Nonlinear Analysis: Modelling and Control, Vol. 24, No. 5, 718–732
https://doi.org/10.15388/NA.2019.5.3
ISSN: 1392-5113
eISSN: 2335-8963

Feedback control and its impact on generalist predator–prey system with prey harvesting

Debabrata Das[a,1], **Tapan Kumar Kar**[b]

[a]Department of Mathematics,
B.P. Poddar Institute of Management and Technology, Kolkata,
Kolkata-700052, West Bengal, India
debuju84@gmail.com

[b]Department of Mathematics,
Indian Institute of Engineering Science and Technology, Shibpur,
Howrah-711103, West Bengal, India
tkar1117@gmail.com

Received: August 13, 2018 / **Revised:** December 31, 2018 / **Published online:** September 27, 2019

Abstract. This article examines the effectiveness of feedback control as a management policy on a generalist predator–prey system with prey harvesting. We discuss the result of implementing feedback control with respect to prey and predator separately. This paper also depicts the effect of exploitations up to maximum sustainable yield (MSY). We observe that with a constant fishing effort MSY policy is a sustainable management policy to protect both the species. However, further increase of fishing effort may cause the extinction of prey species. But considering feedback control of fishing effort may restrict the extinction of prey species. When fishing effort is controlled in terms of prey density, the extinction of prey population can be avoided. In this case, there may be coexistences of prey, predator and fishery or extinction of fishery. But when fishing effort is controlled by predator density, it is difficult to manage the coexistences of prey, predator and fishery.

Keywords: predator–prey, maximum sustainable yield, average yield, feedback control, limit cycle.

1 Introduction

Renewable natural resources are those that have the ability to renew themselves without extinguishing all together. Examples of renewable resources include fishery, agriculture, forestry etc. Practically, it is difficult to manage the exploitation of this kind of resources due to uncertain sustainability of natural system. In particular, fishery management requires a interdisciplinary cooperation between ecological, biological and economic foci to understand its complexity and proper policy making of the system. Some of the desirable

[1]Corresponding author.

objectives of fishery management may be considered as the furnishing of a good biomass yield, the protection of fish biomass, allocation of good economic return and recreation. For more economic return, human impact on marine ecosystem with the use of sophisticated equipments, open access fishery, over-exploitation and unregulated trawling are creating diverse effects. A number of fish stocks have been depleted, and large proportion of fisheries are fully or overexploited, and many are in danger of extinction [5]. In spite of our endeavor, many fisheries have been exhausted resulting a poor harvest or a series of fisheries collapse. This also brought a disastrous economic consequences to the society. Although there is a debate over the right approach to fisheries managements, so many measures have already been taken to protect the species and at the same time to earn some good revenue. The optimal taxation policy [15], the marine protected areas [16], the maximum sustainable yield (MSY) policy [6,9] and ecosystem based fishery management approach [24] are some major process to protect biological resources.

The principal target of fishery management is to operationalize the balancing act between present day benefits and future rewards. For the need to maintain the balance, the concept of "maximum sustainable yield (MSY)" was introduced. In population ecology, maximum sustainable yield is the maximum limit up to which natural resource can be harvested without long term depletion. Schaefer [31] introduced MSY policy for proportional harvesting in a single species model with logistic law of growth. Kar and Matsuda [17] discussed the effects of implementing MSY policy in a single species fishery with strong allee effect. Legović [21] investigated the same with logistic population under proportional harvesting. In traditional predator–prey system with harvesting of prey species at MSY level will cause the extinction of predator species [22] but if there is a strong intraspecific competition in predator species, then harvesting of prey population at MSY level may not leads the predator population to extinction [16]. The classical theory of MSY were very much criticised by Larkin [19] and Clark [7] due to not considering ecological interactions, age classes and economic factors. Estimates of the potential sustainable yield of fishery are common enough, but reviewing a sample of these, Pauly [29] concluded that most of them are simplistic and some are based on previous estimates. So the global catch is close or may exceed the robust estimates of sustainable yield resulting a overexploited or depleted fisheries.

Recently, marine reserve is being accepted widely as a tool for fisheries management, marine conservation and ecotourism [2, 4, 18] because a traditional form of fisheries management seems to be inadequate in stock management and conservation of multi-species fisheries. Conventional method of fisheries management mainly imposes restriction over catch by fixing a target level or by putting limit over fishing effort. This regulation sometimes become undesirable to some fisheries, and that is a big reason of failure to fish conservation. Marine reserve can be introduced in order to protect the fisheries from scientific uncertainty or wrong stock assessments or stock collapse [20]. Marine reserves also have an economic impact on marine ecotourism in terms of non-extractive re-creative activities [1, 28]. But there is a huge controversy for the right planning of marine reserve. Barnes and Sidhu [3] discussed the effect of implementation of reserve over yield and stock mobility. Holland and Brazee [13] has shown by simulation that the effect of reserve is much noticeable in heavily fished fishery due to increment of yield, but in lightly fished fishery, there exist no noticeable changes. Moussaoui et al. [25] prefer to have many small nature reserves over one single large total area, where, as Halpern [10] discussed, the relative effect of reserves in terms of change in biomass is independent of the size of reserve. Hilborn et al. [12] discussed that for single species fisheries, the effect of reserve is very little compared to conventional method, where the reserve has a noticeable potential advantages for multi-species fisheries.

World fisheries are considered to be in crisis. Most of the difficulties come from fluctuating stock dynamics and uncertainty lies in the fisheries management process. Difficulties also lies in the process how we are dealing with uncertainty and risk in harvest decision. So fisheries are now in a stage of flux, and many researchers think that we are at a position, where new policies strive for attention and demand evidence for their usefulness. Recently, adaptive management has become very much acceptable as a management policy as it is strong enough to overcome the uncertainty and fluctuating stock dynamics [23, 33]. Most of the fisheries management agency mainly concentrates over the relationship between stock and amount of catch, but scientists mainly focus on creative harvest policy to protect the population. The lack of connection between management agencies and research team has leads the way to the concept of adaptive process. Adaptive management can be considered as a combination of adaptive learning and feedback control. Conventional research did not take decision without full scientific understanding, but in adaptive learning, sometimes risky choices can also be accepted through careful decision making. The choices may be considered as there is a possibility to know more about the system and has a chance to minimize the uncertainty supposed to be faced by future policy makers. Whereas in feedback control, the management policy changes depending on the current observed estimation of the targeted species and previous statistics. The purpose is not to build a single best policy but to find out a number of alternative hypotheses, which are consistence with historical experiences. A feedback management process was proposed by Tanaka [32] for sustainable fisheries with uncertainty in stock abundance. In this process, the catch was managed to reach a target population size. Harada et al. [11] discusses the stability of the stock-harvesting system taking feedback control on fishing effort instead of catch. Kai and Shirakihara [14] proposed a feedback control model on the basis of size of a marine reserve. Parker [27] discusses optimal population biomass using feedback control over death and birth rate.

Our paper deals with the issue of feedback control and its consequences over a prey and predator system. We desire to build a better population model, which is robust against process and measurement errors by considering feedback. The paper is organized as follows. In Section 2, we introduce a predator–prey logistic model with harvesting of prey and introduce feedback control of fishing effort in terms of resource and predator biomass. Section 3 is a discussion of analytical solution of the model and stability of equilibrium without taking feedback as a management policy. We also investigate MSY policy considering average yield of prey population. Particularly, in Section 4, we consider feedback control in terms of prey biomass to investigate the dynamical properties of predator–prey model and find the condition for coexistences of species under this policy. Different scenarios of the system are shown by numerical simulation depending on variety of target catch per unit effort. Section 5 gives a brief conclusion of this research.

2 Model

In this section, we consider a model of a predator–prey system with generalist predator and harvesting of prey. The discussion of Matsuda and Abrams [23] about the effectiveness of feedback control for ecosystem based fisheries management, considering a predator–prey system with specialist predator, help us to assume the biological and technical assumptions of this model. Murdoch and Bence [26] observed that, due to generalist predator, the dynamic equilibrium become unstable causing local extinction. As Rosenzweig and MacArthur [30] mentioned, the generalist predator may stabilize the dynamical system by suppressing dominant species and restricting the outbreak of prey population. Thus the study of the impact of generalist predator on a predator–prey system is a challenging aspect in this context.

In the above perspective, we consider generalist predator in two species modeling with harvesting of prey as follows:

$$\frac{\mathrm{d}X_1}{\mathrm{d}t} = r_1X_1\left(1-\frac{X_1}{K_1}\right) - \frac{bX_1X_2}{1+bX_1h} - qEX_1, \tag{1}$$

$$\frac{\mathrm{d}X_2}{\mathrm{d}t} = r_2X_2\left(1-\frac{X_2}{K_2}\right) + \frac{mbX_1X_2}{1+bX_1h}, \tag{2}$$

where X_1 and X_2 are the prey and predator biomass, respectively; r_1 and r_2 are the per capita birth rate of prey and predator, respectively; K_1 and K_2 are the carrying capacity of the prey and predator, respectively; m is considered as energy conversion rate from prey to predator; b is a positive constant measure the maximum of predator's functional response to prey; h is the inverse of half saturation abundance of the functional response; E is the fishing effort on the prey; and q is catchability coefficient of prey. Due to fluctuating stock size, we concentrate over the time average yield Y, which is defined as

$$Y = \frac{1}{T}\int_0^T qEX_1(t)\,\mathrm{d}t,$$

where T is the time required to provide the yield Y. While calculating the yield, we neglect the probable time–density dependence of cost and cost-effective rebate of future harvests.

The feedback control of fishing effort in terms of resource biomass and predator biomass is considered as

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \begin{cases} \mu(E)(qX_1 - s_1) & \text{if } E > 0, \\ \max[0,\ \mu(E)(qX_1 - s_1)] & \text{if } E = 0, \end{cases} \tag{3}$$

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \begin{cases} \mu(E)(X_2 - s_2) & \text{if } E > 0, \\ \max[0,\ \mu(E)(X_2 - s_2)] & \text{if } E = 0, \end{cases} \tag{4}$$

where $\mu(E)$ is a function of fishing effort representing the adaptive changes. Here, for simplicity, we have considered $\mu(E) = \mu_0$ as a constant throughout our analysis; s_1 is the target catch per unit effort, and s_2 is the target level of predator biomass. Statistical data on predator population is not always available when prey is the only target. So, in the case of considering feedback control, we ignore measurement errors while collecting data of predator population.

3 Analysis

First, we consider the predator–prey dynamics of system (1), (2) with E as a constant and obtain the equilibrium of these equations analytically. The points of boundary equilibria are: (i) $(0, 0)$, (ii) $(0, K_2)$, (iii) $((K_1/r_1)(r_1 - qE), 0)$. Interior equilibrium of dynamics of (1), (2) is denoted by (X_1^*, X_2^*), where

$$X_2^* = K_2\left(1 + \frac{mbX_1^*}{r_2(1 + bX_1^*h)}\right),$$

and X_1^* is the solution of the equation

$$AX^3 + 3BX^2 + 3CX + D = 0 \tag{5}$$

with

$$A = \frac{b^2h^2r_1}{K_1}, \qquad B = \frac{1}{3}\left(\frac{bhr_1}{K_1}(2 - K_1bh) + b^2h^2qE\right),$$

$$C = \frac{1}{3}\left(\frac{mK_2b}{r_2}(m + r_2h) + \frac{r_1}{K_1} - 2bh(r_1 - qE)\right), \qquad D = mK_2 - (r_1 - qE).$$

The boundary equilibrium (i), (ii) and (iii) exists immediately as $r_1/q > E$, i.e. bio technical productivity(BTP) of prey is greater than the harvesting effort. We consider $B_T = r_1 - qE$, so $B_T > 0$ always.

The general equation (5) can have one positive root and two complex root, two positive roots and one negative roots, two negative and one positive roots or three positive roots. It is observed that for

$$\frac{mK_2}{2}\left(\frac{m}{r_2h} + 1\right) + \frac{r_1}{2K_1bh} < B_T < mK_2,$$

equation (5) has only one positive root, and for

$$\frac{2r_1}{K_1bh} < B_T < \frac{mK_2}{2}\left(\frac{m}{r_2h} + 1\right) + \frac{r_1}{2K_1bh} \quad \text{and} \quad B_T < mK_2, \tag{6}$$

it has exactly two positive roots. Again, for at most three positive roots, $B_T > mK_2$ and $2r_1/(K_1bh) < B_T < r_1/(2K_1bh)$ must be satisfied, which is mathematically impossible. It is also shown numerically in Fig. 1(a) that at equilibrium, the isoclines of equations (1), (2) can intersect at no more than two point in the interior of phase plane, indicating nonexistence of three roots.

The Jacobian Matrix of the system is given by

$$\begin{pmatrix} r_1 - \frac{2r_1X_1}{K_1} - \frac{bX_2}{(1+bX_1h)^2} - qE & -\frac{bX_1}{(1+bX_1h)} \\ \frac{mbX_2}{(1+bX_1h)} & r_2 - \frac{2r_2X_2}{K_2} + \frac{mbX_1}{(1+bX_1h)}. \end{pmatrix}$$

(i) Stability at $P_0(0,0)$. The corresponding eigenvalues are

$$\lambda_1 = B_T \quad \text{and} \quad \lambda_2 = r_2.$$

Since both $\lambda_1 > 0$ and $\lambda_2 > 0$, P_0 is an unstable equilibrium point.

(ii) Stability at $P_1(0, K_2)$. The corresponding eigenvalues are

$$\lambda_1 = B_T - bK_2 \quad \text{and} \quad \lambda_2 = -r_2.$$

Here P_1 is stable if $B_T < bK_2$. If $B_T \geqslant bK_2$, P_1 becomes a saddle point.

(iii) Stability at $P_2(B_TK_1/r_1, 0)$. Here

$$\lambda_1 = -B_T \quad \text{and} \quad \lambda_2 = \frac{r_2 - \frac{mbK_1B_T}{r_1}}{1 + \frac{bhK_1B_T}{r_1}}.$$

This point is stable only when $r_1r_2 < mbK_1B_T$.

(iv) Stability at $P^*(X_1^*, X_2^*)$. The corresponding characteristic equation becomes

$$\lambda^2 - P\lambda + Q = 0,$$

where

$$P = \left(\frac{r_1X_1^*}{K_1} + \frac{r_2X_2^*}{K_2}\right) - \frac{b^2hX_1^*X_2^*}{(1+bX_1^*h)^2}$$

and

$$Q = \left(\frac{b^2hX_1^*X_2^*}{(1+bX_1^*h)^2} - \frac{r_1X_1^*}{K_1}\right)\frac{2r_2X_2^*}{K_2} - \frac{mbX_1^*X_2^*}{(1+bX_1^*h)^3}.$$

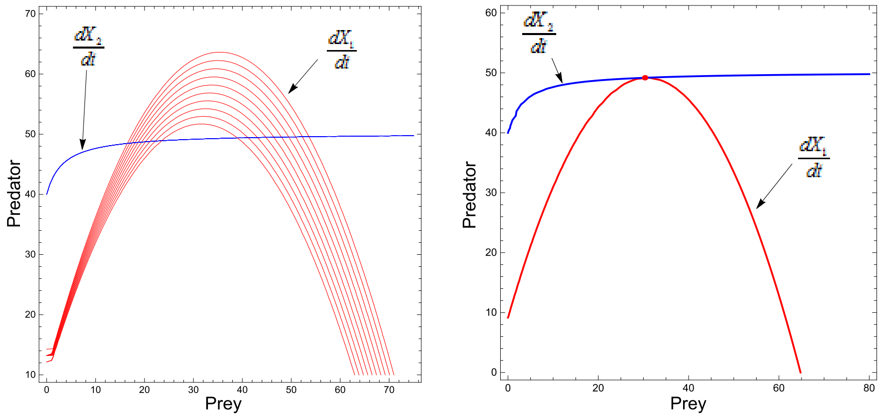
Applying Routh–Hurwitz criterion, the conditions for asymptotic stability are

$$\frac{r_1X_1^*}{K_1} + \frac{r_2X_2^*}{K_2} < \frac{b^2hX_1^*X_2^*}{(1+bX_1^*h)^2}$$

and

$$\frac{2r_2b^2hX_1^*X_2^{*2}}{K_2(1+bX_1^*h)^2} > \frac{2r_1r_2X_1^*X_2^*}{K_1K_2} + \frac{mbX_1^*X_2^*}{(1+bX_1^*h)^3}.$$

In Fig. 1(a), the nearly horizontal blue line is the predator isocline, and the prey isoclines are given by 10 uni-modal red curve, which decreases with the increase of fishing effort E. It is observed that X_2^* did not change significantly with the effort. In our simulation, the parameters are chosen in such a way that equation (6) is satisfied, and we

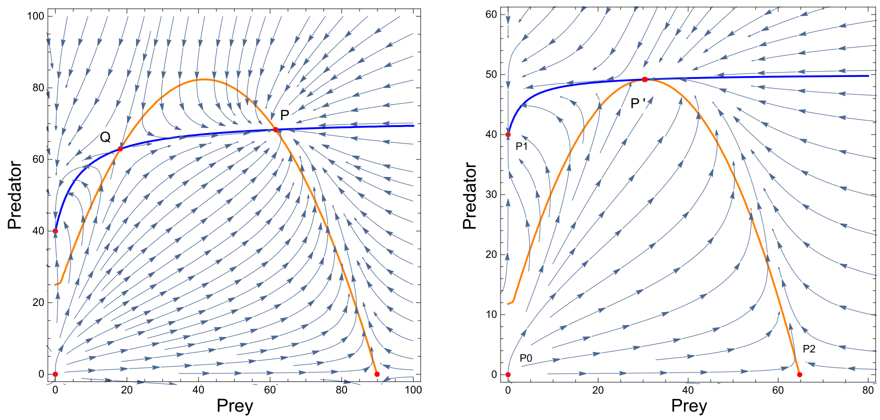


(a) $E = 0.12i$, $i = 1, 2, 3, \ldots, 10$ (b) $E = 1.4431$

Figure 1. Isoclines of system (1), (2). Parameter set is considered as $b = 1$, $r_1 = 10.6$, $r_2 = 3.27$, $h = 0.3$, $K_1 = 75$, $K_2 = 40$, $m = 0.25$, $q = 1$.

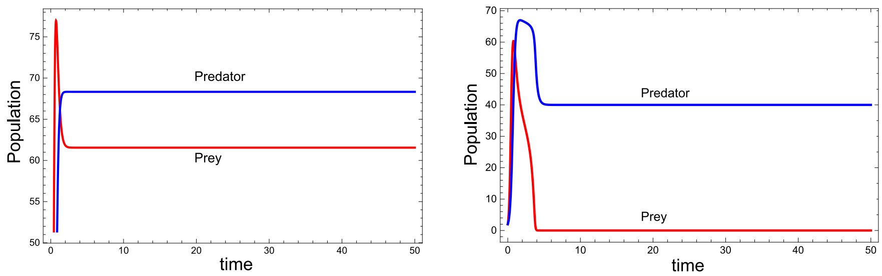
obtain two equilibrium point for a fixed fishing effort. As a result, we get two different value of X_1^*. According to simulation, between these two values of X_1^*, one set of values increases with increasing fishing effort, but the other set decreases for the same condition. When we increase fishing effort gradually, two sets of equilibrium point come close to each other and coincide, then after a threshold effort, there will be no equilibrium point. It is shown in Fig. 1(b) that, with the increase of effort, two equilibrium points merge and the system contains only one equilibrium satisfying the condition of existence of unique solution. With the increase of effort, the equilibrium of the system vanishes. We are interested to find the critical effort E_c for which there may exist a prey free equilibrium. It is observed that if the effort $E \geqslant E_c = (r_1 - bK_2)/q$, the prey population extinct. But as the equilibrium predator abundance X_2^* is always positive for any parameter values, predator free equilibrium is not possible for this system.

For most of the nonlinear autonomous systems, it is impossible to find the explicit solutions. We can take the help of numerical simulation to have an conception about the solutions, but qualitative analysis may be able to answer some questions faster than the numerical technique will do. So, due to the complexity of the system, we take the help of numerical simulation to discuss its stability. In Fig. 2(a), we show the stream plot of system (1), (2), and the red dots represent the equilibrium points. It helps to visualize the trajectories as slope-field for this autonomous equations. So it is very obvious that all the velocity vector in the neighborhood of the point P accumulate over it and produce a locally stable equilibrium, where the point Q become a saddle. Similar phenomena occurred with the unharvested system also. According to simulation (Fig. 2(b)), when the system has only one co-existence equilibrium point, then it is unstable. All the streamlines that come over the point P' move along the separatrix and tend to equilibrium point $P_1(0, K_2)$. As a result, the point $P_1(0, K_2)$ is a stable equilibrium point, the parameter values satisfy the condition of stability of the point P_1. We simulate two time series plot of system (1), (2) considering the parameters set equivalent to Figs. 2(a) and 2(b),



(a) $E = 0.012$ (b) $E = 1.4431$

Figure 2. Stream plot of system (1), (2) for two equilibrium point (a); one equilibrium point (b). Parameter set is considered as $b = 1$, $r_1 = 10.6$, $r_2 = 3.27$, $h = 0.3$, $K_1 = 75$, $K_2 = 40$, $m = 0.25$, $q = 1$.



(a) $E = 0.012$ (b) $E = 1.4431$

Figure 3. Time series plot of system (1), (2). Parameter set is considered as in Fig. 2.

respectively. In the first case, we find a stable system with non extinction of predator and prey population, but in the second cases, the prey population extinct with time leaving behind a constant predator population (K_2) as expected.

Here we consider the case where parameter values did not satisfy the condition of having one or two positive equilibrium of the system. Again, we take the help of numerical simulation to investigate its effect. We take parameter values satisfying the condition $B_T > mK_2$ such that both the conditions of having single solution and two positive solutions of (5) are violated. Under this condition, we notice that the system looses its stability and approaches to a limit cycle as shown in Fig. 4.

Now we study about the effects of implementing maximum sustainable yield (MSY) policy over this system. The MSY policy is one of the simplest tool to maintain sustainable harvesting. It is the maximum level at which a natural resource can be routinely exploited without long term depletion. Recently, Ghosh and Kar [8] discussed the impact of MSY policy on both single species and multi-species fisheries. Matsuda and Abrams [23]

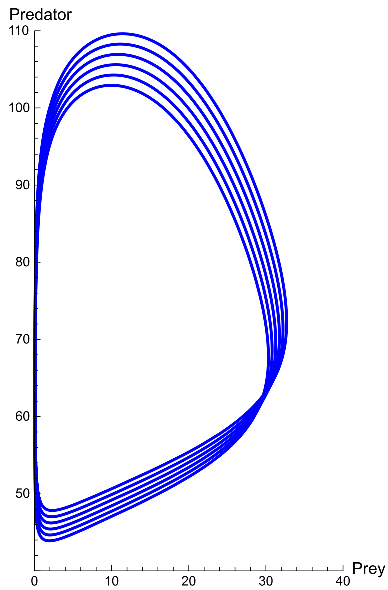
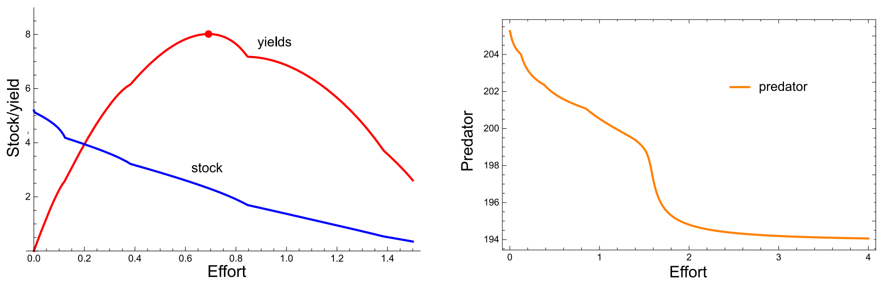


Figure 4. Limit cycle of the predator–prey system (1), (2). Six closed trajectories represents the limit cycle for fishing effort $E = 0, 2, 4, 6, 8, 10$ from outer to inner, respectively. Parameter set is considered as $b = 0.3$, $r_1 = 19.236$, $r_2 = 0.2139$, $h = 0.23$, $K_1 = 50$, $K_2 = 20$, $m = 0.5$, $q = 0.1$.



(a) (b)

Figure 5. Relation of prey abundance and yield with effort (a); predator abundance with effort (b). Parameter set is considered as $b = 2$, $r_1 = 10$, $r_2 = 0.25$, $h = 0.75$, $K_1 = 500$, $K_2 = 200$, $m = 0.8$, $q = 5$.

identify the effects of MSY policy in a predator–prey system with specialist predator. They illustrate that the harvesting of prey at MSY level may lead the predator population to extinction. But in our observation, taking generalist predator–prey system, the harvesting of prey at MSY level guarantee the non-extinction of both prey and predator species. In Fig. 5(a), average yield and average stock abundance are represented as a function of fishing effort. It shows that the average prey abundance is monotonically decreasing with increasing fishing effort and the yield curve is humped and maximized at the point indicated by the red point. Yield curve increases till the effort reaches to 0.69, after that it starts decreasing. From this simulation we obtained maximum sustainable yield 8.016 unit for $E_{MSY} = 0.69$. It is observed that at MSY, condition (6) for existence of two positive equilibrium is also fulfilled. It is identified from Figs. 5(a) and 5(b) that predator and prey abundance at effort E_{MSY} is positive indicating the persistence of both the species under MSY policy.

4 Feedback control in terms of species density

We consider feedback control to change fishing effort with the change of CPUE(qN_1). System (1), (2) with the feedback control (3) has an interior equilibrium (X_1^*, X_2^*, E^*), where

$$\begin{aligned} X_1^* &= \frac{s_1}{q}, \qquad X_2^* = K_2\left(1 + \frac{mbs_1}{r_2(q + bhs_1)}\right), \\ E^* &= \frac{r_1(qK_1 - s_1)}{q^2K_1} - \frac{bK_2(r_2q + bs_1(hr_2 + m))}{r_2 * (q + bhs_1)^2}. \end{aligned} \tag{7}$$

The Jacobian matrix is

$$\begin{pmatrix} r_1 - \frac{2r_1X_1}{K_1} - \frac{bX_1}{(1+bX_1h)^2} - qE & -\frac{bX_1}{(1+bX_1h)} & -qX_1 \\ \frac{mbX_2}{(1+bX_1h)^2} & r_2 - \frac{2r_2X_2}{K_2} + \frac{mbX_1}{(1+bX_1h)} & 0 \\ \mu_0 q & 0 & 0 \end{pmatrix}.$$

At the interior equilibrium (X_1^*, X_2^*, E^*), the corresponding characteristic equation is given by

$$\lambda^3 - (A^* + B^*)\lambda^2 + \lambda(A^*B^* + C^*) - \mu_0 q^2 X_1^* B^* = 0,$$

where

$$\begin{aligned} A^* &= r_1 - \frac{2r_1X_1^*}{K_1} - \frac{bX_2^*}{(1 + bX_1^*h)^2} - qE^* = -X_1^*\left(\frac{r_1}{K_1} - \frac{b^2hX_2^*}{(1 + bhX_1^*)^2}\right), \\ B^* &= r_2 - \frac{2r_2X_2^*}{K_2} - \frac{mbX_1^*}{(1 + bX_1^*h)} = -\frac{r_2X_2^*}{K_2}, \qquad C^* = \frac{mb^2X_1^*X_2^*}{(1 + bX_2^*h)^3} - q^2\mu_0X_1^*. \end{aligned}$$

By Routh–Hurwitz criteria the condition of stability is given by

$$-(A^* + B^*)(A^*B^* + C^*) > -\mu_0 q^2 X_1^* B^*$$

with $-(A^* + B^*) > 0$ and $-\mu_0 q^2 X_1^* B^* > 0$.

It is observed that $-\mu_0 q^2 X_1^* B^* > 0$ is trivially true as $B^* < 0$. Hence, $-(A^*+B^*) > 0$ is satisfied if $r_1/K_1 > b^2hX_2^*/(1 + bhX_1^*)^2$.

Two types of outcome are possible from the given system: coexistence of prey, predator and fishery or the extinction of fishery. The coexistence equilibrium of prey, predator

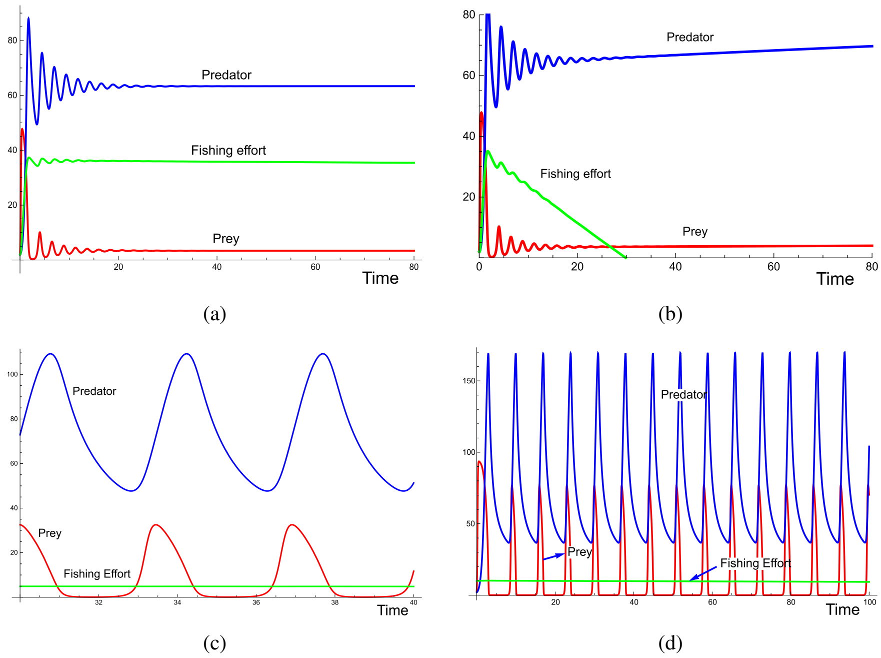


Figure 6. The time series plot of prey population (red line), predator population (blue line), and fishing effort (green line) with various values of target catch per unit effort. (a) Parameter values are same as in Fig. 7(a); (b) parameter values are same as in Fig. 7(b); (c) parameter set is considered as $b = 0.3$, $r_1 = 19.23$, $r_2 = 0.2139$, $h = 0.23$, $K_1 = 50$, $K_2 = 20$, $m = 0.5$, $q = 0.1$, $u = 0.01$, $s_1 = 1.5$; (d) parameter values are $b = 0.3$, $r_1 = 19.23$, $r_2 = 0.2139$, $h = 0.23$, $K_1 = 100$, $K_2 = 26$, $m = 0.5$, $q = 0.1$, $u = 0.01$, $s_1 = 2$.

and fishery is denoted by (X_1', X_2', E'), and the equilibrium without fishery is denoted by (X_1', X_2'). From (7) it is obvious that coexistence equilibrium is possible if

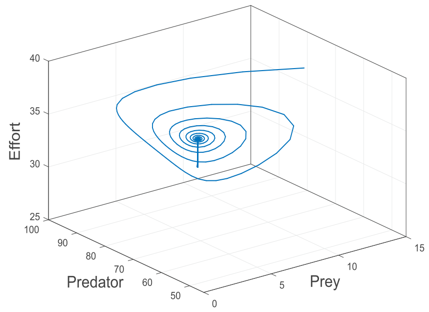
$$\frac{r_1(qK_1 - s_1)}{q^2K_1} > \frac{bK_2(r_2q + bs_1(hr_2 + m))}{r_2 * (q + bhs_1)^2} \quad \text{and} \quad s_1 < qX_1'$$

(see Fig. 6(a)), but when

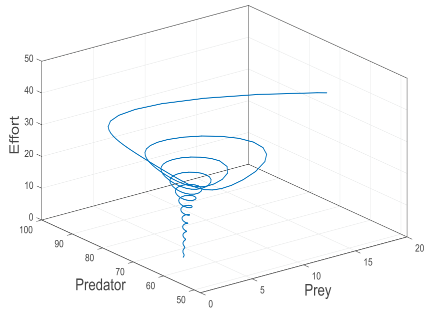
$$\frac{r_1(qK_1 - s_1)}{q^2K_1} \leqslant \frac{bK_2(r_2q + bs_1(hr_2 + m))}{r_2 * (q + bhs_1)^2},$$

system (1), (2) with feedback control (3) possesses the equilibrium (X_1', X_2') without fishery (see Fig. 6(b)). Even if $s_1 > qX_1'$, then also prey, predator and fishery can co-exists with a synchronous cycle as shown in Fig. 6(c). This fact is very similar like coexistence of three species in a one prey and two predator system with a fluctuating biomass from its average abundance.

To check the stability of the system, we take the help of numerical simulation, which shows that system (1), (2) with feedback control (3) is stable. Figure 7(a) shows that predator, prey and fishery coexist and the system tends to a stable focus at the coexisting



(a)



(b)

Figure 7. Stable focus of the predator–prey system (1), (2) with feedback control (3). Parameter set is considered as $b = 0.3$, $r_1 = 19.2294$, $r_2 = 0.2139$, $h = 0.1$, $K_1 = 50$, $K_2 = 20$, $m = 0.5$, $q = 0.19$, $u = 3.7$. (a) $s = 0.65$ and (b) $s = 1$.

equilibrium. In case of equilibrium without fishery, the system undergoes a stable focus as shown in Fig. 7(b).

In Section 3, we find that with the increase of fishing effort when $E > E_c$ prey population extinct. Here from (7) X_1^* is always positive for any parameter values. So, when feedback control (3) is considered over system (1), (2), prey free equilibrium is not possible for any fishing effort. For predator population, the scenario remain same, indicating the non-extinction of the species. So feedback with respect to prey density is a sustainable fishing policy for our system.

Figure 7(a) shows the numerical simulation of predator, prey and fishery coexistence for system (1), (2) with feedback control (3). If the parameter values are taken as $b = 0.3$, $r_1 = 19.2294$, $r_2 = 0.2139$, $h = 0.1$, $K_1 = 50$, $K_2 = 20$, $m = 0.5$, $q = 0.19$, $u = 3.7$, $s = 0.65$, $\mu(E) = 1$, the coexisting equilibrium denoted by $(X_1^*, X_2^*, E^*) = (3.42, 63.52, 33.35)$ is a stable focus. If target CPUE increases, then for the same parameter values except $s = 1$, the system possess a fishery free equilibrium denoted by $(X_1^*, X_2^*) = (5.26, 63.76)$, which is also a stable focus as shown in Fig. 7(b).

In Fig. 6, we use time series plot to compare different scenarios of system (1), (2) with feedback control (3) for variety of target catch per unit effort. When coexisting equilibrium exists, the system with feedback control of fishing effort converges to a stable equilibrium, synchronous cycle or asynchronous cycle with more complex behavior. Figure 6(a) shows the coexistence of predator, prey and fishery taking parameter same as in Fig. 7(a). The coexistence of only predator and prey is illustrate by Fig. 6(b) with parameter values identical to Fig. 7(b) indicating the disappearance of fishing effort. The system remains stable for initial values of target CPUE s_1, but if $s_1 > qX_1'$, then with the increase of CPUE, the system converges to a synchronous or an asynchronous cycle as described by Figs. 6(c) and 6(d), respectively. So we can conclude from these results that the feedback control over prey population can avoid the extinction of prey species while the target catch per unit effort is less than the equilibrium catch per unit effort.

Now we take the feedback control of fishing effort over the fishery in terms of predator biomass. So the predator–prey system (1), (2) with feedback control (4) has the equilib-

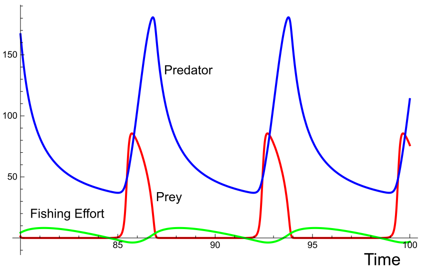


Figure 8. The time series plot of the predator–prey system (1), (2) with feedback control (4). Parameter values are taken as $b = 0.3$, $r_1 = 19.23$, $r_2 = 0.2139$, $h = 0.23$, $K_1 = 100$, $K_2 = 26$, $m = 0.5$, $q = 0.1$, $u = 0.1$, $s_2 = 75$.

rium $(X_1^{**}, X_2^{**}, E^{**})$, where

$$X_1^{**} = \frac{r_2(s_2 - K_2)}{r_2bh(K_2 - s_2) + mbK_2}, \qquad X_2^{**} = s_2,$$
$$E^{**} = \frac{1}{q}\left(r_1\left(1 - \frac{r_2(s_2 - K_2)}{r_2bhK_1(K_2 - s_2) + mbK_1K_2}\right) - \frac{bs_2(r_2bh(K_2 - s_2) + mbK_2)}{r_2bh(K_2 - s_2) + mbK_2 + bhr_2(s_2 - K_2)}\right).$$

The co-existence equilibrium $(X_1^{**}, X_2^{**}, E^{**})$ exist if

$$r_1\left(1 - \frac{r_2(s_2 - K_2)}{r_2bhK_1(K_2 - s_2) + mbK_1K_2}\right) > \frac{bs_2(r_2bh(K_2 - s_2) + mbK_2)}{r_2bh(K_2 - s_2) + mbK_2 + bhr_2(s_2 - K_2)}$$

along with the conditions $mK_2 > r_2h(s_2 - K_2)$ and $K_2 < s_2$ or $mK_2 < r_2h(s_2 - K_2)$ and $K_2 > s_2$. The time series plot shown in Fig. 8 clearly indicate that system (1), (2) with feedback control (4) cannot have any stable dynamics, and hence coexistence equilibrium is not stable in this case. So feedback control of fishing effort in terms of predator biomass may not be a sustainable fishing policy.

5 Conclusion

In this research, adaptive management is considered by building a predictive model of fisheries with feedback control. This was not a decision making model, but a design for maintaining the fisheries to a sustainable state. We discussed feedback control as a management tool to protect the prey species, which has some kind of threat to extinct due to over fishing. We have discussed that with constant fishing effort a predator free equilibrium is not possible, but with continues increment of effort when it exceed some critical value, a prey free equilibrium exist resulting extinction of prey. We also investigate the implementation of MSY policy to this model and observe the non extinction of both prey and predator species. So, for this model, maximum sustainable yield of prey harvesting is a effective management policy to restrict extinction of species due to over fishing. Again, we try to find out some alternative way to restrict the extinction of prey species due to heavy fishing effort in terms of feedback control. Hence, we show that in a generalist predator–prey system, feedback control in terms of resource biomass is a effective policy in avoiding stock collapse and extinction of prey. It is shown numerically that with feedback control in terms of prey biomass the co-existing equilibrium exist and the system approaches to a stable focus. But feedback in terms of predator biomass has no advantages as it did not guarantee the stability of the system.

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