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# Dynamic output nonfragile reliable control for nonlinear fractional-order glucose–insulin system

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**Abstract.** The main intention of this paper is to scrutinize the problem of internal model-based dynamic output feedback nonfragile reliable control problem for fractional-order glucose–insulin system. Specifically, a robust control law that represents the insulin injection rate is designed in order to regulate the level of glucose in diabetes treatment in the existence of meal disturbance or external glucose infusion due to improper diet. By the construction of suitable Lyapunov functional, a novel set of sufficient conditions is derived with the aid of linear matrix inequalities for obtaining the required dynamic output feedback control law. In particular, the designed controller ensures the robust stability and disturbance attenuation performance against meal disturbance of the glucose–insulin system. Numerical simulation results are performed to verify the advantage of the developed design technique. Specifically, the irregular blood glucose level can be brought down to normal level by injecting suitable rate of insulin to the patient. The result exposes that the level of blood glucose is sustained in the identified ranges via the proposed dynamic output feedback control law.

**Keywords:** fractional-order glucose–insulin system, dynamic output feedback, nonlinear reliable control, internal model approach.

## 1 Introduction

In recent years, the applications of fractional calculus play a significant role in the mathematical modeling and control of different kind of real-world problems. In particular, due to the fact that the fractional calculus is the general case of integer calculus, practical systems can be modelled more accurately by using fractional models. Moreover, in most of fractional-order control systems, output tracking, stabilization and stability are the main objectives to be ensured because of the uncertain models caused by variations in system

parameters, disturbances and so on [18]. Over the last few decades, the analysis of filtering problem [9], feedback control [14] and event-triggered scheme [10] for dynamical systems have received increasing interests of researchers. This is because of the fact that the complete information of control system states is not easy to be estimated in experimental process. It is noted that the problem of dynamic output feedback control is somewhat more complicated than the problem of static output feedback. Some interesting analysis based on dynamic output feedback control design have been recently discussed in [2]. Thus, robust stability and stabilization via dynamic output feedback control have become as an exciting important issue for fractional-order control systems [8].

Besides, fractional calculus has been applied to model various control systems such as humanoid robots, electrical systems and bio-medical sciences. Recently, there are several interesting works on the study of qualitative property analysis and existence of solutions for various kinds of fractional models of biological systems [1]. It is well known that high amount of blood glucose level produces a main long-lasting syndrome called as diabetes. Diabetes arises due to the main reason that the fault caused by the abnormal production of insulin or action of insulin or both so that the body could not maintain the particular glucose level in blood. [6, 12]. Generally, the human body must have the particular range of glucose concentration level in blood in order to decrease the effect of diabetes. In particular, either a very greater value or very lower value of glucose level in blood may produce the serious health issues such as hyperglycemia or hypoglycemia, respectively. If the level of glucose concentration in blood is decreasing below $50$ mg/dl, then it will result in damage of brain cells, which is known as brain failure. It should be mentioned that glucagon and insulin are the two hormones, which control or maintain the normal range of glucose level in the blood [16]. Insulin is antibiotic, which generates the activity of glucose uptake in human body and in so doing it reduces the level of glucose in the blood. On the other hand, the glucagon is catabolic, which increases the glucose production in blood. Normally, in an individual, a less amount of insulin is often produced or cleared inside pancreas maintaining the insulin basal value $I_b$. The clearance increases as the insulin level increases above basal value, but basal secretion increases due to decrease in basal concentration $G_b$. Note that for a healthy people, blood glucose level range is identified as $70-120$ mg/dL. If it fails to lie in this range, then many organs on the body, like heart, brain, eyes, legs, kidneys could be damaged, and periodontal disease could attack. Therefore, it is necessary and important to study the dynamical behaviors of the blood glucose concentration or insulin level to regulate the required level of glucose in the blood. Taking this into consideration, various control techniques have been used to control the level of blood glucose or insulin rate and to maintain the range of blood glucose level [3–5, 13, 17].

Recently, N'Doye et al. [15] developed a $H_\infty$ control for the stabilization of fractional-order glucose–insulin system with the use of fractional Lyapunov direct method. The main benefit of the dissipativity approach is that it offers the flexibility by adjusting the parameters to obtain $H_\infty$, passivity, and mixed $H_\infty$ and passivity performances. Moreover, dissipative control technique acts is an unified approach for designing suitable control for glucose–insulin system. Further, the static output feedback control strategy cannot be used always to stabilize an unstable system due to some practical issues, which

will motivate us to consider the control design together with the dynamic output feedback controller. However to the best of authors knowledge, the stabilization of glucose–insulin system subject to the effect of improper diet via the internal model-based dynamic output feedback nonfragile reliable control has not yet been reported. Motivated by this consideration, in this work, we study the stabilization of glucose–insulin system via dynamic output feedback controller. The important features of this work have been listed as follows:

- Stabilization of glucose–insulin system is examined via dynamic output feedback nonfragile reliable control law.
- Developed dynamic output feedback controller is well suited to regulate the glucose level in the diabetes patients in the presence of meal disturbances.
- Precisely, the proposed control law makes the glucose–insulin system to reach the equilibrium point by rapidly rejecting meal disturbances.

## 2 System formulation and preliminaries

In this section, we discuss the stabilization problem for glucose–insulin system described by fractional-order nonlinear systems via internal model-based dynamic output feedback nonfragile reliable controller. Specifically, we consider the state–space equation of glucose–insulin system in the following form:

$$\begin{aligned} {}^{\mathrm{C}}D^{\alpha}x(t) &= \mathcal{A}x(t) + \mathcal{B}u(t) + \mathcal{D}d(t) + \mathcal{F}\big(x(t)\big), \\ y(t) &= \mathcal{C}x(t), \quad \pi(0) = x_0, \end{aligned} \tag{1}$$

where ${}^{\mathrm{C}}D^{\alpha}$ denotes the Riemann–Liouville derivative of commensurate order $\alpha$ with $0 < \alpha < 1$; $\mathcal{F}(x(t))$ is the measurable function satisfying the Lipschitz condition; $x(t) = [x_1^{\mathrm{T}}(t)\ x_2^{\mathrm{T}}(t)\ x_3^{\mathrm{T}}(t)]^{\mathrm{T}}$ denotes the state vector, here $x_1(t)$ and $x_2(t)$ indicate the glucose and insulin concentration in blood, respectively. Also, $x_3(t)$ denotes the insulin excitation; $u(t)$ is the insulin injection rate, which is considered as control variable; $d(t)$ is the meal disturbance; the output $y(t)$ denotes the blood glucose concentration and $\pi_0 = [x_1^{\mathrm{T}}(0)\ x_2^{\mathrm{T}}(0)\ x_3^{\mathrm{T}}(0)]^{\mathrm{T}}$ represents the initial condition. Furthermore, the coefficient matrices of (1) are given by

$$\mathcal{A} = \begin{bmatrix} -p_1 & 0 & 0 \\ 0 & -p_2 & p_3 \\ 0 & 0 & -n \end{bmatrix}, \qquad \mathcal{F}\big(x(t)\big) = \begin{bmatrix} -x_1^{\mathrm{T}}(t)x_2^{\mathrm{T}}(t) & 0 & 0 \end{bmatrix}^{\mathrm{T}},$$

$$\mathcal{B} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^{\mathrm{T}}, \quad \mathcal{D} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^{\mathrm{T}} \quad \text{and} \quad \mathcal{C} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix},$$

where $p_1$ represents the glucose clearance rate independent of insulin; $p_2$ denotes the diminishing level of insulin reaction at $t$; $p_3$ represents the raise in insulin uptake ability; $n$ denotes the decay rate of blood insulin. In order to achieve the stabilization of fractional-order glucose–insulin system, the dynamic output feedback controller is considered as

$$\begin{aligned} {}^{\mathrm{C}}D^{\alpha}x_C(t) &= \mathcal{A}_C x_C(t) + \mathcal{B}_C y(t), \\ y_C(t) &= \mathcal{K}x_C(t), \end{aligned} \tag{2}$$

where $\mathcal{A}_C$, $\mathcal{B}_C$ and $\mathcal{K}$ are the controller parameters to be computed; $x_C(t)$ and $y_C(t)$ represent the state and output vectors of the dynamic output feedback controller, respectively. In particular, the main motive is to construct an appropriate controller, which makes the state trajectories of the closed-loop system should converge to the equilibrium values. In order to do this, the internal model is designed as

$$ {}^{\mathrm{C}}D^{\alpha}x_R(t) = \mathcal{A}_R x_R(t) + \mathcal{B}_R\big(r(t) - x(t)\big), \tag{3} $$

where $x_R(t)$ is the state of the internal model; $\mathcal{A}_R$ and $\mathcal{B}_R$ are known constant parameters; $r(t) = [G_b \ 0 \ I_b]^{\mathrm{T}}$ is the equilibrium value, which has to be traced, where $G_b$ is the glucose basal value and $I_b$ is the insulin basal value. On the other hand, fluctuations in control design may occur during the real process due to external disturbances [20, 21]. Taking this into account, a nonfragile controller is defined by $u(t) = y_C(t) + (\mathcal{K}_R + \Delta\mathcal{K}_R(t))x_R(t) = (\mathcal{K} + \Delta\mathcal{K}(t))x_C(t) + (\mathcal{K}_R + \Delta\mathcal{K}_R(t))x_R(t)$, where $\mathcal{K}$ and $\mathcal{K}_R$ are the gain matrices associated with the dynamic output feedback controller and internal model, respectively; $\Delta\mathcal{K}(t)$ and $\Delta\mathcal{K}_R(t)$ are the uncertain matrices given by $\Delta\mathcal{K}(t) = \mathcal{M}F(t)\mathcal{N}$ and $\Delta\mathcal{K}_R(t) = \mathcal{M}F(t)\mathcal{N}_R$, where $\mathcal{M}$, $\mathcal{N}$, $\mathcal{N}_R$ are known suitable constant matrices. Also, the time-varying matrix $F(t)$ is Lebesgue measurable and the elements of $F(t)$ satisfies $F^{\mathrm{T}}(t)F(t) \leqslant I$. Further, faults in control design (insulin injection rate) occur naturally in real situations, which will affect the performance of the considered fractional-order glucose–insulin system and make it unstable. To tackle this, we choose a reliable controller in the following form:

$$ u^F(t) = G\big(\mathcal{K} + \Delta\mathcal{K}(t)\big)x_C(t) + G(\mathcal{K}_R + \Delta\mathcal{K}_R(t))x_R(t) + g\big(u(t)\big), \tag{4} $$

where $u^F(t)$ is the faulty control input, $G$ is the actuator fault matrix given by $G = \operatorname{diag}\{h_1, h_2, \ldots, h_m\}$, $h_i \in [\underline{h_i}, \overline{h_i}]$, $0 \leqslant \underline{h_i} \leqslant h_i \leqslant \overline{h_i} \leqslant 1$, $i = 1, 2, \ldots, m$, where $h_i$ is an unknown constant; $\underline{h_i}$ and $\overline{h_i}$ denote the lower and upper bounds of $h_i$, respectively. If $h_i$ is taken as 0, then the actuator fails completely. The actuator works without any fault if $h_i = 1$ and fails partially for $h_i \in (0, 1)$. In (4), $g(u(t))$ describes the nonlinear fault term in control input. Then it is obvious that $g^{\mathrm{T}}(u(t))g(u(t)) \leqslant (u^{\mathrm{T}}(t)\Xi_1 u(t))$, where $\Xi_1 = \operatorname{diag}\{\sigma_1, \sigma_2, \ldots, \sigma_m\}$. The stability of the system is independent of basal values. Therefore, we assume that $r(t) = 0$. From (1)–(4), an augmented nonlinear system can be written as

$$ {}^{\mathrm{C}}D^{\alpha}\Omega(t) = \tilde{A}\Omega(t) + \tilde{F}\mathcal{F}\big(x(t)\big) + \tilde{\mathcal{B}}g\big(u(t)\big) + \tilde{D}d(t), \tag{5} $$

where $\Omega(t) = [x^{\mathrm{T}}(t) \ x_C^{\mathrm{T}}(t) \ x_R^{\mathrm{T}}(t)]^{\mathrm{T}}$ and

$$ \tilde{A} = \begin{pmatrix} \mathcal{A} & \mathcal{B}G(\mathcal{K} + \Delta\mathcal{K}(t)) & \mathcal{B}G(\mathcal{K}_R + \Delta\mathcal{K}_R(t)) \\ \mathcal{B}_C C & \mathcal{A}_C & 0 \\ -\mathcal{B}_R & 0 & \mathcal{A}_R \end{pmatrix}, $$

$$ \tilde{F} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \tilde{\mathcal{B}} = \begin{pmatrix} \mathcal{B} \\ 0 \\ 0 \end{pmatrix} \quad \text{and} \quad \tilde{D} = \begin{pmatrix} D \\ 0 \\ 0 \end{pmatrix}. $$

With the aid of the following assumption, we will derive the main result in the forthcoming section.

**Assumption 1.** The nonlinear function $\mathcal{F}(x(t))$ with $\mathcal{F}(0) = 0$ for any $L_1 > 0$ satisfies the Lipschitz condition $\|\mathcal{F}(x(t))\| \leqslant \|L_1 x(t)\|$.

## 3 Main results

In this section, the dissipative and internal model-based dynamic output feedback non-fragile reliable controller will be designed to ensure the stabilization of the fractional-order glucose–insulin system (1). First, we obtain a set of criteria to analyze the stability of the augmented fractional-order nonlinear system (5) with known control gain parameters. Then the result is extended to the case of unknown control gain parameters.

**Theorem 1.** *Let Assumption* 1 *be hold. For given positive scalars* $\alpha_1$, $\beta_1$, $\gamma_1$, $\rho_1$, $\rho_2$, *real constant matrices* $L_1 > 0$, $G > 0$, $\Xi_1 > 0$, $Q$, $S$, $R$ *and gain matrices* $\mathcal{K}$, $\mathcal{K}_R$, *the augmented fractional-order nonlinear system* (5) *with* $0 < \alpha < 1$ *reaches the equilibrium point with strictly* $(QSR)$*-dissipativity if there exist symmetric matrices* $P_1 > 0$, $P_2 > 0$, $P_3 > 0$ *such that the below linear matrix inequality* (*LMI*) *holds*:

$$\Psi = [\Psi]_{9\times 9} < 0, \tag{6}$$

*where*

$$\Psi_{11} = \mathcal{A}^{\mathrm{T}}\frac{P_1}{\alpha_1} + \frac{P_1}{\alpha_1}\mathcal{A}, \qquad \Psi_{12} = \mathcal{C}^{\mathrm{T}}\mathcal{B}_C^{\mathrm{T}}\frac{P_2}{\beta_1} + \frac{P_1}{\alpha_1}\mathcal{B}G\mathcal{K},$$
$$\Psi_{13} = -\mathcal{B}_R^{\mathrm{T}}\frac{P_3}{\gamma_1} + \frac{P_1}{\alpha_1}\mathcal{B}G\mathcal{K}_R, \qquad \Psi_{14} = \frac{P_1}{\alpha_1}, \qquad \Psi_{15} = \frac{P_1}{\alpha_1}\mathcal{B},$$
$$\Psi_{16} = \rho_1 L_1^{\mathrm{T}}, \qquad \Psi_{18} = \frac{P_1}{\alpha_1}\mathcal{D} - \mathcal{C}^{\mathrm{T}}S, \qquad \Psi_{19} = \sqrt{-Q}\mathcal{C}^{\mathrm{T}},$$
$$\Psi_{22} = \mathcal{A}_C^{\mathrm{T}}\frac{P_2}{\beta_1} + \frac{P_2}{\beta_1}\mathcal{A}_C, \qquad \Psi_{27} = -\mathcal{K}, \qquad \Psi_{33} = \mathcal{A}_R^{\mathrm{T}}\frac{P_3}{\gamma_1} + \frac{P_3}{\gamma_1}\mathcal{A}_R,$$
$$\Psi_{37} = -\mathcal{K}_R, \qquad \Psi_{44} = -\rho_1, \qquad \Psi_{55} = -\rho_2^{-1}, \qquad \Psi_{66} = -I,$$
$$\Psi_{77} = -\rho_2\Xi_1^{-1}, \qquad \Psi_{88} = -R, \qquad \Psi_{99} = -I,$$

*and the remaining terms are zero.*

*Proof.* According to [11, Lemma 1], the augmented fractional-order nonlinear system (5) can be modified in the following form:

$$\frac{\partial z(\varpi,t)}{\partial t} = -\varpi z(\varpi,t) + \tilde{A}\Omega(t) + \tilde{F}\mathcal{F}\big(x(t)\big) + \tilde{\mathcal{B}}g\big(u(t)\big) + \tilde{D}d(t),$$
$$\Omega(t) = \int_0^\infty \lambda(\varpi)z(\varpi,t)\,\mathrm{d}\varpi, \tag{7}$$

where $z(\varpi,t)$ is the frequency distributed state. Now, we consider a monochromatic Lyapunov function with respect to the weighting frequency $\lambda(\varpi)$ as follows:

$$\mathcal{W}(\varpi,t)=z^{\mathrm{T}}(\varpi,t)Pz(\varpi,t),$$

where

$$P=P^{\mathrm{T}}=\operatorname{diag}\left\{\frac{P_1}{\alpha_1},\frac{P_2}{\beta_1},\frac{P_3}{\gamma_1}\right\}$$

is positive definite. Let $v(t)$ be the Lyapunov function given by

$$v(t)=\int_0^\infty \lambda(\varpi)\mathcal{W}(\varpi,t)\,\mathrm{d}\varpi=\int_0^\infty \lambda(\varpi)z^{\mathrm{T}}(\varpi,t)Pz(\varpi,t)\,\mathrm{d}\varpi.$$

Taking the time derivative of $v(t)$ along the trajectories of (7), we obtain

$$\begin{aligned}\dot v(t)&=\int_0^\infty \lambda(\varpi)\big\{-\varpi z^{\mathrm{T}}(\varpi,t)+\Omega^{\mathrm{T}}(t)\tilde A^{\mathrm{T}}+\mathcal{F}^{\mathrm{T}}\big(x(t)\big)\tilde F^{\mathrm{T}}+g^{\mathrm{T}}\big(u(t)\big)\tilde{\mathcal{B}}^{\mathrm{T}}+d^{\mathrm{T}}(t)\tilde D^{\mathrm{T}}\big\}\\&\quad\times Pz(\varpi,t)\,\mathrm{d}\varpi\\&\quad+\int_0^\infty \lambda(\varpi)z^{\mathrm{T}}(\varpi,t)\\&\quad\times P\big\{-\varpi z(\varpi,t)+\tilde A\Omega(t)+\tilde F\mathcal{F}\big(x(t)\big)+\tilde{\mathcal{B}}g\big(u(t)\big)+\tilde Dd(t)\big\}\,\mathrm{d}\varpi\\&=\int_0^\infty -2\lambda(\varpi)\varpi z^{\mathrm{T}}(\varpi,t)Pz(\varpi,t)\,\mathrm{d}\varpi+2\Omega^{\mathrm{T}}(t)P\tilde A\Omega(t)+2\Omega^{\mathrm{T}}(t)P\tilde{\mathcal{B}}g\big(u(t)\big)\\&\quad+2\Omega^{\mathrm{T}}(t)P\tilde F\mathcal{F}\big(x(t)\big)+2\Omega^{\mathrm{T}}(t)P\tilde Dd(t).\end{aligned}\tag{8}$$

Since the first term of the above inequality (8) is negative, it is sufficient to prove that $\Lambda<0$, where

$$\begin{aligned}\Lambda&=2\Omega^{\mathrm{T}}(t)P\tilde A\Omega(t)+2\Omega^{\mathrm{T}}(t)P\tilde{\mathcal{B}}g\big(u(t)\big)+2\Omega^{\mathrm{T}}(t)P\tilde F\mathcal{F}\big(x(t)\big)\\&\quad+2\Omega^{\mathrm{T}}(t)P\tilde Dd(t).\end{aligned}$$

It follows from Assumption 1 and (5) that for any $\rho_1>0$, $\rho_2>0$, we can have the following inequalities:

$$\begin{aligned}&-\rho_1(\mathcal{F}^{\mathrm{T}}\big(x(t)\big)\mathcal{F}\big(x(t)\big)+\rho_1(x^{\mathrm{T}}(t)L_1^TL_1x(t))\geqslant 0,\\&\rho_2^{-1}(g^{\mathrm{T}}u(t))g\big(u(t)\big)\leqslant\rho_2^{-1}(u^{\mathrm{T}}(t)\Xi_1u(t)).\end{aligned}\tag{9}$$

By combining (8) and (9) and considering

$$J=-y^{\mathrm{T}}(t)Qy(t)+2y^{\mathrm{T}}(t)Sd(t)+d^{\mathrm{T}}(t)Rd(t),$$

we can get

$$\dot{v}(t) - J \leqslant \chi^{\mathrm{T}}(t)\bar{\Psi}_1\chi(t), \tag{10}$$

where

$$\chi(t) = \begin{bmatrix} x^{\mathrm{T}}(t) & x_C^{\mathrm{T}}(t) & x_R^{\mathrm{T}}(t) & \mathcal{F}^{\mathrm{T}}(x(t)) & g^{\mathrm{T}}(u(t)) & d^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}} \quad \text{and} \quad \bar{\Psi}_1 = [\bar{\Psi}]_{6\times 6},$$

$$\bar{\Psi}_{1,1} = \mathcal{A}^{\mathrm{T}}\frac{P_1}{\alpha_1} + \frac{P_1}{\alpha_1}\mathcal{A} + \mathcal{C}^{\mathrm{T}}Q\mathcal{C} + \rho_1 L_1^{\mathrm{T}}L_1,$$

$$\bar{\Psi}_{1,2} = \mathcal{C}^{\mathrm{T}}\mathcal{B}_C^{\mathrm{T}}\frac{P_2}{\beta_1} + \frac{P_1}{\alpha_1}\mathcal{B}G\mathcal{K}, \qquad \bar{\Psi}_{1,3} = -\mathcal{B}_R^{\mathrm{T}}\frac{P_3}{\gamma_1} + \frac{P_1}{\alpha_1}\mathcal{B}G\mathcal{K}_R,$$

$$\bar{\Psi}_{1,4} = \frac{P_1}{\alpha_1}, \qquad \bar{\Psi}_{1,5} = \frac{P_1}{\alpha_1}\mathcal{B}, \qquad \bar{\Psi}_{1,6} = \frac{P_1}{\alpha_1}\mathcal{D} - \mathcal{C}^{\mathrm{T}}S,$$

$$\bar{\Psi}_{2,2} = \mathcal{A}_C^{\mathrm{T}}\frac{P_2}{\beta_1} + \frac{P_2}{\beta_1}\mathcal{A}_C + \rho_2^{-1}\Xi_1\mathcal{K}^{\mathrm{T}}\mathcal{K},$$

$$\bar{\Psi}_{3,3} = \mathcal{A}_R^{\mathrm{T}}\frac{P_3}{\gamma_1} + \frac{P_3}{\gamma_1}\mathcal{A}_R + \rho_2^{-1}\Xi_1\mathcal{K}_R^{\mathrm{T}}\mathcal{K}_R,$$

$$\bar{\Psi}_{4,4} = -\rho_1, \quad \bar{\Psi}_{5,5} = -\rho_2^{-1} \quad \text{and} \quad \bar{\Psi}_{6,6} = -R,$$

and the remaining terms are zero in $\bar{\Psi}$. By applying Schur complement in (10) and it follows from (6) that

$$\dot{v}(t) - J < 0. \tag{11}$$

To prove that the augmented fractional-order closed-loop system (5) reaches the equilibrium point, we consider $d(t) = 0$. Then it is obtained from (11) that the monotonic decreasing function $v(t) > 0$ satisfies $\|x(t)\|^2 \to 0$ as $t \to \infty$. Therefore, the augmented fractional-order closed-loop system (5) reaches the equilibrium point. To prove the dissipativity of augmented fractional-order closed-loop system (5), we consider $d(t) \neq 0$. Hence, from (11) and Definitions 3.1 and 3.2 in [19] we obtain the dissipativity condition as $\dot{v}(t) + y^{\mathrm{T}}(t)Qy(t) - 2y^{\mathrm{T}}(t)Sd(t) - d^{\mathrm{T}}(t)Rd(t) \leqslant 0$. Thus, the augmented fractional-order closed-loop system (5) reaches the equilibrium point and $(QSR)$-dissipativity.

Next, by assuming the gain matrix to be unknown, a design method of dynamic output feedback nonfragile reliable controller for the considered fractional glucose–insulin system (1) is presented, which admits that the augmented system (5) reaches the equilibrium point and satisfies the desired dissipative performance index. □

**Theorem 2.** *For given scalars $\alpha_1 > 0$, $\beta_1 > 0$, $\gamma_1 > 0$, $\rho_1 > 0$, $\rho_2 > 0$, $\rho_3 > 0$ and real constant matrices $L_1 > 0$, $G > 0$, $\Xi_1 > 0$, $Q$, $S$, $R$, $M$, $N$, $N_R$, the augmented fractional-order closed-loop system* (5) *reaches the equilibrium point and strictly $(QSR)$-dissipativity if there exist symmetric matrices $P_1 > 0$, $P_2 > 0$, $P_3 > 0$ and scalar $\epsilon > 0$, such that the following LMIs hold*:

$$\psi < 0, \tag{12}$$

$$\begin{pmatrix} -\rho_3 & \mathcal{C}W_1 - V_B\mathcal{C} \\ * & -I \end{pmatrix} < 0, \tag{13}$$

*where*

$$\psi = [\psi]_{11\times 11}, \qquad \psi_{1,1} = \alpha_1 \mathcal{A} W_1 + \alpha_1 W_1 \mathcal{A}^{\mathrm{T}}, \qquad \psi_{1,2} = \beta_1 \mathcal{B} G X_2 + \alpha_1 \mathcal{C}^{\mathrm{T}} N_B,$$
$$\psi_{1,3} = -\alpha_1 W_1 \mathcal{B}_R^{\mathrm{T}} + \gamma_1 \mathcal{B} G X_3, \qquad \psi_{1,4} = I, \qquad \psi_{1,5} = \mathcal{B},$$
$$\psi_{1,6} = \mathcal{D} - \alpha_1 \mathcal{C}^{\mathrm{T}} S W_1, \qquad \psi_{1,7} = \alpha_1 \sqrt{-Q} W_1 \mathcal{C}^{\mathrm{T}}, \qquad \psi_{1,8} = \alpha_1 W_1 \sqrt{\rho_1} L_1^{\mathrm{T}},$$
$$\psi_{1,10} = \epsilon \mathcal{B} G \mathcal{M}, \qquad \psi_{2,2} = \mathcal{N}_A + \beta_1 \mathcal{M} W_2, \qquad \psi_{2,9} = -X_2 \beta_1 \mathcal{K}^{\mathrm{T}},$$
$$\psi_{2,11} = \beta_1 W_2 \mathcal{N}^{\mathrm{T}}, \qquad \psi_{3,3} = \gamma_1 \mathcal{A}_R W_3 + \gamma_1 W_3 \mathcal{A}_R^{\mathrm{T}}, \qquad \psi_{3,9} = -\gamma_1 X_3 \mathcal{K}_R^{\mathrm{T}},$$
$$\psi_{3,11} = \gamma_1 W_3 \mathcal{N}_R^{\mathrm{T}}, \qquad \psi_{4,4} = -\rho_1, \qquad \psi_{5,5} = -\rho_2^{-1},$$
$$\psi_{6,6} = -R, \qquad \psi_{7,7} = -I, \qquad \psi_{8,8} = -I, \qquad \psi_{9,9} = -\rho_2 \Xi_1^{-1},$$
$$\psi_{9,10} = \epsilon \mathcal{M}, \qquad \psi_{10,10} = -\epsilon, \qquad \psi_{11,11} = -\epsilon,$$

*and the remaining terms are zero in $\psi$. Also, the control parameters are given by $\mathcal{A}_C = \mathcal{N}_A (W_2)^{-1}$, $\mathcal{B}_C = \mathcal{N}_B (V_B)^{-1}$, $\mathcal{K} = X_2 (W_2)^{-1}$ and $\mathcal{K}_R = X_3 (W_2)^{-1}$.*

*Proof.* The proof is obtained by following the similar procedure as in Theorem 1 with some changes. Then, with the aid of Schur complement, the inequality $\Phi = [\Phi]_{11\times 11} < 0$ can be obtained from Theorem 1 by replacing $\mathcal{K}$ and $\mathcal{K}_R$ with $\mathcal{K} + \Delta\mathcal{K}(t)$ and $\mathcal{K}_R + \Delta\mathcal{K}_R(t)$, respectively, where

$$\Phi_{1,1} = \mathcal{A}^{\mathrm{T}} \frac{P_1}{\alpha_1} + \frac{P_1}{\alpha_1} \mathcal{A}, \qquad \Phi_{1,2} = \mathcal{C}^{\mathrm{T}} \mathcal{B}_C^{\mathrm{T}} \frac{P_2}{\beta_1} + \frac{P_1}{\alpha_1} \mathcal{B} G \mathcal{K},$$
$$\Phi_{1,3} = -\mathcal{B}_R^{\mathrm{T}} \frac{P_3}{\gamma_1} + \frac{P_1}{\alpha_1} \mathcal{B} G \mathcal{K}, \qquad \Phi_{1,4} = \frac{P_1}{\alpha_1}, \qquad \Phi_{1,5} = \frac{P_1}{\alpha_1} \mathcal{B},$$
$$\Phi_{1,6} = \sqrt{\rho_1} L_1^{\mathrm{T}}, \qquad \Phi_{1,8} = \mathcal{D} - \mathcal{C}^{\mathrm{T}} S, \qquad \Phi_{1,9} = \sqrt{-Q} \mathcal{C}^{\mathrm{T}},$$
$$\Phi_{1,10} = \frac{P_1}{\alpha_1} \epsilon \mathcal{B} G \mathcal{M}, \qquad \Phi_{2,2} = \mathcal{A}_C^{\mathrm{T}} \frac{P_2}{\beta_1} + \frac{P_2}{\beta_1} \mathcal{A}_C, \qquad \phi_{2,7} = -\mathcal{K}^{\mathrm{T}},$$
$$\Phi_{2,11} = \frac{P_2}{\beta_1} \mathcal{N}^{\mathrm{T}} \qquad \Phi_{3,3} = \mathcal{A}_R^{\mathrm{T}} \frac{P_3}{\gamma_1} + \frac{P_3}{\gamma_1} \mathcal{A}_R, \qquad \Phi_{3,7} = -\mathcal{K}_R^{\mathrm{T}},$$
$$\Phi_{3,11} = \frac{P_3}{\gamma_1} \mathcal{N}_R^{\mathrm{T}}, \qquad \Phi_{4,4} = -\rho_1, \qquad \Phi_{5,5} = -\rho_2^{-1}, \qquad \Phi_{6,6} = -I,$$
$$\Phi_{7,7} = -\rho_2 \Xi_1^{-1}, \qquad \Phi_{8,8} = -R, \qquad \Phi_{9,9} = -I,$$
$$\Phi_{9,10} = -\epsilon \mathcal{M}, \qquad \Phi_{10,10} = -\epsilon, \qquad \Phi_{11,11} = -\epsilon,$$

and the remaining values of $\Phi$ are zero. In order to obtain the controller parameters, let us choose $W_1 = P_1^{-1}$, $W_2 = P_2^{-1}$, $W_3 = P_3^{-1}$ and $\mathcal{C} W_1 = V_B \mathcal{C}$. Pre- and post-multiplying $\Phi$ by

$$\mathrm{diag}\{\alpha_1 W_1, \beta_1 W_2, \gamma_1 W_3, \underbrace{I, \ldots, I}_{8 \text{ times}}\},$$

condition (12) is obtained. In addition, the assumption $\mathcal{C} W_1 = V_B \mathcal{C}$ cannot be solved by using Matlab LMI toolbox directly. Therefore, by considering the optimization technique,

$\mathcal{C}W_1 = V_B\mathcal{C}$ is rewritten as $\operatorname{trace}[(\mathcal{C}W_1 - V_B\mathcal{C})^{\mathrm{T}}(\mathcal{C}W_1 - V_B\mathcal{C})] = 0$. Then, by applying Schur complement to the above transformation, for any $\rho_3 > 0$, condition (13) is satisfied. Thus, if the conditions (12) and (13) are satisfied, then the augmented fractional-order closed-loop system reaches the equilibrium point with a satisfactory dissipative performance index, which completes the proof. □

## 4 Simulation results

To demonstrate the potential of the proposed internal model-based dynamic output feedback nonfragile reliable controller for glucose–insulin system (1), the numerical example with simulation results are provided. The parameters of glucose–insulin system are taken from [7] as $p_1 = 0.028735$, $p_2 = 0.028344$, $p_3 = 0.00005035$ and $n = 0.22$. It should be noted that excess food intake or irregular diet may be routine habit for some persons, which eventually leads to insufficient production of glucose in the human body. Precisely, excess food intake or irregular diet is taken as the disturbance input. Here, the disturbance input is chosen as a periodic function, which is modeled by using sinusoidal term $\beta\sin(\omega t)$ with amplitude $\beta$ and frequency $\omega$, where $\omega = 2\pi/T$ and $\beta = 10$ mg/dl, $T = 6$ h. The main work of the proposed controller is to maintain blood sugar level in $70{-}110$ mg/dl by forcing the states $x_1(t)$, $x_2(t)$ and $x_3(t)$. Let the basal value of glucose and insulin be $G_b$ and $I_b$, respectively. Let us consider the remaining parameters as

$$\mathcal{A}_R = \begin{pmatrix} -0.01 & 0 & 0 \\ 0 & -0.01 & 0 \\ 0 & 0 & -0.01 \end{pmatrix}, \qquad \mathcal{B}_R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$\rho_1 = 4$, $\rho_2 = 0.001$, $\rho_3 = 0.1$, $\mathcal{M} = 0.0009$, $\mathcal{N} = [0.008\ \ 0.008\ \ 0]$, $\mathcal{N}_R = [0.1\ \ 0\ \ 0.1]$, $G = 0.8$, $Q = -0.00009$, $S = \sqrt{0.09}$, $R = 5$ and $g(u(t)) = 0.5\sin(u(t))$. Then the controller parameters are obtained by solving the Theorem 2. The corresponding controller parameters are given by

$$\mathcal{K} = \begin{bmatrix} -0.0000 & 0.0000 & 0.00054 \end{bmatrix}, \qquad \mathcal{K}_R = \begin{bmatrix} 0.0050 & 0.0068 & 0.0062 \end{bmatrix},$$

$$\mathcal{A}_C = \begin{pmatrix} -0.5174 & 0.0058 & -0.2950 \\ -0.0058 & -0.2351 & 0.0321 \\ -0.2950 & 0.0321 & -0.5182 \end{pmatrix}$$

and

$$\mathcal{B}_C = \begin{bmatrix} 2.2420 & 0.0734 & 1.9709 \end{bmatrix}^{\mathrm{T}} \times 10^3.$$

The initial values of the states are taken as $x(0) = [380\ \ 0.0001\ \ 210]^{\mathrm{T}}$.

Figures 1–3 represent the state responses of the glucose concentration, insulin excitable tissue glucose and insulin concentration, respectively. It is observed from Figs. 1 and 3 that after injecting insulin to a patient, the glucose concentration and insulin concentration drop down to their corresponding basal values $G_b = 200$ and $I_b = 15$, respectively. Precisely, the glucose concentration level of a diabetic patient decreases and reaches the basal value, after a time period of 550 minutes, whichhas been displayed

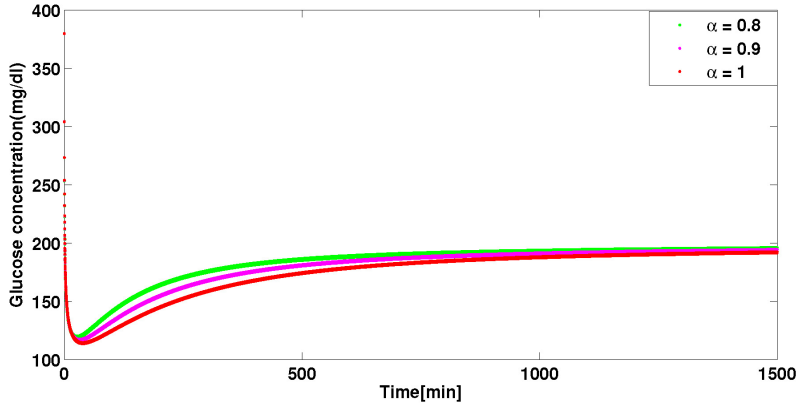


**Figure 1.** State trajectory of the glucose concentration with $\alpha = 0.8, 0.9, 1$.

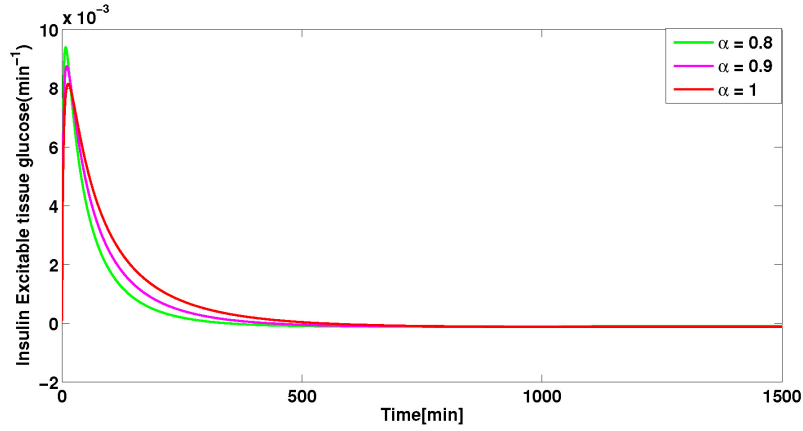


**Figure 2.** State trajectory of insulin-excitable tissue glucose with $\alpha = 0.8, 0.9, 1$.

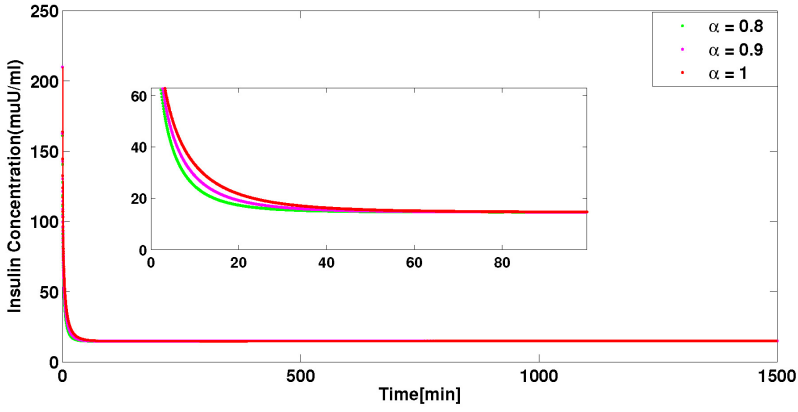


**Figure 3.** State trajectory of the insulin concentration with $\alpha = 0.8, 0.9, 1$.

in Fig. 1. It is also described in Figs. 1–3 that the state responses of the considered system with the fractional-order against the integer-order to analyze the performance of the obtained results. Thus, the simulation results provided through Figs. 1–3 describe the potential of the proposed dynamic output feedback controller for glucose–insulin system. More precisely, the irregular blood glucose level can be brought down to normal level by injecting suitable rate of insulin to the patient. The result eventually concludes that the glucose–insulin control system described by fractional-order model is more suitable one for studying the control of blood glucose levels in diabetic patients.

## 5 Conclusion

In this paper, the stabilization problem of glucose–insulin system in the presence of meal disturbances described by fractional-order nonlinear systems is studied. A novel class of sufficient conditions has been developed as LMIs with the use of Lyapunov approach for the design of dynamic output feedback nonfragile reliable controller of nonlinear fractional-order systems. The performance of the proposed controller has been verified by using numerical simulations. Based on the obtained results, it is concluded that the proposed control strategy can achieve its desired objective in the presence of meal disturbance.

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