

Learning from "transcendental" errors

Aprendizaje a través de errores trascendentales

J. MIRA

Departamento de Física Aplicada, Universidad de Santiago de Compostela, E-15782 Santiago de Compostela, Spain, fajmirap@usc.es

Abstract

An error in an anecdotal calculation that uses physical concepts can be an interesting matter for physics students. It helps in motivation, to grasp some concepts, and creates good scientific attitudes. It is hoped that putting the physics into an unusual context can be both motivational and instructional.

Key words: physics, blackbody radiation, motivation, Bible, Stefan-Boltzmann.

Resumen

Se utiliza un error en un cálculo anecdótico como materia de interés para estudiantes de física. Ayuda en la motivación, a percibir ciertos conceptos, y crea buenas actitudes científicas. Se espera que, situar la física en un contexto inusual, pueda ser motivante e instructivo.

Palabras claves: física, radiación cuerpo negro, motivación, Biblia, STEFAN-BOLTZMANN.

Motivation is the basic ingredient in any learning process. This is even more true in physics, for which the lack of motivation has made the subject an unpleasant experience for many generations of students. It is for this reason that any anecdote or funny story (related to the teaching programme) allowing a distended atmosphere will always be welcome in a classroom. What is presented here is an example with a peculiar matter whose origin can be traced back till the thirties, a period in which scientists devoted part of their attention to the scientific basis of the Bible. In those years somebody called Mr. Wensel of the U.S. Bureau of Standards would have calculated the temperatures of Hell and Heaven, based on paragraphs extracted from the Bible, arriving to the conclusion that Heaven is hotter than Hell. Several decades later, in 1960, the story was first published; but it was not till 1972 when it achieved popularity after the publication in *Applied Optics* (BROMLEY, 1972) and a subsequent comment in the 21 August issue of *Time*.

Data arise from two paragraphs of the Bible. On one hand, Isaiah's 30:26 reads: "Moreover the light of the moon shall be as the light of the sun and the light of the sun shall be sevenfold, as the light of seven days" (it can be seen the typical presence of number 7 in the Bible as a mark of perfection). This is taken as the description of Heaven (although what is described here is not exactly Heaven, but this is not matter for physicists...), and it was originally interpreted that Heaven receives from the moon as much radiation as we do from the sun and in addition seven-times-seven times, i. e. forty-nine, as much as the earth does from the sun (a total of fifty times more). The calculation of the temperature of a body exposed to a certain radiation gives rise to interesting exercises proposed some times in introductory courses or initial lessons of quantum physics. For this purpose it is considered that Isaiah's Heaven is in thermodynamic equilibrium; in such a case the Stefan-Boltzmann fourth-power law for radiation, first obtained empirically in 1879 (Eisberg & Resnick, 1985), gives the relation between the total emitted radiation and the temperature of a system in thermodynamic equilibrium (i.e., the emitted and received radiations are equal)

$$R_r = \sigma T^4 \quad (1)$$

where R_r = radiance (total radiated thermal energy per unit time per unit area, at absolute temperature T), σ = Stefan-Boltzmann constant.

If it is considered that the total radiation falling on earth comes basically from the sun (the light received from the moon is much smaller and can be neglected), and for the temperature of earth a value of 300 K is used, we have that

$$\frac{\sigma T_{\text{Heaven}}^4}{\sigma T_{\text{Earth}}^4} = \frac{R_{\text{Heaven}}}{R_{\text{Earth}}} = 50 \quad (2)$$

giving $T_{\text{Heaven}} = 798$ K [To be physically correct, equation (1) should also include the emissivity of the radiating body, unless it represents a perfect blackbody. The assumption may be made that Heaven and the earth have the same emissivity, so it will drop out when calculating ratios. This is probably more reasonable than assuming they are all perfect blackbodies.].

On the other hand, Hell is described in Apocalypse 21:8: "But the fearful, and unbelieving... shall have their part in the lake which burneth with fire and brimstone". A lake of boiling brimstone means that there the temperature is the boiling point of brimstone or sulphur, which at normal pressure is 717.6 K (CRC HANDBOOK, 1983). Therefore, the authors arrived at the "conclusion" that Heaven is hotter than Hell; this constitutes the beauty of the story. It must be stressed that this holds at normal pressure. Increases of pressure cause increases of boiling points, so, under a high enough pressure the boiling point of sulphur will surpass T_{Heaven} . Kurt Nassau was the first in making this observation in a note (NASSAU, 1972) in which he appended the original idea. To put numbers he used the Clapeyron equation

$$\frac{dP}{dT} = \frac{\Delta H_{\text{vap}}}{T(V_v - V_l)} \quad (3)$$

with V_l = liquid volume, V_v = atomic volume of the gaseous phase, ΔH_{vap} = latent heat of vaporization, P = pressure. He introduced $\Delta H_{\text{vap}} = 10534$ J/g, $V_v = 2.8$ l (assuming S_8 molecules) and neglected V_l . With all this he concluded that under a pressure of more than 4.2 atm. Hell would be hotter than Heaven. By the way, he estimated this new situation as more likely because in such a place both "psychological and physical pressures" are supposed to be high.

It has been recently demonstrated that the entire reasoning was erroneously based from the beginning (MIRA & VIÑA REBOLLEDO, 1998). If the Isaiah's paragraph is more carefully inspected, it is seen that there is a comma with crucial importance, as it states clearly that the light coming on Heaven is only seven times larger than on earth (like the light of seven days), not forty-nine. This reading has been confirmed on texts translated directly from the original manuscripts and the interpretation contrasted with an expert in biblical texts. The omission of the comma in the middle of the aforementioned paragraph is the key of a misunderstanding that has been hidden for almost seventy years and that is indeed decisive. Now, we have that the total radiation falling on Heaven is eight times larger than on earth (seven from the sun plus one from the moon), so

$$\left[\frac{T_{\text{Heaven}}}{T_{\text{Earth}}} \right] = 8 \quad (4)$$

which gives $T_{\text{Heaven}} = 504.5$ K, smaller than the value for Hell. The original error was the key in the final result and had changed the "conclusions" completely.

The educative contribution of this is twofold. Firstly, it provides the students an attractive practical example to grasp the strength of the laws explaining the radiation of a black body (other pedagogical examples are the calculation of temperature of stars; of earth after an hypothetical approach of the sun, etc.). Students also realize the importance of certain premises, like the necessity of thermodynamic equilibrium and the importance of considering all type of variables (pressure in this case) while solving a problem. Secondly, they learn the importance of independence and sense of criticism, that must be present in the analysis of a previous work. Here, the source of the error was that probably nobody paid attention to the paragraphs of the Bible, from which the idea was born, and did not analyze the situation from the beginning, which should be the first reaction of a scientist.

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