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Steady Flow over a Rotating Disk in Porous Medium with Heat Transfer

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Abstract. The steady flow of an incompressible viscous fluid above an infinite rotating disk in a porous medium is studied with heat transfer. Numerical solutions of the nonlinear governing equations which govern the hydrodynamics and energy transfer are obtained. The effect of the porosity of the medium on the velocity and temperature distributions is considered.

Keywords: rotating disk, porous medium, heat transfer, numerical solution.

1 Introduction

The pioneering study of fluid flow due to an infinite rotating disk was carried by von Karman in [1, 1921]. Von Karman gave a formulation of the problem and then introduced his famous transformations which reduced the governing partial differential equations to ordinary differential equations. Cochran [2] obtained asymptotic solutions for the steady hydrodynamic problem formulated by von Karman. Benton [3] improved Cochran's solutions and solved the unsteady problem. The problem of heat transfer from a rotating disk maintained at a constant temperature was first considered by Millsaps and Pohlhausen [4] for a variety of Prandtl numbers in the steady state. Sparrow and Gregg [5] studied the steady state heat transfer from a rotating disk maintained at a constant temperature to fluids at any Prandtl number. The influence of an external uniform magnetic field on the flow due to a rotating disk was studied [6–8]. The effect of uniform suction or injection through a rotating porous disk on the steady hydrodynamic or hydromagnetic flow induced by the disk was investigated [9–11].

In the present work, the steady laminar flow of a viscous incompressible fluid due to the uniform rotation of a disk of infinite extent in a porous medium is studied with heat transfer. The flow in the porous media deals with the analysis in which the differential equation governing the fluid motion is based on the Darcy's law which accounts for the drag exerted by the porous medium [12–14]. The temperature of the disk is maintained at a constant value. The governing nonlinear differential equations are integrated numerically using the finite difference approximations The effect of the porosity of the medium on the steady flow and heat transfer is presented and discussed.

2 Basic equations

Let the disk lie in the plane $z = 0$ and the space $z > 0$ is equiped by a viscous incompressible fluid. The motion is due to the rotation of an insulated disk of infinite extent about an axis perpendicular to its plane with constant angular speed ω through a porous medium where the Darcy model is assumed [14]. Otherwise the fluid is at rest under pressure p_∞. The equations of steady motion are given by

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v^2}{r}\right) + \frac{\partial p}{\partial r} = \mu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2}\right) - \frac{\mu}{K}u, \tag{2}$$

$$\rho\left(u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial z} + \frac{uv}{r}\right) = \mu\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} - \frac{v}{r^2} + \frac{\partial^2 v}{\partial z^2}\right) - \frac{\mu}{K}v, \tag{3}$$

$$\rho\left(u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) + \frac{\partial p}{\partial z} = \mu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right) - \frac{\mu}{K}w, \tag{4}$$

where u, v, w are velocity components in the directions of increasing r, φ, z respectively, P is denoting the pressure, μ is the coefficient of viscosity, ρ is the density of the fluid, and K is the Darcy permeability [12–14]. We introduce von Karman transformations [1],

$$u = r\omega F, \quad v = r\omega G, \quad w = \sqrt{\omega\nu}H, \quad z = \sqrt{\nu/\omega}\zeta, \quad p - p_\infty = -p\nu\omega P,$$

where ζ is a non-dimensional distance measured along the axis of rotation, F, G, H and P are non-dimensional functions of ζ, and ν is the kinematic viscosity of the fluid, $\nu = \mu/\rho$. With these definitions, equations (1)–(4) take the form

$$\frac{\mathrm{d}H}{\mathrm{d}\zeta} + 2F = 0, \tag{5}$$

$$\frac{\mathrm{d}^2 F}{\mathrm{d}\zeta^2} - H\frac{\mathrm{d}F}{\mathrm{d}\zeta} - F^2 + G^2 - MF = 0, \tag{6}$$

$$\frac{\mathrm{d}^2 G}{\mathrm{d}\zeta^2} - H\frac{\mathrm{d}G}{\mathrm{d}\zeta} - 2FG - MG = 0, \tag{7}$$

$$\frac{\mathrm{d}^2 H}{\mathrm{d}\zeta^2} - H\frac{\mathrm{d}H}{\mathrm{d}\zeta} + \frac{\mathrm{d}P}{\mathrm{d}\zeta} - MH = 0, \tag{8}$$

$M = \nu/K\omega$ is the porosity parameter. The boundary conditions for the velocity problem are given by

$$\zeta = 0, \quad F = 0, \quad G = 1, \quad H = 0, \tag{9a}$$

$$\zeta \to \infty, \quad F \to 0, \quad G \to 0, \quad P \to 0, \tag{9b}$$

Equation (9a) indicates the no-slip condition of viscous flow applied at the surface of the disk. Far from the surface of the disk, all fluid velocities must vanish aside the induced axial component as indicated in equation (9b). The above system of equations (5)–(7) with the prescribed boundary conditions given by equations (9) are sufficient to solve for the three components of the flow velocity. Equation (8) can be used to solve for the pressure distribution if required.

Due to the difference in temperature between the wall and the ambient fluid, heat transfer takes place. The energy equation without the dissipation terms takes the form [4, 5];

$$\rho c_p\left(u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}\right)-k\left(\frac{\partial^2 T}{\partial z^2}+\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right)=0, \tag{10}$$

where T is the temperature of the fluid, c_p is the specific heat at constant pressure of the fluid, and k is the thermal conductivity of the fluid. The boundary conditions for the energy problem are that, by continuity considerations, the temperature equals T_w at the surface of the disk. At large distances from the disk, T tends to T_∞ where T_∞ is the temperature of the ambient fluid. In terms of the non-dimensional variable $\theta = (T - T_\infty)/(T_w - T_\infty)$ and using von Karman transformations, equation (10) takes the form;

$$\frac{1}{Pr}\frac{\mathrm{d}^2\theta}{\mathrm{d}\zeta^2}-H\frac{\mathrm{d}\theta}{\mathrm{d}\zeta}=0, \tag{11}$$

where Pr is the Prandtl number, $Pr = c_p\mu_k/k$. The boundary conditions in terms of θ are expressed as

$$\theta(0)=1, \quad \theta(\infty)=0. \tag{12}$$

The system of non-linear ordinary differential equations (5)–(7) and (11) is solved under the conditions given by equations (9) and (12) for the three components of the flow velocity and temperature distribution, using the Crank-Nicolson method [15]. The resulting system of difference equations has to be solved in the infinite domain $0 < \zeta < \infty$. A finite domain in the ζ-direction can be used instead with ζ chosen large enough to ensure that the solutions are not affected by imposing the asymptotic conditions at a finite distance. The independence of the results from the length of the finite domain and the grid density was ensured and successfully checked by various trial and error numerical experimentations. Computations are carried out for $\zeta_\infty = 12$.

3 Results and discussion

Figs. 1–4 present the variation of the profiles of the velocity components G, F, and H and the temperature θ, respectively, for various values of the porosity parameter M and for $Pr = 0.7$. Figs. 1–3 indicate the restraining effect of the porosity of the medium on the flow velocity in the three directions. Increasing the porosity parameter M decreases G, F, and H and the thickness of the boundary layer.

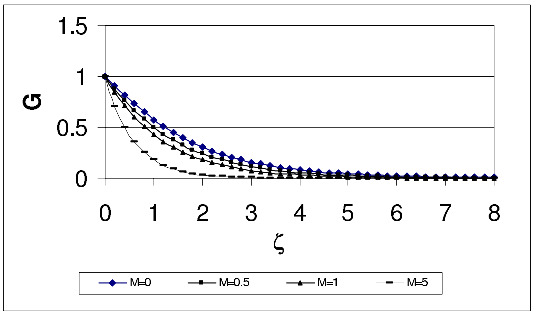

Fig. 1. Effect of the porosity parameter M on the profile of G.

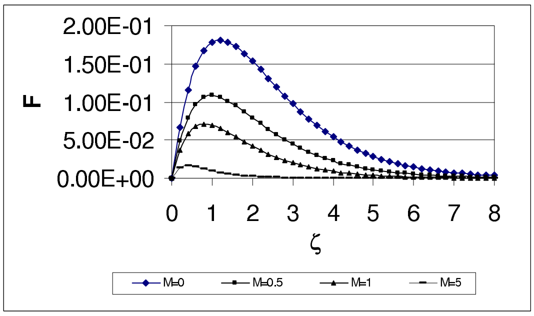

Fig. 2. Effect of the porosity parameter M on the profile of F.

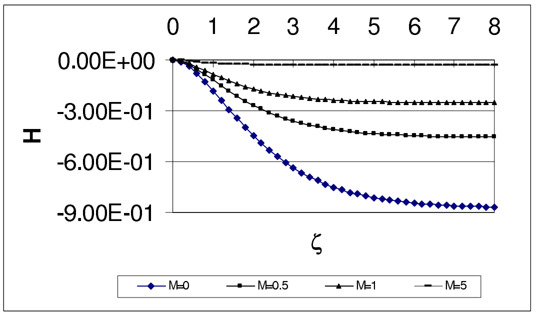

Fig. 3. Effect of the porosity parameter M on the profile of H.

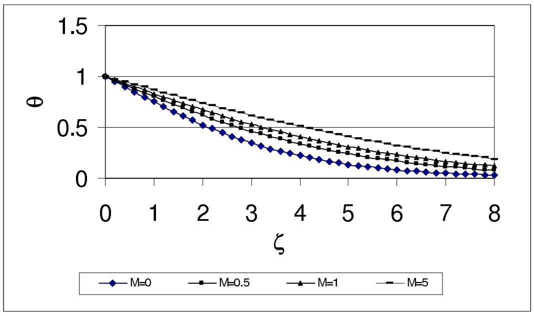

Fig. 4. Effect of the porosity parameter M on the profile of θ.

Fig. 4 presents the influence of the porosity parameter M in increasing the temperature θ as a result of the effect of the porosity in preventing the fluid at near-ambient temperature from reaching the surface of the disk. Consequently, increasing M increases the temperature as well as the thermal boundary layer thickness. The absence of fluid at near-ambient temperature close to the surface increases the heat transfer.

4 Conclusion

In this study the steady flow induced by a rotating disk with heat transfer in a porous medium was studied. The results indicate the restraining effect of the porosity on the flow velocities and the thickness of the boundary layer. On the other hand, increasing the porosity parameter increases the temperature and thickness of the thermal boundary layer.

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