Assignment Problems in Wildfire Suppression: Models for Optimization of Aerial Resource Logistics

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Abstract

Wildfire containment activities involve a combination of important decisions that affect the evolution of the fire and effective resource deployment. When aerial resources (in particular aircraft and helicopters) are used, two tasks are assigned to the aerial coordinator: the allocation of aerial resources to flight routes (circular paths that aerial resources follow such that they have common loading and discharge points) and refueling points.

In this paper, we introduce two models of linear integer programming to execute these tasks. The models are written using AMPL and the Gurobi solver engine and illustrated through examples.

The objective of these models is to provide automatic and rapid support for the coordination of the above mentioned tasks. In order to enhance the robustness of the models, the scheduling times and the characteristics of the aerial resources are also considered. These models aim at minimizing both the containment time of the fire and the total flight hours. The models will reduce the risk of aerial collision of resources by taking into account the maximum number of aerial resources that can simultaneously load water at the same point. Moreover, management of refueling points is also achieved.

Keywords: *wildfire management, aerial resources assignment, flight routes, refueling points, integer linear programming.*

Introduction

There has been extensive research on the propagation and containment of wildfires. It must be kept in mind that the evolution of fires, the ecological impact thereof, the socioeconomic impact, and underlying problems in fire management decisions are interrelated issues. This article addresses the fourth point, and we call attention to the fact that in recent decades, there have been numerous studies on fire management decision making (Miller and Ager, 2013). In terms of fire management, noteworthy aspects are prevention, detection, and management of fire extinguishing resources (Minas *et al.*, 2012).

In wildfires, the presence of several aerial resources working at once (some of them perhaps from other administrations) with different points for loading and discharging water (and other supplies such as foam fire suppressants and water enhancers) lead to an increase in air traffic. This increase in the deployment of aerial resources in wildfires brings with it a risk of air collisions. For example, cases where two flight routes share the same water intake but have different point of discharge or situations where aerial resources exceed the maximum work time as they wait for orders. Therefore, air traffic coordination is imperative to determine aerial resource instructions, flight routes, work duration, and risk of collision.

The design of decision support systems for wildfire containment is an active area of research in modern Operations Research, producing a number of applications. In this study, we focus on two tasks—the assignment of aerial resources to flight routes and the assignment of aerial resources to refueling points. In particular, if we suppose that the number and type of aerial resources have been determined, the first problem is to decide how to maximize the amount of water discharged on the fire affected areas. This is achieved by the appropriate allocation of aerial resources along flight routes subject to the following restrictions: no fronts are left unattended, the maximum number of aerial resources is respected by the flight route (a number determined by the coordinator), and the percentage of water for each front, chosen by the coordinator, is delivered accordingly.

Moreover, when the aerial resources that are assigned to a wildfire begin their resting period, they have to refuel. Assigning these aerial resources to refueling bases is a complex task because of the various factors involved, such as the time of arrival at the refueling points and the amount of fuel available at each point. Therefore, the second problem is to ensure optimal allocation of aerial resources to the refueling points such that the total time taken by all aerial resources in the operation is minimized. In the following sections, we discuss the decision problem that we want to address, the literature in which this problem is framed, and the methodology used for its resolution.

The problems

In Spain, compared to the relatively recent past, the number of aerial resources that can coincide in time and space in a forest fire has increased considerably. It was not until the 1970s that the aerial resources were deployed to fight forest fires. The organization of the air strike did not pose complications and was developed among the aerial resources pilots themselves. However, in the present-day scenario, lack of proper air coordination substantially compromises flight safety as well as the effectiveness of the mission (*cf.* Vélez, 1999).

Consequently, in the past decade, several research projects have been carried out by companies from the public and private sector domains. The main objective of these projects was to develop advanced technologies to fight wildfire fires, thus reducing their number and scope, and to create a safety protocol that significantly reduces the accident rate (technical, brigade, and pilots).

In Spain, the airspace is classified by four 500-ft altitude intervals, depending on the type of aerial resources (*cf.* Couceiro-López, 2007). Aircraft aerial coordinators operate in the fourth interval (more than 1500 ft), which enables them to perform their work more efficiently. The air operations coordinator is responsible for the operations and the organization of air assets in the event of a forest fire. The main objective of the air operations coordination is to ensure the safety and efficacy of the air assets involved in the operations. The altitude interval of 1000–1500 ft comprises the flight routes of heavy aircraft, which are characterized by higher tonnage and cruising speed. Light aircraft fly at altitudes between 500–1000 ft, as they are slower and less powerful, but more economical and with greater maneuverability. Helicopters fly at lower altitudes (less than 500 ft) as they offer greater maneuverability albeit at slower cruising speeds. So, this division of airspace makes it possible for aerial resources with similar characteristics to be grouped together. Moreover, the division of airspace enables the simplification the models, reducing variability in fuel consumption by weight, speed, and altitude; these variables can be considered constant for each flight route.

Specifically, helicopters, both monoturbine and biturbine, are light and easy to maneuver (Bell 407, Eurocopter AS350 Series B3, or Eurocopter AS355N, among others) or heavy and capable of carrying large amounts of water that can be dropped anywhere except the most virulent foci (Ka32 or Kamov AS330J Puma Eurocopter). Airplanes can be classified, as in the case of helicopters, as light (Air Tractor AT802) and heavy (Canadair CL215); each type has different functions. One of the most important aspects of managing flight routes is the homogeneity of the aircraft; grouping similar aerial resources produces uniform cadence of the flight route. Aerial resources work is organized into flight routes, such that groups of aerial resources fly over the fire affected area by forming a circuit pattern, from which each aerial resource has access to a water intake point. Naturally, if these operations are disorganized (*i.e.*, the status of each aerial resource is unknown), they pose a high risk of collision.

The fuel used by the aerial resources is limited, and in large forest fires, more than one instance of refueling per aircraft is necessary on a given day. In Spain, as defined in the regulation *Circular Operativa 16-B (Dirección*

General de Aviación Civil, Ministerio de Fomento de España, 1995), refueling is performed while the aerial resources rest on the ground. As per these regulations, aerial resources must have a minimum 40-minute break between every two hours of consecutive flight. Therefore, to realize efficient operations, it is important that refueling does not exceed the break period.

Theoretical framework

The location of resources and their selection for the so-called initial attack (namely, the actions taken by the first resources to arrive at a wildfire to protect lives and property, and prevent further extension of the fire) have been studied in literature using models from operations research as well as simulation tools. The assignment problem is a classical problem in linear programming, having first appeared in the work of Votaw and Orden (1952) and becoming more widespread with the publication of the Hungarian solution (*cf.* Kuhn, 1955). A recent review of this problem and its generalizations can be found in Pentico (2007).

Determining the optimal flight plan, including the number and type of resources needed to extinguish a forest fire, is not an easy task given the wide range of possibilities. In Islam and Martell (1998), a tool is proposed for designing air tanker dispatch policies that minimize the initial attack response times. They also stress the importance of taking into account the traffic congestion in aerial resources, which is considered in the models presented in this work. The first model we propose is similar to a three-dimensional assignment problem (*cf.* Geetha and Vartak, 1994) because the aerial resources are assigned to flight routes that are in turn allocated to different fire fronts. However, a slightly more general situation arises here because several objectives are simultaneously considered (*cf.* Geetha and Nair, 1993). Given that the purpose is to maximize water discharge on the fronts and minimize distances between aerial resources and flight routes, specific restrictions are introduced in aerial resources and water points related to capacity as well as preferences in terms of the percentage of water received at the fronts.

A tactical decision model, which determines the optimal combination of suppression resources to minimize a certain cost function, was proposed by Donovan and Rideout (2003). The models put forth in this work are similar in certain respects to the abovementioned models. However, this study emphasizes on the problem of maximizing water disbursement and minimizing refueling times.

The quality of data and its availability can limit the subsequent analysis. We believe that this issue is relevant, and in fact, there have been a number of recent works published on this topic, including Calkin *et al.* (2014) and Stonesifer *et al.* (2016). In these works, statistical studies are performed wherein variables such as the number of water downloads by air assets, size of fire, and time of day are considered. They mention the problem of the lack of data regarding the number of downloads and their effectiveness and the importance of finding a solution to this problem. This fits with our work because although our algorithms offer a response to the demand for wildfire coordinators, it is necessary to have sufficient data to reliably verify the effectiveness of operations and to ascertain the need for possible improvements.

In Dimopoulou and Giannikos (2004), an integrated tool comprising a geographic information system, a linear programming model, and simulation tools is proposed to address the problem of allocating land resources to different areas of a fire. With the addition of tools designed by our team, our study can complement the integrated tool because we address additional issues such as air assets, flight routes, and refueling points.

Martell (2015) offers a review of the existing research on recent forest and wildfire management decision support systems. The author also describes a general working procedure for wildfire fires, which states that "In the case of amphibious air tankers, the air attack officer must decide from which body of water each air tanker picks up water and when and where each air tanker drops its load". This is the starting point of our work, and hence it is emphasized.

Methodological approach

We propose two models of linear integer programming to solve the two decision problems described in the introduction and framed in the line of optimal allocation of fire extinction resources.

The first model is designed to maximize the output per hour of aerial resources flight time. As regards the sets aerial resources (helicopters and airplanes), we group these resources such that the resources of the same group can be integrated (all of them or only a proper subset) along the same flight route. We also consider a set of water points and a set of fire fronts. In general, each resource can only be assigned to a certain water point.

With respect to the parameters of interest, the coordinator must know the capacity (in liters) of each water point. The coordinator must also know the maximum number of resources for each group that can be assigned to a single flight route, which consists of a front and water point. In addition, the following are also known: the number of downloads per hour for an aerial resource, the distance from the resource location to each front, the number of active flight routes that share a given water point, and the amount of water (expressed as the relative frequency) as a proportion of the total capacity of resources that is intended for each front.

It should be noted that cruising speeds and topographical features are not explicitly considered. However, the performance on the flight routes implicitly reflects these elements. More precisely, the yield is estimated by statistical regression functions, the details of which we omit here, based on the characteristics of the aerial resources and the distances between the water points and fronts; this allows the coordinator to estimate the time required for an aerial resource to travel the route, which is, in fact, the time between two consecutive water discharges.

Another important aspect is the priority (threat-based ranking) assigned to the different fronts of a wildfire. In Spain, air coordinators allocate more water to the most important fronts. We must emphasize that this model only considers those fronts that are selected by the coordinator for attack.

The second model manages the allocation of aerial resources to refueling bases. The model should take into account the following aspects. First, it should consider the number of aerial resources that can simultaneously refuel at a given base. For example, if a refueling point is a tanker in the middle of an open field, *i.e.*, a single tanker with a single hose, the simultaneous supply of fuel to multiple aerial resources becomes impossible. The amount of fuel at each base and the fuel capacity of each aerial resource are also relevant factors. Another possible scenario is one where the refueling base is close to the fire, but the amount of available fuel is less than that required by the aerial resource. Multiple aerial resources should not be assigned to the base despite its proximity to the fire. Moreover, an aerial resource may prefer to wait in the air while another aerial resource completes refueling as opposed to going to a base further away (thereby losing time), except in the case where the time spent waiting is greater than having to go to another base. Moreover, once the optimal allocation is determined, the aerial resources are issued a warning regarding the new capacity of refueling points, to ensure that fuel is replenished where necessary.

As regards the sets for which we need information, we consider a set of resources (again helicopters and airplanes), a set of refueling bases, and a set of periods of time. As regards the parameters, we take into account the fuel load of each aerial resource, the refueling time required by each aerial resource, the quantity of fuel available in each base, the number of aerial resources that can refuel simultaneously on each base, and the time it takes to move each aerial resource to each base. In addition, we must consider that some aerial resources do not have the capability to refuel at all bases.

It is important to remark that this paper introduces two models to solve common tasks during a wildfire suppression operation. These two tasks are interrelated because they begin at the instance when an aerial resource fleet is assigned a schedule. This is a common and difficult problem for wildfire coordinators to solve. To illustrate the use of the proposed models, we introduce a scheme that a coordinator should follow. However, the time that this decision takes place may be right at the beginning of the fire extinction protocol, or at different times during the extinction process of the said fire. This is because aerial resources are assigned different areas, and need time to refill their water tanks. Hence, when a set of aerial resources are assigned roles in the fire extinction process, the coordinator is faced with the problem of deciding which aerial resources have to attack which front, and their corresponding refill points (*i.e.*, the aim of the allocation model of aerial resources to flight routes). After the allocation of the task by the coordinator, the aerial resources accordingly start working on the fire. Subsequently, a set of aerial resources presently working on the fire will be required to take breaks (due to aviation regulations). At such instances, the aerial resources will perform refueling operations. It is at this moment that a new problem arises, the problem of assigning the aerial resources to the refueling points (*i.e.*, the aim of the allocation model of aerial resources to refueling points). To address this problem, time discretization is essential because of the waiting time of each aerial resource needed to carry out these tasks. Once the refueling task is completed, the aerial resources will return to their previously assigned work plan. Therefore, it is paramount to consider the time needed to fly to the refueling point as well as the time needed to return.

After contextualizing the use of these two models, it is important to emphasize when they should be executed. The first model would be executed/re-executed whenever a new aerial resource enters or abandons the extinguishing protocol as well as when any relevant changes in the evolution of the fronts occur. The resolution time for this model must be low in order to ensure efficient operations. The second model will be executed after the coordinator determines when and where each aircraft would run out of fuel. This will give the coordinator, pilots, and ground crew sufficient time to make the necessary preparations. Hence, it is important to note that even though the two models are related, they work to solve different and independent situations.

Problem formulation

In this section, we formulate our problems as mathematical programming models.

Model for allocation of aerial resources to flight routes

Notation and decision variables

Sets

i, i', $\mathcal{I}$ = indices and current set of aerial resources involved in the extinguishing protocol.

g, $\mathcal{G}$ = index and current set of aerial resources groups.

Each group represents all those aerial resources that can be integrated (all of them or only a proper subset) in the same flight route. We have explained the interest and the construction of flight routes in the previous section called "The problems".

p, $\mathcal{P}$ = index and current set of water recharge points.

k, k', $\mathcal{K}$ = indices and current set of fire fronts.

$\mathcal{P}_i$ = the current set of water points that can be assigned to resource i.

$\mathcal{G}_i$ = the current group of aerial resources to which resource i is assigned.

Parameters

CAP_i = the carrying capacity in liters of resource i.

NUI_{gpk} = the maximum number of resources of group g that can be assigned to the flight route given by fire front k and water recharge point p.

DOI_{gpk} = the number of downloads per hour performed by an aerial resource of group g in the flight route, given by fire front k and water recharge point p.

DIS_{ik} = the distance from the current position of aerial resource i to fire front k.

NUW_p = the number of current flight routes that can share water recharge point p.

PER_k = the percentage of water (expressed as the relative frequency) intended for front k.

Decision variables

We use four sets of decision variables in our formulation.

a_{ipk} = the binary variable that takes value 1 if aerial resource i is assigned to a flight route given by water point p and fire front k, and 0 otherwise.

m_k = the real variable that measures lack of water used in fire front k relative to the amount initially assigned (sometimes this allocation may not be satisfied in full).

f_k = the binary variable that takes value 1 if fire front k is left unattended, and 0 otherwise.

w_{gpk} = the binary variable that takes value 1 if group g is assigned to a flight route given by water point p and fire front k, and 0 otherwise.

Objective function

$$
\begin{aligned}
\max \quad & \sum_{i\in\mathcal{I}} \sum_{g\in\mathcal{G}_i} \sum_{p\in\mathcal{P}_i} \sum_{k\in\mathcal{K}} DOI_{gpk} CAP_i a_{ipk} - \sum_{k\in\mathcal{K}} M \ \ (m_k + f_k) \\
& - \sum_{i\in\mathcal{I}} \sum_{p\in\mathcal{P}_i} \sum_{k\in\mathcal{K}} \frac{DIS_{ik}}{\max\limits_{i'\in\mathcal{I}, k'\in\mathcal{K}} DIS_{i'k'}} a_{ipk}
\end{aligned}
\tag{1}
$$

Here, M is a constant with a sufficiently large value. We use M to give priority to the minimization of the difference between actual and assigned water use for all fire fronts. In addition, in the term involving distances from aerial resources to fronts, a proportion is used instead of using DIS_{ik} directly, so that the distances are rescaled making smaller distances even smaller. Therefore, in case of ties during resource selection, the closest ones will be chosen accordingly.

Constraints

The relationships that describe the real-world model are translated in our formulation via mathematical constraints.

$$
\sum_{p\in\mathcal{P}_i} \sum_{k\in\mathcal{K}} a_{ipk} = 1, \quad \forall i \in \mathcal{I} \tag{2}
$$

$$
\sum_{i\in\mathcal{I}:\, g\in\mathcal{G}_i,\, p\in\mathcal{P}_i} a_{ipk} \leq NUI_{gpk}, \quad \forall g \in \mathcal{G}, \ \forall p \in \mathcal{P}, \ \forall k \in \mathcal{K} \tag{3}
$$

$$
\sum_{i\in\mathcal{I}} \sum_{p\in\mathcal{P}_i} a_{ipk} \geq 1 - f_k, \quad \forall k \in \mathcal{K} \tag{4}
$$

$$\sum_{g\in\mathcal{G}}\sum_{k\in\mathcal{K}} w_{gpk} \leq NUW_p, \quad \forall p \in \mathcal{P} \tag{5}$$

$$w_{gpk} \geq a_{ipk}, \quad \forall i \in \mathcal{I},\ \forall g \in \mathcal{G}_i,\ \forall p \in \mathcal{P}_i,\ \forall k \in \mathcal{K} \tag{6}$$

$$\sum_{i\in\mathcal{I}}\sum_{p\in\mathcal{P}_i} CAP_i a_{ipk} \leq PER_k \sum_{i\in\mathcal{I}} CAP_i + m_k, \quad \forall k \in \mathcal{K} \tag{7}$$

The goal of this model is to assign a current set of aerial resources to a current set of flight routes such that the greatest possible download of water/retardant is achieved (this objective corresponds to equation (1)). In addition, we take into account the distances between air resources and fronts. Also it aims to minimize the total of such distances, but as a result of rescaling used in the third term, this second goal we are giving less weight.

A number of restrictions must be taken into account. First, each aerial resource must be assigned to a single flight route (equation (2)), and the maximum number of aerial resources in a flight route must not be exceeded (equation (3)) [1] The fact that many aerial resources fly along the same route implies that there is little space between them; thus, inefficient management may result in collisions or loss of valuable time. In addition, each of the fronts of the fire must be assigned at least one aerial resource. That is, no front

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[1] Equation (2) specifies that each aerial resource must be assigned to a single flight route. This may lead to an infeasibility in the model when there is insufficient airspace along the flight routes for all considered aerial resources (equation (3) will not be satisfied). We assume that the air coordinator selects a number of aerial resources taking this fact into consideration. In the case of that this infeasibility occurs, a slack variable must be added that represents the violation of equation (2). This would result in a slight variation of the model, which does not pose significant difficulty, to obtain a more clear and concise representation.

considered by the coordinator may be left unattended (equation (4)). It is important to determine which aerial resources can access the water points. If the operation takes place along a coastal region, the aerial resources may recharge at sea simultaneously, if conditions permit. However, if the charging point is a pit, in many cases, it is accessible only by certain aerial resources, such as helicopters equipped with helitanks; in such scenarios, access may be impossible for an airplane. Accordingly, the number of aerial resources that can recharge simultaneously at a single water point will not exceed a certain number, e.g., in the case of a lake, a pit, or a water truck repurposed for fire extinguishing activities. Moreover, when determining the maximum number of aerial resources per flight route, the aerial resources on the same route must not coincide at the same recharge point. Thus, the maximum number of aerial resources that can be recharged at the same point will correspond to the number of flight routes (equations (5) and (6)). The arrangement of aerial resources in the various fronts must conform to the priorities assigned by the aerial coordinator. In the case of Spain, this priority takes into account the sum of the capacities of water and retardant that can be transported by the aerial resources. Because each front is assigned a specific percentage of the total amount of water resources based on its severity, an attempt is made to assign each of these fronts with aerial resources whose capacity is rated for that front, to a feasible extent (equation (7)).

Model for allocation of aerial resources to refueling points

We present the different elements that comprise the second model of operational research.

Notation and decision variables

Sets

i, $\mathcal{I}$ = index and current set of aerial resources involved in the extinguishing protocol.

b, $\mathcal{B}$ = index and current set of refueling bases.

t, $\mathcal{T}$ = index and current set of periods of time (we can take each period equal to five minutes, for example).

$\mathcal{B}_i$ = the current set of refueling bases that can be assigned to resource i.

It is important to note that these sets also allow the decision maker to introduce heterogeneity to the aerial resources fleet because the points at which each aerial resource i can refuel is determined. By using these, it is made clear that not all aerial resources can refuel at the same refueling points. For example, a helicopter may land in a much smaller area than a fixed-wing aircraft.

t', $\mathcal{T}_t$ = index and set of periods of time no later than t, *i.e.*, $\mathcal{T}_t = \{1, \ldots, t\}$.

Parameters

LOI_i = the fuel load of aerial resource i.

REF_i = the refueling time of aerial resource i.

FUE_b = the current quantity of fuel available in base b.

NUM_b = the number of aerial resources that can refuel simultaneously on base b.

TIM_{ib} = the time it takes to move aerial resource i from its current location to base b.

ATI_t = the accumulated time since the start of the refueling planning process (that is, when a set of aerial resources request refueling in the air before they actually run out of fuel) up to t period.

Decision variables

We use two sets of decision variables in our formulation.

s_{ibt} = the binary variable that takes the value 1 when aerial resource i starts refueling in base b in period t, and 0 otherwise.

e_{ibt} = the binary variable that takes the value 1 when aerial resource i ends refueling in base b in period t, and 0 otherwise.

Objective function

$$\min \sum_{i \in \mathcal{I}} \sum_{b \in \mathcal{B}_i} \sum_{t \in \mathcal{T}} (ATI_t + TIM_{ib}) e_{ibt} \tag{8}$$

Constraints

The relationships that describe the real-world model of the allocation of aerial resources to refueling points are formulated by means of the following constraints.

$$\sum_{i \in \mathcal{I} \,:\, b \in \mathcal{B}_i} \sum_{t' \in \mathcal{T}_t} s_{ibt'} - \sum_{i \in \mathcal{I} \,:\, b \in \mathcal{B}_i} \sum_{t' \in \mathcal{T}_t} e_{ibt'} \leq NUM_b, \quad \forall b \in \mathcal{B},\ \forall t \in \mathcal{T} \tag{9}$$

$$\sum_{b \in \mathcal{B}_i} \sum_{t \in \mathcal{T}} s_{ibt} = 1, \quad \forall i \in \mathcal{I} \tag{10}$$

$$\sum_{t\in\mathcal{T}} s_{ibt} = \sum_{t\in\mathcal{T}} e_{ibt}, \quad \forall i \in \mathcal{I},\ \forall b \in \mathcal{B}_i \tag{11}$$

$$\sum_{b\in\mathcal{B}_i}\sum_{t\in\mathcal{T}}(ATI_t - TIM_{ib})s_{ibt} \geq 0, \quad \forall i \in \mathcal{I} \tag{12}$$

$$\sum_{b\in\mathcal{B}_i}\sum_{t\in\mathcal{T}}(ATI_t + REF_i)s_{ibt} - \sum_{b\in\mathcal{B}_i}\sum_{t\in\mathcal{T}} ATI_t e_{ibt} = 0, \quad \forall i \in \mathcal{I} \tag{13}$$

$$\sum_{i\in\mathcal{I}\,:\,b\in\mathcal{B}_i}\ \sum_{t\in\mathcal{T}} LOI_i s_{ibt} \leq FUE_b, \quad \forall b \in \mathcal{B} \tag{14}$$

Depending on the type of the refueling base (tanker, airfield, or airport), the number of aerial resources that can be refueled simultaneously may vary. For example, in the case of a tanker, only a hose can be provided to supply fuel to aerial resources, or serving space may be limited. To make the model consistent with this limitation, a restriction is included that takes into account the availability of fuel supply to an aerial resource in a given period for a given refueling base (equation (9)).

With these elements, we determine the period when each aerial resource starts and ends refueling at the corresponding base (equations (10) and (11)). Refueling begins once the aerial resource has reached the base and this is available for refueling, given that the means employed for the refueling operation may be preoccupied (equation (12)). On the other hand, we consider refueling to be complete when the refueling time associated with each aerial resource is accomplished. The refueling time for each aerial resource is important because, depending on the type and model of aerial resources, the time required may vary (equation (13)).

Because refueling bases often have limited fuel, the capacity to refuel must be considered. Therefore, before assigning a set of aerial resources to a particular base, determining fuel availability is necessary (equation (14)).

Given the above restrictions, we seek an allocation of aerial resources at various refueling points whereby the time spent on the operation (this time includes the actual refueling time as well as the flying times to and from the base, so we are double counting the flying time for the air resource to the refueling base) is minimized (equation (8)). It is worth noting that in the objective function the parameter ATI_t is multiplied by the indicator variable e_{ibt}. It is thus possible that different air resources begin to refuel in different bases at different times. By taking all these elements into account, the efficiency of the refueling operation as well as the management of fuel stocks can be improved, thus avoiding supply problems.

Examples, numerical results, and sensitivity analysis

The above models were programmed using `AMPL` (Fourer *et al.*, 1993) and solved using the Gurobi solver (Gurobi Optimization, 2016). We work with four databases containing information on the wildfire fronts , the water points, the available aerial resources, and the refueling bases. Although, according to the title of the work, our models would be applicable to an extinguishing operation that makes use of both helicopters and airplanes, in the examples and with the intention of simplifying the presentation, we only considered helicopters. In a more general case, the treatment of one type or another would be different; this is reflected by the fact that the type of aerial resources will determine the permissible type of flight route and the permissible type of refueling base selected.

Assignment of aerial resources to flight routes

Prior to the execution of the program, information about the maximum number of aerial resources per flight route and the discharges per hour of every aerial resources is known.

The first model is demonstrated by an example (the data for models of the examples are real, and were obtained from the websites of the helicopters that are used. The data, described in the Appendix (Tables A1-A8), are inspired by a real situation. The availability of two groups of aerial resources is assumed: light helicopters BellB412-1, BellB412-2, BellB212-1, BellB212-2, BellB407-1, BellB407-2, and BellB407-3 (which can enter the same flight route) and heavy helicopters Ka32-1, Ka32-2, and Ka32-3 (which also can enter the same flight route). Further, it is assumed that there are three wildfire fronts, and from each front, ten water loading points can be accessed. Three helicopters can use all the water points, another three helicopters can use four water points, and the remaining four helicopters can use seven water points. In this case, the model has 6 continuous variables and 273 binary variables. Table 1 summarizes the main results concerning the assignment of aerial resources to fronts and water points.

Table 1: Allocation of aerial resources to flight route (fire front and water point).

Aerial resources/Flight route	F1-P3	F1-P6	F2-P3	F2-P7	F3-P1	F3-P6
BellB412-1 (group 1)	0	0	1	0	0	0
BellB412-2 (group 1)	0	0	0	0	1	0
BellB212-1 (group 1)	0	0	0	1	0	0
BellB212-2 (group 1)	0	0	0	1	0	0
BellB407-1 (group 1)	0	1	0	0	0	0
BellB407-2 (group 1)	0	0	0	0	0	1
BellB407-3 (group 1)	0	1	0	0	0	0
Ka32-1 (group 2)	1	0	0	0	0	0
Ka32-2 (group 2)	1	0	0	0	0	0
Ka32-3 (group 2)	0	0	0	0	1	0

In Table 1, 1 indicates that the aerial resource (in rows) is assigned to the flight route that can be formed by joining corresponding front and water point (in columns). Otherwise, the number is assigned is 0. We note that 10 aerial resources are used. In addition, seven flight routes are formed. Two of the flight routes traverse front 1 and use water points 3 and 6, respectively. One of the flight routes is formed by two helicopters in group 2 (Ka32-1 and Ka32-2), and the other one is formed by two helicopters in group 1 (BellB407-1 and BellB407-3). One flight route traverses front 2; it uses water point 7, and it is formed by the two helicopters in group 1 (BellB212-1 and BellB212-2). The remaining flight routes are organized using just one helicopter.

Moreover, the consistency of the results should be noted. For example, two of the helicopters with the most water capacity are assigned to front 1 and they are on the same flight route; however, these helicopters are not the closest to the front. Now, along this flight route, these types of helicopters are very efficient. Front 1 requires the most water because it represents a greater threat. Front 1 also has another flight route, used in this case by two of the lightest helicopters. The other heavy helicopter, the other light helicopter, and one intermediate helicopter are assigned to the front 3, which has the second highest water requirement. Finally, the remaining three medium capacity helicopters are assigned to front 2, which requires the least amount of water. From the results, it can be seen that 44.51 % of capacity is expended at front 1, 25.08 % at front 2, and 30.41 % at front 3, which is very close to the percentages specified in the database corresponding to the fronts, which are 45 %, 25 % and 30 %, respectively.

To perform a sensitivity analysis, while maintaining the characteristics of the fire, the effect of modifying the fleet of helicopters on the allocation is analyzed. We explain some of the results obtained below. When two of the large helicopters are not considered, then two medium helicopter and one small helicopter take over for front 1, while the remaining large helicopter is assigned to front 3, just as in the initial example. However, as regarding the initial example, when two small helicopters are not considered, we see that each of the big helicopters is assigned to a different front. Finally, when two medium helicopters and one small helicopter are not considered, then two large helicopters are assigned to front 1, while front 3 receives one large helicopter and one small helicopter; two medium and one small helicopter are assigned to front 2. In this case, the percentage of water allocated to the fronts is 45.36%, 26.49%, and 28.15%, respectively.

We have also analyzed the effect of modifying the groups. For example, we have replaced three air resources in group 1 with another three in group 2, which are resources with more capacity. The result has been that the amount of water discharged has been increased and the allocations of resources to the flight routes have changed, while the rest of the results have remained similar. We have also been interested in finding out the effect of modifying distances from the current position of air resources to fire or to water points. For example, starting from the initial case, one of the resources with the most capacity has been considered in a current position closer to the fire. The effect has been a change in the allocation of resources to flight routes, a decrease in the total distance and the remaining results remained unchanged.

Allocation of aerial resources to refueling points

To demonstrate the use of the second model by an example, we assumed a single scenario with four helicopters and three refueling points. Two helicopters can access up to two bases, one helicopter has access to one base, and one helicopter has access to all the bases. It is assumed that the helicopters have to rest. The data corresponding to the given scenario can be seen in Appendix (Tables A9-A13).

In Table 2, 1 indicates that the corresponding helicopter is assigned to the corresponding base. It must be noted that two helicopters are assigned to two bases (each helicopter to a base) bases 1 and 2, and two helicopters are assigned to base 3. The latter accommodates the largest helicopter and one of the lightest, and has the most fuel available and the only base that can serve the large helicopter. Moreover, other scenarios are possible wherein all the bases are not used. However, it is not possible to carry out the entire refueling operation in the least amount of time without using all the three bases. On the other hand, the results show that at each base, there is, at most one helicopter in each time period and that the helicopters begin to refuel in different periods.

Table 2: Allocation of aerial resources to bases (refueling points).

	BellB412	BellB212	Ka32	BellB407
B1	0	0	0	1
B2	1	0	0	0
B3	0	1	1	0

The amount of fuel remaining at each base after this first round of refueling is 300, 450, and 2136 liters, respectively. From the results, we can see that all the bases decrease their initial capacity by more than 50 % but less than 75 %. Moreover, we can see that only one aerial resource is forced to wait for the other helicopter to finish refueling.

The optimal value of the objective function is 120 minutes. This value represents the sum of the refueling time of the helicopters, including a round trip to the base and the waiting time. In other words, the plan assigns each helicopter to the closest possible base, and meets the condition for sufficient capacity. As with the first model, a post-optimality analysis was performed with new experiments to illustrate the consistent sensitivity in the model. To cite an example, by eliminating the heaviest helicopter, the allocation for the other three helicopters remains the same as in the initial example, when the helicopters were assigned to the nearest bases. In the new scenario, there are no waiting times and the total time decreases to 80 minutes. If instead of the heaviest one, we eliminate the lighter helicopter, the time decreases to 77.5 minutes. This may be attributed to the fact that the heaviest helicopter, which requires more time to refuel, is closer to the assigned base. This compensation results in a reduction in the total time. Finally, regarding the initial case, we consider an additional small air resource, close to base 1 and suppose also that the available quantities of fuel increase in bases 1 and 3. The result is that the allocation to the bases of the initial resources does not changes and the additional resource is allocated to base 1. With this, the total time of the operation increases.

Table 3: Executing times (in seconds) with Gurobi: Model 1.

Aerial Resources	Water Points	Fire Fronts	Mean Time	S.D. Time	Max. Time
3	3	3	0.0380	0.1274	0.91
3	6	3	0.0265	0.0715	0.48
3	6	6	0.2456	0.3824	1.66
3	9	3	0.0289	0.0604	0.37
3	9	6	0.2622	0.6218	3.28
6	3	3	0.3404	0.4195	1.87
6	6	3	0.3477	0.4006	1.39
6	6	6	1.2028	1.6589	8.18
6	9	3	0.3051	0.3542	1.59
6	9	6	0.8273	0.9696	4.40
9	3	3	0.8799	0.9659	5.65
9	6	3	2.4459	6.2381	44.11
9	6	6	14.5958	25.2978	145.69
9	9	3	0.6675	0.5205	2.20
9	9	6	9.0187	15.8180	89.63
12	3	3	7.0417	18.9782	137.49
12	6	3	13.5069	25.6252	158.60
12	6	6	7.0090	9.3167	44.88
12	9	3	8.1102	15.3645	83.23
12	9	6	13.2312	23.3966	137.64
15	3	3	28.9217	90.0668	795.07
15	6	3	128.4341	713.8449	7072.86
15	6	6	19.7705	37.4855	215.98
15	9	3	30.4158	81.4541	649.52
15	9	6	55.4892	204.7346	1594.88

Algorithms for larger incidents

In this section, we explore the feasibility of solving real-size instances in a reasonable execution time. We apply the two algorithms over instances ranging from 3 to 15 aerial resources, from 3 to 9 water points, from 3 to 6 fire fronts, and from 3 to 15 refueling bases. The model algorithms can be efficiently computed on a modern PC.

Table 3 lists the execution times (in seconds) for solving the first `AMPL` model by using the Gurobi solver for different instances. The first column indicates the number of aerial resources, the second column indicates the number of water refill points, and the third column indicates the number of fronts. Column 4 indicates the mean time and column 5 indicates the standard deviation. Finally, column 6 shows the maximum value obtained for the time of execution. The parameters for the different cases are randomly generated, and for each case, we consider 100 samples.

It can be seen from Table 3 that the average execution time of all cases for Model 1 barely exceeds two minutes for the worst case. This is illustrated in Figure 1, wherein the execution times (in seconds) are plotted on the vertical axis and the number of aerial resources are plotted on the horizontal axis; for different types of lines, different scenarios are represented by the number of fronts and water points. Clearly, the worst computational result (more than 120 seconds) is obtained for 15 aerial resources, 6 water points, and 3 fronts. In other cases, we obtain an average computation time of less than 60 seconds.

Figure 1: Execution time (in seconds) with different scenarios: Model 1.

For higher values of the problem parameters that define the scenario (number of aerial resources equal to or greater than 9), the standard deviation of runtime ranges from half a minute and reaches almost twelve minutes. In only one case, the runtime reached nearly two hours (with 15 aerial resources, 6 water points, and 3 fire fronts).

In a similar manner, Table 4 lists the execution times (in seconds) for solving the second `AMPL` model by using the Gurobi solver for different instances.

As shown in Table 4, the average execution time of all cases for Model 2 barely exceeds twelve minutes in the worst case. This is illustrated in Figure 2, wherein the execution times (in seconds) are plotted on the vertical axis and the number of aerial resources are plotted on the horizontal axis; for different types of lines, different scenarios are represented by different number of refueling bases. Clearly, the worst computational result (more than 756 seconds) is obtained for 12 aerial resources and 3 refueling bases. In other cases, we obtain a computing average of less than 180 seconds.

For a large number of problem parameters that define the scenario (number of aerial resources equal to 12), the standard deviation of runtime ranges between two minutes and more than thirty minutes. In only one case the runtime standard deviation reached a half hour (with 12 aerial resources and 3 refueling bases).

Table 4: Executing times (in seconds) with Gurobi: Model 2.

Aerial Resources	Refueling Bases	Mean Time	S.D. Time	Max. Time
3	3	0.2920	0.4497	3.9554
3	6	1.0027	3.0879	18.8991
3	9	0.9979	3.5243	31.3052
3	12	1.2722	5.6377	57.0593
3	15	1.1700	3.1200	32.0351
6	3	22.8064	31.0903	103.2493
6	6	21.5974	36.2055	185.7048
6	9	13.2956	28.0623	147.1876
6	12	7.0087	21.9127	142.3974
6	15	12.0798	49.4308	433.9180
9	3	114.0099	198.3733	1474.9258
9	6	63.1443	69.2111	262.9560
9	9	66.4848	99.1811	505.6261
9	12	74.4264	106.4668	392.9953
9	15	48.9573	110.3735	628.9864
12	3	756.9109	1948.0024	12 513.7766
12	6	168.1308	179.1913	1097.7551
12	9	179.4069	147.7079	508.8336
12	12	150.0898	186.3166	867.2902
12	15	129.5944	175.3732	663.8251

Figure 2: Execution time (in seconds) with different scenarios: Model 2.

Conclusions and final remarks

In this work, we present two integer linear programming models that solve two real problems experienced by traffic control coordinators when tackling wildfires. In several examples inspired by real situations, we see that the model resolution is fast. These features indicate, in our opinion, that this work can assist air traffic control coordinators in decision making for this type of fire. In fact, by using the tools proposed in this work, such as collision avoidance algorithms, escape route design for ground crews, efficiency measures of water discharges, and control of the spread of fire, this work can be integrated into a more complex, holistic, and user-friendly system for decision support, which also incorporates the modern methods of image processing and presentation.

Although our models have been tested using real parameters and data, the results could not be verified using historical data. It is difficult to obtain specific results for resource allocation, although it is easier to obtain global data. We emphasize the recommendation that aviation agencies consider the importance of data and its availability.

The proposed models perform resource allocation involving flight routes and refueling points. Therefore, it is important to note that once this assignment is made according to certain objectives and restrictions, aerial resources must effectively integrate the task of fire extinguishing; temporal assignments that include pilot work schedules, rest periods, and flight times must be taken into consideration.

Coordinating aerial resources is a challenging task. By monitoring the effectiveness of our approach and making adjustments to maximize the impact on the fronts, the proposed automated approach can benefit aerial coordinators. Given the fronts, available aerial resources, water points, and refueling points in a scenario, aerial resources are allocated to flight routes and refueling points such that the water discharge and refueling times are optimized. The initial planning must be supplemented with the algorithm mentioned above, which is executed based on the pilot work schedule, rest periods, and active fronts. We emphasize that this last model is flexible in the sense that its goals can be modified according to interest and optimal water discharge or other cost/benefit functions. Variation in one of the model parameters can signal the need for a rerun of any of the three algorithms and reallocations.

Other approaches, such as a single model for the problem presented herein or a stochastic approach may make sense. We do not endorse the single model because of the associated computational costs and the natural separation of problems. With respect to stochastic optimization, the consideration might be interesting for the allocation of aerial resources to a flight routes model because the efficiency of aerial resources during a wildfire must be estimated (depending on the climatological factors and the drought situation of the land, among others), and thus, an open problem would be to study the allocation of air resources to fligh routes by means of a stochastic programming model. However, we do not consider the introduction of stochasticity in the second model, because in this case the parameters are deterministic.

It is important to mention that each region has a specific means to fight fires; for example, in Galicia (a region in North-west Spain, with a surface area of 29574 km^2 and a 69% mountain range), the regional public body responsible for fighting forest fires lists 30 aerial resources in 2017, of which 25 are helicopters. These helicopters not only allow for intervention from the air, but also for the transport of land brigades. They are distributed in 21 operational bases spread across the four provinces of Galicia. The rest of the fleet comprise large seaplanes sent by the Central Administration as needed, and they are able to refill from pools or swamps without needing to return to their base.

The models developed in this work could also serve as a starting point for subsequent case studies. Finally, as our first model is currently a multi-objective model with emphasis on water download, creating a Pareto frontier to demonstrate the trade-off between flight distance and water download may be interesting.

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Appendix

A Information of the examples

This appendix describes the data used in examples above. Such data are provided as used to run the model programmed with `AMPL`.

Table A1: Helicopter type.

Light	BellB412	BellB212	BellB407
Heavy	Ka32		

Table A2: Water points useful for helicopter types.

	Water points									
Helicopter type	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
BellB412	✓	✓	✓	✓	✓		✓			✓
BellB212	✓	✓	✓	✓	✓		✓			✓
BellB407	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Ka32	✓	✓	✓		✓					

Table A3: Helicopter type capacities.

Helicopter type	BellB412	BellB212	BellB407	Ka32
Capacity (liters)	2274	2360	1205	5000

Table A4: Number of helicopters per flight route.

Light helicopters	Water points									
Front number	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
K1	2	3	2	6	3	2	2	8	3	3
K2	2	4	2	10	9	2	2	6	3	6
K3	2	4	2	10	3	2	2	4	2	5

Heavy helicopters	Water points									
Front number	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
K1	2	2	2	4	2	2	2	6	2	2
K2	2	3	2	7	7	2	2	4	2	4
K3	2	3	2	7	2	2	2	3	2	4

Table A5: Water downloads per hour depending flight route.

Light helicopters	Water points									
Fronts	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
K1	15	9	15	5	10	22	18	3	8	9
K2	18	7	22	1	3	15	24	5	9	5
K3	18	6	16	3	10	27	18	7	12	5

Heavy helicopters	Water points									
Fronts	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
K1	16	10	16	5	11	24	19	3	8	10
K2	19	7	24	1	3	16	26	5	10	5
K3	19	6	17	3	11	30	19	7	13	5

Table A6: Distance from helicopters to the fronts (hours).

	Fronts		
Helicopters	K1	K2	K3
BellB412-1	0.31	0.31	0.34
BellB412-2	0.35	0.33	0.32
BellB212-1	0.23	0.20	0.20
BellB212-2	0.21	0.22	0.24
BellB407-1	0.75	0.75	0.71
BellB407-2	0.98	0.98	0.82
BellB407-3	0.63	0.62	0.70
Ka32-1	1.36	1.55	1.54
Ka32-2	0.57	0.56	0.57
Ka32-3	0.14	0.14	0.13

Table A7: Number of helicopters charging water in same water point.

Water points	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
Num. of helicopters	5	5	5	3	3	4	5	2	2	4

Table A8: Desired percentage of water in each front.

Fronts	K1	K2	K3
Percentage	45	25	30

Table A9: Refueling points useful for helicopter types.

	Refueling points		
Helicopter type	B1	B2	B3
BellB412		✓	✓
BellB212		✓	✓
BellB407	✓	✓	✓
Ka32			✓

Table A10: Helicopters information.

Helicopters	Fuel charge (liters)	Refueling time (minutes)
BellB412	1050	7.5
BellB212	614	5.0
BellB407	400	2.5
Ka32	2250	12.5

Table A11: Bases information.

Bases	Fuel available (liters)	Maximum number of helicopters refueling simultaneously
B1	700	1
B2	1500	1
B3	5000	1

Table A12: Flight time of helicopters to the bases (minutes).

	Bases		
Helicopters	B1	B2	B3
BellB412	5.0	5.0	15.0
BellB212	25.0	25.0	15.0
Ka32	17.5	12.5	10.0
BellB407	12.5	15.0	15.0

Table A13: Time allocated to each time period.

Time periods	1	2	3	4	5	6	7	8	9	10	11	12	13
Time (minutes)	0	2.5	5	7.5	10	12.5	15	17.5	20	22.5	25	27.5	30