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Fuzzy Sets Theory in Comparison with Stochastic Methods to Analyse Nonlinear Behaviour of a Steel Member under Compression*

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Abstract. The load-carrying capacity of the member with imperfections under axial compression is analysed in the present paper. The study is divided into two parts: (i) in the first one, the input parameters are considered to be random numbers (with distribution of probability functions obtained from experimental results and/or tolerance standard), while (ii) in the other one, the input parameters are considered to be fuzzy numbers (with membership functions). The load-carrying capacity was calculated by geometrical nonlinear solution of a beam by means of the finite element method. In the case (ii), the membership function was determined by applying the fuzzy sets, whereas in the case (i), the distribution probability function of load-carrying capacity was determined. For (i) stochastic solution, the numerical simulation Monte Carlo method was applied, whereas for (ii) fuzzy solution, the method of the so-called α cuts was applied. The design load-carrying capacity was determined according to the EC3 and EN1990 standards. The results of the fuzzy, stochastic and deterministic analyses are compared in the concluding part of the paper.

Keywords: fuzzy set, membership function, stochastic, steel, imperfection.

1 Introduction

In this paper, methods will be presented on behalf of which the indeterminateness can be modelled. The indeterminateness has (at least) two complementary facets:

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randomness [1] and fuzziness [2]. The fuzziness can be modelled by the fuzzy set theory, whereas the randomness, on behalf of the probability theory. With a little exaggeration, it can be said that the fuzzy set theory answers the question, "what set in as a matter of fact" whereas the probability theory answers the question whether "anything set in".

In project practice, the design reliability is basically ensured by standard rules for design. The contemporary approach is based on the method of partial reliability factors of ultimate limit state, which is, in general, newly introduced by the European unified documents (EUROCODES). The steel beam design load-carrying capacity can be calculated deterministically according to the Eurocode 3 Standard [3]. At the calculation, the input characteristics are considered by characteristic or nominal values. Surely, the deterministic load-carrying capacity calculation method cannot be considered to be fully convenient but another approach is not viable in the project practice. Alternatively, the design load-carrying capacity can be determined by statistical calculation, applying the statistical characteristics of input (material and geometrical) random quantities according to the EN1990 Standard [4] procedure. The standard [4] stipulates, for the load-carrying capacity limit state, the determination of the design value as a quantity obtained from several possible distribution types, see [4]. For the target reliability index $\beta = 3.8$, the design load-carrying capacity can be determined as 0.1% quantile.

In the preset paper, the load-carrying analysis is analysed on a simple example of a member under axial compression. The fuzzy analysis result has been compared with the results of stochastic analysis elaborated by applying the numerical simulation Monte Carlo method. Further on, the deterministic load-carrying capacity values are given for the load-carrying capacity ultimate limit state according to the standards [3] and [4].

2 Fuzzy sets

For the first time, the notion "fuzzy" was used by Prof. Lotfi Zadeh in 1962 [5]. In 1965, L. Zadeh published the paper, legendary at present, "Fuzzy sets" [6]. The fuzzy sets theory or the fuzzy logic is based on the idea that each element in a certain system can get one value within the interval 0 to 1. Mathematically, it can be expressed as follows. Let be X a classical set which generates a space, and its elements let be marked x. The membership of the set A, which is the subset of the space X, can be described by the membership function μ_A, which gets the values $\{0; 1\}$, as follows:

$$\mu_A = \begin{cases} 1, & \text{if and only if } x \in A, \\ 0, & \text{otherwise}. \end{cases} \quad (1)$$

If the membership function can get real values within the interval, the set A is called fuzzy set, and expresses the grade of membership of x to the set A. The more the value approximates to the value 1, the more so x belongs to the set A. At the value zero, the element does not belong to the set, at the value 1, it belongs to that fully; in the other cases, it belongs to the set partly. It is admissible for a fuzzy element to belong to more sets, namely to each series with various grade of membership [2].

The grade of membership has nothing in common with the probability. If we wanted to speak about the probability, we would have to study a phenomenon, whether it would or wouldn't take place. On behalf of the fuzzy sets, however, it is possible to describe the vague notions in themselves.

3 Fuzzy number

Fuzzy numbers are the fuzzy sets, defined on the set of real numbers. Usually, they are supposed to have the special form presented in Fig. 1.

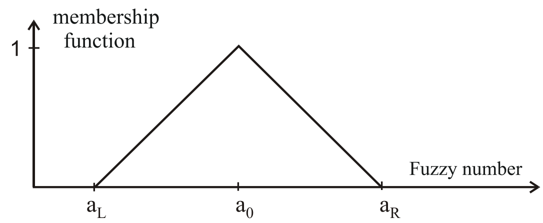

Fig. 1. Membership function of the number "about a_0" (triangle distribution).

A fuzzy number intuitively represents the value which is inaccurate, i.e., the value which can be characterized in words by the expressions "about a_0". Typical examples are "about 5", "roughly 1205", "approximately 1 m", etc. [2]. In practice, we met, quite entirely, the numbers which are fuzzy. When measuring the dimensions of a table by a common metre, the result may be, e.g., 55 cm. However, it means in reality "about 55 cm" because our measurement is rather rough and we cannot be sure whether it might not be, e.g., 55.001 cm, or similar. Let us realize that the measurement results are always inaccurate namely also in the case that we apply the most accurate measuring system which exists.

Let us imagine that, e.g., we will measure the heights of manufactured, hot-rolled beam IPE140, see Fig. 2. When studying the probability, our interest will

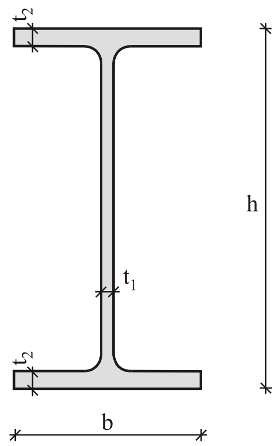

Fig. 2. Geometry of IPE140.

be focused on the occurrence frequency of values within the interval near to the nominal value of 140 mm. On the contrary, the fuzzy set theory informs on to what verity degree it is possible to assume that a hot-rolled IPE140 profile is concerned. The larger the deviation from the nominal value – 140 mm will be, the less it will be true that the IPE140 profile is concerned. The probability informs on the frequency of a phenomenon, whereas the fuzzy set theory determines the phenomenon.

The absolute majority of phenomena in the reality are determined just by vague notions which are dealt with by means of a natural language. In traditional logic, the use of exact notions is assumed which, however, are applicable in case of an ideal idea only. The endeavour at reaching the incessantly better exactness leads to disproportionate increase of definitions, and of the scope of treatises on practically simple things. The limit exactness means the capacity of describing each phenomenon in reality. So, the science gets into the situation of telling always more on an always smaller reality part. Fuzzy words correspond to the reality far better – maybe yes, maybe not, a little, moderately, etc.

4 Input quantities as random and fuzzy numbers

Member with length $L = 1.57$ m was analysed. The corresponding non-dimensional strut slenderness calculated by [3] is $\bar{\lambda} = 1.0$. The loading of a steel strut is demonstrated as an example, see Fig. 3 The load-carrying capacity is limited by geometrical and material characteristics the uncertainty of which conditions also the uncertainty of the load-carrying capacity.

Fig. 3. Member under axial compression.

For the first alternative, the input quantities are assumed to be random [1]. Buckling in the direction of the axis perpendicular to the web plane was taken into account. The initial curvature of the member axis was introduced as one half-wave of the sine function with random amplitude e_0. The Gaussian distribution function of the initiation curvature amplitude e_0 was introduced. Its statistical characteristics were calculated so that the frequency of the occurrence of random realizations within the interval was 95%. For geometrical characteristics of cross-section h (cross-section height), b (flange width), t_1 (web thickness), t_2 (flange thickness), Gaussian distribution is assumed with the mean value equalling the nominal value. The standard deviation S_X has been derived, based on the assumption that 95% of all the realizations (rule $2S_X$) lie within tolerance limits of the Standard [7]. For yield strength of the steel S235, Gaussian distribution with statistical characteristics was considered according to experimental research results [8]. For Young's modulus E, the study was based on the data given in literature [9, 10]. The influence of deviations of physical-mechanical material characteristics (e.g., heterogeneousness), is included in the Young's modulus variability.

For the second alternative, the input characteristics are considered to be fuzzy numbers. The membership functions are assumed to be identical in form with the probability functions, see Tab. 1. It means that the courses of membership functions are nonlinear.

Table 1. Statistical characteristics of input random quantities

No.	Quantity	Name of random quantity	Type of distribution	Dimensions	Mean value	Standard deviation
1.	h	Cross-section height	Gauss	mm	140	1.25
2.	b	Flange width	Gauss	mm	73	1.25
3.	t_1	Web thickness	Gauss	mm	4.7	0.35
4.	t_2	Flange thickness	Gauss	mm	6.9	0.75
5.	e_0	Amplitude of curvature	Lognormal	mm	0.524	0.62
6.	f_y	Yield strength	Gauss	MPa	297.3	16.8
7.	E	Young's modulus	Gauss	GPa	210	12.6

The maximum value of the membership function equals 1, see Figs. 4–10.

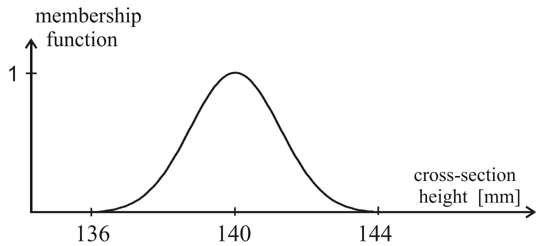

Fig. 4. Membership functions of height h.

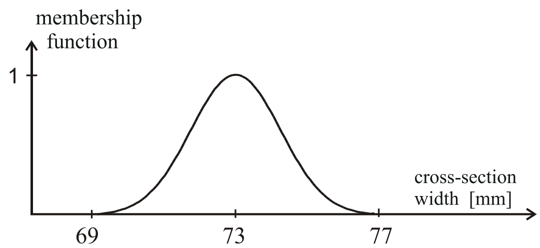

Fig. 5. Membership functions of height b.

If the IPE140 cross-section height equals $140\,$mm, the membership function gets the value 1 (i.e., the statement is absolutely true). For the membership function of the initial curvature amplitude in the form of lognormal distribution, the maximum is not identical with mean value, see Fig. 8.

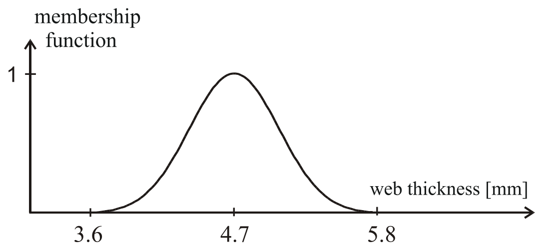

Fig. 6. Membership functions of height t_1.

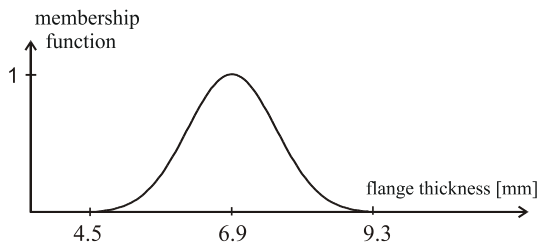

Fig. 7. Membership functions of height t_2.

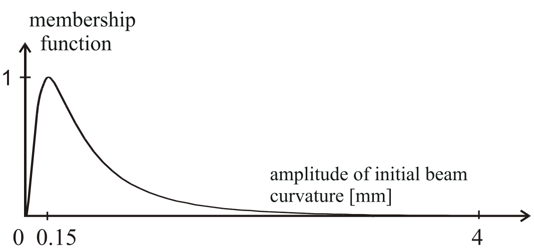

Fig. 8. Membership of amplitude of initial imperfection e_0.

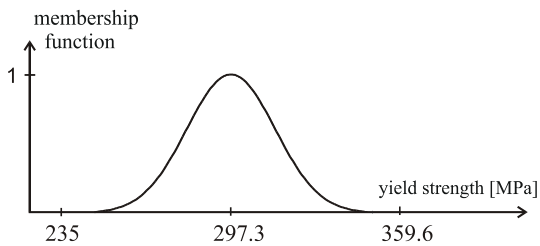

Fig. 9. Membership functions of yield strength f_y.

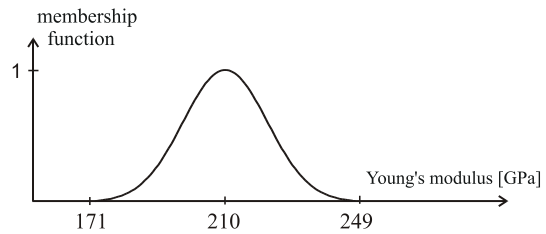

Fig. 10. Membership functions of Young's modulus E.

5 Nonlinear computational model for steel plane beam analysis

Member geometries may be modelled by means of the beam element with initial curvature in the form of a parabola of the 3rd degree [11]. The member was meshed into 10 beam elements. The steel member was solved by the nonlinear Euler incremental method and combined with the Newton-Raphson method. Geometrical and material nonlinearities were considered. The first criterion for the load-carrying capacity is a loading at which plastification of the flange is initiated. The second criterion for the load-carrying capacity is represented by a loading corresponding to a decrease of the determinant to zero. The ultimate one-parametric loading is defined as the lowest value of load-carrying capacity. This phenomenon occurs at high yield point values with small geometrical member imperfections. In each step of the simulation method, the load-carrying capacity was determined to an accuracy of 0.1%. The load-carrying capacity was evaluated for the basic element material only [11].

6 Conclusion

The results obtained by application of the fuzzy set theory and by probability distribution were compared to clear up the difference between the fuzzy distribution and the probability distribution. As both methods applied are based on different assumptions, the comparison of results is difficult. However, the informative value of each method is of a different type.

The full line in Fig. 11 represents the membership function obtained on behalf of the so-called α-cuts [2, 12] for ten layers. The histogram of the relative frequency of random load-carrying capacity was obtained by the Monte Carlo method for 10000 simulation runs. The mean load-carrying capacity of the histogram is 314.8 kN; the standard deviation is 50.8 kN. The value of the membership function for mean load-carrying capacity is 0.93 (in the ascendant part of the diagram), i.e., the verity of the statement that the strut load-carrying capacity is 314.8 kN represents only 93%.

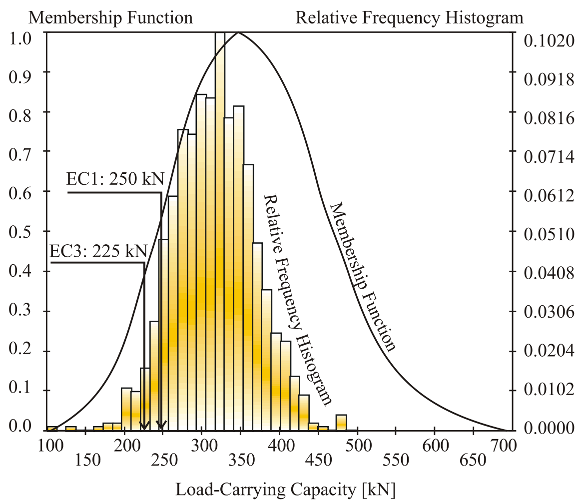

Fig. 11. Comparison of the fuzzy, stochastic and deterministic analysis.

It has been confirmed by the Chi-square test that the Gauss distribution function can be assumed for the random load-carrying capacity. The design load-carrying capacity determined according to [4] from the Gaussian probability distribution for the reliability index $\beta = 3.8$ (as 0.1% quantile) has the value 249.7 kN. This value is by 11% higher than the value 224.56 kN calculated according to the procedures of the Standard for design of steel structures, EC3 [3]. The comparison of design values according to the standards [3, 4] represents one among the possibilities how to calibrate and verify the standard design procedures, or, as the case may be, how to analyse the steel structure reliability by applying the data experimentally found.

The stochastic and fuzzy set theories cannot be considered to be an omnipotent mean which will solve all the problems automatically. They have to be understood as an appropriate instrument for modelling the indeterminateness. As the main objective of fuzzy sets is the modelling of the semantics of a natural language there exist numerous specialisations in which the fuzzy sets can be applied. In the field of the design of building structures, the papers [13, 14] can be mentioned.

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