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Implementing the mathematical model of the throughput of compressor station aggregates

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Abstract. The main aim of the European union energy policy is to ensure a continuous supply of natural gas for the member states. The Russian gas is transported through the Belarus, Ukraine, Slovakia (SR) and Czech Republic to Germany. This article shortly describes the transit system of Slovakia and compressor station KS1 – Vel'ké Kapušany. The throughput of this compressor station plays an important role in gas transport, because this station is the entry to the transit system of SR. To find the best combination of the aggregates used in gas transport a simulation algorithm was devised. We present the architecture of this simulation software together with the description of its development phases. The software was evaluated against data gained from the KS1 compressor station operation. The combinations of aggregates were found as dependent on inlet pressure, ambient temperature and compression ratio.

Keywords: natural gas transport, compressor station, simulator program, refactoring, transit system, aggregate, Russian gas.

1 Introduction

A transit system of natural gas transportation is built to carry large volumes of natural gas over long distances, i.e. from the source to the consumers [2, 5]. The annual capacity of the network is more than 93 billion m³ of natural gas transported, which is about 15 times more than the domestic consumption of natural gas in Slovakia. This system is associated with major European transport networks in a number of interconnection points such as Vel'ké Kapušany at the border with Ukraine, Lanžhot at the border with the Czech Republic, and Baumgarten on the Austrian border. The length of the transport pipeline with four compressor stations is 2270 km. Through the domestic input/output point, there

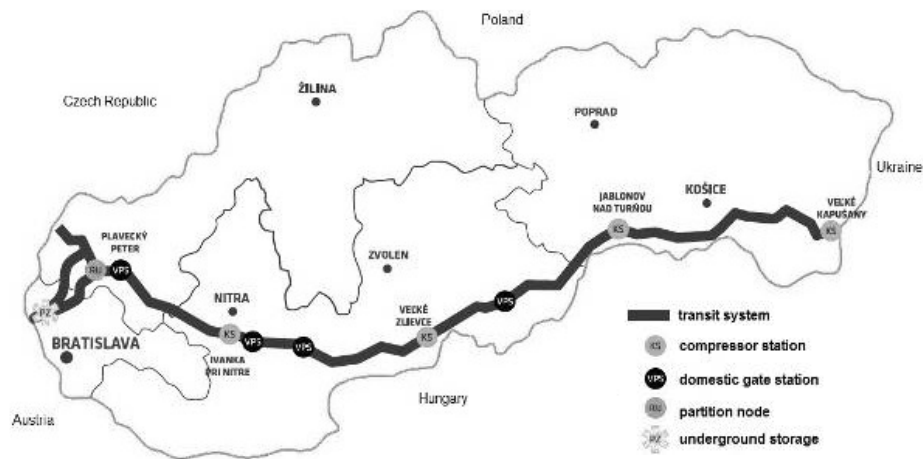


Fig. 1. The transit system of natural gas transportation in Slovak Republic.

is access to the distribution network and natural gas storage facilities on the territory of Slovakia [1] allowed. The transit system of natural gas transportation is shown in Fig. 1.

Natural gas flows through four parallel pipes with the diameter of 1200 and 1400 mm, with operating pressure 73.105 Pa. The fifth line of the system serves as reservoir of natural gas. Depending on the desired volume of transported natural gas, the compressor stations have to be associated with the appropriate serial configuration for the safe, reliable and efficient transport from the entry of the system into the individual outlet points. Many turbo aggregates are parallel connected within the compressor stations, which are to be synchronized with each other and operate in the best possible way [6, 17].

Mathematical model is useful in simulations, which allow to simulate system behavior in usual and/or unexpected situations. Such unusual situations could be reverse flow of gas or pipe failures. The model could serve also for checking whether the actual configuration of aggregates is the optimal one or some aggregates could be removed or stopped. This paper focuses on the reusable and replaceable parts of the system – aggregates and compressor stations. Future work is required to add the pipes to the model.

2 Compressor stations in Slovakia

The transit system in Slovakia consists of the following compressor stations:

- KS1 – Veľké Kapušany,
- KS2 – Jablonov nad Turňou,
- KS3 – Veľké Zlievce,
- KS4 – Ivánka pri Nitre.

Compressor station KS1 in Veľké Kapušany serves to increase the pressure of natural gas in the transport chain, which offsets the pressure loss of the flowing gas in the pipeline [11]. To compress the natural gas in the transit system turbochargers driven by

Table 1. Numbers of aggregates used in different compressor stations in Slovakia.

	TS 6MW ČKD	NP PGT 25	Cooper- Rolls	Electric drive	NP PGT 25+
El. Power [MW]	6	22.67	27.426	25	31.069
KS1	19	1	3	3	2
KS2	21	1	2	0	0
KS3	22	0	2	0	0
KS4	12	0	4	0	0

gas combustion turbines are used at [15]. Aggregates used in this compressor station are shown in Table 1.

The permeability of the compressor station KS1 – Vel'ké Kapušany determines the throughput of the transit system in Slovakia, because this station is the biggest station in the system from the point of its capacity.

The second one is the compressor station KS2 – Jablonov nad Turňou, which one's operation was started on 29th September 1975. The technical equipment is nearby the same as in KS1, but KS2 has a leadership because two turbo aggregates of type Cooper-Rolls with the performance of 28 MW were first time installed here in 1997.

Line division of Rožňava provides maintenance and operation of transit pipelines, input/output facility of KS2 with the pipeline court, national natural gas transfer station, 459 km pipelines with diameters from 1200 up to 1400 mm, 25 stations of the active cathodic protection system, telemetry – remote data transfer and remote control of ball valves. Aggregates used in compressor station KS2 – Jablonov nad Turňou are shown in Table 1.

The parts of compressor station KS3 – Vel'ké Zlievce take four halls with 22 turbo aggregates with the performance of 6 MW, electro halls with 2 Cooper-Rolls aggregates with a performance above 27 MW. Aggregates used in the compressor station KS3 – Vel'ké Zlievce are shown in Table 1.

To optimize the natural gas transit through the Slovak Republic, this compressor station is frequently weaned and re-enabled in a shorter time. However, the large handling, which consists of the start of KS3, shut down, or transition from one stage of two-stage operation, respectively starting or weaning more machines, increases from the year 2000.

The largest supply region of the Eustream Company is the region nearby the smallest compressor station (in terms of performance) KS4 – Ivánka pri Nitre, which overlays the western regions of Slovak Republic. To secure the pressure and the quantities of the transported natural gas is the priority of this compressor station and the diverting hub in Plavecký Peter (where the natural gas is divided into two lines and being transported to Czech Republic and Austria). Aggregates used in compressor station KS4 – Ivánka pri Nitre are also shown in Table 1.

3 The model

3.1 Universal characteristics of the turbo compressors

Method to construct the universal characteristic is as follows:

1. To the horizontal axis is plotted the performance Q ($\text{m}^3 \text{s}^{-1}$, $\text{m}^3 \text{h}^{-1}$) – volume flow re-count to the conditions in the inlet throttle as pressure p_1 and temperature t_1 , respectively mass flow M (kg s^{-1}).
2. To the vertical axis is plotted the outlet pressure p_2 (Pa) from the outlet throttle or ratio of pressures $\varepsilon = p_1/p_2$ respectively pressure differences $\Delta p = p_2 - p_1$ (Pa), and efficiency η .

Construction of the universal characteristics of aggregates is associated with difficulties arising from the lack of documentation to assess the medium flow in the machine out

Table 2. Overview on measurements and units used.

Notation	Measurement	Unit
η, η_k	efficiency of the compressor	[-]
η_{tt}	turbine drive efficiency	[-]
ε	compression ratio	[-]
ε_d	lower optimal compression ratio	[-]
ε_h	upper optimal compression ratio	[-]
ρ	density of natural gas	$[\text{kg m}^{-3}]$
e_k, e_t	nominal gas consumption by the compressor or by the turbine	$[\text{MJ m}^{-3}]$
M	mass flow of the natural gas in compressor	$[\text{kg s}^{-1}]$
\mathcal{M}	molecular weight/molar mass	$[\text{g mol}^{-1}]$
n	operating speed of the compressor	$[\text{s}^{-1}]$
n_t	universal operating speed	$[\text{s}^{-1} \text{K}^{-0.5}]$
n_{tt}	operating speed of the turbine or compressor drive	$[\text{s}^{-1}]$
Δp	pressure difference on the compressor	[Pa]
p_1	inlet pressure to the compressor	[Pa]
p_2	outlet pressure of the compressor	[Pa]
P	compressor power	[W]
P_k	compressor performance	[W]
$P_t, P_{t,\max}$	maximum turbine or compressor performance	[W]
Q	gas flow rate	$[\text{m}^3 \text{s}^{-1}]$
R^2	regression analysis reliability	[-]
T_1, t_1	inlet temperature to the compressor	[K]
t_2	outlet temperature of the compressor	[K]
t_{ok}	ambient temperature	[K]
x, y	regression analysis variables	[-]
Z	compressibility factor	[-]
$a, b, c, d,$ $a_\varepsilon, b_\varepsilon, c_\varepsilon, d_\varepsilon,$ $a_\eta, b_\eta, c_\eta, d_\eta,$ $a_1, b_1, c_1, d_1, \dots$ $a_d, b_d, c_d,$ $a_h, b_h, c_h,$ $a_k, b_k, c_k, d_k,$ $a_n, b_n, c_n, d_n,$ $a_t, b_t, c_t, d_t,$ $A, B, C, D,$ A_1, B_1, C_1, \dots AA_1, BB_1, CC_1, \dots AR_1, AS_1, AT_1, \dots	Coefficients obtained by regression analysis	[-]

of the proposed conditions. Compressor characteristics are designed by the measurement of these variables [4].

A temperature change at the compressor inlet affects the compression change and the efficiency increase. If the temperature decreases at the compressor inlet, then the compression increases and vice versa. The specific work of natural gas at the constant operating speed and volume flow is unchanged for radial and axial compressors. Isoentropic work remains nearby constant at value w_{is} . The larger density of the natural gas that occurred from the temperature T_1 decrease at the constant pressure p_1 affect the mass flow increases through the compressor [3, 9].

3.2 The solution method

The characteristics of the compressors were revised from the graphical form to the mathematical form using regression analysis:

$$\varepsilon = f\left(\frac{M\sqrt{T_1}}{p_1}, \frac{n}{\sqrt{T_1}}\right) \quad P_k = f\left(\frac{M\sqrt{T_1}}{p_1}, \frac{n}{\sqrt{T_1}}\right) \quad \eta_k = f\left(\frac{M\sqrt{T_1}}{p_1}, \frac{n}{\sqrt{T_1}}\right).$$

Specific graphs for each aggregate were the source of information to determine the permeability of the aggregates, where the natural gas flow depends on compression ratio and performance on the operating speed.

The most accurate read of the source graphs, i.e. the original characteristics, was the first step to define the later developed curves. To read these graphs, the Photo Filter Studio software was used at 400 times magnification to ensure accuracy. Data from these characteristics are related to the reference gas with parameters: compressibility factor Z , molecular weight M , inlet temperature T_1 and inlet pressure p_1 . Depend on the volume flow Q at the inlet parameters and operation speed are power of the compressor P , compression ratio ε and efficiency given to the graphs. The high accuracy of reading was proved with the high value of reliability R^2 that is nearby one as shown in Fig. 2.

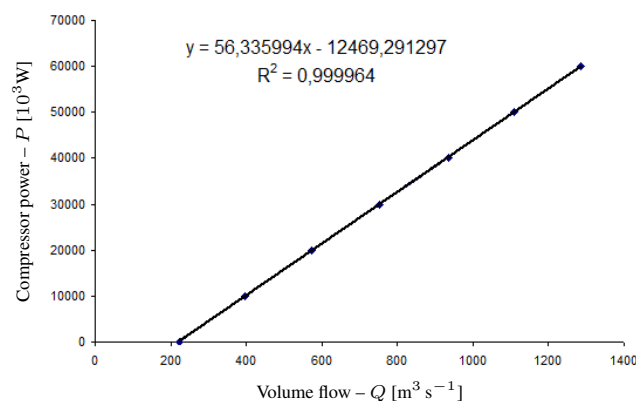


Fig. 2. Example of the original graphs read.

Table 3. The natural gas composition.

Component name	Chemical formula	Volume percent
Methane	CH ₄	98.63%
Ethane	C ₂ H ₆	0.18%
Propane	C ₃ H ₈	0.02%
Butane	C ₄ H ₁₀	0.10%
Carbon dioxide	CO ₂	1.01%
Nitrogen	N ₂	0.12%

Table 4. Re-count operational speed for aggregate Nuovo Pignone PGT 25 MW.

n [rpm]	n_t [s ⁻¹ K ^{-0.5}]
4270	248.1258
4880	283.5724
5490	319.0189
5867	340.9261
6100	354.4655
6405	372.1888

Magnification reduces the display reading error by introducing a more detailed scale on the axes, i.e. pixel size converges to unit size. This means that there will be a lower error in reference point setup in polynomial approximation.

The parameters of natural gas [7, 12] used to calculate the permeability were: temperature $T_1 = 298.5$ K and pressure $p_1 = 5.1$ MPa.

Parameters needed to construct the universal characteristics were calculated from the composition of natural gas shown in Table 3.

Calculations to construct the universal characteristics were realized for all aggregates used for natural gas transport in Slovakia. Results of operational speed for aggregate Nuovo Pignone PGT 25 MW are shown in Table 4.

To determine the universal characteristics of the aggregates a polynomial function was constructed. This function has the following form:

$$y = Ax^3 + Bx^2 + Cx + D,$$

where A , B , C are constants, x – mass flow convert for universal characteristics, D – curves shift parameter.

The constants A , B , C are maintained for all values of operational speed, which are referred for compression ratio. Polynomial functions that are applied in the mathematical model and consequently in the software have the forms shown below:

$$\begin{aligned} \text{compression ratio} & \quad \varepsilon = a_\varepsilon x^3 + b_\varepsilon x^2 + c_\varepsilon x + d_\varepsilon, \\ \text{compressor input} & \quad P_k = a_k x^3 + b_k x^2 + c_k x + d_k, \\ \text{compressor efficiency} & \quad \eta = a_\eta x^3 + b_\eta x^2 + c_\eta x + d_\eta. \end{aligned}$$

Calculated constants for compression ratio using the universal characteristic for aggregate Nuovo Pignone PGT 25 MW are shown in Table 5 and Fig. 3 – six curves for six different values of n_t .

To calculate the permeability of the compressor aggregates the following functions were used:

$$\begin{aligned} \varepsilon &= f(Q, n), & P_k &= f(Q, n), & \eta_k &= f(Q, n), \\ \eta_k &= f\left(\frac{M\sqrt{T_1}}{p_1}, \frac{n}{\sqrt{T_1}}\right) & P_t &= f(P_k, n_{tt}, t_{ok}). \end{aligned}$$

Table 5. Constants for compression ratio.

n_t [$s^{-1} K^{-0.5}$]	a	b	c	d
248.125847	0.175790176	-0.626576936	0.449092571	1.248908449
283.5723965	0.006740254	-0.180298327	0.067344857	1.487545119
319.0189461	-0.06253819	0.03796171	-0.14811455	1.72047954
340.9260759	-0.054293153	0.009844561	-0.115785756	1.827824246
354.4654957	-0.1133	0.260364	-0.45996	2.062477
372.1887704	-0.216284587	0.706997039	-1.084206845	2.458736856

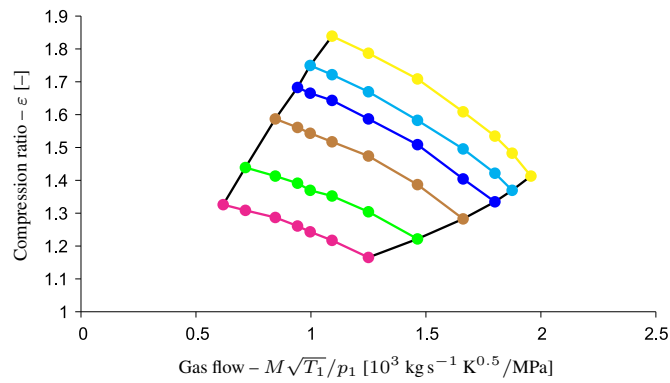


Fig. 3. Universal characteristic dependence graph for n_t values shown in Table 5.

Graphical dependences were converted to the characteristic polynomial form. Influence of the compression ratio:

$$M = Q\rho, \quad n_t = \frac{n}{\sqrt{T_1}}, \quad x = \frac{M\sqrt{T_1}}{p_1},$$

$$\varepsilon = a_1x^3 + b_1x^2 + c_1x + d_1,$$

$$a_1 = A_1n_t^2 + B_1n_t + C_1,$$

$$b_1 = A_2n_t^2 + B_2n_t + C_2,$$

$$c_1 = A_3n_t^2 + B_3n_t + C_3,$$

$$d_1 = A_4n_t^2 + B_4n_t + C_4.$$

Definition of the upper (ε_h) and lower (ε_d) limits of optimal compressor operation:

$$\varepsilon_h = a_hx^2 + b_hx + c_h,$$

$$\varepsilon_d = a_dx^2 + b_dx + c'_d.$$

Polytrophic work and performance definition:

$$P_k = a_2x^3 + b_2x^2 + c_2x + d_2,$$

$$a_2 = AA_1n_t^2 + BB_1n_t + CC_1,$$

$$\begin{aligned}
 b_2 &= AA_2 n_t^2 + BB_2 n_t + CC_2, \\
 c_2 &= AA_3 n_t^2 + BB_3 n_t + CC_3, \\
 d_2 &= AA_4 n_t^2 + BB_4 n_t + CC_4.
 \end{aligned}$$

Efficiency definition:

$$\begin{aligned}
 \eta_k &= a_3 x^3 + b_3 x^2 + c_3 x + d_3, \\
 a_3 &= AR_1 n_t^2 + AS_1 n_t + AT_1, \\
 b_3 &= AR_2 n_t^2 + AS_2 n_t + AT_2, \\
 c_3 &= AR_3 n_t^2 + AS_3 n_t + AT_3, \\
 d_3 &= AR_4 n_t^2 + AS_4 n_t + AT_4.
 \end{aligned}$$

Compressor operating speed definition depends on the efficiency to set up the limit performance of the compressor:

$$P_k = a_k n_{tt}^3 + b_k n_{tt}^2 + c_k n_{tt} + d_k. \quad (1)$$

Compressor efficiency definition depend on the performance to set up the limit performance of the compressor:

$$\eta_{tt} = a_n P_k^3 + b_n P_k^2 + c_n P_k + d_n.$$

Turbine or compressor drive maximum performance definition:

$$P_{t,\max} = a_t n_{tt}^3 + b_t n_{tt}^2 + c_t n_{tt} + d_t.$$

These functions were constructed for all compressor aggregates.

4 Simulations

The calculation equations shown above are used in the selected steps of the simulation workflow. This simulation workflow consists of several steps (see Fig. 4):

1. Starting the first part of simulations using a simulator program.
2. Analyzing the results of the first simulations and modification of these results adding the additionally computed parameters using Excel software.
3. Additional simulation runs based on the previously gained and extended data.
4. Implementing Final evaluation of results exploiting the generation and analysis of graphs and charts.

In this paper, the simulation program for the first simulation will be discussed in detail; the second simulation program is similar by structure. The outputs compared are the outputs of the second program.

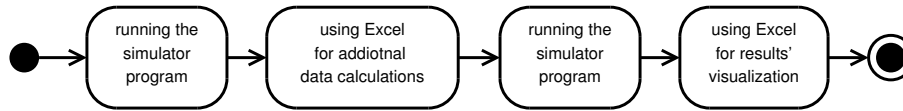


Fig. 4. The simulation workflow.

4.1 Simulation algorithm and its implementation

The simulation program was originally implemented in Pascal language, but the new requirements stemming from the application area called for the implementation in C language, as a language that is capable and fast for calculations and file manipulation and is compatible with the simulation program Matlab too.

There are several possible implementations based on the Pascal version interpretations. In this paper, two alternatives are presented in Figs. 5 and 6. Both presented algorithms express the same calculation flow, but the one shown in Fig. 5 eliminates the parallelism that was introduced in [16]. In fact, there is no need of parallelism because the aggregates are calculated separately and the calculations are not time consuming ones.

The software algorithm shown in Fig. 6 is our choice for investigation and implementation. This is done in C language. Using an open source automated tool for source code conversion fastened the implementation procedure. The tool *p2c* generates C source code corresponding to the given Pascal sources. Because the tool is not perfect [10, 13], additional source code correction was introduced after the code generation process. The major changes to provide were such as implementation of missing I/O routines and/or removal of redundant code. There are still areas of code to refactor [8, 14].

4.2 Simulation program evaluation

Input parameters to the software were pressure p_1 (Pa) and gas temperature t_1 ($^{\circ}\text{C}$) at the inlet to the compressor, compression ratio ε and ambient temperature t_{ok} ($^{\circ}\text{C}$). The output parameters from the software were gas temperature after compression t_2 ($^{\circ}\text{C}$), natural gas flow through the aggregate as depending on the input parameters Q (million m^3/day), nominal gas consumption by the turbine and nominal consumption by the compressor e_k and e_t (MJ m^{-3}), and the compressor type for the inlet flow. The results were displayed in a form of data files (formatted text file human readable and readable for external programs such as Microsoft Excel).

5 Results and discussion

The software can be used to find the combinations of aggregates depending on the gas inlet pressure, nominal consumption of the turbine and nominal consumption of the compressor. The results for the input parameters: inlet gas pressure (p_1) 4.3, 4.7 and 5 MPa, ambient temperature (t_{ok}) 25 $^{\circ}\text{C}$, 15 $^{\circ}\text{C}$ and 35 $^{\circ}\text{C}$, compression ratio (ε) 1.3, 1.4 and 1.35; for gas flow rates (Q) from 100 to 110 mil. m^3 per day were investigated.

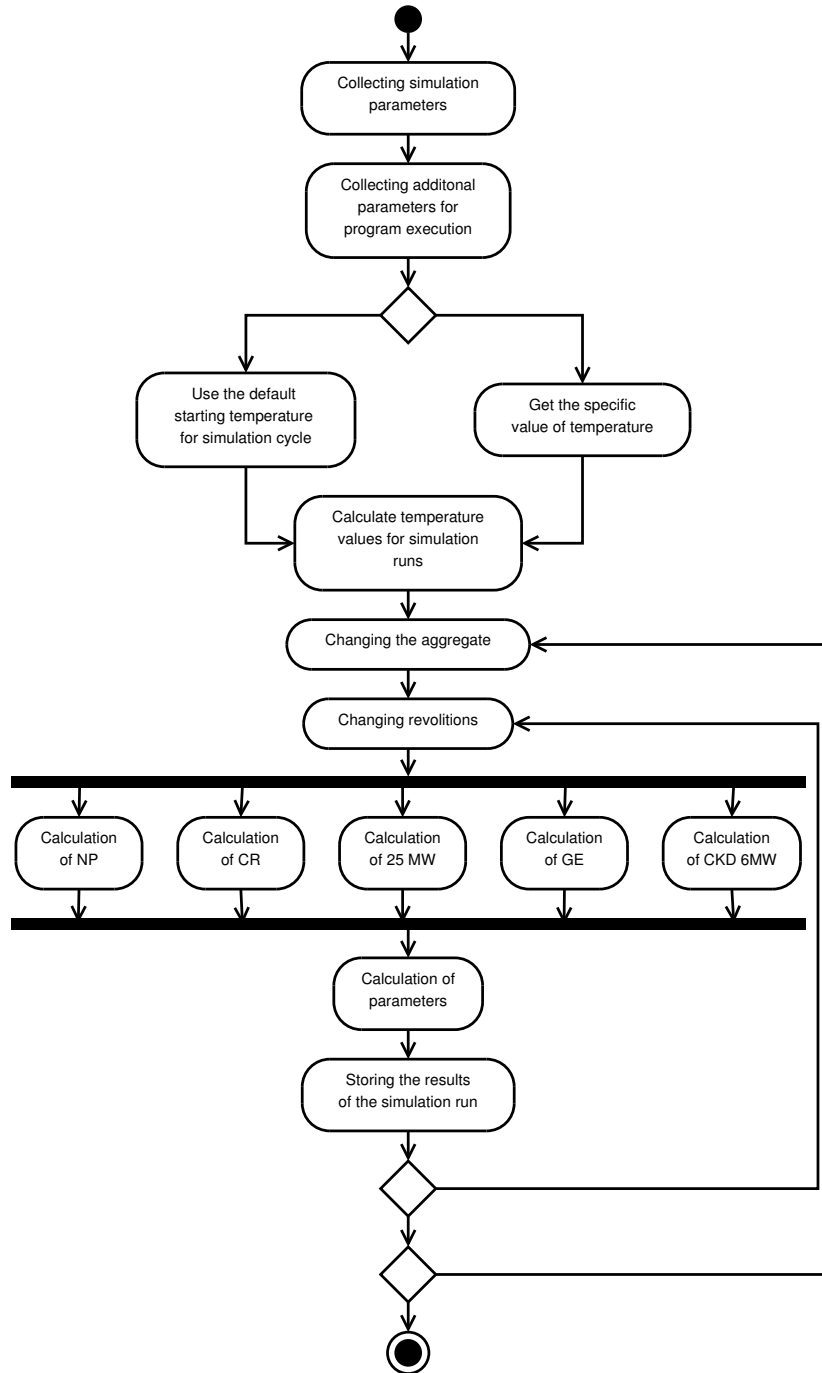


Fig. 5. The simulation algorithm – parallel version.

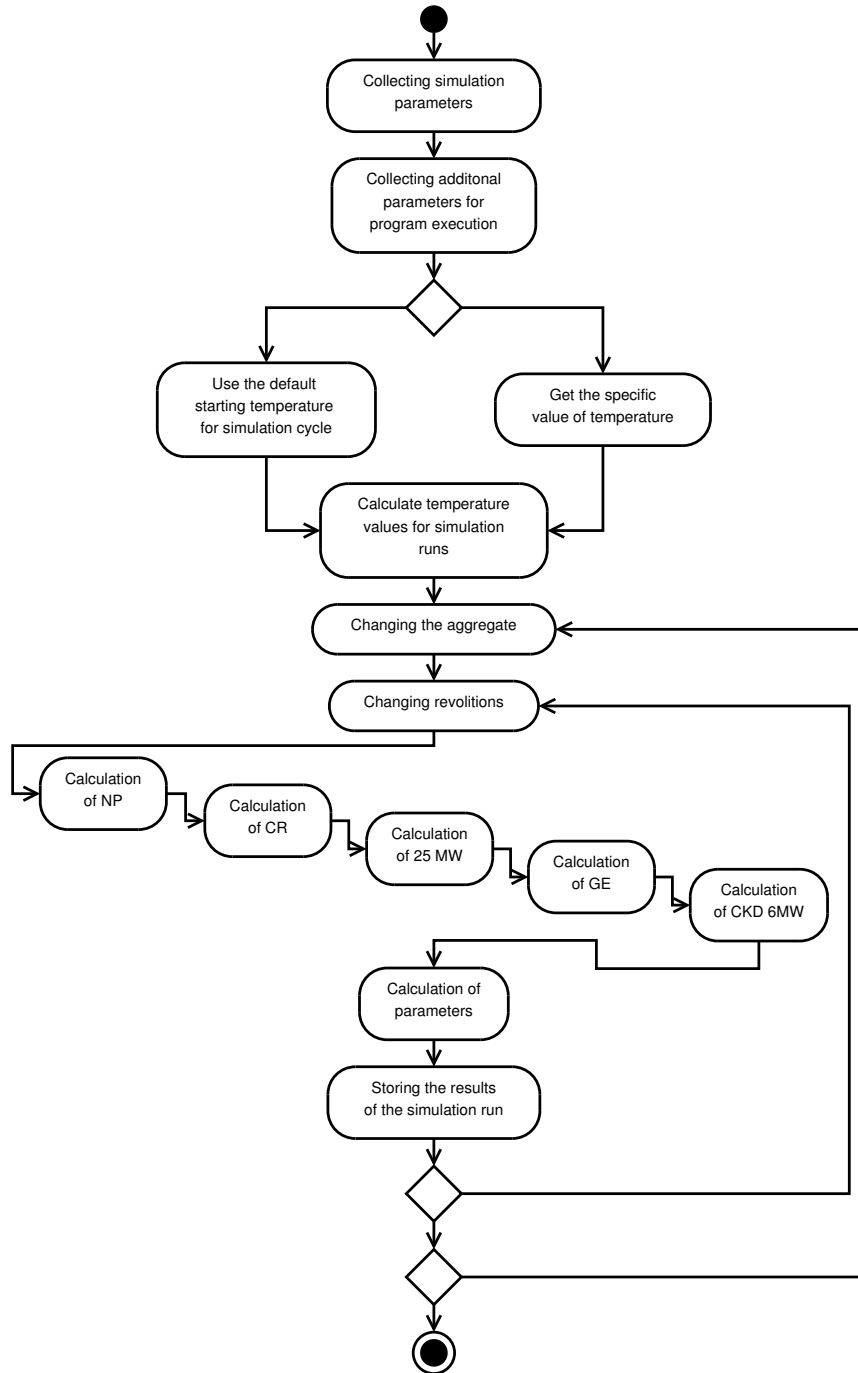


Fig. 6. The simulation algorithm – sequential version.

Table 6. Comparison of the outlet pressure p_2 in MPa from Eustream, a.s. simulations with outlet pressure from the simulation software.

Eustream, a.s.	Our simulation software	Offset [%]
5.640050906	5.59	0.88
6.641419858	6.58	0.92
6.810213109	6.75	0.88

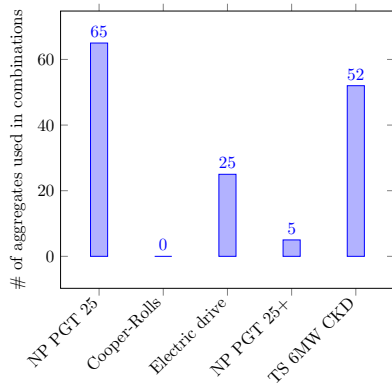


Fig. 7. Simulation data visualization for input parameters: $p_1 = 4.7$ MPa, $\varepsilon = 1.4$, $t_{ok} = 15$ °C.

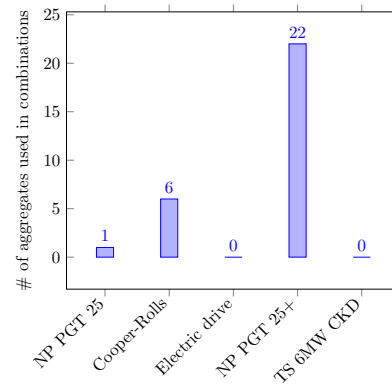


Fig. 8. Simulation data visualization for input parameters: $p_1 = 5$ MPa, $\varepsilon = 1.35$, $t_{ok} = 35$ °C.

The results from this software were compared with the outcome of company (i.e. Eustream a.s.) simulations. The offset of these results is shown in Table 6.

The higher inlet pressure and compression ratio cause that the aggregates TS 6 MW ČKD and Electric drive have not to be used. The newest aggregate type NuovoPignone NP PGT 25+ is the best solution to transport a larger amount of the natural gas.

Visualization of simulation data for values of input parameters: $p_1 = 4.7$ MPa, $\varepsilon = 1.4$, $t_{ok} = 15$ °C and input parameters: $p_1 = 5$ MPa, $\varepsilon = 1.35$, $t_{ok} = 35$ °C are shown in Figs. 7–8.

The discussed part of the software can be used to calculate the throughput of the individual compressor station only and not to calculate the throughput for entire transit system. To calculate the entire transit system throughput the line part of the transit has to be computed. The source code has to be extended by the line part of the transit system and the other three compressor stations should be included as well.

A little different view makes the presented solution more general. The industry white papers are reliable useful in empirical modeling by application of several image and data processing techniques.

The small offset between simulation output and company data indicates that at least the corresponding part of the created model is valid. Based on that, additional simulation steps will be included in the next generation natural gas transit system simulator that will completely include the compressor station throughput simulator software as its part.

The presented mathematical model is useful in simulations of usual and/or unexpected system behavior. Such unusual situations could be reverse flow of gas or pipe failures. The model could serve also for checking whether the actual configuration of aggregates is the optimal one or some aggregates could be removed or stopped. Future work is required to add the pipes to the model to be able to simulate the whole transit system.

References

1. Eustream, a.s., Annual report, 2009.
2. D.J. Evans, R.A. Chadwick, *Underground Gas Storage: Worldwide Experiences and Future Development in the UK and Europe*, Geological Society of London, 2009.
3. L. Gyulai, Sz. Szabó, D.J. De Kock, J.A. Snyman, A study of the feasibility of using mathematical optimisation to minimize the temperature in a smelter pot room, *Journal of Building and Environment*, **42**(6):2268–2278, 2007.
4. K. Kolarčík, J. Kaminský, O. Pumpřla, *Compressors*, VŠB – Technical University of Ostrava, 2004 (in Czech).
5. W. Kostowski, The possibility of energy generation within the conventional natural gas transport system, *Strojárstvo*, **52**(4):429–440, 2010.
6. I. Leššo, P. Flegner, B. Pandula, P. Horovčák, New principles of process control in geotechnics by acoustic methods, *Metallurgija*, **46**(3):165–168, 2007.
7. J. Mačicová, P. Pilát, Emissions from the combustion of fast-growing willow, in *Machine Modeling and Simulations*, Scientific and Technical Society, University of Žilina, 2009, pp. 91–98.
8. N. Pataki et al., Features of C++ template metaprograms, in *Proceedings of the 8th International Conference on Applied Informatics, Eger, Hungary, January 27–30, 2010*, Vol. 2., pp. 451–451.
9. V. Peřková, M. Repášová, Progressive diagnostic methods in gas distribution industry, *Acta Metallurgica Slovaca*, **11**(1):271–276, 2005 (in Slovakian).
10. J. Porubán, M. Forgáč, M. Sabo, M. Běhálek, Annotation based parser generator, *Comput. Sci. Inf. Syst.*, **7**(2):291–307, 2010.
11. K. Potužák, *Gas Haulage and Distribution*, GAS s.r.o., 2001, p. 1116 (in Czech).
12. A. Svoboda, *Encyclopedia of Gas: 150 Years of Natural Gas in Czech and Moravia*, GAS s.r.o., 1997 (in Czech).
13. Cs. Szabó, R. Bazelides, L. Samuelis, Visualization and archiving of object-oriented tests, in *Computer Science and Technology Research Survey*, Vol. 4, 2010, pp. 169–174.
14. Cs. Szabó, T. Fesič, Database refactoring for GIS, in *Electrical Engineering and Informatics*, Faculty of Electrical Engineering and Informatics, Technical University of Košice, 2010, pp. 347–350.
15. A. Varga, *Gas Transport and Distribution*, 1st ed., Faculty of Metallurgy, Technical University of Košice, 1999 (in Slovakian).

16. Zs. Vaszi, A. Varga, Design and verification of the mathematical model for detecting the throughput of the compressor stations, *Acta Metallurgica Slovaca*, **15**(2):117–125, 2009.
17. P. Záhorský, Management of Slovak transit system, *Slovgas*, **2006**(1), http://www.szn.sk/slovgas/Casopis/2006/1/2006_1_04.pdf (in Slovakian).