.41
S4	0.5	0.5	0	3	24	18	X2	7.288	270	96.43
S5	0	0.5	0.5	3	34.46	18	X2	10.465	270	138.45
S6	0.5	0	0.5	3	24	18	X2	7.288	270	96.43
S7	0.33	0.33	0.34	3	24	18	X2	7.288	270	96.43
S8	0	0.1	0.9	3.59	30.46	25	X2	25.552	448.75	243.41
S9	0.1	0	0.9	3.95	31.6	18	X2	16.636	355.5	220.10

These design scenarios are selected with an intent to effectively cover the design space for the exploration of solution space using different combination of weights on goals. The different weights assigned to the goals indicate designer’s interest to achieve target set to the goals. Assigning weight as 1 (Scenarios 1, 2 and 3) to a goal means that the designer’s interest is to achieve target set to the goal as closely as possible while ignoring the other goals. For instance, assigning weight w1=1 to Mass would mean that the designer is interested to achieve the target set to mass as closely as possible while not considering the other two goals. Similarly, assigning 0.5 (Scenarios 4, 5 and 6) to two goals means that the designer is equally interested in achieving the target set to the two goals while not considering the third goal. At last, Scenario 7 means that designer is equally interested in achieving the target set to all three goals and so on. With the solutions obtained for all the scenarios, the designer is now most interested in exploring the solution space to obtain solutions that are of prime importance to the decision maker, that is, the designer of the gearbox in the present context. With the information tabulated in Table 6.8, ternary plots for each goal are created. The axes in the ternary plots indicate the weights assigned to each goal while the colored ternary space in the interior indicate the value achieved for that specific goal. For instance, ternary plot for Mass goal shows the value achieved for Mass goal within the ternary space, when different weights are assigned to each goal. Once the ternary plots for the goals are drawn, an acceptable region within each ternary plot is identified. Finally, acceptable regions identified from each ternary plot are superimposed into one plot to explore feasible solution region considering all 3 goals.

The ternary plot for Mass goal is shown in Figure 6.14. As discussed in Chapter 4, we are interested in achieving a lower value for Mass goal. For Mass goal, our interest is to identify regions where lower values for Mass have been achieved. The blue region comprises the design solutions that achieve the lower value for Mass goal whereas the red region comprises the design solutions that achieve the maximum value for Mass goal. The maximum value achieved for Mass goal is 30.83 Kg while the minimum achieved value is 7.29 Kg. Our interest is now to look for region with lower value of Mass. We now define an acceptable region within the solution space as Mass ≤ 20 Kg identified by the red dashed lines. Any design solutions contained within this region is acceptable for us as it satisfies the requirement for mass.

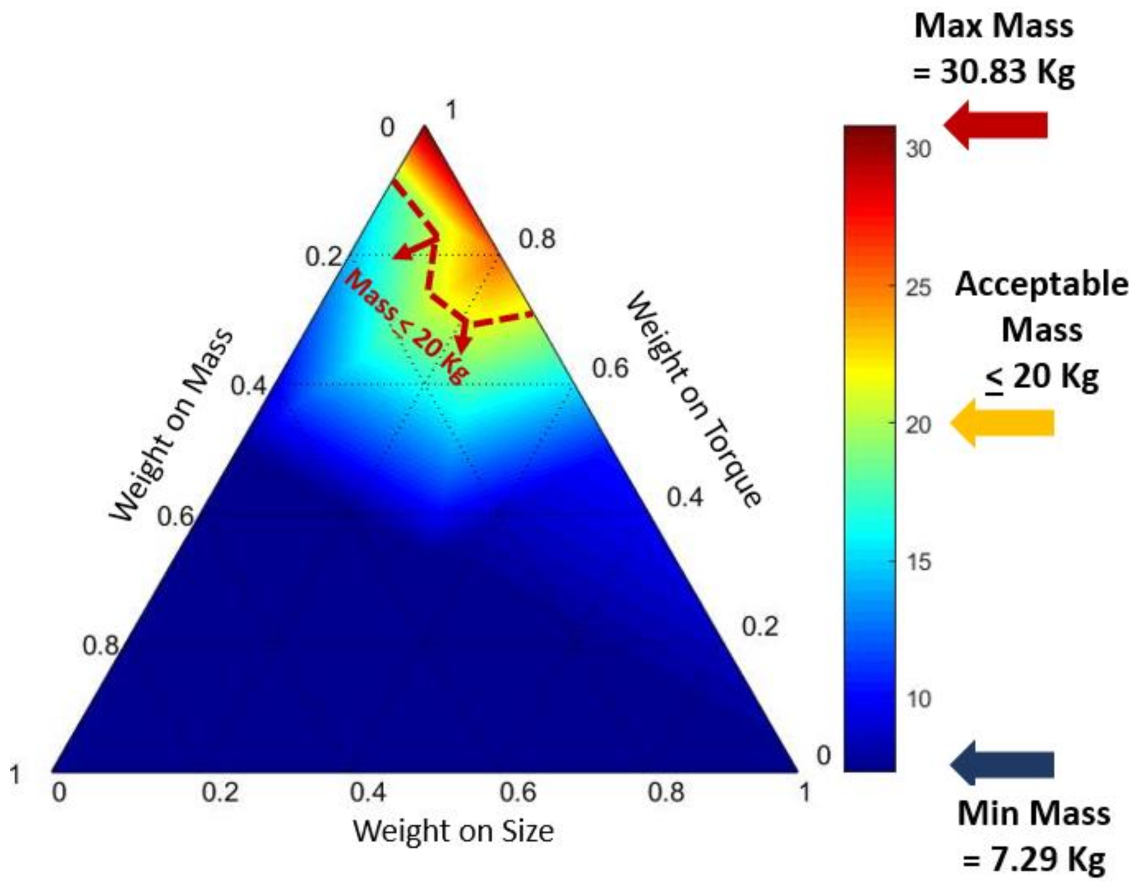


Figure 6.14: Solution Space for Mass

The ternary plot for Size goal is shown in Figure 6.15. As discussed in Chapter 4, we are interested in achieving a lower value for Size goal. For Size goal, our interest is to identify regions where lower values for Size have been achieved. The blue region comprises the design solutions that achieve the lower value for Size goal whereas the red region comprises the design solutions that achieve the maximum value for Size goal. The maximum value achieved for Size goal is 494 mm while the minimum achieved value is 270 mm. Our interest is now to look for region with lower value of Size. We now define an acceptable region within the solution space as Size < 400 mm identified by the black dashed lines. Any design solutions contained within this region is acceptable for us as it satisfies the requirement for Size.

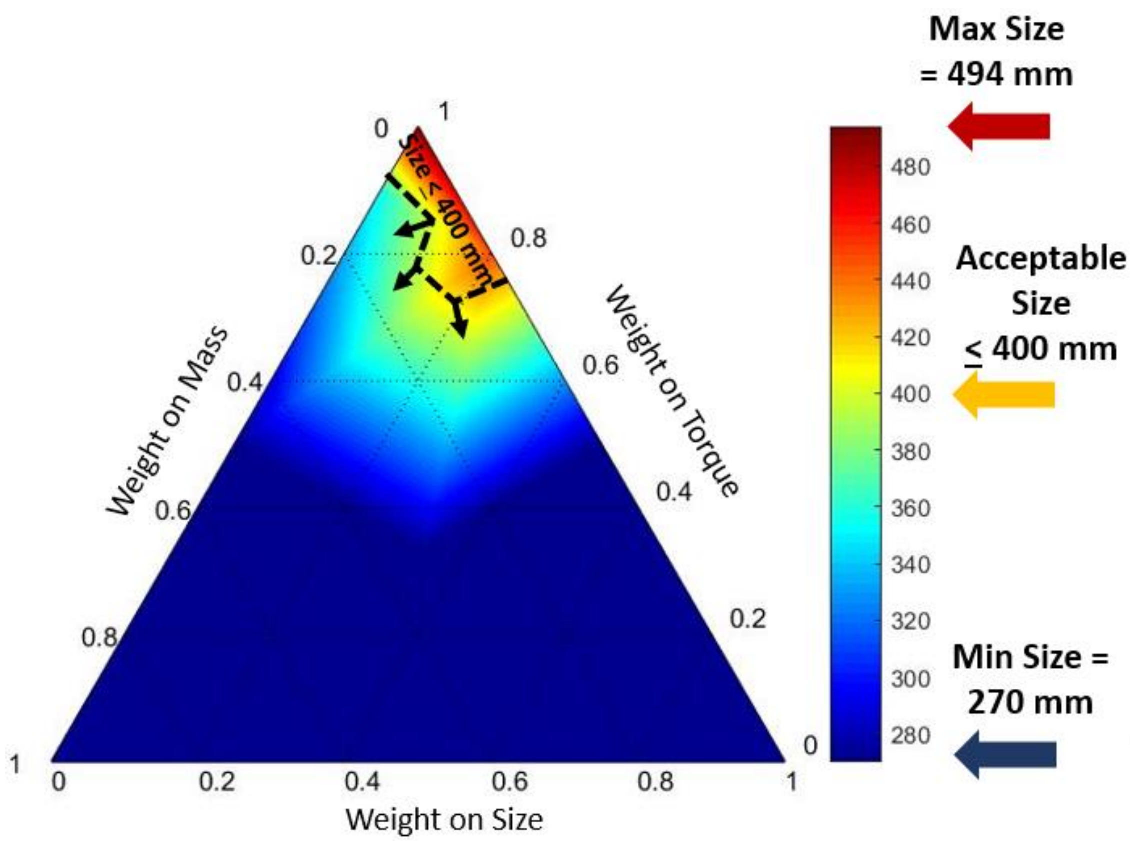


Figure 6.15: Solution Space for Size

The ternary plot for Torque goal is shown in Figure 6.16. As discussed in Chapter 4, we are interested in achieving a higher value for Torque goal. For Torque goal, our interest is to identify regions where higher values for Torque have been achieved. The blue region comprises the design solutions that achieve the lower value for Torque goal whereas the red region comprises the design solutions that achieve the maximum value for Torque goal. The maximum value achieved for Torque goal is 282.41 Nm while the minimum achieved value is 96.43 Nm. Our interest is now to look for region with higher value of Torque. We now define an acceptable region within the solution space as Torque ≥ 200 Nm identified by the white dashed lines. Any design solutions contained within this region is acceptable for us as it satisfies the requirement for Torque.

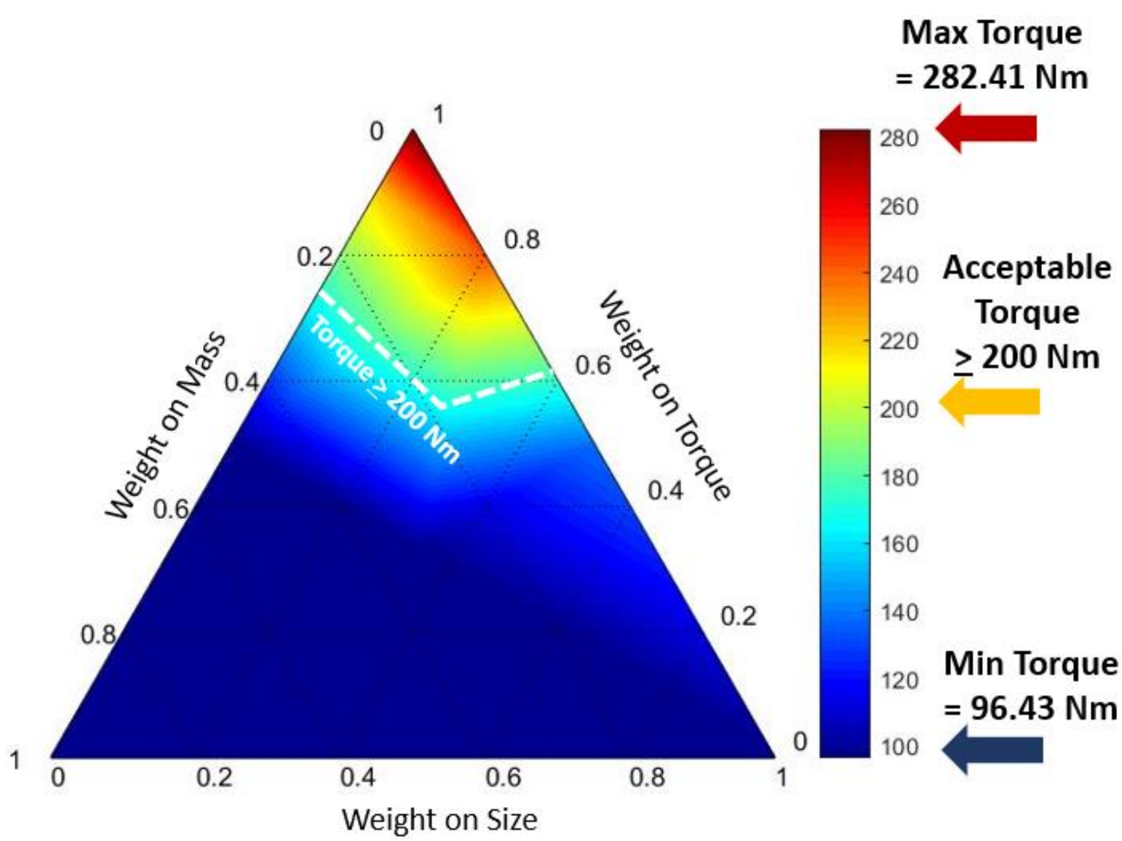


Figure 6.16: Solution Space for Torque

The acceptable solution region identified from all the three individual ternary plots are superimposed in one plot. As our interest lies in identifying a satisficing solution region against multiple conflicting goals, we derive a superimposed solution space as discussed earlier and shown in Figure 6.17. The overlap region in Figure 6.17 is our search space for identifying design solutions that meet our conflicting need of minimizing mass and size while maximizing torque. We identify one design solutions (Scenario 9) to lie within the overlap. The design variables corresponding to these robust solutions are tabulated in Table 6.9.

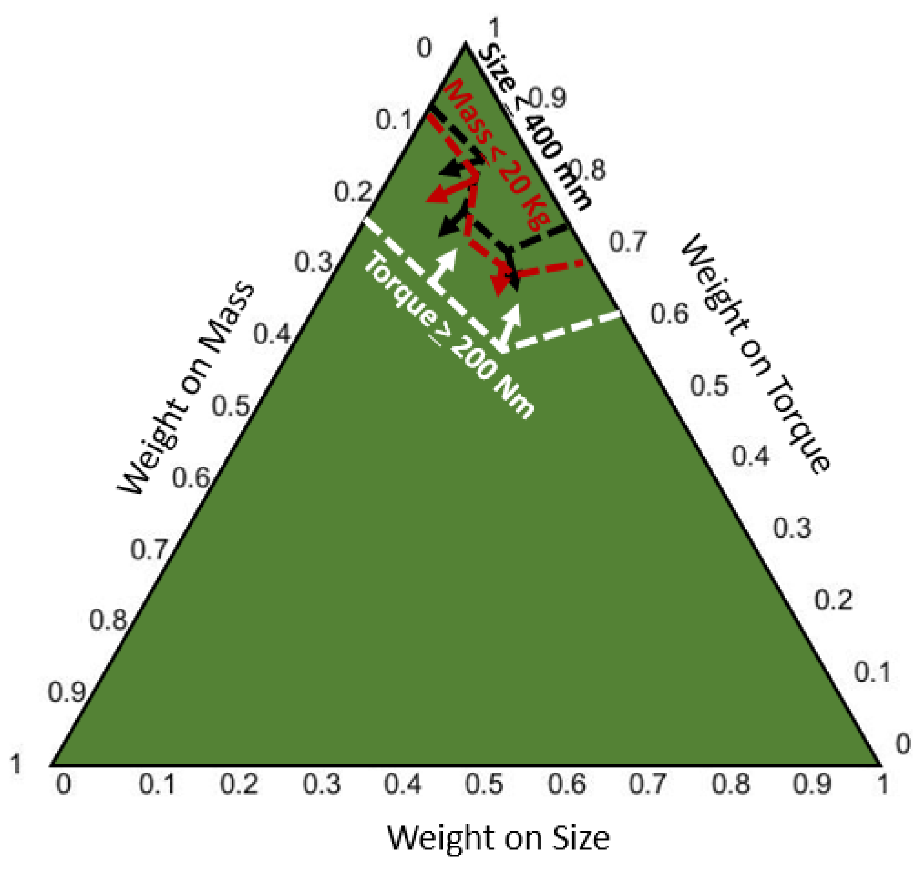


Figure 6.17: Superimposed Satisficing Solution Space

The acceptable solution region identified against each of the goals is kept same as that in Decision Scenario 1. With the idea discussed in Decision Scenario 1, the selected design solution for Scenario 2 is shown in Table 6.9.

Table 6.9: Design Scenario Selected for Gear

Scenarios	Design variables-Gear					Goals		
	m	b	z	T	Material	Mass	Size	Torque
S9	3.95	31.6	18	220.10	X2	16.636	355.5	220.10

Note: X2 = AISI 4140 G1

Table 6.10: Goals and Design Variables Achieved for Different Design Scenarios-Shaft

Scenarios	Weights			Design variables-Shaft			Goals	
	Mass	Size	Torque	Di mm	D0 mm	Material MPa	Mass Kg	Torque Nm
S1	1	0	0	20	30	208.212	1.593	96.43
S2	0	1	0	20	30	208.212	1.593	96.43
S3	0	0	1	31.32	49.72	200.18	4.229	282.41
S4	0.5	0.5	0	20	30	208.212	1.593	96.43
S5	0	0.5	0.5	22.31	35.36	201.027	2.141	138.45
S6	0.5	0	0.5	20	30	208.212	1.593	96.43
S7	0.33	0.33	0.34	20	30	208.212	1.593	96.43
S8	0	0.1	0.9	29.96	47.08	200.775	3.815	243.41
S9	0.1	0	0.9	26.92	42.09	200.387	3.058	220.10

As discussed previously, the decisions (coupled cDSP-sDSP shown in Table 6.9) at Level 1 (coupled cDSP-sDSP) pertaining to gear decisions are coupled to Level 2 (Shaft decisions) functionally. The design variables for shaft in Scenario S9 (highlighted in green in Table 6.10) is the shaft design corresponding to the gear designed.

Decision Scenario 3

Level 1: Selection of gear material with varying weights to all attributes + Compromise of design variables for gear with total weight to torque

Level 2: Compromise of design variables for shaft

Table 6.11: Goals and Design Variables Achieved for Different Scenarios- Gear

Scenarios	Weights				Design variables-Gear				Goals		
	I1	I2	I3	I4	m mm	b mm	z	Material	Mass Kg	Size mm	Torque Nm
S1	1	0	0	0	5.85	59.81	25	X1	133.229	731.25	876.19
S2	0	1	0	0	3.05	24.43	21	X4	10.437	320.25	152.05
S3	0	0	1	0	5.04	40.36	30	X1	96.092	756	526.63
S4	0	0	0	1	5.04	40.36	30	X1	96.092	756	526.63
S5	0.5	0.5	0	0	3.49	30.83	26	X3	26.437	453.7	273.57
S6	0.5	0	0.5	0	5.04	40.36	30	X1	96.092	756	526.63
S7	0.5	0	0	0.5	5.07	40.58	30	X1	97.769	760.5	535.82
S8	0	0.5	0.5	0	3.27	26.14	30	X4	26.199	490.5	267.15
S9	0	0.5	0	0.5	3.39	29.07	25	X3	21.745	423.75	234.02
S10	0	0	0.5	0.5	5.04	40.36	30	X1	96.092	756	526.63

Decision Scenario 3 is solved for 10 different design scenarios. The design scenarios and the results for Level 1 decisions (coupled cDSP-sDSP) pertaining to gear decisions are summarized in the Table 6.11. These scenarios are selected based on designer's aspiration to effectively capture the design space for the exploration of solution space using different combination of weights on selection attributes. Torque goal in compromise DSP was assigned weight equal 1, while ignoring the other two goals in compromise DSP for gears. With the solutions obtained for all the design scenarios, we are now most interested in exploring the solution space to obtain solutions that are of prime importance to the decision maker, that is, the designer of gearbox the in the present context. Various scenarios are generated and presented to the designer. The designer then chooses designs that most fit the designer's aspiration. In the present context, designer's wish is to achieve maximum torque for gears and select material that is durable.

How are spider plots created for solution space exploration?

Spider plot is a two-dimensional form of plot for displaying multivariate data. Each variable has its own axis and all axes are joined in the center of the plot. In Figure 6.18, we have 10 variables as shown by the number on each corner. These variables correspond to the 10 design scenarios (Table 6.11). Each variable takes up a value ranging from 0 to 1 with an interval of 0.2. These value signify the normalized deviation for the goals.

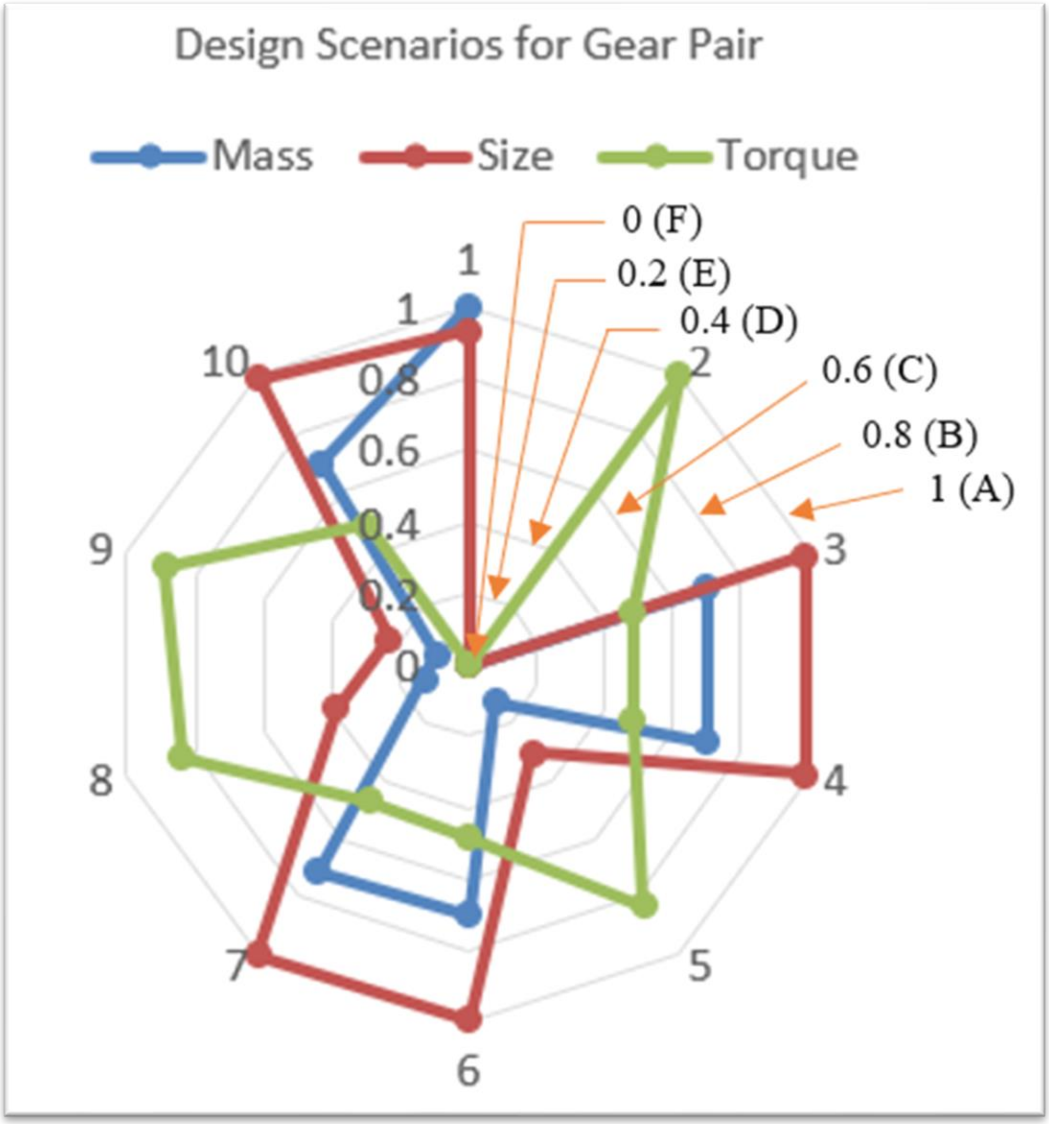


Figure 6.18: Spider Plot for Solution Exploration in Decision Scenario 3

Each corner of the decagon in Figure 6.18 represents the ten design scenarios (Table 6.11). Also, each decagon represents the normalized deviations from the target set for compromise goals with the outermost decagon (A) signifying normalized deviation equal to 1 for the three compromise goals. The decagon second to the outermost decagon (B) signifies normalized deviation equal to 0.8

and so on with center of the decagon signifying normalized deviation equal to 0. Ideally, we want solutions that have 0 deviations and lie nearer to the center of the plot. In Figure 6.18, we find Scenario 5 which have normalized deviation approximately equal to 0.8 has the least deviation (among Scenarios 2, 5, 8 and 9 which are potential design solutions) from torque goal and hence, the highest value achieved for torque goal. Hence, Scenario 5 is the design scenario that closely achieves the torque goal (shown in Table 6.11).

Table 6.12: Goals and Design Variables Achieved for Different Scenarios- Shaft

Scenarios	Weights				Design variables-Shaft			Goals	
	I1	I2	I3	I4	Di mm	D0 mm	Material MPa	Mass Kg	Torque Nm
S1	1	0	0	0	31.11	49.38	390.36	4.173	876.19
S2	0	1	0	0	23.13	36.45	200	2.283	152.04
S3	0	0	1	0	30.24	49.38	393.75	4.107	526.63
S4	0	0	0	1	30.24	49.38	393.75	4.234	526.63
S5	0.5	0.5	0	0	31.37	49.72	221.77	4.107	273.57
S6	0.5	0	0.5	0	30.24	49.38	393.75	4.107	526.63
S7	0.5	0	0	0.5	30.29	49.49	393.75	4.124	535.82
S8	0	0.5	0.5	0	31.19	49.52	200.34	4.196	267.15
S9	0	0.5	0	0.5	30.79	48.89	201.21	4.089	234.02
S10	0	0	0.5	0.5	30.24	49.38	393.75	4.107	526.63

As discussed previously, the decisions (coupled cDSP-sDSP shown in Table 6.11) at Level 1 (coupled cDSP-sDSP) pertaining to gear decisions are coupled to Level 2 (Shaft decisions) functionally. The design variables for shaft in Scenario 5 (highlighted in green in Table 6.12) is the shaft design corresponding to the gear designed.

Table 6.13: Design Goals Achieved for Gears and Shafts in 3 Decision Scenarios

Decision Scenarios	Gear Design Goals			Shaft Design Goals	
	Mass Kg	Size mm	Torque Nm	Mass Kg	Torque Nm
1	10.74	270	202.42	2.91	202.42
2	16.64	355.5	220.10	3.06	220.10
3	26.44	453.7	273.57	4.11	273.57

The design space is explored differently in the three decision scenarios and hence, the results obtained also indicate design solutions that differ from one another (Table 6.13). The least mass is obtained in Decision Scenario 1 where the values attained is 10.74 Kg and 2.91 Kg respectively for the gears and the shafts. Also, the least size equal to 270 mm is obtained in Decision Scenario 1. On the other hand, maximum value for the torque is obtained in Decision Scenario 3 where the value attained is 273.57 Nm (input side of the gearbox) for both the gear and the shaft. The design goal values obtained in Decision Scenario 2 lie in between the design goal values obtained in Decision Scenario 1 and Decision Scenario 3.

6.3 Exploring Solution Space in the Design of Composite Structures

In Chapter 5, the details of the design approach has been discussed. There are two design approaches discussed in the design of composite structure.

Multiscale Approach

Composite material has hierarchical nature as shown in Figure 5.10, that is, the skin and core microstructure (i.e. fiber and matrix in skin and honeycomb in core) influences the macro properties for the sandwich composite. However, material selection in concurrent design approach is carried out using discrete materials mentioned in manufacturer's datasheets. Thus, the approach does not exploit the tailorable nature of composites entirely. Concurrent design solutions can be further improved upon by including this tailorable nature of composites in the design workflow itself. Hence, in this approach, two steps are involved. First, the design space and material space for skin and core are simultaneously explored against the performance requirements by treating skin and core materials as design variables. The second step involves tailoring the microstructures to achieve skin and core properties value achieved in first step and also required for target performance.

Coupled Problem Approach

In this approach, the design problem has been bifurcated as decision making process involving two interacting decisions, that is, selection and compromise decision. Selection decision involves the choice of fiber and matrix combination for design of skin. Compromise decision involves the determination of sizing parameters, that is, the thickness of akin and core material. In this problem, core is considered to have honeycomb structure to be made out of aluminum.

As discussed in Chapter 5, the design of the sandwich composite beam has been considered for 3 load cases. An efficiency factor is defined as ratio of target values to the achieved values $\left(\eta_i = \frac{T_i}{i}\right)_{i=W,\delta}$. In the given test case problem, lower values of weight and deflection are always preferred. The following table (Table 6.14) contains the results for the 3 load case scenarios, that are, LCS1, LCS2 and LCS3 (codes available in Appendix).

Table 6.14: Results for 3 Load Cases

					Efficiency (%)	
Test Problems	Selected Materials	Sizing (mm)	$W\,(N)$	$\delta(mm)$	η_W	η_δ
LCS1	$M\begin{cases} Skin = CL0900 \\ Core = PCGA - XR23003 \end{cases}$	$T1\begin{cases} t_s = 5.03 \\ t_c = 89.16 \end{cases}$	14.98	10.01	93.46	99.93
LCS2	$M\begin{cases} Skin = CL0900 \\ Core = PCGA - XR23003 \end{cases}$	$T2\begin{cases} t_s = 5.13 \\ t_c = 89.75 \end{cases}$	15.36	16.58	91.15	60.31
LCS3	$M\begin{cases} Skin = CL0900 \\ Core = PCGA - XR23003 \end{cases}$	$T3\begin{cases} t_s = 5.16 \\ t_c = 89.68 \end{cases}$	15.42	26.06	90.79	38.37

Table 6.15: Results for 3 Load Cases (Combining Material and Sizing Combination)

A final solution was obtained by selecting material and sizing combination which provides high efficiencies for all the load cases and has been tabulated in the Table 6.15 (highlighted in red).

					Efficiency (%)	
Material	Sizing	Test Problems	$W(N)$	$\delta(mm)$	η_W	η_δ
M	T1	LCS1	14.98	10.01	93.46	99.93
		LCS2	14.98	17.24	93.46	58.00
		LCS3	14.98	27.18	93.46	36.79
M	T2	LCS1	15.36	9.63	91.15	103.80
		LCS2	15.36	16.58	91.15	60.31
		LCS3	14.98	27.18	93.46	36.79
M	T3	LCS1	15.42	9.60	90.79	104.18
		LCS2	15.42	16.52	90.79	60.53
		LCS3	15.42	26.06	90.79	38.37

Multiscale Approach

In this approach, two steps are involved.

1. Design of Structure: In this step, the design space and material space for skin and core are simultaneously explored against the performance requirements by treating skin and core materials as design variables.

2. Design of Microstructure: This step involves tailoring the microstructures to achieve skin and core properties value achieved in first step and also required for target performance.

Design of Structure

Table 6.16: Results for 3 Load Cases - Multiscale Approach

Test Problems	Material $E,\ G: Mpa$ $\rho: (\frac{Kg}{m^3})$	Sizing	$W(N)$	$\delta(mm)$	Efficiency (%) η_W	η_δ
LCS1	$M1 \begin{cases} E_s = 160250 \\ \rho_s = 1595.48 \\ G_c = 150 \\ \rho_c = 24.17 \end{cases}$	$T1 * \begin{cases} t_s = 6.06 \\ t_c = 79.6 \end{cases}$	14.34	9.11	97.63	109.77
LCS2	$M2 \begin{cases} E_s = 202587.34 \\ \rho_s = 1676.17 \\ G_c = 219 \\ \rho_c = 204310 \end{cases}$	$T2 * \begin{cases} t_s = 5.02 \\ t_c = 89.95 \end{cases}$	14.72	10.08	95.11	99.20
LCS3	$M3 \begin{cases} E_s = 204310 \\ \rho_s = 1679.25 \\ G_c = 193 \\ \rho_c = 31.07 \end{cases}$	$T3 * \begin{cases} t_s = 5.36 \\ t_c = 90 \end{cases}$	15.34	15.5	91.26	64.52

As discussed in Chapter 5, the design of the sandwich composite beam has been considered for 3 load cases. An efficiency factor is defined as ratio of target values to the achieved values $\left(\eta_i = \frac{T_i}{i}\right)_{i=W,\delta}$. In the given test case problem, lower values of weight and deflection are always preferred. The table (Table 6.16) contains the results for the 3 load case scenarios, that is, LCS1, LCS2 and LCS3.

A final solution was obtained by selecting material and sizing combination which provides high efficiencies for all the load cases and has been tabulated in the table (Table 6.17) that follows.

Table 6.17: Results for 3 Load Cases - Multiscale Approach (Combining Material and Sizing Combination)

					Efficiency (%)	
Material	Sizing	Test Problems	$W\,(N)$	$\delta\,(mm)$	η_W	η_δ
$\boldsymbol{M1}$	$T1*$	LCS1	14.34	9.11	97.63	109.77
		LCS2	14.34	15.12	97.67	66.14
		LCS3	14.34	24.17	97.63	41.37
$\boldsymbol{M2}$	$T2*$	LCS1	14.72	6.035	95.11	165.70
		LCS2	14.72	10.08	95.11	99.20
		LCS3	14.72	16.08	95.11	62.19
$\boldsymbol{M3}$	$T3*$	LCS1	15.34	5.85	91.25	170.88
		LCS2	15.34	9.68	91.25	103.31
		LCS3	15.34	15.5	91.26	64.52

It can be observed that the solution M3 $(E_s = 204310\ \mathrm{MPa},\ \rho_s = 1679.25\left(\frac{Kg}{m^3}\right),\ G_c = 193.0\ \mathrm{MPa}\ \rho_c = 31.07\left(\frac{Kg}{m^3}\right),\ T3*(t_s = 5.36\ \mathrm{mm},\ t_c = 90\ \mathrm{mm})$ yields the best efficiency of deflection for all load cases.

Design of Microstructure

In this step, our intention is to tailor skin and core microstructure in such a way that we are able to design skin and core materials to extract the material properties (density and modulus) obtained in the first step.

Skin Microstructure

The microstructure is chosen such that it also satisfies the non-functional requirements for given problem. Thus, the target values for skin and core namely $E_s^t = 204309\ MPa,\ \rho_s^t = 1679\ \frac{Kg}{m^3}$ and $G_c^t = 193\ MPa,\ \rho_c^t = 31\ \frac{Kg}{m^3}$ are sought. The one or more microstructures that yield $E_s \geq E_s^t$ and $\rho_s \leq \rho_s^t$ are chosen as suitable microstructures. In this problem, the functional and non-functional requirements for skins are only achieved by unidirectional fiber reinforced composites as it yields high longitudinal specific stiffness as compared to the biaxial and woven composite. The suitable lamina and its constituent are shown in Table 6.18.

Table 6.18: Selection of Skin Microstructure

V_f	$E_s(GPa)$	$\rho_s(\frac{Kg}{m^3})$	$\frac{E_s(GPa)}{\rho_s(\frac{Kg}{m^3})}$	Fiber (Carbon)	Matrix (Epoxy)
70	204	1641	0.125	IM7	3501-6

Core Microstructure

The microstructures that yield $G_C \geq G_c^t$ and $\rho_C \leq \rho_c^t$ are chosen as suitable microstructures. The functional and non-functional requirements for the core are only achieved by aluminum honeycomb as it offers high specific shear stiffness as compared to the open and closed cell foams. The obtained core microstructures are shown in Table 6.19.

Table 6.19: Selection of Core Microstructure

$t(mm)$	$h(mm)$	θ°	$G_c(MPa)$	$\rho_c(\frac{Kg}{m^3})$	$\frac{G_c(MPa)}{\rho_c(\frac{Kg}{m^3})}$
0.11	**23**	**30**	**232**	**27.68**	**8.383**
0.01	2	30	242	28.94	8.382
0.11	22	30	242	28.94	8.382
0.11	24	30	222	26.53	8.381
0.11	25	30	213	25.47	8.381
0.11	17	45	203	28.95	7.014

The microstructure having best specific shear stiffness $(\frac{G_C}{\rho_C})$ is selected.

The design of a sandwich composite beam is carried out using concurrent design approach and multiscale design approach. Design efficiency (η) showing the achievement of target values are computed for each approach. A unique set of material and thicknesses were selected as final solution that achieves better overall efficiencies for all the load cases For the combined loadings (e.g., bending and torsion) multiscale approach has a potential to evolve to find the suitable microstructure such as braided composite or laminated composite with varying stacking sequences. The multiscale approach shows higher design efficiencies as compared to the concurrent design approach. The approach explores large design space to achieve best performance efficiencies. In composite structures, failure is governed by local microstructure behavior, this can also be incorporated in the multiscale approach as a design criterion while obtaining the suitable microstructure. Manufactures can use this method to serve designers better by creating new materials, as the former approach has limited selection options.

6.4 Exploring Robust Solution Space in the Design of Composite Structures

In this section, results pertaining to the robust design of a composite structure is presented. A test case of designing a sandwich composite cantilever beam is performed. The design problem involves sizing and material selection for skin and core of a sandwich composite beam based on the requirements and constraints while including the material and structural uncertainties. The design approach follows two steps. First, the design space and material space for skin and core are simultaneously explored against the performance requirements. In addition to the performance requirement, design exploration is carried out by putting an emphasis on the mitigation of impact on performance due to perturbation in dimensions and properties of skin and core. The second step involves tailoring the microstructures to achieve skin and core properties required for target performance. In this step, the mitigation of impact on skin and core properties due to perturbation in microstructural parameters is also considered. Each step is formulated as a compromise Decision Support Problem (cDSP).

Robust Design of Structure (cDSP1)

The system variables and respective variability considered in this step is in Table 6.20.

Table 6.20: Design Variables Corresponding to Design of Sandwich Beam Structure

S.N.	Design variables (X)	Variability (Δx)
(1)	X1, Skin thickness (Tf)	$\pm$ 0.2 mm
(2)	X2, Density of skin (Rs)	$\pm$ 4.0 kg/m^3
(3)	X3, Elastic Modulus of skin (Es)	$\pm$ 5.0 Mpa
(4)	X4, Core thickness (Tc)	$\pm$ 0.2 mm
(5)	X5, Density of core (Rc)	$\pm$ 1.0 kg/m^3
(6)	X6, Shear modulus of core (Gc)	$\pm$ 5.0 Mpa

The sandwich composite beam is designed to achieve target set to the maximum beam deflection and weight such that the effect of change in design variables on beam deflection and weight is mitigated. 5 different scenarios were considered, the results of which are tabulated in Table 6.21.

Table 6.21: Design Scenarios Corresponding to Design of Sandwich Beam Structure

Scenarios	Weight		Design Variables						Goal Achieved		Performance	
	Deflection	Mass	Tf mm	Tc mm	Es Mpa	Gc Mpa	Rs Kg/m^3	Rc Kg/m^3	DCI 1	DCI 2	Deflection mm	Weight N
S1	1	0	14.96	70	134816	530.57	1406	3.4	7.236	16.096	0.848	31.09
S2	0.25	0.75	14.84	70	94060	512.46	1406	3.4	3.401	16.565	1.043	30.85
S3	0.5	0.5	15	70	94060	463.03	1406	3.4	3.621	15.939	1.086	31.18
S4	0.75	0.25	12.73	70	94060	365.6	1406	3.4	1.344	25.019	1.341	26.49
S5	1	0	12.72	70	94060	464.18	1406	3.4	1.345	25.060	1.191	26.46

Scenario S3 has been selected as an acceptable solution in cDSP formulated for Design of Structure. With the solutions obtained in S3, cDSP for Design of Microstructure will be solved.

Robust Design of Microstructure (cDSP2)

The system variables and respective variability considered in this step is tabulated Table 6.22.

Table 6.22: Design Variables Corresponding to Design of Skin and Core Microstructure

S.N.	Design variables (X)	Variability (Δx)
(1)	X1, Volumetric fraction (Vf)	$\pm$ 0.05
(2)	X2, Wall angle (Θ)	$\pm$ 0.3 ^{0}
(3)	X3, Wall length (h)	$\pm$ 0.3 mm
(4)	X4, Wall thickness (t)	$\pm$ 0.01 mm

The skin and core microstructures are to be designed to achieve the properties obtained in Robust Design of Structure (Scenario S3) such that the effect of change in microstructural design variables on those properties are mitigated. 12 different scenarios were considered, the results of which are tabulated in Table 6.23.

Table 6.23: Design Scenarios Corresponding to Design of Skin and Core Microstructure

Scenarios	Weights				Design variables				Goal Achieved			
	Ds	Es	Dc	Gc	Vf	Θ deg.	H mm	T mm	Rs Kg/m^3	Es Mpa	Rc Kg/m^3	Gc Mpa
1	1	0	0	0	0.428	30	2	0.02	1485.44	119604	57.88	487.67
2	0	1	0	0	0.622	30	2	0.02	1578.56	157046	57.88	487.67
3	0	0	1	0	0.483	30	2.57	0.027	1511.84	130219	60.83	512.53
4	0	0	0	1	0.458	30	2.11	0.028	1499.84	125394	76.95	648.37
5	0.5	0	0.5	0	0.483	30	2.57	0.027	1511.84	130219	60.83	512.53
6	0	0.5	0.5	0	0.483	30	2.15	0.026	1511.84	130219	69.93	589.19
7	0.5	0	0	0.5	0.635	30	2.73	0.029	1584.8	159555	61.59	518.99
8	0	0.5	0	0.5	0.635	30	2.12	0.027	1584.8	159555	73.61	620.21
9	0	0	0.5	0.5	0.402	30	2	0.019	1472.96	114586	54.98	463.28
10	0.25	0.25	0.25	0.25	0.644	30	2	0.019	1589.12	161292	54.98	463.28
11	0	0	0.25	0.75	0.479	30	2.35	0.03	1509.92	129447	74.01	623.62
12	0	0	0.75	0.25	0.402	30	2	0.019	1472.96	114586	54.98	463.28

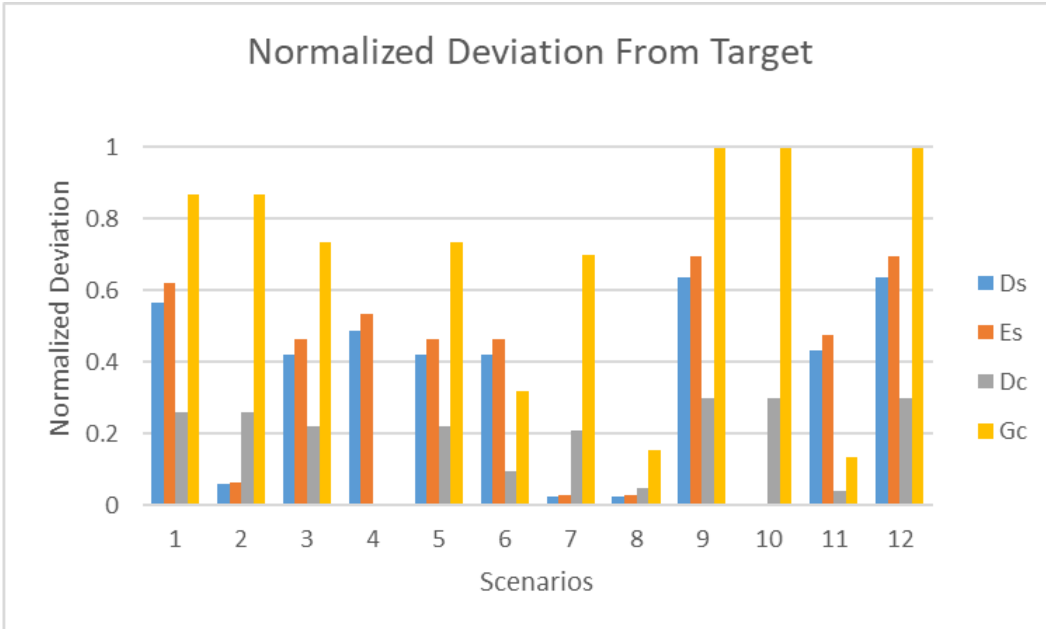


Figure 6.19: Normalized Deviations of Skin and Core Properties

Scenario 9 has been selected as an acceptable solution in cDSP formulated for Design of microstructure as Scenario 9 achieves the target set more closely. The normalized deviation plot (Figure 6.19) has been shown to see which scenario is closer to the target set. The value 1 represents that the solution is nearer to the target set. The target set for properties are the values for skin and core properties obtained in the Design of Structure (Scenario S3).

The following table (Table 6.24) contains the design variables (both at structural and microstructural level) that have been selected for the robust design of a sandwich composite beam for the chosen design problem.

Table 6.24: Design Solution Corresponding to Design of Structure, Skin Microstructure and, Core Microstructure

Design of composite beam		Design of microstructure	
Skin thickness	15 mm	Volumetric fraction	0.402
Core thickness	70 mm	Wall angle	30^0
Elastic modulus (skin)	114586 Mpa	Wall length	2 mm
Density (skin)	1472.96 Kg/m^3	Wall thickness	0.019 mm
Shear modulus (core)	463.28 Mpa		
Density (core)	54.98 Kg/m^3		

What has been demonstrated in the design of composite structure?

- Designing the target material properties to achieve the desired performance objectives of the composite structure.
- Determining the minimum set of material properties that can be used to achieve robust performance.
- Determining the structural integrity by exploring various combination of skin and core materials.
- Tailoring the skin properties by exploring different combination of fiber and matrix for various volumetric fraction.

- Tailoring the core properties by exploring different combination of material and design configuration.
- Designing a robust manufacturing process for composite structures.
- Developing a robust design strategy for composite structures.
- Evaluating the tradeoff between stronger materials vs. higher dimensions of skin and core material.
- Determining the lowest cost strategy for achieving the desired objectives. Evaluating how the increased cost of stronger materials compare with reduced cost of other materials.

Through this study, designer's, manufacturer's and firms working in composite structures will be able to demonstrate

- How robust design strategy for composite structures can be realized and implemented.
- How product design can leverage advances in modeling and simulation of composite materials and manufacturing processes.
- How the available pool of materials can be combined to design composite structures for various applications.
- How products can be made cheaper, lighter, and cost efficient using composite materials.

– How the design exploration platform can be created and utilized for realistic design of composite structures under uncertainty and conflicting requirements.

6.5 Building Confidence in the Results

To build confidence in the results that have been presented. A convergence plot is drawn for design Scenario 1 involving the design of fender. The convergence plot tracks the deviation in goals at each iteration. Three convergence plots are drawn for the design scenario presented in Table 6.1 for different start values. The values of the design variables are given different start values and the deviation variable is tracked at each iteration. The plots with three start values for design variables (lower, middle and upper) are shown in Figure 6.20, Figure 6.21 and, Figure 6.22 respectively.

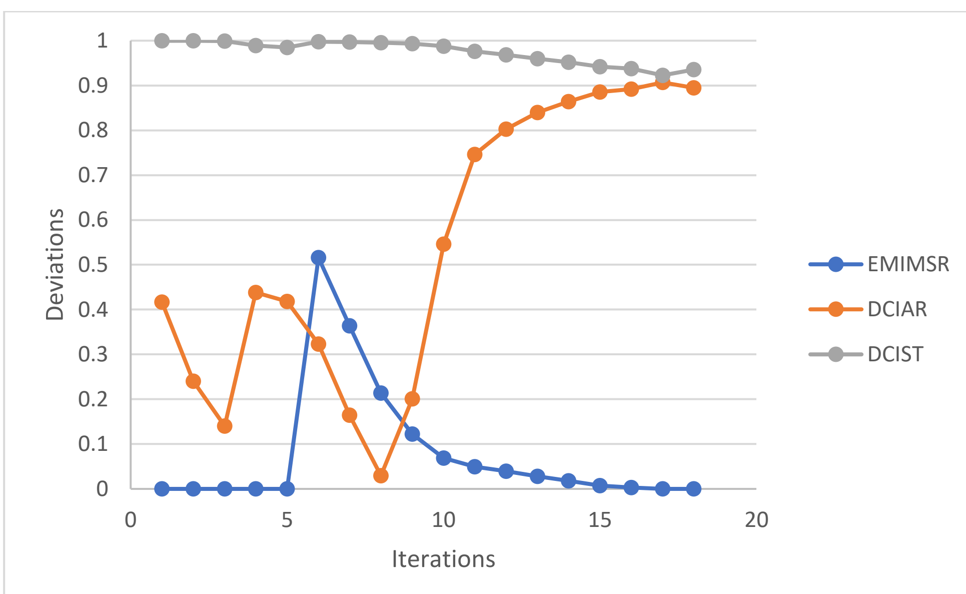


Figure 6.20: Deviation Plotted Against Iteration with Start Value

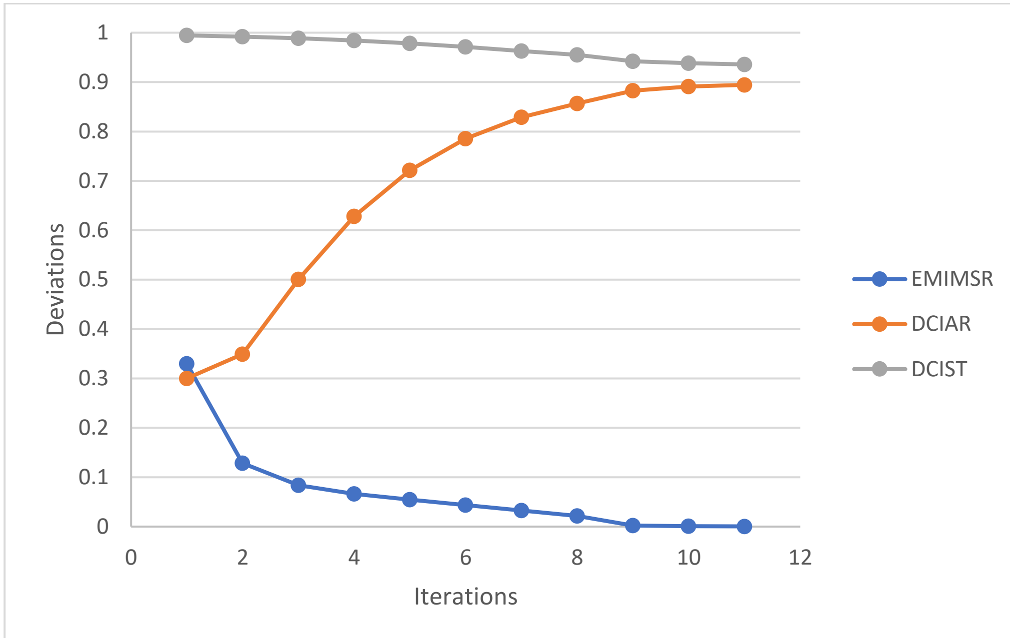


Figure 6.21: Deviation Plotted Against Iteration with Middle Value

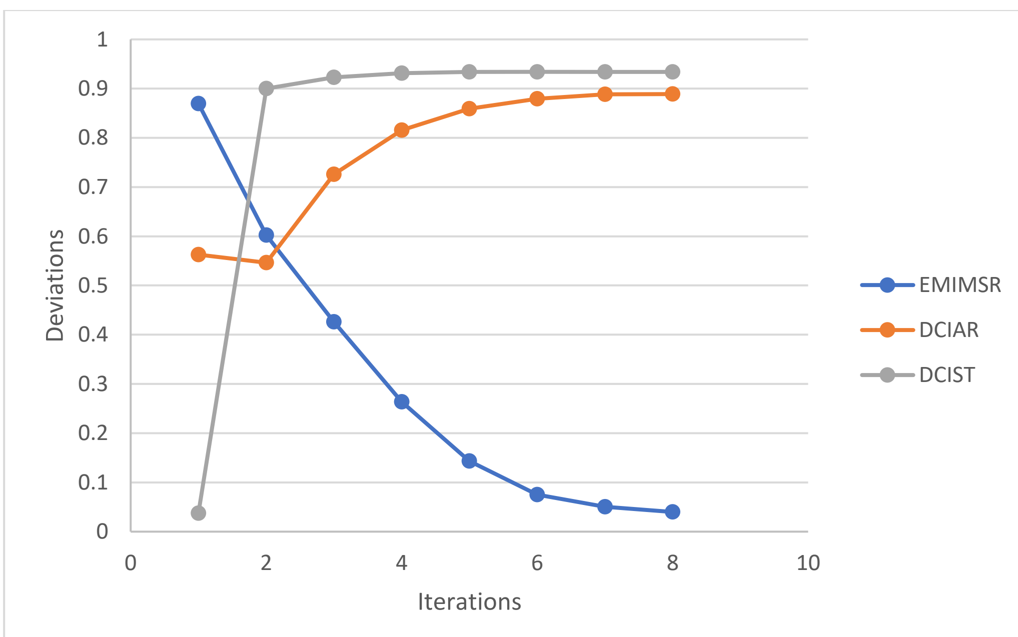


Figure 6.22: Deviation Plotted Against Iteration with End Value

In Figure 6.20, Figure 6.21 and, Figure 6.22, typical convergence of deviation is shown. It is observed that the deviation achieved for each goal settles at the same value irrespective of start value of design variables. The convergence plot is similar for the other scenarios presented in Table 6.1. Having the deviation converged, we gain confidence in the results.

6.6 Answering Research Questions Through Test Problems

6.6.1 *Design of Fender*

Decision Scenario Matrix (DSM) for classifying coupled decisions using DSPs is presented in context of designing a fender. Also, presented is an approach for modeling decision interaction among decisions that are represented using concurrency. Also, presented is an approach for addressing the issue of uncertainty involved in coupled design problems. In terms of managing uncertainty, what has been shown is summarized as:

(i) Robustness against variability in performance due to uncertainty: Our expectation about how the design should perform becomes more accurate if we can identify and manage sources that alter our expectation. It is crucial to identify the variabilities that can impact design performances. In this context, applying EMI to the performance requirement can be effective in managing uncertainty stemming due to variability in design variables and material properties. In the example presented, uncertainty in design variables and material properties have been considered. The aim is not to eliminate all possible deviation in the goals but to minimize any such deviation in goals because of variability emerging from change in our expectation about design variables and material properties. Similarly, DCI have been applied to other two goals , that is, Aspect Ratio (AR) and Stiffness (ST) as the deviations in these goals is expected to occur because of change in the value of design variables.

(ii) Robust solution exploration by treating material properties as system variable: One of the biggest benefits by treating material as a variable is that the design space get enlarged, allowing to have a larger search space with a possibility of finding better quality designs without compromising the performances. In recent years, tremendous research effort has been put on designing material that empowers us to choose materials with properties beyond the standard set.

(iii) Robust solution exploration involving concurrent selection – compromise decision: In case of a material selection the use of selection DSP seems appropriate. Compromise DSPs are more appropriate to determine design variables against multiple conflicting goals. When a decision must been taken when selection and compromise decision are interrelated, coupled DSPs are most appropriate. Here, example is used to demonstrate exploration of robust solutions for a coupled selection-compromise decision.

6.6.2 *Design of Gearbox*

Multi-leveled Decision Scenario Matrix (MDSM) that enables the representation of complex system as set of interacting decisions using DSPs is presented. Also, presented is an approach for modeling such interactions among decisions that are represented using hierarchy and concurrency. Multi-leveled Decision Scenario Matrix (MDSM) for classifying coupled decisions involving concurrency and hierarchy is presented to showcase how MDSM can be applied for representing the design process involved in the design complex engineered system. The notion of horizontal and vertical coupling is introduced to model concurrency and hierarchy, respectively. Further, an approach and mathematics for representing and modeling multiple interacting decisions in the design of a complex engineered system is shown. A test problem involving the design of one-stage reduction gearbox is used to demonstrate the aforementioned claims. The design problem is represented by a set of 3 decisions involving concurrency and hierarchy. Compromise and selection decisions pertaining to the design of gears involve concurrency and lie at the top of the hierarchy. Consequently, decisions pertaining to the design of shafts follow the concurrent decisions and at lie at the bottom of the hierarchy. For the exploration of the design space, 3 different decision scenarios are created. By varying the weights assigned to the goals or attributes in each of the decision scenarios, the solution to multiple design scenarios within each decision scenarios are generated using DSIDES.

6.6.3 *Design of Composite Structures*

A methodology to design composite structures subjected to multiple design loads under various boundary conditions using coupled design approach is presented. Decision Scenario Matrix (DSM) for classifying coupled decisions using DSPs is presented in context of designing composite structures. Also, presented is an approach for modeling decision interaction among decisions that are represented using concurrency. By presenting an approach for addressing the issue of uncertainty involved in coupled design problems, the validation to uncertainty managing technique for coupled problems is validated. Mathematics for modeling interaction and managing uncertainties are presented and validated by designing a composite structure and consequently, microstructures.

6.7 Knowledge Management in the Design of Engineered Systems

The archival of engineering knowledge is critical for supporting the reuse of the knowledge put in designing engineered systems. In context of coupled systems, where there are numerous interacting decisions and can be represented using the elements from Multi-level Decision Scenarios Matrix (MDSM), creating knowledge to capture the decision interaction is vital. Decision interactions are the "glue" to connect different decisions and reach the shared design output. Modeling these interactions is critical to enable the planning of flexible design decision workflows and to explore the design space. One of the challenges in modeling decision interactions is that one must take different decision types into account. In engineering design, a decision can be a choice among multiple alternatives such as design concepts, structures, and materials, etc., it can also be the determination of the values for a set of design variables such as the dimension of a product, the process parameters of a manufacturing system. Through gearbox design example, ontology for representing knowledge of decision interaction in decision-based design is shown in (Ming, Sharma et al. 2020). In the paper, two horizontal interaction patterns, namely, the strong compromise-compromise and strong selection-compromise patterns, are used in formulating the coupling of decisions in gearbox design.

As engineering enterprises are increasingly concerned with meeting the dynamic requirements of the global market and reducing the time for bringing products to the market, closer attention must be paid to the design process. A decision-based design process is embodied by a workflow of decisions that are connected (or interconnected) to generate shared and desired outputs. Carefully designing or planning decision workflows at early design stages is critical for enterprises to produce quality designs and meet the changing requirements. One of the challenges in designing decision workflows is that the decision workflows for the design of complex engineered systems usually involves different types of decisions which are made at multiple levels in a hierarchy and decisions are interacting vertically and horizontally. There is a need for a tool to facilitate designers designing and executing complex decision workflows in the exploration of the solution space at early design stages. This can be addressed by designing a template-based method for the design and execution of decision workflows in the design of engineered systems. The method is based on three basic templates which represent the building blocks of decisions workflows: the compromise Decision Support Problem (cDSP) template, the selection Decision Support Problem (sDSP) template, and the interaction template. Advantages of the method are anchored in that it enables the flexibility, reusability, and executability of decision workflows at early design stages.

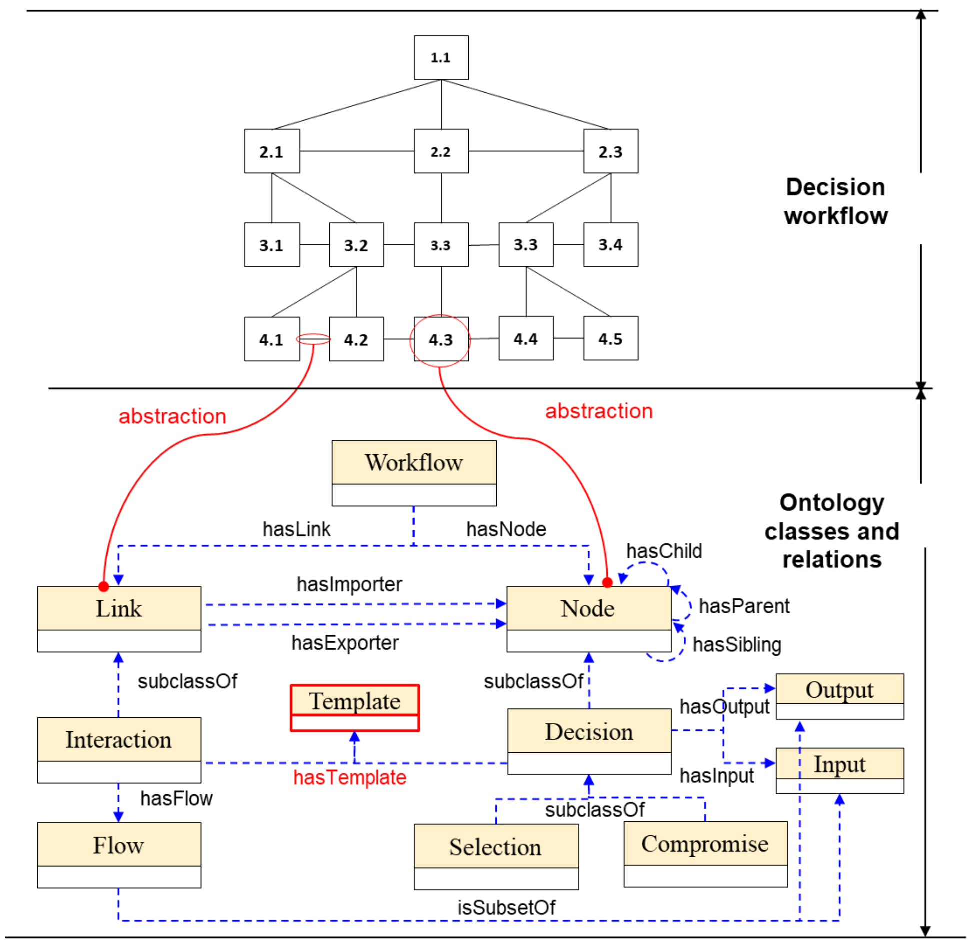


Figure 6.23: An Ontology for Integration Of Decision Workflow Building Blocks

In Figure 6.23, an ontology represent the decision workflows corresponding to the design of complex hierarchical systems is shown. There are two layers in it. The top layer is a decision workflow to be modeled, which reflects the design process of a multilevel hierarchical system with both vertical and horizontal dependencies between subsystems. The bottom layer is the information model, namely, the ontology that represent the decision workflow.

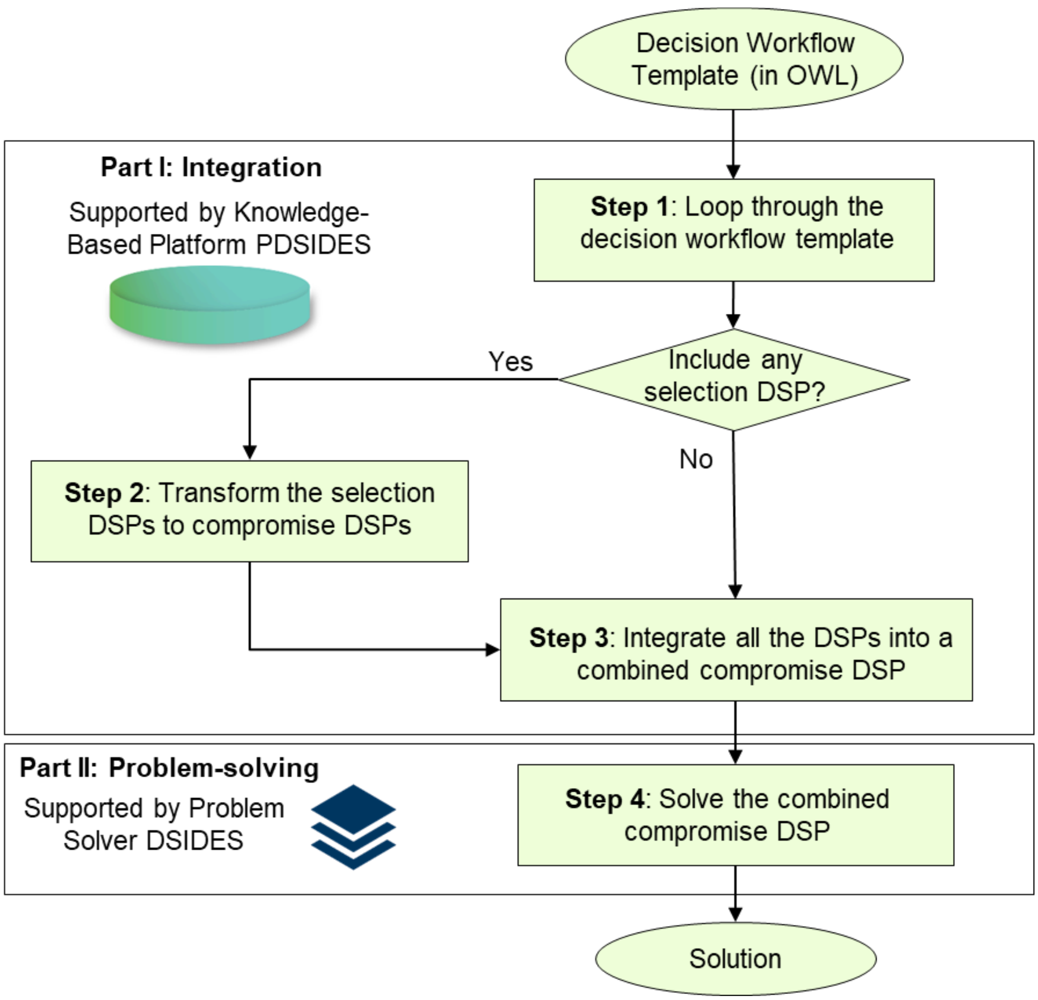


Figure 6.24: Procedure for Execution Of Decision Workflow Templates

In the ontology, Class *Workflow* is the overall abstraction of the decision workflow on the top layer, and Classes *Link* and *Node* are the abstractions of the two basic elements of the decision workflow. The *Workflow* class is related to its element by Relation *hasLink* and has *Node*. To connect to other nodes and form a hierarchical workflow, Class Node is referred to itself by three relations – *hasChild*, *hasParent*, and *hasSibling*, wherein the first wo are essential for vertical interactions and the third is essential for horizontal interactions. Class *Link* is related to Class Node by two object properties – *hasImporter* and *hasExporter*, which capture the direction of information flow on a specific link. Classes *Interaction* and *Decision* are the subclasses of *Link* and *Node* respectively, and both are related (through Relation *hasTemplate*) to Class *Template,* of which the instance structures are specified in Section 3.1 as sDSP template, cDSP template, and interaction template. Class *Interaction* inherits the properties of *hasImporter* and *hasExporter* from its superclass *Link* and is related (through Relation *hasFlow*) to Class *Flow* which captures the information content flows from a decision to another. Through Relation *isSubsetOf*, Class *Flow* is related to Classes *Input* and *Output* (which are properties of Class *Decision*). This is consistent with the fact that a portion of (critical, not all) information is flowing from one decision to another in decision interaction patterns. All the classes and relations of the ontology shown in Figure 6.23 are formally defined using web ontology language (OWL) and are implemented in platform PDSIDES as the knowledge representation scheme for decision workflows. By the ontology, the building blocks are integrated in a semantic and computational environment and form the basis for the composition and execution of decision workflows. In Figure 6.24, the procedure for the execution of decision workflow templates is shown. The details of the work is published and available in (Ming, Sharma et al. 2019).

6.8 Summary of Chapter 6

In this chapter, the results pertaining to the math formulations derived in Chapter 3, Chapter 4 and Chapter 5 are presented. For each test problem, the design solutions are explored, and the results are discussed in detail. Following which, critical evaluation is made in terms of how well the research questions have been answered. Finally, the development of design templates and ontology for archiving engineering knowledge put in designing coupled engineered systems is discussed.

Chapter 7: Closure

Having discussed the elements in the design of coupled engineered systems in previous chapters, the research questions are revisited and discussion on the research questions and hypotheses are made in Chapter 7. The discussion is on contributions made in terms of creating new knowledge in designing coupled engineered systems. The initial section of this chapter contains the summary of the work. It is done in Section 7.1. In Section 7.2, the relevant contributions made and the extent to which the objectives of the work has been achieved is discussed. This will also concentrate on highlighting the answers to the research questions. In Section 7.3 and Section 7.4, the discussion is about the way forward and the future research directions. To conclude the chapter, I-statement is presented at Section 7.5.

7.1 Summary of the Thesis

In this thesis, coupled decisions in the design of engineered systems is dealt. The design solutions are an accumulation of number of design decisions. These design decisions have an influence on one another. Changing one of these decisions is likely to impact other decisions. This is to say that when dealing with the design of engineered systems, coupling in decisions is inevitable. In this thesis, the foundational design perspective is the Decision-Based Design (DBD). One fundamental demonstration of the decision-based design construct is the Decision Support Problem Technique (DSPT). By resting on the premises of Decision

Support Problem Technique (DSPT), the two major decisions in the design of engineered systems, that is, selection decision and compromise decision are identified and classified. In DSPT, all engineering decisions are categorized as selection, compromise, or a combination of these decisions. When there exists an interaction among these decisions in the given engineered system, the engineered system is referred to as coupled engineered system and the corresponding decisions as coupled decisions. These coupled decisions have different interaction strengths and can occur across various levels. Besides, these decisions are open to various kind of uncertainties. Our assertion in this thesis is that the capability in design method to address decision coupling and simultaneously managing the impact of various uncertainties pertaining the design decisions will improve the quality of design decisions. In this thesis, a computational framework adoptable in a coupled and uncertain design environment is presented and demonstrated.

In Chapter 1, a foundation for the thesis is established. The need to address the decision coupling and robust decision making in design of engineered systems is established. Also, the suitability of Decision Support Problem Technique (DSPT) for modeling decisions as DSPs is discussed. The creation and utility of Multi-leveled Decision Scenario Matrix (MDSM) is explained. Finally, the scope of the work, including the research questions posed, hypothesis proposed, and the boundary of the present work is detailed.

Chapter 2 of this thesis contains the detailed discussion about all the tools, techniques, formulation and mathematical framework that will be applied in this work. In particular, the discussion is on coupled decision, robustness, compromise Decision Support Problem (DSP) construct, Design Capability Index (DCI) and Error Margin Index (EMI). This chapter details the fundamental mathematical foundations to be used in Chapter 3, 4 and 5.

In Chapter 3, first demonstrative instance of a coupled design problem is introduced. The coupling in decision in the design of a fender is discussed. The mathematical formulations for solving the fender design problem as (i) a coupled problem approach and, (ii) material design approach is detailed. Following this, mathematical formulations for addressing uncertainties pertaining to the design of fender as a coupled decision problem is presented.

In Chapter 4, design decision making in the design of a gearbox is introduced as a multi-leveled coupled decision problem. This is followed by the DSP based mathematical formulations for solving a multi-level coupled design problem.

In Chapter 5, the overall picture of decision problem in the design of composite structures is presented. First, the DSP based mathematical formulations for the design of composite structures as (i) a coupled problem approach and, (ii) multiscale approach is presented. Following this, the DSP based mathematical formulations for the robust design of composite structures as multiscale approach is presented.

In Chapter 6, the results obtained in Chapter 3, 4 and 5 are respectively presented and discussed. The results pertaining to each mathematical formulations in Chapter 3, 4 and 5 are presented and details regarding the solution exploration approach is discussed. In detail, the discussion about the validity and usefulness of the method is outlined.

In this chapter, a summary of this thesis is given at first. The research questions are then revisited and discussion on the research hypotheses are made. Further, the achievements and contributions made on the thesis are summarized. Finally, the author's vision for opportunities in further research is presented.

7.2 Answering the Research Questions and Validating the Hypotheses

Three research questions addressed in this thesis can be broadly classified into two research areas, that are, (i) Decision Framework for Coupled Engineered Systems and, (ii) Design of Coupled Engineered Systems Under Uncertainty.

7.2.1 *Research Area 1 - Decision Framework for Coupled Engineered Systems*

The primary research question in this thesis deals with modeling coupling among decisions and integrating decision coupling to create a decision framework. The purpose of creating a decision framework is to support the creation of decision templates for designing and analyzing coupled engineered systems. The primary research question that is formulated is as follows,

What are the necessary scientific foundations necessary for designing and analyzing coupled engineered systems in an uncertain environment?

To answer this primary research question, 3 secondary research questions are formulated.

Secondary Research Question Associated with Primary Research Question (RQ1): What is the necessary mathematical foundation for modeling coupling among various design decisions required for designing and analyzing coupled engineered systems?

The hypotheses (H1) for answering these this research question are as follows:

- By establishing a method to represent coupling among decisions lying at the same level and at different levels.
- Establishing the concept of horizontal and vertical coupling to represent coupling among various design decisions.

By stepping on these hypotheses, the method to relate design decisions is established. This involves understanding how decisions can be related and mathematics to study such relationship can be established. The idea about decision coupling to study such relationship is presented in Chapter 2. In Chapters 3, 4 and, 5, the idea presented in Chapter 2 is leveraged to develop mathematical formulation for the test problems.

Another secondary research question that is formulated to answer the primary research question is as,

Secondary Research Question Associated with Primary Research Question (RQ2): What is the necessary foundation for integrating the decision coupling to create a generalized decision framework suitable for designing coupled engineered systems?

The hypotheses (H2) for answering these this research question are as follows:

- Developing a classification scheme for representing coupled design problems.
- By establishing a decision scenario matrix that gives a generalized decision framework for coupled problems with two primary decisions (selection and compromise), varying strength of interaction and multi-level decision using DSPs.

Stepping on these hypotheses allows us to expand on our understanding about decision coupling identified by answering RQ1 to develop a classification scheme for coupled design problems. Classification scheme is built on by establishing classification criteria. Chapter 2 contains details about Multi-leveled Decision Scenario Matrix (MDSM) that is built on by stepping on these hypotheses. In Chapters 3, 4 and, 5, the mathematical formulations for the test problems that represent decision patterns identified in DSM is presented.

Theoretical Structural Validation

Theoretical structural validation involves establishing the logical soundness of constructs (individual and integrated) used in modeling decision coupling and the creation of decision framework altogether.

In Chapter 1, the need for modeling coupling in decision for efficient exploration of design space is established. Further, the creation of decision framework by identifying such decision coupling is also elaborated in Chapter 1. Two primary decisions in Decision Support Problem Construct (DSPT) is highlighted and critical review of literature is done. Following two gaps are identified:

- Modeling decision coupling among decisions in the design of coupled engineered system
- Framework to identify decision pattern for a given design problem

Based on these gaps, requirements for creating a generalized decision framework is established. Different literature are critically reviewed in context of work previously carried out on addressing decision coupling in design. In Section 1.3, the proposed decision framework, called the Decision Scenario Matrix (DSM) is shown.

Empirical Structural Validation

Empirical structural validation involves establishing the appropriateness of the test problems used to verify the performance of the decision framework. The design of a fender (Chapter 3) is taken as a first test problem. The first test problem deals with strong coupling between selection – compromise decision (P9 from the Decision Scenario Matrix). In this first test problem, the horizontal coupling among decisions is considered. In Chapter 2, the mathematical construct to model coupling is introduced and Chapter 3, the mathematical formulation for coupled decision modeling in context of designing a fender is established.

Empirical Performance Validation

Empirical structural validation involves establishing the appropriateness of the comprehensive test problems used to verify the performance of the decision framework. Design of a gearbox (Chapter 4) and Design of composite structures (Chapter 5) is taken as the test problems. The first test problem deals with multi-leveled coupling among decisions. In this first test problem, both the horizontal coupling and vertical coupling among decisions is considered. Horizontal coupling is demonstrated by the strong coupling between selection and compromise decisions (P9 from DSM) for design of gears. Vertical coupling is demonstrated by the coupling between gear decisions and shaft decisions. The second test problem deals with weak coupling between selection – compromise decision (P8 from the Decision Scenario Matrix), but with two selection decisions. The two selection decisions involves simultaneous selection of material for fiber and matrix.

7.2.2 *Research Area 2 – Design of Coupled Engineered Systems Under Uncertainty*

The third research question addressed in this thesis is formulated is as follows,

Secondary Research Question Associated with Primary Research Question (RQ3): What is the mathematical foundation required for designing and analyzing coupled engineered systems under uncertainty?

The hypotheses (H3) for answering these this research question (RQ3) are as follows:

- Developing the mathematical representation for defining the couplings identified by answering RQ1 and RQ2.
- By incorporating robustness metrics in the form of system constraints and goals in coupled DSPs. Depending on the kind of robustness required, different metrics may be applied, namely Error Margin Index (EMI) and Design Capability Index (DCI).

Stepping on these hypotheses allows us to mathematical foundations for managing uncertainty for coupled engineered systems. Chapter 2 contains details on formulating coupled decisions as DSPs. It also deals with the mathematical constructs for addressing uncertainty for coupled DSPs.

Theoretical Structural Validation

Theoretical structural validation involves establishing the logical soundness of constructs (individual and integrated) used in managing uncertainty for coupled decisions in design.

In Chapter 1, the need for managing uncertainty for coupled decisions for robust performance is established. Two major mathematical constructs (DCI and EMI) for managing uncertainty is highlighted and critical review of literature is done in Chapter 2. Following gap is identified:

- Managing uncertainty in the design of coupled engineered system

Based on the gap, requirements for dealing with uncertainty is established. Different literature are critically reviewed in context of work previously carried out on addressing uncertainty in design.

Empirical Structural Validation (ESV)

Empirical structural validation involves establishing the appropriateness of the test problems and validating of individual constructs of error margin index and design capability index for managing uncertainty in design of coupled engineered systems. It involves systematically identifying the scope of the two construct's application, reviewing relevant literature and identifying the research gap that exists. The first test problem (Chapter 3) deals with managing uncertainty for strongly coupled selection – compromise decision (P9 from the Decision Scenario Matrix). In this first test problem, uncertainty management when horizontal coupling among decisions exist is considered. In Chapter 2, the mathematical construct to manage uncertainty is introduced and Chapter 3, the mathematical formulation for coupled decision modeling for managing uncertainty in context of designing a fender is established.

Empirical Performance Validation (EPV)

Empirical structural validation involves establishing the appropriateness of the comprehensive test and validating of individual constructs of error margin index and design capability index for managing uncertainty in design of coupled engineered systems. Design of composite structures (Chapter 5) is taken as the test problems for managing uncertainty in a weakly coupled selection - compromise decision. This test problem deals with weak coupling between selection – compromise decision (P8 from the Decision Scenario Matrix), but with two selection decisions. The two selection decisions involves simultaneous selection of material for fiber and matrix.

7.2.3 *Theoretical Performance Validation (TPV)*

Theoretical performance validation involves establishing the generality of the proposed design method. It involves speculation but is anchored in the foundations that are laid on TSV, ESV and EPV. Verification for TPV comes from all the three quadrants (TSV, ESV and EPV). The validation to TPV comes from the idea that the method can be extended, that is, establishing the utility of the presented method in examples not presented in the thesis. It involves two steps i) demonstrating the usefulness of the design method to solve general class of problems and, ii) building confidence in design method as a generalized approach.

The characteristics of the test problems presented in this thesis are:

- Design decisions can be represented in terms of selection or compromise or combination of these two decisions.
- When only two decisions exist, the decision pattern can take one of the nine patterns shown in DSM.
- When more decisions are involved, such decisions can be modeled as multi-leveled decisions by establishing vertical coupling among decisions to be taken at different levels.
- Decisions are to be taken by accepting that the analysis models are incomplete, inaccurate and not of equal fidelity.

These characteristics allow us to generalize the proposed design method for all the class of problems that satisfy these characteristics.

7.3 Method and Application

Advanced computing technologies are rapidly changing the product design and realization platform. Traditional design methods need to be updated and adapted to support development of powerful design platforms that can address the need of time. Such design platform should possess some characteristics which can be enlisted below:

- Model and analyze decision interaction in design of engineered systems.
- Efficiently and rapidly process the huge amount of data available.
- Support mass collaboration among geographically dispersed population.
- Rapidly create, realize and, validate variant and adaptive designs to support mass customization.

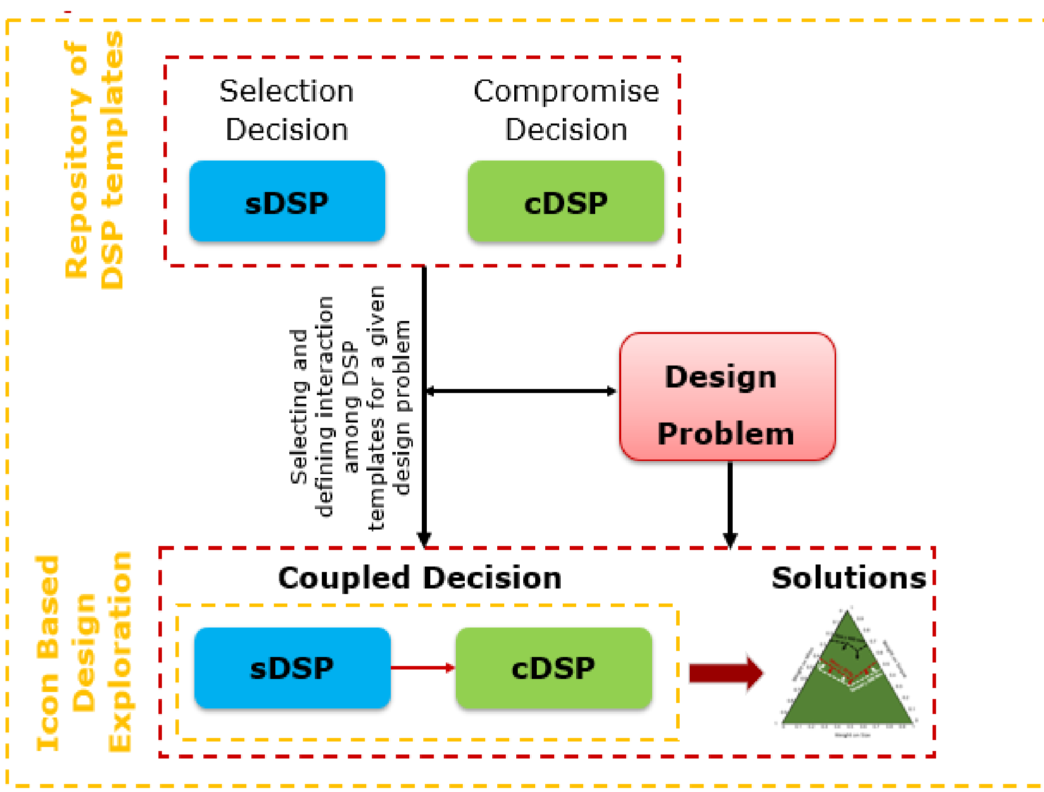


Figure 7.1: Icon Based Robust Design Exploration Framework For Coupled Engineered Systems

Cloud-Based Platform for Decision Support in the Design of Engineered Systems (CB-PDSIDES) possess some characteristics. To improve and infuse better functionality and features into CB-PDSIDES, icon- based design exploration is a way forward. It addresses the issue of modeling decision interactions and efficiently process the huge amount of data, particularly from large material databases. In this context, icon-based robust design exploration framework for coupled engineered systems (shown in Figure 7.1) is proposed as an immediate application of the research presented in the thesis.

7.4 Way Forward

The major focus on this thesis is on creating and validating framework that enable designers to take design decisions in a coupled decision environment under uncertainty. In this section, my intent is to drift a bit and extend the discussion towards a broader aspect of product development. In particular, this section of the thesis is dedicated to the discussion on the future of product development, specifically highlighting on materials, design and manufacturing in the context of promising future technologies: artificial intelligence and, 3D printing. After brief discussion on these technologies, the major focus is to envision how these technologies will drive the future of research on materials, design, and manufacturing and my vision on how these technologies can be exploited to maximize the research efforts in design of engineered systems.

At the core of Artificial Intelligence (AI) is the idea of being able to create machines that can potentially exhibit some form of human intelligence. In essence, AI is anything that empowers machines to make decisions on behalf of a human operator. To this day, the world has already witnessed the disruptions AI is creating in various fields like aerospace, agriculture, finance, medicine, materials, etc. to name a few. In fact, AI is already beginning to impact the everyday lives of millions of people around the world.

3D printing is another potential technology that has already begun to reshape manufacturing by addressing the limitations of conventional manufacturing. As opposed to subtractive manufacturing (removal of material), 3D printing builds the desired part by adding material gradually, one layer after the other. The major advantages of 3D printing over conventional manufacturing is that it offers faster production, reduces material wastage and, can produce complex parts with intricate geometries.

Until recently, the design, materials and, manufacturing aspect of product development processes extensively leveraged the known form of physics-based model complimented by human experience and judgement. With the advent of AI and 3D printing, these powerful tools supplemented by the existing set of tools have equipped designers to create better quality product, considering cost, time and, performance.

Research Need in AI in Context of Design, Materials and, Manufacturing

- Developing powerful algorithms to explore infinite space of geometry exploration, other than the known form of geometries that act as a starting point for any design
- Developing robust algorithms capable of making efficient predictions for wide range of problems in design, materials and, manufacturing
- Metrices to quantify the sensitivity of these algorithms under uncertainty
- Metrices to evaluate and quantify the possible error margins for decisions made by machines
- Validating the correctness of the machine decisions on live-decision environment
- Overcoming the consequences of relying on machines for critical decisions

Research Need in 3D Printing in Context of Design, Materials and, Manufacturing

<u>Combining part printing with part processing requirement:</u> In conventional manufacturing, the part manufacturing involving the process of getting the desired shape and, tuning to desired properties is viewed as being distinct from one another. The actual shape may be obtained from various available techniques such as casting, machining, rolling, etc. while the properties are tuned either before and/or after the final shape is obtained. For instance: In gears, higher hardness along the surface as compared to the core of the gear profile is desired

to prevent surface wear. In making gears, the actual gear profile is obtained by shaping/hobbing and then is treated to enhance the hardness at the surface using various techniques like induction hardening, carburizing, nitriding, etc. Using 3D printing, the possibility to combine these distinct processes seem viable. This will not only revolutionize manufacturing but also bring newer paradigm to design. Often times, designers are forced to use the known geometry or to design multiple parts to achieve some desired performance as a result of manufacturing complication involved. With 3D printing, this no longer is true. **Besides, the ability to design a part with varying properties will enable designers to extract various functionality from a part, thus allowing designers to address multiple conflicting requirements without compromise.**

7.5 I – Statement: Speculation

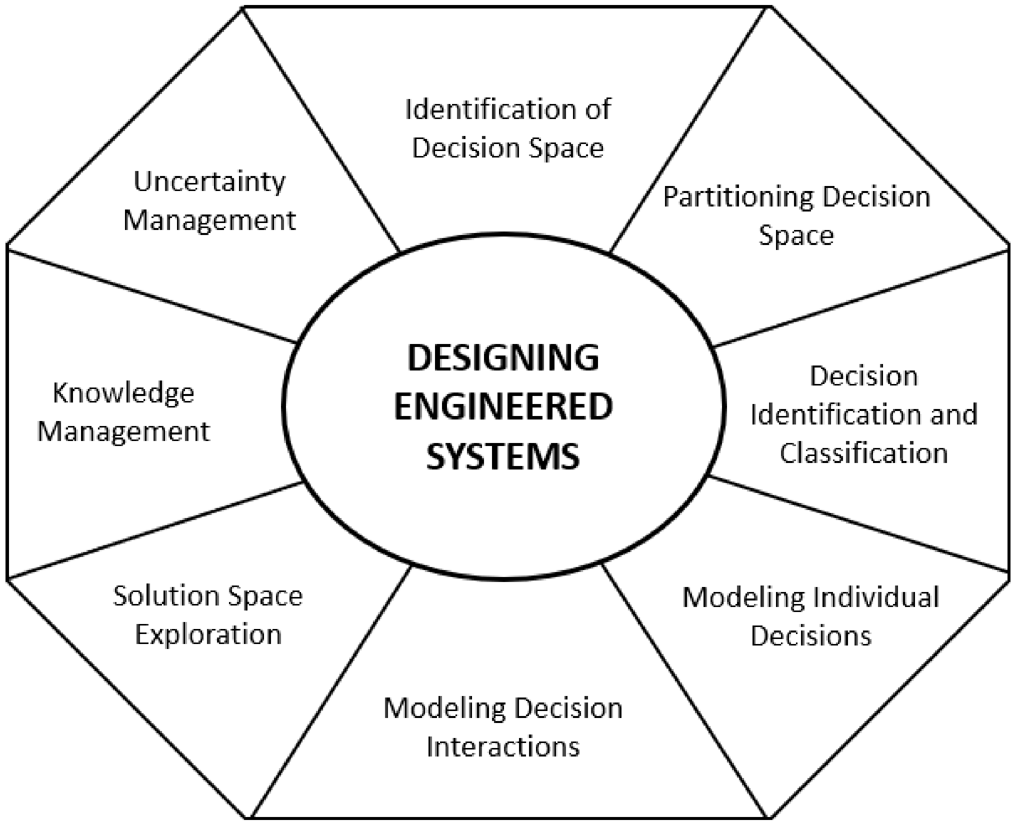


Figure 7.2: Elements in Design of Engineered Systems

Table 7.1: Contributions in this Thesis

Elements	How?	Sections	Contribution
Identification of Decision Space	Design scope and boundary establishment	1.1, 1.3	Problem identification and formulation
Partitioning Decision Space	Use of decision genes, namely selection and compromise	1.3	Simplifying problem realization and solution strategy

Decision Identification and Classification	Decision Scenario Matrix	1.3	Establishing design process for interacting systems
Modeling Individual Decisions	Modeling as Decision Support Problems (DSPs)	2.1, 2.2, and 2.3	Establishing decision making process for compromise and selection decisions
Modeling Decision Interactions	Horizontal and vertical coupling	2.3	Ability to account for influence of one decision over others
Solution Space Exploration	Ternary plots, spider plot and bar charts	6.1, 6.2, 6.3 and 6.4	Identifying satisficing design solutions
Knowledge Management	Design templates	6.7	Archival of engineering knowledge for reuse
Uncertainty Management	Robustness metrics	2.4	Designing solutions that are relative insensitive to uncertainty

In this thesis, I have established the foundations for designing coupled engineered systems by establishing the various elements in the design of engineered systems as shown in Figure 7.2. In Table 7.1, I have highlighted the major contributions made. In particular, my focus in the thesis has been on developing a conceptual decision framework and mathematical foundations required for designing and analyzing coupled engineered systems. For efficient design exploration, the design process associated with such complex engineered systems require designers to decompose the system into subsystem modules and coupling the subsystems to model interaction between these subsystems. Therefore, design of coupled engineered systems require designer to ascertain subsystems and model their interactions. In this thesis, the idea of horizontal and vertical coupling is introduced to model interaction between subsystems. In multi-leveled decisions, horizontal coupling models the interaction between subsystems at same hierarchical level while vertical coupling models the interaction between subsystems at adjacent hierarchical levels.

Leveraging the two foundational axioms in Decision Support Problem Technique (DSPT) that enable designers to formulate design problems in terms of selection, compromise and/or combination of these decisions, I developed a decision framework when these decisions are interacting. Furthermore, I developed mathematical foundation for two crucial decision patterns arising from the framework which is important for designing and analyzing coupled engineered systems under uncertainty. I tested the validity of the decision framework and mathematical foundations with three test problems, namely design of a fender, design of a one-stage reduction gearbox and, design of composite structures. The fundamental contribution is a computational framework that supports human designers in making informed design decisions in a coupled decision environment. In this thesis, I introduce elements in Decision Based Design for developing methods to address complex design problems, wherein design decisions influence each other and are subject to uncertainties. Through the computational framework, I established the foundations for:

1. Designing engineered systems in a coupled and uncertain environment.
2. Developing knowledge-based decision support platform for coupled engineered system.

I have realized and internalized that regardless of domain of application, effective and efficient design of complex engineered systems requires:

1. Decomposition/Partitioning into subsystems and coupling partitioned subsystems (to model their interaction).
2. Multi-leveled coupled representation of subsystems to model concurrent and hierarchical decisions.
3. Managing uncertainties for interacting subsystems that are modeled across various levels.
4. Implementing a multidisciplinary approach.

Research Thrust 1: Designing Complex Engineered Systems Under Uncertainty

In essence, I believe every system in nature is coupled and uncovering how the system interacts with its subsystems and with other systems and/or their subsystems enhances our understanding which is crucial for effective decision making.

Having developed method for designing coupled engineered systems under uncertainty, my understanding on designing engineered systems have augmented. There are some key questions that need to be answered in order to elevate human capability in making effective decisions in design of complex engineered system. What are the fundamental knowledge required in partitioning a system into subsystems and how can we justify the appropriateness of a particular partitioning logic? What makes up a system or how can we create a boundary for defining a system? Having answered these fundamental questions enhances the ability to define system/s with corresponding subsystem/s. At this stage, we are more interested in asking questions like: Can these system or subsystem/s be modeled independently? If not, how can the relationship between these systems and/or subsystems be established? Having answered these questions allow for the creation of system/s and/or subsystem/s that have an established relationship with one another. In design of complex engineered systems, these are likely to be functional and assembly relationship. There are many questions that arise at this stage. How can the decision interaction between these system/s and/or subsystem/s be modeled? How can horizontal and vertical coupling be established between system/s and/or subsystem/s that have an established relationship with concurrency and hierarchy? What are the necessary mathematical foundations for managing uncertainty for such systems with horizontal and vertical coupling across multiple levels?

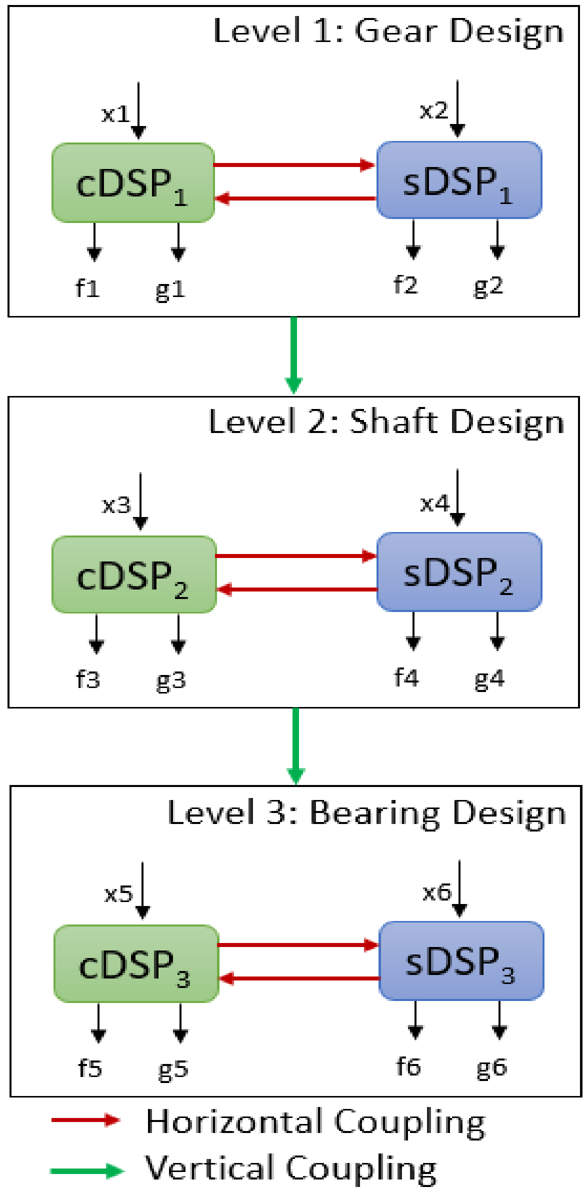


Figure 7.3: Multi-leveled Decisions in Design

Considering a design of an engineered system comprising a gearbox, shafts and bearings, one of the decision pattern that can arise is shown in Figure 7.2. However, answering the question raised earlier will augment the designer's ability to create a decision pattern by systematically partitioning the system, modeling interaction, establishing coupling and creating levels for effective and informed decision-making in the design of engineered systems. This will also enable designers to create boundary for defining subsystems and splitting subsystems or integrating subsystems by expanding the horizon for making informed decisions in the design of complex engineered systems.

Research Plan: To address the challenges associated with design of a complex engineered systems, I plan to establish a systematic approach for dealing with complex systems by disintegrating the system into smaller chunks of decisions which are then integrated together by defining coupling among these decisions. Defining coupling allows for designers to incorporate the influence of one decision on the other. By leveraging the structure from Multi-leveled Decision Scenario Matrix (MDSM), I would develop a method to represent the complex system with a set of multiple decisions that are coupled as a multi-leveled decisions modeled with concurrency and hierarchy. To manage uncertainty, I plan to look at different type of uncertainties, and devise appropriate technique to manage uncertainties associated with individual decisions and uncertainties due to the network of coupled decisions that represent the complex engineered system.

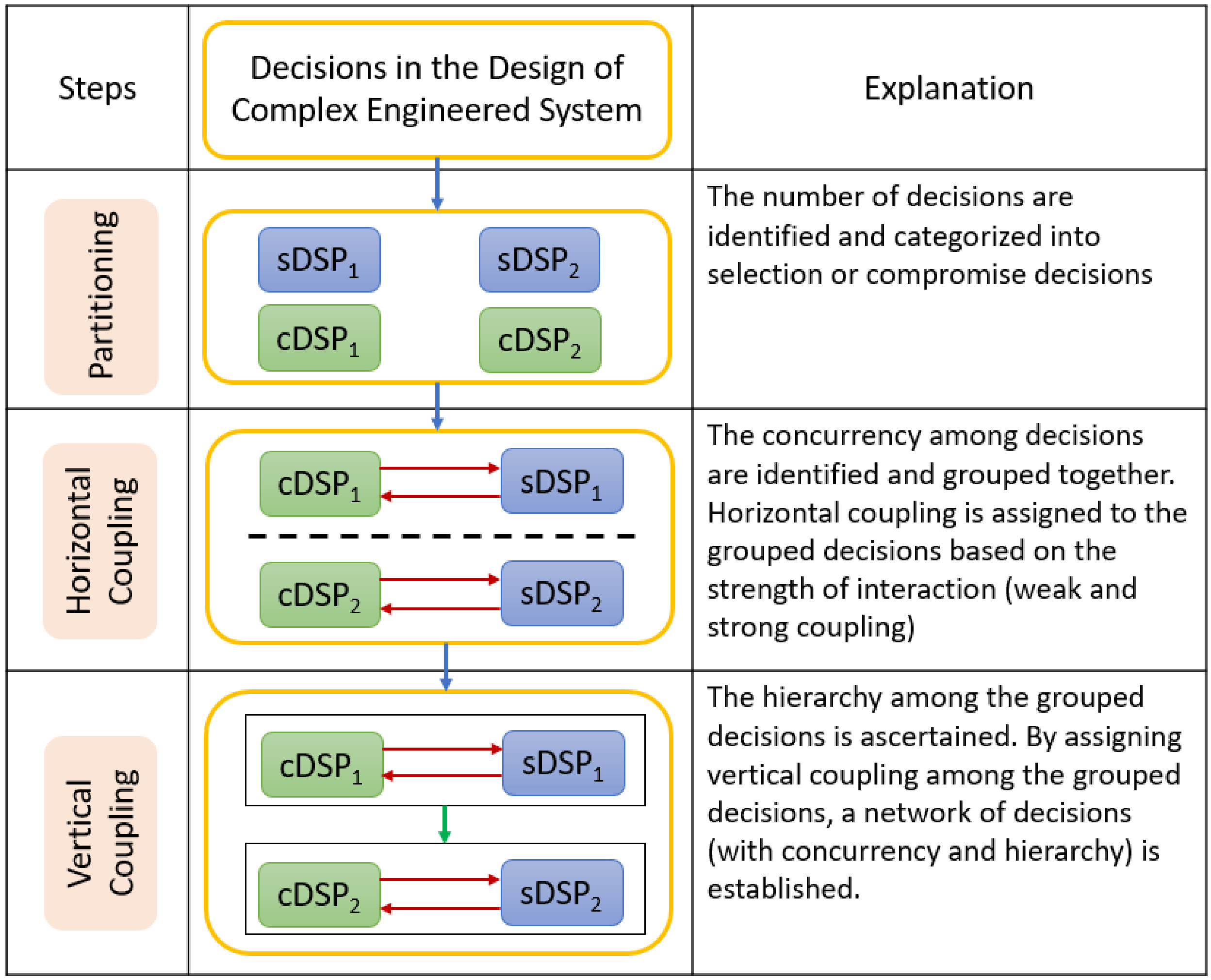


Figure 7.4: Network Of Coupled Decisions in the Design of Complex Engineered Systems

Anticipated Outcome: Mathematical models embedded in a decision framework for designing a complex engineered system under uncertainty.

Broader Impacts: A complex engineered system comprises numerous interacting subsystems and encompasses knowledge across multiple domains. As such, the realization of one true design space for such systems seems unlikely. Even if the design space is realized, efficiently navigating through the design space for better designs become challenging. One way to tackle this issue is to design process associated with such complex engineered systems to be decomposed into subsystem modules which are coupled through transference of output data. However, decomposition into subsystems and coupling subsystems for efficiently traversing through the design space is not straightforward. As an answer to this challenge, the goal is to develop a decision framework with embedded mathematical models for representing complex engineered systems as cluster of interrelated decisions with concurrency and hierarchy defined through coupling. The need to manage uncertainty is also addressed by the mathematical models. The idea is not to give designers one way of decomposing into subsystems and coupling as a system but give a generic framework that allows designers to generate multiple conceptual decision scenarios . The objective is to augment designer's ability in leveraging his experience to exercise better judgement about potential decision scenarios for making informed design decisions.

Through this research, I plan to make study and explore strategies to be able to do the following:

1. Establish a method for representing complex engineered system through a set of interrelated decisions dispersed across various levels of priority and modeled through concurrency and hierarchy.

2. Develop mathematics for designing systems that is represented through a set of interrelated decisions dispersed across various levels of priority.
3. Develop mathematics to manage uncertainties associated with such systems.

Generating knowledge for better understanding of decision interactions in design of complex engineered systems to enable designers in efficiently traversing the design space is at the core of this research.

Research Thrust 2: Designing Complex Engineered Systems for Additive Manufacturing Under Uncertainty

Additive manufacturing is a revolutionary technology that has opened numerous possibilities by addressing the limitations of conventional manufacturing. It is reshaping manufacturing by offering faster production time, reduced material wastages and, producing parts with intricate geometries. The ramifications of this include mass customization, simplified supply chain network, novel designs with improved performance, etc.

Mass Customization: Unlike, conventional manufacturing rearrangement of tooling and production sequences to accommodate different designs are not required in additive manufacturing as 3D printers can produce parts with various geometric configurations without much adjustments. This makes the possibility of producing products that are custom designed without an added cost.

Simplified Supply Chain Network: There exists number of distribution channels that link manufacturing unit to the end users. With easy access to 3D printers, the possibility to produce products when and where required has emerged. This eliminates both the wait time for buyers as well as the longer and complicated distribution channels.

Better Quality Designs: This technology has added more freedom to designers in designing novel products. Designers are no longer constrained by the limitations of convectional manufacturing and are free to explore a wider design space. The possibility to print intricate geometries and different material combination are widening the design space. Hence, with the advent of 3D printing, the possibility to explore disruptive design solutions without compromising performance is viable.

In context of above possibilities, I plan to explore following research areas:

- Uncertainty quantification for different additive manufacturing processes.
- Uncertainty management for different additive manufacturing processes.
- Designing for mass customization.

- Integrated realization of product, materials and additive manufacturing process under uncertainty in a coupled decision environment.
- Generating knowledge required for converting existing designs (designs that are manufactured with existing techniques) into designs that can be manufactured using additive technology.
- Reducing the amount of material use by developing novel strategies to model geometry.

Area 1: Integrated realization of product, materials and additive manufacturing processes under uncertainty in a coupled decision environment.

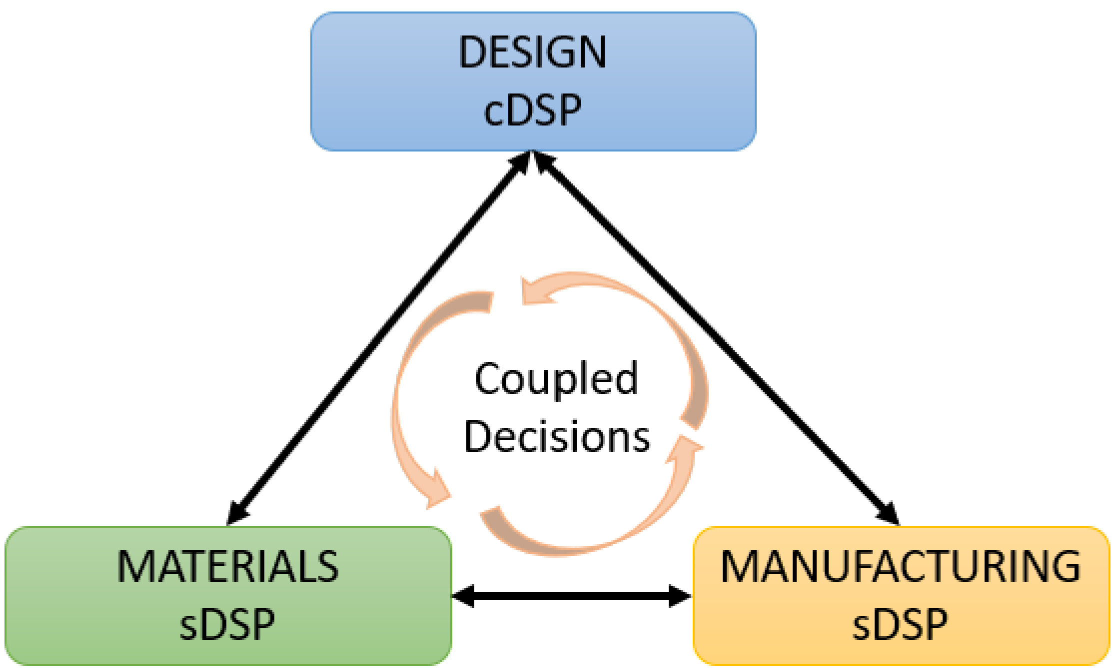


Figure 7.5: Coupled Decisions Environment in Design, Materials and, Manufacturing

For improving product decisions, it is imperative that decisions pertaining to design, materials and, manufacturing are judiciously made. These decisions must be taken in coherence. With 3D printing as a manufacturing technique, the possibility to make decisions about design and material is no longer the same. Further, the need to address uncertainty in this new manufacturing environment is critical for effective decision-making. In this research, I aim to study the decision interactions between decisions in design, materials and, manufacturing and develop methods to carry out product decisions by accounting the interactions in a coupled and uncertain environment.

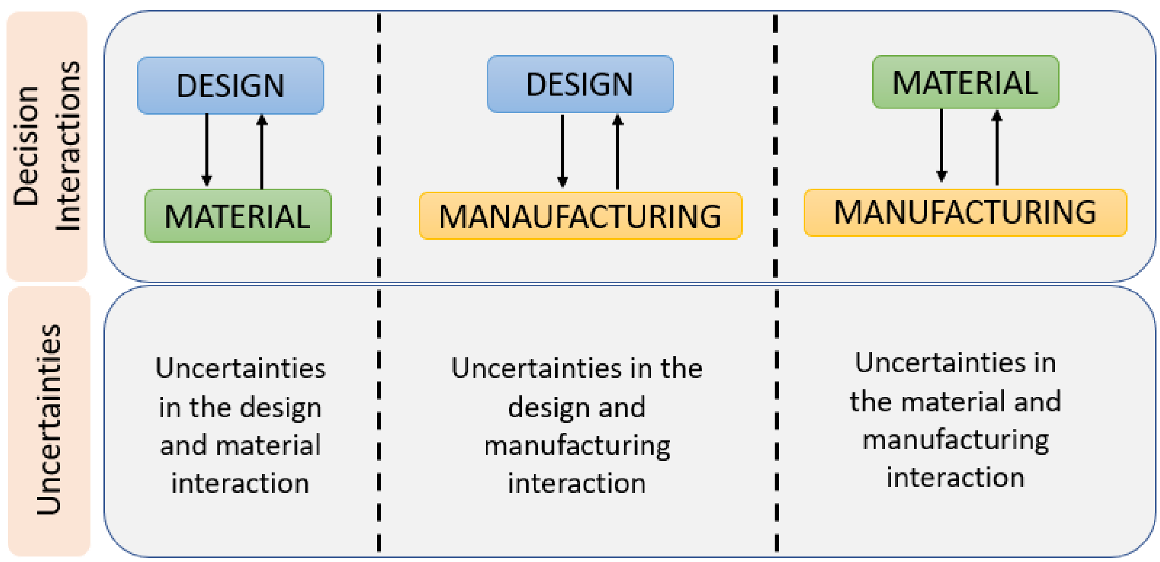


Figure 7.6: Decisions Interactions and Uncertainties

Research Plan: In context of 3D printing, I plan to study and establish the nature of decision interactions between design, materials and, manufacturing decisions.

First, I plan to partition the decision interaction into 3 categories as shown in Figure 7.6:

- Design and Material Decision Interaction
- Design and Manufacturing Decision Interaction
- Material and Manufacturing Decision Interaction

I will then study the nature and type of uncertainties in each interactions. This study will enable me to develop/suggest methods to manage the uncertainties. Consequently, I will establish necessary scientific and mathematical foundations necessary to make effective decisions on design, material and, manufacturing in a coupled and uncertain environment.

Broader Impacts: The ability to address the impact of material decisions on design and manufacturing, manufacturing decisions on design and material and, design decisions on material and manufacturing is critical in developing strategies to make effective decisions. Further, establishing the nature and methods to address uncertainty in these decision interactions play a vital role in devising methods to develop robust decision-making techniques. Through robust decision-making techniques, the cumulative design, material and, manufacturing decisions can be taken where fluctuations in these decisions are less likely to impact product performance. In context of designing for additive manufacturing, I plan to study and establish the scientific foundations for modeling decision interactions between design, material and, manufacturing in a coupled and uncertain environment. Through this research, the knowledge required by designers in making products decisions by accounting the influence of one decision over others is established, which will augment the ability of designers and reduce design iterations.

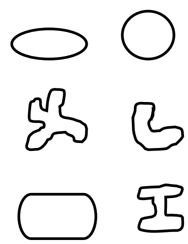
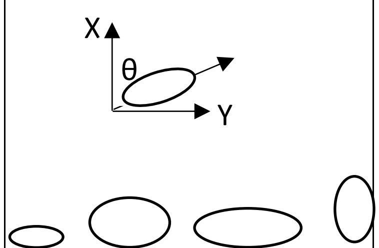
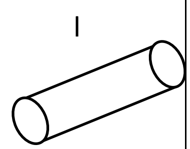
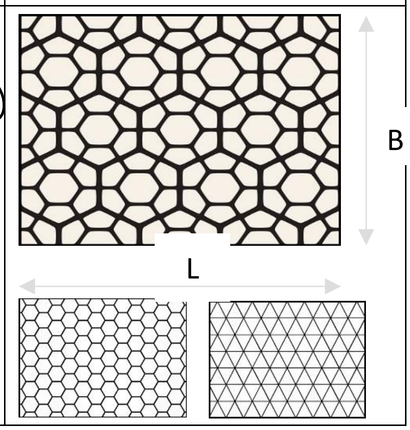
Area 2: Reducing the amount of material use by discovering novel strategies to model geometry

Traditionally, designs are created using standard geometry. Research in recent years have shown design using these geometry do not use material efficiently. As a result, researchers have heavily concentrated on reducing material wastages in design with novel methods like topology optimization and generative design. In topology optimization, the algorithm tries to figure out the necessary material distribution required to maintain the structural integrity under desired performance requirement. The topology optimizer will gradually remove material from sections that are not picking up much stress and have little strain energy. On the other hand, generative design involves an iterative process where computer algorithm attempts to explore all possible permutation of design solutions for a given design problem. The algorithm receives basic design information like weight, size, material, load, etc. to create thousands of potential design solutions.

In this research, our aim is to make studies to develop methods for creating and analyzing organic designs with an aim of discovering disruptive design solutions for a given design problem. By going from traditional shapes to organic shapes, we intend to reduce the excess use of materials while not compromising the performance. With additive manufacturing at our disposal, such unconventional designs can be easily manufactured.

Research Plan: I plan to partition the decision about design geometry into smaller chunks of decisions. Together with material alternatives as a selection decision, the various geometric configurations will be analyzed for improved design performance. Material decision together with these smaller chunks of geometric decisions, I plan to explore the design space in search for disruptive design solutions. The design variables are bifurcated to two, that are, micro design variables and macro design variables. Micro variables include micro elements of various organic shapes, transformation (orientation and scaling) applied to the shapes and, extrusion applied to the shapes. The exploration of solution space for micro design variables results in the decision regarding micro elements configuration, which forms the building block for macro structure. At macro structure design exploration, the design variables at structural level are varied to achieve the required design performance. First, I plan to develop mathematics to represent the design problem with micro and macro design variables that are coupled. Consequently, I will test the mathematics on different design problems with varying design requirements.

Table 1: Partitioning Geometric Decisions

Micro Design Variables			Macro Design Variables
Micro elements	Transformation and orientation of micro elements	Micro elements configuration	Macro Structure
			

Broader Impacts: With the use of 3D printing technology, it has become possible to manufacture designs with complex geometries. How can designers exploit this possibility to explore innumerous designs and systematically traverse through this extended design space for searching disruptive design solutions? As an answer to this question, I plan to create knowledge required for designing complex and intricate geometries with better performance.

Through this research, I plan to achieve the following goals:

1. Develop method to explore unconventional designs by segregating into smaller decisions and coupling these decisions along with the material decision.

2. Study to understand whether change in material is to be adjusted by changing macro design variables (structural level) and/or micro design variables.
3. Study the sensitivity on performance of the resulting designs as a result of deviations in design variables.
4. Develop method to explore wider design space as a result of added design variables (micro and macro design variables).
5. Establish a starting point for developing novel approaches in machine learning and artificial intelligence algorithms for searching better geometric designs.
6. Provide guidelines to CAD software developers to help them create platform that allows designers to easily and quickly model designs with unconventional geometry.

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APPENDIX: Codes for DSIDES

In appendix, the FORTRAN codes that are written to implement math formulations presented in the thesis are included. Specifically, it will include FORTRAN codes (.f file and .dat file) for the math formulations presented in Chapter 3, Chapter 4 and, Chapter 5.

Robust Design of Fender – Material as a variable approach

The codes are for the math formulation for Example 1 presented in Chapter 3 (Table 3.9). There are two files, that are, .f and .dat file.

FORTRAN file (.dat) for Robust Design of Fender (Example 1)

```
PTITLE  : Problem Title
   Design of a Fender
NUMSYS  : Number of system variables
   4   0   0
SYSVAR   : System variable information
THICK     1     0.12     0.75     0.12
DIAM      2     3.0      24.0     3.0
AS        3     30.0     36.0     30.0
E         4     27.5     30.5     27.5
NUMCAG  : Number of constraints and goals
   0   5   0   0   3
DEVFUN  : Deviation function
  1   : level
  1 3  : level 1, 3 terms
 (-1,0.33) (-2,0.33) (-3,0.33)
```

STOPCR : Stopping criteria

1 0 300 0.05 0.05

NLINCO : Names of nonlinear constraints

bstress 1 : bending stress

deflec 2 : maximum deflection

EMI1   3 : Goal 1 constraint

DCI2   4 : Goal 2 constraint

DCI3   5 : Goal 3 constraint

NLINGO : Names of the nonlinear goals

mbeam 1 : mass/strength

aspect 2 : aspect ratio

Stiff 3 : Stiffness

ALPOUT   : Input/output Control

1   1   1   1   1   1   1   1 1 1

USRMOD : Input/Output flags

1   0   0   0

OPTIMP : Optimization parameters

-0.05   0.5 0.05

ENDPRB : Stop reading the data file at this point

FORTRAN file (.f) for Robust Design of Fender (Example 1)

```
      SUBROUTINE USRINP (NDESV, NINP, NOUT, DESVAR)
C
C***  DUMMY ROUTINE.  Not used.
```

```
C
      INTEGER NDESV, NINP, NOUT
      REAL DESVAR(NDESV)
C
      RETURN
      END

      SUBROUTINE USROUT (NDESV, NOUT, DESVAR, LCONDF, LCONSV, LXFEAS)
C
C***  DUMMY ROUTINE.  Not used in the formulation
C
      INTEGER NDESV, NOUT
      REAL DESVAR(NDESV)
      LOGICAL LCONDF, LCONSV, LXFEAS
C
      RETURN
      END

      SUBROUTINE USRANA (NDESV, NOUT, DESVAR)
C
C***  DUMMY ROUTINE.  Not used in the formulation
C
      INTEGER NDESV, NOUT
```

```
      REAL DESVAR(NDESV)
C
      RETURN
      END


      SUBROUTINE USRMON (NDESV, NDEVAR, NMPRI, NNLCON, NNLGOA, NOUT,
     &                 DESVAR, DEVVAR, CONDEV, DEVFUN, GVAL)
C
C***  DUMMY ROUTINE.  Not used in the formulation
C
      INTEGER NOUT, NDESV, NDEVAR, NMPRI, NNLCON, NNLGOA
      REAL DESVAR(NDESV), DEVVAR(NDEVAR),
     &    CONDEV, DEVFUN(NMPRI), GVAL(NNLCON+NNLGOA)
C
      RETURN
      END
C+
C*****************************************************************
******
C
C Subroutine USRSET
C
C Purpose:  Evaluate non-linear constraints and goals.
C          NOTE - Do not specify the deviation variables
```

```
C
C-------------------------------------------------------------------
C Arguments      Name     Type   Description
C ---------      ----     ----   -----------
C Input:         IPATH    int    = 1   Evaluate constraints and goals
C                                = 2   Evaluate constraints only
C                                = 3   Evaluate goals only
C                NDESV    int    Number of design variables
C                MNLNCG   int    Max. number of nonlinear constraints
C                                and goals
C                NOUT     int    Output file/device number
C                DESVAR   real   Vector of current system variables
C
C Output:        CONSTR   real   Vector of constraint values
C                GOALS    real   Vector of goal values
C
C Input/Output:   none
C-------------------------------------------------------------------
C Common Blocks:  none
C
C Include Files:  none
C
C Calls to:       none
```

```
C-------------------------------------------------------------------

C Development History

C

C Modifications:

C

C**************************************************************
******

C-

C

      SUBROUTINE USRSET (IPATH, NDESV, MNLNCG, NOUT, DESVAR,

     &                 CONSTR, GOALS)

C

C--------------------------------------

C     Arguments:

C--------------------------------------

C

      INTEGER IPATH, NDESV, MNLNCG, NOUT

C

      REAL DESVAR(NDESV)

      REAL CONSTR(MNLNCG), GOALS(MNLNCG)

C

C--------------------------------------

C     Local variables:

C--------------------------------------
```

```
C
      REAL THICK, DIAM, P, L, AS, AD, TW, AR, I, E, RHO, PI, VOLUME,TS
      REAL dD,dt, dE, dS, dy1u, dy2, dy3, EMI1, DCI1, DCI2, MSR
      REAL g1D, g1t,g1S, g2D, g2t, g3D, g3t,g3E, cv
C     1.0 Set the values of the local design variables (optional)
C
      THICK  =  DESVAR(1)
      DIAM   =  DESVAR(2)
       AS = DESVAR(3)
       E  = DESVAR(4)
C
C     2.0 Perform analysis relevant to non-linear constraints and goals
C   Design Parameters
      P   = 12000.00
      PI  = 3.1415926
      AD  = 0.025
      L   = 100.00
      RHO = 0.28
      TW  = 6.00
      AR  = 14.00
      TS = 600000
C  Calculation of Moment of Inertia and Volume
      I = ((DIAM**4 - (DIAM-2.0*THICK)**4) * PI) / 64.0
```

```
      VOLUME  = (PI/4.0) * (DIAM**2 - (DIAM-2.0*THICK)**2) * L

c     Defining delta for each design variables

      dD = 0.8
      dt = 0.05
      dE = 0.3
      dS = 0.6

c     Calculating partial differential of each goal wrt design variables

      g1D = RHO*PI*THICK*L/(AS)
      g1t = RHO*PI*(DIAM-2*THICK)*L/(AS)
      g1S = RHO*PI*(DIAM**2 - (DIAM-2.0*THICK)**2) * L/(AS)**2

      g2D = 1/THICK
      g2t = DIAM/THICK**2

      g3D = 3*PI*E*10**6*(DIAM**3 - (DIAM-2.0*THICK)**3)/(L)**3
      g3t = 6*PI*E*10**6*(DIAM-2.0*THICK)**3/(L)**3
      g3E = 3*PI*(DIAM**4 - (DIAM-2.0*THICK)**4)/(4*(L)**3)

c     Calculating delta y for each goal
```

```
c        Defining material variability and manufacturing variability as cumulative
variability factor cv

      cv = (1.05)*1.08**(3/DIAM)

      MSR = (RHO*VOLUME/AS)

      dy1u=cv*MSR-MSR+cv*(g1D*dD+g1t*dt+g1S*dS)

      dy2 = g2D*dD + g2t*dt

      dy3 = g3D*dD + g3t*dt + g3E *dE


c    Evaluating DCIs


      EMI1 = (50 - (RHO*VOLUME/AS))/dy1u

      DCI1 = (200 - (DIAM/THICK))/dy2

      DCI2 = ((48*E*10**6*I/L**3)- 60000)/dy3


C     3.0 Evaluate non-linear constraints

C

      IF (IPATH .EQ. 1 .OR. IPATH .EQ. 2) THEN

C

C         SHEAR BENDING constraint.


         CONSTR(1) = 1.0 - ((P*L*DIAM) / (8.0*I*AS*1000) )

C

C       MAXIMUM DEFLECTION constraint. Calculate the modulus of

C       elasticity for the relative alternative.
```

```
C
      CONSTR(2) = 1.0 - ((P*L**3) / (48.0*E*10**6*I*AD))

c                   Goal 1 constraint

      CONSTR(3) = (50 - (RHO*VOLUME/AS))/dy1u - 1

c                   Goal 2 constraint

      CONSTR(4) = (200- (DIAM/THICK))/dy2 - 1

c                   Goal 3 constraint

      CONSTR(5) = ((48*E*10**6*I/L**3)- 60000)/dy3 - 1

    END IF
C
C    4.0 Evaluate non-linear goals
C
    IF (IPATH .EQ. 1 .OR. IPATH .EQ. 3) THEN
C
C      MASS OF BEAM goal
C
```

```
      GOALS(1) = EMI1/16 - 1.0
C
C      ASPECT RATIO goal
C
      GOALS(2) = DCI1/30 - 1.0

C        Stiffness goal
C
      GOALS(3) = DCI2/8 - 1.0

    END IF
C
C    5.0 Return to calling routine
C
    RETURN
    END
    SUBROUTINE USRLIN (MLINCG, NDESV, NLINCO, NLINGO, NOUT,
   &                DESVAR, COFLIN, RHSLIN)
C
C***  DUMMY ROUTINE.  Not used in the formulation
C
    INTEGER MLINCG, NDESV, NLINCO, NLINGO, NOUT
    REAL DESVAR(NDESV),
```

```
   &    COFLIN(MLINCG,NDESV), RHSLIN(NLINCO+NLINGO)
C
      RETURN
      END
```

Robust Design of Fender – Coupled Problem Approach

The codes are for the math formulation for Example 2 presented in Chapter 3 (Table 3.10). There are two files, that are, .f and .dat file.

FORTRAN file (.dat) for Robust Design of Fender (Example 2)

```
PTITLE  : Problem Title
   Design of a Fender
NUMSYS  : Number of system variables
   2   0   3

SYSVAR   : System variable information
t           1     0.12     0.75    0.12
D        2     3.0      24.0     3.0
X1       3     0.0      1.0     0.0
X2       4     0.0      1.0     1.0
X3       5     0.0      1.0     0.0

NUMCAG  : Number of constraints and goals
   1   5    0   0   4
```

LINCON : Linear constraints

Alt 3 : Selection of one alternative

(3,1.0) (4,1.0) (5,1.0)

== 1.0

ACHFUN : Achievment function

2   : level

1 1 : level 1, 1 term

(-1,1.0)

2 3 : level 2, 3 terms

(-2,0.33) (-3,0.33) (-4,0.33)

STOPCR : Stopping criteria

1 0 300 0.05 0.05

NLINCO : Names of nonlinear constraints

bstress 1 : bending stress

deflec 2 : maximum deflection

EMI1   3 : Goal 1 constraint

DCI2   4 : Goal 2 constraint

DCI3   5 : Goal 3 constraint

NLINGO : Names of the nonlinear goals

Alt 1 : Materials

mbeam 2 : mass/strength

aspect 3 : aspect ratio

Stiff 4 : Stiffness

ALPOUT   : Input/output Control

    1 1 1 1 1 1 1 1 1 1

USRMOD : Input/Output flags

   1 0 0 0

OPTIMP : Optimization parameters

   -0.05   0.5 0.05

ENDPRB : Stop reading the data file at this point

FORTRAN file (.dat) for Robust Design of Fender (Example 2)

```
C
C
      SUBROUTINE USRINP (NDESV, NINP, NOUT, DESVAR)
C
C***  DUMMY ROUTINE.  Not used.
```

```
C
      INTEGER NDESV, NINP, NOUT
      REAL DESVAR(NDESV)
C
      RETURN
      END

      SUBROUTINE USROUT (NDESV, NOUT, DESVAR, LCONDF, LCONSV, LXFEAS)
C
C***  DUMMY ROUTINE.  Not used in the formulation
C
      INTEGER NDESV, NOUT
      REAL DESVAR(NDESV)
      LOGICAL LCONDF, LCONSV, LXFEAS
C
      RETURN
      END

      SUBROUTINE USRANA (NDESV, NOUT, DESVAR)
C
C***  DUMMY ROUTINE.  Not used in the formulation
C
      INTEGER NDESV, NOUT
```

```
      REAL DESVAR(NDESV)
C
      RETURN
      END

      SUBROUTINE USRMON (NDESV, NDEVAR, NMPRI, NNLCON, NNLGOA, NOUT,
     &                  DESVAR, DEVVAR, CONDEV, DEVFUN, GVAL)
C
C***  DUMMY ROUTINE.  Not used in the formulation
C
      INTEGER NOUT, NDESV, NDEVAR, NMPRI, NNLCON, NNLGOA
      REAL DESVAR(NDESV), DEVVAR(NDEVAR),
     &     CONDEV, DEVFUN(NMPRI), GVAL(NNLCON+NNLGOA)
C
      RETURN
      END
C+
C*****************************************************************
******
C
C Subroutine USRSET
C
C Purpose:  Evaluate non-linear constraints and goals.
C         NOTE - Do not specify the deviation variables
```

```
C
C-------------------------------------------------------------------
C Arguments      Name     Type   Description
C ---------      ----     ----   -----------
C Input:         IPATH    int    = 1   Evaluate constraints and goals
C                                = 2   Evaluate constraints only
C                                = 3   Evaluate goals only
C                NDESV    int    Number of design variables
C                MNLNCG   int    Max. number of nonlinear constraints
C                                and goals
C                NOUT     int    Output file/device number
C                DESVAR   real   Vector of current system variables
C
C Output:        CONSTR   real   Vector of constraint values
C                GOALS    real   Vector of goal values
C
C Input/Output:   none
C-------------------------------------------------------------------
C Common Blocks:  none
C
C Include Files:  none
C
C Calls to:       none
```

```
C---------------------------------------------------------------------
C Development History
C
C Modifications:
C
C**************************************************************
******
C-
C
      SUBROUTINE USRSET (IPATH, NDESV, MNLNCG, NOUT, DESVAR,
     &                   CONSTR, GOALS)
C
C--------------------------------------
C     Arguments:
C--------------------------------------
C
      INTEGER IPATH, NDESV, MNLNCG, NOUT
C
      REAL DESVAR(NDESV)
      REAL CONSTR(MNLNCG), GOALS(MNLNCG)
C
C--------------------------------------
C     Local variables:
C--------------------------------------
```

```
c
      REAL THICK, DIAM, P, L, AS, AD, TW, AR, I, E, RHO, PI, VOLUME,TS
      REAL dD,dt, dE, dS, dy1u, dy2, dy3, EMI1, DCI1, DCI2, MSR
      REAL g1D, g1t,g1S, g2D, g2t, g3D, g3t,g3E, cv
      REAL I1,I2,I3,I4,a11,a12,a13,a14,a21,a22,a23,a24,a31,a32,a33,a34
      REAL P1,P2,P3,C1,C2,C3,AS1,AS2,AS3,E1,E2,E3,R1,R2,R3,MF1,MF2,MF3
      REAL SR,SE,SAS, dy1u1, dy1u2, dylu3
C     1.0 Set the values of the local design variables (optional)
C
      t  =  DESVAR(1)
      D  =  DESVAR(2)
      X1  =  DESVAR(3)
      X2  =  DESVAR(4)
      X3  =  DESVAR(5)

C     2.0 Perform analysis relevant to non-linear constraints and goals
C  Design Parameters
      P   = 12000.00
      PI  = 3.1415926
      AD  = 0.025
      L   = 100.00
      RHO = 0.28
      TW  = 6.00
```

```
      AR  = 14.00

       TS = 600000

      AS=30.00

      E=30.00

C   Calculating Moment of Inertia and Volume

      I = ((D**4 - (D-2.0*t)**4) * PI) / 64.0

      VOLUME  = (PI/4.0) * (D**2 - (D-2.0*t)**2) * L

c    Material attributes

      P1   = 3.0

      P2  = 21.3

      P3  = 21.6

      AS1  = 28.00

      AS2= 34.8

      AS3  = 27.5

      E1 = 26.00

      E2 = 15.2

      E3 = 19.00

      R1 = 0.272

      R2 = 0.163

      R3 = 0.298

c    Merit function Calculations

      I1   = 0.1

      I2  = 0.3
```

```
I3  = 0.0

I4  = 0.6

a12 = 0.1

a13 = 0.146

a14 = 0.493

a22 = 0.5

a23 = 0.121

a24 = 0.329

a32 = 0.4

a33 = 0.732

a34 = 0.178


C1=P1*R1*(PI/4.0) * (D**2 - (D-2.0*t)**2) * L

C2=P2*R2*(PI/4.0) * (D**2 - (D-2.0*t)**2) * L

C3=P3*R3*(PI/4.0) * (D**2 - (D-2.0*t)**2) * L


a11 = 0.7- (C1/(C1+C2+C3))

a21 = 0.7- (C2/(C1+C2+C3))

a31 = 0.7- (C3/(C1+C2+C3))


MF1= I1*a11+I2*a12+I3*a13+I4*a14

MF2= I1*a21+I2*a22+I3*a23+I4*a24

MF3= I1*a31+I2*a32+I3*a33+I4*a34
```

```
c    Defining delta for each design variables


     dD = 0.8

     dt = 0.05

     dE = 0.3

     dS = 0.6

c    Calculating partial differential of each goal wrt design variables

     SR = R1*X1+R2*X2+R3*X3

     SE = E1*X1+E2*X2+E3*X3

     SAS = AS1*X1+AS2*X2+AS3*X3


     g1D = (SR)*PI*t*L/(SAS)

     g1t = (SR)*PI*(D-2*t)*L/(SAS)

     g1S=(SR)*PI*(D**2-(D-2*t)**2)*L/(SAS)**2


     g2D = 1/t

     g2t = D/t**2


     g3D = 3*PI*SE*10**6*(D**3 - (D-2.0*t)**3)/(L)**3

     g3t = 6*PI*SE*10**6*(D-2.0*t)**3/(L)**3

     g3E = 3*PI*(D**4 - (D-2.0*t)**4)/(4*(L)**3)
```

```
c   Calculating delta y for each goal

c   Defining material variability and manufacturing variability as cumulative
variability factor cv

    MSR = (SR*VOLUME/SAS)

    dy1u1=0.94+0.32*MSR-0.201*D+1.32*(g1D*dD+g1t*dt+g1S*dS)-0.201*dD

    dy1u2=0.82+0.30*MSR-0.31*D+1.30*(g1D*dD+g1t*dt+g1S*dS)-0.31*dD

    dy1u3=0.88+0.33*MSR-0.22*D+1.33*(g1D*dD+g1t*dt+g1S*dS)-0.22*dD

    dy1u = dy1u1*X1+dy1u2*X2+dy1u3*X3

    dy2 = g2D*dD + g2t*dt

    dy3 = g3D*dD + g3t*dt + g3E *dE

c   Evaluating DCIs

    EMI1 = (50 - (SR*VOLUME/SAS))/dy1u

    DCI1 = (200 - (D/t))/dy2

    DCI2 = ((48*SE*10**6*I/L**3)- 60000)/dy3

C    3.0 Evaluate non-linear constraints

C

    IF (IPATH .EQ. 1 .OR. IPATH .EQ. 5) THEN

C

C       SHEAR BENDING constraint.
```

```
      CONSTR(1) = 1.0 - ((P*L*D) / (8.0*I*AS*1000) )
C
C      MAXIMUM DEFLECTION constraint. Calculate the modulus of
C      elasticity for the relative alternative.
C
      CONSTR(2) = 1.0 - ((P*L**3) / (48.0*E*10**6*I*AD))
c                 Goal 1 constraint
      CONSTR(3) = (50 - (RHO*VOLUME/AS))/dy1u - 1


c                 Goal 2 constraint
      CONSTR(4) = (200- (D/t))/dy2 - 1


c                 Goal 3 constraint
      CONSTR(5) = ((48*E*10**6*I/L**3)- 60000)/dy3 - 1


     END IF
C
C    4.0 Evaluate non-linear goals
C
     IF (IPATH .EQ. 1 .OR. IPATH .EQ. 4) THEN
c      Alternative selection
      GOALS(1) = MF1*X1+MF2*X2+MF3*X3 - 1.0
```

```
C
C       MASS OF BEAM goal
      GOALS(2) = EMI1/16 - 1.0
C
C       ASPECT RATIO goal
      GOALS(3) = DCI1/30 - 1.0


C                    Stiffness goal
      GOALS(4) = DCI2/8 - 1.0


    END IF
C
C    5.0 Return to calling routine
C
    RETURN
    END



    SUBROUTINE USRLIN (MLINCG, NDESV, NLINCO, NLINGO, NOUT,
   &                 DESVAR, COFLIN, RHSLIN)
C
C***  DUMMY ROUTINE.  Not used in the formulation
C
```

```
      INTEGER MLINCG, NDESV, NLINCO, NLINGO, NOUT
      REAL DESVAR(NDESV),
     &     COFLIN(MLINCG,NDESV), RHSLIN(NLINCO+NLINGO)
C
      RETURN
      END
```

Design of Gearbox – Multi-level Design Approach

The codes are for the math formulation for design of one-stage reduction gearbox presented in Chapter 4 (Table 4.4). The different scenarios for exploration are obtained as explained in Section 4.2.3 (Scenarios for Exploration). There are two files, that are, .f and .dat file.

FORTRAN file (.dat) for Multi-level Design of Gearbox

```
PTITLE  : Problem Title
   Design of a Gearbox
NUMSYS  : Number of system variables
   7   0   5
SYSVAR   : System variable information
m          1     3.0      6.0     3.0
b     2     24.0     72.0    24.0
T     3     80.0     1000.0   80.0
Di    4      20.0     40.0    20.0
D0    5      30.0     50.0    30.0
```

Sy 6 200.0 400.0 200.0

z 7 18.0 30.0 18.0

X1 8 0.0 1.0 0.0

X2 9 0.0 1.0 1.0

X3 10 0.0 1.0 0.0

X4 11 0.0 1.0 0.0

X5 12 0.0 1.0 0.0

NUMCAG : Number of constraints and goals

3 4 0 0 5

LINCON : Linear constraints

Alt 5 : Selection of one alternative

(8,1.0) (9,1.0) (10,1.0) (11,1.0) (12,1.0)

== 1.0

bmin 2 : Maximum face width

(1,8.0) (2,-1.0)

LE 0.0

bmax 2 : Maximum face width

(1,12.0) (2,-1.0)

GE 0.05

ACHFUN : Achievment function

3 : level

2 1 : level 2, 1 term

(-1,1.0)

1 3 : level 1, 3 terms

(-2,0.0) (-3,0.15) (-4,0.85)

3 1 : level 3, 1 term

(-5,1.0)

STOPCR : Stopping criteria

1 0 300 0.05 0.05

NLINCO : Names of nonlinear constraints

bstress 1 : bending stress

cstress 2 : contact stress

shear1stress 3 : shear1 stress

shear2stress 4 : shear2 stress

NLINGO : Names of the nonlinear goals

Alt 1 : Materials

mgear 2 : mass

sgear 3 : size

Torque 4 : Torque

mshaft 5 : mass shaft

ALPOUT   : Input/output Control

1   1   1   1   1   1   1   1 1 1

USRMOD : Input/Output flags

1   0   0   0

OPTIMP : Optimization parameters

```
   -0.05   0.5  0.05

ENDPRB  : Stop reading the data file at this point
```

FORTRAN file (.f) for Multi-level Design of Gearbox

```
C
C
      SUBROUTINE USRINP (NDESV, NINP, NOUT, DESVAR)
C
C***  DUMMY ROUTINE.  Not used.
C
      INTEGER NDESV, NINP, NOUT
      REAL DESVAR(NDESV)
C
      RETURN
      END

      SUBROUTINE USROUT (NDESV, NOUT, DESVAR, LCONDF, LCONSV, LXFEAS)
C
C***  DUMMY ROUTINE.  Not used in the formulation
C
      INTEGER NDESV, NOUT
      REAL DESVAR(NDESV)
      LOGICAL LCONDF, LCONSV, LXFEAS
```

```
C
      RETURN
      END
      SUBROUTINE USRANA (NDESV, NOUT, DESVAR)
C
C***  DUMMY ROUTINE.  Not used in the formulation
C
      INTEGER NDESV, NOUT
      REAL DESVAR(NDESV)
C
      RETURN
      END

      SUBROUTINE USRMON (NDESV, NDEVAR, NMPRI, NNLCON, NNLGOA, NOUT,
     &                 DESVAR, DEVVAR, CONDEV, DEVFUN, GVAL)
C
C***  DUMMY ROUTINE.  Not used in the formulation
C
      INTEGER NOUT, NDESV, NDEVAR, NMPRI, NNLCON, NNLGOA
      REAL DESVAR(NDESV), DEVVAR(NDEVAR),
     &    CONDEV, DEVFUN(NMPRI), GVAL(NNLCON+NNLGOA)
C
      RETURN
```

```
      END
C+
C*****************************************************************
******
C
C Subroutine USRSET
C
C Purpose:  Evaluate non-linear constraints and goals.
C          NOTE - Do not specify the deviation variables
C
C------------------------------------------------------------------
C Arguments      Name     Type   Description
C ---------      ----     ----   -----------
C Input:         IPATH    int    = 1   Evaluate constraints and goals
C                                = 2   Evaluate constraints only
C                                = 3   Evaluate goals only
C              NDESV    int    Number of design variables
C              MNLNCG   int    Max. number of nonlinear constraints
C                              and goals
C              NOUT     int    Output file/device number
C              DESVAR   real   Vector of current system variables
C
C Output:        CONSTR   real   Vector of constraint values
C              GOALS    real   Vector of goal values
```

```
C
C Input/Output:   none
C------------------------------------------------------------------
C Common Blocks:  none
C
C Include Files:  none
C
C Calls to:       none
C------------------------------------------------------------------
C Development History
C
C Modifications:
C
C**************************************************************
******
C-
C
      SUBROUTINE USRSET (IPATH, NDESV, MNLNCG, NOUT, DESVAR,
     &                   CONSTR, GOALS)
C
C-------------------------------------
C     Arguments:
C-------------------------------------
C
```

```
      INTEGER IPATH, NDESV, MNLNCG, NOUT
C
      REAL DESVAR(NDESV)
      REAL CONSTR(MNLNCG), GOALS(MNLNCG)
C
C---------------------------------------
C     Local variables:
C---------------------------------------
C
c
      REAL m,b,z,T,X1,X2,X3,X4,X5,Di,D0,Sy, TorC
      REAL C1,C2,C3,C4,C5
      REAL St1,St2,St3,St4,St5,Sc1,Sc2,Sc3,Sc4,Sc5, St,Sc
      REAL I1,I2,I3,I4,a11,a12,a13,a14,a21,a22,a23,a24,a31,a32,a33,a34
      REAL a41,a42,a43,a44,a51,a52,a53,a54,P1,P2,P3,P4,P5
      REAL MF1,MF2,MF3,MF4,MF5

C     1.0 Set the values of the local design variables (optional)
C
      m  =  DESVAR(1)
      b  =  DESVAR(2)
      T  =  DESVAR(3)
      Di =  DESVAR(4)
```

```
      D0 =  DESVAR(5)

      Sy =  DESVAR(6)

      z  =  DESVAR(7)

      X1  =  DESVAR(8)

      X2  =  DESVAR(9)

      X3  =  DESVAR(10)

      X4  =  DESVAR(11)

      X5  =  DESVAR(12)


C     2.0 Perform analysis relevant to non-linear constraints and goals


c     Material attributes (Bending and Contact Strength for 5 alternatives)

      St1 = 184.2

      St2 = 266.9

      St3 = 301.5

      St4 = 342.8

      St5 = 380.0

      Sc1 = 600.0

      Sc2 = 944.0

      Sc3 = 1088.0

      Sc4 = 1034.0

      Sc5 = 1241.0

c     Merit function Calculations
```

I1 = 0.0

I2 = 0.0

I3 = 0.5

I4 = 0.5

P1 = 0.161

P2 = 0.177

P3 = 0.212

P4 = 0.242

P5 = 0.218

a12 = 0.068

a13 = 0.270

a14 = 0.235

a22 = 0.170

a23 = 0.225

a24 = 0.235

a32 = 0.218

a33 = 0.180

a34 = 0.235

a42 = 0.238

a43 = 0.216

a44 = 0.176

a52 = 0.306

a53 = 0.108

a54 = 0.118

C1=P1*(b*m**2*z**2)

C2=P2*(b*m**2*z**2)

C3=P3*(b*m**2*z**2)

C4=P4*(b*m**2*z**2)

C5=P5*(b*m**2*z**2)

a11 = 0.4 - (C1/(C1+C2+C3+C4+C5))

a21 = 0.4- (C2/(C1+C2+C3+C4+C5))

a31 = 0.4- (C3/(C1+C2+C3+C4+C5))

a41 = 0.4- (C4/(C1+C2+C3+C4+C5))

a51 = 0.4- (C5/(C1+C2+C3+C4+C5))

MF1= I1*a11+I2*a12+I3*a13+I4*a14

MF2= I1*a21+I2*a22+I3*a23+I4*a24

MF3= I1*a31+I2*a32+I3*a33+I4*a34

MF4= I1*a41+I2*a42+I3*a43+I4*a44

MF5= I1*a51+I2*a52+I3*a53+I4*a54

c Select material properties (Bending strength and Contact strength)

St = X1*St1+X2*St2+X3*St3+X4*St4+X5*St5

Sc = X1*Sc1+X2*Sc2+X3*Sc3+X4*Sc4+X5*Sc5

```
      TorC=((Sc*m*z)**2*b)/(29810*191**2)

C     3.0 Evaluate non-linear constraints
      IF (IPATH .EQ. 1 .OR. IPATH .EQ. 4) THEN
C
C       BENDING stress constraint.
        CONSTR(1) = 1.0 - ((10760*TorC) / (St*b*m**2*z))
C
C       Contact stress constraint.
        CONSTR(2) = 1.0 - ((191/Sc)*((29810*TorC)/(b*m**2*z**2))**0.5)

C       Input shaft max shear stress
        CONSTR(3) = 1.0 - ((25.46*TorC*1000)/(Di**3*Sy))

C       Output shaft max shear stress
        CONSTR(4) = 1.0 - ((101.86*TorC*1000)/(D0**3*Sy))
      END IF
C     4.0 Evaluate non-linear goals
C
      IF (IPATH .EQ. 1 .OR. IPATH .EQ. 5) THEN

        GOALS(1) = MF1*X1+MF2*X2+MF3*X3+MF4*X4 - 1.0
C
```

```
C       MASS OF gear goal
        GOALS(2) = 7.28*1000000000/(13.35*b*7880*m**2*z**2) - 1.0
C
C       Size goal
        GOALS(3) = 270/(5*m*z) - 1.0


C                   Torque goal
        GOALS(4) = (((Sc*m*z)**2*b)/(29810*191**2))/1000 - 1.0


c                   Mass goal for Shaft
        GOALS(5) = 1.5/(0.001225*(Di**2+D0**2)) - 1.0
      END IF
C
C     5.0 Return to calling routine
C
      RETURN
      END


      SUBROUTINE USRLIN (MLINCG, NDESV, NLINCO, NLINGO, NOUT,
     &                   DESVAR, COFLIN, RHSLIN)
C
C***  DUMMY ROUTINE.  Not used in the formulation
C
```

```
      INTEGER MLINCG, NDESV, NLINCO, NLINGO, NOUT
      REAL DESVAR(NDESV),
     &     COFLIN(MLINCG,NDESV), RHSLIN(NLINCO+NLINGO)
C
      RETURN
      END
```

Robust Design of Composite Structures – Multiscale Design Approach

The codes are for the math formulation for design of composite structures presented in Chapter 5. There are two formulations: one dealing with the design of structure (Table 5.5) and other dealing with the design of microstructures (Table 5.6). Each formulations have two files, that are, .f and .dat file.

FORTRAN file (.dat) for Robust Design of Structure (Table 5.5)

```
PTITLE  : Problem Title
   Design of a Composite Structure
NUMSYS  : Number of system variables
   6    0    0

SYSVAR   : System variable information
Tf      1  5.0 15.0 5.0  : skin thickness
Tc      2  70.0 90.0 70.0  : core thickness
Es      3  94060.0 204310.0 94060.0  :  skin modulus
Gc      4  21.6 536.6 21.6 : core modulus
Rs      5  1406.0 1651.0 1406.0   : skin density
```

Rc   6 3.4 86.3 3.4   : core density

NUMCAG : Number of constraints and goals

0 2 0 0 2 : nlinco,nnlinq,nnlequ,nlingo,nnlgoa

ACHFUN : Achievment function

1   : level

1 2 : level 1, 2 terms

(-1, 0.0) (-2, 1.0)

STOPCR : Stopping criteria

1 0 300 0.05 0.05

NLINCO : Names of nonlinear constraints

defco 1 : Constraint on del

weico 2 : Constraint on weight

NLINGO : Names of the nonlinear goals

Defle 1 : Goal on Deflection

Wts 2 : Goal on Weight

ALPOUT   : Input/output Control

1   1   1   1   1   1   1   1 1 1

USRMOD : Input/Output flags

```
   1   0   0   0
```

OPTIMP : Optimization parameters

```
   -0.05   0.5  0.05
```

ENDPRB : Stop reading the data file at this point

FORTRAN file (.f) for Robust Design of Structure (Table 5.5)

```
C
C
      SUBROUTINE USRINP (NDESV, NINP, NOUT, DESVAR)
C
C***  DUMMY ROUTINE.  Not used.
C
      INTEGER NDESV, NINP, NOUT
      REAL DESVAR(NDESV)
C
      RETURN
      END

      SUBROUTINE USROUT (NDESV, NOUT, DESVAR, LCONDF, LCONSV, LXFEAS)
```

```
C
C***  DUMMY ROUTINE.  Not used in the formulation
C
      INTEGER NDESV, NOUT
      REAL DESVAR(NDESV)
      LOGICAL LCONDF, LCONSV, LXFEAS
C
      RETURN
      END

      SUBROUTINE USRANA (NDESV, NOUT, DESVAR)
C
C***  DUMMY ROUTINE.  Not used in the formulation
C
      INTEGER NDESV, NOUT
      REAL DESVAR(NDESV)
C
      RETURN
      END

      SUBROUTINE USRMON (NDESV, NDEVAR, NMPRI, NNLCON, NNLGOA, NOUT,
     &                  DESVAR, DEVVAR, CONDEV, DEVFUN, GVAL)
C
```

```
C***  DUMMY ROUTINE.  Not used in the formulation
C
      INTEGER NOUT, NDESV, NDEVAR, NMPRI, NNLCON, NNLGOA
      REAL DESVAR(NDESV), DEVVAR(NDEVAR),
     &    CONDEV, DEVFUN(NMPRI), GVAL(NNLCON+NNLGOA)
C
      RETURN
      END
C+
C*****************************************************************
******
C
C Subroutine USRSET
C
C Purpose:  Evaluate non-linear constraints and goals.
C          NOTE - Do not specify the deviation variables
C
C---------------------------------------------------------------------
C Arguments      Name     Type   Description
C ---------      ----     ----   -----------
C Input:         IPATH    int    = 1   Evaluate constraints and goals
C                                = 2   Evaluate constraints only
C                                = 3   Evaluate goals only
C                NDESV    int    Number of design variables
```

```
C           MNLNCG  int   Max. number of nonlinear constraints
C                    and goals
C           NOUT    int   Output file/device number
C           DESVAR  real  Vector of current system variables
C
C Output:      CONSTR  real  Vector of constraint values
C           GOALS   real  Vector of goal values
C
C Input/Output:  none
C-------------------------------------------------------------------
C Common Blocks:  none
C
C Include Files:  none
C
C Calls to:     none
C-------------------------------------------------------------------
C Development History
C
C Modifications:
C
C*************************************************************************
******
C-
C
```

```
      SUBROUTINE USRSET (IPATH, NDESV, MNLNCG, NOUT, DESVAR,
     &                  CONSTR, GOALS)
C
C--------------------------------------
C    Arguments:
C--------------------------------------
C
      INTEGER IPATH, NDESV, MNLNCG, NOUT
C
      REAL DESVAR(NDESV)
      REAL CONSTR(MNLNCG), GOALS(MNLNCG)
C
C--------------------------------------
C    Local variables:
C--------------------------------------
C
c
      REAL Tf, Tc, Rs, Rc, Es, Gc P, Pl, B, L, g, q
      REAL Es1, Es2, Es3, Es4, Gs1, Gs2, Gs3, Gs4, Rs1, Rs2, Rs3, Rs4
      REAL Ec1, Ec2, Ec3, Ec4, Gc1, Gc2, Gc3, Gc4, Rc1, Rc2, Rc3, Rc4
      REAL dTf,dTc,dEs,dGc,dRs,dRc,aTf,aTc,aEs,aGc,bTf, bTc,bEs,bGc
      REAL a1,a2, dy1, dy2,EI, GA, DCI1, DCI2
      REAL g1Tf, g1Tc, g1Es, g1Gc,g2Tf,g2Tc,g2Rs,g2Rc, Def, Wt
```

```
C     1.0 Set the values of the local design variables (optional)
C
      Tf  =  DESVAR(1)
      Tc  =  DESVAR(2)
      Es  =  DESVAR(3)
      Gc  =  DESVAR(4)
      Rs  =  DESVAR(5)
      Rc  =  DESVAR(6)
C
C     2.0 Perform analysis relevant to non-linear constraints and goals
C
      P   = 1000.00
      PI  = 3.1415926
      B  = 50.0
      L   = 1500.00
      g   = 9.81
      q   = 1.5
c    Defining delta for each design variables
      dTf = 0.2
      dTc = 0.2
      dEs = 5.0
      dGc = 5.0
```

```
dRs = 4.0

dRc = 1.0

c    Calculating partial differential of each goal wrt design variables

a1 = 8*Es*B*(Tf**3/6)+(Tc*(2*Tf+Tc)**2/2)

a2 = 2*Gc*B*(2*Tf+Tc)**2/Tc


aTf = 4*Es*B*Tf**2+16*Es*B*Tc*(2*Tf+Tc)

aTc = 4*Es*B*(4*Tf**2+8*Tf*Tc+3*Tc**2)

aEs = (8*B*Tf**3/6)+(4*B*Tc*(2*Tf+Tc)**2)

aGc = 0

bTf = 8*Gc*B*(2*Tf+Tc)/Tc

bTc = 2*Gc*B*(2*Tf+Tc)*(Tc-2*Tf)/(Tc**2)

bEs = 0

bGc = 2*B*(2*Tf+Tc)**2/Tc


g1Tf = q*L**2*(L**2*aTf/a1**2+bTf/a2**2)/10**(3)

g1Tc = q*L**2*(L**2*aTc/a1**2+bTc/a2**2)/10**(3)

g1Es = q*L**4*aEs/a1**2

g1Gc = q*L**2*bGc/a2**2


g2Tf = 2*B*L*Rs*g/10**(9)

g2Tc = B*L*Rc*g/10**(9)

g2Rs = 2*Tf*B*L*g/10**(9)
```

```
      g2Rc = Tc*B*L*g/10**(9)

c     Calculating delta y for each goal

      dy1 = g1Tf*dTf + g1Tc*dTc + g1Es*dEs + g1Gc*dGc
      dy2 = g2Tf*dtf + g2Tc*dtc + g2Rs*dRs + g2Rc*dRc

c     Evaluating DCIs

      EI = (Es*B*Tf**3/6)+(Es*B*Tc*(2*Tf+Tc)**2/2)
      GA = Gc*B*(2*Tf+Tc)**2/Tc

      Def = (q*L**4/(8*EI))+(q*L**2/(2*GA))
      Wt = (2*Tf*B*L*Rs*g+Tc*B*L*Rc*g)/10**(9)

      DCI1 = (30 - Def)/(dy1)
      DCI2 = (40 - Wt)/dy2

C      3.0 Evaluate non-linear constraints
C
      IF (IPATH .EQ. 1 .OR. IPATH .EQ. 2) THEN
C
C            MAXIMUM DEFLECTION constraint.
```

```
      CONSTR(1) = DCI1 - 1.0
C
C        MAXIMUM WEIGHT constraint.

      CONSTR(2) = DCI2 - 1
      END IF
C
C   4.0 Evaluate non-linear goals
C
      IF (IPATH .EQ. 1 .OR. IPATH .EQ. 2) THEN
C
C
C       Deflection goal
C
        GOALS(1) = (30 - Def)/(5*dy1) - 1.0

C          Weight goal
C
        GOALS(2)=(40 - Wt)/(dy2*50) - 1.0

      END IF
C
```

```
C    5.0 Return to calling routine
C
      RETURN
      END
      SUBROUTINE USRLIN (MLINCG, NDESV, NLINCO, NLINGO, NOUT,
     &                  DESVAR, COFLIN, RHSLIN)
C
C***  DUMMY ROUTINE.  Not used in the formulation
C
      INTEGER MLINCG, NDESV, NLINCO, NLINGO, NOUT
      REAL DESVAR(NDESV),
     &     COFLIN(MLINCG,NDESV), RHSLIN(NLINCO+NLINGO)
C
      RETURN
      END
```

FORTRAN file (.dat) for Robust Design of Structure (Table 5.6)

```
PTITLE  : Problem Title, User Name and Date
          Design of a Cantilever beam, Gehendra  June 17, 2019


NUMSYS  : Number of system variables: real,integer,boolean
  4  0  0
SYSVAR   : System variable information
```

Vf 1 0.4 0.7 0.7 : Volume fraction

theta 2 30.0 60.0 30.0 : Angle

h 3 2.0 25.0 2.0 : Wall length

t 4 0.001 0.11 0.01 : Wall thickness

NUMCAG : Number of constraints and goals

0 4 0 0 4 : nlinco,nnlinq,nnlequ,nlingo,nnlgoa

ACHFUN : Achievment function

1 : level

1 4 : level 1, 4 terms

(-1,0.0) (-2,0.0) (-3,1.0) (-4,0.0)

STOPCR : Stopping criteria

1 0 100 0.02 0.02 : perfm cal, prt intereslts, Mcyles,sta dev, sta var

NLINCO : Names of nonlinear constraints

DCIds 1 : Constraint on DCIds

DCIes 2 : Constraint on DCIes

DCIdc 3 : Constraint on DCIdc

DCIgc 4 : Constraint on DCIgc

NLINGO : Names of nonlinear goals

Ds 1 : Goal on skin density

Es 2 : Goal on skin modulus

Dc 3 : Goal on core density

Gc 4 : Goal on core shear modulus

ALPOUT : Output Controls

1 1 1 1 0 1 0 1 1 1

USRMOD : User module flags

1 0 0 0

OPTIMP : Optimization parameters

-0.05 0.05 0.005 : VIOLIM, REMO, STEP

ADPCTL

1

ENDPRB :**STOP reading the data file at this point**

FORTRAN file (.f) for Robust Design of Structure (Table 5.6)

```
      SUBROUTINE USRINP (NDESV, NINP, NOUT, DESVAR)
```

```
C

C ***  DUMMY ROUTINE.  Not used
C
      INTEGER NDESV, NINP, NOUT
      REAL DESVAR(NDESV)
C
C
C
      RETURN
      END
      SUBROUTINE USROUT (NDESV, NOUT, DESVAR, LCONDF, LCONSV, LXFEAS)
C
C***  DUMMY ROUTINE.  Not used.
C
      INTEGER NDESV, NOUT
      REAL DESVAR(NDESV)
      LOGICAL LCONDF, LCONSV, LXFEAS
C
      RETURN
      END
      SUBROUTINE USRANA (NDESV, NOUT, DESVAR)
C
```

```
C***  DUMMY ROUTINE.  Not used.
      INTEGER NDESV, NOUT
      REAL DESVAR(NDESV)
C
      RETURN
      END

      SUBROUTINE USRMON (NDESV, NDEVAR, NMPRI, NNLCON, NNLGOA, NOUT,
     &                   DESVAR, DEVVAR, CONDEV, DEVFUN, GVAL)
C
C***  DUMMY ROUTINE.  Not used.

      INTEGER NOUT, NDESV, NDEVAR, NMPRI, NNLCON, NNLGOA
      REAL DESVAR(NDESV), DEVVAR(NDEVAR),
     &    CONDEV, DEVFUN(NMPRI), GVAL(NNLCON+NNLGOA)
C
      RETURN
      END
C+
C*****************************************************************
******
C
C Subroutine USRSET
C
```

```
C Purpose:  Evaluate non-linear constraints and goals.
C          NOTE - Do not specify the deviation variables
C
C-------------------------------------------------------------------
C Arguments      Name     Type   Description
C ---------      ----     ----   -----------
C Input:         IPATH    int    = 1   evaluate constraints and goals
C                                = 2   evaluate constraints only
C                                = 3   evaluate goals only
C              NDESV    int    number of design variables
C              MNLNCG   int    maximum number of nonlinear
C                              constraints and goals
C              NOUT     int    unit number of output data file
C              DESVAR   real   vector of design variables
C
C Output:        CONSTR   real   vector of constraint values
C              GOALS    real   vector of goal values
C
C Input/Output:   none
C-------------------------------------------------------------------
C Common Blocks:  none
C
C Include Files:  none
```

```
C
C Called from:    GCALC
C
C Calls to:       none
C--------------------------------------------------------------------
C Development History
C
C Author: BHARAT PATEL
C Date:   13 MARCH, 1992.
C
C Modifications:
C
C*****************************************************************
******
C-
C
    SUBROUTINE USRSET (IPATH, NDESV, MNLNCG, NOUT, DESVAR,
   &                  CONSTR, GOALS)
C
C-------------------------------------
C    Arguments:
C-------------------------------------
C
    INTEGER IPATH, NDESV, MNLNCG, NOUT
```

```
C
      REAL DESVAR(NDESV)
      REAL CONSTR(MNLNCG), GOALS(MNLNCG)
C
C---------------------------------------
C     Local variables:
C---------------------------------------

      REAL Vf,theta,h,t,a
      REAL Df,Dm,Ef,Em,D,G

C     Derivates
      REAL dsv,esv,dct,dch,dca,gct,gch,gca
      REAL delv,dela,delh,delt
      REAL delds,deles,deldc,delgc
      REAL lrlds,lrles,lrldc,lrlgc
      REAL ds,es,dc,gc,gi,cs,PI
      REAL DCIds,DCIes,DCIdc,DCIgc

C     Target
      REAL tds,tes,tdc,tgc
C
C     1.0 Set the values of the local design variables (optional)
```

```
C
      Vf = DESVAR(1)
      theta = DESVAR(2)
      h =  DESVAR(3)
      t =  DESVAR(4)

C     2.0 Perform analysis relevant to non-linear constraints and goals
C     a     = Angle in radian
C     den     = (1+cosa)*sina*h
C     D   = Density
C     E    = Modulus
C     f   = fibre
C     m    = matrix
      PI=3.1415
      a=PI*theta/180
      den =(1+cos(a))*sin(a)*h

C Set targets

      tds= 18.0
      tes = 9.0
      tdc = 7.0
      tgc = 3.0
```

```
C Properties of fibre and matrix

      Df= 1760

      Ef = 230000

      Dm = 1280

      Em = 37000


C Properties of core material

      D = 2700

      G = 26000


C Calculation of derivatives

C Calculation of derivatives - SKIN


      dsv = (Df-Dm)

      esv = (Ef-Em)


C Calculation of derivatives - Core


      dct = (2*D)/den

      dch = (2*t*D)/(den*h)

      dca = (2*h*t*D)*(cos(a)+(cos(a)**2)-sin(a)**2)/(den)**2

      cs = (1+cos(a)**2)

      gct = (cs)*G/den
```

```
      gch = ((cs)*t*G)/(den*h)
      gi=(1+cos(a))*sin(a)*sin(2*a)+(cs)*(cos(a)+(cos(a)**2)-sin(a)**2)
      gca=((t*h*G*gi)/den**2)
C
C Variation in design variables considered
      delv = 0.05
      dela = 0.3
      delh = 0.3
      delt = 0.01

C Calculation for change in goals for the variations considered

      delds = dsv*delv
      deles = esv*delv

      deldc = 0.1*(dct*delt+dch*delh+dca*dela)
      delgc = 0.1*(gct*delt+gch*delh+gca*dela)
C Lower Requirement limit for skin and core properties

      lrlds = 1200
      lrles = 80000
      lrldc = 2
      lrlgc = 450
```

```
C Calculation of robustness metrics

      ds = Df*Vf+Dm*(1-Vf)
      es = Ef*Vf+Em*(1-Vf)

      dc = 2*t*D/den
      gc = (cs)*t*G/den

      DCIds = (1600-ds)/delds
      DCIes = (es-lrles)/deles
      DCIdc = (dc-lrldc)/deldc
      DCIgc = (gc-lrlgc)/delgc

C     3.0 Evaluate non-linear constraints
C
      IF (IPATH  .EQ.  1  .OR.  IPATH  .EQ.  4) THEN

C   Robustness metrics

      CONSTR(1) = DCIds - 1.0

      CONSTR(2) = DCIes - 1.0
```

```
      CONSTR(3) = DCIdc - 1.0


      CONSTR(4) = DCIgc - 1.0
C
      END IF
C     4.0 Evaluate non-linear goals
C
      IF (IPATH .EQ. 1 .OR. IPATH .EQ. 4) THEN
        GOALS(1) = DCIds/20 -1.0


        GOALS(2) = DCIes/12 -1.0


        GOALS(3) = DCIdc/12 -1.0


        GOALS(4) = DCIgc/4 -1.0
C
      END IF
C
C
C     5.0 Return to calling routine
C
      RETURN
```

```
      END

      SUBROUTINE USRLIN (MLINCG, NDESV, NLINCO, NLINGO, NOUT,
     &                  DESVAR, COFLIN, RHSLIN)
C
C***  DUMMY ROUTINE.  Not used.
C
      INTEGER MLINCG, NDESV, NLINCO, NLINGO, NOUT
      REAL DESVAR(NDESV), COFLIN(MLINCG,NDESV), RHSLIN(NLINCO+NLINGO)

C---------------------------------------
C     Local variables:
C--------------------------------------

      RETURN
      END
```