Overcoming the Realization Problems of Wideband Matching Circuits

Balázs Matolcsy and Attila Zólomy

Abstract—During the analytical design process of wideband impedance matching major problems may arise, that might lead to non-realizable matching networks, preventing the successful impedance matching. In this paper two practical design rules and a simplified equation is presented, supporting the design of physically realizable impedance matching networks. The design rules and calculation technique introduced by this paper is summarized, and validated by microwave circuit simulation examples.

Index Terms—physical matching limits, wideband impedance matching, realizable matching networks

I. INTRODUCTION

Analytical wideband impedance matching techniques have been thoroughly discussed in previous many studies [1], but most of these only focus on the theoretical limits of the matching techniques, by issuing an infinite number of passive L-C elements for the matching circuit [2]. Several approaches have been shown to be successful for matching complex impedances [3] [4], but hardly any of them discuss the physical realization problems, and practical limitations of the finite length matching networks [5] [6]. Due to the high calculation complexity, of the wideband matching networks, mostly only third-order matching networks are used due to practical reasons (higher order matching networks have various problems, such as weak parameter tolerance margins, inhibiting manufacturing processes), thus this paper only discusses third-order lossless matching networks.

This paper presents two of the practical realization limits of the analytical wideband complex impedance matching technique, presented by R. M. Fano [1], and H. W. Bode [2]. Utilising the proposed limitation factors, and simplified calculations presented in this paper, matching optimization goals are easier to define, and a wide range of practically unrealizable solutions are excluded before the complex calculation process. The rest of this paper is organized as follows: first the Bode-Fano matching technique is presented in detail followed by the practical parameter restrictions in Section IV. and Section V. Later on in Section VI. the modified matching algorithm and a simplified caluclation for a certain matching parameter is introduced as well. Finally the proposed design rules are validated by two simulation examples in Section VIII.

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II. ANALYTICAL MATCHING TECHNIQUE (BODE-FANO)

The analytical wideband complex impedance matching methods are based on the Darlington-theorem, which states, that a complex load can be substituted, with a passive reactant network that is terminated in a unity value resistance [7]. This principle allows that the complex wideband impedance matching task can be redefined as a double-terminated filter synthesis problem. In most situations complex impedances are matched (on the largest possible bandwidth) to a purely real valued generator impedance, thus the matching network's purpose is to completely cancel out the imaginary part of the load impedance, and match the remaining real part to the generator at the same time. Well-known examples for analytical matching methods are: Bode-Fano matching [2], and Youla's matching technique based on complex normalization [11]. In this paper an in-depth analysis is presented discussing the Bode-Fano method for complex terminations, matched to purely real 50 Ω source impedance.

Within the design equations lies a problem which partially inhibits the realization of matching networks, at certain initial parameters. Furthermore the upper, and lower matched frequency should be very carefully chosen, otherwise analytical matching can result in matching networks that are physically unrealizable. In the following section the detailed equations and restrictions are presented for realizable matching networks (where matching networks are constructed from shorted quarter wavelength stubs, that can only represent purely real valued impedances). Shortly thereafter, the physical design limitations are taken into consideration during the calculations, highly restricting the range of complex impedances where the Bode-Fano analytical method provides adequate matching. Obeying these design rules during the design process may help designing load impedances (where allowed), at which the Bode-Fano method results in acceptable matching (e.g. where $|S_{11}|$ is less than -10 dB).

III. THE ANALYTICAL MATCHING PROCESS

An important aspect of the Bode-Fano matching method is that it can only be used for terminations where the impedance-frequency dependency resembles a single-reactance load's impedance or admittance. Thereby the load shall be substituted with a well chosen single-reactance circuit model, i.e. a series or parallel R-C, R-L impedance. Substitution model validation methods are omitted here. All matching results will be compared to the ideal infinite matching networks limits, discussed

for single-reactance models by Bode and Fano in [1]. The problems that may arise during the matching process is shown through a practical example. The goal is to match a series R-L impedance to a purely real 50 Ω generator impedance. First, the series capacitance (C_{series}) should be calculated through which the series R-L can be turned into a resonant R-L-C structure. (for parallel terminations, parallel resonance may be required, and design equations are slightly different)

$$C_{\text{series}} = \frac{1}{4\pi^2 f_c^2 L}, \tag{1}$$

where f_c represents the center frequency of the desired matched band. The quality factor (Q) for this R-L-C structure is defined as

$$Q = \frac{2\pi f_c L}{R}. \tag{2}$$

The theoretical matching quality limit can be calculated using the decrement factor,

$$\delta = \frac{1}{Q} \frac{\sqrt{4\pi^2 f_h f_l}}{2\pi(f_h - f_l)}, \tag{3}$$

where f_h is the upper, and f_l is the lower frequency limit. Using the decrement factor and the minimum of the maximal reflection coefficient on the desired bandwidth is expressed as

$$\Gamma_{\text{max}} = 20 \log_{10}\left(e^{-\pi\delta}\right), \tag{4}$$

where Γ_{max} stands for the best available reflection coefficient in case of an ideal matching network consisting of an infinite number of L-C ladder components. This limit is also known as the Bode matching limit. This parameter is used for the comparison of theoretical and the finite third-order matching quality. In order to point out the critical parameters during the matching process, the detailed process of the single reactance matching example is presented. The following equations will lead to the solution of the low pass filter prototype, for the matching circuit,

$$d = \sinh\frac{\sinh^{-1}\sqrt{\frac{1}{10^{(r/10)}-1}}}{n}, \tag{5}$$

$$D = \frac{d}{\delta \sin\left(\frac{\pi}{2n}\right)} - 1, \tag{6}$$

where n represents the order of the matching circuit, and r stands for the maximal allowed Chebyshev ripple factor in the matched frequency band. Later on in Section VII., a simplified parameter calculation method is presented for d. These parameters are used for evaluating the coupling coefficients for the low pass filter prototype network, provided by Green's equations [10]:

$$k_{1,2} = \sqrt{\frac{3}{8}\left(1 + \left(1 + \frac{D^2}{3}\right)\delta^2\right)}, \tag{7}$$

$$k_{2,3} = \sqrt{\frac{3}{8}\left(1 + \left(\frac{1}{3} + D^2\right)\delta^2\right)}. \tag{8}$$

Based on [8] and [10], the low pass filter prototype component values for the double-terminated filter are

$$g_0 = 1, \tag{9}$$

$$g_1 = \frac{1}{\delta}, \tag{10}$$

$$g_2 = \frac{1}{g_1 \cdot k_{1,2}^2}, \tag{11}$$

$$g_3 = \frac{1}{g_2 \cdot k_{2,3}^2}, \tag{12}$$

$$g_4 = \frac{1}{D \cdot \delta \cdot g_3}. \tag{13}$$

As seen in (9), the generator impedance (R_g) is determined as the synthesis result. This overrides the original generator impedance (50 Ω), which is unacceptable. Furthermore this is a low-pass filter prototype circuit, hence filter transformation steps are required for a passband configuration. For overcoming these synthesis problems, admittance inverters are used. The auxilary parameters used for defining the admittance inverter's parameters are

$$d_p \approx 1, \tag{14}$$

$$\omega_m = \frac{(f_h - f_l)}{f_c}, \tag{15}$$

$$\Theta_1 = \frac{\pi}{2}\left(1 - \frac{\omega_m}{2}\right) \tag{16}$$

and the admittance inverter parameters are calculated using equations in [10] and [9]:

$$C_2 = g_2, \tag{17}$$

$$C_3 = g_0 g_3 g_4 \frac{R_L}{R_g}, \tag{18}$$

$$C_2' = g_2(1 - d_p), \tag{19}$$

$$C_2'' = d_p g_2, \tag{20}$$

$$C_3' = C_2'', \tag{21}$$

$$C_3'' = C_3 - C_3', \tag{22}$$

$$J_{2,3} = \frac{1}{R_L}\sqrt{\frac{C_2 C_3}{g_2 g_3}}, \tag{23}$$

$$N_{2,3} = \sqrt{(J_{2,3} R_L)^2 + \left(\frac{C_2'' \tan(\Theta_1)}{g_0}\right)^2}. \tag{24}$$

The construction of the matching network is based on quarter wavelength (at the center frequency), shorted stubs. The shorted stubs only have a single free parameter in this case: transmission line admittance (or impedance). These admittance values are obtained by the following equations:

$$Y_2 = \frac{1}{g_0 R_L}C_2' \tan(\Theta_1) + \frac{1}{R_L}(N_{2,3} - J_{2,3} R_L), \tag{25}$$

$$Y_3 = \frac{1}{g_0 R_L}C_3'' \tan(\Theta_1) + \frac{1}{R_L}(N_{2,3} - J_{2,3} R_L), \tag{26}$$
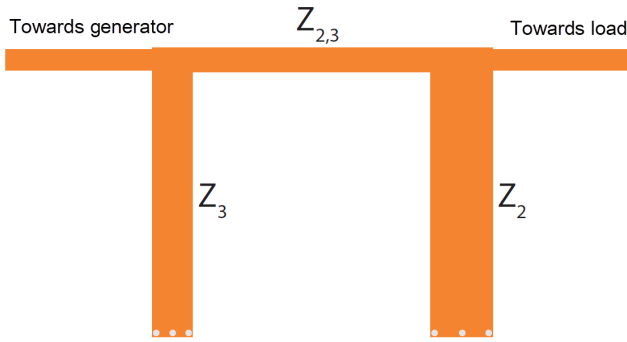
$$Y_{2,3} = J_{2,3}. \tag{27}$$

Fig. 1. Third order matching network with $\lambda/4$ shorted stubs.

The impedance values are the reciprocal, of the given admittances respectively ($Z_2, Z_3, Z_{2,3}$). The third-order matching network structure is shown in Fig. 1. However, this method does not always provide purely real transmission line impedance values. In several cases, if the input parameters are not well chosen the equations may lead to complex transmission line immittances. Complex transmission lines are not realizable with lossless components. For effective impedance matching, only lossless circuit elements are allowed (with purely real impedance values), otherwise the required power would not reach the load, but dissipate in the lossy elements. In the following section mandatory parameter restriction is presented, used for avoiding these unwanted complex transmission line impedances.

IV. AVOIDING COMPLEX TRANSMISSION LINE IMPEDANCES

Matching network transmission line impedances are expressed as the reciprocal of the admittance values. As known from linear network theory, these impedance values can be solely purely real (assuming lossless transmission lines). However there are multiple cases where the results are complex impedance values. Complex impedance value may arise first during the design if

$$\text{Im}\left(J_{2,3}\right) \neq 0. \tag{28}$$

It is also worth mentioning that $J_{2,3}$ can be purely real, or pure imaginary due to the nature of the square root function, in (24). In order to avoid the pure imaginary impedance value it is necessary to point out which parameter values affect this. If

$$\frac{C_2 C_3}{g_2 g_3} < 0, \tag{29}$$

then $J_{2,3}$ is pure imaginary. As the parameters are multiplied together, an odd number of negative coefficients in the expression can result in a value less than zero. Due to the fact that the decrement δ, and Green-coefficients $k_{1,2}$ and $k_{2,3}$ are always greater than or equal to zero, g_0, g_1, and g_2 can only be positive. Thus a negative value in Eq. (29) can solely occur

if $C_3 < 0$, leading to $g_4 < 0$. The g_4 parameter is negative if, $D < 0$, resulting

$$D < 0 \quad \text{iff.} \quad \frac{d}{\delta \sin\left(\frac{\pi}{2n}\right)} < 1, \tag{30}$$

assuming that $n = 3$ (third-order matching network) and $\delta > 0$:

$$\text{Im}\left(J_{2,3}\right) = 0 \quad \text{iff.} \quad 2d > \delta. \tag{31}$$

In order to avoid complex transmission line impedance values, the mandatory rule is to set,

$$d > \frac{\delta}{2}. \tag{32}$$

On one hand the d parameter can be set by modifying r, which stands for the amount of allowed Chebyshev-ripple in the matched frequency band, on the other hand the decrement factor (δ) is pre-determined by the quality factor, and the frequency band. If the matching task allows the modification of either r, or f_l or f_h, this mandatory rule can be satisfied (in some cases), as shown later in Section VIII-B.

V. PHYSICAL TRANSMISSION LINE IMPEDANCE LIMITATIONS

As most of the matching circuit designs are based on microstrip transmission lines, one should always avoid extreme line impedances. As a basic rule e.g. on a printed circuit board, transmission line impedances should be $15\,\Omega \leq Z \leq 150\,\Omega$. This is due to the copper structure manufacturing tolerance limits. The matching network realization fails, if any of these transmission line impedances do not obey this rule. As an example let's assume a purely real $Z_{2,3}$ that satisfies the rule

$$15\,\Omega < R_L \sqrt{\frac{g_2 g_3}{C_2 C_3}} < 150\,\Omega. \tag{33}$$

Substituting into this expression,

$$R_L \sqrt{\frac{g_2 g_3}{C_2 C_3}} = R_L \sqrt{\frac{D R_g k_{1,2}^2}{R_L k_{2,3}^2}}, \tag{34}$$

where R_L is the real part of the load impedance, and R_g is the generator impedance. If the condition,

$$15\,\Omega < R_L \sqrt{\frac{D R_g k_{1,2}^2}{R_L k_{2,3}^2}} < 150\,\Omega, \tag{35}$$

does not apply, the matching network is not realizable, due to the aforementioned physical limits. This short expression requires only five parameters, and can be used to exclude non-realizable matching networks at an early phase of the network design. The same limit calculation can be applied to Z_2 and Z_3, however the expression is more complex, and irrelevant, considering the fact, that if at least one of the transmission line impedances is not realizable, the matching network is non-realizable as well.

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Algorithm 1 Modified matching algorithm

task $\rightarrow$ initial parameters$(R_\mathrm{L}, R_\mathrm{g}, L, f_\mathrm{c}, b, r)$
while $|S_{11}| > -10\,\mathrm{dB}$ **do**
 calculate d and δ parameters
 if $2\,d > \delta$ **then**
  non-complex matching network
  calculate $D, k_{1,2}, k_{2,3}$
  if $15\,\Omega < R_\mathrm{L}\sqrt{\dfrac{D\,R_\mathrm{g}\,k_{1,2}^2}{R_\mathrm{L}\,k_{2,3}^2}} < 150\,\Omega$ **then**
   physically realizable, non-complex network
   calculate network impedances $(Z_2, Z_3, Z_{2,3})$
   recalculate $|S_{11}|$
  else
   physical realization problem, modify r or b
  end if
 else
  if $2\,d < \delta$ **then**
   complex matching network, modify r or b
  end if
 end if
end while
successful matching is achieved

VI. ALGORITHMIC IMPLEMENTATION OF THE MODIFIED MATCHING METHOD

A modified matching algorithm (Algorithm 1) is defined herein for avoiding the non-realizable matching network solutions, by using the rules given in previous sections. As it can be seen in Algorithm 1, the matching network impedances should only be calculated, if both physical limitations are satisfied. The complexity of these calculations can be further reduced, if the calculation of parameters d and δ is faster. The aim of the upcoming section is to show a possible faster approximate expression to calculate the required parameter d from r, thus speeding up the iteration process.

VII. SIMPLIFIED CALCULATION OF THE d PARAMETER

The exact calculation of the parameters d and D for Green's coupling coefficients were shown in Section III. As the hyperbolic, and the inverse hyperbolic functions may be difficult to evaluate, this paper will introduce a simplified approximate expression for the parameter d. As
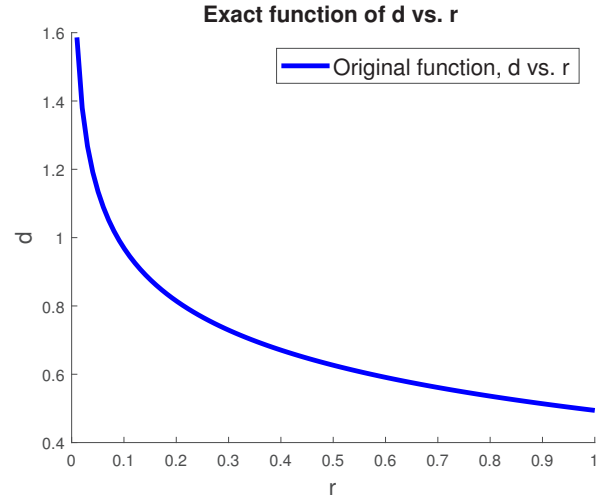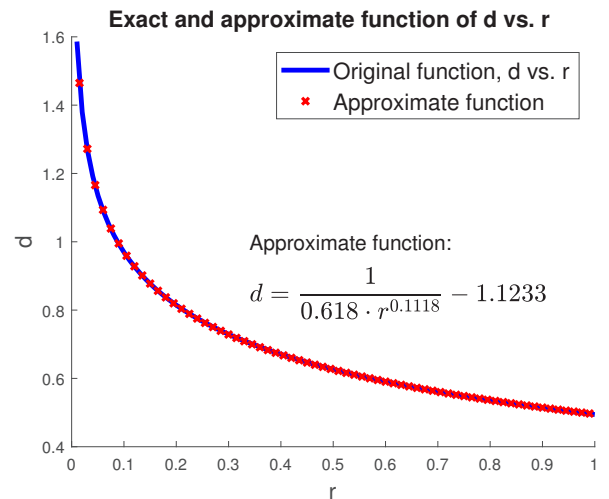
$$\sinh x = \frac{e^x - e^{-x}}{2}, \tag{36}$$

$$\sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right), \tag{37}$$

these expressions can be used to transform hyperbolic equations to their exponential and logarithmic forms. By substituting

$$u = \sqrt{\frac{1}{10^{(r/10)} - 1}},$$

the parameter d can be rewritten in the form:

$$d = \frac{1}{2}\left(e^{\ln\left(u + \sqrt{u^2+1}\right)^{\frac{1}{3}}} - e^{\ln\left(u + \sqrt{u^2+1}\right)^{-\frac{1}{3}}}\right) =$$
$$= \frac{1}{2}\left(\sqrt[3]{u + \sqrt{u^2+1}}\right) - \frac{1}{2}\left(\sqrt[3]{u + \sqrt{u^2+1}}\right)^{-1}.$$

Fig. 2. The original function of d vs. r

Fig. 3. The original and the approximate function of d vs. r

As can be seen from Fig. 2, if d vs. r is plot in the Cartesian coordinate-system, the function resembles a monotonic decending hyperbolic function. These embedded square, and third-root functions are computationally extensive, thus an acceptable approximate function is proposed. The approximation (i.e the curve fitting task) was carried out in MATLAB 2017b. As mentioned before the proposed initial function is a custom hyperbolic function which has the symbolic form

$$\hat{d}(r) = \frac{1}{a \cdot r^b} + c,$$

where a, b, c are arbitrary constants. Utilizing the Least-Square Method based curve fitting, these constants were determined

$$a = 0.618, \quad b = 0.1118, \quad c = -1.1233.$$

Based on these constants, the approximate function for d is

$$\hat{d}(r) = \frac{1}{0.618 \cdot r^{0.1118}} - 1.1233. \quad (38)$$

This approximate and the original exact function is presented in Fig. 3. The curve-fitting approximation exhibited acceptable results, with an RMSE (Root Mean Squared Error) value of $8.86 \cdot 10^{-4}$, and R-square factor of 1.00. On this basis it can be safely stated that

$$d \approx \frac{1}{0.618 \cdot r^{0.1118}} - 1.1233. \quad (39)$$

During the iterative solution to the best matching network, in Algorithm 1 the parameter d should be reevaluated as soon as r changes. This evaluation occurs everytime the algorithm runs into an unrealizable matching solution. With this new hyperbolic formula introduced here in Eq. (39), the resulting algorithm exhibits a reduced processing time.

VIII. MATCHING EXAMPLES (SIMULATIONS)

In this section two matching examples are introduced. One where the $2d > \delta$ condition is initially satisfied, and another where it is not. These simulation setups were created in *AWR Microwave Design Studio 2010*. The simulated impedance matching networks are designed for matching series R-L loads to a $R_g = 50\,\Omega$ generator, on a specific European ISM UHF frequency band, i.e. 868 MHz.

A. Impedance matching example 1.

The impedance matching example parameters are shown in Table I,

TABLE I
IMPEDANCE MATCHING EXAMPLE 1. - INITIAL PARAMETERS

f_c	b	R_g	R_L	L	r	δ	d
868 MHz	25%	50 Ω	30 Ω	20 nH	0.066	**0.5366**	**1.068**

where values in bold, are parameters calculated from the six initial parameters: center frequency, relative matching bandwidth, generator source impedance, load resistance, load inductor value, and maximal Chebyshev-ripple factor respectively. Regarding the values in Table I, the fundamental rule for non-complex transmission line impedances (presented in Section IV.) and the practical impedance realization limit are both satisfied (see Eq. (35)).

The matching network consists of the following transmission line impedances, and series capacitance.

Fig. 4 presents the input reflection coefficient as a function of frequency, for the matching network. As it is highlighted with the markers, an acceptable matching ($|S_{11}| \leq -10\,\text{dB}$) is reached on the $738\ldots1116\,\text{MHz}$ frequency band. The physical line impedance realization limit is satisfied as well,

$$15\,\Omega < 50\,\sqrt{1.153} < 150\,\Omega.$$

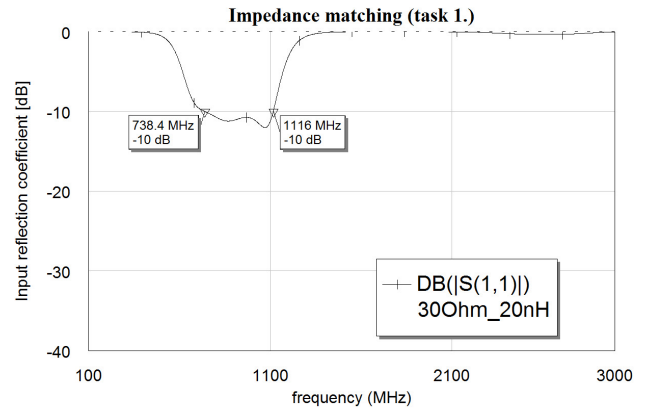Fig. 4. Wideband matching result, for example 1.

TABLE II
MATCHING NETWORK COMPONENT PARAMETERS (EX. 1.)

$Z_{2,3}$	Z_2	Z_3	C_{series}
51.36 Ω	26.75 Ω	70.9 Ω	1.68 pF

As a conclusion, this example clearly states, that by abiding the rules introduced in Section IV. and Section V. one can avoid matching networks that are practically impossible to implement.

B. Impedance matching example 2.

In this example the initial parameters do **not** satisfy the fundamental realization rule (see Eq. (32)), therefore resulting in a non-realizable matching network. Overcoming this problem is presented hereby.

TABLE III
IMPEDANCE MATCHING TASK 2. - INITIAL PARAMETERS

f_c	b	R_g	R_L	L	r	δ	d
868 MHz	10%	50 Ω	50 Ω	20 nH	0.066	**2.281**	**1.068**

One might presume, that this matching task is easier, due to the smaller bandwidth, and higher real part of the load impedance, however the Bode-Fano method basically does not respect the realization rule shown in Section IV., hence the synthesis results in complex transmission line impedances.

TABLE IV
MATCHING NETWORK COMPONENT PARAMETERS (EX. 2.)

$Z_{2,3}$	Z_2	Z_3	C_{series}
$-18.86\,\text{j}\Omega$	$7.31 + 3.472\,\text{j}\Omega$	$-1.21 + 0.0668\,\text{j}\Omega$	1.68 pF

The solution for this problem is to either modify the matched relative bandwidth, or the Chebyshev-ripple factor, if allowed. This time, by setting $r = 0.0001$ the results have become acceptable (Fig. 5).

TABLE V
MODIFIED NETWORK PARAMETERS (EX. 2.)

$Z_{2,3}$	Z_2	Z_3	C_{series}
$53.17\,\Omega$	$27.18\,\Omega$	$104.1\,\Omega$	$1.68\,\mathrm{pF}$

By modifying r, d is also modified ($d = 3.668$) and Eq. (32) is thereby satisfied, thus the matching task is solvable exclusively utilizing lossless components (impedance values are in Table V). The results in Fig. 5 show that the matching quality is better than expected, and the matched bandwidth is almost 50%, contrary to the predefined $b = 10\%$. This result is due to the outstandingly low, maximal Chebyshev-ripple factor, prescribed as the fix for avoiding complex impedances. As seen in this second matching example varying the Chebyshev-ripple factor has beneficiary effects on avoiding transmission line impedances. If the matching task allows, modifying the upper and lower frequency limits may have the same effect on avoiding unrealizable networks.
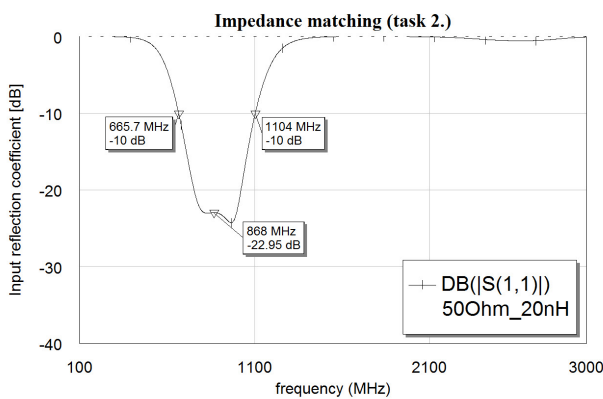

Fig. 5. Modifying r factor results in successful impedance matching and realizable network

CONCLUSION

The presented physical realization problems and solutions introduced in Section IV. and Section V. were successfully validated with simulation examples. The modified wideband matching algorithm was presented, and successfully applied for multiple matching tasks. If the realization limiting equations presented in this paper are satisfied, avoiding non-realizable matching networks becomes possible before complex calculations and optimalization steps were made. Furthermore the approximation for the parameter d reasonably reduces the calculation time at the iterative stage, thus speeding up the complete matching process.

ACKNOWLEDGMENT

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