Short communication

Adaptive detection using randomly reduced dimension generalized likelihood ratio test

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A B S T R A C T

We address the problem of detecting a signal of interest in the presence of Gaussian noise with unknown statistics when the number of training samples available to learn the noise covariance matrix is less than the size of the observation space. Following an idea by Marzetta, a series of K random semi-unitary matrices are applied to the data to achieve dimensionality reduction. Then, the K corresponding generalized likelihood ratios are computed and their median value provides the final detector. We show that the semi-unitary matrices can be replaced by random Gaussian matrices without affecting the final test statistic. The new detector avoids eigenvalue decomposition and is easily amenable to parallel implementation. It is compared to conventional techniques based on diagonal loading of the sample covariance matrix or based on rank reduction through eigenvalue decomposition and is shown to perform well.

Keywords:
Adaptive detection
Dimensionality reduction
Generalized likelihood ratio
Random semi-unitary matrices

1. Problem statement

Many radar systems are required to operate in an uncertain environment where the number of data available to learn the environment is smaller than the number of space and/or time channels [1–3]. This is typically the case with space-time adaptive processing where the size of the observations is large or in heterogeneous environments where a limited number of cells are deemed to share the same disturbance covariance matrix as the cell under test. The problem of detecting a target with signature $\boldsymbol{v}$ can be formulated as a composite hypothesis testing problem, namely

$$\begin{aligned} H_0 &: \boldsymbol{x} \overset{d}{=} \mathcal{CN}(\mathbf{0}, \boldsymbol{R}), \; \boldsymbol{z}_t \overset{d}{=} \mathcal{CN}(\mathbf{0}, \boldsymbol{R}), t = 1, \ldots, T \\ H_1 &: \boldsymbol{x} \overset{d}{=} \mathcal{CN}(\alpha\boldsymbol{v}, \boldsymbol{R}), \; \boldsymbol{z}_t \overset{d}{=} \mathcal{CN}(\mathbf{0}, \boldsymbol{R}), t = 1, \ldots, T \end{aligned} \quad (1)$$

where α stands for the target amplitude, $\boldsymbol{R}$ is the disturbance (clutter and noise) covariance matrix and $\mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{R})$ denotes the circularly symmetric complex Gaussian distribution with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{R}$. In (1) $\boldsymbol{x} \in \mathbb{C}^{M\times 1}$ corresponds to the data under test while $\boldsymbol{z}_t$ are training samples used to learn the disturbance which affects $\boldsymbol{x}$. When $T \geq M$, the generalized likelihood ratio test (GLRT) was derived by Kelly [4] who showed that it enjoys the constant false alarm rate property and who derived analytic expressions for the probability of detection. Kelly's GLRT is considered as the reference detector for the problem in (1). A second reference detector is the so-called adaptive matched filter (AMF) [5] which is indeed a two-step GLRT: first the GLRT for known $\boldsymbol{R}$ is derived and then $\boldsymbol{R}$ is substituted for the sample covariance matrix (SCM) $\hat{\boldsymbol{R}} = T^{-1}\sum_{t=1}^{T} \boldsymbol{z}_t \boldsymbol{z}_t^H$. Both detectors involve the inverse of the SCM and therefore need that $T \geq M$.

However, in a number of situations, one has to deal with $T < M$ and yet solve (1). When some additional information about $\boldsymbol{R}$ is available, e.g., it is persymmetric [6–8] or it possesses some specific structure [9,10] the number of actual unknown parameters describing $\boldsymbol{R}$ is somewhat reduced, and a low sample support can be addressed properly. When $\boldsymbol{R}$ is arbitrary, which is the case we consider herein, two main approaches can be advocated. The first approach consists in regularizing the SCM, generally by using diagonal loading [11,12], i.e., replace $\hat{\boldsymbol{R}}$ by $\hat{\boldsymbol{R}} + \nu \boldsymbol{I}_M$ where ν is the loading level. This technique leads to the loaded GLRT [13] or the loaded AMF [14] whose performance is very close to that of the matched filter even in low sample support, especially if the matrix $\boldsymbol{R}$ is close to a low-rank matrix plus a scaled identity matrix. The second approach consists in dimensionality reduction, also referred to as partially adaptive processing [1,15]. The basic idea is to use a data transformation $\boldsymbol{x} \to \boldsymbol{T}^H\boldsymbol{x}$, $\boldsymbol{z}_t \to \boldsymbol{T}^H\boldsymbol{z}_t$ where $\boldsymbol{T}$ is a $M \times R$ matrix with $R < M$ and to operate in an R-dimensional subspace. These techniques can be classified either as reduced-dimension methods (in this case $\boldsymbol{T}$ is fixed, see e.g., [16,17]) or rank-reducing methods where $\boldsymbol{T}$ depends on the data. Usually, the matrix $\boldsymbol{T}$ is constructed from the principal eigenvectors of the SCM, see e.g. [18–20] for the most well-known methods using this

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https://doi.org/10.1016/j.sigpro.2019.107265

principle. For detection purposes, this results in a low-rank AMF where the inverse of the SCM is replaced by the projector onto the subspace orthogonal to the principal eigenvectors.

In [21,22], Marzetta proposed a beautiful and original idea where dimensionality reduction is achieved through an ensemble of K isotropically random unitary matrices. More precisely, a column of $\boldsymbol{T}$ is aligned with $\boldsymbol{v}$ (which guarantees that the signal of interest goes through the data transformation), while the other columns are drawn at random in the subspace orthogonal to $\boldsymbol{v}$. Processing is then done in the reduced-dimension space and the outputs are subsequently combined (averaged). Marzetta provided a theoretical analysis of such technique, provided insightful results about its relation with shrinkage of the SCM eigenvalues, and applied it successfully to direction of arrival estimation and covariance matrix estimation. In this communication, we propose to use and to adapt this idea in the framework of detection, in order to solve (1) when $T < M$.

2. Randomly reduced dimension GLRT

Let us assume with no loss of generality that $\boldsymbol{v}$ is unit-norm and let $\boldsymbol{V}_\perp$ be a $M \times (M-1)$ semi-unitary matrix ($\boldsymbol{V}_\perp^H \boldsymbol{V}_\perp = \boldsymbol{I}_{M-1}$) whose columns are orthogonal to $\boldsymbol{v}$, i.e., $\boldsymbol{V}_\perp^H \boldsymbol{v} = \boldsymbol{0}$. Let $\boldsymbol{\Psi}_k$ ($k = 1, \ldots, K$) be a $(M-1) \times N$ matrix uniformly distributed on the Stiefel manifold [23]: such a matrix can be generated from a complex Gaussian distributed matrix $\boldsymbol{N}_k$ as [21,23]

$$\boldsymbol{\Psi}_k = \boldsymbol{N}_k \left(\boldsymbol{N}_k^H \boldsymbol{N}_k\right)^{-H/2} \tag{2}$$

where $\boldsymbol{N}_k \overset{d}{=} \mathcal{CN}(\boldsymbol{0}, \boldsymbol{I}_{M-1}, \boldsymbol{I}_N)$. Let us consider the $M \times (N+1)$ matrix $\boldsymbol{Q}_k = \begin{bmatrix} \boldsymbol{V}_\perp \boldsymbol{\Psi}_k & \boldsymbol{v} \end{bmatrix}$ and the transformed data $\tilde{\boldsymbol{x}}_k = \boldsymbol{Q}_k^H \boldsymbol{x}$ and $\tilde{\boldsymbol{Z}}_k = \boldsymbol{Q}_k^H \boldsymbol{Z}$ where $\boldsymbol{Z} = \begin{bmatrix} \boldsymbol{z}_1 & \ldots & \boldsymbol{z}_T \end{bmatrix}$. The first N components of the transformed data $\tilde{\boldsymbol{x}}_k, \tilde{\boldsymbol{Z}}_k$ correspond to $\boldsymbol{\Psi}_k$ times the coordinates of $\boldsymbol{x}$, $\boldsymbol{Z}$ in the subspace orthogonal to $\boldsymbol{v}$ while the last component corresponds to the output of a conventional beamformer steered towards $\boldsymbol{v}$, a structure which is reminiscent of a sidelobe canceler structure. Now, from (1) one has

$$\begin{aligned} H_0 &: \tilde{\boldsymbol{x}}_k \overset{d}{=} \mathcal{CN}\left(\boldsymbol{0}, \tilde{\boldsymbol{R}}_k\right), \ \tilde{\boldsymbol{Z}}_k \overset{d}{=} \mathcal{CN}\left(\boldsymbol{0}, \tilde{\boldsymbol{R}}_k, \boldsymbol{I}_T\right) \\ H_1 &: \tilde{\boldsymbol{x}}_k \overset{d}{=} \mathcal{CN}\left(\alpha \boldsymbol{e}_{N+1}, \tilde{\boldsymbol{R}}_k\right), \ \tilde{\boldsymbol{Z}}_k \overset{d}{=} \mathcal{CN}\left(\boldsymbol{0}, \tilde{\boldsymbol{R}}_k, \boldsymbol{I}_T\right) \end{aligned} \tag{3}$$

where $\boldsymbol{e}_{N+1} = \begin{bmatrix} 0 & \ldots & 0 & 1 \end{bmatrix}^T$ and $\tilde{\boldsymbol{R}}_k = \boldsymbol{Q}_k^H \boldsymbol{R} \boldsymbol{Q}_k$. Therefore the GLRT for the composite hypothesis problem (3) amounts to comparing

$$t\left(\tilde{\boldsymbol{x}}_k, \tilde{\boldsymbol{Z}}_k\right) = \frac{\left| \boldsymbol{e}_{N+1}^H \tilde{\boldsymbol{S}}_k^{-1} \tilde{\boldsymbol{x}}_k \right|}{\left(1 + \tilde{\boldsymbol{x}}_k^H \tilde{\boldsymbol{S}}_k^{-1} \tilde{\boldsymbol{x}}_k\right)\left(\boldsymbol{e}_{N+1}^H \tilde{\boldsymbol{S}}_k^{-1} \boldsymbol{e}_{N+1}\right)} \tag{4}$$

to a threshold, with $\tilde{\boldsymbol{S}}_k = \tilde{\boldsymbol{Z}}_k \tilde{\boldsymbol{Z}}_k^H$. Similarly to what was done in [21,22] the next step is to combine these K test statistics. For the application considered herein, we need to construct a single test statistics to be compared against a threshold in order to decide between H_0 and H_1. A natural and intuitively appealing approach is to use the *median value* of the $t_k = t\left(\tilde{\boldsymbol{x}}_k, \tilde{\boldsymbol{Z}}_k\right)$ as the final test statistic. Note that the average value could also be investigated. It turns out that the two approaches yield approximately the same performance in terms of detection. However, since we are dealing with ratios, the median seems more appropriate. The proposed random reduced-dimension GLR test is thus displayed in Fig. 1.

Some comments are in order regarding this detector. First, it is amenable to parallel implementation as suggested by the structure in Fig. 1. Next, we observe that

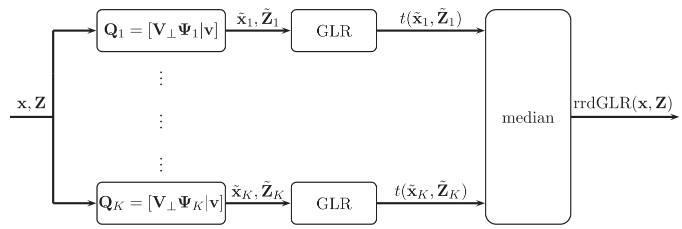

Fig. 1. Structure of the proposed random reduced-dimension generalized likelihood ratio test.

$$\begin{aligned} \boldsymbol{Q}_k &= \begin{bmatrix} \boldsymbol{V}_\perp \boldsymbol{\Psi}_k & \boldsymbol{v} \end{bmatrix} \\ &= \begin{bmatrix} \boldsymbol{V}_\perp \boldsymbol{N}_k \left(\boldsymbol{N}_k^H \boldsymbol{N}_k\right)^{-H/2} & \boldsymbol{v} \end{bmatrix} \\ &= \begin{bmatrix} \boldsymbol{V}_\perp \boldsymbol{N}_k & \boldsymbol{v} \end{bmatrix} \begin{bmatrix} \left(\boldsymbol{N}_k^H \boldsymbol{N}_k\right)^{-H/2} & \boldsymbol{0} \\ \boldsymbol{0} & 1 \end{bmatrix} \\ &= \bar{\boldsymbol{Q}}_k \boldsymbol{A}_k \end{aligned} \tag{5}$$

It follows that $\tilde{\boldsymbol{x}}_k = \boldsymbol{A}_k^H \bar{\boldsymbol{Q}}_k^H \boldsymbol{x}$ and $\tilde{\boldsymbol{Z}}_k = \boldsymbol{A}_k^H \bar{\boldsymbol{Q}}_k^H \boldsymbol{Z}$ so that

$$\begin{aligned} \boldsymbol{e}_{N+1}^H \tilde{\boldsymbol{S}}_k^{-1} \tilde{\boldsymbol{x}}_k &= \boldsymbol{e}_{N+1}^H \left[\boldsymbol{A}_k^H \bar{\boldsymbol{Q}}_k^H \boldsymbol{Z} \boldsymbol{Z}^H \bar{\boldsymbol{Q}}_k \boldsymbol{A}_k\right]^{-1} \boldsymbol{A}_k^H \bar{\boldsymbol{Q}}_k^H \boldsymbol{x} \\ &= \boldsymbol{e}_{N+1}^H \boldsymbol{A}_k^{-1} \left[\bar{\boldsymbol{Q}}_k^H \boldsymbol{Z} \boldsymbol{Z}^H \bar{\boldsymbol{Q}}_k\right]^{-1} \bar{\boldsymbol{Q}}_k^H \boldsymbol{x} \\ &= \boldsymbol{e}_{N+1}^H \left[\bar{\boldsymbol{Q}}_k^H \boldsymbol{Z} \boldsymbol{Z}^H \bar{\boldsymbol{Q}}_k\right]^{-1} \bar{\boldsymbol{Q}}_k^H \boldsymbol{x} \end{aligned} \tag{6}$$

since

$$\begin{aligned} \boldsymbol{A}_k^{-H} \boldsymbol{e}_{N+1} &= \begin{bmatrix} \left(\boldsymbol{N}_k^H \boldsymbol{N}_k\right)^{1/2} & \boldsymbol{0} \\ \boldsymbol{0} & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{0} \\ 1 \end{bmatrix} \\ &= \boldsymbol{e}_{N+1} \end{aligned} \tag{7}$$

Similarly,

$$\begin{aligned} \boldsymbol{e}_{N+1}^H \tilde{\boldsymbol{S}}_k^{-1} \boldsymbol{e}_{N+1} &= \boldsymbol{e}_{N+1}^H \left[\boldsymbol{A}_k^H \bar{\boldsymbol{Q}}_k^H \boldsymbol{Z} \boldsymbol{Z}^H \bar{\boldsymbol{Q}}_k \boldsymbol{A}_k\right]^{-1} \boldsymbol{e}_{N+1} \\ &= \boldsymbol{e}_{N+1}^H \boldsymbol{A}_k^{-1} \left[\bar{\boldsymbol{Q}}_k^H \boldsymbol{Z} \boldsymbol{Z}^H \bar{\boldsymbol{Q}}_k\right]^{-1} \boldsymbol{A}_k^{-H} \boldsymbol{e}_{N+1} \\ &= \boldsymbol{e}_{N+1}^H \left[\bar{\boldsymbol{Q}}_k^H \boldsymbol{Z} \boldsymbol{Z}^H \bar{\boldsymbol{Q}}_k\right]^{-1} \boldsymbol{e}_{N+1} \end{aligned} \tag{8}$$

and

$$\begin{aligned} \tilde{\boldsymbol{x}}_k^H \tilde{\boldsymbol{S}}_k^{-1} \tilde{\boldsymbol{x}}_k &= \tilde{\boldsymbol{x}}_k^H \bar{\boldsymbol{Q}}_k \boldsymbol{A}_k \left[\boldsymbol{A}_k^H \bar{\boldsymbol{Q}}_k^H \boldsymbol{Z} \boldsymbol{Z}^H \bar{\boldsymbol{Q}}_k \boldsymbol{A}_k\right]^{-1} \boldsymbol{A}_k^H \bar{\boldsymbol{Q}}_k^H \boldsymbol{x} \\ &= \tilde{\boldsymbol{x}}_k^H \bar{\boldsymbol{Q}}_k \left[\bar{\boldsymbol{Q}}_k^H \boldsymbol{Z} \boldsymbol{Z}^H \bar{\boldsymbol{Q}}_k\right]^{-1} \bar{\boldsymbol{Q}}_k^H \boldsymbol{x} \end{aligned} \tag{9}$$

Therefore, the test statistic is left *unchanged* if $\boldsymbol{Q}_k = \begin{bmatrix} \boldsymbol{V}_\perp \boldsymbol{\Psi}_k & \boldsymbol{v} \end{bmatrix}$ is replaced by $\bar{\boldsymbol{Q}}_k = \begin{bmatrix} \boldsymbol{V}_\perp \boldsymbol{N}_k & \boldsymbol{v} \end{bmatrix}$ or equivalently if $\boldsymbol{\Psi}_k$ is replaced by $\boldsymbol{N}_k$. This means that *it is not necessary to orthonormalize the columns of $\boldsymbol{N}_k$ and one just needs to generate matrices with i.i.d. complex Gaussian entries* $\mathcal{CN}(0, 1)$. This fact, together with the possible parallelization and the fact that one deals with matrices of reduced dimensions makes this detector rather simple from a computational point of view.

A final remark concerns the distribution of the test statistic under H_0. Despite the fact that the marginal distributions of all t_k do not depend on $\boldsymbol{R}$ under H_0, this does not necessarily imply that the joint distribution of $(t_1, \ldots, t_K)$ is independent of $\boldsymbol{R}$, and therefore the proposed detector does not possess the constant false alarm rate property. However, this is also not the case of the diagonally loaded or the low-rank adaptive matched filters.

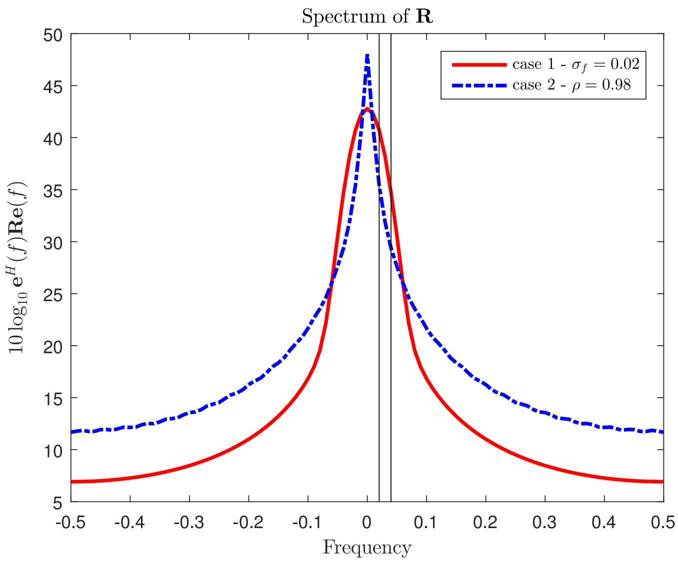

Fig. 2. Spectrum of $\boldsymbol{R}$ for the two cases considered. The vertical lines show the frequency of the signal of interest.

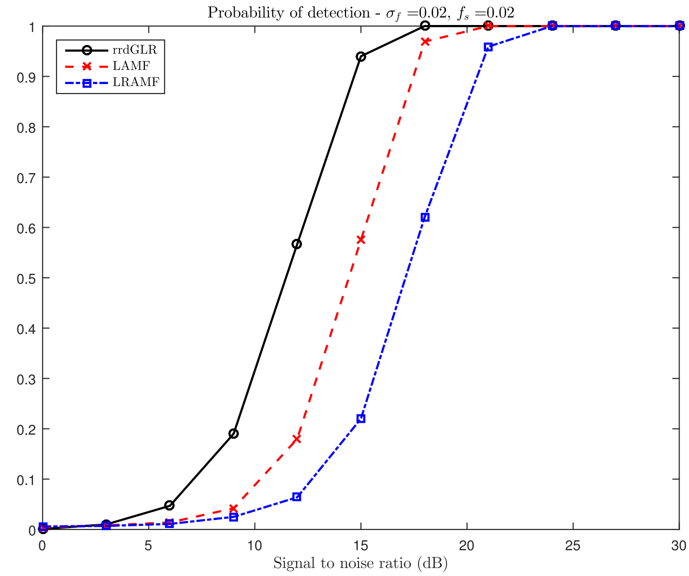

Fig. 3. Probability of detection in case 1. $P_{\text{fa}} = 10^{-3}$, $f_s = 0.02$, $\sigma_f = 0.02$, $N = 11$ and $T = 22$.

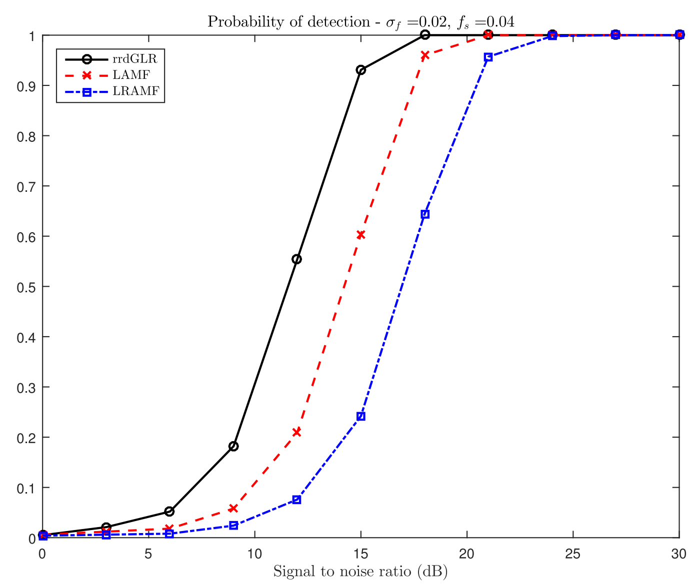

Fig. 4. Probability of detection in case 1. $P_{\text{fa}} = 10^{-3}$, $f_s = 0.04$, $\sigma_f = 0.02$, $N = 11$ and $T = 22$.

3. Performance analysis

In this section we investigate the performance of the proposed detector and compare it with state of the art detectors. We consider a scenario where $M = 128$. The disturbance covariance matrix is of the form $\boldsymbol{R} = \boldsymbol{R}_c + \boldsymbol{I}_M$ which corresponds to a colored clutter plus thermal noise model. Two cases will be considered. In the first case the (k, ℓ) element is $\boldsymbol{R}_c(k, \ell) = Pe^{-0.5(2\pi\sigma_f|k-\ell|)^2}$ with $\sigma_f = 0.02$, while in the second case $\boldsymbol{R}_c(k, \ell) = P\rho^{|k-\ell|}$ with $\rho = 0.98$. The clutter to noise ratio $\text{CNR} = 10\log_{10} P$ is set to $\text{CNR} = 30\text{dB}$. The signal of interest is $\boldsymbol{v} = \boldsymbol{e}(f_s)$ where $\boldsymbol{e}(f) = 1/\sqrt{M}\begin{bmatrix} 1 & e^{2i\pi f} & \dots & e^{2i\pi(M-1)f} \end{bmatrix}^T$. We consider low frequencies $f_s = 0.02$ or $f_s = 0.04$ so that the signal of interest is strongly buried in noise. For illustration purposes, Fig. 2 shows the spectrum of $\boldsymbol{R}$, that is $\boldsymbol{e}^H(f)\boldsymbol{Re}(f)$.

The proposed detector, which is referred to as rrdGLR in the figures below, is compared to benchmark competitors, namely the loaded AMF and the low-rank AMF

$$\text{LAMF} = \frac{|\boldsymbol{v}^H(\hat{\boldsymbol{R}} + \nu\boldsymbol{I}_M)^{-1}\boldsymbol{x}|^2}{\boldsymbol{v}^H(\hat{\boldsymbol{R}} + \nu\boldsymbol{I}_M)^{-1}\boldsymbol{v}} \tag{10}$$

$$\text{LRAMF} = \frac{|\boldsymbol{v}^H\boldsymbol{P}^{\perp}\boldsymbol{x}|^2}{\boldsymbol{v}^H\boldsymbol{P}^{\perp}\boldsymbol{v}} \tag{11}$$

where $\boldsymbol{P}^{\perp}$ stands for the projector onto the subspace orthogonal to the N principal eigenvectors of $\hat{\boldsymbol{R}}$. For LAMF the diagonal loading level was fixed at 15dB above the white noise level. Note that we also tested the loaded GLRT but its performance is identical to that of LAMF, so we only plot the results of the latter. For both rrdGLR and LRAMF, N is chosen as the "effective rank" of $\boldsymbol{R}_c$ which is defined as the lowest integer for which $\sum_{m=1}^{N}\lambda_k(\boldsymbol{R}_c) \geq 0.95\sum_{m=1}^{M}\lambda_k(\boldsymbol{R}_c)$ where $\lambda_k(\boldsymbol{R}_c)$ are the eigenvalues of $\boldsymbol{R}_c$. In other words, at least 95% of the energy in $\boldsymbol{R}_c$ is contained in the first N eigenvectors. For both cases described above, this results in $N = 11$. The number of training samples is set to $T = 2N$ and the probability of false alarm is $P_{\text{fa}} = 10^{-3}$. Through preliminary simulations, we investigated the influence of K on the probability of detection of rrdGLR, varying from $K = 20$ to $K = 80$. It turned out that there is almost no improvement for larger K so, to decrease computational load, we fix $K = 20$.

In Figs. 3–6 we plot the probability of detection versus signal to noise ratio, which is defined as $\text{SNR} = |\alpha|^2\boldsymbol{v}^H\boldsymbol{R}^{-1}\boldsymbol{v}$. As can be noticed from these figures, the rrdGLRT performs very well and is shown to outperform both the LAMF and the LRAMF, especially when the frequency f_s is small which in radar could correspond to slowly moving targets. The improvement is more pronounced in case 1 than in case 2. In simulations not reported here, we observed that the improvement is less important when σ_f decreases or when ρ increases, i.e., when noise is more lowpass and the effective rank of $\boldsymbol{R}$ decreases. However, it is remarkable that such technique performs so well.

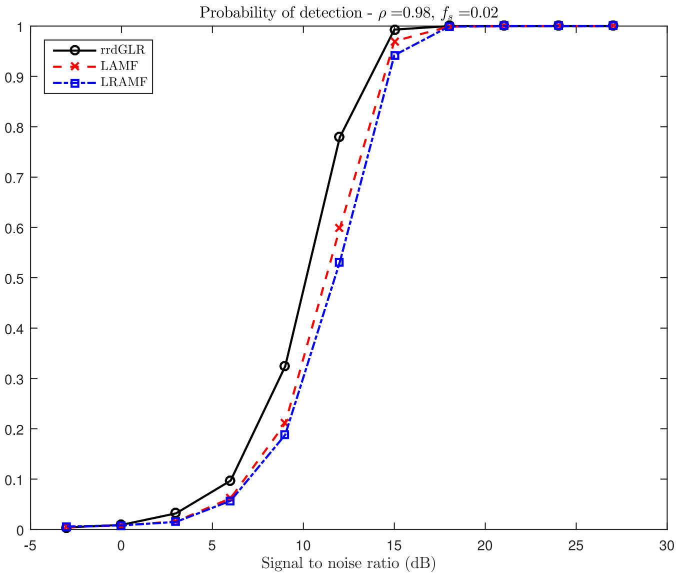

Fig. 5. Probability of detection in case 2. $P_{fa} = 10^{-3}$, $f_s = 0.02$, $\rho = 0.98$, $N = 11$ and $T = 22$.

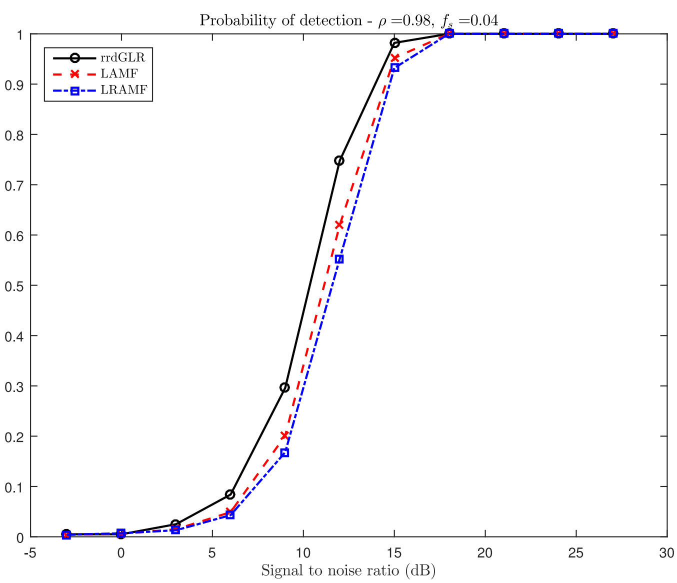

Fig. 6. Probability of detection in case 2. $P_{fa} = 10^{-3}$, $f_s = 0.04$, $\rho = 0.98$, $N = 11$ and $T = 22$.

4. Conclusions

In this communication, we considered detection in Gaussian noise with unknown statistics when the number of target-free training samples is smaller than the size of the observation space. We adapted an idea originally developed by Marzetta which relies on a set of random semi-unitary matrices to achieve dimensionality reduction, processing in reduced dimension and recombination. For our detection problem, we proposed to use the median value of the reduced dimension generalized likelihood ratios. This technique avoids eigenvalue decomposition, is easily amenable to parallel implementation and we showed that one does not need to generate semi-unitary matrices but only independent and identically distributed Gaussian random matrices. The new detector was shown to perform very well, compared to state of the art detectors.

Declaration of Competing Interest

I acknowledge that there is no conflict of interest regarding the paper entitled “Adaptive detection using randomly reduced dimension generalized likelihood ratio test”.

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