A simple mathematical model for the spread of two political parties

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Abstract. In this paper, a non-linear mathematical model for the spread of two political parties has been proposed and analyzed by using epidemiological approach. The whole population is assumed to be a constant and homogeneously mixed. Equilibria have been obtained analytically and their local and global stability have been discussed. Conditions for the co-existence of both the political parties have been obtained. Numerical simulation is also performed to support the analytical results.

Keywords: mathematical model, equilibria, stability, political parties, epidemic approach.

1 Introduction

Democracy is a system of government by which political independence is retained by the people and exercised directly by citizens. The unique characteristic of democracy is the source of power for a government by the people and for the people. The essential process in democracies is competitive elections. An election is a decision-making process by which a population chooses an individual to hold formal office. This is the usual mechanism by which democracy fills offices in the legislature and parliament [1].

The Constitution of India provides single and uniform citizenship for the whole nation and grants the right to vote every person who is a citizen of India and who acquires 18 years of age. In the parliamentary and legislature elections India follows the single vote method, in which each voter votes for one candidate and the candidate who receives maximum votes wins, even if he receives less than a majority of votes.

In most of the democratic countries, the voting system is single vote method and generally two major political parties exist. If any other minor party exists then that party joins any one of the major parties based on their ideology and circumstances. Both the political parties contact the voters class for joining their party. Now a days the shifting of individuals from one political party to other has become a common trend. As some of the party members/workers who do not get proper weight/positions in their party or they are not very much satisfied with the changes occurring in the party's ideology, they leave their original party and join the second party.

Some modeling studies have been conducted regarding the growth of political parties and voters [2–7]. In most of the modeling studies statistical methodology is being used by considering that two political parties are competing for the voters class. In particular, Belenky and King [2] in their model considered two political parties, which compete for the votes of undecided voters who do not vote in US Federal elections. They have considered the role of election campaigns by all competing candidates on these voters. Khan [6] studied a multiparty political system by using nonlinear differential equations with time delay. In this model, three political parties and a class of people who are not member of any political party have been considered. In this study a predator-prey type model for the interactions between the parties have been considered and the conditions for Hopf-bifurcation have also been derived. Huckfeldt and Kohfeld [5] have presented two mathematical models, one linear and another nonlinear regarding the electoral stability and the decline of class in democratic politics by considering two political parties. In the linear model they have considered behavioral independence within the electorate and in the nonlinear model behavioral interdependence has been considered. In both the models the movement from one political party to other has also been considered.

In the above models, the epidemic approach has not been used in the modeling process, however Calderon et al. [3] have studied a nonlinear mathematical model for the spread of political third parties in American politics by using an epidemiological approach. As during the elections, the party members make door-to-door campaigning and convince the persons to vote for their party so it is more appropriate to use epidemic approach in the modeling process of political parties. During the elections party members also appeal to the voters to join their party based on the ideology of their party. By using epidemiological approach, the study of Calderon et al. [3] was limited and main focus has been given only to the expansion of the third party. They have not considered the role of two other major parties on the political system. In the simplified model they have considered three classes namely susceptible class (those people who vote, but do not vote to third party), voters class (those people who vote for third party) and members class (party officials, donors etc.). In the general model they have divided the susceptible class and voters class into two parts on the basis of affinity (high or low) to the third party's ideology. In the modeling process they have assumed that the total population participating in the system is constant. Many dynamical social phenomenon may be modelled by using the epidemiological type differential equations [8, 9]. In the present study, we assume that individuals of voters class are susceptible to both the political parties, like in epidemics, where two infective class and one susceptible class have been considered [10].

Keeping above in view, in this paper, we model the growth of two political parties by using the co-infection type epidemiological differential equations.

2 Mathematical model

Let N be the total population considered in the system, which is assumed to be a constant and homogeneously mixed. This total population has been divided into three classes:

(i) Voters class, (ii) Political party B class, and (iii) Political party C class. Due to the assumption of a constant population size we assume that per capita exit rate is equal to the per capita entering rate into the political system. As we know that most world-wide constitutions gives the right of voting to individuals when they become 18 years old, hence we assume that individuals enter into the voters class automatically with the rate μN, where μ is the rate at which individuals enter or leave the voting system. Due to the inactiveness and death, individuals exit from the political system from each class with the rates μV, μB and μC respectively. In the modelling process, it is considered that the individuals of voters class (V) are susceptible to both the political parties B and C. The members of both the political parties B and C contact the individuals of the V class and try to convince them to join their party on the basis of party's ideology. Let k_1 be the average number of contacts of members of political party B with voters per unit time, and p_1 be the probability of convincement per contact by a voter with a member of party B then the per capita recruitment rate in party B is $\beta_1 = p_1 k_1$. Thus the individuals in V class may decide to join the party B at a rate $\beta_1 V(B/N)$, where B/N is the chance of coming into contact with the members of party B. Similarly we may find the per capita recruitment rate β_2 in party C as $p_2 k_2$, where k_2 is the average number of contacts of members of party C with voters per unit time, and p_2 be the probability of convincement per contact by a voter with a member of party C. So the individuals in V class may decide to join the party C at a rate $\beta_2 V(C/N)$, where C/N is the chance of coming into contact with the members of party C.

In the modeling process it is also assumed that members shift from one political party to another when they are being contacted by the members of the other party. Let θ_1 and θ_2 be the per capita recruitment from party B to C and from C to B respectively. Thus the members of party B leave their party with the rate $\theta_1 B(C/N)$ and join the party C with the same rate. Similarly members of party C leave their party with the rate $\theta_2 C(B/N)$ and join the party B with the same rate. The flow diagram of the model is shown in Fig. 1.

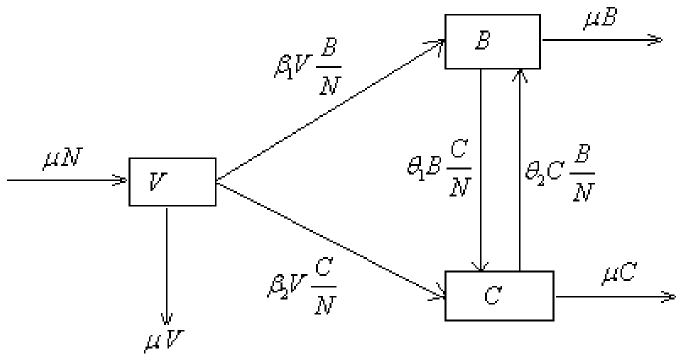

Fig. 1. Block diagram of the model.

In view of the above assumptions and considerations, the governing equations of the model may be written as follows:

$$\begin{aligned}\frac{\mathrm{d}V}{\mathrm{d}t} &= \mu N - \beta_1 V\frac{B}{N} - \beta_2 V\frac{C}{N} - \mu V,\\ \frac{\mathrm{d}B}{\mathrm{d}t} &= \beta_1 V\frac{B}{N} - \theta_1 B\frac{C}{N} + \theta_2 C\frac{B}{N} - \mu B,\\ \frac{\mathrm{d}C}{\mathrm{d}t} &= \beta_2 V\frac{C}{N} + \theta_1 B\frac{C}{N} - \theta_2 C\frac{B}{N} - \mu C,\end{aligned} \tag{1}$$

where $V(0) > 0$, $B(0) \geq 0$, $C(0) \geq 0$.

Now adding all the three equations of model system (1), we obtain that $\mathrm{d}N/\mathrm{d}t = 0$ showing that the total population N is constant over time and $V + B + C = N$. Here we may easily observe that the net shifting of members will be either from party B to party C or vice versa. Thus, let us put $\theta_1 - \theta_2 = \theta$. With this simplification, the above model system (1) reduces to the following system:

$$\begin{aligned}\frac{\mathrm{d}V}{\mathrm{d}t} &= \mu N - \beta_1 V\frac{B}{N} - \beta_2 V\frac{C}{N} - \mu V,\\ \frac{\mathrm{d}B}{\mathrm{d}t} &= \beta_1 V\frac{B}{N} - \theta B\frac{C}{N} - \mu B,\\ \frac{\mathrm{d}C}{\mathrm{d}t} &= \beta_2 V\frac{C}{N} + \theta B\frac{C}{N} - \mu C.\end{aligned} \tag{2}$$

Without loss of generality, we may assume that $\theta > 0$. This assumption implies that net rate of shifting of party members is from political party B to political party C.

Now we proportionalize our system by defining new variable v, b, c as proportions of the total voting population N. Let $v = V/N$, $b = B/N$ and $c = C/N$ denotes the proportionate variables of our system. Then the above system reduces to the following form:

$$\begin{aligned}\frac{\mathrm{d}v}{\mathrm{d}t} &= \mu - \beta_1 vb - \beta_2 vc - \mu v,\\ \frac{\mathrm{d}b}{\mathrm{d}t} &= \beta_1 vb - \theta bc - \mu b,\\ \frac{\mathrm{d}c}{\mathrm{d}t} &= \beta_2 vc + \theta bc - \mu c.\end{aligned} \tag{3}$$

Let (v^*, b^*, c^*) be the equilibrium of the above model then $v^* = V^*/N$, $b^* = B^*/N$ and $c^* = C^*/N$, where (V^*, B^*, C^*) be the equilibrium of the unreduced system (2). As we know that total voting population is constant over time and $V+B+C = N$, this gives $v + b + c = 1$ for the reduced system (3). Using this fact, the reduced model system (3) will be given by the following two differential equations:

$$\begin{aligned}\frac{\mathrm{d}b}{\mathrm{d}t} &= \beta_1(1 - b - c)b - \theta bc - \mu b,\\ \frac{\mathrm{d}c}{\mathrm{d}t} &= \beta_2(1 - b - c)c + \theta bc - \mu c.\end{aligned} \tag{4}$$

The study of this model (4) is equivalent to the study of model system (1). Keeping this in view we study the model system (4). The governing equations in the model system (4) are ecological type equations. Now we analyze the above system (4) like ecological models and interpret the results in terms of political science.

3 Equilibrium analysis

The model system (4) has four equilibria, which are as follows:

(i) *Party free equilibrium* $E_0(0,0)$. This equilibrium always exists without any condition. In terms of political science this situation corresponds to a nonpartisan. In a nonpartisan system, no official political parties exist. In nonpartisan elections, each candidate is eligible for office on his/her own merits and capabilities. By putting the legal restrictions on political parties, this situation may arise [11]. For example, in Pakistan there was no political parties from 2001–2008. This situation arose because of restrictions on all political parties in the above said period.

(ii) *Single party equilibria* $E_1(1-\mu/\beta_1, 0)$ *and* $E_2(0, 1-\mu/\beta_2)$. In this case there exist two equilibria. The equilibrium $E_1(1-\mu/\beta_1, 0)$ exists if the condition $\beta_1 > \mu$ holds, whereas the equilibrium $E_2(0, 1-\mu/\beta_2)$ exists if $\beta_2 > \mu$.

In the equilibrium $E_1(1-\mu/\beta_1, 0)$, political party B exists whereas members in the political party C are zero, i.e. political party B exists and political party C does not exist. Similar explanation also holds for the equilibrium $E_2(0, 1-\mu/\beta_2)$, but in reverse order.

In single-party systems, one political party is legally allowed to hold effective power. In this kind of system one political party has large number of members whereas the other party has negligible members. The single-party system is generally corresponds to dictatorship and may be taken as dominant-party systems. Examples of dominant party systems include the "People's Action Party" in Singapore and the "African National Congress" in South Africa [11].

(iii) *Interior/endemic equilibrium* $E_3(b^*, c^*)$. This equilibrium may be obtained by solving the following two algebraic equations:

$$\beta_1(1-b-c) - \theta c - \mu = 0, \tag{5}$$

$$\beta_2(1-b-c) + \theta b - \mu = 0. \tag{6}$$

From the above equations (5) and (6), the values of b^* and c^* may be obtained as follows:

$$b^* = \frac{\mu(\beta_1-\beta_2) - \theta(\beta_2-\mu)}{\theta(\beta_1-\beta_2+\theta)} \quad \text{and} \quad c^* = \frac{\theta(\beta_1-\mu) - \mu(\beta_1-\beta_2)}{\theta(\beta_1-\beta_2+\theta)}.$$

Here we note that $v^* = 1-b^*-c^* = \theta/(\beta_1-\beta_2+\theta)$. As we already know that $v^* \in (0,1]$, this implies that $\beta_1 \geq \beta_2$. If we take $\beta_1 = \beta_2$, then $v^* = 1$ and $b^* = c^* = 0$. This is the case of party free equilibrium and has been discussed earlier. Thus for the existence of interior equilibrium we take $\beta_1 > \beta_2$.

Keeping in mind that $\beta_1 > \beta_2$, the equilibrium $E_3(b^*, c^*)$ exists provided parameters of the model system (4) satisfy the following two more conditions:

$$\mu(\beta_1 - \beta_2) - \theta(\beta_2 - \mu) > 0, \tag{7}$$

$$\theta(\beta_1 - \mu) - \mu(\beta_1 - \beta_2) > 0. \tag{8}$$

In most of the worldwide countries mainly two political parties exist. In this case the two political parties dominate the whole population of the country to such an extent that electoral success under the banner of any other party is extremely difficult. If there exist other parties then they may be minor parties and they collaborate to any one of the major party. For example, in India there exists many regional (minor) parties but they are not getting proper success. At the time of elections these regional parties collaborate to one of the major parties. The other examples of the two-party systems are United States, Jamaica, United Kingdom, etc. The United Kingdom is widely considered a two-party state, as historically power alternates between two dominant parties, namely the Labour party and the Conservative party [11].

4 Stability analysis

The general Jacobian matrix J of the model system (4) is given as follows:

$$J = \begin{pmatrix} \beta_1(1 - 2b - c) - \theta c - \mu & -\beta_1 b - \theta b \\ -\beta_2 c + \theta c & \beta_2(1 - b - 2c) + \theta b - \mu \end{pmatrix}.$$

Now we investigate the stability of various equilibria by finding eigenvalues of Jacobian matrix J corresponding to each equilibria. Let J_i be the Jacobian matrix evaluated at the equilibria E_i $(i = 0, 1, 2, 3)$.

It is easy to check that the eigenvalues of the matrix J_0 are $\beta_1 - \mu$ and $\beta_2 - \mu$. Since for the existence of equilibrium $E_1(1 - \mu/\beta_1, 0)$, $\beta_1 > \mu$ and for the existence of equilibrium $E_2(0, 1 - \mu/\beta_2)$, $\beta_2 > \mu$. Thus $E_0(0, 0)$ is unstable whenever equilibrium E_1 or E_2 exists.

The eigenvalues of the matrix J_1 are $-(\beta_1 - \mu)$, which is negative and $(\theta(\beta_1 - \mu) - \mu(\beta_1 - \beta_2))/\beta_1$, which is positive for the existence of equilibrium E_3 (see condition (8)). Thus $E_1(1 - \mu/\beta_1, 0)$ is unstable whenever interior equilibrium $E_3(b^*, c^*)$ exists.

Similarly the eigenvalues of the matrix J_2 are $(\mu(\beta_1 - \beta_2) - \theta(\beta_2 - \mu))/\beta_2$, which is positive for the existence of equilibrium E_3 (see condition (7)) and $-(\beta_2 - \mu)$,which is negative. Since one eigenvalue of matrix J_2 is positive, thus $E_2(0, 1 - \mu/\beta_2)$ is unstable whenever interior equilibrium $E_3(b^*, c^*)$ exists.

The local stability behavior of $E_3(b^*, c^*)$ of model (4) is established by finding the roots of characteristic equation of the Jacobian matrix evaluated at $E_3(b^*, c^*)$. The global stability result has been proved by using the divergency criterion [12].

Theorem 1. *The equilibrium $E_3(b^*, c^*)$ whenever exists, is locally asymptotically stable without any condition.*

Proof. The matrix J_3 (J, evaluated at E_3) is given as follows:

$$J_3 = \begin{pmatrix} -\beta_1 b^* & -(\beta_1+\theta)b^* \\ -(\beta_2-\theta)c^* & -\beta_2 c^* \end{pmatrix}.$$

From the above matrix, we observe that trace(J_3) = $-(\beta_1 b^* + \beta_2 c^*)$, which is negative and $|J_3|$ = $\theta(\beta_1 - \beta_2 + \theta)b^* c^*$, which is positive (since $\beta_1 > \beta_2$). This implies that both the eigenvalues of the matrix J_3 will be either negative or having negative real part. Thus $E_3(b^*, c^*)$ whenever exists, is locally asymptotically stable without any condition. □

Theorem 2. *Model system* (4) *does not have any limit cycle in the interior of the positive quadrant of the* bc*-plane.*

Proof. Let us consider the following function

$$H(b,c) = \frac{1}{bc},$$

which is positive in the interior of the positive quadrant of the bc-plane.

Define

$$h_1(b,c) = \beta_1 b(1-b-c) - \theta bc - \mu b,$$
$$h_2(b,c) = \beta_2 c(1-b-c) + \theta bc - \mu c.$$

Then we have

$$\Delta(b,c) = \frac{\partial}{\partial b}(h_1 H) + \frac{\partial}{\partial c}(h_2 H) = -\left(\frac{\beta_1}{c} + \frac{\beta_2}{b}\right) < 0.$$

This shows that $\Delta(b,c)$ does not change sign and is not identically zero in the interior of the positive quadrant of the bc-plane. Therefore, by Bendixson–Dulac criterion, it follows that there is no closed trajectory and hence no limit cycle in the interior of the positive quadrant of the bc-plane. □

In the following theorem we are able to show that the model system (4) is uniformly persistent [13, 14]. By the persistence of a system we mean that both the political parties are present and none of them will go to extinction.

Theorem 3. *The model system* (4) *is uniformly persistent if the conditions of existence of the equilibrium* $E_3(b^*, c^*)$ *holds.*

Proof. Let the average Lyapunov function for model system (4) be $\sigma(X) = b^p c^q$, $p > 0$, $q > 0$. Then $\sigma(X)$ is a nonnegative C^1 function in $\Re^2_+$. We note that

$$\Psi(X) = \frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}t} = p\big[\beta_1(1-b-c) - \theta c - \mu\big] + q\big[\beta_2(1-b-c) + \theta b - \mu\big].$$

Since there is no periodic orbit in the interior of the positive quadrant of the bc-plane (see Theorem 2). Thus, to show the uniform persistence of the system it is sufficient to show that $\Psi(X) > 0$ for all equilibria $X \in bd\Re^2_+$, for a suitable choice of $p, q > 0$, i.e. the following conditions must be satisfied for the persistence of the system:

$$\Psi(E_0) = p[\beta_1 - \mu] + q[\beta_2 - \mu] > 0, \tag{9}$$

$$\Psi(E_1) = \frac{q}{\beta_1}\big[\theta(\beta_1 - \mu) - \mu(\beta_1 - \beta_2)\big] > 0, \tag{10}$$

$$\Psi(E_2) = \frac{p}{\beta_2}\big[\mu(\beta_1 - \beta_2) - \theta(\beta_2 - \mu)\big] > 0. \tag{11}$$

Keeping in mind that for the feasibility of the model system (4), we have $\beta_1 > \mu$. Now by increasing p to a sufficiently large value, $\Psi(E_0)$ can be made positive and thus condition (9) is satisfied. Again the conditions of existence of equilibrium $E_3(b^*, c^*)$ implies that $\Psi(E_1) > 0$ and $\Psi(E_2) > 0$, and thus conditions (10) and (11) are also satisfied. This proves the theorem. □

Theorem 4. *If the equilibrium $E_3(b^*, c^*)$ is locally asymptotically stable, then it is globally asymptotically stable.*

Proof. Let $\Gamma(t) = (b(t), c(t))$ be any one non-trivial periodic orbit of system (4) with period $T > 0$. To prove the theorem it is enough to show that

$$\int_0^T \operatorname{tr}\big(J\big(b(t), c(t)\big)\big)\,\mathrm{d}t < 0,$$

where J is the Jacobian matrix.

Since

$$\int_0^T \big[\beta_1\big(1 - b(t) - c(t)\big) - \theta c(t) - \mu\big]\,\mathrm{d}t = \int_0^T \frac{b'(t)}{b(t)}\,\mathrm{d}t = 0$$

and

$$\int_0^T \big[\beta_2\big(1 - b(t) - c(t)\big) + \theta b(t) - \mu\big]\,\mathrm{d}t = \int_0^T \frac{c'(t)}{c(t)}\,\mathrm{d}t = 0.$$

So we have

$$\int_0^T \operatorname{tr}\big(J\big(b(t), c(t)\big)\big)\,\mathrm{d}t = \int_0^T \big[-\beta_1 b(t) - \beta_2 c(t)\big]\,\mathrm{d}t < 0.$$

Hence the divergency criterion [12] implies that all the periodic solutions must be orbitally stable. This is impossible, since $E_3(b^*, c^*)$ is locally asymptotically stable. Thus, the system (4) has no periodic orbit in $\Re^2_+$. Since the equilibria $E_0(0, 0)$, $E_1(1 - \mu/\beta_1, 0)$, $E_2(0, 1 - \mu/\beta_2)$ are unstable equilibria, the theorem follows immediately. □

5 Numerical example

To check the feasibility of our analysis regarding existence conditions and stability of various equilibria, we have conducted some numerical computation by choosing the values of parameters in model (1) as given in (12). Since the realistic data is not available regarding movement of members from one political party to another, we have intuitively assumed the hypothetical parameter values so as the existence conditions are satisfied at least for a set of parameter values.

As we know that the constitution of most of the countries all around the world gives the right of voting to a person when he acquires the age of 18 years. A person also loses his interest in politics i.e. in voting to any party when he crosses 78 years of age due to his old age and health problems. With these reasons, we may assume that persons are involved in politics only between the age of 18–78 years old, thus we may take $\mu \approx 1/60$. Because voters are susceptible to both political parties, thus each political party pays much more attention on the recruitment from this voters class. Let the per capita recruitment rate from the voters class to the party B is $1/24$. This implies that approximately 24 members of political party B are able to convince 01 voter in a year to join their party. Let us assume that the per capita recruitment rate from voters class to the political party C is $1/36$. At the same time it is also assumed that the per capita recruitment rate from the political party C to B is $7/720$ and from B to C is $17/720$. The above data implies that political party B pays much attention on the new recruitment from the voters class, however does not pay sufficient attention on their existing members. In contrast political party C pays some less attention (in comparison to party B) on the recruitment from voters class but more attention on their members. The per capita rate of leaving party B and taking the membership of party C is more than that of leaving party C and joining party B.

Thus for the above situation we have the following set of parameter values in model system (1):

$$\beta_1 = \frac{1}{24}, \quad \beta_2 = \frac{1}{36}, \quad \theta_1 = \frac{17}{720}, \quad \theta_2 = \frac{7}{720}, \quad \mu = \frac{1}{60}. \tag{12}$$

For the above set of parameter values, we note that $\theta_1 > \theta_2$ and $\theta_1 - \theta_2 = \theta = 1/72$. It may also be checked that for these parameter values, the conditions of existence of interior equilibrium $E_3(b^*, c^*)$ of model system (4) are satisfied. The values of b^* and c^* are obtained as 0.2 and 0.3 respectively.

In this case, we have numerically calculated eigenvalues of Jacobian matrix using above parameter values. The eigenvalues of the Jacobian matrix corresponding to the equilibrium $E_3(b^*, c^*)$ are obtained as, $(-3 + \sqrt{6})/360$ and $(-3 - \sqrt{6})/360$. It may be noted that these are negative, hence $E_3(b^*, c^*)$ is locally stable.

We have made the simulation analysis of our model system (4) for the above set of parameter values. In Fig. 2, we have shown the variations of political party B and C with respect to time t. From this figure it is easy to note that if both parties have started with same number of members then for some time political party B will lead over C but after some time result will be reversed i.e party C will lead over B.

In Fig. 3, we have shown the global stability of interior equilibrium $E_3(b^*, c^*)$. We have made the trajectories with different initial starts and it may be noted that all the trajectories are approaching to the interior equilibrium, which shows the global stability of the equilibrium $E_3(b^*, c^*)$.

In model system (4), θ is a very sensitive parameter. In Fig. 4 and Fig. 5, we made the graphs of variation of political party B and C with respect to time t for different values of θ. From these figures it is clear that as the value of θ increases the members of political party B decreases whereas members in party C increases.

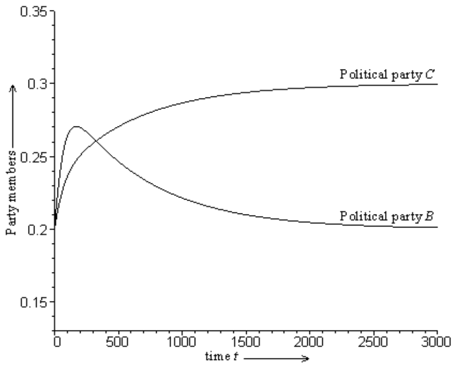

Fig. 2. Variation of political parties with respect to time t.

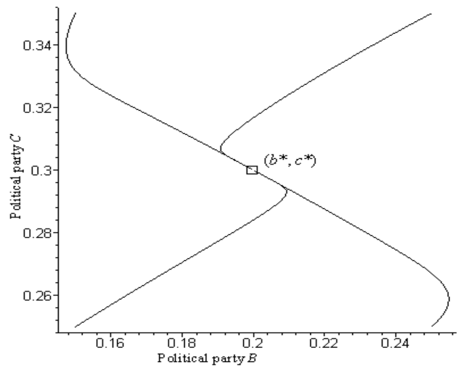

Fig. 3. Global stability of equilibrium $E_3(b^*, c^*)$.

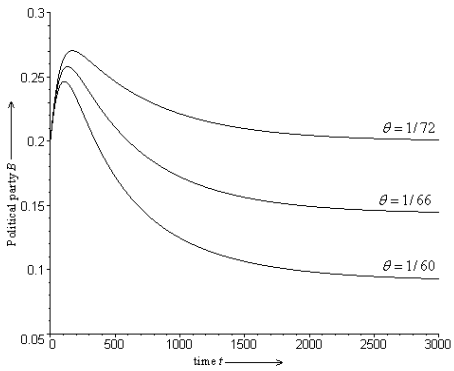

Fig. 4. Variation of party B with respect to time t for different values of θ.

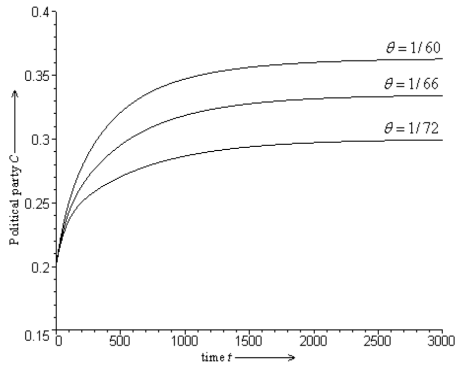

Fig. 5. Variation of party C with respect to time t for different values of θ.

6 Conclusion

In this paper, we have proposed and analyzed an epidemiological type nonlinear mathematical model for two political parties. Each party depends on its solid core of supporters/members to sustain it through difficult times. A party may die out if it does not have solid members/supporters. The analysis of model shows that if the net rate of shifting of members from political party B to C (i.e. θ) is greater than $\mu(\beta_1 - \beta_2)/(\beta_2 - \mu)$, then political party B will die out. On the other hand as we have taken that recruitment from voters class to political party B is greater than political party C (i.e. $\beta_1 > \beta_2$) so if $\theta < \mu(\beta_1 - \beta_2)/(\beta_1 - \mu)$, then political party C will die out. So for the existence of both the political parties θ must lie in the open interval $(\mu(\beta_1 - \beta_2)/(\beta_1 - \mu), \mu(\beta_1 - \beta_2)/(\beta_2 - \mu))$. This suggests that for the survival of any political party; retaining the existing members is more important than new recruitment of members from voters class. If the party members/workers leave the party and join another party, then in this case the survival of the party may be difficult. In fact party should make changes time to time in their ideology according to the existing members of the party. The changes should be made in the party's ideology by considering the various factors like, socioeconomic, demographic, religion, race, gender, etc., in such a way that individuals from voters class get attracted to join the party and existing members also be satisfied. Numerical simulation also suggests that a small change in the movement of one party to another may change the results very fast. For the real democracy both the parties should pay much attention on their members as well as on the contacts to the voters. Uniform persistence of the model implies that if both parties exist then will exist under certain conditions.

This model may be generalized by considering a third political party, which may affect the dynamics of the political scenario of any country. The another generalization of the above model is considering the effect of time delay involved in moving from one political party to another. These are left for future research.

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