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A comparison of estimation methods for fitting Weibull, Johnson's S_B and beta functions to *Pinus pinaster, Pinus radiata* and *Pinus sylvestris* stands in northwest Spain

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Abstract

The purpose of this study was to compare the accuracy of the Weibull, Johnson's S_B and beta distributions, fitted with some of the most usual methods and with different fixed values for the location parameters, for describing diameter distributions in even-aged stands of *Pinus pinaster*, *Pinus radiata* and *Pinus sylvestris* in northwest Spain. A total of 155 permanent plots in *Pinus sylvestris* stands throughout Galicia, 183 plots in *Pinus pinaster* stands throughout Galicia and Asturias and 325 plots in *Pinus radiata* stands in both regions were measured to describe the diameter distributions. Parameters of the Weibull function were estimated by Moments and Maximum Likelihood approaches, those of Johnson's S_B function by Conditional Maximum Likelihood and by Knoebel and Burkhart's method, and those of the beta function with the method based on the moments of the distribution.

The beta and the Johnson's S_B functions were slightly superior to Weibull function for *Pinus pinaster* stands; the Johnson's S_B and beta functions were more accurate in the best fits for *Pinus radiata* stands, and the best results of the Weibull and the Johnson's S_B functions were slightly superior to beta function for *Pinus sylvestris* stands. However, the three functions are suitable for this stands with an appropriate value of the location parameter and estimation of parameters method.

Key words: probability density function; moments; maximum likelihood; conditional maximum likelihood, Knoebel and Burkhart's method.

Resumen

Comparación de métodos de estimación de ajuste de las funciones Weibull, S_B de Johnson y beta a masas de *Pinus pinaster, Pinus radiata* y *Pinus sylvestris* en el noroeste de España

El objetivo de este estudio fue comparar la precisión de las distribuciones Weibull, S_B de Johnson y beta, ajustadas por alguno de los métodos más habituales y fijando diferentes valores para los parámetros de localización, para describir distribuciones diamétricas en masas regulares de *Pinus pinaster*, *Pinus radiata* y *Pinus sylvestris* en el noroeste de España. Se midieron un total de 155 parcelas permanentes en masas de *Pinus sylvestris* en Galicia, 183 parcelas de *Pinus pinaster* en Galicia y en Asturias y 325 parcelas de *Pinus radiata* en ambas regiones para describir sus distribuciones diamétricas. Los parámetros de la función Weibull fueron estimados por las aproximaciones de los Momentos y Máxima Verosimilitud, los de la función S_B de Johnson por los estimadores condicionados de Máxima Verosimilitud y por el método de Knoebel y Burkhart, y los de la función beta por el método basado en los Momentos de la distribución.

Las funciones beta y S_B de Johnson fueron ligeramente superiores a la función Weibull en las masas de *Pinus pinaster*; las funciones S_B de Johnson y beta fueron más precisas en los mejores ajustes en las masas de *Pinus radiata*, y los mejores resultados de las funciones Weibull y S_B de Johnson fueron ligeramente superiores a los de la función beta en las masas de *Pinus sylvestris*. No obstante, las tres funciones son apropiadas para estas masas siempre que se elija un valor de localización y método de estimación de los parámetros apropiado.

Palabras clave: función de densidad; momentos; máxima verosimilitud; máxima verosimilitud condicionada; método de Knoebel y Burkhart.

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Introduction

Maritime pine (*Pinus pinaster* Ait.), Monterrey pine (*Pinus radiata* D. Don) and Scots pine (*Pinus sylvestris* L.) stands are important natural resources in northwest Spain (in the autonomous regions of Galicia and Asturias). These species are mainly present in pure stands, but sometimes also in mixed stands. The climate in this area is the most suitable for timber production in Spain, with an annual harvest volume of almost 50% of the total for the entire country. *Pinus* spp. and *Eucalytus globulus* Labill. are the most commonly used species in productive stands in this area of Spain.

According to the third Spanish National Forest Survey, pure stands of Maritime pine cover 383,632 ha in the region of Galicia and 22,499 ha in the adjoining region of Asturias; these stands are mainly derived from natural regeneration and occasionally from plantations. Exotic Monterrey pine covers 59,198 ha in Galicia and 17,167 ha in Asturias, always in plantations. Scots pine stands cover 63,196 ha and 5,565 ha in Galicia and Asturias respectively, also in plantations (DGCN 2006). The mean annual harvest volume for the period 1995-2002 in Galicia was 2,261,305 m^3 from *Pinus pinaster* stands (39% of the total for Galicia), 597,606 m^3 (10.5% of the total) from *Pinus radiata* stands, and 59,303 m^3 from *Pinus sylvestris* stands (1% of the total) (Dans *et al.,* 2005). In Asturias, the total annual harvest volume for the three species in the period 2007 and 2008 was 110,854 m^3 (17% of the total for Asturias) (SADEI 2010).

The use of growth models for the species enables promotion of the productive aspects of these species in northwest Spain. Diameter class models enable planning of various uses and provide data about stand structure. These types of models are used to estimate stand variables and their structure with a probability density function (PDF) or a cumulative distribution function (CDF), which is fitted to diameter distributions at breast height (1.3 m). Stand variables as the number of stems, stand basal area, total stand volume, the volumes of different timber assortments, etc. can also be computed with this models. Further, nonparametric methods are also used for describing mainly multimodal distributions (Maltamo and Kangas, 1998).

Forest managers may be interested in estimating the number of trees in different diameter classes in a stand, because the size of the diameter partly determines the industrial use of the wood and thus the price of the different products. Diameter distributions also provide information about stand structure, age structure, stand stability, etc. and enable planning of silvicultural treatments. Furthermore, tree diameter is an important factor in harvesting because it determines the type of machines used and how they perform during felling and transportation of the wood.

An array of diameter distribution models has been published in the relevant literature. These are largely applied to single-species, even-aged stands or plantations. The Weibull function (Bailey and Dell, 1973; Maltamo *et al.,* 1995; Zhang *et al.,* 2003), Johnson's S_B function (Johnson and Kitchen, 1971; Knoebel and Burkhart, 1991; Kamziah *et al.,* 1999) and the beta function (Zöhrer, 1969; Loetsch *et al.,* 1973) are the most commonly used distributions in forest research, and comparisons have shown the Weibull function to be the most suitable for estimating diameter distributions in many cases (e.g. Maltamo *et al.,* 1995; Borders *et al.,* 1987). In other studies, Johnson's S_B was superior to the beta function (Hafley and Schreuder, 1977) and was slightly more accurate than the Weibull distribution (Siipilehto, 1999; Zhang *et al.,* 2003).

In Spain authors such as Condés (1997), Álvarez-González (1997), Gorgoso (2003), and Palahí *et al.* (2007) have used different functions (Weibull, truncated Weibull, beta, Johnson's S_B and Charlier -type A) for fitting and modelling diameter distributions of different species.

The purpose of this study was to compare the accuracy of the Weibull, Johnson's S_B and beta distributions, fitted with some of the most usual methods and with different fixed values for the location parameters, for describing diameter distributions in even-aged stands of *Pinus pinaster*, *Pinus radiata* and *Pinus sylvestris* in the regions of Galicia and Asturias (NW Spain).

One step in future studies can be modelling the parameters of the functions obtained in the best fits with stand or site variables using parameters prediction models (PPM) or parameters recovery models (PRM) (Hyink and Moser, 1983).

Materials and methods

Data description

A total of 155 permanent plots in *Pinus sylvestris* stands throughout Galicia, 183 permanent plots in

Pinus pinaster stands throughout Galicia and Asturias (101 plots in Galicia and 82 plots in Asturias) and 325 permanent plots in *Pinus radiata* stands in both regions (212 plots in Galicia and 113 in Asturias) were measured to describe the diameter distributions. The size of the plots ranged from 400 to 1,200 m^2, depending on the stand density, in order to achieve a minimum of 30 trees per plot. The plots were established in even-aged and pure stands, and were installed to cover the existing range of combinations of age, number of trees per hectare and site quality.

In each plot, all trees were labelled with a number. Diameter at breast height (dbh, 1.3 m above ground level) was measured twice (the measurements were made at right angles to each other) to the nearest 0.1 cm, and the arithmetic mean of the two measurements was calculated. The minimum diameter inventoried was 5 cm. A total of 12,826, 25,818 and 16,854 diameter measurements were available for analysis in the study of *Pinus pinaster*, *Pinus radiata* and *Pinus sylvestris*, respectively.

The following stand and distribution variables were calculated from each plot-age combination: age, quadratic mean diameter (d_g), number of trees per hectare (N), stand basal area (G), dominant height (H_0), minimum diameter (d_{min}), maximum diameter (d_{max}), range, skewness and kurtosis. Summary statistics including mean, maximum and minimum values and standard deviation of the main stand variables are shown in Table 1.

Table 1. Summary statistics for the three types of stands under study

		Mean	Maximum	Minimum	Std. dev.
Pinus pinaster	*Age*	26.4	61.0	8.0	11.9
	d_g	22.2	41.5	6.8	8.1
	N	1,107.3	3,031.0	363.0	566.5
	G	36.1	75.7	7.1	15.0
	H_0	15.0	30.6	5.1	5.3
	d_{min}	9.4	21.7	5.0	4.5
	d_{max}	31.2	63.2	8.6	12.7
	Range	21.9	51.1	3.5	10.2
	Skewness	0.124	2.451	–0.848	0.438
	Kurtosis	–0.090	6.571	–1.274	0.902
Pinus radiata	*Age*	22.5	54.0	5.0	9.4
	d_g	21.9	48.4	5.7	9.1
	N	972.6	4,864	200	485.9
	G	32.7	87.1	4.9	14.5
	H_0	19.4	39.8	5.9	6.3
	d_{min}	8.6	31.6	5.0	5.3
	d_{max}	37.4	72.9	11.1	13.8
	Range	28.8	57.6	7.6	10.9
	Skewness	0.278	1.684	–0.975	0.362
	Kurtosis	–0.267	5.559	–1.376	0.810
Pinus sylvestris	*Age*	33.0	48.0	12	7.8
	d_g	17.4	27.9	7.46	4.4
	N	1,495.4	3,650	620	470.7
	G	34.2	74.2	4.2	14.4
	H_0	12.1	22,6	4.0	4.3
	d_{min}	7.3	17.1	5.0	2.7
	d_{max}	28.0	49.2	10.8	7.0
	Range	20.7	37.2	5.8	5.7
	Skewness	0.119	1.921	–0.531	0.348
	Kurtosis	–0.133	6.889	–1.134	0.986

d_g (cm): quadratic mean diameter, (N): number of trees per hectare, G (m^{2}ha^{-1}): basal area, H_0 (m): dominant height, d_{min}: minimum diameter, d_{max}: maximum diameter.

Model fitting

The Weibull function

The three-parameter Weibull CDF is obtained by integrating the Weibull PDF, and has the following expression for a continuous random variable x:

$$F(x)=\int_0^x\left(\frac{c}{b}\right)\cdot\left(\frac{x-a}{b}\right)^{c-1}\cdot e^{-\left(\frac{x-a}{b}\right)^c}\cdot dx=1-e^{-\left(\frac{x-a}{b}\right)^c} \quad [1]$$

where $F(x)$ is the cumulative relative frequency of trees with diameter equal to or smaller than x, a is the location parameter, b is the scale parameter and c is the shape parameter. Two methods of estimating the parameters of the Weibull distribution were compared: Maximum Likelihood and Moments.

– *Maximum Likelihood (ML):*

The ML estimation method used by Condés (1997), Nanos and Montero (2002), and Eerikäinen and Maltamo (2003) enables calculation of the distribution parameters with the following equations:

$$\frac{\sum_{i=1}^{n}(x_i-a)^c\ln(x_i-a)}{\sum_{i=1}^{n}(x_i-a)^c}-\frac{1}{c}=\frac{1}{n}\sum_{i=1}^{n}(x_i-a) \quad [2]$$

$$b=\left(\frac{1}{n}\sum_{i=1}^{n}(x_i-a)^c\right)^{\frac{1}{c}} \quad [3]$$

where n equals the number of sample observations in a Weibull distribution and x_i (cm) the diameter of each tree. To obtain Maximum Likelihood estimators, iterative methods are required (e.g., Harter and Moore, 1965; Bain and Antle, 1967; Ek *et al.*, 1975). In the present study, the LIFEREG procedure in SAS/STAT™ (SAS Institute Inc., 2003) was used to calculate parameters b and c of the Weibull distribution.

– *Method of Moments (MMW):*

The method of moments (Shifley and Lentz, 1985; Nanang, 1998; Río, 1999; Stankova and Zlatanov, 2010) is based on the relationship between the parameters of the Weibull function and the first and second moments of the diameter distribution (arithmetic mean diameter and variance, respectively):

$$b=\frac{\bar{d}-a}{\Gamma\left(1+\frac{1}{c}\right)} \quad [4]$$

$$\sigma^2=\frac{(\bar{d}-a)^2}{\Gamma^2\left(1+\frac{1}{c}\right)}\cdot\left[\Gamma\left(1+\frac{2}{c}\right)-\Gamma^2\left(1+\frac{1}{c}\right)\right] \quad [5]$$

where $\bar{d}$ is the arithmetic mean diameter of the distribution, σ^2 the variance and $\Gamma(i)$ is the Gamma function. Equation [5] was resolved by a bisection iterative procedure (Gerald and Wheatley, 1989).

Both methods of parameter estimation require knowledge of the location of parameter a of the Weibull distribution. In this study, four values of this parameter were compared: zero (in this case for the two parameter Weibull distribution), 50% of minimum observed diameter in each distribution, the minimum diameter observed, and the estimator proposed by Zanakis (1979), which has the following expression for the location parameter:

$$\begin{cases}\dfrac{x_1x_n-x_2^2}{x_1+x_n-2x_2} & \text{if} \quad x_2-x_1<x_n-x_2\\ x_1 & \text{otherwise}\end{cases} \quad [6]$$

where x_1, x_2 (cm) are the smallest diameters of the plot considered and x_n (cm) is the maximum diameter.

The Johnson's S_B function

The model of the S_B PDF (Johnson, 1949) has the following expression for a continuous random variable x:

$$f(x)=\frac{\delta}{\sqrt{2\pi}}\cdot\frac{\lambda}{(\varepsilon+\lambda-x)(x-\varepsilon)}\cdot e^{-\frac{1}{2}\left[\gamma+\delta\cdot\ln\left(\frac{x-\varepsilon}{\varepsilon+\lambda-x}\right)\right]^2} \quad [7]$$

where $f(x)$ is the probability density associated with diameter x, $\varepsilon < x < \varepsilon + \lambda$, $-\infty < \varepsilon < \infty$, $-\infty < \gamma < \infty$, $\lambda > 0$, and $\delta > 0$.

The model is characterized by the location parameter ε, the scale parameter λ, and the shape parameters γ and δ (asymmetry and kurtosis parameters, respectively). Two methods of estimating the Johnson's S_B parameters were compared: Conditional Maximum Likelihood (CML) and Knoebel and Burkhart's method (1991).

– *Conditional Maximum Likelihood (CML):*

The CML estimation method for the shape parameters γ and δ of Johnson's S_B PDF with predetermined values of ε and λ was computed by Johnson (1949), Hafley and Schreuder (1977) and Siekierski (1992). The values of the parameters are obtained with the following expressions:

$$\delta = \frac{1}{S_i} \quad [8] \qquad \gamma = \frac{-\bar{f}_i}{S_i} \quad [9] \qquad \bar{f}_i = \sum_{i=1}^{n} \frac{f_i}{n} \quad [10]$$

$$S_i^2 = \frac{\sum_{i=1}^{n} \left(f_i - \bar{f}_i \right)^2}{n} \quad [11] \qquad f_i = \ln\left(\frac{x_i - \varepsilon}{\varepsilon + \lambda - x_i} \right) \quad [12]$$

where x_i ($i = 1, 2, \ldots, n$) are the tree diameters. Parameter ε is predetermined with different values: zero, 25%, 50% and 75% of the minimum diameter of the plot. Zhang *et al.* (2003) and Scolforo *et al.* (2003) compared different values of this parameter of the S_B distribution fitted by this method. Parameter λ is established as the maximum diameter of the plot in all cases.

– *Knoebel and Burkhart's method (KB):*

This method, developed by Knoebel and Burkhart (1991) and used by, e.g., Zhou and McTague (1996) and Zhang *et al.* (2003), estimates the parameters of the Johnson's S_B PDF with the following equations:

$$\delta = \frac{Z_{95}}{\ln\left(\frac{D_{95} - \varepsilon}{\varepsilon + \lambda - D_{95}} \right) - \ln\left(\frac{D_{50} - \varepsilon}{\varepsilon + \lambda - D_{50}} \right)} \quad [13]$$

$$\gamma = -\delta \cdot \ln\left(\frac{D_{50} - \varepsilon}{\varepsilon + \lambda - D_{50}} \right) \quad [14]$$

$$\varepsilon = d_{\min} - 1.3 \quad [15] \qquad \lambda = d_{\max} - \varepsilon + 3.8 \quad [16]$$

where $d_{\min}$ (cm) is the minimum observed diameter of the plot, $d_{\max}$ (cm) is the maximum diameter of the plot, Z_{95} represents the standard normal value corresponding to the cumulative percentile of 95%, and D_{50} and D_{95} are estimates of the 50th and 95th data percentiles of the observed diameter distribution.

The beta function

The general expression of the beta PDF for a continuous random variable x is as follows:

$$f(x) = c \cdot (x - L)^{\alpha} \cdot (U - x)^{\gamma} \quad \text{if} \quad L \le x \le U$$
$$f(x) = 0 \quad \text{otherwise} \quad [17]$$

where $f(x)$ is the probability density associated with diameter x, U and L are the upper and lower limits of the beta PDF, c is the scaling factor of the function, and α and γ are the first and the second exponents that determine the shape of the distribution, respectively.

Only one method of estimating the beta parameters has been computed in forestry studies, and is based on the moments of the distribution.

– *Method of moments (MMB):*

The method of moments for the beta function (MMB) was first used by Loestch *et al.* (1973) and more recently by, e.g., Maltamo *et al.* (1995) and Palahí *et al.* (2007). Parameters of the function are obtained from the first and second moments of the distributions (the arithmetic mean diameter $\bar{d}$ and the variance (s^2), respectively) with the following expressions:

$$\gamma = \frac{\frac{Z}{s_{rel}^2 \cdot (Z+1)^2} - 1}{Z+1} - 1 \quad [18] \qquad \alpha = Z \cdot (\gamma + 1) - 1 \quad [19]$$

where:

$$Z = \frac{x_{rel}}{1 - x_{rel}} \quad [20] \qquad x_{rel} = \frac{\bar{d} - L}{U - L} \quad [21] \qquad s_{rel}^2 = \frac{s^2}{(U - L)^2} \quad [22]$$

The upper limit U is considered the maximum diameter inventoried in each plot, while several values were tested for the lower limit L: zero, 50% of the minimum observed diameter in each plot, and the minimum diameter of the plot.

The parameter c is estimated as:

$$c = \frac{1}{\int_L^U (x - L)^{\alpha} \cdot (U - x)^{\gamma} \, dx} = \quad 1$$

$$= \frac{1}{\frac{(-L+U)^{1+\gamma} \cdot \Gamma(1+\alpha) \cdot \Gamma(1+\gamma)}{\left(\frac{1}{-L+U} \right)^{\alpha} \cdot \Gamma(2+\alpha+\gamma)}} \quad [23]$$

where $\Gamma(i)$ is the Gamma function in the point i.

Model comparison

The consistency of the model and the fitting method used was evaluated by the bias, mean absolute error (MAE), and mean square error (MSE), with the following expressions:

$$\text{Bias} = \frac{\sum_{i=1}^{N} Y_i - \hat{Y}_i}{N} \quad [24] \qquad \text{MAE} = \frac{\sum_{i=1}^{N} \left| Y_i - \hat{Y}_i \right|}{N} \quad [25]$$

$$\text{MSE} = \frac{\sum_{i=1}^{N} \left(Y_i - \hat{Y}_i \right)^2}{N} \quad [26]$$

where Y_i is the relative frequency of trees observed value in each diameter class, $\hat{Y}_i$ is the theoretical value predicted by the model, and N is the number of data points.

The Bias, MAE and MSE values were calculated for each fit in the mean relative frequency of trees for all diameter classes (1 cm was considered) and plots combinations. For correct comparison of results, the Weibull PDF was considered instead of the Weibull CDF. The Kolmogorov-Smirnov (KS) statistic (D_n) for a given cumulative distribution function *F(x)* also was used in the evaluation and comparison of results: $D_n = \sup_x \left| F_n(x) - F(x) \right|$, where sup *x* is the supremum of the set of distances.

Results and discussion

The mean values of bias, mean absolute error (MAE), mean square error (MSE) in relative frequency of trees, and the mean value and the standard deviation of the Kolmogorov-Smirnov statistic (D_n) for the fits in *Pinus pinaster* stands (N = 183 plots) are shown in Table 2.

The corresponding statistics for *Pinus radiata* stands (N = 325 plots) are shown in Table 3, and those for *Pinus sylvestris* stands (N = 155 plots) in Table 4.

Figure 1 (a, b, c, d, e and f) shows the changes in bias and MSE in the relative frequency of number of trees in each diameter class for Maritime pine, Radiata pine and Scots pine in the fits with the lowest value of the MSE.

Figure 2 (a, b, c, d, e and f) shows the diameter distribution in number of trees per ha observed and fitted in two plots of *Pinus pinaster* stands, in two plots of *Pinus radiata* stands and in two plots of *Pinus sylvestris* stands.

The results did not reveal any major differences between the suitability of the beta, Weibull and Johnson's S_B functions when the best fits of these functions were compared for each species. However, the Weibull function clearly provided poorer results than the best fits with the Johnson's S_B and beta in terms of the KS statistic, although only in the *Pinus pinaster* stands. The bias and MSE for the fits with the three functions in each diameter class followed similar trends (Fig. 1), although the Weibull function fitted by the moments approach and fixing the situation parameter as 50% of minimum observed diameter provided the poorest results for the smallest diameter classes. The results obtained with all three functions were similar in terms of mean square error (MSE) for the most suitable values of the situation parameters of the functions and with the most accurate estimation of parameters methods.

The three functions provided the best results when applied to *Pinus sylvestris* data in terms of bias, MAE, MSE and the KS statistic value, except the Weibull function, which provided the lowest value of this statistic for *Pinus radiata* stands; the poorest results were those obtained for the *Pinus pinaster* stands. The lack of silvicultural treatments in this stands which are mainly derived from natural regeneration could be the reason because the skewness is the most heterogeneous of the three species. Further, the mean of the quadratic mean diameter also is the highest (see Table1).

The beta function was slightly superior to the Johnson's S_B and Weibull functions in terms of MSE in the best fit for *Pinus pinaster* stands, although the Johnson's S_B function was better in terms of the KS statistic according with Fonseca (2004) in Maritime pine in Portugal using the documented error index, and the Weibull function was the best in terms of MAE in these stands; the Johnson's S_B and beta function were more accurate in terms of MSE and the KS statistic in the best fits for *Pinus radiata* stands, although the Weibull function was also the best in MAE, and the best fit of the Weibull function was slightly superior to Johnson's S_B and beta function for *Pinus sylvestris* stands in MSE and MAE, although the Johnson's S_B function provided the best result for the KS statistic. The trees were smaller and data were less heterogeneous in these stands (see Table 1). The mean total value of bias may result in compensations of different sign errors and may be

Table 2. Mean values of bias, mean absolute error (MAE), mean square error (MSE) in relative frequencies of number of trees and mean value and standard deviation of the Kolmogorov-Smirnov (KS) statistic (D_n) for the fits with the Weibull, Johnson's S_B and beta functions in *Pinus pinaster* stands ($N = 183$ plots)

	Parameter *a*		Bias	MAE	MSE	D_n
Weibull (ML)	Zero		0.001943	0.017716	0.000609	0.189301 (0.077218)
	$0.5 \cdot d_{min}$		0.001804	0.017489	0.000579	0.180582 (0.078234)
	d_{min}		0.001371	0.017649	0.000567	0.178985 (0.077550)
	Zanakis (1979)		0.001691	0.018245	0.000626	0.150808 (0.073652)
Weibull (MMW)	Zero		0.001903	0.017740	0.000615	0.192473 (0.081696)
	$0.5 \cdot d_{min}$		0.001790	0.017470	0.000578	0.179934 (0.079871)
	d_{min}		0.001571	0.017806	0.000585	0.164778 (0.077225)
	Zanakis (1979)		0.001492	0.017807	0.000586	0.165670 (0.077239)
	Parameter ε	**Parameter λ**	**Bias**	**MAE**	**MSE**	D_n
Johnson's S_B (CML)	Zero	d_{max}	0.000333	0.019886	0.000733	0.228584 (0.072905)
	$0.25 \cdot d_{min}$	d_{max}	0.000860	0.018080	0.000607	0.176929 (0.075373)
	$0.5 \cdot d_{min}$	d_{max}	0.000922	0.017639	0.000566	0.147710 (0.068900)
	$0.75 \cdot d_{min}$	d_{max}	0.000962	0.017722	0.000565	0.127253 (0.063602)
Johnson's S_B (KB)	$d_{min} - 1.3$	$d_{max} - \varepsilon + 3.8$	0.001277	0.017905	0.000601	0.149869 (0.068316)
	Parameter *L*	**Parameter *U***	**Bias**	**MAE**	**MSE**	D_n
beta (MMB)	Zero	d_{max}	0.001336	0.017974	0.000614	0.181210 (0.075957)
	$0.5 \cdot d_{min}$	d_{max}	0.000958	0.017819	0.000586	0.166512 (0.074542)
	d_{min}	d_{max}	–0.00011	0.017892	0.000561	0.128995 (0.047864)

d_{min} (cm): minimum observed diameter; d_{max} (cm): maximum observed diameter; ML: maximum likelihood; MMW: method of moments for the Weibull function; CML: Conditional Maximum Likelihood; KB: Knoebel and Burkhart's method; MMB: method of moments for the beta function.

less important in the comparison of the results than the MSE, MAE and the KS statistic.

The best fit of the Johnson's S_B provided the best results in terms of the KS statistic in all three species, according with the idea of Fonseca *et al.* (2009) that theoretical knowledge shows and empirical studies corroborate that the four-parameter Johnson's S_B PDF provides greater generality in fitting diameter distributions than many of the commonly applied PDFs in forestry, such as the beta, gamma, and Weibull PDFs.

For all three species, the accuracy of the methods of moments and maximum likelihood for fitting the Weibull distribution were similar in terms of bias, MAE and MSE, and the best fits depended on the location parameter of the function. However, the lowest values

Table 3. Mean values of bias, mean absolute error (MAE), mean square error (MSE) in relative frequencies of number of trees and mean value and standard deviation of the Kolmogorov-Smirnov (KS) statistic (D_n) for the fits with the Weibull, Johnson's S_B and beta functions in *Pinus radiata* stands (N = 325 plots)

	Parameter *a*		**Bias**	**MAE**	**MSE**	D_n
Weibull (ML)	Zero		0.001915	0.015186	0.000446	0.155089 (0.046078)
	$0.5 \cdot d_{min}$		0.001831	0.015058	0.000436	0.146754 (0.046090)
	d_{min}		0.001673	0.015268	0.000438	0.147092 (0.045302)
	Zanakis (1979)		0.001991	0.015475	0.000454	0.118503 (0.042531)
Weibull (MMW)	Zero		0.001974	0.015128	0.000446	0.154579 (0.049656)
	$0.5 \cdot d_{min}$		0.001869	0.015014	0.000433	0.143547 (0.046668)
	d_{min}		0.001858	0.015305	0.000442	0.135774 (0.044917)
	Zanakis (1979)		0.001834	0.015297	0.000441	0.1360380 (0.044945)
	Parameter *ε*	**Parameter *λ***	**Bias**	**MAE**	**MSE**	D_n
Johnson's S_B (CML)	Zero	d_{max}	-0.00011	0.016989	0.000521	0.206931 (0.066404)
	$0.25 \cdot d_{min}$	d_{max}	0.000938	0.015452	0.000448	0.159089 (0.051500)
	$0.5 \cdot d_{min}$	d_{max}	0.001045	0.015146	0.000430	0.131295 (0.048113)
	$0.75 \cdot d_{min}$	d_{max}	0.001103	0.015188	0.000429	0.111041 (0.051211)
Johnson's S_B (KB)	$d_{min} - 1.3$	$d_{max} - \varepsilon + 3.8$	0.001219	0.015169	0.000449	0.126778 (0.052380)
	Parameter *L*	**Parameter *U***	**Bias**	**MAE**	**MSE**	D_n
beta (MMB)	Zero	d_{max}	0.001387	0.015276	0.000452	0.164744 (0.052223)
	$0.5 \cdot d_{min}$	d_{max}	0.000992	0.015163	0.000437	0.146358 (0.049756)
	d_{min}	d_{max}	0.000113	0.015301	0.000430	0.110342 (0.048228)

d_{min} (cm): minimum observed diameter; d_{max} (cm): maximum observed diameter. ML: maximum likelihood; MMW: method of moments for the Weibull function; CML: Conditional Maximum Likelihood; KB: Knoebel and Burkhart's method; MMB: method of moments for the beta function.

of the KS statistic with the Weibull distribution were obtained with the maximum likelihood fits in which the location parameter was fixed to the approximation of Zanakis (1979), for all three species. Similar values of bias, MAE and MSE were obtained with both methods (MV and MMW) for the same value of the location parameter for *Pinus radiata* and *Pinus sylvestris* stands. In *Pinus pinaster* stands, differences in accuracy between both methods (MV and MMW) of fit were generally higher.

The maximum likelihood method (ML) has been successfully applied with the two-parameter Weibull model of Nanos and Montero (2002) in *Pinus pinaster* stands in Spain, and by Eerikäinen and Maltamo (2003) in *Pinus kesiya* stands in Zambia and Zimbabwe. Zhang *et al.* (2003) obtained better results with this method

Table 4. Mean values of bias, mean absolute error (MAE), mean square error (MSE) in relative frequencies of number of trees and mean value and standard deviation of the Kolmogorov-Smirnov (KS) statistic (D_n) for the fits with the Weibull, Johnson's S_B and beta functions in *Pinus sylvestris* stands (N = 155 plots)

	Parameter *a*		Bias	MAE	MSE	D_n
Weibull (ML)	Zero		0.001513	0.011968	0.000325	0.170819 (0.049427)
	$0.5 \cdot d_{min}$		0.001298	0.011843	0.000315	0.161997 (0.049159)
	d_{min}		0.001045	0.012367	0.000331	0.153353 (0.048404)
	Zanakis (1979)		0.000894	0.012795	0.000363	0.129571 (0.043655)
Weibull (MMW)	Zero		0.001588	0.011887	0.000326	0.172764 (0.053303)
	$0.5 \cdot d_{min}$		0.001299	0.011753	0.000312	0.160586 (0.050792)
	d_{min}		0.001024	0.012474	0.000339	0.146158 (0.046715)
	Zanakis (1979)		0.001018	0.012457	0.000338	0.146434 (0.046765)
	Parameter ε	**Parameter λ**	**Bias**	**MAE**	**MSE**	D_n
Johnson's S_B (CML)	Zero	d_{max}	0.000678	0.013796	0.000407	0.184710 (0.054656)
	$0.25 \cdot d_{min}$	d_{max}	0.000878	0.012315	0.000331	0.147386 (0.046012)
	$0.5 \cdot d_{min}$	d_{max}	0.000718	0.012031	0.000314	0.123018 (0.041382)
	$0.75 \cdot d_{min}$	d_{max}	0.000490	0.012417	0.000332	0.105708 (0.034352)
Johnson's S_B (KB)	$d_{min} - 1.3$	$d_{max} - \varepsilon + 3.8$	0.000678	0.013796	0.000407	0.135175 (0.041212)
	Parameter *L*	**Parameter *U***	**Bias**	**MAE**	**MSE**	D_n
beta (MMB)	Zero	d_{max}	0.001423	0.012178	0.000332	0.150910 (0.047014)
	$0.5 \cdot d_{min}$	d_{max}	0.001018	0.012095	0.000319	0.136493 (0.043814)
	d_{min}	d_{max}	0.000200	0.012359	0.000319	0.111233 (0.029571)

d_{min} (cm): minimum observed diameter; d_{max} (cm): maximum observed diameter. ML: maximum likelihood; MMW: method of moments for the Weibull function; CML: Conditional Maximum Likelihood; KB: Knoebel and Burkhart's method; MMB: method of moments for the beta function.

than the moments and percentiles approaches in conjunction with the three-parameter function for mixed spruce-fir stands in northeastern North America. Nanang (1998) found both methods (moments and maximum likelihood) more accurate than the method of percentiles for neem plantations in Northern Ghana.

The most suitable value for the situation parameter of the Weibull distribution must be considered as 50% of the minimum observed diameter for the three species because this value provided the lowest values of MSE and MAE, except in the fit by ML in *Pinus pinaster* stands, in which the minimum observed diameter provided better results in terms of MSE. These values for the situation parameter have been found to be the most suitable in several studies (Hawkins *et al.,* 1988; Río, 1999; Gorgoso, 2003;

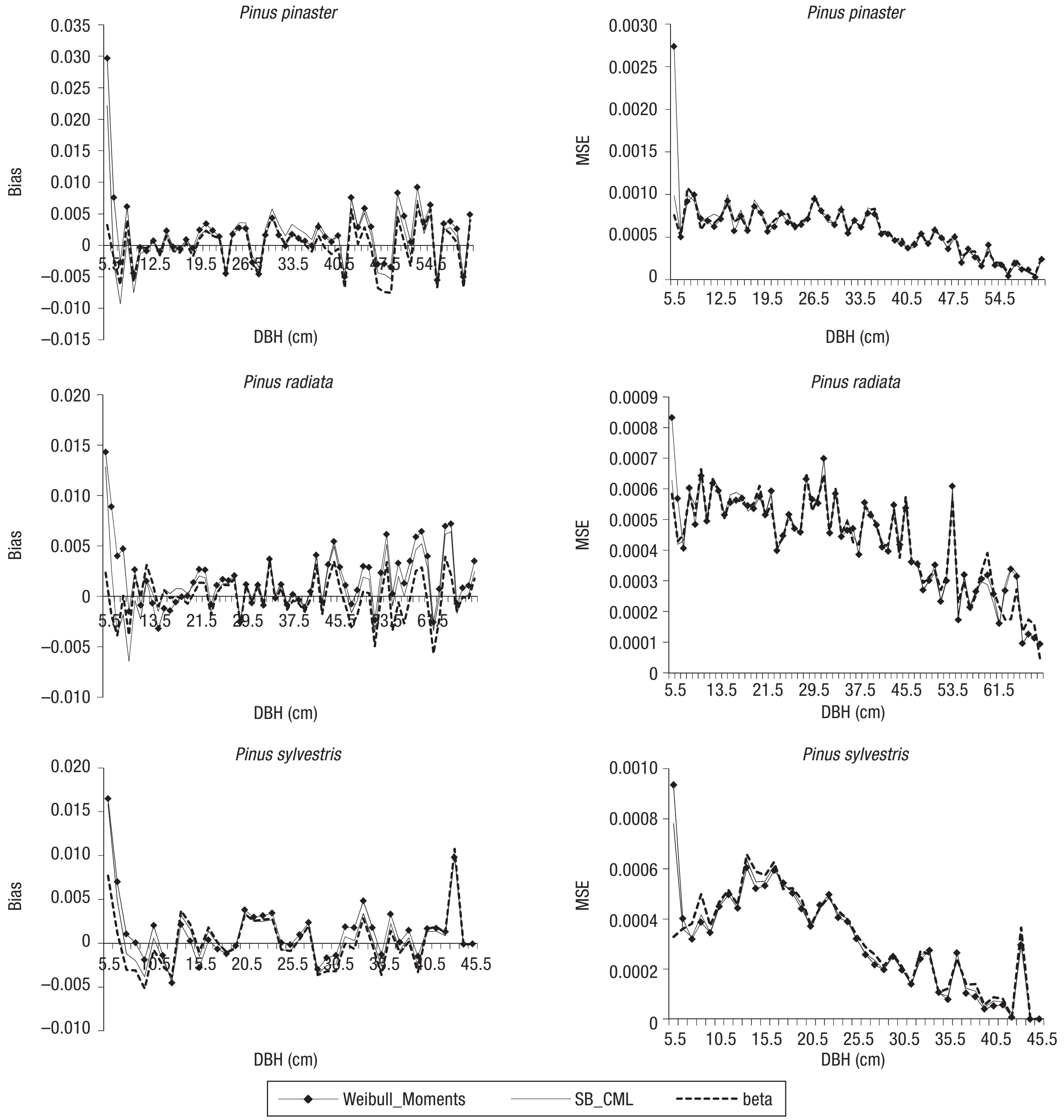

Figure 1. (a, b, c, d, e and f). Changes in bias and MSE in mean relative frequency of number of trees in each diameter class for the three species in the best fits with the Weibull distribution (Moments approach fixing the situation parameter *a* as 50% of minimum diameter in *Pinus pinaster*, *Pinus radiata* and *Pinus sylvestris* stands), Johnson's S_B distribution (fitted by Conditional Maximum Likelihood, fixing the situation parameter ε as 75% of minimum diameter in *Pinus pinaster* and *Pinus radiata* stands) and 50% of the minimum diameter in *Pinus sylvestris* stands, and the beta distribution with the lower limit of the function *L* as the minimum diameter observed in the distributions.

Cao, 2004; Liu *et al.,* 2004). However, the KS statistic provided the best results when the location parameter was fixed as the approximation reported by Zanakis (1979). A large number of truncated distributions were used in this study, and the results maybe be improved by using the truncated Weibull distribution model, as suggested by Palahí *et al.* (2007), but in the present study the part of the distributions between zero and the minimum diameter observed was ignored in analyses.

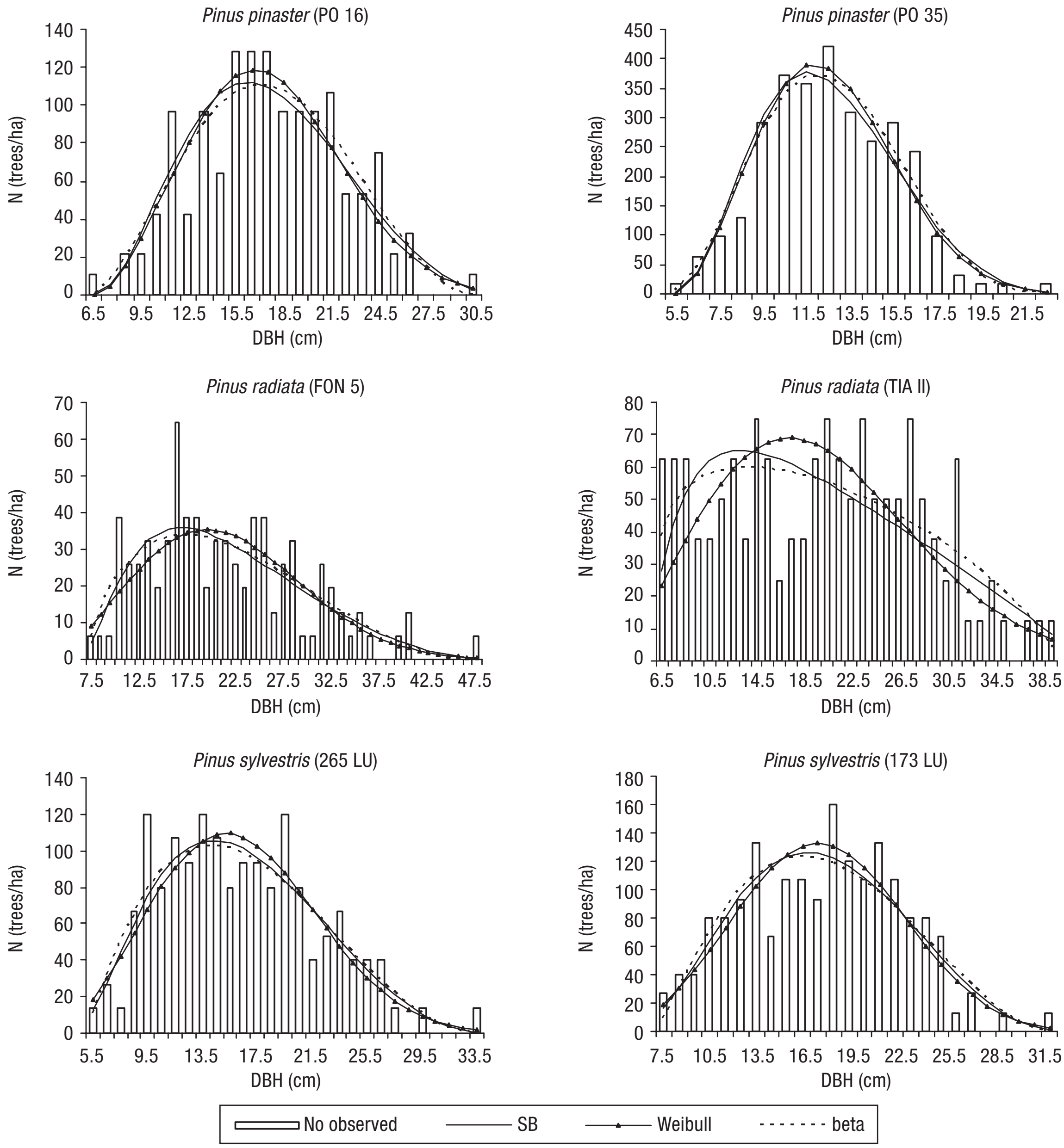

Figure 2. (a, b, c, d, e and f). Diameter distributions in number of trees per ha in 2 plots of *Pinus pinaster* stands observed and fitted by the S_B (CML with $\varepsilon = 0.75 \cdot d_{min}$), Weibull (Moments with $a = d_{min}$) and beta (with $L = d_{min}$) functions; in 2 plots of *Pinus radiata* observed and fitted by the S_B (CML with $\varepsilon = 0.75 \cdot d_{min}$), Weibull (Moments with $a = 0.5 \cdot d_{min}$) and beta (with $L = d_{min}$) functions and in 2 plots of *Pinus sylvestris* observed and fitted by the S_B (CML with $\varepsilon = 0.5 \cdot d_{min}$), Weibull (Moments with $a = 0.5 \cdot d_{min}$) and beta (with $L = d_{min}$) functions, with d_{min} the minimum diameter observed.

The two parameter model of the Weibull distribution was inferior to the best fits with the three parameter model for the three species. Nevertheless, in *Pinus sylvestris* stands, the two parameter model was the second best in terms of MSE and MAE in the accurate rank of fits with the Weibull distribution, with better results than when $a = d_{min}$ or a = Zanakis (1979). The smaller size of trees and lower heterogeneity in the data, except the kurtosis coefficient in case of *Pinus sylvestris* stands due to they have the highest number of trees per ha and the lowest range, appeared to improve the results with the Weibull function and concretely with the two parameter model because the lowest mean value of the minimum diameter also correspond to *Pinus sylvestris* stands (see Table 1). Maltamo *et al.* (1995) found that the two-parameter approach of the Weibull distribution gave better results than the three-parameter approach in the fits by maximum likelihood for modelling basal area diameter distributions in non-truncated stands of *Pinus sylvestris* and *Picea abies* in Finland.

Results with the Weibull distribution were less accurate in *Pinus pinaster* stands than the results reported by Cao (2004) in terms of the Kolmogorov-Smirnov (KS) statistic for loblolly pine in USA. However, the best result obtained by Cao (2004) in the KS statistic, was 0.142, obtained by fitting the Weibull distribution with the method Cumulative Distribution Function (CDF) Regression and computing the location parameter as 50% of the minimum diameter in the stand. This value is higher than the obtained with the Weibull distribution in the best fits in *Pinus radiata* stands (in the fits by ML (0.118) and MMW (0.136) in which the situation parameter were fixed as in Zanakis (1979) and MMW with $a = d_{min}$ (0.136)) and in *Pinus sylvestris* stands with the fits by ML (0.130) considering the location parameter as suggested by Zanakis (1979). However, the results for the three species in the fits by ML and MMW computing the location parameter as 50% of the minimum diameter are within the range obtained by Cao (2004) using 6 methods of fit. The best results for the Johnson's S_B and the beta functions were also better than the fits described by Cao (2004) in terms of KS statistic.

The results for the fits with the Johnson's S_B distributions showed that the best fits by Conditional Maximum Likelihood (CML) approach were better than those obtained with Knoebel and Burkhart's (KB) method. The poorest results were obtained when the situation parameter ε was zero, which may not be recommended for truncated distributions. This is consistent with the findings of Scolforo *et al.* (2003) for *Pinus taeda* stands in Brasil, and Zhang *et al.* (2003), who compared five and four methods of fitting Johnson's S_B distributions, respectively, and found the CML to be more accurate than Knoebel and Burkhart's method.

The best results in terms of the MSE and the KS statistic were obtained with the CML approach when the situation parameter ε of the S_B distribution was 75% of the minimum observed diameter in *Pinus pinaster* and *Pinus radiata* stands. However, in *Pinus sylvestris* stands the best results for MSE and MAE were obtained when ε was $0.5 \cdot d_{min}$. This value generally provided better results in terms of MAE and bias. Results with different values of this parameter of Johnson's S_B function have been compared in several studies. For example, Knoebel and Burkhart (1991) proposed $\varepsilon = d_{min} - 1.3$, Zhang *et al.* (2003) fitted Johnson's S_B distribution by fixing $\varepsilon = d_{min} - c$, with c equal to 0.5, 1, 1.5 and 2, and obtained the best results with c = 0.5; Scolforo *et al.* (2003) obtained the best results with the location parameter equal to 25% of the minimum diameter of the distributions, Palahí *et al.* (2007) used the minimum diameter of the distribution or zero in non-truncated distributions and Parresol (2003) 0.8 of minimum diameter. On the other hand, parameter λ can be obtained by maximizing a log-likelihood function (Mønness, 1982; Siipilehto, 1999; Palahí *et al.,* 2007) or can be predetermined as in Knoebel and Burkhart (1991), Zhou and McTague (1996) and Scolforo *et al.* (2003).

The beta distribution was also suitable for all three species. The values of MSE and MAE for *Pinus radiata* stands were similar to those obtained by Gorgoso *et al.* (2008) for *Betula alba* and *Quercus robur* stands in northwest Spain. Results for the MSE and the KS statistic show that the optimum value for the lower limit L of the distributions was the minimum diameter of the distributions in *Pinus pinaster* and *Pinus radiata* stands, with the upper limit U the maximum diameter inventoried. Zöhrer (1969, 1970) and Loetsch *et al.* (1973) proposed similar values for truncated distributions. However, similar results were obtained for MSE in *Pinus sylvestris* stands when parameter L was equal to 50% of the minimum observed diameter. Palahí *et al.* (2007) fixed this parameter at zero. A good fit of the function to the smallest diameter classes was observed when the lower limit of the function L is the minimum diameter inventoried (Fig. 1).

As conclusions the beta and the Johnson's S_B functions were slightly superior to Weibull function for *Pinus pinaster* stands; the Johnson's S_B and beta functions were more accurate in the best fits for *Pinus radiata* stands, and the best results of the Weibull and the Johnson's S_B functions were slightly superior to beta function for *Pinus sylvestris* stands. The three functions are suitable for this stands with an appropriate value of the location parameter and estimation of parameters method. The three functions provided the best results when applied to *Pinus sylvestris* data in terms of bias, MAE, MSE and the KS statistic value, except the Weibull function, which provided the lowest value of this statistic for *Pinus radiata* stands; the poorest results were those obtained for the *Pinus pinaster* stands.

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