# Studying the neutron orbital structure by coherent hard exclusive processes off ${ }^{3} \mathrm{He}$ 

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Summary. - Hard exclusive processes, such as Deeply Virtual Compton Scattering (DVCS), allows to access generalized parton distributions (GPDs). By means of an Impulse Approximation (IA) calculation, it is shown here how, in the low momentum transfer region, the sum of the GPDs $H$ and $E$, is dominated by the neutron contribution. Thanks to this property, ${ }^{3} \mathrm{He}$ could open a new way to access the neutron structure information. In this work, a simple and efficient extraction procedure of the neutron GPDs, able to take into account the nuclear effects included in IA analysis, is proposed.

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In the last 20 years, many studies have been performed to measure the helicty quark contributions to the nucleon spin, obtained in DIS or SiDIS experiments, and the orbital angular momentum (OAM) of the partons, crucial steps to solve the so called "Spin Crisis". Generalized parton distributions [1] (GPDs), which parametrize the nonperturbative hadron structure in hard exclusive processes, allow to access important information, such as the OAM of the partons inside the nucleon [2]. The cleanest process to access GPDs is Deeply Virtual Compton Scattering (DVCS), i.e. $e H \longmapsto e^{\prime} H^{\prime} \gamma$ when $Q^{2} \gg M^{2}$, where $Q^{2}=-q \cdot q$ is the momentum transfer between the leptons $e$ and $e^{\prime}, \Delta^{2}$ the one between hadrons $H$ and $H^{\prime}$ with momenta $P$ and $P^{\prime}$, and $M$ is the nucleon mass. Another relevant kinematical variable is the skewedness, $\xi=-\Delta^{+} /\left(P^{+}+P^{\prime}+\right)\left({ }^{1}\right)$. The DVCS cross-section dependence on the GPDs is quite complicated; despite of this fact, experimental data have been obtained and analyzed from proton and nuclear targets, see refs. [3, 4].

[^0]For nuclear targets, the measurement of GPDs is relevant to unveil medium modifications of bound nucleons and to distinguish between different models, possibilities excluded e.g. in DIS experiments. Nuclear targets are also necessary to access the neutron information, crucial to obtain, together with the proton data, a flavor decomposition of GPDs. At this purpose, ${ }^{3} \mathrm{He}$ is very promising thanks to its spin structure (see, e.g. ref. [5]). In fact, among the light nuclei, ${ }^{3} \mathrm{He}$ is the only one for which the combination $\tilde{G}_{M}^{3, q}\left(x, \Delta^{2}, \xi\right)=H_{q}^{3}\left(x, \Delta^{2}, \xi\right)+E_{q}^{3}\left(x, \Delta^{2}, \xi\right)$, whose second moment at $\Delta^{2}=0$ gives the total angular momentum of the parton of $q$ flavor, could be dominated by the neutron contribution. To this aim ${ }^{2} \mathrm{H}$ and ${ }^{4} \mathrm{He}$ are not useful, as discussed in ref. [6]. To what extent this fact can be used to extract the neutron information, is shown in refs. [6, 7], and summarized here. The formalism used in ref. [8], where a convolution formula for GPD $H_{q}^{3}$ of ${ }^{3} \mathrm{He}$ was found in IA, has been extended to obtain $\tilde{G}_{M}^{3, q}$ :

$$
\begin{equation*}
\tilde{G}_{M}^{3, q}\left(x, \Delta^{2}, \xi\right)=\sum_{N} \int \mathrm{~d} E \int \mathrm{~d} \vec{p} \tilde{P}_{N}^{3}\left(\vec{p}, \vec{p}^{\prime}, E\right) \frac{\xi^{\prime}}{\xi} \tilde{G}_{M}^{N, q}\left(x^{\prime}, \Delta^{2}, \xi^{\prime}\right) \tag{1}
\end{equation*}
$$

where $\tilde{P}_{N}^{3}\left(\vec{p}, \vec{p}^{\prime}, E\right)$ is a proper combination of components of the spin dependent, one body off diagonal spectral function:

$$
\begin{equation*}
P_{S S^{\prime}, s s^{\prime}}^{N}\left(\vec{p}, \vec{p}^{\prime}, E\right)=\frac{1}{(2 \pi)^{6}} \frac{M \sqrt{M E}}{2} \int \mathrm{~d} \Omega_{t} \sum_{s_{t}}\left\langle\vec{P}^{\prime} S^{\prime} \mid \vec{p}^{\prime} s^{\prime}, \vec{t} s_{t}\right\rangle_{N}\left\langle\vec{p} s, \vec{t} s_{t} \mid \vec{P} S\right\rangle_{N}, \tag{2}
\end{equation*}
$$

where $x^{\prime}$ and $\xi^{\prime}$ are the variables for the bound nucleon GPDs, $p\left(p^{\prime}=p+\Delta\right)$ and $S, S^{\prime}\left(s, s^{\prime}\right)$ are the 4-momentum and spin projections in the initial (final) state, and $E=E_{\text {min }}+E_{R}^{*}$, with $E_{R}^{*}$ is the excitation energy of the two-body recoiling system. The most important quantity appearing in the definition eq. (2) is the intrinsic overlap integral

$$
\begin{equation*}
\left\langle\vec{p} s, \vec{t} s_{t} \mid \vec{P} S\right\rangle_{N}=\int \mathrm{d} \vec{y} e^{i \vec{p} \cdot \vec{y}}\left\langle\chi_{N}^{s}, \Psi_{t}^{s_{t}}(\vec{x}) \mid \Psi_{3}^{S}(\vec{x}, \vec{y})\right\rangle \tag{3}
\end{equation*}
$$

between the wave function of ${ }^{3} \mathrm{He}, \Psi_{3}^{S}$, with the final state, described by two wave functions: i) the eigenfunction $\Psi_{t}^{s_{t}}$, with eigenvalue $E=E_{\text {min }}+E_{R}^{*}$, of the state $s_{t}$ of the intrinsic Hamiltonian pertaining to the system of two interacting nucleons with relative momentum $\vec{t}$, which can be either a bound or a scattering state, and ii) the plane wave representing the nucleon $N$ in IA. In order to estimate the nucleon contributions to eq. (1), a numerical evaluation is needed. To this aim, a simple nucleonic model of GPDs [9], which fulfills the main properties, together with the overlaps, eq. (3), calculated with the wave function [10] of Av18 [11] potential and corresponding to the analysis of ref. [12], have been used. In ref. [8] the main properties of the ${ }^{3} \mathrm{He}$ GPD $H_{q}^{3}$ have been verified, such as the forward limit and the first moment. The only possible check for $\tilde{G}_{M}^{3, q}$ is its integral
$G_{M}^{3}\left(\Delta^{2}\right)=\sum_{q} \int \mathrm{~d} x \tilde{G}_{M}^{3, q}\left(x, \Delta^{2}, \xi\right)$, yielding the magnetic form factor (ff) of ${ }^{3} \mathrm{He}$, due the fact that the forward limit of $E_{M}^{3, q}$ is not defined. Our result is consistent with the Av18 one-body calculation of ref. [13]. It is clear that for $-\Delta^{2} \ll 0.15 \mathrm{GeV}^{2}$, where DVCS off nuclei can be performed, our results compare well also with data [14] (see fig. 1). Now it comes the main result of the analysis. The neutron contribution dominates


Fig. 1. - The magnetic ff of ${ }^{3} \mathrm{He}, G_{M}^{3}\left(\Delta^{2}\right)$, with $\Delta^{\mu}=\sqrt{-\Delta^{2}}$. Full line: the present IA calculation, obtained as the $x$-integral of $\sum_{q} \tilde{G}_{M}^{3, q}$ (see text). Dashed line: experimental data [14]; square points: one-body direct calculation, using the Av18 wave function only.
eq. (1), in particular in the forward limit, necessary to obtain the OAM; but increasing $-\Delta^{2}$, the proton one grows up, see fig. 2a, in particular for $u$ flavor $[6,7]$. Because of the behavior in $-\Delta^{2}$ and the complicated convolution formula, an extraction procedure of the neutron information is necessary. To this aim, one can see that eq. (1) can be written introducing the function $g_{N}^{3}\left(z, \Delta^{2}, \xi\right)$, the "light cone spin dependent off-forward momentum distribution":

$$
\begin{equation*}
\tilde{G}_{M}^{3, q}\left(x, \Delta^{2}, \xi\right)=\sum_{N} \int_{x_{3}}^{\frac{M_{A}}{M}} \frac{\mathrm{~d} z}{z} g_{N}^{3}\left(z, \Delta^{2}, \xi\right) \tilde{G}_{M}^{N, q}\left(\frac{x}{z}, \Delta^{2}, \frac{\xi}{z},\right) \tag{4}
\end{equation*}
$$



Fig. 2. - (a): The quantity $x_{3} \tilde{G}_{M}^{3}\left(x, \Delta^{2}, \xi\right)$, where $x_{3}=M_{3} / M x$ and $\xi_{3}=M_{3} / M \xi$, shown at $\Delta^{2}=0 \mathrm{GeV}^{2}$ and $\xi_{3}=0$ (stars and lines) and $\Delta^{2}=-0.1 \mathrm{GeV}^{2}$ and $\xi_{3}=0.1$, together with the neutron (dashed) and the proton (dot-dashed) contribution. (b): The quantity $x_{3} \tilde{G}_{M}^{n, q}\left(x, \Delta^{2}, \xi\right)$ for the neutron at $\Delta^{2}=-0.1 \mathrm{GeV}^{2}$ and $\xi_{3}=0.1$ with $u, d$ and $u+d$ contributions (full lines), compared with the approximation $x_{3} \tilde{G}_{M}^{n, q, \text { extr }}\left(x, \Delta^{2}, \xi\right)$, eq. (6), (dashed).


Fig. 3. - (a) The ratio $r_{n}\left(x, \Delta^{2}, \xi\right)=\tilde{G}_{M}^{n, e x t r}\left(x, \Delta^{2}, \xi\right) / \tilde{G}_{M}^{n}\left(x, \Delta^{2}, \xi\right)$, in the forward limit (full), at $\Delta^{2}=-0.1 \mathrm{GeV}^{2}$ and $\xi_{3}=0$ (dashed) and at $\Delta^{2}=-0.1 \mathrm{GeV}^{2}$ and $\xi_{3}=0.1$ (dot-dashed). (b) $r_{n}\left(x, \Delta^{2}, \xi\right)=\tilde{G}_{M}^{n, \text { extr }}\left(x, \Delta^{2}, \xi\right) / \tilde{G}_{M}^{n}\left(x, \Delta^{2}, \xi\right)$, at $\Delta^{2}=0.1 \mathrm{GeV}^{2}$ and $\xi_{3}=0$, using the model of ref. [9] for the nucleon GPDs (dashed), the one of ref. [16] (full) and the one of ref. [17] (full and stars).

Since $g_{N}^{3}\left(z, \Delta^{2}, \xi\right)$ is strongly peaked around $z=1$, with $x_{3}=\left(M_{A} / M\right) x \leq 1$, one has

$$
\begin{align*}
\tilde{G}_{M}^{3, q}\left(x, \Delta^{2}, \xi\right) & \simeq \operatorname{low} \Delta^{2} \simeq \sum_{N} \tilde{G}_{M}^{N, q}\left(x, \Delta^{2}, \xi\right) \int_{0}^{\frac{M_{A}}{M}} \mathrm{~d} z g_{N}^{3}\left(z, \Delta^{2}, \xi\right)  \tag{5}\\
& =G_{M}^{3, p, p o i n t}\left(\Delta^{2}\right) \tilde{G}_{M}^{p}\left(x, \Delta^{2}, \xi\right)+G_{M}^{3, n, p o i n t}\left(\Delta^{2}\right) \tilde{G}_{M}^{n}\left(x, \Delta^{2}, \xi\right)
\end{align*}
$$

where the magnetic pointlike ff has been introduced: $G_{M}^{3, N, p o i n t}\left(\Delta^{2}\right)=\int_{0}^{\frac{M_{A}}{M}} \mathrm{~d} z g_{N}^{3}\left(z, \Delta^{2}\right.$, $\xi$ ), which represents the ff of the nucleus if nucleons were point-like particles with their physical magnetic moments. These quantities are theoretically well known and their dependence on the nuclear potential is rather weak [6]. From eq. (5) the neutron contribution can be extracted:
(6) $\tilde{G}_{M}^{n, e x t r}\left(x, \Delta^{2}, \xi\right) \simeq\left\{\tilde{G}_{M}^{3}\left(x, \Delta^{2}, \xi\right)-G_{M}^{3, p, p o i n t}\left(\Delta^{2}\right) \tilde{G}_{M}^{p}\left(x, \Delta^{2}, \xi\right)\right\} / G_{M}^{3, n, p o i n t}\left(\Delta^{2}\right)$.

Figure 2b shows our main achievement: the procedure works even beyond the forward limit, since $G_{M}^{n, \text { extr }}$ compares perfectly with $\tilde{G}_{M}^{n}$, evaluated in the same model used for the complete calculation of $\tilde{G}_{M}^{3}$. It is important to remark that, for this check, the only theoretically ingredient is the magnetic point-like ff, which is under control. This crucial result can be analyzed in details looking at fig. 3a, where the ratio $r_{n}\left(x, \Delta^{2}, \xi\right)=$ $\tilde{G}_{M}^{n, \text { extr }}\left(x, \Delta^{2}, \xi\right) / \tilde{G}_{M}^{n}\left(x, \Delta^{2}, \xi\right)$ is shown in different kinematical regions and in fig. 3b, where $r_{n}$ is shown using different nucleonic models in the calculation. It is evident that, for $x<0.7$, where data are expected from JLab, the procedure works and depends weakly on the nucleonic (see fig. 3b and ref. [6]) and nuclear models used in the calculation. In summary, DVCS off ${ }^{3} \mathrm{He}$ allows to access the neutron information at low $\Delta^{2}$ and if data were taken at higher $\Delta^{2}$, a relativistic treatment [15] and/or the inclusion of many body currents, beyond the present IA scheme, should be implemented.

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    $\left({ }^{1}\right)$ In this paper, $a^{ \pm}=\left(a^{0} \pm a^{3}\right) / \sqrt{2}$.

