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Application of the Total Approximation Method for the Investigation of the Temperature Regime of a Polychromatic Solid-State Lamp*

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Abstract. In this paper the application of the total approximation method for polychromatic solid-state lamp was considered. The main goal of this work was to present an investigation method for the temperature regime of n LEDs based on the investigation of the temperature regime of one LED. There were presented numerical results of the investigated problem.

Keywords: light-emitting diode, lighting source, temperature regime, total approximation method.

1 Introduction

Nowadays there has been huge interest in alternative lighting sources. Most of the electric energy used in lighting can be saved by swiching from standard to efficient and cold solid-state lighting sources. The creation of efficient sources of white light is the ultimate goal of the solid-state lighting technology [1]. The polychromatic solid-state lamps which produce white light by additive mixing of the emissions from primary colored light emitting diodes [2] are used in various applications (infrared LEDs in remote controls (for TVs, VCRs etc), clusters in

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traffic signals that replace ordinary bulbs behind colored glass, car indicator lights and lightings for bicycles and pedestrians, calculator and measurement instrument displays, although now mostly replaced by LCDs).

LEDs offer benefits in terms of maintenance and safety. The typical working lifetime of the device, including the bulb, is ten years, which is much longer than the lifetimes of most other light sources. Further, LEDs fail by dimming over time, rather than the abrupt burn-out of incandescent bulbs. LEDs give off less heat than incandescent light bulbs and are less fragile than fluorescent lamps. Since an individual device is smaller than a centimetre in length, LED-based light sources used for illumination and outdoor signals are built using clusters of tens of devices.

The creation of the lighting sources producing white light was investigated by A. Žukauskas, R. Gaška, R. Vaicekauskas *et al.* in [3, 4]. They solved the optimization problem of multi color LED which allowed them to design and buil Versatile Solid State Lamps with adjustible spectrum for treating affective disorder [5–7].

The main goal of this work was to present an investigation method for the temperature regime of n LEDs, based on the investigation of the temperature regime of one LED. This simplification was necessary because of huge amount of LEDs and quite complex nonlinear area structure with nonlinear variation area of variables. The principal idea was to split a solid-state lamp with many light-emitting diodes (see Fig. 1(a)) to one LED lamp (see Fig. 1(b)). Also the area of the whole lamp can be decreased because the temperature far from LED is uniform (Fig. 1(c)) and there is no necessity to do needless calculations. After splitting one LED lamp and solving its temperature regime problem such number

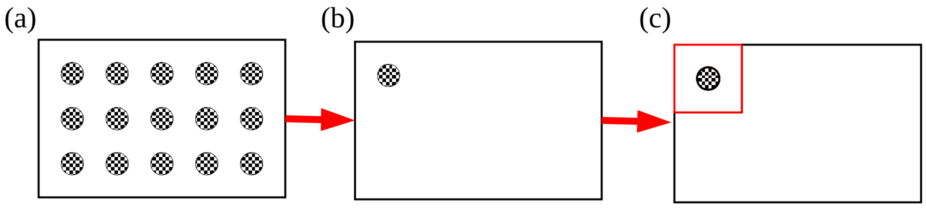

Fig. 1. (a) Light-emitting diode polychromatic lamp; (b) the same lamp as in (a), but with one LED after applying splitting method; (c) decreasing of the investigation lamp area.

of time as we have different kind of light-emitting diodes we can freely use a total approximation method for summing up the solutions of these problems. We prove by an example that the total approximation method gives the same solution as if we solve the whole LEDs polychromatic lamp problem.

2 Mathematical model

It has to be mentioned that light-emitting diodes were placed on the iron plate and covered by air (Fig. 2).

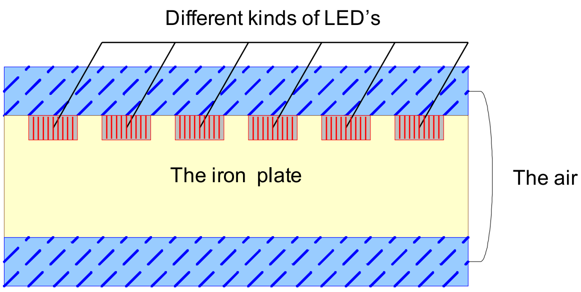

Fig. 2. Solid-state lamps system section: on the iron plate is the array of light emitting diodes and the iron plate is covered by air.

The beginning of the description of the mathematical method let start with the definition of the inside and boundary areas of the investigating system. Let $\bar{\Omega} = \Omega \cup \Gamma$ be the finite closed area of a system with boundary Γ ($\Omega = \Omega_1 \cup \Omega_2 \cup \Omega_3$ the whole inside area, Ω_1 – the air area, Ω_2 – the iron area, Ω_3 – the LEDs area).

The temperature regime of the investigating LEDs system consists of a system of the following differential equations:

$$\begin{aligned} \frac{\partial u}{\partial t} &= \operatorname{div}(D_1 \operatorname{grad} u), \quad x \in \Omega_1, \\ \frac{\partial u}{\partial t} &= \operatorname{div}(D_2 \operatorname{grad} u), \quad x \in \Omega_2, \\ \frac{\partial u}{\partial t} &= \operatorname{div}(D_3 \operatorname{grad} u) + f, \quad x \in \Omega_3, \quad t > 0, \quad x = (x_1, x_2, x_3). \end{aligned} \tag{1}$$

The initial condition

$$u(0, x) = u_0, \quad x \in \bar{\Omega}. \tag{2}$$

The boundary condition

$$u(t,x) = u_0, \quad x \in \Gamma, \quad t > 0, \tag{3}$$

where u is the temperature, $D_l = k_l(\rho_l Q_l)$ $(l = 1,2,3)$ is the constant diffusion rate of materials, k_l is the thermal conductivity coeficient of materials, Q_l is the specific heat of each material, ρ_l is the density of air, iron and LED, f is the light source parameter and t is time.

To investigate such a system is quite difficult because the investigated area is difficult, with a huge amount of LEDs, and has complex nonlinear structure, that is why the problem was simplified into one (Fig. 3) light emitting diode problem.

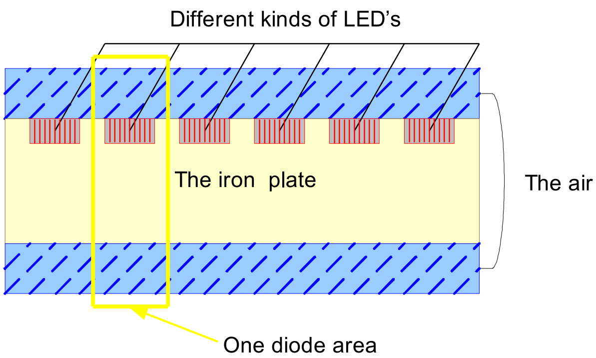

Fig. 3. The separation of the system (one LED system).

Denote $u = \widetilde{u} + u_0$, $\widetilde{u} = \sum_{k=1}^{K} u_k$ and $f = \sum_{k=1}^{K} f_k$, where K – the amount of light emitting diodes, u_0 – constant (room temperature), then the first task is

$$\begin{aligned} \frac{\partial u_k}{\partial t} &= \operatorname{div}(D_1 \operatorname{grad} u_k), \quad x \in \Omega_{1k}, \\ \frac{\partial u_k}{\partial t} &= \operatorname{div}(D_2 \operatorname{grad} u_k), \quad x \in \Omega_{2k}, \\ \frac{\partial u_k}{\partial t} &= \operatorname{div}(D_3 \operatorname{grad} u_k) + f_k, \quad x \in \Omega_{3k}, \end{aligned} \tag{4}$$

with initial condition $u_k(0,x) = 0$, $x \in \bar{\Omega}_k$ and boundary condition $u_k(t,x) = 0$, $x \in \Gamma_k,\ t > 0,\ x = (x_1, x_2, x_3)$. The sum of these k problems and u_0 gives the approximate solution of (1)–(3) problem. We solve (4) problem so many times as we have a different kind of light-emitting diodes and then we duplicate the solutions of the same power diodes.

First of all we denote that $\Omega_{1k} = \Omega_{11k} \cup \Omega_{12k}$ be the inside air area above (Ω_{12k}) and under (Ω_{11k}) the iron plate, and $\Gamma_{1k} = \Gamma_{11k} \cup \Gamma_{12k}$ be the boundary of air area above (Γ_{12k}) and under (Γ_{11k}) the iron plate. Γ_{11k}^{top} denotes the top of Γ_{11k}, $\Gamma_{11k}^{top} \subset \Gamma_{11k}$, Γ_{11k}^{bot} – the bottom of Γ_{11k}, $\Gamma_{11k}^{bot} \subset \Gamma_{11k}$, Γ_{12k}^{top} – the top of Γ_{12k}, $\Gamma_{12k}^{top} \subset \Gamma_{12k}$ and Γ_{12k}^{bot} – the bottom of Γ_{12k}, $\Gamma_{12k}^{bot} \subset \Gamma_{12k}$.

Let Ω_{2k} be the inside area and Γ_{2k} the boundary of the iron plate. Γ_{2k}^{top} denotes the top of Γ_{2k}, $\Gamma_{2k}^{top} \subset \Gamma_{2k}$, Γ_{2k}^{bot} – the bottom of Γ_{2k}, $\Gamma_{2k}^{bot} \subset \Gamma_{2k}$.

Let Ω_{3k} be the k^{th} LED inside area, Γ_{3k} – the boundary. Γ_{3k}^{top} denotes the top of Γ_{3k}, $\Gamma_{3k}^{top} \subset \Gamma_{3k}$.

Beside available boundary conditions additional flow conditions have to be defined:

$$
\begin{aligned}
& D_1 \frac{\partial u_k}{\partial n}\Big|_{\Gamma_{11k}^{top}} = D_2 \frac{\partial u_k}{\partial n}\Big|_{\Gamma_{2k}^{bot}}, \quad D_3 \frac{\partial u_k}{\partial n}\Big|_{\Gamma_{3k} \setminus \Gamma_{3k}^{top}} = D_2 \frac{\partial u_k}{\partial n}\Big|_{\Gamma_{3k} \setminus \Gamma_{3k}^{top}}, \\
& D_3 \frac{\partial u_k}{\partial n}\Big|_{\Gamma_{3k}^{top}} = D_1 \frac{\partial u_k}{\partial n}\Big|_{\Gamma_{3k}^{top}}, \quad D_2 \frac{\partial u_k}{\partial n}\Big|_{\Gamma_{2k}^{top}} = D_1 \frac{\partial u_k}{\partial n}\Big|_{\Gamma_{12k}^{bot}},
\end{aligned}
$$

where $\partial u / \partial n\big|_{\Gamma}$ is the derivative of u with respect to the internal normal direction to the surface Γ.

3 Numerical solution

The finite difference technique [8] was used for mathematical model discretization because of the investigated system difficulty which makes an analytical solution impossible. Let U_{ijs}^{n} is the solution of difference scheme, which approximates (4) problem. The discrete mesh was constructed in such a way (Fig. 4):

$$
\begin{aligned}
& x_{1i} = i \cdot h_1, \quad i = 0, 1, \ldots, N_{a_1}, \ldots, N_{a_2}, \ldots, N_a, \\
& h_1 = \frac{a}{N_a}, \quad a_1 = h_1 \cdot N_{a_1}, \quad a_2 = h_1 \cdot N_{a_2};
\end{aligned}
$$

$$
\begin{aligned}
&x_{2j} = j \cdot h_2, \quad j = 0, 1, \ldots, N_{b_1}, \ldots, N_{b_2}, \ldots, N_{b_3}, \ldots, N_b,\\
&h_2 = \frac{b}{N_b}, \quad b_1 = h_2 \cdot N_{b_1}, \quad b_2 = h_2 \cdot N_{b_2}, \quad b_3 = h_2 \cdot N_{b_3};\\
&x_{3s} = s \cdot h_3, \quad s = 0, 1, \ldots, N_{c_1}, \ldots, N_{c_2}, \ldots, N_c,\\
&h_3 = \frac{c}{N_c}, \quad c_1 = h_3 \cdot N_{c_1}, \quad c_2 = h_3 \cdot N_{c_2};\\
&t_n = n \cdot \tau, \quad n = 0, \ldots, M.
\end{aligned}
$$

Time step τ was chosen to fulfil the solution stability condition $\tau \leq \frac{h^2}{6D}$, here $h = \min\{h_1, h_2, h_3\}$, $D = \max\{D_1, D_2, D_3\}$.

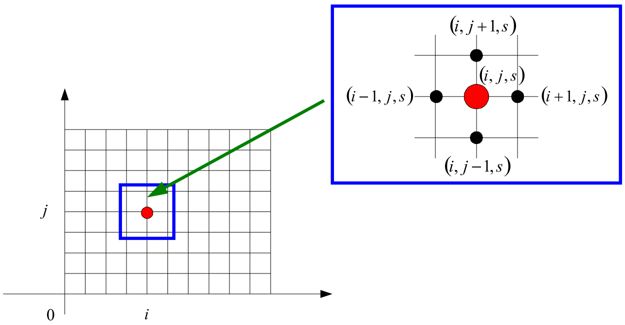

Fig. 4. Mesh construction.

Let consider the approximation of differential equations and initial and boundary conditions by difference equations. After using explicit scheme (4) the system of differential equations was changed by following difference equations:

$$
\begin{aligned}
U_{ijs}^{n+1} &= \frac{\tau D_1}{h_1^2}(U_{i+1,j,s}^n + U_{i-1,j,s}^n) + \frac{\tau D_1}{h_2^2}(U_{i,j+1,s}^n + U_{i,j-1,s}^n)\\
&\quad + \frac{\tau D_1}{h_3^2}(U_{i,j,s+1}^n + U_{i,j,s-1}^n) + \Big(1 - 2\frac{\tau D_1}{h_1^2} - 2\frac{\tau D_1}{h_2^2} - 2\frac{\tau D_1}{h_3^2}\Big)U_{ijs}^n,\\
U_{ijs}^{n+1} &= \frac{\tau D_2}{h_1^2}(U_{i+1,j,s}^n + U_{i-1,j,s}^n) + \frac{\tau D_2}{h_2^2}(U_{i,j+1,s}^n + U_{i,j-1,s}^n)\\
&\quad + \frac{\tau D_2}{h_3^2}(U_{i,j,s+1}^n + U_{i,j,s-1}^n) + \Big(1 - 2\frac{\tau D_2}{h_1^2} - 2\frac{\tau D_2}{h_2^2} - 2\frac{\tau D_2}{h_3^2}\Big)U_{ijs}^n,
\end{aligned}
$$

$$
\begin{aligned}
U_{ijs}^{n+1} &= \frac{\tau D_3}{h_1^2}(U_{i+1,j,s}^n + U_{i-1,j,s}^n) + \frac{\tau D_3}{h_2^2}(U_{i,j+1,s}^n + U_{i,j-1,s}^n) \\
&+ \frac{\tau D_3}{h_3^2}(U_{i,j,s+1}^n + U_{i,j,s-1}^n) + \Big(1 - 2\frac{\tau D_3}{h_1^2} - 2\frac{\tau D_3}{h_2^2} - 2\frac{\tau D_3}{h_3^2}\Big)U_{ijs}^n \\
&+ \tau \cdot f_k.
\end{aligned}
$$

After denoting

$$
\begin{aligned}
&I = U_{i+1,j,s}^n + U_{i-1,j,s}^n, \quad J = U_{i,j+1,s}^n + U_{i,j-1,s}^n, \quad S = U_{i,j,s+1}^n + U_{i,j,s-1}^n, \\
&o_m = \frac{\tau D_1}{h_m^2}, \quad g_m = \frac{\tau D_2}{h_m^2}, \quad d_m = \frac{\tau D_3}{h_m^2}, \quad m = 1, 2, 3,
\end{aligned}
$$

difference equations are:

$$
\begin{aligned}
U_{ijs}^{n+1} &= o_1 \cdot I + o_2 \cdot J + o_3 \cdot S + (1 - 2o_1 - 2o_2 - 2o_3)U_{ijs}^n, \\
U_{ijs}^{n+1} &= g_1 \cdot I + g_2 \cdot J + g_3 \cdot S + (1 - 2g_1 - 2g_2 - 2g_3)U_{ijs}^n, \\
U_{ijs}^{n+1} &= d_1 \cdot I + d_2 \cdot J + d_3 \cdot S + (1 - 2d_1 - 2d_2 - 2d_3)U_{ijs}^n + \tau \cdot f_k.
\end{aligned}
$$

After initial and boundary conditions approximation we have:

$$
\begin{aligned}
&U_{i,j,s}^0 = 0, \quad i = \overline{0, N_a}, \quad j = \overline{0, N_b}, \quad s = \overline{0, N_c}; \\
&U_{0,j,s}^0 = 0, \quad U_{N_a,j,s}^n = 0, \quad j = \overline{1, N_b - 1}, \quad s = \overline{1, N_c - 1}; \\
&U_{i,j,0}^0 = 0, \quad U_{i,j,N_c}^n = 0, \quad i = \overline{1, N_a - 1}, \quad j = \overline{1, N_b - 1}; \\
&U_{i,0,s}^0 = 0, \quad U_{i,N_b,s}^n = 0, \quad i = \overline{1, N_a - 1}, \quad s = \overline{1, N_c - 1}; \\
&U_{i,N_{b_1},s}^n = \frac{D_2 U_{i,N_{b_1}+1,s}^n + D_1 U_{i,N_{b_1}-1,s}^n}{D_1 + D_2}, \\
&\qquad i = \overline{1, N_a - 1},\ s = \overline{1, N_c - 1}; \\
&U_{i,N_{b_3},s}^n = \frac{D_1 U_{i,N_{b_3}+1,s}^n + D_2 U_{i,N_{b_3}-1,s}^n}{D_2 + D_1}, \\
&\qquad i = \overline{1, N_a - 1},\ s = \overline{1, N_{c_1} - 1}; \\
&U_{i,N_{b_3},s}^n = \frac{D_1 U_{i,N_{b_3}+1,s}^n + D_2 U_{i,N_{b_3}-1,s}^n}{D_2 + D_1}, \\
&\qquad i = \overline{1, N_a - 1},\ s = \overline{N_{c_2} + 1, N_c - 1}; \\
&U_{i,N_{b_3},s}^n = \frac{D_1 U_{i,N_{b_3}+1,s}^n + D_2 U_{i,N_{b_3}-1,s}^n}{D_2 + D_1}, \\
&\qquad i = \overline{1, N_{a_1} - 1},\ s = \overline{N_{c_1} + 1, N_{c_2} - 1};
\end{aligned}
$$

$$U^n_{i,N_{b_3},s} = \frac{D_1 U^n_{i,N_{b_3}+1,s} + D_2 U^n_{i,N_{b_3}-1,s}}{D_2 + D_1},$$
$$i = \overline{N_{a_2}+1, N_a - 1},\ s = \overline{N_{c_1}+1, N_{c_2}-1};$$
$$U^n_{i,N_{b_2},s} = \frac{D_3 U^n_{i,N_{b_2}+1,s} + D_2 U^n_{i,N_{b_2}-1,s}}{D_2 + D_3},$$
$$U^n_{i,N_{b_3},s} = \frac{D_1 U^n_{i,N_{b_3}+1,s} + D_3 U^n_{i,N_{b_3}-1,s}}{D_3 + D_1},$$
$$i = \overline{N_{a_1}+1, N_{a_2}-1},\ s = \overline{N_{c_1}+1, N_{c_2}-1};$$
$$U^n_{N_{a_1},j,s} = \frac{D_3 U^n_{N_{a_1}+1,j,s} + D_2 U^n_{N_{a_1}-1,j,s}}{D_2 + D_3},$$
$$U^n_{N_{a_2},j,s} = \frac{D_2 U^n_{N_{a_2}+1,j,s} + D_3 U^n_{N_{a_2}-1,j,s}}{D_3 + D_2},$$
$$j = \overline{N_{b_2}+1, N_{b_3}-1},\ s = \overline{N_{c_1}+1, N_{c_2}-1};$$
$$U^n_{i,j,N_{c_1}} = \frac{D_3 U^n_{i,j,N_{c_1}+1} + D_2 U^n_{i,j,N_{c_1}-1}}{D_2 + D_3},$$
$$U^n_{i,j,N_{c_2}} = \frac{D_2 U^n_{i,j,N_{c_2}+1} + D_3 U^n_{i,j,N_{c_2}-1}}{D_3 + D_2},$$
$$i = \overline{N_{a_1}+1, N_{a_2}-1},\ j = \overline{N_{b_2}+1, N_{b_3}-1}.$$

4 Results of calculations

For numerical calculations the investigated (4) system was constructed from 5 by 5 by 5 mm of light emitting diodes, 15 by 15 by 15 mm of an iron plate, 15 by 15 by 5 mm of the air layer above and under the iron plate and the diode system.

The solution to (4) model with two different kinds of LEDs is achieved with the following values of the parameters (Table 1):

Table 1. Materials parameters

Material	Density g/mm^3	Thermal conductivity W/(mmK)	Specific heat J/(gK)	Diffusion coefficient Wmm^2/J
Air	$1.29 \cdot 10^{-6}$	$2.41 \cdot 10^{-2}$	1.02	18315.8
Iron	$7.88 \cdot 10^{-3}$	$7.40 \cdot 10^{-2}$	0.44	21.34

The lighting source parameter was chosen to be the constant depending on power.

The results of the calculations of the temperature regime of problem (4) are shown in Fig. 5 (till the stationary process). (a) and (b) shows different tempera-

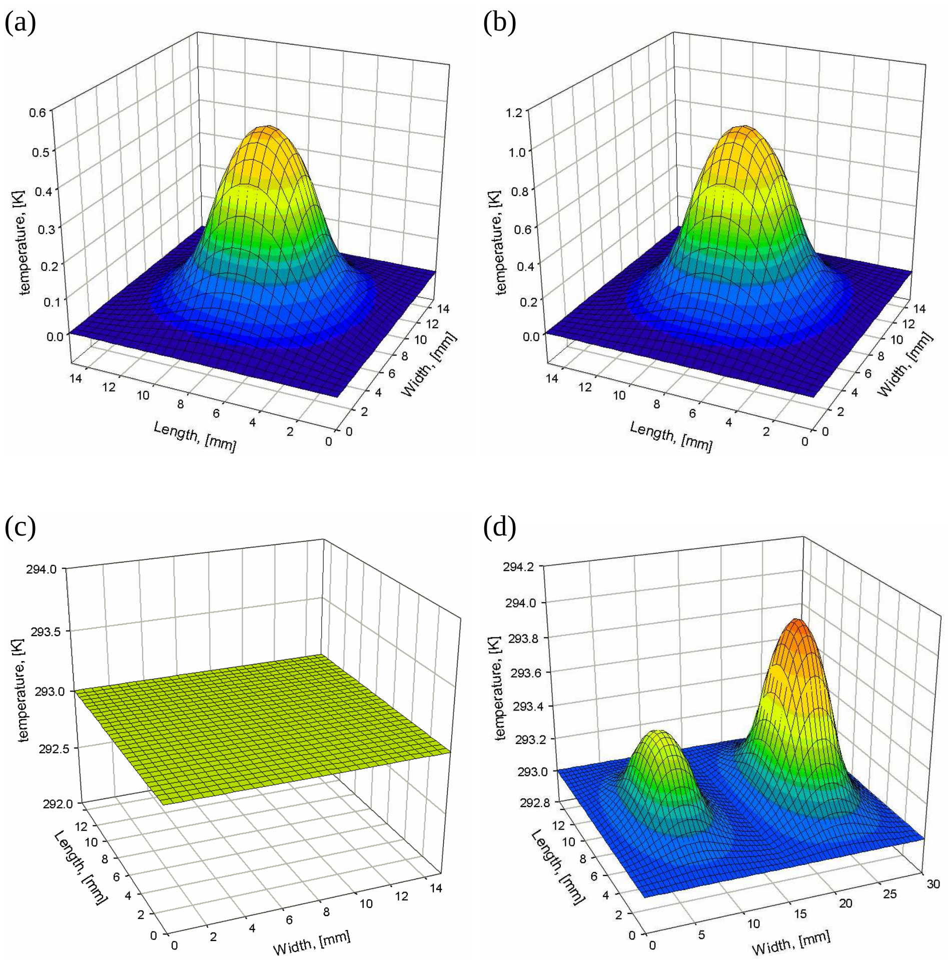

Fig. 5. Temperature distributions: (a) obtained by (4) with the lighting source parameter of 6.46 K/s, (b) also by (4) with the lighting source parameter of 12.92 K/s, (c) u_0 – constant room temperature, (d) after summing up (a), (b) and (c) solutions.

ture distributions when the lighting source parameter was 6.46 K/s and 12.92 K/s. (c) shows u_0 – the constant temperature and (d) is the sum of the previous three solutions. Fig. 6 shows the solution of (1)–(3) with the same values as in (4). As shown in pictures (Fig. 5(d) and Fig. 6), the solutions are similar. The reliability of numerical results was verified by decreasing difference steps values.

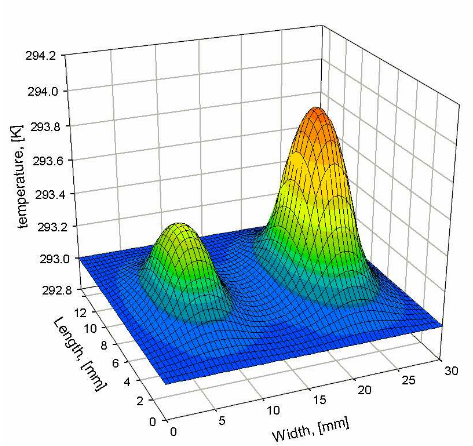

Fig. 6. The temperature distribution obtained by (1)–(3).

For more mathematical proofs we calculated temperature gradients in both cases:

1. Task (4) with the lighting source parameter of 6.46 K/s, temperature gradients are $\max\left|\frac{\partial U}{\partial x}\right| = 0.16$ K/s, when y and z are fixed on the maximum temperature section, and the $\sum_z \left|\frac{\partial U}{\partial x}\right| = 25.63$ K/s is in the same section.
2. Task (4) with the lighting source parameter of 12.92 K/s, temperature gradients are $\max\left|\frac{\partial U}{\partial x}\right| = 0.31$ K/s, when y and z are fixed on the maximum temperature section, and the $\sum_z \left|\frac{\partial U}{\partial x}\right| = 51.26$ K/s is in the same section.
3. Task (1)–(3) with the lighting source of parameters 6.46 K/s and 12.92 K/s, temperature gradients are $\max\left|\frac{\partial u}{\partial x}\right| = 0.31$ K/s, when y and z are fixed on the maximum temperature section, and the $\sum_z \left|\frac{\partial u}{\partial x}\right| = 72.17$ K/s is in the same section.

As seen from the above remarks the temperature gradient $\max\left|\frac{\partial u}{\partial x}\right|$ (1)–(3) coincides with the maximum temperature gradient of two different tasks (4), and the sum of $\sum_z \left|\frac{\partial U}{\partial x}\right|$.

5 Conclusions

The proposed total approximation method for investigating the temperature regime of the polychromatic solid-state lamp consisting of n light-emitting diodes is an equivalent of the whole system temperature regime.

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