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Generalized Adaptive Backstepping Synchronization for Non-identical Parametrically Excited Systems

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Abstract. In this paper, we investigate the synchronization of chaotic systems consisting of non-identical parametrically excited oscillators. The backstepping design, which is a recursive procedure that combines the choice of a Lyapunov function with the design of a controller is generalized and employed so as to achieve global chaos synchronization between a parametrically excited gyroscope and each of the parametrically excited pendulum and Duffing oscillator. Numerical simulations are implemented to verify the results.

Keywords: synchronization, chaos, parametrically excited systems, backstepping.

1 Introduction

Chaotic behaviour is a well-known phenomenon in physics, engineering, biology, and many other scientific disciplines. Recently, it has received much attention [1, 2]. The control and synchronization of chaotic systems, represents a challenge, since a chaotic system is extremely sensitive to small pertubations. Notwithstanding, the possibility of control and synchronization of chaotic systems under certain conditions have been established [1–8]. Due to the connection between control and synchronization, recent studies cast the problem of synchronization in the framework of control theory.

In this light, various techniques have been proposed for achieving synchronization between identical and non-identical systems. For instance, the active control scheme proposed by Bai and Lonngren [9] has received considerable attention in the last few years due to its simplicity and robustness. This scheme has been modified over time, but recently, Lei et al. [10] applied Lyapunov stability theory and Routh-Hurwitz criteria to synchronize identical parametrically excited system by using the active control technique. The backstepping design scheme which can guarantee global stability, tracking and transient performance for a broad class of strict-feedback nonlinear systems, has been widely employed for controlling, tracking and synchronizing many chaotic systems [11–16] as well as hyperchaotic systems [18]. The advantages of backstepping include applicability to a variety of chaotic systems with or without external excitation, need for only one controller to realize synchronization, and have requirement of less control effort in comparison with other control methods [12, 13, 18].

In [12], Tan et al. proposed an adaptive backstepping design for synchronizing identical chaotic systems. However, in practice most system are non-identical. Therefore, it is very necessary to synchronize two non-identical chaotic systems. Since non-identical chaotic systems have different nonlinear functions, different number of equilibrium points, different phase maps and shapes, synchronization of non-identical chaotic systems is difficult to achieve and hence has received less attention [19–21, 23, 24].

Ge et al. [19] constructed control functions based on linear coupling of the state variables of the drive and response systems. However, the state variables need to be seperated from the others and coupled into a linear coupling term to add into the synchronized systems. This is difficult to realize in practice. In [20], the controller was designed by constructing the Lyapunov function or calculating Lyapunov exponent to realize synchronization, but the calculations of Lyapunov exponent is usually difficult. To address the problems associated with the applications of the controllers in [19, 20], Lü et al. [21] presented a nonlinear feedback control strategy for synchronizing different chaotic systems. In addition, the method of active control has been applied by Njah and Vincent [22] to synchronize between single and double well Duffing-Van der Pol oscillator and Vincent in [23] applied the method to achieve synchronization between different 4-D chaotic systems while Zhang et al. [18] proposed an active-backstepping method to solve this problem. All the approaches described in [18–21, 23, 24], including the active-sliding mode control [24] employed control functions which are numerically equal to the dimension of the system. This requirement makes the controllers very complex for practical applications. A recent analysis in [16, 17] shows that the adaptive backstepping design [12], besides its efficiency would also reduce considerably the controller complexity, since only one control function is required to achieve the synchronization goal. Thus, in this paper we proposed a generalized adaptive backstepping strategy for synchronizing non-identical chaotic systems. This problem has not been treated previously in the literature to the best of our knowledge.

Specifically, we illustrate this approach using non-identical parametrically excited systems. Parametrically excited systems have been widely explored for modeling the dynamic behaviour of many engineering systems such as offshore platforms, buildings under earthquakes, orientation information [25–28] and so on. However, not much attention has been given to the study of synchronization of parametrically excited systems. A few reports, can be found in [10, 28–32]. In [10] the synchronization of identical parametrically excited pendulum and the Duffing oscillators were considered separately by Lei et al., while in [29–31], the synchronization of identical nonlinear gyroscope were treated. In all of the above reports, synchronization between non-identical system were not treated except in [32], where we considered the synchronization of different parametrically driven oscillators using active control method.

The rest of the paper is organized as follows: in the following section, we formulate the problem, while in Section 3, we give a brief description of the systems under consideration. The adaptive synchronization between the parametric gyroscope and pendulum are presented in Section 4 and that between the parametric gyroscope and Duffing oscillator is presented in Section 5. Numerical simulations are also given to verify the results. Finally, the paper is concluded in Section 6.

2 Problem formulation

Consider a chaotic system described by

$$\dot{x} = A(t)x + f(x), \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is an n-dimensional state vector of the system, $A(t) \in \mathbf{R}^n$ is a time-periodic matrix for the system parameter, and $f\colon \mathbf{R}^n \to \mathbf{R}^n$ is the nonlinear part of the drive system which is continuously differentiable and satisfies the global Lipschitz condition,

$$\|f(x_1) - f(x_2)\| \le c\|x_1 - x_2\|, \quad \forall x_1, x_2 \in \mathbf{R}^n \text{ for some } c > 0. \tag{2}$$

The response system is given by

$$\dot{y} = B(t)y + g(y) + u(t), \tag{3}$$

where $y(t) \in \mathbf{R}^n$ denotes the state vector of the responding system, $B(t) \in \mathbf{R}^n$ is the matrix of the response system parameter, and $g\colon \mathbf{R}^n \to \mathbf{R}^n$ is the nonlinear part of the responding system. $u(t) \in \mathbf{R}^n$ is a controller which is to be designed.

If $A(t) = B(t)$ and $f(x) = g(y)$, then x and y are the states of two identical systems. If $A(t) \neq B(t)$ or/and $f(x) \neq g(y)$, then x and y are the states of two different chaotic systems. The case for two different chaotic systems is what we treat here.

By properly choosing u, synchronization between the drive and response system can be achieved. The dynamics of the synchronization errors can be obtained as

$$\dot{e} = C(t)e + g(y) - f(x) + u(t), \tag{4}$$

where $C(t) = B(t) - A(t)$ is the matrix of the linear part of the error dynamics parameter and $e = y - x$. Hence, the synchronization goal is to make $lim_{t\to+\infty}\|e(t)\| = 0$.

In the absence of the control $u(t)$, the error system (4) would have an equilibrium at $(0, 0)$. If a control $u(t)$ is chosen such that the equilibrium $(0, 0)$ remains unchanged, then the synchronization problem can be transformed to that of realizing asymptotic stabilization of system (4) about $(0, 0)$. Thus our objective is to design an adaptive feedback controller for system (3) that guarantees global stability at the origin.

The backstepping design procedure contains n steps. At first an intermediate control function α_i shall be developed using an appropriate Lyapunov function, V_i. Next an update for the parameter estimate is designed, thereafter, the stabilizing function α_i and an update law are designed to render the derivative of the chosen Lyapunov function negative definite. We illustrate the approach with examples in Sections 4 and 5.

3 System description

Detailed description of the systems under study can be found in [27, 29, 31, 33] and references therein.

The equation governing the motion of the *parametrically excited gyro* is given by [29]

$$\begin{aligned} \dot{x_1} &= y_1, \\ \dot{y_1} &= g(x_1) - ay_1 - by_1^3 + \beta \sin x_1 + f \sin \omega t \sin x_1, \end{aligned} \tag{5}$$

where $g(x_1) = -\alpha^2 \frac{(1-\cos x_1)^2}{\sin^3 x_1}$.

The nonlinear gyro given by equation (5) is taken to be the driver system. It exhibits varieties of dynamical behaviour including chaotic motion – displayed in Fig. 1 for the following parameters $\alpha^2 = 100,\ \beta = 1,\ a = 0.5,\ b = 0.05,\ \omega = 2$, and $f = 35.5$.

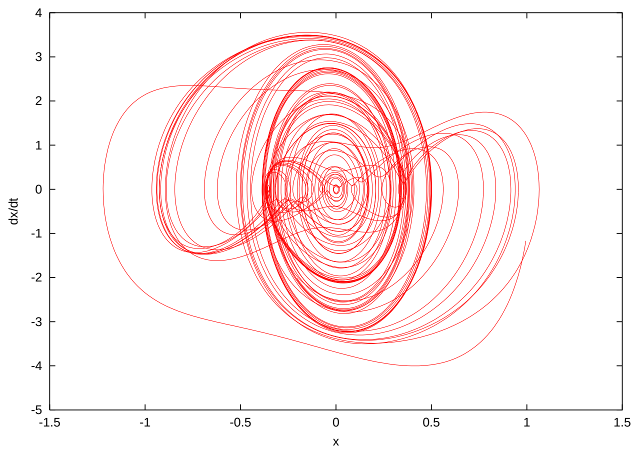

Fig. 1. The phase portrait of the chaotic gyro.

The first slave system under consideration is a *parametrically excited chaotic pendulum*, which can be described by [27]

$$\begin{aligned} \dot{x_2} &= y_2, \\ \dot{y_2} &= -hy_2 - \sin x_2 - \rho \cos \omega t \sin x_2 + u(t), \end{aligned} \tag{6}$$

where u is a control input to be determined. The chaotic attractor of this pendulum (6) for the following parameters $h = 0.1, \rho = 2.0$ and $\omega = 2.0$ is displayed in Fig. 2.

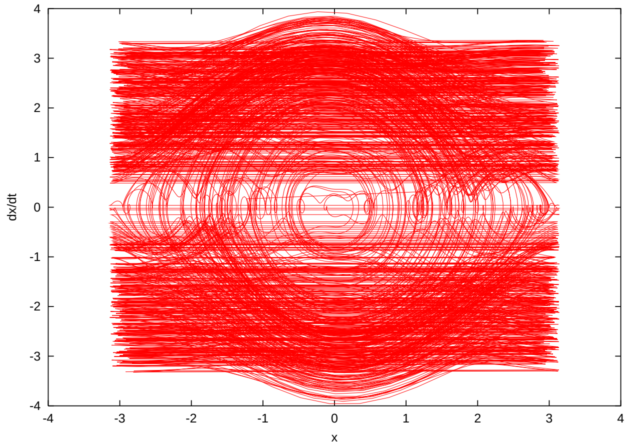

Fig. 2. The phase portrait of the chaotic pendulum.

The second slave system is a *parametrically excited Duffing system* subject to harmonic parametric excitation in the form [33]

$$\begin{aligned} &\dot{x}_2 = y_2, \\ &\dot{y}_2 = -\gamma y_2 + x_2 - {x_2}^3 + \mu x_2 \sin \Omega t + u(t), \end{aligned} \tag{7}$$

where u is a control input to be determined.

The phase portrait of the chaotic attractor associated with Duffing system (7) for $\gamma = 0.2, \mu = 0.5$ and $\Omega = 1.0$ is given in Fig. 3.

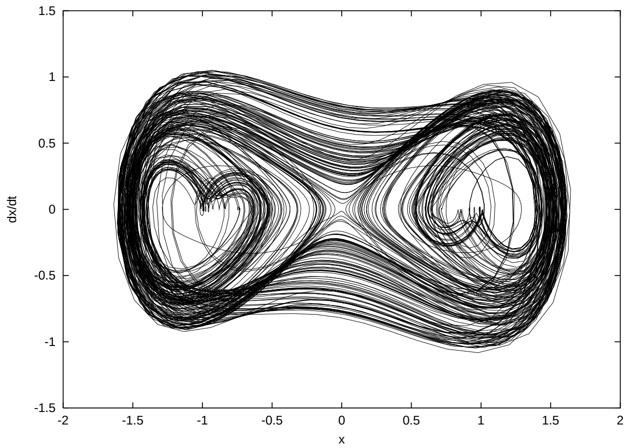

Fig. 3. The phase portrait of the chaotic Duffing oscillator.

4 Adaptive synchronization between the gyroscope and pendulum

Here we synchronize the parametric gyroscope (5) with the parametric pendulum (6), where the gyro is the drive system and pendulum is the response system.

Let the error state between (5) and (6) be $e_x = x_2 - x_1$ and $e_y = y_2 - y_1$.

Using the above definition, we have the following error dynamics for the drive-response system as:

$$\begin{aligned} \dot{e_x} &= y_2 - y_1, \\ \dot{e_y} &= -hy_2 - \sin x_2 - \rho \cos \omega t \sin x_2 - g(x_1) + ay_1 \\ &\quad + b{y_1}^3 - \beta \sin x_1 - f \sin \omega t \sin x_1 + u(t). \end{aligned} \tag{8}$$

The objective is to find a control law so that system (8) is stabilized at the origin. Starting from the first equation of system (8), an estimative stabilizing function $\alpha_1(e_x)$ has to be designed for the virtual control e_y in order to make the derivative of $V_1(e_x) = \frac{1}{2}{e_x}^2$, negative definite when $\alpha_1(e_x) = -e_x$. Define the error variable w_2 as

$$w_2 = e_y - \alpha_1(e_x). \tag{9}$$

Considering the (e_x, w_2) subspace given by

$$\begin{aligned} \dot{e_x} &= w_2 - e_x, \\ \dot{w_2} &= -he_y + y_1(a - h + b{y_1}^2) - g(x_1) - \sin(e_x + x_1)(1 + \rho \cos \omega t) \\ &\quad - \sin x_1(\beta + f \sin \omega t) + w_2 - e_x + u(t) \end{aligned} \tag{10}$$

which form the complete system.

Choosing the Lyapunov function

$$V_2(e_x, w_2) = V_1(e_x) + \frac{1}{2}w_2^2. \tag{11}$$

The derivative of equation (11) along the error dynamics (10) is

$$\begin{aligned} \dot{V_2} = -e_x^2 + w_2\big[&- he_y + y_1\big(a - h + b{y_1}^2\big) - g(x_1) - \sin(x_2)(1 + \rho \cos \omega t) \\ &- \sin x_1(\beta + f \sin \omega t) + w_2 + u(t)\big]. \end{aligned} \tag{12}$$

If

$$\begin{aligned} u(t) = -\big[&- he_y + y_1\big(a - h + b{y_1}^2\big) - g(x_1) - \sin(x_2)(1 + \rho \cos \omega t) \\ &- \sin x_1(\beta + f \sin \omega t) + 2w_2\big], \end{aligned} \tag{13}$$

then

$$\dot{V_2} = -e_x^2 - w_2^2 < 0 \tag{14}$$

is negative definite and according to Lasalle-Yoshizawa theorem [34], the error dynamics will converge to zero as $t \to \infty$, while the equilibrium $(0, 0)$ remains global asymptotically stable. Thus, the synchronization between two non-identical parametrically excited system is achieved via the adaptive backstepping design.

We performed numerical simulations for the systems with parameters as stated earlier and the initial conditions are $(x_1, y_1) = (1, -1)$, $(x_2, y_2) = (1, 1)$. In Fig. 4 and Fig. 5, we display a situation where the control was de-activated and activated at $t = 0$ respectively. It is very clear that the synchronization has been achieved since the error dynamics (e_1, e_2) between the drive and the response systems approaches zero as $t \to \infty$.

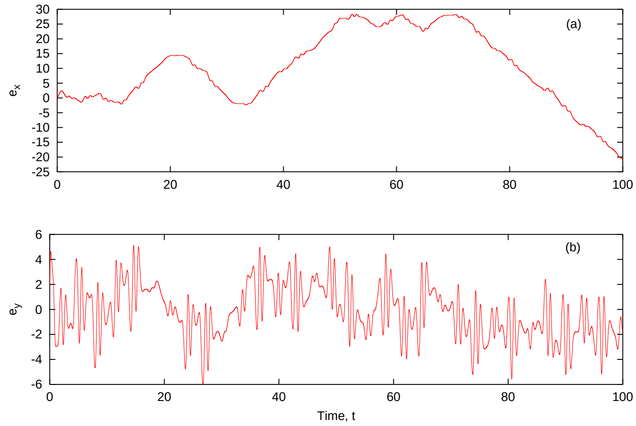

Fig. 4. Error dynamics of the coupled system when the active controller is de-activated for a parametric gyro and pendulum: (a) e_x, (b) e_y.

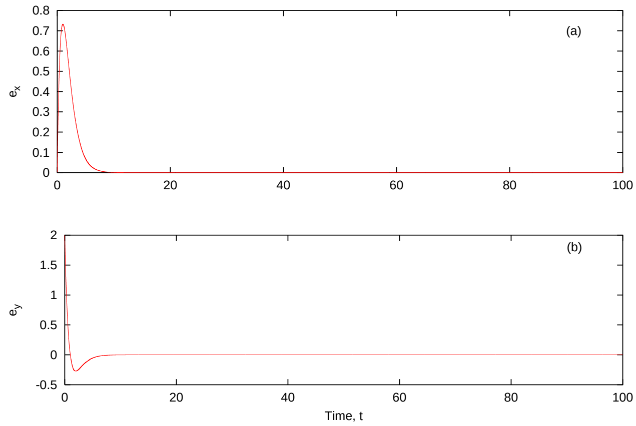

Fig. 5. Error dynamics of the coupled system when the active controller is activated for a parametric gyro and pendulum: (a) e_x, (b) e_y.

5 Adaptive synchronization between the gyroscope and Duffing oscillator

Here we synchronize the parametric gyroscope (5) with the parametric Duffing oscillator (7), where the gyro is the drive system and Duffing oscillator is the response system.

Let the error state between (5) and (7) be $e_x = x_2 - x_1$ and $e_y = y_2 - y_1$.

Using above definition, we have the following error dynamics for the drive-response system as:

$$\begin{aligned} \dot{e_x} &= y_2 - y_1, \\ \dot{e_y} &= -\gamma y_2 + x_2 - x_2^3 + \mu x_2 \sin \Omega t - g(x_1) + a y_1 \\ &\quad + b y_1^3 - \beta \sin x_1 - f \sin \omega t \sin x_1 + u(t). \end{aligned} \tag{15}$$

The objective is to find a control law so that system (15) is stabilized at the origin. Starting from the first equation of system (15), an estimative stabilizing function $\alpha_1(e_x)$ has to be designed for the virtual control e_y in order to make the derivative of $V_1(e_x) = \frac{1}{2}{e_x}^2$, negative definite when $\alpha_1(e_x) = -e_x$. Define the error variable w_2 as

$$w_2 = e_y - \alpha_1(e_x). \tag{16}$$

Considering the (e_x, w_2) subspace given by

$$\begin{aligned} \dot{e_x} &= w_2 - e_x, \\ \dot{w_2} &= -\gamma e_y + y_1\big(a + b y_1^2 - \gamma\big) + e_x(1 - e_x^2 - 3 e_x x_1 - 3 x_1^2 + \mu \sin \Omega t) \\ &\quad + x_1\big(1 - x_1^2 + \mu \sin \Omega t\big) - g(x_1) - \sin x_1(\beta + f \sin \omega t) + w_2 + u(t), \end{aligned} \tag{17}$$

which form the complete system.

Choosing the Lyapunov function

$$V_2(e_x, w_2) = V_1(e_x) + \frac{1}{2} w_2^2. \tag{18}$$

The derivative of equation (18) along the error dynamics (17) is

$$\begin{aligned} \dot{V_2} = -e_x^2 + w_2\big[&-\gamma e_y + y_1\big(a + b y_1^2 - \gamma\big) \\ &+ e_x\big(1 - e_x^2 - 3 e_x x_1 - 3 x_1^2 + \mu \sin \Omega t\big) \\ &+ x_1(1 - x_1^2 + \mu \sin \Omega t) - g(x_1) \\ &- \sin x_1(\beta + f \sin \omega t) + w_2 + u(t)\big]. \end{aligned} \tag{19}$$

If $u(t)$ is chosen such that

$$\begin{aligned} u(t) = -\big[&-\gamma e_y + y_1\big(a + b y_1^2 - \gamma\big) + e_x\big(1 - e_x^2 - 3 e_x x_1 - 3 x_1^2 + \mu \sin \Omega t\big) \\ &+ x_1\big(1 - x_1^2 + \mu \sin \Omega t\big) - g(x_1) - \sin x_1(\beta + f \sin \omega t) + 2 w_2\big], \end{aligned} \tag{20}$$

then

$$\dot{V_2} = -e_x^2 - w_2^2 < 0 \tag{21}$$

is negative definite and according to Lasalle-Yoshizawa theorem [34], the error dynamics will converge to zero as $t \to \infty$, while the equilibrium $(0,0)$ remains global asymptotically stable. Thus, the synchronization between the parametric gyroscope and the parametric Duffing oscillator is achieved via the adaptive backstepping design.

We performed numerical simulations for the systems with parameters as stated earlier and the initial conditions are $(x_1, y_1) = (1, -1)$, $(x_2, y_2) = (1, 2.1)$. In Fig. 6 and Fig. 7, we display a situation where the control was de-activated and activated at $t = 0$. It is very clear that the synchronization has been achieved since the error dynamics (e_1, e_2) between the drive-response system approaches zero as $t \to \infty$.

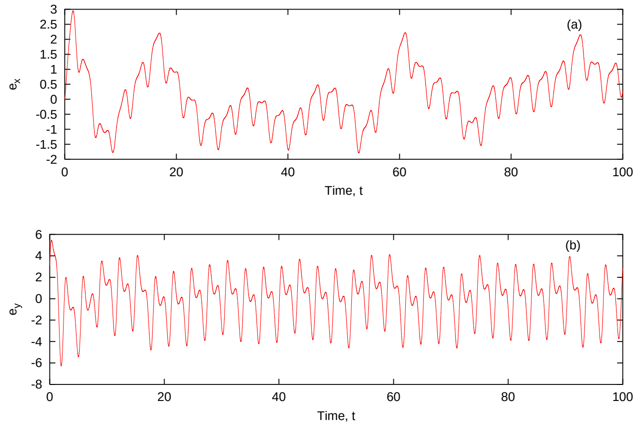

Fig. 6. Error dynamics of the coupled system when the active controller is de-activated for a parametric gyro and Duffing oscillator: (a) e_x, (b) e_y.

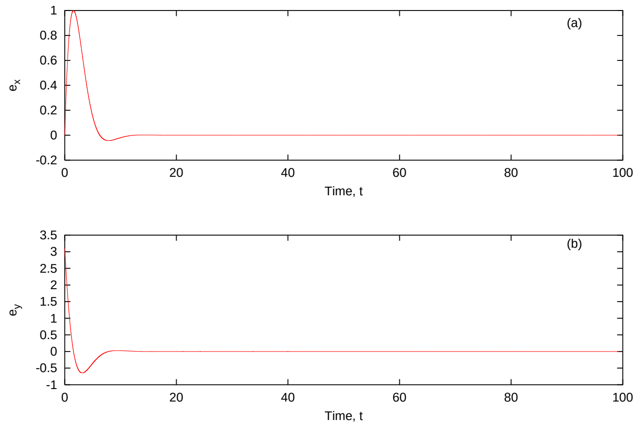

Fig. 7. Error dynamics of the coupled system when the active controller is activated for a parametric gyro and Duffing oscillator: (a) e_x, (b) e_y.

6 Conclusions

Backstepping is a systematic Lyapunov method to design control algorithms which stabilize nonlinear systems. In this paper, we have been able to synchronize non-identical parametrically excited systems via adaptive backstepping design for the first time. The work by Ge et al. [35], Tan et al [12] etc have been extended to achieve the set goal. To the best of our knowledge, previous authors have not utilized the adaptive backstepping design method to synchronize non-identical systems, of which the active control have been found valuable. The adaptive method implemented here allows for flexibility in the controller design and global stability based on the appropriate choice of Lyapunov functions, thus, it can readily be extended to other non-identical chaotic systems other than those with parametric excitation as well as higher dimensional chaotic systems. Our results, complimented with numerical simulations, show that the method is effective and feasible.

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