Generation of spherical aberration with axially translating phase plates via extrinsic aberration

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Abstract: We show that spherical aberration of all orders can be generated as an extrinsic aberration in a system of axially translating plates. Some practical examples are provided. In particular for two phase plates that are 10 *mm* in diameter it is possible to generate from −10 to 10 waves of fourth-order spherical aberration with an axial displacement of +/− 0.65 *mm*. We also apply the phenomenon of extrinsic aberration for the generation of a conical wavefront and other non-axially symmetric wavefronts, in other words we propose what can be called a *generalized zoom plate.*

OCIS codes: (220.0220) Optical design and fabrication; (220.1000) Aberration compensation.

References and links

1. L. W. Alvarez and W. E. Humphrey, "Variable power lens and system," U.S. Patent 3,507,565 (1970).
2. L. W. Alvarez, "Two-element variable-power spherical lens," U.S. Patent 3,305,294 (1967).
3. A. W. Lohmann, "A new class of varifocal lenses," Appl. Opt. **9**(7), 1669–1671 (1970).
4. N. López-Gil, H. C. Howland, B. Howland, N. Charman, and R. Applegate, "Generation of third order spherical and coma aberrations by use of radially symmetrical fourth order lenses," J. Opt. Soc. Am. A **15**(9), 2563–2571 (1998).
5. I. A. Palusinski, J. M. Sasián, and J. E. Greivenkamp, "Lateral-shift variable aberration generators," Appl. Opt. **38**(1), 86–90 (1999).
6. T. Hellmuth, A. Bich, R. Börret, A. Holschbach, and A. Kelm, "Variable phase plates for focus invariant optical systems," Proc. SPIE **5962**, 596215 (2005).
7. E. Acosta and S. Bará, "Variable aberration generators using rotated Zernike plates," J. Opt. Soc. Am. A **22**(9), 1993–1996 (2005).
8. B. M. Pixton and J. E. Greivenkamp, "Spherical aberration gauge for human vision," Appl. Opt. **49**(30), 5906–5913 (2010).
9. E. Acosta and J. Sasián, "Phase plates for generation of variable amounts of primary spherical aberration," Opt. Express **19**(14), 13171–13178 (2011).
10. M. T. Chang and J. Sasian, "Variable spherical aberration generators" Proc. SPIE **3129**, 217 (1997).
11. J. P. Mills, T. A. Mitchell, K. S. Ellis, and P. K. Manhart, "Conformal dome aberration correction with counter-rotating phase plates," Proc. SPIE **3705**, 201 (1999).
12. J. Sasian, "Extrinsic aberrations in optical imaging systems," Adv. Opt. Technol. **2**(1), 75–80 (2013).
13. P. S. Tsai, B. Migliori, K. Campbell, T. N. Kim, Z. Kam, A. Groisman, and D. Kleinfeld, "Spherical aberration correction in nonlinear microscopy and optical ablation using a transparent deformable membrane," Appl. Phys. Lett. **91**(19), 191102 (2007).
14. E. Theofanidou, L. Wilson, W. Hossack, and J. Arlt, "Spherical aberration correction for optical tweezers," Opt. Commun. **236**(1–3), 145–150 (2004).
15. A. Kołodziejczyk and Z. Jaroszewicz, "Diffractive elements of variable optical power and high diffraction efficiency," Appl. Opt. **32**(23), 4317–4322 (1993).
16. Anees Ahmad, "High resonance adjustable mirror mount," U.S. Patent 4,726,671, (1988).
17. K. Kato, A. Ono, W. Inami, and Y. Kawata, "Plasmonic nanofocusing using a metal-coated axicon prism," Opt. Express **18**(13), 13580–13585 (2010).
18. K. Petelczyc, S. Bará, A. C. Lopez, Z. Jaroszewicz, K. Kakarenko, A. Kolodziejczyk, and M. Sypek, "Imaging properties of the light sword optical element used as a contact lens in a presbyopic eye model," Opt. Express **19**(25), 25602–25616 (2011).
19. B. Chebbi, S. Minko, N. Al-Akwaa, and I. Golub, "Remote control of extended depth of field focusing," Opt. Commun. **283**(9), 1678–1683 (2010).
20. F. M. Dickey and J. D. Conner, "Annular ring zoom system using two positive axicons," Proc. Soc. Photo Opt. Instrum. Eng. **8130**, 81300B (2011).

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1. Introduction

After the seminal work of Alvarez [1,2] a variety of papers have been published about the generation of aberration by the relative translation, or rotation of two complementary phase plates [3–11]. The basic theory is that the subtraction of the wavefront deformation introduced by one plate from the wavefront introduced by the other plate results in a particular type of aberration.

While the phenomenon of extrinsic (or induced) aberration is discussed in higher order aberration theory [12], it has not been fully exploited. Rather that translating or rotating two phase plates for generating aberration, it also possible to generate aberration by the axial displacement of two phase plates due to extrinsic aberration. Extrinsic aberration results when there is previous aberration incoming to an element that contributes aberration. Extrinsic aberration can be considered as a cross term, or synergy, or the interaction of an aberrated beam with an element that contributes aberration.

We first discuss extrinsic aberrations and then present some application for generating aberration. In particular we highlight the generation of fourth-order spherical aberration which is currently a subject of interest. This paper follows our previous work on phase plates [9]. The system of two phase plates presented here is quite a simple and can be used in ophthalmology, optical testing, microscopy, optical alignment, and other applications [13,14]. While the concept is implemented using refractive optics, it can be also implemented with diffractive optics as well [15].

2. Extrinsic aberrations

We assume a system of two phase plates in air where a light beam is transmitted and aberrated by the first plate by $W_1(\vec{\rho})$ and then the beam is again transmitted and aberrated by a second plate by $W_2(\vec{\rho})$. The total wavefront aberration $W(\vec{\rho})$ can be written as the sum of the individual aberration contributions and an extrinsic term

$$W(\vec{\rho}) = W_1(\vec{\rho}) + W_2(\vec{\rho}) + W_{12}(\vec{\rho}), \tag{1}$$

where $W_{12}(\vec{\rho})$ is the extrinsic aberration and $\vec{\rho}$ is the normalized aperture vector that specifies where a ray intersects the first plate in the system.

The ray error $\Delta\vec{\rho}$ at the second plate is given by

$$\Delta\vec{\rho} = \frac{d}{a}\vec{\nabla} W_1(\vec{\rho}), \tag{2}$$

where d is the distance between the plates and a is the physical radius of the plates. A given ray that intersects the first plate at the point defined by $a\vec{\rho}$ will intersect the second plate at the point defined by $a(\vec{\rho}+\Delta\vec{\rho})$. We assume that the incoming beam is collimated.

The aberration introduced by the second plate will change due to the change of ray position $\vec{\rho}+\Delta\vec{\rho}$. Then we will have $W_2(\vec{\rho}+\Delta\vec{\rho})$ rather than $W_2(\vec{\rho})$ for the aberration introduced by the second plate. The total aberration introduced by both plates is then

$$\begin{aligned} W_2(\vec{\rho}+\Delta\vec{\rho}) &= W_1(\vec{\rho}) + W_2(\vec{\rho}+\Delta\vec{\rho}) \cong W_1(\vec{\rho}) + W_2(\vec{\rho}) + \frac{1}{a}\vec{\nabla} W_2(\vec{\rho})\cdot\Delta\vec{\rho} \\ &= W_1(\vec{\rho}) + W_2(\vec{\rho}) + \frac{d}{a^2}\vec{\nabla} W_2(\vec{\rho})\cdot\vec{\nabla} W_1(\vec{\rho}) \end{aligned} \tag{3}$$

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where $W_{12}(\vec{\rho}) = \frac{d}{a^2}\vec{\nabla}W_2(\vec{\rho})\cdot\vec{\nabla}W_1(\vec{\rho})$ is the extrinsic aberration. If the intrinsic aberration contributed by each plate is equal but opposite in sign, $W_1(\vec{\rho}) = -W_2(\vec{\rho})$, we have that the total aberration is simply the extrinsic aberration

$$W_2(\vec{\rho}+\Delta\vec{\rho}) \cong \frac{d}{a^2}\vec{\nabla}W_2(\vec{\rho})\cdot\vec{\nabla}W_1(\vec{\rho}). \tag{4}$$

Let us first analyze the case of having the plates contributing an axially symmetric wavefront deformation in the form

$$W_1(\vec{\rho}) = -W_2(\vec{\rho}) = W_{3/2}\rho^{3/2} + W_2\rho^2 + W_3\rho^3 + W_4\rho^4 + W_5\rho^5. \tag{5}$$

Table 1 presents the total aberration when only on term is present at a time. Note that all aberration generated is positive in sign.

Table 1. Extrinsic Aberration Terms

$W_1(\vec{\rho})$	$\frac{d}{a^2}\vec{\nabla}W_2(\vec{\rho})\cdot\vec{\nabla}W_1(\vec{\rho})$
$W_{3/2}\rho^{3/2}$	$\frac{d}{a^2}\frac{9}{4}W_{3/2}^2\rho$
$W_2\rho^2$	$4\frac{d}{a^2}W_2^2\rho^2$
$W_3\rho^3$	$9\frac{d}{a^2}W_3^2\rho^4$
$W_4\rho^4$	$16\frac{d}{a^2}W_4^2\rho^6$
$W_5\rho^5$	$25\frac{d}{a^2}W_5^2\rho^8$

Noteworthy is that it is possible to generate a conical wavefront deformation similar to the aberration introduced by an axicon, focus, fourth, sixth, and eight-order spherical aberration as well. The amount of aberration is proportional to the axial displacement d between the plates.

Let us second analyze the case of having the plates contributing a non-axially symmetric wavefront deformation in the form

$$\begin{aligned} &W_1(\vec{\rho}) = -W_2(\vec{\rho}) = . \\ &W_{focus}(\vec{\rho}\cdot\vec{\rho}) + W_{astigmatism}(\vec{i}\cdot\vec{\rho})^2 + W_{coma}(\vec{i}\cdot\vec{\rho})(\vec{\rho}\cdot\vec{\rho}) + W_{line\ coma}(\vec{i}\cdot\vec{\rho})^3 \end{aligned} \tag{6}$$

Where $\vec{i}$ is a unit vector which defines a particular direction to define the aberration symmetry. Table 2 presents the total aberration when only on term is present at a time. The second order terms, focus and astigmatism, contribute also focus and astigmatism respectively as an extrinsic aberration. Coma and line coma produce fourth-order terms. Other aberration forms can be developed using Cartesian coordinates rather than polar coordinates.

Table 2. Extrinsic Aberration Terms

$W_1(\vec{\rho})$	$\frac{d}{a^2}\vec{\nabla}W_2(\vec{\rho})\cdot\vec{\nabla}W_1(\vec{\rho})$
$W_{focus}(\vec{\rho}\cdot\vec{\rho})$	$4\frac{d}{a^2}W_{focus}^2(\vec{\rho}\cdot\vec{\rho})$
$W_{astigmatism}(\vec{i}\cdot\vec{\rho})$	$4\frac{d}{a^2}W_{astigmatism}^2(\vec{i}\cdot\vec{\rho})$
$W_{coma}(\vec{i}\cdot\vec{\rho})(\vec{\rho}\cdot\vec{\rho})$	$\frac{d}{a^2}W_{coma}^2(8(\vec{i}\cdot\vec{\rho})^2(\vec{\rho}\cdot\vec{\rho})+(\vec{\rho}\cdot\vec{\rho})^2)$
$W_{line\ coma}(\vec{i}\cdot\vec{\rho})^3$	$9\frac{d}{a^2}W_{line\ coma}^2(\vec{i}\cdot\vec{\rho})^4$

The assembly of two phase plates as indicated above could be named as a *generalized zoom plate* due to the fact that axial displacements provide a variable amount of both axial and non-axial wavefront aberrations.

3. Examples

3.1 Spherical aberration

The case of fourth order spherical aberration is of practical interest as there are applications in ophthalmology, optical testing, and optical alignment [8,13,14] where having the ability to introduce a desired amount of spherical aberration would be of value. Here we optimize two phase plates as shown in Fig. 1 for generating fourth-order spherical aberration. One plate is plano concave and the other is plano convex. Both are optically strong. The design of the plates involves also specifying not only cubic, but fourth, and fifth order terms as to obtain as much as possible pure fourth-order spherical aberration, and for having no aberration for a spacing of 0.75 *mm*. We use a wavelength of 500 *nm*.

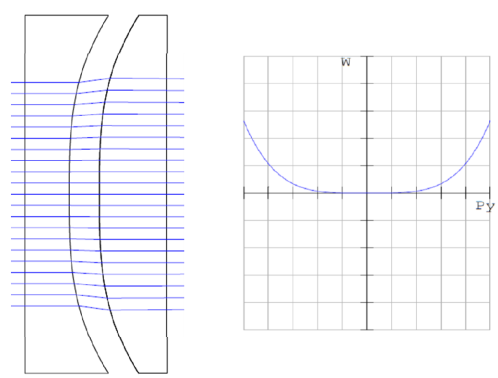

Fig. 1. Phase plates for the generation of +/− 10 waves of fourth-order aberration in their 1.35 *mm* displacement position. Note the strong plate asphericity. The beam diameter is 10 *mm*.

The relationship between the wavefront deformation $W_1(\vec{\rho})$ and the surface asphericity is

$$W_1(\vec{r}) = (n-1)(S_3r^3 + S_4r^4 + S_5r^5) \quad (7)$$

where n is the index of refraction, $r=\sqrt{x^2+y^2}=a\sqrt{\rho_x^2+\rho_y^2}$, is the radial coordinate in Cartesian coordinates, and S stands for the aspheric surface coefficients which are presented in Table 3.

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The axial displacement range is from 0.1 *mm* for −10 waves, 0.75 *mm* for zero waves, and 1.35 *mm* for + 10 waves. The maximum error in producing fourth-order spherical aberration is less than 0.05 waves at a displacement of 1.35 *mm*. The material used for the design of the plates is acrylic plastic $n \approx 1.49$.

Table 3. Surface Definition Aspheric Coefficients

	First plate	Second plate
S_3	3.3754e-3 mm^{-2}	3.3794e-3 mm^{-2}
S_4	5.3521e-5 mm^{-3}	3.0974e-4 mm^{-3}
S_5	−6.575e-6 mm^{-4}	−6.165e-6 mm^{-4}

This system of phase plates is sensitive to lateral misalignment. For an error of 0.001 *mm* about 0.25 waves of coma aberration is introduced. The tilt of one plate of 1 arc-minute produces 0.1 waves of coma which is not too large. Fortunately, some motion mechanisms based on tangential flexures can be designed to provide a high level of precision motion [16]. Most importantly, any coma present in the beam from misalignment of the phase plates can be corrected by moving both plates with respect to the light beam. For a relative displacement of the plates of 0.005 *mm*, there are about 1.25 waves of coma. This can be corrected by laterally moving both plates with respect to the beam by 0.15 *mm* which is not a demanding alignment.

This follows the relationship for the coma, W_{coma} , generated by an element that introduces spherical aberration, $W_{spherical}$, when there is a lateral displacement Δy :

$$W_{coma} = 4\frac{\Delta y}{a} W_{spherical} \tag{8}$$

3.2 Conical wavefront

The generation of a wavefront deformation in the shape of a cone is of interest. Axicons are optical elements shaped as cones and produce a conical wavefronts. They can be used in a variety of applications from optical alignment to the generation of special beams [17,18]. Recently the control of the light distribution in the focal region has been proposed by using two [19] and three [20] axicons. In this work, the extrinsic aberration would be a linear term proportional to displacement if we generate terms of the form $W_{3/2}\rho^{3/2}$ according to theory. Figure 2 (Left) shows a system of two plates that generate a conical wavefront as shown in Fig. 2 (Center). The aperture of these plates is 10*mm*, the surface coefficient is $S_{3/2} = 0.02 mm^{1/2}$, and the amount of conical wavefront generated is about 1.23 waves per millimeter of axial displacement.

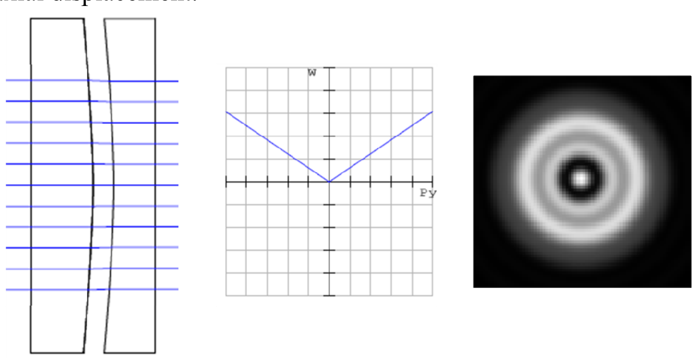

Fig. 2. Left, phase plates for generating a conical wavefront; center, wave fan for an axial displacement of 1 *mm* corresponding to 1.24 waves of wavefront amplitude; Right, point spread function when the beam is focused by a perfect lens. Note the strong plate asphericity. The beam diameter is 10 *mm*.

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One feature of this system of plates is that the conical wavefront is generated with smooth surfaces. This should be useful in providing a precise wavefront near the optical axis.

4. Conclusions

We have presented a *generalized zoom*, a system of phase plates that are optically strong for generating optical aberration by the phenomenon of extrinsic aberration. The basic theory for these plates is discussed and spherical aberration of all orders can be generated, including a linear term which represents a conical wavefront. Two specific examples are provided that illustrate the design and working of such phase plates. The generation of variable spherical aberration is of interest for a variety of applications, as well as the generation of variable amounts of a conical wavefront.

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