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Fully differential Monte Carlo generator dedicated to TMDs and Bessel-weighted asymmetries

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Summary. — We present studies of double longitudinal spin asymmetries in semi-inclusive deep inelastic scattering using a new dedicated Monte Carlo generator, which includes quark intrinsic transverse momentum within the generalized parton model based on the fully differential cross section for the process. Additionally, we apply Bessel-weighting to the simulated events to extract transverse momentum dependent parton distribution functions and also discuss possible uncertainties due to kinematic correlation effects.

PACS 13.60.-r – Photon and charged-lepton interactions with hadrons.
PACS 13.87.Fh – Fragmentation into hadrons.
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1. – Fully differential SIDIS cross section

The transverse momentum dependent (TMD) partonic distribution and fragmentation functions play a crucial role in measuring and interpreting information towards a true 3-dimensional imaging of the nucleons. TMD PDFs can be accessed in several experiments, but the main source of information is semi-inclusive deep inelastic scattering (SIDIS) of polarized leptons off polarized nucleon. For SIDIS, the theoretical formalism is described in a series of papers [1,2] using a tree level factorization [3] where the standard momentum convolution integral [4] relates the quark intrinsic transverse momentum in a nucleon to the transverse momentum of the produced hadron P_{hT}.

In this work we present a model independent extraction of the ratio of polarized, g_{1L}, and unpolarized, f_1, TMD distributions using a Monte Carlo (MC) generator based on the fully differential cross section, in which we reconstruct the transverse momentum of the final hadron after MC integration over the quark intrinsic transverse momentum. In the MC generator we used the SIDIS cross section described in ref. [1]. The Bessel-weighted asymmetry, providing access to the ratio of Fourier transforms of g_{1L}

and f_1 [5], has been extracted. The uncertainty of the extracted TMDs was estimated using unintegrated, transverse momentum *non*-factorized distribution and fragmentation functions.

The MC generator software employs the general-purpose, self-adapting MC event generator Foam [6] for drawing random points according to an arbitrary, user-defined distribution in the n-dimensional space.

We consider the following SIDIS process

$$\ell(l) + N(P) \rightarrow \ell(l') + h(P_h) + X, \tag{1}$$

where ℓ is the scattered lepton, N is the proton target and h is the observed hadron (four-momenta notations given in parentheses). The virtual photon momentum is defined $q = l - l'$ and its virtuality $Q^2 = -q^2$. Following the Trento conventions [7], we use the virtual photon-nucleon center of mass system, where the virtual photon momentum q is along the z axis and the proton momentum P is in the opposite direction. The detected hadron h has momentum P_h. In the parton model the virtual photon scatters off an on-shell quark. The initial quark momentum k and scattered quark momentum k' have the same intrinsic transverse momentum component $k_\perp$ with respect to the z axis. The initial quark has a fraction x of the proton's light-cone momentum, while the produced hadron h has a light-cone momentum fraction z and transverse momenta component $p_\perp$ with respect to the scattered quark's momentum k' (see ref. [1]). The fully differential cross section used in the MC generator is then given by [1]:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}^2\mathbf{p}_\perp\mathrm{d}^2\mathbf{k}_\perp} = K(x,y)J(x,Q^2,k_\perp)\Bigg[\sum_q f_{1,q}(x,k_\perp)D_{1,q}(z,p_\perp) + \lambda\sqrt{1-\varepsilon^2}g_{1L,q}(x,k_\perp)D_{1,q}(z,p_\perp)\Bigg], \tag{2}$$

where the summation runs over the quark flavors in the target. The kinematic factors $K(x,y)$ and ε, and the Jacobian $J(x,Q^2,k_\perp)$ are defined in [1]. λ represents the product of lepton and nucleon helicities ($\lambda = \pm 1$). $f_{1,q}(x,k_\perp)$ and $g_{1L,q}(x,k_\perp)$ are the quark TMD distributions and $D_{1,q}(z,p_\perp)$ is the TMD fragmentation function for an unpolarized quark q.

In many phenomenological studies of the semi-inclusive deep inelastic scattering, the transverse momentum dependence of distribution and fragmentation functions is factorized from the light-cone momentum dependence x and z, respectively. In our MC generator we use the Bessel-weighting method [5] to alleviate this approximations in the extraction of TMD distributions from SIDIS measurements. We discuss the following modified Gaussian (MG) expressions for the TMD distributions and fragmentation function, inspired by AdS/QCD [8, 9], in which x and $k_\perp$ (z and $p_\perp$) are not factorized:

$$f_1(x,k_\perp) = f_1(x)\frac{e^{-\frac{k_\perp^2}{x(1-x)\langle k_\perp^2\rangle_{f_1}}}}{x(1-x)\langle k_\perp^2\rangle_{f_1}}, \qquad g_{1L}(x,k_\perp) = g_{1L}(x)\frac{e^{-\frac{k_\perp^2}{x(1-x)\langle k_\perp^2\rangle_{g_1}}}}{x(1-x)\langle k_\perp^2\rangle_{g_1}} \tag{3}$$

$$D_1(z,p_\perp) = D_1(z)\frac{e^{-\frac{p_\perp^2}{z(1-z)\langle p_\perp^2\rangle}}}{z(1-z)\langle p_\perp^2\rangle}. \tag{4}$$

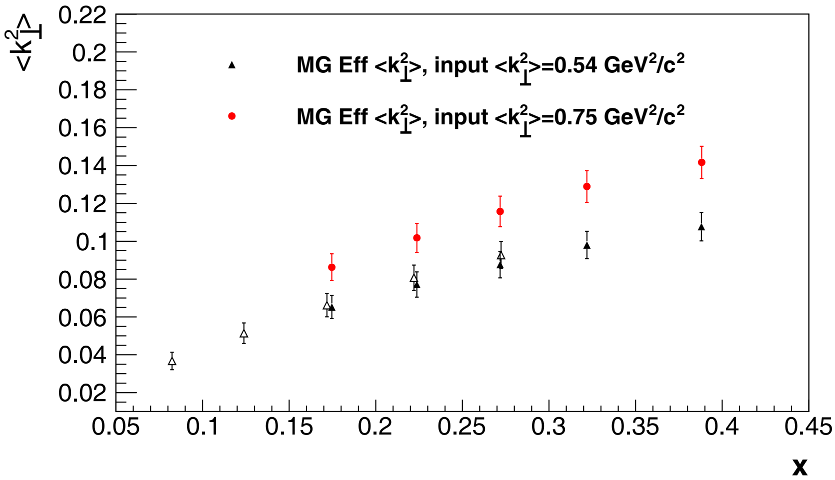

Fig. 1. – (Color online) The effective $\langle k_\perp^2(x)\rangle$ in the modified Gaussian (MG) ansatz for the TMD distributions (see eq. (3)) as function of x for two different input parameters $\langle k_\perp^2\rangle$ obtained from MC events simulated within the range $0.50 < z < 0.52$ and using 6 GeV (solid symbols) and 11 GeV (empty symbols) lepton beam energies.

TMD distributions non-factorized in x and $k_\perp$ are also suggested by the diquark spectator model [10] and the NJL model [11]. For a test extraction, the x and z dependence in eqs. (3), (4) we use the parametrizations $f_1(x) = (1-x)^3\, x^{-1.313}$, $g_{1L}(x) = f_1(x)x^{0.7}$ and $D_1(z) = 0.8\,(1-z)^2$, with widths: $\langle k_\perp^2\rangle_{f_1} = 0.54\,\text{GeV}^2$ [8], $\langle k_\perp^2\rangle_{g_1} = 0.8\langle k_\perp^2\rangle_{f_1}$ and $\langle p_\perp^2\rangle = 0.14\,\text{GeV}^2$. Figure 1 illustrates the average effective transverse momentum $\langle k_\perp^2\rangle = \langle k_\perp^2\rangle_{f_1}(1-\langle x\rangle)\langle x\rangle$ as a function of x from MC events for 6 GeV (solid symbols) and 11 GeV (empty symbols) incoming electron beam energies and produced hadrons within $0.5 < z < 0.52$. Note that the value for $\langle k_\perp^2\rangle_{f_1}$ obtained from the MC events is always smaller than the implemented value due to energy and momentum conservation [12]. A similar non-flat dependence of the average quark transverse momenta *versus* x and z is also observed in NJL-jet model [11].

2. – Bessel-weighted extraction of the double spin asymmetry A_{LL}

We present the extraction of the double spin asymmetry A_{LL}, defined as the ratio of the difference and the sum of electroproduction cross sections for antiparallel, σ^+, and parallel, σ^-, configurations of lepton and nucleon spins, using the Bessel-weighting procedure described in [5] and applied in [13]. Within this approach, one can extract the Fourier transform of the double spin asymmetry, $A_{LL}^{J_0(b_T P_{hT})}(b_T)$, defined as

$$(5)\quad A_{LL}^{J_0(b_T P_{hT})}(b_T) = \frac{\tilde{\sigma}^+(b_T) - \tilde{\sigma}^-(b_T)}{\tilde{\sigma}^+(b_T) + \tilde{\sigma}^-(b_T)} = \frac{\tilde{\sigma}_{LL}(b_T)}{\tilde{\sigma}_{UU}(b_T)} = \sqrt{1-\varepsilon^2}\,\frac{\sum_q \tilde{g}_1^q(x, z^2 b_T^2)\tilde{D}_1^q(z, b_T^2)}{\sum_q \tilde{f}_1^q(x, z^2 b_T^2)\tilde{D}_1^q(z, b_T^2)},$$

using measured double spin asymmetries as functions of P_{hT} [14], for fixed x, y, and z bins. Here b_T is the Fourier conjugate of the P_{hT}. The Fourier transforms of the helicity dependent cross sections, $\sigma^\pm(b_T)$, can be extracted by integration (analytic models) or

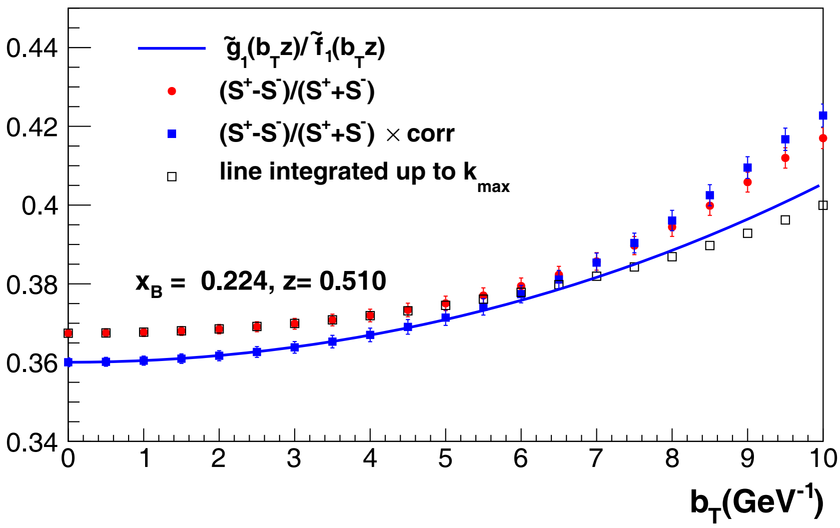

Fig. 2. – (Color online) Bessel-weighted asymmetry vs b_T with and without the correction, together with analytical and numerical comparison from the MC with PDFs and FFs from eq. (3). See the text for more details.

summation (for data and MC) over the hadronic transverse momentum, weighted by a Bessel function J_0,

$$\tilde{\sigma}^{\pm}(b_T) \simeq S^{\pm} = \sum_{i=1}^{N^{\pm}} J_0(b_T P_{hT,i}). \qquad (6)$$

The Bessel-weighted asymmetry obtained from the simulated events is shown in fig. 2 as a function of b_T with filled (red) circles, while the analytic expression $\frac{\tilde{g}_1(x,zb_T)}{\tilde{f}_1(x,zb_T)}$ using $\langle k_\perp^2\rangle_{g_1}$ and $\langle k_\perp^2\rangle_{f_1}$ from the fits to $k_\perp^2$ distributions from the same MC sample is depicted by the (blue) full line. For values $b_T < 6\,\text{GeV}^{-1}$, which corresponds to about 1 fm, the Bessel-weighted asymmetries could be extracted with an accuracy of 2.5%, although with a systematic shift. This clear systematic shift between the extracted and calculated asymmetries is due to the kinematic restrictions introduced by energy and momentum conservation([1]) as well as binning effects, which deform the Gaussian shapes of the $k_\perp$ and $p_\perp$ distributions. In experiments, there is always a cutoff at high P_{hT} due to acceptance and the small cross section, as well as a cutoff at small P_{hT} where the azimuthal angles are not well defined due to the experimental resolution. These restrictions in P_{hT} directly affect the extracted $k_\perp$ and $p_\perp$ distributions and yield to the mentioned distortion of Gaussian shapes which result in the systematic shift between the extracted and the calculated asymmetries. Obviously, this shift depends on experimentally introduced restrictions for the accessible P_{hT} range.

([1]) In the light-cone coordinate system the transverse component is less than or equal to the momentum component along the light-cone vector.

We discuss two approaches which take these conditions into account, one corrects the data (simulated MC events) the other applies limits on the integration range for the intrinsic transverse parton momenta when calculating the asymmetry. In order to correct the data, the contributions from the missing P_{hT} ranges outside the accessible values is estimated by choosing a certain model for the parton transverse momentum dependence, *e.g.* a Gaussian distribution. This model dependent correction of data is shown in fig. 2 by the (blue) filled squares. The extracted asymmetry does now match the theoretical curve for values $b_T < 6\,\mathrm{GeV}^{-1}$. Alternatively, limited numerical integration over intrinsic transverse momenta in the calculation of the asymmetry, where the integration limits correspond to the accessible experimental P_{hT} range, yields a calculated asymmetry that describes correctly the experimental situation without introducing a model dependence. This is shown in fig. 2 by the (black) open squares.

In summary, we used a fully differential Monte Carlo event generator to study how the implemented quark transverse momentum distributions $k_\perp$ and $p_\perp$ are being changed due to the energy and momentum conservation low and kinematic correlations within the generalized parton model. We then use the obtained $k_\perp$ and $p_\perp$ distributions to apply the Bessel-weighting strategy for the extraction of TMD distributions. As example we calculate the Bessel-weighted double spin asymmetry $A_{LL}^{J_0(b_T P_{hT})}(b_T)$. Within the b_T range of $b_T < 6\,\mathrm{GeV}^{-1}$, which corresponds to about 1 fm, the Bessel-weighted asymmetries could be extracted within 2.5% accuracy.

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