

New exact traveling wave solutions for DS-I and DS-II equations

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Abstract. In this present work, the simplest equation method is used to construct exact solutions of the DS-I and DS-II equations. The simplest equation method is a powerful solution method for obtaining exact solutions of nonlinear evolution equations. This method can be applied to nonintegrable equations as well as to integrable ones.

Keywords: the simplest equation method, DS-I and DS-II equations.

1 Introduction

In this paper, we consider the Davey–Stewartson (DS) equations [1–3]

$$\begin{aligned}iq_t + \frac{1}{2}\delta^2(q_{xx} + \delta^2q_{yy}) + \lambda|q|^2q - \phi_xq &= 0, \\ \phi_{xx} - \delta^2\phi_{yy} - 2\lambda(|q|^2)_x &= 0.\end{aligned}\tag{1}$$

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The case $\delta = 1$ is called the DS-I equation, while $\delta = i$ is the DS-II equation. The parameter λ characterizes the focusing or defocusing case. The Davey–Stewartson I and II are two well-known examples of integrable equations in two space dimensions, which arise as higher dimensional generalizations of the nonlinear Schrödinger (NLS) equation [3].

Davey and Stewartson first derived their model in the context of water waves, from purely physical considerations. In the context, $q(x, y, t)$ is the amplitude of a surface wave packet, while $\phi(x, y, t)$ represents the velocity potential of the mean flow interacting with the surface wave [3].

The Davey–Stewartson equations are also reduced to Hamiltonian ODEs [4], and so exact solutions could be furnished by the integrability [5] of finite-dimensional Hamiltonian systems.

The research area of nonlinear evolution equation has been very active for the past few decades. There are various kinds of nonlinear evolution equations that appear in various areas of physical and mathematical sciences. Much effort has been made on the construction of exact solutions of nonlinear equations, for their important role in the study of nonlinear physical phenomena. Nonlinear wave phenomena appears in various scientific and engineering fields, such as fluid mechanics, plasma physics, optical fibers, biology, solid state physics, chemical kinematics, chemical physics and geochemistry. Nonlinear wave phenomena of dispersion, dissipation, diffusion, reaction and convection are very important in nonlinear wave equation. In recent years, the powerful and efficient methods to find analytic solutions of nonlinear equations have drawn a lot of interest by a diverse group of scientists such as simplest equation method [6–10], tanh method [11, 12], multiple exp-function method [13], Backlund transformation method [14], Hirota's direct method [15, 16], transformed rational function method [17] and so on.

In 1996, Ma and Fuchssteiner proposed a powerful approach for finding exact solutions to nonlinear differential equations [18]. Their key idea is to expand solutions of given differential equations as functions of solutions of solvable differential equations, in particular, polynomial and rational functions. This idea is so important that many types of nonlinear equations can be solved by it. A more systematical theory on decompositions and transformations is presented very recently in [17] and [19]. Ma and his coauthors' theory unifies many existing approaches to exact solutions such as the tanh-function methods, the homogeneous balance method, the exp-function method and the Jacobi elliptic function method.

The simplest equation method is a very powerful mathematical technique for finding exact solutions of nonlinear ordinary differential equations. It has been developed by Kudryashov [6–10] and used successfully by many authors for finding exact solutions of ODEs in mathematical physics [20, 21].

The aim of this paper is to find exact solutions of DS-I and DS-II equations by using the simplest equation method.

The paper is arranged as follows. In Section 2, we describe briefly the simplest equation method. In Sections 3, we apply this method to DS-I and DS-II equations.

2 The simplest equation method

Step 1. We first consider a general form of nonlinear equation

$$P(u, u_x, u_t, u_{xx}, u_{xt}, \dots) = 0. \quad (2)$$

Step 2. To find the traveling wave solution of Eq. (2) we introduce the wave variable $\xi = x - ct$ so that

$$u(x, t) = y(\xi).$$

Based on this we use the following changes:

$$\frac{\partial}{\partial t}(\cdot) = -c \frac{\partial}{\partial \xi}(\cdot), \quad \frac{\partial}{\partial x}(\cdot) = \frac{\partial}{\partial \xi}(\cdot), \quad \frac{\partial^2}{\partial x^2}(\cdot) = \frac{\partial^2}{\partial \xi^2}(\cdot) \quad (3)$$

and so on for other derivatives.

Using (3) changes the PDE (2) to an ODE

$$Q\left(y, \frac{\partial y}{\partial \xi}, \frac{\partial^2 y}{\partial \xi^2}, \dots\right) = 0, \quad (4)$$

where $y = y(\xi)$ is an unknown function, Q is a polynomial in the variable y and its derivatives.

Step 3. The basic idea of the simplest equation method consists in expanding the solutions $y(\xi)$ of Eq. (4) in a finite series

$$y(\xi) = \sum_{i=0}^l a_i z^i, \quad a_l \neq 0, \quad (5)$$

where the coefficients a_i are independent of ξ and $z = z(\xi)$ are the functions that satisfy some ordinary differential equations.

These ordinary differential equations are called the simplest equations. The simplest equation is characterized by the fact that it is of a lesser order than Eq. (4) and, the general solution of this equation is known (or we know the way of finding its general solution, or at least we know some particular solutions of this equation). This means that the exact solutions $y(\xi)$ of Eq. (4) can be presented by a finite series (5) in the general solution $z = z(\xi)$ of the simplest equation.

As examples of simplest equations used in the literature, we can cite the Riccati equation, the equation for the Jacobi elliptic function and the equation for the Weierstrass elliptic function.

In this paper, we use the Riccati equation as simplest equation

$$\frac{dz}{d\xi} = k + az(\xi) + bz^2(\xi), \quad (6)$$

where k , a and b are independent on ξ . When $k = 0$ and $a, b \neq 0$, we obtain the Bernoulli equation

$$\frac{dz}{d\xi} = az(\xi) + bz^2(\xi). \quad (7)$$

We found that the use of the Bernoulli equation leads to new traveling-wave and wavefront solutions of Eq. (1). Equation (7) admits the following exact solutions:

$$z(\xi) = \frac{a \exp[a(\xi + \xi_0)]}{1 - b \exp[a(\xi + \xi_0)]}, \quad (8)$$

for the case $a > 0, b < 0$ and

$$z(\xi) = -\frac{a \exp[a(\xi + \xi_0)]}{1 + b \exp[a(\xi + \xi_0)]}, \quad (9)$$

for the case $a < 0, b > 0$, where ξ_0 is a constant of integration.

When $k = \beta \neq 0$ and $a = 0, b = \alpha \neq 0$ we obtain the Riccati equation

$$\frac{dz}{d\xi} = \beta + \alpha z^2(\xi). \quad (10)$$

Equation (10) admits the following exact solutions [22]:

$$z(\xi) = -\frac{\sqrt{-\alpha\beta}}{\alpha} \tanh\left(\sqrt{-\alpha\beta}\xi - \frac{\varepsilon \ln \xi_0}{2}\right), \quad \xi_0 > 0, \quad \varepsilon = \pm 1, \quad (11)$$

when $\alpha\beta < 0$, and

$$z(\xi) = \frac{\sqrt{\alpha\beta}}{\alpha} \tan(\sqrt{\alpha\beta}\xi + \xi_0), \quad \xi_0 = \text{const},$$

when $\alpha\beta > 0$.

Remark 1. l is a positive integer, in most cases, that will be determined. To determine the parameter l , we usually balance the linear terms of highest order in the resulting equation with the highest order nonlinear terms.

Step 4. Substituting (5) into (4) with (6), then the left hand side of Eq. (4) is converted into a polynomial in $z(\xi)$, equating each coefficient of the polynomial to zero yields a set of algebraic equations for a_i, c .

Step 5. Solving the algebraic equations obtained in Step 4, and substituting the results into (5), then we obtain the exact traveling wave solutions for Eq. (2).

Remark 2. This method is a simple case of the method in [18].

3 DS-I and DS-II equations

To find exact solutions of DS-I and DS-II Eqs. (1), first we make the transformation

$$q(x, y, t) = u(\xi)e^{i(\alpha x + \beta y + \gamma t)}, \quad \phi(x, y, t) = v(\xi),$$

where $\xi = i\mu(x + y - ct)$, we have a relation $c = \alpha\delta^2 + \beta\delta^4$ and reduce system (1) to the following system of ordinary differential equations:

$$-\left(\gamma + \frac{1}{2}\alpha^2\delta^2 + \frac{1}{2}\beta^2\delta^4\right)u - \frac{\mu^2\delta^2}{2}(\delta^2 + 1)u_{\xi\xi} + \lambda u^3 - i\mu v_{\xi}u = 0, \quad (12a)$$

$$\mu(\delta^2 - 1)v_{\xi\xi} - 2i\lambda(u^2)_{\xi} = 0. \quad (12b)$$

Integrating Eq. (12b) once with respect to ξ and setting the constant of integration to be zero, we obtain

$$v_{\xi} = \frac{2i\lambda}{\mu(\delta^2 - 1)}u^2. \quad (13)$$

Substituting (13) into Eq. (12a) we have

$$\frac{M}{2}(\delta^2 - 1)u + \frac{\mu^2\delta^2}{2}(\delta^4 - 1)u_{\xi\xi} - \lambda(\delta^2 + 1)u^3 = 0, \quad (14)$$

where $M = 2\gamma + \alpha^2\delta^2 + \beta^2\delta^4$.

For the solutions of Eq. (14), we make the following ansatz:

$$u(\xi) = \sum_{i=0}^l a_i z^i, \quad a_l \neq 0, \quad (15)$$

where a_i are all real constants to be determined, l is a positive integer which can be determined by balancing the highest order derivative term with the highest order nonlinear term after substituting ansatz (15) into Eq. (14), where z satisfies Eq. (7).

Balancing $u_{\xi\xi}$ with u^3 in (14) gives

$$l + 2 = 3l,$$

so that

$$l = 1.$$

This suggests the choice of $u(\xi)$ in Eq. (14) as

$$u(\xi) = a_0 + a_1 z(\xi). \quad (16)$$

Substituting (16) along with (7) in Eq. (14) and then setting the coefficients of z^j ($j = 3, 2, 1, 0$) to zero in the resultant expression, we obtain a set of algebraic equations involving a_0, a_1, a and b as

$$\begin{aligned} \mu^2\delta^2(\delta^4 - 1)b^2a_1 - \lambda(\delta^2 + 1)a_1^3 &= 0, \\ \frac{3}{2}\mu^2\delta^2(\delta^4 - 1)aba_1 - 3\lambda(\delta^2 + 1)a_0a_1^2 &= 0, \\ \frac{1}{2}\mu^2\delta^2(\delta^4 - 1)a^2a_1 + \frac{M}{2}(\delta^2 - 1)a_1 - 3\lambda(\delta^2 + 1)a_0^2a_1 &= 0, \\ \frac{M}{2}(\delta^2 - 1)a_0 - \lambda(\delta^2 + 1)a_0^3 &= 0. \end{aligned}$$

Using Maple gives two sets of solutions:

$$a_0 = \sqrt{\frac{M(\delta^2 - 1)}{2\lambda(1 + \delta^2)}}, \quad a_1 = \frac{b\delta\mu}{\lambda} \sqrt{\lambda(\delta^2 - 1)}, \quad a = \frac{1}{\delta\mu} \sqrt{\frac{2M}{1 + \delta^2}}, \quad (17)$$

where α, β, γ and b are arbitrary constants;

$$a_0 = -\sqrt{\frac{M(\delta^2 - 1)}{2\lambda(1 + \delta^2)}}, \quad a_1 = -\frac{b\delta\mu}{\lambda} \sqrt{\lambda(\delta^2 - 1)}, \quad a = -\frac{1}{\delta\mu} \sqrt{\frac{2M}{1 + \delta^2}}, \quad (18)$$

where α, β, γ and b are arbitrary constants.

Assuming $a > 0$ and choosing $b < 0$ in case (17). Therefore, using solution (8) of Eq. (7), ansatz (16), we obtain the following traveling-wave solution of Eq. (14):

$$u_1(\xi) = \sqrt{\frac{(\delta^2 - 1)M}{2\lambda(1 + \delta^2)}} \left\{ 1 + 2 \frac{\exp\left[\frac{1}{\delta\mu} \sqrt{\frac{2M}{1 + \delta^2}} (\xi + \xi_0)\right]}{1 - b \exp\left[\frac{1}{\delta\mu} \sqrt{\frac{2M}{1 + \delta^2}} (\xi + \xi_0)\right]} \right\}. \quad (19)$$

Assuming $a < 0$ and choosing $b > 0$ in case (18). Therefore, using solution (9) of Eq. (7), ansatz (16), we obtain the following traveling-wave solution of Eq. (14):

$$u_2(\xi) = -\sqrt{\frac{(\delta^2 - 1)M}{2\lambda(1 + \delta^2)}} \left\{ 1 + 2 \frac{\exp\left[-\frac{1}{\delta\mu} \sqrt{\frac{2M}{1 + \delta^2}} (\xi + \xi_0)\right]}{1 + b \exp\left[-\frac{1}{\delta\mu} \sqrt{\frac{2M}{1 + \delta^2}} (\xi + \xi_0)\right]} \right\}. \quad (20)$$

By using (13) and (19), (20) we have

$$\begin{aligned} v_1(\xi) &= \frac{2i\lambda}{k(\delta^2 - 1)} \int u_1^2 d\xi \\ &= \sqrt{-\frac{\delta^2 M}{2(1 + \delta^2)}} \left\{ \frac{4}{b^2} \ln \left(b \exp \left[\frac{1}{\delta\mu} \sqrt{\frac{2M}{1 + \delta^2}} (\xi + \xi_0) \right] - 1 \right) \right. \\ &\quad - \frac{4}{b} \ln \left(b \exp \left[\frac{1}{\delta\mu} \sqrt{\frac{2M}{1 + \delta^2}} (\xi + \xi_0) \right] - 1 \right) \\ &\quad \left. + \frac{1}{\delta\mu} \sqrt{\frac{2M}{1 + \delta^2}} (\xi + \xi_0) - \frac{4}{b^2 (b \exp \left[\frac{1}{\delta\mu} \sqrt{\frac{2M}{1 + \delta^2}} (\xi + \xi_0) \right] - 1)} \right\}, \\ v_2(\xi) &= \frac{2i\lambda}{k(\delta^2 - 1)} \int u_2^2 d\xi \\ &= \sqrt{-\frac{\delta^2 M}{2(1 + \delta^2)}} \left\{ \frac{4}{b^2} \ln \left(b \exp \left[-\frac{1}{\delta\mu} \sqrt{\frac{2M}{1 + \delta^2}} (\xi + \xi_0) \right] + 1 \right) \right. \end{aligned}$$

$$\left. \begin{aligned} & + \frac{4}{b} \ln \left(b \exp \left[-\frac{1}{\delta\mu} \sqrt{\frac{2M}{1+\delta^2}} (\xi + \xi_0) \right] + 1 \right) \\ & - \frac{1}{\delta\mu} \sqrt{\frac{2M}{1+\delta^2}} (\xi + \xi_0) + \frac{4}{b^2 (b \exp \left[-\frac{1}{\delta\mu} \sqrt{\frac{2M}{1+\delta^2}} (\xi + \xi_0) \right] + 1)} \end{aligned} \right\}.$$

Thus, we obtain the following traveling-wave solutions of DS-I and DS-II equations:

$$q_1(x, y, t)$$

$$= \sqrt{\frac{(\delta^2 - 1)M}{2\lambda(1 + \delta^2)}} \left\{ 1 + 2 \frac{\exp \left[\frac{1}{\delta\mu} \sqrt{\frac{2M}{1 + \delta^2}} (i\mu(x + y - (\alpha\delta^2 + \beta\delta^4)t) + \xi_0) \right]}{1 - b \exp \left[\frac{1}{\delta\mu} \sqrt{\frac{2M}{1 + \delta^2}} (i\mu(x + y - (\alpha\delta^2 + \beta\delta^4)t) + \xi_0) \right]} \right\} \\ \times e^{i(\alpha x + \beta y + \gamma t)},$$

$$\phi_1(x, y, t)$$

$$= \sqrt{-\frac{\delta^2 M}{2(1 + \delta^2)}} \left\{ \frac{4}{b^2} \ln \left(b \exp \left[\frac{1}{\delta\mu} \sqrt{\frac{2M}{1 + \delta^2}} (i\mu(x + y - (\alpha\delta^2 + \beta\delta^4)t) + \xi_0) \right] - 1 \right) \right. \\ \left. - \frac{4}{b} \ln \left(b \exp \left[\frac{1}{\delta\mu} \sqrt{\frac{2M}{1 + \delta^2}} (i\mu(x + y - (\alpha\delta^2 + \beta\delta^4)t) + \xi_0) \right] - 1 \right) \right. \\ \left. + \frac{1}{\delta\mu} \sqrt{\frac{2M}{1 + \delta^2}} (i\mu(x + y - (\alpha\delta^2 + \beta\delta^4)t) + \xi_0) \right. \\ \left. - \frac{4}{b^2 (b \exp \left[\frac{1}{\delta\mu} \sqrt{\frac{2M}{1 + \delta^2}} (i\mu(x + y - (\alpha\delta^2 + \beta\delta^4)t) + \xi_0) \right] - 1)} \right\};$$

$$q_2(x, y, t)$$

$$= -\sqrt{\frac{(\delta^2 - 1)M}{2\lambda(1 + \delta^2)}} \left\{ 1 + 2 \frac{\exp \left[-\frac{1}{\delta\mu} \sqrt{\frac{2M}{1 + \delta^2}} (i\mu(x + y - (\alpha\delta^2 + \beta\delta^4)t) + \xi_0) \right]}{1 + b \exp \left[-\frac{1}{\delta\mu} \sqrt{\frac{2M}{1 + \delta^2}} (i\mu(x + y - (\alpha\delta^2 + \beta\delta^4)t) + \xi_0) \right]} \right\} \\ \times e^{i(\alpha x + \beta y + \gamma t)},$$

$$\phi_2(x, y, t)$$

$$= \sqrt{-\frac{\delta^2 M}{2(1 + \delta^2)}} \left\{ \frac{4}{b^2} \ln \left(b \exp \left[-\frac{1}{\delta\mu} \sqrt{\frac{2M}{1 + \delta^2}} (i\mu(x + y - (\alpha\delta^2 + \beta\delta^4)t) + \xi_0) \right] + 1 \right) \right. \\ \left. + \frac{4}{b} \ln \left(b \exp \left[-\frac{1}{\delta\mu} \sqrt{\frac{2M}{1 + \delta^2}} (i\mu(x + y - (\alpha\delta^2 + \beta\delta^4)t) + \xi_0) \right] + 1 \right) \right. \\ \left. - \frac{1}{\delta\mu} \sqrt{\frac{2M}{1 + \delta^2}} (i\mu(x + y - (\alpha\delta^2 + \beta\delta^4)t) + \xi_0) \right. \\ \left. + \frac{4}{b^2 (b \exp \left[-\frac{1}{\delta\mu} \sqrt{\frac{2M}{1 + \delta^2}} (i\mu(x + y - (\alpha\delta^2 + \beta\delta^4)t) + \xi_0) \right] + 1)} \right\}.$$

Substituting (16) along with (10) in Eq. (14) and then setting the coefficients of z^j ($j = 3, 2, 1, 0$) to zero in the resultant expression, we obtain a set of algebraic equations involving a_0, a_1 and k as

$$\begin{aligned}\mu^2 \delta^2 (\delta^4 - 1) a_1 - \lambda (\delta^2 + 1) a_1^3 &= 0, \\ -3\lambda (\delta^2 + 1) a_0 a_1^2 &= 0, \\ \mu^2 \delta^2 (\delta^4 - 1) k a_1 + \frac{M}{2} (\delta^2 - 1) a_1 - 3\lambda (\delta^2 + 1) a_0^2 a_1 &= 0, \\ \frac{M}{2} (\delta^2 - 1) a_0 - \lambda (\delta^2 + 1) a_0^3 &= 0.\end{aligned}$$

Solving these under-determined algebraic equations, we get the following result:

$$a_0 = 0, \quad a_1 = \pm \frac{\delta \mu}{\lambda} \sqrt{\lambda (\delta^2 - 1)}, \quad k = -\frac{M}{2\mu^2 \delta^2 (\delta^2 + 1)},$$

where α, β and γ are arbitrary constants.

Therefore, using solution (11) of Eq. (10), ansatz (16), we obtain the following traveling-wave solution of Eq. (14):

$$u_3(\xi) = \mp \sqrt{\frac{(\delta^2 - 1)M}{2\lambda(1 + \delta^2)}} \tanh \left[\sqrt{\frac{M}{2\mu^2 \delta^2 (\delta^2 + 1)}} (\xi + \xi_0) \right].$$

By using (13) we have

$$\begin{aligned}v_3(\xi) &= \frac{2i\lambda}{k(\delta^2 - 1)} \int u_3^2 d\xi \\ &= \sqrt{-\frac{2\delta^2 M}{1 + \delta^2}} \left\{ \frac{1}{2} \ln \left(\tanh \left[\sqrt{\frac{M}{2\mu^2 \delta^2 (\delta^2 + 1)}} (\xi + \xi_0) \right] - 1 \right) \right. \\ &\quad \left. - \frac{1}{2} \ln \left(\tanh \left[\sqrt{\frac{M}{2\mu^2 \delta^2 (\delta^2 + 1)}} (\xi + \xi_0) \right] + 1 \right) \right. \\ &\quad \left. - \tanh \left[\sqrt{\frac{M}{2\mu^2 \delta^2 (\delta^2 + 1)}} (\xi + \xi_0) \right] \right\}.\end{aligned}$$

Then exact solutions to DS-I and DS-II equations can be written as

$$\begin{aligned}q_3(x, y, t) &= \mp \sqrt{\frac{(\delta^2 - 1)M}{2\lambda(1 + \delta^2)}} \tanh \left[\sqrt{\frac{M}{2\mu^2 \delta^2 (\delta^2 + 1)}} (ik(x + y - (\alpha\delta^2 + \beta\delta^4)t) + \xi_0) \right] \\ &\quad \times e^{i(\alpha x + \beta y + \gamma t)},\end{aligned}$$

$$\begin{aligned} \phi_3(x, y, t) &= \sqrt{-\frac{2\delta^2 M}{1+\delta^2}} \left\{ \frac{1}{2} \ln \left(\tanh \left[\sqrt{\frac{M}{2\mu^2 \delta^2 (\delta^2 + 1)}} (ik(x+y - (\alpha\delta^2 + \beta\delta^4)t) + \xi_0) \right] - 1 \right) \right. \\ &\quad - \frac{1}{2} \ln \left(\tanh \left[\sqrt{\frac{M}{2\mu^2 \delta^2 (\delta^2 + 1)}} (ik(x+y - (\alpha\delta^2 + \beta\delta^4)t) + \xi_0) \right] + 1 \right) \\ &\quad \left. - \tanh \left[\sqrt{\frac{M}{2\mu^2 \delta^2 (\delta^2 + 1)}} (ik(x+y - (\alpha\delta^2 + \beta\delta^4)t) + \xi_0) \right] \right\}. \end{aligned}$$

4 Conclusion

In this work, we obtained exact solutions of DS-I and DS-II equations by using the simplest equation method. The efficiency of this method was demonstrated. The new complex solution of the DS-I and DS-II equations were obtained. The solutions obtained may be significant and important for the explanation of some practical physical problems. The method may also be applied to other nonlinear partial differential equations.

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