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Synchronization of decentralized event-triggered uncertain switched neural networks with two additive time-varying delays*

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Abstract. This paper addresses the problem of synchronization for decentralized event-triggered uncertain switched neural networks with two additive time-varying delays. A decentralized event-triggered scheme is employed to determine the time instants of communication from the sensors to the central controller based on narrow possible information only. In addition, a class of switched neural networks is analyzed based on the Lyapunov–Krasovskii functional method and a combined linear matrix inequality (LMI) technique and average dwell time approach. Some sufficient conditions are derived to guarantee the exponential stability of neural networks under consideration in the presence of admissible parametric uncertainties. Numerical examples are provided to illustrate the effectiveness of the obtained results.

Keywords: event-triggered scheme, switched neural networks, Lyapunov–Krasovskii functional, exponential stability, average dwell time.

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1 Introduction

In recent years, neural networks (NNs) have become the active research field, and their successive application used in various areas, such as image processing and optimization problems [3,6,26]. Time delays, which always cause instability and degrade performance, are ubiquitously present in many NNs due to the signal transmission (see [1, 12] and references therein). Switched systems, which provide a unified framework for mathematical modeling of many physical or man-made systems displaying switching features, such as power electronics, flight control systems, network control systems, have been widely studied recently [5, 22, 23]. The switched system consists of a collection of subsystems and a switching signal governing the switching among them. In addition, the average dwell time method introduced in [7] has been recognized to be flexible and efficient in finding a suitable switching signal to guarantee the stability of switched systems or improve the system performance [4, 20].

In the recent years, event-triggered control has received increasing attention in real time control systems. Especially to the case of battery-powered wireless devices, reducing the number of network transmissions has an important effect on the battery lifespan [14, 15]. Therefore, how to saving limited network resource is a significant and challenging task. The main task to design a decentralized event-triggered scheme for saving the limited communication resources while guaranteeing that the drive response system is synchronous. As a result, event-triggered scheme particularly decentralized event-triggered scheme has received a lot of research interest, and some important results have been published [9, 16]. In [18], network-based event-triggered filtering for Markovian jump systems is studied. In this paper, the model of decentralized event-triggered is for saving the limited communication resources while assure that the switched neural networks with additive time-varying delay is exponentially stable.

Recently, a new type of systems with two additive time-varying delays were proposed in [8], and the stability problem was further discussed in [11]. In networked systems, signals are communicated from one point to another may experience two network segments, which can possibly produce two time-varying delays with different properties by cause of variable network transmission conditions [13,23,28]. Moreover, synchronization has been extensively studied due to its strong potential applications in engineering, such as secure communication, robot queue, and chemical reaction [10, 27]. Synchronization strategies can have communication between nodes, which cause the network congestion and waste the network resources. In order to overcome the conservativeness of synchronization strategies, the event-triggered strategy is proposed. Many results have been reported in the literature for synchronization-based event-triggered problem [17, 19, 25]. In addition, to the best of our knowledge, synchronization of uncertain switched neural networks with two additive delay components via decentralized event-triggered scheme has not been completely investigated, which motivates the study of this paper.

Based on the above discussions, in this paper, the problem of synchronization of decentralized event-triggered uncertain switched neural networks with two additive delay components is considered. By utilizing a novel Lyapunov–Krasovskii functional, integral techniques, some sufficient conditions are expressed in terms of linear matrix inequalities

(LMIs), which can be easily checked by using MATLAB LMI Control toolbox. Finally, numerical examples are given to verify the effectiveness of the obtained criteria. The main contributions of this paper are listed as follows: (i) a sufficient condition of exponential stability with two additive time-varying delays for uncertain switched neural networks is established by using average dwell time method, Lyapunov functional method, and some mathematical techniques; and (ii) A decentralized event-triggered scheme is proposed to synchronize the drive-response uncertain switched neural networks.

Notations. Throughout this manuscript, $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote the n-dimensional Euclidean space and the set of all $n\times n$ real matrices, respectively. The superscript T denotes the transposition, and the notation $P > 0$ means that P is real, symmetric and positive definite; diag denotes the block-diagonal matrix; $\|\cdot\|$ refers to the Euclidean vector norm. The notation $*$ always denotes the symmetric block in one symmetric matrix. $\lambda_{\min}(A)$ or $\lambda_{\max}(A)$ denotes the maximum eigenvalue or the minimum eigenvalue of matrix A, respectively. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2 Problem statement and preliminaries

Consider the following uncertain drive-response neural networks with two additive time-varying delay components:

$$\begin{aligned}\dot{y}(t) &= -\big(W_{\beta(t)} + \varDelta W_{\beta(t)}(t)\big)y(t) + \big(A_{\beta(t)} + \varDelta A_{\beta(t)}(t)\big)f\big(y(t)\big)\\ &\quad + \big(B_{\beta(t)} + \varDelta B_{\beta(t)}(t)\big)f\big(y\big(t-\hat{h}_1(t)-\hat{h}_2(t)\big)\big)\\ &\quad + \big(C_{\beta(t)} + \varDelta C_{\beta(t)}(t)\big)\int\limits_{t-\hat{\rho}(t)}^{t} f\big(y(s)\big)\,\mathrm{d}s + \nu(t), \qquad (1)\end{aligned}$$

$$\begin{aligned}\dot{z}(t) &= -\big(W_{\beta(t)} + \varDelta W_{\beta(t)}(t)\big)z(t) + \big(A_{\beta(t)} + \varDelta A_{\beta(t)}(t)\big)f\big(z(t)\big)\\ &\quad + \big(B_{\beta(t)} + \varDelta B_{\beta(t)}(t)\big)f\big(z\big(t-\hat{h}_1(t)-\hat{h}_2(t)\big)\big)\\ &\quad + \big(C_{\beta(t)} + \varDelta C_{\beta(t)}(t)\big)\int\limits_{t-\hat{\rho}(t)}^{t} f\big(z(s)\big)\,\mathrm{d}s + \nu(t) + u(t), \qquad (2)\end{aligned}$$

where $y(t) = [y_1(t), y_2(t), \dots, y_n(t)]^{\mathrm{T}} \in \mathbb{R}^n$, $z(t) = [z_1(t), z_2(t), \dots, z_n(t)]^{\mathrm{T}} \in \mathbb{R}^n$ is the neuron state vector in the drive system (1) and the response system (2), respectively; $W_{\beta(t)} = \mathrm{diag}\{w_{\beta(t)i}\}$ with $w_{\beta(t)i} > 0$ $(i = 1, 2, \dots, n)$; $A_{\beta(t)}, B_{\beta(t)}, C_{\beta(t)} \in \mathbb{R}^{n\times n}$ are the connection weight matrix, discretely delayed connection weight matrix and the distributed delayed connection weight matrix, respectively; $\nu(t) = [\nu_1(t), \nu_2(t), \dots, \nu_n(t) \in \mathbb{R}^n]$ is an external input vector; $u(t) \in \mathbb{R}^n$ is the control input of the response system (2); $f(\cdot) = [f_1(\cdot), \dots, f_n(\cdot)]^{\mathrm{T}} \in \mathbb{R}^n$ is the neuron activation function; $\varDelta W_{\beta(t)}(t)$, $\varDelta A_{\beta(t)}(t)$, $\varDelta B_{\beta(t)}(t)$, $\varDelta C_{\beta(t)}(t)$ are time-varying parametric uncertainties, where $\beta(t)$ denotes the switching signal and takes the values in the finite set $M = \{1, 2, \dots, m\}$, which means that the matrices $W_{\beta(t)}$, $A_{\beta(t)}$, $B_{\beta(t)}$, $C_{\beta(t)}$ are allowed to take values in the finite set $[(W_1, A_1, B_1, C_1)], \dots, [(W_m, A_m, B_m, C_m)]$. Corresponding to the switching signal $\beta(t)$, we have the switching sequence $\{x_{t_0}; (l_0, t_0), \dots (l_k, t_k), \dots, |\ l_k \in M, k = 0, 1, 2 \dots\}$, it means that lth subsystem is active when $t \in [t_k, t_{k+1})$. In the drive-response neural networks, $\hat{\rho}(t)$ represents the distributed delay, and $\hat{h}_1(t)$ and $\hat{h}_2(t)$ are additive time-varying delays that satisfy

$$0 \leqslant \hat{h}_1(t) \leqslant \hat{h}_1, \qquad \dot{\hat{h}}_1(t) \leqslant \mu_1, \qquad 0 \leqslant \hat{h}_2(t) \leqslant \hat{h}_2, \qquad \dot{\hat{h}}_2(t) \leqslant \mu_2,$$
$$0 \leqslant \hat{\rho}(t) \leqslant \hat{\rho}, \qquad \dot{\hat{\rho}}(t) \leqslant \mu_3,$$

where $\hat{h}_1$, $\hat{h}_2$, $\hat{\rho}$, μ_1, μ_2 and μ_3 are known constants. Also, we denote $\hat{h}(t) = \hat{h}_1(t) + \hat{h}_2(t)$, $\hat{h} = \hat{h}_1 + \hat{h}_2$ and $\mu = \mu_1 + \mu_2$. We assume that the matrices $\varDelta W_l(t)$, $\varDelta A_l(t)$, $\varDelta B_l(t)$, $\varDelta C_l(t)$ are norm bounded and satisfy

$$\big[\varDelta W_l(t)\ \ \varDelta A_l(t)\ \ \varDelta B_l(t)\ \ \varDelta C_l(t)\big] = G_l(t) F_l(t) \big[X_{1l}\ \ X_{2l}\ \ X_{3l}\ \ X_{4l}\big],$$

where $G_l(t)$, X_{1l}, X_{2l}, X_{3l}, X_{4l} are known real constant matrices with appropriate dimensions. The uncertain matrix $F_l(t)$ satisfies $F_l^{\mathrm{T}}(t) F_l(t) \leqslant I$, $t \geqslant 0$. In order to simplify the equations, we write $[\varDelta W_l(t)\ \varDelta A_l(t)\ \varDelta B_l(t)\ \varDelta C_l(t)]$ as $[\varDelta W_l\ \varDelta A_l\ \varDelta B_l\ \varDelta C_l]$.

Assumption 1. Each neuron activation function $f_z(\cdot)$ is bounded and there exist constants l_z^-, l_z^+ such that $l_j^- \leqslant l_j^+$ and

$$l_z^- \leqslant \frac{f_z(\rho_1) - f_z(\rho_2)}{\rho_1 - \rho_2} \leqslant l_z^+$$

for all $\rho_1, \rho_2 \in \mathbb{R}$, $\rho_1 \neq \rho_2$, $z \in \mathbb{N}$. For simplicity of presentation, we denote $H_1 = \operatorname{diag}\{l_1^- l_1^+, l_2^- l_2^+, \dots, l_n^- l_n^+\}$, $H_2 = \operatorname{diag}\{(l_1^- + l_1^+)/2,\ (l_2^- + l_2^+)/2,\ \dots,\ (l_n^- + l_n^+)/2\}$. Combining (1) and (2) with $r(t) = y(t) - z(t)$, the synchronization error system can be obtained as

$$\begin{aligned} \dot{r}(t) = {} & -\big(W_l + \varDelta W_l(t)\big) r(t) + \big(A_l + \varDelta A_l(t)\big) g\big(r(t)\big) + \big(B_l + \varDelta B_l(t)\big) \\ & \times g\big(r\big(t - \hat{h}_1(t) - \hat{h}_2(t)\big)\big) + \big(C_l + \varDelta C_l(t)\big) \int\limits_{t-\hat{\rho}(t)}^{t} g\big(r(s)\big)\,\mathrm{d}s + u(t), \end{aligned} \tag{3}$$

where $g(r(t)) = f(z(t)) - f(y(t))$, and it can be checked that the function $g_z(\cdot)$ satisfies the following condition:

$$l_z^- \leqslant \frac{g_z(\rho)}{\rho} \leqslant l_z^+, \quad g_z(0) = 0 \quad \forall z \in \mathbb{R},\ \rho \neq 0,\ z \in \mathbb{N}.$$

The decentralized event-triggered scheme is initiate to decrease the communication burden. In this design, the (m) entries and the measurement errors $r(t)$ are collected into $\hat{v}$ nodes, therefore the signals corresponding to node $j \in \{1, 2, \dots, \hat{v}\}$ are denoted by

$k_j(t) \in \mathbb{R}^{n_j}$ for $\sum_{j=1}^{\hat{v}} n_j = n$. A decentralized event-triggered condition embedded in the event generators (EGs) is used to decide whether the sampled data should be released to the controller or not. We denote the release instants of the $j^{t\bar{h}}$ event generator by $[t^j_{k_j}\bar{h}]^{\infty}_{K_j} = 0$, and the next release instant $t^j_{k_j+1}\bar{h}$ of event generator j is determined by

$$t^j_{k_j+1}\bar{h} = t^j_{k_j}\bar{h} + \min\bigl\{i\bar{h} \mid \omega_j^{\mathrm{T}}\bigl(t^j_{k_j}\bar{h} + i\bar{h}\bigr)\bar{\phi}_{jl}\omega_j\bigl(t^j_{k_j}\bar{h} + i\bar{h}\bigr) > \kappa_j r_j^{\mathrm{T}}\bigl(t^j_k\bar{h}\bigr)\bar{\phi}_{jl} r_j\bigl(t^j_k\bar{h}\bigr)\bigr\}, \tag{4}$$

where κ_j, $\phi_j > 0$, $j \in \mathbb{N}$, and also defined the error between the current sampling vector and the latest transmission is

$$\omega_j\bigl(t^j_{k_j}\bar{h} + i\bar{h}\bigr) = r_j\bigl(t^j_{k_j}\bar{h} + i\bar{h}\bigr) - r_j\bigl(t^j_{k_j}\bar{h}\bigr).$$

We will consider a decentralized event-triggered scheme in this paper, which is one of the most important components for designing every control framework and reducing the communication burden in the network. Generally, from (4), the set of release instants $\{t^j_{k_j}\bar{h}\}$ is a subset of $\{0, \bar{h}, 2\bar{h}, \dots\}$, i.e., all the sampled signals are transmitted to the controller. The decentralized event-triggered communication scheme is designed to reduce some unwanted data transmissions. Therefore, the real-time detection hardware is no longer needed. In this paper, we are interested in designing the following controller:

$$u(t) = K_{\beta(t_k\bar{h})}\bigl[r_1^{\mathrm{T}}\bigl(t^1_{k_1}\bar{h}\bigr),\ r_2^{\mathrm{T}}\bigl(t^2_{k_2}\bar{h}\bigr),\ \dots,\ r_{\hat{v}}^{\mathrm{T}}\bigl(t^{\hat{v}}_{k_{\hat{v}}}\bar{h}\bigr)\bigr], \quad t \in [t_k\bar{h}, t_{k+1}\bar{h}),$$

where $K_{\beta(t_k\bar{h})}$ is the gain matrix to be determined. When $\beta(t_k\bar{h}) = \beta(t_{k+1}\bar{h}) = l$, the lth subsystem is activated on $[t_k\bar{h}, t_{k+1}\bar{h})$ and

$$t_k\bar{h} = \max_{j=1,2,\dots,\hat{v}}\bigl\{t^j_{k_j}\bar{h}\bigr\}, \quad t_{k+1}\bar{h} = \min_{j=1,2,\dots,\hat{v}}\ \max_{i=1,2,\dots,\hat{v}}\bigl\{t^j_{k_j+1}\bigr\}\bar{h} \bigm| t^j_{k_j+1} \geqslant t^i_{k_i} + 1.$$

Let $\hat{v}_k = t_{k+1} - t_k$. Then the interval $[t_k\bar{h},\, t_{k+1}\bar{h})$ can be shown as $[t_k\bar{h}, t_{k+1}\bar{h}) = \bigcup_{i=0}^{\hat{v}_k-1}\lambda_i$, where $\lambda_i = [t_k\bar{h} + i\bar{h},\, t_k\bar{h} + i\bar{h} + \bar{h})$. Define $\varsigma_1(t) = t - t_k\bar{h} - i\bar{h}$ for $t \in \lambda_i$. It is easy to understand that $\varsigma_1(t)$ is a piecewise-linear function satisfying

$$0 \leqslant \varsigma_1(t) \leqslant \bar{h}, \quad t \in \lambda_i, \qquad \dot{\varsigma}_1(t) = 1, \quad t \neq t_k\bar{h} + j\bar{h}.$$

Therefore, the threshold error $\omega_j(t_k\bar{h} + j\bar{h})$ can be rewritten as

$$\omega_j\bigl(t - \varsigma_1(t)\bigr) = r_j\bigl(t - \varsigma_1(t)\bigr) - r_j\bigl(t^j_{k_j}\bar{h}\bigr),$$

Denote $\omega(t - \varsigma_1(t)) = \{\omega_1(t - \varsigma_2(t)), \omega_2(t - \varsigma_2(t)), \dots, \omega_{\hat{v}}(t - \varsigma_n(t))\}$. Then the control input $u_l(t)$ can be obtained as

$$u_l(t) = K_l\bigl(r\bigl(t - \varsigma_1(t)\bigr) - \omega\bigl(t - \varsigma_1(t)\bigr)\bigr). \tag{5}$$

Substituting (5) in system (3), which gives

$$\begin{aligned}\dot r(t) &= -\big(W_l + \varDelta W_l(t)\big)r(t) + \big(A_l + \varDelta A_l(t)\big)g\big(r(t)\big)\\&\quad + \big(B_l + \varDelta B_l(t)\big)g\big(r\big(t - \hat h_1(t) - \hat h_2(t)\big)\big) + \big(C_l + \varDelta C_l(t)\big)\int_{t-\hat\rho(t)}^{t} g\big(r(s)\big)\,\mathrm{d}s\\&\quad + K_l\big(r\big(t - \varsigma_1(t)\big) - \omega\big(t - \varsigma_1(t)\big)\big) \quad \text{for } t \in \lambda_i.\end{aligned} \tag{6}$$

The following definitions and lemmas will play an important role in the derivation of our result.

Definition 1. (See [22].) System (3) is said to be robustly exponentially stable under switching signal $\beta(t)$ if there exist some scalars $k \geqslant 0$ and $\eta \geqslant 0$ such that

$$\big\|x(t)\big\| < k\mathrm{e}^{-\eta(t-t_0)}\big\|x(t_0)\big\|,$$

where $\|x(t_0)\| = \sup_{-h\leqslant s\leqslant 0}(\|x(s+t)\|, \|\dot x(s+t)\|)$.

Definition 2. (See [22].) For any $T_2 > T_1 \geqslant 0$, let $N_\beta(T_1, T_2)$ denote the number of switching $\beta(t)$ over (T_1, T_2). If $N_\beta(T_1, T_2) \leqslant N_0 + (T_2 - T_1)/T_a$ holds for $T_a > 0$, $N_0 \geqslant 0$, then T_a is called the average dwell time. It is commonly assumed in the literature that $N_0 = 0$.

Lemma 1 [Wirtinger-based integral inequality]. (*See* [12].) *For any constant matrix $\bar N > 0$, the following inequality holds for all continuously differentiable function $\hat\theta$ in $[a, b] \to \mathbb{R}^n$; $(b-a)\int_a^b \hat\theta^{\mathrm T}(\lambda)\bar N\hat\theta(\lambda)\,\mathrm{d}\lambda \geqslant (\int_a^b \hat\theta(\lambda)\,\mathrm{d}\lambda)^{\mathrm T}\bar N(\int_a^b \hat\theta(\lambda)\,\mathrm{d}\lambda) + 3\varrho^{\mathrm T}\bar N\varrho$, where $\varrho = \int_a^b \hat\theta(\lambda)\,\mathrm{d}\lambda - (2/(b-a))\int_a^b\int_a^b \hat\theta(\lambda)\,\mathrm{d}u\,\mathrm{d}\lambda$.*

Lemma 2. (*See* [3].) *For scalars $\hat a$ and $\hat b$ satisfying $\hat a \leqslant \hat b$ and a matrix $R \in \mathbb{S}^n_+$, the following inequality holds*: $\int_{\hat a}^{\hat b} \dot r^{\mathrm T}(s)R\dot r(s)\,\mathrm{d}s \geqslant (1/(\hat b - \hat a))\sum_{k=0}^{3}(2k+1)\rho_k^{\mathrm T}R\rho_k$.

Lemma 3. (*See* [26].) *For a scalar $\alpha \in (0, 1)$, matrices $R_1, R_2 \in \mathbb{S}^n_+$ and $Y_1, Y_2 \in \mathbb{R}^{n\times n}$, the following matrix inequality*

$$\begin{bmatrix}\frac{1}{\alpha}R_1 & 0\\ * & \frac{1}{1-\alpha}R_1\end{bmatrix} \leqslant \begin{bmatrix}R_1 + (1-\alpha)X_1 & Y(\alpha)\\ * & R_2 + \alpha X_2\end{bmatrix}$$

holds, where $X_1 = R_1 - Y_1R_2^{-1}Y_1^{\mathrm T}$, $X_2 = R_2 - Y_2R_1^{-1}Y_2^{\mathrm T}$ and $\alpha Y_1 + (1-\alpha)Y_2$.

3 Main results

In this section, our aim is to study the exponential stability of switched neural networks with additive time-varying delays using the decentralized event-triggered design. First, we consider the following nominal system:

$$\begin{aligned}\dot r(t) &= -W_lr(t) + A_lg\big(r(t)\big) + B_lg\big(r\big(t - \hat h_1(t) - \hat h_2(t)\big)\big) + C_l\int_{t-\hat\rho(t)}^{t} g\big(r(s)\big)\,\mathrm{d}s\\&\quad + K_l\big(r\big(t - \varsigma_1(t)\big) - \omega\big(t - \varsigma_1(t)\big)\big),\end{aligned} \tag{7}$$

In order to make the presentation more sententious, we define

$$\tilde r_i = [0_{n(i-1)n}\ I_n\ 0_{n(34-i)n}]; \quad i = 1, 2, \dots, 34,$$

$$\theta_{0^m} = \int_a^b \int_a^{s_1} \cdots \int_a^{s_{m-1}} x(s_m)\,\mathrm{d}s_m \cdots \mathrm{d}s_2\,\mathrm{d}s_1,$$

$$\begin{aligned}\varGamma_1 &= \mathrm{col}\{\tilde r_1 - \tilde r_3,\ \tilde r_2 + \tilde r_3 - 2\tilde r_{24},\ \tilde r_2 - \tilde r_3 - 6\tilde r_{24} + 12\tilde r_{25},\\ &\qquad \tilde r_2 + \tilde r_3 - 12\tilde r_{24} + 60\tilde r_{25} - 60\tilde r_{26}\},\\ \varGamma_2 &= \mathrm{col}\{\tilde r_1 - \tilde r_2,\ \tilde r_1 + \tilde r_2 - 2\tilde r_{21},\ \tilde r_1 - \tilde r_2 - 6\tilde r_{21} + 12\tilde r_{22},\\ &\qquad \tilde r_1 + \tilde r_2 - 12\tilde r_{21} + 60\tilde r_{22} - 60\tilde r_{23}\},\end{aligned}$$

$$\begin{aligned}\xi_1^{\mathrm T}(t) = \Bigg[& r^{\mathrm T}(t),\ r^{\mathrm T}\big(t-\hat h(t)\big),\ r^{\mathrm T}(t-\hat h),\ r^{\mathrm T}\big(t-\hat h_1(t)\big),\ r^{\mathrm T}(t-\hat h_1),\\ & r^{\mathrm T}\big(t-\hat h_2(t)\big),\ r^{\mathrm T}(t-\hat h_2),\ \dot r(t),\ g^{\mathrm T}\big(r(t)\big)g^{\mathrm T}\big(r\big(t-\hat h(t)\big)\big),\\ & \int_{t-\hat h}^{t} r^{\mathrm T}(s)\,\mathrm{d}s,\ \int_{t-\hat h}^{t}\int_{\beta}^{t} r^{\mathrm T}(s)\,\mathrm{d}s\,\mathrm{d}\beta,\ \int_{-\hat h}^{0}\int_{t+\beta}^{t} \dot r^{\mathrm T}(s)\,\mathrm{d}s\,\mathrm{d}\beta,\ \int_{t-\hat\rho(t)}^{t} g^{\mathrm T}\big(r(s)\big)\,\mathrm{d}s,\\ & \int_{t-\hat\rho}^{t}\int_{\beta}^{t} g^{\mathrm T}\big(r(s)\big)\,\mathrm{d}s\,\mathrm{d}\beta,\ r^{\mathrm T}\big(t-\varsigma_1(t)\big),\ r^{\mathrm T}(t-\bar h),\ \omega^{\mathrm T}\big(t-\varsigma_1(t)\big)\Bigg],\end{aligned}$$

$$\begin{aligned}\xi_2^{\mathrm T}(t) = \Bigg[& \frac{1}{\hat h_1(t)} \int_{t-\hat h_1(t)}^{t} \dot r^{\mathrm T}(s)\,\mathrm{d}s,\ \frac{1}{\hat h_1 - \hat h_1(t)} \int_{t-\hat h_1}^{t-\hat h_1(t)} \dot r^{\mathrm T}(s)\,\mathrm{d}s,\ \frac{1}{\hat h_2(t)} \int_{t-\hat h_2(t)}^{t} \dot r^{\mathrm T}(s)\,\mathrm{d}s\\ & \frac{1}{\hat h_2 - \hat h_2(t)} \int_{t-\hat h_2}^{t-\hat h_2(t)} \dot r^{\mathrm T}(s)\,\mathrm{d}s\Bigg], \qquad \alpha_1 = \frac{\hat h - \hat h(t)}{\hat h}, \qquad \alpha_2 = \frac{\bar h - \varsigma_1(t)}{\bar h},\end{aligned}$$

$$\begin{aligned}\varGamma_3 &= \mathrm{col}\{\tilde r_{16} - \tilde r_{17},\ \tilde r_{16} + \tilde r_{17} - 2\tilde r_{32},\ \tilde r_{16} - \tilde r_{17} - 6\tilde r_{32} + 12\tilde r_{33},\\ &\qquad \tilde r_{16} + \tilde r_{17} - 12\tilde r_{32} + 60\tilde r_{33} - 60\tilde r_{34}\},\\ \varGamma_4 &= \mathrm{col}\{\tilde r_1 - \tilde r_{16},\ \tilde r_1 + \tilde r_{16} - 2\tilde r_{29},\ \tilde r_1 - \tilde r_{16} - 6\tilde r_{29} + 12\tilde r_{30},\\ &\qquad \tilde r_1 + \tilde r_{16} - 12\tilde r_{29} + 60\tilde r_{30} - 60\tilde r_{31}\},\end{aligned}$$

$$\tilde R = \mathrm{diag}\{R_{3l},\ 3R_{3l},\ 5R_{3l},\ 7R_{3l}\}, \qquad \tilde M = \mathrm{diag}\{M_{3l},\ 3M_{3l},\ 5M_{3l},\ 7M_{3l}\}$$

$$\xi_3^{\mathrm{T}}(t) = \Big[\dot{r}^{\mathrm{T}}\big(t-\hat{h}(t)\big),\ \dot{r}^{\mathrm{T}}(t-\hat{h}),$$
$$\frac{\nu_0(t)}{\hat{h}(t)},\ \frac{\nu_1(t)}{\hat{h}(t)},\ \frac{\nu_2(t)}{\hat{h}(t)},\ \frac{\hat{\delta}_0(t)}{\hat{h}-\hat{h}(t)},\ \frac{\hat{\delta}_1(t)}{\hat{h}-\hat{h}(t)},\ \frac{\hat{\delta}_2(t)}{\hat{h}-\hat{h}(t)}\Big],$$
$$\xi_4^{\mathrm{T}}(t) = \Big[\dot{r}^{\mathrm{T}}\big(t-\varsigma_1(t)\big),\ \dot{r}^{\mathrm{T}}(t-\bar{h}),$$
$$\frac{\gamma_0(t)}{\varsigma_1(t)},\ \frac{\gamma_1(t)}{\varsigma_1(t)},\ \frac{\gamma_2(t)}{\varsigma_1(t)},\ \frac{\hat{\varrho}_0(t)}{\bar{h}-\varsigma_1(t)},\ \frac{\hat{\varrho}_1(t)}{\bar{h}-\varsigma_1(t)},\ \frac{\hat{\varrho}_2(t)}{\bar{h}-\varsigma_1(t)}\Big]$$
$$\varpi_1^{\mathrm{T}}(t) = \big[r^{\mathrm{T}}(t),\ r^{\mathrm{T}}\big(t-\hat{h}(t)\big),\ r^{\mathrm{T}}\big(t-\hat{h}\big),\ \xi_3^{\mathrm{T}}(t)\big],$$
$$\varpi_2^{\mathrm{T}}(t) = \big[r^{\mathrm{T}}(t),\ r^{\mathrm{T}}(t-\varsigma_1(t)),\ r^{\mathrm{T}}(t-\bar{h}),\ \xi_4^{\mathrm{T}}(t)\big].$$

3.1 Exponential stability analysis using decentralized event-triggered scheme

Theorem 1. *For given scalars $\hat{h}_u(u=1,2)$, ρ, $\hat{h}$, μ, $\bar{h}$, μ_j $(j=1,2,3)$, the equilibrium point of system* (7) *is exponentially stable if there exist symmetric positive-definite matrices P_l, Q_{il} $(i=1,2,\dots,7)$, R_{jl} $(l=1,2,3,4)$, S_{1l}, S_{2l}, W_{1l}, M_{1l}, M_{2l}, positive diagonal matrices $\Lambda_{fl}>0$ $(f=1,2)$ and the matrix $\bar{\phi}_l>0$, any matrices $\hat{\alpha}_1$, $\hat{\alpha}_2$, $\hat{\alpha}_3$, $\hat{\alpha}_4$, $\hat{\alpha}_5$, L_{1l}, T_l, U_{sl} $(s=1,2)$ with appropriate dimensions and $Y_1,Y_2,Y_3,Y_4\in\mathbb{R}^{4n\times 4n}$ such that the following linear matrix inequalities hold*:

$$\hat{\Pi}_{\hat{S}} = \begin{bmatrix} \hat{\Pi}_\alpha & -\hat{h}_1\hat{\alpha}_{\hat{I}}\mathrm{e}^{-\alpha\hat{h}_1} & -\hat{h}_2\hat{\alpha}_{\hat{L}}\mathrm{e}^{-\alpha\hat{h}_2} \\ * & -\hat{h}_1R_1\mathrm{e}^{-\alpha\hat{h}_1} & 0 \\ * & * & -\hat{h}_2R_2\mathrm{e}^{-\alpha\hat{h}_2} \end{bmatrix} \leqslant 0, \tag{8}$$

where

$$\hat{\Pi}_\alpha = \eta^{\mathrm{T}}(t)\hat{\Pi}\eta(t), \qquad \hat{S}=1,\dots,4, \qquad \hat{I}=1,3, \qquad \hat{J}=2,4, \tag{9}$$

and

$$\begin{aligned}
\hat{\Pi} &= \tilde{r}_1^{\mathrm{T}}\bigg\{\alpha P_l + Q_{1l}+Q_{2l}+Q_{3l}+Q_{4l}+Q_{5l}+Q_{6l}+\hat{h}^2R_{4l}+M_{1l} \\
&\quad - \frac{\pi^2}{4}W_{1l}\mathrm{e}^{-\alpha\bar{h}} - H_1\Lambda_{11} + \hat{\alpha}_1 + \hat{\alpha}_1^{\mathrm{T}} + \hat{\alpha}_2 + \hat{\alpha}_2^{\mathrm{T}} + \hat{h}\hat{\alpha}_5 + \hat{h}^{\mathrm{T}}\hat{\alpha}_5^{\mathrm{T}} - 2L_lW_l\bigg\}\tilde{r}_1 \\
&\quad + \tilde{r}_3^{\mathrm{T}}\big\{-\mathrm{e}^{-\alpha\hat{h}}Q_{4l}\big\}\tilde{r}_3 + \tilde{r}_1^{\mathrm{T}}\{-\hat{\alpha}_1+\hat{\alpha}_3\}\tilde{r}_4 - \tilde{r}_1^{\mathrm{T}}\{\hat{\alpha}_3\}\tilde{r}_5 + \tilde{r}_1^{\mathrm{T}}\{-\hat{\alpha}_2+\hat{\alpha}_4\}\tilde{r}_6 \\
&\quad - \tilde{r}_1^{\mathrm{T}}\{\hat{\alpha}_4\}\tilde{r}_7 + \tilde{r}_1^{\mathrm{T}}\big\{P_l - L_{1l} - W_l^{\mathrm{T}}L_{1l}^{\mathrm{T}}\big\}\tilde{r}_8 + \tilde{r}_1^{\mathrm{T}}\{L_{1l}A_l + H_2\Lambda_{1l}\}\tilde{r}_9 \\
&\quad + \tilde{r}_1^{\mathrm{T}}\{L_{1l}B_l\}\tilde{r}_{10} - \tilde{r}_1^{\mathrm{T}}\{\hat{\alpha}_5\}\tilde{r}_{11} - \tilde{r}_1^{\mathrm{T}}\{\hat{\alpha}_5\}\tilde{r}_{13} + \tilde{r}_1^{\mathrm{T}}\{L_{1l}C_l\}\tilde{r}_{14} \\
&\quad + \tilde{r}_1^{\mathrm{T}}\bigg\{T_l + \frac{\pi^2}{4}W_{1l}\bigg\}\tilde{r}_{16} - \tilde{r}_1^{\mathrm{T}}\{T_l\}\tilde{r}_{18} + \tilde{r}_2^{\mathrm{T}}\big\{-(1-\mu_1)\mathrm{e}^{-\alpha\hat{h}}Q_{1l} - H_1\Lambda_{21}\big\}\tilde{r}_2 \\
&\quad + r_2^{\mathrm{T}}\{H_2\Lambda_{2l}\}\tilde{r}_{10} - \tilde{r}_4^{\mathrm{T}}\{(1-\mu_1)\mathrm{e}^{-\alpha\hat{h}_1}Q_{2l}\}\tilde{r}_4 - \tilde{r}_5^{\mathrm{T}}\big\{\mathrm{e}^{-\alpha\hat{h}_1}Q_{5l}\big\}\tilde{r}_5
\end{aligned}$$

$$
\begin{aligned}
&- \tilde{r}_6^{\mathrm{T}}\big\{(1-\mu_2)\mathrm{e}^{-\alpha\hat{h}_2}Q_{3l}\big\}\tilde{r}_6 - \tilde{r}_7^{\mathrm{T}}\big\{\mathrm{e}^{-\alpha\hat{h}_2}Q_{6l}\big\}r_7 \\
&+ \tilde{r}_8^{\mathrm{T}}\bigg\{\hat{h}_1R_{1l} + \hat{h}_2R_{2l} + \hat{h}^2R_{3l} + \frac{\hat{h}^4}{4}S_{1l} + \bar{h}^2M_{2l} + \bar{h}^2W_l - 2L_1\bigg\}\tilde{r}_8 \\
&+ \tilde{r}_8^{\mathrm{T}}\{L_{1l}A_l\}\tilde{r}_9 + \tilde{r}_8^{\mathrm{T}}\{L_{1l}B_l\}\tilde{r}_{10} + \tilde{r}_8^{\mathrm{T}}\{L_{1l}C_l\}\tilde{r}_{14} + \tilde{r}_8^{\mathrm{T}}\{T_l\}\tilde{r}_{16} - \tilde{r}_8^{\mathrm{T}}\{T_l\}\tilde{r}_{18} \\
&+ \tilde{r}_9^{\mathrm{T}}\big\{Q_{7l} + \rho^2S_{2l} - \Lambda_{1l}\big\}\tilde{r}_9 + \tilde{r}_{10}^{\mathrm{T}}\big\{-(1-\mu_1)\mathrm{e}^{-\alpha\hat{h}}Q_{7l} - \Lambda_{2l}\big\}\tilde{r}_{10} \\
&- \tilde{r}_{11}^{\mathrm{T}}\big\{4\mathrm{e}^{-\alpha\hat{h}}R_{4l}\big\}\tilde{r}_{11} + \tilde{r}_{11}^{\mathrm{T}}\bigg\{\frac{6}{\hat{h}}R_{4l}\mathrm{e}^{-\alpha\hat{h}}\bigg\}\tilde{r}_{12} - \tilde{r}_{12}^{\mathrm{T}}\bigg\{\frac{12}{\hat{h}^2}R_{4l}\mathrm{e}^{-\alpha\hat{h}}\bigg\}\tilde{r}_{12} \\
&- \tilde{r}_{13}^{\mathrm{T}}\{S_{1l}\}re_{13} - \tilde{r}_{14}^{\mathrm{T}}\big\{2(1-\mu_3)\mathrm{e}^{-\alpha\hat{\rho}}S_{2l}\big\}\tilde{r}_{14} - \tilde{r}_{15}^{\mathrm{T}}\bigg\{\frac{12}{\hat{\rho}^2}(1-\mu_3)S_{2l}\mathrm{e}^{-\alpha\hat{\rho}}\bigg\}\tilde{r}_{15} \\
&+ \tilde{r}_{16}^{\mathrm{T}}\bigg\{-\frac{\pi^2}{4}W_{1l}\mathrm{e}^{-\alpha\bar{h}} + \kappa\bar{\phi}_l\bigg\}\tilde{r}_{16} - \tilde{r}_{16}^{\mathrm{T}}\{\kappa\bar{\phi}_l\}\tilde{r}_{18} + \tilde{r}_{18}^{\mathrm{T}}\{\kappa\bar{\phi}_l - \bar{\phi}_l\}\tilde{r}_{18} \\
&- (2-\alpha_1)\Gamma_1^{\mathrm{T}}\tilde{R}\Gamma_1 - (1+\alpha_1)\Gamma_2^{\mathrm{T}}\tilde{R}\Gamma_2 + \tilde{r}_{14}^{\mathrm{T}}\bigg\{\frac{6}{\hat{\rho}}(1-\mu_3)S_{2l}\mathrm{e}^{-\alpha\hat{\rho}}\bigg\}r_{15} \\
&- \mathrm{sym}\big\{\Gamma_1^{\mathrm{T}}\big(\alpha_1Y_1 + (1-\alpha_1)Y_2\big)\Gamma_2\big\} + (1-\alpha_1)\Gamma_1^{\mathrm{T}}Y_1\tilde{R}^{-1}Y_1^{\mathrm{T}}\Gamma_1 \\
&+ \alpha_1\Gamma_2^{\mathrm{T}}Y_2^{\mathrm{T}}\tilde{R}^{-1}Y_2\Gamma_2 - (2-\alpha_2)\Gamma_3^{\mathrm{T}}\tilde{M}\Gamma_3 - (1+\alpha_2)\Gamma_4^{\mathrm{T}}\tilde{M}\Gamma_4 \\
&- \mathrm{sym}\big\{\Gamma_3^{\mathrm{T}}\big(\alpha_2Y_3 + (1-\alpha_2)Y_4\big)\Gamma_3\big\} + (1-\alpha_2)\Gamma_3^{\mathrm{T}}Y_3\tilde{M}^{-1}Y_3^{\mathrm{T}}\Gamma_3 \\
&+ \alpha_2\Gamma_4^{\mathrm{T}}Y_4^{\mathrm{T}}\tilde{M}^{-1}Y_4\Gamma_4. \qquad (10)
\end{aligned}
$$

Then system (7) *is exponentially stable for any switching signal with average dwell time*

$$T_a > T_a^* = \ln\frac{\hat{\mu}}{\alpha}.$$

Moreover, the estimator of state decay is given by

$$\big\|x(t)\big\| < \sqrt{\frac{\tilde{m}}{\tilde{n}}}\mathrm{e}^{-\eta(t-t_0)}\big\|x(t_0)\big\|,$$

where for all $l, j \in M$, $\hat{\mu} \geqslant 1$ *satisfies*

$$
\begin{array}{llll}
P_l \leqslant \hat{\mu}P_j, & Q_{1l} \leqslant \hat{\mu}Q_{1j}, & Q_{2l} \leqslant \hat{\mu}Q_{2j}, & Q_{3l} \leqslant \hat{\mu}Q_{3j}, \\
Q_{4l} \leqslant \hat{\mu}Q_{4j}, & Q_{5l} \leqslant \hat{\mu}Q_{5j}, & Q_{6l} \leqslant \hat{\mu}Q_{6j}, & Q_{7l} \leqslant \hat{\mu}Q_{7j}, \\
R_{1l} \leqslant \hat{\mu}R_{1j}, & R_{2l} \leqslant \hat{\mu}R_{2j}, & R_{3l} \leqslant \hat{\mu}R_{3j}, & R_{4l} \leqslant \hat{\mu}R_{4j}, \\
S_{1l} \leqslant \hat{\mu}S_{1j}, & S_{2l} \leqslant \hat{\mu}S_{2j}, & M_{1l} \leqslant \hat{\mu}M_{1j}, & M_{2l} \leqslant \hat{\mu}M_{2j}, \\
W_l \leqslant \hat{\mu}W_j, & & &
\end{array} \qquad (11)
$$

$$\eta = \frac{1}{2}\frac{\alpha - \ln\hat{\mu}}{T_a}, \qquad \tilde{n} = \min_{l\in M}\lambda_{\min}P_l,$$

$$\begin{aligned}
\tilde{m} = {} & \max_{l\in M}\lambda_{\max}P_l + \hat{h}\max_{l\in M}\lambda_{\max}Q_{1l} + \hat{h}_1\max_{l\in M}\lambda_{\max}Q_{2l} + \hat{h}_2\max_{l\in M}\lambda_{\max}Q_{3l} \\
& + \hat{h}\max_{l\in M}\lambda_{\max}Q_{4l} + \hat{h}_1\max_{l\in M}\lambda_{\max}Q_{5l} + \hat{h}_2\max_{l\in M}\lambda_{\max}Q_{6l} + \hat{h}\max_{l\in M}\lambda_{\max}Q_{7l} \\
& + \hat{h}_1^2\max_{l\in M}\lambda_{\max}R_{1l} + \hat{h}_2^2\max_{l\in M}\lambda_{\max}R_{2l} + \frac{\hat{h}^3}{2}\max_{l\in M}\lambda_{\max}R_{3l} + \frac{\hat{h}^3}{2}\max_{l\in M}\lambda_{\max}R_{4l} \\
& + \frac{\hat{h}^6}{6}\max_{l\in M}\lambda_{\max}S_{1l} + \frac{\hat{\rho}^3}{2}\max_{l\in M}\lambda_{\max}S_{2l} + \bar{h}\max_{l\in M}\lambda_{\max}M_{1l} + \frac{\bar{h}^3}{2}\max_{l\in M}\lambda_{\max}M_{2l} \\
& + \bar{h}^3\max_{l\in M}\lambda_{\max}W_l - \bar{h}\max_{l\in M}\lambda_{\max}W_l,
\end{aligned}$$

and the controller gain is $K_l = L_{1l}^{-1}T_l$.

Proof. Choose the LKF for system (7) as

$$V\big(r(t)\big) = \sum_{\iota=1}^{8} V_\iota\big(r(t)\big), \tag{12}$$

where

$$\begin{aligned}
V_1\big(r(t)\big) = {} & r^{\mathrm{T}}(t)P_l r(t), \\
V_2\big(r(t)\big) = {} & \int_{t-\hat{h}(t)}^{t} \mathrm{e}^{\alpha(s-t)} r^{\mathrm{T}}(s)Q_{1l}r(s)\,\mathrm{d}s + \int_{t-\hat{h}_1(t)}^{t} \mathrm{e}^{\alpha(s-t)} r^{\mathrm{T}}(s)Q_{2l}r(s)\,\mathrm{d}s \\
& + \int_{t-\hat{h}_2(t)}^{t} \mathrm{e}^{\alpha(s-t)} r^{\mathrm{T}}(s)Q_{3l}r(s)\,\mathrm{d}s + \int_{t-\hat{h}}^{t} \mathrm{e}^{\alpha(s-t)} r^{\mathrm{T}}(s)Q_{4l}r(s)\,\mathrm{d}s \\
& + \int_{t-\hat{h}_1}^{t} \mathrm{e}^{\alpha(s-t)} r^{\mathrm{T}}(s)Q_{5l}r(s)\,\mathrm{d}s + \int_{t-\hat{h}_2}^{t} \mathrm{e}^{\alpha(s-t)} r^{\mathrm{T}}(s)Q_{6l}r(s)\,\mathrm{d}s \\
& + \int_{t-\hat{h}(t)}^{t} \mathrm{e}^{\alpha(s-t)} g^{\mathrm{T}}(r(s))Q_{7l}g\big(r(s)\big)\,\mathrm{d}s \\
V_3\big(r(t)\big) = {} & \int_{-\hat{h}_1}^{0}\int_{t+\beta}^{t} \mathrm{e}^{\alpha(s-t)}\dot{r}^{\mathrm{T}}(s)R_{1l}\dot{r}(s)\,\mathrm{d}s\,\mathrm{d}\beta + \int_{-\hat{h}_2}^{0}\int_{t+\beta}^{t} \mathrm{e}^{\alpha(s-t)}\dot{r}^{\mathrm{T}}(s)R_{2l}\dot{r}(s)\,\mathrm{d}s\,\mathrm{d}\beta, \\
V_4\big(r(t)\big) = {} & \hat{h}\int_{-\hat{h}}^{0}\int_{t+\beta}^{t} \mathrm{e}^{\alpha(s-t)}\dot{r}^{\mathrm{T}}(s)R_{3l}\dot{r}(s)\,\mathrm{d}s\,\mathrm{d}\beta + \hat{h}\int_{-\hat{h}}^{0}\int_{t+\beta}^{t} \mathrm{e}^{\alpha(s-t)} r^{\mathrm{T}}(s)R_{4l}r(s)\,\mathrm{d}s\,\mathrm{d}\beta, \\
V_5\big(r(t)\big) = {} & \frac{\hat{h}^2}{2}\int_{-\hat{h}}^{0}\int_{\beta}^{0}\int_{t+\lambda}^{t} \mathrm{e}^{\alpha(s-t)}\dot{r}^{\mathrm{T}}(s)S_{1l}\dot{r}(s)\,\mathrm{d}s\,\mathrm{d}\beta\,\mathrm{d}\lambda,
\end{aligned}$$

$$V_6\big(r(t)\big) = \hat{\rho}(t) \int_{-\hat{\rho}(t)}^{0} \int_{t+\beta}^{t} \mathrm{e}^{\alpha(s-t)} g^{\mathrm{T}}(r(s)) S_{2l} g\big(r(s)\big)\,\mathrm{d}s\,\mathrm{d}\beta,$$

$$V_7\big(r(t)\big) = \int_{t-\varsigma_1(t)}^{t} \mathrm{e}^{\alpha(s-t)} r^{\mathrm{T}}(s) M_{1l} r(s)\,\mathrm{d}s + \bar{h} \int_{-\bar{h}}^{0} \int_{t+\beta}^{t} \mathrm{e}^{\alpha(s-t)} \dot{r}^{\mathrm{T}}(s) M_{2l} \dot{r}(s)\,\mathrm{d}s\,\mathrm{d}\beta,$$

and $V_8(r(t))$ is the discontinuous functional of the form

$$\begin{aligned} V_8\big(r(t)\big) = {}& \bar{h}^2 \int_{t-\varsigma_1(t)}^{t} \mathrm{e}^{\alpha(s-t)} \dot{r}^{\mathrm{T}}(s) W_{1l} \dot{r}(s)\,\mathrm{d}s \\ & - \frac{\pi^2}{4} \int_{t-\varsigma_1(t)}^{t} \mathrm{e}^{\alpha(s-t)} \big(r(s)-r\big(t-\varsigma_1(t)\big)\big)^{\mathrm{T}} W_{1l} \big(r(s)-r\big(t-\varsigma_1(t)\big)\big)\,\mathrm{d}s. \end{aligned} \tag{13}$$

The time derivative of $\sum_{\iota=1}^{8} V_\iota(r(t))$ along the trajectory of system (7) is given by:

$$\dot{V}_1\big(r(t)\big) = 2r^{\mathrm{T}}(t) P_l \dot{r}(t),$$

$$\begin{aligned} \dot{V}_2\big(r(t)\big) \leqslant {}& r^{\mathrm{T}}(t) Q_{1l} r(t) - (1-\mu)\mathrm{e}^{-\alpha\hat{h}} r^{\mathrm{T}}\big(t-\hat{h}(t)\big) Q_{1l} r\big(t-\hat{h}(t)\big) + r^{\mathrm{T}}(t) Q_{2l} r(t) \\ & - (1-\mu_1)\mathrm{e}^{-\alpha\hat{h}_1} r^{\mathrm{T}}\big(t-\hat{h}_1(t)\big) Q_{2l} r\big(t-\hat{h}_1(t)\big) + r^{\mathrm{T}}(t) Q_{3l} r(t) \\ & - (1-\mu_2)\mathrm{e}^{-\alpha\hat{h}_2} r^{\mathrm{T}}\big(t-\hat{h}_2(t)\big) Q_{3l} r\big(t-\hat{h}_2(t)\big) + r^{\mathrm{T}}(t) Q_{4l} r(t) \\ & - (1-\mu)\mathrm{e}^{-\alpha\hat{h}} r^{\mathrm{T}}(t-\hat{h}) Q_{4l} r(t-\hat{h}) + r^{\mathrm{T}}(t) Q_{5l} r(t) \\ & - \mathrm{e}^{-\alpha\hat{h}_1} r^{\mathrm{T}}(t-\hat{h}_1) Q_{5l} r(t-\hat{h}_1) + r^{\mathrm{T}}(t) Q_{6l} r(t) \\ & - \mathrm{e}^{-\alpha\hat{h}_2} r^{\mathrm{T}}(t-\hat{h}_2) Q_{6l} r(t-\hat{h}_2) - \alpha V_2(t) + g^{\mathrm{T}}\big(r(t)\big) Q_{7l} g\big(r(t)\big) \\ & - (1-\mu)\mathrm{e}^{-\alpha\hat{h}} g^{\mathrm{T}}\big(r\big(t-\hat{h}(t)\big)\big) Q_{7l} g\big(r\big(t-\hat{h}(t)\big)\big), \end{aligned}$$

$$\begin{aligned} \dot{V}_3\big(r(t)\big) = {}& \hat{h}_1 \dot{r}^{\mathrm{T}}(t) R_{1l} \dot{r}(t) - \int_{t-\hat{h}_1}^{t} \mathrm{e}^{-\alpha\hat{h}_1} \dot{r}^{\mathrm{T}}(s) R_{1l} \dot{r}(s)\,\mathrm{d}s + \hat{h}_2 \dot{r}^{\mathrm{T}}(t) R_{2l} \dot{r}(t) \\ & - \int_{t-\hat{h}_2}^{t} \mathrm{e}^{-\alpha\hat{h}_2} \dot{r}^{\mathrm{T}}(s) R_{2l} \dot{r}(s)\,\mathrm{d}s - \alpha V_3(t), \end{aligned} \tag{14}$$

$$\begin{aligned} \dot{V}_4\big(r(t)\big) = {}& \hat{h}^2 \dot{r}^{\mathrm{T}}(t) R_{3l} \dot{r}(t) - \hat{h} \int_{t-\hat{h}}^{t} \mathrm{e}^{-\alpha\hat{h}} \dot{r}^{\mathrm{T}}(s) R_{3l} \dot{r}(s)\,\mathrm{d}s + \hat{h}^2 r^{\mathrm{T}}(t) R_{4l} r(t) \\ & - \hat{h} \int_{t-\hat{h}}^{t} \mathrm{e}^{-\alpha\hat{h}} r^{\mathrm{T}}(s) R_{4l} r(s)\,\mathrm{d}s - \alpha V_4(t), \end{aligned} \tag{15}$$

$$\dot{V}_5\big(r(t)\big) = \frac{\hat{h}^4}{4}\dot{r}^{\mathrm{T}}(t)S_{1l}\dot{r}(t) - \left(\int_{-\hat{h}}^{0}\int_{t+\beta}^{t}\dot{r}(s)\,\mathrm{d}s\,\mathrm{d}\beta\right)^{\mathrm{T}} S_{1l}\left(\int_{-\hat{h}}^{0}\int_{t+\beta}^{t}\dot{r}(s)\,\mathrm{d}s\,\mathrm{d}\beta\right) - \alpha V_5(t), \tag{16}$$

$$\dot{V}_6\big(r(t)\big) \leqslant \rho^2 g^{\mathrm{T}}\big(r(t)\big)S_{2l}g\big(r(t)\big) - (1-\mu_3)\hat{\rho}(t)\int_{t-\hat{\rho}(t)}^{t}\mathrm{e}^{-\alpha\hat{\rho}}g^{\mathrm{T}}(r(s))S_{2l}g\big(r(s)\big)\,\mathrm{d}s - \alpha V_6(t), \tag{17}$$

$$\dot{V}_7\big(r(t)\big) = r^{\mathrm{T}}(t)M_{1l}r(t) + \bar{h}^2\dot{r}^{\mathrm{T}}(t)M_{2l}\dot{r}(t) - \bar{h}\int_{t-\bar{h}}^{t}\mathrm{e}^{-\alpha\bar{h}}\dot{r}^{\mathrm{T}}(s)M_{2l}\dot{r}(s)\,\mathrm{d}s - \alpha V_7(t), \tag{18}$$

$$\dot{V}_8\big(r(t)\big) = \bar{h}^2\dot{r}^{\mathrm{T}}(t)W_{1l}\dot{r}(t) - \frac{\pi^2}{4}\big[\big(r(t) - r\big(t-\varsigma_1(t)\big)\big)^{\mathrm{T}}W_{1\beta(t)}\big(r(t) - r\big(t-\varsigma_1(t)\big)\big)\big] - \alpha V_8(t).$$

Using Lemmas 1–3 in (15), (17) and (18), we have

$$\begin{aligned}
-\hat{h}\int_{t-\hat{h}}^{t}\mathrm{e}^{-\alpha\hat{h}}r^{\mathrm{T}}(s)R_{4l}r(s)\,\mathrm{d}s \leqslant \mathrm{e}^{-\alpha\hat{h}}\Bigg[&-\left(\int_{t-\hat{h}}^{t}r^{\mathrm{T}}(s)\,\mathrm{d}s\right)R_{4l}\left(\int_{t-\hat{h}}^{t}r(s)\,\mathrm{d}s\right)\\
&- 3\left[-\int_{t-\hat{h}}^{t}r^{\mathrm{T}}(s)\,\mathrm{d}s + \frac{2}{\hat{h}}\int_{t-\hat{h}}^{t}\int_{\beta}^{t}r^{\mathrm{T}}(s)\,\mathrm{d}s\,\mathrm{d}\beta\right]\\
&\times R_{4l}\left[-\int_{t-\hat{h}}^{t}r(s)\,\mathrm{d}s + \frac{2}{\hat{h}}\int_{t-\hat{h}}^{t}\int_{\beta}^{t}r(s)\,\mathrm{d}s\,\mathrm{d}\beta\right]\Bigg],
\end{aligned}$$

Also,

$$\begin{aligned}
&-\hat{h}\int_{t-\hat{h}}^{t-\hat{h}(t)}\mathrm{e}^{-\alpha\hat{h}}\dot{r}^{\mathrm{T}}(s)R_{3l}\dot{r}(s)\,\mathrm{d}s - \hat{h}\int_{t-\hat{h}(t)}^{t}\mathrm{e}^{-\alpha\hat{h}}\dot{r}^{\mathrm{T}}(s)R_{3l}\dot{r}(s)\,\mathrm{d}s\\
&\quad\leqslant \varpi_2^{\mathrm{T}}(t)\mathrm{e}^{-\alpha\bar{h}}\big(-(2-\alpha_2)\Gamma_2^{\mathrm{T}}\tilde{M}\Gamma_2 - (1+\alpha_2)\Gamma_3^{\mathrm{T}}\tilde{M}\Gamma_3\\
&\qquad - \mathrm{sym}\big\{\Gamma_2^{\mathrm{T}}\big(\alpha_2 Y_2 + (1-\alpha_2)Y_2\big)\Gamma_3\big\}\\
&\qquad + (1-\alpha_2)\Gamma_2^{\mathrm{T}}Y_1\tilde{M}^{-1}Y_2^{\mathrm{T}}\Gamma_2 + \alpha_2\Gamma_3^{\mathrm{T}}Y_3^{\mathrm{T}}\tilde{M}^{-1}Y_3\Gamma_3\big)\varpi_2(t).
\end{aligned}$$

From the Newton–Leibniz formula, the following equations are true for any matrices $\hat{\alpha}_1$, $\hat{\alpha}_3$, $\hat{\alpha}_2$, $\hat{\alpha}_4$ and $\hat{\alpha}_5$ with appropriate dimensions:

$$2\eta^{\mathrm{T}}(t)\mathcal{Z}\hat{\alpha}_1\left[r(t) - r\big(t - \hat{h}_1(t)\big) - \hat{h}_1(t)\frac{1}{\hat{h}_1(t)}\int\limits_{t-\hat{h}_1(t)}^{t} \dot{r}(s)\,\mathrm{d}s\right] = 0,$$

$$2\eta^{\mathrm{T}}(t)\mathcal{Z}\hat{\alpha}_3\left[r\big(t - \hat{h}_1(t)\big) - r(t - \hat{h}_1) - \big(\hat{h}_1 - \hat{h}_1(t)\big)\frac{1}{\hat{h}_1 - \hat{h}_1(t)}\int\limits_{t-\hat{h}_1}^{t-\hat{h}_1(t)} \dot{r}(s)\,\mathrm{d}s\right] = 0,$$

$$2\eta^{\mathrm{T}}(t)\mathcal{Z}\hat{\alpha}_2\left[r(t) - r\big(t - \hat{h}_2(t)\big) - \hat{h}_2(t)\frac{1}{\hat{h}_2(t)}\int\limits_{t-\hat{h}_2(t)}^{t} \dot{r}(s)\,\mathrm{d}s\right] = 0,$$

$$2\eta^{\mathrm{T}}(t)\mathcal{Z}\hat{\alpha}_4\left[r\big(t - \hat{h}_2(t)\big) - r(t - \hat{h}_2) - \big(\hat{h}_2 - \hat{h}_2(t)\big)\frac{1}{\hat{h}_2 - \hat{h}_2(t)}\int\limits_{t-\hat{h}_2}^{t-\hat{h}_2(t)} \dot{r}(s)\,\mathrm{d}s\right] = 0,$$

$$2\eta^{\mathrm{T}}(t)\mathcal{Z}\hat{\alpha}_5\left[\hat{h}r(t) - \int\limits_{t-\hat{h}}^{t} r(s)\,\mathrm{d}s - \int\limits_{-\hat{h}}^{0}\int\limits_{t+\theta}^{t} \dot{r}(s)\,\mathrm{d}s\,\mathrm{d}\theta\right] = 0.$$

Also,

$$\eta^{\mathrm{T}}(t) = \big[\xi_1^{\mathrm{T}}(t),\ \xi_3^{\mathrm{T}}(t),\ \xi_4^{\mathrm{T}}(t),\ \xi_2^{\mathrm{T}}(t)\big] \quad \text{and} \quad \mathcal{Z} = \mathrm{col}\big[\overbrace{1,\ 1,\ 1}^{18\text{ times}},\ \overbrace{0,\ 0,\ 0}^{20\text{ times}}\big].$$

On the other hand, we have the following equality for any real matrix L_{1l}:

$$\begin{aligned} &2\big[r^{\mathrm{T}}(t)L_{1l} + \dot{r}^{\mathrm{T}}(t)L_{1l}\big]\Bigg[-\dot{r}(t) - W_l r(t) + A_l g\big(r(t)\big) + B_l g\Big(r\big(t - \hat{h}_1(t) - \hat{h}_2(t)\big)\Big) \\ &\quad + C_l\int\limits_{t-\hat{\rho}(t)}^{t} g\big(r(s)\big)\,\mathrm{d}s + K_l\Big(r\big(t - \varsigma_1(t)\big) - \omega\big(t - \varsigma_1(t)\big)\Big)\Bigg] = 0. \end{aligned} \tag{19}$$

Combining and adding (12)–(19), we obtain

$$\dot{V}\big(r(t)\big) \leqslant \eta^{\mathrm{T}}(t)\bar{\Theta}\eta(t),$$

where

$$\bar{\Theta} = \begin{bmatrix} \hat{\Pi}_\alpha & -\hat{h}_1(t)\hat{\alpha}_1\mathrm{e}^{-\alpha\hat{h}_1} & -(\hat{h}_1 - \hat{h}_1(t))\hat{\alpha}_3\mathrm{e}^{-\alpha\hat{h}_1} & a_1 & a_2 \\ * & -\hat{h}_1(t)R_{1l}\mathrm{e}^{-\alpha\hat{h}_1} & 0 & 0 & 0 \\ * & * & -(\hat{h}_1 - \hat{h}_1(t))R_{1l}\mathrm{e}^{-\alpha\hat{h}_1} & 0 & 0 \\ * & * & * & b_1 & 0 \\ * & * & * & * & b_2 \end{bmatrix} < 0,$$

$$a_1 = -\hat{h}_2(t)\hat{\alpha}_2 \mathrm{e}^{-\alpha\hat{h}_2}, \qquad a_2 = -\big(\hat{h}_2 - \hat{h}_2(t)\big)\hat{\alpha}_4 \mathrm{e}^{-\alpha\hat{h}_2},$$
$$b_1 = -\hat{h}_2(t)R_{2l} \mathrm{e}^{-\alpha\hat{h}_2}, \qquad b_2 = -\big(\hat{h}_2 - \hat{h}_2(t)\big)R_{2l} \mathrm{e}^{-\alpha\hat{h}_2}.$$

There exists a scalar $\epsilon > 0$, such that

$$\dot{V}\big(r(t)\big) \leqslant -\epsilon\eta^{\mathrm{T}}(t)\eta(t) \leqslant -\epsilon r^{\mathrm{T}}(t)r(t) < 0 \quad \forall r(t) \neq 0. \tag{20}$$

Then the following linear matrix inequality leads for $(\hat{h}_1(t) \to \hat{h}_1)$ and for $\hat{h}_1(t) \to 0$:

$$\bar{\Theta}_1 = \begin{bmatrix} \bar{\Sigma}_{11} & \bar{\Sigma}_{12} \\ * & \bar{\Sigma}_{22} \end{bmatrix} < 0, \tag{21}$$

where

$$\bar{\Sigma}_{11} = \begin{bmatrix} \hat{\Pi}_\alpha & -\hat{h}_1\hat{\alpha}_1 \mathrm{e}^{-\alpha\hat{h}_1} \\ * & -\hat{h}_1 R_{1l} \mathrm{e}^{-\alpha\hat{h}_1} \end{bmatrix}, \qquad \bar{\Sigma}_{12} = \begin{bmatrix} -\hat{h}_2(t)\hat{\alpha}_2 \mathrm{e}^{-\alpha\hat{h}_2} & -(\hat{h}_2 - \hat{h}_2(t))\hat{\alpha}_4 \mathrm{e}^{-\alpha\hat{h}_2} \\ 0 & 0 \end{bmatrix},$$
$$\bar{\Sigma}_{22} = \begin{bmatrix} -\hat{h}_2(t)R_{2l} \mathrm{e}^{-\alpha\hat{h}_2} & 0 \\ * & -(\hat{h}_2 - \hat{h}_2(t))R_{2l} \mathrm{e}^{-\alpha\hat{h}_2} \end{bmatrix}.$$

$$\bar{\Theta}_2 = \begin{bmatrix} \bar{\Xi}_{11} & \bar{\Xi}_{12} \\ * & \bar{\Xi}_{22} \end{bmatrix} < 0, \tag{22}$$

where

$$\bar{\Xi}_{11} = \begin{bmatrix} \hat{\Pi}_\alpha & -\hat{h}_1\hat{\alpha}_3 \mathrm{e}^{-\alpha\hat{h}_1} \\ * & -\hat{h}_1 R_{1l} \mathrm{e}^{-\alpha\hat{h}_1} \end{bmatrix}, \qquad \bar{\Xi}_{12} = \bar{\Sigma}_{12}, \qquad \bar{\Xi}_{22} = \bar{\Sigma}_{22}.$$

It is easy to see that $\bar{\Theta}_2$ results from $\bar{\Theta}|_{\hat{h}_1(t)=\hat{h}_1}$. Also, for our convenience we neglected the zero row and zero column. The LMIs (21) and (22) imply (20) because

$$\frac{\hat{h}_1(t)}{\hat{h}_1}\eta_1^{\mathrm{T}}(t)\bar{\Theta}_1\eta_1(t) + \frac{\hat{h}_1 - \hat{h}_1(t)}{\hat{h}_1}\eta_2^{\mathrm{T}}(t)\bar{\Theta}_2\eta_2(t) = \eta^{\mathrm{T}}(t)\bar{\Theta}\eta(t) < 0.$$

Define

$$\eta_1^{\mathrm{T}}(t) = \Bigg[\xi_1(t), \ \frac{1}{\hat{h}_1(t)}\int_{t-\hat{h}_1(t)}^{t} \dot{r}^{\mathrm{T}}(s)\,\mathrm{d}s, \ \frac{1}{\hat{h}_2(t)}\int_{t-\hat{h}_2(t)}^{t} \dot{r}^{\mathrm{T}}(s)\,\mathrm{d}s$$
$$\frac{1}{\hat{h}_2 - \hat{h}_2(t)}\int_{t-\hat{h}_2}^{t-\hat{h}_2(t)} \dot{r}^{\mathrm{T}}(s)\,\mathrm{d}s\Bigg],$$

$$\eta_2^{\mathrm{T}}(t) = \left[\xi_1(t),\ \frac{1}{\hat{h}_1 - \hat{h}_1(t)} \int\limits_{t-\hat{h}_1}^{t-\hat{h}_1(t)} \dot{r}^{\mathrm{T}}(s)\,\mathrm{d}s,\ \frac{1}{\hat{h}_2(t)} \int\limits_{t-\hat{h}_2(t)}^{t} \dot{r}^{\mathrm{T}}(s)\,\mathrm{d}s\right.$$
$$\left.\frac{1}{\hat{h}_2 - \hat{h}_2(t)} \int\limits_{t-\hat{h}_2}^{t-\hat{h}_2(t)} \dot{r}^{\mathrm{T}}(s)\,\mathrm{d}s\right],$$

and $\bar{\Theta}$ is convex in $\hat{h}_1(t) \in [0, h_1]$. In $\bar{\Theta}_1$, for $\hat{h}_2(t) \to \hat{h}_2$ and $\hat{h}_2(t) \to 0$, gives (9). Similarly, for $\hat{h}_2(t) \to \hat{h}_2$ and $\hat{h}_2(t) \to 0$ in $(\bar{\Theta}_2)$, gives the LMI (16). Also $\hat{\Pi}_1$ and $\hat{\Pi}_2$ can be get from $(\bar{\Theta}_1)$ because

$$\frac{\hat{h}_2(t)}{\hat{h}_2}\eta_3^{\mathrm{T}}(t)\hat{\Pi}_1\eta_3(t) + \frac{\hat{h}_2 - \hat{h}_2(t)}{\hat{h}_2}\eta_4^{\mathrm{T}}(t)\hat{\Pi}_2\eta_4(t) = \eta_1^{\mathrm{T}}(t)\bar{\Theta}_1\eta(t)_1(t) < 0,$$

and $\bar{\Theta}_1$ is convex in $\hat{h}_2(t) \in [0, \hat{h}_2]$.

Define

$$\eta_3^{\mathrm{T}}(t) = \left[\xi_1(t),\ \frac{1}{\hat{h}_1(t)} \int\limits_{t-\hat{h}_1(t)}^{t} \dot{r}^{\mathrm{T}}(s)\,\mathrm{d}s,\ \frac{1}{\hat{h}_2(t)} \int\limits_{t-\hat{h}_2(t)}^{t} \dot{r}^{\mathrm{T}}(s)\,\mathrm{d}s\right],$$

$$\eta_4^{\mathrm{T}}(t) = \left[\xi_1(t),\ \frac{1}{\hat{h}_1(t)} \int\limits_{t-\hat{h}_1(t)}^{t} \dot{r}^{\mathrm{T}}(s)\,\mathrm{d}s,\ \frac{1}{\hat{h}_2 - \hat{h}_2(t)} \int\limits_{t-\hat{h}_2}^{t-\hat{h}_2(t)} \dot{r}^{\mathrm{T}}(s)\,\mathrm{d}s\right],$$

where $\hat{\Pi}_1$ and $\hat{\Pi}_2$ are defined in (16). Similarly, $\hat{\Pi}_3$ and $\hat{\Pi}_4$ can get from $(\bar{\Theta}_2)$ because

$$\frac{\hat{h}_2(t)}{\hat{h}_2}\eta_5^{\mathrm{T}}(t)\hat{\Pi}_3\eta_5(t) + \frac{\hat{h}_2 - \hat{h}_2(t)}{\hat{h}_2}\eta_6^{\mathrm{T}}(t)\hat{\Pi}_4\eta_6(t) = \eta_2^{\mathrm{T}}(t)\bar{\Theta}_2\eta(t)_2(t) < 0,$$

and $\bar{\Theta}_2$ is convex in $\hat{h}_2(t) \in [0, \hat{h}_2]$. Define

$$\eta_5^{\mathrm{T}}(t) = \left[\xi_1(t),\ \frac{1}{\hat{h}_1 - \hat{h}_1(t)} \int\limits_{t-\hat{h}_1}^{t-\hat{h}_1(t)} \dot{r}^{\mathrm{T}}(s)\,\mathrm{d}s,\ \frac{1}{\hat{h}_2(t)} \int\limits_{t-\hat{h}_2(t)}^{t} \dot{r}^{\mathrm{T}}(s)\,\mathrm{d}s\right],$$

$$\eta_6^{\mathrm{T}}(t) = \left[\xi_1(t),\ \frac{1}{\hat{h}_1 - \hat{h}_1(t)} \int\limits_{t-\hat{h}_1}^{t-\hat{h}_1(t)} \dot{r}^{\mathrm{T}}(s)\,\mathrm{d}s,\ \frac{1}{\hat{h}_2 - \hat{h}_2(t)} \int\limits_{t-\hat{h}_2}^{t-\hat{h}_2(t)} \dot{r}^{\mathrm{T}}(s)\,\mathrm{d}s\right],$$

where $\hat{\Pi}_3$, $\hat{\Pi}_4$ are defined in (8).

Furthermore, when $t \in [t_k, t_{k+1})$, (8) result in

$$V(t) = V_{\beta(t)} \leqslant \mathrm{e}^{-\alpha(t-t_k)} V_{\beta(t_k)}(t_k). \tag{23}$$

Using (11), at the switching instant t_l, we have

$$V(t) = V_{\beta(t_l)}(t_l) \leqslant \hat{\mu} V_{\beta(t_l^-)}(t_l^-), \quad l = 1, 2, \dots . \tag{24}$$

Together with (23) and (24), with the relation $k = N_\beta(t, t_0) \leqslant (t - t_0)/T_a$,

$$\begin{aligned} V(t) &\leqslant \mathrm{e}^{-\alpha(t-t_k)} \hat{\mu} V_{\beta(t_k^-)}(t_k^-) \leqslant \dots \leqslant \mathrm{e}^{-\alpha(t-t_0)} \hat{\mu} V_{\beta(t_0^-)}(t_0^-) \\ &\leqslant \mathrm{e}^{\alpha - \ln(\hat{\mu}/T_a)}(t - t_0) V_{\beta(t_0)}(t_0). \end{aligned}$$

Moreover, for any $t \in [0, h]$, from the definition of LKFs in (12), the following inequalities hold:

$$\begin{aligned} \lambda_{\min} P_l \|r(t)\|^2 &\leqslant r^{\mathrm{T}}(t) P_l r(t) \leqslant V(t) \leqslant \mathrm{e}^{-\alpha - \ln(\hat{\mu}/T_a)}(t - t_0) V_{\beta(t_0)}(t_0) \\ &\leqslant \bar{m} \|r(t_0)\|^2. \end{aligned}$$

Therefore, we have

$$\|r(t)\|^2 \leqslant \frac{1}{\bar{n}} V(t) \leqslant \frac{\bar{m}}{\bar{n}} \mathrm{e}^{-(\alpha - \ln(\hat{\mu}/T_a)}(t - t_0) \|r(t_0)\|^2.$$

This implies

$$\|r(t)\| \leqslant \sqrt{\frac{\bar{m}}{\bar{n}}} \mathrm{e}^{-(\alpha - \ln(\hat{\mu}/T_a))/2}(t - t_0) \|r(t_0)\|.$$

Therefore, from Definition 1, the synchronization error system (7) is exponentially stable. □

Based on Theorem 1, we presents the exponential stability for uncertain switched neural networks using decentralized event-triggered scheme.

Theorem 2. *For given scalars $\hat{h}_u (u = 1, 2)$, ρ, $\hat{h}$, μ, $\bar{h}$, μ_j ($j = 1, 2, 3$), the equilibrium point of system* (6) *is exponentially stable if there exist symmetric positive-definite matrices P_l, Q_{il} ($i = 1, 2, \dots, 7$), R_{jl} ($j = 1, 2, 3, 4$), S_{1l}, S_{2l}, M_{1l}, M_{2l}, positive diagonal matrices $\Lambda_{fl} > 0$ ($f = 1, 2$), scalars $\epsilon > 0$, the matrix $\bar{\phi}_l > 0$ and any matrices $\hat{\alpha}_1$, $\hat{\alpha}_2$, $\hat{\alpha}_3$, $\hat{\alpha}_4$, $\hat{\alpha}_5$, L_{1l}, T_l, U_{sl} ($s = 1, 2$) with appropriate dimensions and $Y_1, Y_2, Y_3, Y_4 \in \mathbb{R}^{4n \times 4n}$, such that the following linear matrix inequalities hold*:

$$\hat{\Pi}_{\hat{S}} = \begin{bmatrix} \hat{\Pi}_\alpha & -\hat{h}_1 \hat{\alpha}_{\hat{I}} \mathrm{e}^{-\alpha \hat{h}_1} & -\hat{h}_2 \hat{\alpha}_{\hat{L}} \mathrm{e}^{-\alpha \hat{h}_2} & \Upsilon \\ * & -\hat{h}_1 R_1 \mathrm{e}^{-\alpha \hat{h}_1} & 0 & 0 \\ * & * & -\hat{h}_2 R_2 \mathrm{e}^{-\alpha \hat{h}_2} & 0 \\ * & * & * & -\epsilon I \end{bmatrix} \leqslant 0, \tag{25}$$

where

$$\hat{\Pi}_\alpha = \eta^{\mathrm{T}}(t) \hat{\Pi} \eta(t), \qquad \Upsilon = \mathrm{col}\big[L_1 G_l, \overbrace{0, 0, 0}^{6 \text{ times}}, L_1 G_l, \overbrace{0, 0, 0}^{26 \text{ times}}\big],$$
$$\hat{S} = 1, \dots, 4, \qquad \hat{I} = 1, 3, \qquad \hat{J} = 2, 4,$$

and

$$
\begin{aligned}
\hat{\varPi} = {} & \tilde{r}_1^{\mathrm{T}}\bigg\{\alpha P_l + Q_{1l} + Q_{2l} + Q_{3l} + Q_{4l} + Q_{5l} + Q_{6l} + \hat{h}^2 R_{4l} + M_{1l} - \frac{\pi^2}{4} W_{1l}\mathrm{e}^{-\alpha\bar{h}} \\
& - H_1 \varLambda_{11} + \hat{\alpha}_1 + \hat{\alpha}_1^{\mathrm{T}} + \hat{\alpha}_2 + \hat{\alpha}_2^{\mathrm{T}} + \hat{h}\hat{\alpha}_5 + \hat{h}^{\mathrm{T}}\hat{\alpha}_5^{\mathrm{T}} - 2L_l W_l\bigg\}\tilde{r}_1 \\
& + \tilde{r}_3^{\mathrm{T}}\big\{-\mathrm{e}^{-\alpha\hat{h}} Q_{4l}\big\}\tilde{r}_3 + \tilde{r}_1^{\mathrm{T}}\{-\hat{\alpha}_1 + \hat{\alpha}_3\}\tilde{r}_4 - \tilde{r}_1^{\mathrm{T}}\{\hat{\alpha}_3\}\tilde{r}_5 + \tilde{r}_1^{\mathrm{T}}\{-\hat{\alpha}_2 + \hat{\alpha}_4\}\tilde{r}_6 \\
& - \tilde{r}_1^{\mathrm{T}}\{\hat{\alpha}_4\}\tilde{r}_7 + \tilde{r}_1^{\mathrm{T}}\{P_l - L_{1l} - W_l^{\mathrm{T}} L_{1l}^{\mathrm{T}}\}\tilde{r}_8 + \tilde{r}_1^{\mathrm{T}}\{L_{1l}A_l + H_2\varLambda_{1l}\}\tilde{r}_9 \\
& + \tilde{r}_1^{\mathrm{T}}\{L_{1l}B_l\}\tilde{r}_{10} - \tilde{r}_1^{\mathrm{T}}\{\hat{\alpha}_5\}\tilde{r}_{11} - \tilde{r}_1^{\mathrm{T}}\{\hat{\alpha}_5\}\tilde{r}_{13} + \tilde{r}_1^{\mathrm{T}}\{L_{1l}C_l\}\tilde{r}_{14} \\
& + \tilde{r}_1^{\mathrm{T}}\bigg\{T_l + \frac{\pi^2}{4}W_{1l}\bigg\}\tilde{r}_{16} - \tilde{r}_1^{\mathrm{T}}\{T_l\}\tilde{r}_{18} + \tilde{r}_2^{\mathrm{T}}\big\{-(1-\mu_1)\mathrm{e}^{-\alpha\hat{h}}Q_{1l} - H_1\varLambda_{21}\big\}\tilde{r}_2 \\
& + r_2^{\mathrm{T}}\{H_2\varLambda_{2l}\}\tilde{r}_{10} - \tilde{r}_4^{\mathrm{T}}\big\{(1-\mu_1)\mathrm{e}^{-\alpha\hat{h}_1}Q_{2l}\big\}\tilde{r}_4 - \tilde{r}_5^{\mathrm{T}}\big\{\mathrm{e}^{-\alpha\hat{h}_1}Q_{5l}\big\}\tilde{r}_5 \\
& - \tilde{r}_6^{\mathrm{T}}\big\{(1-\mu_2)\mathrm{e}^{-\alpha\hat{h}_2}Q_{3l}\big\}\tilde{r}_6 - \tilde{r}_7^{\mathrm{T}}\big\{\mathrm{e}^{-\alpha\hat{h}_2}Q_{6l}\big\}r_7 \\
& + \tilde{r}_8^{\mathrm{T}}\bigg\{\hat{h}_1 R_{1l} + \hat{h}_2 R_{2l} + \hat{h}^2 R_{3l} + \frac{\hat{h}^4}{4}S_{1l} + \bar{h}^2 M_{2l} + \bar{h}^2 W_l - 2L_1\bigg\}\tilde{r}_8 \\
& + \tilde{r}_8^{\mathrm{T}}\{L_{1l}A_l\}\tilde{r}_9 + \tilde{r}_8^{\mathrm{T}}\{L_{1l}B_l\}\tilde{r}_{10} + \tilde{r}_8^{\mathrm{T}}\{L_{1l}C_l\}\tilde{r}_{14} + \tilde{r}_8^{\mathrm{T}}\{T_l\}\tilde{r}_{16} - \tilde{r}_8^{\mathrm{T}}\{T_l\}\tilde{r}_{18} \\
& + \tilde{r}_9^{\mathrm{T}}\{Q_{7l} + \rho^2 S_{2l} - \varLambda_{1l}\}\tilde{r}_9 + \tilde{r}_{10}^{\mathrm{T}}\big\{-(1-\mu_1)\mathrm{e}^{-\alpha\hat{h}}Q_{7l} - \varLambda_{2l}\big\}\tilde{r}_{10} \\
& - \tilde{r}_{11}^{\mathrm{T}}\big\{4\mathrm{e}^{-\alpha\hat{h}}R_{4l}\big\}\tilde{r}_{11} + \tilde{r}_{11}^{\mathrm{T}}\bigg\{\frac{6}{\hat{h}}R_{4l}\mathrm{e}^{-\alpha\hat{h}}\bigg\}\tilde{r}_{12} - \tilde{r}_{12}^{\mathrm{T}}\bigg\{\frac{12}{\hat{h}^2}R_{4l}\mathrm{e}^{-\alpha\hat{h}}\bigg\}\tilde{r}_{12} \\
& - \tilde{r}_{13}^{\mathrm{T}}\{S_{1l}\}re_{13} - \tilde{r}_{14}^{\mathrm{T}}\big\{2(1-\mu_3)\mathrm{e}^{-\alpha\hat{\rho}}S_{2l}\big\}\tilde{r}_{14} + \tilde{r}_{14}^{\mathrm{T}}\bigg\{\frac{6}{\hat{\rho}}(1-\mu_3)S_{2l}\mathrm{e}^{-\alpha\hat{\rho}}\bigg\}r_{15} \\
& - \tilde{r}_{15}^{\mathrm{T}}\{\frac{12}{\hat{\rho}^2}(1-\mu_3)S_{2l}\mathrm{e}^{-\alpha\hat{\rho}}\}\tilde{r}_{15} - \tilde{r}_{16}^{\mathrm{T}}\{\kappa\bar{\phi}_l\}\tilde{r}_{18} + \tilde{r}_{16}^{\mathrm{T}}\bigg\{-\frac{\pi^2}{4}W_{1l}\mathrm{e}^{-\alpha\bar{h}} + \kappa\bar{\phi}_l\bigg\}\tilde{r}_{16} \\
& + \tilde{r}_{18}^{\mathrm{T}}\{\kappa\bar{\phi}_l - \bar{\phi}_l\}\tilde{r}_{18} - (2-\alpha_1)\varGamma_1^{\mathrm{T}}\tilde{R}\varGamma_1 - \mathrm{sym}\big\{\varGamma_1^{\mathrm{T}}\big(\alpha_1 Y_1 + (1-\alpha_1)Y_2\big)\varGamma_2\big\} \\
& + (1-\alpha_1)\varGamma_1^{\mathrm{T}}Y_1\tilde{R}^{-1}Y_1^{\mathrm{T}}\varGamma_1 + \alpha_1\varGamma_2^{\mathrm{T}}Y_2^{\mathrm{T}}\tilde{R}^{-1}Y_2\varGamma_2 - (2-\alpha_2)\varGamma_3^{\mathrm{T}}\tilde{M}\varGamma_3 \\
& - (1+\alpha_2)\varGamma_4^{\mathrm{T}}\tilde{M}\varGamma_4 - \mathrm{sym}\big\{\varGamma_3^{\mathrm{T}}\big(\alpha_2 Y_3 + (1-\alpha_2)Y_4\big)\varGamma_3\big\} \\
& + (1-\alpha_2)\varGamma_3^{\mathrm{T}}Y_3\tilde{M}^{-1}Y_3^{\mathrm{T}}\varGamma_3 + \alpha_2\varGamma_4^{\mathrm{T}}Y_4^{\mathrm{T}}\tilde{M}^{-1}Y_4\varGamma_4 - (1+\alpha_1)\varGamma_2^{\mathrm{T}}\tilde{R}\varGamma_2.
\end{aligned}
$$

System (6) *is exponentially stable for any switching signal with average dwell time* $T_a > T_a^* = \ln(\hat{\mu}/\alpha)$. *Moreover, the estimator of state decay is given by* $\|x(t)\| < \sqrt{\tilde{m}/\tilde{n}}\mathrm{e}^{-\eta(t-t_0)}\|x(t)\|$, *where, for all* $l, j \in M$, $\hat{\mu} \geqslant 1$ *satisfies*

$$
\begin{aligned}
& P_l \leqslant \hat{\mu}P_j, \quad Q_{1l} \leqslant \hat{\mu}Q_{1j}, \quad Q_{2l} \leqslant \hat{\mu}Q_{2j}, \quad Q_{3l} \leqslant \hat{\mu}Q_{3j}, \quad Q_{4l} \leqslant \hat{\mu}Q_{4j}, \\
& Q_{5l} \leqslant \hat{\mu}Q_{5j}, \quad Q_{6l} \leqslant \hat{\mu}Q_{6j}, \quad Q_{7l} \leqslant \hat{\mu}Q_{7j}, \quad R_{1l} \leqslant \hat{\mu}R_{1j}, \quad R_{2l} \leqslant \hat{\mu}R_{2j}, \\
& R_{3l} \leqslant \hat{\mu}R_{3j}, \quad R_{4l} \leqslant \hat{\mu}R_{4j}, \quad S_{1l} \leqslant \hat{\mu}S_{1j}, \quad S_{2l} \leqslant \hat{\mu}S_{2j}, \quad M_{1l} \leqslant \hat{\mu}M_{1j}, \\
& M_{2l} \leqslant \hat{\mu}M_{2j}, \quad W_l \leqslant \hat{\mu}W_j;
\end{aligned}
$$

$$\eta = \frac{1}{2}\left(\alpha - \ln\frac{\hat{\mu}}{T_a}\right), \qquad \tilde{n} = \min_{l\in M}\lambda_{\min}P_l,$$

$$\begin{aligned}
\tilde{m} = {} & \max_{l\in M}\lambda_{\max}P_l + \hat{h}\max_{l\in M}\lambda_{\max}Q_{1l} + \hat{h}_1\max_{l\in M}\lambda_{\max}Q_{2l} \\
& + \hat{h}_2\max_{l\in M}\lambda_{\max}Q_{3l} + \hat{h}\max_{l\in M}\lambda_{\max}Q_{4l} + \hat{h}_1\max_{l\in M}\lambda_{\max}Q_{5l} \\
& + \hat{h}_2\max_{l\in M}\lambda_{\max}Q_{6l} + \hat{h}\max_{l\in M}\lambda_{\max}Q_{7l} + \hat{h}_1^2\max_{l\in M}\lambda_{\max}R_{1l} \\
& + \hat{h}_2^2\max_{l\in M}\lambda_{\max}R_{2l} + \frac{\hat{h}^3}{2}\max_{l\in M}\lambda_{\max}R_{3l} + \frac{\hat{h}^3}{2}\max_{l\in M}\lambda_{\max}R_{4l} \\
& + \frac{\hat{h}^6}{6}\max_{l\in M}\lambda_{\max}S_{1l} + \frac{\hat{\rho}^3}{2}\max_{l\in M}\lambda_{\max}S_{2l} + \bar{h}\max_{l\in M}\lambda_{\max}M_{1l} \\
& + \frac{\bar{h}^3}{2}\max_{l\in M}\lambda_{\max}M_{2l} + \bar{h}^3\max_{l\in M}\lambda_{\max}W_l - \bar{h}\max_{l\in M}\lambda_{\max}W_l.
\end{aligned}$$

Moreover, the controller gain is $K = L_1^{-1}T$, and the other parameters are same as in Theorem 1.

Proof. Replacing W_l, A_l, B_l, C_l in (10) with $W_l + G_lF_l(t)X_{1l}$, $A_l + G_lF_l(t)X_{2l}$, $B_l + G_lF_l(t)X_{3l}$, $C_l + G_lF_l(t)X_{4l}$, respectively, and following the similar line in the proof of Theorem 1, we obtain

$$\hat{\Pi}_\alpha + \Psi_aF(t)\Psi_b + \left(\Psi_aF(t)\Psi_b\right)^{\mathrm{T}} \leqslant 0, \tag{26}$$

where

$$\begin{aligned}
\Psi_a &= \operatorname{col}[L_1G_l,\ \underbrace{0,\ 0,\ 0}_{6\text{ times}},\ L_1G_l,\ \underbrace{0,\ 0,\ 0}_{26\text{ times}}], \\
\Psi_b &= \operatorname{col}[-X_{1l},\ \underbrace{0,\ 0,\ 0}_{7\text{ times}},\ X_{2l}X_{3l},\ 0,\ 0,\ 0,\ X_{4l},\ \underbrace{0,\ 0,\ 0}_{20\text{ times}}].
\end{aligned}$$

By Lemma in [1], the necessary and sufficient condition satisfy inequality (26), and there exists a scalar $\epsilon > 0$ such that

$$\hat{\Pi}_\alpha < \epsilon^{-1}\Psi_a^{-1}\Psi_a + \epsilon\Psi_b^{-1}\Psi_b. \tag{27}$$

Now, by applying the Schur complement lemma in (27), we get (25), which guarantees that the drive system (1) and the response system (2) are synchronous. This completes the proof. □

Remark 1. In the absence of switching signal, control input and distributed delays the nominal system (7) is reduced to the following neural networks:

$$\dot{r}(t) = -Wr(t) + Ag\big(r(t)\big) + Bg\big(r\big(t - \hat{h}_1(t) - \hat{h}_2(t)\big)\big). \tag{28}$$

Then by Theorem 1, it is easy to have the following corollary.

Corollary 1. *For given scalars* $\hat{h}_u$ $(u = 1,2)$*,* $\hat{\rho}$*,* $\hat{h}$*,* μ*,* $\bar{h}$*,* μ_j $(j = 1,2,3)$*, system* (28) *is asymptotically stable if there exist symmetric positive-definite matrices* P*,* Q_i $(i = 1,2,\dots,7)$*,* R_j $(l = 1,2,3,4)$*,* S_1*, positive diagonal matrices* Λ_{fl} $(f = 1,2)$*, and any matrices* $\hat{\alpha}_1$*,* $\hat{\alpha}_2$*,* $\hat{\alpha}_3$*,* $\hat{\alpha}_4$*,* $\hat{\alpha}_5$*,* L_1*,* U_1*, with appropriate dimensions and* $Y_1, Y_2 \in \mathbb{R}^{4n\times 4n}$*, such that the following linear matrix inequalities hold*:

$$\hat{\Pi}_{\hat{S}} = \begin{bmatrix} \hat{\Pi}_\alpha & -\hat{h}_1\hat{\alpha}_{\hat{I}} & -\hat{h}_2\hat{\alpha}_{\hat{L}} \\ * & -\hat{h}_1 R_1 & 0 \\ * & * & -\hat{h}_2 R_2 \end{bmatrix} \leqslant 0,$$

where $\hat{\Pi}_\alpha = \eta^{\mathrm{T}}(t)\hat{\Pi}\eta(t)$*,* $\hat{S} = 1,\dots,4$*,* $\hat{I} = 1,3$*,* $\hat{J} = 2,4$*, and the remaining terms are defined in Theorem* 1*.*

Proof. The LKF can be chosen as in Theorem 1. Then applying the same procedure of Theorem 1 we can get that system (28) is asymptotically stable. □

Remark 2. It will be mentioned that condition (9) in Theorem 1 is dependent on the additive time-varying delay $\hat{h}_1(t)$, $\hat{h}_2(t)$, which cannot be solved directly by LMI tool. Noted that $\hat{\Pi}$ is a linear function on the variable $\hat{h}_1(t)$, $\hat{h}_2(t)$ it is easy to show that condition (9) is satisfied for all $0 \leqslant \hat{h}_1(t) \leqslant \hat{h}_1$, $0 \leqslant \hat{h}_2(t) \leqslant \hat{h}_2$ if $\hat{\Pi}_{\hat{h}_1(t),\,\hat{h}_2(t)=0} < 0$ and $\hat{\Pi}_{\hat{h}_1(t)=\hat{h}_1,\,\hat{h}_2(t)=\hat{h}_2} < 0$.

Remark 3. In the available existing literature, the stability problem has been discussed for various NNs through different techniques. In [5], the authors studied the stability problem of switched Hopfield NNs of neutral type with additive time-varying delay using Jensen integral inequality and Finser lemma. In [11], the authors discussed the problem of generalized neural networks with additive time-varying delay by using integral inequality technique (IIT), Writinger double integral inequality (WDII). Furthermore, some pioneering works have been done to transform the event-triggered control for synchronization of switched neural networks in [19], and event-triggered synchronization strategy for complex dynamical networks was studied in [17]. The model considered in the present study is more practical than that proposed by [5, 19], because in this paper we consider event-triggered switched neural networks with additive time-varying delay. The scheme of event-triggered sampling is an effective way within the electronic chips with limited capacity and energy for their great abilities to reduce the data transmission and power consumption. which is the main contribution and motivation of our work. Hence, the results presented in this paper are essentially new.

Remark 4. In general, if size of the LMIs increase then the computation burden will also increase. However, large size of LMIs yield better system performances. In this paper, the proposed criteria are employed by the several integral inequalities; as a result, some computational complexity can occur in the proposed criterion. To avoid the computational burden, in future the Finsler's lemma was applied in the proof of the main results, which in turn to reduce the computational burden. Moreover, in the future work, we will focus on lower computational complexity of the stability problems while maintaining the desired system performances.

4 Numerical examples

Example 1. Consider the uncertain switched neural networks (6) with parameter as follows:

$$W_1 = \begin{bmatrix} 5 & 0 \\ 0 & 4 \end{bmatrix}, \qquad W_2 = \begin{bmatrix} 3 & 0 \\ 0 & 7 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} 1 & 0.4 \\ -2 & 0.1 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 0.5 & 0.7 \\ 0.7 & 0.4 \end{bmatrix}, \qquad C_1 = \begin{bmatrix} 0.5 & -0.3 \\ 0.2 & 1.2 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.3 & 0.2 \\ 0.4 & 0.1 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} -0.5 & 0.2 \\ 0.1 & -0.2 \end{bmatrix}, \qquad C_2 = \begin{bmatrix} 0.1 & 0.4 \\ -0.3 & -0.1 \end{bmatrix},$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad G_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad G_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix},$$

$$X_{k1} = \operatorname{diag}\{0.1, 0.1\}, \quad X_{k2} = \operatorname{diag}\{0.2, 0.2\}, \quad k = 1, 2, 3, 4.$$

Also let $\hat{h}_1 = 1.2$, $\hat{h}_2 = 1.5$, $\hat{\rho} = 0.1$, $\mu_1 = 0.2$, $\mu_2 = 0.3$, $\bar{h} = \kappa = 0.1$, $\mu_3 = 0.1$, $\alpha = 0.1$, $\hat{\mu} = 1.05$, $H_1 = 0$, $H_2 = 0.5I$. By using the Matlab LMI control Toolbox, we solve the linear matrix inequalities in Theorem 2, we get the following control gain matrices and the event triggered parameters. Due to page limitation we omitted the feasible solutions:

$$K_1 = \begin{bmatrix} 0.0273 & -0.0004 \\ -0.0000 & 0.0302 \end{bmatrix}, \qquad K_2 = \begin{bmatrix} -0.5038 & -0.0902 \\ -0.0301 & -0.1652 \end{bmatrix},$$

$$\bar{\phi}_1 = \begin{bmatrix} 0.1000 & 0.0120 \\ 0.0120 & 0.0410 \end{bmatrix}, \qquad \bar{\phi}_2 = \begin{bmatrix} 0.6111 & 0.1089 \\ 0.1089 & 0.8148 \end{bmatrix}, \qquad \epsilon = 0.0023.$$

For given $\hat{\mu} = 1.05$, Theorem 2 gives $T_a^* = \ln \hat{\mu}/\alpha = 0.4879$. Then by using Theorem 2 and taking $T_a = 0.5$, $\eta = 0.0012$, we obtain

$$\|x(t)\| < 19.5261\mathrm{e}^{-0.0012(t-t_0)}\|x(t_0)\|.$$

In the simulation Fig. 1, (a) and (b) depict the synchronization state trajectories $y_1(t)$, $y_2(t)$ of system (6) and $z_1(t)$, $z_2(t)$ of its response system, respectively. Figure 1(c) describes the synchronization error curves of the state variables $r_1(t)$, $r_2(t)$ between the drive system and the corresponding response system, the switching signal is plotted in Fig. 1(d). It can also be verified that the LMI (25) are even feasible for larger upper delay bounds $\hat{h}_1$ and $\hat{h}_2$, which indicates that the system is exponentially synchronized.

Example 2. Consider the neural network (46) with the following parameters:

$$W = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \qquad A = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}, \qquad B = \begin{bmatrix} 0.88 & 1 \\ 0 & 1 \end{bmatrix}.$$

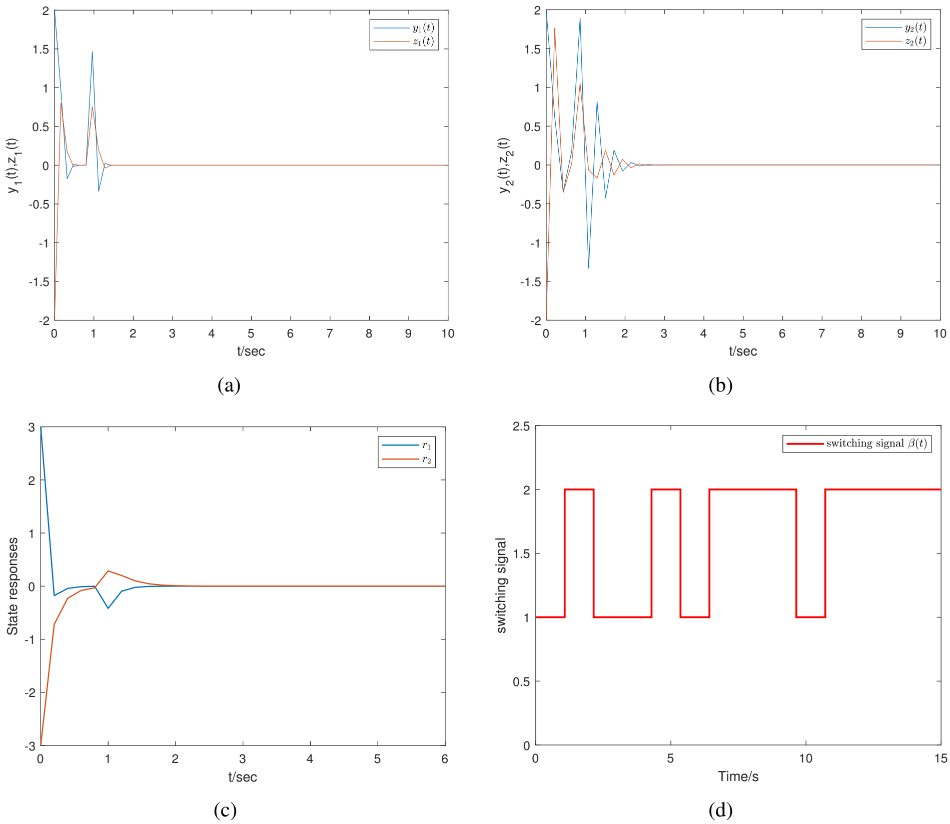

Figure 1. Panels (a)–(d) contain the simulation results of the synchronization state trajectories, error curves and switching signal of system (6).

Table 1. Comparison of the number of the decision variables involved in Corollary 1.

Methods	number of decision variables involved
[28]	$36.5n^2 + 8.5n$
[13]	$32.5n^2 + 8.5n$
[2]	$21.5n^2 + 8.5n$
Corollary 1	$13.5n^2 + 8.5n$

Table 2. Allowable upper bounds of $\hat{h}_2$ for different values of $\hat{h}_1$ with $\mu_1 = 0.7, \mu_2 = 0.1$.

$\hat{h}_1$	0.8	1.0	1.2
$\hat{h}_2$ by [28]	0.8831	0.6832	0.4043
$\hat{h}_2$ by [13]	1.5666	1.3668	1.1664
$\hat{h}_2$ by [21]	1.9528	1.7992	1.6441
$\hat{h}_2$ by [24]	1.9666	1.8351	1.6803
$\hat{h}_2$ by Corollary 1	2.0147	1.9215	1.7361

In this example, the activation functions are given as $g_1(r) = 0.4\tanh(r)$ and $g_2(r) = 0.8\tanh(r)$. It is easy to check that Assumption 1 holds with $H_2 = \operatorname{diag}\{0.4, 0.8\}$ when $\mu = 0.8$ ($\mu_1 = 0.7$, $\mu_1 = 0.1$) and $\mu = 0.9$ ($\mu_1 = 0.7$, $\mu_1 = 0.2$). Let $\hat{h}_1$ and $\hat{h}_2$ be the upper bounds of time-varying delays $\hat{h}_1(t)$ and $\hat{h}_2(t)$, respectively, and $\dot{\hat{h}}_1(t) \leqslant \mu_1$, $\dot{\hat{h}}_2(t) \leqslant \mu_2$. Solving LMIs in Corollary 1 by using Matlab LMI toolbox, we can calculate allowable upper bounds of $\hat{h}_1(t)$ and $\hat{h}_2(t)$, which are given in Table 2.

5 Conclusion

The problem of synchronization of decentralized event-triggered has been addressed for uncertain switched neural networks with two additive time-varying delays. To reduce the communication burden in the network, the novel decentralized event-triggered communication scheme has been proposed. By using the Lyapunov functional method, convex combination techniques and Wirtinger-based integral inequality some sufficient conditions in terms of linear matrix inequalities have been derived to guarantee the exponential stability of the neural networks under consideration. Numerical examples are provided to demonstrate the effectiveness of the obtained results. In future work, we will utilize the proposed method to deal with stochastic neural networks, fuzzy neural networks, or other types of neural networks with time-varying delays.

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