On the equilibrium of the Egyptian game Senet

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Abstract

In this work we compute the probabilities of obtaining the values associated with the throwing of four rods in the Egyptian game Senet. Our purpose is to, among several sets of rules obtained by some historians and archeologists, deduce which ones provide the most balanced game.

In the course of this study it was also possible to infer the most likely format for the rod's cross section.

Keywords: Senet, Centre of Mass, Probability

MSC: 97A20, 01A16, 60-03

Historical Introduction

Senet, a member of the board game family to which modern Backgammon and Tabula belong, gained much popularity in all social classes of ancient Egypt over a period of two thousand years. This happened not only due to its ludic character – it was a racing game with a strategic component – but also because of the religious interpretation that was associated with it, with consequent inclusion in their mythology.

Senet's oldest complete board ever found was discovered in a tomb in Saqqara (south of the Nile delta) dating from the 17th century BC. Nevertheless, in the grotto of Abydos and in Saqqara, similar boards, dating from the pre-dynastic period and the First dynasty (3500 BC–3100 BC) were discovered; surely it would be some forerunner of the game.

On the walls of several *mastabas* (i.e. tombs), located in Giza, Saqqara, Abu Sir and Meir, dating from the Ancient Kingdom (2686-2160 BC), amongst painted scenes of daily life of ancient Egypt, several murals were found

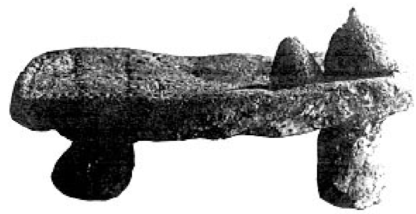

Figure 1: Ancestor of Senet (≈ 3300 BC - el-Mahasna, Egypt).

that depict people playing Senet during the rituals and celebrations of the festival in honor of the goddess Hathor.

Recent research (analysis of various boards, murals and interpretation of sacred writings) seem to prove that, in the light of Egyptian religion, the strategies used in the game reflect the strategies of the gods of ancient Egypt (at least, according to ancient religious texts) and that Senet highlights some key aspects of religious beliefs of the Egyptian people. It enables a mystical union of the player with the sun god Re-Horakhty, or at least attempts to influence the judgment of the soul when the moment of its departure to eternal life arrives.

Figure 2: Playing Senet

It is believed that, in the beginning, Senet had emerged as a simple amusement activity devoid of any religious significance. But it did not take long before people began to assign it a mystical role, which quickly evolved. During the 5th dynasty the deceased was sometimes represented witnessing two people alive playing Senet, but in the course of the 6th dynasty he reached the main role becoming the adversary of someone who was still alive. It was during this period that major changes occurred in the way this game was regarded. Due to the religious significance that it was acquiring, it is conjectured that its board assumed a role of a bridge that physically connects the world of the living to the realm of the dead. During the Middle Kingdom, Senet was associated with contact with the dead, as well as the free movement and passage of the *ba*, the soul of the individual (his spiritual essence, which defines his identity and personality) to the other world. The beliefs of the Egyptians were changing and at the end of the 12th dynasty (≈ 1900 BC), this game came to be associated with a spiritual rebirth of the person after death, made possible by the mobility of his soul which, from that moment, acquired the ability to freely cross the border between life and death.

Senet then became known as the «Game of passage of the soul to another world». Quoted in the well-known «Book of the Dead», it symbolizes the struggle of the player's soul against Evil or against enemy forces that wander in the underworld.

Figure 3: A board of Senet with its pieces

When the 18th dynasty was coming to an end (≈ 1293 BC), this game had already become a «simulation» of the other world, with its squares representing the major Egyptian deities and the main trials of «life after death». At that time its status was completely changed. The «society game» had given rise to a ritual that recreated the crossing to the underworld by the person playing it, as if he were impersonating a deceased journeying to the other world.

This game was so important that, during the first dynasty, several *mastabas* (tombs) were built that clearly emulate a Senet grid, and later, during the entire pharaonic period it remained an important funerary gift for the rich and the poor. Some humble graves were found in which, aside from some pottery and a few beads, a Senet board and its pieces were the only other objects buried with the dead.

During the 20th dynasty (≈ 1180 BC) the ritual of Senet was recorded on a papyrus, entitled «Text of the Great Game»; three copies have survived until modern times. One of them, which was preserved in the Turin Papyrus 1775, is accompanied by illustrations of various Senet boards. It explains that the ritual should only be performed on special boards, where each square (or «house») was associated with a particular deity. Before playing, permission to the gods of the squares should be asked, clearly stating the purpose of the play.

The highest purpose an Egyptian could aim at was to achieve immortality. That was the symbolic purpose of Senet and of some aspects of Egyptian culture.

The rules of the game

Senet was often referred to as the game of thirty squares, because it was usually played on boards with 3×10 squares. In the following these squares will be numbered from 1 to 30. Note that, although boards with $3 \times 11, 3 \times 6$ squares and other configurations have also been found, historians do not have enough information to allow them to decide whether they were different games or just slight variations of the same game.

Some squares of the board are important and thus usually marked with special decorations:

1. The *House of Rebirth* (15), decorated with an *Ankh*.
2. The *House of Beauty* or *House of Happiness* (26), usually decorated with a circle.
3. The *House of Water* or *House of Humiliation* (27), usually decorated with some wave drawings.
4. The *House of the Three Truths* (28), usually decorated with the symbol *III*.
5. The *House of the Sun* (29), usually decorated with the symbol *II*.
6. The last square of the board (30), usually decorated with the symbol *I*.

Until the end of the 18th dynasty the role of dice was played by four small rods (or wooden sticks) roughly shaped like a straight cylinder, truncated longitudinally by a plane parallel to its axis. The number of faces facing up would be related with the number of squares the player could move one of his pieces. The player's aim was, after following a path in the form of

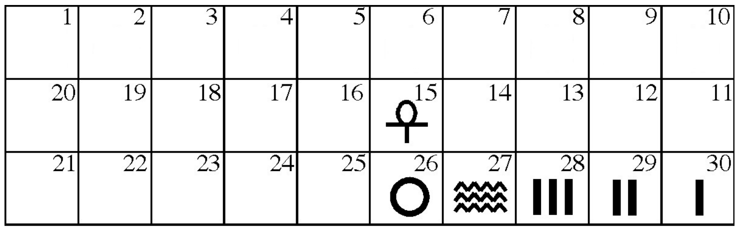

Figure 4: The squares of the Senet board

an inverted S (from square 1 through square 30) during which he should overcome his opponent's attacks and prevent his pieces from landing on some harmful squares, to be the first to take all his pieces off the board or reach a certain predetermined configuration.

What were the exact rules of this game? We do not know. Those that were followed in antiquity, are unknown to us. One might think that describing the rules of a game that lasted more than two millennia is somewhat meaningless because, instead of remaining static, they surely underwent many changes, not only over time, but also geographically; note that due to oral transmission, some local variations must surely have occurred. But, in fact, such apparent obstacles only reflect the richness of the theme. It is fascinating to seek information on the rules that remained stable (i.e., those that characterized the essence of the game), those that were used at certain times or places (probably related with local culture or with some events) and try to interpret the changes that have occurred from a sociological point of view.

The sets of rules that currently exist were inferred by some researchers (Timothy Kendall, historian; Robert Bell, expert in board games; Gustave Jéquier, archaeologist and John Tait, historian), based on information gleaned from scenes depicted in murals and texts alluding to the game, kept in papyrus. However, there is no consensus among experts on which ones are correct. Our purpose here is to investigate some characteristics found in these sets of rules (that were used until the end of the 18th dynasty) and to check if their study gives us evidence enough to ascertain which of them enable the most balanced game.

Let's compare these sets of rules. According to Jéquier and Tait, each player starts with 5 pieces while Kendall believes that this number was 7 and Bell suggests 10. These two sets of pieces, painted one in black and the other in white, were sometimes carved with the shape of an animal.

Kendall and Jéquier assumed that the players would put their pieces in the

Figure 5: Senet board with its rods

first squares of the board, starting with white and alternating with black. The other two researchers conjectured that the players started with their pieces off the board. In Bell's version, the game is played backwards with each piece entering one of the last five squares of the board, as long as the player gets 1 to 5 in the rods and the corresponding square (counting from the last square of the board) is empty; in Tait's version each piece enters the board on a throw of 4 or 6 and goes to square 4 or 6, respectively, unless it is already occupied by a piece of the same color.

According to Bell each player takes turns throwing the rods and one of his pieces may move the number of squares indicated by the wooden rods. Tait allows the player to make an extra throw after obtaining a 6 and, according to Jéquier and Kendall, each contestant continues to play until a throw of 2 or 3 is obtained; after making his move, he passes his turn to the opponent.

We must emphasize now the fact that after simultaneously throwing the four rods, for each flat face facing up a chosen piece would be able to move one square; but if no flat faces were facing up the piece could move 6 squares, according to Jéquier and Tait, or 5, if we follow the rules conjectured by the other researchers.

There are some limitations concerning the movement of the pieces. Following Kendall and Jéquier no piece can land on a square already occupied by a piece of the same color or attack an opponent's piece if protected (that is, when it stands next to another of the same color); but if it is unprotected, i.e., isolated, the two pieces change positions. Jéquier adds that the pieces can jump over the opponent's pieces if the latter do not form a block of at least three consecutive pieces. In addition, two pieces cannot occupy the same square. Bell and Tait allow the attack of any of the opponent's pieces;

whenever that happens the piece is bumped off and must reenter the board again.

Figure 6: Senet special squares

Some squares are important: the House of Beauty (26) is a mandatory stop (Kendall); the House of Water, or Humiliation (27) sends the piece back to square 15 (Kendall), or off the board (Tait); a piece that had fallen in the House of the Three Truths (28), or Re-Akoum (29), can only get out of it (and off the board) with an exact throw of 3 or 2, respectively (Kendall); if a piece lands on squares 26, 28 or 29, it cannot be attacked (Jéquier).

Finally, there are some differences concerning the end of the game. While Bell asks the player to lead his pieces, moving backward, to squares odd-numbered 1 to 19, or even-numbered 2 to 20, the other researchers require that a piece can be off the board only if the throw gives the exact number of squares needed to attain square 30, plus one; Jéquier adds the additional rule that a piece can get off the board only if all the pieces of the same color are already on the last line. As expected, the first player who successfully removes all his pieces from the board is the winner of the game.

The above comparison allowed us to highlight the main similarities and differences between these four sets of rules, but there are more subtleties that were not mentioned; additional information concerning this game can be obtained, for example, in [1], [2], [3] and [4].

About the rules and the rods

Until the end of the 18th dynasty the role of the dice in Senet was played by four small wooden rods, as shown in Figure 5.

Note that the curved face and the rectangular flat face of the rods were painted with different colors and that, during the 18th dynasty, these

Figure 7: A Senet board

wooden rods would gradually be replaced by the *Astragali*, small bones of sheep and ox.

A small discussion is now required concerning the ludic aspects of the sets of rules that were explained in section 2. Although all are viable, those that use a rather large number of pieces are most unlikely to have been used, due to the difficulty of devising winning strategies for games whose boards are overcrowded with pieces. Thus, a game that follows the rules proposed by Bell and Kendall does not appear to be a very balanced one.

According to the Senet rules suggested by Jéquier and Tait, after simultaneously throwing the four rods, each flat face facing up was worth one point.

Thus, the possibilities were:

1 flat face up - 1 point

2 flat faces facing up - 2 points

3 flat faces facing up - 3 points

4 flat faces facing up - 4 points

0 flat faces facing up - 6 points.

There does not seem to be a clear reason explaining the absence of the 5 points score, which seems to contradict some images and texts showing that, in addition to being a result that could actually be obtained, was also an excellent move. As the other sets of rules accepted the 5 points score, we wonder why these two researchers hypothesized its absence. Maybe the game evolved and a replacement of the 5 by the 6 had taken place with the aim of making the game more exciting.

Additionally, both attached similar importance to specific pairs of numbers. Jéquier conjectured that the scores 2 and 3 had a special common role: after the player had moved that number of squares, the turn was given to the opponent. On the other hand, Tait conjectured that the numbers 4 and 6 were relevant: each piece would enter the board only after a throw of any of the mentioned numbers.

These two situations are unexpected and deserve further investigation. It seems natural to suppose that these rules aren't arbitrary, but instead, there is some deeper reason for both, probably related with the shape of the rods. Thus, one may enquire what format the rod should have if one assumes that:

(a.1) the numbers 2 and 3 should occur with the same probability, $P(2) = P(3)$, or

(a.2) with a probability ratio proportional to the score ratio: that is, $P(2)/P(3)$ equals 2/3;

(b.1) the numbers 4 and 6 should occur with the same probability, $P(4) = P(6)$, or

(b.2) with a probability ratio proportional to the score ratio: that is, $P(4)/P(6)$ equals 4/6.

The computation of the probabilities of the throw of the Senet rods has already been addressed, from an experimental point of view, in [5]. In this work we will consider that when a player throws a rod, whether it lands with the curved face or the rectangular flat face upwards depends ultimately on the properties of the rod's cross section, that is, we suppose that the cross-section radius R is much smaller than the length L (see Figure 8). In addition, it is admitted that the rod is made of a homogeneous and isotropic material.

With these hypotheses it is expectable that the probability of a rod falling on one of the bases is negligible; thus, it is sufficient to analyze the relationship between the shape of the rod's cross-section and the probability that the rectangular flat face or the curved face is facing upwards. That does not mean that the three-dimensional case should not be considered. In fact, it can be studied using the notion of *Solid Angle*, which is suitable for that situation. It can be shown that, when the ratio R/L tends to zero, the limit of each of the expressions obtained for the probabilities of when the rod rests on one of the bases or on the curved face or on the rectangular plane face, are exactly the expressions that were obtained, for the corresponding cases, in the two-dimensional analysis. As these computations are very technical they will be omitted here and published elsewhere.

Probabilities for the two-dimensional case

We start this discussion by supposing that the rod is roughly shaped like a straight cylinder, truncated longitudinally by a plane parallel to its axis, as indicated schematically in Figure 8:

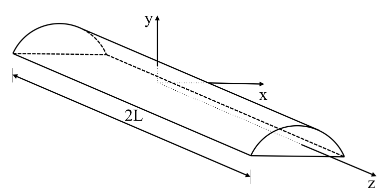

Figure 8: Three-dimensional rod

Now, we consider the rod's cross section as in the following figure:

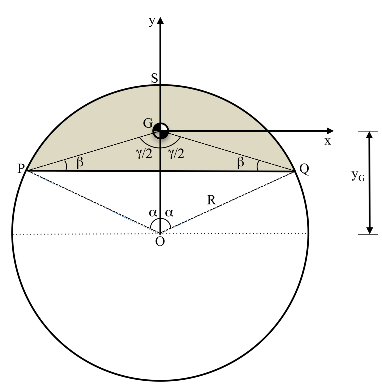

Figure 9: Rod's cross section (shaded)

Let P_P be the probability that the rod lands with the rectangular flat face upwards and by $P_C = 1 - P_P$ the probability that it lands with the curved face upwards.

With four rods, the probabilities of obtaining the various scores are:

Score	Probabilities
1	$4P_P P_C^3$
2	$6P_P^2 P_C^2$
3	$4P_P^3 P_C$
4	P_P^4
6	P_C^4

Note that the (cumulative) probability of all events is $(P_P + P_C)^4 = 1$.

Now, we may ask what the relationship between P_P and P_C is, if we want the probability of obtaining 4 or 6 points to be the same (according to Tait this happens when the game begins). For this, we must have $P_P^4 = P_C^4$, that is, $P_P = P_C = 1/2$.

However, if we want the probability ratio $P(4)/P(6)$ to be proportional to the score ratio 4/6, then $6P_P^4 = 4P_C^4$, that is $3^{1/4}P_P = 2^{1/4}P_C$; thus

$$P_P = \frac{(2/3)^{1/4}}{1+(2/3)^{1/4}} \approx 0.47 \text{ and } P_C = \frac{1}{1+(2/3)^{1/4}} \approx 0.53.$$

Similarly, we can ask what is the relationship between P_P and P_C if we want the probability of passing the game to the opponent to be the same, that is, according to Jéquier, when we get 2 or 3 points. For this, it is needed that $6P_P^2 P_C^2 = 4P_P^3 P_C$, that is $3P_C = 2P_P$, and so $P_P = 0.6$ and $P_C = 0.4$.

However, if we want the probability ratio $P(2)/P(3)$ to be proportional to the score ratio 2/3, then $9P_C = 4P_P$; thus $P_P = 9/13 \approx 0.69$ and $P_C = 4/13 \approx 0.31$.

The following table shows the probabilities of obtaining points for the cases mentioned above.

Score	$P(4) = P(6)$	$6P(4) = 4P(6)$	$P(2) = P(3)$	$3P(2) = 2P(3)$
1	$4/16 = 0.25$	≈ 0.22	$96/625 = 0.1536$	$2304/28561 \approx 0.08$
2	$6/16 = 0.375$	≈ 0.37	$216/625 = 0.3456$	$7776/28561 \approx 0.27$
3	$4/16 = 0.25$	≈ 0.28	$216/625 = 0.3456$	$11664/28561 \approx 0.41$
4	$1/16 = 0.0625$	≈ 0.05	$81/625 = 0.1296$	$6561/28561 \approx 0.23$
6	$1/16 = 0.0625$	≈ 0.08	$16/625 = 0.0256$	$256/28561 \approx 0.01$

There is no direct archaeological evidence of the importance of these issues but their consideration will lead to rods with different cross sections.

Computation of the rod's mass centre

In order to compute the values of P_P and P_C, as a function of the shape of the rod's cross section, we start by computing the geometric properties of the rod's cross section as a function of angle α. We remark that whether the rod lands on the rectangular flat face or on the curved face is related solely on the position of the centre of mass, G, of its cross section.

Consider again Figure 9. The area A of the rod's cross section, obtained from the area of the sector with circular arc $\widehat{PQ}$, is given by:

$$A = R^2 \left(\alpha - \frac{\sin 2\alpha}{2} \right), \quad \alpha \in]0, \pi[.$$

The variation of $\frac{A}{R^2}$ with respect to the angle α is shown in the following figure:

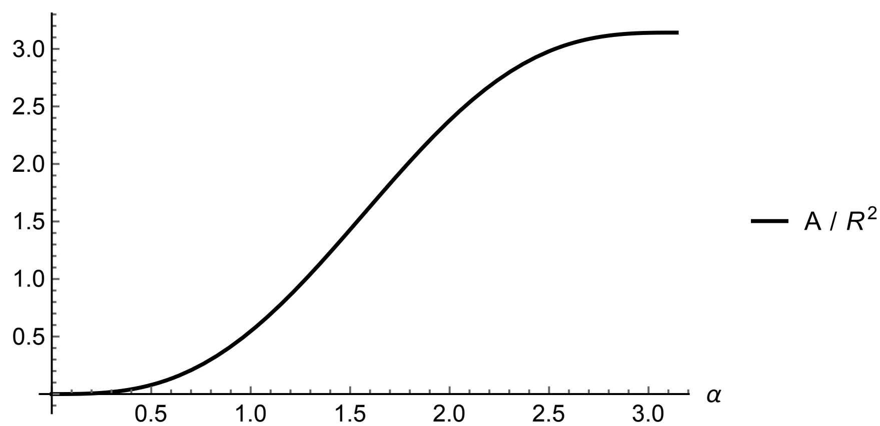

Figure 10: Cross section area as a function of angle α

The coordinates (x_G, y_G) of the centre of mass G (with respect to a system of axes parallel to the system Gxy and passing through the point O), are given by:

$$x_G = 0, \quad y_G = R\,\frac{2}{3}\,\frac{\sin^3 \alpha}{\alpha - \sin\alpha\cos\alpha}, \quad \alpha \in]0, \pi[.$$

The variation of $\frac{y_G}{R}$ with respect to the angle α is shown in the following figure:

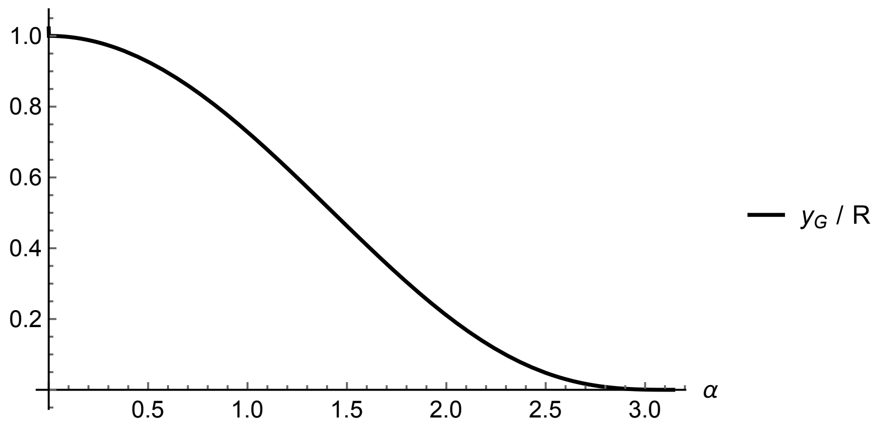

Figure 11: Centre of mass as function of the angle α

The centre of mass lies inside the rod, bearing in mind that $y_G - R\cos\alpha \geq 0$, as shown in the following figure:

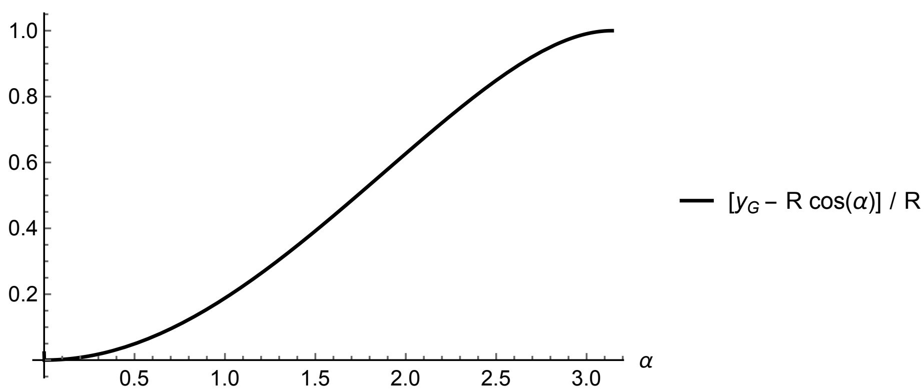

Figure 12: Centre of mass in the rod

Computation of P_P and P_C probabilities for the two-dimensional case

Consider again Figure 9. The angle $\beta \in]0, \frac{\pi}{2}[$ is given by:

$$\beta = \arctan\left(\frac{\frac{2}{3}\frac{\sin^3\alpha}{\alpha - \sin\alpha\cos\alpha} - \cos\alpha}{\sin\alpha}\right), \quad \alpha \in]0, \pi[.$$

This expression is represented graphically in the following figure:

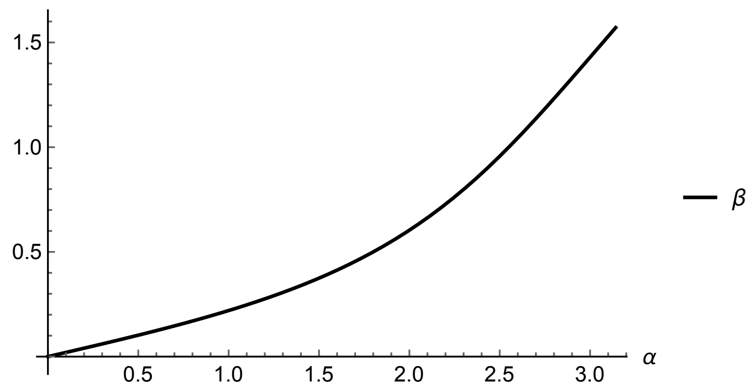

Figure 13: Angle β as a function of angle α.

In the following, angle γ will be given by $\pi - 2\beta$ and its value, as a function of α, is represented in the following figure:

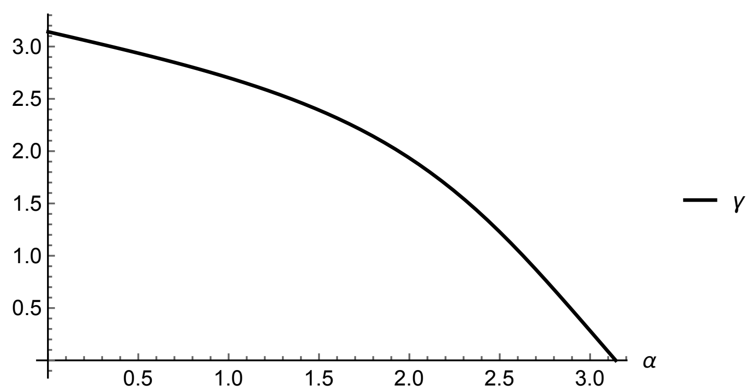

Figure 14: Angle γ as a function of angle α

From the previous discussion, the probabilities of the rod falling with the flat face upwards (P_P) or with the circular face upwards (P_C) are related to the position of its centre of mass relative to the vertical axis, as shown in Figure 15, and are respectively:

$$P_P = \frac{2\pi - \gamma}{2\pi} = \frac{1}{2} + \frac{\beta}{\pi}, \quad P_C = \frac{\gamma}{2\pi} = \frac{1}{2} - \frac{\beta}{\pi}.$$

These results are shown in the figure:

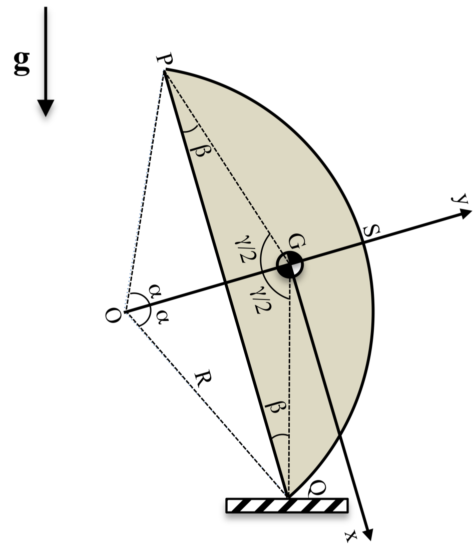

Figure 15: Influence of the centre of mass in the calculation of P_P and P_C

Some special cases

Using the results and computations of the previous section we may obtain the values of α in the cases referred to in section 3.

In the first case (a.1), that is $P(2) = P(3)$, we must have $2P_P = 3P_C$ so $P_P = 0.6$ and $P_C = 0.4$; this situation occurs when $\alpha \approx 1.32 \approx 78^{\text{o}}$.

In the second case (a.2), which occurs when $P(2)/P(3)$ is proportional to the score ratio 2/3, we must have $4P_P = 9P_C$ so $P_P = 0.69$ and $P_C = 0.31$; for this situation to happen we must have $\alpha \approx 1.99 \approx 114^{\text{o}}$.

In the third case (b.1), which is $P(4) = P(6)$, we have $P_P = P_C = 0.5$, so $\alpha = 0^{\text{o}}$. This is a degenerate case because, in this situation, the rod reduces to a line.

Finally, the fourth case (b.2), that is $6P_P^4 = 4P_C^4$, has no solution. In section 4, we obtained $P_P \approx 0.47$ and $P_C \approx 0.53$ but in Figure 16 we see that $P_P \geq 0.5$ and $P_C \leq 0.5$.

The case $\alpha = \pi/2$ (semicircular cross section) is also relevant. In this case we have:

$$A = \frac{\pi}{2}\, R^2, \quad y_G = \frac{4}{3}\, \frac{R}{\pi}, \quad \beta = \arctan\left(\frac{4}{3\pi}\right) \approx 0.4 \approx 23^{\text{o}},$$

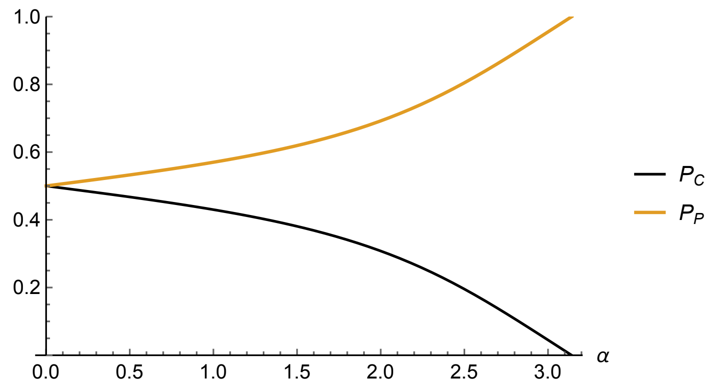

Figure 16: P_P and P_C as functions of angle α

where

$$P_P = \frac{1}{2} + \frac{\arctan\left(\frac{4}{3\pi}\right)}{\pi} \approx 0.63, \quad P_C = \frac{1}{2} - \frac{\arctan\left(\frac{4}{3\pi}\right)}{\pi} \approx 0.37,$$

and therefore the probabilities associated with the scores $1, 2, 3, 4$ e 6 are as follows:

Score	Probability
1	≈ 0.13
2	≈ 0.33
3	≈ 0.37
4	≈ 0.16
6	≈ 0.02

Conclusion

We have analyzed four sets of rules of the Egyptian game Senet, whose main characteristics were explained in section 2, in order to determine which of these sets make possible a more balanced game. The interpretation of some of these rules, from a probabilistic point of view, led us to ask some questions concerning the study of the shape of the rods used as dice.

The unlikeliness that the rules of this game would have remained static over time, as well as geographically, led us to look for one or several sets of rules that would make the game sufficiently balanced and therefore attractive; we are assuming that a game only becomes popular and survives long enough if it is exciting and, in particular, if it has a reasonable difficulty level.

Except for the rules deduced by Bell, the others (suggested by Kendall, Tait and Jéquier) share many common elements (the way the pieces move as well as the special squares, are the same; the number of pieces, the beginning of the game and the way of passing the game to the opponent have similarities). Kendall's rules were also quickly set aside because if they were followed, the board would be crowded with pieces and, for quite a while, their movements would be very limited.

In this work we computed the probabilities of obtaining the values associated with the throwing of the four rods of Senet in several, very particular, situations that occur in Jéquier and Tait's rules. According to results obtained in section 7 the ones suggested by Tait, that is throwing a 4 or a 6, have the flaw of proposing an unbalanced way of entering the game (this is important enough because, in addition to the pieces that enter the board when the game starts, there are those that have to reenter it, after falling into the House of Water); but, in return, the rules suggested by Jéquier seem adequate, because passing the game to the opponent with a 2 or a 3 is balanced as long as the angle α of the rods is close to 78º (according to the table in the end of section 7, an angle between 78º and 90º also seems acceptable).

This study leads us to believe that the rules Jéquier recommended seem to be the most balanced ones, among those that have been suggested. Additionally the rods that best suit Jéquier rules seem to be shaped like a straight cylinder truncated longitudinally by a plane so that the result is a little smaller than half a cylinder; this also makes sense from a practical point of view.

Finally, we believe that this work shows an interesting application of mathematics to the unrelated fields of history and archeology; this can be considered as some sort of Mathematics applied to Archeology.

Acknowledgements

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References

[1] R. C. Bell, *Board and table games from many civilizations*, Dover Publications Inc. (1979).

[2] T. Burrett, P. Van Delft, C. Splunteren, *The World of Games: Their Origins and History, How to Play Them, and How to Make Them*, Jack Botermans (ed.) (1989).

[3] T. Kendall, *Passing through the Netherworld: the meaning and play of Senet, an ancient egyptian funerary game*, The Kirk Game Company (1978).

[4] H. Murray, *History of games other than chess*, Oxbow Books (2000).

[5] P. A. Piccione, *The egyptian game of Senet and the migration of the soul* in *Ancient board games in perspective*, I. L. Finkel (ed.), pp. 54-63 (2007).

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