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Fuzzy Hammerstein Model of Nonlinear Plant

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Abstract. This paper presents the synthesis and analysis of the enhanced predictive *fuzzy Hammerstein* model of the water tank system. *Fuzzy Hammerstein* model was compared with three other fuzzy models: the first was synthesized using *Mamdani* type rule base, the second – *Takagi-Sugeno* type rule base and the third – composed of *Mamdani* and *Takagi-Sugeno* rule bases. The synthesized model is invertible so it can be used in the model based control. The *fuzzy Hammerstein* model was synthesized to eliminate disadvantages of the other *fuzzy* models. The advantage of the *fuzzy Hammerstein* model was experimentally proved and presented in this paper.

Keywords: fuzzy modeling, nonlinear modeling, predictive modeling, Hammerstein model, fuzzy Hammerstein model, level modeling.

1 Introduction

A critical step in synthesizing model based control systems is the development of suitable model which could sufficiently approximate dynamic characteristics of nonlinear plant. Recently fuzzy modeling of nonlinear dynamic systems has drawn a great deal of attention [1].

Nonlinear autoregressive models with exogenous inputs (NARX) [2] are often used with many nonlinear identification algorithms [3]. As most system identification strategies [4–8], NARX has its own disadvantages: problems with parameters estimation in high dimensions are caused by the course of dimensionality [9], exponential increasing memory usage and the prior information requirements. These problems make the NARX method unpractical for the modeling of the high level dynamic processes. As an alternative the block-oriented fuzzy models can be used. The well-known members of this class of the models are *fuzzy Hammerstein* and *Wiener* models.

The aim of this paper is to describe the synthesis of a *fuzzy Hammerstein* model for the nonlinear water level plant. As the main advantage of the proposed *fuzzy Hammerstein* model is that the model can be described with less parameters, is invertible (it can be used in the model based predictive control), and is adequate to the real plant.

2 Hammerstein class models

2.1 Hammerstein class models

Enhanced modeling can be obtained by using a *Hammerstein* class model [10] where linearization is straightforward by the inversion of a static input-nonlinearity. The *Hammerstein* models are suitable for the gray box modeling, where the static process behavior is known in advance. The *Hammerstein* model consists of two parts: a static nonlinearity part, that describes nonlinearities of a plant, and a linear dynamics part, as shown in Fig. 1 [11], where the intermediate signal $x(k)$ is not available. Such a model structure has shown to be appropriate for the modeling of the behavior of a wide range of systems such as distillation processes [12], friction dynamics [13], water level or air pressure and etc.

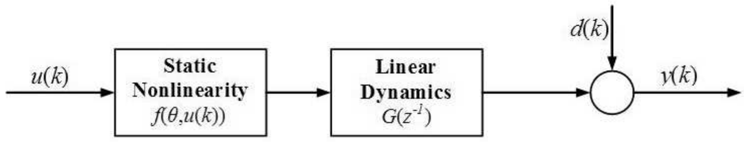

Fig. 1. The structure of Hammerstein model.

The *Hammerstein* model is represented by the following equations:

$$y(k) = \frac{B\big(q^{-1}\big)}{A\big(q^{-1}\big)}x(k) + d(k), \tag{1}$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + \ldots + b_m q^{-m}, \tag{2}$$

$$A(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_n q^{-n}, \tag{3}$$

the non-measured intermediate variable $x(k)$ is given by

$$x(k) = f\big(\Theta, u(k)\big), \tag{4}$$

where $q - 1$ is the unit delay operator, $u(k)$ is the input, $y(k)$ is the output, $d(k)$ is the measurement noise, $(m; n)$ is the order of the linear part, $f(.)$ is any nonlinear function and Θ is a set of parameters, that describe the nonlinearity [11].

Hence MIMO (multi input multi output) *Hammerstein* model can be written using equation [14]:

$$\widehat{y}(k) = \sum_{i=1}^{n_a} A_i y(k-i) + \sum_{i=1}^{n_b} B_i f\big(u(k-i-n_d)\big), \tag{5}$$

where $\widehat{y}(k), \ldots, \widehat{y}(k-n_a+1)$ is the predicted output of the plant, $u(k-n_d), \ldots, u(k-n_b-n_d+1)$ – the input to the plant, n_a, n_b – the rank of polynomials A and B respectfully, n_d – the delay of the plant. $A_1, \ldots, A_{n_a}$ and $B_1, \ldots, B_{n_b}$ are matrices of polynomials coefficients. The size of matrix A is $n_y \times n_y$ where n_y is the number of outputs of the plant and the size of matrix B is $n_y \times n_u$, where n_u is the number of inputs to the plant.

2.2 Fuzzy Hammerstein model

The *fuzzy Hammerstein* model is a special case of the NARX model, which is the combination of series of nonlinearities and linear dynamics but its structure is simpler than the structure of the general NARX fuzzy model. The *fuzzy Hammerstein* model approximates nonlinearities of a plant and provides the predictions of outputs with a smaller error than general NARX fuzzy models. The *fuzzy Hammerstein* model consists of the series of nonlinearities, expressed by a fuzzy system as a non linear function and a linear dynamical part with the transfer function G (as shown in Fig. 1), where $y = [y_1, \ldots, y_{n_y}]^T$ is the output vector, $u = [u_1, \ldots, u_{n_u}]^T$ – the input vector, and $v = [v_1, \ldots, v_{n_u}]^T$ – transformed input variables.

Fuzzy logic is chosen here because of its property to convert complex problems into simpler problems using approximate reasoning and to allow to model uncertainties and non-linearity of the plant. The nonlinear part of the *fuzzy Hammerstein* model is usually approximated with the fuzzy system where zero-order *Takagi-Sugeno* fuzzy rules of the form

$$R_j^h: \text{ IF } u_1 \text{ is } T_{1,j} \text{ and } \ldots \text{ and } u_{n_u} \text{ is } T_{n_u,j} \text{ THEN } v_h = p_j^h \tag{6}$$

are used [4]. Here T_{ij} are the membership functions (gaussian, triangular, or trapezoidal shape), that cover the universes of discourse of the input variables. Usually symmetric triangular membership functions are used as they are simple to calculate. In case of the singleton defuzzification [14] the output of the fuzzy system is calculated according to the equation:

$$\nu_h = \frac{\sum_{j=1}^{N_r} \beta_j(u) p_j^h}{\sum_{j=1}^{N_r} \beta_j(u)}, \tag{7}$$

where j is the truth value of the j-th rule's premise. Product operator is used to represent the premise of the rules:

$$\beta_j = \prod_{i=1}^{n} T_{i,j}. \tag{8}$$

If symmetric triangular membership functions are used, then

$$\sum_{j=1}^{n_r} \beta_j(u) = 1. \tag{9}$$

The *fuzzy Hammerstein* model is nonlinear in its B_j and p_j parameters, where B_j are polynomial coefficients and p_j is zero order polynomial coefficient of the fuzzy sub system's j-th rule. The *fuzzy Hammerstein* model is described with the equation:

$$\widehat{y}(k) = \sum_{i=1}^{n_a} A_i y(k-i) + \sum_{i=1}^{n_b} \sum_{j=1}^{n_r} B_i p_j \beta_j(k-i-n_d) \tag{10}$$

To easy the identification of the model parameters, the product of the parameters B_i and p_j is used instead, $B_i^j = B_i p_j$. The generalized *fuzzy Hammerstein* model then can be described using equation [14]:

$$\widehat{y}(k) = \sum_{i=1}^{n_a} A_i y(k-i) + \sum_{i=1}^{n_b}\sum_{j=1}^{n_r} B_i^j \beta_j(k-i-n_d) \tag{11}$$

If $\frac{b_i^k}{b_j^k}\frac{b_j^l}{b_i^l} = 1, \forall i,j,k,l$ then generalized *fuzzy Hammerstein* model becomes the original *fuzzy Hammerstein* model [14].

The advantage of the *fuzzy Hammerstein* model is that with the fuzzy logic it is quite easy to approximate any non-linearity. The model is clear and linguistically interpretable. The *fuzzy Hammerstein* model is classified as low complexity model because it can be synthesized with the smaller number of parameters comparing to the other fuzzy models. Besides, simple fuzzy models do not incorporate previous state information in their rule base [15]. The quality of the *fuzzy Hammerstein* model mainly depends on the identification of its parameters.

3 Fuzzy Hammerstein model identification algorithms

The identification of the block-oriented model is a complex task. Different identification algorithms are available for the parameter estimation of *Hammerstein* class models. Hammerstein model identification methods usually use either parametric, like least squares, recursive least squares [16, 17] and gradient method [4], or nonparametric methods, like Bayesian regression which describes the unknown map as multidimensional stochastic process which statistically summarizes the prior information that is available about the map [6].

The aim of the nonparametric methods is to relax assumptions on the form of an underlying nonlinear characteristic, and to let the training data decide which characteristic fits those best [18]. Also, in the non parametric approach the nonlinearity is assumed to be a continuous function, or a measurable function. In this case, the non-linear element is represented by an approximation of a truncated series or an orthogonal function. But, the choice of type and the length of series are not straightforward [19].

Alternative methods for the estimation of the nonlinear model parameters are available when the model is synthesized using polynomials with unknown coefficients or by a piecewise constant function [5]. This approach is preferable in control applications, especially when piecewise linearization is feasible. Then the parameter estimation can be solved by using regression techniques, iterating algorithms or combinations of these.

The least square parameter estimation algorithm, first used by Gauss in 1795, identifies unknown parameters using technique where measurement data are fitted to the underlying governing equations such that the identified parameter values minimize the squared error (where error is, for example, measurement data minus the ideal measurement data that would occur with zero noise and using the identified parameter values).

In this paper the recursive least square (RLS) method is used for the identification of fuzzy Hammerstein model parameters. This method was chosen because of several its advantages:

- It can be applied in the real time, because it is not necessary to use all input-output data pairs to estimate parameters, and method uses earlier estimated parameters as initial conditions or to specify previous estimates (using non recursive method we need to recalculate parameters from all data) if plant conditions changes.
- The method does not require to have input output data which cover all possible input output set.
- The method works faster because it does not use operations with matrices.
- The method faster converges if forgetting factor techniques is used [20].
- The method is simple for the implementation [21].
- The algorithm is able to learn very good policies using only a small number of samples compared to conventional learning approaches [21], such as Q-learning [22].
- The algorithm requires little or no modification to adapt it to various situations [21].

The basic idea behind a RLS algorithm is to compute the parameter update at time instant k by adding a correction term to the previous parameter estimate once the new information becomes available. Such reformulation has reduced the computational requirement significantly, making the RLS extremely attractive in the last three decades for on-line parameter estimation applications. It can be seen that due to its recursive nature, the complexity of the RLS has been reduced considerably from O(N3) in the batch least squares (BLS) to O(N2) in each estimate update [23, 24].

In case of the *fuzzy Hammerstein* model parameters identification, RLS algorithm searches for the best estimates of the model parameters A and B, taking into consideration that the other parts of the model (the number of fuzzy sets, the centers of membership functions) are chosen correctly in advance. The parameters of the *fuzzy Hammerstein* model are identified from the linguistic rules and the process input-output data. The parameters of the nonlinear static part B and p are multiplied, making them linear in their product as the recursive least squares method is linear, so the estimate is the product of nonlinear parameters, used to calculate the output of the model. The non restricted weighted recursive least squares method is described using equation [4, 17]:

$$\Theta(k) = \Theta(k-1) + \frac{P(k-2)\varphi^T(k-1)\left[\widehat{y}(k) - \varphi^T(k-1)\Theta(k-1)\right]}{\alpha + \varphi^T(k-1)P(k-2)\varphi(k-1)}, \tag{12}$$

$$P(k-1) = \frac{1}{\alpha}\left[p(k-2) + \frac{p(k-2)\varphi(k-1)\varphi^T(k-1)P(k-2)}{\alpha + \varphi^T(k-1)P(k-2)\varphi(k-1)}\right], \tag{13}$$

where $\alpha = [0.9; 1]$ is a forgetting factor (in this paper the value 0.99 was chosen), P is a covariance matrix [4, 17]:

$$\begin{aligned}\varphi(k-1) = \big\{&y(k-1), \dots, y(k-n_y),\\ &\beta_1\big(u(k-n_d-1), x\big), \dots, \beta_{N_r}\big(u(k-n_d-1), x\big), \dots,\\ &\beta_1\big(u(k-n_d-n_u), x\big), \dots, \beta_{N_r}\big(u(k-n_d-n_u), x\big)\big\},\end{aligned} \tag{14}$$

$(k-1) = \{a_1, \dots, a_{n_y}, p_{1,1}, \dots, p_{n_u,N_r}\}$ – parameters vector, $\widehat{y}(k)$ – measured process output. If the forgetting factor is smaller, then the *fuzzy Hammerstein* model may become unstable or its prediction may become weaker if predicting more complex signal for a longer time period. If forgetting factor is not used, then it is not possible to track time-varying parameter variation since the algorithm gain converges to zero when $k \to \infty$. Further, the RLS algorithm converges very slowly, at rate of $\frac{1}{k}$ [25].

In this paper the RLS method is used for the estimation of polynomial A coefficients and the product of polynomial B coefficients with the fuzzy rule base coefficients. During the process of identification the cost function

$$E = \big[\widehat{y}(k) - \varphi^T(k-1)\Theta(k-1)\big]^2 \tag{15}$$

is minimized [17], the RLS criterion is

$$J(\Theta, k) = \sum_{j=1}^{k} a^{k-j}\big[y(j) - \varphi^T(j-1)\Theta\big]^2, \tag{16}$$

where $y(j)$ is measured process output, is forgetting factor [20].

4 Nonlinear plant

The laboratory plant used for the modeling is shown in Fig. 2. Its central part is a close tank with the adjustable water level within the range from 0 to 25 cm. The "level" variable of the process can be varied using water pump (item 1 in Fig. 2). The pump is the actuator

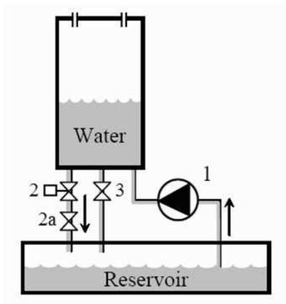

Fig. 2. The plant's structure.

and has an electrical input-range of 0 to 10 V. The tank has two outlets for water flow. The manual valve (item 3 in Fig. 2) and/or the combination of the magnetic valve (item 2 in Fig. 2) and manual valve (item 2a in Fig. 2) control the exit water flow. These valves and the control of the water pump manipulate the stationary condition of water flow. The water flows in and out of the tank through rubber hoses, what are circled in rings. This water flow peculiarity increases plant's nonlinear characteristics. The pumps have dead zones of different magnitudes and saturation non-linearity; they intro-duce electrical noises and delays into the system. The water flow also depends on the water temperature and its softness, what makes the modeling task more difficult.

5 Fuzzy Hammerstein model of nonlinear water plant

Fig. 3 shows the scheme of the two input one output fuzzy Hammerstein model.

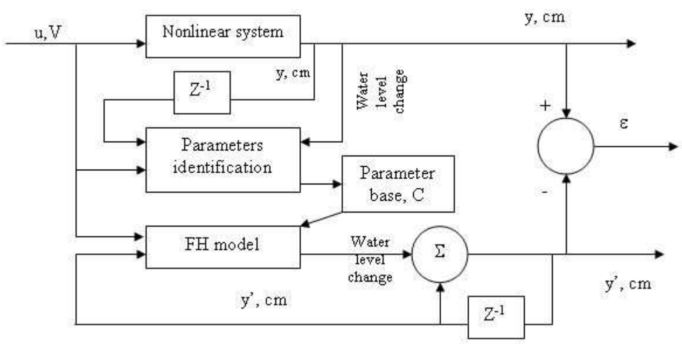

Fig. 3. The scheme of fuzzy Hammerstein model.

The inputs are the control signal and the actual water level. The output is the change of water level in reservoir. The predicted water level at the time moment t is calculated as the sum the previously calculated water level and the predicted water level change. The generalized *fuzzy Hammerstein* model was synthesized for the nonlinear plant. The structure of the model is shown in Fig. 4.

The input linguistic variables are described using symmetric triangular membership functions, equally spread across the universes of discourses. The universe of discourse of the control signal is an interval $[0; 10]$ and the universe of discourse of the water level is $[0; 20]$. The first linguistic variable is composed of 6 membership functions, the second with 21. Fuzzy sub-model uses 126 zero order *Takagi-Sugeno* fuzzy rules, product for rules implication and singleton defuzzification. The generalized *fuzzy Hammerstein* model is described using parameter vector, containing polynomial A coefficients, the order of the polynomials A and B, the *Takagi-Sugeno* fuzzy rule base the meaning of which is explained in [26] and coefficient C. The order of the polynomial A was experimentally chosen to be 3 as higher order makes the model unstable. The order of the polynomial B was experimentally chosen to be 6.

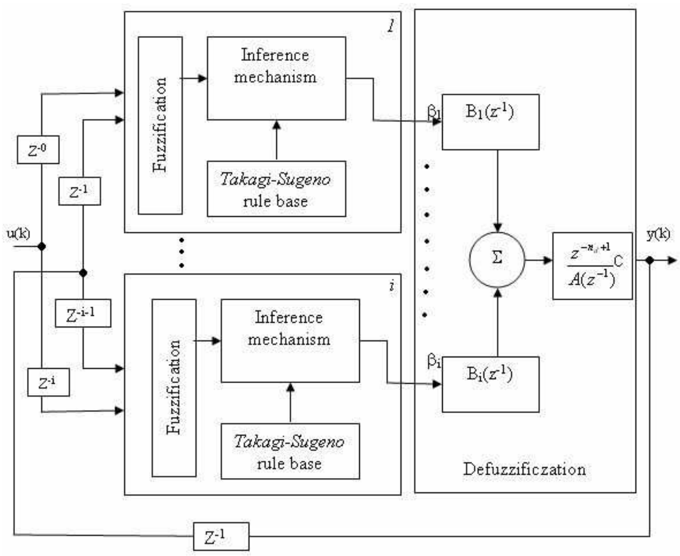

Fig. 4. The structure of generalized fuzzy Hammerstein model.

In order to increase the prediction rate of the *fuzzy Hammerstein* model, the parameter grouping was introduced. The parameters were estimated for the five different groups of the model input (the plant control signal) values: $[0\ldots2]$, $[2\ldots4]$, $[4\ldots6]$, $[6\ldots8]$, and $[8\ldots10]$ volts. Subject to the input values of the *fuzzy Hammerstein* model different parameter vectors were used. Besides, the polynomial A coefficients are multiplied by the coefficient C:

$$\widehat{y}(k) = \sum_{i=1}^{n_a} CA_i y(k-i) + \sum_{i=1}^{n_b}\sum_{j=1}^{n_r} B_i^j \beta_j(k-i-n_d). \tag{17}$$

The coefficient C was introduced in the equation noting from the experiments that the model is more accurate when the roots of the polynomial A are closer to 0. The influence of the coefficient C was also analyzed changing the order of the polynomial B. It was noticed that the values of the coefficient are symmetric in regard to the input signal of each group and is different when the polynomial B order changes (higher values if polynomial B order is higher and lower values if its order is lower). The coefficient C has always value 1 at the ends of group intervals because the model parameters are identified at these points. Table 1 presents the experimentally determined values of coefficient C using which the prediction error is the smallest.

An increase of the order of the polynomial A in most cases makes the model unstable so the relationship to the values of the coefficient C was not found.

Table 1. Coefficient C values for group $[4 \dots 6]$

Control signal (V)	4	4.1	4.5	5	5.5	5.9	6
The order of Polynomial B							
2	1	0.06	0.4	0.37	0.4	0.06	1
4	1	0.3	0.46	0.5	0.46	0.3	1
6	1	0.52	0.68	0.75	0.68	0.52	1

6 The analysis of fuzzy Hammerstein model

The synthesized *fuzzy Hammerstein* model was tested with the real data from the plant and the results were compared with the *Mamdani* type fuzzy model, *Takagi-Sugeno* type fuzzy model and the hybrid fuzzy model [26].

The experiments were done in the real time. The step form input signal with the steps of 5 volts, 4.1 volts and 6 volts was passed to the plant ant to the models at the same time. The data were acquired at 1 second intervals.

Fuzzy models were compared according to the following criteria:

- The number of parameters that need to be identified.
- The number of times the re-identification of the parameters was used expressed in % (the process of re-identification of model parameters is applied when the model's prediction error exceeds the defined limit).
- The accuracy of the prediction (the mean, the mean quadratic deviation, the standard deviation and the relative error of prediction were calculated).

The results of the experiments are presented in the Table 2 and the Figs. 5–8. From the figures it can be seen that the best prediction is achieved with the *fuzzy Hammerstein* model. This model is defined with the smallest amount of the parameters (185 parameters) and it predicts more accurate than the Mamdani type model, defined by 2892 parameters. Another advantage of the *fuzzy Hammerstein* model is that it never re-identifies its parameters as the other models do, so it is $100\,\%$ predictive.

Table 2. Performance analysis of fuzzy models

Fuzzy model	No. of parameters	No. of online re-ident (%)	Prediction error			
			Relative	Mean	Mean quadric deviat.	Stand. deviation
Mamdani	2892	4.5	0.2016	0.2113	0.0413	0.2032
Takagi-sugeno	18	10.60	0.3472	0.3154	0.0893	0.2989
Hybrid	908	5.15	0.2470	0.2204	0.0281	0.1677
Hammerstein	185	0.0	0.1280	0.2066	0.0476	0.2183

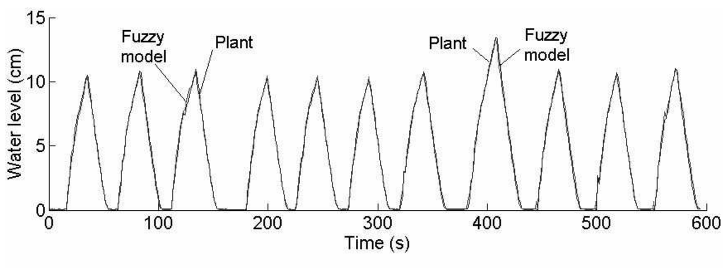

Fig. 5. Outputs of the plant and the Mamdani type fuzzy model.

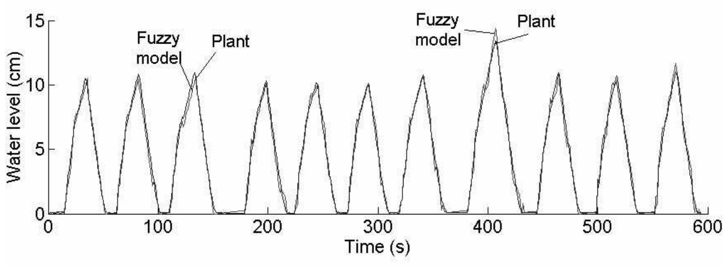

Fig. 6. Outputs of the plant and the Takagi-Sugeno type fuzzy mode.

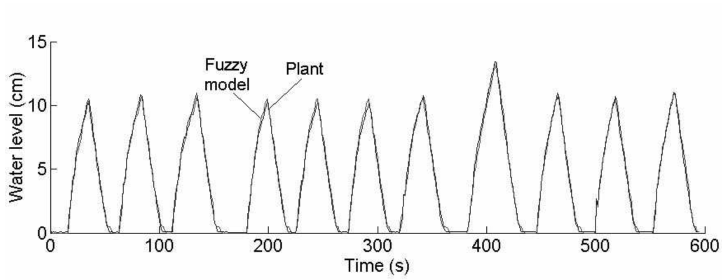

Fig. 7. Outputs of the plant and the Hybrid fuzzy model.

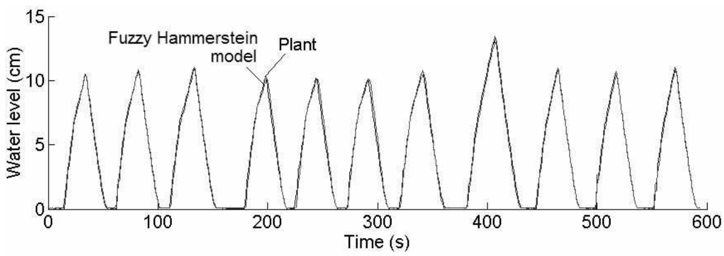

Fig. 8. Outputs of the plant and the fuzzy Hammerstein model.

7 Conclusions

In this paper the synthesis of the *fuzzy Hammerstein* model for the nonlinear and non-static water level plant has been introduced. The *fuzzy Hammerstein* model was experimentally compared with the *Mamdani* type, *Takagi-Sugeno* type and Hybrid fuzzy models. It was experimentally proved, that the *fuzzy Hammerstein* model is more adequate to the real plant than the other fuzzy models and it can be described using less parameters than the other models, analyzed in this paper. For the identification of the model's parameters the recursive least square algorithm was used. In order to increase the quality of the model, the parameter grouping during the process of the parameter identification was introduced to the *fuzzy Hammerstein* model. It was experimentally proved that the *fuzzy Hammerstein* model with the parameter grouping is more precise than the model without it. It was also experimentally proved that once identified the *fuzzy Hammerstein* model is quite precise for any operating mode of the plant and did not need additional parameter re-identification as the other analyzed models did.

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