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Sensitivity Analysis of Fatigue Behaviour of Steel Structure under In-Plane Bending*

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Abstract. The aim of the present paper is to analyse the influence of some input quantities on the fatigue resistance of a steel structure. The steel element subjected to many times repeated in-plane bending moment was analysed. The fatigue resistance was defined as the number of cycles causing the initial crack size propagation up to the critical size. The variability influence of input random quantities on the fatigue resistance variability was studied by means of the stochastic sensitivity analysis. All input imperfections were considered to be random quantities. The Latin Hypercube Sampling (LHS) numerical simulation method (Monte Carlo type method) was used. It has been found by the stochastic sensitivity analysis that the fatigue resistance variability is most sensitive to the initial crack variability. The paper presented draws attention to the necessity of identifying the statistical characteristics of the initial crack size as exactly as possible because their random variability can largely influence the failure probability of a structure. Large diversity in applying the statistical characteristics of initial crack size by numerous specialists is illustrated by the list of international publications at the end of the present paper.

Keywords: fatigue, crack, steel, sensitivity, randomness, correlation.

1 Introduction

When designing a new structure or when evaluating an existing structure stressed by many times repeated loading (bridges and so on), it is necessary to take into

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consideration fatigue degradation of the structure. There exit numerous hypotheses, theories and calculation methodologies for the analysis of the cumulative character of fatigue failure in steel structures [1, 2]. All the theories are based on results of special experiments, see, e.g., [3]. Basic information on the structure fatigue behaviour is supplied by the experimental analysis of steel structures having real sizes, see, e.g., [4]. In theoretical analysis, two basic approaches are applied to the structure fatigue evaluation: the Wöhlerian approaches and the linear mechanics method.

When evaluating the fatigue, general valid standard approaches are applied [5], based on the well elaborated Wöhlerian approach. The basic requirement of the Wöhlerian approach is neither the occurrence of defects and cracks connected with manufacturing and assembling, nor the initiation of fatigue cracks with measurable dimensions, even during the structure working time. The objective of the presented paper is an analysis of the influence of some input quantities on the growth analysis of fatigue cracks which occur in real structures. Provided that the Wöhlerian approach does not admit the possibility of fatigue crack growth, it is necessary to apply another approach which enables to describe the fatigue crack propagation process.

If a small initial crack is supposed in a new structure, then crack growth can be observed by using the linear fracture mechanics. There are many computing models which give a good correspondence between computed and measured values [6]. Although the principles of linear fracture mechanics have been known for decades, they have not been elaborated, in comparison with the Wöhlerian approach, in such detail, that they could be applied in Standards.

Therefore the quantities entering the evaluation process by means of the linear fracture mechanics method cannot be considered by constant numerical values deterministically due to their random character. The approach to taking the random character of input quantities into account (material characteristics, geometrical characteristics, and loading effect characteristics) leads to an application of principles and calculation methods of the probability theory.

In general, it corresponds to the present state of classification of probability methods into three basic levels elaborated by the International Committee for the Security of Structures of the JCSS [7]. In the application fields, numerical simulation methods of the Monte Carlo type above all showed intensive development and use [8]. The description of the method in more detail can be found in numerous publications, see, e.g., [9]. In this connection, the applications of reliability methods are supported by intensive development of computer technology in the last decades. Among the frequently applied improved simulation methods of the Monte Carlo type, there is to be mentioned the method Latin Hypercube Sampling (LHS). The method LHS provides, in comparison with the classic Monte Carlo method (more exactly termed “Simple Random Sampling”), very good assessment of mean value, standard deviation, skewness, kurtosis, and of the distribution function see, e.g., [10]. This method provides important possibility for research workers to draw statistical conclusions even on the basis of relatively low number of simulations.

In addition to a simple statistical and/or probabilistic analysis, it can be also interesting to determine in what manner an input quantity influences the output one. If the information on the variability of input and output quantities is at our disposal, the sensitivity of an input quantity to the output one can be determined in a quantified manner. In this connection, it is spoken about the so-called stochastic sensitivity analysis [11]. The stochastic sensitivity analysis enabled us to assess the relative sensitivity of random variability of the phenomenon studied to the random variability of individual input quantities [12].

The aim of the presented study is a sensitivity analysis of the effect of factors influencing the fatigue resistance in a steel structure under in-plane bending moment, see Fig. 1. The linear elastic fracture mechanics based on Paris-Erdogan’s formula was used. According to the results obtained, it can be recommended which input quantities, because of their maximum influence on the fatigue resis-

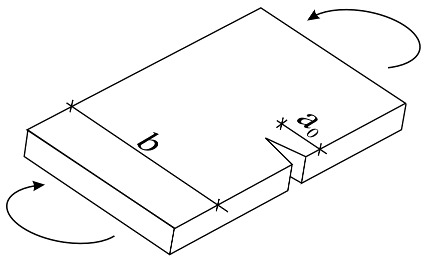

Fig. 1. Element with initial crack.

tance, should be controlled, both in the manufacturing process and by revisions of existing structures, with increased attentiveness and accuracy.

2 Linear elastic fracture mechanics

The methodology of calculation algorithm applied in the present paper is based on the generally most used and recognized model describing the fatigue crack growth. According to the Paris-Erdogan's equation, the crack propagation rate is as follows:

$$\frac{da}{dN} = C(\Delta K)^n \tag{1}$$

where a – crack size, N – number of cycles, C, n – material constants. C, n are material constants which can be determined by statistical processing from a set of experimentally determined data pairs $(da/dN, \Delta K)$. The amplitude of the stress intensity coefficient ΔK is defined by the relation:

$$\Delta K = K_{max} - K_{min} = F(a)\Delta\sigma\sqrt{\pi a} \tag{2}$$

where $\Delta\sigma$ – constant stress amplitude, $F(a)$ – calibration function. $F(a)$ is the so-called calibration function depending on the geometry both of the member and crack. By rearrangement and integration of the Paris-Erdogan's equation (1) and by considering the initiation crack propagation from the value a_1 to the final one, a_2, and the corresponding number of cycles N_1 a N_2, we get the relation:

$$\int_{a_1}^{a_2} \frac{da}{\left[F(a)\sqrt{\pi a}\right]^n} = \int_{N_1}^{N_2} C\Delta\sigma^n dN. \tag{3}$$

When assembling a bridge structure (welding, cutting, drilling), the fatigue crack can initiate and propagate still starting with the first loading cycle. For bridge structures, it is therefore justified to consider the number of cycles at the time of fatigue crack initiation by the value $N_0 = 0$. Provided that the initial crack size value is introduced to be $a_1 = a_0$, and the final one, $a_2 = a_{cr}$, the relation (3) can be written in the form:

$$\int_{a_0}^{a_{cr}} \frac{da}{\left[F(a)\sqrt{\pi a}\right]^n} = CN\Delta\sigma^n \tag{4}$$

where a_0 – initial crack size, a_{cr} – critical crack size, N – number of cycles, C, n – material constants. N is the total number of cycles at crack growth from a_0 to a_{cr}. The mathematical model describing the fatigue crack growth of an element stressed by in-plane bending moment is defined, according to [13, 14], by the calibration function:

$$F(a) = 1.12 - 1.39\,\frac{a}{b} + 7.32\Big(\frac{a}{b}\Big)^2 - 13.08\Big(\frac{a}{b}\Big)^3 + 14\Big(\frac{a}{b}\Big)^4 \tag{5}$$

where a – crack size, b – size of element, sets, however, it is possible to describe the vague notions in themselves.

3 Input random quantities

In the linear elastic fracture mechanics, the initial crack size is one among the basic input random quantities.

The initial crack size was modelled by lognormal distribution [15, 16]. The problem is how to define the mean value and the standard deviation [17]. According to the published experimental results [18, 19], obtained based on crack propagating from the surface of weld joints, it is possible, for an initiating crack, to consider the lognormal distribution with mean value $m_{a0} = 0.526$ mm and standard deviation $S_{a0} = 0.504$ mm, see Fig. 2.

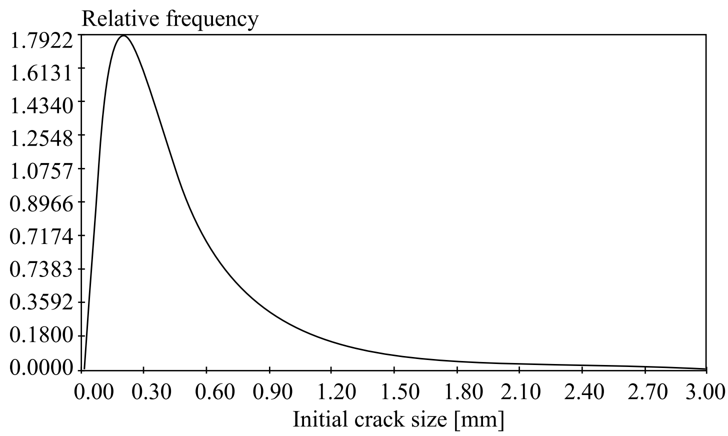

Fig. 2. Lognormal distribution function of initial crack size.

The plate width b and critical length a_{cr} to which the crack propagates without the rise of macroplastic instability were considered as the other random quantities.

The coefficient n which is the function of many factors [14] was introduced randomly, as well. The exponent n increases with decreasing fracture toughness. In our study, the parameter n was supposed, in a simplified way, with the Gaussian distribution to have the mean value $m_n = 3$ and the variation coefficient $v_n = 0.01$.

The strong correlation between the parameters C and n [14] was confirmed experimentally. Provided that the exponent n is not any universal constant, it follows from the dimensional analysis of the Paris-Erdogan's equation (1) that also the physical dimension of the constant C gets changed in general. According to [14], the mutual relation between C and n can be expressed as follows:

$$\log(C) = c_1 + c_2 n \tag{6}$$

where $c_1 < 0$, $c_2 < 0$ are the parameters constants for the given material grade.

In our problem, we considered, in compliance with [14, 20], $c_1 = -11.141$, $c_2 = -0.507$ for the steel grade S235. The input random quantities are clearly given in Table 1.

Table 1. Input random quantities

	Distribution	Mean	Standard deviation
Initial crack size	Lognormal	0.526	0.504
Parameter n	Normal	3	0.030
Thickness	Normal	400	25
Critical crack size	Normal	200	20

4 Sensitivity analysis

According to the relation (4), the number of loading cycles N was analysed at which there took place an increase of crack initiation a_0 to critical size a_{cr}. The variability influence of the maximum number of reached loading cycles N (fatigue resistance) on the variability of input random quantities was studied by means of the sensitivity analysis.

The objective of this paper is therefore a stochastic sensitivity analysis which provides more extended information about the problem studied, see [11, 12]. The first method can be applied practically for all numerical simulation methods of the Monte Carlo type. The method described is based on the assumption that there will be higher correlation degree with the output in case of the quantities relatively more sensitive to the output. The so-called Spearman rank-order correlation r_i is frequently applied within the framework of a simulation method. The Spearman rank-order correlation can be defined as:

$$r_i = 1 - \frac{6 \sum_j (k_{ji} - l_j)^2}{N(N^2 - 1)}, \quad r_i \in [-1; 1] \tag{7}$$

where r_i is the order representing the value of random variable X_i in an ordered sample among N simulated values applied in the j-th simulation (the order k_i equals the permutation at LHS), l_j is the order of an ordered sample of the resulting variable for the j-th run of the simulation process ($k_{ji} - l_j$ is the difference between the ranks of two samples). If the coefficient r_i had the value near to 1 or -1, it would suggest a very strong dependence of the output on the input. Opposite to this, the coefficient with its value near to zero will signalise a low influence.

The second method is based on the comparison of sensitivity coefficients p_i, defined on behalf of variation coefficients by the relation:

$$p_i = 100 \frac{v_{yi}^2}{v_y^2} \ [\%]. \tag{8}$$

v_{yi} is the variation coefficient of the output quantity, assuming that all the input quantities except the i-th one ($i = 1, 2, \ldots, M$; where M is number of input variables) are considered to be deterministic ones (during the simulation, they are equal to the mean value). v_y is the variation coefficient of the output quantity, assuming that all the input quantities are considered to be random ones.

The realizations of input random quantities were generated by means of the LHS method for 400 simulation runs. The LHS method is a method of the Monte Carlo type which makes it possible to simulate the realizations of input random quantities as if they were obtained by measurements [10]. Within the framework of each run of the LHS method, the maximum reached loading cycles N was found out by the formula (4). Dependence of the sensitivity coefficients (7) and (8) on the parameter of constant stress amplitude (deterministic quantity) are studied, see Figs. 3 and 4.

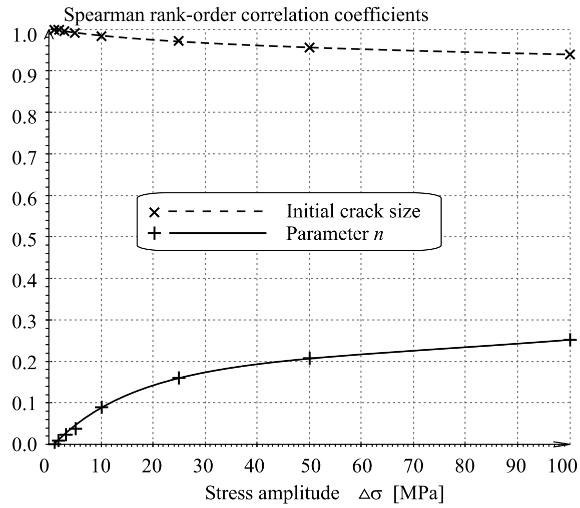

Fig. 3. Results of sensitivity analysis according to (7).

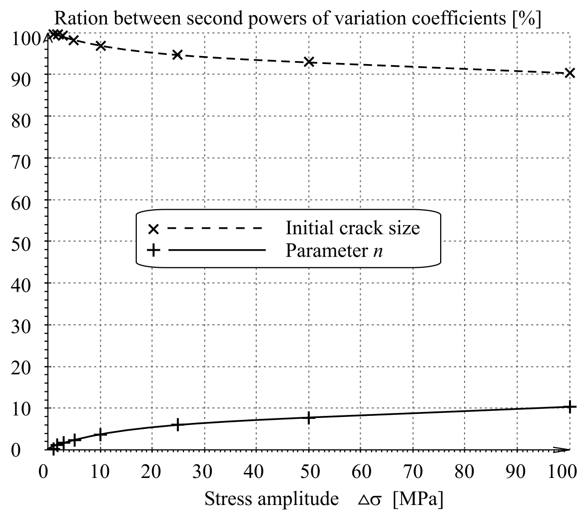

Fig. 4. Results of sensitivity analysis according to (8).

5 Conclusion

It is evident from the results given in Fig. 3 and Fig. 4 that the maximum permissible number of cycles N is the most sensitive to the crack initiation size a_0. The variability of this quantity influences the output quantity variability (number of cycles up to failure) in dominant manner. The attention is to be drawn to the lack of knowledge of statistical characteristics of initial crack size. The statistical characteristics and the distribution function type of the quantity must be known very exactly for their application in probabilistic calculations. The only possible method is the statistical evaluation of a larger quantity of experimental results. In opposite case, the failure probability calculation can be afflicted with a big mistake [20].

Although the importance of initial crack size for fatigue resistance decrease is generally known, the definition of the initial crack size as a random quantity is not a clear affair. At present, only little knowledge exists of the probability distribution function. According to one among few experimental results of 718 welded elements [20], it is possible to substitute the histogram of initiation crack by the lognormal distribution function with mean value $m_{a0} = 0.526$ mm and standard deviation $S_{a0} = 0.504$ mm. This distribution is the most frequently used one applied to modelling the initial crack as a random variable. In addition to this, also the exponential [21] and Weibull [22] distribution functions are applied. Jiao [22] has found that there is only a small difference between initial crack modelling by the logarithmic distribution and the Weibull one The lognormal distribution is sometimes termed as the aggressive one because due to it, the probability of the occurrence of larger cracks is more frequent.

The opinions of specialists differ very much from each other, as far as the distribution function application and its statistical characteristics are concerned. Albrecht a Yazdani [23] recommend the lognormal distribution with mean value 0.5 mm. In [16], the lognormal distribution with mean value 0.508 mm and standard deviation 0.254 mm was applied. In [24], there was used the lognormal distribution for two alternatives of fatigue crack propagation – in the flange of a welded beam, and in that of a hot-rolled beam. In the first case, the initial crack was introduced with mean value 0.468 mm, and the standard deviation 0.021 mm; in the second case, with mean value 0.03 mm, and standard deviation 0.00072 mm.

For initial crack size, the paper [25] gives the lognormal distribution with mean value 0.6 mm and standard deviation 0.03 mm, further on, in [26] the lognormal distribution is applied with mean value 0.1 mm and standard deviation 0.02 mm. It is evident from the papers mentioned that the intervals of mean value (0.03 mm to 0.5 mm) and of variation coefficient (0.02 to 0.5) are quite large. This disunity of applied statistical characteristics can have a fatal effect on the disunity of the results of statistical and probabilistic analyses.

It can be recommended without any doubt that initial cracks have to be controlled and measured with increased accuracy. In this connection, there lack publications containing the results of statistical analysis, e.g., in form of histograms. The sensitivity coefficient value of the crack initiation size a_0 decreases with increasing stress amplitude $\Delta\sigma$.

The second dominant quantity is the parameter n. The finding of satisfactorily accurate statistical characteristics of the parameter n is of major importance for further application in probability analyses; see, e.g., [9]. The sensitivity coefficient of the parameter n increases with increasing value $\Delta\sigma$. The variability of parameter n is initiated, e.g., by the surrounding temperature variability, which, for bridge structures, can undergo relatively important changes during a year. The material fracture toughness decreases with decreasing temperature as well, and due to this fact, the exponent n of the Paris-Erdogan's equation (1) increases.

The critical crack size a_{cr} and the element thickness b are these quantities the variability of which influences the fatigue resistance variability only to a negligible extent. The determination of the critical crack size is irrelevant, because in real applications the longest part of crack propagation life is spent in the case of small cracks [1]. The value of these sensitivity coefficients has nowhere exceeded the value 0.06, therefore they are not presented in Fig. 3 and in Fig. 4, either, for the clearness' sake. For the quantities the sensitivity of which was relatively low, it is not necessary to identify their statistical characteristics too exactly but professional assessments are sufficient. Provided that they are applied in probabilistic calculation, the mean value can be considered deterministically.

It follows from the relation (8) that, for the constant value, the sum of sensitivity coefficients of all the input quantities must be 100 %. Practically, it means that the incrementation gradient of the sensitivity coefficient n is the same as the decrementation gradient a_0, see Fig. 4. For values $\Delta\sigma = 10, 50, 100$ MPa, the sensitivity coefficients according to (8) are presented also in form of pie graphs, see Fig. 5.

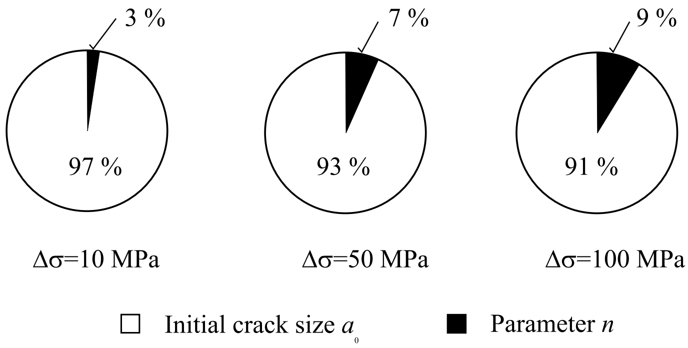

Fig. 5. Results of sensitivity analysis according to (8).

According to our results obtained, the initial crack size a_0 is one among the important factors influencing the structure service life. As it was shown, the comprehensive information on statistical characteristics and distribution type must be identified by experimental research carried out on larger number of specimens. In future, the definition of statistical dates by a non-commercially oriented experimental research could then contribute to the solution of the problem of generally valid, and therefore also codifiable statistical data in an important manner.

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