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Nonlinearities in Artificial Neural Systems Interpreted as an Application of Ising Physics

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Abstract. In this review, the nonlinearities in different processes such as spin glasses, finite field models, Hamiltonian functions, learning and storing capabilities, mean field systems and others in the area of physics related to the artificial neural networks namely the main brain structure interpreted as Ising spin systems are discussed. It is shown that nonlinearities serve as exclusive role in the applied physics field.

Keywords: spin glass, ising spin systems, nonlinearities, Hamiltonian function.

1 Introduction

Nonlinearities in the nature, especially in the biology or neuroanatomy, as well as in artificial technical systems and even in social life play a marked role in the behavior either small separate particles or large-scale, massive, strongly interconnected systems. The neuroanatomy systems included the central nervous system with massive interconnected neural networks of the cerebral cortex matter belong to latter.

The neural networks approximately reflect the natural neurophysiological system and they are used by neurophysiologists and modelers-cybernetists to study real nature objects or construct new artificial systems more precisely copying a natural being's behavior.

In this review, I would like to pay attention to different nonlinearities that influence to the processes in neural networks as the brain main structure on the one hand and as an Ising spin system on the other hand.

In the statistical physics as well as in the ferromagnetism or even in crystal physics, the simplified model, which is wide used, is based on an Ising spin system. The original work on the theory of the Ising spins related the ferromagnetism is the paper of Nowell and Montroll published in 1953 [1]. I would like to pay attention to applying this theory to the artificial neural systems or, in general, to the brain science, especially underlying the role of aspects of nonlinearities referring to the works of Little [2–4]. Though these issues are devoted to the specific problem related to the memory storage capacity in the brain, the Hebb rule determines the basis of learning in the neural networks and defines the behavior of the networks by the nonlinear laws.

The spin glass phase of neural networks as an Ising spin system with some moments of nonlinearities in Section 2 is discussed. In Sections 3 and 4 the finite-field nonlinear models as well as the results of the practical experiments are considered. The Hamiltonian function as a main description of a behavior of complex systems in Section 5 is represented. The exponential, learning, and storing nonlinearities in Sections 6–8 are characterized. The proposed formalism of the mean field description is analysed in Section 9. The brief discussion is presented in the separate Section 10.

2 Spin glass nonlinearities

One of the important and difficult to forecast phases of the Ising spin system is the spin glass phase [5–7]. The spin glass systems with macroscopic magnetic moments according to EA (Edwards and Anderson [5] hereafter referred to as EA) are dilute magnetic alloys CuMn or AuFe with weak magnetic concentration. They are able to show the surprising properties, one of which is a susceptibility having the cusp as the existence of preferred orientation of the spins at the critical temperature T_c. This property is result of a change of the sign between magnetic atoms due to the so-called the Rudermann-Kittel-Kasuya-Yosida (RKKY) [8–10] interaction. According to the RKKY interaction the sign depends on the distance between the atoms. Note that Mn challenges a slight anisotropy and it should therefore correspond to the Heisenberg spin glasses. Under the spin-fluctuation effects at zero temperature, the effective impurity moments are vanishing at a lower magnetic concentration. At a high concentration (ferromagnetic or antiferromagnetic), the spin glass properties are impaired.

Spin glasses have a sharp thermodynamic phase transition at temperature $T = T_{sg}$, such that for $T < T_{sg}$ the spins freeze in some random-looking orientation. The spin glasses susceptibility is defined as

$$\chi_{sg} = 1/N \sum_{ij} [\langle s_i s_j \rangle_T^2]_{av}, \tag{1}$$

where s_i, s_j are the spins at sites i and j, $\langle \cdots \rangle_T$ denotes a thermal average, $[\cdots]_{av}$ indicates averaging over a Gaussian distribution of exchange interactions, N is the number of spin elements.

The susceptibility has a consequent cusp, which has been found experimentally [11]. The EA susceptibility (1) diverges. Since the susceptibility is nonlinear, it can be defined by the coefficient at h^3 in the expansion of the magnetization m,

$$m = \chi - \chi_{nl} h^3 + O(h^5), \tag{2}$$

where h is an external magnetic field. It is expected that the nonlinear susceptibility, $\chi_{nl} = \frac{\partial^2 m}{\partial h^2}$ diverges less strongly because of cancelations at T_{sg} such as

$$\chi \sim (T - T_{sg})^{-\gamma}, \tag{3}$$

where γ is a critical exponent. This divergent behavior has been observed in the experiment on the alloy with Mn in Cu mentioned in [5].

The dynamics in the spin glasses at low temperature below T_{sg} is never in-complete equilibrium because the energy function landscape is very complicated. It has many valleys separated by barriers. The values of free energy of the valleys can be similar while the spin configurations rather different. Since the brain systems are large-scale, spin glass energy excitation becomes low.

3 Practical experiments of susceptibility

A spin-glass transition was studied on the basis of practical experiments and Monte-Carlo simulation with alloys for $\mathrm{CuGa_2O_4}$ [12]. The magnetic susceptibility, magnetization measurements in the fields up to 50 kOe, specific heat and muon-spin relaxation (μSR) measurements were carried out in the cubic spins of

$\mathrm{CuGa_2O_4}$. Transition undergoing from the paramagnetic to the spin glass phase was established at $T_f = 25$ K. As a result of experiments the phenomena of nonlinearities in results of experiments are preferred everywhere.

The magnetization dependence of $\mathrm{CuGa_2O_4}$ as a single crystal versus temperature is nonlinear and even the curves diverge below the bifurcation point. The bifurcation point tends nonlinearly to a lower temperature by increasing a bias field.

The ac susceptibility dependence on temperature in the absence of a bias field (H $= 0$ kOe) is strongly nonlinear, while with an increase in the bias field (H $= 5$ KOe, H $= 10$ kOe), the dependence becomes almost linearly decreasing versus temperature. The nonlinearities disappear because the magnetic field suppresses the cusp of susceptibility. It has been also shown that the character of nonlinearities of susceptibility does not change dependent on different frequencies, only the cusp moves to higher temperatures. Specific heat and muon-spin relaxation curves do not distinguish in mere nonlinearities.

Analogous results of studying the spin-glass behavior have been obtained for an ordered transition of metal alloy $\mathrm{FeAl_2}$ [13]. The ac susceptibility (at the ac field amplitude of 1 Oe and frequency of 125 Hz) versus temperature is represented by the curve with a cusp at lower temperatures. The inverse curve follows the Curie-Weiss law $\chi = C/(T - \Theta)$, where Θ is the Weiss temperature, C is a constant defined by the nonlinear part and the straight-line.

4 Finite field nonlinear models

Nonlinearities play an important role in the main characteristics used for evaluating the behavior of spin glass states and the transition line called an Almeida-Thouless (AT) line [14] in the infinite-range (mean field) Sherrington-Kirkpatrick model [15] and in finite field models (as more realistic one) with short range interactions. In the latter, it is confirmed that the SOPT is valid for infinite range interactions as well as for finite-range ones, though the latter has not proved. The proves of an existence of the SOPT line for finite-range interactions were performed in [14]. The Almeida-Thouless (AT) line separates the paramagnetic phase from the spin glass phase (Fig. 1(a)).

About existence AT line in finite range there is a controversial point of view.

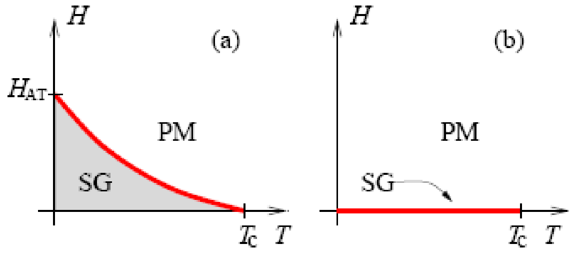

Fig. 1. (a) SOPT according to RSB scenarios (SG means spin glass, PM means paramagnetic phase), (b) SOPT according to droplet picture, a SG achieves only for $H = 0$ (pictures from [16] modified).

The RSB approach presented by Parisi [17, 18], and others postulates that the AT line occurs at infinite – range (SK) interaction as well as at finite-range ones (Fig. 1(a)), while the droplet picture approach followers Fischer [19], Moore [20] claim that for finite size scaling AT line occurs only at the zero field (Fig. 1(b)).

The AT line, which divides the spin glass domain from the paramagnetic phase or the ferromagnetic one, is nonlinear as a complicated phase transition limit. The line existence and form the mostly depends on the strength of the external field, for example, even in small fields there is no AT line in one- or three-dimensional spin glasses [16, 21]. In short range interaction studies, the couplings J_{ij} are given by

$$J_{ij} = c(\sigma)\frac{e_{ij}}{r_{ij}^2}, \tag{4}$$

where e_{ij} are the random values and are chosen, as a rule, according to the Gaussian distribution with zero mean and the standard deviation unity, $r_{ij} = (L/\pi)sin[(\pi|i-j|)/L]$ and represents the geometric distance between the spins on the ring of length L, $c(\sigma) \sim L^{-(1-2\sigma)/2}$, for the large L, where σ is the exponent which the range of changes defines whole list of different models [21]. All these values distinguish themselves by strong nonlinearities determined different complex spin glass laws.

The next important nonlinear function in this area of investigation is correlation length divided by the system size ζ/L. It satisfies the finite-size scaling form

$$\zeta/L = x\big(L^{1/\gamma}T - T_c(h)\big), \tag{5}$$

where x is a scaling function, $T_c(h)$ is the critical (transition) temperature for the external field strength h, and γ is the correlation length exponent [16]. The presence of nonlinearities in (5) leads to the simplified method of defining the critical point of the second order transition depending on temperature, where the changing data of different size L causes intersection of the functions $\zeta/L(T)$. The two quantities are mostly connected with overlap in the finite size scaling spin glass systems. The AT line existence and form the mostly depends on the strength of the external field, for example, even in small fields there is no AT line in one- or three-dimensional spin glasses [16, 21].

The next important nonlinear function in this area of investigation is correlation length divided by the system size. It satisfies the finite-size scaling form, where the critical (transition) temperature for the external field strength is the correlation length exponent [16, 21]. The presence of nonlinearities here leads to the simplified method of defining the critical point of the second order transition depending on temperature, where the changing data of different size causes intersection of the correlation functions (Fig. 2).

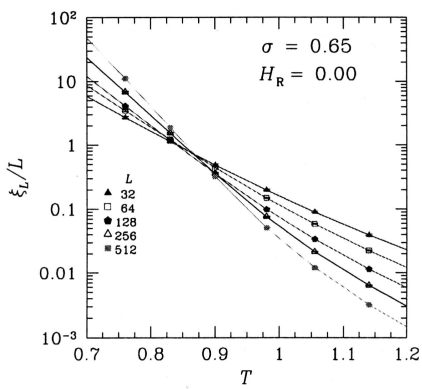

Fig. 2. Relative correlation length depends on temperature for $H_r = 0$ and exponent parameter $\sigma = 0.65$ at different sizes L (from Katzgraber and Young [21] modified).

The same authors [4] present the modeling results for 1D finite-range interaction which shows that it also possible the crossing of the correlation lengths for the magnetic field $H_r = 0.1$ in mean field case. The marginal behavior is achieved for power-law exponent $\sigma = 0.65$. It is necessary to underly, that short range models are more realistic than the infinite range one.

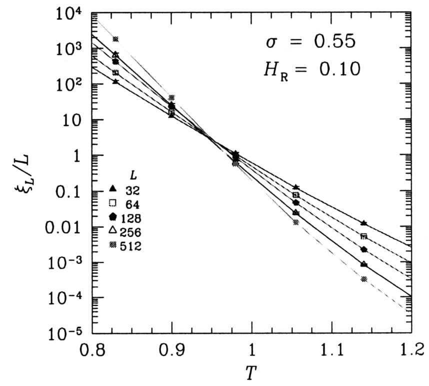

Fig. 3. Relative correlation length depends on temperature for $H_r = 0.1$ and exponent parameter $\sigma = 0.55$ at different sizes L (from Katzgraber and Young [21] modified).

The overlapping, q, defined by the formula

$$q = \frac{1}{N} \sum_{i=1}^{N} S_i^{(1)} S_i^{(2)}, \tag{6}$$

where N is the number of spins, "(1)" and "(2)" refer to two replicas of the system with the state value S_i for the same spin i, and the link overlapping, q_l, defined as

$$q_l = \frac{1}{M} \sum_{i,j}^{N} S_i^{(1)} S_j^{(1)} S_i^{(2)} S_j^{(2)}, \tag{7}$$

where M is the number of bonds and the sum of i and j connected by bonds. These nonlinear quantities characterize the following: q links with the volume of the cluster, q_L characterizes the surface of the cluster [22].

5 Nonlinear Hamiltonian function

The next source of nonlinearities is the Hamiltonian function or the energy function, as usual, or the effective energy function for the system of Ising spins

$$H = -\frac{1}{2}\sum_{i\neq j}^{N} J_{ij}S_iS_j + \sum_{i=1}^{N} h_iS_i \tag{8}$$

and the effective energy function which characterizes the growth of the energy

$$\widehat{H} = -\frac{1}{\beta}\sum_{i}^{N} \ln\Big[2\cosh\Big(\beta\sum_{j}^{N} J_{ij}S_j\Big)\Big]. \tag{9}$$

If to interpret the Ising system as the neural one, the states, $S_i, S_j = \pm 1$, represent two levels of activity of the i, j-th neurons, the couplings J_{ij} are the synaptic efficiencies of pairs of the neurons, h_i is the external field on the i site, N is the number of neurons, and β is the inverse parameter of temperature.

The surface of the Hamiltonian, under an influence of nonlinearities of J_{ij} and the product S_iS_j of vectors of states, is very complicated. There are very many valleys, barriers, complex boundary conditions, and other hyperplane phenomena.

The Hamiltonian function is even more complicated under the influence a random-anisotropy of the mixed-spin Ising model [23]. According to [23], the mixed-spin Ising model is represented by a two-sublattice system with variables $\sigma = \pm 1$ and $S = 0, \pm 1$ on sublattices A and B, respectively. The most general spin Hamiltonian in the spin configuration space is described as

$$H_0 = -J\sum_{i\in A,\, j\in B} \sigma_iS_j + D\sum_{j\in B} S_j^2, \tag{10}$$

where J is a parameter of ferromagnetic exchange, the second member of (10) characterizes the crystal field with the parameter $D > 0$, A and B are the sets of sublattices.

The competition between ferromagnetic exchange and anisotropy leads to the appearance of critical lines and a tricritical point (as a point at which three phases simultaneously become identical) location.

Nonlinear transition lines for the Curie-Weiss version of the mixed-spin Ising model are given as a division of the ferro- and paramagnetic fields on behalf of the phase diagrams [23].

An analogous study of the tricritical points for the spin-3/2 Ising model was performed applying the ternary fluid mixtures by the authors in [24]. Here the strong nonlinearities challenge a nonsymmetrical model and Landau expansion from the fourth order to the eighth order which allows us to study the tricritical points and behavior of the multicomponent fluid mixtures.

6 Exponential nonlinearities

Most ideas of the statistical mechanics, as mentioned above, apply to the neural dynamics with quenched random couplings and typical exponential nonlinearities. The long-time behavior of the dynamic models is governed by infinite-range Ising spin glasses and monotonically decreases the value of H (9), (10) with a decrease of temperature T, and leads eventually to the stationary state which is the local minimum of H. If we take into account the random couplings and states with noise, the distribution of states will converges to the Gibbs distribution

$$P(S) \propto \exp\big(-\beta H[S]\big) \tag{11}$$

with H in (9) or (10).

It is interesting to investigate these models not only in the context of memory images, but also in the context of nonlinear disordered statistical mechanics or magnetic systems.

To solve such tasks, as usual, the Boltzmann distribution

$$P\big(\{J\},\{S\}\big) = \frac{1}{Z\big(\{J\}\big)} e^{-\beta H(\{J\},\{S\})}, \tag{12}$$

where the normalization constant is the partition function

$$Z\big(\{J\}\big) = \sum_{\{S\}} e^{-\beta H(\{J\},\{S\})}, \tag{13}$$

is used as well as Boltzmann machine algorithm.

The typical choice of the J_{ij} for $P(\{J\},\{S\})$ is a Gaussian distribution with zero mean and the standard deviation unity, and of the S_{ij} for $P(\{S\})$ is the uniform or Gaussian distribution, if we consider them separately. It is easy to show that thermodynamic quantities, like the free energy $f(\{J\}) = -\frac{1}{\beta} \ln Z(\{J\})$, are selfaveraging. It means that the free energy density $f(\{J\})/N$ reaches the limit for large N. As usual, the solutions are found by partial differential equations in searching for the saddle point values. The nondeterministic nature of the statistical systems can be replaced by a large system of deterministic systems and solved by a system of determined nonlinear algebraic equations [25].

Some of authors [5, 6] represent the Boltzmann law equations as linearized ones in order to build analytic expressions and to investigate stability problems. Here one may put a question to what extent the analysis of the linearized model is relevant to the full nonlinear problem, which is much more complicated than the 3D Ising problem. On the other hand, certain computer simulations indicate a loose link between the behavior of the full nonlinear model. However, these arguments are insufficient to prove that behavior of the linear model with the N^2 transitions has a set of N^2 classes of transitions for the general nonlinear model.

7 Learning nonlinearities

Another area of the nonlinearities is displayed in the field of learning mechanisms of massive connectionistic neural networks in the brain by interpreting them as a physical Ising model modification. The main sub-system of the cerebral cortex matter is the synapse-dendrite-soma-axon chain. Experiments demonstrate that all components of the chain are characterized by nonlinearities, some of which are strongly nonlinear as neuron cells, others as synaptic excitatory receptors or inhibitory ones are weakly nonlinear. At first let us characterize the synaptic nonlinearities. Note that synapses are not randomly distributed on the dendrites surface. Second, the synapses both excitatory and inhibitory typically operate by changing the conductance of postsynaptic membrane opening ion channels. The time course of synaptic conductance changes and, as a consequence, the electrical current changes are different and depend on the type of synapses. Fast excitatory (non-NMDA) and inhibitory (GABAa) synapses operate within 1ms and peak conductance on the other of 1nS. The conductance is up to 10 times higher than slow excitatory (NMDA) and inhibitory (GABAb) within a time scale of 10–100 ms. There is a domain where the slope is of negative conductance.

The learning processes in neural networks are connected with the modification of synapses, as a rule, of the type introduced by D. Hebb [26]. Here is the Hebb's neurophysiological postulate: "When an axon of cell A is near enough to excitate a cell B and repeatedly or persistently takes part in firing it, some growth process of metabolic change takes place in one or both cells such that A's efficiency as one of the cells firing B is enhanced".

It is known that, when the input potential of the neuron achieves the threshold, a series of impulses is generated by the output (axon) with some firing rate. Thus, the firing rate of each output neuron is forced to the value determined by the input. This means that, for any neuron i,

$$S_i = f(e_i), \tag{14}$$

which indicates that the firing rate is a function of the dendrite activation e_i. This function is as a sigmoidal one, i.e., strongly nonlinear with saturated areas and its precise form is irrelevant at least during the learning phase. The function (14) is frequently approximated by the discrete function like the Ising spin with $S_i = \pm 1$.

The Hebb rule can be then represented as follows

$$\Delta W_{ij} = k S_i S_j, \tag{15}$$

where ΔW_{ij} is the change of the synaptic weight W_{ij} which depends on the conjunctive presence of the presynaptic firing S_j and the postsynaptic one S_i, and k is the learning rate which characterize how many the synapses alter on any pairing. The Hebb rule is expressed in the product (nonlinear) form to reflect the Hebb postulate above.

In the Ising spin system with energy function (8), the bonds J_{ij} are the synaptic efficiency of the pair (ij) of neurons for the neural networks. Now the Hebb learning rule is represented as the accumulated effect of learning [27], which after a some changes is as follows

$$T_{ij} = \frac{1}{p} \sum_{\mu}^{p} \zeta_i^{\mu} \zeta_j^{\mu}, \tag{16}$$

where p is the number of patterns $\{\zeta_i\}, \{\zeta_j\}$ as the embedded memories, besides the patterns are random with equal probability for $\zeta_i^{\mu} = \pm 1$. Then, according to the work [27], the networks defined by the Hamiltonian (8) are characterized with the nonlinear modified synapses

$$J_{ij} = cf(T_{ij}) + \nu_{ij}, \tag{17}$$

where $f(T_{ij})$ is a nonlinear function, ν_{ij} is a possible noise, and c is a constant. It should be noted that ionic current of synapses is also nonlinear versus potential as shown in the works [25, 28].

If we compare the nonlinear function with the linearized one, the first on is preferred in the sense that it provides a narrower range of exchange J_{ij} and so is more reasonable because the Hamiltonian surface, in this case, has more expressed local minima. According to [27], when the number of patterns is not large, nonlinear learning rules can be used to increase the computation capabilities of neural networks as the area of Ising spins applied physics.

8 Storing nonlinearities

The capacity of memory in a neural network as an Ising spin system depends on the number of synapses rather than on increasing the number of neurons with the same percentage [3]. It is important to show, that apart from increasing the number of synapses or neurons, the synapse nonlinearities influence the capability of memory because of much more expressed local minima of the Hamiltonian function. According to the work [29] the percentage of retrieval errors decreases nonlinearly and very rapidly to zero with decreasing of the relation between implemented patterns and the number of neurons, $\alpha = p/N$. The similar law was noticed in [29] for the average of number of errors.

In the paper [25], I have tried to include a nonlinear synaptic function and a strong nonlinear current-voltage relation of the neuron soma and have done some computational experiments. The modeling results have shown that the nonlinear synapse strength provides a smaller number of errors than the linearized one, and the nonlinear synapse strengths crucially decrease the number of errors in the retrieval processes of the neural network systems.

9 Mean field nonlinearities

I will demonstrate the last episode of nonlinearities in the context of the statistical mechanics of disordered magnetic systems. The Hamiltonian defined by (8) is a case of infinite spin glasses, where each coupling of spins is connected via quenched random J_{ij} and the external field is not included for simplicity. I will follow the work [29], however, the coupling J_{ij} will be changed to more realistic and closer to the main idea of the Hebb's postulate above, i.e.,

$$J_{ij} = \frac{1}{p} \sum_{\mu=1}^{p} \zeta_i^\mu \zeta_j^\mu, \tag{18}$$

which is as that of (16) only with the new indication of couplings. Then, after substituting (18) into (8) without the second member, the Hamiltonian becomes

$$H = -\frac{1}{2p} \sum_{i \neq j}^{N} \Big[\sum_{\mu=1}^{p} \zeta_i^\mu \zeta_j^\mu \Big] S_i S_j, \tag{19}$$

where $\zeta_i^\mu \zeta_j^\mu$ are independent random variables with zero mean. The system will be considered in the thermodynamic limit $N \to \infty$ and finite p. The free energy function is defined by the partition function, for a given realization of $\zeta_i^\mu \zeta_j^\mu$, as follows

$$Z = Tr_s \exp(-\beta H) = Tr_s \exp\Big(\frac{\beta}{2p} \sum_{i \neq j} \Big[\sum_{\mu=1}^{p} \zeta_i^\mu \zeta_j^\mu \Big] S_i S_j \Big). \tag{20}$$

Using the identity

$$\sum_{\mu=1}^{p} \big[\sum_{ij}^{N} \zeta_i^\mu \zeta_j^\mu S_i S_j \big] = \frac{1}{2} \Big[\sum_{\mu=1}^{p} \sum_{i=1}^{N} S_i \zeta_i^\mu \Big]^2 - \frac{Np}{2}, \tag{21}$$

the equation (20) becomes as follows:

$$\begin{aligned} Z &= \exp\Big(-\frac{\beta N}{4} \Big) Tr_s \exp\Big(\frac{\beta}{2p} \sum_{\mu}^{p} \Big[\sum_{i}^{N} S_i \zeta_i^\mu \Big]^2 \Big) \\ &= (\beta N)^{1/2} e^{\frac{-\beta N}{4}} \int \Pi_\mu \frac{dm^\mu}{(2\pi)^{1/2}} \exp\Big[-\frac{\beta N^2}{2p} \vec{m}^2 + \sum_{i}^{N} \ln 2 \cosh(\beta \vec{m} \vec{\zeta}_i) \Big]. \end{aligned} \tag{22}$$

Regarding to

$$f = -\frac{\ln Z}{N\beta} = \lim_{N\to\infty} \big[N^{-1}\big\langle\!\big\langle \ln Tr_s \exp(-\beta H)\big\rangle\!\big\rangle\big] \tag{23}$$

and substituting (22) into (23), the free energy density becomes the mean field equation as follows

$$f = \frac{N}{2p}\vec{m}^2 - \frac{1}{N\beta}\sum_i \ln 2\cosh(\beta\vec{m}\vec{\zeta}_i). \tag{24}$$

The order parameter vector $\vec{m}$ is defined by the saddle point equations for each of components m^μ

$$\frac{\partial f}{\partial m^\mu} = 0. \tag{25}$$

After finding partial derivatives, the vector

$$\vec{m} = \frac{p}{N^2}\big[\vec{\zeta}_i \sum_i \tanh(\beta\vec{m}\vec{\zeta}_i)\big] \tag{26}$$

or

$$\vec{m} = \frac{p}{N}\vec{\zeta}_i \big\langle\!\big\langle \tanh(\beta\vec{m}\vec{\zeta}_i)\big\rangle\!\big\rangle, \tag{27}$$

where $\langle\!\langle \cdots \rangle\!\rangle$ is the average over $\{\zeta_i\}$.

Equations (26), (27) have been obtained in the such a form first, and they include the fraction p/N which is the parameter α. On the other hand, equation (26) is strong nonlinear.

Thus, the phenomena of nonlinearities in the field of the artificial brain functions, interpreted as an extension of the Ising spin physics, play an exclusive role in thermodynamic investigations.

10 Discussion

It needs to remark, that the progress in understanding and qualifying the spin glass problem has used an artificial replica theory. Indeed, the physical macroscopic measurements on equivalent random systems are dominated by their mean values (23). For finite-range interactions the effective Hamiltonian expression (9) with replicas cannot be evaluated exactly. By analogy with the magnetism, it is better to consider first a mean field approximation in which an iteraction problem is replaced by the effective non-interacting systems with the self-consistently determined solutions. For infinite-range systems with scaling of J_0, J with the numbers of spins N such a consideration can be performed exactly. The infinite-range models are usually proved by the procedure of mapping to macroscopic variables with dominated generating functionals.

As concerns phenomenon of nonlinearities that is great interesting phenomena in the technical systems, physics, biophysics, neurobiology, ecology, medicine and other scientific fields. This phenomenon frequently arises even chaotic processes influences to the attractor structures and behavior of the dynamic system states, self-organized topological structures as the dissipative ordered ones. Many physical as well as biophysical systems are characterized by nonlinearities. They frequently have a control parameter dependent on its value in evolution of states, the systems must be stable or unstable.

Many questions connected with phenomenona of nonlinearities and their influence in the different areas of the applied Ising physics have been considered in the review, however, the topic of nonlinearities in the technical, physical, and other systems is rather wide and, of course, cannot consider in one article. We hope that this review though partially fills the insufficient discussed scientific area of the nonlinear applied physics.

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