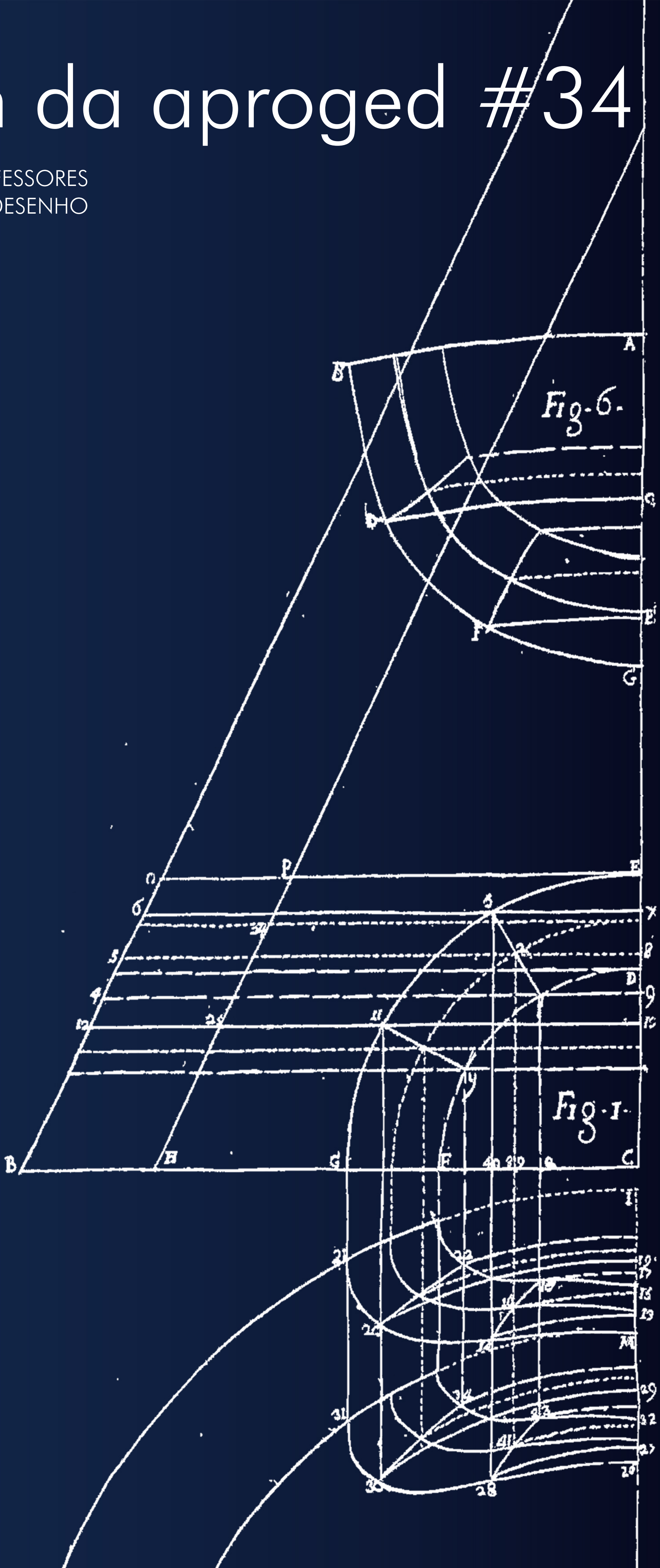
boletim da aproged #34

ASSOCIAÇÃO DOS PROFESSORES
DE GEOMETRIA E DE DESENHO

DEZEMBRO 2018
ISSN 2184 - 4933



APROGED

APROGED
ASSOCIAÇÃO DOS PROFESSORES
DE GEOMETRIA E DE DESENHO
Escola Artística de Soares dos Reis
Rua Major David Magno, 139
4000-191 Porto, Portugal
Telemóvel: + 351 91 627 02 79
E-mail: aproged@aproged.pt | Site: www.aproged.pt

Capa/Cover: Guarino Guarini (1737)
Architettura civile del Padre D. Guarino Guarini Cherico Regolare Opera Postuma Dedicata a Su Sacra Reale Maestá Turin, Italy.
Appresso Gianfrancesco Mairesse all'Insegnai Santa Teresa di Gesù, TAV. XXX

Edição literária | Literary edition: Vera Viana, Vítor Murtinho e João Pedro Xavier.
Edição gráfica | Graphic edition: Vera Viana.

Viana, V., Murtinho, V. & Xavier, J. (Eds.). (2018). *Boletim da Aproged, 34*. Porto, Portugal: Aproged (ISSN 2184-4933)
Disponível em: http://www.aproged.pt/geometrias17/boletim34digital.pdf
Retrieved from: http://www.aproged.pt/geometrias17/boletim34digital.pdf

APROGED
ASSOCIAÇÃO DOS PROFESSORES
DE GEOMETRIA E DE DESENHO

BOLETIM DA APROGED #34

Dezembro de 2018

ISSN 2184-4933

INDEX 1/2

INDEX 2/2

EDITORIAL

Comissão Organizadora da
Conferência Internacional Geometrias'17[1]

A Conferência Geometrias'17, que decorreu no Departamento de Arquitectura da Universidade de Coimbra durante os dias 16, 17 e 18 de Junho de 2017 foi, não apenas um corolário, mas (mais) uma prova de que as problemáticas da geometria que envolvem o ensino, a prática profissional e a investigação científica são assuntos candentes que mobilizam a atenção de diversos académicos, investigadores, especialistas e estudantes, constituindo um desafio permanente sendo centro das suas preocupações, influenciando as suas abordagens e investigações científicas.

É uma realidade que, gradualmente, os meios informáticos têm vindo a tomar posição de destaque nas diferentes práticas que envolvem a arquitectura, as artes e as engenharias, mas nunca como hoje existiu tão plena consciência da importância dos processos de representação para alavancar os avanços científicos nos mais diversos domínios. Neste sentido, o tema aglutinador "*Pensar, Desenhar, Modelar*" constituiu um interessante pretexto para envolver um grupo de especialistas cujo trabalho se relaciona com diversas e diferentes questões que gravitam em torno da Geometria.

Este evento foi, inequivocamente, um forte testemunho, quer da importância das metodologias de formalização, quer dos processos de desenvolvimentos dos objectos, para efeito da criação e desenvolvimento de novas materialidades e diferentes conceitos artísticos. Na realidade, muito da produção actual nas áreas de engenharia, do *design* ou da arquitectura tem sido alavancada pelo aprofundamento das tecnologias associadas à representação. Este ambiente, com cunho particularmente inovador, tem propiciado novas dinâmicas, dando azo ao aparecimento e desenvolvimento de formas inovadoras que, sustentadamente, encorajam a definição de novos rumos e diferentes formas. Perante esta nova energia, torna-se difícil ignorar o impacto que os ambientes virtuais assumem, quer nos processos de projecto, quer nas acções da sua concretização no espaço.

EDITORIAL

Organizing Committee of the
International Conference Geometrias'17[1]

The Conference Geometrias'17, held in the Department of Architecture of the University of Coimbra, between the 16th and the 18th of June 2017 was, not only a corollary, but one (more) proof that geometry stands still as a matter of the utmost importance, through which scholars, researchers, specialists and students are challenged and motivated in their professional procedures, teaching practices and scientific investigations.

Although nowadays, the prominence of digital technologies in every practice related to architecture, arts and engineering is an undeniable factor, there has never been, as much as today, such an awareness for the knowledgeable reasoning on the representational procedures as an essential requirement to ensure conscious developments in scientific and technological research. In this regard, "*Thinking, Drawing, Modelling*" as leitmotiv revealed itself as a successful strategy to bring together many scholars and investigators actively working on these matters, with geometry, in its broader sense, as common concern.

The conference itself was a firm testimony of the importance of form-finding traditional and innovative methodologies as much as the procedures involved in the conceptualization of objects as creative outcomes of new materialities and artistic concepts. In fact, much of nowadays production in architecture, arts or engineering is anchored in technologies firmly entwined with the science of representation. And it is precisely within this innovative milieu that new dynamics are being generated, inspiring ground-breaking ideas, which, through its development, stimulate more challenges, new synergies, different frameworks and inventive forms. Challenged by this new energy, the impact that virtual environments outline not only in project methodologies, but also in its concretization in space is not to be undermined.

The Geometrias'17' Scientific Committee gathered authors that have been producing some of the best scientific practices concerning geometry, drawing

---

[1] João Pedro Xavier, Faculdade de Arquitectura da Universidade do Porto (jpx@arq.up.pt);
Vera Viana, Aproged, Centro de Estudos de Arquitectura e Urbanismo, FAUP (veraviana@veraviana.net);
Vítor Murtinho, Departamento de Arquitectura da Universidade de Coimbra (vmurtinho@uc.pt).

A Comissão Científica da Geometrias'17 reuniu, no seu seio, autores que têm produzido nos últimos tempos alguma da melhor ciência que envolve as questões da geometria, do desenho e do ambiente digital. Tal ponto de partida confirmou esta conferência internacional como uma notável alavanca para a qualidade do conjunto de artigos apresentados. E, a valorar este encontro em Coimbra, estiverem as indagações intelectuais trazidas pelos oradores convidados, designadamente Gunter Weiss, Lino Cabezas, José Pedro Sousa, Giuseppe Fallacara e Soraya Genin, que nas suas diversas especialidades, conseguiram prender atenções e gerar a produção de conhecimento muito inovador.

A presente edição do Boletim da Aproged reúne artigos de vários autores de países da Europa e da América atina que foram aprovados para apresentação pela Comissão Científica da Geometrias'17. Os parágrafos seguintes apresentam uma descrição muito sumária do conteúdo de cada artigo. Os restantes artigos que foram apresentados durante esta conferência internacional serão objecto de outra publicação que será oportunamente anunciada[2].

Gunter Weiss, com o tema "*Geometry, What Else!?*", defendeu o princípio de que a geometria se encontra disseminada por todo o lado, sendo tal particularmente evidente na definição das formas, quer no mundo natural, quer no universo dos objectos fabricados pelo Homem. Tendo como base alguns exemplos históricos ou contemporâneos, Weiss explicitou uma lógica de relação entre a geometria e a realidade, tendo como fundamento o facto de ser na análise da realidade que se faz a revelação da geometria, como, concomitantemente, é pelo recurso à geometria que se cria, normalmente, a realidade.

A sessão co-organizada pela Aproged, a *Associação dos Professores de Geometria e de Desenho* e a *Polish Society for Geometry and Engineering Graphics* compreendeu a apresentação de seis artigos de autores de origem polaca. Esta sociedade científica, fundada em 1994, tem desempenhado um papel importante no país, no que diz respeito às questões concernentes com a geometria e a visualização.

O leque de artigos seleccionado abordou assuntos bastante variados, mas o enfoque recaiu sobretudo nas questões de desenho de modelação e algoritmos. Nesta publicação, incluem-se os artigos de Anita Pawlak-Jakubowska, "P*arametric Modeling of Class II*

and digital knowledge and this, in itself, was a notable starting point that settled the pace for the quality of the contributions presented, confirming this as a unique event. Adding an enormous value to this conference were the intellectual concerns shared by five notable keynote speakers: Gunter Weiss, Lino Cabezas, José Pedro Sousa, Giuseppe Fallacara and Soraya Genin, who grabbed everyone's attention with their ground-breaking and inspirational presentations.

This edition of Aproged's Bulletin includes a number of papers that have been selected for presentation by the Geometrias'17, Scientific Committee. authored by many scholars, researchers and students from European and Latin-American countries. The following paragraphs briefly summarize the content of each research. The remaining papers that were presented during Geometrias'17 will be included in another publication, soon to be announced[3].

In the paper "*Geometry! What Else!?*", Gunter Weiss addresses the principle through which geometry is present in everything, revealing itself in the definition of all forms created, not only by nature, but also by human beings. Weiss exposes the logic of the connexion between geometry and reality through the analysis of natural forms and from historical and contemporaneous examples from architecture, confirming that it is through the analysis of reality that geometry is revealed, being the opposite also true, given that our reality is created from geometry.

The session co-organized between Aproged, the Portuguese *Geometry and Drawing Teachers Association* and the *Polish Society for Geometry and Engineering Graphics* corresponded to six papers authored by Polish scholars. This scientific society, founded in 1994, has a relevant presence in its country, especially in the educational and scientific frameworks of geometry and visualization elated issues. The papers in this session focused on various themes, spanning from the importance of spatial perception for the understanding of representational methods, as in Piotr Udzik, Ewa Terczynsk and Krzysztof Tytkowski's paper, "*Modeling as the Way of Acquiring Knowledge*"; to virtual three-dimensional modelling for the analysis of complex forms related to architecture and engineering, as in Anita Pawlak-Jakubowska's paper, "*Parametric Modeling of Class II Mechanisms Applied in Movable Structures*"; and the exploration of algorithmic software for the resolution of geometrical problems and surfaces

---

[2] Mais informações a partir de www.aproged.pt/geometrias17proceedingsen.html#proceedings

[3] More information in www.aproged.pt/geometrias17proceedingsen.html#proceedings

Mechanisms Applied in Movable Structures"; Piotr Dudzik, Ewa Terczynska e Krzysztof Tytkowski com "*Modeling as the Way of Acquiring Knowledge*"; Michal Nessel e Szymon Filipowski com dois artigos: um primeiro, denominado "*Examples of Genetic Algorithms Usage in Geometry and Algorithmic Design*" e um segundo, "*Algorithmic Approach in the Design of Repetitive Patterns on Architectural Surfaces*".

Daniela Velichová, explorando a modelação de linhas espaciais, apresentou o artigo "*Lace Curves*", com incursões na matemática que explicitam uma metodologia de modulação de curvas com um enorme potencial de variação de forma.

Benjamino Polimeni estabelece uma aproximação prática ao tema "*Producing Design Objects from Polyhedra*", através de uma investigação com grande aparato imagético, ilustrando um método que permite uma ampla variação de formas e de configurações espaciais, utilizando como ponto de partida poliedros convexos regulares.

Catia Ramos desenvolveu o tema "*Guarda's Representation Laboratory: Researching, Interpreting, Modeling and Visualising a City's Growth*". Este texto explora a possibilidade de implementação de modelos dinâmicos tridimensionais, em suporte informático, como processo de estudo evolutivo das cidades. Como caso de estudo, é utilizada a cidade de Guarda em Portugal, defendendo a autora o princípio de que esta metodologia constitui um laboratório de representação.

Isidora Duric, Ratko Obradovic e Nebojsa Ralevic desenvolveram o tema de "*A Review of Augmented Reality for Architecture Visualization*". O enfoque deste artigo prende-se com o potencial de criação de modelos virtuais e sua incorporação em ambientes diferenciados do original ou a possibilidade de sobreposição de imagens virtuais em espaços reais permitindo criar situações em que se procede a simulações muito próximas da realidade, situações muito vantajosas para a disciplina da arquitectura.

Filipa Crespo Osório, Alexandra Paio e Sancho Oliveira abordaram o tema "*Origami Tesselations: Folding Algorithms, from Local to Global*". Com o *software Rhinoceros* e o *plug-in Grasshopper*, os autores exploram em ambiente digital o potencial de superfícies com dobras, sendo apresentados vários exemplos e metodologias para o seu desenvolvimento, com particular destaque em arquitectura.

Samanta Aline Teixeira e Thaís Regina Ueno Yamada abordaram "*Estudo do Design de Origami Tessellations: Análise de Compactação e Complexidade Estrutural de*

analysis as well, as depicted by Michal Nessel e Szymon Filipowski, respectively, in "*Examples of Genetic Algorithms Usage in Geometry and Algorithmic Design*" and "*Algorithmic Approach in the Design of Repetitive Patterns on Architectural Surfaces*".

Daniela Velichová presented the paper "*Lace Curves*", in which the mathematical background for the methodology employed to model these spatial curves, whose variation on its geometry foreshadow an enormous potential, is clarified.

Beniamino Polimeni formulated a practical approach to the theme "*Producing Design Objects from Polyhedra*" in a highly visual fulfilling research that illustrates a large spectrum of form variations and complex spatial configurations through geometric transformations starting from regular convex polyhedra as points of departure.

Cátia Ramos presented her research entitled "*Guarda's Representation Laboratory: Researching, Interpreting, Modeling and Visualising a City's Growth*" that explores possible applications of three-dimensional virtual modelling in the analysis of the evolutionary process of the cities. The city of Guarda, in Portugal, was elected as case study and the methodology is regarded by the author as a laboratory of representation.

Isidora Duric, Ratko Obradovic and Nebojsa Ralevic addressed the theme "*A Review of Augmented Reality for Architecture Visualization*", demonstrating the potential of photogrammetry and augmented reality models and their virtual integration in simulated and real environments, a relevant field of research for architectural visualisation, especially in situations with a certain degree of formal complexity.

Filipa Osório, Alexandra Paio and Sancho Oliveira presented the paper "*Origami Tesselations: Folding Algorithms, from Local to Global*", through which the exploration in *Rhinoceros* and *Grasshopper* of folded surfaces and its development into more complex forms with possible applications in architectural design are highlighted.

Samanta Teixeira and Thaís Yamada, through "*Estudo do Design de Origami Tessellations: Análise de Compactação e Complexidade Estrutural de Seis Crease Patterns*", illustrated their studies on the patterns of different folding procedures and the corresponding forms outlined by their compression and its structural space-saving advantages.

Victor Izquierdo presented "*Structural Analysis of a Parametric Geometric Design*", electing the Foyer of Stuttgart's Casino as case-study and detailing the

Seis Crease Patterns". Este artigo pretende evidenciar o estudo de vários padrões, apresentando imensa potencialidade criativa com vantagens intrínsecas quer em termos de economia de espaço, quer no dinamismo das formas obtidas.

Victor Rodriguez Izquierdo, elaborou uma análise estrutural do *foyer* do Casino de Estugarda. Com o tema "*Structural Analysis of a Parametric Geometric Design*", o autor detalhou a resplandecente solução para a fachada do edifício referido, num contexto de celebração dos 20 anos do espaço. Este artigo descreve desde o processo de projecto, os sistemas digitais utilizados e os recursos disponibilizados até à sua construção.

Maria João Pinto, em "*Form's Age*", aborda questões filosóficas que estão muitas vezes na génese da arquitectura. Para esse efeito, e numa lógica de contemporaneidade, discorre sobre qual o papel e a importância da introdução do desenho assistido por computador, enquanto poderosa ferramenta, no meio arquitectónico.

Ana Paula Rocha, Debora Mariane Fantinato, Renata Maria Geraldini Beltramin e Daniel Moreira de Carvalho desenvolveram o tema do "*Desenho de Conceção em Arquitetura: o Papel do diagrama no processo de projeto*". Este tema, com particular interesse para a arquitectura foi desenvolvido tendo como foco a *Seattle Central Library*, da autoria de Rem Koolhaas. Aqui se concretizaram análises gráficas deste espaço, ajudando a compreender alguns dos problemas de projecto e da organização de informação.

Teresa Pais, desenvolveu o tema de "*The Mastering of Perspective in Observational Drawing*", no qual, partindo de uma experiência pedagógica bastante consolidada, aborda alguns dos problemas no ensino do desenho aos estudantes de arquitectura. Fazendo comparações entre esboços e desenhos de contorno, explicita alguns dos problemas que se colocam aquando da elaboração de trabalhos no espaço exterior, sendo esta abordagem muito importante para quem desenvolve actividade pedagógica relacionada com as questões da representação no âmbito universitário.

Constantino Rodrigues, abordou o tema "*Thought modelling in Descriptive Geometry*". Este artigo, elaborado num registo bastante abstracto, discorre sobre possíveis relações conceptuais entre o pensamento e o discurso. Não descurando a legitimidade relacional entre a ciência a geometria descritiva, são propostas algumas situações onde a problemática de modelação pode induzir a determinados enviesamentos.

Luísa Mendes Tavares e Danielle Spada Tavares, desenvolveram o tema "*Sketchbook: Exercício de Expressão*

resplendent solution for the building's façade in commemoration of its 20th anniversary. The paper describes the project procedures and the digital systems and resources employed until its actual construction.

Maria João Pinto, in "*Form's Age*"" addressed some philosophical concerns inherent to the genesis of architecture, although from a contemporaneous perspective that reflects upon the importance of the introduction of computer assisted drawing in the architectonic milieu, its present-day pervasiveness and the possibility of it partially discouraging some the architect's creative procedures.

Ana Rocha, Debora Fantinato, Renata Beltramin and Daniel Carvalho focused on the graphical analysis of Seattle Central Library in the paper "*Desenho de Conceção em Arquitetura: o Papel do Diagrama no Processo de Projeto*", highlighting some of the problems in this Rem Koolhaas' project and its data organization, a theme of particular interest for architects.

In "*The Mastering of Perspective in Observational Drawing*", Teresa Pais reflected upon the predicaments identified in drawings authored by her undergraduate architecture students, which become clear through the comparison of sketches with contour drawings. Some of the problems inherent to drawing practices developed in the exterior space are discussed, outlining an interesting viewpoint on the teaching practices of representation in higher education.

In the paper "*Thought Modelling in Descriptive Geometry*", Constantino Rodrigues discussed possible conceptual connexions between thinking and discourse. Without disregarding the rational legitimacy between science and descriptive geometry, the author proposes situations in which the problematic of modelling may lead to unusual standpoints.

In "*Sketchbook: Exercício de Expressão para Alunos de Design*", Luísa Tavares and Danielle Tavares debated upon some of the works developed in classroom environment in relation to an experience in which students were invited to explore an application conceived to improve their practices as students and future specialists in areas connected to design, art and architecture.

Professor Vera de Spinadel meant to present a research entitled "*Application of the Proportion Theory to Form Design*" but, sadly, she was taken from us in January 2017. In what was probably one of her last scientific writings, Spinadel explored the relations between geometric progressions and numerical sequences, a theme to which she dedicated a major part of her studies.

para Alunos de Design". Este texto, corresponde a um testemunho de práticapedagógica com recurso a um determinado *software*. Neste artigo são desenvolvidos e apresentados alguns dos trabalhos desenvolvidos em contexto de sala de aula e estabelecida a sua contextualização tendo em linha de conta o seu valor e importância para efeitos de prática futura em arte, na arquitetura ou no *design*.
Vera de Spinadel começou a desenvolver um artigo sobre o tema "*Application of the Proportion Theory to Form Design*". Infelizmente, à data da conferência, esta notável professora já não estava entre nós, tendo aquele sido talvez um dos seus últimos artigos escritos que, infelizmente, não chegou a ser enviado para publicação. Os seus amigos Graciela Colagreco, Gunter Weiss e Manuel Couceiro da Costa fizeram uma apresentação *in memoriam* de Vera de Spinadel, recordando a importância da sua obra e destacando, do resumo alargado que não pôde concluir (mas que se inclui nesta publicação), a sua perspectiva matemática sobre situações relacionais entre progressões geométricas e sequências numéricas, um tema caro a Spinadel que ocupou uma boa parte dos seus estudos.

Assim, *pensar, desenhar e modelar* são inevitavelmente acções que estão na génese de todo e qualquer acto que vise a criação da forma e da sua construção. Pelo que reunir num mesmo espaço um conjunto tão ilustre de personalidades com trabalho reconhecido, em torno das questões da geometria, como aquele que esteve em Coimbra e com o apoio incondicional da APROGED, é circunstância digna de júbilo que se espera poder continuar ciclicamente a repetir. Se assim vier a acontecer é sinal que a geometria, o desenho e as ciências da representação são assuntos candentes, pertinentes e com futuro. Na prática, a modelação tridimensional e as aplicações aramétricas são certamente propostas que abrem abrangentes esperanças no domínio dos ambientes virtuais, mas certamente não poderemos ignorar que a estereotomia, o desenho, a ciência da representação ou as didácticas da geometria serão assuntos sempre inesgotáveis e que a todo o tempo constituem um cadinho tão essencial como incontornável. E nesse caso essa é uma prova e um testemunho do valor e pertinência destes temas e da necessidade de com regularidade os discutir e colocar no centro dos nossos pensamentos.

A Comissão Organizadora da Geometrias'17,
João Pedro Xavier, Vera Viana e Vítor Murtinho

The extended abstract that Professor Vera presented to the Scientific Committee is included in this edition, as well as this reference to a special *in memoriam* session held during the conference and conducted by her friends, Graciela Colagreco, Gunter Weiss and Manuel Couceiro da Costa.

In conclusion, and according to what all the papers combined in this book demonstrate, thinking, drawing and modelling are fundamental activities that stand in the origin of every act leading to the creation of form and its materialization. Bringing together such an outstanding group of individuals with recognized works and researches concerning geometry matters as in the conference held in Coimbra, through the efforts of a small teachers' association, has been a jubilant circumstance, that we hope might regularly continue. If this turns out to be the case, then geometry, drawing and the sciences of representation prevail as pertinent matters, whose discussion has a great future in sight. Three-dimensional modelling and algorithmic software are magnificent proposals that have been paving the way to great challenges in our world, but one cannot ignore that stereotomy, drawing, the science of representation and geometric literacy alike remain as limitless inspiring ources of knowledge, so fundamental as much as inevitable for ever professional and scholar committed to the investigation and innovation in the contexts related to geometry.

The Geometrias'17 Organizing Committee,
João Pedro Xavier, Vera Viana e Vítor Murtinho

Acknowledgement from the Organizing Committee

João Pedro Xavier, Vera Viana and Vítor Murtinho formally express their enormous gratitude for the contribution of all the Scientific Committee members and the additional reviewers which, through their experience and knowledge, certified the scientific quality of the Geometrias'17 Conference and this edition of Aproged's Bulletin.

SCIENTIFIC COMMITTEE

ALEXANDRA PAIO, ISCTE - Instituto Universitário de Lisboa Portugal
ANDREAS KRETZER, Technical University of Kaiserslautern, Germany
ANTONIO LUIS AMPLIATO, Universidad de Sevilla, Spain
ANTÓNIO ORIOL TRINDADE, Faculdade de Belas Artes da Universidade de Lisboa, Portugal
CORNELIE LEOPOLD, Technical University of Kaiserslautern, Germany
ELIANA MANUEL PINHO, Centro de Estudos de Arquitectura e Urbanismo, Fac. Arquitectura Univ. Porto
ENCARNACIÓN REYES IGLESIAS, Universidad de Valladolid, Spain
FILIPE GONZALEZ, Escola de Arquitectura e Artes da Universidade Lusíada de Lisboa, Portugal
GIUSEPPE AMORUSO, Dipartimento di Design - Politecnico Milano, Italy
GIUSEPPE FALLACARA, Politecnico di Bari, Italy
GONÇALO CANTO MONIZ, Faculdade de Ciências e Tecnologia da Univ. Coimbra, Portugal
GRACIELA COLAGRECO, Universidad Nacional de La Plata, Argentina
GUNTER WEISS, Technische Universitat Dresden, Germany
HELENA MENA MATOS, Faculdade de Ciências da Universidade do Porto, Portugal
HELLMUTH STACHEL, Technische Universität Wien, Austria
JAVIER BARRALLO, University of the Basque Country, Spain
JOÃO PEDRO XAVIER, Faculdade de Arquitectura da Universidade do Porto, Portugal
JOSÉ IGNACIO ROJAS SOLA, University of Jaen, Spain
JOSÉ PEDRO SOUSA, Faculdade de Arquitectura da Universidade do Porto, Portugal
LINO CABEZAS, Facultat de Belles Arts, Universitat de Barcelona, Spain
LUÍS MARQUES PINTO, Universidade Lusíada do Porto, Portugal
LUÍS MATEUS, Faculdade de Arquitectura da Universidade de Lisboa, Portugal
MANUEL COUCEIRO DA COSTA, Faculdade de Arquitectura da Universidade de Lisboa, Portugal
MARIA FRANCISCA BLANCO, Universidad de Valladolid, Spain
PEDRO DE AZAMBUJA VARELA, Faculdade de Arquitectura da Universidade do Porto, Portugal
RATKO OBRADOVIC, University of Novi Sad, Serbia
RICCARDO MIGLIARI, La Sapienza, Universitá di Roma, Italy
RINUS ROELOFS, Netherlands
SORAYA GENIN, ISCTE-Instituto Universitário de Lisboa, Portugal
TERESA PAIS, Faculdade de Ciências e Tecnologia da Universidade de Coimbra, Portugal
VERA DE SPINADEL, Mathematics and Design Association's, Argentina
VÍTOR MURTINHO, Faculdade de Ciências e Tecnologia da Universidade de Coimbra, Portugal

ADDITIONAL REVIEWERS

FRANCISCO GONZÁLEZ QUINTIAL, E. Técnica Superior de Arquitectura - Universidad del País Vasco, Spain
JOÃO VENTURA LOPES, ISCTE - Instituto Universitário de Lisboa
PEDRO MARTINS, Faculdade de Arquitectura da Universidade do Porto, Portugal
VASCO CARDOSO, Faculdade de Belas Artes da Universidade do Porto, Portugal

GEOMETRY. WHAT ELSE !? - MORE OF “ENVIRONMENTAL GEOMETRY”.

Gunter Weiss[1]

ABSTRACT

This paper is an *addendum* to a previous article [01] in which several examples demonstrate that “all natural or artificial objects have a shape or form resulting from a natural (bio-physical) or technical (design) process, and therefore have an intrinsic (immanent) geometric constituent”, focusing on the fact that “reality reveals geometry and geometry creates reality”. Since many objects are metaphors for geometric and mathematical content and the starting point for mathematical abstraction, one can conclude that geometry is simply everywhere. This sort of “Appendix” focuses on the symbiotic terms “grasping via senses” and “meaning” in connection with geometry and its visualisation and interpretation, from objects found in our usual environment. A real object that we see or recognize may even gain spiritual meaning, because it is extraordinary and rare and has, therefore, besides its somehow practical purpose, a symbolic one. Here, simplicity, symmetry, smoothness and regularity play an essential role beyond simple aesthetics. In our mainly secular culture, the aesthetic point of view stands in the foreground.

KEYWORDS: elementary geometry, intuitive geometry, right angle, cross and square, proofs without words.

INTRODUCTION

Geometry is “defined” as the science of structure and form , for example, in [01] and [02]. There is no doubt about the fact that Geometry plays an important role in technical applications as a problem-solving tool (e.g. in machine engineering and architecture), but Geometry also helps to understand phenomena of our environment via more or less simplified models. Thereby, “modelling” involves Geometry at least as a tool to visualize abstractions of these phenomena. Abstraction and logic are therefore basic key-concepts for Geometry: Euclid’s “definitions” of the abstract concepts of “point”, “line”, “plane” and his description on how to deal with these concepts can be seen as a metaphor for modelling processes in general. Modern modelling is based on the rich toolbox provided by all branches of mathematics, but the geometric concepts “point” and “space”, as well as their representation in our imagination, remain crucial as elsewhere. The imagination of simple objects of the so-called Platonic world could be called “common sense abstractions”, which we all take as granted without consideration. This makes it possible to call a “circle” the mere graphite dust on a paper physically drawn by a compass or similar, as much as a special set of pixels on a laptop screen calculated via a graphics software. There are differences, though, between these physical models: while the physically drawn circle reminds the designer of the definition of the circle as the set of points that, in the plane, are equidistant from the circle’s centre and of the invariance of distances with respect to motions, the screen circle received by the corresponding CAD command does not directly show the circle’s definition.

There is a close relationship between geometric abstractions and linguistic notions: similar to, for instance, the abstract concept “door” that comprises the set of realizations fulfilling the abstract property of a door, the concept “circle” is used to designate every individual realisation of different sizes that, somehow, fulfil the abstract definition of a circle. The concept circle automatically leads us to the concept of (abstract) similarity, a term that is also used in common language but with a quite different meaning. A teacher having this in mind, will introduce pupils carefully to technical language, where words of common speech get a well-defined, abstract meaning. We emphasize that speaking in terms of Geometry involves a special

---

[1] TU Vienna, Austria and University of Technology Dresden, Germany. weissgunter@gmx.at

technical language through which words of common speech get an abstract meaning by precise definitions still using common speech words! Here the necessity of schooling occurs, as much as a second language learning.

Finally, we mention that there are also psychological and aesthetic effects connected with geometric abstractions and their realizations, that cannot be neglected as, for instance, the individual stars of the Big Dipper are connected by (virtual) straight lines. There are sketch books for children, where they connect numbered points consecutively by more or less straight segments of lines to obtain a certain final image. Drawing a proper straight line by freehand is therefore trained and recurrently used when learning to write block capitals, and good results such as symmetry, iteration, and regularity are highly estimated in this practice. One can also experience that children at preschool age, learning to handle a ruler, express great enthusiasm. Drawing of pre-schoolers starts with (straight) lines and zigzag-bands, ovals and spirals and its repetitions. Here, the basic idea of symmetry, iteration, and regularity is already visible, and leads to ornamentations and paintings made in prehistoric times, by people from autochthonous cultures or children (Fig. 1). We might note that the aesthetic properties of geometric objects can be summarized by their simplicity, smoothness, symmetry, and modularity. Intuitively, we connect these properties with such abstract objects as straight lines and circles, for example. But a great lot of complicated geometric objects show them as well, and some, in addition, can even be aesthetically appealing.

Fig. 1 - Cartoon of a man together with a freeze of notes and hearts made by a girl at the age of five. The drawing roughly repeats the symmetry of the depicted person. One can easily recognize the intention of the authoress from the abstract image of the person. (G. Weiss).

In the following, we shall deal with the properties mentioned above by presenting and discussing some (abstract) geometric objects in respect to their realisations. The conclusion will be that, even if there is a natural and inborn basis to acquire geometric knowledge and understanding, some sort of schooling is strongly advisable for a correct interpretation of (technical) images.

1. THE STRAIGHT LINE - A FULCRUM FOR MORE

A straight line is visualized as a segment, but we imagine it as arbitrarily extendable (by the way, ‘arbitrarily extendable’ is the formulation used by Euclid!). Here, we meet the abstract concept of infinity, and we wonder if infinity is more than just a shortcut for the formulation of ‘arbitrarily extendable’? Is ‘arbitrarily small’ the same as ‘infinitely small’? The well-known paradox of Achill and the turtle treats this difference, but we get used to identify these two concepts because of facts that we experience in the real world. How fruitful this naïve identification is and how it shows the classical problems of limit calculations, integration, and differentiation! To mention elementary examples, we refer to the Cavalieri principle, arc-length and area calculation, as well as to the concepts of instantaneous direction and speed. Moreover, and despite the explicit realisations of a line segment that, in the end, consist of distant molecules, one gets an idea of a continuum, especially, when drawing the line segment by hand.

The descriptions and axioms of Euclid seem to describe ‘obvious observations of the real world’, but even so, the discovery of incommensurability by the Pythagoreans caused a collapse of the view of the world in ancient Hellas, based on a composition of the world by finite elements called ‘atoms’ (which were imagined as Platonic solids). Here, a change of the view of the world caused by geometric (!) reasoning, occurs, probably for the first time.

2. THE CROSS AND THE SQUARE - ARCHETYPICAL FORMS

Lines occur embedded into planes or into the space of perception. A line segment is not only smooth, it can be mirrored into itself, i.e. it is symmetric and it has a midpoint. A line segment in the plane, together with a chosen segment of a mirror line form a cross, which is not only a widely used symbol, but also an abstract model of the pair of horizontal and vertical directions. In our world, we, as human beings, naturally keep track of these clearly distinguished two directions. Common spruce grows vertically, for instance, and lakes represent horizontal planes. The concept of direction relates to the abstract geometric concept of parallel: a spruce forest consists of trees growing in the same vertical direction and their trunks are parallel. The symmetry of a cross introduces the concept of orthogonality.

Other distinguished pairs of mutually orthogonal directions are connected with oneself: up-down, front--back and left-right. By this, we assume a natural, but abstract, '3D-moving frame' of crosses connected with most animals, including humans. Repetitions of crosses occur as warp and weft in weaving and basketry (Fig. 2) and give rise to the archetypical form of the rectangle. Its 3D-variant is the box, which became a dominating form on our civilized environment (Figs. 3 and 4). There are practical reasons why we can neither omit the key directions vertical and horizontal in architecture, nor the use of boxes for packing purposes. Floors should be horizontal, the rotation axis of doors should be vertical, as practical and static reasons determine(d) that walls were vertically positioned rectangles. Door and window openings must rather be rectangular and optimized and standardised furniture consists also of box-shaped forms. The classical roof forms even give way to platform roofs thus emphasizing the box shape of an optimised, modern house. But when a form becomes omnipresent, it loses its aesthetic attractivity. For instance, around 1950, municipal buildings had naked front walls with uniform rectangular window holes but, nowadays, this type of building is an example of rather boring architecture. There were and are two ways for architects and designers to surpass this fact: one is to decorate the faces of the box - the vertical front wall gets e.g. plastering ornaments, the upper part of windows is replaced by arcs and the planarity of the façade is broken by adding oriels (c.f. baroque and classicistic buildings); the second is to hide the necessary box structure behind a skew or broken wall and, through this, give the building the appearance of a huge sculpture (c.f. e.g. [03] and Fig. 5).

In nature, the occurrence of straight lines and planes is more or less restricted to crystals, and therefore is a rare happening. Such objects have the properties of smoothness, simplicity and symmetry and thus seem aesthetically appealing to us (Fig. 6). Salt, a mineral of great importance for humans and stimulus for cultural development, crystallizes in cubes, a box form with congruent faces and "many" symmetries. Looking carefully at such a salt crystal and studying its symmetries, one might derive the octahedron (as the dual solid to the cube) as well as the pair of tetrahedrons

Fig. 2 - Detail of a leather bag with orthogonal warp and weft wickerwork (left), Basket showing right angles and circles around the square-like basis (right), (G.Weiss).

Fig. 3 - Box shaped architecture: buildings in Oslo (G.Weiss).

Fig. 4 - Box shaped architecture with strict symmetry: private houses in Oslo (G.Weiss).

Fig. 5 - T-Mobile Austria building, architectural abstraction of a cruise ship with considerable static requirements and dead space because of the contrast between the inner and outer structure of the building (Vienna, Austria; G. Weiss).

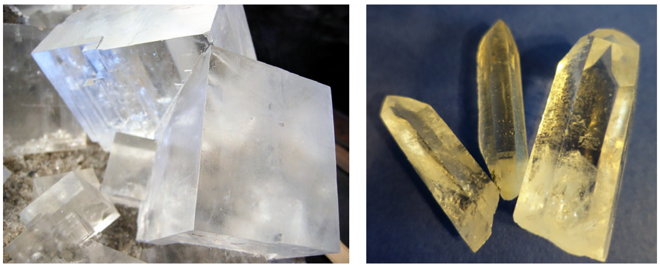


Fig. 6 - Salt crystal (left [04]) and quartz crystals (right; G.Weiss).

(the *stella octangula*) inscribed into the cube. But the concept square as a special rectangle surely was coined first. It is worth mentioning that in ancient Greece, the property of a solid body seemed so self-evident that it was not even necessary to coin individual names for these Platonic bodies. This, in itself, is evidence of a very high level of abstraction, since those objects were only distinguished by their number of faces.

The symmetry axes of a square can be ordered in two pairs forming two concentric crosses, one halving the right angles formed by the other. From this, one might derive the concept of rotation by halve of a right angle, and end up with a regular octagon, a form widely used in ancient and medieval architecture (e.g. Fig. 7). With three coplanar segments of equal length, one can build an equilateral triangle and quartz crystals (Fig. 6, right) and show regular hexagons as its cross sections, which contain equilateral triangles themselves. Many flowers' species are realizations of n-gons. It seems possible that, by discovering the underlying properties of some real objects and by 'inventing' decorative patterns, one can deduce such abstract concepts as regular polygon, rotation, translation, rosette symmetry and frieze symmetry (Figs. 7 and 8) One might also stumble over abstract questions as the constructability of regular polygons, or the geometric equivalences within the set of frieze ornaments.

The equilateral triangle, the square and the regular hexagon occur as alchemistic symbols and they had (and still have) a theological meaning, too. This shows, again, how simplicity and regularity have impressed people of different cultures. Nowadays, regular hexagons and regular octagons occur, for instance, in jam glasses

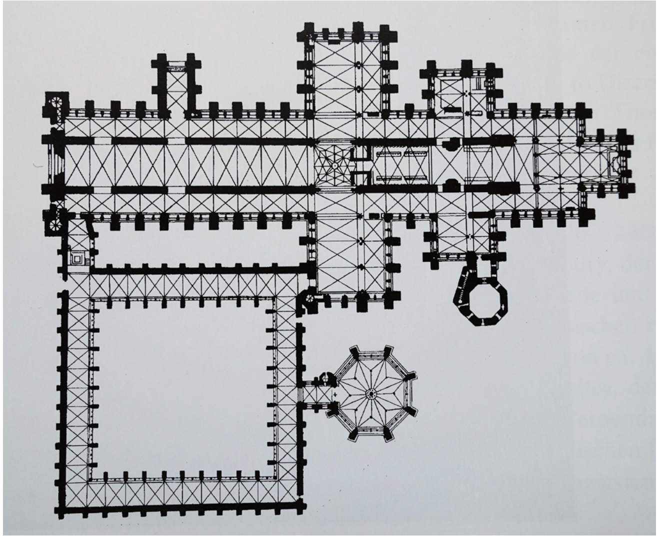


Fig. 7 - The square as fundamental design module for medieval churches and monasteries. Outline of the Cathedral of Salisbury (1220-1266). (see [05], p. 119).

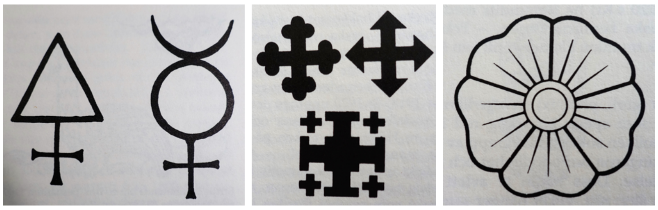


Fig. 8 - Alchemistic symbols for sulphur and mercury (left), Heraldic crosses (middle), symbolic image of a plum flower (right) (see [06]).

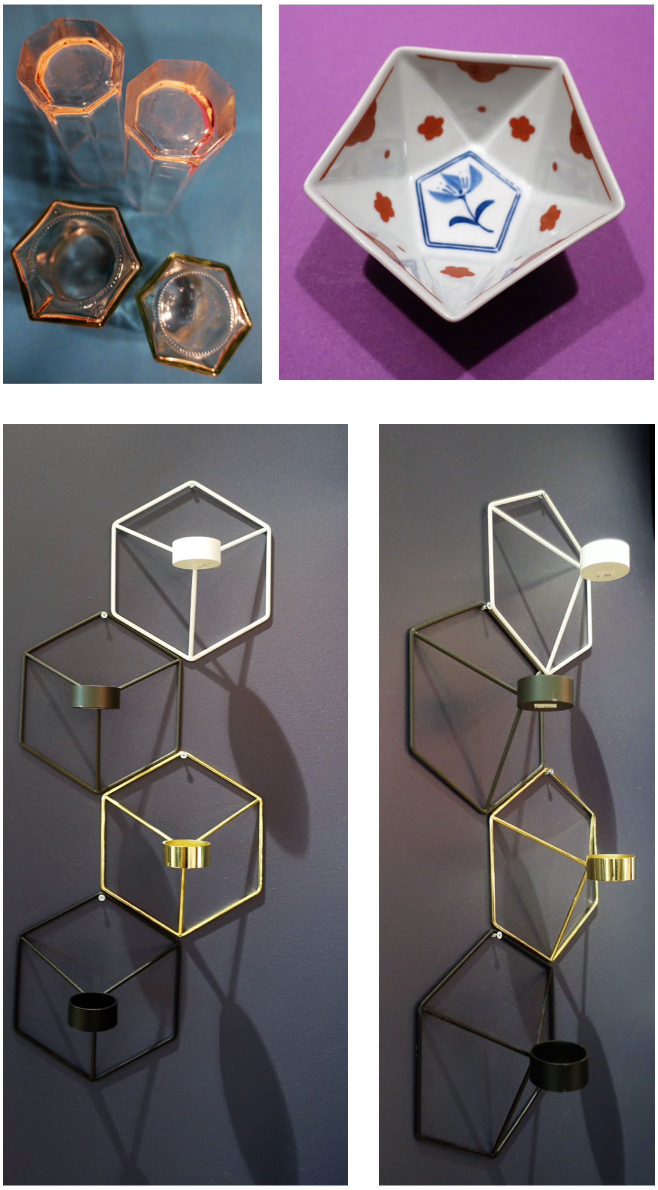


Fig. 9 - Polygonal cups and jam glasses (upper left), Japanese pentagonal bowl (upper right), planar hexagons counterfeiting the outline of a cube (bottom) (G.Weiss).

and flower pots (Fig. 9 left), but also as optical illusions (Fig. 9, right). In addition to their many realisations as flowers and knots, the regular pentagon and the regular heptagon receive an extraordinary symbolic meaning in philosophy and art. (Fig. 10).

As mentioned in the Introduction, the discovery of the incommensurability of side and diagonal of the regular 5-gon caused, not only a change of the view of the world, but also turned the concept of (general) proportion into a fundamental concept of geometry, architecture, and art. The concept of proportion, together with the cross of cardinal directions and the measurement of lengths has brought numbers into geometry. We should note here that the classical functions sin and cosine were (and are) formulated as proportions, and it was via proportions and similarity rules and practical logic that Eratosthenes (~276 -194 b.C.) could calculate the radius of the earth with incredible accuracy.

Remark: The wish to visualize the topography of the earth by globes made it necessary to describe the locus of a point by astronomic data (i.e. numbers) based on certain reference systems. This went hand in hand with the invention of the Cartesian frame and analytic geometry by R. Descartes (1596-1650) during the Pre-Enlightenment, and it caused the second big revolution of our view of the world. In the meantime, Mathematics developed as an abstract science and, more recently, started to become the background for technical development and engineering. This, in itself, justifies nowadays the need to enhance visualisations of abstract mathematical structures and concepts.

3. THE CIRCLE:
A MAGIC ARCHETYPICAL FORM

Many plant species have trunks and fruits showing realizations of rotational symmetry and sometimes they even have (at least locally) a smooth surface (Fig. 11). Putting such a fruit in one's hand, one can feel this rotational symmetry and is therefore nearly forced to coin this abstract concept. Pottery and basketry are man-made realizations of this concept as well and, of course, the rotation of a segment (a piece of a rope) around one end in a plane lets the other end generate a circle. The observation of astronomic motion surely has added to materialize the concept of (continuous) rotation and render it more precise.

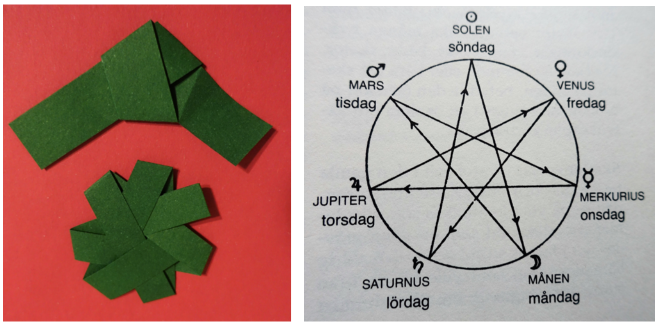


Fig. 10 - Knotted pentagon and heptagon (left), (G.Weiss); star shaped heptagon, symbol for the 7 days of a week (right), (see [06]).

A very natural way to get in contact with the abstract concepts of the cross and the circle is basketry (see Fig. 2). Together, these two concepts are loaded with symbolic and transcendent meaning, as e.g. the female gender symbol, the Atlantis cross, the cross of consecration, medieval Islamic city maps, mandalas, and many more. The combination of a circle with an inscribed polygon often has symbolic meaning, as e.g. the compass rose or the Buddhistic Dharma Chacra (Fig. 12).

Circles and circular disks alone stand for abstractions of the Ouroboros Snake and the Fortuna's wheel, but circles also occur meaningfully as wedding rings. We meet circular disks as beermats, manhole covers, plates, and buttons, and very often, these objects are decorated. For many centuries, architects have been applying circular arcs, especially half-circles, for arches. The flat world viewpoint uses the circular "cosmological disk model" of the earth, which also acts as a metaphor for (plane) hyperbolic geometry.

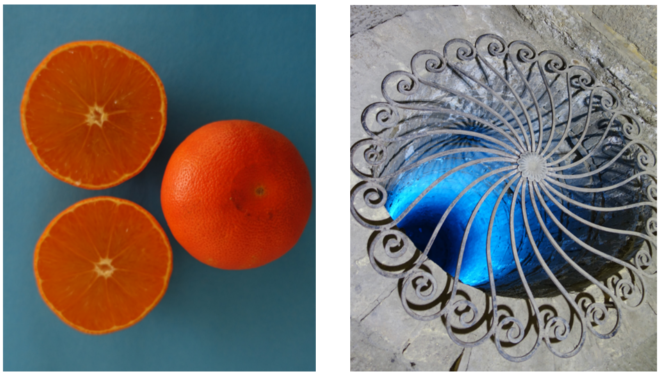


Fig. 11 - Cross section of a rotational symmetric fruit, showing the natural realisation of a rosette (left); spherical cistern cover, Isfahan, Iran, (right). (G.Weiss).

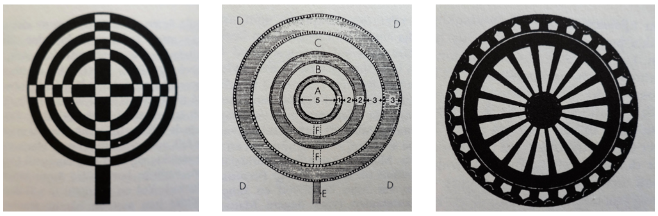


Fig. 12 - The "Atlantis cross", symbol for esoteric research (left), "map" of Atlantis city according to Platon (middle); *Dharma Chacra*, a Buddhist wisdom's symbol (right). (see [06]).

Remark: Even now, the hyperbolic plane is visualized as a Euclidean circular disk, although unnecessarily. Due to F. Klein (1849-1925), the hyperbolic plane is the inner part of a conic section in a real projective plane (see e.g. [07]). The use of models with Euclidean overlay stems from pre-computerized graphic times, where one had to adapt the model to ruler and compass use. Here, we meet a visualisation of an abstract mathematical structure in the sense of the Remark in Section 2. We might also note that, by inventing explicit models for a geometry that does not fulfil Euclid's parallel postulate, it was possible to accept a non-Euclidean geometry as being of the same value as Euclidean geometry. Without non-Euclidean geometries in the sense of B. Riemann (1826 -1866), F. .Klein and H. Minkowski (1864 - 1909), the current development of modern physics had not been possible! This development, still based on geometric imagination, caused a third fundamental reform of our view of the world.

The 3D-variants of a circle are cylinders of revolution and spheres. Cylinders occur as columns and towers, as parts of bottles and many other technical objects. In addition to all types of balls and half-spherical cupolas, there is a great lot of spherical objects in our environment as well (Fig. 13).

When we think of a column consisting of cylindrical parts as in Fig. 14, we meet astonishing exactness of the right angle between the base plane and the axis of the cylinder. This exactness is of course a condition *sine qua non* without which, one would not get a proper result. When looking at ancient Egyptian temple buildings and obelisks, one guesses that geometry had been started with the carpenter's hook and compass, rather than with measuring land, where a thin layer of mud could never have hid borders. By the way, the carpenter's hook and compass were the symbols for medieval architects, and they still are the main symbols in freemasonry. And with the carpenter's hook, one is also able to solve the cubic problem of trisecting an angle.

4. CURVES AND SURFACES: SMOOTHNESS BY MOTION

Every solid body has an outer surface and, by touching it, we can feel its roughness or smoothness as well as any sharp edges and corners. To generate a smooth body's surface as, for instance, in sculpture, we use cutting tools and sandpaper in the end, to polish it. Thereby, we move along the surface so that the prototype of such motions become smoother and what we get is - abstractly speaking - a locally smooth surface. A reverse situation occurs, when we keep the cutting tool fixed and rotate the object, as we find in classical pottery or turnery.

Generating a smooth curve means continuously moving a point in space or in a plane. We already mentioned the generation of a circle by a special continuous motion.

Fig. 13 - Modern coffee cans, (above);
half spherical cover over an underground bus station,
Marrakesh, Morocco. (G.Weiss).

Fig. 14 - Cylindrical shaped column parts at Pasargadae,
Iran (~550 b.C.). Cross sections are exactly orthogonal
to cylinders' axis. (G.Weiss).

To receive curves and surfaces, which are somehow aesthetically appealing, a certain “simplicity” of the motion in respect to the generation rules seems to be an important feature. For example, besides the practical applicability of a helix motion for screws, the double helix acts also as a metaphor for chromosomes and their genes. Architects apply spiral stairs even at airports (Fig. 15). In his book *Medicina mentis* [08] E. W. v. Tschirnhaus (1651-1708) proposes rope constructions of curves based on a given set of focal points, generalizing the gardener construction of an ellipse, which he recognizes already as a generalisation of the circle (Fig. 16). By using the arguments of forces within the ropes’ segments, he could also find the tangents of the curves. This example shows very well the didactical principle to start with obvious geometric facts to walk on, step by step, to more advanced problems. Similarly, de Casteljau’s algorithm starts with the well-known generation of a parabola and generalizes the construction to more than three control points.

The concept of motion can be generalized as well: we already mentioned similarities and, similarly to a helix, which is the only space curve that allows self-motions, spirals are the single curves that are fixed under a one-parameter group of similarities. Shells of molluscs are natural realisations of (parts of) such spiral surfaces, and nobody will deny that they are beautiful, e.g. Fig. 17. As a last example of a rather simple smooth and rotational symmetric object, we present a pebble stone in Fig. 18. As such stones naturally occur very seldom, they are seen as of divine origin and therefore loaded with great symbolic meaning.

5. PLATONIC SOLIDS: PARENTS OF A BIG FAMILY

In Section 2, we already mentioned the cube as being the grandfather of the octahedron and the tetrahedron. But the remaining two Platonic solids, the icosahedron and the pentagonal dodecahedron, can also be related to the cube. For instance, by adding hipped roofs to the faces of a cube in a specific manner, one gets the dodecahedron; adding five-sided pyramids with equilateral faces to the pentagonal faces of that dodecahedron results in the icosahedron so, therewith, this polyhedron is also subscribed to the initial cube. The enchantment of the Platonic solids is still ongoing and has resulted in a rich and vast set of publications. A glassware producing firm, which can be found in many airports, has a set of Platonic glass bodies in its catalogue (Fig. 19)!

Fig. 15 - Spiral staircase, 3D-visualisation of a helical motion. Oslo airport. (G.Weiss).

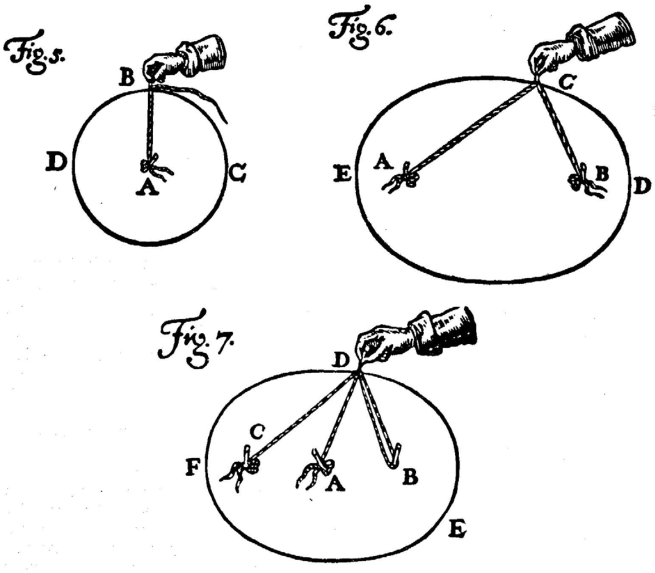


Fig. 16 - E. W. v. Tschirnhaus‘ generalisations of conics by rope constructions, which allow to derive tangents without differentiation (see [08]).

Fig. 17 - Fossil Nautilus shell and snail houses, almost perfect spiral forms. (G.Weiss).

Fig. 18 - Naturally smooth, rotational symmetric pebble from the sacred river Narmada, (World Museum Vienna, Austria). Geometrically, it is almost an outer parallel surface to an ellipsoid of revolution. For Hindus it is a representation of the God Shiva. (G.Weiss).

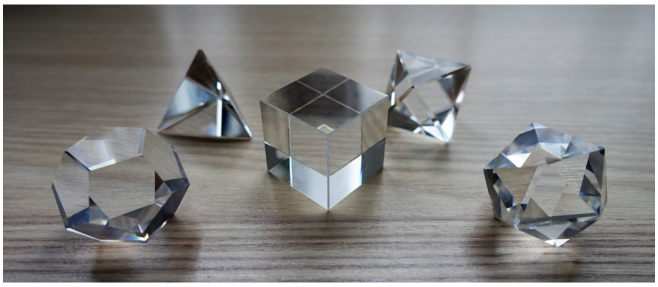


Fig. 19 - A realisation of the 5 Platonic solids in glass by Swarovski, Austria. (G.Weiss).

There are many reasons why these objects are of special interest, but we pick out only two of their properties: they are convex and uniform with respect to their corner figures and faces; and they generalize regular n-gons to three dimensions and the rosette symmetry groups to discrete spherical symmetry groups.

Moreover, they are adequate for a naïve and intuitive perception of the concept dimension. But passing from the relation between the (equilateral) triangle and the square to the relation between the (regular) tetrahedron and the cube carries already the germ for further abstractions, namely the simplex and hypercube, at first, in a four-dimensional space and then in a general n-space. Visualisation of such higher-dimensional counterparts makes it necessary to develop corresponding mapping methods, i.e. to develop a descriptive geometry from 4D to 2D (see [09]).

The symbolic meaning of the Platonic solids ranges from being of a divine origin over being basic elements for all that is matter, to an identification with the planets of our solar system. J. Kepler (1571-1630) ”mapped“ the planets to a concentric set of Platonic solids that, in a specific order, are consecutively subscribed to the circum-sphere of the former (see [10] and Fig. 20).

Modifying properties of the five Platonic solids result in sets of polyhedrons that are of great interest for mathematics, as well as for art and architecture. For instance, if we allow two or more types of regular faces and keep the convexity and symmetry groups of the Platonic solids, we obtain the 13 Archimedean solids, to which we might add the infinite set of prisms and antiprisms. The Archimedean solids are obtainable from "truncation" of vertices and edges of the Platonic solids. If we, in addition, skip their (full) symmetry groups and just focus on regular faced convex polyhedrons, the result is (besides prisms and antiprisms) the set of 92 Johnson polyhedrons [11]. Here belongs also Miller's pseudo-rhombicuboctahedron (Johnson-number *J37*),

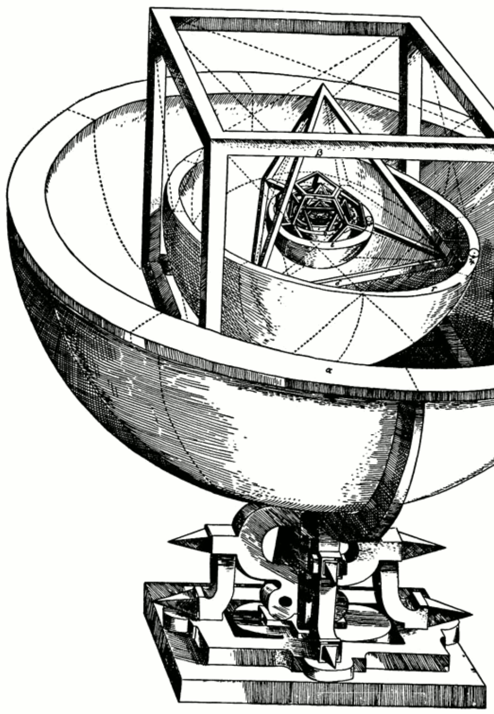


Fig. 20 - According to Kepler, the average orbit radii of the planets (known at his time) suit well to the set of spheres inscribed and circumscribed to concentric platonic solids in a certain sequential arrangement ([10]).

an interesting variant of the Archimedean rhombicuboctahedron re-discovered in 1930 (Fig. 21 and [12]).

The midpoints of the cube's faces are vertices of an octahedron and again, the midpoints of the octahedron's faces are vertices of a cube. By recognizing this fact and abstracting it to a somehow involutoric mapping from planes to points, one is almost forced to coin a concept for it: it is named duality and, as a general procedure, it can be applied to all regular-faced polyhedrons. Dualizing the Archimedean solids results in the Catalan solids (from E. C. Catalan 1814-1894), whose faces are general triangles, rhombuses, deltoids and irregular but symmetric pentagons [13]. The symmetry groups, however, are still those of the Platonic solids and they are still convex.

One could, of course, also dualize Johnson's solids and would receive convex polyhedrons with irregular face-polygons similarly to those of Catalan solids, see e.g. [14]. A systematic treatment of Johnson duals seems not to have been done yet. Now, the seemingly open problem arises: how can we "dualize" these dual Johnson's solids? Catalan solids or general polyhedrons? What should be taken as "midpoints" of irregular faces?

Among many generalizations of the cube, we mention parallelotopes [15]. Cubes and hypercubes are space-filling polyhedrons. By parallel projecting such a hypercube of a d-space ($d > 3$) into a 3-space, we receive a parallelotope, which can also act as a brick to fill 3-space, (see e.g. [16] and [17]). The question arises: to characterize all space-filling polyhedrons (also, such with irregular faces), generalizing the problem of characterizing planar tilings to three dimensions.

Truncation of vertices means to cut away regular pyramids from a Platonic solid. Another sort of dualization process would be to add regular pyramids of arbitrarily chosen height h to the faces of a platonic solid. Doing so and rotating the added pyramids by an arbitrarily chosen angle ρ, we end up with a two-parameter family of polyhedrons, that comprise the Catalan solids as well as many other already well-known polyhedrons(Fig. 22).

6. PROOFS WITHOUT WORDS:
WHAT THEY NEED INSTEAD

In the above-mentioned, we emphasised a naïve approach to geometric facts. Things should be obvious and belonging to the personal field of experience; and abstractions should be self-evident and the result of elementary logic reasoning. There occur also very

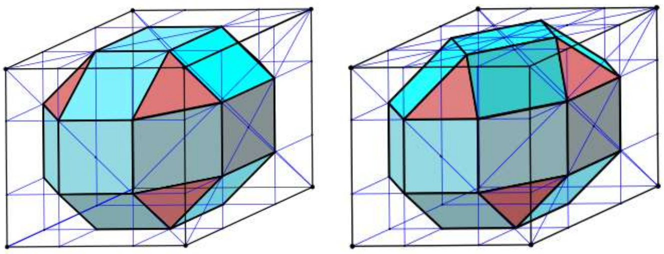


Fig. 21 - Rhombicuboctahedron (left) and J.C.P. Miller's Pseudo-Rhombicuboctahedron (right).

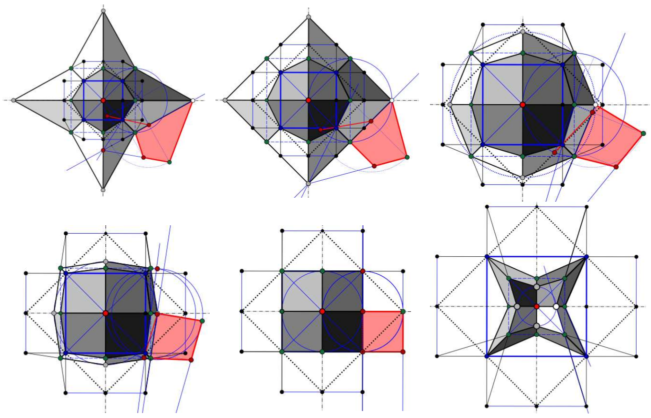


Fig. 22 - Four-sided pyramids with altitude h added to the faces of a cube of edge length 1 and rotated by the fixed angle ρ=45°. The intersection of the 6 pyramids results in a polyhedron with 24 deltoids as faces. Among these polyhedrons, occur the octahedron (h=1), the Catalan's deltoidal-icositetrahedron (h=1/2), and the cube itself (h=0).

elementary questions, which are basic for all types of science: what is the object (or structure) in consideration good for, and what can we do with it? If one can abstract a property of it, does this property characterize the object as being unique? Are there more objects with this property? Is it possible to describe all of them? How to generalize the property (and, therewith, the object) to get a family of related objects? Can one put the object in sort of a phylogenetic tree of objects?

An example: If we exploit natural numbers to the end by basic arithmetic operations, we see that we need negative integers, too, and that halving (and division) makes a further extension to rational numbers necessary. These new structures open up for further abstractions and, finally, a finetuning among the set of real numbers, as well as further extensions to general number fields and rings.

Concluding from this is a process of branching up during the development of geometry and mathematics that follows rules of biological evolution. It is obvious that, the further we step onwards this branching process, the more and specialised schooling is necessary to be able to follow up with the degree of abstraction. When we, in the following, discuss so-called "proof without words" as presented e.g. in [18] and [19], we notice that such proofs deal with elementary geometric (and mathematic) theorems, and that a written sequential arrangement of logic arguments is replaced by a figure. Thereby it seems to be clear what is meant by a proper mathematical proof, but what about "proper" figures? Every visualisation, be it a naturalistic painting or technical drawing, is already an abstraction. It is a mapping from reality or virtuality, to another reality. That a geometric or mathematical fact becomes obvious from a figure is not a matter of the banal statement that "an image says more than a thousand words". We collect some essential needs that the viewer of a didactically and psychologically clever--made visualisation of a geometric theorem should fulfil, referring what the viewer should be able to do:

- relate the figure to a specific subject matter (he/she needs broad expert knowledge)
- recognize the visualized problem (he/she needs special expert knowledge)
- guess and understand the idea of the visualized problem (he/she needs the capability to interpret the figure the way the presenter packed it in)
- draw conclusions (he/she needs creativity and phantasy).

This means that the viewer needs cultural literacy and education.

The standard example of a proof without words is the theorem of Pythagoras, usually visualized as in Fig. 23. When the author showed the figure to grown-ups with academic education, many of them meant that some special ornamented tiles were depicted, because they did not relate the figure to Pythagoras theorem, in spite of having given that topic during their high school education. After a hint - which used some words -, some of them could then relate the figure to Pythagoras' theorem, but without immediately recognizing its proof. The author concludes that, for such visualisations to be interpreted correctly, a first glance can only remind the viewer of facts he/she already internalized and knows by heart. Maybe a concentrate examination of that figure can lead the unencumbered viewer to the right proof.

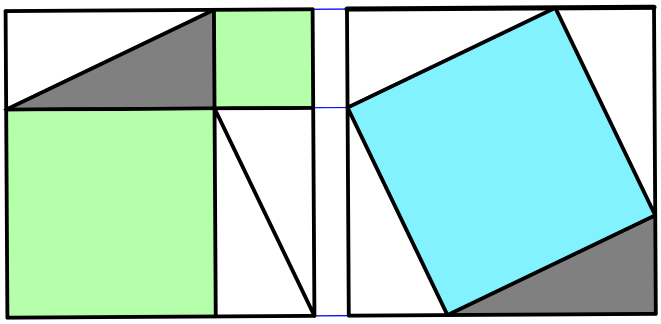


Fig. 23 - Pythagoras' theorem "proofed without words".

As a second example, we present the well-known "butterfly theorem" illustrated in Fig. 24. It reads as follows: Given a non-convex quadrangle inscribed to a circle; the line orthogonal to the connection of the circle's centre with the intersection point P of the crossing sides of the quadrangle intersects the remaining sides in points having the same distance to P.

The elementary geometric proof is rather tricky and not at all obvious to read off from the figure. But having Projective Geometry as knowledge background, the figure shows not only the theorem itself, but it is at the same time indeed a "proof without words" (one has to interpret the circle and the quadrangle as two elements of a pencil of conics. Each line g intersects the set of conics in pairs of an involutoric projectivity called Desargues Involution. As the line in our figure passes through P, this involution has P as fixed point and therefore is hyperbolic with a second fixed point Q. In an involution, each pair of points is harmonic to the two fixed points. In our figure, the intersections of the given line with the circumcircle of the quadrangle is symmetric to P, therefore Q must be the ideal point of the line and all conics (including the singular ones) of the pencil intersect the line in points symmetric to P, *qed*).

We realize that there is a need of the pre-knowledge one would only get via a very specialized study in geometry, in order to let this figure finally act as proof without words. Note that the butterfly theorem is a purely Euclidean statement. Using Projective Geometry for its proof is a trick demanding creativity of the viewer, who is trained to use Euclidean tools for Euclidean facts.

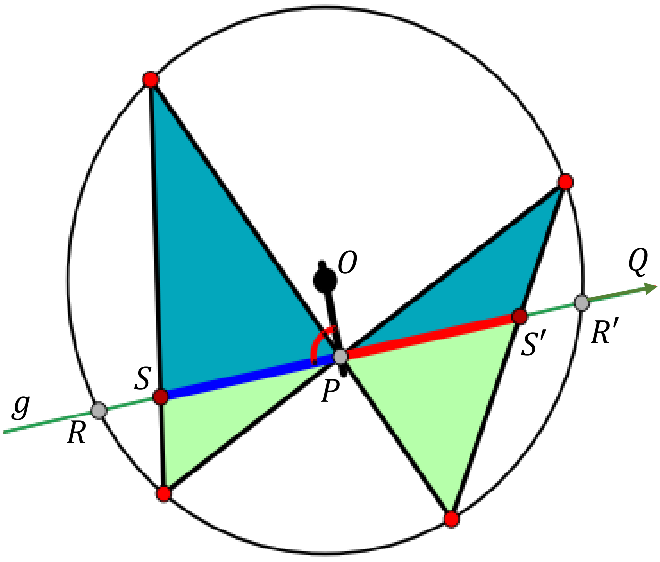


Fig. 24 - Visualisation of the Euclidean butterfly theorem.
The segments between front and back wing have equal length.

Very often, a geometric statement can be proved by a "trick" and, by knowing the trick, the visualisation of the statement might act as proof without words. But the visualisation can already reveal the trick.

As a third example of this type, we mention the following problem: given an acute-angled triangle *ABC*, find the inscribed triangle *XYZ* with minimal circumference (Fig. 25). This is, in fact, a trivariate extreme value problem, but it can be solved by simple reflections. Thereby, the extreme value problem is hidden by the obvious fact that the shortest Euclidean distance between two points is the segment.

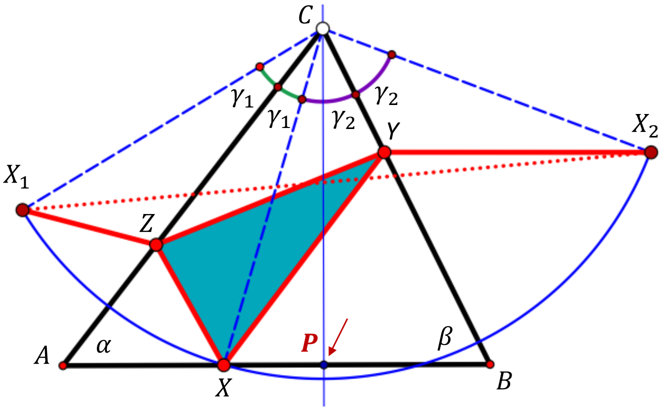


Fig. 25 - Visualisation of the "trick" to find the triangle *XYZ* with minimal circumference inscribed to a given triangle *ABC*. For any choice of $X \in AB$, the triangle X_1CX_2 is isosceles with fixed angle 2γ at *C*. Therewith, segment $\overline{X_1X_2}$ becomes minimal, if *X* coincides with the altitude foot *P* on *AB*.

SUMMARY AND CONCLUSION

By using reflection, we are back to the very beginning, when we emphasized the naïve approach to abstract concepts in geometry. All sorts of visualisation are, in fact, already abstractions. They map objects, structures, statements of the real or Platonic world into other objects of the real world. Very often, these images are loaded with symbolic meaning, too, and this meaning depends on the cultural context. Therewith, any "correct" interpretation presumes knowledge of that cultural context, as well as broad and specialized expert knowledge. An X-ray photography is such an example of a visualisation, the interpretation of which is a matter of a highly specialized physician. Similarly, this means that there is no silver bullet to geometric knowledge, and schooling cannot be omitted.

Geometry has the advantage to deal with abstractions of objects and facts of our usual environment in such form that (basic) geometry becomes as natural as breathing. A third and important property of geometry is its inner beauty, that, via visualisation, leads to appealing realisations as ornaments and architecture. How important Geometry was (and is) for developing our actual view of the world can e.g. be read in [20].

Nowadays, geometry education in high schools and technical universities makes use of computer aided visualisation tools. One should have in mind that "to grasp or comprehend something" (German "*begreifen*") has to do with taking that something into one's hand. One should also be conscious about the fact that the intellectual development in which children grow is (was?) essentially stimulated by a skilful hand controlled by the eye. In the opinion of the author, education in primary and secondary schools should not omit hand drawing and writing and should confront pupils not only with 2D-geometry, but also with 3D-models of geometric objects.

Gunter Weiss, ret. Prof.

ACKNOWLEDGEMENT

The author thanks Vera Viana for her valuable support and suggestions.

REFERENCES:

[01] Weiss, W. (2017) Geometry. What else ?!. *Boletim da Aproged # 33*. APROGED: Porto. p. 41-49 ISSN 2183-1939

[02] Cocchiarella, L. (ed.) (2015) *The Visual Language of Technique* (Vol. 1: History and Epistemology). Springer Intern. Publ: Switzerland, ISBN 987-3-319-05349-3

[03] Weiss, G.& Colagreco, G. (2015) Naïve architecture, its ornamentation and geometry. *Geometrias & Graphica 2015 Proceedings*, p.37-51, Lissabon, APROGED: Porto ISBN 978-989

[04] Salt crystal: https://gustoflight.files.wordpress.com/2011/11/rock_salt_crystal.jpg

[05] Toman, R. (ed.) (1998) *Die Kunst der Gotik*. Könemann: Köln, ISBN 3-89508-313-5

[06] Biedermann, H. (1993) *Symbol Leksikon - Dictionary of Symbolism: Cultural icons and the Meaning behind them*. Cappelen: Oslo, ISBN 8202135931

[07] Greenber, M. J. (2008) *Euclidean and non-Euclidean geometries – development and history*. Freeman: New York, ISBN 978-0-7167-9948-1

[08] Tschirnhaus, E. W. v. (1693) *Medicina mentis, sive artis inveniendi praecepta generalia*. Lipsiae (German translation: J. Haussleiter. J.A.Barth--Verlag, Leipzig

[09] Lindgren, C. E. S. & Slaby, S. M. (1968) *Four-dimensional descriptive geometry.* McGraw-Hill: New York

[10] Caspar, M. & Dyck, W. v. (Eds.) (1938/1993) *J. Kepler's collected works.* Vol.1: Mysterium cosmographicum. De stella nova. Beck: München, ISBN 3-406-0169-1 (Figure retrieved from https://de.wikipedia.org/wiki/Johannes_Kepler#/media/File:Kepler-solar-system-1.png)

[11] Johnson, N. W. (1966) Convex Solids with Regular Faces. In: *Canadian Journal of Mathematics*. Band 18, ISSN 0008-414X, S. 169–200.

[12] Cuboctahedron: https://en.wikipedia.org/wiki/Rhombicuboctahedron

[13] Catalan solids: https://en.wikipedia.org/wiki/Catalan_solid

[14] Webb, R. http://www.software3d.com/Stella.php

[15] Grünbaum, B. (2003) *Convex polytopes* (*Graduate Texts in Mathematics*), 2nd ed., Springer: New York, ISBN 0-387-00424-6

[16] Vörös, L. (2009) Rhombic Structures. Geometry and Modeling from Crystals to Space Stations (Rombikus Szerkezetek. Geometria és modellezés a kristályoktól az űrállomásig). *UNICONSTANT* (ed. S. Kabai), Pecs, ISBN 978-963-87767-3-0

[17] Vörös, L. (2005) Reguläre Körper und mehrdimensionale Würfel (Regular Bodies and Hypercubes). *KoG* No.9, p. 21 – 27. ISSN 1331-1611

[18] Nelsen, R. B. (1997) *Proofs without Words: Exercises in Visual Thinking*, Mathematical Association of America, p. 160, ISBN 978-0-88385-700-7

[19] Miller, R. L. (n/d) On Proofs Without Words. https://www.whitman.edu/Documents/Academics/Mathematics/Miller.pdf

[20] Scriba, C. J. & Schreiber, P. (2015) *5000 Years of Geometry. Mathematics in History and Culture*. Birkhäuser-Springer: Basel. ISBN 978-3-0348-0897-2

PARAMETRIC MODELLING OF CLASS II MECHANISMS APPLIED IN MOVABLE STRUCTURES

Anita Pawlak-Jakubowska [1]

ABSTRACT
Movable engineering structures are an important topic of modern scientific issues connected with civil engineering. Working on solutions to these issues requires an interdisciplinary action in fields such as geometry, construction, machine theory and mechanics or automatics. The author work considers class II mechanisms in existing solutions that occur in moving engineering, such as retractable roofs and bridges. Movement realization has a major influence on the behaviour of the structure during displacement. The analysis of the movability of existing engineering objects supported by parametric modelling facilitates the study of movement and allows for new concepts of solutions.
KEYWORDS: retractable roof, movable bridge, class II mechanisms, quadrangles.

Innovative solutions in the field of geometry of movable engineering structures, such as bridges or retractable roofs, are currently one of the main challenges for designers, constructors or mechanics. Such constructions require a cooperation of specialists from several scientific fields. Calculation of kinematics of a structure in movement is associated with the use of knowledge in the field of geometry and constitutes a basis for the formulation of research problems. We may find applications of class II mechanisms in existing solutions (quadrangles) with turning or sliding kinematic pairs that ensure the movement of a roof or a bridge. In order to calculate the change in the location of the structure, an analytic research is supported by digital technologies. The conducted research and results acquired for models which imitate movement may serve as a basis to investigate new solutions of movable structures for their geometry, structure and kinematics.

RESEARCH

The subject of research of this paper is second-class flat mechanisms (quadrangles) used in mobile engineering structures, such as bridges or retractable roofs. The way in which quadrangles move influence the behaviour of the structure during motion. These mechanisms comprise four elements (one immobile and three mobile) and four kinematic pairs (rotational or sliding). An example of this type of mechanism is shown in Fig. 1, in which a schematic record of the mechanism was used - segments mark mobile parts, which are numbered; the base is marked with a rectangular area covered in lines and numbered 0; otational kinematic pairs are marked with circles, while sliding ones are marked with segments and given successive letters of the Latin alphabet; the arrow marks the driver.

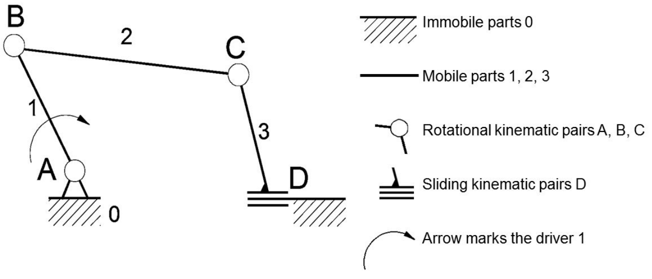


Fig. 1 - Second-class flat mechanisms (quadrangles) composed of four elements (one immobile (0) and three mobile (1, 2, 3)) and four kinematic pairs (on the schematic: rotational A, B, C or sliding D), source: own study.

---

[1] Silesian University of Technology, Faculty of Civil Engineering, Poland. anita.pawlak@polsl.pl

Using computer programs to seek new solutions hastens and upgrades the determination of the boundary conditions, for example, by the elimination of singular placements (unusual placements - extreme[2], and dead centre[3]). Additionally, it allows the observation of the mechanism work in motion and the determination of its work envelope. In Fig. 2, the frame indicates the cases of quadrangles used in existing realisations.

In bridge constructions, second-class mechanisms can be found in the crane bridge type. In the construction of these bridges, variable parallelograms can be separated geometrically, as in Fig. 3: A, B, C and D. The structure consists of a horizontal girder stocked in counterbalance (drive - 1) with the support articulated in the pole (base - 0), stiffened by a diagonal bar.

In the bridge of Auckland (Fig. 4), while lifting the landing, pylons (3) tilt. The movement construction takes place by winches installed on the abutments (A) and gear wheels mounted on top of both pylons (C).

Among roof structures, second-class mechanisms in form of a four–bar linkage were used in structures such as the roof of the Reliant Stadium in Houston, USA (Fig. 5b), and the roof of Marlins Park Florida, USA. Each panel of the mobile covering leans on a quadrangle, which absorbs rapid blasts of wind. Due to that, each roof panel can move into the direction transverse to the direction of movement of the covering (Fig. 5).

There are two second-class mechanisms in the roof of Wimbledon Centre Court in the UK (Fig. 6a). In one of them, there are two roof panels located between two carriages, as shown in Fig. 6b: its elements 2 and 3 are connected to each other with lattice constructions in a rotational way (kinematic pairs B, C, and D). The movement of the roof panels is connected to the motion of one of the carriages, in Fig. 6b, marked as element 1, while the second carriage is blocked. The carriage is the driver of both roof panels and a mechanism determining maximal spreading of a roof segment (elements 4 and 5). Movement of the driver (1) causes the motion of elements 2 and 3 until they reach the position that ensures the minimal inclination of the panel in order to drain precipitation water.

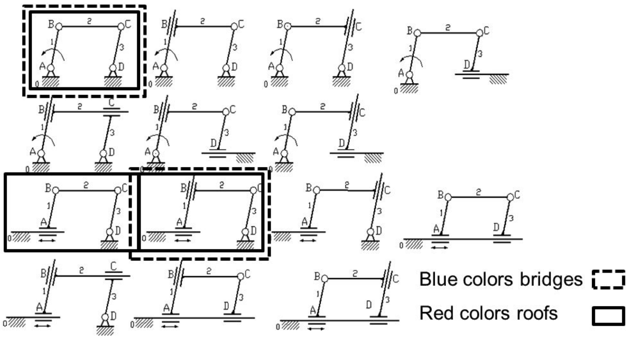


Fig. 2 - Kinematic diagrams of second-class mechanisms with rotational and sliding kinematic pairs, source: own study.

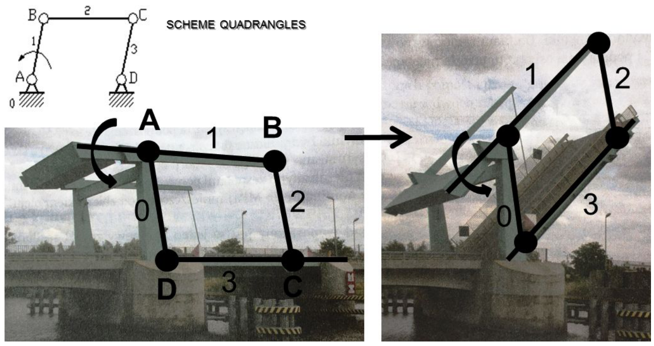


Fig. 3 - Quadrangles - geometrically variable parallelogram in Drewnica Bridge (Poland), own study based on [01].

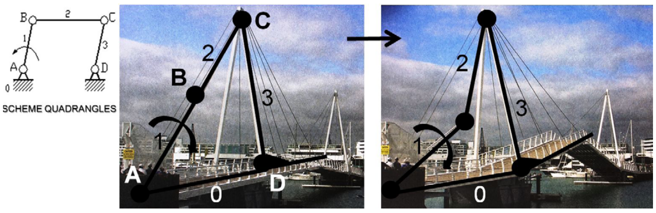


Fig. 4 - Quadrangles in Auckland Bridge (New Zealand), own study based on [01].

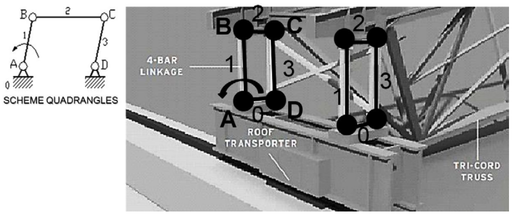


Fig. 5 - Retractable roof - second-class mechanisms:
a) four-bar linkage mechanism connected to roof panels,
b) quadrangles in Reliant Houston Stadium (USA),
own study based on uni-systems.

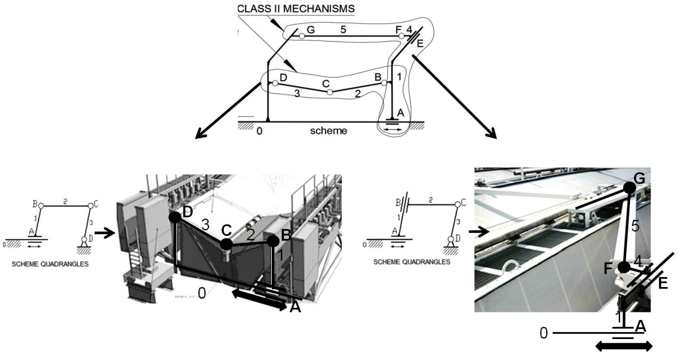


Fig. 6 - Retractable roof of Wimbledon Centre Court in the UK:
a) two second-class mechanisms, b) diagram of mechanisms,
c) equal distances between lattice girders by stabilizing arm (5),
source: own study based on [02].

---

[2] Dead centre placement is a placement of elements in a mechanism that can't be changed by applying force, no matter how large, to the driver.

[3] Extreme placement is a placement of elements in a mechanism in which motion of the driver changes the sense of speed of, at least, one of the elements of the mechanism.

The purpose of the second mechanism is to maintain equal distances between lattice constructions. One of its elements is called stabilizing arm (5) and its length determines the maximal distance between lattice girders (Fig. 6c). The stabilizing arm (5) is rotationally connected with the lattice girder and the element 4 (kinematic pairs F and G). Element 4 makes a sliding motion along the lattice girder (kinematic pair E).

The knowledge of existing realisations using second-class mechanisms enables us to seek for new solutions which would use them as well. A quick realisation of a diagram with the computer program *AutoCAD 2016* and the support of parametric modelling enhances this process, that has four phases. The first one is the creation of a mechanism diagram. The second phase is the layering of geometrical (here the immobile element is chosen, and the relations of parallelism and perpendicularity between particular elements are determined) and dimensional bindings (in which the lengths of particular elements and the angles between them are determined). The next phase is the modification of lengths of the elements and angles through an edition in dimensional binding or a change of a particular system component in the manager. The last phase is propelling the driver, observing how the position changes for other elements, and determining the work envelope of the mechanism.

Gathered data can be used to select possible solutions. The knowledge of structural and kinematic solutions was inspired by the search for new ideas supported by parametric modelling. Fig. 7 shows the proposed lifting the landing using a class II mechanism with a horizontal sliding driver (1). Its movement causes the displacement of the remaining members (2, 3) and 4 (pylon with landing). The displacement of the driving member (1) can be made by hydraulic cylinders or by the movement trolley driven by an electric motor following the linear track as in gantry cranes.

Another possibility to implement this solution is the displacement of the driving member (1) vertically (Fig. 8). Its movement causes the dislocation of the remaining members (2, 3) and 4 (pylon with landing). The displacement of the driving member (1) can be made by hydraulic cylinders or ropes as it does in drawbridges.

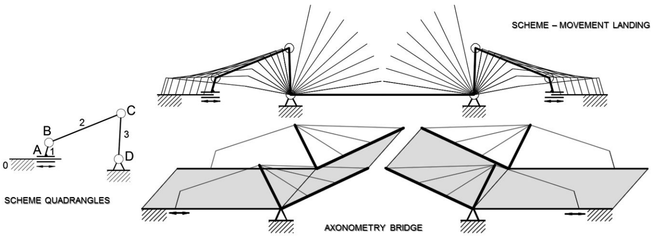


Fig. 7 -Lifting the landing using a class II mechanism with a horizontal sliding driver (1), source: own study.

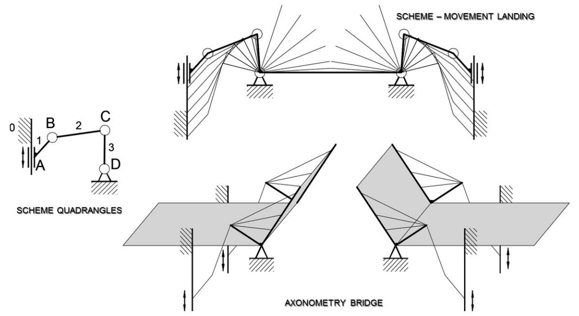


Fig. 8 - Lifting the landing using a class II mechanism with a vertically sliding driver (1), source: own study.

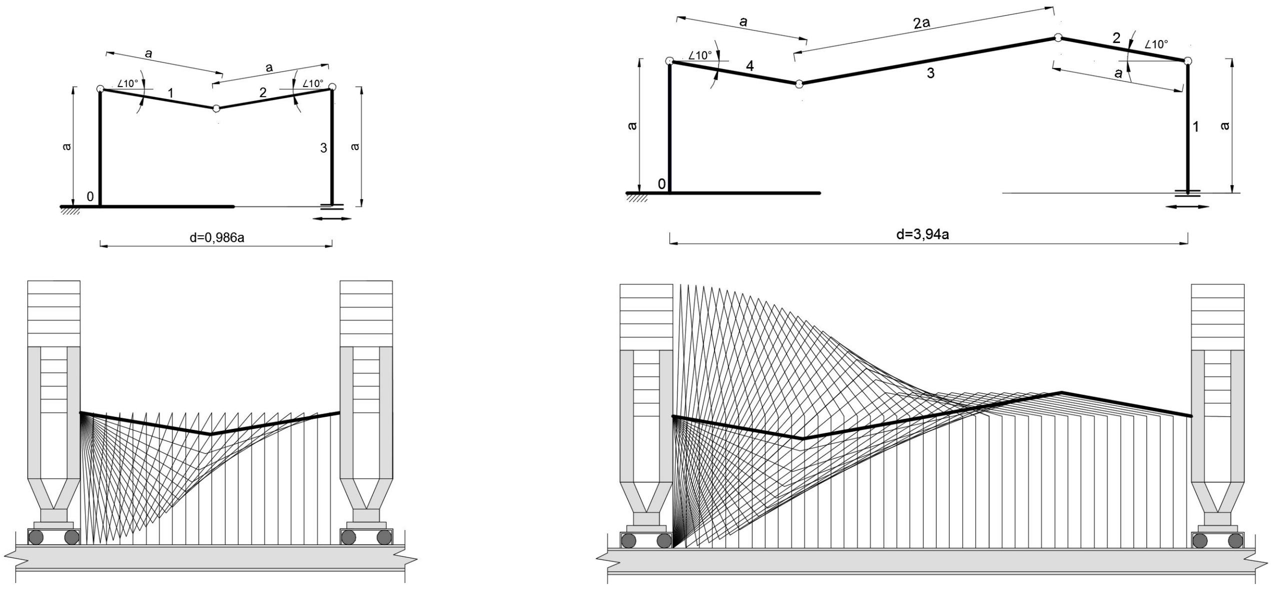


Fig. 9 - Roof of Wimbledon Center Court (UK) realised on the basis of parametric modelling:
a) current solution, b) alternative solution, source: own study.

In roofs with panels directly connected to the elements, the range of their motion must guarantee fulfilling of the roof panel function, and the inclination angle of the roof panels must ensure the draining of precipitation water. One of the examples is the roof of Wimbledon Centre Court, UK, for which the analysis of the movement of particular elements was performed using parametric modelling in *AutoCAD 2016* (Fig. 9a). After performing the structure analysis, an alternative solution to the existing one was proposed. The number of elements (roof panels) was increased by one. The panels fold, creating a so-called harmonica, shown in Fig. 9. Sliding driver 1 consists of a carriage and, connected to it, a lattice girder. The opposite lattice girder, along with the blocked second carriage, creates base 0.

Between girders, roof panels 2, 3, and 4 have been installed in a rotational way. The first phase of the movement is connected to the blocking possibility of motion of element 4. The movement of carriage 1 causes the motion of panels 2 and 3 until they reach a position that ensures minimal inclination of the panel in order to drain precipitation water. For panels 2, 3, and 4, a 10° angle was assumed. After reaching the proper position, element 2 is blocked and element 4 gets unblocked. Then, the movement of the carriage causes the motion of panels 3 and 4 into a position determined by the inclination of the roof panels.

CONCLUSION

The structural analysis of the solutions used in existing roofs offers opportunities for their improvement. In the case of Wimbledon Centre Court, UK, a possibility of covering the same area with a lower number of lattice girders and carriages was raised (Fig. 10b). Using the same number of lattice girders and carriages as the existing solution, it is possible to cover an area of sigificantly larger dimensions (Fig. 10c).

Thus, innovative solution may lead to lighter and cheaper roofs, since fewer elements equals smaller costs in its construction. Additionally, reducing the number of lattice girders leads to the reduction of the garage that is parking space for the roof in an open state. In the example considered, nearly a double spread of the covered area (Fig. 10c) was achieved in comparison to the existing solution (Fig. 10a).

In conclusion, digital technologies can be used as a basis to investigate new solutions for mobile engineering structures.

Anita Pawlak-Jakubowska

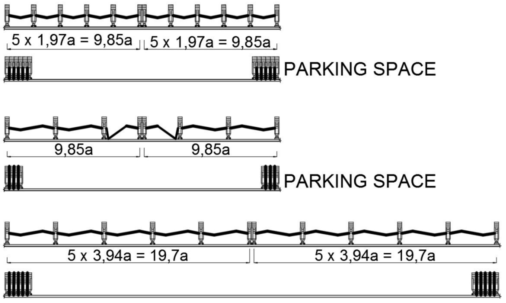


Fig. 10 - Roof of Wimbledon Center Court (UK):
a) current state, b) proposed solution maintaining the same spread of the roof, c) proposed solution maintaining the same number of lattice girders, source: own study.

REFERENCES:

[01] Al-Khafaji, T.; Zobel, H. (2015). *Mosty ruchome*. Warszawa : Wydawnictwo Naukowe PWN SA.

[02] www.architectmagazine.com - accessed 5th September 2017.

MODELING AS THE WAY OF ACQUIRING KNOWLEDGE

Piotr Dudzik [1], Ewa Terczynska [2] and Krzysztof Tytkowski [3]

ABSTRACT

Changes taking currently place in upbringing and teaching, force teachers to constantly search for newer methods of transmitting knowledge to their students. Young people beginning their studies are brought up in different ways of gaining recognition skills than they were a few years ago. Children play with blocks, building various structures and learning the shapes and features of single blocks, but they must be able to imagine the overall effects in their work. During our geometry classes, we refer to wooden models of solids like pyramids and prisms, so that building models from blocks becomes more natural and less hard. With this in mind, we came up with an idea to prepare a number of models that students could cut from a single piece of paper and then fold it to confront a 2D illustration with the corresponding 3D model. In this paper, assumptions according to the model that we made, as well as solutions and some examples, are presented.

KEYWORDS: modelling, 3D model.

INTRODUCTION

Nowadays, young people who enter higher education are raised on other sources of knowledge than before. Pervasive digitization (computers, tablets, smartphones) causes, *inter alia*, the slow disappearance of handwriting and drawing, and the use of modern recording technologies instead of handwritten notes is becoming increasingly popular. However, will such tendencies have any side effects in the future? Is considering that the computer will do everything for us correct? The use of modern technology is desirable and its advantages shall not be questioned here, but to make teaching more effective and connect it better to real life experience, one should use different sources of information.

DESCRIPTIVE GEOMETRY IN POLAND

Nowadays, descriptive geometry and engineering graphics are only taught within the higher education *curriculum*. Most frequently, the classes are conducted in the first semester of the first year of studies. At some faculties, the subject is taught in the second semester, and sometimes even in the third semester (year two). Classes most often concern only on advanced applications of geometry and the use of proper graphic design software. The curricular program nowadays at universities is just a slight extension of what used to be taught in secondary schools.

It is difficult to ascertain when descriptive geometry ceased to be a secondary school subject; however, it is known that it was a part of the *curriculum* before 1939 [01] and just after World War II [02]. The last known secondary school descriptive geometry textbook was published in 1971 for students of elective (optional) courses [03]. Therefore, it can be stated that descriptive geometry stopped being a part of the Polish secondary school *curriculum* in the 1970s.

Between the end of the 1980s and the beginning of the 1990s, the scope of the material and number of teaching hours related to geometry at the university level were reduced step by step. Certain geometry topics were covered during technical drawing classes. For example, the minimum *curriculum* requirements for Architecture

---

[1] Adiunkt (Lecturer), Silesian University of Technology, Poland. piotr.dudzik@polsl.pl

[2] Adiunkt (Lecturer), Silesian University of Technology, Poland. ewa.terczyska@polsl.pl

[3] Senior Lecturer, Silesian Univ. of Technology, Poland. krzysztof.tytkowski@polsl.pl

and Urban Planning in Poland were two semesters of descriptive geometry, comprising 30 hours of lectures and 30 hours of project classes. A minimum scope of materials was defined for all majors. For Environmental Engineering, for instance, the minimum *curriculum* included two semesters of technical drawing in the form of 45 hours of project classes and one semester of descriptive geometry accomplished in 30 hours of lectures and 45 hours of project classes. Most of Mechanics and Machine Design as well as Robotics majors involved one semester of descriptive geometry in 30 hours of lectures and 30 hours of project classes. The project group comprised 12-16 students. Today, classes are often run in groups of more than 30 students. At the Silesian University of Technology, where each faculty decides the number of hours of a given subject, the geometry course for the Environmental Engineering major includes 15 hours of lectures and 15 hours of practice classes, in the second semester alone. Since so little time is devoted to this subject, and due to the fact that students usually start learning from the subject's absolute basis, it is necessary to find adequate methods of quick and independent understanding of selected geometric issues. This refers in particular to those that are crucial for engineers, such as interpreting and preparing technical drawings.

RESEARCH

There is no better age to start training spatial perception than early childhood. An active involvement of the senses and the imagination when exploring space, leads us to automatically learn how to manipulate objects in space by rotations, translations and other geometricoperations. Most often, this happens when children build with blocks, lay puzzles and make models.

Young children, when making different "structures" with blocks, learn the shapes and properties of the component solids. This way, they exercise their imagination according to the outcome of their actions. These blocks, however, should not be confined to basic solids as cuboids. Other elements of previously designed structures (for example, *Lego* blocks) should be manipulated as well. A multitude of shapes and universal combining elements make these shapes easy to incorporate on new structures, created by children themselves. Of course, you can use them to build other things, besides those that the manufacturer imposes by combining shapes in a specific way.

To emphasize the need to develop the imagination when working with the model while preparing and reading drawings as well, students will certainly understand the flexibility and versatility of the model itself. Building a model from blocks is becoming increasingly less natural for children and therefore more difficult. Our observations indicate that more and more we move into the virtual world, an evolution that, in theory, should facilitate our education. But is this true?

During the course of geometry and engineering graphics, we refer to models in the form of ordinary blocks of wood (Fig. 1), such as prisms and pyramids. It turns out that, every year, the number of students who claim never to have played with classic blocks in their childhood increases.

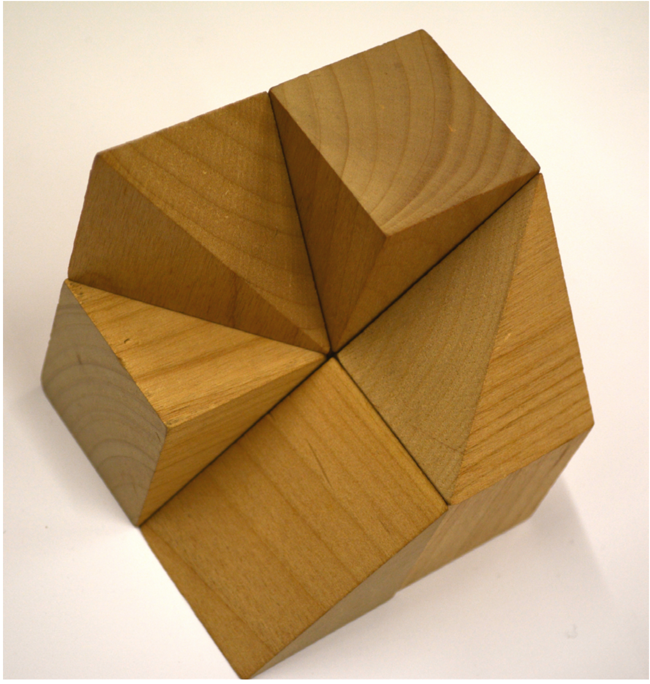


Fig. 1

TASKS

While developing tasks for students in the field of geometry and engineering graphics, and taking into account the three basic areas of their applicability (the students' own work, exercises and tests), one must remember a few aspects:

- each student should be given the possibility to practice working with the material until he/she believes mastering it;
- tasks should have a similar degree of difficulty in groups such, that they can be used to compare the gained knowledge,;
- there should be a large number of tasks with the same level of difficulty that can be explored through

distance learning methods or data-sharing platforms. This way, the probability of redrawing the same task significantly decreases, an opportunity for students to check if their own solution should be provided;
- it should be easy to construct projections and axonometries from the solids chosen, by accepting the fact that the resulting solid consists of basic solids with characteristic points in the edges' midpoints or in the faces' centroids (basic solids are cubes cut by planes sectioning edges or containing vertices);
- all axonometries should be similar, in order to facilitate its analysis and comparability;
- to facilitate inspection, and for a better comparability between the tasks' difficulties, one should keep to frontal axonometry, instead of using the six standard projections of technical drawings;
- one must use a certain predefined class of difficulties.

Taking the above in consideration, an appropriate system to generate this type of tasks [04] was developed. For issues related to multi-projection and axonometry, a solution was proposed in [05, 06] and the idea, developed in [07]. It was a group of basic solids, from which a composed solid is generated (Fig. 2). This approach allows students to understand the solid's components. Thanks to this, in the initial stages of learning, students look for these components in a composed solid. On the other hand, the use of component solids allows teachers to generate tasks with a similar scale of difficulty. The fact that basic solids are rotated around different axes allows teachers and students to generate different composed solids, from exactly the same components.

In both types of tasks, we deal with axonometry and the composed solid is built on the same basis - beginning with the information on the geometric form of the solid and its solution throught multi-view projection. This may be one or three cubes (Fig. 3). It should be noted that the solids can rotate around the axes x, y and z, which gives the three possible orientations for the same basic solid.

Random distribution of basic solids can be made according to two schemes. One is the possibility of a repetition of one of the elements.

The number of possible complex solid Z_r

$$Z_r = p^s$$

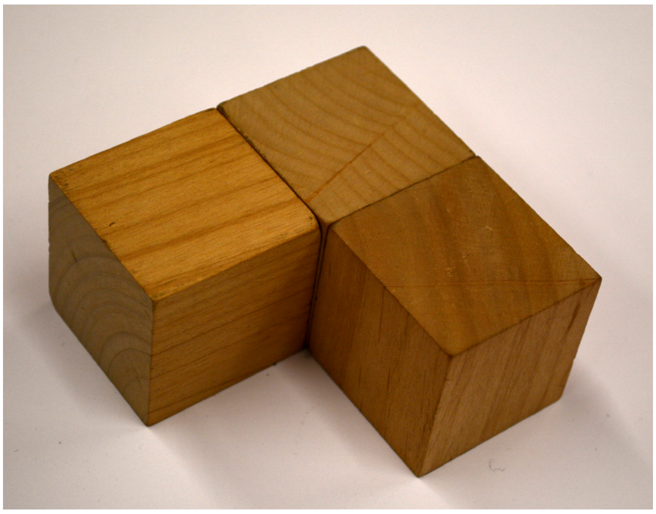


Fig. 2

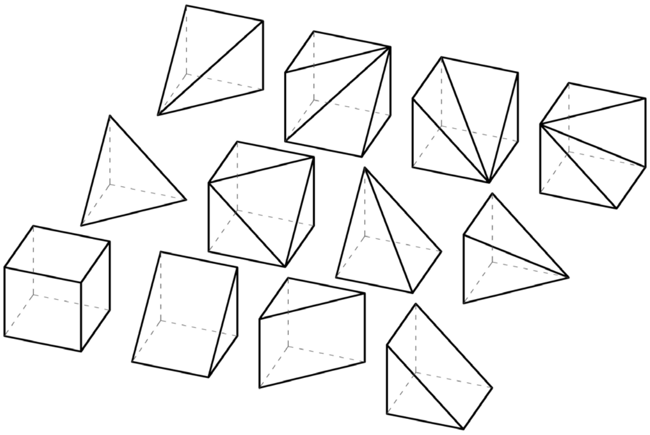


Fig. 3

where:

p – number of available elementary solids,

s – number of randomly drawn solids.

The second scheme is a random drawing without repetition of the component solids. Then the number of possible composed solids will amount to Z_{nr}

$$Z_{nr} = (p - 1) \dots (p - s + 1).$$

Table 1 shows the number of possible composed solids, assuming that s = 4, as in the deployment of 4 basic solids explored in this paper. The argument for the scheme without repetitions is the fact that the composed solids are more comparable in terms of difficulty, and such a scheme should be applicable to test the students' knowledge. The superpositions of the same basic solids may be surprising, given the fact that some of the faces of different basic solids lie in one plane and, occasionally in addition, have a common edge.

Table 1 - Complex solids

p	4	5	6	7	8	9	10	11	12
with repetitions Z_r	256	625	1296	2401	4096	6561	10000	14641	20736
without repetitions Z_{nr}	24	120	360	840	1680	3024	5040	7920	11880

For other purposes, one can take the scheme of composing basic solids through repetitions, which results in a higher number of possible composed solids. This leads to the achievement of a greater diversity though, sometimes, a solid with a lower level of difficulty might occur.

Due to the representation in axonometry and its uniqueness, certain basic solids formed by rotation should be excluded, as they can cause trouble in interpretation. In the applied axonometry of Fig. 4, a face of a base solid is not in a vertical position and is only partially visible.

Tasks of this type have been explored for several years. Changes in the preparation of students led the authors to seek effective methods of transferring the information on the geometric form of the solid. In the tasks in which the input information is an axonometry, a gradual increase in the difficulty of interpretation of its information was observed, especially in subsequent years. Hence, we got the idea to prepare nets of composed solids with increased complexity (Fig. 4).

The students' task is to develop and cut out these nets, glue them into 3D objects and then draw its projections (Figs. 5 and 6). Contrary to appearances, the preparation of such cut-out sheets turned out to be a demanding task and so, in the process of developing a polyhedron's net, one should take into account several factors such as:

- the fact that the size of the entire development should fit in a sheet of a particular size for every task;
- the transparency when gluing (bending) the model;
- the minimum amount of glued edges and the maximum amount of bended edges;
- a well-thought order of gluing;
- the selection of the bending direction: "outside" and "inside".

The preparation of the polyhedron's net, assuming a specific colour for all the elements with the orientation of a given plane, would certainly facilitate the identification of these elements and planes [08].

However, the costs of preparation and realisation of such a net increases significantly, so, before the final development, one should compare the results obtained with a coloured model and with a monochrome model. Further developments and cost reduction of 3D printers can also be a solution, because, along with a set of tasks, students can have access to the prepared files, that will be used to print a particular model.

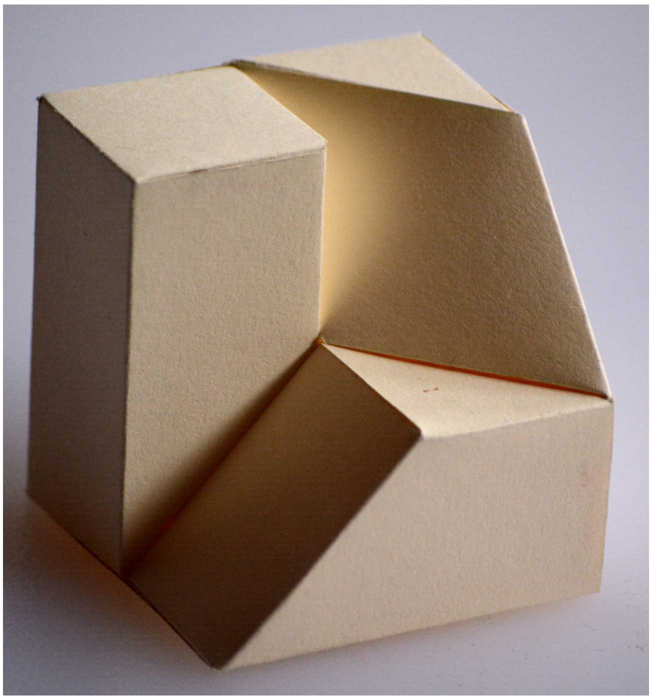


Fig. 4

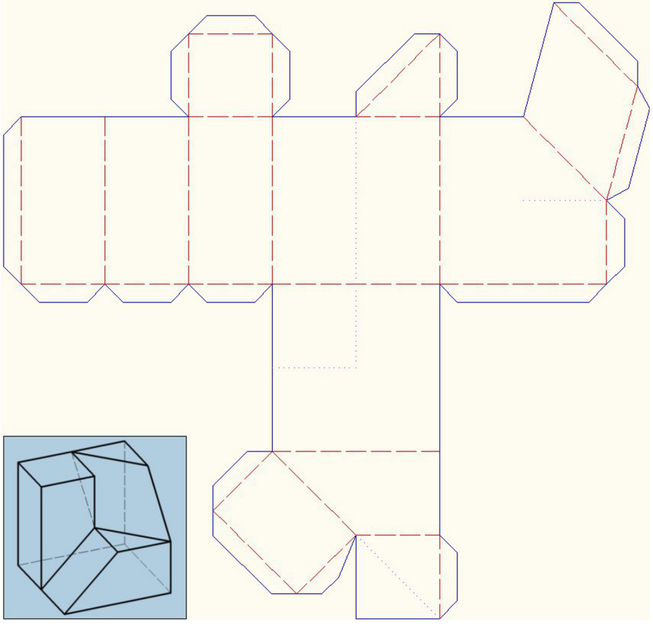


Fig. 5

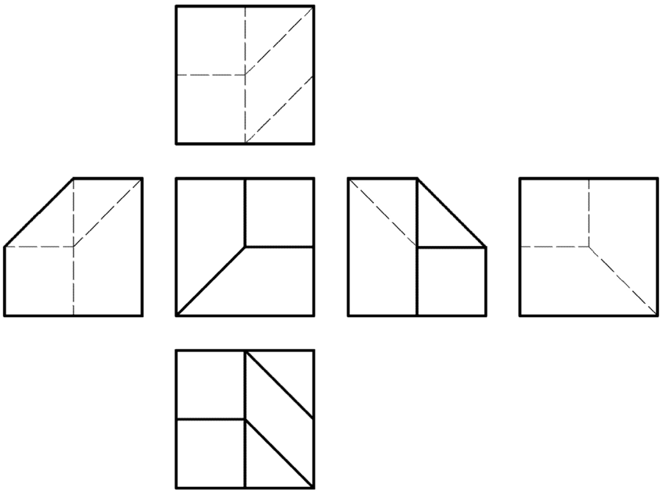


Fig. 6

The big disadvantage is the effort to prepare the net and make the construction of the 3D model. For those without manual skills, this task can be tedious, due to the required accuracy but, on the other hand, to develop his manual skills is not an irrelevant task.

The models will allow to train an engineer-to-be in other desirable ability, namely, the prediction of successive assembly stages on the basis of imagination.

Of course, one can sustain that the assessment of the level of difficulty for each task is subjective, so, in order to objectify the assessment, the key to the number of planes' orientation in the composed solid should be applied by splitting tasks into sub-tasks. In the tasks in which a solid can be repeated, it was also taken into account the information on repetitions (4, 3 or 2 of the same basic solid, and the case where two different basic solids were repeated, sorting these cases from the easiest 4, 3, 2+2, 2).

CONCLUSION

Further developments on the net of a given composed solid gives the opportunity to build the model by gluing. If the student cannot distinguish the component solids, there is an opportunity for him/her to build a solid composed of blocks. Then, drawing a multi-projection becomes easier, because the process of restitution based on projections can be omitted in the initial stage of the exercise. The model glued through this process facilitates the recognition of the configuration of faces in the model: whether they are triangles, rhomboids or other polygons. Becoming familiar with particular faces in the process of cutting and bending affects the process of restitution of the solid. The authors predict a confirmation of their previous observations on a representative group of students, admitting the need for any adjustments that might turn out to be necessary.

Dudzik Piotr, Terczynska Ewa and
Tytkowski Krzysztof

REFERENCES:

[01] Szerszeń S.(1939) *Geometria wykreślna. Cz. 2, Dla II klasy liceum matematyczno-fizycznego* ["Descriptive Geometry. Part 2, for the 2nd Year of Secondary Schools in Mathematics and Physics"], Wydawnictwo Zakładu Narodowego im. Ossolińskich, Lviv, 1939

[02] Straszewicz S., Kulczycki S. (1947) *Geometria wykreślna: dla II klasy liceum ogólnokształcącego, wydział matematyczno-fizyczny* ["Descriptive Geometry for the 2nd Year of Secondary School, Class of Mathematics and Physics"], Książnica-Atlas, Warsaw 1947

[03] Straszewicz S. (1971) *Geometria wykreślna: zajęcia fakultatywne w grupie matematyczno-fizycznej* ["Descriptive Geometry: Optional Course, Group of Mathematics and Physics"], Państwowe Zakłady Wydawnictw Szkolnych, Warsaw 1971

[04] Tytkowski K. (2002 October) Solids generators in teaching engineering graphics. *Proceedings of Symposium on Computational Geometry*. SCG'2002, Kocovce, October 2002. pp. 79-82.

[05] Dudzik P., Terczynska E., Tytkowski K. (2014 June) KPE platform - realization - engineering graphics tests. *Proceedings of 21st Conference Geometry Graphics Computer*. CGGC, Gdansk - Sopot, 25th - 27th June 2014, pp. 15-16

[06] Dudzik P., Terczynska E., Tytkowski K. (2014 June) KPE platform as the answer to the changes in the process of educating engineers, *Proceedings of 21st Conference Geometry Graphics Computer*. CGGC, Gdansk - Sopot, 25th - 27th June 2014, pp. 11-12

[07] Dudzik P., Terczynska E., Tytkowski K. (2015 July) Are the students aware of their knowledge? - an experiment, *Proceedings of 22nd Conference Geometry Graphics Computer*. CGGC, Cracow, 1st - 3rd July 2015, pp. 16-19

[08] Terczynska E., Tytkowski K. (2015 September) E-learning - descriptive geometry and engineering graphics. *Proceedings of the Slovak-Czech Conference on Geometry and Graphics. 24th Symposium on Computer Geometry SCG'2015. 35th Conference on Geometry and Graphics*, pp. 235-240

EXAMPLES OF GENETIC ALGORITHMS USAGE IN GEOMETRY AND ALGORITHMIC DESIGN

Michał Nessel [1] and Szymon Filipowski [2]

ABSTRACT

In this paper, the authors test genetic algorithms as geometric and design issues' solvers to explain and explore the possibilities of this computing technique in design and research. The tests and explanations are based on three classical geometry problems of the Ancient Greece and on a pattern distribution algorithm, created by the authors and inspired by the definition of Lebesgue covering dimension. The basic tools for research are: *Rhinoceros*, the *Grasshopper* plug-in and the *Galapagos* tool. The tests prove that, as a result based on computing techniques, genetic algorithms can be used to find solutions without the implementation of analytic methods. This advantage of evolutionary computing can be very useful in case of complex issues, where implementation of analytic methods reveals itself difficult or even impossible.

KEYWORDS: genetic algorithms, evolutionary computing, algorithmic design, descriptive geometry, *Grasshopper.*

INTRODUCTION

According to Patrik Schumacher's paper "*The Parametricist Epoch: Let the Style Wars Begin*" parametricism is the unified style in design for the 21st century [01]. Maria Helenowska-Peschke uses the term "parametric--algorithmic design" to describe the design process with the use of algorithms [02]. Intense development of visual programming plug-ins and tools like *Grasshopper* for *Rhinoceros* (first appeared in 2007), *Dynamo* for *Revit* and *Max Creation Graph* for *3Ds Max* prove that parametric and algorithmic design techniques are really important nowadays. The aim of this paper is to analyse the possibilities of genetic algorithms' usage to solve geometric and design issues. This specialist computing technique was developed by John Holland in 1975 [03] and extended and described by David E. Goldberg in the book "*Genetic Algorithms in Search, Optimization and Machine Learning*" [04]. The technique was initially available for professional programmers for a long time. Thanks to visual programming languages, evolutionary computing techniques are now available for other professions, like designers 'and researchers. In this paper, the authors test genetic algorithms as geometric and design issues' solvers to explore the possibilities of this computing technique. The basic tools for research are: *Rhinoceros*, developed and provided by *Robert McNeel & Associates* [05], *Grasshopper,* the algorithmic modelling plug-in for *Rhinoceros* created by Scott Davidson [06] and *Galapagos*, the evolutionary computing tool for *Grasshopper* created by David Rutten [07]. In the first part of this paper, we examine and explain computing techniques based on genetic algorithms to find the solution for basic geometric issues involved in the three classical geometry problems of the Ancient Greece: trisecting an angle, doubling the cube and squaring the circle. The second part of this paper presents more advanced possibilities of evolutionary computing usage in geometry and design. We analyse and test several possibilities to create evolutionary computing based algorithm inspired by the definition of Lebesgue covering dimension for covering curves, planar shapes and surfaces with the minimum number of given circles and spheres.

---

[1] M.Sc.Arch., Cracow University of Technology, Poland. mnessel@pk.edu.pl

[2] M.Sc.Arch., Cracow University of Technology, Poland. sfilipowski@pk.edu.pl

THE THREE CLASSICAL GEOMETRY PROBLEMS OF THE ANCIENT GREECE

Squaring the circle, doubling the cube and trisecting an angle are the tree classical geometry problems that cannot be solved using compass-and-straightedge construction. In this paper, we do not try to search for geometric solutions to those problems. The authors understand the fact that it has been proven that they are unsolvable with the use of classic geometry and that they have already been solved with analytic techniques. However, we use them as examples for testing purposes of certain evolutionary computing techniques.

Evolutionary computing techniques as described by Krzysztof Krawiec are based on the following steps: initialization of population, evaluation, selection, parent recombination and mutation [08]. All the steps, with the exception of the first, are repeated until a good result is found. The described process is an integral part of *Galapagos* tool that we use for the analysis.

The example of squaring the circle presents a very simple usage of evolutionary computing. We create a circle with constant radius and a rectangle where the *X* and *Y* dimensions are controlled by a number slider, which is a default way of creating numerical inputs in *Grasshopper*. The next step is to measure the area of the circle and the rectangle. The area of the circle is then subtracted from the area of the rectangle. The absolute value of the subtraction is used to evaluate the results and is called "fitness value". For the presented algorithm, the lower fitness values the best solution. If the areas of the circle and the rectangle equals the fitness value 0, it means that the best result was found. The final step in the creation of the algorithm is the addition of the *Galapagos* tool. The "Fitness" input in *Galapagos* tool is connected to the part of the algorithm where the fitness value is calculated. The "*Genome*" input is connected to the slider that controls the rectangle's dimensions (Fig. 1).

Once the algorithm is created, we can start the evolutionary computing with the *Galapagos* tool. The tool is set to search for the result with the minimum fitness value. The process of computing starts with the generation of a first random population of parameters, that controls the final result. The *Galapagos* tool generates random values for the slider that controls the dimensions of the rectangle. Each of the results is evaluated by the fitness function described above. The best results are chosen for reproduction and the worse are eliminated from the population. During the reproduction process, a new generation of parameters is generated. The process continues until we stop it or until an optimal solution is found. The algorithm finds the result where the areas of the circle and the rectangle are equal. The accuracy of computing is very high and controlled by the accuracy settings of *Rhinoceros,* the accuracy of Pi and the slider that controls the dimensions of the rectangle.

The second algorithm is based on the doubling of the cube example and is very similar to the one described above. The difference is that the algorithm compares the volumes of two cubes: the double value of a cube with constant dimensions and the value of a cube, whose dimensions are controlled by the slider connected to the *Galapagos* tool (Fig. 2).

In both examples, the fitness function was easy to define, because there are very clear criteria of evaluation and the fitness value depends on only one parameter that is represented by the corresponding slider.

The case of the angle trisection is more complex. The algorithm generates an arc and two division points that belongs to the arc. The position of the points is controlled by two sliders, one for each of the points. Both sliders are connected to the *Genome* input in the *Galapagos* tool. The fitness function is based on the distances between points. The following distances are then calculated: from the start point of the arc to the first division point, from the first division point to the second division point and from the second division point to the

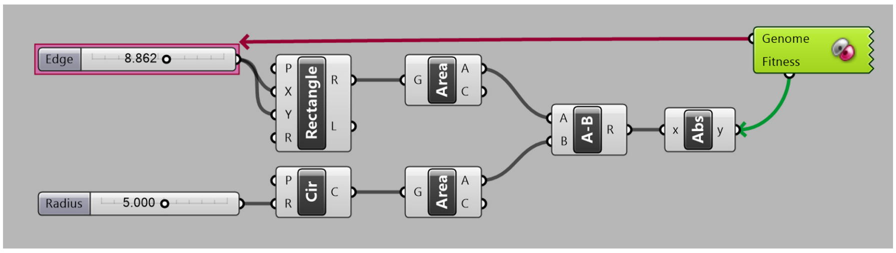


Fig. 1 - The example of a squaring the circle algorithm.

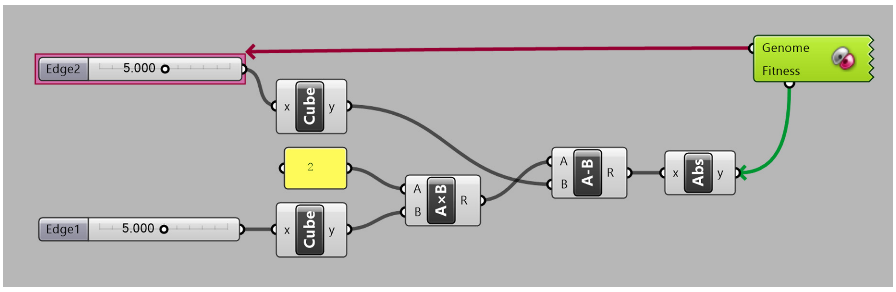


Fig. 2 - The example of a doubling the cube algorithm.

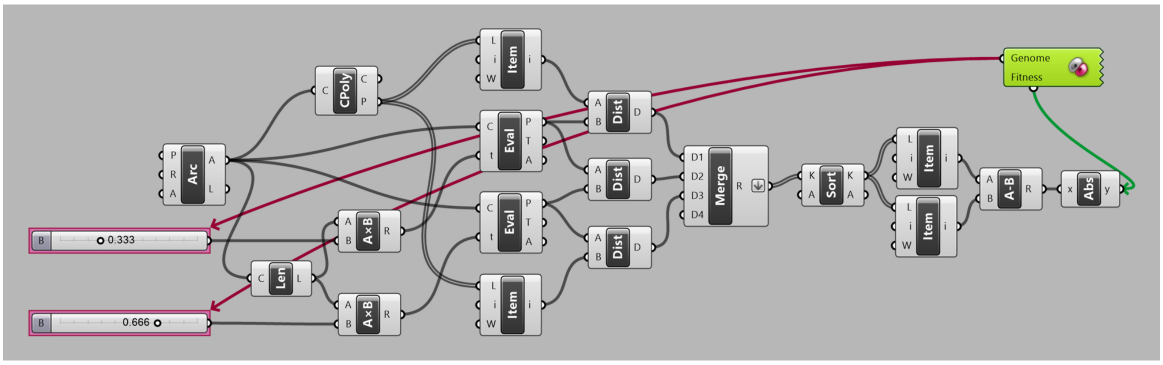


Fig. 3 - The example of an angle trisection algorithm.

endpoint of the arc. The distances are placed in a list of values that is sorted out with the use of an incremental sorting. The first distance from the list (minimum value) is subtracted from the last distance (maximum value). The result of the subtraction is used as a fitness value for evaluation of the results. As in the previous cases, the smaller fitness value is the best solution. *Galapagos* finds the best result but the calculations take more time than in previous cases, due to the fact that the fitness function depends not on one, but on two parameters (Fig. 3).

The examples present the way how evolutionary algorithms can be implemented in certain geometric problems. It should be emphasized that the algorithms are not searching analytical or geometric solutions to problems, but only a result that meets the criteria we have previously defined. Tests proves that the accuracy of genetic algorithms is very high in every problem in which the criteria for evaluation are clear and the fitness function depends on a limited number of parameters.

LEBESGUE COVERING DIMENSION AND PATTERN DISTRIBUTION

Pattern distribution and surface subdivision are important design and geometric issues. The techniques designed for those purposes are based, for example, in triangulation, tessellation or packing dimension. The definition of Lebesgue covering dimension was an inspiration for the authors to consider a possibility for a pattern distribution based on the definition of covering dimension and evolutionary computing. The topological explanation of covering dimension can be found in [09].

Following the interpretation of covering dimension explained in [10], it is possible to cover a curve with circles in such way that the intersection of any three circles is empty. It is also possible to cover a two-dimensional surface with circles in such way that the intersection of any 4 circles is empty (Fig. 4).

The tests starts with several examples of covering two-dimensional curves. The first algorithm covers a base-circle with a given number of circles. The centres of the covering circles belong to the main circle and the radiuses and distances between their centres are equal. The *Galapagos* tool searches for the smallest radius of covering circles, so that the base-circle is fully covered with circles. The fitness function detects the intersections between each of the covering circles and the base-circle. The lengths of the arcs, that result from the intersections, are summed and compared with the total length of the base-circle with the use of the absolute value of subtraction (Fig. 5).

The second algorithm covers a curve with equal circles and a given radius. The difference is that, this time, it is not the radius but the number of covering circles to be optimized. The number of circles is controlled by the number of division points, that are generated by the "*Divide Curve into equal parts*" tool. The points define the centres of the circles (Figs. 6 and 7).

In the second part of the tests, we analyse the possibilities of covering planar surfaces with circles. It was observed that the optimal covering of an infinite planar surface is such that the centres of the circles create a regular triangular grid. We considered the possibility of covering a planar surface with such pattern but the distribution of the pattern is not optimal at the edges of the surface, even after evolutionary optimization of position and rotation of the grid. For that reason, the authors decided to focus on the

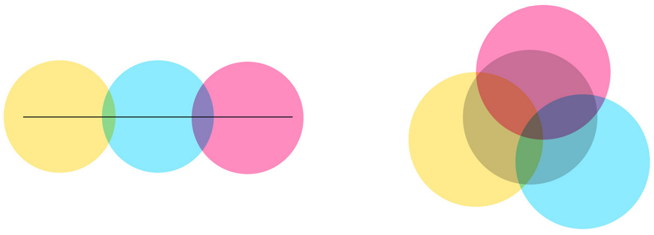


Fig. 4 - A graphic interpretation of the definition of covering dimension at the example of linear curve and circular surface.

covering that relates to the shape that limits the surface. The starting point of this algorithm is similar to the algorithm that covers a curve with circles with a given radius. The difference is that the centres of the circles are not located on the curve that represents the boundary of the surface. The algorithm creates an inner curve by offsetting the boundary by a distance equal to the distance between rows of the circles' centres in the case of the triangular grid described above. The inner curve contains the centres of the circles and the number of circles is optimized by the *Galapagos* tool to cover the boundary of the surface. Next step is to detect the intersections between the covering circles and to create a curve that contains them. The curve is a boundary of the part of the surface that has not yet been fully covered. The process of offsetting the boundary and covering it with the circles whose centres belong to the inner curve is repeated until the whole surface is covered. Due to the fact that recursive algorithms are forbidden in the *Grasshopper* plug-in, the process of evolutionary optimization of the curves' covering is divided into separate steps, one for each curve (Fig. 8).

The final test was to cover a three-dimensional surface. The algorithm is similar to the one used for covering a planar surface, however, not circles but spheres are now used for covering. The intersections of curves and spheres are used for the evolutionary optimization process (Fig. 9).

As a result of computing, the surfaces are fully covered with circles or spheres whose centres can be used for pattern distribution and for surface subdivision as well. The distribution of the pattern is very uniform and regular, with the exception of the center of the surface, in which the distribution is distorted. It needs to be mentioned that for surfaces, the covering does not exactly meet the definition of covering dimension.

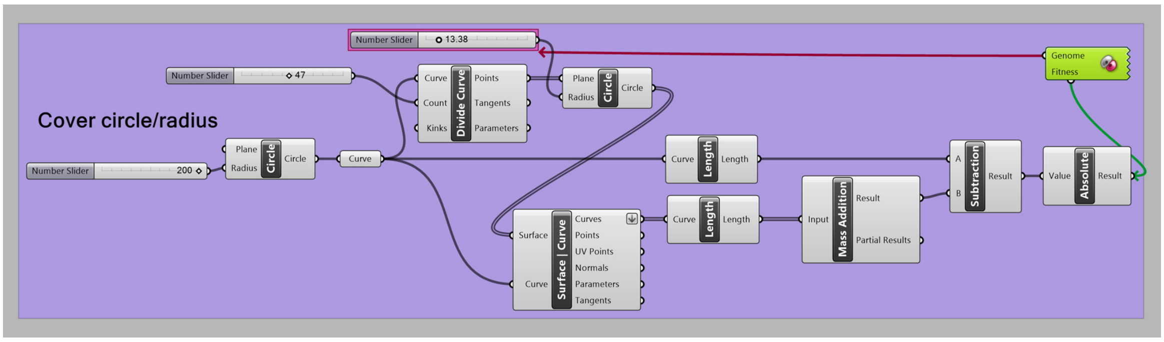


Fig. 5 – Example of an algorithm covering a circle with a given number of circles.

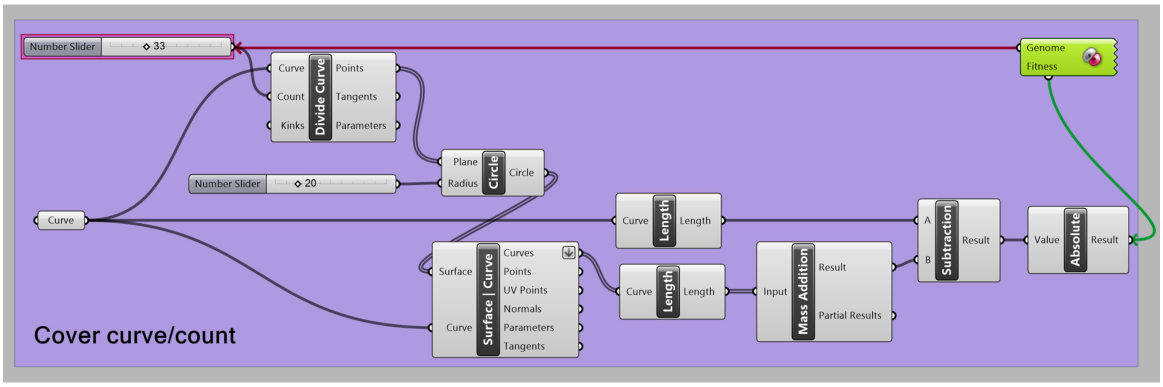


Fig. 6 – Example of an algorithm covering a curve with circles with given radius.

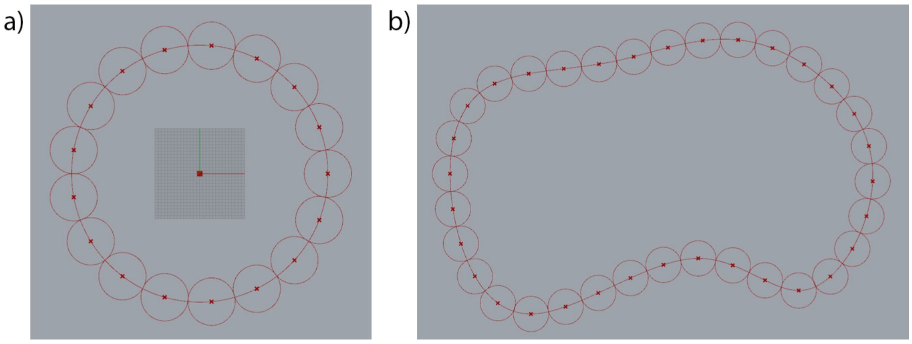


Fig. 7 - The results of computing: a) circle covered with a given number of circles. b) curve covered with circles with given radius.

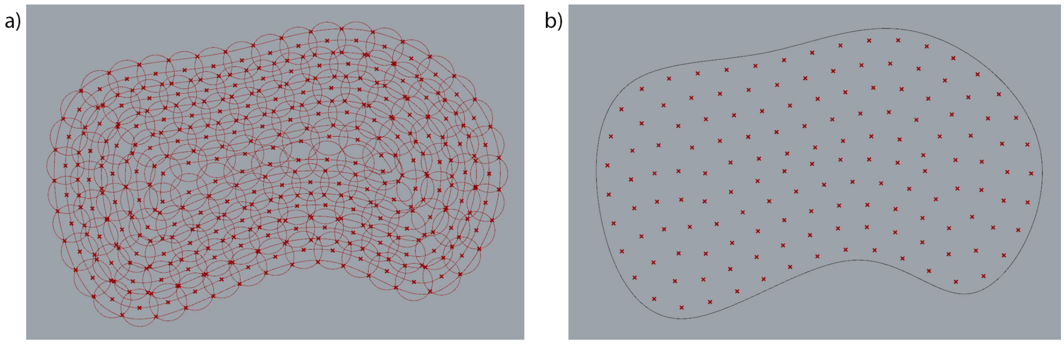


Fig. 8 - The results of computing:
a) two-dimensional surface covered with circles, b) two-dimensional surface and the centres of the covering circles.

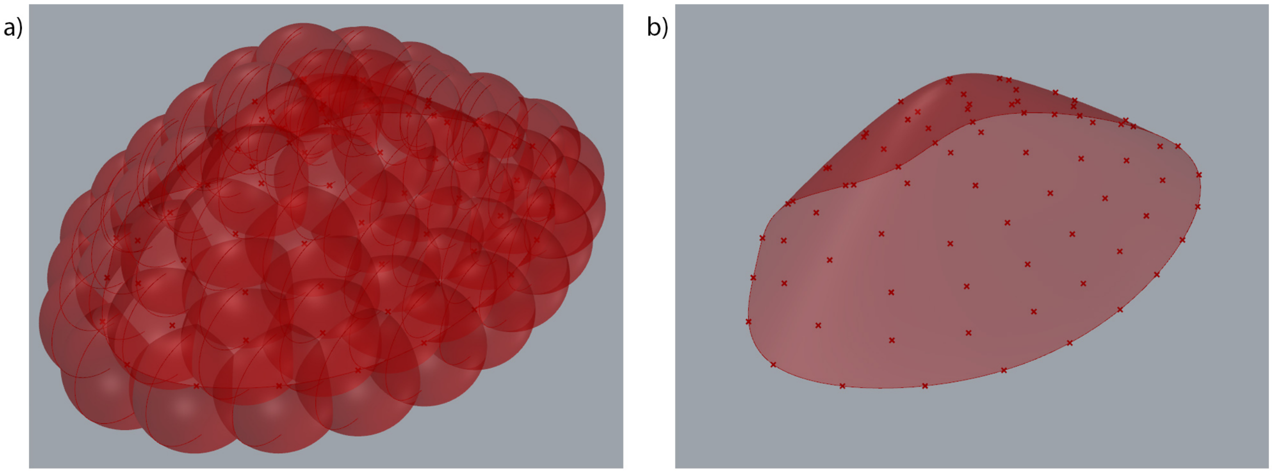


Fig. 9 – The results of computing:
a) three-dimensional surface covered with spheres, b) three-dimensional surface and the centres of the covering spheres.

In some cases, it violates the rule that any four of covering curves has an empty intersection, particularly close to the center of the surface. However, the rule is respected in the case of curves that are generated and covered during the process of covering the surface.

The usage of genetic algorithms was the inspiration and an effective way of working with the pattern distribution based on covering dimension. Thanks to the nature of evolutionary computing, the research was focused not on finding the analytical solution to problems, but on the final results. This allowed to skip time-consuming survey focused on the methods and to test the basic idea of pattern distribution inspired by covering dimension in a shorter time.

CONCLUSIONS

Genetic algorithms and evolutionary computing technique can be effective tools for solving geometric and design issues. As a result-based computing technique, evolutionary computing can be used to find the solution without the implementation of analytic methods. This advantage of genetic algorithms can be very useful for more complex issues, where the implementation of analytic methods is difficult or impossible.

The tests based on three classical geometry problems of the Ancient Greece prove that geneticalgorithms may be very effective and accurate for well implemented simple problems. For more complex tests inspired by Lebesgue covering dimension, the usage of this computing technique led the authors to proper results. However, it needs to be mentioned that authors, being aware of the nature of evolutionary computing, paid much attention to the proper implementation of this technique.

The idea of pattern distribution inspired by the definition of covering dimension and evolutionary computing is an additional value for the research and further development of this technique.

Michał Nessel and Szymon Filipowski

REFERENCES:

[01] Schumacher, P. (2009, July/August). Parametricism - A New Global Style for Architecture and Urban Design. *AD Architectural Design - Digital Cities*, Number 4, Volume 79.

[02] Helenowska-Peschke, M. (2014). *Parametryczno-algorytmiczne projektowanie architektury*. Gdańsk: Politechnika Gdańska.

[03] Holland, J. H. (1975). Adaptation in natural and artificial systems. Michigan, USA: The University of Michigan Press.

[04] Godlberg, D. E. (1989). *Genetic Algorithms in Search, Optimization, and Machine Learning*. Boston, MA: Addison-Wesley Longman Publishing Co., Inc.

[05] Rhino 5 Features. Retrieved in August, 2017 from https://www.rhino3d.com/features

[06] Grasshopper algorithmic programming for Rhino. Retrieved in August, 2017 from http://www.grasshopper3d.com/

[07] Robert McNeel & Associates. (2016). Galapagos [online]. Retrieved from http://www.grasshopper3d.com/group/galapagos

[08] Krawiec, K. (2004). *Evolutionary feature programming, Cooperative learning for knowledge discovery and computer vision*. Poznań: Wydawnictwo Politechniki Poznańskiej.

[09] Engelking, R. (1977). *General Topology*, Warsaw, Poland: PWN.

[10] Peitgen, H. , Jürgens, H., Saupe, D. (1992). *Fractals for the Classroom, Part One Introduction to Fractals and Chaos*. New York, USA: Springer.

All images are from the authors.

Boletim da Aproged # 34 - Dezembro 2018 | ISSN 2184-4933

ALGORITHMIC APPROACH IN THE DESIGN OF REPETITIVE PATTERNS ON ARCHITECTURAL SURFACES

Szymon Filipowski [1] and Michał Nessel [2]

ABSTRACT

Nowadays, smooth, curved surfaces are very common in architectural solutions, and proper design of repetitive patterns on such surfaces is a real challenge for architects and designers. This paper presents various algorithmic approaches to pattern design and distribution, based on examples of existing solutions and on the authors' designed algorithms and didactic experiences. Algorithmic studies are based on *Rhinoceros* for *NURBS* (Non-Uniform B-Spline) modelling and on the *Grasshopper* plugin for graphic algorithm editing. This approach to the topic combines geometry, aesthetics, flexibility and *IT* techniques, that are very useful in case of pattern distribution. The examples present algorithms that help in clear definitions and the realization of the designer's intentions to achieve accurate results.

KEYWORDS: pattern, surface, mapping, tessellation.

INTRODUCTION

Nowadays, thanks to advanced software, it is possible to create digital models of any surface we can imagine. Using a computer, designers and theorists can even generate geometries that exceeds the limits of human mind due to the number of elements, complicated relationships between them or smooth, curved surfaces, that otherwise can be difficult to imagine. However, even in such cases, architects have to meet spatial, aesthetic and structural expectations. Smooth, curved surfaces are very common in architectural solutions nowadays and a proper design for this type of structures is a real challenge for architects and designers. Even when they appear to be absolutely smooth, in most of the cases, they consist of smaller parts and the pattern of divisions is an important element of its design. This paper presents various algorithmic approaches in the design of repetitive patterns on architectural surfaces and is based on examples of existing solutions and on the authors' designed algorithms and didactics.

The first part of this research is focused on literature studies and the analysis of exiting solutions. The authors present techniques carried out by mathematicians, programmers and graphic designers and selected examples of their implementation in historic architecture, folk art, contemporary graphics, 3D mesh subdivision and design. The second part of the paper presents algorithms designed by the authors whit the use of *Rhinoceros* software and the *Grasshopper* plug-in and students' works that were made during the courses prepared and taught by the authors.

HISTORICAL AND MODERN APPROACHES TO PATTERN DISTRIBUTION

Paweł Rubinowicz in "*Chaos and Geometric Order in Architecture and Design*" describes the kinds of complexities in architecture:

> "*It is relatively easy to distinguish between geometric order and chaos in architectural compositions, but the definition of these concepts is difficult. The following definitions can be assumed: The geometric order is represented by ideal mathematical forms (in 2D: e.g. line, circle, quarter, or 3D: e.g. plane, sphere, cube) and ideal relationships (e.g.*

---

[1] M.Sc.Arch., Cracow University of Technology, Poland. sfilipowski@pk.edu.pl

[2] M.Sc.Arch., Cracow University of Technology, Poland. mnessel@pk.edu.pl

> *perpendicularly, parallelism, symmetry, rhythm/ regularity). Chaos is the opposite of geometric order; it is represented by forms and relationships that are complex and difficult to describe with the language of classic mathematics*". [01]

Rubinowicz also explains a typical approach to architecture from a philosophical point of view:

> "*Since the beginning of human history, geometric order has been applied in architectural structures. This order emphasises their unusualness and confers the high importance, monumentality and even the sacral dimension. The natural world is constructed according to more complex rules. The ideal forms and relationships distinguish architecture against the background of nature. Platon's (427–347 B.C.) philosophy observed the world differently from the real world. His model of the world was idealised. Such philosophy emphasises human supremacy over the natural world and ideal forms over more complex forms. This is typical for many phases of the history of architecture.*" [01]

He denotes that: "Geometric order and chaos exist in architecture together" [01]. Shapes in architecture are also a field of the authors' investigations, for example in [02].According to Popper's book "*The Open Universe*", another interpretation of shape is explained:

> *"it can be seen that shape belongs to World 3, because a shape exists in a person's imagination, can be drawn with the use of theories coined by humans."* [03]

Two main kinds of complex structures are generally recognized by the human perception: ordered and random. In their researches, the authors observed that repetitive, continuous shapes based on simple geometry are characteristic for architectural forms in the case of most modern objects and almost always in historical objects.

Geometry plays an important role in pattern design, especially when basic geometric assumptions affect the character of the whole design. Mirek Majewski in "*Geometric Ornament in Art and Architecture of Western Cultures*" writes widely about geometrical rules:

> "*Mathematics, in particular geometry, always played a major role in architecture. In early civilizations the tombs of leaders had shapes derived from a prism with a square base or half sphere. A real sophistication of geometric forms in architecture can be found in ancient Chinese, Indian or Greek architecture.*" [04]

He investigates precisely the geometry of western ornaments and in the case of Byzantium, he observes that:

> "*We can easily see that the structure of this ornament is based on the rectangular grid of circles with the same radius, say R. In further calculations and constructions we will use R as a base unit for this ornament. All of referenced examples are constructed of rectangular grid and arcs.*" [04]

Geometrical roots of architectural patterns are not characteristic of the western culture alone. Yahya Abdullahi and Mohamed Rashid Bin Embi in "*Evolution of Islamic geometric patterns*" describes that:

> "*The grids were used as constructive bases for the simplest regular and semi-regular tiling with equilateral triangles, squares, hexagons, and octagons. Then, contemporary to rise of Persian philosophers and cosmologists from Abu Sahl Al-Tustari to Sohravadi who had debates and important contribution to nature of numbers and their relation with that of nature (Critchlow, 1989), mystical Tetractys motifs and symbol merged to traditional geometric patterns. The result was the invention of abstract 6-point geometrical patterns based on the Tetractys symbol and 12-point star patterns that are associated with 12 zodiacal sectors.*" [05]

They present both flat and spatial patterns in the cases of domes, ceilings and arches.

Andrew Harris in [06] explains, that from a mathematical point of view, a true tessellation with the use of regular polygons is possible only in the case of triangles, tetragons and hexagons. Frei Otto roofings are a leading example of a surface subdivision with the use of the tetragonal grid in architectural design, especially the exterior and interior of the Multihalle in Mannheim, Germany (1975).

Nowadays, new theories and design tools improve human ability to interpret reality and to create new objects. Matthias Rippmann in "*Funicular Shell Design Geometric Approaches to Form Finding and Fabrication of Discrete Funicular Structures*" following the studies of Mario Carpo and Bernadetta Addis, states that:

> "*The way architecture is designed, planned, built and operated has changed dramatically with the increasing use of computers in architectural and engineering offices. Software for computer-aided design (CAD) was already commercially available in the 1970s and 1980s but its use only became common practice starting in the late 1980s with the availability of personal computers at low cost…*

> *With this digital turn novel computerised tools for architecture, structural and civil engineering have gained influence, enabling the design and construction of buildings with complex, doubly-curved geometry.*" [07].

In one of the chapters of his Ph.D. thesis, Rippmann presents methods for a surface subdivision with the use of triangular, quadrilateral and pentagonal patches.

Milena Stavric and Slavic Jablan in "*Advanced Geometry of Modular Tiles*" [08] indicate the importance of modern computing methods and their great influence on architectural patterns. They present flat and spatial ornaments created with the use of mathematical rules and advanced software tools. Most of those examples are based on surface coverings with the use of tetragonal, especially rectangular, grid. They also present possibilities of hexagonal and triangular grids usage.

The principles of aesthetics seem to be immutable and are confirmed by ancient researches as well as by modern scientists. An orderly based of regular arrays of elements, symmetry, specific proportions and characteristic trends, such as level and plumb, is preferred by our perception. As human beings, we easily recognize patterns

based on rectangular, triangular and hexagonal grids and the combination of these figures is used for tessellations. Another very characteristic feature of our perception is to synthesize spatial geometry to planar images. Those tendencies are very important in a proper design of repetitive patterns and were our starting point for an algorithmic approach to this process.

ALGORITHMIC APPROACH TO PATTERN DISTRIBUTION

These algorithmic studies are based on *Rhinoceros* for *NURBS* (Non-Uniform B-Spline) modelling and on the *Grasshopper* plugin for graphic algorithm editing. This set of tools allows a direct usage of *NURBS,* thanks to the implementation of Bezier patches and fully editable curves. It also provides a set of additional tools for analytical purposes that enable the implementation of geometry principles in an algorithmic manner. The main research is focused on covering any surface with a flat pattern and to reduce the amount of deformation caused by the surface curvature. Direct usage of the "map to surface" tool implemented in *Grasshopper* causes deformations of the patterns and affects the final aesthetic expression of the design. The surfaces that are used for tests are implemented in *Rhino* as tetragonal Bezier patches, that optionally can be cropped. During the process of mapping, the coordinates of planar source surface are directly converted to the coordinates of a point on a target surface. Using the example of a sphere, the problem is clearly visible (Fig. 1).

A sphere is defined as a cropped tetragonal Bezier patch where two opposite sides are reduced to a 0 length curve. After mapping, the dimensions of the pattern depend on the distance from the poles of sphere (the dimensions of the elements close to the pole are significantly reduced). This process causes undesirable effects in the case of mapping other types of smooth surfaces, too. This paper presents such test and examples of algorithms that reduce the amount of deformation. The first one is based on the "map to surface" tool and allows manual calibration of pattern distribution, through manipulation of the control points of the source surface. It is an intuitive technique, however it allows to reduce the amount of deformation significantly (Figs. 2 and 3).

The second solution is more complex and based on the horizontal and vertical directions at a specific point of the surface. Two perpendicular lines are projected onto the surface and the intersection of the lines is located at a specific point. The distribution of the pattern is manipulated by the location of the lines' intersection and by the direction of projection. Using the offset tool and projected lines, the algorithm creates the pattern. The most important advantage of this algorithm is that the distances between lines on the mapped surface are very accurate on the entire surface and the directions and angles between lines are accurate close to the specified point. This feature can be very useful in pattern distribution in architectural design. The disadvantage of this solution is that directions and angles between lines are distorted away from specified point. However, due to the accurate distances between lines, the quality of the pattern remains proper (Figs. 4 and 5).

PATTERN DISTRIBUTION IN DIDACTICS

The presented approach is included in the course for Erasmus Students taught by the authors. This part of the course starts with the *NURBS* theory and freeform, smooth surfaces modelling. Also, default tools, like "map to surface" are presented. The students test pattern distribution using their own examples and default tools to analyse pattern deformations. The next stage of the classes is an algorithmic pattern distribution with the use of techniques presented in this paper, based on "map to surface" *Grasshopper*

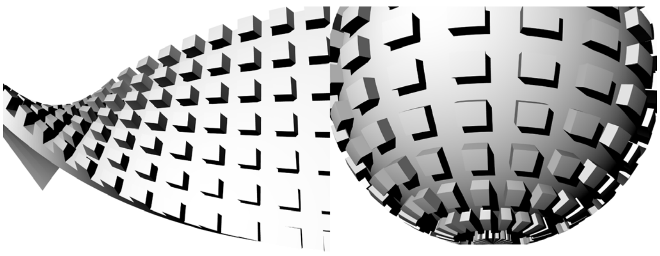


Fig. 1 - Examples of automatic mapping of a rectangular pattern to surfaces: tetragonal (left) and spherical (right).

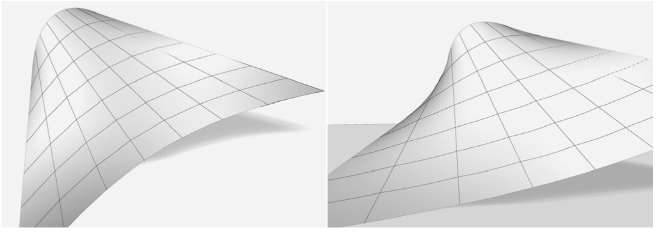


Fig. 2 - A surface covered with a regular pattern by the algorithm based on the “map to surface” *Grasshopper* tool.

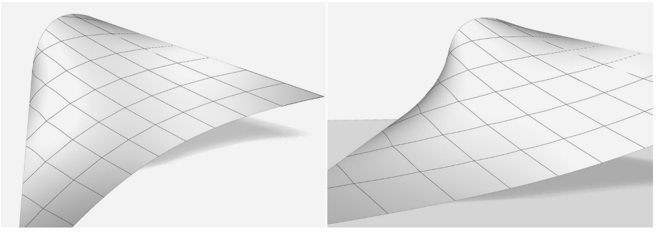


Fig. 3 - A surface covered with a regular pattern by the algorithm based on the “map to surface” *Grasshopper* tool and the source surface manipulation, for intuitive calibration of a pattern distribution.

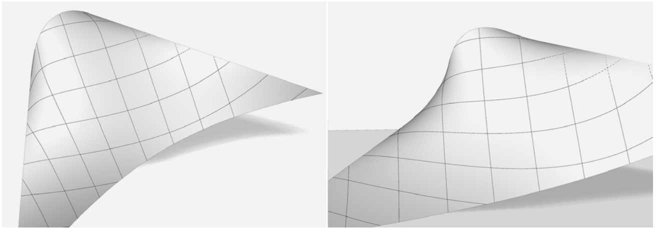


Fig. 4 - A Surface covered with a regular pattern by the algorithm based on two perpendicular lines projected onto the surface and distributed with constant distances between the lines.

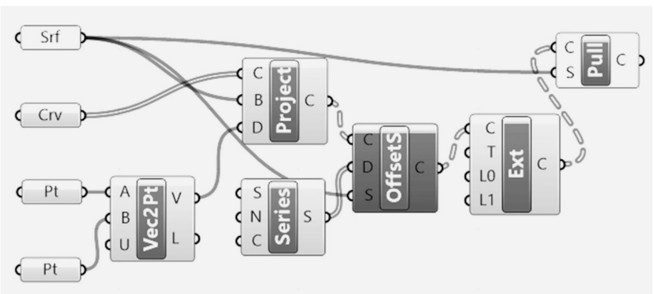


Fig. 5 - An example of the algorithm based on two perpendicular lines projected onto the surface and distributed with constant distances between the lines.

tool and the source surface manipulation for intuitive calibration of pattern distributions.

The students unroll a symmetrical surface that represents a module of a dome with the use of *Rhinoceros* and create the rectangle that circumscribes the unrolled surface. Finally, they create a pattern inside the rectangle. The students’ task is to map a dome with their own design pattern with the use of the *Grasshopper* plugin. Students create algorithms which transform the data of a pattern to fit curved target surfaces of the dome. The source-surface is tetragonal, but the target surface is triangular, and this reduces an edge of the source-surface to a point during mapping. As a result, the pattern is squeezed close to that point. To prevent such deformations, students create a tetragonal surface with two points at the same position and replace the original source-surface with the new one. By changing input parameters, students manipulate the final output of the algorithm, that indicates the flexibility of the algorithmic-parametric design and the pattern distribution (Figs. 6 and 7).

CONCLUSIONS

Design of repetitive patterns on architectural surfaces is a complex process that depends on specific conditions and factors. The role of the designer is to receive an order that determinates the final expression of the pattern

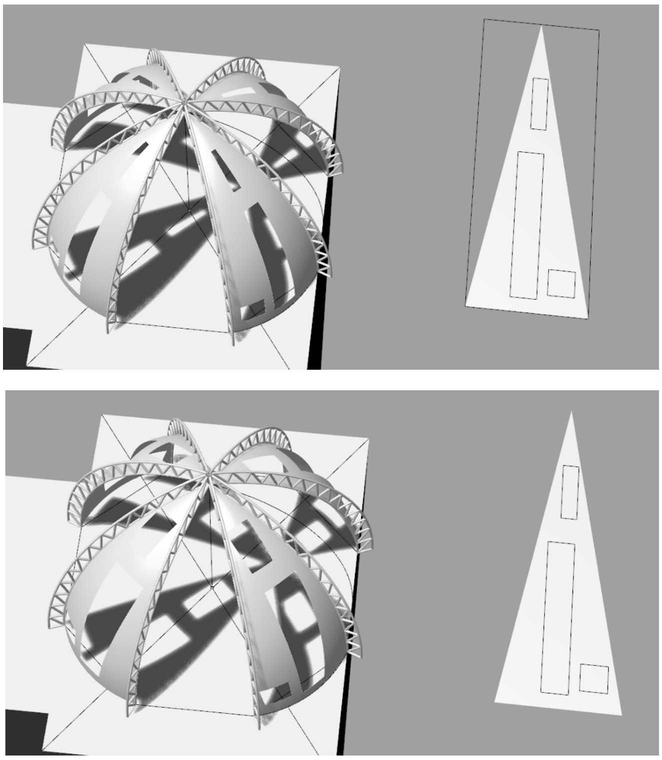


Fig. 6 - Examples from didactics: mapping of a rectangular surface (left) mapping of a transformed surface as target surface - two points of the tetragon in one place resulting in a triangle (right).

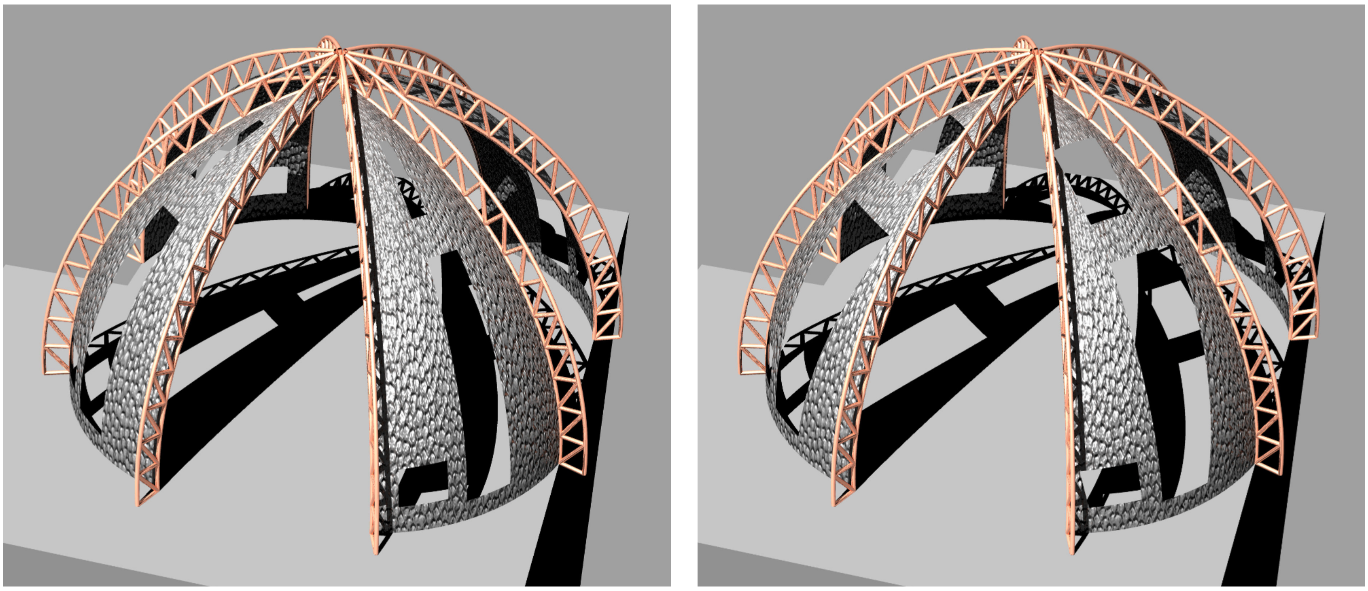


Fig. 7 - A textured rendering of examples from didactics: mapping of a rectangular surface (left) mapping of a transformed surface (right).

distribution. As Milena Stavric and Slavic Jablan writes:

> “*Through the history of architecture the role and denotation of ornament was shaped by cultural, intellectual and technical development.*” [08].

An algorithmic approach to this topic combines geometry, aesthetics, flexibility and *IT* techniques that are very useful in this type of design. The examples present that algorithms helps in the clear definition and realization of the designer’s intentions that lead to accurate and expected results.

Szymon Filipowski
and Michał Nessel

REFERENCES:

[01] Rubinowicz, P. (2000). Chaos and Geometric Order in Architecture and Design. *Journal for Geometry and Graphics*, Volume 4, No. 2.

[02] Filipowski, S. (2015). Natural And Artificial Shapes, And Their Interpretation. *Technical Transactions Architecture 3-A*, Cracow, Poland.

[03] Popper, K. R. (1996). *Open Universe. An Argument for Indeterminism*. Znak, Cracow, Poland.

[04] Majewski, M. (2012). *Geometric Ornament in Art and Architecture of Western Cultures*. NYIT, UAE.

[05] Abdullahi, Y., Rashid Bin Embi, M. (2013). Evolution of Islamic geometric patterns. *Frontiers of Architectural Research*, Volume 2, Issue 2, Elsevier.

[06] Harris, A. (2000). *The Mathematics of Tessellation*. St martin’s College.

[07] Rippmann, M. (2016). *Funicular Shell Design Geometric approaches to form finding and fabrication of discrete funicular structures*. A thesis submitted to attain the degree of Doctor of Sciences of ETH Zurich (Dr. sc. ETH Zurich)

[08] Stavric, M., Jablan, S. (2011). Advanced Geometry of Modular Tiles. *14th Generative Art Conference GA2011*

All images are from the authors.

LACE CURVES

Daniela Velichová [1]

ABSTRACT

Two families of lace curves generated as partial Minkowski sum or product of pairs of curves in Euclidean 3D-space are presented in this paper. Some of their basic geometric properties are derived from the properties of Minkowski operation basic components. Illustrations of various interesting shapes resulting from the combinations of simple curve segments are included.

KEYWORDS: Minkowski point set operations, modelling curves, laces.

INTRODUCTION

Minkowski sum and product of point sets are point-wise operations based on the definitions of sum and product of 2 points in arbitrary geometric space. Minkowski sum defined as the sum of points' position vectors with respect to a fixed reference point is used for modelling offsets and calculating the trajectory of a robot motion working in a space with obstacles. Minkowski product can be utilised for generating shapes as geometric figures in various spaces, e.g. in complex plane when defined as product of complex numbers representing points in plane (see [01], [02]) or in four-dimensional geometric space through definition by means of quaternions, as in [03]. The definition by means of outer (wedge) product of position vectors of points presented in this paper provides a powerful algorithm for modelling geometric objects, mainly curves or surfaces in geometric space of arbitrary dimension, as introduced from [04] to [05]. Minkowski sum or product of two differentiable manifolds, smooth curves or surfaces in the space E_n, is again a differentiable manifold, whose properties are predictable, as these can be determined by means of representations of the two components of the respective Minkowski operation. Partial Minkowski operations are used for generation of two different families of curves (laces) unexpectedly rich in forms and shapes, whose differential properties are inherited, partially, from the original two curves used as operands in the two point set operations, and determined by the properties of generating operations. We willl begin presenting definitions of basic concepts, such as operations of Minkowski sum and product of point sets, partial Minkowski operations and Minkowski point set combinations.

BASIC CONCEPTS

Let the fixed reference point be located at the origin of Cartesian coordinate system in E_n, and each point in the space $a = [a_1, a_2, ..., a_n]$ be attached position vector $\mathbf{a} = (a_1, a_2, ..., a_n)$. Sum of points a and $b = [b_1, b_2, ..., b_n]$ in E_n is point $c \in E_n$ represented by the position vector

$$\mathbf{c} = (a_1 + b_1, a_2 + b_2, ..., a_n + b_n).$$

Minkowski sum of two point sets $A, B \subset E_n$ is a point set $S \subset E_n$ defined as sum of all points from set A with all points from set B

$$S = A \oplus B = \{a + b, a \in A, b \in B\}.$$

---

[1] Slovak University of Technology in Bratislava, Slovakia. daniela.velichova@stuba.sk

Product of two points $a, b \in E_n$ is point

$$c \in E_d,\ d = n\,(n - 1)/2$$

represented by position vector $\mathbf{c} = \mathbf{a}\wedge\mathbf{b}$, which is the wedge product of vector **a** and **b** defined as

$$c = \left(\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}, \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}, \ldots, \begin{vmatrix} a_1 & a_n \\ b_1 & b_n \end{vmatrix}, \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}, \ldots, \begin{vmatrix} a_{n-1} & a_n \\ b_{n-1} & b_n \end{vmatrix}\right)$$

Minkowski product of two point sets $A, B \subset E_n$ is a point set $P \subset E_d$ defined as product of all points from set A with all points from set B

$$P = A \otimes B = \{a \wedge b,\ a \in A,\ b \in B\}.$$

In the case of $n = 3$ then, $d = 3$ and the wedge product of two vectors can be regarded as equivalent to the cross product of the two vectors. The resulting set generated as Minkowski product of two subsets in E_3 is therefore set P in the same space E_3.

Scalar multiple of a point set A and non-zero real number $k \neq 0$, is set A_k defined as follows

$$A_k = k.A = \{k.a,\ a \in A,\ k \in R\},$$

where point $k.a$ is represented by position vector

$$k.\mathbf{a} = k\,(a_1, a_2, \ldots, a_n) = (ka_1, ka_2, \ldots, ka_n).$$

Based on the above operations, we can define Minkowski linear combination of point sets A, B in E_n as Minkowski sum of different scalar multiples of point sets A and B

$$L(k, l) = k.A \oplus l.B = A_k \oplus B_l =$$
$$= \{k.a + l.b,\ a \in A,\ b \in B,\ k, l \in R\}.$$

Similarly, multiplicative combination of point sets A and B can be defined by means of the product operation, while due to properties of wedge product,

$$k.a \wedge l.b = k.l.\,(a \wedge b),$$

coefficients k and l influence just the size of the resulting set, not its shape and, therefore,

$$LP(k, l) = k.A \otimes l.B = A_k \otimes B_l =$$
$$= \{k.a \wedge l.b,\ a \in A,\ b \in B,\ k, l \in R\} = k.l.(A \otimes B).$$

Minkowski triple of point sets A, B and C can be defined as combination

$$T(k, l, m) = (k.A \oplus l.\,B) \otimes m.C =$$
$$= \{(k.a + l.b) \wedge m.c,\ a \in A,\ b \in B,\ c \in C,\ k, l, m \in R\}.$$

Considering two smooth curve segments in space E_n with vector maps

$$A{:} = {}^1\mathbf{r}(u),\ u \in I \subset R,\ B{:} = {}^2\mathbf{r}(v),\ v \in K \subset R,$$

their Minkowski sum and product are surface patches with different properties in spaces of different dimensions. The partial Minkowski operations can be performed for equally parameterised curve segments by the same parameter

$$u \in I \subset R,\ A{:} = {}^1\mathbf{r}(u),\ B{:} = {}^2\mathbf{r}(u),$$

resulting in curve segments positioned on the respective surface patches generated by Minkowski operations of these curves, if differently parameterised.

LACES

Minkowski linear combinations $A_k \oplus B_l$ of two equally parameterised curves in E_n is curve S in the same space E_n determined by vector function

$$\mathbf{s}(u) = k.{}^1\mathbf{r}(u) + l.{}^2\mathbf{r}(u),\ u \in I \subset R,$$

where $k, l \in R$, $k^2 + l^2 \neq 0$.

Thus, a two-parametric family of curves denoted as summative laces can be determined, with two shaping parameters k and l enabling swift shape change and modelling, in addition to shaping by super-positioning of the two respective curves in the space. Examples of laces generated as Minkowski linear combinations of circle and shamrock curve located in perpendicular planes in E_3 are presented in Fig. 1.

Minkowski multiplicative combination $A_k \otimes B_l$ of two equally parameterised curve segments is curve segment P in space E^d, $d = n(n - 1)/2$, defined by vector map

$$\mathbf{p}(u) = k.{}^1\mathbf{r}(u) \wedge l.{}^2\mathbf{r}(u),$$
$$u \in I \subset R \text{ and } k, l \in R,\ k^2 + l^2 \neq 0,$$

while coefficients k and l are scaling factors of the resulting curve segments forming family of multiplicative laces. Different forms of Minkowski product of the same pair of curves as in Fig. 1 are presented in Fig. 2.

Singular points of summative laces of degree 1 can be determined as points represented by position vectors

$$\mathbf{s}(u_0) = k.{}^1\mathbf{r}(u_0) + l.{}^2\mathbf{r}(u_0),$$
$$\text{while } u_0 \in I \subset R \text{ and } k, l \in R,\ k^2 + l^2 \neq 0,$$

which are solutions of vector equation satisfied by such points on the resulting curve segment, at which the tangent vector is a zero vector

$$\mathbf{s}'(u) = k.{}^1\mathbf{r}'(u) + l.{}^2\mathbf{r}'(u) = 0,$$

where ${}^1\mathbf{r}'(u)$ and ${}^2\mathbf{r}'(u)$ are derivatives of the vector functions representing the original curve segments determining their tangent vectors at the points with curvilinear coordinate u. Solutions can be as follows:

1. ${}^1\mathbf{r}'(u_0) = 0$ and ${}^2\mathbf{r}'(u_0) = 0$,

where ${}^1\mathbf{r}(u_0)$ and ${}^2\mathbf{r}\,(u_0)$ are singular points on curves A and B

2. ${}^1\mathbf{r}'(u_0) = -\,l.\,{}^2\mathbf{r}'(u_0)/k$,

where ${}^1\mathbf{r}(u_0)$ and ${}^2\mathbf{r}(u_0)$ are such points on curves A and B at which tangent vectors are collinear.

Singular points of degree n, $n \geq 2$ can be similarly defined by vanishing respective higher order derivatives at the corresponding points on the two original curve segments.

Double (or similarly multiple) points on summative laces are points whose position vectors satisfy equation $\mathbf{s}(u_1) = \mathbf{s}(u_2)$, while $u_1, u_2 \in I \subset R$ are solutions of an equation (or system of equations)

$$k.{}^1\mathbf{r}(u_1) + l.{}^2\mathbf{r}(u_1) = k.{}^1\mathbf{r}(u_2) + l.{}^2\mathbf{r}(u_2),$$
$$k.\,({}^1\mathbf{r}(u_1) - {}^1\mathbf{r}(u_2)) = l.\,({}^2\mathbf{r}(u_2) - {}^2\mathbf{r}(u_1)),$$

giving thus condition for collinearity of points' position vectors ${}^1\mathbf{r}(u_1)$ and ${}^2\mathbf{r}(u_1)$.

Illustration is in Fig. 3, where particular Minkowski linear combination of shamrock curve and leaf of Descartes is presented for selected values of shaping parameters *k* and *l* with coinciding position vectors of double point on resulting summative lace curve for different curvilinear coordinates *t*.

DIFFERENTIAL PROPERTIES OF LACES

Intrinsic differential properties of both families of lace curves can be represented by means of derivatives of vector maps defining original curve segments, which appear as operands in the respective Minkowski combinations. Let two piecewise smooth curve segments *A* and *B* be given in the space E_3 determined by their vector parameterisations,

$$A: = {}^1\mathbf{r}(u),\ B: = {}^2\mathbf{r}(u),\ u \in I \subset R.$$

Partial Minkowski sum of the two curve segments is a smooth curve segment *S* given in E_3 determined parametrically as

$$S: = \mathbf{p}(u) = {}^1\mathbf{r}(u) + {}^2\mathbf{r}(u),\ u \in I \subset R.$$

Considering the existence of the continuous non-zero differentiable derivatives of both curve vector representations up to order 3 denoted as

$${}^1\mathbf{r}'(u),\ {}^1\mathbf{r}''(u),\ {}^1\mathbf{r}'''(u),\ {}^2\mathbf{r}'(u),\ {}^2\mathbf{r}''(u),\ {}^2\mathbf{r}'''(u),\ u \in I \subset R$$

one can derive formulas for the first and second curvatures, ${}^1\kappa$ and 1τ, for the summative lace curve *S* segments in the form

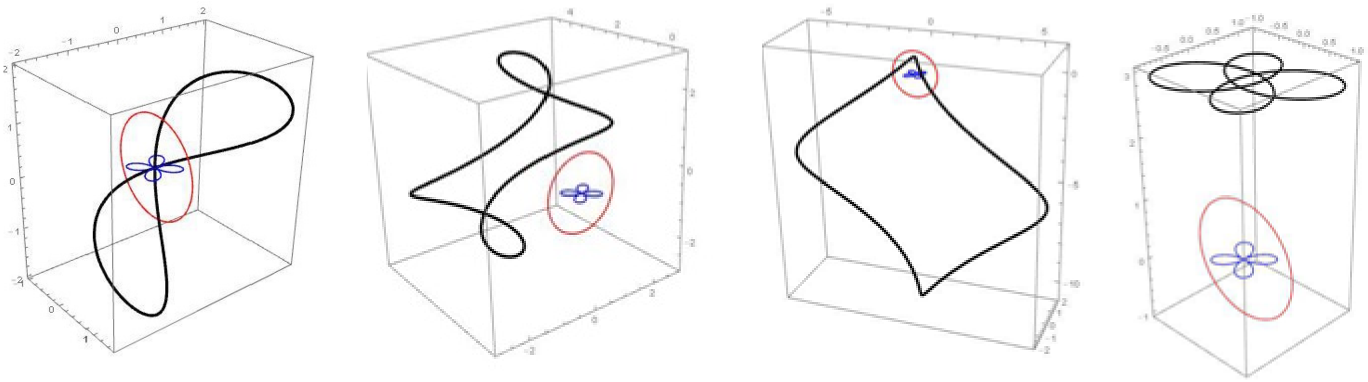


Fig. 1 - Forms of laces generated as Minkowski linear combinations.

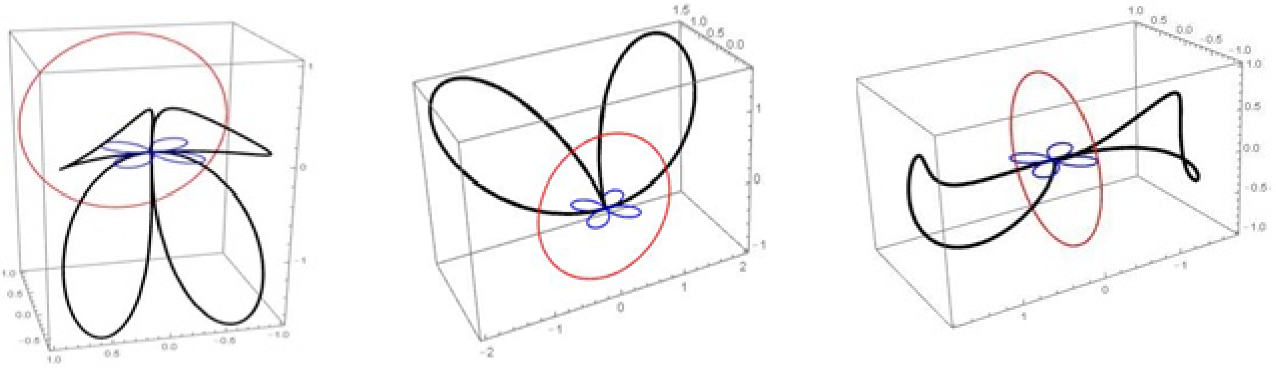


Fig. 2 - Forms of laces generated as Minkowski multiplicative combinations.

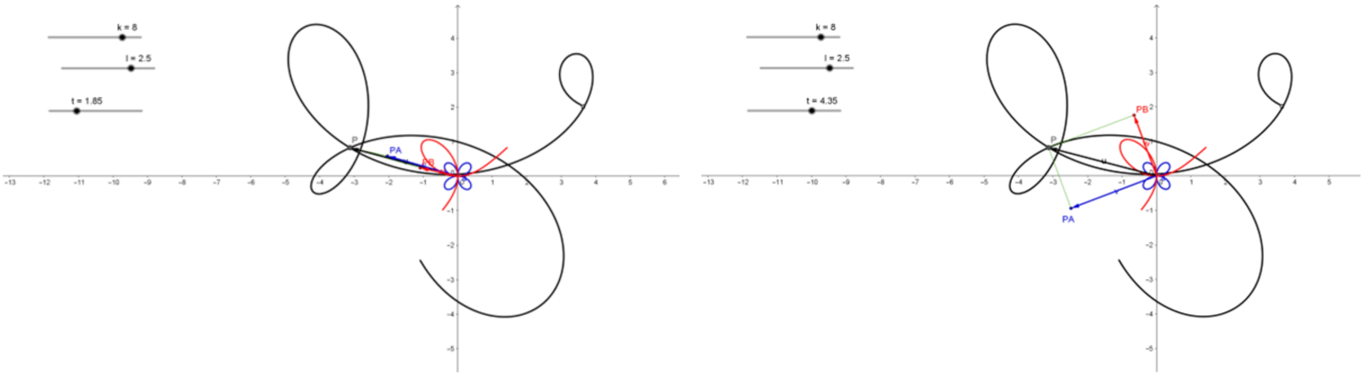


Fig. 3 - Singular points on a summative lace curve singular points on multiplicative laces can be traced in a similar way.

$$ {}^{1}\kappa = \frac{\left|\mathbf{s}'(u)\times\mathbf{s}''(u)\right|}{\left|\mathbf{s}'(u)\right|^{3}} = \frac{\left|\left({}^{1}\mathbf{r}'(u)+{}^{2}\mathbf{r}'(u)\right)\times\left({}^{1}\mathbf{r}''(u)+{}^{2}\mathbf{r}''(u)\right)\right|}{\left|{}^{1}\mathbf{r}'(u)+{}^{2}\mathbf{r}'(u))\right|^{3}} $$

$$ {}^{1}\tau = \frac{\left[\mathbf{s}'(u),\mathbf{s}''(u),\mathbf{s}'''(u)\right]}{\left|\mathbf{s}'(u)\times\mathbf{s}''(u)\right|^{2}} = \frac{\left(\left({}^{1}\mathbf{r}'(u)+{}^{2}\mathbf{r}'(u)\right)\times\left({}^{1}\mathbf{r}''(u)+{}^{2}\mathbf{r}''(u)\right)\right)\cdot\left({}^{1}\mathbf{r}'''(u)+{}^{2}\mathbf{r}'''(u)\right)}{\left|\left({}^{1}\mathbf{r}'(u)+{}^{2}\mathbf{r}'(u)\right)\times\left({}^{1}\mathbf{r}''(u)+{}^{2}\mathbf{r}''(u)\right)\right|^{2}} $$

These differential characteristics of curve *S* could be also represented by means of flections and torsions of the original curves used as components of the generating Minkowski sum.

Partial Minkowski product $P = A \otimes B$ of two equally parameterised piecewise smooth curve segments *A* and *B* in E_3 determines a smooth curve segment *P* in E_3 that is defined by vector function

$$ P{:} = \mathbf{p}(u) = {}^{1}\mathbf{r}(u) \wedge {}^{2}\mathbf{r}(u),\ u \in I \subset R. $$

To derive formulas for its first and second curvature, the following relations should be considered

$$ \mathbf{p}'(u) = \left[{}^{1}\mathbf{r}(u)\times{}^{2}\mathbf{r}(u)\right]' = {}^{1}\mathbf{r}'(u)\times{}^{2}\mathbf{r}(u)+{}^{1}\mathbf{r}(u)\times{}^{2}\mathbf{r}'(u) $$

$$ \begin{aligned} \mathbf{p}''(u) &= \left[{}^{1}\mathbf{r}(u)\times{}^{2}\mathbf{r}(u)\right]'' = \left[{}^{1}\mathbf{r}'(u)\times{}^{2}\mathbf{r}(u)+{}^{1}\mathbf{r}(u)\times{}^{2}\mathbf{r}'(u)\right]' = \\ &= {}^{1}\mathbf{r}''(u)\times{}^{2}\mathbf{r}(u)+2\,{}^{1}\mathbf{r}'(u)\times{}^{2}\mathbf{r}'(u)+{}^{1}\mathbf{r}(u)\times{}^{2}\mathbf{r}''(u) \end{aligned} $$

$$ \begin{aligned} \mathbf{p}'''(u) &= \left[{}^{1}\mathbf{r}''(u)\times{}^{2}\mathbf{r}(u)+2\,{}^{1}\mathbf{r}'(u)\times{}^{2}\mathbf{r}'(u)+{}^{1}\mathbf{r}(u)\times{}^{2}\mathbf{r}''(u)\right]' = \\ &= {}^{1}\mathbf{r}'''(u)\times{}^{2}\mathbf{r}(u)+3\,{}^{1}\mathbf{r}''(u)\times{}^{2}\mathbf{r}'(u)+3\,{}^{1}\mathbf{r}'(u)\times{}^{2}\mathbf{r}''(u)+{}^{1}\mathbf{r}(u)\times{}^{2}\mathbf{r}'''(u) \end{aligned} $$

Then, formulas for flection ${}^{2}\kappa$ and torsion ${}^{2}\tau$ are as follows

$$ \begin{aligned} {}^{1}\kappa &= \frac{\left|\mathbf{p}'(u)\times\mathbf{p}''(u)\right|}{\left|\mathbf{p}'(u)\right|^{3}} = \\ &= \frac{\left|\left({}^{1}\mathbf{r}'(u)\times{}^{2}\mathbf{r}(u)+{}^{1}\mathbf{r}(u)\times{}^{2}\mathbf{r}'(u)\right)\times\left({}^{1}\mathbf{r}''(u)\times{}^{2}\mathbf{r}(u)+2\,{}^{1}\mathbf{r}'(u)\times{}^{2}\mathbf{r}'(u)+{}^{1}\mathbf{r}(u)\times{}^{2}\mathbf{r}''(u)\right)\right|}{\left|{}^{1}\mathbf{r}'(u)\times{}^{2}\mathbf{r}(u)+{}^{1}\mathbf{r}(u)\times{}^{2}\mathbf{r}'(u)\right|^{3}} \end{aligned} $$

$$ \begin{aligned} {}^{1}\tau &= \frac{\left[\mathbf{p}'(u),\mathbf{p}''(u),\mathbf{p}'''(u)\right]}{\left|\mathbf{p}'(u)\times\mathbf{p}''(u)\right|^{2}} = \\ &= \left(\left({}^{1}\mathbf{r}'(u)\times{}^{2}\mathbf{r}(u)+{}^{1}\mathbf{r}(u)\times{}^{2}\mathbf{r}'(u)\right)\times\left({}^{1}\mathbf{r}''(u)\times{}^{2}\mathbf{r}(u)+2\,{}^{1}\mathbf{r}'(u)\times{}^{2}\mathbf{r}'(u)+{}^{1}\mathbf{r}(u)\times{}^{2}\mathbf{r}''(u)\right)\right) \\ &\cdot\left({}^{1}\mathbf{r}'''(u)\times{}^{2}\mathbf{r}(u)+3\,{}^{1}\mathbf{r}''(u)\times{}^{2}\mathbf{r}'(u)+3\,{}^{1}\mathbf{r}'(u)\times{}^{2}\mathbf{r}''(u)+{}^{1}\mathbf{r}(u)\times{}^{2}r'''(u)\right). \\ &.\left|\left({}^{1}\mathbf{r}'(u)\times{}^{2}\mathbf{r}(u)+{}^{1}\mathbf{r}(u)\times{}^{2}\mathbf{r}'(u)\right)\times\left({}^{1}\mathbf{r}''(u)\times{}^{2}\mathbf{r}(u)+2\,{}^{1}\mathbf{r}'(u)\times{}^{2}\mathbf{r}'(u)+{}^{1}\mathbf{r}(u)\times{}^{2}\mathbf{r}''(u)\right)\right|^{-3} \end{aligned} $$

CONCLUSIONS

Presented abstract algebraic operations of Minkowski sum and Minkowski product of two point sets, namely their partial forms, can be used as modelling tools of families of curves, summative or multiplicative laces, displaying a great potential of shape variation. Two shaping parameters enable easy manipulation and modification of the resulting curve segment form. Many applications of these algorithms can be seen in computer graphics, computer modelling, and in design and art. Minkowski combinations of curve segments parameterized by different parameters lead to a powerful tool for modelling surfaces with possible smooth manipulation of their form by means of 2 parameters. Examples of rich potential in forms and shapes can be seen in Fig. 4, where various possible modifications of surface patches generated as Minkowski triples of three circles are presented. Modelling of solids determined as Minkowski triples of curve segments can be regarded as well. Practical applications of the described method can be seen in modelling trajectories of robot motion in a space with obstacles, in finding offsets of given objects, in the modification of shapes satisfying certain conditions or in generating point mosaics, but also in graphical coding and data encryption, in various possible physical interpretations, and many others.

Daniela Velichová

ACKNOWLEDGMENTS

The author gratefully acknowledges the Scientific Grant Agency APVV of the Slovak Republic for supporting this work under the Grant No. APVV-0161-12.

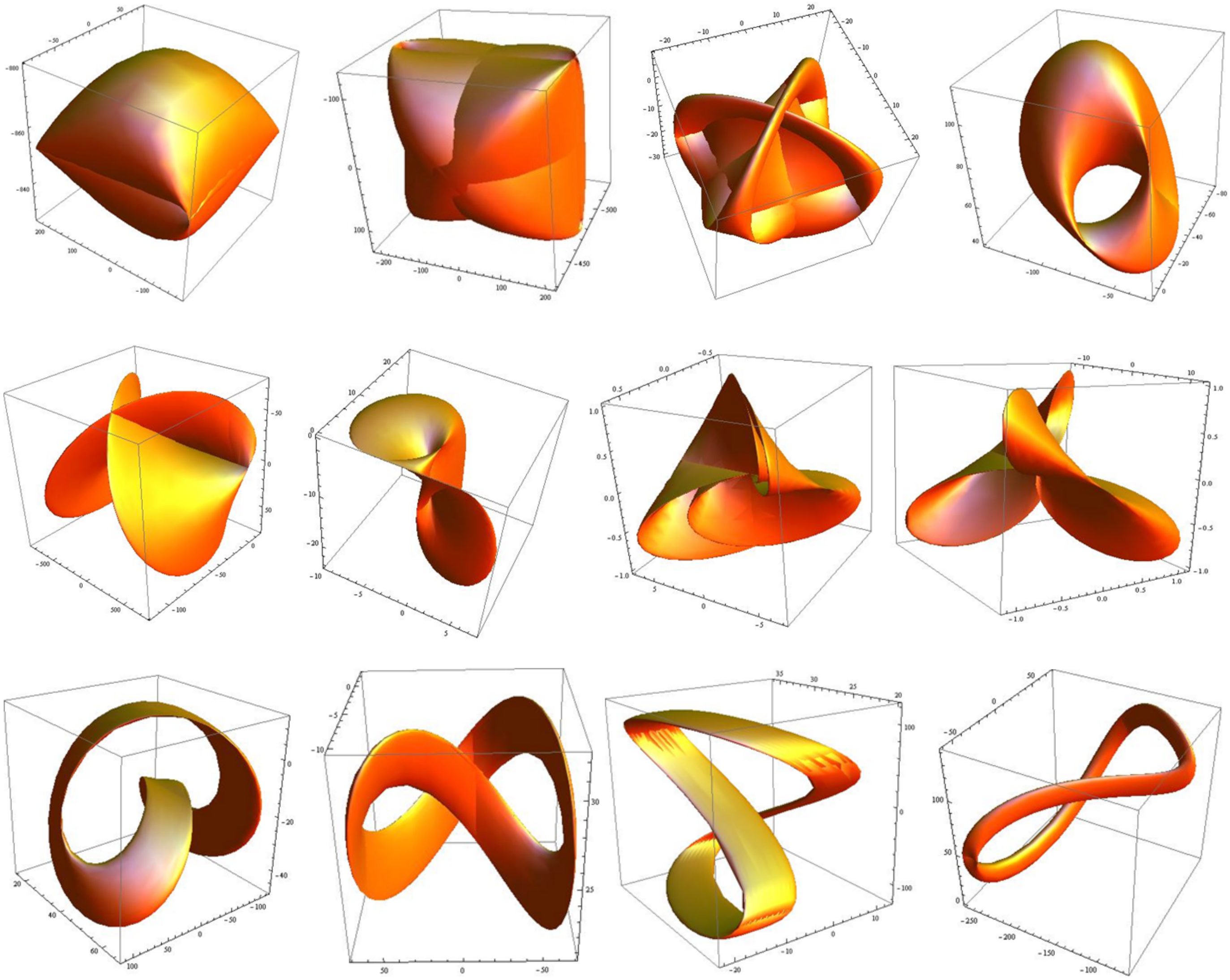


Fig. 4 - Forms of Minkowski triples of circles.

REFERENCES:

[01] Kaul, A., & Farouki, R. T. (1995). Computing Minkowski Sums of Planar Curves. In *International Journal of Computational Geometry and Applications*, 5, 413-432.

[02] Lee, I. K., & Kim, M. S., & Elberg, G. (1998). The Minkowski Sum of 2D Curved Objects. *Proceedings of Israel-Korea Bi-National Conference on New Themes in Computerized Geometrical Modelling,* Tel-Aviv Univ. 155-164.

[03] Weiner, I., & Gu, W. (2002). Minkowski Geometric Algebra of Quaternion Sets. In *Journal of Pure and Applied Mathematics*, 3 (4), 385-411.

[04] Velichová, D. (2013). The Art of Geometry. Proceedings 2013, 16th Conference Bridges, Enschede. Tessalations, Phoenix, Arizona, USA. 143-150.

[05] Velichová, D. (2015). Minkowski operations in shape modeling. In *Mathematics and Art III*. Cassini, France.

PRODUCING DESIGN OBJECTS FROM REGULAR POLYHEDRA: A PRACTICAL APPROACH

Beniamino Polimeni [1]

ABSTRACT

In the last few years, digital modelling techniques have played a major role in architecture and design, influencing, at the same time, the creative process and the fabrication of objects. This revolution has produced a new productive generation of architects and designers focused on the expanding possibilities of material and formal production, reinforcing the idea of architecture as an interaction between art and artisanship. This original perspective inspires this paper, which illustrates the contemporary scenario and provides some practical guidance about tools and technologies the designers most often use for creating geometric sculptures with 3D printing. Creative possibilities of topological mesh modelling are used to generate complex geometries from regular polyhedra. This process explores how combining different geometric operations can activate architectural inquiry and generate fascinating shapes with creative flexibility.

KEYWORDS: regular polyhedra, mesh modelling, topmod, 3D, design procedures

INTRODUCTION

From design practice to academic studies, the definition of space through surface manipulation has produced various design ideas and has been used to design an extensive selection of industrial products. The possibility of creating complex shapes and geometries with these new methods has fostered many interactions between designers, architects, mathematicians, and artists. This innovative interdisciplinary collaboration has given birth to a series of fascinating experiments in which geometry is explored to model and fabricate an extraordinary variety of artworks and design creations that turn topological and geometric abstractions into real objects. Artists such as George Hart, David and Harriet Brisson [01], Helaman Ferguson and Bathsheba Grossman [02], have combined art and geometry to create sculptures and design objects otherwise impossible to create using traditional techniques (Figs.1 and 2). A group of contemporary scholars and academics, including Andrew Kudless, Marjan Colletti, and Michael Hansmeyer have been conducting investigations into

Fig. 1 - Andrew Kudless, Spore Lamp [03].

Fig. 2 - Bathsheba Grossman, "Quin" Lamp [04].

---

[1] De Montfort University, Leicester School of Architecture, UK. beniamino.polimeni@dmu.ac.uk

geometries, exploring the use of algorithms and computation to generate original forms inspired by nature. The work of these artists inspired this piece of research, which illustrates a practical method to design and fabricate a wide variety of complex geometric shapes starting from regular convex polyhedra.

PREVIOUS EXPERIENCES

For thousands of years, there has been a keen interest in geometric plane shapes and solid forms, especially for those possessing regular features of proportion and symmetry [05]. Among the solid forms that have attracted the curiosity of scientists and artists, regular polyhedra have served as art motifs since ancient times [06].

The regular convex polyhedra, or Platonic Solids, discovered by the Pythagoreans and described by Plato in the *Timaeus*, consist of regular, convex polyhedra defined by the same number of congruent regular polygonal faces meeting at each vertex (Table 1). The faces of this group of solids are congruent equilateral triangles (tetrahedron, octahedron, and icosahedron), congruent squares (cube), or congruent regular pentagons (dodecahedron).

Table1 - Number of faces (F), vertices (V), and edges (E) associated with the Platonic solids

	Tetra-hedron	Cube	Octa-hedron	Dodeca-hedron	Icosa-hedron
F	4	6	8	12	20
V	4	8	6	20	12
E	6	12	12	30	30

Plato associated each of the four classical elements (earth, air, water, and fire) with a regular solid. The Earth was associated with the cube, air with the octahedron, water with the icosahedron, and fire with the tetrahedron. Of the fifth Platonic solid, the dodecahedron, Plato remarks that "...God used it for arranging the constellations on the whole heaven" [07].

The correlation between form and scientific knowledge is the basis for all the graphic and visual explorations published between the 16th and the 17th century. Several books written during the *Renaissance* discuss perspective treatments of the regular solids and many other figures. Among these, one of the most significant is the treatise of Luca Pacioli, *De Divina Proportione* (1509) [09], which contains 60 plates of the Platonic solids and other peculiar polyhedra illustrated by Leonardo da Vinci. Leonardo's sketches are in all probability the first illustrations of skeletonic solids that allowed a clear distinction between front and back.

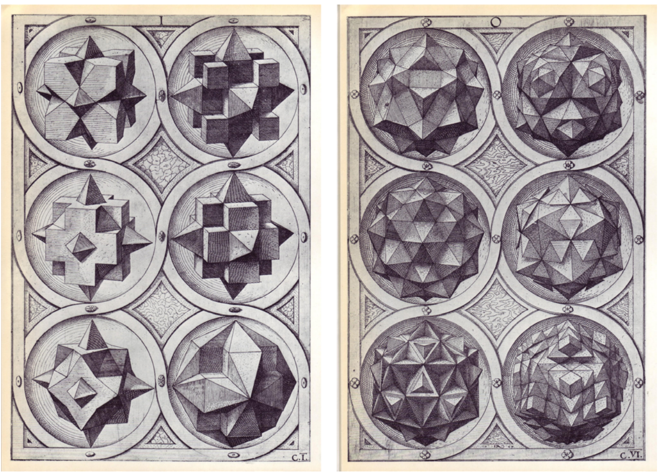


Fig. 3 - An example of W. Jamnitzer's Geometric Study (1568) [08].

Perspectiva Corporum Regularium ("Perspective of regular solids") is another masterpiece of geometric design [10]. Published in 1568 by the German goldsmith and printmaker Wenzel Jamnitzer, it is a study in shapes that contains 120 forms inspired by the Platonic solids. Jamnitzer rotates, cuts, and combines each of the solids to demonstrate how they might function as the building blocks of the world. The resulting three-dimensional objects, illustrated by detailed engravings, are often so varied it can be hard to distinguish from which regular solid they have originated (Fig.3).

As examples of geometric transformations of basic shapes, these experiments became a source of inspiration for architects and designers who studied the possibilities to combine various primitive forms to create systems in which a few standardised components are used to build a structure on a scale larger than the single elements [11].

A remarkable application of this approach is the work of Zvi Hecker and his "Polyhedric Architecture", a definition that has been coined to describe buildings in which polyhedral geometry, incorporating a combination of different polyhedra, is a primary aspect of the design. The *Ramot Housing Complex* (Fig. 4) is a model of a spatial configuration in which the use of interlocking arrays of cubes and dodecahedra create an arrangement of apartment units distributed according to the geometry of the existing rocky terrain [12].

Several well-known artists investigated the theme of Regular Convex Polyhedra, creating unique and fascinating works that explored and exhibited a broad range of mathematical ideas. In particular, Maurits Cornelis Escher demonstrated a keen interest in this theme, which was the primary subject of many of his illustrations and artworks [14]. In the famous engraving *Stars* (Fig. 5),

Fig. 4 - Zvi Hecker, *The Ramot Housing Complex* [13].

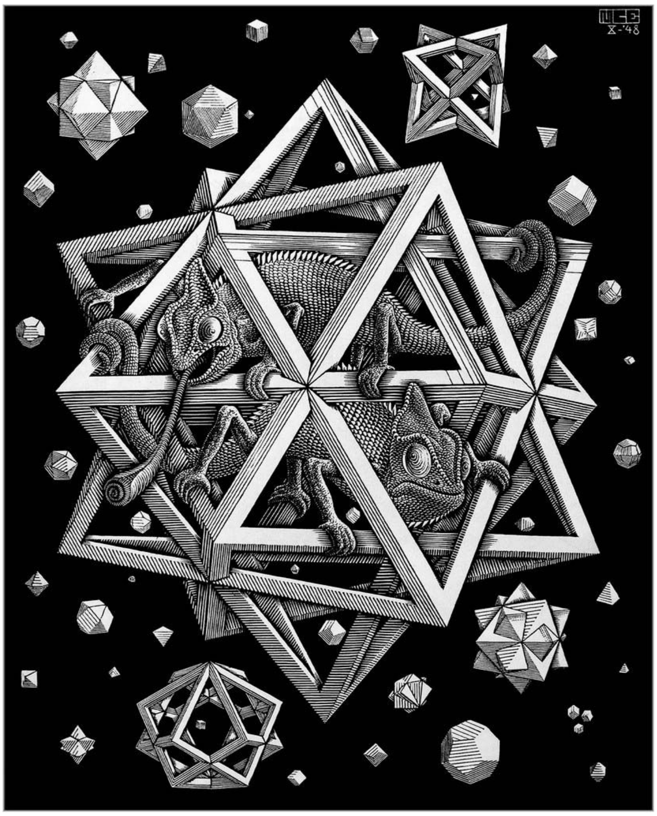


Fig. 5 - M. C. EScher, *Stars* [18].

a polyhedral form composed of three regular octahedra containing two chameleons floats in space. Several other polyhedra and polyhedral compounds float in the background defining a composition suggesting the artist's familiarity with the illustrations drawn by Leonardo da Vinci for Luca Pacioli's book, *De Divina Proportione* [15]. Up until recently, form-making has been limited to direct processes, wherein the designer of the structure manually applied geometrical transformations on different types of solids to achieve the desired result. Today's computational power allows us to write complex scripts and codes to generate shapes based on programmatic and operational goals rather than only simple aesthetic ideas. The *Platonic Solids Project*, accomplished by Michael Hansmeyer in 2008, explores how a purely operations-based geometric process can produce complex forms (Fig. 6). Instead of studying the possibilities for combining different solids, this project examined the potential inherent in a single primitive given an applicable process. It takes the most primitive forms, such as regular polyhedra, and repeatedly applies the same operation - the division of the faces of a solid into smaller faces - until a new form is produced [16].

Other contemporary experiments are based on topological mesh modelling to create high-genus shapes [17]. The three-dimensional objects generated are printed using metal printing technology to produce sculptures in bronze and stainless steel, jewellery, or housewares when using different materials. Some of the most common volumetric manipulations used in the previous examples will be described in the next chapter. Each operation will be included in a group of six images, each containing twenty 3D objects.

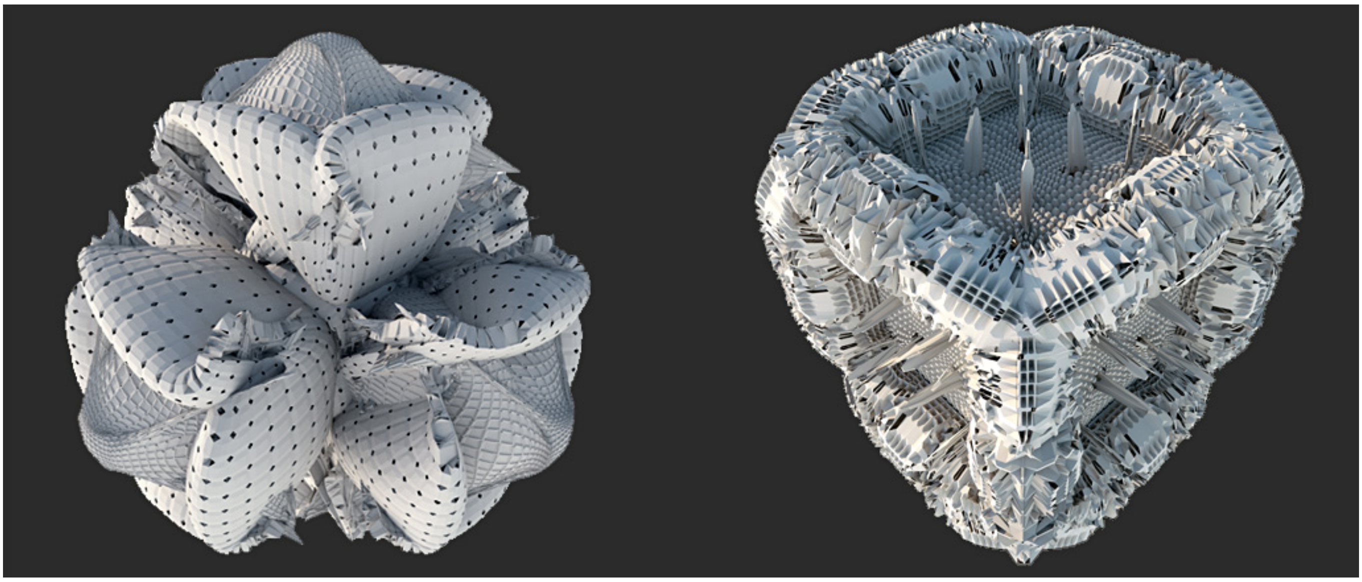


Fig. 6 - Michael Hansmeyer, Tetrahedron and Hexahedron [19].

RESEARCH

Using the above examples as a reference, we propose an investigation based on the exploration of a catalogue of geometric operations as a tool for spatial interpretation. Starting from regular convex polyhedra as base-volumes, we define a set of design procedures based on a collection of mesh modelling operators. These design processes consist of different topological and standard operations implemented with an existing 2-manifold mesh modelling system, called *TopMod*. *TopMod3D* is a freely available topological mesh modeller, developed by the *Department of Computer Science at Texas A&M University*, which also provides a significant modelling example that allows users to produce complex shapes efficiently [20]. In our study, this software is used to undertake design research into spatial, formal, and material expression. We incorporated the categories, extrusions (Figs. 7 and 8), remeshing schemes (Fig. 9), and a set of high-genus modelling operators (Fig. 10), to create two reproducible design guidelines that start from basic regular shapes and extend to design of sculptural objects perceived as belonging to the same family. After defining this catalogue of operators and applying each singularly on every regular solid analysed, some of them have been chosen and implemented with the aim of offering practical design guidance to produce sculptural objects for 3D printing.

The procedural steps of the first guideline are the following (Fig.11):

- Stage 1: Original Shapes: The process starts with the five regular convex polyhedra: the tetrahedron with four triangular faces, the cube with six square faces, the octahedron with eight triangular faces, the dodecahedron with twelve pentagonal faces, and the icosahedron with twenty triangular faces.
- Stage 2: Remeshing: This operation is applied to multiply the original faces of the solids to be connected using other geometric transformations.
- Stage 3: Creating Handles: This operation is implemented to connect symmetrically-related faces using multi-segment curved handles. Creating holes and handles may be used for both aesthetic and functional purposes.
- Stage 4: Final Remeshing: This final operation is essential to create a smooth surface. Although many schemes may be selected this operation, we use the Catmull-Clark subdivision algorithm with a recursive process.

In the second design procedure, we introduce extrusions and wire modelling operators (Fig. 12):

- Stage 1: Original Shapes: As done previously, the process starts with the five regular solids as basic shapes.
- Stage 2: Extrusions: Extrusions or stellations are applied to all the faces of the solid, adding twists and rotations, creating additional effects. Extrusions based on regular polyhedra may also be applied to the basic primitives. In our case, an icosahedral extrusion is used to increase the complexity of the basic shapes.
- Stage 3: Wire Frame Modelling: Wire modelling converts a wireframe mesh into a 3D model in which every edge is replaced by a 3D pipe of determinate thickness and volume. This operation may be used by varying the diameter of the pipe and its cross section [21].
- Stage 4: Final Remeshing: As previously discussed, this operation is used to create smooth surfaces. In our case, the Catmull-Clark scheme is used recursively.

These proposed guidelines are not absolute nor prescriptive and are meant to encourage creativity and diversity within a defined range of operations. Therefore, every single action of the process may be repeated, postponed, or anticipated by combining the suggested operations creatively, which introduces the idea of design as an active and fluid process. The result of this process is a set of complex shapes that would otherwise be difficult to produce with conventional 3D modelling application.

A group of six graphic elaborations is presented in rendered models to illustrate their spatial qualities (Figs. 7-12). The first group of images defines the set of possible operations performed in the procedure and the second group describes the process used in the design guidelines. Every form generated has been 3D printed to analyse the changes in geometry occurring to the objects throughout the procedure. The goal is to combine technical possibilities with an adequate working knowledge to bring new energy to the study of architectural forms.

CONCLUSIONS

Architecture and design have been traditionally conceived using additive and subtractive processes in which parts are combined and arranged to generate different configurations. With additive operations, individual components are joined to form a construction.

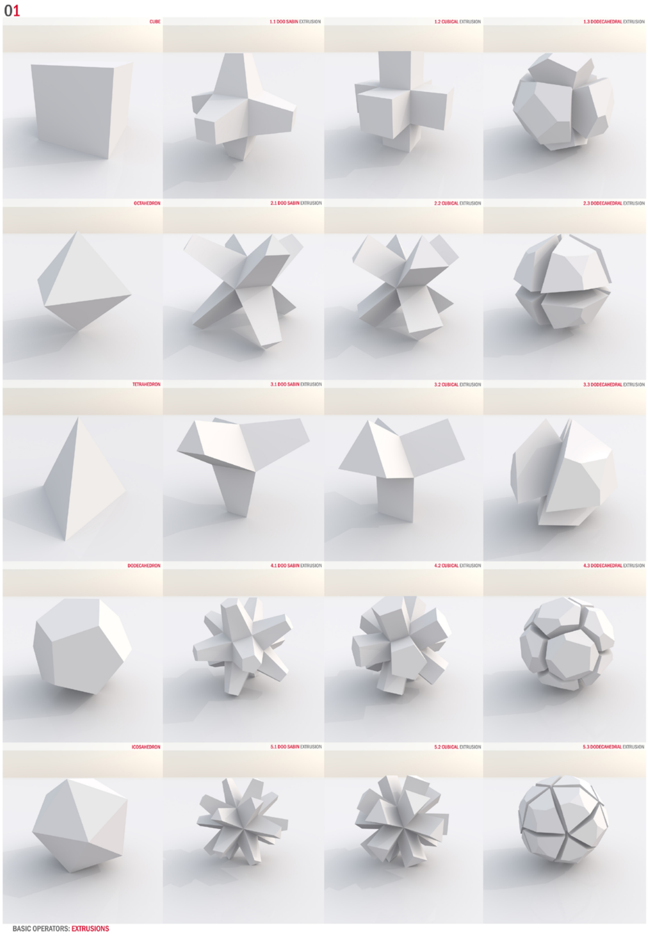


Fig. 7 - Different types of extrusions.

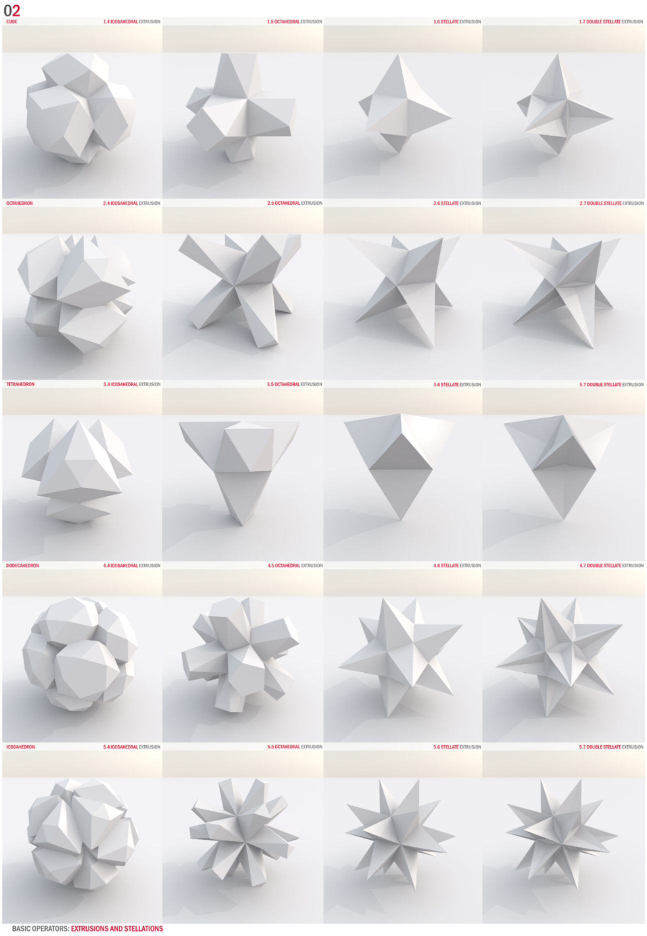


Fig. 8 - Different types of extrusions and stellations.

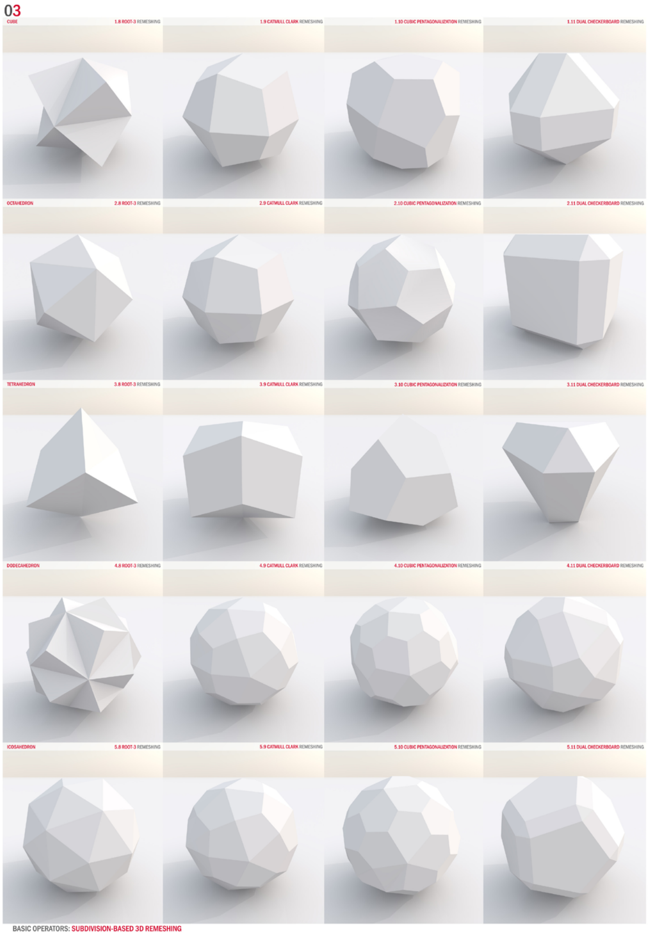


Fig. 9 - Subdivision and remeshing operators.

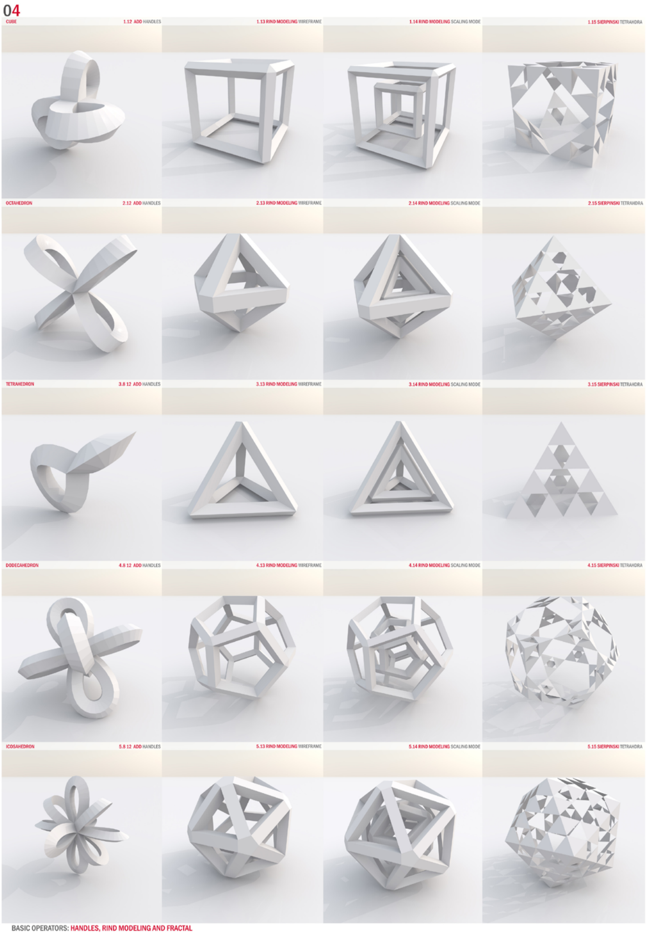


Fig. 10 - Handles, rind modelling, and fractal operators.

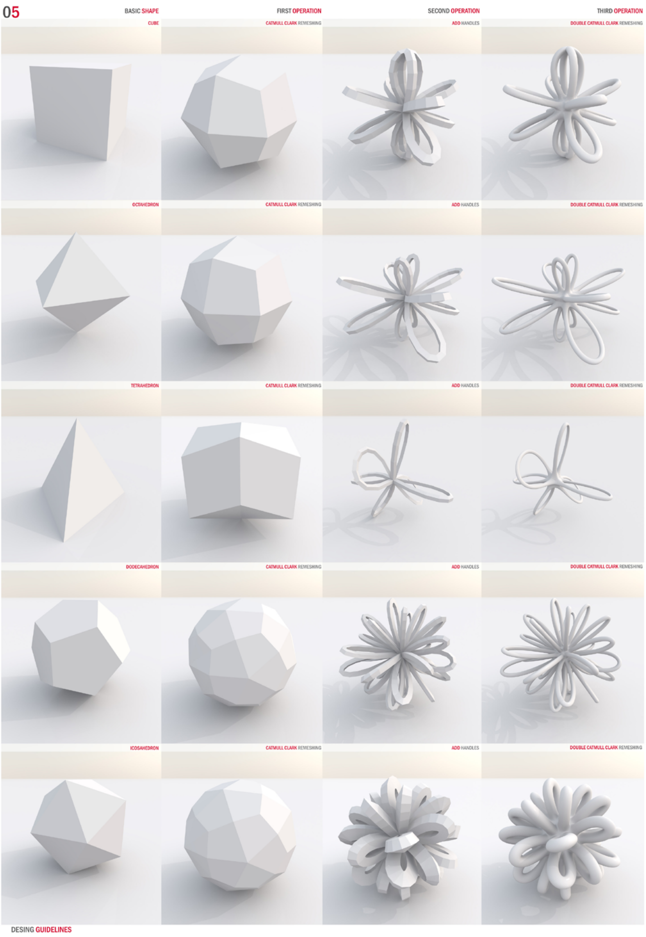


Fig. 11 - First design procedure.

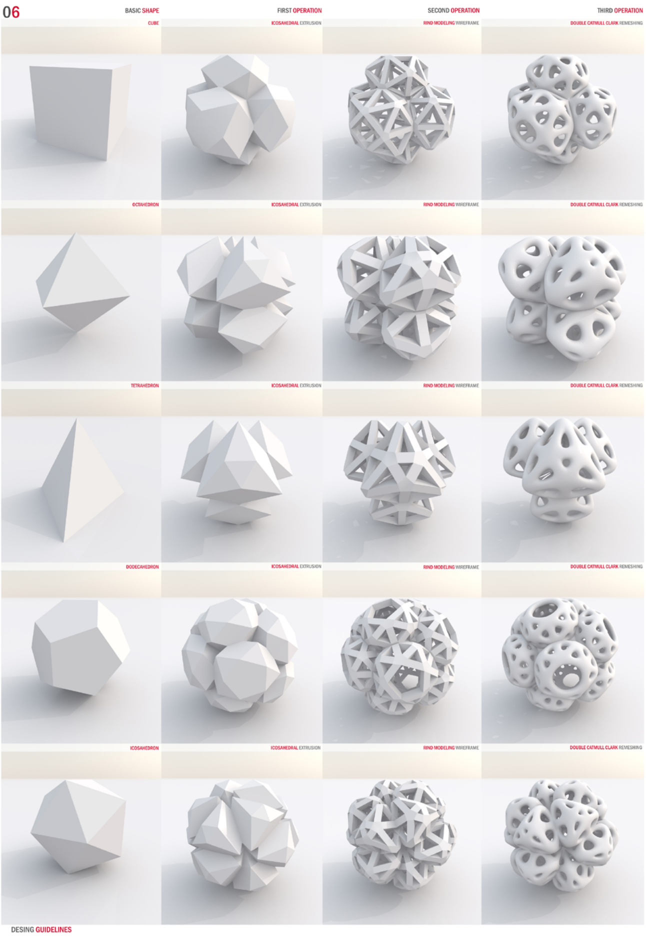


Fig. 12 - Second design procedure.

On the other hand, with subtractive operations, pieces are removed to arrive at the desired design. Alongside these core operations, other volumetric manipulations are applied to modify the primary and secondary shapes to explore different solutions. To define and rationalise these processes, catalogues of "design operations" and "spatial verbs" have been proposed by scholars and artists. The work of Dimary and Yoo [22], which starts from the acquisitions of Richard Serra [23], is particularly significant in this respect since it defines a lexicon of starting points for the creation of space.

In our research, we explored the spatial formation through a similar approach, applying a limited set of geometric transformations to each regular convex polyhedron. The final result is a collection of varied forms to be used for generating new spatial configurations through a straightforward and intuitive process. Our goal is, therefore, to encourage abstract spatial thinking and inspire architects, designers, and scholars to understand and create complex geometries by incorporating user-friendly and flexible tools. Finally, the operations presented in our study may also be used as an interpretive guide for reading and understanding works and space that already exist.

Beniamino Polimeni

REFERENCES:

[01] Brisson, H. (1992). Visualization in Art and Science. *Leonardo,* 25(3/4), 257-257.

[02] Akleman E., Ozener O., & Yuksel C. (2006). On a Family of Symmetric, Connected and High Genus Sculptures. In *Bridges London: Mathematics, Music, Art, Architecture, Culture* (pp. 145-150). London, UK: Southwestern College.

[03] Spore Lamp. [Online image]. Retrieved July 11, 2017, from http://archinect.com/people/cover/36571395/andrew-kudless

[04] Quin. [Online image]. Retrieved July 11, 20157, from https://www.bathsheba.com/gallery/mgx/quin/.

[05] Emmer, M. (1982). Art and Mathematics: The Platonic Solids. *Leonardo,* 15(4), 277-277.

[06] Calter, P. (1998). The Platonic Solids. Retrieved July 04, 2017, from https://www.dartmouth.edu/~matc/math5.geometry/unit6/unit6.html.

[07] Platonic solid. (2017, June 30). In Wikipedia. Retrieved July 04, 2017, from https://en.wikipedia.org/wiki/Platonic_solid.

[08] Perspectiva Corporum Regularium (1568) [Online image].Retrieved July 11, 2017, from http://www.graphicine.com/the-geometry-of-an--art-wenzel-jamnitzer/

[09] A copy of the Original text of Pacioli can be found at https://archive.org/stream/divinaproportion00paci#page/n0

[10] Swetz, F. (2013). Mathematical Treasure: Wenzel Jamnitzer's Platonic Solids. Retrieved May 29, 2015, from http://www.maa.org/publications/periodicals/convergence/mathematical-treasure-wenzel-jamnitzers-platonic-solids

[11] Taalman, L., & Hunsicker E., (2005). Simplicity is not Simple. *Tassellations and Modular Architecture.* Math Horizons, 10(1), 5-9.

[12] Hecker, Z. (1980). The Cube and the Dodecahedron in My Polyhedric Architecture. *Leonardo*, 13(4), 272.

[13] The Ramot Housing Complex [Online image]. Retrieved July 11, 2017, from http://architectuul.com/architecture/ramot-polin-apartments

[14] Smith, B. Sidney (2014). The Mathematical Art of M.C. Escher. Retrieved 13 Mar 2014 from Platonic Realms Minitexts: http://platonicrealms.com/minitexts/Mathematical-Art-Of-M-C-Escher/.

[15] Coxeter, H. S. M. (1985), A special book review: M. C. Escher: His life and complete graphic work. *The Mathematical Intelligencer* 7 (1), 59–69.

[16] Hansmeyer, M. (2008). Computational Architecture: Platonic Solids. Retrieved May 1, 2015, from http://www.michael-hansmeyer.com/projects/.

[17] Akleman E., Chen J., & Srinivasan V. (2001). An Interactive Shape Modeling System for Robust Design of Functional 3D Shapes. In *Proceedings* of ACADIA 2001 (pp. 248-257). Buffalo, NW

[18] Stars (1948) [Online image]. Retrieved October 19, 2018, from https://www.pinterest.pt/pin/250090585542778746/?lp=true

[19] Topological Mesh Modeling. (2008). Retrieved July 04, 2017, from https://www.viz.tamu.edu/faculty/ergun/research/topology/download.html

[20] Tetrahedron and Hexahedron. [Online image]. Retrieved May 1, 2015, from http://www.graphicine.com/the-geometry-of-an-art-wenzel--jamnitzer/

[21] Mandal, E., Akleman, E., & Srinivasan, V. (2003). Wire modeling. In P*roceedings of the SIGGRAPH 2003 Conference on Sketches & Applications in Conjunction with the 30th Annual Conference on Computer Graphics and Interactive Techniques - GRAPH '03.*

[22] Dimari A., & Yoo N. (2012). *Operative Design. A Catalogue of Spatial Verbs*, Amsterdam, Netherlands: Bis Publishers.

[23] Richard Serra's Verb List (1967–68). (n.d.). Retrieved July 04, 2017, from https://unprofessionaldevelopment.org/portfolio/verb--list-1967-68/.

GUARDA'S REPRESENTATION LABORATORY (100-2010): RESEARCHING, INTERPRETING, MODELING AND VISUALIZING A CITY'S GROWTH

Cátia Ramos [1]

ABSTRACT

This article focuses on the construction of the city of Guarda between the years of 100 and 2010, assessing the importance of computer technology as a tool for historical research. In urban research, computer technology can be a tool with potential for a more democratic opening for research findings. Although three-dimensional models and other visual means can help the historian in his interpretation process, nowadays, he must also be concerned with the means of communication that help to expand the dissemination of his work. By explaining the process of research, interpretation, modelling and visualization of Guarda's growth, we will introduce the steps and the difficulties found in the project of building a representation laboratory of Guarda. As a tool for historians, architects and citizens that interpret Guarda's growth and critically design its future, the laboratory is a first step for providing citizens and researchers the comprehension of Guarda's growth through democratic access tools.

KEYWORDS: three-dimensional modelling, computer simulation, representation laboratory, architecture and urban history, Guarda.

THE SIGNIFICANCE OF RESEARCH AND VISUALIZATION OF A CITY'S GROWTH: A RETROSPECTIVE ON THE 19TH CENTURY

The Industrial Revolution brought with it a radical transformation of the city *per se*, accompanied by an impressive demographic growth and unprecedented urban development. New cities were being built and in the older ones, sharp divisions were created between the city centre and the peripheral areas of factories, which then were enclosed by slums. This deep transformation of the city resulted in a new spatial order devoid of its traditional richness of meaning. Therefore, the need to understand this phenomenon and following the objective way in which Man started to observe the material world, a comprehension of the city through a visual experience, became important to 19th century scholars.

In Vienna, at the end of the 19th century, the architect and city theorist Camilo Sitte (1843-1903) published his work "*Der Städtebau nach seinen künstlerischen Grundsätzen*". At the backdrop of Vienna's physical transformation of the Ringstraße, Sitte was not only criticizing modern city planning, but also analyzing and questioning the physical qualities of the medieval inner city. He pursued a conception of a city regarded as a 'natural' historical product, where the city's squares were an expression of community values [01]. Sitte used plans and drawings of Europe's squares to unveil the 'artistic principles' behind them. The public spaces portrayed with the position of monuments in relation to other buildings served him as tools to construct an alternative to the design of the Ringstraße.

At the beginning of the 20th century, the urban historian Marcel Poëte (1866-1960) started his research on the iconography of the city of Paris at the *Bibliothèque Historique de la Ville de Paris*. The creation of this library was bound to the major urban renewal set forth by Haussmann, the loss of the city's memory and the need to keep the studies of urban history alive. In this library, Poëte organized lectures and exhibitions on the history of Paris addressed to experts and concerned citizens. His research work resulted in a four volume book on the history of Paris, where drawings, illustrations and photography became indispensable tools.

These tools were paramount to his thesis developed in "*Introduction à l'urbanisme: l'évolution des villes, la leçon de l'Antiquité*" (on the city as a collective work that is built and developed in time) but also to his audience,

---

[1] Ph.D. Student, Department of Architecture, Faculty of Science and Technology, University of Coimbra, Portugal. catiaramos00@gmail.com

architects, engineers and citizens. Poëte put forward a method for the backward restitution of different phases of the city transformation [02], whilst acknowledging the historical correspondence between a city's shape and the shape of its institutions.

Meanwhile, in the United Kingdom, the biologist and city planner Patrick Geddes (1854-1932) soon understood the need of visual experience for the comprehension and planning of a city's development, while sharing knowledge on urban history. Evolutionists' theories covered his studies and writings on cities, namely in his book "*Cities in Evolution*" [03]. Geddes' socio-biological background led him to the study of cities based on a survey method that should look at the specificity of each context by understanding every viewpoint. In 1892, he created the *Edinburgh Outlook Tower*, which was a laboratory and civic museum directed at the studies of Edinburgh in relation to its region, country and the world, providing a visualization of the city's transformations from the outside-in. The *Outlook Tower* was a place destined to deal with the practices of citizenship, providing citizens the knowledge needed to intervene in the community's life. This museum administered what Geddes called an *Encyclopaedia Graphica,* where the scientific and artistic aspects built upon surveys, maps, drawings, art and photography, set a basis for an interpretation of the city's course of evolution until present days [04]. By opening the Tower to all citizens, Geddes created a cultural and democratic approach, where an "educated citizenship" could take part in the forecast and endeavours for his city's future possibilities [05].

Although Sitte's, Pöete's and Geddes's reaction to the transformations of the 19th century city resulted in different solutions, they established the importance of applying different methods for their research, visualization and interpretation of the city's transformation. One of the key aspects for my research on the urban and architectural history of the city of Guarda in general is this ontological importance of the visual experience. These 19th century scholars presented, for the first time, new tools that turned the city into something readable. Moreover, their surveys had a common goal: the creation of knowledge that served as a platform for interpreting the city's past, using institutions with the intention to provide a basis of information for a quality city design.

Today, in a digital universe, the significance of the graphic reconstruction of urban space remains central. Although material evidences are undeniably significant to the construction of the city narrative, documents, drawings and images considered as research matter are also taken into consideration. By engaging with new multimedia tools to clarify the process of a city's growth, architects and urban historians are aiming for a diversified audience to extend the comprehension of their research findings and overcome its academic enclosure, just like Sitte, Poëte and Geddes aimed in the past [06]. By doing so, I strongly believe we are transforming the quality of urban research. As the results on the history of the city are made comprehensible and available for all, we are culturally and democratically empowering citizens, allowing everyone to intervene in the debate of the city's future design.

METHODOLOGY: RESEARCHING, INTERPRETING, MODELLING AND VISUALIZING GUARDA'S GROWTH

This article examines the construction of the physical reality of the city of Guarda between the years 100 and 2010, focusing on the importance of computer technology as a tool for historical research on the built environment using three-dimensional (3D) models and multimedia tools. As prevously outlined, 19th century scholars resorted to drawings and other visual materials on the city to comprehend its growth and building institutions, so that they and others had tools to rely on, while planning and building alternatives for the city's development. In conclusion, we can state that it is possible today, with the assistance of computer technology, to work for a more democratic opening of urban research. Hereupon follow comments on the politics of representation methods as well as other communication tools, which are presenting new paths for researchers to clarify the process of a city's formation and growth. Lastly, the process of research, interpretation, modelling and visualization of Guarda will be unraveled, thus introducing the steps and difficulties met to build, what I have called a *Representation Laboratory of Guarda*. The progress achieved in this representation laboratory, that adopts a 3D outlook, is shown through a video that includes a temporal coordinate. This timeline seeks to enhance the reading of Guarda's urban progression in time. The laboratory is conceived as a dynamic device for historians, architects, citizens and other city stakeholders to interpret Guarda's growth and, ultimately, a tool to critically design its future. Finally, and after setting some conclusions on the work done so far, I shall look at the possibilities through which this representation laboratory adresses our goal of providing citizens with a better comprehension of Guarda's growth. Additionally, the representation laboratory is a first step towards the assembly of a comprehensive database of the city, which will be open to further developments.

THE POLITICS OF REPRESENTATION AND COMMUNICATION: THEIR IMPORTANCE FOR URBAN HISTORY

For quite some time, several architecture and urban historians have been conducting their researches using new techniques of representation. As in several known architectural studies, in urban studies, it is necessary to take into account a variety of documents, i.e. construction plans, texts, drawings, photographs and descriptions, among others, in order to demonstrate a historic overview. Thus, it becomes clear that, ontologically, distinctive methods are implicit in historical representations. These documents support the historical undertaking of getting to know a variety of realities. And once one is involved in writing history, like me in Guarda's case, we consciously realise, as the historian Spiro Kostof did for architecture, that the city consists of buildings that were "born of images and live on images" [07], images that are inherently and intrinsically linked with urban research. In addition, for the historian whose task consists in organising piecemeal information and exerting judgement on which sources are the most reliable, there exists an overlapping urgency in constructing accurate representations for the urban space as well. In "*Cairo: Histories of the City*", Nezar Alsayyad calls this the politics of representation: the urgency of constructing accurate representations at the same time in which History is written. So, here, the use of computer modeling, computer simulation, engravings, photographs and drawings contribute to the quality and strength of the historical narrative, whereas its use is counterbalanced within History as a science of interpretation [08].

Now I would like to extend the concept of politics of representation to what I call the politics of communication, or more precisely, on how these are intertwined. In the past, architecture and urban history books, museums or other city related institutions where the vehicles for transmission of knowledge about the city. Today, however, the specific tasks inherent to the politics of representation in history research have to consider the way in which the representation techniques and the means of knowledge transmission rapidly change. This implies an adherence of urban and architecture historians to new forms of communication. Here, I am referring to the use of computer simulation and other multimedia tools, as, for instance, the digital supports used to create, manipulate, store and research content. Regarding this matter, in an article introducing the work for a research project[2], Donatella Calabi picks up the thoughts of the French philosopher Michael Serres on the importance of rethinking teaching methods when confronted with a young population that absorbs images and sounds between rapid swifts of thumbs [05]. She calls on the importance of making knowledge available to a wider audience, through the researcher's involvement with "Digital Humanities"[3]. Whereas the researcher assimilates other languages, he, at the same time, is also providing new research input and thus making it possible to see things from other perspectives [05].

Therefore, besides the importance addressed to representation, the work of the historian must also be concerned with the politics of communication. When confronted with the dynamic way in which traditional tools of representation and communication are changing, the researcher must acquire new skills and knowledge. Hence, he is more capable to help students, institutions and citizens to interpret and visualize changes of the ongoing city transformation. Thus, scholarly work can more easily overcome barriers created by academia, becoming more culturally and democratically available to all city players and stakeholders.

To show examples in which research findings are exposed beyond scholarly work, I shall present two projects involved with this politics of communication. Since the 1990's, Nezar Alsayyad's research focus has been the transformation of Cairo since its earliest days. As he was unveiling the transformations of Cairo's high medieval age, he came across the difficulties imposed by the two-dimensional abstract models, and decided to explore the third dimension in order to create an experimental depiction of space [08]. This resulted in a multimedia tool called '*Virtual Cairo: An Urban Design History*' [09], in which viewers are able to tour

---

[2] The project is called "*Built City, Design City, Virtual City. The Museum of the City.*".

[3] Digital Humanities is an area of scholarly activity that intersects disciplines of humanities with computing or digital technologies. Digital Humanities has a collaborative nature, usually involving a multidisciplinary team or researchlab composed of faculty staff, graduate and undergraduate students, information technology specialists, and other partners.

the buildings as they were in the 10th and 15th centuries. In Europe, the digital humanities project called '*Visualizing Venice*' [10] has an educational and experimental focus. Firstly, it is dedicated to the understanding of the ongoing transformation of the city, the lagoon and the region of Venice. Secondly, it focuses on new ways to disseminate knowledge through its website and other applications and data bases that explain the evolving process of the urban environment. Besides the involvement of researchers with digital devices, city institutions, such as city councils and museums, are recognising the role which urban research has for the local community and its visitors. Thanks to this engagement with urban research and technologies, city institutions are building websites, digital archives and other portable applications. See for instance the website of the *Chicago History Museum*, that provides online exhibitions [11], and the website of the Barcelona's History Museum (MUHBA), where one can see the city's development through cartographic means, scrolling back and forward through a time line [12].

In the first stages of my Ph.D. dissertation on Guarda, one of the first goals was to reach the intelligibility of its transformations through time. To accomplish this, I felt, as with the development of any architectural project, that it was necessary to comprehend its history beforehand, by means of a closer scrutiny of Guarda's shapes, structures, social and political forces which had driven its development. I started to question which visualization tools were more appropriate for that inquiry. Therefore, and within the scope of a historical work, investing in digital content was unavoidable. Although I am still involved with the politics of representation, I believe that some communication tools can derive from this first approach towards the development of digital content, from what I call representation laboratory.

GUARDA 100-2010: RESEARCHING, INTERPRETING, MODELLING, VISUALIZING THE CITY'S GROWTH. BUILDING A REPRESENTATION LABORATORY.

Located in the hinterland of Portugal, Guarda's origins go back to the Roman Empire, although the city is best known as a former medieval city. Some significant aspects of Guarda's evolution are that it started as a roman city, located a mile away from its later medieval core and the evolution of its medieval core, which occurred due to a continuous process of transformation, starting with the obliteration of roman structures and the transformation of its Romanic fortress since the 10th century. In recent years, its development is marked, like that of many other cities, by urban sprawl. In the face of these changes, the historical research of the city had partially explored the importance of architectural representation by tracing its transformations. Originating in the works of the historian Rita Gomes [13], the drawing of the city's shape between the 12th and 15th century has been widely reproduced by other researchers. Also, the archaeologist Victor Pereira resorted to 2D and 3D plans and models, to make the roman occupancy of the city intelligible: its territory, roads and the city's Roman Bath [14]. The disciplinary specificity of these two works are focused in detailed time-frames of the city's development, however, within the scope of my study, it was necessary to make an effort using architectural representation methods, which could facilitate a reflective and critical inquiry on Guarda's growth from its early stages to the present. I needed to create drawings that could contribute to the quality of the research narrative, a translation of each lasting spatial and political order of the city (e.g. roman occupancy, early high and late medieval ages, 19th century, dictatorship, democracy, etc.).

The first step made to build the city's history was the research about the human processes which were responsible for the construction of spaces at a particular time and determined the city's form. To achieve this, it was vital to study sources, their analysis and interpretation, and make use of all the available historical cartography. Sources on Guarda's work included archaeological, medieval and other historical studies, official city records, maps of the city at different times, artistic depictions of spaces, buildings, and photographs. Next task was the interpretation of these sources, confronting and judging their relative value in relation to socio-economical events, the existing urban form and contemporaneous cartography. This allowed organizing several sources, constructing new 2D cartography, and grasping data gaps in some time-frames. For instance, in the interpretation of the 16th century city transformation drawing on the accounts of other Portuguese cities at the time. At that point, the concerns with city aesthetics, salubrity and safety were expressed and regulated by King D. Manuel I (1469-1521). These concerns coincided with the modernization and concentration of the local power, accompanied by a prosperous intellectual and economical conjuncture caused by the Portuguese ultramarine expansion.

The new 'rossios', 'terreiros' and market squares, where transforming the cities outside their walls. However, in Guarda's case and cities such as Setúbal and Nisa, the transformation of the city square and the construction of a new functional program occurred inside the walls. Without strong documental and material sources, the comprehension of the transformation of Guarda's market square in the 16th century relied on a comparative interpretation between the evidences of other Portuguese cities at that time, such as Coimbra's Old Square and Lisbon's 'Rua Nova' (Figs. 1 and 2).

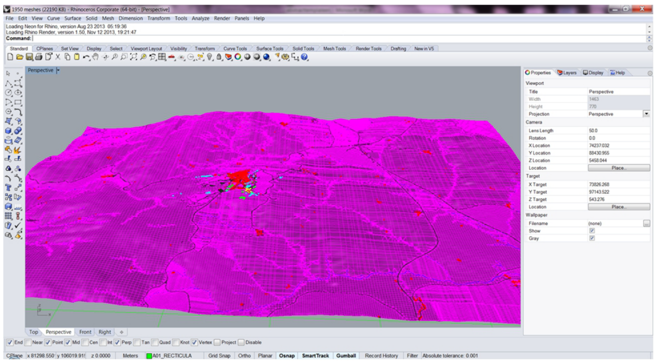


Fig. 1 - Three-dimensional modeling.

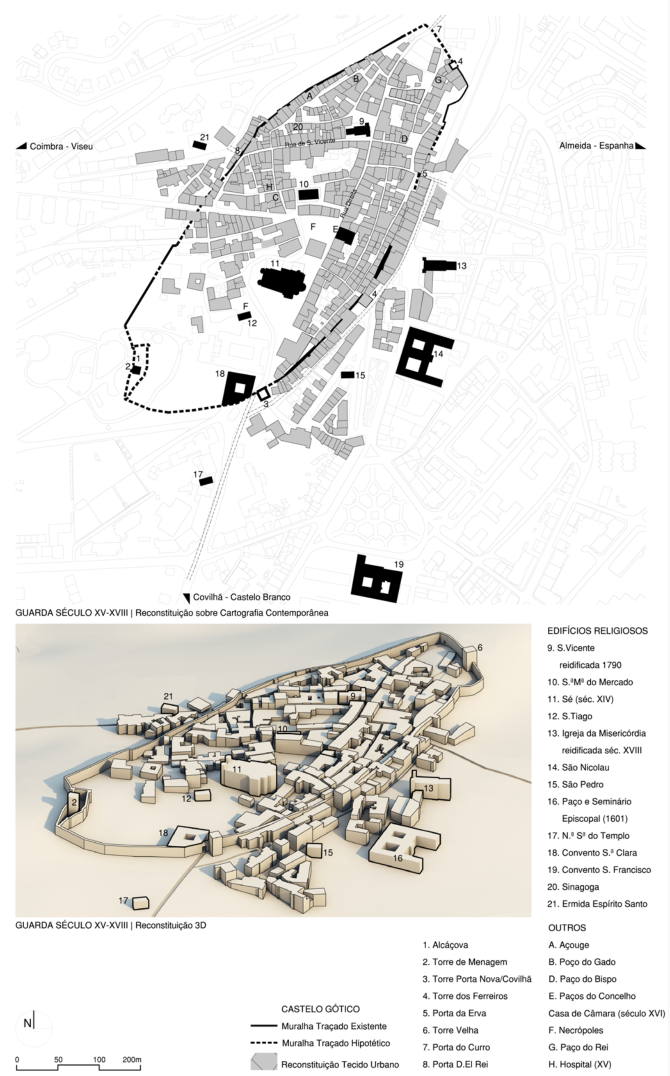


Fig. 2 - Guarda between the 16th and 18th centuries: 2D and 3D Representation.

The interpretation of sources resulted in 16 plan drawings at various time-intervals of the city's history, four of them prior to the 20th century and one for each decade until 2010. Within the context of a Portuguese and European city, the definition of a chronology for each plan drawing was based on the documentation that was available and secondly, depended on the socio-political and material changes of the city. Therefore, the first four plans express the changes of the city and its administrative organization from the late antiquity to the end of medieval ages, and between the 19th and the 20th century, when the city walls were demolished. In the 20th century it was possible to develop a more refined interpretation of the city's changes, because of the available cartography data. Thus, it was possible to comprehend the moments of stagnation and acceleration of Guarda's growth from the early 20th century until today. During this process, the efforts to construct sixteen 3D models of these time-intervals allowed for an interpretation of urban patterns in relation to the city's geographic specificity and to observe the relations between buildings and their settings (Fig. 3).

Nonetheless, the three-dimensional models do not convey movement or a continuous movement, which would allow for an interpretation and visualization of the growth of the built environment through time. Here, computer simulation proved to be a procedure capable of emulating movement, enabling an understanding of the city's dynamics; contrarily to the real city's evolution, which is understood through the synchrony that embraces all historical moments. Thanks to multimedia production and video editing, it was possible to understand the evolution of city structures through a temporal coordinate. The virtual simulation of the city growth reasserted the most dynamic and feeble

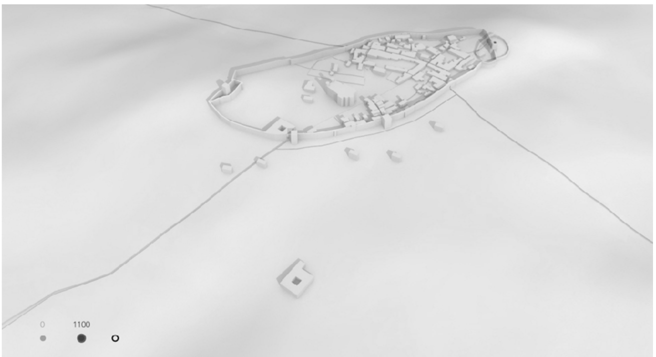


Fig.3 - Video-frame of the form of transition between Guarda's Romanic and Gothic fortress.

Fig. 4 - Video-frame of the decade of 2010.

moments of the city's development (Fig. 4). For instance, as it reaches the 20th century, the video portrays the city's swift growth, marked by profound changes in infrastructures, patterns, buildings and its programmes. As I was decoding these space-time relations, I concluded that I was building a ground-tool which was deciphering the city's palimpsest, and so I called it a representation laboratory. This representation laboratory stands for the interpretation of Guarda's ongoing urban transformations.

CONCLUSION: BEYOND GUARDA'S REPRESENTATION LABORATORY. ASSIMILATING OTHER LANGUAGES AND EXPANDING KNOWLEDGE TRANSMISSION

The work done so far allowed to establish a representation laboratory built on 2D and 3D geo-referenced scale models. However, those still rely on a great level of abstraction and were paramount to a politics of representation. Though the video is a first attempt to enter the realm of a politics of communication, this multimedia approach can be understood as a pedagogical tool open to all with an interest in Guarda's history and growth. As for architectural design, it will be possible to critically inquire the city, proposing alternatives to its development.

Moving beyond to what Guarda's representation laboratory means, it is possible that it will become a basis for the construction of less abstract, more realistic, comprehensive and complete depictions of urban space. Further developments can be made in order to assimilate other languages, by including other fields of knowledge that are also concerned with the comprehension of the city. A more common approach is the imprint of material scale models that depict each stage of the city's development. Other consist in the development of a GIS database that can include text, iconographic (e.g. infographics, photos, ortophotomaps, etc.) and demographic data of buildings and urban transformations. Another hypothesis relies on the investment of realistic depictions of urban space, by exploring new information or communication technologies, such as virtual or augmented reality, since computer-generated sensory inputs are able to deliver computer tools that can lead the average citizen to understand the city's transformation as he goes online. As research and computer technology provides new and diverse forms of communication and visualization on city growth, one should not forget - much the same way as 19th scholars did not - the role of citizens and city institutions in disseminating and transmitting knowledge. It is through their involvement with city history that we can aspire to stay updated and continue to receive and deliver democratic responses concerning Guarda's present and future.

Cátia Ramos

REFERENCES:

[01] Schorske, C. (1981). *Fin-De-Siècle Vienna Politics and Culture*. New York: Vintage Books, Random House.

[02] Poëte, M. (2011). I*ntroducción al urbanismo, La evolución de las ciudades: la lección de la Antigüedad* (Fund.Caja de Arquitectos). Barcelona.

[03] Geddes, P. (1915). *Cities in Evolution: An Introduction to the Town Planning Movement and to the Study of Civis.* (pp. 313-318)). (Nabu Plubic Domain Reprints). London: Williams & Nortgate.

[04] Meller, H. (1994). *Patrick Geddes: Social Evolutionist and City Planner*. Taylor & Francis.

[05] Calabi, D. (2013). On the 'History of the City': its study and related institutions. In D. Calabi (Ed.), *Built City, Design City, Virtual City, The Museum of the City*. (pp. 7–22). Roma: Centre for Studies of Rome (CROMA).

[06] Kostof, S. (1995). *A History of Architecture: settings and rituals* (2nd ed). New York: Oxford Universty Press.

[07] Alsayyad, N. (2011). *Cairo: Histories of a City*. Cambridge, Massachusetts and London: The Belknap Press of Harvard University Press.

[08] Alsayyad, N. (1999). Virtual Cairo: An Urban Historian's View of Computer Simulation. *Leonardo*, 32(2), 93–100.

[09] 'Outreach Videos | IASTE', Retrieved 20 May 2017, http://iaste.berkeley.edu/outreach-videos/.

[10] 'Visualizing Venice', Retrieved 20 May 2017, http://www.visualizingvenice.org/visu/.

[11] 'Chicago History Museum - See Chicago's History Unfold', Chicago History Museum, Retrieved 20 May 2017, https://www.chicagohistory.org/.

[12] 300.000Km/s, 'Carta Històrica de Barcelona', Carta Històrica de Barcelona, Retrieved 20 May 2017, http://cartahistorica.muhba.cat.

[13] Gomes, R. (1987). A Guarda Medieval: Posição, Morfologia e Sociedade (1200-1500). *Revista de História Económica e Social,* (9–10), 201.

[14] Pereira, V. (2012). O Sítio Romano da Póvoa do Mileu (Guarda). *Povoamento, Estruturas e Materiais Arqueológicos de uma Pequena Ciuitas Localizada nos Confins Ocidentais do Império Romano*. (Dissertação de Doutoramento em História, especialidade de Arqueologia). FLUC, Coimbra.

A REVIEW OF AUGMENTED REALITY FOR ARCHITECTURE AND CULTURAL HERITAGE VISUALIZATION

Isidora Đurić [1], Ratko Obradović [2] and Nebojša Ralević [3]

ABSTRACT

This paper presents an overview of the use of Augmented Reality in the area of architecture and cultural heritage visualization. The subject of this research are objects that have significant cultural and historical values which, for different reasons, cannot be perceived in their real environment. Using two case-studies, the processes of 3D reconstruction, optimization and AR presentation of the reality models are described in detail. The 3D models are created by using photogrammetry and the AR visualization is performed by using an existing platform for the AR presentation. The aim of the paper is to emphasize the importance of incorporating photogrammetry and augmented reality for the visualization of different types of object features.

KEYWORDS: augmented reality, architecture visualization, photogrammetry

INTRODUCTION

Augmented reality (AR) is a contemporary technology which is capable of linking or overlaying virtual content and information with the user's view of the real world scene. The three main characteristics developed by Azuma [01] define an AR system more precisely: (1) it combines real and virtual world, (2) it is interactive in real time and (3) it is registered in 3D.

Many surveys on AR technology and applications have already been published, starting from the first one provided by Azuma in 1997 [01] and later in 2001 [02], up to more recent researches that identified different AR application domains and discussed limitations and future trends of AR technology [03][04][05][06]. In most of the cases, the main focus is on the evaluation of the technology, as well as on the outlining potentials of the AR implementation in different areas, which implies that AR represents still a technology in development and an active topic among various multi-disciplinary researches.

The potentials and challenges for application of AR in architecture and design are discussed in [07], with a focus on the technical and technological issues and challenges, whereas the preparation of the reality model is addressed as one of the three main issues in the design and implementation of AR in these areas.

The importance of AR can certainly be related to virtual heritage, especially when the object cannot be visualized in the real scene due to its size or accessibility issues [08]. In addition, 3D reconstruction of the virtual heritage has become the most important method which can be used to present objects that have been partially lost or do not exist anymore, as well as to preserve and collect cultural and historical data related to objects or urban environments and specific locations [09].

The subjects chosen for this research are two objects that have significant cultural and historical value and that, for different reasons, cannot be perceived in their real environment. Such objects are usually characterized by a specific form, construction, materialization or location constraints, and therefore cannot be recreated by traditional CAD (computer-aided design) modeling techniques. Thus, the employment of photogrammetry has a crucial role in virtual model reconstruction. In addition, the application of the AR presentation is significant, as it provides real-time visualization.

---

[1] Faculty of Technical Sciences, University of Novi Sad, Serbia. isidoradjuric@uns.ac.rs

[2] Faculty of Technical Sciences, University of Novi Sad, Serbia. obrad_r@uns.ac.rs

[3] Faculty of Technical Sciences, University of Novi Sad, Serbia. nralevic@uns.ac.rs

The importance of the synergy between AR and photogrammetry is emphasized in [10] through the creation of an outdoor mobile AR application, as well as in [11], which highlights the benefits of the AR application in the field of the cultural heritage. However, as authors noted, the main limitations of their approaches are related to the technology required for the real-time tracking, since the usage of different wearable devices, such as laptop computer, HMD and web camera, affected the overall user experience. The approach similar to ours, that uses a low-cost AR-media software for the creation of AR visualization was implemented by Pejić et al. [12], for the purpose of presenting a 3D model of the newly designed facade. In this case, the 3D model of a newly designed facade was created especially for the purpose of augmented reality visualization. If, on the other hand, already existing objects (such as an object of architectural and cultural heritage) needs to be visualized by AR, photogrammetry is usually employed and the whole process of a virtual reconstruction of an object and its preparation for AR presentation is more complex.

PRACTICAL EXPERIENCES OF IMPLEMENTATION OF PHOTOGRAMMETRY AND AUGMENTED REALITY FOR CULTURAL HERITAGE VISUALIZATION

The proposed process of creating a 3D model of existing objects for the purpose of AR visualization is split into the following main procedure phases, based on combining several modeling techniques with different software:

- 3D reconstruction of the existing objects by using image-based modeling technique. The 3D models are automatically generated in *Agisoft PhotoScan* [13].
- Optimization of the automatically reconstructed 3D models, in order to prepare models for exporting and further AR visualization. In this case, the 3D modeling software *3ds Max* is used.
- Connection of the optimized 3D model with an appropriate marker or a location, by using an *AR-media Plugin* for the *3ds Max* software.
- AR visualization of the 3D model, which is conducted by using *AR-media Viewer* for computer, and *AR-media Player* [14] for *IOS/Android* mobile devices or tablets. In this case, the AR visualization is performed in real environment with *Samsung Galaxy Tab 5*.

The following case studies describe the process of 3D model creation, optimization and finally visualization with augmented reality.

CASE-STUDY OF THE GRAVESTONE OF THE ARMENIAN FAMILY "ČENAZI"

The first example is an object that has recently been removed from its original location due to the construction of the new residential and commercial building. According to new urban plans for that site, it will be moved to another location. The gravestone of the Armenian family "Čenazi" is the only material evidence which testifies the existence and life of Armenians in Novi Sad, and as such, represents a cultural heritage of the city. Fig. 1 shows the real stateof the gravestone of the Armenian family "Čenazi" before it was removed from its original location. In this case, the aim of the AR presentation is to preserve cultural heritage in the form of virtual 3D reconstruction, and to allow a visualization of the removed object within the real environment of its future location.

IMAGE-BASED MODELING APPROACH

The 3D model of the object is created by using image-based modeling approach. The object is recorded by moving a camera around it and reconstructed by using photogrammetry, as a primary technique for the processing of image data. The surveying was conducted using *SAMSUNG*, model *SM-G900F camera*, with the

Fig. 1 -The gravestone of the Armenian family "Čenazi" before it was removed from its location.

following parameters: F-stop – 2.2, ISO speed – ISO-40, Exposure time - 1/498sec, and according to the previously calculated camera orientation and position relative to the object. Afterwards, the photographs, gained by previously applied surveying method and masks, previously applied to the photographs, are imported into photogrammetric modeling software, *Agisoft PhotoScan* and the 3D model of the gravestone is automatically generated. Also, all textures from the images are extracted automatically. In that way, generated 3D mesh with textures from the *Agisoft PhotoScan* software was exported to the .obj format required for the 3D modeling software *3ds Max*, where it was further optimized. Fig. 2 shows the reconstructed 3D model of the gravestone as well as estimated camera positions, generated by the *Agisoft PhotoScan* software. However, certain limiting factors during the surveying process have affected the final result of the reconstructed model. Due to the size and height of 2 meters of the object and, at the same time, the lack of adequate equipment such as an adequate platform that would ensure recording the object with two heights, the reconstructed top areas resulted in completely open surfaces. In addition, an incomplete structure and a lack of clearly visible texture is clearly distinguishable in one of the sides of the reconstructed model. This is caused by location constraints, namely the position of a wall near the object, due to which photographs from that side of the object could not be taken.

OPTIMIZATION OF THE RECONSTRUCTED 3D MODEL

In this step, the reconstructed 3D model of the gravestone with textures was imported into the *3ds Max* software, where it was prepared for further AR visualization. In order to clean the model of redundant faces, the produced mesh was manually filtered by removing polygons in *3ds Max*.

Afterwards, since the large open surfaces on top of the model and the noticeably visible lack of texture in one of the sides presented major problems with the reconstructed model, the main task of the 3D model preparation for AR presentation was closing the holes in an appropriate way. Due to the size of the open areas, automatic options for closing holes in *3ds Max* produced results that were undesirable. For that reason, the model was manually fixed using several techniques. The smaller holes of the 3D model were closed with *3ds Max* functions - Weld vertices and Fill holes - while the large open areas were filled with new polygons by creating planes and attaching them to the entire object. In addition, the new plane with previously created texture was added on the side of the 3D model which was not entirely reconstructed, due to surveying issues. The comparison of the reconstructed 3D model before and after the optimization is shown in Fig. 3.

Table 1 - Optimization parameters for the reconstructed 3D model of the gravestone of the Armenian family “Čenazi”.

The gravestone of the Armenian family “Čenazi”	
Modifier for an automatic optimization	not used
Polygon count before optimization	499,949 triangles
Polygon countafter optimization	487,351 triangles

Table 1 shows that the number of polygons of the imported model was close to 500 000, but slightly reduced after deleting redundant polygons and cleaning duplicating vertices. This way, the improved model was now ready for the AR presentation.

AUGMENTED REALITY VISUALIZATION

After the model was fixed, the *AR-media Plugin* for *3ds Max* was included. The *AR-media Plugin* allows the user to create augmented reality files that can be further displayed with *AR-media Player*. The created 3D model is linked to the marker and the view of augmented reality scene is tested by using the *AR-media*

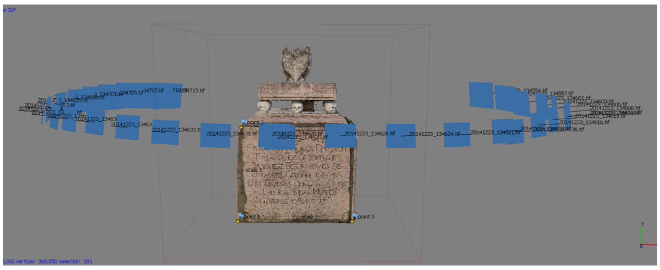


Fig. 2 -The reconstructed 3D model of the gravestone obtained by *Agisoft PhotoScan* software.

Fig. 3 – The comparison of the reconstructed 3D model: a) before and b) after the optimization.

Viewer for the computer. Since the virtual 3D model, with a high level of detail, appeared properly in real time, it was concluded that the 3D reconstructed model did not require further optimization. Afterwards, the model was geolocated and exported for *IOS/Android* view. As shown in Figs. 4 and 5, the real-time visualization of the virtual model of the gravestone of the Armenian family "Čenazi" in its future location is enabled. The AR visualization is performed in the real environment by using Samsung *Galaxy Tab 5* and the *AR-media Player* for *Android*.

CASE STUDY OF THE COACH OF THE METROPOLITAN OF KARLOVCI

In this case study, the AR for real environment presentation is applied to an object which cannot be moved from its current location. An example of this is the Coach of the *Metropolitan of Karlovci* which was used in the 18th century as a wheeled vehicle. The real state of the Coach of the *Metropolitan of Karlovci* in the *Museum of Vojvodina* in Novi Sad is shown in Fig. 6.

Fig. 4 - The gravestone of the Armenian family "Čenazi" visualized on its future location.

Fig. 5 - Print screen from the *AR-media Player* viewport.

It represents the only preserved coach from that period on the territory of former Yugoslavia and it is located in the *Museum of Vojvodina* in Novi Sad, as a part of its permanent exhibition. As the visualization of the coach outside of the area of the museum is impossible, the AR presentation will provide to modern viewers the possibility to imagine and interact with this particular scene from the past.

IMAGE-BASED MODELING APPROACH

The main challenge, in this case, was the creation of the 3D model of the coach, as it represents an object with a highly complex structure. The realistic 3D model of the complex geometric form of the *Coach of the Metropolitan of Karlovci* was reconstructed by combining different surveying and image-based modeling techniques.

Considering the complex form of the coach, as well as the location constraints, the object was logically divided into main parts: the front and back wheels, the cabin, and the front and back part of the coach, which were separately surveyed. Each single part was surveyed with two different camera heights, by using the tripod and a *NIKON D7000* camera (pixel size - 4.78μm, sensor size - 23.6x15.6mm, focal length - 18mm), with the manual settings of the following parameters: F-stop, ISO speed, Exposure time [15]. The series of photographs obtained for single parts of the coach and masks were separately imported into the photogrammetric modeling software *Agisoft PhotoScan*, where each part of the coach was automatically reconstructed [15]. The final models of the parts of the coach were then exported to the .obj format required for the 3D modeling software, *3ds Max*, where they were manually connected into a whole object.

OPTIMIZATION OF THE RECONSTRUCTED 3D MODEL

Since the fully automated process for 3D reconstruction of the objects has files of very large size as result, the main technical issue was the optimization of the 3D model for the augmented reality visualization.

After importing single meshes of individual parts of the coach into *3ds Max*, the total polygon count was over three million triangles, which cannot be performed by any existing augmented reality server. As the 3D model of the coach was obtained by manually connecting single meshes into a whole object (resulting in lots of overlapping surfaces), the first step of the optimization was to delete the overlapping elements of each single 3D model, removing redundant polygons.

Also, *MultiRes modifier,* a useful tool whenever a model needs to be exported for use outside *3ds Max*, was employed in order to further decrease the number of polygons and vertices. *MultiRes modifier* was applied to each single mesh and the exact percentage of polygons' reduction for each individual element specified. For each 3D model, an individual multi-resolution mesh was generated, i.e. a different vertices percentage was used aiming to decrease memory, while keeping at the same time a certain level of detail. This way, the total polygon count was reduced to nearly 700000 triangles, which can be seen in Table 2.

Table 2 - Optimization parameters for the reconstructed 3D model of the coach of the Metropolitan of Karlovci

Coach of the Metropolitan of Karlovci	
Modifier for an automatic optimization	MultiRes*
Polygon count before optimization	3,786,069 triangles
Polygon countafter optimization	685,029 triangles

* Vertices percentage: 10% to 80% depending on the individual mesh)

In addition, since the transparent glass areas cannot be reconstructed by *Agisoft PhotoScan* those areas were removed from the 3D model and the new surfaces with applied standard transparent material within the *3ds Max* software were added. The optimized 3D model of the coach is shown in Fig. 7.

It can be seen that, despite the overall optimization process and the large reduction of the polygon count, the resultant 3D model still maintains a high level of detail for the purpose of AR presentation.

AUGMENTED REALITY VISUALIZATION

As in the previous case study, the *AR-media Plugin* was used for the creation of the augmented reality visualization. Since the polygon limit is not specified by the AR-media server, the 3D model has been optimized until its display in the *AR-media Viewer* software becomes possible.

In addition, it has been noticed that transparent materials of the object were displayed well, while in the case of the flat surfaces without thickness, only one side of the surface was apparent in the *AR-media Viewer*. Consequently, the *Shell modifier* tool was added to all such surfaces so that they could be displayed properly, as in the viewport of the *3ds Max* program.

Fig. 6 -The Coach of the Metropolitan of Karlovci in the Museum of Vojvodina, Novi Sad.

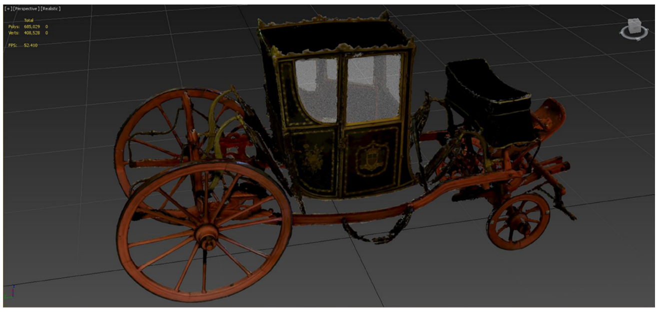


Fig. 7 - The optimized 3D model of the coach.

Fig. 8 - The coach of the Metropolitan of Karlovci visualized on the real location of the city centre.

Fig. 9 - Print screen from the *AR-media Player* viewport.

Fig. 10 - 3D model of the gravestone previewed in the a) *Agisoft PhotoScan*, b) *3ds Max* and c) *AR-media Player.*

The 3D model prepared in that way was then linked to the marker, as well as to the real location and exported for IOS/Android view. AR visualization of the coach was performed in the location of the city center, where it was originally used in the past, and the presentation was conducted using *Samsung Galaxy Tab 5* and *AR-media Player for Android*, which is shown in Figs. 8 and 9. Since the location accuracy is estimated at 12 meters by the AR-media server, the exact position of the 3D model in the real environment was manually adjusted by using the world coordinate system.

ANALYSIS AND DISCUSSION

The previously described case studies gave an overview of the detailed process of the creation, optimization and preparation of the 3D model and finally its AR visualization by using the AR-media platform. The transformations of the models and their appearance in different software are displayed in Figs. 10 and 11. It can be seen that, in both cases, 3D modeling presents a fundamental step in the process of developing an AR visualization of each object. In addition, since a limited number of polygons can effectively be displayed in the existing augmented reality servers, and given that a certain level of detail of the reconstructed 3D model is required to achieve a realistic visualization of the model, the optimization of the model itself has a critical role in the whole process. In the case of the gravestone, the level of detail has been increased by adding new surfaces and textures, while in the second example of the coach, the 3D model was significantly optimized, while preserving a satisfactory level of detail for the purpose of augmented reality presentation. It can be seen that in the whole processes, optimization presents

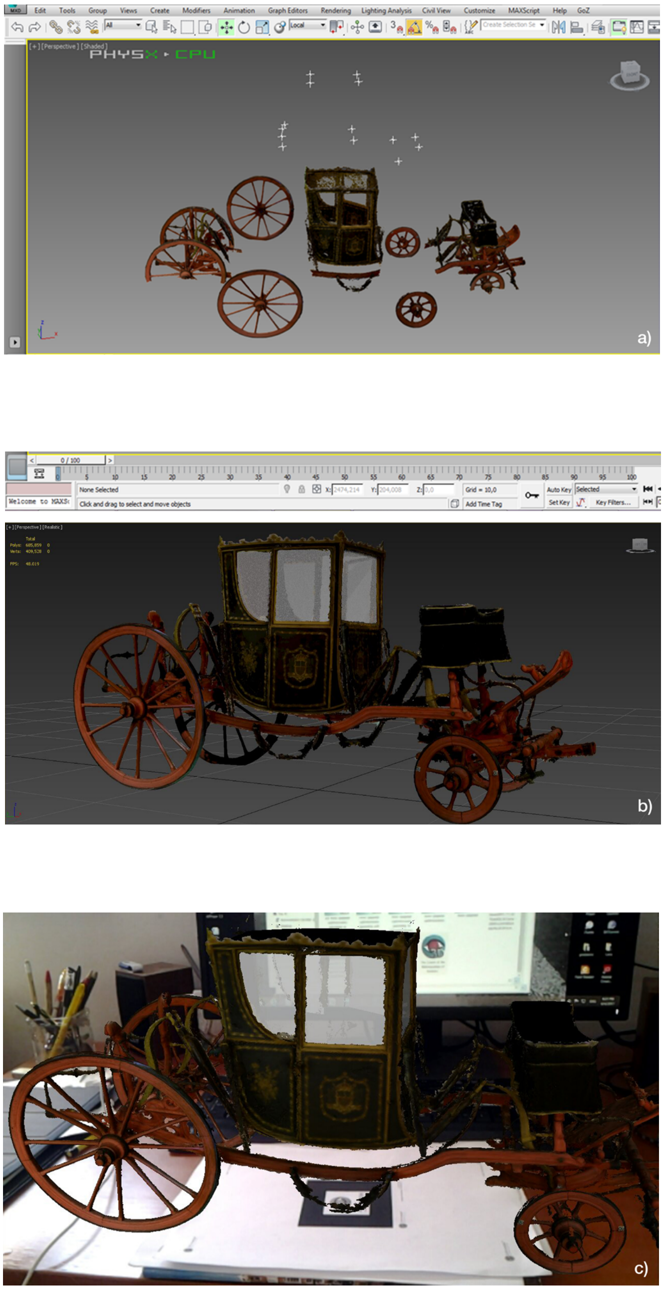


Fig. 11 - a) Individual parts of the coach in *3ds Max*, b) connected and optimized 3D model in *3ds Max* and c) 3D model of the coach previewed in *AR-media Player*.

the most demanding and time-consuming task. On the other hand, while the model is being prepared, the process of developing an AR presentation by using an existing AR platform is quite a simple assignment. Basically, that is mostly so because the existing third-party platforms for creating AR experiences, such as *AR-media*, *Augment*, *BuildAR* and others, do not provide a lot of possibilities for further development, and are, in fact, intended mainly for visualization of a model extracted from the digital workspace.

CONCLUSION

In this paper, the case studies of applying AR visualization to objects of architectural and cultural heritage are presented. The complete overview of the processes of creation, optimization and preparation of the 3D models are explained. Subsequently, the previously prepared 3D models are visualized, by using an existing platform for AR presentation. It can be seen that, although optimization is a demanding task, in specific cases, it may not be required. On the other hand, the creation of a 3D model presents a fundamental and important step in the process of developing an AR visualization. Hence, photogrammetry, as a technique capable of producing detailed and photorealistic 3D models, presents the most appropriate method for the reconstruction of objects which cannot be recreated by traditional CAD modeling techniques. Based on this, it can be concluded that the integration of augmented reality and photogrammetry is of great importance for different types of visualization.

Isidora Đurić, Ratko Obradović
and Nebojša Ralević

ACKNOWLEDGEMENT
This work was supported by the Ministry of Education, Science and Technological Development of the Republic of Serbia, within the Projects No. ON174009 and No. TR34014.

REFERENCES:

[01] Azuma, R. (1997). A Survey of Augmented Reality. *Teleoperators and Virtual Environments,* 355-385.

[02] Azuma, R., Baillot, Y., Behringer, R., Feiner, S., Julier, S., &MacIntyre, B. (2001). *Recent Advances in Augmented Reality. Computer Graphics and Applications*, 21(6), 34-47.

[03] Manuri, F., & Sanna, A.(2016). A Survey on Applications of Augmented Reality. *ACSIJ Advances in Computer Science: an International Journal*, 5(1), 18-27.

[04] Mekni, M., & Lemieux, A. (2014). Augmented Reality: Applications, Challenges and Future Trends. *Applied Computational Science,* 205-214.

[05] Zhou, F., Duh, H.B.-L., & Billinghurst, M. (2008) Trends in Augmented Reality Tracking, Interaction and Display: A review of ten years of ISMAR. In M. A. Livingston, O. Bimber, and H. Saito, editors, ISMAR'08: *Proc. 7 th Int'l Symp. on Mixed and Augmented Reality* (193–202). Cambridge: IEEE CS Press.

[06] Freitas, M. R., & Ruschel, R. C. (2013). What is happening to virtual and augmented reality applied to architecture? In *Proceedings of the 18th international conference on computer-aided architectural design research in Asia*, (CAADRIA 2013), Hong Kong (407–416).

[07] Wang, X. (2009). Augmented reality in architecture and design: potentials and challenges for application. *International Journal of Architectural Computing*, 7(2), 309–326.

[08] Noh, Z., Sunar, M., & Pan, Z. (2009). A Review on Augmented Reality for Virtual Heritage System. *4th International Conference on E--Learning and Games*, Edutainment 2009.(50-61). Banff, Canada: Springer-Verlag Berlin Heidelberg 2009.

[09] El-Hakim, S. F., Beraldin, J.-A., Picard, M. and Godin, G. (2004). Detailed 3D reconstruction of large-scale heritage sites with integrated techniques. *IEEE Computer Graphics and Applications*, 24(3), 21–29.

[10] Portalés, C., Lerma, J.L., Navarro, S. (2010). Augmented reality and photogrammetry: A synergy to visualize physical and virtual city environments. ISPRS J. Photogramm. Remote Sens. 65, 134–142.

[11] Portalés, C., Lerma, J. L., & Pérez, C. (2009). Photogrammetry and augmented reality for cultural heritage applications. *The Photogrammetric Record*, 24(128), 316–331.

[12] Pejić, P., Krasić, S., Petković, D., & Veljković, M. (2015). Application of augmented reality in facade redesign presentation. *Journal of industrial design and engineering graphics*, Special Issue, Vol. 10, 45-48.

[13] Agisoft PhotoScan User Manual (Professional edition) Version 1.0.0. Retrieved in July, 2017 fromhttp://www.agisoft.com/

[14] AR-media Player, Installation & user guide (Inglobe Technologies). Version 2.3. Retrieved in July, 2017 from https://www.inglobetechnologies.com/en/new_products/arplayer/download/_current_/UsersGuideWin.pdf

[15] Đurić, I., & Letić, J. (2016). Image-based modeling of complex geometric forms in restricted surveying conditions – a case study of the Coach of Metropolitan of Karlovci in the Museum of Vojvodina. Between Computational Models and Performative Capacities, *Proceedings of the 4th eCAADe International Regional Workshop* (pp. 62-74). Novi Sad: eCAADe (Education and Research in Computer Aided Architectural Design in Europe) and Faculty of Technical Sciences, University of Novi Sad.

ORIGAMI TESSELATIONS: FOLDING ALGORITHMS, FROM LOCAL TO GLOBAL

Filipa Crespo Osório [1], Alexandra Paio [2] and Sancho Oliveira [3]

ABSTRACT

Rigid Origami folding surfaces have very interesting qualities for Architecture and Engineering given their geometric, structural and elastic qualities. The ability to turn a flat element, isotropic, without any structural capacity, into a self-supporting element strictly through folds in the material opens the door to a multitude of uses. Besides that, the intrinsic geometry of the crease pattern may allow the surface to assume doubly curved forms while the flat element, before the folding, could never do it without the deformation of the material [01][02].

The main goal of this Ph.D. research is to reach a workflow that allows for the design and implementation of kinetically reconfigurable *Origami* Surfaces. In this paper, we will address mainly the parameterization of certain folded geometries, illustrating our method, simulating the folding of regular crease patterns through geometric operations on the smallest set of faces (local) that can be reproduced to simulate the whole group (global).

KEYWORDS: *Origami* geometry; *Grasshopper* for *Rhinoceros* definitions; architectural simulation; folding surfaces; kinetic surfaces.

INTRODUCTION

Origami is widely present in our everyday life, from objects like pastry paper boxes, paper bags, animal models done for or by children, art works, temporary buildings, mountable and demountable shelters, solar panels for space satellites and even in biomedical devices, summing up, on every planar material that is pleated to achieve other objects.

Despite its presence for hundreds of years, ever since paper was invented, it was only in the 80's of the 20th century that *Origami*'s mathematical and geometrical properties started to be thoroughly studied, granting it the category of a research subject. The *Origami, Science, Mathematics and Education* (OSME) Meetings were instrumental for the acknowledgement of the *Origami*'s importance as a mathematical subject and allowed for an easy dissemination of *Origami* research and connections between *Origami* scholars.

It was on these meetings that were published the *Huzita-Hatori Axioms*, very similar to the Euclidean axioms for constructions with straightedge and compass. The first six were defined by Huzita in 1986 and the seventh was defined by Hatori in 2002, although it had already been formulated by Justin in 1996. The seven axioms are usually known as Huzita-Hatori or Huzita-Justin [03].

Origami models belong to different categories, depending on the type of creases (straight lines or curves), the type of geometries that the faces can assume (planar or curved) and the folded result. This research focuses especially on *Rigid* and *Flat-foldable Origami*.

Rigid Origami has several characteristics that make it particularly well-suited to be used on kinetic foldable surfaces. A surface folded from a planar, rigid material gains structural abilities and can assume a wide range of forms from the unfolded state, planar, to the completely folded state.

Rigid Origami always starts with a tessellated planar surface. The creases must be straight as each face of the tessellation remains plan and cannot stretch during the folding process. The folding happens when adjacent faces rotate around their common edge, that acts like an hinge, towards each other. If the faces rotate upwards, then the crease is called a *valley-fold*, if they rotate downwards, the crease is a *mountain-fold*.

---

[1] ISCTE - Instituto Universitário de Lisboa, ISTAR-IUL, Portugal. filipa_osorio@iscte-iul.pt

[2] ISCTE - Instituto Universitário de Lisboa, ISTAR-IUL, Portugal. alexandra.paio@iscte.pt

[3] ISCTE - Instituto Universitário de Lisboa, IT-IUL, Portugal. sancho.oliveira@iscte.pt

The *Flat-foldable Origami* subset, in which every *Rigid Origami* model that can be flatten without creating new creases belongs, is a subset of *Rigid Origami*.

The foldable surfaces that respect the rules of *Rigid* and *Flat-foldable Origami* have very important qualities to be used in an architectural kinetic context, since they can cover a big space and collapse over themselves in a 2D shape, leaving it open. Roughly speaking, and disregarding the thickness of the material, it is possible to say that these surfaces have a compression capacity of 100%.

In order to understand the transformations that occur in these surfaces during the folding process, this research proposes the use of digital parametric simulations.

The use of digital parametric tools allows the designer to try and test all the desirable solutions and choose the most appropriate for a particular building site or function. These tools allow also to make changes and optimize the chosen solution before its construction. In the particular cases of *Rigid* and *Flat-Foldable Origami* to be used in a kinetic context, these tools reveal to be even more important, since they make possible the simulation of the movement that the surface will undertake from the unfolded to any folded state, as well as every state in between.

In this sense, we are developing a system in *Grasshopper* (Robert McNeel and Associates, version August 2014) for *Rhino 5*, for the folding simulation of any regular *Rigid Origami* pattern.

FOLDING SIMULATION METHOD

On the parametric system in development for this research, the goal is, from the crease pattern design and the definition of the mountain and valley folds, that the system should be able to simulate the entire range of forms that a given pattern can produce from the plan state to the completely folded and, inherently, the path that each face and vertex follows during folding.

There is already a very extensive work on this matter, especially from authors like Robert Lang [03], that uses spherical trigonometry for the simulation; Tomohiro Tachi [04] who uses the angles between edges and between faces as variables; Ron Resch and Christiansen [05], who use a combination of analysis and elastic constraints between the connections and truss elements; and, also, Casale and Valenti [01] that use *Rhinoceros* and *Grasshopper* to simulate the folding of different crease patterns, each one with a different approach.

This research's method is more similar to the one used by Casale and Valenti [01], but these authors create their definitions to fold the entire crease pattern at once, while our intention is to define the local rules for the folding of the minimal possible module of the regular tessellation, and then reproduce that module with vectorial copies allowing thus the crease pattern to extend as far as we want.

The experiences accomplished so far are not yet a system that can fold any crease pattern but rather directed to specific crease patterns. The *Grasshopper* definitions that will be explained further are very tangible and closely related to the observation that can be made with the physical manipulation of paper.

This research method to create the *Grasshopper* definitions comprises 3 steps:

1. Analysis of the regular tessellation in order to define the base faces;
2. Simulation of the folding of the base faces, from the unfolded state to the completely folded state;
3. Generation of the complete tessellation through vectorial copies of the base faces.

In the *Grasshopper* definitions, it is always chosen one point or crease that does not change during folding. This element behaves as the attachment to the *XYZ* referential as the centre of all transformations in relation to which every other element moves.

In this paper, we will explain two examples of our folding simulations. These are the *Miura* pattern, that folds on the plane , and the *Yoshimura* pattern, that starts from a planar form and folds into a cylinder. The definitions are purely geometrical and based on the rotation of the faces, that constitute the base-set around their common edges and then the reproduction of these base-faces through vectors that are continuously redefined through the folding process.

MIURA PATTERN

In this case, it is possible to observe that the whole pattern can be described by two simple translations in the horizontal and vertical direction of the base-faces. The chosen base-faces are composed of four identical quadrilaterals with a symmetry relation between them (Figs. 1 to 3).

First, points *A* (at the origin), *B* and *D* are defined in *Grasshopper*, all of them in the first quadrant, so the *XYZ* coordinates are simplified. Point *E* is defined by the translation of the line *AD* to point *B* (vector *AB*). The vertices of the edge *AD* and the translated line define the

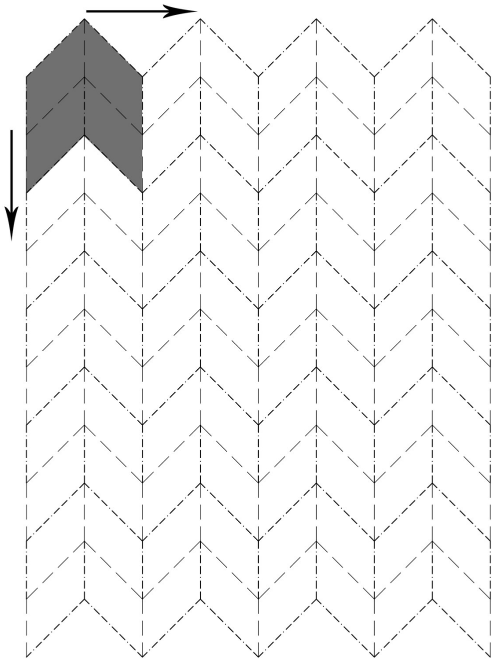


Fig. 1 - *Miura-Ori* Pattern, base faces and vectors of translation.

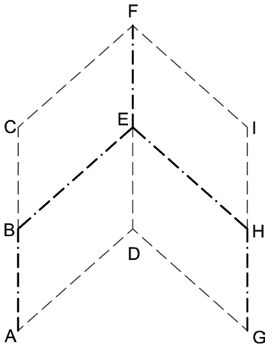


Fig. 2 - Base-faces of the *Miura* Pattern and vertices.

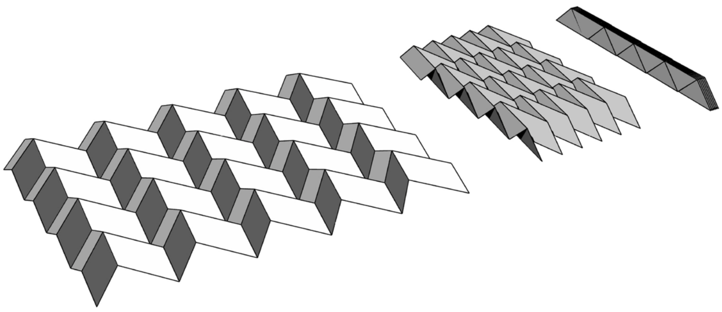


Fig. 3 - *Miura* surface, 3 folding states.

face *ADEB*. Then, line *AD* is translated again, twice the vector *AB*, thus defining line *CF* and consequently face *BEFC*. Face *ADEB* rotates from 0º to 90º around the axis *AD*. This will be the fixed geometry, the reference to the movement of the whole surface. Face *BEFC* then rotates around the axis *BE* twice and contrary to the angle of the rotation of face *ADEB*. The rotational movement of both faces is controlled with the same slider, which creates a synchronous motion as it happens with the rigid physical models of the *Miura-Ori* pattern. To generate faces *DGHE* and *EHIF*, it is enough to use the mirror component with the plane defined by the points *D, E and F*. To create the complete surface, we use the Rectangular Array tool, where the array cell is the quadrilateral defined by *A*, *C*, *I* and G. By defining the cell with these points, it is possible to guarantee that it adapts constantly to their movement, therefore creating a closed geometry for any number of columns and lines (Fig. 4).

YOSHIMURA PATTERN

In this pattern, we have chosen a group of eight faces to act as base-faces.

Less faces could also be the base of this pattern, but by setting this group will make the act of folding into a cylinder, the definition of the translation vectors and the assembly of different units much easier (Figs. 5 to 7).

Folding starts when the creases *AG* and *AF* rotate in the plane defined by them, having *A* as centre of rotation. At the same time, the arcs (red, in Fig. 6) follow the folding of lines *AG* and *AF*. The arc on the left defines the path where points *B* and *E* can exist, while the arc on the right defines the path for points *C* and *D*.

The complete possible movement for each point during the folding is remapped from 0 to 1, no matter how long is the length of the curve where they exist. This way, all the points go from the unfolded to the completely folded state in the same period of time.

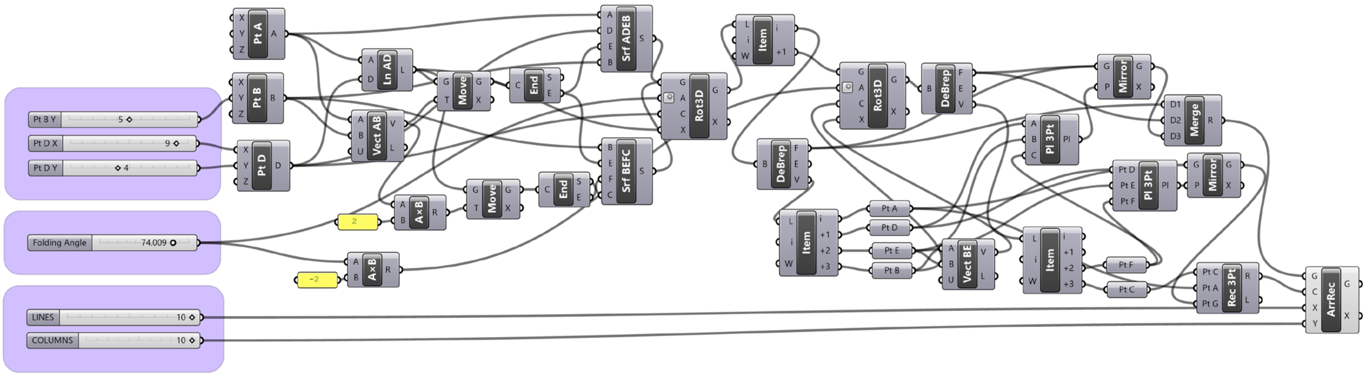


Fig. 4 – G*rasshopper* definition for the *Miura* Pattern.

After all the faces are set, copies are made first in a linear way, according to vector *BE*. Then this set is selected to be moved according to a vector that starts in the centre of the circle defined by *F*, *A* and *G* and that, with its tip at point F, transforms itself into a vector with the same start point but with endpoint *G* (red, in Fig. 7). This system only starts to work when folding is bigger than 0.

CONCLUSIONS

This paper explains the potential of *Rigid* and *Flat-foldable Origami* Surfaces to be used in an architectural kinetic context. It proposes the use of digital parametric tools in order to simulate the movement of these kind of surfaces from the unfolded to the completely folded state, particularly by using *Rhinoceros 5* and its plug-in *Grasshopper*.

Although there are several works about *Grasshopper* definitions on folded surfaces, these are mainly disseminated through open source channels and explained roughly (or not at all). Instead of being a tutorial for specific patterns, this paper explains a method to consider regular *Origami* patterns and subdivide them into the base-faces of each pattern, aiming to understand the transformations that happen to those faces while folding, and explaining how to replicate them in order to generate the surface.

In this paper, two examples are used to demonstrate the use of this method of thinking: the *Miura* and *Yoshimura* patterns. The *Grasshopper* definitions are based on the *Rigid Origami* rule, through which all faces must maintain their geometric configuration, be flat at all times and rotate on their shared edges. It is also explained

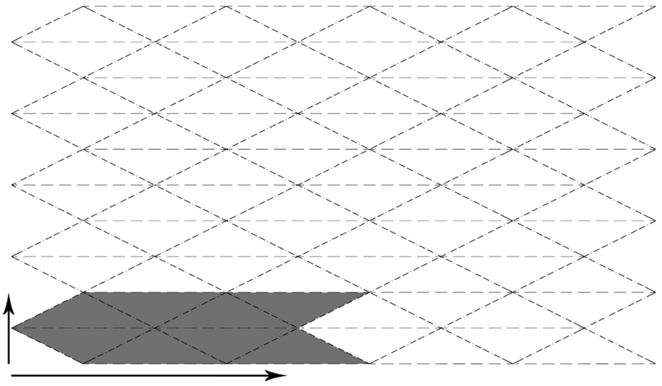


Fig. 5 - *Yoshimura* Pattern, base faces and vectors of translation.

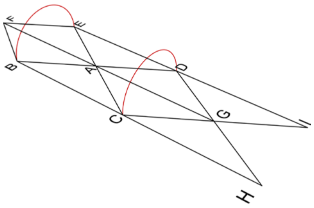


Fig. 6 - Base faces of the *Yoshimura* Pattern and vertices.

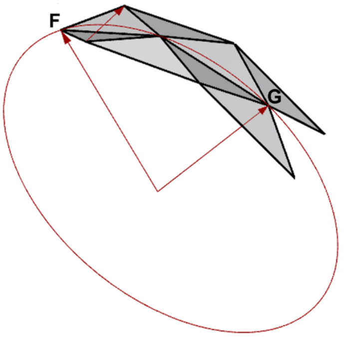


Fig. 7 - Translation vectors.

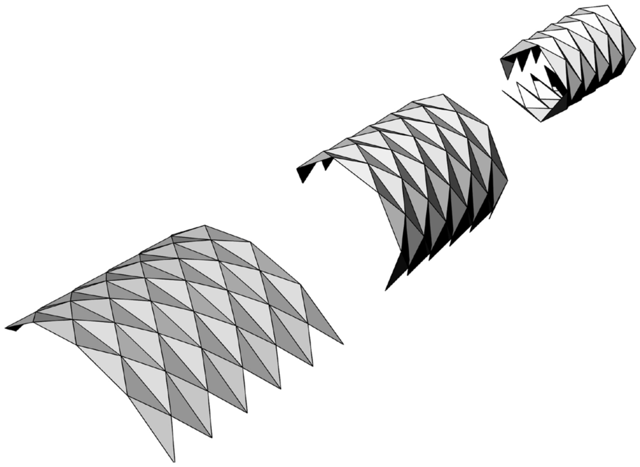
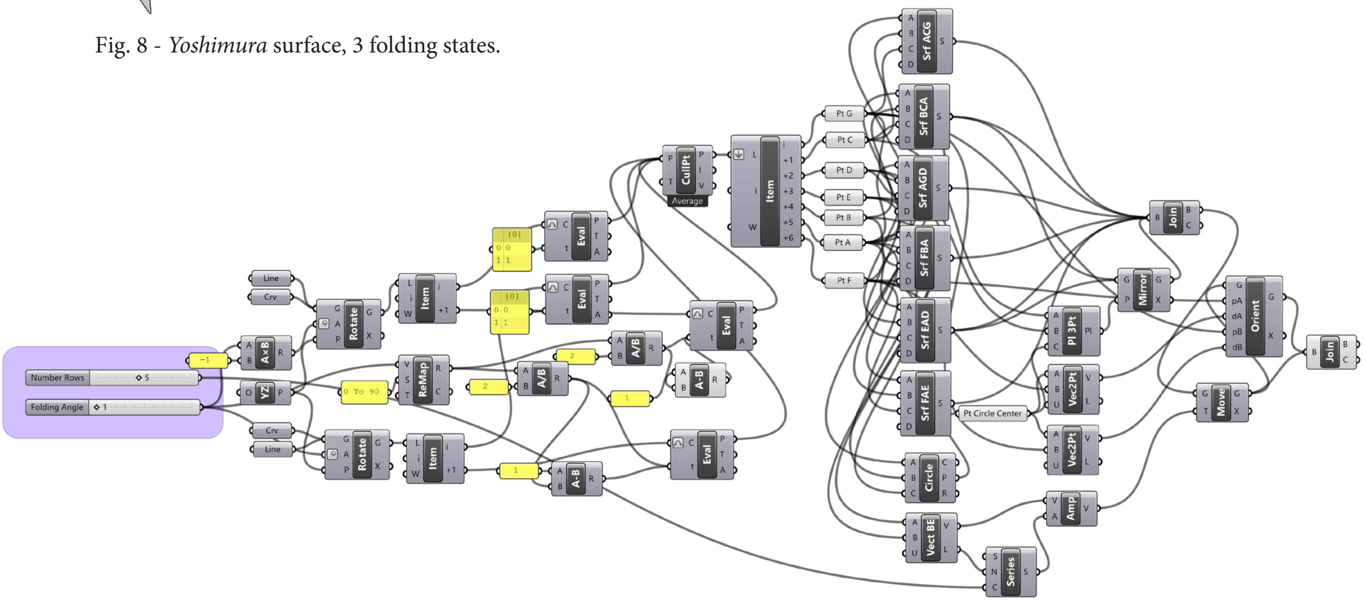


Fig. 8 - *Yoshimura* surface, 3 folding states.



Number Rows 5

Folding Angle 1

Fig. 9 - *Grasshopper* definition for the *Yoshimura* Pattern.

how the base geometry was defined on *Grasshopper* and all the transformations, mainly rotations, symmetries and translations, that happen to the base-faces.
Filipa Crespo Osório, Alexandra Paio, Sancho Oliveira

ACKNOWLEDGMENTS
This research is funded by a scholarship with reference SFRH/BD/100818/2014 from Fundação para a Ciência e Tecnologia (FCT).

REFERENCES:

[01] Casale, A., & Valenti, G. M., (2012) Architettura delle Superfici Piegate, Le Geometrie che Muovono gli Origami, *Nuovi quaderni di Applicazioni della Geometria Descrittiva*, vol.6, Edizioni Kappa, 2012

[02] Demaine, E., Demaine, M., Hart, V., Price, G., & Tachi, T. (2011) (Non)existence of Pleated Folds: How Paper Folds Between Creases, in Graphs and Combinatorics, Volume 27, Issue 3, pp 341-351, Springer Japan

[03] Lang, R. (2010) Origami and Geometric Constructions, Retrieved from www.langorigami.com

[04] Tachi, T. (2011) Rigid-Foldable Thick Origami, *Origami5 - Fifth International Meeting of Origami Science, Mathematics, and Education*, CRC Press by Taylor and Francis Group

[05] Resch, R., & Christiansen, H. (1970) Design and Analysis of Kinematic Folded Plate Systems, *Proceedings of IASS Symposium on FoldePlates and Prismatic Structures*, Viena, Austria

ESTUDO DO DESIGN DE *ORIGAMI TESSELLATIONS*: ANÁLISE DE COMPACTAÇÃO E COMPLEXIDADE ESTRUTURAL DE SEIS *CREASE PATTERNS*

Samanta Aline Teixeira [1] e Thaís Regina Ueno Yamada [2]

RESUMO

Este estudo analisa o *design* de seis *Origami Tessellations*, levando em conta seus níveis de compactação, complexidade estrutural e de reprodução por meio de seus *crease patterns* ou padrão de dobras. Foram construídos seis padrões geométricos em *Origami* com o propósito de compreender as principais características projetuais de cada *crease pattern*, os diferentes níveis de economia de espaço por meio da compactação estrutural, suas limitações e possibilidades projetuais. A contribuição para a diversidade de pesquisas do *Design Adaptado do Origami*, bem como os pontos positivos e negativos de cada padrão de dobras são discutidos.

PALAVRAS-CHAVE: *design*, *origami tessellations*, *crease pattern*, *design adaptado do origami*.

INTRODUÇÃO

Atualmente, a arte japonesa do *Origami* é uma fonte de estudos e aplicações tecnológicas das mais diversas naturezas [01]. A pesquisa científica do *design* de *Origami* atinge tanto escalas macroscópicas, como os painéis solares e estruturas espaciais [02] quanto escalas microscópicas, como a criação de novos materiais nanométricos [03]. Estão surgindo conceitos mais técnicos e específicos em prototipagem no chamado *Design Adaptado do Origami* [04]. Dentro desta vasta área, o *Origami Tessellations* - dobradura que possui repetidos padrões geométricos [05] - ganha especial atenção dos pesquisadores e *designers* por conta de suas formas naturalmente adaptáveis. Este artigo busca explorar o *design* de *Origami Tessellations*, focando em uma de suas técnicas projetuais, o *crease pattern* (ou mapeamento de dobras), e descobrir quais são os níveis de compactação de determinados padrões e sua intrínseca complexidade de reprodução.

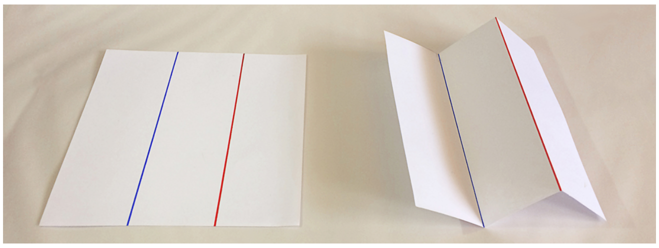


Fig. 1 - Dobra-montanha e Dobra-vale.

INVESTIGAÇÃO: O PRINCÍPIO DO *CREASE PATTERN*

O *Origami* é um papel plano que se transforma em um objeto bi ou tridimensional. De acordo com [06], essa transformação se dá por duas dobras primárias: dobra-vale e dobra-montanha. Nos papéis da Fig. 1, a linha azul é a dobra-vale e a linha vermelha é a dobra-montanha. O conjunto dessas linhas é chamado de *crease pattern*, literalmente traduzido como padrão de vincos. O *crease pattern* constitui-se como o mapeamento de dobras, ou seja, é uma técnica de planificação onde se determina a localização das dobras de forma precisa para reproduzir determinados modelos de *Origami*, como mostra a Fig. 2.

Além das tradicionais formas de ensino do *Origami* (com aulas demonstrativas, vídeos e diagramas passo

---

[1] Estudante de doutorado, UNESP - Univ. Estadual Paulista, Brasil. laranjasat@gmail.com

[2] Professora, UNESP - Univ. Estadual Paulista, Brasil. thaisueno@faac.unesp.br

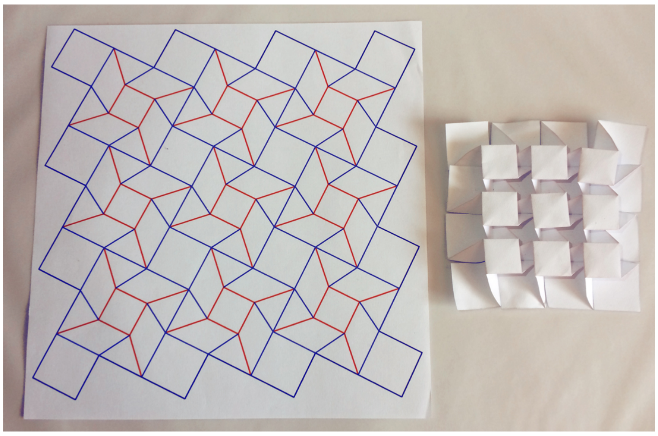


Fig. 2 - *Origami* de Yoshinobu Miyamoto e seu *crease pattern*.

a passo), o padrão de vincos tem sido amplamente utilizado pelos pesquisadores e origamistas para explorarem a fundo os limites da mutação do papel e, em consequência, de outros tipos de materiais com o *Design Adaptado do Origami*. Na Fig. 2, a dobradura se compacta ao ser dobrada. Isso quer dizer que há um potencial de inovação nas técnicas do *Origami*, tendo aplicações na medicina cirúrgica (*stents* dobráveis), na indústria automobilística (*software* de simulação de abertura de *airbags*), na engenharia espacial (painéis solares e telescópios compactáveis), no *design* de moda com roupas adaptáveis a diferentes tipos de pessoas, entre outras áreas. Até esse ponto, cabem estudos mais aprofundados na técnica do *crease pattern*, quer dizer, nos diferentes desempenhos de mapeamento, e tendo como variáveis determinados tipos de *Origami tessellations* e a relação entre os níveis de compactação com a complexidade dos processos de reprodução.

CONSTRUÇÃO E ANÁLISE DE *ORIGAMI TESSELLATIONS* E SEUS *CREASE PATTERNS*

Existem diversos tipos de *Origami* atualmente, sendo que diferentes técnicas levam a diferentes categorias [07][08][09]. O *Origami Tessellations* é escolhido para este estudo, pois sua principal característica é possuir padrões geométricos que se repetem no papel. Essa repetição torna possível uma padronização de construção e análise de diferentes *crease patterns*. São analisados seis tipos de padrões de dobras. De início, um *grid*-base foi montado para construir os *crease patterns* e fazer com que as dobras tenham, aproximadamente, o mesmo tamanho. Adotou-se essa estratégia em busca da comparação de cada tipo de *Origami* de modo mais igualitário possível. Na Fig. 3, o *grid* é desenhado com linhas pretas e as linhas tracejadas vermelhas são um dos padrões de dobras inscrito nesse *grid*.

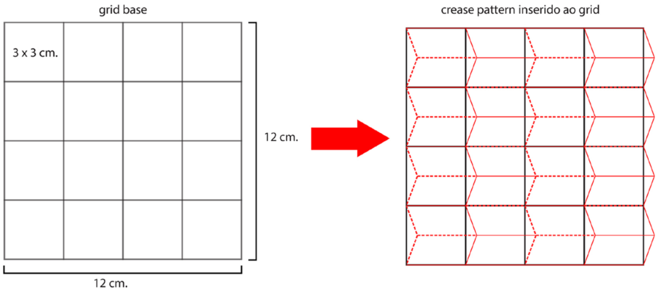


Fig. 3 - *Grid* construído para o estudo de *crease patterns*.

Para diminuir a margem de erro dos tamanhos, priorizou-se que todas as dobras do padrão tenham pelo menos a altura igual a três centímetros, podendo variar a largura e a disposição desses padrões. A Fig. 4 ilustra os seis padrões construídos para análise dentro do *grid*-base. O padrão de dobras *Shumakov* foi desenvolvido por Yuri e Katrin Shumakov e o padrão *Miura* (ou *Miura-Ori*) com *design* de Koryo Miura, foi criado para ser aplicado em membranas espaciais [10]. O padrão *Gjerde* foi desenvolvido inicialmente por Ronald Resch, sendo adaptado por Eric Gjerde. O padrão *Resch* também foi criado por Ronald Resch e o padrão *Mitani* foi feito pelas autoras através do *software Ori-Revo*, criado por

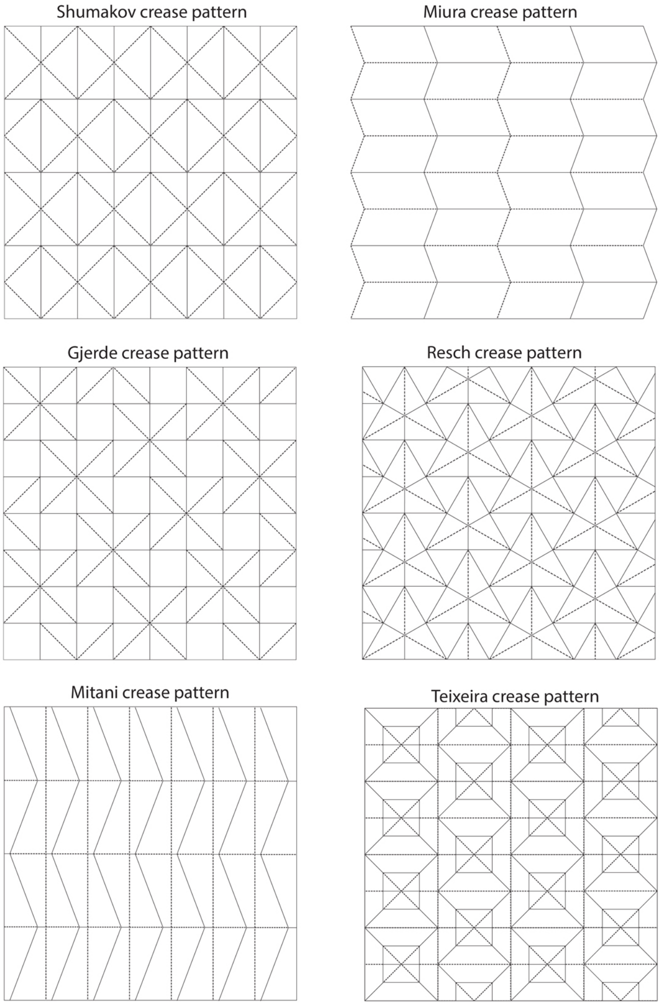


Fig. 4 - Os *Crease Patterns* escolhidos para análise.

Jun Mitani. O Teixeira é um *Origami* criado durante o processo de busca dos padrões existentes. A Fig. 5 mostra os *Origami* e seus *crease patterns*.

Depois de dobrado, cada *Origami* foi manipulado para que fosse possível medir suas formas finais compactas sob os eixos *x* (comprimento), *y* (altura) e *z* (profundidade). Durante o processo de medição, foram considerados os maiores tamanhos que cada figura alcançou em cada eixo. Em seguida, calculou-se o volume total de cada *Origami*, esclarecendo níveis maiores e menores de compactação. A Tabela 1 elenca esses dados.

Tabela 1 - Medidas de compactação de cada *crease pattern*.

Crease Pattern	Comprimento	Altura	Profundidade	Volume Final
Gjerde	6,0 cm	1,5 cm	7,2 cm	64,8 cm^3
Resch	7,5 cm	1,0 cm	6,8 cm	51,0 cm^3
Mitani	3,5 cm	0,5 cm	3,7 cm	6,47 cm^3
Miura	0,5 cm	2,2 cm	5,5 cm	6,05 cm^3
Shumakov	0,5 cm	1,5 cm	7,2 cm	5,4 cm^3
Teixeira	0,5 cm	0,7 cm	7,4 cm	2,59 cm^3

Percebe-se que o padrão menos compacto é o *Gjerde* e o mais compacto é o *Teixeira*. Os que possuem valores de volume final próximos entre si são os *Mitani*, *Miura* e *Shumakov*. Os padrões que menos variam de forma são os *Gjerde* e *Resch*, sendo estes os cinematicamente mais rígidos dos seis considerados (Fig. 6).

Os *crease patterns* possuem tamanhos particulares em cada eixo quando estão no estado compacto. Para melhor observar esse comportamento, o Gráfico I foi construído para comparar as medidas nos eixos *x*, *y* e *z*. Alguns padrões possuem mais economia de espaço que outros, se considerar a altura, largura ou profundidade isoladamente. O padrão *Mitani*, por exemplo, possui a menor profundidade e a menor altura, além de ser um padrão que parte de um quadrado plano para se transformar em uma espécie de cilindro (Fig. 7). O padrão *Miura* é o modelo que mais ganha altura depois de dobrado, por conta do *grid*-base. Para alcançar o grau máximo de compactação, que é próprio desse *Origami*, é necessário que o padrão se repita muitas vezes. No entanto, essa característica não foi alcançada plenamente dentro do *grid*-base (Fig. 8).

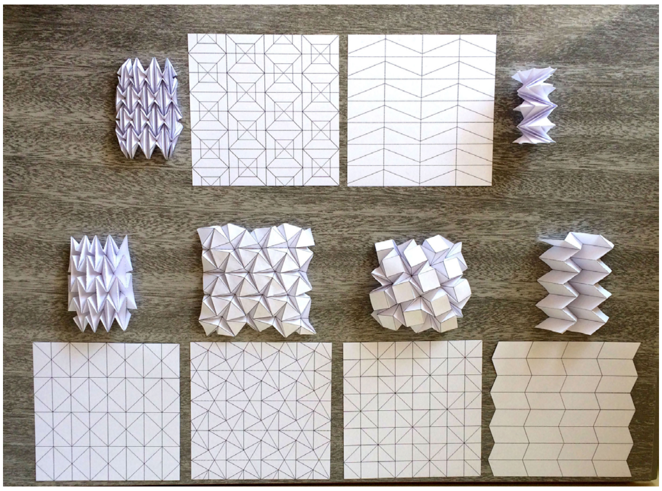


Fig. 5 - Os *Origami* dobrados a partir de seus *crease patterns*.

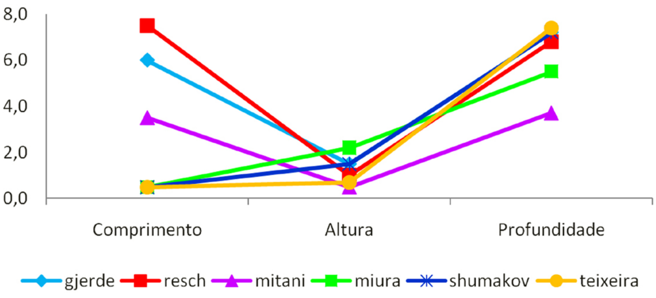


Gráfico 1 - Comparação das medidas
de cada eixo dos *crease patterns*.

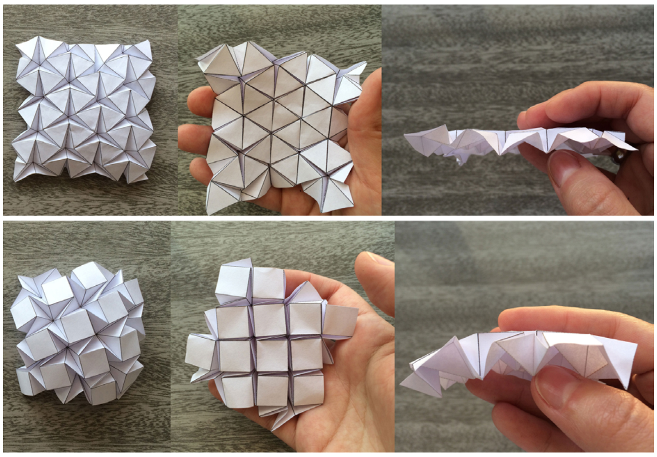


Fig. 6 - Compactação do padrão *Resch* (em cima).
Compactação do padrão *Gjerde* (em baixo).

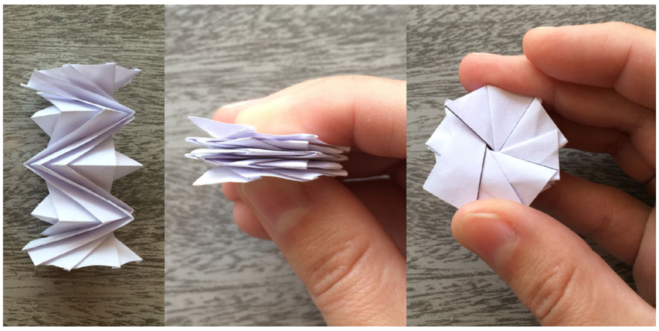


Fig. 7 - Compactação do padrão *Mitani*.

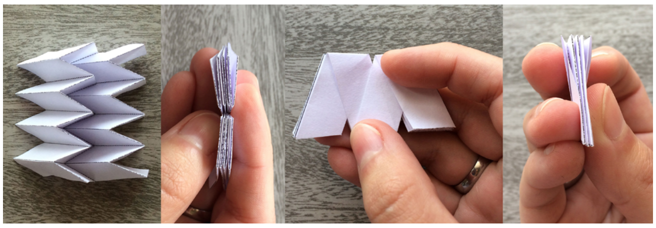


Fig. 8 - Compactação do padrão *Miura*.

De uma maneira geral, nota-se que a forma compacta e o grau de economia de espaço dos *crease patterns* também são influenciados pela disposição e quantidade de repetições ao longo do plano. Considerando a Tabela 1, os padrões *Shumakov* e *Teixeira* possuem desempenhos de compactação muito semelhantes, sendo por isso considerados os modelos mais eficazes, em termos de economia de espaço, dos seis considerados (Fig. 9).

O próximo passo (Tabela 2) foi a contagem de linhas dos *crease patterns* planificados e do número de dobras construídas nos modelos finais. Essa etapa investiga o nível de complexidade necessário para a reprodução de cada *crease pattern,* hipoteticamente, em âmbito industrial.

Tabela 2 - Números de linhas e dobras de cada *crease pattern*.

Crease Pattern	Número de Linhas	Dobra--Vale	Dobra--Montanha	Total de Dobras
Teixeira	114	130	188	318
Resch	81	58	162	220
Gjerde	45	51	112	163
Shumakov	58	64	66	130
Mitani	37	45	49	94
Miura	31	22	30	52

Na Tabela 2, o número de linhas não é o mesmo que o número de dobras, ou seja, uma linha pode construir duas ou mais dobras, dependendo do conjunto. Com exceção dos padrões *Gjerde* e *Shumakov,* os que menos possuem linhas também são os que menos possuem dobras e vice versa. Considerando uma produção em larga escala e os valores de dobras totais, os padrões mais simples de serem confeccionados são o *Miura* e o *Mitani*; ao passo que os mais complexos são o *Teixeira* e o *Resch*. Apesar do padrão *Teixeira* ser o modelo que mais possui economia de espaço, é o mais difícil de ser reproduzido. Por outro lado, a premissa "quanto mais compacto, mais complexo" não permeia por todos os modelos considerados. O *Shumakov*, por exemplo, é um dos mais compactos e possui um número total de dobras e linhas em nível mediano.

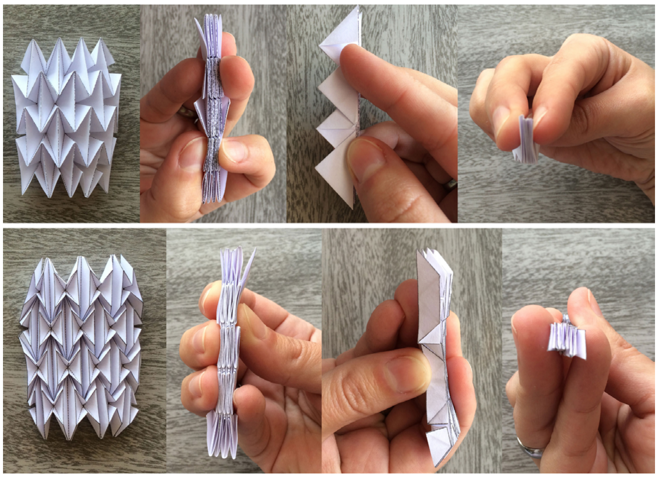


Fig. 9 - Compactação do padrão *Shumakov* (em cima).
Compactação do padrão *Teixeira* (em baixo).

CONCLUSÃO

A análise de medidas de compactação e o número de dobras dos seis *Origami Tessellations* considerados demonstrou que alguns padrões atingiram um nível maior de compactação ao mesmo tempo em que o número de repetição de dobras aumentou. Contudo, tal característica não permeou por todos os padrões, o que indica que o potencial de um *crease pattern* não depende apenas de suas medidas e números de dobras, mas também das suas formas geométricas e a maneira como elas são dispostas sob um plano. Observou-se que, apesar de não possuir grande potencial de economia de espaço, outros padrões possuem relevante aplicação dadas as situações apropriadas. Foi observado que os diferentes *crease patterns* possuem especificidades que podem auxiliar os processos criativos, possibilitando tanto a economia de espaço quanto o dinamismo e a adaptação de formas. Espera-se que estudos futuros com o mapeamento de dobras encontrem mais benefícios para o *Design Adaptado do Origami*.

Samanta Aline Teixeira
e Thaís Regina Ueno Yamada

REFERÊNCIAS:

[01] Lang, R. J., Iverson, P. W., & Yim, M. (2011). *Origami 5: Fifth International Meeting of Origami Science, Mathematics, and Education*. Massachusetts: A K Peters.

[02] Zirbel, S. A., Lang, R. J., Thomson, M. W., Sigel, D.A., Walkemeyer, P. E., Trease, B. P., Magleby, S. P. & Howell, L. L. (2013). Accommodating thickness in origami-based deployable arrays. In *Proceedings of ASME 2013 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference*. doi: 10.1115/1.4025372.

[03] Xu, L. Z., Shyu, T. C., & Kotov, N. A. (2017). Origami and Kirigami Nanocomposites. ACS Nano, 8, 7587-7599. doi: 10.1021/acsnano.7b03287.

[04] Francis, K. C., Morgan, D. C., Rupert, L. T., Magleby, S. P., Lang, R. J., & Howell L. L. (2014). From crease pattern to product: considerations to engineering origami-adapted designs. In: *Proceedings of ASME 2014 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference*. doi:10.1115/DETC2014-34031.

[05] Verrill, H. (1998). Origami Tessellations. *Bridges: Mathematical Connections in Art, Music, and Science*, 1, 55-68.

[06] Dureisseix, D. (2012). An overview of mechanisms and patterns with origami. *International Journal of Space Structures*, 27, 1-14. doi 10.1260/0266-3511.27.1.1.

[07] Teixeira, S. A. (2017). Design do origami: um estudo sobre técnicas projetuais com dobras. Master's dissertation, São Paulo State University, Bauru, Brazil.

[08] Yamada, T. R. U. (2016). Estruturas flat foldable em Bambu Laminado Colado baseadas em técnicas de dobra e corte do origami e do kirigami. PhD thesis, São Paulo State University, Bauru, Brazil.

[09] Gjerde, E. (2008). *Origami Tessellations: Awe-inspiring geometric designs*. Massachusetts: A K Peters.

[10] Miura, K. (1985). Method of Packaging and Deployment of Large Membranes in Space. In *31 st Congress of the International Astronautical Federation*. Tokyo, Japan: The Institute of Space and Astronautical Science report.

As figuras, tabelas e gráfico são da autoria de Samanta Aline Teixeira e Thaís Regina Ueno Yamada.

STRUCTURAL ANALYSIS OF A PARAMETRIC FORMFINDING. CASE STUDY: CASINO IN STUTTGART

Víctor Rodríguez Izquierdo [1]

ABSTRACT

In projects with complex geometries, digital production interface to structural analysis is enormously benefitted through parametrization. A design and structural analysis linked platform is seen as a necessary step contributing to a fully integrated digital construction process.

KEYWORDS: parametric, *grasshopper*, digital fabrication, structural analysis.

INTRODUCTION

A new, gold-shining, 3D overhanging façade for the Casino in Stuttgart was designed to renovate and celebrate the 20th anniversary of the construction. The architectural firms from Stuttgart, *SL-Rasch* and *Designtoproduction* were commissioned for the project. The Stuttgart office of *Mayr | Ludescher | Partner* completed the team providing the structural engineering as sub-consultants from *SL-Rasch* (Fig. 1).

METHODOLOGY

Made out of "gold" pieces as a symbol of nights of luxury and profit and a "dice"-shape, it emulates the casino games. The intervention has a clear goal, a sign transversely to the road that will claim the attention of the drivers. Almost 1700 (477m^2) gold-plated stainless steel elements, each one different but all of only 1.5 millimeters thickness were installed on the wall, which extends 46 meters from the outer-most point of the facade over the foyer to the end of the large playroom. In order to support the geometry, a unique and complex sculptural stainless steel structure had to be developed (Fig. 2).

Not only element dimensions change but also, the game is actually pending on the angle of the panels with the horizontal plane. It goes from almost 90° to less than 20°. This makes that the composition changes, from the inner downer to the outer upper corner, from almost flat vertical to a maximum cantilevering "rock" system. Digital manufacturing was accomplished, despite of the

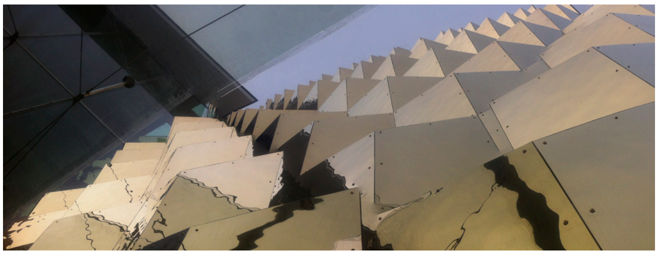


Fig. 1 - Outer gold-elements.

Fig. 2 - Inner and outer gold-elements.

---

[1] *Mayr | Ludescher | Partner*, Spain. victor.rodriguez@mayr-ludescher.de

complexity of the global image, through a geometry parametrization game developed by the architects of *Designtoproduction*, a renowned company specialized in complex geometries. Workshop drawings were fabricated as a direct input to the steel manufacturer, using the 3D-modelling software *Rhinoceros*, its plug-in *Grasshopper* and the script-editor *Phyton* (Fig. 3).

With high time pressure as one of the biggest pre--conditions, the project needed to be ready for the 20th anniversary of the Casino, and because of its limited budget, the concept has to be carefully "standardized" to be feasible. Stainless steel panels had to be 1.5 millimeters; thicker gold elements were not possible because of the delivery time. Approximately 14.000 screws were needed only for the connections between plates (Fig. 4).

Every three-elements dice was pre-assembled and fixed in a two-way cantilevered U-beam. They were hanged on horizontal, 12 millimeters thick, flat stainless steel profiles and connected to each other through two different connection types: a 6- and a 2-panels connection pieces. The horizontal 1078 lasercutted stainless steel parts of the substructure were then punctually fixed at the wall through L-plates and anchor bolts (Fig. 5).

The fact that around one third of the elements are outside of the building's envelope makes a clear distinction for the later on structural analysis, as wind and temperature actions have to be considered. A second topographical distinction is made through a single row of elements disposed on the roof, as these elements have higher wind loads (wind coming from both sides) and less supporting points; they had to be structurally considered as a different group and system. More connecting pieces were needed so that elements could maintain still their thickness (Fig. 6). Since all the panels are different from each other, the connections in between were standardized through 7712 laser-edge plates. The connection between twopanels, which is placed in the middle of every dice side, has 1.5 millimeters for the inner elements and 2 millimeters for the outer ones. Pieces were laser-perforated along their bending axes so each independent connection could be brought to their final angle manually on site by the mounting team. The "master" piece of the system is the connection between six panels located in every panel corner where no U-beam ends. They are all 1.5 millimeters and composed with two parts of 4 "leaves" and were laser-perforated along the bending axes.

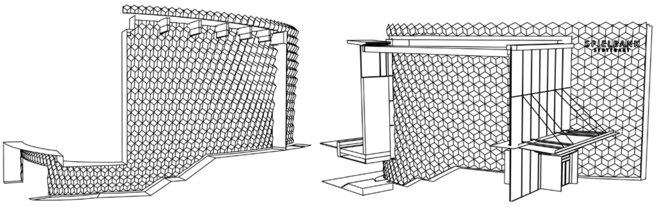


Fig. 3 - 3D-Rhino Model.

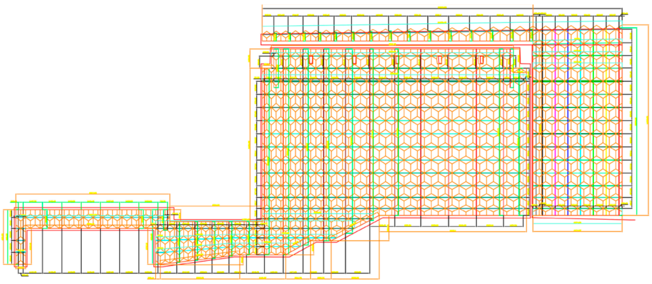


Fig. 4 - Developed Geometry (intermediate stand).

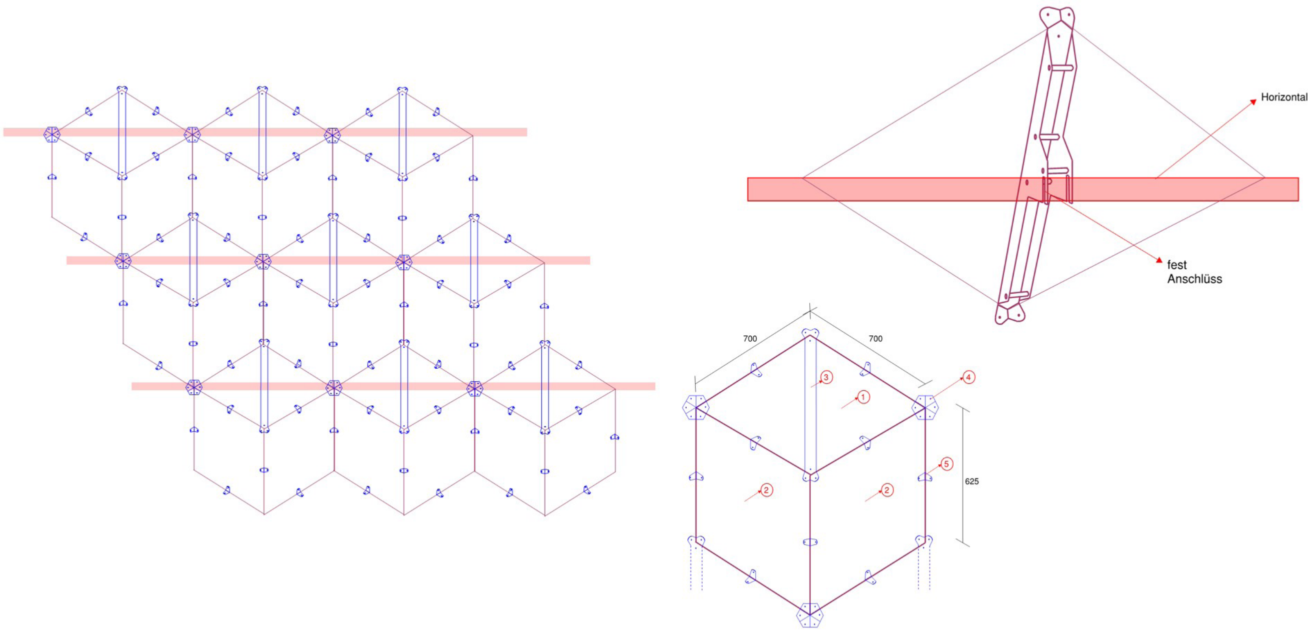


Fig. 5 - Assembly process.

Connections were structurally modelled in the FEM software *Axis VM12*, and in order to reproduce the same supporting conditions, structural engineers from *Mayr | Ludescher | Partner* had to re-built, as finite elements in 3D-modelling, the whole exterior system and not only the single connections or the worst element cases. Circular washes of 2 millimeters were added to every mechanical fixation, not only to increase the thickness of the connections but also to create a fixed connection to the panel (Fig. 7).

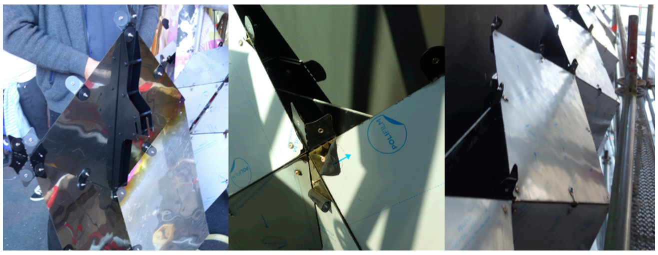


Fig. 6 - Simplified system.

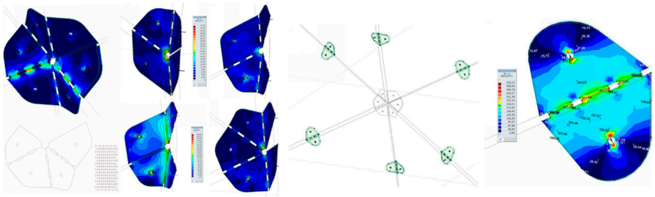


Fig. 7 - 6-, 3- and 2-panels connection; FEM Model.

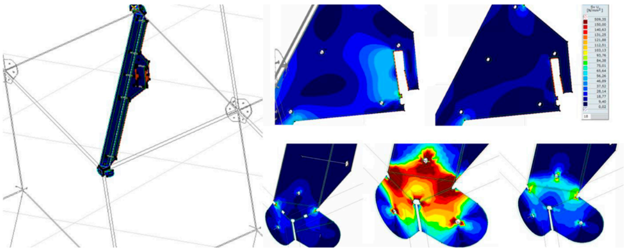


Fig. 8 - U-beam FEM Model.

As every connection angle is different, and in order to be able to perforate the gold elements at the same distance to the edges, every connection piece perforation was located at a different place along the stainless steel plate. All this complex system would not have been possible without a digital programming of the workshop drawings.

U-beams were also perforated to allow manual bending: thickness are 1.5 millimeters inside, 2 millimeters in general outside and 3 millimeters in the upper outer rows where not only the wind loads are higher but also bending moments increase as the dices cantileverate the most (Fig. 8).

Another example of how parametric modelling adapts to different conditions is given at the edges of the intervention. The same principle was adopted to solve the connection of the gold panels and the perimeter stainless steel black plates (Fig. 9).

CONCLUSIONS

There were several prototypes built that were determined for different stainless steel qualities (EN 1.4301 and 1.4571) the maximum thickness and the relation in between perforations and material that will still allow the edge manually bending on site. Cost had an important role here as not only it was also about reducing the amount of material, it was about reducing the number of independent perforations in the plates as

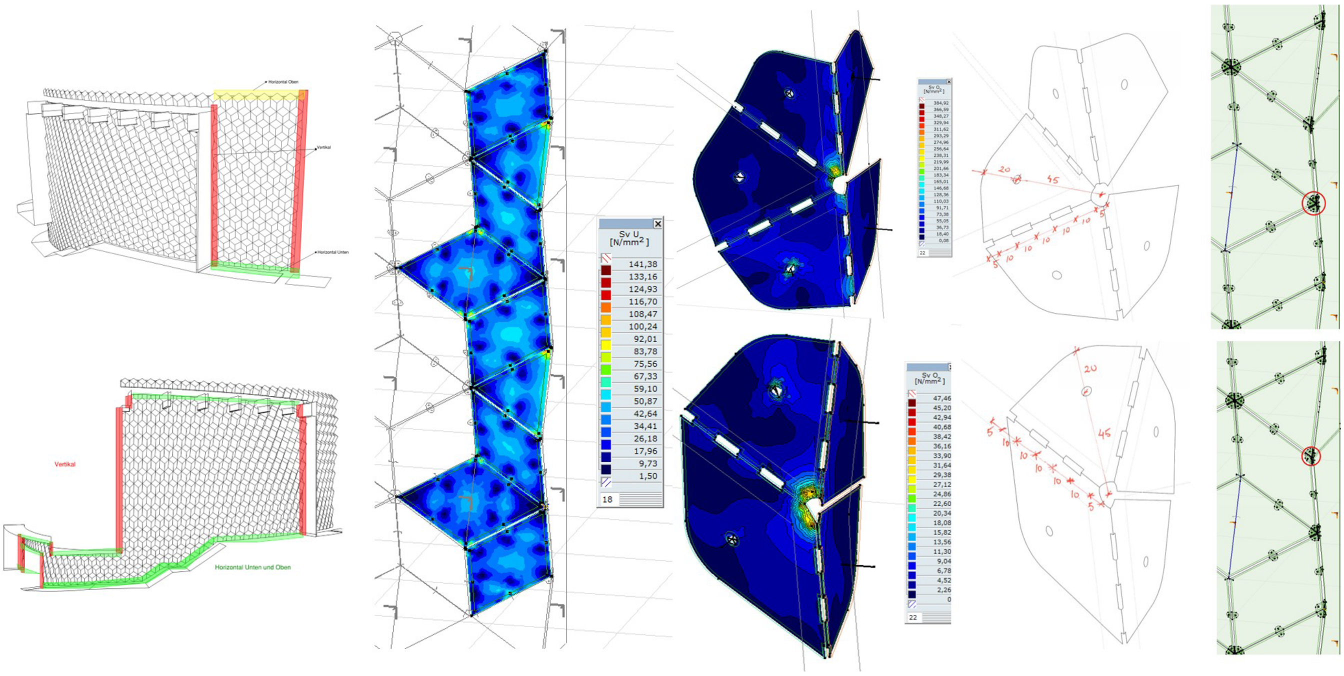


Fig. 9 – Edge conditions; FEM Model.

the laser would need to stop for each of them and make the whole process more expensive.

Under special conditioning of time pressure and budget, parametric models, with a huge amount of different elements, suppose not only an extra effort in terms of production time, which becomes impossible without automatic digital direct input to the manufacturer, but also in pre-assembling and mounting time. Simplification measures, as manual bending of the connection pieces to their final angle, have to be taken to equal the time balance. Building testing prototypes and structural engineering analyses play a very important role to test and develop this measures prior to the final mounting of the 'sculpture'.

Víctor Rodríguez Izquierdo

ACKNOWLEDGMENTS

The author is grateful to Mr. Guido Ludescher for the opportunity to learn from this awesome project, his support and helpful advices and extends his sincere appreciation to the architect Mr. Arnold Walz, (*Design2Production*, Stuttgart) for his awesome geometry lead and collaboration.

Figs. 1, 2, 5, 6, 7, 8 and 9: ©*Mayr | Ludescher | Partner*

Figs. 3, 4: ©*Designtoproduction*

FORM'S AGE
Maria João de Oliveira Sousa Pinto [1]

ABSTRACT

The need to control Forms has been gradually becoming a 21st Century obsession, not only to architects but also among several other areas. The Form has to be perfect according to predefined stereotypes: bodies, cars, gadgets, buildings must have a perfect form to be loved. This deep control about all forms that surround us, make us part of an era that can be called Form's Age.

Form's control is a structural concern when it comes to architecture, and architects deal with it in their everyday work as it is its core since the first sketches until the building on site. Being able to control form is the core, as without it, architects aren't capable of mastering it in order to make it buildable. The question is therefore how can form be controlled and designed to be constructed has it was effectively planned.

KEYWORDS: architecture, form, geometry, Oscar Niemeyer, Zaha Hadid.

QUESTIONS OF FORM

The question of Form and how to represent it has its origins in Antiquity, and appeared before the capability of using technical drawing as representation. The approaches to representing and controlling Form (conceived to be built) was firstly done by written descriptions and informal drawings as, for instance, in the Classical Order Standards' definition by *Vitruvius Threat*, 1st Century B.C.. In this case, the Form was described in a text, without any specific technical drawing support.

When Man started to control technical drawing, such as perspective, it became much easier to represent and explain what was supposed to be built, and architects could start building new original forms. At the same time, with the easiness of controlling simple geometrical volumes, the architect could also improve and focus on the design of interior spaces.

The research on space designing also developed and spread to public spaces and to the building of new landscapes (Fig. 1).


Fig. 1 - Plaza of St. Peter in Vatican, Bernini. 17th Century.

The architect gets more and more aware of the magnificence of designing the interior space, although controlling it meant, quite often, having to move and live on the site, as it happened with Gaudi in *Sagrada Família* (Fig. 2). In this case, construction was controlled through some drawings, but mostly by models (some of them 1:1). The *Sagrada Família*'s project was so complex and used such organic geometries that the architect could not use only technical drawings to support the construction on site.

---

[1] Portugal. mjoaopinto@pfarquitectos.com

Fig. 2 - Sagrada Família in Spain, Gaudi 19th Century.

The sumptuosity of the interior space created make us understand that this is a unique building, with several complex geometries. Taking into account the primitive facilities used by Gaudi, we can say that Form was controlled specially in the architect's mind.

Later, in the 19th Century, with the Industrial Revolution and with materials' evolution, the design of form begins to reach new goals and attach new areas of knowledge. Materials started to be tested and used to their limits, as did form and space (e.g. the Eiffel Tower as an example of iron and steel architecture) and space.

In fact, the use of those new materials became so important that, together with the appearance of Modernism, originated new thoughts, such as the Functionalism (Fig. 3) and Minimalism thinking, where form was taken on has a direct result of function. The non-straight lines and angles were "banned" giving place to the famous Mies Van der Rohe's "*Less in More*" (Fig. 4).

The approach to form is a consequence of the huge technological evolution that widespread with Modernism. Before that, form followed strict rules defined by architectonical styles, but now it is facing new directions. The loss left by the disappearance of architectonical styles among with the constructive technics evolution allows architects to focus on the essence of the building structure and its function, denying the use of ornament that supports Minimalism.

As a natural evolution, architects curiosity leads them to new attempts, and if we take a look at contemporary architecture, we realize that form isn´t at all just a direct result of function. The continuous technological development together with materials and constructive processes evolution, move architects towards other research themes. Furthermore, the constant presence of visual information leads to other goals and points of view that take an important role in the building's design. Nowadays, architects are taking form to its limits having in consideration the domain of the materials that allow forms to be materialized. Within this context, discussions about form and its role in the process of designing are taking place in several worldwide conferences.

So what is in charge? Function or form? Definitely, nowadays form isn't a direct result of function, as we can see for instance in architectural competitions - with the same program (and function), buildings proposed by different architects are totally different one from another. This proves that the relationship between function and form is not as direct as minimalists used to defend.

Contemporary architects move along with all those evolutions, searching for forms with different basis, sometimes influenced by their School and local environment, and others, influenced by globalization. This research lead architects to complex geometries. This geometric complexity makes it harder to have complete control of the form, and architects are faced with the question of how they can properly control it. Should they base themselves on geometry, on drawing, on computer aided design, on models…? Is it a matter of being expedient when it comes to working with computers or is it still necessary to control form by sketching or by defining its geometrical principles? Can the architect control the form or is it in charge of the design process?

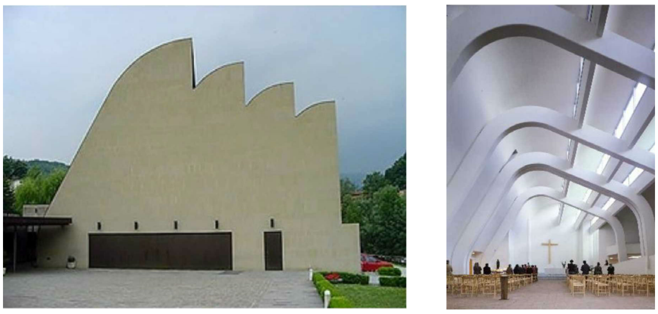


Fig. 3 - The Assumption of Mary in Italy, Alvar Aalto. 20th Century.

Fig. 4 - Crown Hall in Chicago, Mies Van Der Rohe. 20th Century.

And does function still have any influence in the final form? These questions are extremely important when it comes to design consistently and they origin form's questions on form's Era.

Eduardo Souto Moura says in an interview that "…the architecture that most of the architects studied no longer exists."[2] [01], but contemporary consistent architecture is only possible if it bases itself on the same principles that guide ancient and modern architects: the goal of designing Space (which is of course limited in its interior and exterior by form itself). Space is still the leading figure in the architect's work. To help us understand the evolution of form that controls and surrounds space, we took in consideration two architects who won the *Pritzker Prize*: Oscar Niemeyer (1988's *Pritzker*) and Zaha Hadid (2004's *Pritzker*). These two architects designed in different decades, but both had, apparently, no prejudice towards using and experiment forms. The reason for choosing these two architects has to do with their work, that is strictly related with the control of non-simplex geometric forms, and due to the quotation of the *Pritzker Prize* jury himself, that mentioned form:

> "*Recognized as one of the first to pioneer new concepts in architecture in this hemisphere, his designs are artistic gesture with underlying logic and substance. His pursuit of great architecture linked to roots of his native land has resulted in new plastic forms and a lyricism in buildings, not only in Brazil, but around the world.*" [02]
>
> "*Clients, journalists, fellow professionals are mesmerized by her dynamic forms and strategies for achieving a truly distinctive approach to architecture and its settings. Each new project is more audacious than the last and the sources of her originality seem endless.*" [03]

The connection between these architects and the questions about form is quite an assumed commitment, and therefore this was taken as the correct choice.

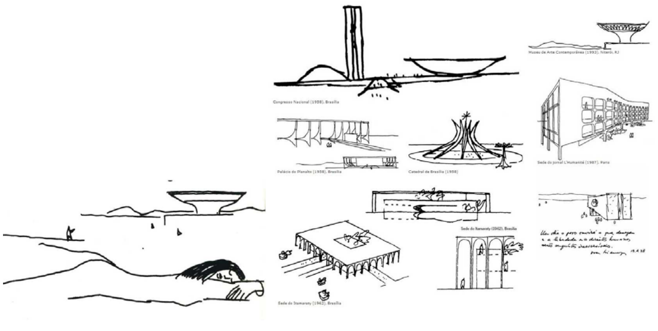


Fig. 5 - Oscar Niemeyer's sketches .

To develop this theme, firstly, we analyzed the architects' thoughts towards form principles, and then, by analyzing their projects, design processes, and used geometries, we could come to some conclusions about how final form is influenced by the process of design: sketching *vs* computer aided design.

OSCAR NIEMEYER

Oscar Niemeyer is a Brazilian Modernist architect known for his experiments in concrete to create new volumes and forms. He was a "*bon vivant*", in love with his city, sea and life in general, which gave him the pleasure to design. By analyzing his way through the design process, we can conclude that his buildings started to be designed as a little sketch in a simple paper, followed by drawn images of parts of the building imagined. In Niemeyer's process, the volume is thought as a series of views along a path that is sketched in order to show what his mind had imagined (Fig. 5).

As mentioned before, Niemeyer usually started his projects by sketching small hand drawings in a 1:500 scale, showing a drive through his buildings with several perspectives. Then, those drawings would be given to his collaborators so that they could test his idea by making models of the designed volumes. If the architect

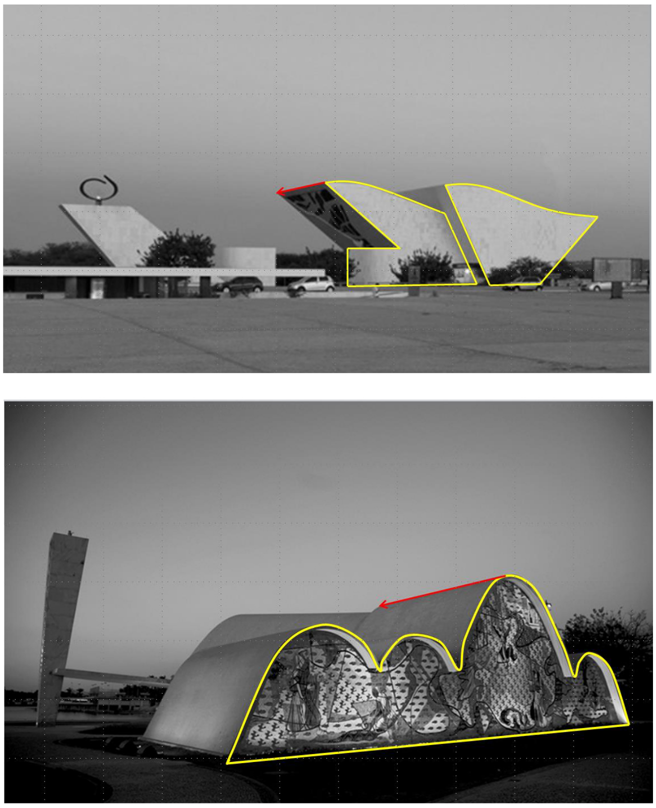


Fig. 6 - Tancredo Neves' Pantheon and
São Francisco de Assis Church, Oscar Niemeyer.
Extrusion of the yellow line. Analysis by the author.

---

[2] Original quote:
"…a arquitetura que muitos dos arquitetos estudaram acabou."

accepted the volume, the final technical drawings were made and the project was prepared to be built.

The main material used by Niemeyer was concrete, which allowed him to construct curved forms. We can assess that the geometry of the form is based on the definition of a closed line that was then extruded or revolved according to the purpose (Figs. 6 and 7).

ZAHA HADID

Zaha Hadid was born in Iraq and after studying mathematics in Beirut, she went to London to study architecture. Her first experience as an architect was working with Rem Koolhaas but soon she started working on her own. Her approach to design was based on the use of complex geometries, starting to design focusing on the form of the building and gradually giving more importance to the interior spaces created in the process.

Her deconstructive thoughts made her experience new design approaches. Her buildings started to be drawn and painted in a very personal way (Fig. 8). but soon she based the buildings' form on the definition of parametric design and complex geometries, done by an algorithm that generates the shape's geometry. After having had a plastic approach, she would try out her forms with 3D physical and virtual models. The guidelines were generated mathematically according to geometrical principles, allowing the control of the organic curves, so that they would become buildable (Fig. 9).

In Zaha Hadid's case, the control of form is made by defining guide lines with complex geometries, most of them as a result of a mathematical formula, which was developed in her mind (in canvas or in the computer). Forms were created with parametric design, and needed a strong team work to be developed. With this parametric approach, Zaha was capable of bringing to construction some apparently non-buildable volumes (Fig. 10).

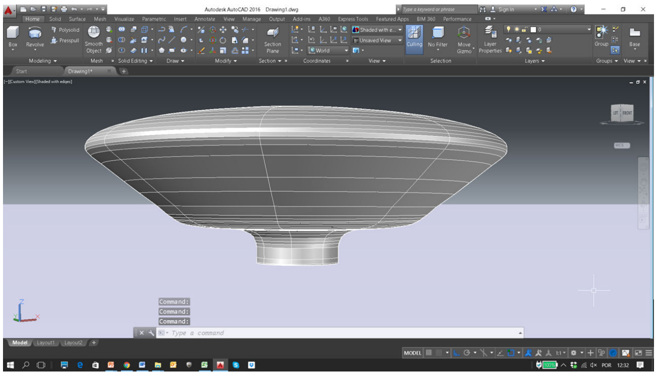


Fig. 7 - Niteroi's Art Museum, Oscar Niemeyer. Section revolved. Analysis by the author.

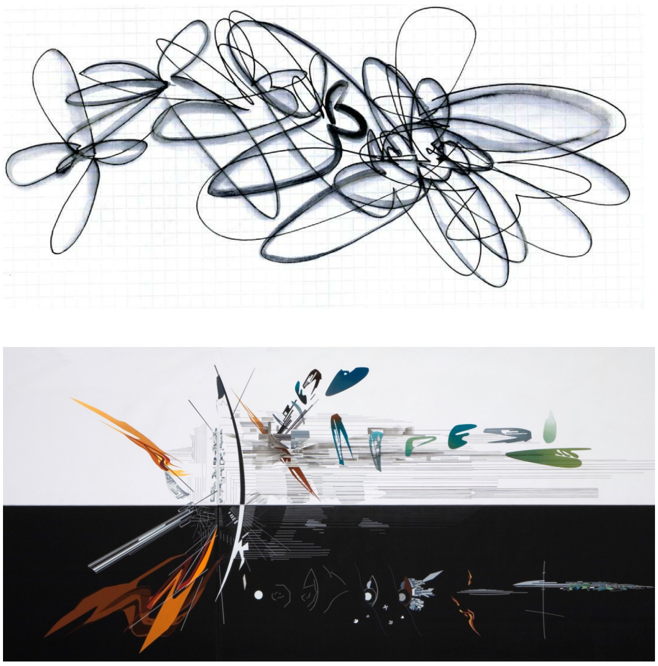


Fig. 8 - Drawing and Painting , Zaha Hadid.

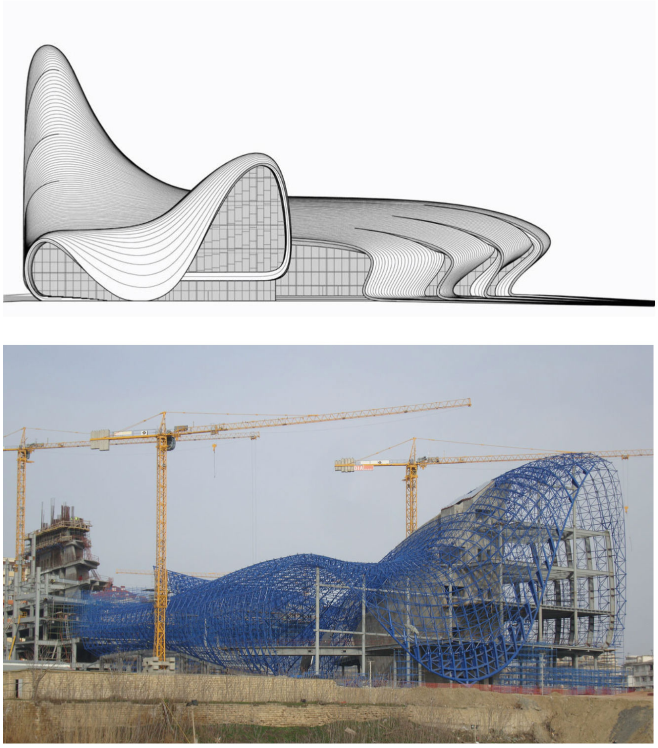


Fig. 9 - Heydar Aliyev Center's, Zaha Hadid. Parametric Design + Virtual Model + Site + Concluded building photos.

CONCLUSIONS

The paradox between these two architects' approaches to design and to forms' control has enlarged throughout the years, supported by all the technological developments (materials, aided design, hardware …).

Comparing the two architects chosen (Oscar Niemeyer and Zaha Hadid) we verify that the processes of creating the resultant forms are different. Niemeyer used an

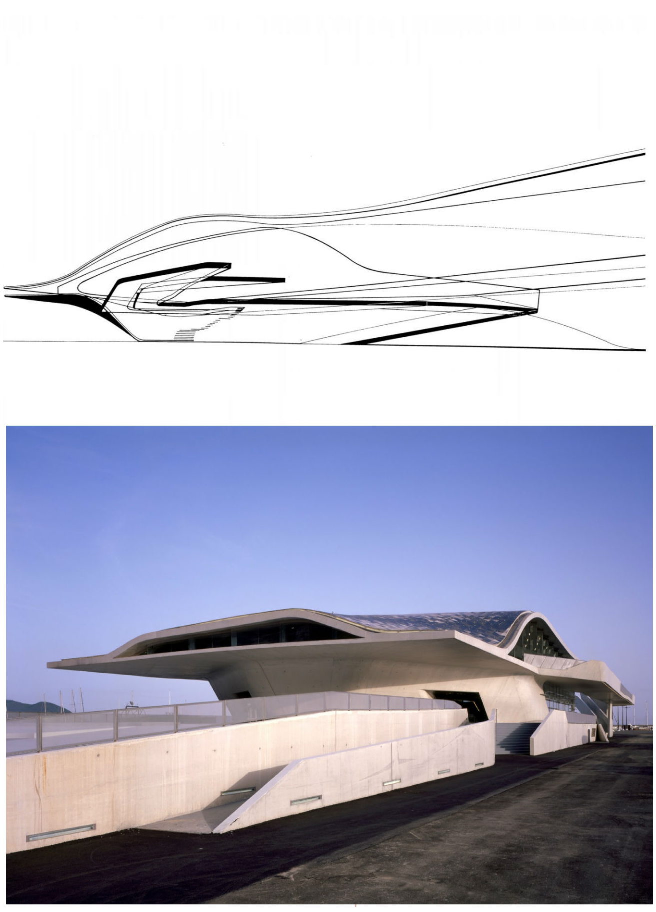


Fig. 10 - Maritime Terminal in Salerno, Zaha Hadid
Guidelines + Virtual Model + Building.

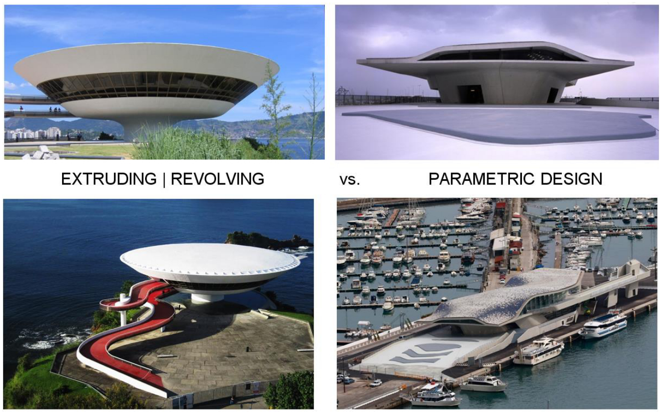


Fig. 11 – Niteroi's Art Museum by Oscar Niemeyer
and Maritime Terminal in Salerno by Zaha Hadid.

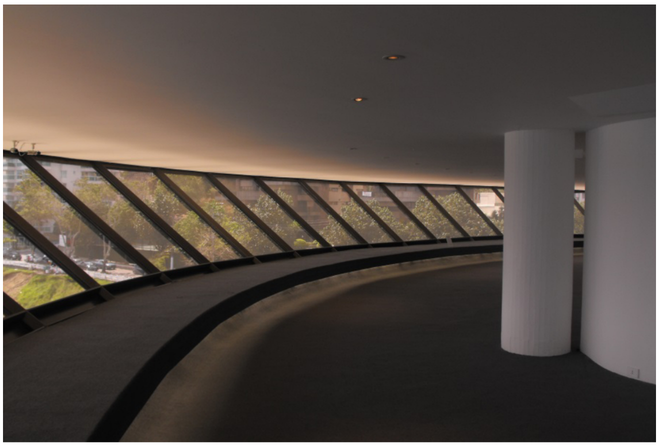


Fig. 12 - Niteroi's Art Museum by Oscar Niemeyer.

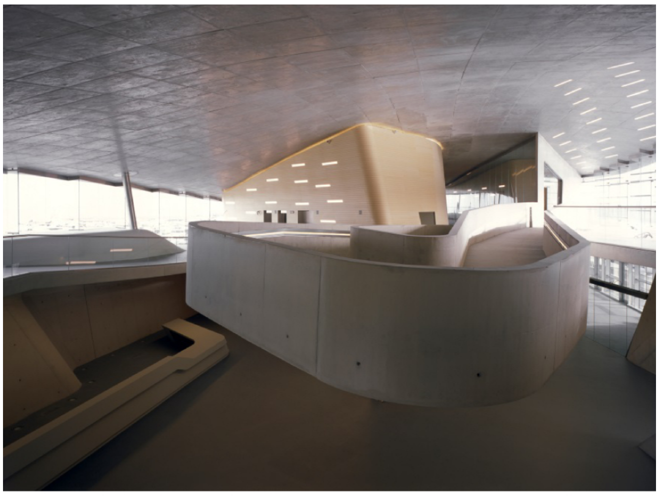


Fig. 13 - Maritime Terminal in Salerno by Zaha Hadid.

extruding/revolving method and Zaha based forms on the use of parametric design, leading us to the conclusion that the buildings' forms are, among other things, a result of the era in which they are produced. Even though those architects had no preconceptions about shapes, the tools through which they developed their projects were significant to the results they obtained (Fig. 11).

Zaha could evolve to more complex forms due to parametric design, which solved, apparently, all the forms by making them buildable and helped define the structure and skin of what was going to be built. Oscar Niemeyer's works show that "playing" with forms doesn't have to be strictly done with technological advanced software or hardware. But if we look at Zaha Hadid's works, we realize that the use of these technological tools helps the designing process especially when it comes to building them in site (Figs. 12 and 13).

Nevertheless, it is clear that although the resulting form is influenced by computer tools and that design can be based on parametric formulas, geometry, sketching and the choice of the materials to be used have an extremely important role on forms' result. However, the use of such complex forms takes a greater effort to preconceive the interior spaces which are going to be created, and architects should keep this fact in their minds.

Architects design spaces, packed in forms and, therefore, they need to have total control of these forms, in order to perceive the desired spaces. In order to control shape, architects need to sketch to test forms and then, necessarily, they have to transform them into geometric principles, given by simple geometries or parametric design.

Geometry is still in the essence of forms' control but the technological evolution influences, more than the form, the way through which we control it and the way we explain it to others. Having all this in mind, the Form's Era means mostly that architects can now focus on the coherent spaces they create, as all the available tools are an excellent support to do so.

Maria João Pinto

REFERENCES:

[01] Souto Moura, E. (2016). A arquitetura que eu estudei acabou. Retrieved in March, 2017 from http://www.diarioimobiliario.pt/Entrevistas/A-arquitectura-que-eu-estudei-acabou

[02] Pritzker Prize (2016), Announcement. Retrieved in February, 2017 from https://www.pritzkerprize.com/laureates/1988-niemeyer

[03] Pritzker Price (2017), Purposes. Retrieved in February, 2017 from https://www.pritzkerprize.com/laureates/2004

SOURCES OF THE FIGURES:

Fig. 01 - http://fgsaraiva.blogspot.com/2015/09/praca-de-sao-pedro-sera-um.html, retrieved in 01/06/2017

Fig. 02 - https://pt.wikipedia.org/wiki/Templo_Expiat%C3%B3rio_da_Sagrada_Fam%C3%ADlia, retrieved in 01/06/201.
Maria João Pinto, 08/2016.
https://pt.wikipedia.org/wiki/Templo_Expiat%C3%B3rio_da_Sagrada_Fam%C3%ADlia, retrieved in 01/06/2017

Fig. 03 - https://artchist.wordpress.com/2016/06/28/church-of-the-assumption-of-mary-in-riola-di-vergato-by-alvar-aalto/, retrieved in 01/06/2017

Fig. 04 - https://www.metalocus.es/en/news/restoring-mies-van-der-rohe-sr-crown-hall, retrieved in 01/06/2017

Fig. 05 - http://arquitetesuasideias.com.br/2016/11/10/arquitetos-e-seus-croquis/, retrieved in 01/06/2017
https://diariodearquiteto.wordpress.com/tag/oscar-niemeyer/, retrieved in 01/06/2017

Fig. 06 - https://www.tripadvisor.com.br/LocationPhotoDirectLink-g303322-d4393172-i168410874-Praca_dos_Tres_Poderes-Brasilia_Federal_District.html, retrieved in 01/06/2017.
https://www.guiadasemana.com.br/turismo/noticia/relembre-algumas-obras-de-oscar-niemeyer, retrieved in 01/06/2017.

Fig. 07 - Drawing by the author in *AutoCad*

Fig. 08 - https://frieze.com/article/zaha-hadid-1950%E2%80%932016, retrieved in 01/06/2017
https://www.archdaily.com/798362/the-creative-process-of-zaha-hadid-as-revealed-through-her-paintings, retrieved in 01/06/2017

Fig. 09 - https://www.pinterest.pt/pin/351632683378301003/?lp=true, retrieved in 01/06/2017
https://www.slideshare.net/renurajbahak/presentation-on-architect-zaha-hadid-and-her-work, retrieved in 01/06/2017
https://www.adelto.co.uk/construction-of-heydar-aliyev-center-by-zaha-hadid/, retrieved in 01/06/2017
https://www.archdaily.com.br/br/01-154169/centro-heydar-aliyev-zaha-hadid-architects, retrieved in 01/06/2017

Fig. 10 - https://www.metalocus.es/en/news/salerno-maritime-terminal-zaha-hadid-first-posthumous-project, retrieved in 01/06/2017

Fig. 11 - https://www.archdaily.com/417751/ad-classics-niteroi-contemporary-art-museum-oscar-niemeyer/52128a03e8e44e2030000219-ad-classics-niteroi-contemporary-art-museum-oscar-niemeyer-photo, retrieved in 01/06/2017
https://www.metalocus.es/en/news/salerno-maritime-terminal-zaha-hadid-first-posthumous-project, retrieved in 01/06/2017

Fig. 12 - https://www.archdaily.com/417751/ad-classics-niteroi-contemporary-art-museum-oscar-niemeyer/52128a03e8e44e2030000219-ad-classics-niteroi-contemporary-art-museum-oscar-niemeyer-photo, retrieved in 01/06/2017

Fig. 13 - https://www.metalocus.es/en/news/salerno-maritime-terminal-zaha-hadid-first-posthumous-project, retrieved in 01/06/2017

Boletim da Aproged # 34 - Dezembro 2018 | ISSN 2184-4933

DESENHO DE CONCEPÇÃO EM ARQUITETURA: O PAPEL DO DIAGRAMA NO PROCESSO DE PROJETO

Ana Paula Rocha [1], Debora Mariane Fantinato [2],
Renata Maria Geraldini Beltramin [3] e Daniel de Carvalho Moreira [4]

RESUMO

Pautada na associação entre forma e conteúdo, a arquitetura de Rem Koolhass é ícone da prática diagramática, característica evidenciada na concepção de seus projetos. Dada a importância do diagrama enquanto ferramenta programática na obra de Rem Koolhaas, este artigo teve como objetivo investigar a presença do diagrama na concepção do projeto da *Seattle Central Library* sob a ótica dos princípios estabelecidos pela teoria do *information design*. Este estudo foi realizado mediante um levantamento bibliográfico sobre teoria da informação e programa de necessidades, a produção geral do arquiteto e uma análise gráfica dos diagramas de concepção através do procedimento de desconstrução de camadas de informação. Este estudo não apenas reafirmou o caráter programático do arquiteto Rem Koolhaas como também permitiu a verificação de como a representação gráfica pode ampliar a compreensão do problema de projeto, sobretudo por possibilitar a organização sistemática da informação.

PALAVRAS-CHAVE: arquitetura contemporânea; processo de projeto; programa de necessidades; diagrama arquitetônico; Rem Koolhaas.

INTRODUÇÃO

O arquiteto e teórico holandês Rem Koolhaas - do escritório *OMA* - possui uma importância significativa na produção da arquitetura atual, abrindo eixos de discussão relacionados tanto à teoria quanto à prática arquitetônica contemporânea, e levantando discussões sobre o processo de projeto em arquitetura: “Koolhaas produz uma arquitetura em que conteúdo é forma” [01, p.20]. Sob tal premissa, é conferido à etapa de elaboração do programa de necessidades um destaque estratégico em relação ao contexto geral do processo de projeto do arquiteto. Desse modo, a arquitetura de Rem Koolhaas tem como característica notável a concepção inicial do projeto através da construção de diagramas que definem, dimensionam e distribuem os elementos do programa, evidenciando seus limites e suas inter-relações. A prática diagramática do arquiteto ilustra uma tendência no processo de projeto contemporâneo, solidificando o papel do diagrama como um instrumento útil e efetivo de análise de dados para gerar a forma. Diante da vitalidade do diagrama enquanto ferramenta gráfica de concepção programática na obra de Rem Koolhaas, este artigo tem como objetivo fazer uma investigação da representação gráfica e do processo de projeto da *Seattle Central Library*, importante obra no contexto da produção do escritório *OMA*. Tal estudo é realizado a partir da análise dos diagramas de concepção do projeto sob a ótica dos princípios contemporâneos de representação gráfica de dados estabelecidos pela teoria do *information design*.

INVESTIGAÇÃO

O estudo foi iniciado por um levantamento bibliográfico sobre teoria da informação, processo de projeto, programa de necessidades e da produção arquitetônica do arquiteto Rem Koolhaas.

A sistematização da pesquisa foi realizada recorrendo a um estudo teórico sobre o processo de projeto de Koolhaas aplicado à obra selecionada - *Seattle Central Library* - e a uma análise gráfica dos diagramas de concepção do arquiteto, baseada nos princípios do *information design*.

---

[1] Universidade Estadual de Campinas, Brasil. apaularocha@yahoo.com.br
[2] Universidade Estadual de Campinas, Brasil. fantinatodm@gmail.com
[3] Universidade Estadual de Campinas, Brasil. renelens05@gmail.com
[4] Universidade Estadual de Campinas, Brasil. damore@fec.unicamp.br

Esta investigação justifica-se pela pertinência da análise do processo de projeto contemporâneo pelo viés da representação gráfica da informação, que determina e configura a conformação e a forma arquitetônica.

O PROCESSO DE PROJETO EM ARQUITETURA

A década de 1960 foi um importante período para o pensamento do processo de projeto. Fatores como as crescentes inovações tecnológicas e as transformações sociais, econômicas e culturais conferiram um significativo aumento da complexidade no processo produtivo do edifício, o que implicou a necessidade de repensar o processo de projeto. O movimento dos *design methods* surge neste período como estratégia de organização e otimização do processo de projeto. Diante dessa nova realidade, autores como Jones [02], Asimow [03] e Broadbent [04], discutem a constituição e a dinâmica do processo de projeto a partir da elaboração de métodos de projeto. Embora existam diferenças conceituais e estruturais notáveis entre os métodos propostos por cada autor, o processo de projeto, enquanto conjunto de atividades destinadas à produção do objeto arquitetônico, é essencialmente o mesmo: parte-se sempre de um problema de projeto composto por uma série de aspectos e variáveis; tal problema deve ser minuciosamente analisado, compreendido e registrado para constituir a diretriz da elaboração da solução - ou seja, o projeto arquitetônico. A solução concebida deve, então, ser avaliada e medida, de modo que o projetista ou o grupo de projetistas ateste o atendimento a todos (ou quase todos, conforme os conflitos e restrições identificados) os requisitos de projeto listados na formulação do problema.

Lawson [05] apresenta um mapeamento generalizado do processo de projeto, ressaltando que

> *"o que o mapeamento faz é nos dizer que os projetistas têm de reunir informações sobre o problema, estudá-lo, imaginar uma solução e desenhá-la, não necessariamente nessa ordem"* e que os *"mapeamentos tendem a ser, ao mesmo tempo, teóricos e normativos. Parecem derivar mais do pensamento sobre o ato de projetar do que da observação experimental"* [05, pp, 43-48].

De acordo com o autor, o processo de projeto ocorre, essencialmente, em três etapas: análise, síntese e avaliação. A primeira etapa (a análise) consiste no momento em que as principais metas e objetivos do projeto são definidos, bem como o seu contexto de implantação; nesta etapa, são coletadas, caracterizadas e organizadas, conforme o escopo definido, todas as informações necessárias ao início do desenvolvimento do projeto. O principal produto da etapa de análise é o programa de necessidades, documento que dispõe ao arquiteto e aos demais agentes do processo todos os requisitos de desempenho e informações necessários ao início da elaboração da solução de projeto. Na segunda etapa (a síntese) são concebidas possíveis soluções que satisfaçam os requisitos, as restrições, as oportunidades e os parâmetros de desempenho identificados na etapa de análise.

Uma grande variedade de técnicas pode ser utilizada nesta etapa, tais como *brainstorming*, emprego de soluções precedentes e métodos sistemáticos. Os projetistas devem retornar à etapa de análise sempre que necessário, seja pela inclusão de novas informações coletadas, ou pela identificação de conflitos entre requisitos de desempenho e/ou soluções parciais, dentre outros. O principal produto da etapa de síntese é o próprio projeto arquitetônico. Por fim, a terceira etapa (a avaliação) compreende a verificação da solução escolhida em relação aos requisitos, restrições e critérios de desempenho primariamente estabelecidos. O objetivo desta etapa é detectar deficiências no projeto antes do encaminhamento para a construção, momento a partir do qual eventuais alterações tornam-se progressivamente lentas e caras. Para o atendimento a este objetivo, deve ser possível retornar a qualquer uma das etapas anteriores. A avaliação pode ocorrer através de métodos específicos.

O PROGRAMA ARQUITETÔNICO E SEU PAPEL ESTRATÉGICO NO PROCESSO DE PROJETO

Produto principal da etapa de análise, o programa de necessidades é um componente fundamental do processo de projeto, visto que, enquanto formulação detalhada do problema de projeto, influencia diretamente a qualidade e a acurácia da solução de projeto a ser desenvolvida. Duerk [06] descreve o programa como um método sistemático de investigação para delineamento do contexto onde o projeto deve ser desenvolvido e definição dos requisitos a que um projeto bem-sucedido deve atender. A autora define a programação arquitetônica como um processo de gerenciamento da informação que possibilita que o tipo certo de informação esteja disponível na etapa certa do processo de projeto e que as melhores decisões possam ser tomadas na formação do resultado do projeto do edifício. Para ela, a programação também é o processo que cria a estrutura para a realização dos sonhos, expectativas, desejos e aspirações dos futuros usuários do edifício. A autora refere ainda que a programação é o plano de procedimento e organização de todos os recursos (equipes, informação, orçamento, etc.) necessários para o desenvolvimento de um projeto diante de um contexto e de requisitos específicos.

Uma vez que constitui o conjunto de diretrizes que conduz todo o processo de projeto, o programa de necessidades é a fonte de informações na qual as soluções de projeto são apoiadas, tendo, portanto, o próprio projeto arquitetônico completo como resultado. Lawson [05] permite concluir que, a partir da observação de práticas de projeto, é comum a identificação de entraves ao longo da concepção de soluções arquitetônicas em decorrência da má formulação dos problemas de projeto.

ORGANIZAÇÃO E CARACTERIZAÇÃO DE DADOS: A ESTRUTURA DO PROGRAMA DE NECESSIDADES

Visto que o programa de necessidades tem como objetivo o levantamento e a organização de informações necessárias ao desenvolvimento do projeto, torna-se necessário que sua elaboração compreenda alguma estrutura organizacional, através da qual as informações são dispostas sob determinados critérios. Trata-se de uma estrutura conceitual, sob a qual não apenas as informações do contexto são organizadas como também podem ser compreendidas as relações funcionais entre o contexto e o espaço físico.

> "*Uma estrutura conceitual para o programa arquitetônico é um procedimento para orientar o raciocínio e estabelecer uma conduta de trabalho no levantamento das informações sobre o contexto*" [07, p.43].

A respeito dos critérios de organização de dados no programa, identificá-los conforme suas naturezas tem sido o caminho mais adequado. É a partir dessa premissa que Lawson [05] busca uma estrutura conceitual de programação baseando-se nas naturezas básicas de problemas, estrutura esta que ele intitula como "modelo de problemas de projeto" e que pretende ser uma diretriz de entendimento da natureza dos problemas, e não um método de projeto. O autor caracteriza o problema de projeto como multidimensional, o que implica a necessidade de uma solução integrada, que atenda simultaneamente a todos os componentes do problema. O modelo de [05] compreende uma configuração tridimensional matricial, composta por três variáveis: os agentes do processo (projetistas, clientes, usuários e legisladores), os tipos de restrições (internas e externas) e as naturezas de restrições (radical, prática, formal e simbólica).

O *INFORMATION DESIGN* COMO INSTRUMENTO DE ANÁLISE

Ao longo das últimas décadas, as questões acerca do tratamento gráfico da informação deram origem a um campo específico de conhecimento: o *information design* (ou *design* da informação). Embora não haja um consenso sobre a existência de uma prática particular em *information design*, teorias e estratégias de *design* da informação vêm sendo discutidas e estabelecidas por estudiosos de *design*, sobretudo desde a década de 1990, conforme aponta Jacobson [08]. Um dos exemplos mais expressivos dos estudos sobre *design* da informação é a obra "*Envisioning Information*", de Edward Tufte [09], que traz uma discussão acerca de técnicas essenciais de representação gráfica da informação de acordo com suas qualidades. As técnicas descritas por Tufte [09] são:

- O aumento em número e densidade das representações que podem ser dispostas em superfícies planas, bem como sua organização em estruturas de diagramas;
- As micro e macro leituras, quando a partir de um único desenho ou diagrama é possível compreender um todo, os detalhes individuais desse todo em diversas escalas ou as relações presentes entre os detalhes;
- A organização em camadas e segmentações, para que se possam revelar detalhes e complexidades sem que haja excesso de informações e, consequentemente, incompreensão por parte do espectador;
- A disposição de pequenos elementos múltiplos, que permitem a visualização comparativa de objetos;
- O emprego da cor como elemento de informação, dado através da atribuição das cores certas aos locais adequados;
- As narrativas de espaço e tempo, para registro de informações relativas à realidade mundana em suas qualidades de tridimensionalidade e temporalidade.

No âmbito deste estudo, o *information design* tem como foco tratar a relação entre o diagrama e seu observador. Sendo assim, os principais aspectos tratados pela literatura classificam a qualidade gráfica como a capacidade do diagrama de simplificar, elucidar e acelerar a captura de dados por parte deste observador; ou seja, "um bom diagrama tem como função tornar a absorção da informação mais rápida em um espaço reduzido" [09]. Já na arquitetura, e principalmente neste estudo de caso, o diagrama pode ser utilizado como um gerador de forma. Trata-se, neste caso, da relação entre o diagrama e o arquiteto, observador de seu próprio produto, atribuindo ao diagrama uma maior responsabilidade no projeto resultante.

OS DIAGRAMAS DA BIBLIOTECA CENTRAL DE SEATTLE

Utilizando os diagramas da Biblioteca Central de Seattle para exemplificar a característica do diagrama como gerador de forma, a análise permite evidenciar sua relação com o programa. Koolhaas parte de um programa padrão de biblioteca para em seguida desconstruí-lo a fim de atingir melhor desempenho às funções pré-estabelecidas. Os usos do edifício foram separados em blocos independentes de atividades e interligados através de escadas rolantes e elevadores, focando-se em uma circulação verticalizada.

Como apresentado na Fig. 1, o arquiteto utiliza um gráfico de barras ou colunas empilhadas (*Stacked Bar Chart*) para reorganizar e intercalar os ambientes, apresentando soluções que correspondem às exigências listadas no programa de necessidades (Fig. 1). Essa forma de organização de programa é utilizada para fundamentar a linha de raciocínio executada. Um mesmo diagrama pode conter inúmeras informações. Cada uma dessas informações corresponde a uma camada (Fig. 2), um conteúdo a ser transmitido. Esse procedimento adotado separa e analisa essas camadas isoladamente, de forma qualitativa e quantitativa dentro do recorte de estudo. Qualitativamente, as camadas foram enquadradas, dentro de sistemas separados, em: sistemas de projeção, sistemas de composição e sistemas de conteúdo, descritos da seguinte forma:

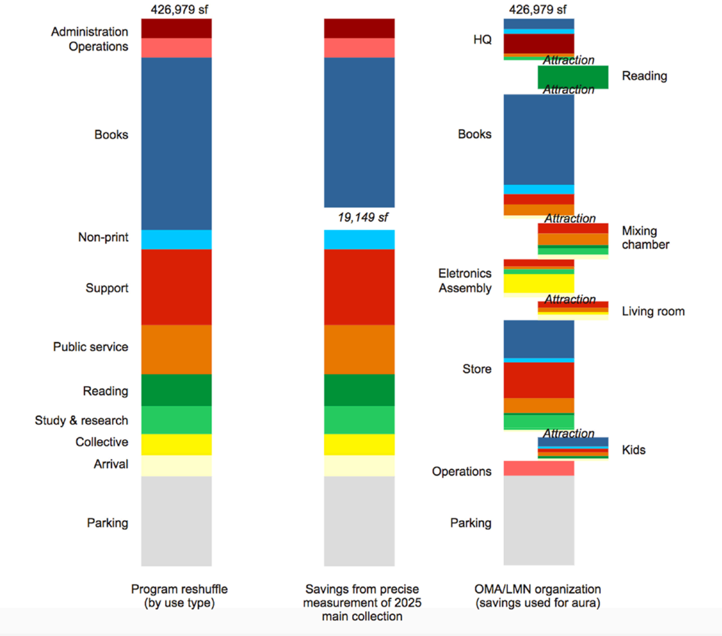


Fig. 1 - Diagrama de síntese do programa
Fonte: Imagem adaptada de Cecília [10, p.72].

- Sistemas de projeção: correspondem ao plano em que as representações são projetadas. Abrangem a visão do observador sobre o objeto representado (como por exemplo, vista planimétrica, seccional ou perspectiva).
- Sistemas de composição: compreende a disposição do diagrama na folha. Dependendo de como as informações podem ser representadas, ele pode ser único, ou estar agrupado a figuras complementares, e ainda assim, constituir um único diagrama. As relações entre essas figuras seguem princípios de agrupamento por proximidade, continuidade e semelhança, que expressam o potencial da organização para a compreensão das informações transmitidas através do diagrama. Diagrama sequencial é um expressivo exemplo de tipo pertencente a este sistema.
- Sistema de conteúdo: são as representações carregadas de informação, geralmente apresentam signos (cores, flechas, pontilhados, etc). Revelam o assunto de que se trata o diagrama. Em maior diversidade de tipos, um diagrama pode ter várias camadas dentro da classificação deste sistema. Diagramas de circulação, programáticos e sensoriais são exemplos de tipos dentro do sistema de conteúdo.

ANÁLISE GRÁFICA

Os dados levantados no estudo qualitativo direcionaram-nos para uma investigação mais concreta das estratégias projetuais do arquiteto. A linha de diagramas analisados permitiu identificar os seguintes pontos:

- Diagrama de síntese de programa (como o da Fig. 1): Numa primeira etapa, é utilizado um pensamento independente de um sistema de projeção, ou seja, desligado da representação arquitetônica como planta ou corte, o que caracteriza o forte caráter programático. É evidenciado que o problema a ser resolvido é a distribuição do programa.

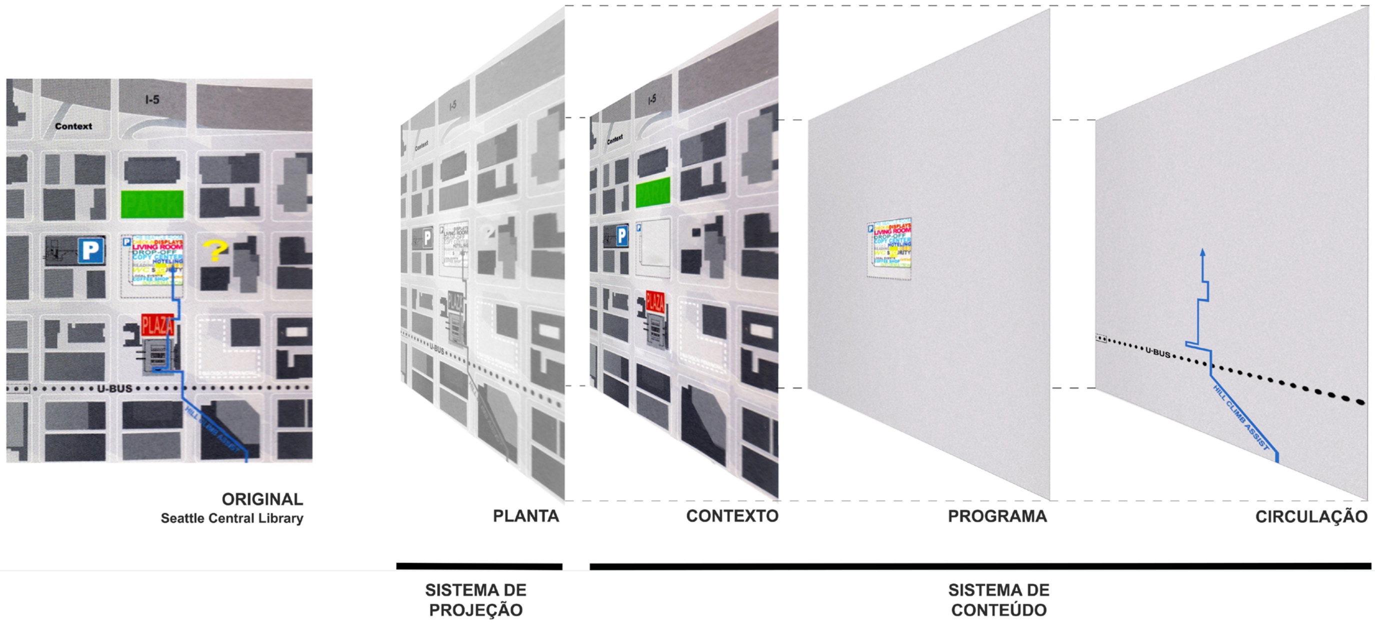


Fig. 2 - Desconstrução do diagrama do projeto da Seattle Central Library
Fonte: Imagem produzida pelos autores, a partir de imagem adaptada [10, p.76].

- Diagramas de contexto (Fig. 2): Neste caso, as interferências de entorno - acessos, contexto, campos visuais, etc. - são postas na projeção horizontal, demonstrando maior preocupação na implantação e delimitando uma área para que o programa aconteça, ainda com uma instabilidade formal.
- Diagramas formais (como o da Fig. 3): Uma vez que o entorno é definido, a projeção de representação passa a ser vertical, o que caracteriza que sua preocupação seja a organização programática. A circulação é pensada verticalmente e pode-se entender que Koolhaas distingue a circulação vertical em duas condições diferentes: uma que conecta apenas os espaços livres (instáveis) e a outra que dá acesso a todos os pavimentos do edifício.

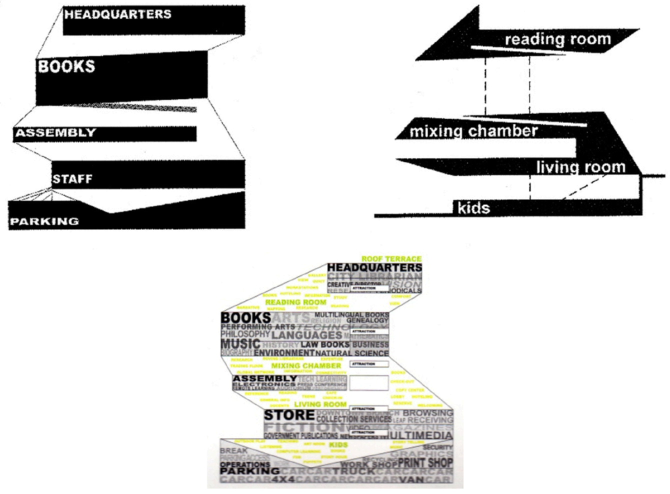


Fig. 3 - Diagramas dos grupos programáticos estáveis e instáveis e diagrama do programa da *Seattle Central Library* Fonte: Imagem adaptada de [10, p.72].

O levantamento permitiu identificar a ligação entre os diagramas e a forma final. Como resultado, é possível reconhecer o padrão de raciocínio dentro do processo de projeto: a análise, definição das principais metas e objetivos do projeto são definidos, bem como a implantação, corresponde aos diagramas de síntese de programa e de contexto (Figs. 1 e 2), realizados, ou sem uma projeção, ou dentro da projeção horizontal e as informações de entorno. Já os diagramas formais (Fig. 3), pertencentes à etapa de síntese, em que são concebidas possíveis soluções, são representados dentro da projeção vertical, sem que em nenhum desses diagramas a camada de programa desapareça.

CONCLUSÃO

O programa arquitetônico é crucial para incorporar as necessidades do usuário em um projeto de arquitetura. Esta investigação reafirma o caráter programático do arquiteto Rem Koolhaas, uma vez que os diagramas de concepção evidenciam a importância do programa no desenvolvimento das estratégias de projetos arquitetônicos complexos.

A experimentação gráfica em questão permitiu a verificação de como a representação gráfica pode ampliar a compreensão do problema de projeto a partir da disposição de uma grande quantidade de informações, de diversas naturezas, muitas da quais ambíguas, e, em desenhos e diagramas capazes de demonstrar relações, hierarquias, pesos, contradições e tantos outros aspectos impossíveis de serem detectados em uma simples disposição de dados. A análise gráfica da *Seattle Central Library* possibilitou uma maior compreensão dos problemas de projeto através de uma organização sistemática da informação, apresentando um levantamento detalhado das possibilidades e desafios inerentes à incorporação sistemática da representação gráfica à elaboração do programa de necessidades.

Ana Rocha, Debora Fantinato,
Renata Beltramin e Daniel Moreira

REFERÊNCIAS:

[01] Eisenman, P. (2013). *Supercrítico: Peter Eisenman, Rem Koolhaas*. São Paulo: Cosac Naify.

[02] Jones, J. (1963). A method of systematic design. In: Jnes, D.; Thornley, D. (Ed.). *Conference on Design Methods. Oxford*: Pergamon Press.

[03] Asimow, M. (1962). *Introduction to design*. Englewoods Cliffs: Prentice Hall.

[04] Broadbent, G. (1973). *Design in architecture*. Nova York: John Wiley.

[05] Lawson, B. (2011). *Como arquitetos e designers pensam.* São Paulo: Oficina de Textos.

[06] Duerk, D. (1993). *Architectural Programming: information management for design*. Nova York: John Wiley & Sons.

[07] Moreira, D.; Kowaltowski, D. (2009). Discussão sobre a importância do programa de necessidades no processo de projeto em arquitetura. *Ambiente Construído*, 9(2), p. 31-45.

[08] Jacobson, R. (ed). (1999). *Information Design*. Cambridge: MIT Press.

[09] Tufte, E. (1993). *The visual display of quantitative information*. 3. ed., Cheshire: Graphics Press.

[10] Cecilia, F.; Levene, R. (2007). Biblioteca Central de Seattle. *El croquis*, 134/135, p. 62-117.

MASTERING PERSPECTIVE IN OBSERVATIONAL DRAWING
Teresa Pais [1]

ABSTRACT

The exercise of spatial representation poses specific difficulties to students of architecture. To identify and understand such setbacks, several students were asked to make a modelled drawing and a contour drawing of three urban spaces with highly differentiated features.

The examination of the students' drawings has enabled the identification of the most recurrent imprecisions, their corresponding spatial position and in which type of drawing they more frequently occur. The data collected suggest that, despite inaccuracies occurring more frequently and clearly in contour drawings, this type of exercise allows the credible representation of a place, thereby stimulating the observation and consideration of aspects related to the materiality of the surfaces and the constructive definition of the elements that are part of that space.

Modelled drawings and contour drawings - the latter preferably developed under given coordinates - are key and complementary exercises for developing the ability to control shape and space.

KEYWORDS: observational drawing, spatial representation, perspective, visual perception.

INTRODUCTION

In most architecture courses, Drawing is included in the group of subjects that constitute the first year of training for future architects. In this subject, the topic of spatial representation is generally approached in a systematic way throughout the school year and it is one of the most motivating for the students (Fig. 1).

Fig. 1 - Photograph taken at the College of Arts in October 2017, in the first class of Drawing I of the Integrated Master's Course in Architecture of the Faculty of Sciences and Technology of the University of Coimbra, focusing on the representation of space.

Nearly twenty years of pedagogical practice has shown that most students struggle with common difficulties, which translate into recurrent imprecisions that may vary, depending on the mode of drawing adopted. In order to understand those failures and their context, a comparative examination was conducted, focusing on a group of drawings representing a number of highly differentiated spaces, that attempted to identify which mode of drawing (modelled drawing or contour drawing) more frequently occurs, which inaccuracies are more prevalent and in which spatial situation they arise.

The option for modelled drawing as a term of comparison with contour drawing was chosen because, first of all, they stand as opposite modes in perceptive and procedural attitudes and features: from the general to the specific in modelled drawing, and from the specific to the specific, in contour drawing; for trial and error and correction in modelled drawing, as mistakes are not advisable in contour drawing; for the plastic diversity of a graphic register in modelled drawing and the linear exclusivity of the contour drawing. Secondly, because the characteristics of these two modes allow decisive common denominators to be established, such

---

[1] Professora Auxiliar, Faculdade de Ciências e Tecnologia da Universidade de Coimbra, Portugal. tpais@darq.uc.pt

[2] Based on the definition by Vieira [01], drawing mode should be understood as "the attitude which we assume or which conditions the graphic act. The basic or elementary modes are four: sketch, detail drawing, contour drawing and modelled drawing." [01, p. 36].

as the scale of drawings and the possibility of each being carried out in an identical length of time and in similar conditions of the support format.

METHODOLOGY

Twenty-four students were requested to make, sequentially and in the same period of time, a modelled drawing and a contour drawing of three spaces in the city of Coimbra: an indoor public space, the entrance hall of the Faculty of Medicine; an outdoor public space of linear development, the access way to the Botanic Garden; access way to the Botanic Garden; and an outdoor public space of distribution and permanence, the Largo do Romal (Fig. 2).

RESULTS

A comparative examination of the assignments showed that most inaccuracies occurred in both modelled drawings and contour drawings, but more frequently in the latter. When comparing the attitude and process involved for each mode, one easily understands the reason why inaccuracies are more frequent in contour drawing. Modelled drawing allows students to evolve by trial and error and correction, enabling them to rectify their records so that the final image may be as consistent as possible with both the perspective principle, which they are theoretically familiar with, and the reality they are observing. On the contrary, in contour drawing, since the image is built from the specific to the specific, and configurations are considered separately, the process leads to the accentuation of imprecisions in the alignments of the compositional architectural structures. The desired regularity of elements undergoes inconsistent changes; shapes are sometimes distorted and other times disproportionate, and the geometrical progression of perspective is, thus, compromised. In addition, the drawing student is procedurally prevented from using a support structure that would ensure, for example, the convergence of lines to a given point. Given that the basic principles which govern the representation of space have not been fully assimilated yet, it is not surprising that in pairs of modelled and contour drawings sequentially made, imprecisions in the inclinations drawn are obvious in the former, whilst inconsistencies in perspective are more noticeable in the latter. Some of the imprecisions occurring more frequently are the accentuation of the angles of almost horizontal lines and the smoothing of larger angles. Fig. 3 shows, from left to right, a modelled drawing and a contour drawing made by the same student, as well as a photograph taken by him from the place where he was standing while drawing. Comparing the images, one may see that, in both modes, the blue line, of a larger amplitude, was smoothed, whereas the red ones were accentuated. In most cases, the horizontal lines which the students tend to accentuate correspond, in general, to conditions of proximity to the horizon line, that is, the contact line of vertical planes with the ground. On the other hand, the steeper lines, drawn with smoother angles, coincided with aspects located at higher levels or closer to the

Fig. 2 - Images of spaces where the exercises done by the students took place.

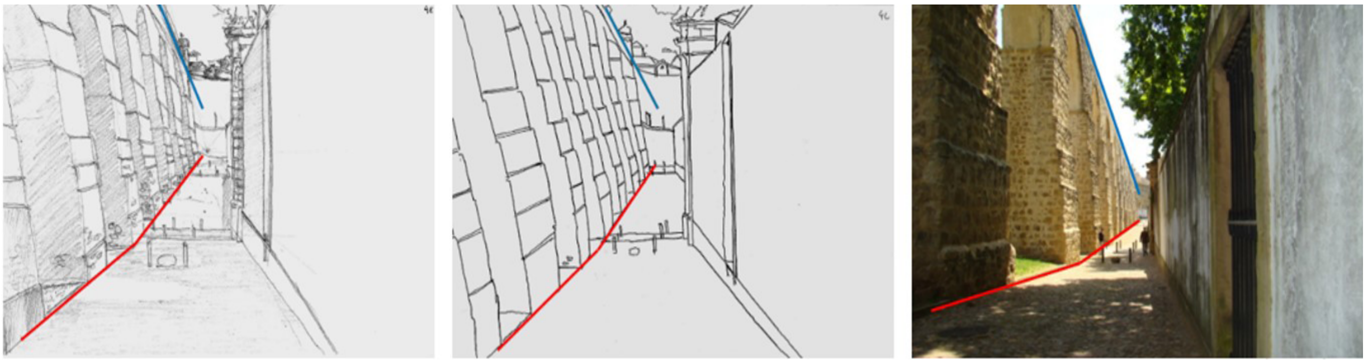


Fig. 3 - Side entrance to the Botanic Garden: modelled drawing (A4, 40 minutes, graphite), contour drawing (A4, 40 minutes, marker) and photograph taken by the student from where he was when drawing.
In the representation of space, one of the most common tendencies is the accentuation of the angle of contact of the vertical planes (walls or buildings) with the ground, giving the illusion that the observer is located on a higher level.

observer. These imprecisions resulted in the elevation of the horizon line and, thus, the feeling that the observer was on a higher level.

Associated with the propensity to accentuate and smooth certain lines, another frequent inaccuracy was detected: the tendency to draw lines of very different angular values and contrary inclinations with relatively close, sometimes almost symmetrical, amplitudes. This is illustrated in Fig. 4, in which the red and blue lines have similar angular values that are symmetrical in relation to the horizontal dotted line. This tendency was observed in both modes and results from the fact that lines assuming very small or very great angular values do notseem credible to most students. Consequently, they tend to smooth them. This propensity is mentioned by Sommers [02], as students (and people in general) avoid the difference - the very high and the very low, the very wide and the very narrow, etc. - by tending to normalise it.

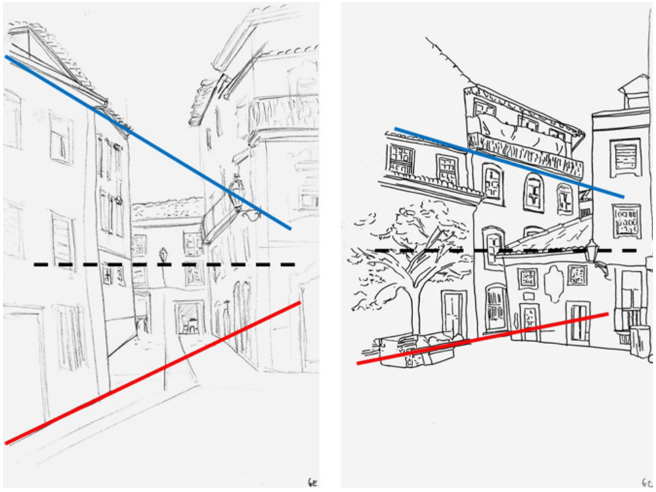


Fig. 4 - Largo do Romal: modelled drawing and contour drawing (A4, 45 minutes, graphite and marker, respectively). In these examples, lines were marked whose angles with the horizon line should have been clearly different. However, amplitudes tend to assume values which are relatively close, almost in symmetry.

Another frequent inaccuracy was the convergence of lines, not to a vanishing point but to several ones. This fragility was also identified in both modes, as shown in Fig. 5. On comparing the processes, it comes as no surprise that this flaw has been detected in contour drawings due to the absence of an auxiliary support structure. In modelled drawings, the persistence of this inaccuracy may be justified by the difficulty in understanding the concepts involved in the perspective system, which causes students to pay little attention to the surroundings. Instead, they make sure that the geometrical coherence of the set is assured to the detriment of the observation of each spatial element. The perspective structure was often recorded "by heart", but disconnected from perception. In some cases, students only incorporated information gathered through observation after they had seen the structural web which organises the image established. For example, let's have a look at Fig. 6. What were the students looking at, when the photograph was taken? Only one of them (at the top on the left) was actually looking at the object which was the focus of his attention as he stretched out his arm to measure. Of the others, three were absent-minded and the remaining ones were looking at their drawings.

Looking for one minute and drawing for half a minute: this is the great requirement for those who wish to draw from life. Few students can do it at the early stage of learning. At this point, the eagerness to draw is greater than the will to look, and this is the tendency which must be fought. Another flaw, also very common in both modes, is the tendency to draw lines which are parallel between them when they should be convergent. The drawings in Fig. 7 are examples of this. It was noticed that they are often located outside the focused central image that characterises the visual field. In the drawings collected, these lines frequently coincide with the more slanted ones, and the propensity is not

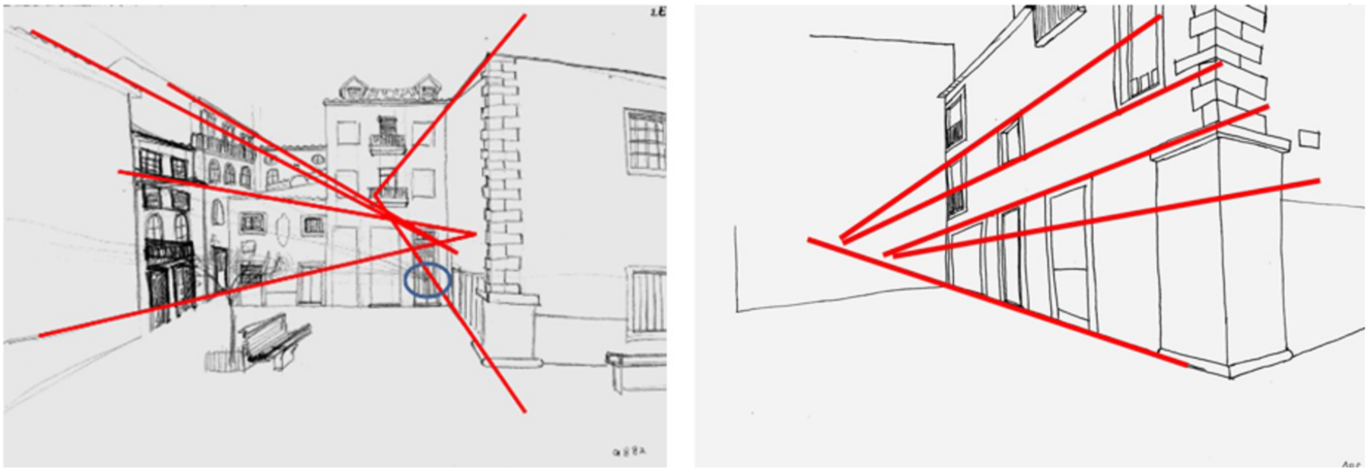


Fig. 5 - Largo do Romal: modelled drawing and contour drawing (A4, 45 minutes, graphite and maker respectively). Example of a modelled drawing in which the marking in the drawing of a “central convergence point” (marked in blue) failed to avoid lines converging to other points. In the contour drawing, the convergence to a point (not marked) was not considered either.

towards accentuating the angular values, as it should be, but rather to reduce them. The persistence of this inaccuracy in both modes may be related to the fact that, in modelled drawing, these lines are located in the periphery of the drawing sheet, being, as such, more difficult to relate to the vanishing points marked on the sheet or the horizon line. The more distant they are from these, the subtler the differences between angular values to be recorded seem, and less evident the convergence of those lines becomes. On the other hand, in contour drawing, students did not have any reference which might guide them to be more accurate in transcribing the perceived angular amplitudes in the paper, for which reason they found it difficult to do so correctly. Goldstein [03] provides an example associated with the representation of the human figure to explain this propensity:

> “*When confronted by a foreshortening situation, he [the beginner] will often substitute remembered notions of the subject’s forms for what he actually sees. He may see a foreshortened leg, but will draw it as if it were fully extended.*” [03, p.127].

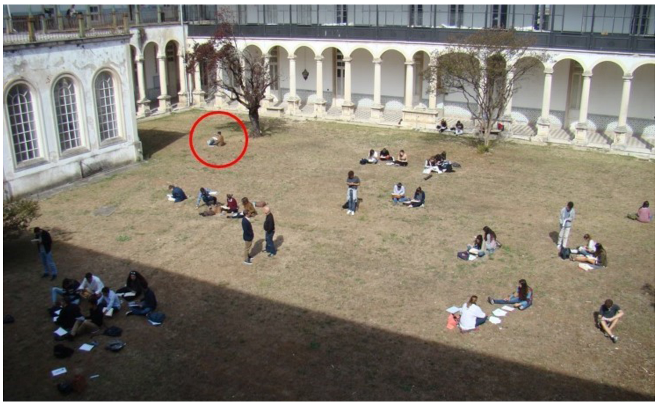


Fig. 6 - In the first class of Drawing I focusing on spatial representation, only one student (circled in red) is looking at the object that was the focus of his attention.

In the drawings of spatial representation, the same has happened. The converging lines were there, before the students’ eyes, but because convergence was neither obvious nor plausible, they “made the lines out” parallel and drew them with the bias provided by the knowledge they had about their location in space.

The tendency to draw parallel lines instead of converging ones may also be explained by the perceptive posture in both modes that, although different, led to the same flaw. In modelled drawing, despite the location of one or more vanishing points being, as a rule, primarily established in the drawings, the look upon the subject is fleeting, swift and not really focused. As Almeida [04] explains, the kind of perception required involves peripheral vision with a predisposition to apprehend generic information of an imprecise nature. In contour drawing, the kind of perception involved is focused on small consecutive areas of the subject. The look becomes static and concentrates primarily on the surface’s features, whilst focusing on each alteration perceived and recording, through lines, each one of those changes. At a given moment, one fails to be aware of the image as a representation of the subject itself, as one’s attention concentrates on very specific aspects; the drawing student “records what he sees, not because he understands it but because he is seeing it” [01 p.34]. In drawings of spaces, the tendency to draw parallel lines becomes harder, due to the scale of representations and the predominance of straight lines.

The most common faults appeared both in modelled drawings and contour drawings, with the difference that the gravity and number of inaccuracies were slightly higher in the latter. This asymmetry results from the various aspects related to the characteristics of the contour mode: the lack of an auxiliary support structure

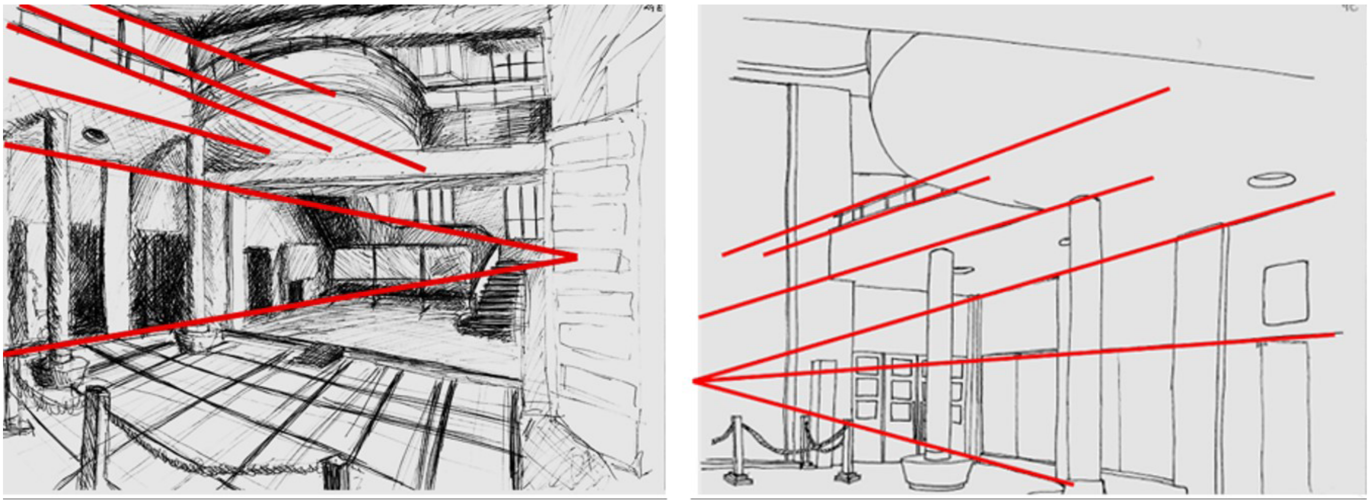


Fig. 7 - Entrance hall of the Faculty of Medicine: modelled drawing and contour drawing (A4, 40 minutes, ballpoint pen and marker, respectively). This pair of contour drawings is one of many cases in which the convergence of lines to a vanishing point is unequivocally at the level of the observer. But in terms of higher levels, in addition to not converging to the point, they are virtually parallel between them.

with lines and reference points, the impossibility of making corrections, the kind of perception involved, resulting from the fact that vision is focused on isolated elements, and a consequent tendency for what one knows about the object to interfere in the drawing, because of the scale and predominance of straight lines in the subject depicted. However, close comparative examinations show a crucial difference: despite the flaws, the students' attention was directed to specific aspects of shape and space, with the drawing showing details connected with the material characterisation of surfaces and the formal and constructive definition of space (Fig. 8).

Given the seriousness of inaccuracies identified in some contour drawings, an attempt was made to find if any specific circumstance common to the drawings might have taken place. And, in fact, some circumstances occurred when the boundaries of the visual field were less comprehensive (Fig. 9). This suggests that increasing the representation scale may translate into added value for a good performance by the students in the practice of perspective drawing, thus minimising the tendency shown in this mode to accentuate or reduce the inclination of certain angles.

CONCLUSION

The comparison drawn between modelled drawings and contour drawings shows that the most frequent inaccuracies are common in both modes, being especially visible in the accentuation of angular values, the inversion of the angles' directions and the lack of convergence of lines in vanishing points. The spatial situations in which these failures occur are the same in both modes, because students either accentuate less steep lines or smooth lines of greater amplitude.

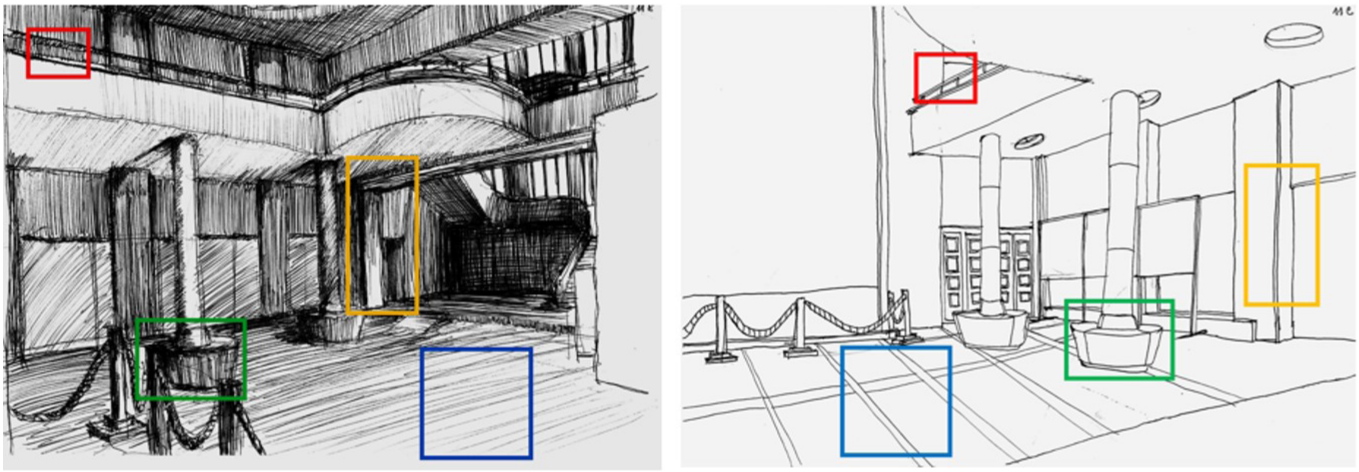


Fig. 8 - Entrance hall of the Faculty of Medicine: modelled drawing and contour drawing (A4, 40 minutes, ballpoint pen and marker, respectively). Despite the inaccuracies, the student's attention in the contour drawing was directed to specific aspects of shape and space, noticing details more related to the material characterisation of surfaces and its formal and constructive definition.

Fig. 9 -Entrance hall of the Faculty of Medicine (A4, 40 minutes, ballpoint pen); Largo do Romal (A4, 45 minutes, marker).
In the contour drawings in which the limits of the visual field were less comprehensive, the angular imprecisions were of little relevance.

Although most inaccuracies occur more frequently and more evidently in contour drawings, the conclusion may be that, despite this being a process that involves the construction of the image from specificity to specificity and does not allow trials or corrections, it allows students to obtain credible representations of a place.

Therefore, it encourages careful observation and the consideration of aspects related to the materiality of surfaces and the constructive definition of the elements which participate in the depicted space. Thus, modelled drawings and contour drawings, the latter made preferably under given coordinates, constitute essential and complementary exercises for developing the ability for students to control shape and space.

Data also suggest that a space represented in contour drawing will be more coherent and supported if the scale of the elements represented is greater and, consequently, if the visual field is more limited.

Teresa Pais

REFERENCES:

[01] Vieira, J. (2009). *As matérias do desenho: teorias e métodos da disciplina de Desenho I*. Retrieved in October, 2016 from http://pintovieiraensinodesenho.blogspot.pt/p/programas-da-disciplina.html

[02] Sommers, P. (2009). *Drawing and cognition: descriptive and experimental studies of graphic production processes*. Cambridge: Cambridge University Press.

[03] Goldstein, N. (1977). *The art of responsive drawing*. New Jersey: Prentice Hall.

[04] Ameida, P. (2008). *Mancha Directa: desenho como percepção e expressão de valores tonais*. Guimarães: [S.n.]. Doctoral thesis presented at University of Minho.

THOUGHT MODELLING IN DESCRIPTIVE GEOMETRY

Constantino Rodrigues [1]

ABSTRACT

This paper investigates, analyses, reflects upon, and draws conclusions on the regulation of thought/ discourse in science and in descriptive geometry. After stating the synonymy of thought and discourse, as advanced by Júlio Fragata, this study identifies Michel Foucault's *Procedures for Controlling and Delimiting Discourse*, Louis Althusser's *Ideology*, and Bento Caraça's vision of science, and their effects on the former. We also observe the divergences in knowledge and truth, ratified by Karl Popper.

Based on Foucault´s categories, we aim to, first, question the discourses on the genesis of both Analytical and Descriptive Geometry, where the ontological, the logical, and/or the discipline truths don't always conform and, secondly, question the actions on discourse/thought, which are related with Foucault's categories and subcategories and direct them towards Geometry.

We conclude by affirming the existence of the aforementioned procedures, of interferences in the ontological truth, while recognizing the need for surveillance.

KEYWORDS: thought, modelling, discourse, truth, descriptive geometry.

INTRODUCTION

Considering how important scientific thought and discourse are to Geometry, we aim to study both the phenomena and the agents that limit them. We base this intent on social conditioning and in the idea that the scientific community itself cannot escape neither these actions nor their consequences.

Thought Modelling in Descriptive Geometry thus investigates, analyses, reflects upon and draws conclusions on the regulation of thought/discourse in science, and contiguously in Descriptive Geometry as a singular science, from the structuring theme of the Conference Geometrias'17: "*Thinking, Drawing, Modelling*".

DEVELOPMENT/THESIS

The three nuclear terms from Geometrias'17: "*Thinking, Drawing, Modelling*" exhort us to reflect upon their action concerning didactics, as well as representation in Geometry. The first questions proposed by these terms are related with the understanding of a) their meaning; b) their action range and; c) their interdependence.

The ability to *Think*, which is assumed to be human, is noble and crucial to human reflection. The importance of thought is also evident as it is *conditio sine qua non* to every discourse, as Júlio Fragata suggests, by stating that

> "*Writing is communicating your own thoughts through signals which have been recorded and represent words and that can be read and interpreted by others. Writing is therefore one of the fixed ways to communicate thought.*"[2] [2] [01, p.19].

The author's theory allows us to replace *writing* with *drawing* - at the action level - and move simultaneous and inevitably to the *problematics of discourse*.

Michel Foucault, in *The Order of Discourse*, refers to the latter as "a thing pronounced or written" [02, p.52] and rarely as thought; yet, thought is the condition to all symbolic-graphical, symbolic-gestural and symbolic-verbal discourses.

---

[1] Universidade de Lisboa, Faculdade de Belas-Artes, Centro de Investigação e Estudos em Belas-Artes (CIEBA); Universidade Lusófona de Humanidades e Tecnologias, ECATI, Lisboa. constantino.rodrigues@gmail.com

[2] Translated from the Portuguese edition:
"Escrever é comunicar o próprio pensamento por meio de sinais, gravados e representativos de palavras, que possam ser lidos ou interpretados por outros. A escrita é portanto uma das maneiras fixas de comunicação do pensamento".

The faculty of human beings to *draw* allows them to exhale their emotions and their ethereal thoughts; thus, the nature of drawing is consequently beautiful. Drawing, from the Italian *disegno* [03, p.1444] and *to draw*, also from the Italian *disegnare* [03, p.1282] and from the Latin *designo*, *ās*, *āvi*, *ātum*, *āre*[3], do not survive apart from their meanings[4]. Their etymological correspondence are shown in Table 1.

We understand, from what has been exposed, that the act of *drawing* implies *thought, intention, recording* and *signification*. On the other hand, modelling involves the act of *regulating, guiding* and *conditioning*, which in turn necessarily implies *planning* and *thinking*. Modelling is the operation that turns something into an agreed model or example. This means stereotype, which carries the possibility of multiple collateral damage. From our current understanding, the terms *Thinking, Drawing* and *Modelling* are discursive and interdependent in Descriptive Geometry. So, *thought, discourse, thought as discourse*, as well as *discourse as a way of thinking*, are paramount to science in its broader sense and to the sciences of representation, which house Synthetic Geometry in its Descriptive form.

Thought/discourse modelling may occur strictly upon individual *reflection* - through which it would be considered an *internal matter* - and may also be the act of the *Individual* over the *Other* - considered as an external or externalizable phenomenon. From the premise that every individual is a social being, we may conclude that, to every individual, we are necessarily the *Other*. This perspective makes us think about the conditioning effect the *Other*'s thought/discourse has in the way we construct ourselves as individuals and the way we perceive the world, science and art.

From now on, in this study, thought/discourse modeling, is understood as an inner phenomenon of the Being, i.e., it occurs at his innermost self as a result not only of his reflection, but also of the complex individual network that is permeable to the outer world, and therefore surrounds him. This means that the phenomenon occurs at his inner core, although its hypocentre is external. It is very important to consider the phenomena which condition and regulate him, that, in a word, model him. So, we move towards the “procedures for controlling and delimiting discourse”, referred by Foucault [02, p.56]. Accepting the existence of *agents* and *phenomena* that model the thought/discourse implies validating the presence of *ideology* in them (and we consider them in this study) and, simultaneously, the existence of shades in the *Truth*.

In what concerns *ideology*, Louis Althusser says:

> “*All ideology hails or interpellates concrete individuals as concrete subjects, by the functioning of the category of the subject.*
>
> *This is a proposition which entails that we distinguish for the moment between concrete individuals on the one hand and concrete subjects on the other, although at this level concrete subjects only exist insofar as they are supported by a concrete individual. I shall then suggest that ideology ‘acts’ or ‘functions’ in such a way that it ‘recruits’ subjects among the individuals (it recruits them all), or ‘transforms’ the individuals into subjects (it transforms them all) by that very precise operation which I have called interpellation or hailing, and which can be imagined along the lines of the most commonplace everyday police (or other) hailing: ‘Hey, you there!’*” [05, p.47-48]

Table 1 - Terms | Evolution | Meaning

Term	Language	Evolution	Decomposition / Meaning
Desenho	Portuguese	From the Latin *designo*	*de+signo*, where *signo* means sign or signal
Designo	Latin	-	to designate, to mark, to indicate, to draw, to note, to show, to define, to signal
Desígnio	Portuguese	From the late Latin *designĭu*	intent, intention, purpose, project, plan, idea, thought
Designar	Portuguese	From the late Latin *designare*	to point out, to signal, to name, to determine
Disegnare	Italian		*di+segnare*, meaning, to draw, to outline, to plan, to project, to stress
Segnare	Italian		*assignar*, to mark, to indicate, to write down, to scribble, to blot, to register, to draw

---

[3] Cf. Instituto Antônio Houaiss de Lexicografia [03, p.1278]. According to the *Diccionario Encyclopedico* ou *Novo Diccionario da Lingua Portugueza* (…) [04, p.916] by José Maria de Almeida & Araujo Corrêa de Lacerda, the term *desenho* comes from the Italian *dissegno*, and *desenhar* derives simultaneously from the Italian *dissegnare* and from the Latin *designo*, *are*.

[4] Without its signification/concept, according to Charles Peirce.

Althusser also says that individual subjects in general live in the chimera in which everyone thinks himself as impenetrable and exterior to ideology. At the same time, the author emphasizes the need of individuals to understand their position towards Science. Althusser finishes, however, with the idea that ideology is omnipresent except to the ontological reality:

> "*What really takes place in ideology seems therefore to take place outside it. That is why those who are in ideology believe themselves by definition outside ideology: one of the effects of ideology is the practical denegation of the ideological character of ideology by ideology: ideology never says, 'I am ideological'. It is necessary to be outside ideology, i.e. in scientific knowledge, to be able to say: I am in ideology (a quite exceptional case) or (the generalcase): I was in ideology. As is well known, the accusation of being in ideology only applies to others, never to oneself (unless one is really a Spinozist or a Marxist, which, in this matter, is to be exactly the same thing). Which amounts to saying that ideology has no outside (for itself), but at the same time that it is nothing but outside (for science and reality)*." [05, p.49]

Although Bento de Jesus Caraça does not exactly adress ideology, he understands that individuals and disciplines should survey certain levels of dogmatism and staticity, since attitudes towards science may be diverse. The author states that "science may be looked at through two different perspectives." [5] [06, p.xiii].

In his text of paramount importance, entitled "*Duas Atitudes em Face da Ciência*" (whose English translation could be "*Two Attitudes Towards Science*"), the author identifies two possible positions, namely; (i) looking at science as it is exposed in science textbooks,"as a built thing" [6] [06, p.xiii], that is harmonious, well-defined and without contradiction; (ii) observing the way science evolved: hesitantly, uncertaintly and, at certain moments, with contradictons. It is therefore left to the individual and to disciplines the choice to stagnate or to question, in which the former is a reductionist attitude, whereas the latter is enhancing. Quoting:

> "(…) i*n the first situation, science seems to be self-sufficient, the formation of concepts and theories seems to obey to inner needs only; on the contrary, the influence that the social atmosphere exerts on the development of science is very clear in the second situation*" [7] [06, p.xiii]

Moreover, the *truth* appears stratified and more or less stable before the individual, hence its several classifications. The *ontological truth*, meaning the *objective truth* or the *truth of the objects*, defined by Charles Lahr as "what is" [8] [07, p.430], isn't always coincidental with the *logical truth* or the *truth of knowledge* and/or with the *disciplinary truth*.

Karl Popper comes to help in this topic by stating that "Knowledge consists in the search for truth - the search for objectively true, explanatory theories." [08, p.4], so *knowledge* does not always reach or correspond to the *truth*. Foucault also explains that the *disciplinary truth* is subject to a discursive policing to which the subject individual should obey; to the author, therefore, discoursing the "*truth*" [02, p.61] is not synonym to *entering* or *being in the truth*. According to this conception, *entering* or *being in the truth* corresponds to a pre-established truth by a group for itself:

> "*It is always possible that one might speak the truth in the space of wild exteriority, but one is 'in the true' only by obeying the rules of discursive 'policing' which one has to reactivate in each of one's discourses*." [02, p.61]

It is agreed common sense that there are limits and regulations to the discourse/thought. Foucault says on the subject:

> "*We know quite well that we do not have the right to say everything, that we cannot speak of just anything in any circumstances whatever; and that not everyone has the right to speak of anything whatever*." [02, p.52].

There are effectively social modelling, ordering, scrutiny and exclusion practices at all phases of the production and exposition of discourse/thought which influence the ontological truth. These phenomena are identified, named and organized by Foucault under three large categories, distinguished through their performing principles: (i) *exclusion of discourse*; (ii) *rarefaction of discourse;* (iii) *rarefaction of the subject who utters the discourse*[9].

---

[5] Translated from the Portuguese edition:
"a ciência pode ser encarada sob dois aspetos diferentes."

[6] Translated from the Portuguese edition:
"como coisa criada".

[7] Translated from the Portuguese edition:
"(…) no primeiro aspeto, a ciência parece bastar-se a si própria, a formação dos conceitos e das teorias parece obedecer só a necessidades interiores; no segundo, pelo contrário, vê-se toda a influência que o ambiente da vida social exerce sobre a criação da Ciência".

[8] Translated from the Portuguese edition:
"o que é".

[9] Foucault will define subcategories later.

The criterium of truth is thus conditioned/modelled by the thought/discourse that lives in such phenomena i.e. the thought/discourse exterior to the individual conditions his thought/discourse. So, being aware of the existence and of the operationality of the procedures for controlling and delimiting discourse is of paramount importance.

This path brings us to Geometry and makes us rethink our own behaviour and/or keep a close eye on it.

As we move first to the genesis of both Analytical and Descriptive Geometry, it is clear that neither René Descartes nor Gaspard Monge, respectively, built them from the unknown, nor absolutely determined the theoretical *corpus* which we are nowadays familiar with. When talking about the geometry founded by Descartes, Heinrich Wieleitner says that the designation "*Géométrie Analytique*" is first so registered in the volume IV of *Cours de mathématiques* (Paris, 1798), by Silvestre Lacroix[10], and appears in 1808 as the title of a book by Juan Garnier[11]. Wieleitner also mentions that the initial system lacked a third axis[12]. Furthermore, this author gives priority to Girolamo Cardano in what concerns the *rule of signs*, although Descartes is credited for it. As we can see:

> "*In fact, not only did Descartes add to Algebra with his intelligent graphic solutions, since he enunciated the rule of signs (without demonstrating it) which was named after him and to which Cardano had contributed with some evidence, but he came up with a new solution to the fourth grade equation using (…)*" [13] [09, p.162]

From what has been exposed, it is clear that the system credited to Descartes in 1637 has been enhanced throughout the last centuries. The same type of phenomenon happens with the Monge indexed Descriptive Geometry, whereas Hyppolyte Sonnet testifies about the appropriation of the basic principles from several practice in workshops throughout the ages, and also from the work by Amédée Frézier. However, Monge can't be denied the art and the ingenuity of the *Géométrie Descriptive*:

> "*Considering its methods, this science is not as new as we are led to believe. The idea of projecting the bodies on a horizontal and a vertical plane is so natural, that it must have been applied in workshops from time immemorial. The method of projections was used with currency by Frézier, engineer to the king, in the work entitled Théorie et pratique de lacoupe des pierres et des bois (Theory and Practice of the Cutting of Stone and Wood), published in Strasbourg in 1737.*
>
> *The method of switching planes had already been employed by Girard Desargues (1591 - 1661), and applied to the apparatus of arches, in the work entitled Pratique du trait à preuves, published in 1643 by Abraham Brosse. In general, however, the art of modelling had not been practiced but "in a shadowy manner, by poorly educated persons, who knew not how to communicate the results of their meditations" (Monge, Programme de Géométrie descriptive). It was left to the illustrious Monge to join, in one doctrinal work, the graphic procedures employed in the art up to that time, bringing to his work a spirit of order, clarity, and generality that characterise his talent. Presented in this entirely new manner, the art of modelling appeared to be a quite new science; we can say that Monge is the creator of Descriptive Geometry - despite not having formulated all the foundational elements - as he was the one who coordinated and joined them to a general method, of which he demonstrated its great utility.*" [14] [10, p.572]

---

[10] Cf. [09, p.301]

[11] Cf. [09, p.301]

[12] Cf. [09, p.155]

[13] Translated from the Spanish edition:

> "En realidad no se limitó solamente Descartes a enriquecer el Álgebra con sus hábiles soluciones gráficas, sino que enunció la regla de los signos que lleva su nombre (sin demostrarla), sobre la cual había hecho ya Cardano algunas indicaciones, y dió una nueva solución para la ecuación de cuarto grado, valiéndose de (…)"

[14] Translated from the French edition:

> "Considérée dans ses méthodes, cette science n'est pas non plus aussi nouvelle qu'on se le persuade d'ordinaire. L'idée de projeter les corps sur un plan horizontal et sur un plan vertical est si naturelle, qu'elle a dû être appliquée de temps immémorial dans les ateliers. La méthode des projections est employée d'une manière courante par Frézier, in¬génieur du roi, dans l'ouvrage intitulé Théorie et pratique de la coupe des pierres et des bois, publié à Strasbourg en 1737.
>
> La méthode des changements de plans avait déjà été employée par Desargues, et appliquée à l'appareil des voûtes, dans l'ouvrage ayant pour titre Pratique du trait à preuves de M. Desargues, publié en 1643 par Abraham Brosse. Mais, en général, l'art du trait n'avait été pratiqué que «d'une manière obscure, par des personnes dont l'éducation n'avait pas été assez soignée, et qui ne savaient pas communiquer les résultats de leurs méditations» (Monge, Programme de Géométrie descriptive). Il était réservé à l'illustre Monge de réunir en un corps de doctrine les procédés graphiques employés jusqu'alors dans les arts, en apportant à ce travail l'esprit d'ordre, de clarté et de généralité qui caractérise son talent. Présenté ainsi sous un jour tout nouveau, l'art du trait parut une science toute nouvelle; on peut dire, en effet, que Monge est le créateur de la Géomé¬trie descriptive, bien qu'il n'en ait pas créé de toutes pièces tous les éléments, parce que c'est lui qui les a coordonnés et rattachés à une méthode générale, dont il a montré toute la fécondité."

Years later, Gabriel-Marie [11, p.1] will agree with Sonnet concerning the genesis of Descriptive Geometry in 1795. Another important quality of the Descriptive is undoubtedly its close connection with the Analytical. Some current scientific communities seem to lack understanding of the proximity of both Geometries and even goes so far as to promote their distance, which is contradictory to the scientific principles of promotion of interdisciplinarity and the congregation of the sciences that develop in the same direction. Analytical Geometry and Descriptive Geometry have grown together - despite having different ages, they complement one another and this quality has long been recognised. Rodrigo Pinto says:

> "*The object of Descriptive Geometry is to represent parts of the extension that exist in space on a plan so that this representation makes it easy to infer the most remarkable qualities of the parts, their dimensions and their relative position.*
>
> *In the research work of this Science it is convenient to make use of Analytical Geometry as its complement: the former offers general and expeditious means to solve questions, whereas the latter renders the results prompted by solutions sensitive and shows their possible applications to the arts. This is the reason why in our observations we shall commit to compare graphical with analytical solutions.*" [15] [12, p.1]

The same approach is taken by Luiz Porfirio Motta Pegado [13, p.4], and Michel Chasles [14, p.585], corroborated by Charles Dupin [15, p.11] (who was close to Monge) and, inevitably, Gaspard Monge himself:

> "*It is not without reason then, that we compare Descriptive Geometry to Algebra; these two sciences are intimately linked. There is no Descriptive Geometry construction that cannot be translated into Analysis; and when the questions involve no more than three unknowns, each analytic operation can be seen as the authoring of a performance in Geometry. It is to be desired that these two sciences be cultivated together: Descriptive Geometry would bring, in the most complicated analytic operations, the evidence that characterises it, and, Analysis would, in its turn, bring to Geometry its generality*" [16] [16, p.13]

Thought/discourse modelling occurs not only over the genesis and the proximity of the aforementioned Geometries, but also over the theoretical *corpus* of the Synthetic branch. This is, therefore, the core concern of our study; however, for length reasons, we will analyse solely the performing and modelling categories of discourse/thought in this epistemological field - *descriptive* and *non-descriptive* branches, with slight incursions into Mathematics -, by exposing examples of their action. We won't be exhaustive, since we aim to lay the foundations for the promotion of questioning of thought/discourse about Synthetic/ Descriptive Geometry.

Within the procedures of thought/discourse exclusion, we may identify Foucault's subcategories [02] such as "*taboo on the object of speech*", "*ritual of the circumstances of speech*", "*privileged or exclusive right of the speaking subject*", "*division and rejection*", "*true vs. false*" and expose examples of their performance:

i. Exclusion of the analysis on solutions proposed to certain problems, as done by the *Royal Academy of Sciences* in Paris in 1778;
ii. Exclusion of proposed papers to conferences;
iii. Exclusion of eligibility of discourse for lack of the privilege of right recognised/granted to the orator;
iv. Exclusion of cases whose topic or proposed solution might be considered *naive* or *insane*, such as any solution proposing to solve the Delian problem, of doubling the cube, with compass and straightedge;
v. Prevalence of the *will to achieve the truth* that exerts pressure and coercive power and usually receives institutional support. Here we would like to high-light the calculation of the amplitude of the 'angle' formed by two skew lines.

---

[15] Translated from the Portuguese edition:
"O Objecto da Geometria descriptiva é representar sobre um plano as partes da extensão existentes no espaço, de modo que seja facil deduzir desta representação as propriedades mais notaveis das mesmas partes, as suas dimensões, e as relações de posição que entre si guardam.
No estudo desta Sciencia convem empregar a Geometria analytica como complemento della: porque, se uma offerece meios geraes e expeditos de resolver as questões; a outra torna sensiveis os resultados a que as soluções conduzem, e mostra as applicações que dos mesmos resultados se podem fazer ás artes. Por este motivo nos empenharemos especialmente, nas observações de que vamos occupar-nos, em comparar as soluções graphicas com as analyticas."

[16] Translated from the French edition:
"Ce n'est pas sans objet que nous comparons ici la Géométrie descriptive à l'Algèbre; ces deux sciences ont les rapports les plus intimes. Il n'y a aucune construction de Géométrie descriptive, qui ne puisse être traduite en Analyse; et lorsque les questions ne comportent pas plus de trois inconnues, chaque opération analytique peut être regardée comme l'écriture d'un spectacle en Géométrie. Il serait à désirer que ces deux sciences fussent cultivées ensemble: la Géométrie descriptive porterait dans les opérations analytiques les plus compliquées, l'évidence qui est son caractère, et, à son tour, l'Analyse porterait dans la Géométrie la généralité qui lui est propre."

In what concerns the rarefaction of thought/discourse procedures we can identify the subcategories "*normative discipline*", "*inhibiting discipline*", "*stimulating discipline*" and observe situations of their respective actions:

i. According to Foucault, it acts under a "principle which is itself relative and mobile; which permits construction, but within narrow confines." [02, p.59]. As an example, we mention the relation between the curriculum, the textbooks and teaching practices.
ii. It has an inhibiting and/or dissuasive function, like the standpoint of Francisco Gomes Teixeira, who claims the impossibility of solving some specific Geometry problems by the use of ruler and compass alone [17, p.viii], as well as the behaviour of Fernando Bensabat, who claims the vehement rejection of the "*Aguilar problem*" [18, p.12].
iii. It operates according to the principle of stimuli to investigation. In this area, personal initiatives stand out, like David Hilbert's solemn text "*The future problems of mathematics*" uttered in 1900[17], as well as the institutional initiatives like the *Clay Mathematics Institute,* who established *The Millennium Prize Problems* [20]. In the field of Descriptive Geometry, Leonildo de Aguilar took the worthy initiative to present the scientific community the problem meanwhile named "*The Aguilar Problem*". [18, p.9].

CONCLUSIONS

Our previous path allowed us to understand a parallelism between *thought* and *discourse,* whether it is graphical, gestural, and/or verbal. Therefore, we state that *Thinking*, *Drawing* and *Modelling* are discursive and interdependent in Synthetic [Descriptive] Geometry and that there are phenomena and ideological procedures that model thought/discourse and have an impact in the construction of the *ontological truth* and in the coherence between it and the logical and/or the *disciplinary truth*. We observe that the agents that model thought/discourse may act according to diverse principles, operating on the narrative relative to the genesis of a science as well as on its theoretical *corpus*. We have also affirmed the existence of exclusion and rarefaction procedures in the discourse of Synthetic [Descriptive] Geometry (with incursions in Mathematics), additionally exposing variants and showing examples that ratify this work.

This short essay aims to raise awareness in the scientific community relative to Synthetic Geometry, both Descriptive and Non-descriptive, concerning the existence of discourse modelling, controlling and delimiting/restricting procedures. Aditionally, we aim to alert the scientific community to surveil the action range of these procedures in obtaining the ontological truth.

Constantino Rodrigues

---

[17] This text was initially published in the *Göttinger Nachrichten*, 1900, pp. 253-297, and in the *Archiv der Mathematik und Physik*, 3rd ser., vol. 1 (1901), pp. 44-63 and 213-237. It was later translated for the *Bulletin of the New York Mathematical Society*, vol. 8, No 10, [19, pp.437-479], the latter is available online at the official site of the *American Mathematical Society* (AMS): http://www.ams.org/journals/bull/1902-08-10/

REFERENCES:

[01] Fragata, J. (1980). *Noções de Metodologia* (3.ª ed.). Porto: Livraria Tavares Martins.

[02] Foucault, M. (1981). The Order of Discourse. In R. Young (Ed.) *Untying the Text: A Post-Structuralist Reader* (51-78). Boston | London & Hendley: Routledge & Kegan Paul.

[03] Instituto Antônio Houaiss de Lexicografia. (2003). *Dicionário Houaiss da Língua Portuguesa (Tomo III)*. Lisboa: Círculo de Leitores.

[04] Almeida, J. M., & Lacerda, A. C. (1878). *Diccionario Encyclopedico ou Novo Diccionario da Lingua Portugueza* (…) *Seguido do Diccionario de Synonymos com Reflexões Criticas* (5.ª ed., Vol. I). Lisboa: No Escriptorio de Francisco Arthur da Silva.

[05] Althusser, L. (1984). *Essays on Ideology*. London: Verso Editions.

[06] Caraça, B. J. (1978). *Conceitos Fundamentais da Matemática* (7.ª ed.). Lisboa.

[07] Lahr, C. (1941). *Manual de Filosofia, resumido e adaptado do Cours de Philosophie*. Porto: Apostolado da Imprensa.

[08] Popper, K. R. (2000). *In Search of a Better World: Lectures and Essays from Thirty Years. London* | New York: Routledge

[09] Wieleitner, H. (1932). *Historia de las Matemáticas* (2.ª ed.). Barcelona: Editorial Labor.

[10] Sonnet, H. (1895). *Dictionnaire des Mathématiques Appliquées* (5è éd.). Paris: Librairie Hachette et Cie

[11] Gabriel-Marie (1927). *Exercices de Géométrie Descriptive* (6è éd.). Paris: Mame Éditeur.

[12] Pinto, R. (1853). *Complementos de Geometria Descriptiva de M. de Fourcy*. Coimbra: Imprensa da Universidade.

[13] Pegado, L. P. M. (1899). *Curso de Geometria Descriptiva da Escola Polytechnica* (Tomo I). Lisboa: Typographia da Academia Real das Sciencias.

[14] Chasles, M. (1880). *Traité de Géométrie Supérieure* (2è éd.). Paris: Gauthier-Villars Imprimeur-Libraire.

[15] Dupin, C. (1819). *Essai Historique sur les services et les travaux scientifiques de Gaspard Monge*. Paris: Bachelier - Libraire quai des Augustins.

[16] Monge, G. (1820). *Géométrie Descriptive* (4è éd.). Paris: MME VR Courcier Imprimeur - Libraire pour les Sciences.

[17] Teixeira, F. (1915). *Obras sobre Mathematica, Traité des Courbes Spéciales Remarquables planes et Gauches – Tome III* (Vol. VII). Coimbra: Imprensa da Universidade.

[18] Bensabat, F. (1999). O problema de Aguilar. *Boletim da aproged*, 9, 9-12. Porto: APROGED.

[19] Hilbert, D. (1902). Mathematical Problems. *Bulletin of the New York Mathematical Society*, 10, vol. 8, 437-479.

[20] Clay Mathematics Institute. (n.d.). Millennium Problems. Retrieved from http://www.claymath.org/millennium-problems

SKETCHBOOK: EXERCÍCIO DE EXPRESSÃO PARA ALUNOS DE *DESIGN*

Luísa Mendes Tavares [1] e Danielle Spada Tavares [2]

RESUMO

Há uma demanda crescente de profissionais de *design* para trabalhar com *software* de representação virtual. A alta tecnologia e a popularização das ferramentas digitais contribuíram para que os alunos cada vez mais conhecessem e aproveitassem essas técnicas. No entanto, esses *software* são apenas um instrumento para implementar os projetos desenvolvidos pelos alunos e exigem muitas habilidades, incluindo manuais, para serem utilizados de forma integral. O artigo discute a instrumentalização de um recém-chegado ao ensino superior em *design*, a partir do estudo de caso de práticas didáticas utilizadas na disciplina Laboratório de Criação da Universidade Veiga de Almeida, no Rio de Janeiro, Brasil.

O laboratório se concentra no desenvolvimento do potencial criativo dos alunos. Para isso, é incentivada a utilização do caderno de rascunhos, reunindo as ideias e imagens daquele que o constrói, fundando um repertório e prática para o futuro *designer*.

PALAVRAS-CHAVE: *Sketchbook*, desenho, criatividade, didática e *design*.

LABORATÓRIO DE CRIAÇÃO

A formação do aluno em *design* pressupõe, entre outras características, autonomia, criatividade e expressão. Para a aquisição de tais habilidades, são necessárias metodologias de aprendizagem que se diferenciem do tradicional modo de lecionar, no qual os alunos reproduzem de forma deficitária os discursos dos mestres. Com o intuito de desenvolver as capacidades referidas, a disciplina de Laboratório de Criação é realizada no primeiro semestre dos cursos de *design* gráfico, interiores e moda da Universidade Veiga de Almeida, RJ, Brasil. O laboratório visa identificar os diversos modelos mentais e reconhecer suas aplicabilidades no universo criativo, possibilitando a decodificação das barreiras, a expressão e a representação criativa do estudante através de uma linguagem instrumentalizada.

As três palavras que conduzem a base da disciplina são criatividade, desenho e *design*. A criatividade é trabalhada a partir de jogos lúdicos, o aluno é desafiado a interagir com uma dinâmica de trabalhos que conduzem a um pensamento criativo para solucionar as demandas propostas. Enquanto o desenho é estimulado para o desenvolvimento da experimentação visual do aluno, o foco está na prática de uma técnica que permite ao estudante expressar suas próprias ideias. Dos jogos surgem os desenhos, pinturas e colagens que não só incitam a elaborações formais como desencadeiam criações que despertam os princípios de *design*.

Segundo Torrence [01], o sujeito criativo é aquele que tem sensibilidade aos problemas, possui fluência e flexibilidade em suas ações e capacidade de elaborar de forma original. Para desenvolver tais características, a disciplina acontece através de aulas práticas, na qual o aluno, individualmente ou em grupo, é convidado a realizar atividades que serão apresentadas em sala, para debates e discussões com os demais colegas.

Durante as aulas, procura-se mostrar que o aluno deve ter autonomia para construir suas próprias visões estéticas e para jogar com o pré-estabelecido sem ter medo de errar e sabendo lidar com as ambiguidades que são impostas durante a execução das tarefas. Considera-se que o período da escola é frutífero para experimentações que floresçam a criatividade, sem estar preso as questões impostas pelo mercado de trabalho. Durante a disciplina, o aluno é convidado a realizar os desafios, que apesar de propostos pelos professores, possibilitam ao mesmo a escolha de suas referências, do tema, do material com

---

[1] Universidade Veiga de Almeida, Brasil. luisa.tavares@uva.br

[2] Universidade Veiga de Almeida, Brasil. danielle.spada@uva.br

que vai trabalhar. A meta é desconstruir as ideias normativas introduzidas maioritariamente pelos meios mediáticos, que impõem padrões de beleza e gosto, proporcionando a criação de modelos mentais rígidos e inflexíveis. Para tal, o espírito de iniciativa é requerido, forçando o aluno a se colocar no lugar do outro e enxergar além do óbvio, prevendo situações e gerindo sua autoestima e a autocrítica. É necessário buscar um equilíbrio entre estas duas forças para se possa interagir em aula compreendendo o seu espaço e o dos demais alunos. A coragem para se expor e mostrar suas soluções é outra característica trabalhada durante o período em sala e nos desdobramentos nas plataformas digitais.

A disciplina pretende apresentar ao estudante outras formas de se posicionar diante das aulas na universidade e, consequentemente, gera reflexos para sua postura frente ao mundo, um modo no qual sua participação é cobrada não só pela execução das tarefas, mas por sua pró-atividade.

SKETCHBOOK

Para atingir os objetivos traçados acima, que vão além dos conteúdos programados para a disciplina, verificou-se a necessidade de um trabalho que avançasse além do tempo de aula. As características desenvolvidas pelos alunos fazem parte de um investimento global em sua atitude, não ficando restrita ao seu desempenho em sala. Não é possível, por exemplo, exercer um papel de liderança durante as aulas, sem que isso faça parte da própria vida do estudante.

Em uma área como o *design*, o prazer de trabalhar é crucial para o bom desenvolvimento do projeto. Esta afirmação está conectada com as ideias de *flow* criativo utilizadas por Csikszentmihalyi [02]. Para o autor, apenas através de uma tarefa autotélica se é capaz de chegar ao *flow*, um estado de imersão na qual a produção tem mais chances de ser bem-sucedida. Nestes casos, é necessário o enorme interesse do aluno na concretização das tarefas. A realização destas se devem dar por conta da sua própria satisfação, e não por estar cursando a disciplina. Diante das evoluções pedagógicas, acreditamos que a palavra *ensinar* não se aplica na prática de sala de aula. Essa forma foi substituída pelo compartilhamento e troca de conhecimento, e com isso pode se propor uma metodologia que transforme o hábito dos estudantes, incorporando este modo de trabalho em seu cotidiano.

Em um campo como o *design*, exige-se a repetição para que a técnica seja usada ao seu favor e não se torne um determinante do trabalho pela falta de domínio. A cobrança de um fazer diário se torna mais uma vez um requisito do trabalho, atribuindo outras vantagens, como pensar os erros como molas motoras para o aprendizado, aceitando-os como parte da prática.

Percebida a relevância da motivação pessoal e de uma prática extraclasse, concomitantemente às aulas, desde o primeiro dia em sala, é proposto que o aluno tenha um *sketchbook*. Este será produzido ao longo do semestre e caracterizará um diário visual, nele devendo conter tanto os esboços dos trabalhos pedidos pelos professores, como também registros, desenhos e escritas relacionados ao universo do estudante, sendo livre a escolha de temas, materiais e técnicas. O aluno deverá possuir uma relação de empatia com o caderno, para que o mesmo o acompanhe durante o período letivo e seja utilizado em diferentes situações do seu dia-a-dia, fazendo com que o objeto reflita a trajetória do estudante.

O *sketchbook* é um instrumento amplamente utilizado por profissionais da área criativa para esboços rápidos, ideias inesperadas, rascunhos, diários de viagem e etc., é um memorial do indivíduo que está atento ao redor.

> "*The notebooks of Leonardo da Vinci (...) shows us visual thinking of the highest quality; you can almost follow his thinking by the drawings he made and the inventions he came up with. Whether he really kept a notebook with him all day, like we propose to our students, is not known but the results are still astonishing.*" [03, p. 64].

A ideia de estar e usar o *sketchbook* em todos os momentos, como por exemplo na rua, para desenho de observação rápido em situações corriqueiras é uma das suas principais finalidades. Porém, com a proliferação dos tipos de caderno e consequentemente de fins, tornou-se um suporte que pode ser utilizado de diversas maneiras, e por isso vem cada vez mais fazendo parte da vida dos profissionais criativos, como dos arquitetos, *designers*, artistas e outros que trabalham com a plataforma do desenho.

Por ser um exercício acelerado, para poder ser utilizado em qualquer lugar, com facilidade de manuseio, o *sketchbook* tende a ser composto em sua maioria por desenhos, ou seja, o desenho é que realiza a efetividade do caderno através de traçados, diagramas e *croquis*. Muitos alunos têm dificuldade de se expressar através do desenho, se sentindo inibidos e sem confiança para colocar o lápis sobre o papel e desenhar, logo. Além de ser uma ferramenta para reunir as imagens sentimentais e referenciais do indivíduo que o produz, o *sketchbook* mobiliza o sujeito a uma prática quase diária de desenho que é fundamental para o aprimoramento e expressividade do seu trabalho. O uso do *sketchbook* favorece a quebra do modelo mental "*eu não sei desenhar*", possibilitando que o estudante compreenda que o desenho é o processo de dar forma ao que antes estava no mundo das ideias, é apresentar visualmente um pensamento, para tal precisa do hábito de desenhar.

> "*O desenho não consiste num processo mecânico e estável, exige pelo contrário, o habitus para se tornar expressão de autenticidade, de autonomia crítica e de intencionalidade.*" [04, p.29].

Dessa forma, o *sketchbook* reforça o importante hábito do desenho, estimula a curiosidade do olhar na recolha das impressões do universo que está ao redor, registra o processo criativo do sujeito, desenvolvendo sua linguagem visual e cria uma coletânea de referências autorais que pode ser utilizado por ele em qualquer momento e projeto, além de ser uma atividade que só se realiza com uma relação de afeto entre o autor, o *sketchbook* e o repertório gerado. Devido aos seus aspectos positivos, vem sendo sucessivamente introduzido nas universidades de artes e *design* como um suporte para a realização de disciplinas e projetos.

101 IDEIAS PARA O *SKETCHBOOK*

Apesar das intenções positivas baseadas em estudos e compartilhamentos acadêmicos, quando defrontadas com a realidade, questões que até então eram ignoradas se mostraram determinantes para o êxito da proposta. No Brasil, os alunos, em sua maioria, chegam sem nenhum contato com *design*, desenho ou arte no ensino superior, ficando evidente a falta de intimidade com as técnicas e o desconhecimento das ferramentas. É minoria aqueles que já estão aptos a desenhar e manter uma rotina. Mesmo essas características facilitadoras não indicam que o aluno vai ter êxito na disciplina. Porém, grande parte deles sente dificuldade ao chegar na universidade, já que estão acostumados ao sistema do ensino básico e médio que em sua maioria não favorece a autonomia de escolhas e a pró-atividade de realização. A maioria dos alunos cai em um estado de procrastinação, na qual não conseguem começar o trabalho, devido principalmente ao seu caráter amplo. A alta expectativa se confronta com a realidade das possibilidades do aluno. A dificuldade de lidar com as críticas e a supervalorização da opinião alheia contribuem para que a preguiça e o desinteresse se instalem.

Portanto, os *sketchbooks* recém-comprados, passam dias sem ser tocados. A página em branco é temida, seu harmônico vazio ameaça aqueles que não estão habituados a romper com a estabilidade imposta. A maioria tem medo de fazer o primeiro traço, de errar, de estragar aquele caderno de folhas brancas de alto custo para os estudantes brasileiros. É fundamental deixar claro para o aluno que aquele caderno, ainda que seja avaliado pelo professor, é dele. Ele é o autor e responsável pela própria produção. A ideia do erro naquele suporte é relativa, já que todas as marcas fazem parte do caminho e podem ser constantemente transformadas. Os registros realizados têm caráter processual e não de um trabalho finalizado. Como os alunos estão no primeiro período, exige-se a experimentação e a abundância de soluções. O trabalho será bem-sucedido quando transparecer um esforço do estudante na confecção das imagens.

Ainda assim, muitos alunos apresentam dificuldade e se optou por construir uma tabela para estimulá-los a utilizarem seus *sketchbooks* de variadas formas. A Tabela 1 nomeada "*101 ideias para o sketchbook*" enumera diferentes estímulos para a concretização do caderno e funciona como um jogo de combinação aleatória. A atividade foi baseada em diferentes teorias, cada uma contribuiu com algum aspecto na confecção da tabela. Os "princípios de partida" de Purcell e Gero [05] são pontos orientadores fixados em uma lista na qual deve incentivar a concepção e realização das formas. Além disso, os conceitos de associação também foram utilizados pelo caráter multiplicador do suporte, que possibilita diferentes combinações, uma vez que, "ideias criativas são com frequência produto de uma associação de duas referências aparentemente estranhas entre si" [06, p.177]. A própria ideia do jogo foi ratificada pela teoria de Caillois [07], trazendo o tom lúdico e autotélico ambicionados desde o princípio da disciplina. O aluno durante a atividade atingiria um grau de concentração que só é possível em atividades prazerosas e o apelo das distrações diminuiria consideravelmente.

Os pontos de partida foram divididos entre 'Material', 'Ferramenta', 'Técnica', 'Fundamentos' e, 'Aluno'. No último item, o estudante deve propor uma ou mais ideias para o *sketchbook*, completando a tabela e realizando sua sugestão em seu diário visual. Não havia nenhuma outra imposição para o uso da tabela, apenas motivar os alunos que não conseguiam partir de forma autónoma. No início foi difícil, pois os alunos, ainda com pensamento sequencial e cumpridor de tarefas se assustavam ao ver tantas possibilidades em uma tabela. Relatavam as dificuldades para fazer todas as técnicas. Neste momento, era esclarecido que havia autonomia para combinar técnicas, recursos e materiais de acordo com sua necessidade e expressividade - a autonomia era

UVA Escola do Design – Laboratório de Criação - Professora Luísa Tavares

101 Ideias para o Sketchbook

Material	38- Desenho de silhueta	**Escrituras**
1-Lápis	39- Desenho Musical	75- Poesia
2-Lápis de cor	40- Mandala	76- Textos, frases, palavras
3-Caneta	41- Máquina de desenhar	77- Flipbook (animação)
4-Caneta hidrocor	**Pintura**	78- História em quadrinho
5-Carvão	42- Pintura realista/abstrata	79- Ilustração
6- Tinta Aquarela	43- Pintura impressa	80- Lettering
7- Tinta Guache	44- Pintura espelhada	81- Caligrafia
8- Tinta Acrílico	45- Pintura amassada	82-História
9- Tinta spray	46- Pintura Café	83- Diário/ Notícias
10- Nanquim	47- Pintura papel alumínio	84-Carta
11- Lápis de cera	48- Pintura Corporal	**Registros**
12- Pastel Seco	49- Pintura com a mão	85- Coleção
13- Pastel Oleoso	50- Pintura bolinha de gude	86- Taxionomia
14- Fita adesiva colorida	51- Pontilhismo	87- Mapas/Cartografias
15- Cola/ Cola Colorida	52- Impressionismo	88- Postal
16- Barbante/Fio/ Linha	53- Cubismo	89- Composição
17-Arenosos-areia, purpurina,...	54- Giz de cera derretido	90- Paisagem
18- Folhas, flores, legumes...	55- Marmorização	91- Cenas
19- Papeis, papeis coloridos, ...	**Recorte**	92- Objetos, animais,...
20- Revistas, Jornal, Filipeta,...	56- Recorte	93- Pessoas
Ferramenta	57- Colagem	94- Texturas, cores e linhas
21- Pincel	58- Silhueta vazada	**Jogo de Ideias**
22- Rolinho	59- Mosaico	95- Brainstorming
23- Esponja	60- Sobreposição de imagens	96- Mapa Mental
24- Estilete	61-Sopa de letras/palavras	**Fundamentos**
25- Compasso	62- Montagem	97- Amor e afeto
26- Tesoura	63- Dobraduras e origamis	98- Sonho
27- Régua	64- Papel tridimensional	99- Imaginação
Técnica	**Matriz**	100-Criatividade
28- Encadernação	65- Carimbo	**Aluno**
Desenho	66- Frotagem	101-
29- Desenho de observação	67- Xilogravura (isopor)	
30- Desenho cego	68- Estêncil	
31- Desenho contínuo	69- Adesivo	
32- Luz e sombra	70- Estampa	
33- Alto Contraste	**Fotografia**	
34- Desenho da sombra	71- Fotografia realista/abstrata	
35- Desenho coletivo	72- Fotomontagem	
36- Garatuja	73- Foto 360°	
37- Croquis (esboço)	74- Foto sequencia	

Tabela 1 - *101 ideias para o sketchbook*.
Arquivo pessoal, 2016.

a palavra chave no desenvolvimento processual em Laboratório de Criação.

A tabela era apenas um guia e podia ser usada de várias formas pelo seu caráter modular, permitindo que cada aluno criasse um *sketchbook* singular e diferente dos outros colegas. O aluno não era estimulado a comprar todos os materiais, ferramentas e suportes, deveria usar as que lhe eram possíveis, de acordo com sua habilidade e situação financeira. A mesma não caracterizaria um futuro *deficit* no percurso, já que havia o estímulo para o uso de materiais alternativos. Poderiam fazer as combinações que lhe atraíam, juntando elementos que fossem do seu interesse e também jogar com a aleatoriedade, escolhendo ao acaso os itens que iria juntar para a execução das suas imagens. Entre inúmeras combinação possíveis, cito alguns exemplos: 25 (compasso) + 40 (mandala), ou seja, construir uma mandala com o compasso; 7 (guache) + 22 (rolinho) + 94 (textura, cores e linhas), ou seja, uma pintura com rolinho de texturas diversas; ou até apenas com um número, como 41 (Máquina de desenhar), em que o aluno devia construir sua máquina de desenhar.

Após passar a tabela para os alunos, notou-se que possuíam dificuldade em compreender alguns dos itens que eram enunciados. As palavras escolhidas não eram capazes de revelar todo o seu significado. Mesmo quando havia pesquisas na *Internet*, a subjetividade das técnicas distorcia os objetivos que tinham sido pensados originalmente. Desta forma, optou-se por construir um documento que relacionava as palavras a imagens que tinham sido produzidas a partir da técnica nomeada. Não havia nenhuma explicação sobre a técnica para que não houvesse por um lado uma influência sobre o modo de fazer o trabalho e por outro não trouxesse uma complexidade que desestimularia o aluno.

O que aconteceu com regularidade foi o aluno criar, a partir de seu próprio repertório e desejo, e depois conferir se havia indicações na tabela sobre o que ele havia realizado. Os estudantes também podiam compartilhar, no ambiente virtual da disciplina, seu trabalho, fotografando e descrevendo a técnica utilizada. A troca do material nas plataformas digitais foi muito estimulante, já que, além de perceberem o resultado de uma determinada técnica, sentiam-se motivados a seguir nas experimentações.

Entre os diversos trabalhos que cada aluno realizou em seu *sketchbook*, seguem três exemplos que refletem não só como o trabalho foi realizado, mas as possibilidades geradas pela tabela que incentiva um trabalho autoral ainda que partindo de uma matriz comum.

Em uma das páginas de seu *sketchbook* (Fig. 1), Rachel Sathler combina a técnica de silhueta vazada com a pintura de bolinha de gude, caracterizando a combinação: 58 + 50. Como tema ela escolhe o casal Peter Pan e Wendy, popularmente conhecidos através dos desenhos da Disney. A aluna usa a pintura obtida através do rolamento das bolas como fundo das silhuetas. Seus personagens são popularmente conhecidos, mas aparecem aqui com outras texturas, em um processo que representa o início da reelaboração dessas referências.

Fig. 1 -Página do *sketchbook* de Rachel Sathler.
Arquivo pessoal, 2016.

Ao chegar na universidade, o aluno está impregnado dos modelos midiáticos que, se não devem ser combatidos, ao menos merecem ser questionados, tencionando ao aluno para olhares mais desviantes que a normatividade. A aluna Beatriz Soares de Meirelles transforma as folhas de seu *sketchbook* (Fig. 2) em uma página de livro através da colagem. A partir deste cenário, ela elabora uma pintura em aquarela de uma composição presente no imaginário infantil, formando a soma 79 (61 + 6) + 20, ilustração de cena com aquarela sobre colagem de páginas de livro. Os elementos casa, árvore, sol e menina são trabalhados com maturidade através de uma rigidez que nos coloca em um lugar árido e seco. A cena parece ter saído dos contos dos irmãos Grimm e mostra como a junção de diferentes técnicas complexifica o trabalho e acrescenta camadas que desdobram os significados e sentidos para outras direções.

Por último, Acácio Costa parte de um exercício de desenho de observação para compor sua página no *sketchbook* (Fig. 3). O aluno desenha e redesenha um isqueiro e, durante o processo, o reelabora. Completa o trabalho queimando as laterais da folha e traz para a tabela uma técnica que não estava presente, o uso do fogo, formando a soma 82 + 29 + 92 +101= composição com desenho de observação de objeto com fogo.

CONCLUSÃO

Proporcionar a expansão da capacidade criativa e expressão em alunos recém-chegados no ensino superior tem se mostrado uma tarefa tanto estimulante quanto desafiadora. Com a informatização de nossas vidas cotidianas, é cada vez mais necessário estimular as práticas manuais dos futuros profissional de arte, arquitetura e *design*. Trabalhar com os alunos a partir do *sketchbook* é uma prática que desenvolve habilidades básicas exigidas para estas áreas, além de permitir ao professor acompanhar o desenvolvimento do aluno de forma simples ao folhear o caderno. Com a utilização da tabela a partir do primeiro semestre de 2016, foi possível perceber uma grande melhoria dos alunos em relação aos semestres anteriores. Além de produzirem nos seus *sketchbooks* de forma autónoma, complementava-se a pluralidade de linguagens presentes com a exploração de um estilo singular próprio. A diversidade de combinações permitia que cada aluno tivesse um trabalho único e incentivava a continuação do uso do *sketchbook* nos semestres seguintes.

Fig. 2 - Página do *sketchbook* de Beatriz Soares de Meirelles.
Arquivo pessoal, 2016.

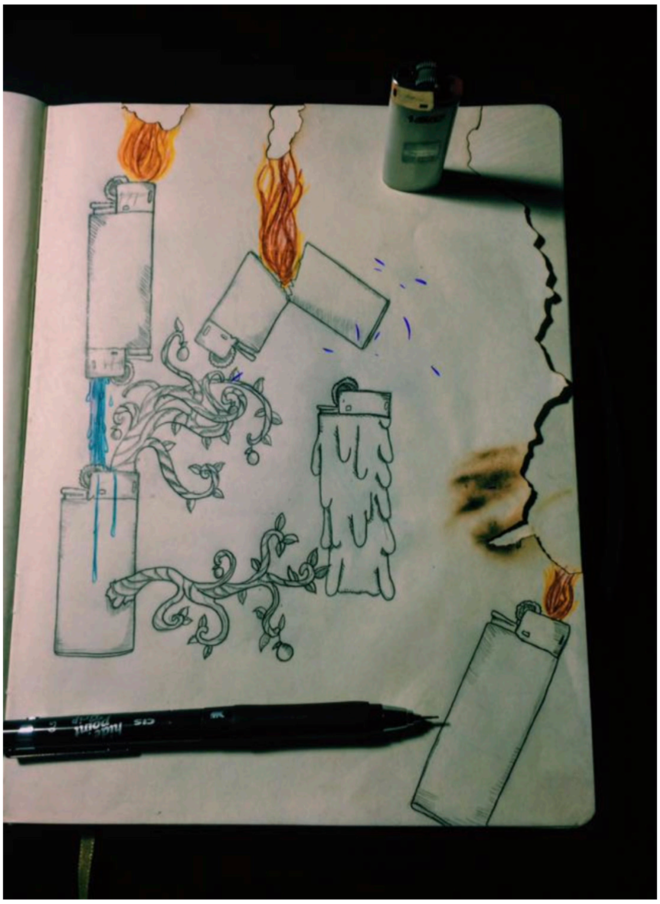


Fig. 3 - Página do *sketchbook* de Acácio Costa.
Arquivo pessoal, 2016.

A tabela realiza uma *interface* entre o *sketchbook* e o universo do aluno, permitindo que a relação com o caderno seja conduzida de maneira mais cuidadosa e atenciosa neste primeiro contato do estudante com o ensino superior em *design*.

Luísa Mendes Tavares e
Danielle Spada Tavares

REFERÊNCIAS:

[01] Torrance, E. P. (1977) *Creativity in the classroom*. Washington, DC: National Education Association.

[02] Csikszentmihalyi, M. (1999) *A descoberta do fluxo – a psicologia do envolvimento na vida cotidiana*. Rio de Janeiro: Rocco.

[03] Toorn, M. van den, R. H (2012). Notebooks as Didactic Tool in Design Education. *International Conference on Engineering and Mathematics*, p. 64-71. Disponível em: <http://repository.tudelft.nl/assets/uuid:ff31ebde-6b10-4676-abb9a84efa307482/282070.pdf> Acesso em: 19 jun 2016.

[04] Rodrigues, L. F (2005). O desenho para a construção do eu numa dialética interior/exterior in *Psiax n. 4 - Estudos e Reflexões sobre Desenho e Imagem*. p. 27-32. Porto: Edição conjunta da UM e da FAUP.

[05] Purcell, A. T., Gero, J. S. (1998) Drawings and the design process: A review of protocol studies in design and other disciplines and related research in cognitive psychology" in *Design Studies*, vol 19-4. p.389-430. Sydney: University of Sydney.

[06] Predebon, J. (1997) *Criatividade: abrindo o lado inovador da mente - um caminho para o exercício prático dessa potencialidade, esquecida ou reprimida quando deixamos de ser criança*. São Paulo: Atlas.

[07] Caillois, R. (1990) *Os Jogos e os Homens*. Lisboa: Edições Cotovia.

APPLICATION OF THE PROPORTION THEORY TO FORM DESIGN[1]

Vera de Spinadel[2]

ABSTRACT

Throughout the evolution of human culture, starting from the early Prehistory, following with the sacred art of Egypt, India, China, Islam and other traditional civilizations, the designers had tried to produce harmonic forms that simultaneously were particularly beautiful.

This objective dominated Greek and Roman art and Architecture, persisting in the movements of the Gothic Middle Ages and later on, in the Renaissance.

RESEARCH

From a mathematical point of view, they looked for numerical sequences that could be used as a base to create a proportion system with additive properties and be, at the same time, geometric progressions such that, “adding” or “subtracting” two consecutive terms of the sequence, another term of the sequence, was found.

Remember that a geometric progression is a sequence of numbers such that each element is equal to the previous one multiplied by a constant factor that is called the “reason” of the progression.

It is well known that geometric progressions do not have additive properties. But in the case of the Fibonacci secondary sequences, we have found an infinite set of geometric progressions that possess additive properties. They form part of a new family of quadratic irrational numbers that the author has called the *Family of Metallic Means* [01] and its more prominent member is the well known Golden Mean. Among its relatives let us mention the Silver Mean, the Bronze Mean, the Copper Mean, the Nickel Mean, etc. What do they have in common, apart from carrying the name of a metal? The answer is that they enjoy common mathematical properties that are fundamental on the actual research on the stability of micro - and macro-physical systems, going from the DNA internal structure to the astronomical galaxies.

The members of the Family of Metallic Means intervene in the determination of the quasi-periodical behavior of non linear dynamical systems, being therefore an invaluable key in the search of universal ways on the roads to chaos.

The numerical sequences based on the members of this family satisfy many additive properties and simultaneously are geometric sequences, offering therefore the possibility of being used as the base of many new systems of proportions.

CONCLUSIONS

Once we have proved that any member of the Family of Metallic Means could be used as a base to introduce a proportion system, we begun with the simplest of all: the Golden Mean Proportion system. This system has

---

[1] Extended abstract submitted by Vera de Spinadel on November, the 26th, 2016 in response to the Geometrias'17 Call for Papers.

[2] Vera Martha Winitzky de Spinadel (August 22, 1929 - January 26, 2017) was an argentinian mathematician.

the enormous advantage of being defined by a positive quadratic irrational number $\phi = 1+\surd 52$, which is the most irrational of all. Besides, it is associated to pentagonal geometry because if you draw a regular pentagon of unitary side, its diagonal has the value of ϕ. The Golden Mean dominated Greek and Roman art and architecture, it persisted on the monuments of the Gothic Middle Ages and later on, in the Renaissance.

Following this $\phi : 1$ proportion, we found another proportion, based on the Silver Mean $\sigma_{ag} = 1+\surd 2$ that was present in the design of at all scales, from the overall dimensions of the courtyard to the individual buildings of Roman houses to the rooms within each building and even to the tapestries in the walls. It was also found in musical proportions. The Silver Mean is associated to octagonal geometry because if you draw a regular octagon of unitary side, its second diagonal has the value of σ_{ag}.

Maybe the rest of the members of the Metallic Means Family have also an association of some sort of geometrical configurations, but this subject has not been considered yet.

Vera de Spinadel

REFERENCES:

[01] Spinadel, V. (2010). *From the Golden Mean to Chaos*" Nueva Librería, Buenos Aires, Argentina, 3rd. edition, June 2010.

ADDENDUM
AUTHORS' SHORT BIOGRAPHIC NOTES

GUNTER WEISS

Gunter Weiss (1946) graduated in Descriptive Geometry & Mathematics at Vienna University and iversity of Techno-logy in Vienna.
Between 1967 and 1995, he was Assistant Professor at the University of Technology in Vienna, Institute for Geometry; between 1995 and 2011, Full Professor for Geometry at the University of Technology in Dresden (Germany). Between 1995 and 2006, Head of the Institute for Geometry, TU-Dresden; from 2006 to 2010, Vice-Dean for Mathematics, TU-Dresden and from 2001 to 2008, resident and Vice-President of the ISGG (International Society for Geometry and Graphics). Gunter Weiss research interests include differential geometry, line geometry and kinematics, elementary geometry, projective and non-Euclidean geometry, technical applications of geometry, bio-geometry, didactics of geometry and engineering graphics. Married since 1971 and retired since 2011, Gunter Weiss has 3 daughters and 7 grandchildren.

ANITA PAWLAK - JAKUBOWSKA

Employee of Geometry and Engineering Graphics Centre of Silesian University, currently appointed as assistant professor. Teaches classes on descriptive geometry, engineering drawing, and CAD engineering drawing. Defended a Ph.D. thesis on Faculty of Architecture of Silesian University of Technology in July 2016. Title of the thesis: “Retractable roofs – geometric structure, kinematics, technology”. Member of Organising Committee of annual conferences of Geometry and Engineering Graphics Centre of Silesian University of Technology since 2007, and member of Polish Society for Geometry and Engineering Graphics since 2010.

PIOTR DUDZIK

Piotr Dudzik has a PhD of technical sciences in civil engineering and he’s an employee of the Geometry and Engineering Graphics Centre of Silesian University of Technology (Poland), where he teaches issues of the descriptive geometry and technical drawing. Prepares teaching aids in the form of presentations and other materials available on the Moodle platform.His research interests are related to the geometrical compactness on single-family houses which is confirmed by numerous publications and doctoral dissertation. In addition, he participate in Polish and international conferences. For many years he’s a member of the organizing committee of the conference CGGC

EWA TERCZYNSKA

PhD in technical sciences, architect. Study: Silesian University of Technology, Faculty of Architecture, Poland, Gliwice. PhD 2007 : Kindergarten Architecture. Development and contemporary trends. Work: Silesian University of Technology, Geometry and Engineering Graphics Centre, Poland, Gliwice since 1996. Member of: Polish Society for Geometry and Engineering Graphics and Chairperson of the organizing committee of the Conference Geometry Graphics Computer Research activities and scientific: geometry, descriptive geometry, engineering graphics, perception of space, spatial imagination, solid modelling, CAD

KRZYSZTOF TYTKOWSKI

Krzysztof Tytkowski is a graduate of Mechanical-Technical Department and has been working in The Geometry and Engineering Graphics Centre since 1989. He obtained a doctorate with honours in 1997. Scientific interests: project geometry, CAD systems (AutoCAD since 1985, Catia 2001, Inventor 2010), visualisation (3D Studio since 1992, 3D Studio MAX since 1996), geomet-rical modelling, computer graphics. Member of: Polish Society of Mechanical Engineers and Technicians (since 1987), Polish Society for Geometry and Engineering Graphics

MICHAŁ NESSEL

Michał Nessel was born on February 4, 1982 in Częstochowa, Poland. He studied architecture at Cracow University of Technology. He joined the Division of Descriptive Geometry, Technical Drawings and Engineering Graphics, Institute of Construction Design, Faculty of Architecture at Cracow University of Technology in 2015. In his research, he is focused on parametric and algorithmic design, genetic algorithms and evolutionary computing.

SZYMON FILIPOWSKI

Szymon Filipowski was born on March 19, 1983 in Cracow, Poland. He studied architecture at Cracow University of Technology. He joined the Division of Descriptive Geometry, Technical Drawings and Engineering Graphics, Institute of Construction Design, Faculty of Architecture at Cracow University of Technology in 2012. In his research he is focused on geometry, mathematics and algoriyhmic design.

DANIELA VELICHOVÁ

Daniela Velichová is professor of applied mathematics at Mechanical Engineering Faculty of Slovak University of Technology in Bratislava, Slovakia, currently head of Institute of Mathematics and Physics. Her scientific interests cover fields of geometric modelling, differential geometry of curves and surfaces, higher dimensional geometric spaces, visualisation of mathematical concepts and connections between mathematics and art. Author of many papers published in scientific journals and proceedings of international conferences, she is involved in many European scientific and research projects. Her current research is focused on modelling geometric objects by means of Minkowski point set operations. More information is available at http://www.evlm.stuba.sk/~velichova.

BENIAMINO POLIMENI

Beniamino is an Italian architect and architectural conservator. Since 2007 he has been cooperating with several architecture firms as consultant and designer and participating in European and International design competitions. In 2008 he won the Italian Prize for digital architecture, promoted by "National Association of Young Architects", on exhibition in the XXII UIA World Congress of Architecture in Torino. In 2010 he received his Ph.D. in Architectural Representation from the School of Architecture of the Università Mediterranea of Reggio Calabria, (National School for Doctorates in Representation Sciences and Survey, University of Florence). In 2012 he was a post-doctoral Fellow at the AKPIA at M.I.T. He is currently a Research Fellow at De Montfort University, Leicester School of architecture, United Kingdom. His main research areas include Architectural Representation, 3D Modelling, Architectural Survey, Islamic Architecture.

CÁTIA RAMOS

Cátia is an Architect and Ph.D. student on the Ph.D. program Architecture and Urban Culture in the Department of Architecture of the Faculty of Science and Technology - University of Coimbra with a Doctoral Grant from the Portuguese Foundation for Science and Technology. She is advised by Ph.D., Architect José António Bandeirinha and Ph.D., Architect Mauro Costa Couceiro. Her research contributes to the debate of the city in present day, focusing on the historical construction of the Portuguese city of Guarda through the understanding of its architecture as a construction that establishes the social and political fabric.

ISIDORA ĐURIĆ

Isidora Đurić is a Ph.D. student and a researcher at the Department of Computer Graphics - Engineering Animation, at the Faculty of Technical Sciences, University of Novi Sad. She obtained her Bachelor's degree in 2013 and Master's degree in 2014, both in Architecture and Urban Planning from the Faculty of Technical Sciences, University of Novi Sad. She was awarded a Scholarship of the Ministry of Education, Science and Technological Development of the Republic of Serbia for PhD students for the period of the 2015-2018. Her research interests include Architectural Visualization, Computer Graphics, Virtual and Augmented Reality, Photogrammetry, Image-based Modeling.

RATKO OBRADOVIĆ

Ratko Obradović is Full Professor at the Faculty of Technical Sciences, University of Novi Sad, Serbia. He is the founder and the head of Computer Graphics - Engineering Animation studies (at the Undergraduate, Master, and Doctoral level) since their establishment at the Faculty of Technical Sciences in 2011. He obtained his Bachelor degree in Mechanical Engineering in 1993 (at the Faculty of Technical Sciences), MSc degree in 1997 and a Ph.D. degree in 2000, both in Computer Graphics (Department of Computer Science, Faculty of Sciences, University of Novi Sad, Serbia). His research interests include Computer Graphics, Computer Geometry, Computer Animation, CAD, Scientific Visualization, Virtual and Augmented Reality, Higher Education. He is the author or co-author of four books and has published 40 journal articles and more than 40 papers in conference proceedings. He is a head of New Silhouette Studio with which he created two CG animated films. He is a member of the ICGG (International Society for Geometry and Graphics) and from 2010 to 2012 he was the president of SUGIG (Serbian Society for Geometry and Graphics). He is also a member of the Portuguese Descriptive Geometry and Drawing Teachers Association (APROGED), Association for Computing Machinery (ACM) and ACM SIGGRAPH.

NEBOJŠA RALEVIĆ

Nebojša Ralević is Full Professor at the Department for Mathematics, at the Faculty of Technical Sciences, University of Novi Sad, Serbia. He obtained his MSc degree in 1994 and a Ph.D. degree in 1997, both in Mathematics, at the Faculty of Sciences, University of Novi Sad, Serbia. He has published 16 books and 15 collections of tasks. He is the author or co-author of25 journal articles and more than 60 papers in conference proceedings. His research interests include Image Processing, Pattern Recognition, Medical Imaging, Algorithm Development in Image Processing, Probability and Measure Theory, Fuzzy Systems, Numerical Analysis, Optimization and Nonlinear Equations.

FILIPA CRESPO OSÓRIO

Filipa Crespo Osório nasceu em Coimbra em 1981. É Arquitecta licenciada pelo Departamento de Arquitectura da Univ. de Coimbra (DARQ-FCTUC) desde 2006. Trabalhou em ateliers de Arquitectura em Barcelona e Lisboa e em 2012/13 concluiu o Curso de Estudos Avançados em Arquitectura Digital do ISCTE-IUL. Actualmente, desenvolve a tese de Doutoramento no ISCTE-IUL com o tema de Superfícies Cinéticas Dobráveis em *Origami* para Grandes Vãos com bolsa financiada pela Fundação para a Ciência e Tecnologia (Ref. SFRH/BD/100818/2014).

ALEXANDRA PAIO

Arquiteta (UL, 1993), Mestre em Desenho Urbano (ISCTE-IUL, 2002) e Doutorada em Arquitectura e Urbanismo (ISCTE-IUL, 2011). Professora auxiliar no ISCTE - Instituto Universitário de Lisboa, Coordenadora do Laboratório de Fabricação Digital - Vitruvius Fablab-IUL, Cocoordenadora do CEAAD: Curso de Estudos Avançados em Arquitectura Digital ISCTE-IUL| FAUP, Coordenadora da especialização em Arquitetura Digital do Doutoramento em Arquitetura dos Territórios Metropolitanos em Arquitetura Digital e Coordenadora do Curso de Especialização em Territórios Colaborativos: Processos, Projeto, Intervenção e Empreendedorismo. Leciona na área de projeto de arquitectura e da computação no Mestrado Integrado em Arquitectura e no CEAAD. Coordenadora regional do projeto OIKOnet. A global multidisciplinar network on housing research and learning (financiado pela União Europeia) e coordenadora do projeto EMERG.CITIES4ALL (ISCTE-IUL). Investigadora no Projeto TEL@FTELa - Technology Enhanced Learning at Future Teacher Education Lab (PTDC/MHC-CED/0588/2014). Coordena e orienta trabalhos de investigação na área da arquitectura digital: design computacional, processos e ferramentas digitais de apoio ao projeto criativo, fabricação digital e métodos tradicionais em arquitectura, habitação emergente, espaço público e processos participativos e arquitectura interativa. Participa e coordena workshops e formação contínua com apoio de empresas da indústria portuguesa e Câmaras Municipais nas temáticas da fabricação digital e processos participativos. Publica artigos em revistas especializadas e trabalhos em atas de eventos na área da arquitectura digital.

SANCHO OLIVEIRA

Assistant professor at University Institute of Lisbon (ISCTE-IUL), Portugal, founder of BioMachines Lab, member at Institute of Telecommunications, and member of VFabLab ISCTE-IUL. My research focuses on autonomous robots, multirobot systems, swarm intelligence, evolutionary computation, communication in large-scale systems, complex systems and high performance computing. I have a PhD and Master in Physics from Universidade de Lisboa. I am graduated in Computer Engineering from Instituto Superior Técnico. I have 14 years of experience in teaching subjects like object-orientated programming, parallel and distributed programming, software for intelligent systems, computer architecture and physical computation.

SAMANTA ALINE TEIXEIRA

Com Mestrado (2017) e Graduação em Design (2012) pela UNESP - Universidade Estadual Paulista, unidade FAAC, atualmente cursa o Doutorado em Design pela mesma instituição. Fez Iniciação Científica (2011), com bolsa FAPESP, e desenvolve oficinas de *origami* desde 2009 até os dias atuais. Sua principal área de pesquisa é o design adaptado de *origami* para produtos.

THAÍS REGINA UENO YAMADA

Com Doutorado (2016), Mestrado (2003) e Graduação em Design (1997) pela Universidade Estadual Paulista, é, atualmente, professora assistente doutora no Departamento de Artes e Representação Gráfica da FAAC - Faculdade de Arquitetura, Artes e Comunicação - UNESP, lecionando Desenho e Geometria nos cursos de Graduação em Design e Licenciatura em Matemática. articipa do Grupo de Pesquisa em Psicologia da Educação Matemática e em Linguagem do Espaço e da Forma, pesquisando nas áreas de expressão e educação gráfica e aplicações de técnicas do *origami* e do *kirigami* em projetos de design gráfico e de produto.

VÍCTOR RODRÍGUEZ IZQUIERDO

Víctor Rodríguez Izquierdo was born in Las Palmas G.C., Spain, in 1983. He holds two Masters of Science in Civil Engineering (U. of Cantabria, Spain; U. of Bristol U.K.) and Architecture (U.L.P.G.C. and U.A.X, Spain). After a successful professional period in Madrid, working for Ove Arup, he moved to Stuttgart where he lives since over five years. Currently works as Senior Structural Engineer at Mayr | Ludescher | Partner, develops his Ph.D. at the University of Seville and is lecturer at the Institute of Construction and Design and at the Institute of Computational Design belonging both to the University of Stuttgart.

MARIA JOÃO PINTO

Maria João Pinto, nascida em 1975, é licenciada em Arquitetura pela Universidade de Coimbra, no ano 2000. Em 2011 completou o Mestrado em Ensino de Artes Visuais no 3º Ciclo e Secundário na Universidade de Aveiro e encontra-se atualmente a frequentar o Doutoramento Co*imbra Studio* no Departamento de Arquitetura da Universidade de Coimbra. Tem vindo a lecionar Geometria Descritiva em várias escolas secundárias e profissionais e encontra-se atualmente a colaborar com o corpo docente de Geometria, unidade curricular do primeiro ano do Mestrado Integrado em Arquitetura, na Universidade de Coimbra.

ANA PAULA ROCHA

Ana Paula Rocha possui graduação em Arquitetura e Urbanismo pela Universidade Federal de Uberlândia (2003). Obteve o Mestrado em Arquitetura, Tecnologia e Cidade pela Universidade Estadual de Campinas (2015) com a dissertação "A-temporalidade do instantâneo: o tempo e o comportamento na Arquitetura Contemporânea". Atualmente é Doutoranda no Programa de Pós-Graduação Arquitetura, Tecnologia e Cidade da Universidade Estadual de Campinas. Atua na linha de pesquisa Teoria e Metodologia do Projeto e da Cidade na temática Teoria, Processo e Análise de Projeto Arquitetônico e Urbanístico.

DEBORA MARIANE FANTINATO

Debora Mariane Fantinato possui graduação em Arquitetura e Urbanismo pela Universidade Estadual de Campinas (2015). Atualmente é Mestranda no Programa de Pós-Graduação Arquitetura, Tecnologia e Cidade da Universidade Estadual de Campinas (início em 2016). Atua na linha de pesquisa Teoria e Metodologia do Projeto e da Cidade na temática Teoria, Processo e Análise de Projeto Arquitetônico e Urbanístico.

RENATA MARIA GERALDINI BELTRAMIN

Renata Maria Geraldini Beltramin possui graduação em Arquitetura e Urbanismo pela Universidade Estadual de Campinas (2010) e mestrado em Arquitetura, Tecnologia e Cidade pela Universidade Estadual de Campinas (2015). Atualmente é estudante de pós-graduação da Universidade Estadual de Campinas, onde desenvolve pesquisa de doutorado nas áreas de processo de projeto, programa de necessidades e information design. Tem experiência e interesse nas seguintes áreas de pesquisa: representação gráfica, análise gráfica de projetos, linguagem arquitetônica, processo de projeto, processo criativo, programa arquitetônico, gerenciamento de projetos, information design e comunicação visual.

DANIEL DE CARVALHO MOREIRA

Daniel de Carvalho Moreira é Arquiteto e Urbanista pela Pontifícia Universidade Católica de Campinas (1994), Mestre em Multimeios pela Universidade Estadual de Campinas (2000) e Doutor em Engenharia Civil pela Universidade Estadual de Campinas (2007). Atualmente é Professor Doutor II (MS-3.2) em Regime de Dedicação Integral à Docência e à Pesquisa (RDIDP) na Faculdade de Engenharia Civil, Arquitetura e Urbanismo (FEC) da Universidade Estadual de Campinas (UNICAMP). Tem experiência na área de Arquitetura e Urbanismo, com ênfase em Projeto e Desenho, atuando principalmente nos seguintes temas: Programa Arquitetônico, Projeto Arquitetônico e Desenho Arquitetônico.

TERESA PAIS

Natural de Alcantarilha, concelho de Silves. Licenciada em Arquitetura pela Faculdade de Ciências e Tecnologia da Universidade de Coimbra. Mestre em Práticas e Teorias do Desenho pela Faculdade de Belas Artes da Universidade do Porto. Doutorada em Arquitetura na especialidade Expressão Plástica e Arquitetura pela Universidade de Coimbra com a tese "O desenho de contorno no processo de aprendizagem do desenho de observação". Professora Auxiliar de Desenho e de Geometria no Departamento de Arquitetura da Faculdade de Ciências e Tecnologia da Universidade de Coimbra. Autora de vários artigos sobre o ensino do Desenho na formação do arquiteto.

CONSTANTINO RODRIGUES

Constantino Rodrigues, nascido a 27 de fevereiro de 1971, natural de Algoz, concelho de Silves, Portugal. Bacharel em Artes Plásticas (Pintura/Escultura), pela ESTGAD. Licenciado em Artes Plásticas, pela ESAD. Mestre em Ensino de Artes Visuais no 3.º Ciclo do Ensino Básico e no Ensino Secundário, pela ULHT. Doutorando em Belas-Artes, especialidade de Geometria, na FBAUL. Professor Assistente Convidado na Universidade Lusófona de Humanidades e Tecnologias, Lisboa, e no Colégio Rainha D. Leonor, Caldas da Rainha. Tem como interesses, no âmbito académico-cientifico, o estudo epistemológico da Geometria Descritiva.

LUÍSA MENDES TAVARES

Mestre em Artes pela Universidade do Estado do Rio de Janeiro (2013). Estudante de Doutorado em Artes pela Universidade Estadual do Rio de Janeiro (2014-2018). Atualmente realiza estágio de pesquisa sanduíche na Universidade De Coimbra (2016/2017). Graduação em Desenho Industrial pela Pontifícia Universidade Católica do Rio de Janeiro (2008) e graduação em Design de Moda pelo Centro de Tecnologia da Indústria Química e Têxtil (2005). Atuante como professora da Universidade Veiga de Almeida, em Laboratório de Criação e Projetos de Moda.

DANIELLE SPADA TAVARES

Mestre em Psicanálise, Saúde e Sociedade da Universidade Veiga de Almeida. Especialização em Desenvolvimento e Liderança pelo INEXH (Instituto Nacional de Excelência Humana). Graduação em Pintura pela Universidade Federal do Rio de Janeiro. Atuante na área de criação, professora de Laboratório de Criação, História da Arte Moderna e Contemporânea e cenografia na UVA - Universidade Veiga de Almeida. Estudos com ênfase na subjetividade, criatividade e processo criativo, atuando principalmente nos seguintes temas: arte e design, linguagem e subjetividade, criatividade e inovação.

VERA MARTHA WINITZKY DE SPINADEL

Has got her PhD as first woman in Mathematic at the University of Buenos Aires, Argentina, in 1958. Between 2010 and 2017, she was Full Emeritus Professor at the Faculty of Architecture, Design and Urban Planning of the University of Buenos Aires, Argentina. In 1995, she was designed Director of the Centre of Mathematics and Design. In April 2005, she inaugurated the Laboratory of Mathematics & Design, University Campus in Buenos Aires. Since 1998 to her death she was the President of the International Mathematics and Design Association, that organizes international congresses every 3 years and publishes a Journal of Mathematics & Design. She was the author of more than 10 books and published more than 100 research papers. Dr. Spinadel was a leader in the field of Metallic mean in the development of the classical Golden Ratio got wide international recognition. (adapted from https://verasmathematicworld.org/about/)

Nenhuma transcrição, cópia, tradução ou transmissão é permitida sem autorização dos Autores.
As opiniões expressas são da exclusiva responsabilidade dos respectivos autores e não reflectem, necessariamente, as opiniões da Aproged, da Comissão Científica ou da Comissão Organizadora da Conferência Internacional Geometrias'17.

Todos os Associados da Aproged que queiram publicar artigos originais em futuras edições deste Boletim deverão enviá-los, em suporte digital, para aproged@aproged.pt. Serão publicados todos os artigos que forem revistos e aprovados pela Comissão dos Boletins da Aproged.

No transcription, copy, translation or transmission is allowed without the Authors' permission.
The opinions expressed herein are the sole responsibility of the Authors and do not necessarily reflect the opinions of Aproged, the Scientific Committee or the Organizing Committee of the International Conference Geometrias'17.

All Aproged Associates willing to publish original papers in future editions of this Bulletin, should send its digital version to aproged@aproged.pt. All papers reviewed and approved by the Aproged's Bulletins Committee will be published.

This publication is licensed under a
Creative Commons Attribution - NonCommercial - No Derivatives 4.0 International (CC BY -NC-ND 4.0):
You are free to share, copy and redistribute the material in any medium or format, as long as you follow the license terms, giving appropriate credit and providing a link to the license. You may do so in any reasonable manner, but not in any way that suggests the editor or author(s) endorses you or your use.
You may not use the material for commercial purposes. You are free to:
Share: copy and redistribute the material in any medium or format;
Adapt: remix, transform, and build upon the material,
under the following terms:
Attribution: You must give appropriate credit, provide a link to the license, and indicate if changes were made. You may do so in any reasonable manner, but not in any way that suggests the licensor endorses you or your use.
NonCommercial: You may not use the material for commercial purposes.
ShareAlike: If you remix, transform, or build upon the material, you must distribute your contributions under the same license as the original.

If you wish to remix, transform or build upon the material, you must have the author(s)'s permission.
The license's text is available at http://creativecommons.org/licenses/by-nc-nd/4.0/.
If you want to do anything listed as not allowed, please contact the editors (aproged@aproged.pt) or the authors of each paper.

boletim da aproged #34

Este Boletim da Aproged é uma das publicações decorrentes da Conferência Internacional Geometrias'17, subordinada ao tema "Pensar, Desenhar, Modelar", que se realizou no Departamento de Arquitectura da Faculdade de Ciências e Tecnologia da Universidade de Coimbra, entre 16 e 18 de Junho de 2017.

This Aproged's Bulletin is one of the publications resulting from the International Conference Geometrias'17, with "Thinking, Drawing, Modelling" as leitmotif, that was held in the Department of Architecture of the Sciences and Technology Faculty of Coimbra's University, between the 16th and the 18th of June, 2017.