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TRANSPORT AND FLUCTUATIONS IN NONLINEAR DISSIPATIVE SYSTEMS: ROLE OF INTERPARTICLE COLLISIONS

(Review article)

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Abstract

The goal of the paper is to overview contemporary theoretical and experimental research of the microwave electric noise and fluctuations of hot carriers in semiconductors, revealing sensitivity of the noise spectra to *non-linearity* in the applied electric field strength and, especially, *in the carrier density*. During the last years, investigation of electronic noise and electron diffusion phenomena in *doped* semiconductors was in a rapid progress. By combining analytic and Monte Carlo methods as well as the available experimental results on noise, it became possible to obtain the electron diffusion coefficients in the range of electric fields where inter-electron collisions are important and Price's relation is not necessarily valid. Correspondingly, a special attention to the role of *inter-electron collisions* and of the *non-linearity in the carrier density* while shaping electric noise and diffusion phenomena in the non-equilibrium states will be paid. The basic and up-to-date information will be presented on methods and advances in this contemporary field - the field in which methods of non-linear analytic and computational analysis are indispensable while seeking coherent understanding and interpretation of experimental results.

Key words: fluctuations in non-linear systems, noise, hot electrons, inter-electron collisions

CONTENTS

1. INTRODUCTION

Fluctuation processes have been intensively investigated during the last three decades. Owing to the developments in theory, instrumentation, computers and other advanced techniques, a profound understanding of various fluctuation phenomena in non-equilibrium systems has been achieved. It has become evident that phenomena

such as the noise of hot carriers in semiconductors, light scattering from non-equilibrium carriers, or from a gas or fluid with gradients, non-equilibrium diffusion in these systems, as well as a number of other non-equilibrium phenomena have major similarities from a fundamental kinetic point of view. A unified description of all these physical processes on the basis of the currently available kinetic theory of non-equilibrium fluctuations could be presented but would be beyond the scope of this review paper. The goal of the paper is to overview contemporary theoretical and experimental research of the spectra of electric noise and fluctuations of *hot carriers* in semiconductors in the microwave region, revealing sensitivity of these spectra to *non-linearity* in the applied electric field strength and, especially, *in the carrier density.* Correspondingly, a special attention to the role of *inter-electron collisions* while shaping electric noise phenomena in the non-equilibrium states will be paid.

Fluctuation phenomena in non-equilibrium steady state of a dissipative system are known to be quite sensitive to details of kinetic processes in the system, often being much more sensitive than average values of physical quantities. In particular, data on electric noise in semiconductors subjected to high electric field contain valuable information on charge carrier scattering processes and band structure of the crystal not easily obtainable from charge transport characteristics (Bareikis et al. 1992, 1994). The productive area of modern physics which can be called fluctuation spectroscopy, or *noise spectroscopy*, of solid state plasma (Bareikis et al. 1992), especially of effects non-linear in applied electric field strength and carrier density, is based on this distinctness of fluctuation phenomena. The aim of the present review article is to give a coherent summary of the state of the art of this contemporary field. Basic and up to date information about the methods and advances in the field will be presented, providing the necessary analytical tools needed for a competent understanding and use of these developments.

In particular, sensitiveness to *non-linearity in carrier density* of electronic noise in *doped* semiconductors was revealed during the last years. It was shown (Katilius et al. 1996, Matulionis et al. 1997) that noise properties of a doped semiconductor at moderate applied electric fields can be remarkably influenced by inter-electron scattering, even provided its current-voltage characteristic is not. In particular, the available experimental results (Aninkevicius et al. 1993, Bareikis et al. 1994, Katilius et al. 1999) on microwave noise in doped n-type GaAs channel (impurity density exceeding 10^{17} cm^{-3}) could not be interpreted, even qualitatively, within the framework of models neglecting inter-electron collisions. The comparison with the results of Monte Carlo simulation (Katilius et al. 1996, Matulionis et al. 1997) showed that

inter-electron scattering causes an essential (over a hundred times) increase in the field strength required for the excess electric noise to manifest itself. This example, in which the influence of inter-electron scattering on noise was well controlled, provoked detail investigation in terms of the earlier developed kinetic theory of fluctuations.

The theory (Gantsevich et al. 1969 a,b, 1970) (see also Gantsevich et al. 1979, Bareikis et al. 1992, Kogan 1996, Katilius 1997) predicts, among other effects, appearance, in non-equilibrium, of *correlation between occupancies of electronic states* (even provided the individual collisions between electrons remain uncorrelated). Moreover, it was demonstrated that the *analytical* treatment of the "kinetic" (or "additional") correlation created by inter-particle collisions in non-equilibrium is possible in cases where frequent enough inter-electron collisions control the shape of electron distribution in energy (Shulman 1970; see also Gantsevich et al. 1979, Bareikis et al. 1992), or both in energy and momentum (Barkauskas and Katilius 1979; see also Bareikis et al. 1992). Recently (Katilius et al. 1999) the available analytic expressions for noise temperature and additional correlation contribution were used for interpretation of the mentioned-above and newly obtained experimental results in the range of applied fields where the measured noise spectrum is remarkably influenced by inter-electron collisions.

An investigation of the influence of inter-electron scattering on noise spectrum performed by combining analytic and Monte Carlo methods in the case where experimental results are available was called for also by practical reasons. For rare inter-electron collisions, Price's fluctuation-diffusion relation (Price 1965) connects the spectral intensities of current fluctuations, in a uniform stationary electron gas, with the diffusion coefficients entering the expressions for the current induced by a small spatial gradient of the electron density. For weakly doped semiconductors, Price's noise-diffusion relation proved to be very useful while providing information on the hot-electron diffusivity from noise measurements performed in spatially homogeneous states (Bareikis et al. 1992, 1994). One of the predictions of the kinetic theory of fluctuations is the violation of Price's relation in a non-equilibrium electron gas in the case when inter-electron collisions cannot be neglected (Gantsevich et al. 1969b; see also Gantsevich et al. 1979, Bareikis et al. 1992). On the other hand, direct measurements of the carrier diffusion coefficients in doped semiconductors, being possible in principle, are difficult in practice and up to now were not performed. Effective methods of Monte Carlo simulation of the carrier diffusion process in the case of the concentration-dependent distribution function are absent (Reggiani et al. 1989, Thobel et al. 1997). As a result, quantitative data on hot-electron diffusion

coefficients were lacking in doped semiconductors at high densities of electrons. Only recently, by combining analytic and Monte Carlo methods as well as the available experimental results on noise, it became possible (Katilius et al. 1999) to obtain the electron diffusion coefficients in the range of electric fields where inter-electron collisions are important and Price's relation is not necessarily valid.

Hence, during the last years, investigation of noise and carrier-diffusion phenomena in doped semiconductors was in a rapid progress. In what follows the basic and up to date information will be presented on methods and advances in this contemporary field - the field in which methods of non-linear analytic and computational analysis are indispensable while seeking coherent understanding and interpretation of experimental results.

The review is organised as follows. Basic concepts concerning noise and fluctuations in non-equilibrium states are introduced in Section 2. Theoretical results on noise and electron diffusion in the effective electron temperature approximation are presented and discussed in detail in Section 3. Available experimental and Monte Carlo results on noise in doped semiconductors under non-equilibrium conditions are presented in Section 4. An interpretation on the lines of the theory exposed in Section 2 of the results described in Section 4 is given in Section 5. Field-dependent electron-diffusion coefficients in doped n-type GaAs are determined in Section 6. The results of calculation of diffusion and correlation at intermediate (lower than in Sections 3-6) densities of carriers are reviewed in Section 7. Analytic solutions valid in the special case of inter-electron scattering more effective than scattering of the quasi-momentum of the system of carriers (case of "drifted" Maxwellian distribution of carriers) are presented in Section 8. The article concludes in Section 9 with a look at the open problems and a discussion of some likely future directions for research.

2. FLUCTUATIONS IN NON-EQUILIBRIUM: BASIC CONCEPTS

The goal of this Section is to introduce main concepts, to give a short historical survey, and to present a theoretical background. We begin with a reasonable definition of what is called an electric noise. "When one tries to measure or amplify small signals, one usually arrives at a lower limit set by the spontaneous fluctuations in current, voltage, and temperature of the system under test. These spontaneous fluctuations are referred to as noise" (Van der Ziel 1986). Noise is commonly viewed by researchers as the limiting factor: "Noise is an important problem in science and

engineering, since it sets lower limits to the accuracy of any measurement and to the strength of signals that can be processed electronically" (Van der Ziel 1986).

But there is also another aspect of the problem, now fully realised and widely exploited. Fluctuations in macroscopic observables result from microscopic random processes; the measurement of noise enables to obtain information about these processes. Indeed, every source of fluctuations is associated with some microscopic mechanism accompanied by dissipation. Moreover, measuring fluctuations *out of equilibrium* provides new information about the system - new as compared to that obtainable while measuring the average values of the observables. Measuring fluctuations is often a way of measuring quantities that are much harder to obtain in other ways.

A typical measured quantity is a *spectrum of noise power*, i.e., the noise power in a given frequency range. Accordingly, the field in question can be called *fluctuation spectroscopy*.

2.1. Equilibrium and non-equilibrium noise spectra

Since the beginning of the 20th century, when A. Einstein and M. von Smoluchowski developed the theory of Brownian motion, fluctuation science has been one of the most important integral parts of statistical physics and physical kinetics. Fluctuations in a physical system take place due to its discrete nature and the thermal motion in it. Investigation of fluctuations is indispensable if one seeks a perfect understanding of the connection between the microscopic and macroscopic properties of the physical system.

In equilibrium, thermal noise is related to energy through the equipartition theorem which states that every physical system at not too low temperature T_0 contains an average amount of kinetic energy of $k_B T_0 / 2$ per degree of freedom, where k_B is the Boltzmann constant. The theorem is true for macroscopic degrees of freedom as well, the macroscopic thermal motion at thermal equilibrium in average being possessed of ''microscopic'' amount of energy $k_B T_0 / 2$. The transformation from total average energy (including the fluctuation components at all frequencies) governed by equipartition and considered in thermodynamics, to the power in a given frequency range was achieved by Nyquist (1928). In Nyquist's derivation, some trace may be seen of Rayleigh's (1900) application of the equipartition theorem to the standing-wave modes of black-body radiation. In some sense, the available noise power is a special low-frequency case of the black-body radiation (cf. Bell 1985). At thermal equilibrium

the power spectrum of noise has no features. The Nyquist theorem (or the more general fluctuation-dissipation theorem (Callen and Welton 1951; see also Lifshitz and Pitaevskii 1980) states that, while every source of fluctuation is associated with a mechanism of dissipation, the available noise power of a thermal-noise source at the temperature T_0 in a frequency interval Δf is a *universal* function of T_0. The fluctuation-dissipation theorem relates noise in some variables to the admittance of the system for these same variables, so that the measurement of fluctuations in equilibrium gives the same information as the measurement of a related transport coefficient. For example, the Nyquist theorem enables to relate the *spectral intensity of current fluctuations in a conductor* and the *real part of the conductivity*.

However, this is not true for systems that are not in thermal equilibrium. In the case of an open system subjected to a continuous energy flow, some energy being added from the external world and then dissipated out to the external world, the whole argument in terms of thermodynamic equilibrium collapses. In the typical non-equilibrium conditions, the system is being displaced from thermal equilibrium with the 'environment' (the thermal bath), so that the system does not have the energy distribution corresponding to the temperature of the thermal bath. The situation can be induced by the "driving force" transferring energy from outside to the system. The system may be driven into a state stationary in time for an interval long enough for some measurements to be made, provided the energy is steadily passed on the thermal bath. Fluctuations in such a virtually stationary state of the system substantially displaced from the equilibrium with the thermal bath do not obey the fluctuation-dissipation theorem. The noise produced in such a non-equilibrium state of the system will be different from that in the equilibrium state.

In the case of a semiconductor or semiconductor structure, the fluctuation-dissipation theorem is applicable only as long as the charge carriers are in thermal equilibrium with the crystal lattice. When a sample has a biasing voltage and current, the non-equilibrium distribution of carriers can be realized with comparative ease. The gas of carriers, which determines electrical properties of the structure including its electric noise characteristics, in such a current-carrying state can be pretty far from the equilibrium with the lattice provided the electric field is of sufficiently high strength. The losses of the carrier energy and momentum due to interaction with the thermal bath (the lattice) are compensated by the work done by the external electric field. The customary name for this situation is *hot carriers* (or *hot electrons*; we shall often write "electrons" meaning in fact "charge carriers", i.e., electrons or holes indifferently). The non-equilibrium distributions of carriers maintained by the electric field have been

widely investigated both theoretically and experimentally during the number of decades and are realized in a great number of semiconductor devices. Taking into account the present tendencies in micro- and nano-electronics, one can predict a further growth of the role of non-equilibrium states, especially of those very far from equilibrium, and a growing importance of noise characteristics of micro- and nano-structures.

Since there is no necessary connection between the electric noise and the impedance of the non-equilibrium system, one must examine the noise in terms of its internal mechanism. On the other hand, it is the failure of fluctuation-dissipation theorem that makes the fluctuation spectroscopy a valuable tool for the diagnostics of a non-equilibrium system. This tool has a fundamental as well as a practical aspect: the mechanisms of momentum, energy, and inter-valley relaxation, as well as free-carrier number relaxation, reflect themselves in the noise spectrum pattern of a semiconductor having a biasing current. In the case of semiconductor structures (structures with quantum wells, etc.), the inter-sub-band, real-space transfer and other kinetic processes impresses their fingerprints on the non-equilibrium noise spectra of the structures. The hot-electron noise measurements - the fluctuation spectroscopy of hot electrons - is a very active field at the moment.

2.2.Theoretical background

The statistical description of a many-particle system in a thermal equilibrium state is based on a few very general properties or principles, such as the Gibbs distribution, possibility to introduce thermodynamic potentials, and validity of fluctuation-dissipation theorems. For non-equilibrium states, these principles do not work (in particular, thermodynamic potentials cannot be introduced). Contrary to the universal functional form of the Gibbs distribution, the non-equilibrium distribution function depends on both the external forces creating the bias from thermal equilibrium and on details of the interaction of the system with the thermal bath. Accordingly, the distribution function is much more dependent on the experimental situation and should be found for each case. In other words, instead of thermodynamics, other concepts and theoretical apparatus had to be and were invented and used.

What was said applies equally to fluctuation phenomena. Fluctuation-dissipation theorems fail in a non-equilibrium state, the correlation functions depend on external forces and details of the interaction of the system with the thermal bath. Calculation, or measurement, of the spectral intensity of noise proves to be an independent problem

which cannot be reduced to the problem of calculation, or measurement, of the response of the system to the external perturbation. Fluctuation characteristics of a non-equilibrium system in fact are new independent kinetic (transport) characteristics of the system.

The quantitative description of non-equilibrium states is accessible provided the standard kinetic theory is applicable to the system (see, e.g., Lifshitz and Pitaevskii 1981). The macroscopic quantities then are expressible in terms of the distribution function obeying the *kinetic*, or *transport*, equation introduced by Ludwig Boltzmann to describe a neutral gas with pair, or two-particle, collisions and then adapted to a gaseous plasma as well as a gas of excitations (free electrons, holes, phonons, magnons, etc.) in solids. Later, the apparatus of the kinetic theory was generalized to describe fluctuations: kinetic equations for two-particle correlation functions were derived (Gantsevich et al. 1969 a,b, 1970; see also Gantsevich et al. 1979, Lifshitz and Pitaevskii 1981, Bareikis et al. 1992, Kogan 1996, Katilius 1997). Also an alternative - Langevin - approach was made suitable for non-equilibrium conditions (Kogan and Shulman 1969; see also Kogan 1996). These achievements have shaped the modern theory of fluctuations in a non-equilibrium state. The criteria of applicability of the theory coincide with those of the Boltzmann kinetic equation: if the kinetic equation for the one-particle distribution function can be worked out and solved, then the kinetic equations for fluctuations can also be written and, though more complicated, successfully solved. These results, in principle, complete the theory of classical fluctuations in a weakly interacting many-particle system - in the same sense in which the Boltzmann equation exhausts the theory of transport in such a system. This review article is devoted to *applications* of the general theory of fluctuations to a gas of mobile ("free") electrons in semiconductors, especially of "hot" electrons, at rather high free electron densities, when the most non-trivial consequences of the theory take a shape allowing comparison with the experimental results.

The frequencies associated with the characteristic relaxation times of hot electrons in semiconductors and semiconductor structures lie in the frequency range from 10^9 to 10^{12} Hz. Therefore, it is quite natural that the investigation of the noise spectrum in the *microwave* frequency range has proved to be a powerful diagnostic tool of the hot-electron state realized by creating in a semiconducting channel the electric field of sufficiently high strength (see Bareikis et al. 1992, 1994). The remarkable progress has been made, and continues to be being made, in the theoretical and experimental investigations of noise and fluctuation phenomena in semiconductors and semiconductor structures under hot-electron conditions at high frequencies, including

microwave frequencies. The current, or voltage, fluctuation spectral intensity when a biasing voltage creates a current in the structure proved to be quite sensitive to the details of the scattering mechanisms and band structure. The satisfactory agreement with the microscopic interpretation has enabled an improved understanding of the different scattering mechanisms that mobile charge carriers undergo in their motion. Furthermore, when carrier-carrier interaction is negligible, the current-fluctuation spectral intensity, as noticed in Introduction, can be simply related to the *diffusion coefficient* of hot carriers. The relation has provided a useful method for the experimental determination of the diffusion coefficient through noise-conductivity measurements: it provides the possibility to determine the hot-carrier diffusion coefficient from noise measurement in homogeneous materials *without* producing a carrier density gradient. This is a good example illustrating what was said above: measuring fluctuations can be a way of measuring quantities that are much harder to obtain in other ways.

2.3. Available power. Equivalent noise temperature

A typical measurement is that of the maximal power $\Delta P_{n\alpha}(\omega,\mathbf{E}) = \dfrac{S_{j\alpha}(\omega,\mathbf{E})V_0}{4\,\mathrm{Re}\,\sigma_{\alpha\alpha}(\omega,\mathbf{E})}\Delta f$, dissipated by the noise source (a two-terminal network) on the matched output impedance at a *frequency* $\omega = 2\pi f$ in a frequency interval Δf. The power fed into a matched load is usually called the power available at the noise source, or the available noise power (Van der Ziel 1986), and is expressible as follows:

$$\Delta P_{n\alpha}(\omega,\mathbf{E}) = \frac{S_{j\alpha}(\omega,\mathbf{E})V_0}{4\,\mathrm{Re}\,\sigma_{\alpha\alpha}(\omega,\mathbf{E})}\Delta f\,, \tag{2.1}$$

Here $\Delta P_{n\alpha}(\omega,\mathbf{E}) = \dfrac{S_{j\alpha}(\omega,\mathbf{E})V_0}{4\,\mathrm{Re}\,\sigma_{\alpha\alpha}(\omega,\mathbf{E})}\Delta f$, is the small-signal AC conductivity, at the angular frequency ω, in the presence of a steady electric field $\Delta P_{n\alpha}(\omega,\mathbf{E}) = \dfrac{S_{j\alpha}(\omega,\mathbf{E})V_0}{4\,\mathrm{Re}\,\sigma_{\alpha\alpha}(\omega,\mathbf{E})}\Delta f$, and V_0 is the sample volume. The quantity

$\Delta P_{n\alpha}(\omega, \mathbf{E}) = \dfrac{S_{j\alpha}(\omega, \mathbf{E}) V_0}{4\,\mathrm{Re}\,\sigma_{\alpha\alpha}(\omega, \mathbf{E})}\,\Delta f$, is the *spectral intensity of fluctuations* of the current density along the direction α in the presence of the steady electric field $\Delta P_{n\alpha}(\omega, \mathbf{E}) = \dfrac{S_{j\alpha}(\omega, \mathbf{E}) V_0}{4\,\mathrm{Re}\,\sigma_{\alpha\alpha}(\omega, \mathbf{E})}\,\Delta f$,

$$S_{j\alpha}(\omega) \equiv 2(\delta j_\alpha^2)_\omega .$$

The quantity $(\delta j_\alpha^2)_\omega$ is a diagonal element of the tensor constituted by the Fourier transforms of the corresponding time-displaced correlation functions,

$$(\delta j_\alpha \delta j_\beta)_\omega \equiv \int_{-\infty}^{\infty} \overline{\delta j_\alpha(t'+t)\delta j_\beta(t')} \exp(i\omega t)dt ,$$

$\delta\mathbf{j}(t)$ being the current density fluctuation at the moment t ,

$$\delta\mathbf{j}(t) = \mathbf{j}(t) - \bar{\mathbf{j}} .$$

The bar designates the ensemble average, or time average. Equation (2.1) is rather transparent having in mind the text-book expression for the power as the current squared times resistance.

At thermal equilibrium the spectral intensity of the current fluctuations and the small-signal AC conductivity, $S_{j\alpha}^{eq}(\omega) \equiv S_{j\alpha}(\omega; E=0)$ and $\tilde{\sigma}_{\alpha\alpha}^{eq}(\omega) = \tilde{\sigma}_{\alpha\alpha}(\omega; E=0)$, are related by the Nyquist relation (see Lifshitz and Pitaevskii 1980):

$$S_{j\alpha}^{eq}(\omega) = (4k_B T_0 / V_0)\,\mathrm{Re}\,\tilde{\sigma}_{\alpha\alpha}^{eq}(\omega) \qquad (2.2)$$

Here k_B is the Boltzmann constant and T_0 is the absolute temperature; the quantum correction factor is neglected, i.e., the inequality $\hbar\omega << $ is assumed. From Eq.(2.2) it follows that the expression for the available noise power in the absence of the biasing voltage, when the charge carriers are in thermal equilibrium with the crystal lattice, is

$$\Delta P_{n\alpha}(\omega; E = 0) = k_B T_0 \Delta f \,. \tag{2.3}$$

Equation (2.3) means that the available power per unit bandwidth is related to energy through the equipartition law. It follows from Eq.(2.3) that, in principle, the noise power available at the equilibrium source can serve for the establishment of the absolute scale of temperature (see Bell 1985, p.21).

Equation (2.2) means that, for thermal equilibrium, an independent determination of the spectral intensity of current fluctuations, as was mentioned above, does not add information that is not available from the conductivity. Under *non-equilibrium* conditions, the *equivalent noise temperature* $T_{n\alpha}(\omega, \mathbf{E})$ can be defined (Van der Ziel 1986) as

$$T_{n\alpha}(\omega, \mathbf{E}) = \frac{\Delta P_{n\alpha}(\omega, \mathbf{E})}{k_B \Delta f} = \frac{V_0}{4k_B} \frac{S_{j\alpha}(\omega, \mathbf{E})}{\mathrm{Re}\,\sigma_{\alpha\alpha}(\omega, \mathbf{E})}\,, \tag{2.4}$$

or, according to Eq.(2.1), as

$$T_{n\alpha}(\omega, \mathbf{E}) = \frac{\Delta P_{n\alpha}(\omega, \mathbf{E})}{k_B \Delta f} = \frac{V_0}{4k_B} \frac{S_{j\alpha}(\omega, \mathbf{E})}{\mathrm{Re}\,\sigma_{\alpha\alpha}(\omega, \mathbf{E})}\,. \tag{2.4a}$$

In equilibrium, by virtue of the fluctuation-dissipation theorem (2.2),

$$T_n^{eq} = T_0\,, \tag{2.5}$$

while under non-equilibrium conditions the noise temperature T_n is a convenient way of expressing the noise power. Its macroscopic meaning is related to the measurable quantity $\Delta P_{n\alpha} \equiv k_B T_n \Delta f$, which is the maximal noise power at frequency *w*, which can be displayed by the network in an output circuit (see Van der Ziel 1986). In conductors, the noise temperature T_n represents a property of the ensemble of mobile carriers which, in general, differs from both its "energy temperature" (conveniently

defined as $2/3 \cdot <\varepsilon> / k_B$, $<\varepsilon>$ being the carrier average energy) and from the thermal-bath temperature T_0. The hot-electron condition leads to a frequency-, field-, and direction-dependence of the noise power and is therefore responsible for the introduction of the frequency-, field-, and direction-dependent equivalent noise temperature.

2.4. Noise and diffusion

At thermal equilibrium, three basic kinetic coefficients are interrelated: the free electron mobility μ, the spectral intensity of current fluctuations S_j, and the free-electron diffusion coefficient D. The Nyquist relation (2.2) expresses the spectral intensity of current fluctuations in terms of the conductivity and the thermal equilibrium temperature. The so-called Einstein relation expresses the carrier diffusion coefficient in terms of their mobility μ^{eq} and thermal-equilibrium temperature T_0 For carrier densities far from degeneracy this is given by

$$D^{eq} = k_B T_0 \mu^{eq} / e \,, \tag{2.6}$$

e being the carrier charge. Thus, in the linear-response regime an independent determination of the diffusion coefficient and noise temperature, in addition to the drift velocity, does not add any particular information about the transport properties of the material as compared to the knowledge obtained from a determination of the Ohmic mobility.

Under hot-electron conditions, not only Nyquist's relation but also Einstein's relation no longer hold in general. Therefore an independent determination of conductivity, diffusion coefficient, and noise temperature provides new information. However, when carrier-carrier scattering can be neglected, an exact relationship exists between noise in the low-frequency region, associated with the velocity fluctuations of carriers, *and* the diffusion of carriers. The *low-frequency region* means here the region of the noise spectrum for which the frequency w is *high* enough to neglect $1/f$ noise as well as generation-recombination noise but sufficiently *low* to satisfy the condition

$$\omega\tau << 1 \,, \tag{2.7}$$

τ_i standing for the carrier relaxation times characterising different kinetic processes inside the conduction band (processes conserving the number of mobile carriers; inequality (2.7) should hold not only for carrier momentum relaxation, but also for the slower relaxation processes in the conduction band, such as energy relaxation, inter-valley relaxation, etc.).

Provided carrier-carrier scattering is neglected, then, as mentioned in the Introduction, the low-frequency spectral intensity of current fluctuations is related to the tensor of the carrier diffusion coefficients by the relation originally proposed by Price (1965):

$$S_{j\alpha}(\omega, \mathbf{E})\big|_{\omega\tau<<1} = (4e^2 n_0 / V_0) D_{\alpha\alpha}(\mathbf{E}) \tag{2.8}$$

where n_0 is the average carrier density, $D_{\alpha\beta}(\mathbf{E})$ is the diffusion constant determining the flux of carriers, $\mathbf{j}^D$, resulting from a small gradient of the carrier density n according to

$$j_\alpha^D = -D_{\alpha\beta}\, \partial n / \partial x_\beta \ . \tag{2.9}$$

Price's noise-diffusion relation proved to be very useful. Due to this relation, the noise measurements in a spatially-homogeneous steady state provide information about the response of the non-equilibrium system to the induced gradient, and *vice versa*.

2.5. Additional correlation tensor

In the cases where inter-electron collisions must be taken into account, the Boltzmann equation is non-linear in electron density. Due to non-linearity of the Boltzmann equation and the extra correlation created by inter-electron collisions in the non-equilibrium state, it is impossible to express the low-frequency spectral intensity of current fluctuations through the diffusion coefficient as was done by neglecting inter-electron collisions in the previous Subsection (see Eq.(2.8)). In the general case (Gantsevich et al. 1969 b)

$$(\delta j_\alpha \delta j_\beta)_{\omega\tau<<1} = (e^2 n_0 / V_0)(D_{\alpha\beta} + D_{\beta\alpha} - \Delta_{\alpha\beta}), \tag{2.10}$$

where $D_{\alpha\beta}$ is the tensor entering the expression for the diffusion current (Eq.(2.9)) while $\Delta_{\alpha\beta}$ can be called the *tensor of additional correlation*. It describes the specific influence of inter-electron collisions on electric noise, being a characteristic of the degree of correlation in the electron gas (or of *non-ideality* of the electron gas) and of significance of non-linearity, in electron concentration, of the distribution function of the non-equilibrium electron gas.

The additional correlation tensor *vanishes in thermal equilibrium* (Gantsevich et al. 1969a):

$$\Delta^{eq}_{\alpha\beta}(\omega) = 0\,, \tag{2.11}$$

So, in the *thermal equilibrium* the *Price relation*

$$(\delta j_\alpha \delta j_\beta)^{eq}_{\omega\tau<<1} = (e^2 n_0 / V_0)(D^{eq}_{\alpha\beta} + D^{eq}_{\alpha\beta})\,. \tag{2.12}$$

holds independently of frequency of inter-electron collisions.

3. FLUCTUATION SPECTRA, CORRELATION, AND DIFFUSION IN EFFECTIVE ELECTRON TEMPERATURE APPROXIMATION

As mentioned in the Introduction and Section 2, in the framework of the kinetic theory of fluctuations in a non-equilibrium electron gas (Gantsevich et al. 1969a,b, 1970; see also Gantsevich et al. 1979, Lifshitz and Pitaevski 1981, Bareikis et al. 1992, Katilius 1997) it has been shown that the pair collisions lead to the additional correlation between the electrons as a result of which the fluctuation-diffusion relation is violated. It was demonstrated that under non-equilibrium conditions the contribution of the additional correlation to the current noise in general is not parametrically small. We shall present the explicit results for the spectral intensities of the current fluctuations and for the diffusion and correlation tensors for few simple but rather realistic cases, making possible interpretation of the available and future experimental results on noise in semiconductors and semiconductor structures with moderate (moderately high) carrier density.

A detailed study is possible (Shulman 1970; see also Gantsevich et al. 1974, 1979) of the case where collisions between electrons govern the electron distribution in energy but not in quasi-momentum:

$$\tau \ll \tau_{ee} \ll \tau_{en} . \tag{3.1}$$

Here τ and τ_{en} are the characteristic electron momentum and energy relaxation times due to electron collisions with the lattice (i.e., with impurities, phonons, etc.), and τ_{ee} is the characteristic time for energy and quasi-momentum transfer *within* the system of electrons. While the rates of quasi-momentum and energy transfer to the thermal bath can differ essentially, those for quasi-momentum and energy exchange within the electron system are known not to differ drastically from each other. Hence, in some range of electron densities, the rate at which inter-electron collisions redistribute energy within the electron system can happen to be larger than the rate at which the electron system transfers the energy to the thermal bath:

$$\tau_{ee} \ll \tau_{en} , \tag{3.2}$$

while the fast relaxation, due to scattering on the thermal bath, of the *odd part* of the electron distribution function can remain unaffected by still comparatively rare inter-electron collisions:

$$\tau \ll \tau_{ee} . \tag{3.3}$$

This particular region of electron densities (the typical values for bulk semiconductors are $n_0 \sim (10^{15} - 10^{17})$ cm^{-3}) is the subject of the investigation in this Section. In this case, the analytic expressions are available both for the spectral intensities of current fluctuations (Kogan and Shulman 1967, Shulman 1970) and for the diffusion tensor (Gulyaev 1969; see also Gantsevich et al. 1974, 1979). A comparison of these expressions demonstrates the degree of violation of the fluctuation-diffusion relation for the particular case.

3.1 Effective electron temperature and its relaxation

The theoretical investigation of effects of carrier-carrier scattering on high-electric-field properties of semiconductors goes back to Compton (1923), Davydov (1937), Fröhlich and Paranjape (1956), and Stratton (1958). Inter-electron collisions, though conserving energy and quasi-momentum in the electron system, have an indirect effect on high-field transport. When the rate at which the inter-electron collisions redistribute energy within the electron system is larger than the rate at which the electron system transfers the energy to the lattice, the energy distribution, irrespective of the initial distribution, after a time interval of the order of τ_{ee} turns out to be nearly Maxwellian:

$$F(\varepsilon) \propto \exp(-\varepsilon / T_e). \tag{3.4}$$

Correspondingly, the situation where the inequalities (3.1) hold is referred to as the *case of effective electron temperature*. The *effective electron temperature* T_e differs, in general, from the temperature T_0 of the thermal bath and changes comparatively slowly in time due to the gain of energy by the electron system from the external field and the transfer of energy to the lattice:

$$(3n_0/2)\partial T_e/\partial t = \mathbf{j}\cdot\mathbf{E} - P(T_e, T_0), \tag{3.5}$$

where

$$j_\alpha = \sigma_{\alpha\beta}(T)E_\beta \tag{3.6}$$

is the current density and $P(T_e, T_0)$ is the rate of energy loss by the electron system. The explicit form of the functions $\sigma_{\alpha\beta}(T)$ and $P(T_e, T_0)$ - the "chord" conductivity and the rate of energy loss by the electron system - depends on the details of electron interaction with the thermal bath. The *steady-state value of the effective electron temperature* T_e is determined from the condition that the rate at which the electron system gains energy from the field, $\sigma_{\alpha\beta}(T)E_\alpha E_\beta$, equals the rate $P(T_e, T_0)$ at which the electron system loses energy through scattering by the thermal bath:

$$\sigma_{\alpha\beta}(T)E_\alpha E_\beta = P(T,T_0). \tag{3.7}$$

For *small* deviations from the steady state,

$$\Delta T_e(t) \equiv T_e(t) - T_e << T_e, \tag{3.8}$$

equation (3.5) takes the form

$$\partial \Delta T_e / \partial t = -\Delta T_e / \tau_T, \tag{3.9}$$

the *differential time for the electron temperature relaxation* τ_T being given by the expression

$$\tau_T = \frac{3n_0 / 2}{\partial P(T,T_0) / \partial T - E_\alpha E_\beta \partial \sigma_{\alpha\beta}(T) / \partial T} \tag{3.10}$$

(Kogan 1962). The values of derivatives in Eq.(3.10) should be taken at the stationary value of the electron temperature found from Eq.(3.7).

3.2. Small-signal conductivity

From what was said it follows that at not too high frequencies, namely, at those less than the characteristic frequency of inter-electron collisions:

$$\omega\tau_{ee} << 1, \tag{3.11}$$

the frequency dependence of the kinetic coefficients is conditioned by time evolution of the electron temperature. The tensor of AC small-signal conductivity at frequencies complying with inequality (3.11) is given by the expression (see, e.g., Gantsevich et al. 1979).

$$\sigma_{\alpha\beta}(\omega) = \sigma_{\alpha\beta} + \frac{2E_\gamma E_\delta}{3n_0} \frac{\partial \sigma_{\alpha\gamma}}{\partial T} \frac{\sigma_{\beta\delta} + \sigma_{\delta\beta}}{-i\omega + 1/\tau_T}. \tag{3.12}$$

In the case of isotropic medium, where

$$\mathbf{j} = \sigma(T) \cdot \mathbf{E}, \tag{3.13}$$

the longitudinal and transverse, with respect to the direction of the applied electric field, AC small-signal conductivities are given by the expressions (Lifshits et al. 1966):

$$\sigma_{\perp}(\omega) = \tilde{\sigma}(T), \tag{3.14}$$

$$\sigma_{\parallel}(\omega) = \sigma\left(1 + \frac{4E^2 \ \partial\sigma / \partial T}{3n_0(-i\omega + 1/\tau_T)}\right). \tag{3.15}$$

In particular, the *static* longitudinal differential conductivity $\sigma_{\parallel}$ is given by the expression (see Eq.(3.13))

$$\sigma_{\parallel} \equiv \frac{d}{dE}(\sigma(T) \cdot E) = \sigma(T) + 2\frac{d\sigma}{dT} \cdot \frac{dT}{dE^2} E^2 \tag{3.16}$$

or, in the equivalent form following from expression (3.15),

$$\sigma \frac{\partial P / \partial T + E^2 \partial\sigma / \partial T}{\partial P / \partial T - E^2 \partial\sigma / \partial T}. \tag{3.17}$$

Below we shall make use of these expressions in the limiting case of comparatively low electric fields (the case of *weakly heated electron gas*, also referred to as the *warm electrons*). This is the case where the effective electron temperature T_e only slightly exceeds the lattice temperature T_0:

$$T_e - T_0 << T_0, \tag{3.18}$$

the difference $T_e - T_0$ being proportional to the field strength squared:

$$T_e - T_0 \propto E^2 \ . \tag{3.19}$$

Neglecting terms of the order higher than E^2 in the expression (3.16), we have:

$$\sigma_{\parallel} = \sigma\left(1 + \frac{2(T - T_0)}{T_0}\left(\frac{d \ln \sigma}{d \ln T}\right)_{T=T_0}\right). \tag{3.20}$$

The dimensionless quantity $d\ln\sigma / d\ln T_e$ can be referred to as the *coefficient of electric sensitivity to electron heating* of the system in question.

3.3. Expressions for diffusion tensor

In the effective electron temperature case, side by side with the expressions for the small-signal conductivity, the expression for the tensor of electron diffusion coefficients ("tensor of diffusivities", see (2.9)) is available (Gantsevich et al. 1979):

$$D_{\alpha\beta} = \frac{T}{e^2 n_0}\left(\sigma_{\alpha\beta} + \frac{2\tau_T}{3n_0} E_\gamma E_\delta \frac{\partial \sigma_{\alpha\gamma}}{\partial T}(\sigma_{\beta\delta} + \sigma_{\delta\beta} + T\frac{\partial \sigma_{\beta\delta}}{\partial T})\right). \tag{3.21}$$

In the case of an isotropic medium, the expressions for the longitudinal and transverse, with respect to the direction of the external electric field, electron diffusion coefficients are (Gulyaev 1969, Gantsevich et al. 1974):

$$D_{\perp} = T\tilde{\sigma} / e^2 n_0 , \tag{3.22}$$

$$D_{\parallel} = D_{\perp}\left[1 + \frac{4\tau_T}{3n_0} E^2 \frac{\partial \tilde{\sigma}}{\partial T}\left(1 + \frac{1}{2}\frac{\partial \ln \tilde{\sigma}}{\partial \ln T}\right)\right]. \tag{3.23}$$

By comparing Eqs. (3.15) and (3.23) we rewrite the latter as follows:

$$D_{\parallel} = D_{\perp}\left[1+\left(\frac{\sigma_{\parallel}}{\sigma}-1\right)\left(1+\frac{1}{2}\frac{\partial \ln \sigma}{\partial \ln T}\right)\right]. \qquad (3.24)$$

- the magnitude of the coefficient of electric sensitivity to electron heating, $d\ln\sigma / d\ln T_e$, determines to what extent the anisotropy of the electron diffusivity differs from that of differential conductivity.

3.4. Expressions for additional correlation tensor

We have noticed in Section 2.5 (see Eq.(2.11)) that the contribution of the additional correlation, created by the inter-electron collisions, into current fluctuations *vanishes* provided the electron distribution is Maxwellian. In the case of comparatively frequent inter-electron collisions we are now investigating, the *main part* of the distribution function is Maxwellian. May we conclude that the extra correlation is unimportant in the case where inequalities (3.1) hold, and that the spectral intensities of the current fluctuations at low frequencies in this case are proportional to the electron diffusion coefficients?

It would be incautious to jump to such a conclusion without a detailed investigation of the problem. Inter-electron collisions play a double role, shaping the distribution function and creating the additional correlation. The more frequent the inter-electron collisions are the more considerable the additional correlation should be. On the other hand, in the case of more frequent inter-electron collisions the energy distribution is Maxwellian with greater accuracy, and so the extra correlation should more effectively vanish. As was shown by Shulman (1970), these opposing tendencies counterbalance each other, and the extra correlation is, generally speaking, as important in the formation of the longitudinal current fluctuations in a heating electric field as the electron temperature fluctuations are.

To obtain a comparatively simple explicit expression for the additional correlation tensor $\ddot{\mathbf{A}}$ entering Eq.(2.10), one should assume, side by side with inequalities (3.1), a *weak inelasticity* of the scattering of electrons by the lattice ($\Delta\varepsilon << \bar{\varepsilon}$, where $\Delta\varepsilon << \bar{\varepsilon}$ is the characteristic change of the electron energy upon collision). The following expression was derived for $\omega\tau_{ee} << 1$ (see Gantsevich et al. 1979):

$$\Delta_{\alpha\beta}(\omega) = \frac{4T^2\tau_T E_\gamma E_\delta}{3n_0^2 e^2 (1+\omega^2\tau_T^2)} \frac{\partial\sigma_{\alpha\gamma}}{\partial T} \frac{\partial\sigma_{\beta\delta}}{\partial T}\left(1 - \frac{2\tau_T}{3n_0}\frac{P}{T - T_0}\right) \qquad (3.25)$$

from which in the case of an isotropic medium the expressions obtained by Shulman (1970) follows:

$$\Delta_\perp(\omega) = 0 \qquad (3.26)$$

$$\Delta_\parallel(\omega) = -\frac{D_\perp}{1+\omega^2\tau_T^2}\left(\frac{\sigma_\parallel}{\sigma} - 1\right)\left[\frac{T}{2(T - T_0)}\left(\frac{\sigma_\parallel}{\sigma} - 1\right) - \frac{\partial \ln\sigma}{\partial \ln T}\right]. \qquad (3.27)$$

- the extra correlation contributes only to the longitudinal current fluctuations. The contribution is of Lorentz form shaped by the electron temperature relaxation time - as far as the inequalities (3.1), (3.11) hold, only the extra correlation of electron energies play a role.

It is quite easy to see that, thanks to Eq.(3.20) valid up to linear in $T_e - T_0$ terms, the right-hand side of Eq.(3.27) *vanishes in the linear, with respect to* $T_e - T_0$, *approximation*, the additional correlation term $\Delta(\omega)$ appearing only as a correction of the order of $(T_e - T_0)^2 \propto E^4$ (cf. Gantsevich et al. 1979, Bareikis et al. 1992, Dedulevich 1989). In other words, for *warm* electrons in the effective electron temperature approximation the additional correlation contribution vanishes. On the contrary, when $T_e - T_0$ is comparable to T_0 or even larger, the longitudinal additional-correlation term $\Delta_\parallel(\omega)$ in general is of the same order of magnitude as the diffusion coefficients are.

3.5. Expressions for spectral intensities of current fluctuations

The formula for the tensor of spectral intensities of current fluctuations in non-equilibrium has been derived by Gantsevich et al. (1979). In the general form it sounds as follows:

$$(\delta j_\alpha \delta j_\beta)_\omega =$$

$$= \frac{T}{V_0}\left\{\sigma_{\alpha\beta}+\sigma_{\beta\alpha}+\frac{2E_\gamma E_\delta}{3\eta_0}\left[\frac{\partial\sigma_{\alpha\gamma}}{\partial T}\cdot\frac{\sigma_{\beta\delta}+\sigma_{\beta\delta}}{-i\omega+\tau_T^{-1}}+\frac{\partial\sigma_{\beta\gamma}}{\partial T}\cdot\frac{\sigma_{\alpha\delta}+\sigma_{\alpha\alpha}}{i\omega+\tau_T^{-1}}+\frac{\partial\sigma_{\alpha\gamma}}{\partial T}\frac{\partial\sigma_{\beta\delta}}{\partial T}\cdot\frac{4\tau_T^2/3\eta_0}{1+\omega^2\tau_T^2}\cdot\frac{TP}{T-T_0}\right]\right\}. \quad (3.28)$$

In the case of an isotropic medium this expression reduces to those obtained by Kogan and Shulman (1967) (see also Shulman (1970)):

$$(\delta j_\perp^2)_\omega = 2T\tilde{\sigma}/V_0\,, \quad (3.29)$$

and

$$(\delta j_\parallel^2)_\omega = \frac{2T\tilde{\sigma}}{V_0}\left\{1+\frac{\sigma_\parallel/\tilde{\sigma}-1}{1+\omega^2\tau_T^2}\left[1+\frac{T}{4(T-T_0)}\left(\frac{\sigma_\parallel}{\tilde{\sigma}}-1\right)\right]\right\}. \quad (3.30)$$

The frequency dependence of the spectral intensity of the longitudinal current fluctuations reflects the way in which fluctuations of the electron temperature die down, the second term in the curl brackets in Eq.(3.30) having the Lorentz shape. In the low-frequency limit $\omega\tau_T << 1$

$$(\delta j_\parallel^2)_{\omega\tau_T<<1} = \frac{2T\sigma}{V_0}\left\{1+\left(\frac{\sigma_\parallel}{\sigma}-1\right)\left[1+\frac{T}{4(T-T_0)}\left(\frac{\sigma_\parallel}{\sigma}-1\right)\right]\right\} \quad (3.31)$$

- in the effective electron temperature case, the spectral intensities of current fluctuations in the isotropic medium in the low-frequency limit $\omega\tau_T << 1$ are expressible in terms of the longitudinal differential conductivity $\tilde{\sigma}_\parallel$, the chord conductivity $\tilde{\sigma}$, the electron temperature T_e and the ambient temperature T_0 only (this is not true for the diffusivity D and additional correlation Δ taken separately, see Eqs. (3.23), (3.27)).

3.6. Expressions for noise temperature

Now we are in position to calculate the noise temperature, defined by Eq.(2.4), in the effective electron temperature case. It follows from Eqs. (3.4) and (3.29) that, in the case of an isotropic medium, the *transverse noise* temperature *is equal* to the *electron* temperature:

$$T_{n\perp}(\omega) = T, \tag{3.32}$$

while the frequency dependence of the longitudinal noise temperature is given by the expression (Gantsevich et al. 1979)

$$T_{n\parallel}(\omega) = T\left[1 + \frac{T}{4(T - T_0)}\left(\frac{\sigma_\parallel}{\tilde{\sigma}} - 1\right)^2 \frac{1}{(\sigma_\parallel / \tilde{\sigma}) + \omega^2 \tau_T^2}\right] \tag{3.33}$$

so that

$$T_{n\parallel}(\omega) \geq T_{n\perp}(\omega) = T. \tag{3.34}$$

- the longitudinal noise temperature exceeds (or, in special cases, is equal to) the transverse one, the latter, in its turn, being equal to the electron temperature. At low frequencies, where $\omega\tau_T << 1$,

$$T_{n\parallel}(\omega) \geq T_{n\perp}(\omega) = T. \tag{3.35}$$

$$T_{n\parallel} = \frac{T}{T - T_0}\left\{\frac{T}{4}\left[\left(\frac{\sigma_\parallel}{\tilde{\sigma}}\right)^{1/2} + \left(\frac{\tilde{\sigma}}{\sigma_\parallel}\right)^{1/2}\right]^2 - T_0\right\}. \tag{3.36}$$

Let us remind that $T_e(E)$ is the electron temperature in the electric field E, the lattice temperature is T_0; the steady current density is $j = \sigma(T_e)E$, the longitudinal differential conductivity is $\tilde{\sigma}_{\|} \equiv dj/dE$. The expressions (3.33), (3.35), (3.36) are valid provided the inequalities $\tau << \tau_{ee} << \tau_{en}$ hold, τ and τ_{en} being, respectively, the electron momentum and the electron energy relaxation times caused by the electron collisions with the lattice (with impurities, phonons, etc.), τ_{ee}^{-1} being the frequency of the inter-electron collisions.

It follows from Eqs. (3.35), (3.36) that the degree of anisotropy of the low-frequency noise temperature is the function *only* of the anisotropy of the differential conductivity,

$$\underline{\frac{\sigma_{\|}}{\sigma_{\perp}} = \frac{\sigma_{\|}}{\sigma}}\ , \tag{3.37}$$

and of the degree of heating of the electron system, being characterised by the ratio T_e/T_0 :

$$\underline{\frac{T_{n\|}}{T_{n\perp}} = \frac{T_{n\|}}{T} = 1 + \frac{1}{4(1+T_0/T)}\frac{\sigma}{\sigma_{\|}}\left(\frac{\sigma_{\|}}{\sigma} - 1\right)^2} \tag{3.38}$$

Relations (3.38) enable one to verify, to some extent, the validity of the approximations leading to those relations. If the quantities $\underline{\sigma_{\perp}}$, $\underline{\sigma_{\|}}$ and $T_{n\perp}$, $T_{n\|}$ are measured and/or computed through a simulation procedure, the validity of relation (3.38) can be checked up. Remarkable deviations would definitely mean that the electron energy distribution is rather far from the Maxwellian one.

In the case of weakly heated electron gas, where the effective electron temperature T_e only slightly exceeds the lattice temperature T_0 (i.e., $T_e - T_0 << T_0$, see Eqs. (3.18), (3.19)), we have

$$\underline{\frac{T_{n\|}}{T_{n\perp}} = 1 + \frac{T - T_0}{T_0}\left(\frac{\partial \ln \sigma}{\partial \ln T}\right)_{T=T_0}} \tag{3.39}$$

i.e., in the warm-electron case the sensitivity coefficient *squared* enters the expression for the anisotropy of the noise temperature.

The theoretical results presented in this Section will be used in the following Sections while seeking an interpretation of the available experimental and Monte Carlo simulation results.

4. ELECTRONIC NOISE IN DOPED *n*-TYPE GaAs: EXPERIMENT AND MONTE CARLO SIMULATION

During the last years the progress has been achieved in experimental investigation, simulation, and theoretical interpretation of noise properties of doped semiconductors at moderate electric fields. Noise characteristics of silicon-doped n-type gallium arsenide, free electron density $n_0 = 3\ 10^{17}$ cm^{-3}, at 80 K lattice temperature have been obtained experimentally (Aninkevièius et al. 1993, Bareikis et al. 1994, Katilius et al. 1999). The performed Monte Carlo calculations (Matulionis et al. 1997) have showed that the obtained experimental results at moderate fields of few hundreds volts per cm cannot be even qualitatively interpreted within a framework of the model neglecting inter-electron collisions. Thus, microscopic simulation at moderate fields for a realistic model of an electron gas in a doped semiconductor with necessary electron-lattice and inter-electron scattering mechanisms taken into account was called for in order to fit the available experimental data. On these lines, effects of inter-electron collisions on observables have been resolved (Matulionis et al. 1997). Moreover, as will be demonstrated in the following Section, interpretation in terms of the effective electron temperature and its fluctuations - in terms of the theory presented in the preceding Section - was achieved.

In this Section, the procedure of simulation of current fluctuations for a realistic model of an electron gas in a doped semiconductor (Matulionis et al. 1997) is described and the results of computation are compared to the experimental data (Aninkevièius et al. 1993, Bareikis et al. 1994).

4.1. Properties of velocity-correlation functions. Monte Carlo technique

Let us introduce the time-dependent drift velocity of N free electrons weakly interacting among themselves and with an unperturbed thermal bath, i.e., the velocity of the mass centre of the electron system:

$$v_d(t) = \frac{1}{N}\sum_n^N v_i(t) \tag{4.1}$$

where $v_i(t)$ is the instantaneous velocity of the *i*-th electron. Under a steady-state (at equilibrium as well), the drift velocity of the electron system fluctuates around its average (over time, or over an ensemble of the systems) value

$$V_d = \overline{v_d(t)}\,. \tag{4.2}$$

The total energy and quasi-momentum of the electrons being conserved during an inter-electron collision, the fluctuations of the drift velocity,

$$\delta v_d(t) = v_d(t) - V_d\ , \tag{4.3}$$

are caused only by electron interaction with the thermal bath (phonons and impurities in the case of a semiconductor). On the other hand, the instantaneous velocity of the *i*th electron $v_i(t)$ in respect to its long-time average V_d ,

$$\delta v_i(t) = v_i(t) - V_d\ , \tag{4.4}$$

is influenced by all scattering mechanisms in action. In this Subsection, as far as electron velocity fluctuations are considered along a chosen direction (that in which a constant electric field is applied), the vector indices are omitted.

The time-displaced drift-velocity to drift-velocity correlation function is

$$\Phi(t) = N\ \overline{\delta v_d(t_1 + t)\delta v_d(t_1)}\,, \tag{4.5}$$

where the average is taken over t_1 with the time interval between two observations, t, being kept fixed. The function $\Phi(t)$ can be presented as:

$$\Phi(t) = \Phi_{auto}(t) + \Phi_{cross}(t)\,, \tag{4.6}$$

where

$$\Phi_{auto}(t) = \frac{1}{N}\sum_i \overline{\delta v_i(t_1 + t)\delta v_i(t_1)}\,, \tag{4.7}$$

and

$$\Phi_{cross}(t) = \frac{1}{N}\sum_{i\neq j} \overline{\delta v_i(t_1 + t)\delta v_j(t_1)}\,, \tag{4.8}$$

will be referred to as auto-correlation and cross-correlation function, respectively.

The main features of time-displaced correlation of the electron velocities in the presence of inter-electron collisions can be illustrated (Katilius et al. 1996) by the (rather artificial) case when the interaction of electrons with the thermal bath is weak as compared to that between themselves, i.e., when the inter-electron relaxation time τ_{ee} is shorter than that of the electron quasi-momentum relaxation time τ conditioned by an interaction with the thermal bath: $\tau_{ee} << \tau$. In this case the auto-correlation function, starting from its equal-time ($t = 0$) value,

$$\Phi_{auto}(0) = \overline{v_i^2}\,, \tag{4.9}$$

decreases with t mainly due to inter-electron collisions: any collision causes a loss of one-electron velocity auto-correlation, and the shortest time constant, τ_{ee}, dominates the rate of decay of the auto-correlation function $\Phi_{auto}(t)$ in a short time scale.

In equilibrium there is no equal-time cross-correlation: $\Phi_{cross}(0) = 0$. On the other hand, any inter-electron collision, conserving energy and quasi-momentum, cause the correlation of velocities of the two electrons involved, and, for small t, the cross-correlation function $\Phi_{cross}(t)$ grows proportionally to t. The opposite

tendencies in the evolution of $\Phi_{auto}(t)$ and $\Phi_{cross}(t)$ counterbalance each other, and the resultant total (drift-velocity to drift-velocity) correlation function $\Phi(t)$ changes slowly, its decay being caused only by the interaction of the electrons with the thermal bath. So, frequent inter-electron collisions tend to redistribute the correlation between the diagonal and off-diagonal terms in favour of the cross-correlation, the total correlation function for $t << \tau$ being kept approximately constant.

At $t >> \tau_{ee}$ the auto-correlation function becomes small enough, and the cross-correlation function $\Phi_{cross}(t)$ follows closely the total correlation function $\Phi(t)$, both decaying with the time constant τ. So, the cross-correlation function passes over the maximum, its maximum value being under $k_B T_0 / m$ at equilibrium (Katilius et al. 1996).

Let us simulate motion in a uniform electric field of N electrons undergoing different types of scattering events. All electrons move without scattering for the time between two successive "events in the electron system". By the event we mean either a scattering of an electron by the thermal bath or a mutual collision between two electrons. The time between two successive events in the electron system will be referred to as the "time of free flight of the system".

For independent scattering events the time of free flight of the system is defined by the combined scattering rate (Matulionis et al. 1997):

$$\lambda_{comb}(\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_N) = \sum_{i=1}^{N} \lambda_i(\mathbf{v}_i) + \frac{1}{N-1} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \lambda_{ij}^{ee}(\mathbf{v}_i, \mathbf{v}_j), \tag{4.10}$$

where λ_i and λ_{ij}^{ee} are the integral rates of scattering of the *i*-th electron by the thermal bath and by the *j*-th electron, respectively. The factor $(N-1)^{-1}$ normalises the inter-electron scattering rate, so that each electron under simulation is weighted by n_0 / N where n_0 is the electron density. Equation (4.10) reduces to that written down by Hasegawa et al. (1988) for $N = 2$.

The combined scattering rate λ_{comb} depends on the electron velocity distribution. The Monte Carlo procedure deals with the instantaneous velocity distribution rather

than its time-average function. Thus the fluctuations of electron distribution are not ignored. In order to cope with the time-dependent combined scattering rate the Rees self-scattering procedure (see Jacoboni and Lugli (1989)) is applied: a fictitious "scattering rate" is added to the instantaneous combined scattering rate to make the resultant total scattering rate independent of time.

Now, the simulation of synchronous motion of N electrons is straightforward. The procedure starts from a chosen electron velocity distribution. A random number is generated to simulate a realisation of the "time of free flight of the system" determined by the total scattering rate. Since all electrons move without any scattering during the time of free flight, the velocities of all electrons before the scattering event are available, and this is sufficient to calculate the integral scattering rates of each possible scattering event, to make up the combined scattering rate, and to determine the self-scattering rate. Now, another random number is generated to select a type of the scattering event in proportion to its integral scattering rate. Provided the choice falls on the self-scattering, nothing happens in the system, and another free flight is simulated. Provided the choice falls on the *i*-th electron to be scattered by one of the lattice-related mechanisms (phonon, impurity) the consequences of the collision are simulated in the standard way (Jacoboni and Lugli 1989). If the pair collision of the *i*-th electron with the *j*-th electron is selected, then a random number is generated to choose the scattering angle in the $(\mathbf{v}_i, \mathbf{v}_j)$ plane according to the differential inter-electron scattering rate, and the final velocities of the two electrons involved are determined respecting the energy and quasi-momentum conservation.

The final velocity of the electron, or those of the electrons of the pair are used to renew the set of initial velocities for the next free flight. So, the velocities of all electrons are known at any time, and the simulation continues as long as required.

The simulated realisation of the events in the electron system contains information on fluctuations around the steady state. The velocity correlation functions, $\Phi(t)$, $\Phi_{auto}(t)$ and $\Phi_{cross}(t)$ [Eqs. (4.6)-(4.8)] are obtained as averages over the simulation time t_1 for any fixed time difference t.

The proposed "combined scattering rate" technique avoids the short-time-step procedure inherent to conventional Ensemble Monte Carlo methods (Jacoboni and Lugli 1989, Lugli and Ferry 1983, Brunetti et al. 1985, Moško and Mošková 1991). Since the time step in the Ensemble Monte Carlo technique should be chosen essentially shorter than the mean time of free flight of the system, while each step is accompanied by a selection of a type of scattering event (the latter procedure is the

same in both techniques), the combined scattering rate technique seems to be beneficial.

Evidently, Eq.(4.10) can be modified to consider inter-particle collisions of different quasi-particles: electrons and holes, light and heavy holes, Γ and X electrons, etc.

4.2. Comparison to experimental data

It remained to perform calculations for a realistic model of an electron gas in a doped semiconductor in the case where experimental data on microwave noise were available (Aninkevicius et al. 1993, Bareikis et al. 1994), namely, for silicon-doped *n*-type GaAs, $n = 3 \cdot 10^{17}$ cm^{-3}, at an 80 K lattice temperature. Since inter-electron collisions were expected to be important at not too high electric fields (as is well known, the Coulomb scattering mechanisms gradually switches off with an increase of electron energies), calculations were performed (Matulionis et al. 1997) within a framework of a parabolic one-valley (Γ-valley) model, thus ignoring inter-valley transitions. Non-elastic acoustic and optical scattering by phonons (acoustic deformation potential, polar optical) was taken into account, the phonons being supposed to remain in thermal equilibrium. The ionised-impurity scattering and inter-electron pair collisions were taken into account in the screened Coulomb approach. The effect of electron heating on the screening was neglected.

The experimental data on microwave noise were available (Aninkevicius et al. 1993, Bareikis et al. 1994) for silicon-doped n-type GaAs demonstrating zero-field mobility of electrons, μ = 4000 cm^2/Vs at 80 K, essentially lower than that predicted for uncompensated n-type GaAs (see Kuphal et al. (1978)) indicating acceptor contribution. Compensation of donors in silicon-doped n-type GaAs can be important (silicon is an amphoteric impurity in GaAs, and silicon atoms in Ga sites act as donors while those occupying As sites act as acceptors). The technique to evaluate the degree of compensation in GaAs and other compound semiconductors is based on the low-field mobility measurements (see Blakemore (1982), Anderson and Apsley (1986)). The degree of compensation of the doped GaAs samples for which the experimental data on noise were available (Aninkevièius et al. 1993, Bareikis et al. 1994) was determined by Matulionis et al. (1997) from comparison of the measured dependence of current on electric field with those calculated at different densities of ionised impurities for the same experimentally determined free electron density, $n_0 = 3 \cdot 10^{17}$ cm^{-3}. A reasonable fit to the experimental current-voltage characteristics showed by the

samples in question was achieved for the ionised-impurity density $N_I \approx 7.5 \cdot 10^{17}$ cm^{-3} (see Fig.1). Later, experimental data on noise were obtained also for other degrees of compensation (Katilius et al. 1999).

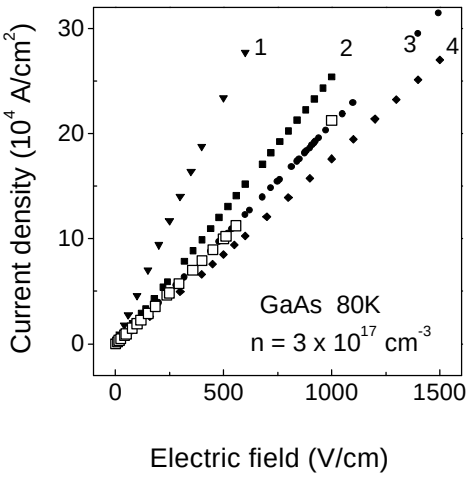

Fig. 1. Measured current-voltage characteristic of the GaAs sample (open squares, Katilius et al. 1999) compared with those calculated, taking into account inter-electron collisions, $n_0 = 3 \cdot 10^{17}$ cm^{-3}, for different ionized impurity densities N_I: 1 - $3 \cdot 10^{17}$ cm^{-3}, 2 - $6 \cdot 10^{17}$ cm^{-3}, 3 - $7.5 \cdot 10^{17}$ cm^{-3}, 4 - 9 $3 \cdot 10^{17}$ cm^{-3}.

The results on the spectral intensity of drift-velocity fluctuations are presented in Fig.2. The spectral intensity of current fluctuations was measured by Aninkevièius et al. (1993), Bareikis et al. (1994) at 10 GHz frequency. The frequency was high enough to avoid $1/f$ and generation-recombination noise, but it was low in comparison to the inverse time constants of the kinetic processes related to electron scattering in the conduction band. The experimental data on the spectral intensity of drift-velocity fluctuations (Fig.2, open squares) were obtained from the current fluctuation data through normalization at zero field by using mobility data and the Nyquist formula.. The closed circles in Fig.2 give the electric field dependence of the spectral intensity resulting from the calculated total correlation function. For comparison, the results of the simulation neglecting inter-electron collisions are shown (Fig.2, diamonds).

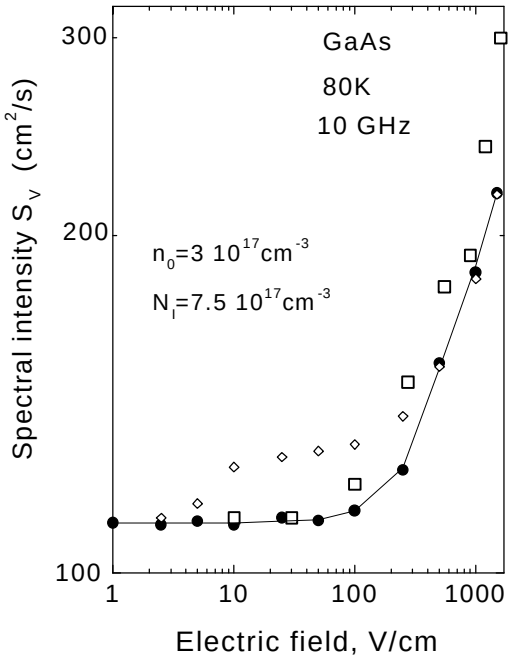

Fig. 2. Dependence of the spectral density of electron drift velocity fluctuations at 10 GHz for Si-doped *n*-type GaAs at 80 K (n_0= 3·10^{17} cm^{-3}, N_I= 7.5·10^{17} cm^{-3}). Monte Carlo simulation: with phonon, impurity, and inter-electron scattering taken into account (closed circles, Matulionis et al.1997), and without inter-electron scattering (diamonds). Experimental data - open squares (Aninkevièius et al. 1993). Solid and dotted lines are guides to the eye.

One can see that the inter-electron collisions have little influence on the fluctuation spectra at very low electric fields and at high electric fields (cf. diamonds and closed circles in Fig.2). The most pronounced effect is obtained at intermediate fields ranging from 5 V/cm to 500 V/cm. The inter-electron collisions cause an essential increase (up to two decades) in the field strength required for the excess hot-electron fluctuations to manifest themselves. For example, one can see from Fig.2 that the field value for 5% enhancement in the spectral intensity of current fluctuations shifts from about 10 V/cm predicted neglecting inter-electron collisions to about 200 V/cm obtained from measurements as well as from Monte Carlo calculation taking into account inter-electron collisions.

The inter-electron collisions being taken into account, the calculated dependence (Fig.2, closed circles) fits the experimental one (Fig.2, open squares), indicating an important role of the inter-electron Coulomb scattering mechanisms in the formation of the spectral intensity of current fluctuations at the intermediate electric fields.

The spectral intensity of velocity fluctuations remains nearly constant at fields up to 200 V/cm (Fig.2, closed circles and open squares). This behaviour can be explained by enhanced energy loss by electrons on optical phonons in the presence of inter-electron scattering (Levinson and Mazhuolyte 1966): the inter-electron-collision-dependent energy losses include spontaneous optical phonon emission by the electrons having acquired enough energy in the result of collisions with other electrons. The role of inter-electron scattering diminishes at higher fields: an increase of electron energy causes the inter-electron scattering rate to decrease and the spontaneous optical phonon emission to become the dominant scattering mechanism.

So, the proposed Monte Carlo procedure was demonstrated to be an efficient tool for studying hot-electron noise in the presence of carrier-carrier scattering. The fluctuation properties of non-equilibrium electron gas in a semiconductor were shown to be sensitive to the presence of inter-electron collisions. Taking them into account is crucial for an explanation of experimental data on microwave noise in doped gallium arsenide at 80K at moderate electric fields (5 ... 500 V/cm).

5. ELECTRONIC NOISE IN DOPED n-TYPE GaAs: ANALYTIC APPROACH

In this Section, experimental results on microwave noise in *n*-type GaAs ($n_0 = 3 \cdot 10^{17}$ cm^{-3}, 80K) at electric fields up to few hundreds V/cm will be interpreted combining analytic approach and Monte Carlo simulation. The analytic approach is based on applicability, due to high frequency of inter-electron collisions, of the electron temperature approximation described in Section 3. Monte Carlo simulation shows (see Fig.1) a rather low sensitivity of the conductance to the electron heating, accompanied by small deviations from the Ohm law. These circumstances were shown to favour a nearly isotropic noise temperature and - as will be demonstrated in the subsequent Section - an approximate validity of the Price fluctuation-diffusion relation.

In the preceding Section, it was demonstrated that noise properties of a doped semiconductor can be remarkably influenced by inter-electron collisions. Using a model which takes into account inter-electron collisions was shown to be indispensable while seeking to fit the results of Monte Carlo calculations to the experimental data on

microwave noise in typical silicon-doped *n*-type GaAs channels (donor density exceeding 10^{17} cm^{-3}) in the field range below 400 V/cm.

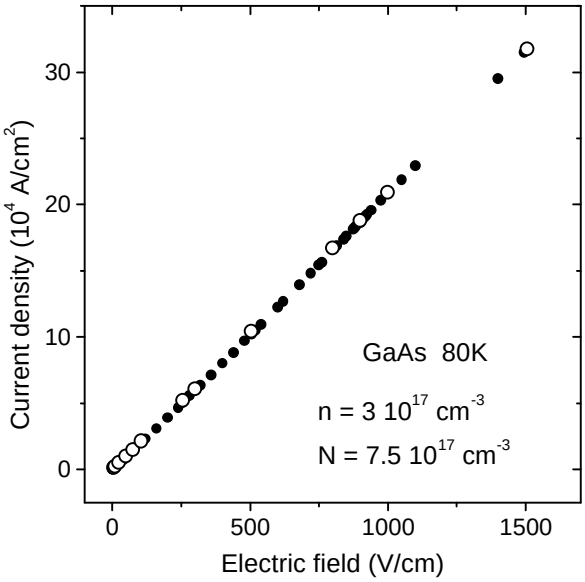

Fig. 3. Current-voltage characteristic of the GaAs sample, $n_0 = 3 \cdot 10^{17}$ cm^{-3}, $N_I = 7.5 \cdot 10^{17}$ cm^{-3}, calculated taking into account (solid circles, see Fig.1) and neglecting (open circles, Katilius et al. 1999) inter-electron collisions.

The obtained agreement of the experimental results and those of Monte Carlo simulation stimulated an *analytic* investigation of the situation (Katilius et al. 1998, 1999). Indeed, the situation in the doped n-GaAs channels is suitable for an analytic treatment. The energy distribution at low and intermediate electric fields was proved to be quite close to a Maxwellian one. This allowed, for the first time, an analytic treatment of the *experimental results* on noise in the non-equilibrium in the situation where inter-electron collisions make difference. On the lines described in Section 3, the role of theoretically predicted inter-electron-collision-born correlation between occupancies of electronic states in non-equilibrium was estimated quantitatively for typical doped GaAs channels (see the subsequent Section).

5.1. Main features of transport and noise characteristics of doped n-type GaAs

To guarantee the experimental basis for the analysis, the measurements of the noise as well as of the current-voltage characteristic in *n*-type GaAs channels containing different density of ionised impurities and essentially the same density of free electrons (around $3 \cdot 10^{17}$ cm^{-3}) were performed by Katilius et al. (1999). The density of electrons was high enough for a well-pronounced effect of inter-electron collisions. On the other hand, the density was low enough for degeneracy effects in the electron gas not to influence the results remarkably. Insignificant degeneracy effects on noise in the case was demonstrated by direct Monte Carlo simulation.

Two features of the typical channel current-voltage characteristic will prove to be quite important for what follows. The experimental results and those of Monte Carlo simulation presented in Fig.1 demonstrate a low sensitivity of conductance to electric field strength -- the deviations from the Ohm law are rather small in the considered range of electric fields. Another important circumstance is a weak dependence of the current-voltage characteristic on the frequency of inter-electron collisions demonstrated by Fig.3, cf. solid circles and open circles in Fig.3.

Figure 4 presents (open squares) the field dependence, measured at 80 K ambient temperature, of the longitudinal noise temperature of the sample, $T_{n\|}$, the "longitudinal" referring to the quantity determined in the direction of the steady current caused by the applied electric field. The same figure presents the longitudinal noise temperature and the transverse one (determined in the direction perpendicular to the steady current) as simulated by Monte Carlo technique neglecting and taking into account inter-electron collisions. The most important features of the noise characteristics of Fig.4 are:

(i) the experimental results on the noise temperature (contrary to those on the current-voltage characteristic) cannot be explained reasonably without taking into account inter-electron collisions;

(ii) in the entire field range where the inter-electron collisions are essential the electron heating remains small enough for the experimental dependence $T_{n\|}(E)$ to be almost parabolic, i.e., the hot electron problem to be treatable in the framework of the "warm electron" approach (this is not the case when fluctuations are simulated neglecting inter-electron collisions, see Fig.4);

(iii) the simulated transverse noise temperature almost coincides with the longitudinal one; in other words, the noise temperature is found to be almost *isotropic* (the *same* in the longitudinal and the transverse directions). The obtained near-

isotropy of the noise temperature is in contrast to the pronounced anisotropy of the noise in non-equilibrium electron gas observed/calculated in the majority of cases investigated earlier (see Gantsevich et al. (1979), Bareikis et al. (1992), Katilius and Miliushyte (1980), Barkauskas and Katilius (1979), Dedulevich et al. (1989)).

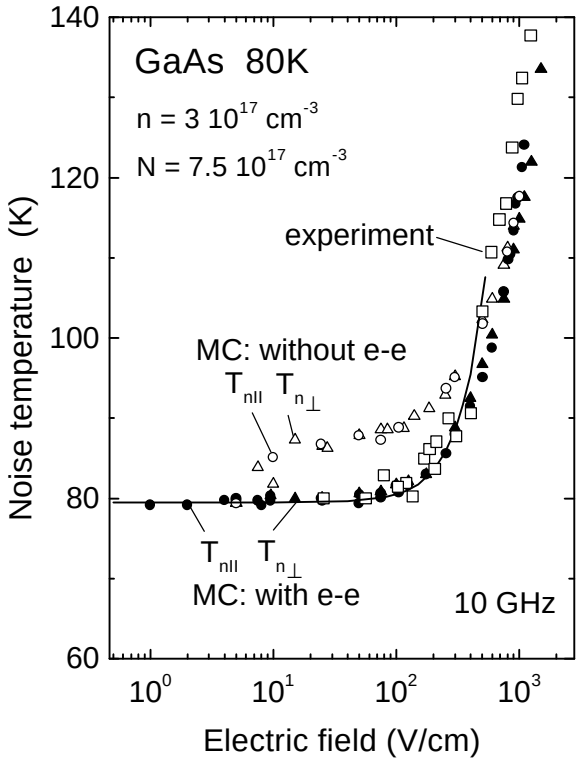

Fig. 4. Measured longitudinal noise temperature $T_{n||}$ (open squares, Katilius et al. 1999 and Liberis et al., private communication) compared with that calculated taking into account (solid circles) and neglecting (open circles) inter-electron (e-e) collisions (Katilius et al. 1999). Transverse noise temperature $T_{n^\wedge}$ calculated taking into account (solid triangles) and neglecting (open triangles) inter-electron collisions is also presented. Measurements were performed under DC bias (up to 260 V/cm) and under pulsed bias (over 100 V/cm). Solid line is a parabolic approximation of the experimental data.

Below we use the analytic approach presented in Section 3 to reveal how these features are interrelated. In particular, it will be shown that the near-isotropy of noise

is related to the observed relatively weak dependence of the conductivity on electric field.

5.2. Electron energy distribution in doped n-type GaAs. Validity of effective electron temperature approximation. Relation to noise temperature

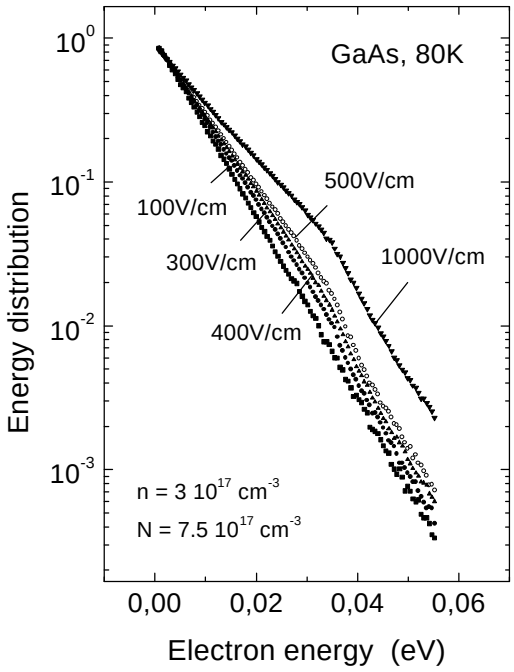

Fig. 5. Electron energy distribution function at different applied electric fields, calculated by Monte Carlo procedure (Raguotis et al., private communication).

An understanding, on the microscopic level, of the inter-electron effects was achieved in terms of the electron temperature and its fluctuations as described in Section 3. At high electron densities, the shape of electron energy distribution at low/moderate fields is controlled by inter-electron collisions (except, may be, the high energy tail). Free-electron density $n_0 = 3 \cdot 10^{17}$ cm^{-3} is high enough to expect, at not too high electric fields, the electron energy distribution to be close to Maxwellian. Direct Monte Carlo simulation (Matulionis et al. 1997) in the framework of the model described in the previous Section confirmed this conjecture for fields up to 400-500

V/cm – see Fig.5. Indeed, inter-electron collisions set a good one-temperature distribution at 100 V/cm, a rather good one-temperature distribution is established at higher fields, up to 400-500 V/cm, while a kink gradually develops at the optical phonon energy (0.036 eV), being quite well detectable at 1000 V/cm. So, for fields up to 400-500 V/cm, the effective electron temperature exists, its dependence on the electric field strength being found from the slope of curves of Fig.5.

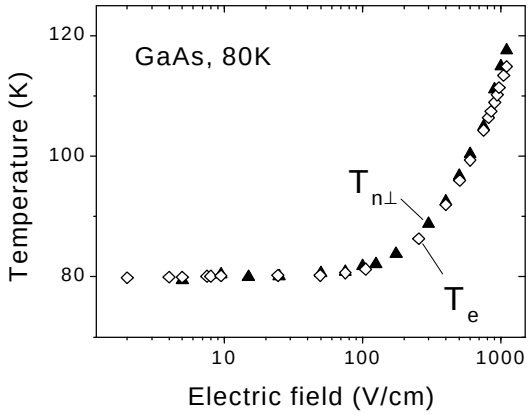

Fig. 6. Field dependence of effective electron temperature T_e (diamonds, cf. Fig. 5) in comparison with the directly simulated transverse noise temperature $T_{n\wedge}$ (triangles).

As one may conclude from Fig.6, the effective electron temperature T_e coincides fairly well with the transverse noise temperature $T_{n\perp}$ obtained simulating the fluctuation process directly (Fig.6, diamonds and triangles). This is not unexpected: according to Eq.(3.35), in the "electron temperature case" the transverse noise temperature of an isotropic semiconductor at sufficiently low frequencies (of microwave range) is equal to the effective electron temperature

$$T_{n\perp} = T_e\,, \tag{5.1}$$

while the longitudinal noise temperature is expressible [Eq.(3.36)] in terms of the conductivity tensor components, the electron temperature, and the lattice temperature. Here we rewrite Eq.(3.36) in an equivalent form:

$$T_{n\|} = T_e\left[1+\frac{T_e}{4(T_e-T_0)}\frac{s}{s_\|}\left(\frac{s_\|}{s}-1\right)^2\right]. \qquad (5.2)$$

According to Eqs. (5.1), (5.2), the longitudinal and the transverse noise temperatures are interrelated through the conductivity components σ and $\tilde{\sigma}_\|$. As noted in Subsection 3.6, this enables one to verify independently the validity of the electron temperature approximation in the situation of interest. Supposing that the quantities $T_{n\perp}$, $T_{n\|}$ and σ, $\tilde{\sigma}_\|$ are measured and/or computed through the simulation procedures, the validity of the relation between $T_{n\perp}$ and $T_{n\|}$ can be checked up. Remarkable deviations would definitely mean that the energy distribution is rather far from a Maxwellian one.

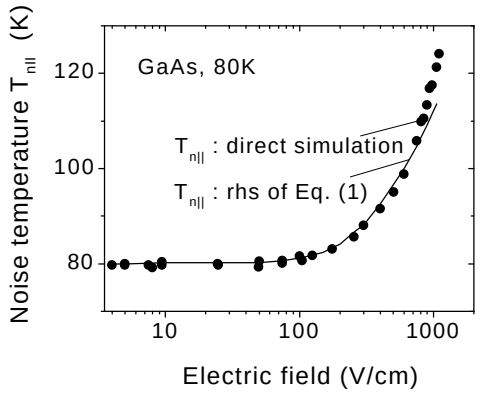

Fig. 7. Directly simulated longitudinal noise temperature taken from Fig.4 (closed circles) in comparison with that calculated as the right-hand side of Eq.(5.2) (solid line).

The obvious confirmation of validity of the electron temperature approximation is presented by the already mentioned fulfilment in practice of relation (5.1) (Fig.6). Further confirmation would be offered by coincidence of data on the longitudinal noise temperature $T_{n\|}$ obtained in two different ways: (i) from the right-hand side of Eq.(5.2) as calculated using the already available data on electron temperature T_e (Fig.6), on σ and on $\tilde{\sigma}_\|$ (Figs. 1, 3), and (ii) from the direct Monte Carlo simulation

of the fluctuation process. One can see that these data (Fig.7, solid line and dots) are pretty close to each other reaffirming the self-consistence of the electron temperature approach at the moderate electric fields where the inter-electron collisions manifest themselves.

5.3. Near-isotropy of noise temperature

The most specific property to be explained by the theory is a rather unexpected nearly-isotropic behaviour of the noise temperature throughout the field range in question, evidenced by Fig.4. To find a reason, another already mentioned feature of the noise-voltage and the current-voltage characteristic was exploited. At fields up to 400--500 V/cm the electron temperature and the noise temperature of the sample increases by less than 20 K, i.e., within 25%, while the conductivity changes even less. As already noticed, the electron energy loss, more efficient than that in absence of the inter-electron collisions, is known to result from the "combined" scattering (Levinson and Mazhuolyte 1966): the inter-electron collisions open an additional electron energy relaxation channel through emission of optical phonons (one of two colliding electrons can gain enough energy to emit an optical phonon).

The field dependencies of the measured noise temperatures, as well as those calculated taking into account the inter-electron collisions, are almost parabolic for the investigated samples at the low and moderate fields. This enables one to expand expression (5.2) for $T_{n\|}$ and the corresponding expression for $\tilde{\sigma}_{\|}$ in powers of electric field strength E to the second order, neglecting terms of the order higher than E^2. In other words, the entire region where the inter-electron collisions make difference can be treated in terms of "warm electrons". Unlike this, if the inter-electron collisions were neglected, the warm electron region would shrink dramatically -- down to about 10 V/cm (see Fig.4).

The expansion of $\sigma(T_e)$ in powers of the increase in electron temperature, $\Delta T_e = T_e - T_0$, the latter being proportional to E^2, in the linear approximation in ΔT_e led to the expression (3.20). As mentioned above, the electric sensitivity to electron heating is rather low at the low/moderate fields -- see Figs. 1, 3. The coefficient of sensitivity ($d\ln\sigma / 2d\ln T_e$) does not exceed 0.1 (see Fig.8, upper curve). Thus, according to Eq.(3.20), the ratio of the differential conductivity components contains the small term resulting into small values of the differential conductivity's anisotropy at the moderate electric fields (see Fig.8, lower curve):

$$\frac{s_{\parallel}}{s} - 1 \approx 0.4 \frac{\Delta T}{T_0} < 0.1$$

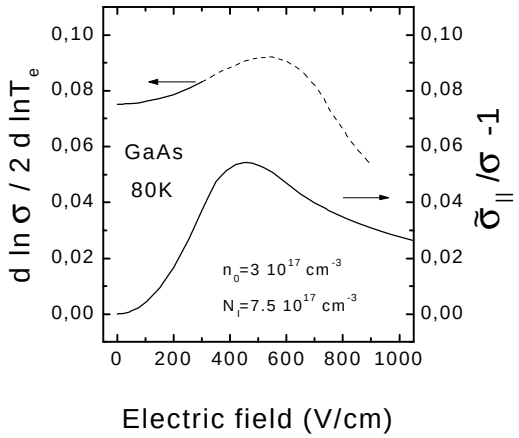

Fig. 8. Sensitivity of conductance to electron heating, $d\ln\sigma / 2\, d\ln T_e$ (left), and anisotropy of differential conductivity $\tilde{\sigma}_{\parallel}/\sigma - 1$ (right), as functions of electric field, fitting the Monte Carlo simulation data (see Katilius et al. 1999).

With this in mind, let us compare the longitudinal and the transverse noise temperatures of the warm electrons. According to Eq.(3.39), in the warm electron region the sensitivity coefficient *squared* decides the noise temperature anisotropy. As a result, the noise temperature anisotropy is extremely low, lower than that of the differential conductivity. Expression (3.39) tells that the noise temperature anisotropy for the typical samples should not exceed 0.01 - 0.04. Figure 4 demonstrates that this important prediction is fulfilled quite well.

5.4. Synopsis

Inter-electron collisions can indirectly but significantly influence the energy losses of the electron system in GaAs channels subjected to electric field at liquid nitrogen temperature. The resultant enhancement of the electron temperature is relatively small in the range of fields where the inter-electron collisions are of importance. This leads

to the changes in fluctuation and transport characteristics which are, with sufficient accuracy, *quadratic in the field strength* at the fields of interest (“warm electrons”). Furthermore, the results of Monte Carlo simulation compared with those of noise measurements show that the electron energy distribution at fields in question is Maxwellian with sufficient accuracy. These circumstances, being typical for the partially compensated *n*-type GaAs channels containing a high density of electrons, ensure applicability of the developed combined analytic and Monte Carlo approach to fluctuation phenomena leading to the following conclusion: a characteristic feature of the GaAs channels at moderate fields being the rather weak sensitivity of conductance on electron heating, the anisotropy of the noise temperature, proportional to the sensitivity coefficient *squared*, is extremely small at the low/moderate electric fields.

6. ELECTRON DIFFUSION IN DOPED *n*-TYPE GaAs

6.1. Field-dependent electron diffusion coefficients in doped GaAs

The investigation of the inter-electron effects is of a special interest for electron diffusivity. As described in Section 2, the fluctuation--diffusion relation has been proposed by Price (1965) to connect the tensor of spectral intensities of current fluctuations in a uniform and stationary electron gas with that of the electron diffusion coefficients, i.e., with the coefficients entering the expressions for the electric current induced by a small electron density gradient. This relation has proved to be very useful in providing information on hot-electron diffusivity from noise measurements performed in spatially homogeneous electron gas in weakly doped semiconductors (see Bareikis et al. 1992, 1994). However, as stated in Subsection 2.5, the kinetic theory of fluctuations predicts that Price's relation in a non-equilibrium electron gas should be violated provided inter-electron collisions were not negligible (Gantsevich et al. 1969b). On the other hand, direct measurements of diffusion coefficients from spreading of a cloud of carriers in doped semiconductors have never been performed: the application of this technique is hindered at a high background density of electrons since the spreading is controlled by dielectric relaxation rather than diffusion. Efficient techniques of Monte Carlo simulation of diffusion process in case of concentration-dependent distribution function are absent (Reggiani et al. 1989, Thobel et al. 1997). So, a determination of the field-dependent coefficient entering Fick's law for the diffusion current for a long time remained an unsolved problem, and quantitative data on hot-electron diffusion coefficients in doped semiconductors at high densities of electrons were lacking. Only recently it was demonstrated (Katilius et al. 1998, 1999)

that sometimes it is possible to determine electron diffusion coefficients from the noise measurements even where inter-electron collisions are essential. The aim of this Section is to present these results.

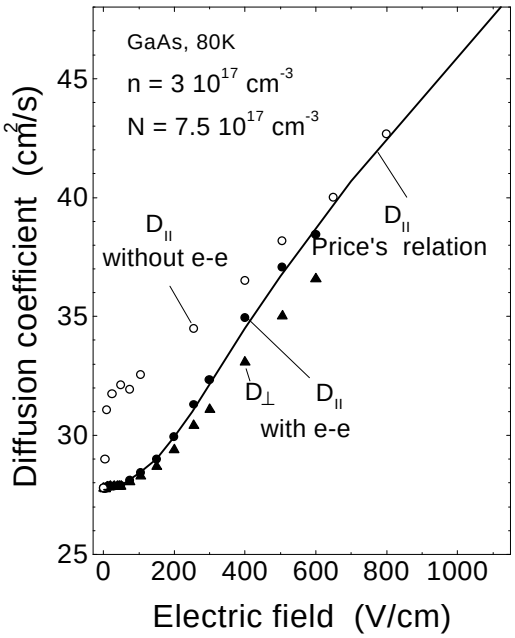

Fig. 9. Field dependence of longitudinal and transverse Fick's diffusion coefficients, $D_{||}$ and $D^{\wedge}$, as predicted by Eqs. (3.22) and (3.23) (solid circles and solid triangles). Solid line presents diffusion coefficient $D_{||}$ as predicted by Price's relation, $D_{||}=(\delta j^2)\omega\tau_{<<1}V_0/2e^2n_0$, current fluctuations being directly simulated (inter-electron collisions included). For comparison, longitudinal diffusion coefficient obtained from noise simulation neglecting inter-electron collisions is also presented (open circles).

In the electron temperature case, side by side with the expressions for the differential conductivity and the noise temperature, the expressions are available for the electron diffusion coefficients D_{ik} entering Fick's law, i.e., the expression for the diffusion current $j_i = -eD_{ik}\P n / \P x_k$. The corresponding expressions have been presented in Subsection 3.3 (see Eqs. (3.22), (3.23)). These expressions enable one to calculate the field dependence of the electron diffusion coefficients from those of the conductivity provided the electron temperature is known from the transverse noise

temperature measurement or from Monte Carlo simulation. In absence of experimental data on the transverse noise temperature, equation (5.2) can be used to extract the electron temperature from the experimental data on the longitudinal noise temperature and the current--voltage characteristics.

In particular, we notice that the quantity $d\ln\sigma / 2d\ln T_e$ determines to what extent the anisotropy of the electron diffusivity is not described by the degree of anisotropy of the differential conductivity, in other words -- to what extent the so-called "Robson conjecture" (see Katilius and Miliushyte (1980)) is not valid. For the typical sample, as already noticed, the coefficient of electric sensitivity to electron heating $d\ln\sigma/(2d\ln T_e)$ does not exceed 0.1 (see the upper curve of Fig.8). Such is the predicted degree of violation of Robson's conjecture in this particular case.

Figure 9 presents the field dependence of the longitudinal and transverse diffusion coefficients obtained from Eqs. (3.22), (3.23) using data of Figs. 3, 6, and 8. For comparison, the longitudinal diffusion coefficient calculated without taking into account the interelectron collisions is also presented in Fig.9.

6.2. Degree of violation of Price relation

We know that the inter-electron collisions in non-equilibrium state create correlation between occupancies of electronic states (Section 2). This leads to violation of the fluctuation--diffusion relation, known as Price's relation, between the current fluctuations and the carrier diffusivity. The spectrum of the current intensity fluctuations in the low-frequency limit, according to Eq.(2.10), is expressible in terms of the electron diffusion coefficients D_{ik} *and* the above-mentioned contribution of the additional (inter-electron-collision-born) correlation Δ_{ik}. In the electron temperature case, the transverse contribution due to the inter-electron-collision-born correlation vanishes (see Eq.(3.26)): $\Delta_{\perp} = 0$, while the longitudinal contribution is expressible (see Eq.(3.27)) in terms of the current-voltage characteristic and the field dependence of the electron temperature:

$$\frac{\Delta_{\parallel}}{2D_{\parallel}} = -\left(\frac{s_{\parallel}}{s} - 1\right)\frac{\dfrac{T_e}{2(T_e - T_0)}\left(\dfrac{s_{\parallel}}{s} - 1\right) - \dfrac{d\ln s}{d\ln T_e}}{2\dfrac{s_{\parallel}}{s} + \left(\dfrac{s_{\parallel}}{s} - 1\right)\dfrac{d\ln s}{d\ln T_e}}. \qquad (6.1)$$

Now we see to what extent Price's relation -- the relation connecting, in absence of inter-electron collisions, the spectral intensities of current fluctuations and the electron diffusion coefficients -- is violated by frequent inter-electron collisions in the electron temperature case. The Price relation survives in the directions perpendicular to the steady current. The degree of violation of the relation between the longitudinal diffusion coefficient and the longitudinal current fluctuation spectral intensity in the case under consideration can be estimated using Eq.(6.1).

As already noticed, the warm electron region in the case under consideration almost coincides with the range of fields where the inter-electron collisions make difference. So, in the entire field range in question we can expand Eq.(6.1) in powers of $\Delta T_e = T_e - T_0$. It is easy to verify that, thanks to Eq.(3.20), valid up to linear in ΔT_e terms, the right-hand side of Eq.(6.1) *vanishes in the linear, with respect to* ΔT_e, *approximation*, the inter-electron-collision-born correlation term $\Delta_{\|}$ as given by Eq.(6.1) appearing only as a correction of the order of $\Delta T_e^{\,2}$ (i.e., E^4) (see Subsection 3.4). At higher fields, the dimensionless contribution given by Eq.(6.1) changes its sign, nevertheless its absolute value remains below $0.15\ \%$ in the entire range of fields of interest (Katilius et al. 1999). It was shown by Katilius et al. (1998, 1999) that a rather weak contribution of the inter-electron-collision-born correlation into noise is typical for partially compensated GaAs channels with electron mobility of about 4000 $\mathrm{cm}^2/(\mathrm{V\ s})$ at $80\,\mathrm{K}$.

In other words, Price's relation between the spectral intensities of current fluctuations and Fick's diffusion coefficients is found to hold quite well in the given situation, despite of the fact that the inter-electron collisions are proved to be important in shaping the electron energy distribution. At high fields, where the inter-electron collisions become ineffective, Price's relation works by definition. We come to an important conclusion that, with a sufficiently high accuracy, *Price's relation in the given situation is valid in the entire range of fields*. Consequently, the electric field dependence of the Fick's diffusion coefficients of the electron system in the considered GaAs channels in a wide range of fields is attainable from noise calculations/measurements. It is important to stress that the Price relation between the spectral intensities of current fluctuations and Fick's diffusion coefficients is found to hold quite well *despite of the fact* that the inter-electron collisions are important in shaping the electron energy distribution and opening the effective channel for the energy relaxation.

The Price fluctuation-diffusion relation is valid with a rather good accuracy since the contribution of the additional correlation created by the inter-electron collisions in the warm electron region is proportional to the enhancement of the electron temperature *squared*. The degree of violation of the Price relation inside and outside the warm electron region is estimated not to exceed 0.01 for the investigated GaAs channels. Due to the demonstrated validity of the Price relation, the electric field dependence of the Fick diffusion coefficients is attainable from noise calculations/measurements over a wide range of fields.

The validity of Price's relation with high accuracy even in the range of fields where in other respects the inter-electron collisions make difference is the specific feature of the GaAs channels containing a high density of electrons. The result is interesting since it regenerates the possibility to obtain information about hot-electron diffusion coefficients from noise measurements even in cases when the inter-electron collisions influence them. This possibility is illustrated by the solid lines (Fig.9*)* obtained from Price's relation, the lines being in agreement with the independently calculated longitudinal diffusion coefficient data in the corresponding ranges of electric fields (*solid circles at the low/moderate fields and open circles at the high fields*). The possibility to exploit Price's relation can prove to be important since a direct measurement or calculation of Fick's diffusion coefficients when inter-electron collisions influence them, as was already mentioned, is by no means straightforward (cf. Reggiani et al. (1989), Thobel et al. (1997)).

The extremely small inter-electron-collision-born correlation is specific to warm electrons in the electron temperature case. Outside the warm electron region the higher terms in the expansion become of importance, and, in general, the ratio $\Delta_{\|} / 2D_{\|}$ may not remain small. This does not happen in our case: the ratio $\tilde{\sigma}_{\|} / \sigma$ entering Eq.(6.1) remains close to unity until the inter-electron collisions cease to influence the noise, and the ratio $\Delta_{\|} / 2D_{\|}$ remains small in the entire field range. Supposing that the non-linearity of the current-voltage characteristic were well-pronounced, the contribution of the inter-electron-collision-born correlation would not be necessarily small in the entire field range where the noise is affected by the inter-electron collisions.

On the other hand, even for warm electrons, the exceptional smallness of the additional correlation contribution cannot be guaranteed beyond the limits of validity of the electron temperature approximation, $\tau << \tau_{ee} << \tau_{en}$. In the following Subsection we present the results obtained by Dedulevich et al. (1989) showing that in

the intermediate cases (e.g., $\tau_{ee} \sim \tau_{en}$) a substantial violation of Price's relation can be conditioned by a non-linear dependence on the electron density of the electron momentum/energy distribution. This possibility of violation of Price's relation is excluded from the very beginning in the electron temperature case in which the shape of the distribution function does not depend on the electron density.

7. FLUCTUATIONS IN WEAKLY HEATED ELECTRON GAS OF INTERMEDIATE DENSITY

Detailed calculations of the diffusion tensor D_{ik} and the correlation tensor Δ_{ik} in the case of "warm" electrons were performed by Dedulevich et al. (1989).

In the general case the correlation tensor Δ_{ik} consists of two different parts:

$$\Delta_{ik} = A_{ik} + B_{ik} \tag{7.1}$$

One of them, A_{ik}, is born by the equal-time electronic-state-occupancy cross-correlation generated by the inter-electron collisions. Up to this moment, we have paid principal attention only to this part of the tensor Δ_{ik}, since in the electron temperature approximation (see inequality (3.1)) the electron distribution function is a *linear function of concentration*, the part B_{ik} vanishes. But in the general case the part B_{ik} can be essential, also being conditioned by inter-electron collisions, but not through the correlation created by them. Inter-electron collisions violate the Price relation not only creating the correlation between the electrons but also changing the dependence of the form of the stationary distribution function on the electron density. Vanishing in the electron temperature case as well as in the drifted Maxwellian case, the term B_{ik} could play an essential role at *intermediate* carrier densities, as was shown by Dedulevich et al. (1989), in the case when inter-electron scattering and the energy relaxation via electron-phonon scattering is of the same efficiency, i.e., the conditions

$$\tau_p << \tau_{ee} \sim \tau_\varepsilon , \tag{7.2}$$

are satisfied, the term B_{ik} in the warm-electron region is even more essential than the pure extra-correlation term A_{ik}.

In accordance with the main idea of the warm-electron theory, the longitudinal and transverse components of the diffusion and correlation tensors were expanded into a Taylor series with respect to the strength of the electric field up to the quadratic terms:

$$D_{\perp,\parallel} = D^{eq}\left[1+\gamma^{D}_{\perp,\parallel}\,(E/E_0)^2\right], \tag{7.3}$$

$$\Delta_{\perp,\parallel} = D^{eq}\gamma^{\Delta}_{\perp,\parallel}\,(E/E_0)^2, \tag{7.4}$$

where D^{eq} is the equilibrium isotropic diffusion coefficient, and the dimensionless coefficients γ^{D} and γ^{Δ} characterise the non-equilibrium correction to the diffusion and correlation tensors, correspondingly.

It was found that the tensor Δ_{ik} in the weak-heating region in the case (7.2) receives an *isotropic* contribution proportional to E^2 (i.e., $\gamma^{\Delta}_{\parallel} = \gamma^{\Delta}_{\perp} = \gamma^{\Delta}$). It was shown that in the weak-heating region in the case of quasi-elastic scattering it is necessary to retain only the term B_{ik} in the tensor Δ_{ik}, the term A_{ik} being parametrically small. In the case of effective electron temperature where the inequality (3.1) holds, the additional correlation term for warm electrons was shown to vanish in Section 3. Consequently, under weak-heating conditions, violation of the Price relation in the case of quasi-elastic scattering is possible only due to the dependence of the *shape* of the stationary distribution function on the electron density at intermediate electron densities, where the inter-electron relaxation time τ_{ee} and the energy relaxation on the lattice time τ_{en} have the same order of magnitude.

For an actual estimate of the extent to which the Price relation can be violated in the weak-heating region, we cite some numerical-calculation results of Dedulevich et al. (1989). The coefficients $\gamma^{D}_{\parallel,\perp}$ and $\gamma^{\Delta} = \gamma^{\Delta}_{B}$ were calculated for quasi-elastic interactions, when the equations for the symmetric parts of the distribution and correlation functions contain the inter-electron-collision term and the Davydov-type term of electron collisions with the heat bath (Davydov 1937). The asymmetric parts of the collision terms were written in the momentum-relaxation time approximation

$\tau(\varepsilon) \propto \varepsilon^{-1/2}$. The Davydov-type operator is a differential operator. The inter-electron collision operator was used in the so-called Landau form, which takes suitable account of the scattering with small momentum change, which is most significant in the case of weak heating. In case (7.2), only the symmetric part of this operator is essential, its integral part being a Volterra operator. Consequently, the equations can be reduced to a system of differential equations, and these were solved numerically by the Runge-Kutta method.

For this model, the coefficients $\gamma_{\parallel}^{D}$, $\gamma_{\perp}^{D}$ and γ_{B}^{Δ} were shown to depend on the only parameter W characterising the relative intensity of the inter-electron scattering compared with the thermal-bath mechanisms of energy relaxation. In the limit $W \to 0$, the coefficients $\gamma_{\parallel}^{D}$ and $\gamma_{\perp}^{D}$ go over into the values obtained earlier by neglecting inter-electron collisions (Gurevich and Katilius 1965), while as $W \to \infty$ the values approach those obtained in the electron-temperature approximation. The coefficient γ_{B}^{Δ}, as it should be, vanishes in both limiting cases. It has a maximum when the rate at which inter-electron collisions redistribute energy within the electron system is of the same order as the rate at which the electron system transfers energy to the thermal bath: it was shown that the time τ_{ee} is equal to τ_{en} for $W \approx 6$, and the maximum of γ_{B}^{Δ} corresponds approximately to the same value of W. The maximal value of the coefficient γ_{B}^{Δ} does not exceed 0.05, so a violation of the Price relation occurs, but is not strongly pronounced. The warm-electron coefficients $\gamma_{\perp}^{D}$ and $\gamma_{\parallel}^{D}$ also were shown to be sensitive to the presence of inter-electron collisions.

As mentioned in the previous Sections, in the case of electron scattering by optical phonons at low lattice temperatures the inter-electron collisions provide a new energy-relaxation channel ("composite" scattering sets in). As a result, inter-electron collisions effectively influence both the energy relaxation itself and the form of the steady-state distribution function. In this situation, the coefficients $\gamma_{\parallel,\perp}^{D}$ and γ_{B}^{Δ} were expected to be more sensitive to the presence of inter-electron collisions, and the

coefficient γ_A^{Δ} should not vanish since the collision operator is not quasi-elastic in the presence of scattering by optical phonons.

The computation of these coefficients was performed by Dedulevich et al. (1989) for a compensated semiconductor with parameters corresponding to the heavy-hole band in *p*-Ge at $T_0 = 80\,\mathrm{K}$. It was demonstrated that hole-hole collisions at intermediate densities ($p \sim 10^{14}$ - 10^{16} cm^{-3}) influence the diffusion coefficients substantially. In the region of intermediate densities the coefficient γ_A^{Δ} is still rather small compared with γ_B^{Δ}, apparently an indirect indication that the inequality $\tau_\varepsilon >> \tau_p$ (but not $\bar{\varepsilon} >> \Delta\varepsilon$) still holds. The ratio of the coefficient γ_B^{Δ} and the non-equilibrium correction to the diffusion coefficient $\gamma_\perp^D$ was found to be substantial (exceeding the value 2.5) at $p \sim 10^{14}$ -10^{15} cm^{-3}. This leads to the hope that there is a possibility of observing experimentally violations of the Price relation in semiconductors at intermediate densities under conditions of weak as well as intermediate heating.

8. FREQUENT INTER-ELECTRON COLLISIONS: ELECTRONIC NOISE IN DRIFTED MAXWELLIAN APPROXIMATION

Analytic expressions are also available in the case where inter-electron collisions are so frequent that they control the electron distribution both in energy and in momentum:

$$\tau_{ee} << \tau_p, \tau_\varepsilon \tag{8.1}$$

where τ and τ_{en} are the characteristic electron momentum and energy relaxation times due to electron collisions with the lattice, and τ_{ee} is the characteristic time for energy and quasi-momentum transfer within the system of electrons.

It is known that in some semiconductors, especially in lead chalcogenides, the screening, owing to lattice polarisation, of the static Coulomb potential of ionised impurities is more effective than the same type of screening of the inter-electron Coulomb interaction (Ravich et al. 1971). Therefore, in uncompensated lead-

chalcogenide samples at high carrier densities and low temperatures one can expect inequality (8.1) to be realised, i.e., the inter-electron interaction to be effective enough to control the electron distribution both in energy and in momentum. Under these conditions, the non-equilibrium noise was investigated by Barkauskas and Katilius (1979).

When inequality (8.1) is fulfilled, inter-electron collisions shape the distribution function, making it close to the *drifted Maxvellian distribution*

$$F_p^{M} \propto \exp\left[-\left(\varepsilon_p - \boldsymbol{p}\cdot\boldsymbol{V}\right)T\right]. \tag{8.2}$$

The stationary values of the temperature T_e and of the drift velocity V of the electron gas are obtainable from the energy and momentum balance equations that follow from the Boltzmann equation. The diffusion-coefficient tensor in the case of distribution (8.2) can be easily shown to be proportional to the differential-conductivity tensor. This means that, under conditions (8.1), the spectral intensity of the current fluctuations is not proportional to the AC small-signal conductivity (and thus the noise temperature is anisotropic) unless the correlation tensor $\Delta_{\alpha\beta}$ vanishes. In fact, the influence of the additional correlation created by inter-electron collisions on the current fluctuations in the case (8.1) turned out to be substantial (Barkauskas and Katilius 1979). Under the condition $\tau \sim \tau_{en}$ the additional correlation affects *not only the longitudinal but also the transverse current fluctuations*. The influence of the additional correlation as well as the anisotropy of the noise temperature takes place in a wide range of frequencies up to $\omega \sim \tau_{ee}^{-1}$.

If the energy of the electron system relaxes more slowly than the momentum:

$$\tau_{ee} << \tau_p << \tau_\varepsilon, \tag{8.3}$$

the additional correlation affects only the longitudinal current fluctuations, and only at frequencies $\omega \le \tau_\varepsilon^{-1}$. In the case of nearly elastic scattering by the lattice ($\Delta\varepsilon << \bar{\varepsilon}$), the longitudinal spectral intensity of the current fluctuations and the longitudinal noise temperature can be expressed in terms of the electron and lattice temperatures, the static differential conductivities, and the electron temperature relaxation time, just as

in the case of effective electron temperature (3.1), i.e., *Eqs. (3.30) and (3.33) also hold in the case (8.3*). However, the contribution of the additional correlation between the occupation numbers of the single-electron states due to the inter-electron collisions is different in cases (3.1) and (8.3). In the latter case,

$$\Delta_{\|}(\omega) = \frac{T^2(\sigma_{\|} - \tilde{\sigma})^2}{2e^2 n_0 \tilde{\sigma}(T - T_0)(1+\omega^2\tau_T^2)} \tag{8.4}$$

while in case (3.1) Eq.(3.27) holds. One can easily see from Eq.(8.4) that for "warm" electrons the correlation contribution in the drifted Maxwelian case does not vanish, contrary to the electron temperature case (3.1) (see Subsection 3.4).

In the drifted Maxwelian case the relative contribution of the additional correlation, i.e., the ratio

$$\frac{\Delta_{\|}}{2D_{\|}} = -\frac{1}{4(1-T_0/T)}\left[\left(\frac{\sigma_{\|}}{\tilde{\sigma}}\right)^{1/2} - \left(\frac{\tilde{\sigma}}{\sigma_{\|}}\right)^{1/2}\right]^2, \tag{8.5}$$

depends *only* on the degree of heating of the electron system and on the non-linearity of the current-voltage characteristic (in case (3.1) it depended, through $\partial\sigma/\partial T$, also on the details of scattering mechanisms - see Eq.(3.27)).

Contrary to Eq.(3.27), the sign of $\Delta_{\|}(\omega) \le 0$ as given by Eq.(8.4) is fixed independently of the details of the scattering mechanisms,

$$\Delta_{\|}(\omega) \le 0 \tag{8.6}$$

provided the scattering on the thermal bath is nearly elastic. We conclude that in the case of frequent inter-electron collisions, Eq.(8.1), and quasi-elastic scattering by the lattice the additional correlation does not reveal itself in the transverse current fluctuations and enhances the longitudinal current fluctuations. The inequality

$$T_{n\parallel}(\omega) \geq T_{n\perp}(\omega) = T \tag{8.7}$$

holds being ensured in this case exclusively by the additional correlation created by inter-electron collisions.

To conclude the Subsection we note that, in the case of the drifted Maxwellian distribution (8.2) ensured by the inequalities $\tau_{ee} << \tau_p, \tau_\varepsilon$, the spectral intensity of the current fluctuations $(\delta j_\alpha \delta j_\beta)_\omega$ contains two characteristic terms: a term proportional to a.c. small-signal conductivity $\mathrm{Re}\,\sigma_{\alpha\beta}(\omega)$ (in the low-frequency limit, also to the symmetrized diffusion tensor $D_{\alpha\beta} + D_{\beta\alpha}$), and a term proportional to the correlation tensor $\Delta_{\alpha\beta}(\omega)$. Contrary to this, in the electron temperature case (3.1) the spectral intensity of the current fluctuations $(\delta j_\alpha \delta j_\beta)_{\omega\tau_\varepsilon<<1}$ consisted of three characteristic terms: that proportional to $\sigma_{\alpha\beta}$, that "transforming" $\sigma_{\alpha\beta}$ into $D_{\alpha\beta}$, and the correlation tensor $\Delta_{\alpha\beta}$.

Therefore the experimental separation of an interesting physical effect - the contribution made to the current fluctuations by the additional correlation due to collisions between the electrons - may turn out to be relatively simple in the case of frequent collisions between the electrons. If it is established independently that the case $\tau_{ee} << \tau_p, \tau_\varepsilon$ is realised in experiment, then to investigate the additional correlation it is sufficient to compare $(\delta j^2)_\omega$ with $\mathrm{Re}\,\sigma(\omega)$ (i.e., it is not mandatory, as in other cases, to have independently measured $(\delta j^2)_\omega$ and D). This makes an experimental investigation of the current fluctuations in the case $\tau_{ee} << \tau_p, \tau_\varepsilon$ quite enticing.

9. CONCLUSIONS

Our task was to demonstrate a remarkable progress achieved during the last years in studies of microwave noise properties of doped semiconductors, or, in more

theoretical terms, of the fluctuation phenomena in the systems which are non-linear with respect to the electric field strength *and* the free-electron density. The systems are quite interesting from physical – theoretical and experimental – point of view. They are of great importance for micro- and nano-electronics as well. The progress resulted from the experimental, Monte Carlo and analytic activities which led to a coherent understanding of the noise phenomena in doped GaAs. The understanding is expected to accelerate a progress in the wider field. One of possible future directions of research is experimental investigation and theoretical interpretation of electronic noise and electron diffusion in other doped semiconductors (Si, InP, GaN, etc.). The approaches developed during the last years and outlined in this review open possibilities to treat fluctuations and electronic noise at a microscopic level also in highly confined electron gas. Next in turn can be investigation of electronic noise in up-to-date semiconductor structures and systems with quantum wells and wires, containing two- and one-dimensional electron gas of high density.

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Perdavimai ir fliuktuacijos netiesinëse disipatyviose sistemose. Tarpdaleliniø kolizijø rolë

(apž valginis straipsnis)

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Šio straipsnio tikslas – apž velgti šiuolaikinius teorinius ir eksperimentinius mikrobanginiø elektrinës kilmës triukðmø ir fliuktuacijø puslaidininkiuose tyrinëjimus. Suderinus analitinius ir Monte Karlo metodus, taip pat þinomus eksperimentinius rezultatus apie triukðmus, tapo ámanoma gauti elektronø difuzijos koeficientus tokiems elektriniams laukams, kuriuose yra reikðmingas elektronø tarpusavio kolizijø áskaitymas, be to Price'o sàryðis nebûtinai tenkinamas. Straipsnyje pateikiama svarbiausia informacija, áskaitant naujausius rezultatus, apie metodus ir pasiekimus ðioje ðiuolaikinëje srityje - srityje, kurioje bûtent netiesinë analitinë ir skaitinë analizë padeda darniai suprasti ir interpretuoti eksperimentinius rezultatus.

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