RADIO SCIENCE, VOL. 44, RS4001, doi:10.1029/2008RS003991, 2009



# Impulse regime CRLH resonator for tunable pulse rate multiplication

J. S. Gómez-Díaz,[1] A. Alvarez-Melcon,[1] S. Gupta,[2] and C. Caloz[2]

Received 8 August 2008; revised 29 March 2009; accepted 21 April 2009; published 2 July 2009.

[1] A novel tunable microwave broadband resonator, inspired from optical laser systems, is presented. In contrast to usual harmonic resonators, the proposed device is based on broadband composite right/left handed metamaterial lines. This line, configured as a resonator, provides nonuniform spectral resonances due to the nonlinear nature of its dispersion curve. This is exploited in the impulse regime, where the input pulse spectrum is discretized inside the resonator, with different spectral separation as a function of the carrier frequency. This discretization leads to a pulse periodicity in time, with a tunable output time period. On the basis of the new broadband resonator features, a pulse rate multiplication device is proposed. This device provides an increase in the repetition rate of a periodic input pulse, with the additional advantage of repetition rate tunability.

**Citation:** Gómez-Díaz, J. S., A. Alvarez-Melcon, S. Gupta, and C. Caloz (2009), Impulse regime CRLH resonator for tunable pulse rate multiplication, *Radio Sci.*, *44*, RS4001, doi:10.1029/2008RS003991.

## 1. Introduction

[2] Resonators have been widely used in microwaves, with applications ranging from filters or oscillators to tuned amplifiers [see *Pozar*, 2005]. Conventionally, purely right-handed (PRH) distributed elements are used, providing resonances ($\omega_m$) at the frequencies where the physical length of the structure is multiple of half-wavelength. These devices have been mainly analyzed in the harmonic regime, but little work has been done in the impulse regime, required in recent developments for ultra wideband (UWB) systems [see, e.g., *Ghavami et al.*, 2007].

[3] Composite right/left handed (CRLH) transmission line metamaterials [see *Caloz and Itoh*, 2005] may be used as distributed resonators, providing unusual characteristics such as the presence of resonance frequencies out of harmonic ratios, or the zeroth-order resonance. In the latter case, the condition of resonance is independent of the physical length of the structure, as stated by *Sanada et al.* [2003] and *Abielmonaa et al.* [2006]. The dispersive nature of the CRLH lines may provide interesting broadband solutions. However, the study of this line configured as a resonator has been reported only in the harmonic regime to date.

[4] In this paper, a broadband CRLH based resonator is proposed. It is shown that the CRLH resonator can support nonuniform discrete spectral resonances due to the nonlinear nature of the CRLH dispersion curve [see *Caloz and Itoh*, 2005]. Therefore, the pulse spectral components (broadband excitation with continuous spectrum) inside the resonator are discretized, resulting in periodic temporal signal, with time period being a function of the modulation frequency. The time period of the generated train of pulses depends on the spectral separation of the discrete spectral components, which in turn depends on the modulation frequency. This process can be viewed as a resonant cavity in pulsed lasers in optics, where only discrete modes are being supported, leading to generation of optical pulses [*Saleh and Teich*, 2007]. Moreover, since a CRLH supports nonuniform resonances, the time period of the generated temporal periodic signal can be tuned externally to obtain various output repetition rates.

[5] Inspired from optical pulsed laser systems and resonant cavities, this paper introduces these concepts in the microwave domain using impulse regime CRLH transmission line metamaterials for the first time. Resonator phenomena and various applications are further discussed.

[6] All of the results presented in this paper were computed by a quasi-analytical time domain Green's func-

[1]Department of Information and Communication Technologies, Technical University of Cartagena, Cartagena, Spain.
[2]Ecole Polytechnique de Montréal, Montreal, Quebec, Canada.


0048-6604/09/2008RS003991$11.00

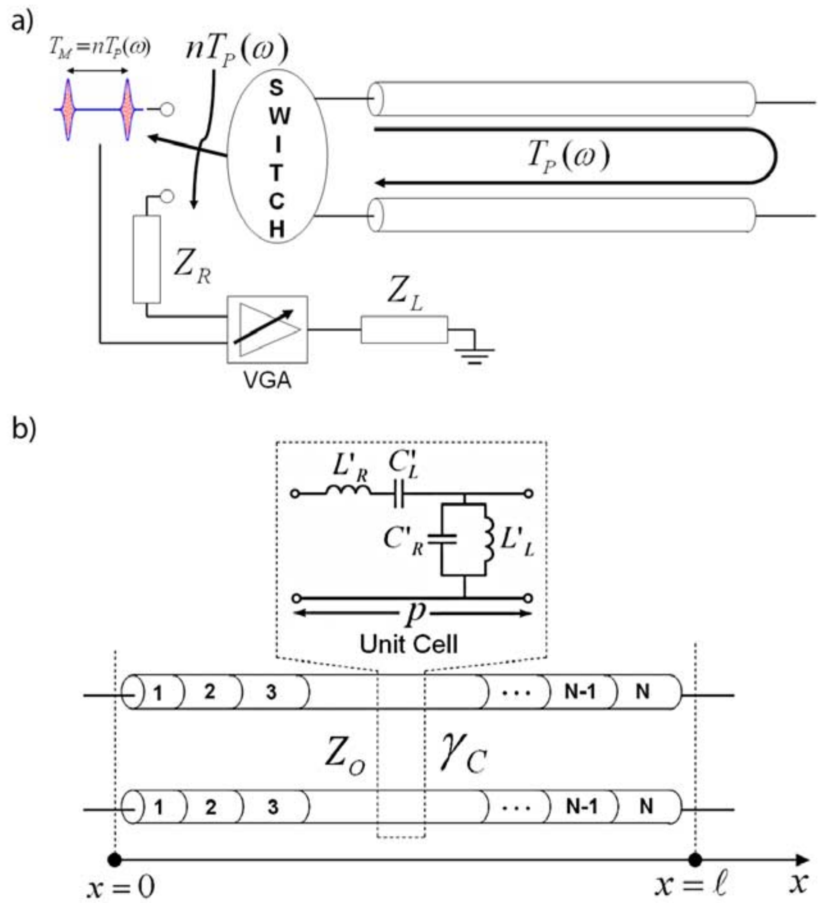


**Figure 1.** Proposed impulse regime CRLH resonator and pulse rate multiplicator. (a) Operation principle. (b) CRLH resonator, constituted of $N$ unit cells of length $p$, with its propagation constant $\gamma_C$, characteristic impedance $Z_0$, and total length $\ell = N\,p$.

tion method, presented by *Zhoua et al.* [2005], and adapted to transmission line structures by *Gómez-Díaz et al.* [2009].

## 2. Proposed Resonator

[7] In this section, the principle and implementation of the proposed UWB resonator based on CRLH transmission line is presented. Although the resonator principle holds in general for different transmission lines technologies, we propose to use a CRLH transmission line, because it provides additional functionalities, such as pulse delay tunability, resulting in variable output repetition rates. This behavior cannot be obtained with conventional nondispersive transmission lines.

### 2.1. Principle

[8] The proposed impulse regime resonator system is sketched in Figure 1a. The resonator is realized by terminating the transmission line at both ends by an open circuit [see *Pozar*, 2005]. Initially, a fast switch injects the input pulse into the resonator [see, e.g., *Jin and Nguyen*, 2005]. Once the pulse has been injected into the resonator, the switch is set to a high impedance $Z_R$, which reflects most of the energy back into the line, and transmits only a small amount of energy, which is then amplified, into the load $Z_L$. The resonator acts as a cavity and therefore supports multiple natural resonances. Owing to the broadband nature of the source, all of the cavity resonances lying within the pulse spectrum are excited simultaneously, which results into discretization of the input pulse spectrum.

[9] In time domain, the pulse travels back and forth between the cavity walls (high impedance port terminations, analogous to reflecting mirrors in case of lasers). The signal takes a total time of $T_P$ for completing a single round trip inside the resonator. One of the line termination impedance is designed to be such that when the pulse

reaches the end of the line, the high $Z_R$ impedance only transmits a small amount of energy, reflecting the rest of the pulse back to the resonator. This is analogous to having partially reflecting mirrors in optical lasers, with typically high reflectivity around 99%. In order to compensate for the line losses, at the output of the resonator a variable gain amplifier (VGA) can be used [see, e.g., *Lee et al.*, 2007], which is synchronized with the input pulse generator. After the pulse has covered $n$ round trips inside the resonator corresponding to a time $T_M = nT_P$, $n$ number of pulses with similar pulse features as that of the input are obtained. Therefore, the system acts as a 1 : $n$ pulse burst generator. Moreover, after the $n^{th}$ round trip, the switch at the input changes its position again to connect the generator, in order to introduce a new pulse inside the resonator to produce another burst of $n$ pulses. As this process repeats itself (synchronized with the output), a constant pulse train at the output is obtained. Consequently, the system also acts as a 1 : $n$ repetition rate multiplier.

[10] If the resonator in the system is implemented using conventional right-handed transmission line technologies, the round trip time $T_P$ is fixed (dispersionless). As a result only one repetition rate at the output is obtained from a given resonator. Therefore, a new line will have to be designed if a different repetition rate is required. To enhance the system features in order to obtain variable repetition rates from the same resonator, a CRLH transmission line can be used. Because of the nonuniform phase response of the CRLH lines, the output time period of the signal can be controlled externally using the modulation frequency of the input signal. The new proposed system, using CRLH lines, is demonstrated and described in the following section.

### 2.2. CRLH-TL Implementation

[11] A CRLH-TL resonator is depicted in Figure 1b, for an ideal open-ended case. The resonant frequencies ($\omega_m$) of the resonator correspond to these frequencies where the physical length ($\ell$) of the line is multiple ($m$) of half guided wavelength. Since the CRLH-TL is able to provide negative and zero values of the propagation constant ($\beta$), the number of resonant modes ($m \in Z$) are symmetrically defined around $m = 0$ [see *Caloz and Itoh*, 2005]. Therefore,

$$\ell = |m|\frac{\lambda}{2} \qquad \text{or} \qquad \beta_m = \frac{m\pi}{\ell}. \tag{1}$$

In addition, the field distribution of a particular resonant mode $m$ presents $|m|$ zeros in the standing wave pattern, at the positions

$$x_k = k\frac{\ell}{m+1}, \qquad k = 1 \ldots m. \tag{2}$$

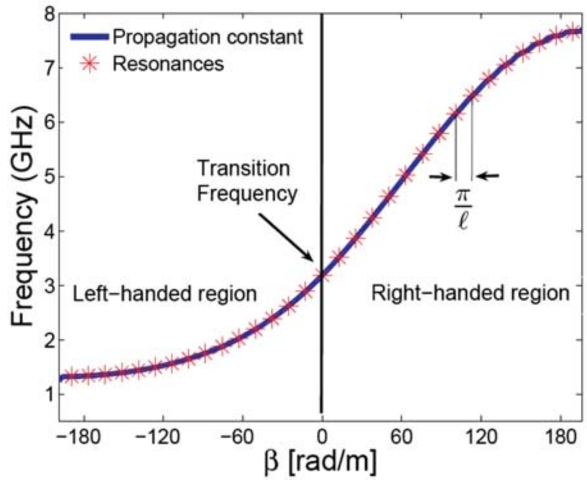


**Figure 2.** Dispersion relation for the CRLH transmission line resonator of Figure 1b and its resonant frequencies $\omega_m$. The line includes $N$ = 16 unit cells of length $p$ = 1.56 cm, which leads to $2N - 1 = 31$ resonances. The circuit parameters are $C_R = C_L = 1.0$ pf and $L_R = L_L = 2.5$ nH.

[12] From equation (1), the resonant frequencies are obtained by sampling the dispersion curve [$\beta(\omega)$] with a sampling rate of $\pi/\ell$. Owing to the nonuniform nature of the CRLH dispersion curve, the resonant frequencies are out of the harmonic ratios. Specifically, a compression in the resonant frequencies is obtained in the left-handed region. This can be observed in Figure 2, which shows the dispersion relation for a particular CRLH line, composed of $N$ unit cells, along with their ($2N - 1$) associated resonances.

[13] In the impulse regime case, the balanced condition of the CRLH (equal and mutually canceling series and shunt resonances leading to gapless transition from left-handed to right-handed frequency ranges) is required. When the CRLH resonator is excited with a broadband pulse signal, all the resonances falling within the spectral band of the pulse are excited. As a result, the continuous spectrum of the input pulse gets discretized. It is well known from basic signal processing concepts, that this pulse spectrum discretization consequently leads to a pulse periodicity in time. It should be noted that depending on the modulation frequency of the input pulse, variable number of resonances can be excited. Since the spectral separation between these resonances also depends on frequency, different sampling rates can be applied, and the corresponding repetition rate in time will be tunable.

[14] From the time domain point of view, the tunable periodicity of the pulse can be explained through the

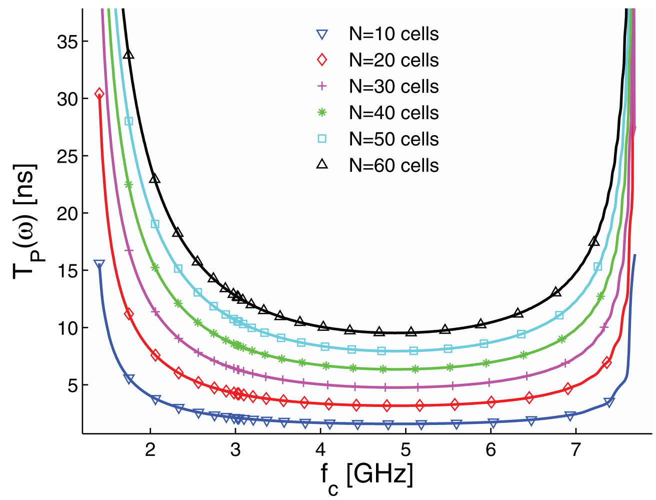


**Figure 3.** Round trip time $T_P(\omega)$ (equation (5)) along the CRLH resonator of Figure 2 for different numbers of cells $N$, versus the carrier frequency $f_c$, computed with equation (7).

temporal pulse propagation inside the CRLH line, which in the infinitesimal limit ($p \rightarrow 0$) can be written as

$$v_g(\omega) = \frac{\omega^2 \omega'_R}{\omega^2 + \omega'_R \omega'_L}, \qquad (3)$$

where

$$\omega'_R = \frac{1}{\sqrt{L'_R C'_R}} \quad \text{and} \quad \omega'_L = \frac{1}{\sqrt{L'_L C'_L}}. \qquad (4)$$

In this last expression $L'_R$, $C'_R$ and $L'_L$, $C'_L$ are per-unit-length and times-unit-length parameters of the right-handed and left-handed CRLH contributions, and ($p$) is the unit cell size, following the notation of *Caloz and Itoh* [2005]. The single round trip time inside the resonator for a pulse is given by

$$T_P(\omega) = \frac{2\ell}{v_g(\omega)} \qquad (5)$$

where $\ell$ is the total length of the physical structure. To determine the controlling parameters of $T_P(\omega)$, the derivative over the modulation frequency is given by

$$\frac{\partial T_P(\omega)}{\partial \omega} = -4\ell \frac{\omega_L}{\omega^3}, \qquad (6)$$

which shows that the two controlling parameters are:

[15] 1. The length of the structure ($\ell$). Larger structures provide higher ranges of $T_P(\omega)$. This is shown in Figure 3, where the variation of $T_P(\omega)$ as a function of the number of cells (i.e., $\ell$) is presented for a particular CRLH line.

[16] 2. The variable $\omega_L$, which provides the dispersion features of the CRLH line.

[17] In order to obtain the accurate expression of the group velocity inside the finite line, the ABCD transmission matrix analysis may be used [see *Caloz and Itoh*, 2005]. In this case the propagation velocity reads

$$v_g = \frac{p \sin[p\,\beta(\omega)]}{\omega/\omega_R^2 + \omega_L^2/\omega^3}. \qquad (7)$$

Note that this expression is valid in all frequency ranges, whereas equation (3) is only valid near the transition frequency between the RH and LH regions.

## 3. Demonstration With Modulated Gaussian Pulses

[18] In this section, the response of an ideal CRLH resonator excited by modulated Gaussian pulses is presented. We have chosen Gaussian pulses because they are convenient to characterize broadband systems, and are easy to generate in practice. Mathematically, a single modulated Gaussian pulse may be expressed as

$$J(t) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{j\omega_0 t} e^{-\frac{1}{2}\left(\frac{t-t_0}{\sigma}\right)^2}, \qquad (8)$$

where $\omega_0$ is the modulation frequency, $\sigma$ is the temporal width of the pulse and $t_0$ is the center of the pulse.

[19] Figure 4 shows the spectral components of a modulated Gaussian pulse as a function of space inside a CRLH transmission line. The pulse (Figure 4a) is modulated at the CRLH line transition frequency to study the resonances at both, the right and left handed

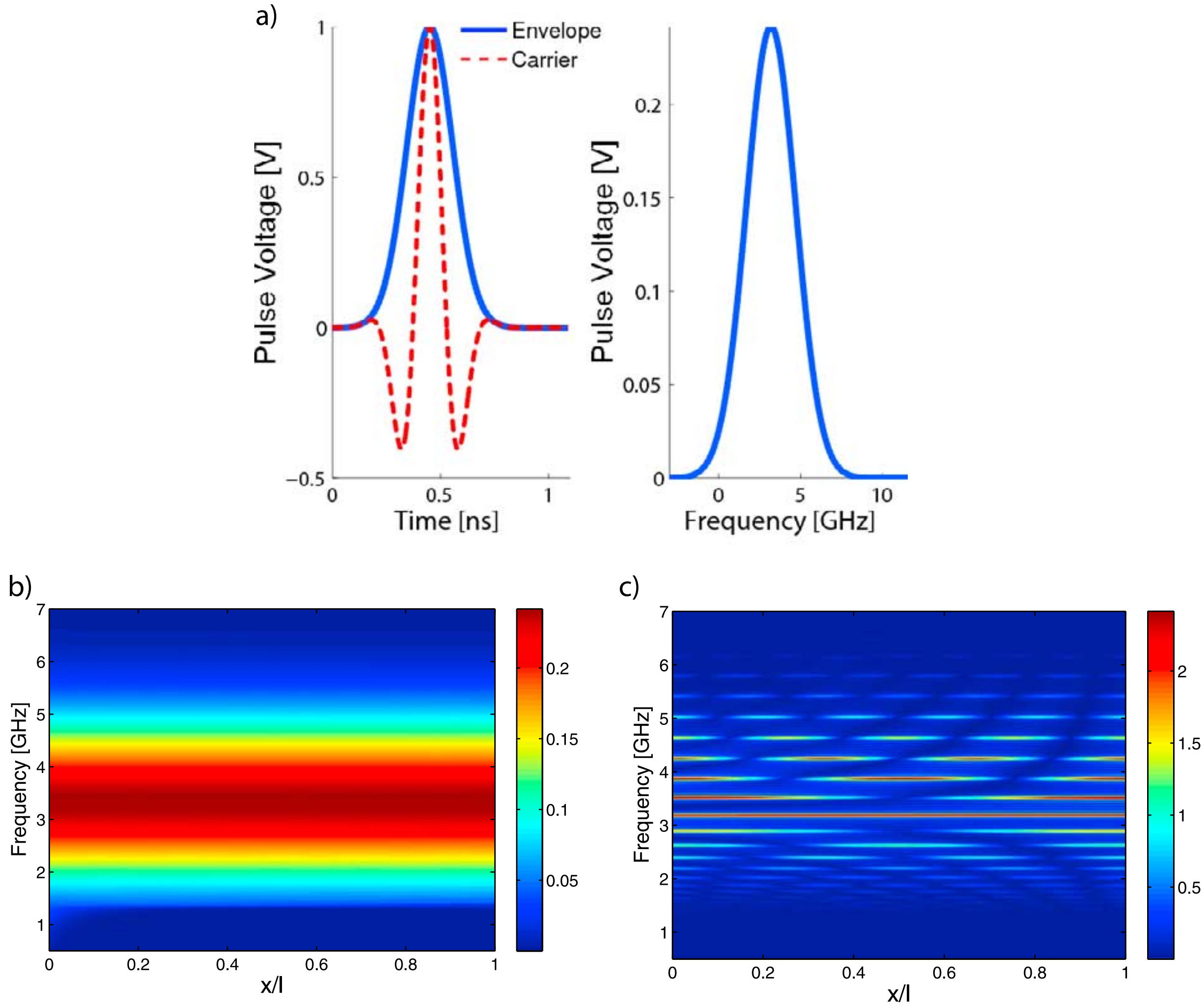


**Figure 4.** Spectrum evolution of a modulated Gaussian pulse propagating along the CRLH structure of Figure 2. (a) Modulated Gaussian pulse at the input, with carrier frequency $f_c = 3.183$ GHz and temporal width $\sigma = 0.15$ ns. (left) The pulse envelope and the carrier. (right) The spectrum of the pulse. (b) Spectrum evolution along the CRLH structure terminated by matched load (transmission line regime). (c) Spectrum evolution along the CRLH structure terminated by an open circuit at both ends (resonator regime).

regions. In addition, the pulse duration ($\sigma = 0.15$ ns) is chosen to produce a very wideband pulse, in order to excite a large number of resonances. In the spectrum of Figure 4a we can observe some aliasing at low frequencies. Note that the bandpass characteristic of the CRLH line will eliminate this aliasing effect.

[20] First, Figure 4b shows the pulse spectral components along the CRLH line, when it is configured as a transmission line. As expected, all frequencies are allowed and can propagate inside the line. On the other hand, Figure 4c represents the pulse spectral components when the line is configured as a resonator. In this case, only discrete frequencies are allowed inside the line, and the pulse spectrum is discretized at the resonant frequencies of the CRLH line. It can be seen that the discrete components are nonuniformly distributed (see Figure 2). It is also noted that at the transition frequency ($f = 3.183$ GHz), the resonance mode $m = 0$ provides a uniform voltage behavior, whereas the rest of the higher-order resonances exhibit zeros at the spatial positions indicated by equation (2).

[21] Figure 5 shows the temporal evolution of a modulated Gaussian pulse ($f_c = 5.0$ GHz, $\sigma = 0.25$ ns) as a function of space along a CRLH line. Figure 5a

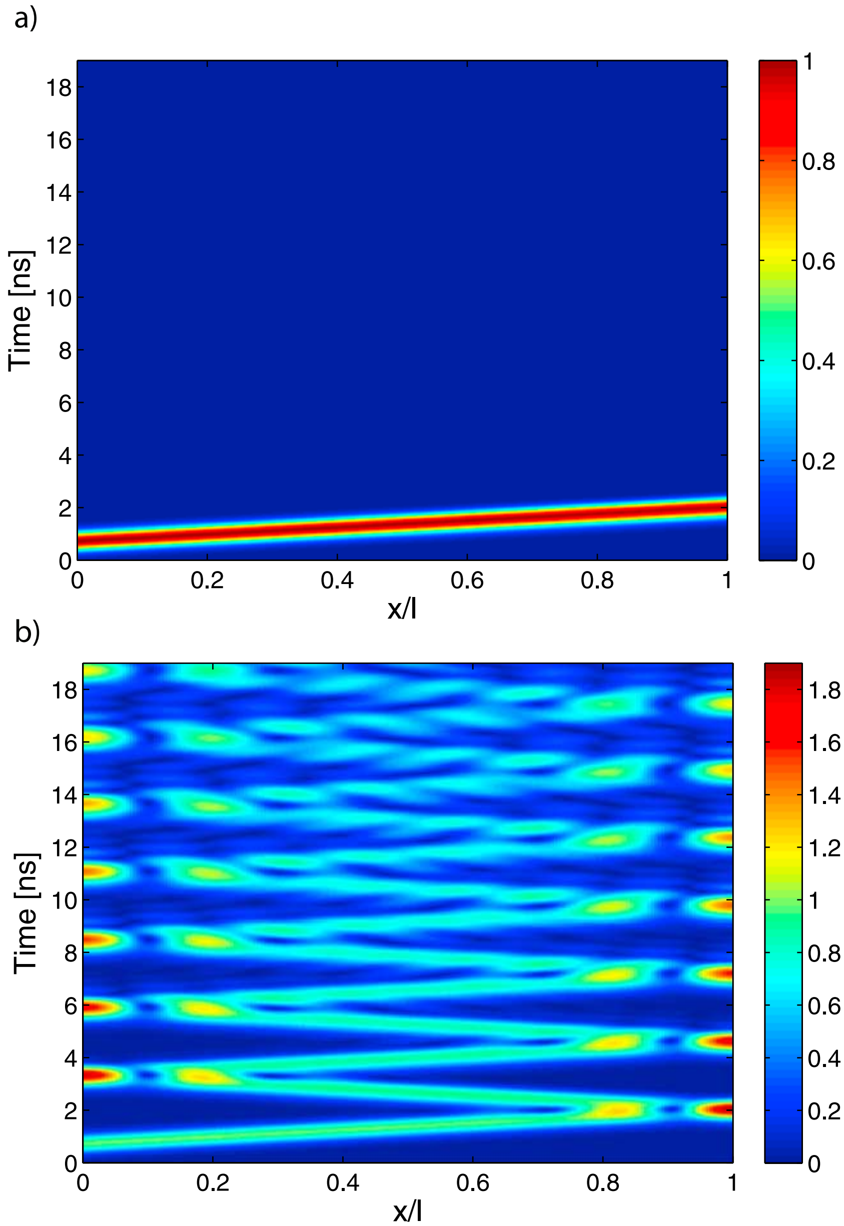


**Figure 5.** Propagation of a modulated Gaussian pulse ($f_c$ = 5.0 GHz, $\sigma$ = 0.25 ns) along the CRLH transmission line of Figure 2. (a) Matched line. (b) Open-ended line resonator.

presents the pulse propagation when the line is matched. In this case, no reflections occur at the end of the line and all the energy is transmitted. On the other hand, Figure 5b shows the pulse propagation when the CRLH line is configured as a resonator. In this case, multiple reflections can be observed inside the line. It is noted that as the pulse bounce back and forth in the line, there is a gradual decrease in the amplitude level of the pulse due to line losses, which is consistent with the energy conservation.

## 4. Application: Pulse Rate Multiplication

[22] The proposed CRLH resonator can be configured to extract a small amount of energy, for instance, using a high impedance ($Z_R$) connected to another circuit (see

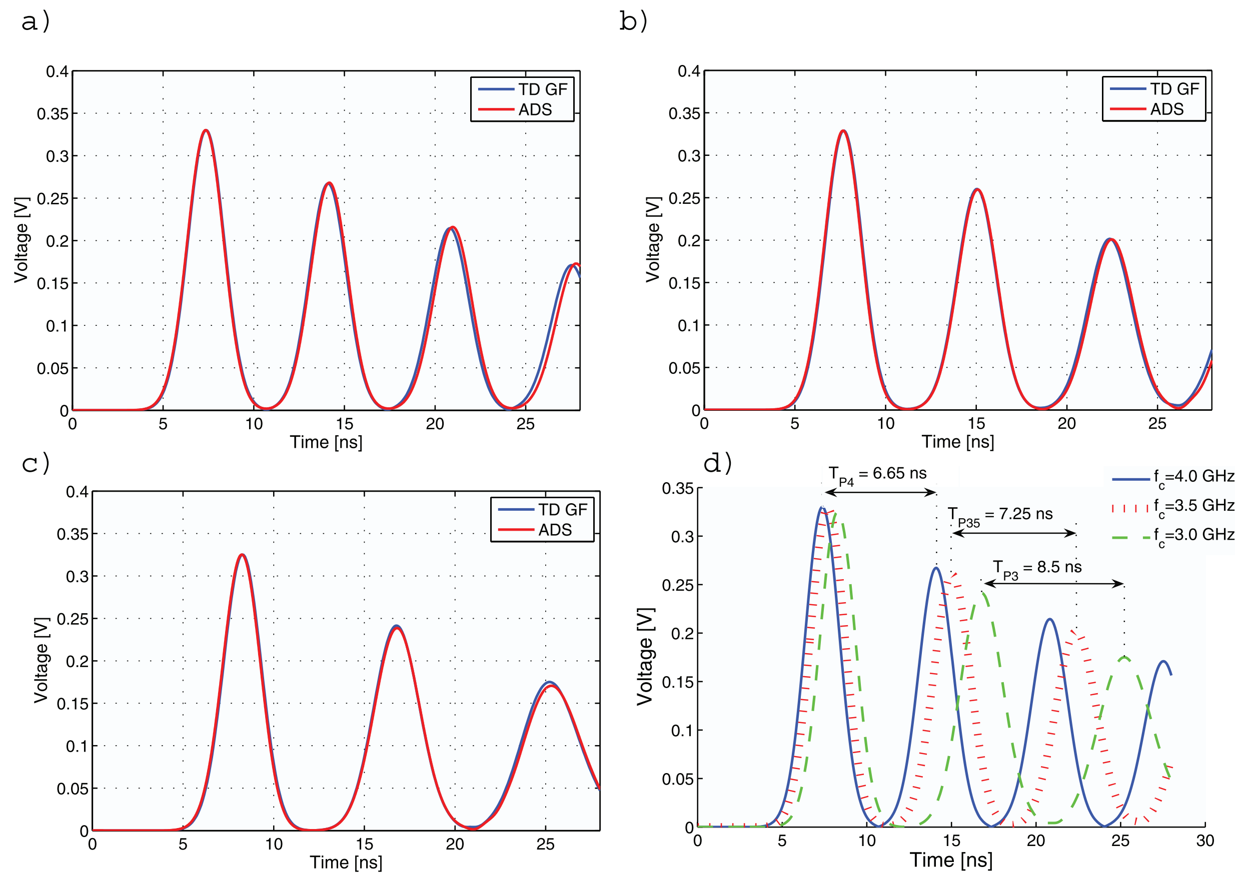


**Figure 6.** Gaussian waveforms ($\sigma$ = 1.0 ns) at the output ($Z_R$) of the CRLH resonator (Figure 1a) for different carrier frequencies ($f_0$) showing the tunability of the system. The CRLH line is composed of 40 unit cells with the same circuit parameters as in Figure 2. The generator impedance is $Z_g \approx \infty\ \Omega$, and the load impedance is $Z_R = 500\ \Omega$. Simulation data from the commercial software ADS© are included as validation. (a) $f_0$ = 4.5 GHz. (b) $f_0$ = 3.5 GHz. (c) $f_0$ = 3.0 GHz. (d) Results from the three carrier frequencies together, to show the tunability effect.

Figure 1a), or even using a broadband coupled line coupler [see, e.g., *Mongia et al.*, 1999]. Using such small-energy extraction mechanism, a train of pulses with similar features as the input is obtained at the output. Note that the frequency components of the generated pulses are exactly the same as in the input pulse, since the resonator is based on linear systems, and therefore no new frequency components are generated in the process [see *Saleh and Teich*, 2007]. The maximum pulse bandwidth inside the resonator is related to the features of the CRLH line, which are scalable in frequency as shown by *Caloz and Itoh* [2005]. Specifically, to allow for a good reconstruction of the periodic train of pulses, maintaining the tunability capabilities of the resonator, the number of resonances which should be simultaneously excited by the input pulse must be around one third of the total number of resonances within the operational frequency bandwidth of the line. Note that due to the frequency scalability properties of the CRLH lines, a CRLH resonator may be designed to operate at the desired frequency range, and with the desired maximum pulse bandwidth. As it was previously mentioned, the proposed resonator may also be implemented using nondispersive right-handed lines. In this case, the resonator could allow shorter pulses (with higher bandwidth), but losing the tunable behavior of the device.

[23] At the resonator output, the decrease in the pulse amplitudes can easily be compensated using a variable-gain amplifier, for instance that proposed by *Lee et al.* [2007], synchronized with the input pulse generator. The main advantage of this approach is that the temporal distance between two consecutive pulses [$T_P(\omega)$] is controlled by the modulation frequency. This is demonstrated in Figure 6, where the pulse waveforms at the

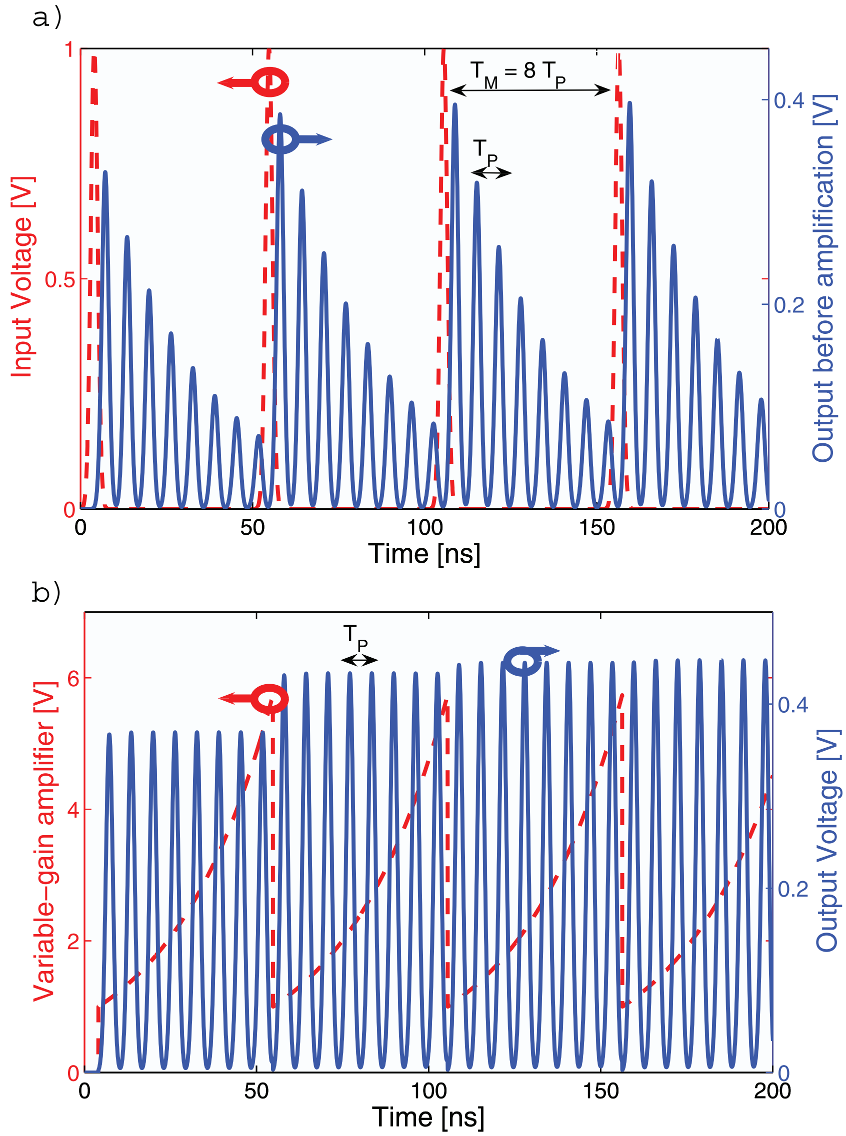


**Figure 7.** Gaussian waveforms at the output of the CRLH resonator of Figure 6d for a pulse train excitation ($f_c$ = 5.0 GHz, $\sigma$ = 1.0 ns, $T_P$ = 6.35 ns and $T_M = 8T_P$ ns). (a) Input pulse train (dashed) and output pulse train (solid) before amplification (at $Z_R$). (b) Amplifier gain (dashed) and output pulse train (solid) after amplification (at $Z_L$).

output of the CRLH resonator (just before the amplifier) are depicted for different carrier frequencies, showing the tunability property of the resonator. These results have been obtained using a time domain Green's function approach [*Zhoua et al.*, 2005; *Gómez-Díaz et al.*, 2009] and validated using the commercial software ADS©.

[24] After several round trips of the pulse, the energy inside the resonator decreases, due to both the losses and the energy extracted from the line. To maintain the same pulse rate [$T_P(\omega)$] at the output, a train of pulses is required in the input, with periodicity $T_M(\omega) = nT_P(\omega)$ ($n \in N$). Therefore, the CRLH resonator may be seen as a 1 : $n$ pulse rate multiplication device. This is demonstrated in Figure 7, which shows the output of a CRLH resonator when it is excited by a modulated train of pulses, before (Figure 7a) and after (Figure 7b) amplification. Note that the voltage-gain amplifier behavior is well defined, because the exponential decay of the voltage due to the losses inside the resonator occurs in a relative large and previously known time interval $T_M$. In practical

cases, the number of pulses generated ($n$) depends on the type of CRLH employed and on the features of the input pulse. In the dispersive (or left-handed) region, the modulation frequency controls the dispersion that the pulse will suffer in time domain. When this dispersion is important, the final shape of the pulse varies, limiting the repetition rate of the resonator. However, this can be totally compensated using the approach presented by *Schwartz et al.* [2008]. In addition, the losses introduced by the line limit the repetition rate in the entire frequency band (both left and right handed regions). Nevertheless, the practical values of ($n$) are high enough to provide a considerable increase in the repetition rate for most applications, with the additional advantage of tunability.

## 5. Conclusion

[25] In this paper, a novel tunable microwave broadband resonator, inspired from optical laser systems, has been presented. In contrast to usual harmonic resonators, the new scheme is based on broadband composite right/left handed (CRLH) metamaterial lines. When this line is configured as a resonator, it provides nonuniform spectral resonances due to the nonlinear nature of its dispersion curve. This has been exploited in the impulse regime, where the input pulse spectrum is discretized inside the resonator, with different spectral separations as a function of the carrier frequency. In time domain, this discretization leads to a pulse periodicity, with a tunable output time period. To show the practical value of the proposed resonator, a pulse rate multiplication device has been presented. This device provides an increase in the repetition rate of a periodic input pulse, with the additional advantage of repetition rate tunability.

[26] **Acknowledgments.** This work was partially supported by the Spanish Ministry of Education and Science under grant FPU-AP2006-015, and national research project with reference TEC2007-67630-C03-02.

## References

Abielmonaa, S., H. Nguyen, and C. Caloz (2006), CRLH zeroth order resonator (ZOR): Experimental demonstration of insensitivity to losses and to size, in *Proceedings of Asia-Pacific Microwave Conference, Yokohama, Japan*, IEEE Press, Piscataway, N. J.

Caloz, C., and T. Itoh (2005), *Electromagnetic Metamaterials: Transmission Line Theory and Microwave Applications*, John Wiley, Hoboken, N. J.

Ghavami, M., L. B. Michael, and R. Kohno (2007), *UWB Signals and Systems in Communication Engineering*, John Wiley, Hoboken, N. J.

Gómez-Diaz, J. S., S. Gupta, A. Álvarez-Melcón, and C. Caloz (2009), Investigation on the phenomenology of impulse-regime metamaterial transmission lines, *IEEE Trans. Antennas Propag.*, in press.

Jin, Y., and C. Nguyen (2005), A 0.25-mm CMOS T/R switch for UWB wireless communications, *IEEE Microwave Wireless Compon. Lett.*, *15*, 502–504.

Lee, H. D., K. A. Lee, and S. Hong (2007), A wideband CMOS variable gain amplifier with an exponential gain control, *IEEE Trans. Microwave Theory Tech.*, *55*(6), 1363–1373.

Mongia, R., I. Bahl, and P. Bhartia (1999), *RF and Microwave Coupled-Line Circuits*, 1st ed., Artech House, Norwood, Mass.

Pozar, D. (2005), *Microwave Engineering*, 3rd ed., John Wiley, Hoboken, N. J.

Saleh, B. E. A., and M. C. Teich (2007), *Fundamentals of Photonics*, 3rd ed., Wiley-Interscience, Hoboken, N. J.

Sanada, A., C. Caloz, and T. Itoh (2003), Novel zeroth-order resonance in composite right/left-handed transmission line resonators, in *Proceedings of Asia-Pacific Microwave Conference, Seoul, Korea*, IEEE Press, Piscataway, N. J.

Schwartz, J. D., I. Arnedo, M. A. G. Laso, T. Lopetegui, J. Azaa, and D. Plant (2008), An electronic UWB continuosly tunable time-delay system with nanosecond delay, *IEEE Microwave Wireless Compon. Lett.*, *18*(2), 103–105.

Zhoua, L., X. Huanga, and C. T. Chan (2005), A time-dependent Green's function approach to study the transient phenomena in metamaterial lens focusing, *Photonics Nanostruct. Fundam. Appl.*, *3*(6), 100–106.

A. Alvarez-Melcon and J. S. Gómez-Díaz, Department of Information and Communication Technologies, Technical University of Cartagena, E-30202 Cartagena, Spain. (jsebastian_gomez@ono.com)

C. Caloz and S. Gupta, Ecole Polytechnique de Montréal, 2500, Chemin de Polytechnique, Montreal, QC H3T 1J4, Canada.