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A Design Parameter for Reef Beach Profiles—A Methodology Applied to Cadiz, Spain

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Abstract: The southwestern coast of Spain is in a tidal zone (mesotidal) which causes the equilibrium profile to be developed in two different sections: the breakage section and the swash section. These two sections give rise to the typical bi-parabolic profile existing in tidal seas. The existence of areas with reefs/rocks which interrupt the normal development of the typical bi-parabolic profile causes different types of beach profiles. The objective of this article is designing an easy methodology for determining new formulations for the design parameters of the equilibrium profile of beaches with reefs in tidal seas. These formulations are applied on 16 profiles to quantify the error between the real profile data and the modelling results. A comparative analysis is extended to the formulations proposed by other authors, from which it is found that better results are obtained with the new formulations.

Keywords: beaches; equilibrium profile; mono-parabolic; bi-parabolic; reef; tide

1. Introduction

A beach regeneration project requires identifying the fundamental parameters that define the equilibrium profile. As some authors have previously stated [1], we can define the equilibrium profile as the situation or state that a beach profile reaches in a constant wave situation for a sufficient time. Different authors [2–10] have defined the concept of an equilibrium profile. Dean [11] defined it as "an idealization of the conditions that occur in nature for particular sediment characteristics and stable wave conditions"; later, Dean [12] added another definition, such as "the balance between constructive and destructive forces".

Bruun [13] and, later, Dean [14] obtained the mathematical formulation that defines the balance profile in beaches, which is of the form:

$$h = A \cdot x^{\frac{2}{3}}. \quad (1)$$

However, the formulation proposed by Bruun and Dean is only valid for the submerged part of the beach profile, not including the intertidal zone [15]. This is the reason why some other authors [16–19] have developed formulations of the equilibrium profile in two sections—the emerged and submerged profiles—which, when taken together, has been called a bi-parabolic profile (for beaches with a tidal range from 1.80 to 4.92 m). To improve the adjustment of bi-parabolic profiles, Contreras et al. [20] proposed new design parameters in the Southwest of Spain. However, these authors did not consider the effects of reef platforms on the bi-parabolic equilibrium profile.

In coastal areas where rocky slabs or reefs play a predominant role, from the point of view of coastal morphodynamics, knowledge of reef profiles is important. Muñoz-Perez [21] was the first researcher who approached, in a theoretical–practical way (using data from Gomez-Pina [22]), a model that allowed the practical design of supported beaches in a relatively simple way. The studies by Muñoz-Perez et al. [23] mainly emphasized the relationship between the Dean's shape parameter with (A_{CL}) and without (A_{SL}) a reef:

$$A_{CL} = 1.48\, A_{SL}. \tag{2}$$

Gomez-Pina [1] proposed a bi-parabolic model for beach profiles in tidal seas, based on field data from the Spanish coast, that allows for the quantification of a series of parameters related to the beach profile. They also proposed a model for the treatment with a reef. The formulations and recommendations proposed are applicable to the southern coast of Spain, but the methodology can be applied to any worldwide beach. Gomez-Pina expanded the formulation of Muñoz-Perez, introducing the parameter of "relative wave height" (H/F_L)—or its equivalent term of "relative freeboard" (F_L/H)—one of the classic parameters of greater influence on energy flow transmission in submerged waterproof dykes (such as rocky slabs), as follows:

$$A_L^{Measured} = A_{Dean}\, f(B_L/F_L, H/F_L), \tag{3}$$

where B_L is the reef width (m), F_L is the freeboard (m), and H is the wave height (m). These parameters are represented in Figure 1.

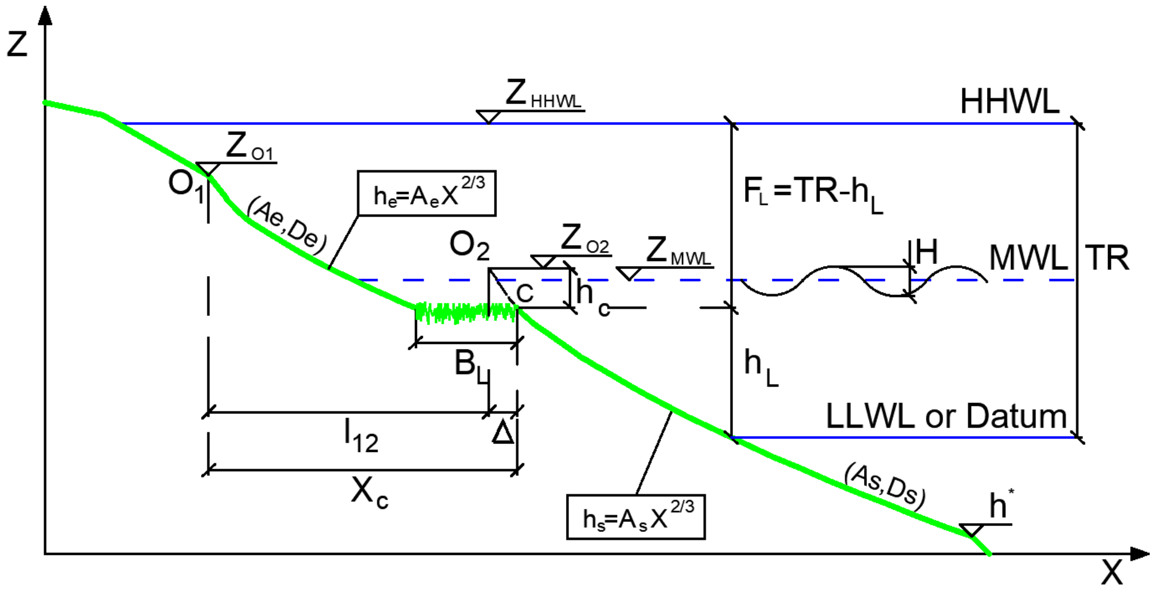


Figure 1. Sketch of a typical bi-parabolic profile in a reef coast. Different parameters are shown. Maritime climate: Highest High Water Level (HHWL), Level of the HHWL (Z_{HHWL}), Mean Water Level (MWL), Level of the MWL (Z_{MWL}), Lowest Low Water Level (LLWL) or Datum, Tidal Range (TR), wave height (H). Morphological parameters: Origin of the emerged (O_1) and submerged (O_2) parables, O_1 Level (Z_{O1}), O_2 Level (Z_{O2}), horizontal distance between O_1 and O_2 (I_{12}), emerged (Ae) and submerged (As) shape parameter, D_{50} or average diameter of the emerged (D_e) and submerged (D_s) zones, beginning of the submerged parabola (C), horizontal distance between O_1 and C (X_C), Length of the reef flat (B_L), freeboard (F_L), reef level (h_L), vertical distance between O_2 and reef level (h_c), closure depth (h^*), level of the profile (Z), and horizontal distance of the profile (X).

The equilibrium profile extends to a "seaward limit", or "closure depth", beyond which there is no significant material transport and which determines the limit of profile validity.

For reef profiles, Muñoz-Perez [21] and Gomez-Pina [1] defined the shape parameters of the emerged parabola. These parameters have been determined in very different coasts, which gives us a good approximation for any other beach. Nevertheless, significant errors have been detected when applying these methods in the Gulf of Cádiz, due to the diversity of tidal range, energy conditions, and sedimentological variations, together with the low number of beaches studied at the time.

Therefore, the objective of this article is determining a new methodology which can be applied to any coast worldwide, by producing formulations which minimize errors for the design parameters of a bi-parabolic profile in reef coastal areas.

2. Methodology

The study area included 71 beaches between the Guadalquivir River and the Strait of Gibraltar (Figure 2). However, only 25 beaches had a reef flat and were, therefore, selected for analysis. A beach profile spacing of 800 m was decided and, thus, 39 profiles were chosen in total [24]. From NW to SE, the tidal range decreases from 3.8 to 1.4 m and the wave energy, $Hs_{,12}$ (wave height exceeded 12 hours a year), varies from 3.5 to 5 m [25]. With respect to sediment, as a general rule, D_{50} increases from NW (0.25 mm) to SE (0.40 mm).

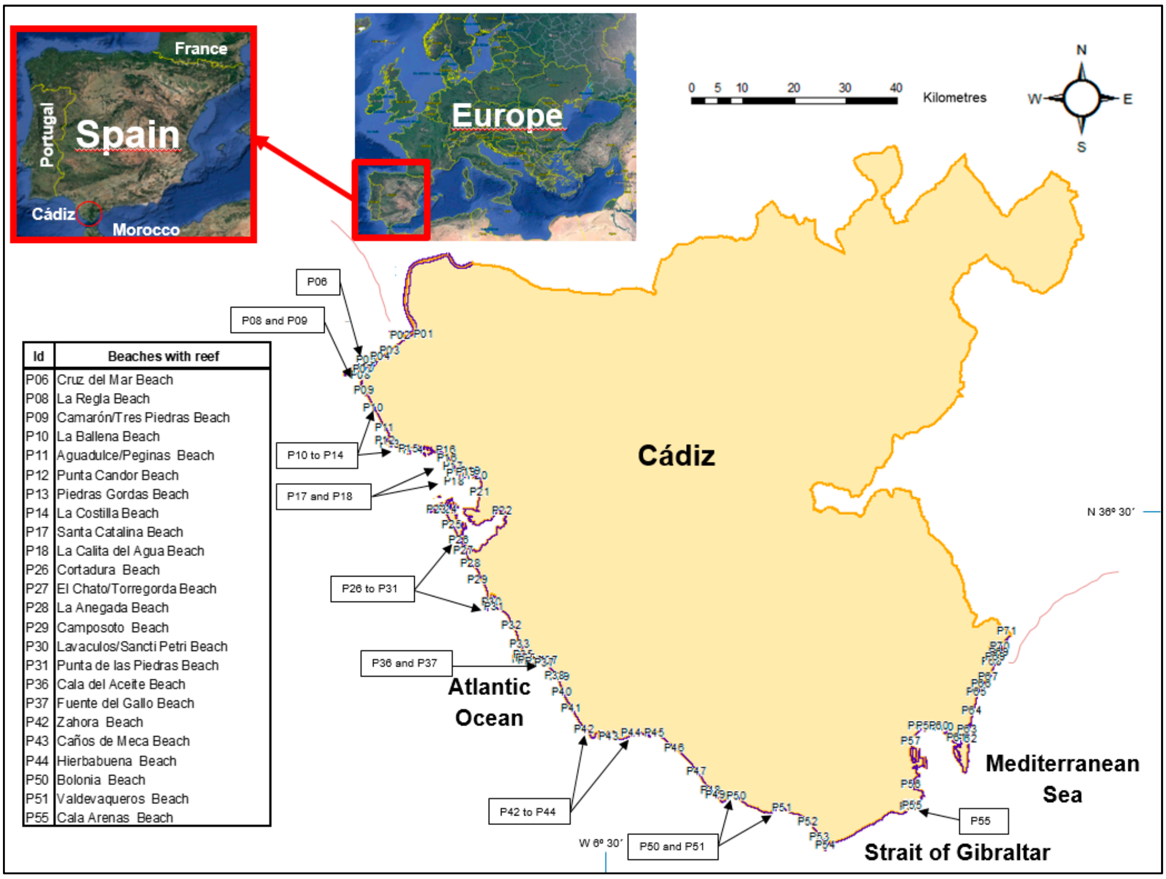


Figure 2. Location of the 25 reef flat beaches (from a total of 71) analysed in this study.

The maritime climate was obtained using the real (REMRO) and virtual (SIMAR) buoy data sets available at (http://www.puertos.es). Moreover, cartographic, bathymetric, morphologic, and sediment data were obtained from the "Ecocartographic Study of the Province of Cádiz" [25] and from other studies carried out previously on the SW coast of Spain [26–29].

With the tool "Arctoolbox" in the ArcMap software, the topography, bathymetry, datum, and location of the cross-section profiles were exported to a CAD file. Once the beach profile was obtained, the geometry of the profile was edited in CAD, obtaining the different points in the coordinates (x, h) that define it. The list of points (x, h) were integrated into the database. Sediment data, through ArcMap tables, were also edited and moved into the database.

For the application of our methodology, it was necessary to define the origin of the emerged (O_1) and submerged (O_2) parables, the emergent (A_e) and submerged (A_s) shape parameters, the "I_{12}" adjustment parameter (horizontal distance between O_1 and O_2), the D_{50} or average diameter of the emerged (De) and submerged (Ds) zones, length of the reef flat (BL), closure depth, and freeboard (F_L).

Figure 1 shows the design parameters of the bi-parabolic profiles supported by a reef flat in a tidal sea.

The dimensionless coefficient of the origin of the emerged parabola (CO_1) is defined as the relationship between the level of the origin of the emerged parabola (Z_{O1}) and the level of the Z_{HHWL} (Highest High Water Level) of each of the profiles studied. Thus, if the coefficient CO_1 is greater than "1", the origin (O_1) is above the HHWL. If the coefficient (CO_1) has a value of "1", the origin of the emerged parabola (O_1) coincides with the HHWL. If its value is "0", the origin of the emerging parabola would be placed at the LLWL (Lowest Low Water Level).

Similarly, the coefficient of the origin of the submerged parabola (CO_2) is defined as the relationship between the level of the submerged parabola (Z_{O2}) and the Mean Water Level (Z_{MWL}). Thus, CO_2 is "1" when the origin of the submerged parabola is exactly at the Mean level of the sea. Meanwhile, CO_2 will be "0" if the origin of the submerged parabola is found on the LLWL or Datum.

The expressions obtained for $A_{e,s} = f\left(D^{50}_{e,s}\right)$ are fundamental in correctly defining the nourished profile (the subscripts *e* and *s* are those referring to the emerged and submerged diameters).

The parameter I_{12} indicates the distance between the poles of the emerged and submerged parables. This parameter was initially defined by Gonzalez [18] and Bernabeu [30] (see I^B_{12} in Equation (4)). The fitness of this formula has been improved and generalized by adding a correction factor "C", depending on the zone (see Equation (5)):

$$I^B_{12} = \left(\frac{h_c + TR}{A_e}\right)^{\frac{3}{2}} - \left(\frac{h_c}{A_s}\right)^{\frac{3}{2}}, \quad (4)$$

$$I_{12} = I^B_{12} * C. \quad (5)$$

The different parameters that define the profile (see Figure 1) were adjusted to find the best fit to the real profiles. This best fit was achieved by minimizing the mean square error (MSE) between the real profile and the modelled profile.

The design parameters resulting from the present research were compared with those presented by previous authors. Table 1 summarizes the formulations for the different design parameters, according to each author.

Table 1. Formulation used for the different design parameters of the models proposed by Dean [14], Muñoz-Perez [21], Gonzalez [18], and Gomez-Pina [1].

Parameters	Gomez-Pina [1]	Gonzalez [18]	Muñoz-Perez [21]	Dean [14]
Profile	Bi-parabolic	Bi-parabolic	Mono-parabolic	Mono-parabolic
Emerged parabola origin "O_1"	HHWL	HHWL	HHWL	HHWL
Emerged parabola	$h = A_e\, x^{\frac{2}{3}}$	$h = A_e\, x^{\frac{2}{3}}$	$h = A_{rf}\, x^{\frac{2}{3}}$	$h = A\, x^{\frac{2}{3}}$
Shape parameter emerged "Ae" (m^3)	$A_e = 0.44\, D_e$	$A_e = 0.65\, D_e^{0,44}$	$A_{rf} = 0.31\, D_e^{0,44}$	$A = 0.214\, D_m^{0,484}$
Submerged parabola origin "O_2"	LLWL	LLWL	-	-
Submerged parabola	$h = A_s\, x^{\frac{2}{3}}$	$h = A_s\, x^{\frac{2}{3}}$	-	-
Shape parameter submerged "A_s" (m^3)	$A_s = 0.44\, D_s$	$A_e = 0.55\, D_s^{0,44}$	-	-

The MSE between the proposed model and the real profile was determined for comparison of the different formulations (see Equation (6)):

$$MSE = \frac{1}{n} \sum_{i=1}^{n} \frac{\left(h_i^{real} - h_i^{model}\right)^2}{h_i^{real}}, \quad (6)$$

where

n is the number of profile points,

h_i^{real} ground level at the point "i" of the real profile, and

h_i^{model} ground level at the point "i" of the modelled profile.

A new error was designed as the quotient between the MSE calculated for each model and the MSE calculated using Dean's formula. This result, which was already dimensionless, indicates the improvement of the proposed model with respect to that proposed by Dean. Therefore, a dimensionless error of "1" indicates that the model did not improve anything with respect to that proposed by Dean; while a dimensionless error of "0" indicates that the model fit perfectly to the real profile.

Once the parameters were validated, they can be applied to future beach regeneration projects on the SW coast of Spain.

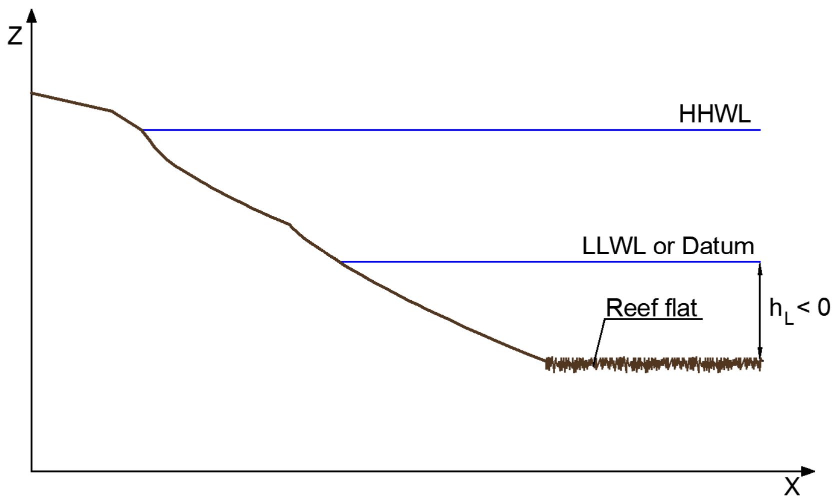
3. Results and discussion

After analysing 25 beaches (39 profiles) on the SW coast of Spain, three different kind of profiles supported on a reef flat were identified (see Figure 3):

The Submerged Reef Bi-parabolic Profile (SRBP), a bi-parabolic profile where the reef flat is always submerged under the LLWL or Datum ($h_L < 0$);

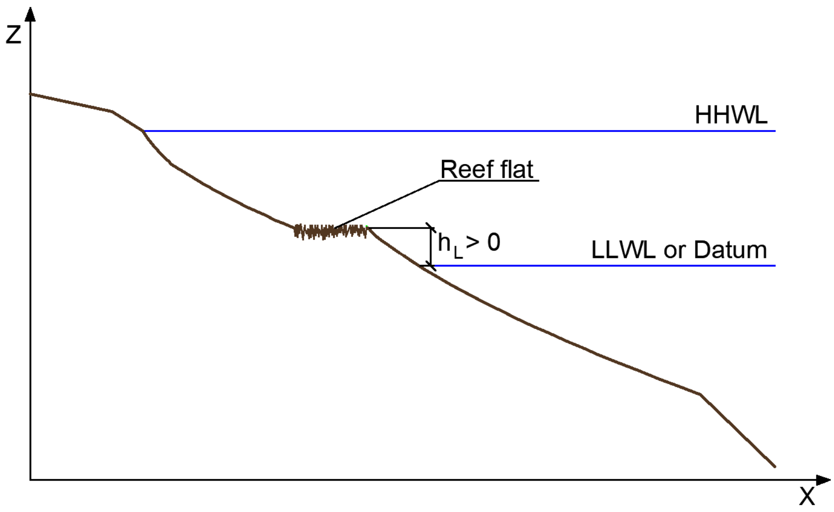
The Emerged Reef Bi-parabolic Profile (ERBP), another bi-parabolic profile where the reef flat sometimes emerges over the LLWL ($h_L > 0$); and

The Reef Mono-parabolic Profile, a mono-parabolic profile where the reef flat is sometimes emerged and sometimes submerged, but is always around the LLWL ($h_L \approx 0$). If the reef flat were deeply submerged, then a SRBP would appear. Meanwhile, if the reef flat was highly emerged and the length of the reef flat was not too much ($B_L < 10 \cdot F_L$, according to Muñoz-Perez et al. [23]), then an ERBP would appear.

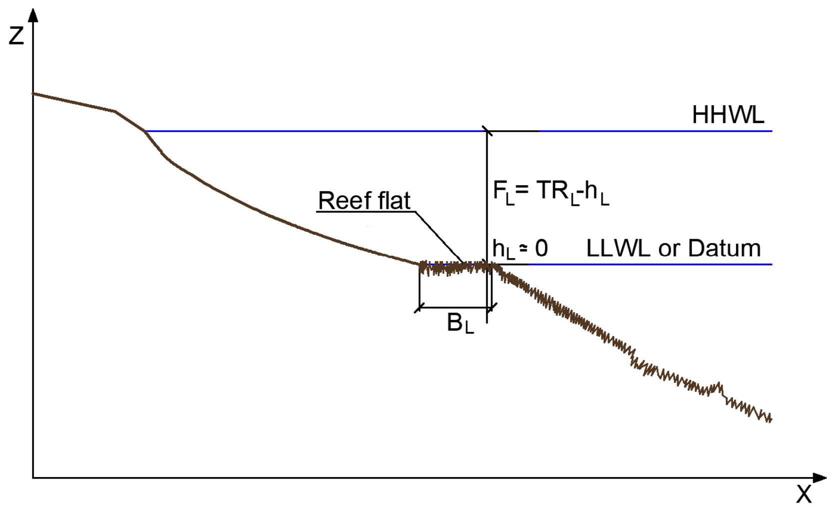


(**A**) SRBP (58% of the profiles)

Figure 3. *Cont.*



(**B**) ERBP (21% of the profiles)



(**C**) ERBP (21% of the profiles)

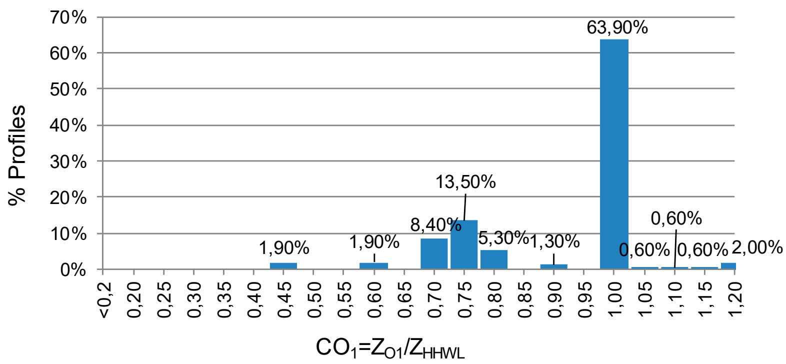
Figure 3. Definition of the three different types of profile supported on a reef flat in the SW coast of Spain: (**A**) Submerged Reef Bi-parabolic Profile (SRBP), (**B**) Emerged Reef Bi-parabolic Profiles (ERBP), and (**C**) Reef Mono-parabolic Profile (RMP).

3.1. Origin (O_1) of the Emerged Parabola

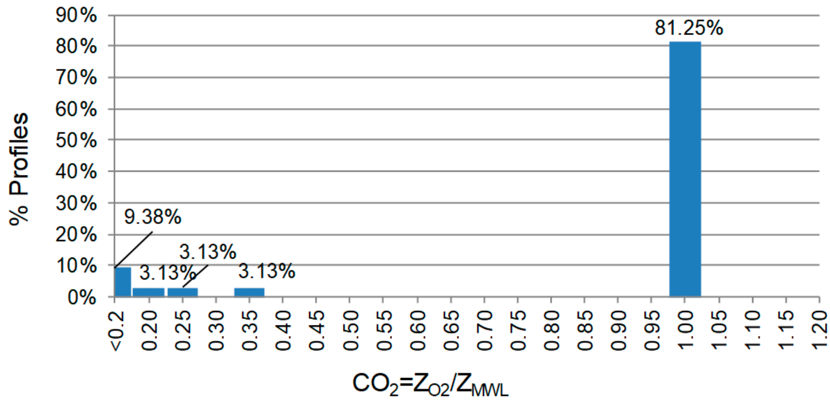
After studying the 39 profiles (mono- and bi-parabolic profiles), the coefficients of the origin of the emerged parabolas (CO_1) are shown in Figure 4 (A). It can be seen that most of the profiles had the origin of the parabola at the HHWL. Thus, the origin of the emerged parabola (O_1) is typically located (63.9%) about the HHWL, and the rest of the profiles are grouped between CO1, ranging from 0.7–0.9 (27.2%). Other positions are negligible. The location of the origin of the emerged parabola at the HHWL confirms what has already been noted by other authors (see Table 1).

3.2. Origin (O_2) of the Submerged Parabola

On the other hand, based on the adjustment of real profiles, the origins of the submerged parabolas (O_2) were mostly located (81.3%) at the MWL (see Figure 4B). In the rest of the profiles (<20%), the origin was dispersed around the LLWL, with CO_2 coefficients ranging between 0.00 and 0.40. Thus, the identification of the MWL as the origin of the submerged parabola found in Figure 4B is different to the former statements of other authors, such as Gomez-Pina [1] and Gonzalez [18]. This difference may have important repercussions in the nourishment design of the bottom part of the intertidal beach profile.



(**A**)



(**B**)

Figure 4. Coefficient (CO_1) of the origin (O_1) of the emerged parabola with respect to the HHWL for mono-parabolic and bi-parabolic profiles (**A**); and Coefficient (CO_2) of the origin (O_2) of the submerged parabola with respect to the MWL for bi-parabolic profiles only (**B**).

3.3. Relationship Between Sediment Size and Profile Typology

The mean D_{50} of the emerged and submerged samples are presented in Figure 5.

As can be seen in Figure 5, the D_{50} ranged from 0.22 to 0.39 mm, where submerged samples had a higher proportion of fine particles than emerged samples. Moreover, the maximum difference was observed on the ERBP profile, probably because the location of the reef flat hinders the ability of fine particles to "climb up" to the emerged beach.

3.4. Relationship between Sediment Size and Dean's Shape Parameter "A"

3.4.1. Emerged Shape Parameter "Ae" for Reef Flat Profiles

As was explained in the methodology, the shape parameters of the emerged parabolas or A_e were adjusted to the different types of profiles by minimizing the MSE. Figure 6 shows a comparative analysis for bi-parabolic (ERBP and SRBP) and mono-parabolic (RMP) models versus Dean's, as a function of diameter (D_e). The correlation coefficient was very high and similar for the three typologies (0.978, 0.977, and 0.999, respectively). As can be seen, the A_e of Dean's formula was always smaller than for any other adjustment to the real topographic data. Moreover, the A_e for SRBP (Figure 3A) was the largest, while A_e for MRP (Figure 3C) was the smallest (closer to Dean's formula); the A_e for ERBP was in the middle (Figure 3B). As is well-known, the larger the shape parameter A_e, the bigger the slope of the beach. Therefore, the beach slope of the emerged parabola is bigger when the reef flat is submerged, is smaller when the reef flat is emerged, and is smallest when the reef flat is located at about the LLWL. This may be as aerial location of the reef flat ($h_L \geq 0$) makes the landward cross-shore transport of the sand by swell waves difficult. Otherwise, the deeper the reef flat ($h_L < 0$), the lesser the barrier effect.

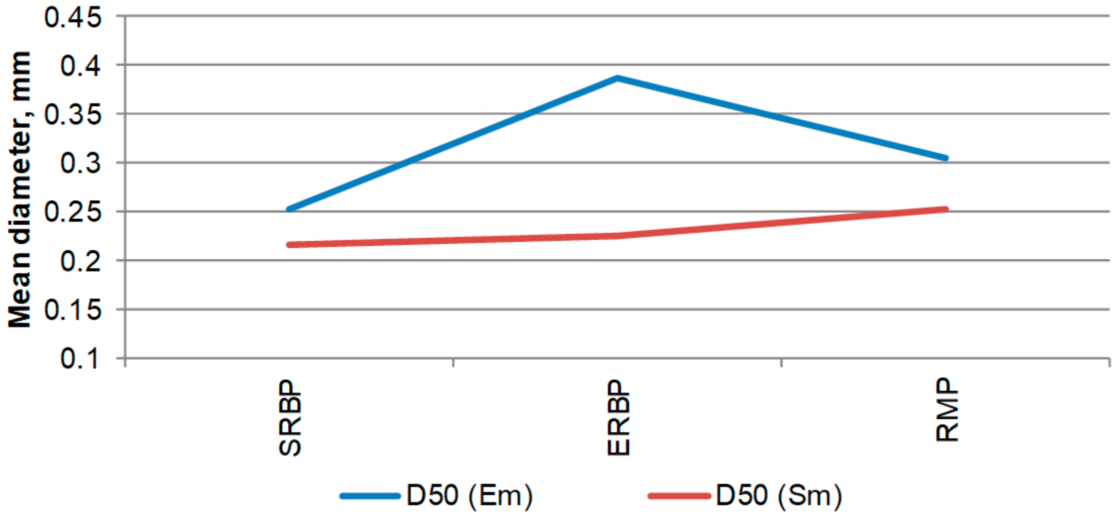


Figure 5. Emerged (Em) and submerged (Sm) mean diameter (D_{50}) of the sand samples for the SRBP, ERBP, and RMP types of reef flat profiles.

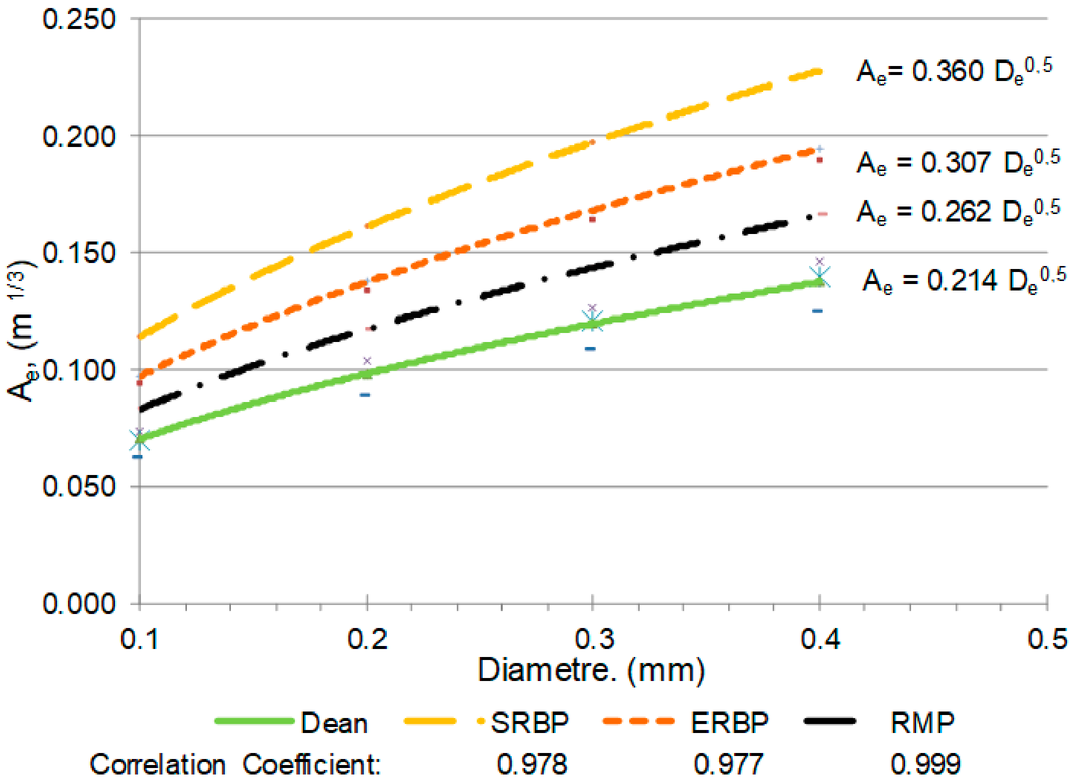


Figure 6. Relationship between the shape parameter of the emerged parabola vs. the average emerged diameter for the three different types of profile (ERBP, SRBP, and RMP). Comparison with Dean's formula.

3.4.2. Submerged Shape Parameter "A_s" for Reef Flat Rofiles

Similarly, the shape parameters of the submerged parabola (or A_s) were also adjusted to the two types of profiles by minimizing the MSE. Remember that the mono-parabolic type (RMP) does not have a submerged parabola (see Figure 3C). Figure 7 shows a comparative analysis for SRBP and ERBP (bi-parabolic profiles) versus Dean's, as a function of the diameter (D_s). The correlation coefficient was very high and similar, again, for these two typologies (0.983 and 0.999, respectively). As shown, the A–D relationship is identical for both SRBP (Figure 3A) and ERBP (Figure 3B). Moreover, there was no difference with Dean's formula. Thus, the methodology presented herein does not provide any significant improvement for the calculation of the submerged parabola.

For all typologies, the emerged shape parameters A_e (Figure 6) were higher than the submerged shape parameter A_s (Figure 7). This means that the emerged parabola has a more vertical profile.

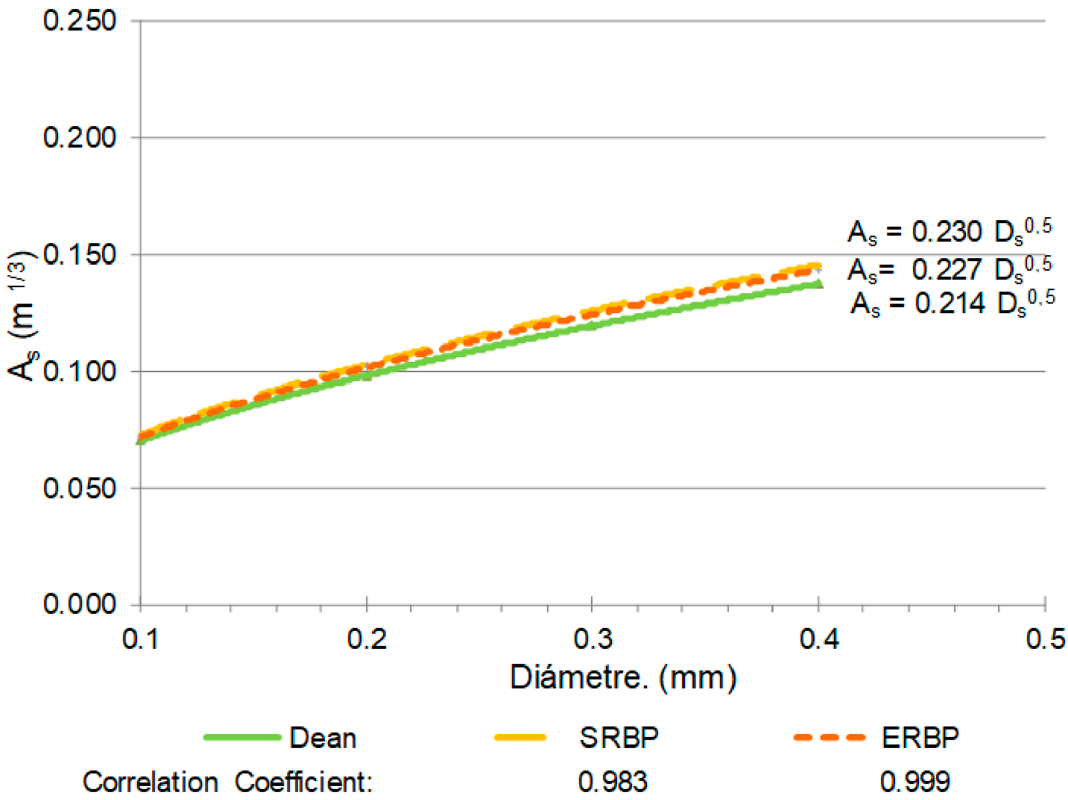


Figure 7. Relationship between the shape parameter of the submerged parabola vs. the average submerged diameter for the two different types of profile (only ERBP and SRBP; obviously, the mono-parabolic type does not have any submerged parabola). Comparison with Dean's formula.

3.5. Parameter "I_{12}"

For the ERBP typology (see Figure 3B), the parameter I_{12} depends on the reef flat length, which will be (see Figure 1):

$$I_{12} = X_c - \Delta = X_c - \left(\frac{h_c}{A_s}\right)^{\frac{3}{2}}. \tag{7}$$

With the following formula for X_c,

$$X_c = B_L + \left(\frac{Z_{o1} - Z_{MWL} + h_c}{A_e}\right)^{\frac{3}{2}}, \tag{8}$$

we obtain

$$I_{12} = B_L + \left(\frac{Z_{o1} - Z_{MWL} + h_c}{A_e}\right)^{\frac{3}{2}} - \left(\frac{h_c}{A_s}\right)^{\frac{3}{2}}. \quad (9)$$

We can simplify the former formulae by using the results presented above (see Figure 4A,B). The origins of emerged and submerged parabolas are approximately the levels of the HHWL and the MWL, respectively:

$$Z_{o1} \approx Z_{HHWL}, \quad (10)$$

$$Z_{o2} \approx Z_{MWL} \approx \frac{Z_{o1}}{2} \approx \frac{Z_{HHWL}}{2}. \quad (11)$$

Therefore,

$$I_{12} = B_L + \left(\frac{\frac{Z_{HHWL}}{2} + h_c}{A_e}\right)^{\frac{3}{2}} - \left(\frac{h_c}{A_s}\right)^{\frac{3}{2}} = B_L + \left(\frac{\frac{TR}{2} + h_c}{A_e}\right)^{\frac{3}{2}} - \left(\frac{h_c}{A_s}\right)^{\frac{3}{2}}. \quad (12)$$

Thus, we can obtain the distance between the origins of the emerged and submerged parabolas (I_{12}), using the values of:

B_L (reef flat length), which is easily measurable;

h_c (vertical distance from the reef flat to the origin of the submerged parabola O_2) approximately to the MWL, as was shown in Figure 4B; and

A_e and A_s, which are obtained from Figures 6 and 7 by introducing D_e and D_s, respectively (ERBP typology).

For the SRBP typology, the correction coefficient "C" with respect to the parameter $I_{12}{}^B$ (the theoretical I_{12} proposed by Bernabeu [30] was already mentioned in the methodology) has been analysed, determining a "C" value of 0.965. Thus, substituting and combining Equations (4) and (5), we obtain:

$$I_{12} = I_{12}^B \cdot C = 0.965 \cdot I_{12}^B = 0.965 \cdot \left(\left(\frac{TR + h_c}{A_e}\right)^{\frac{3}{2}} - \left(\frac{h_c}{A_s}\right)^{\frac{3}{2}}\right). \quad (13)$$

The RMP typology obviously does not have the parameter I_{12} (see Figure 3C), as it is a mono-parabolic profile (i.e., there is no submerged parabola).

3.6. Application of Design Parameters on Real Profiles

The parameters previously determined were validated using the real profiles. These real profiles were modelled to obtain results of quite acceptable adjustments. The equations and parameters used for each of the typologies are shown in Table 2.

Table 2. Summary of equations and formulae to calculate (for each typology) the parameters used to model nourished profiles.

Equations and Parameter	Typologies		
	Bi-parabolic		Mono-parabolic
	Emerged Reef Flat	Submerged Reef Flat	Reef Flat about the LLWL
	ERBP	SRBP	RMP
Emerged parabola origin "O_1"	HHWL	HHWL	HHWL
Emerged parabola	$h = A_e\, x^{\frac{2}{3}}$	$h = A_e\, x^{\frac{2}{3}}$	$h = A_e\, x^{\frac{2}{3}}$
Emerged shape parameter "A_e" ($m^{1/3}$)	$0.307\, D_e^{0.5}$	$0.360\, D_e^{0.5}$	$0.262\, D_e^{0.5}$
Submerged parabola origin "O_2"	MWL	MWL	-
Submerged parabola	$h = A_s\, x^{\frac{2}{3}}$	$h = A_s\, x^{\frac{2}{3}}$	-
Submerged shape parameter "A_s" ($m^{1/3}$)	$0.227\, D_s^{0.5}$	$0.230\, D_s^{0.5}$	-
Distance between poles "I_{12}" (m)	Eq (12)	Eq (13)	-

3.7. Comparative Analysis with other Authors

The study was completed with a comparative analysis between the different theories stated by the authors presented in Table 1 versus the models presented herein, as shown in Table 2.

The comparative analysis was carried out with respect to the theories proposed by Dean [14], Muñoz-Perez [21], Gonzalez [18], and Gomez-Pina [1]. The dimensionless MSE, determined as the quotient between the MSE calculated for each model (Equation (6)) and the MSE calculated using Dean's formula, was estimated for each of the authors and typologies, as summarized in Table 3.

Table 3. MSE (dimensionless), calculated for the different authors and for each profile type.

Typology	Contreras et al.	Gomez-Pina [1]	Gonzalez [18]	Muñoz-Perez [21]
SRBP	0.33	0.70	0.33	-
ERBP	0.16	1.09	1.21	-
RMP	0.19	0.57	-	0.37

Thus, in brief, as can be seen in Table 3, the model that best fit the real profile data was always that proposed in this article. It must be highlighted that the improvement was particularly significant for ERBP; that is, when the reef flat is located above the LLWL.

4. Conclusions

The SW coast of Spain has a wide variety of profiles supported on reef flats. These reef flats interrupt the normal development of the typical profiles in tidal seas. In this paper, three types of profiles were identified, depending on the situation of the reef flat (over, below, or at the LLWL): Emerged Reef Bi-parabolic Profile (ERBP), Submerged Reef Bi-parabolic Profile (SRBP), and Reef Mono-parabolic Profile (RMP).

The emerged and the submerged parts of the profile can be modelled with Dean's parabolic formulae ($h_e = A_e\ x^{2/3}$ and $h_s = A_s\ x^{2/3}$, respectively), where the emerged and submerged shape parameters (A_e and A_s) are directly related to the sediment diameter (D_e and D_s). However, its adjustment to real data is not optimal. Thus, based on the analysis of 39 profiles supported on reef flats chosen from 25 beaches located in the SW of Spain, a new model has been presented herein, in order to improve the modelling of the profile.

Among the findings, it should be noted that the origin "O_1" of the emerged parabola was confirmed to be located at the HHWL. On the other hand, the origin "O_2" of the submerged parabola was placed at the Mean Water Level (MWL), while other authors have established it using the LLWL. Some new and easy-to-apply formulas have been derived, which allow for calculation of the horizontal distance "I_{12}" between "O_1" and "O_2".

As for the emerged parabola, the shape parameters found for SRBP, ERBP, and RMP typologies were $A_e = 0.360\ D_e^{0.5}$, $A_e = 0.307\ D_e^{0.5}$, and $A_e = 0.262\ D_e^{0.5}$, respectively, all of which were higher than that in the original Dean's formula ($A_e = 0.214\ D_e^{0.5}$). Therefore, the beach slope of the emerged parabola is bigger when the reef flat is submerged, smaller when the reef flat is emerged, and smallest when the reef flat is located at about the LLWL. This may be explained by the aerial location of the reef flat ($h_L \geq 0$) making it difficult for swell waves to transport sand landward cross-shore. Otherwise, the deeper the reef flat ($h_L < 0$), the lesser the barrier effect.

As for the submerged parabola, there were no appreciable differences between the shape parameters. Moreover, there was no difference from Dean's formula. Thus, the methodology presented herein does not provide any notorious improvement for the calculation of the submerged parabola.

Finally, it must be highlighted that the improvement is really significant when the adjustment to real profile data of the design parameters proposed in this article is compared to the parameters proposed by other authors; especially when the reef flat is located above the LLWL.

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