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Status of hadronic light-by-light scattering in the muon $g-2$

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Summary. — We give an update on the status of the hadronic light-by-light scattering contribution to the muon $g-2$. We review recent work by various groups, list some of the open problems and give an outlook on how to better control the uncertainty of this contribution. This is necessary in order to fully profit from planned future muon $g-2$ experiments to test the Standard Model. Despite some recent developments, we think that the estimate $a_\mu^{\mathrm{HLbL}} = (116 \pm 40) \times 10^{-11}$ still gives a fair description of the current situation.

PACS `14.60.Ef` – Muons.
PACS `13.40.Em` – Electric and magnetic moments.
PACS `12.38.Lg` – Other nonperturbative calculations.

1. – Introduction

The anomalous magnetic moment of the muon has served over many years as an important test of the Standard Model, see the reviews [1, 2]. It is also sensitive to potential contributions from New Physics. The current status of the muon $g-2$ is summarized in table I where we list the different contributions in theory (QED, weak, hadronic) from various recent sources and compare with the experimental value. More references to earlier work can be found in the quoted papers and in refs. [1, 2]. The experimental world average is dominated by the final result of the Brookhaven muon $g-2$ experiment [7], corrected for a small shift in the ratio of the magnetic moments of the muon and the proton [8]. We observe a difference between experiment and theory of more than three standard deviations:

$$a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{th}} = (293 \pm 88) \times 10^{-11} \quad (3.3\sigma). \tag{1}$$

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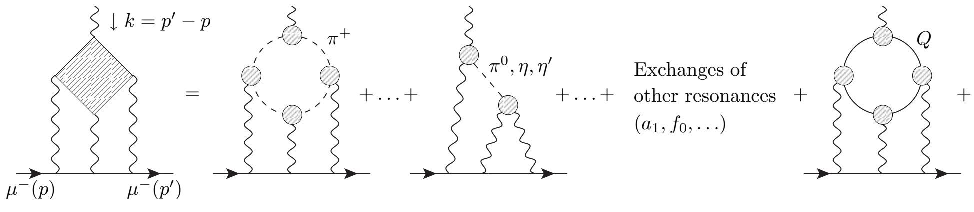
Table I. – *Standard Model contributions to* $a_\mu \times 10^{11}$ *and comparison of theory and experiment.*

Contribution	Value	Error	Reference
QED	116 584 718.853	0.036	[3]
Weak	153.6	1.0	[4]
Leading order HVP	6 907.5	47.2	[5]
Higher order HVP	−100.3	2.2	[5]
HLbL	116	40	[6, 1]
Theory (total)	116 591 796	62	–
Experiment	116 592 089	63	[7]
Experiment - Theory (3.3σ)	293	88	–

Unfortunately, the theoretical uncertainties [1, 2, 9] from hadronic vacuum polarization (HVP) and hadronic light-by-light scattering (HLbL) make it difficult to interpret this discrepancy as a clear sign of New Physics. Most recent evaluations [1, 5, 10, 2, 9], which differ slightly in the treatment of the hadronic contributions, obtain deviations of 3–4σ. In ref. [11] the hadronic cross-section data below 1 GeV was fitted to a (broken) Hidden Local Symmetry (HLS) model and a discrepancy in the muon $g-2$ between 3.7σ and 4.9σ was observed, depending on the selected data.

The HLbL contribution to the muon $g-2$ involves the Green function of four electromagnetic currents, connected to off-shell photons, see fig. 1 and ref. [1] for details and references. The relevant scales for the off-shell photons in HLbL are about 0–2 GeV, *i.e.* larger than the muon mass, and therefore a pure low-energy effective field theory approach with muons, photons and pions fails [12]. In contrast to the HVP contribution, HLbL cannot be directly related to experimental data and therefore various models have been employed to estimate HLbL. One uses some hadronic model with exchanges and loops of resonances at low energies and some form of (dressed, constituent) quark-loop at high energies as short-distance complement of the low-energy hadronic models. The dependence on several momenta leads, however, to a mixing of long and short distances and makes it difficult to avoid a double counting of quark-gluon and hadronic contributions. In ref. [13] a classification of the different contributions to HLbL based on the



Contribution:	pion-loop (dressed)	pseudoscalar exchanges		quark-loop (dressed)
Chiral counting:	p^4	p^6	p^8	p^8
N_c counting:	1	N_c	N_c	N_c

Fig. 1. – The different contributions to HLbL scattering and their chiral and large-N_c counting.

TABLE II. – *Summary of selected estimates for the different contributions to* $a_\mu^{\mathrm{HLbL}} \times 10^{11}$. *For comparison, the last line shows some results when no form factors are used.*

π, K-loops	π^0, η, η'	Axial-Vectors	Scalars	Quark-Loop	Total	Reference
$-4.5(8.1)$	82.7(6.4)	1.7(1.7)	–	9.7(11.1)	89.6(15.4)	[15]
$-19(13)$	85(13)	2.5(1.0)	$-6.8(2.0)$	21(3)	83(32)	[16]
–	83(12)	–	–	–	80(40)	[17]
0(10)	114(10)	22(5)	–	0	136(25)	[18]
–	–	–	–	–	110(40)	[19]
$-19(19)$	114(13)	15(10)	$-7(7)$	2.3 [c-quark]	105(26)	[20]
$-19(13)$	99(16)	22(5)	$-7(2)$	21(3)	116(40)	[6, 1]
–	81(2)	–	–	107(2)	188(4)	[21]
–	–	–	–	–	118–148	[22]
–	68(3) [π^0]	–	–	82(6)	150(3)	[23]
–	–	–	–	–	76(4)–125(7)	[24]
$-(11–71)$	–	–	–	–	–	[25]
$-20(5)$	–	–	–	–	–	[26]
-45	$+\infty$	–	–	60	–	undressed

chiral and large-N_c counting was proposed, see fig. 1. In general, all the interactions of the hadrons or the quarks with the photons are dressed by some form factors, *e.g.* via ρ-γ mixing. Note that in the Feynman diagrams in fig. 1 form factors with off-shell photons and off-shell hadrons enter [14]. Constraints on the models can be obtained from experimental data, *e.g.* on the various form factors, and from theory, *e.g.* chiral perturbation theory at low energies and short-distance constraints from perturbative QCD and the Operator Product Expansion (OPE) at high momenta.

2. – Current status of HLbL and recent developments

A selection of estimates for HLbL is presented in table II. Note that the refs. [15, 16] are the only full calculations of HLbL to date, using, as much as possible, one model for all the contributions (HLS model in ref. [15], Extended Nambu-Jona-Lasinio (ENJL) model in ref. [16]). Both calculations showed that the exchanges of the lightest pseudoscalar states, π^0, η, η', dominate numerically, which can be understood from the large-N_c counting. The contributions from the (dressed) pion-loop and the (dressed) quark-loop are subdominant, but not negligible, and they happen to largely cancel each other numerically. The final results for the total HLbL contribution were rather close in both models. In ref. [27] an ansatz for the pion-photon transition form factor with a minimal number of narrow vector resonances in large-N_c QCD (lowest meson dominance (LMD, LMD+V)) was matched to short-distance constraints from the OPE. The reevaluation of the pion-pole contribution to HLbL in ref. [17] with the ansatz from ref. [27] then revealed a sign error in the earlier calculations [15, 16]. Furthermore, a two-dimensional integral representation for $a_\mu^{\mathrm{HLbL};\pi^0}$ was derived in ref. [17] and the relevant momentum region was found to be 0–1.25 GeV. Later, ref. [18] derived new short-distance constraints from the four-point function on the pion-pole and axial-vector-pole contributions, which do not allow for any form factors at the external vertex. Reference [18] also included the mixing of two axial-vector nonets and studied the pion-loop within the HLS model in more detail. All this lead to a substantial enhancement of these contributions to HLbL. More recently, a new short-distance constraint on the off-shell form factor at the external vertex in pion-exchange was derived in ref. [6], which yielded, again in the framework of the lowest meson dominance approximation to large-N_c QCD, a value for this contribution about half-way between the results in refs. [15-17] and those in ref. [18]. Note that the compilations [19, 20, 1] and ref. [6] are largely based on the full calculations [15, 16], with revised or newly calculated values for some of the contributions. More recent estimates, mostly for the pseudoscalar contribution, can be found in ref. [28]. While most of these evaluations agree at the level of 15%, if one takes the extreme values, there is a spread of $a_\mu^{\mathrm{HLbL;PS}} = (59\text{–}107) \times 10^{-11}$.

Until 2010, a consensus had been reached about the central value $a_\mu^{\mathrm{HLbL}} \approx 110 \times 10^{-11}$, but there was a discussion on how to estimate the error, more progressively, $\pm 26 \times 10^{-11}$, in ref. [20] and more conservatively, $\pm 40 \times 10^{-11}$, in refs. [6, 1]. In view of the precision goal of future $g-2$ experiments at Fermilab and J-PARC [29] with $\delta a_\mu = 16 \times 10^{-11}$ and the continued progress in improving the error in HVP, the HLbL contribution might soon be the main uncertainty in the theory prediction, if it cannot be brought under better control [1, 2, 9].

In the last few years, several works have appeared which yield much larger (absolute) values for some of the contributions, see table II. In ref. [21] the quark-loop was studied using a Dyson-Schwinger equation approach. In contrast to refs. [15, 16], no damping compared to the bare constituent quark-loop result was seen, when a dressing was included. Note that this calculation of the quark-loop is not yet complete and that earlier, very large results for the quark-loop seem to have been affected by some errors in the numerics in certain parts. The large size of the quark-loop contribution in ref. [21] was questioned in the papers [22, 23], using different quark-models and approaches, see also the ballpark prediction for HLbL in ref. [24]. The pion-loop contribution was analyzed in ref. [25]. The authors stressed the importance of the pion-polarizability effect and the role of the axial-vector resonance a_1, which are not included in the models used in refs. [15, 16]. Depending on the value of the pion-polarizability and the model for the a_1 resonance used, a large variation was seen. The issue was taken up in ref. [26] where different models for the pion-loop were studied. The inclusion of the a_1 resonance was attempted, but no finite result for $g-2$ could be achieved. With a cutoff of 1 GeV, a result close to the earlier estimate in ref. [16] was obtained. Reference [26] also pointed out that the very small (absolute) value for the pion-loop in ref. [15] could be due to the fact that the HLS model used in ref. [15] has a wrong high-energy behavior and that there is some cancellation between positive and negative contributions in the pion-loop in HLbL.

3. – Outlook

Concerning the future, maybe lattice QCD will provide a reliable calculation of HLbL at some point, see ref. [30] for some promising recent results. In the meantime, only a close collaboration between theory and experiment can lead to a better controlled estimate for HLbL. On the theory side, the hadronic models can be improved by short-distance constraints from perturbative QCD to have a better matching at high momenta. One can also use dispersion relations to connect the theory with experimental data, *e.g.* in $\gamma\gamma \to \pi\pi$ [31]. Also the issue about whether the dressing of the bare constituent quark-loop leads to a suppression or an enhancement needs to be studied further. This problem is also related to the question whether there is any double counting involved.

On the experimental side, the information on various processes (decays, form factors, cross-sections) of hadrons interacting with photons at low and intermediate momenta, $|q| \leq 2\,\mathrm{GeV}$, can help to constrain the models. Important experiments which should be pursued include more precise measurements of the (transition) form factors of light pseudoscalars with possibly two off-shell photons in the process $e^+e^- \to e^+e^-P$ ($P = \pi^0, \eta, \eta'$) and the two-photon decay width and the (double) Dalitz decays of these mesons. This could further reduce the error of the dominant pseudoscalar exchange contribution [32]. Concerning the pion-loop contribution, in addition to studying $\gamma\gamma \to \pi\pi$, measurements of the pion-polarizability in various processes, *e.g.* in radiative pion decay $\pi^+ \to e^+\nu_e\gamma$, in radiative pion photoproduction $\gamma p \to \gamma'\pi^+n$ or with the hadronic Primakoff effect $\pi A \to \pi'\gamma A$ or $\gamma A \to \pi^+\pi^-A$ (with some nucleus A), can help to improve the models [25]. For the development of models with the axial-vector resonance a_1 and estimates of the sizable axial-vector contribution, information about the decays $a_1 \to \rho\pi, \pi\gamma$ would be useful as well. Finally, to extract the needed quantities from experiment will also require the development of dedicated Monte Carlo programs for the relevant processes [33].

4. – Conclusions

If the recent results for the quark-loop and pion-loop are taken at face value, one obtains the range $a_\mu^{\mathrm{HLbL}} = (64\text{–}202) \times 10^{-11}$. While the new approaches raise some important issues and point to potential shortcomings in the previously used models, these calculations are also still preliminary and further studies are needed. Therefore, the estimate

$$a_\mu^{\mathrm{HLbL}} = (116 \pm 40) \times 10^{-11} \tag{2}$$

from refs. [6, 1] still seems to give a fair description of the current situation.

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