View metadata, citation and similar papers at core.ac.uk
brought to you by CORE
provided by Nonlinear Analysis: Modelling and Control

Nonlinear Analysis: Modelling and Control, Vol. 20, No. 4, 469–486 ISSN 1392-5113
http://dx.doi.org/10.15388/NA.2015.4.1

Convergence analysis of estimated parameters for parametric nonlinear strict feedback system with unknown control direction

Jun Min Li, Chao He

School of Mathematics and Statistics, Xidian University,
Shaanxi, Xi'an, 710071, China

jmli@mail.xidian.edu.cn; xidianhechao@126.com

Received: August 4, 2013 / **Revised:** August 9, 2014 / **Published online:** July 27, 2015

Abstract. In this paper, the adaptive control and parameters identification problems are investigated for a class of linearly parametric strict feedback system with unknown control direction. Firstly, by using backstepping design procedure, the adaptive tracking control scheme combined with Nussbaum gain function is proposed. In the controller, the adaptive law of estimated parameters is derived from Lyapunov stability theorem and Nussbaum-type function. All the signals in closed-loop system are proved to be bounded. Secondly, the identification of unknown parameters in the strict feedback system with unknown control direction is studied. By constructing a novel Lyapunov function, a sufficient condition (PE condition), which can guarantee that the parameters estimation converge to the actual values of parameters, is obtained for the first time. Also, it is more simplified than the existing results on PE. Under the PE condition proposed here, it is shown that the parameters estimation errors are convergent to zero asymptotically by using Nussbaum function technique and Barbalat's lemma. Finally, illustrated examples are given to demonstrate the main results.

Keywords: unknown control direction, Barbalat lemma, Nussbaum gain, persistency excitation condition, convergence of estimated parameters.

1 Introduction

Since the early stages of control methods development, adaptive control of nonlinear systems has been an active area of research with the appearance of recursive backstepping design [7,8,9]. Backstepping is an effective method for adaptive nonlinear control because it can guarantee global stability and asymptotic tracking for parametric strict-feedback systems. These papers are proposed to tackle the nonlinear systems with unknown constant parameters and known control coefficients. In the last two decades, a great deal of attention has been paid to systems with unknown control directions [3, 12, 15, 16, 18, 20, 22]. One effective method is using the Nussbaum gain approach to remove the basic assumptions in many existing studies that the control direction is known and invariant.

© Vilnius University, 2015

[2, 4, 24, 26] extended the adaptive backstepping technique to parametric strict-feedback systems with unknown virtual control coefficients using Nussbaum gain. However, the identification problem of unknown parameters in the adaptive control for nonlinear strict feedback systems with unknown control direction has not been considered, yet.

Recently, the convergence problem of parameter estimation has been widely studied for adaptive systems. For many adaptive schemes, the relationship between parameter identification and the persistency of excitation condition (PE) has been widely studied. PE established a necessary (and sometimes sufficient) condition for parameter identification with the reference trajectory [6]. For large classes of systems, PE implied that tracking error converges to zero only when the adaptation law identifies the actual parameters [21]. Recently, PE was shown to be necessary and sufficient for uniform global asymptotic stability of a class of nonlinear systems that includes the manipulator dynamics [10, 11]. Ref. [14] proposed an adaptive control method for a class of nonlinear systems with unknown high-frequency gains, in which the tracking error and estimation error of unknown parameters were convergent to zero asymptotically with the assumption that the signs of unknown high-frequency gains are known. In [5], a simple PE condition was proposed with an adaptive backstepping controller for parametric strict-feedback systems. In more recent years, the parameters identification has been considered with consensus of multi-agents and synchronization of dynamical complex networks, chaotic networks. From this perspective, some novels results on parameters identification have been presented (see [1, 13, 17, 19, 23, 25]). However, for nonlinear strict-feedback system, the parameters identification problem is more complicated, due to the complex structure of system and complex controller design procedure. Moreover, the parameters identification problem becomes more practical in real world, with unknown control direction.

Inspired by the aforementioned discussions, in this paper, we address the control and parameters identification problem for linearly parametric strict feedback systems with unknown control direction. An adaptive controller is designed by using backstepping procedure and Nussbaum function technique, where the tracking error is proved to be convergent to zero asymptotically. By constructing a novel Lyapunov function, a simple PE condition for system functions is obtained to guarantee that the estimation of parameters is convergent to the actual values with rigorous analysis. Also, the boundedness of all signals in the closed-loop system is guaranteed. The main contributions of this paper are summarized as follows. 1) An adaptive controller for linearly parametric strict feedback system is proposed to force the system to track the specified trajectory, by using Nussbaum functions technique. 2) A novel Lyapunov function is constructed to analysis the convergence of parameters estimation for linearly parametric strict feedback system with unknown control direction for the first time, which is different with the works in [14]. 3) By rigorous analysis, a PE condition is derived, which is simpler than the existing results. As a result, the PE condition proposed in this paper can be used widely in convergence analysis of parameters with adaptive laws.

The rest of this paper is organized as follows. In Section 2, the problem and controller designing are introduced simply. Based on a novel Lyapunov function, Section 3 shows the convergence analysis of parameters identification under PE condition. In Section 4, an illustrated example is introduced to demonstrate the PE condition and convergence of estimated parameters. Finally, Section 5 concludes the paper. For notations convenience, $|\cdot|$ denotes a suitable norm.

2 Problem formulation and preliminary

2.1 System description and basic assumptions

In this section, we consider the following strict feedback system:

$$\begin{aligned} \dot{x}_i &= x_{i+1} + \theta^{\mathrm{T}} f_i(\bar{x}_i) \\ \dot{x}_n &= b_n u + \theta^{\mathrm{T}} f_n(x), \end{aligned} \tag{1}$$

where the trajectory of system is $x = [x_1, x_2, \dots, x_n]^{\mathrm{T}} \in R^n$ and $\bar{x}_i = [x_1, x_2, \dots, x_i]^{\mathrm{T}} \in R^i$. $f_i : R^i \to R^p$ ($1 \leqslant i \leqslant n$) is continuous known nonlinear functions vector, $u \in R$ is the input of control, $b_n \in R$ is the unknown control coefficient whose value and sign are completely unknown, $\theta \in R^p$ is the vector of unknown parameters.

The control objective is to design an adaptive controller to force system (1) such that x_1 tracks a reference trajectory x_r asymptotically and the adaptive law identifies the actual values of θ with PE condition which will be specified later.

For practical systems in real world, system functions may be completely unknown. To study this problem, the systems can be modeled as linearly parametric systems. In this paper, we consider the linearly parametric nonlinear strict feedback system. From the linearly parametric perspective, the unknown functions such as $f(x)$ can be represented as a unknown parameters vector multiplied by a known functions vector: $f(x) = \theta^{\mathrm{T}} \phi(x)$, where θ is a unknown parameters vector and $\phi(x)$ is a known functions vector. Therefore, the linearly parametric nonlinear strict feedback system is a widely used model. Parameters identification problem is a basic problem in the adaptive control. Although many adaptive control methods have been presented, that the estimation of parameters converges to the real value of unknown parameters is not guaranteed. In practical case, due to the disturbance of uncertainties or the bias of modeling, the control direction will be completely unknown. Therefore, the parameters identification method based on known control direction will be invalid. Nussbaum gain method can adapt the control direction on-line and resists the disturbance of variation on control direction. In this paper, a parameters identification problem is considered by using the Nussbaum gain method to adapt the variation of control direction and to overcome the limitation of identification problem presented before.

To show the main results, several assumptions are given as follows.

Assumption 1. The reference trajectory $x_r, x_r^{(i)}$, $i = 1, 2, \dots, n$, exist and $x_r^{(n)}$ is piecewise continuous. Then there is a positive constant B such that $\max_{0 \leqslant i \leqslant n}\{|x_r^{(i)}|_\infty\} \leqslant B$.

Assumption 2. (See [6].) For all $t \in R$, there exist a constant $T > 0$ and a known constant $\mu > 0$ such that $\int_{t-T}^{t} f_1(x_r(\tau)) f_1^{\mathrm{T}}(x_r(\tau))\,\mathrm{d}\tau \geqslant \mu I$, where I is the identity matrix.

Definition 1. (See [20].) A function $N(\xi)$, is called a Nussbaum-type function if it has the following properties:

$$\lim_{s\to\infty}\sup\frac{1}{s}\int_{s_0}^{s} N(\xi)\,\mathrm{d}\xi = +\infty,$$

$$\lim_{s\to\infty}\inf\frac{1}{s}\int_{s_0}^{s} N(\xi)\,\mathrm{d}\xi = -\infty.$$

For instance, in this paper, $N(\xi) = \xi^2\cos(\xi)$ is considered.

Lemma 1. (*See* [24].) *Let* $V(\cdot)$ *and* $\xi(\cdot)$ *be smooth functions defined on* $[0, t_f)$ *with* $V(t) \geqslant 0$ *for all* $t \in [0, t_f)$, *and* $N(\cdot)$ *be an even smooth Nussbaum-type function. If the following inequality holds*:

$$V(t) \leqslant c_0 + \int_0^t \big(bN(\xi) + 1\big)\dot{\xi}\,\mathrm{d}\tau \quad \forall t \in [0, t_f),$$

where b *is a nonzero constant and* c_0 *represent some suitable constant, then* $V(t)$, $\xi(t)$ *and* $\int_0^t (bN(\xi)+1)\dot{\xi}\,\mathrm{d}\tau$ *must be bounded on* $[0, t_f)$.

2.2 The backstepping design with Nussbaum gain

To study the identification problem of parameters in parametric strict feedback system, firstly we will give the control strategy. In this subsection, for system (1), a backstepping approach combined with Nussbaum gain technique is proposed.

As the traditional backstepping design procedure, introduce the new coordinates such that $z_1 = x_1 - x_r$, $z_i = x_i - \alpha_{i-1}$ with $i = 2, 3, \dots, n$.

Following [24], the virtual controllers are given as follows:

$$\alpha_1 = -k_1 z_1 - \hat{\theta}^{\mathrm{T}} f_1(x_1) + \dot{x}_r, \tag{2}$$

$$\alpha_i = -z_{i-1} - k_i z_i - \hat{\theta}^{\mathrm{T}}\omega_i + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}x_{j+1} + \sum_{j=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_r^{(j)}}x_r^{(j+1)} + \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}^{\mathrm{T}}}\Gamma\tau_i + \sum_{k=1}^{i-2} z_{k+1}\frac{\partial\alpha_k}{\partial\hat{\theta}^{\mathrm{T}}}\Gamma\omega_i, \tag{3}$$

where $\omega_1 = f_1(x_1)$, $\omega_i = f_i(\bar{x}_i) - \sum_{j=1}^{i-1}(\partial\alpha_{i-1}/\partial x_j) f_j(\bar{x}_j)$, $\tau_1 = \omega_1 z_1$, $\tau_i = \tau_{i-1} + \omega_i z_i$ $(i = 2, 3, \dots, n)$ and $\hat{\theta}$ is the estimation of θ.

For the nonlinear system with unknown control direction in (1), we design the actual controller as

$$u = N(\xi)\beta, \tag{4}$$

$$\beta = z_{n-1} + k_n z_n + \hat{\theta}^{\mathrm{T}}\omega_n - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}x_{j+1} - \sum_{j=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_r^{(j)}}x_r^{(j+1)} - \frac{\partial\alpha_{n-1}}{\partial\hat{\theta}^{\mathrm{T}}}\Gamma\tau_n - \sum_{k=1}^{n-2} z_{k+1}\frac{\partial\alpha_k}{\partial\hat{\theta}^{\mathrm{T}}}\Gamma\omega_n, \tag{5}$$

where $N(\xi)$ is an even smooth Nussbaum-type function.

From (2)–(5), the tracking error dynamic is:

$$\begin{aligned}
\dot{z}_1 &= -k_1 z_1 + z_2 + \tilde{\theta}^{\mathrm{T}} f_1(x_1),\\
\dot{z}_i &= -z_{i-1} - k_i z_i + z_{i+1} + \tilde{\theta}^{\mathrm{T}}\omega_i - \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}^{\mathrm{T}}}(\dot{\hat{\theta}} - \Gamma\tau_i) + \sum_{k=1}^{i-2} z_{k+1}\frac{\partial\alpha_k}{\partial\hat{\theta}^{\mathrm{T}}}\Gamma\omega_i,\\
&\vdots\\
\dot{z}_n &= \big(b_n N(\xi) + 1\big)\beta - z_{n-1} - k_n z_n + \tilde{\theta}^{\mathrm{T}}\omega_n - \frac{\partial\alpha_{n-1}}{\partial\hat{\theta}^{\mathrm{T}}}(\dot{\hat{\theta}} - \Gamma\tau_n)\\
&\quad + \sum_{k=1}^{n-2} z_{k+1}\frac{\partial\alpha_k}{\partial\hat{\theta}^{\mathrm{T}}}\Gamma\omega_n.
\end{aligned} \tag{6}$$

The control scheme will be clear if we give the adaptive law on the unknown estimated parameters. Here, we design the adaptive laws as

$$\dot{\hat{\theta}} = \Gamma\tau_n, \tag{7}$$

$$\dot{\xi} = \beta z_n. \tag{8}$$

Theorem 1. *Consider nonlinear system* (1) *with the virtual controller* (2), (3), *actual controller* (4), (5) *and adaptive laws* (7), (8). *Then, one has*:

1) *The tracking errors z_i $(i = 1, 2, \dots, n)$ are convergent to zero asymptotically.*
2) *There exist a positive constant Ω such that $V_n \leqslant \Omega$ for all $t \geqslant 0$. Furthermore, all the signals in the close-loop system, including ξ, the estimation of parameter $\hat{\theta}$, the control u and the trajectory of system* (1) *x are bounded in $t \in [0, \infty)$.*

Proof. Consider the Lyapunov function candidate as

$$V_n = \frac{1}{2}\sum_{i=1}^{n} z_i^2 + \frac{1}{2}\tilde{\theta}^{\mathrm{T}}\Gamma^{-1}\tilde{\theta}, \tag{9}$$

where $\tilde{\theta}$ is the estimation error defined as $\tilde{\theta} = \theta - \hat{\theta}$ and $\Gamma = \Gamma^{\mathrm{T}} > 0$.

As shown in [9], due to the "skew-symmetry" property in the error system (6), the derivate of (9) is calculated as follows:

$$\dot{V}_n \leqslant -\sum_{i=1}^{n} k_i z_i^2 + \left(\sum_{k=1}^{n-2} z_{k+1}\frac{\partial\alpha_k}{\partial\hat{\theta}^{\mathrm{T}}}\right)(\Gamma\tau_n - \dot{\hat{\theta}}) + \tilde{\theta}^{\mathrm{T}}\big(\tau_n - \Gamma^{-1}\dot{\hat{\theta}}\big) + \big(b_n \mathrm{N}(\xi) + 1\big)\beta z_n. \tag{10}$$

Substituting the adaptive laws (7), (8) into (10), results in

$$\dot{V}_n \leqslant -\sum_{i=1}^{n} k_i z_i^2 + \big(b_n \mathrm{N}(\xi) + 1\big)\beta z_n. \tag{11}$$

Integrate both sides of (11), we have

$$V_n(t) \leqslant V_n(0) - \sum_{j=1}^{n} k_j \int_0^t z_j^2 d\tau + \int_0^t \big(b_n \mathrm{N}(\xi) + 1\big)\dot{\xi}\, \mathrm{d}\tau. \tag{12}$$

For the positive property of $\sum_{j=1}^{n} k_j \int_0^t z_j^2\, \mathrm{d}\tau$, the following inequality is induced from (12):

$$V_n(t) \leqslant V_n(0) + \int_0^t \big(b_n \mathrm{N}(\xi) + 1\big)\dot{\xi}\, \mathrm{d}\tau. \tag{13}$$

According to Lemma 1, we can conclude that $V(t)$, $\xi(t)$ and $\int_0^t (b_n N(\xi) + 1)\dot{\xi}\, \mathrm{d}\tau$ are bounded for all $t \in [0, \infty)$. Then there exists a positive constant Ω such that $V_n(t) \leqslant \Omega$. Following (12), $\sum_{j=1}^{n} k_j \int_0^t z_j^2\, \mathrm{d}\tau$ is bounded for all $t \geqslant 0$. From the boundedness of $V(t)$ and $\xi(t)$, all the signals in close-loop system are bounded for all $t \in [0, \infty)$. Then, the derivative of z_i^2 $(i = 1, 2, \dots, n)$ is bounded. Based on the Barbalat's lemma, we have that $z_i \to 0$, $i = 1, 2, \dots, n$. □

3 Convergence analysis of parameters estimation

Throughout the discussion above, under the adaptive controller designed with Nussbaum gain, the identification problem of estimated parameters in equation (7) has not been considered till now. In this section, we will address the convergence problem of parameters estimation under the PE condition in Assumption 2 from the Lyapunov perspective.

To analyze the convergence of the adaptive law on the estimated parameters θ in (7) by using a Lyapunov method, some auxiliary functions are needed to construct the Lyapunov function. Firstly, define the following increasing functions:

$$\bar{f}_1(s) \triangleq \max\big\{\big|f_1(l)\big| \colon |l| \leqslant s\big\}, \tag{14}$$

$$\tilde{f}_1(s) \triangleq \sup\left\{\left|\frac{\mathrm{d}f_1(\sigma(t))}{\mathrm{d}t}\right|_\infty \colon \sigma(t) \in c^1,\ \max\big\{\big|\sigma(t)\big|_\infty, \big|\dot{\sigma}(t)\big|_\infty\big\} \leqslant s\right\}. \tag{15}$$

From (14), (15) and Assumption 1, it is easy to see that

$$\big|f_1(x_r)\big| \leqslant \bar{f}_1(B), \qquad \left|\frac{\mathrm{d}f_1(x_r(t))}{\mathrm{d}t}\right|_\infty \leqslant \tilde{f}_1(B). \tag{16}$$

According to Theorem 1, all the signals in close-loop system are bounded for $t \in [0, \infty)$. Then there exists positive constants M_i, $A_{i,j}$ such that

$$\left|f_i(\bar{x}_i)\right| \leqslant M_i, \qquad \left|\frac{\partial \alpha_i}{\partial x_j}\right| \leqslant A_{i,j} \tag{17}$$

with $i = 2, 3, \dots, n$ and $j = 1, 2, \dots, i$.

Based on the Theorem 1, the boundedness of all signals is guaranteed. Thus, it is easy to construct the following auxiliary functions by using equation (14)–(17):

$$\begin{aligned}
g_{0,1} &= \max\bigl\{2c_1\bar{f}_1(B), 2\tilde{f}_1(B)\bigr\},\\
g_{0,2} &= \bar{f}_1(B),\\
g_{1,1}(s) &= |\Gamma|\bar{f}_1(B)\bar{f}_1(\sqrt{2s}+B),\\
g_{1,2}(s) &= |\Gamma|\bar{f}_1(B)\bigl[M_2 + A_{1,1}\bar{f}_1(\sqrt{2s}+B)\bigr],\\
g_{1,i}(s) &= |\Gamma|\bar{f}_1(B)\left[M_i + \sum_{j=2}^{i-1} A_{i-1,j}M_j + A_{i-1,1}\bar{f}_1(\sqrt{2s}+B)\right],\\
g_{2,1}(s) &= \sqrt{2sp}\sup_{|q|\leqslant\sqrt{2l}+B}\left\{\max_j\left|\frac{\partial f_{1,j}(q)}{\partial q}\right|\right\}\bar{f}_1(B),\\
g_{3,1}(s) &= |\Gamma|T\bar{f}_1^2(B)\bar{f}_1(\sqrt{2s}+B),\\
g_{3,2}(s) &= |\Gamma|T\bar{f}_1^2(B)\bigl[M_2 + A_{1,1}\bar{f}_1(\sqrt{2s}+B)\bigr],\\
g_{3,i}(s) &= |\Gamma|T\bar{f}_1^2(B)\left[M_i + \sum_{j=2}^{i-1} A_{i-1,j}M_j + A_{i-1,1}\bar{f}_1(\sqrt{2s}+B)\right],\\
g_{4,1}(s) &= \frac{nT}{2\mu}\left\{\bigl[g_{0,1} + g_{2,1}(s) + g_{3,1}(s)\bigr]^2 + \sum_{j=2}^{n} g_{1,j}^2(s)\right\} + g_{1,1}(s) + k,\\
g_{4,2}(s) &= \frac{nT}{2\mu}\bigl[g_{0,2} + g_{3,2}(s)\bigr]^2 + \frac{\mu}{2nT},\\
g_{4,i}(s) &= \frac{nT}{2\mu}g_{3,i}^2(s) + \frac{\mu}{2nT}
\end{aligned}$$

with $3 \leqslant i \leqslant n$ and $k = \min\{k_1, k_2, \dots, k_n\}$.

Synthesizing the increasing functions proposed above, the components of Lyapunov function can be obtained:

$$V_{a,1} = \tilde{\theta}^{\mathrm{T}} f_1(x_r) z_1, \tag{18}$$

$$V_{a,2} = \frac{1}{T}\tilde{\theta}^{\mathrm{T}} \int_{t-T}^{t}\int_{m}^{t} f_1\bigl(x_r(\tau)\bigr) f_1^{\mathrm{T}}\bigl(x_r(\tau)\bigr)\,\mathrm{d}\tau\,\mathrm{d}m\tilde{\theta}, \tag{19}$$

$$V_a = -V_{a,1} + V_{a,2}. \tag{20}$$

According to Theorem 1, there exists a positive constant $\Omega > 0$ such that $V_n \leqslant \Omega$ for all $t \in [0, \infty)$. Considering the parameters estimation problem based on the adaptive law in (7), we have the following theorem.

Theorem 2. *Under Assumptions* 1–2*, consider the nonlinear system* (1) *together with controllers* (4), (5) *and adaptive laws* (7), (8). *Construct a Lyapunov function such as*

$$V(z_1, z_2, \ldots, z_n, \tilde{\theta}) = V_a + \left[\frac{1}{k}\sum_{i=1}^{n} g_{4,i}(\Omega) + b\bar{f}_1(B) + 1\right] V_n,$$

where b is a constant defined as $b = \max\{1, 1/\lambda_{\min}(\Gamma^{-1})\}$. Then, the following properties hold:

1) *There exist functions $\alpha_1, \alpha_2 \in K_\infty$ such that*
$$\alpha_1\bigl(|z_1, z_2, \ldots, z_n, \tilde{\theta}|\bigr) \leqslant V \leqslant \alpha_2\bigl(|z_1, z_2, \ldots, z_n, \tilde{\theta}|\bigr).$$
2) *Under the PE condition in Assumption* 2*, the tracking error z_i and estimation error of parameters $\tilde{\theta}$ are convergent to zero asymptotically, when $t \to \infty$.*

Proof. 1) From the Lyapunov function (9) in Theorem 1 and the function definition in (14), using Young's inequality, results in

$$\left|\tilde{\theta}^{\mathrm{T}} f_1(x_r) z_1\right| \leqslant b\bar{f}_1(B) V_n. \tag{21}$$

Since $V_{a,2}$ in (19) and $\sum_{i=1}^{n} g_{4,i}(\Omega)$ are nonnegative, one can conclude that

$$V = V_a + b\bar{f}_1(B)V_n + \frac{1}{k}\sum_{i=1}^{n} g_{4,i}(\Omega)V_n + V_n \geqslant \frac{1}{2}\sum_{i=1}^{n} z_i^2 + \lambda_{\min}(\Gamma^{-1})|\tilde{\theta}|^2. \tag{22}$$

Based on the integral mean value theorem and inequalities (16), equation (19) has the following property:

$$\left|\tilde{\theta}^{\mathrm{T}} \int\limits_{t-T}^{t} \int\limits_{m}^{t} f_1\bigl(x_r(\tau)\bigr) f_1^{\mathrm{T}}\bigl(x_r(\tau)\bigr)\,\mathrm{d}\tau\,\mathrm{d}m\tilde{\theta}\right| \leqslant \frac{T^2}{2}\bar{f}_1^2(B)\tilde{\theta}^{\mathrm{T}}\tilde{\theta}. \tag{23}$$

Using (21) and (23), results in

$$\begin{aligned}
V &\leqslant \bar{f}_1(B)|\tilde{\theta}||z_1| + \frac{T}{2}\bar{f}_1^2(B)\tilde{\theta}^{\mathrm{T}}\tilde{\theta} + \frac{1}{k}\sum_{i=1}^{n} g_{4,i}(\Omega)V_n + b\bar{f}_1(B)V_n + V_n \\
&\leqslant \left[2b\bar{f}_1(B) + bT\bar{f}_1^2(B) + \frac{1}{k}\sum_{i=1}^{n} g_{4,i}(\Omega) + 1\right] V_n \\
&\leqslant \left[2b\bar{f}_1(B) + bT\bar{f}_1^2(B) + \frac{1}{k}\sum_{i=1}^{n} g_{4,i}(\Omega) + 1\right] \\
&\quad \times \left(\frac{1}{2}\sum_{i=1}^{n} z_i^2 + \lambda_{\max}\bigl(\Gamma^{-1}\bigr)\tilde{\theta}^{\mathrm{T}}\tilde{\theta}\right).
\end{aligned} \tag{24}$$

From (22) and (24), it is easy to see that there exist functions $\alpha_1, \alpha_2 \in K_\infty$ such that

$$\alpha_1\big(|z_1, z_2, \ldots, z_n, \tilde{\theta}|\big) \leqslant V \leqslant \alpha_2\big(|z_1, z_2, \ldots, z_n, \tilde{\theta}|\big).$$

2) Similarly to the proof in [5], using the auxiliary functions (14)–(17) and Young's inequality, the derivate of V_a along with the tracking error dynamics (6) is

$$\begin{aligned}
\dot{V}_a &\leqslant \frac{nT}{2\mu}\big[g_{0,1} + g_{2,1}(V_n) + g_{3,1}(V_n)\big]^2 z_1^2 + \frac{\mu}{2nT}\tilde{\theta}^{\mathrm{T}}\tilde{\theta} \\
&\quad + \frac{nT}{2\mu}\big[g_{0,2} + g_{3,2}(V_n)\big]^2 z_2^2 + \frac{\mu}{2nT}\tilde{\theta}^{\mathrm{T}}\tilde{\theta} + \frac{nT}{2\mu}\sum_{i=3}^{n} g_{3,i}^2(V_n) z_i^2 \\
&\quad + (n-2)\frac{\mu}{2nT}\tilde{\theta}^{\mathrm{T}}\tilde{\theta} + g_{1,1}(V_n) z_1^2 + \frac{nT}{2\mu} g_{1,2}^2(V_n) z_1^2 + \frac{\mu}{2nT} z_2^2 \\
&\quad + \frac{nT}{2\mu}\sum_{i=3}^{n} g_{1,i}^2(V_n) z_1^2 + \frac{\mu}{2nT}\sum_{i=3}^{n} z_i^2 - \frac{\mu}{T}\tilde{\theta}^{\mathrm{T}}\tilde{\theta} \\
&\leqslant \sum_{i=1}^{n} g_{4,i}(V_n) z_i^2 - \frac{\mu}{2T}\tilde{\theta}^{\mathrm{T}}\tilde{\theta}.
\end{aligned} \tag{25}$$

Due to the monotonicity of functions $g_{4,i}(\cdot)$ $(i = 1, 2, \ldots, n)$, (25) can be rewritten as

$$\dot{V}_a \leqslant \sum_{i=1}^{n} g_{4,i}(\Omega) z_i^2 - \frac{\mu}{2T}\tilde{\theta}^{\mathrm{T}}\tilde{\theta}, \tag{26}$$

where Ω is a positive constant such that $V_n \leqslant \Omega$, which has been proved in Theorem 1.

Synthesizing (11), (26), one gets

$$\begin{aligned}
\dot{V} &= \dot{V}_a + \left[\frac{1}{k}\sum_{i=1}^{n} g_{4,i}(\Omega) + b\bar{f}_1(B) + 1\right]\dot{V}_n \\
&\leqslant \sum_{i=1}^{n} g_{4,i}(\Omega) z_i^2 - \frac{\mu}{2T}\tilde{\theta}^{\mathrm{T}}\tilde{\theta} \\
&\quad + \left[\frac{1}{k}\sum_{i=1}^{n} g_{4,i}(\Omega) + b\bar{f}_1(B) + 1\right]\left(-\sum_{j=1}^{n} k_j z_j^2 + \big(b_n \mathrm{N}(\xi) + 1\big)\dot{\xi}\right) \\
&\leqslant -\sum_{j=1}^{n} k_j z_j^2 - \frac{\mu}{2T}\tilde{\theta}^{\mathrm{T}}\tilde{\theta} + \left[\frac{1}{k}\sum_{i=1}^{n} g_{4,i}(\Omega) + b\bar{f}_1(B) + 1\right]\big(b_n \mathrm{N}(\xi) + 1\big)\dot{\xi} \\
&\leqslant -\sum_{j=1}^{n} k_j z_j^2 - \frac{\mu}{2T}\tilde{\theta}^{\mathrm{T}}\tilde{\theta} + g_0\big(b_n \mathrm{N}(\xi) + 1\big)\dot{\xi},
\end{aligned} \tag{27}$$

where g_0 is a constant defined as $g_0 \triangleq (1/k)\sum_{i=1}^{n} g_{4,i}(\Omega) + b\bar{f}_1(B) + 1$.

Integrate both sides of (27), yields

$$V(t) \leqslant V(0) - \sum_{j=1}^{n} k_j \int_0^t z_j^2 \,\mathrm{d}\tau - \frac{\mu}{2T} \int_0^t \tilde{\theta}^{\mathrm{T}} \tilde{\theta} \,\mathrm{d}\tau + \int_0^t g_0\big(b_n \mathrm{N}(\xi) + 1\big)\dot{\xi} \,\mathrm{d}\tau \leqslant V(0) + \int_0^t g_0\big(b_n \mathrm{N}(\xi) + 1\big)\dot{\xi} \,\mathrm{d}\tau. \tag{28}$$

Based on Lemma 1, one can conclude that $V(t)$, ξ and $\int_0^t g_0(b_n \mathrm{N}(\xi) + 1)\dot{\xi} \,\mathrm{d}\tau$ are bounded for all $t \geqslant 0$. From (28), one has that $\sum_{j=1}^{n} k_j \int_0^t z_j^2 \,\mathrm{d}\tau$ and $(\mu/(2T)) \int_0^t \tilde{\theta}^{\mathrm{T}} \tilde{\theta} \,\mathrm{d}\tau$ are bounded for all $t \in [0, \infty)$ and the derivate of $\tilde{\theta}^{\mathrm{T}} \tilde{\theta}$ is $\mathrm{d}(\tilde{\theta}^{\mathrm{T}} \tilde{\theta})/\mathrm{d}t = -2\tilde{\theta}^{\mathrm{T}} \Gamma \tau_n$. Due to the boundedness of $V(t)$, we can conclude that all the signals in close-loop system are bounded. Then, $\mathrm{d}(\tilde{\theta}^{\mathrm{T}} \tilde{\theta})/\mathrm{d}t$ is bounded for all $t \geqslant 0$. Using the Barbalat's lemma, we can conclude that $z_i \to 0$ and $|\tilde{\theta}| \to 0$, when $t \to \infty$. □

4 Simulation study

In this section, we demonstrate the effectiveness of approach proposed in this paper. Actually, linearly parametric strict feedback system is widely used such as the pendulum system. In this section, we consider a more complex third-order nonlinear system as following:

$$\begin{aligned} \dot{x}_1 &= x_2 + \theta x_1, \\ \dot{x}_2 &= x_3 + \theta x_1 \sin x_2, \\ \dot{x}_3 &= bu + \theta x_2 x_3, \end{aligned} \tag{29}$$

where θ, b are unknown constants. $f_1(x_1) = x_1$, $f_2(\bar{x}_2) = x_1 \sin x_2$, $f_3(x) = x_2 x_3$. The reference trajectory is $x_r = \sin(t)$. Obviously, Assumption 1 holds.

Defining $z_1 = x_1 - x_r$,$z_2 = x_2 - \alpha_1$ and $z_3 = x_3 - \alpha_2$, from (2), the virtual controller is designed as

$$\begin{aligned} \alpha_1 &= -k_1 z_1 - \hat{\theta} f_1(x_1) + \dot{x}_r, \\ \alpha_2 &= -z_1 - k_2 z_2 - \hat{\theta}\omega_2 + \frac{\partial \alpha_1}{\partial x_1} x_2 + \sum_{j=0}^{1} \frac{\partial \alpha_1}{\partial x_r^{(j)}} x_r^{(j+1)} + \frac{\partial \alpha_1}{\partial \hat{\theta}} \Gamma \tau_2. \end{aligned}$$

According to (4) and (5), the actual controller is

$$\begin{aligned} u &= N(\xi)\beta, \\ \beta &= z_2 + k_3 z_3 + \hat{\theta}\omega_3 - \sum_{j=1}^{2} \frac{\partial \alpha_2}{\partial x_j} x_{j+1} - \sum_{j=0}^{2} \frac{\partial \alpha_2}{\partial x_r^{(j)}} x_r^{(j+1)} - \frac{\partial \alpha_2}{\partial \hat{\theta}} \Gamma \tau_3 - z_2 \frac{\partial \alpha_1}{\partial \hat{\theta}} \Gamma \omega_3, \end{aligned}$$

where $\omega_1 = f_1(x_1)$, $\omega_2 = f_2(\bar{x}_2) - (\partial\alpha_1/\partial x_1)f_1(x_1)$, $\omega_3 = f_3(x) - \sum_{j=1}^{2}(\partial\alpha_2/\partial x_j)f_j(\bar{x}_j)$, $\tau_1 = \omega_1 z_1$, $\tau_2 = \tau_1 + \omega_2 z_2$, $\tau_3 = \tau_2 + \omega_3 z_3$. From the adaptive laws (7), (8), the estimated algorithm on unknown parameter are designed as $\dot{\hat{\theta}} = \Gamma\tau_3$, $\dot{\xi} = \beta z_3$. We choose control coefficients $k_1 = 1$, $k_2 = 1$, $k_3 = 1$, $\Gamma = 0.2$.

It is easily seen that there exist constants $T = 2\pi$ and $0 < \mu < \pi$ such that $\int_{t-2\pi}^{t} x_r^2(\tau)\,\mathrm{d}\tau \geqslant \mu$ for all $t \in R$. Thus, Assumption 2 holds. As shown in Theorem 1, under Assumptions 1 and 2, we have proved that the estimation error of parameters is convergent to zero for system (1) with the controller (4), (5) and adaptive laws (7), (8).

Without variation of control direction, we choose $\theta = 2$ and $b = -1$ for all $t \in [0, \infty)$. Figure 1 shows the curves of x_1, x_r and curves of x_2, α_1. While, the curves of x_3, α_2 and the convergence of estimation of θ is presented in Fig. 2. In Fig. 3, the variation of ξ is given. In Figs. 1 and 2, the convergence of z_1, z_2 and z_3 is shown, respectively. From Fig. 2, it is easily seen that the estimation of θ is convergent to the actual value of θ without variation of control direction.

To test the effectiveness of Nussbaum gain method, we try several times control direction switches. Let $\theta = 2$ and $b = -1$ for all $t \in (0s, 20s]$, $b = 1$ for all $t \in (20s, 40s]$, $b = -1$ for all $t \in (40s, 60s]$, $b = 1$ for all $t \in (60s, 80s]$, we have the following simulation results. Figure 4 shows the curves of x_1, x_r and curves of x_2, α_1. Curves of x_3, α_2 and the convergence of estimation of θ is shown in Fig. 5. Also, Fig. 6 gives the variation of ξ. From Figs. 4–6, we have that the Nussbaum gain method can adapt the control direction effectively. In Fig. 5, the convergence of estimation of θ is guaranteed with the switches of control direction.

In this paper, θ is a vector of unknown parameters. It means that there should be some different θ_i in the plant system. From the main conclusion in this paper, it is easily seen that the estimations of unknown parameters in the plant system which satisfies

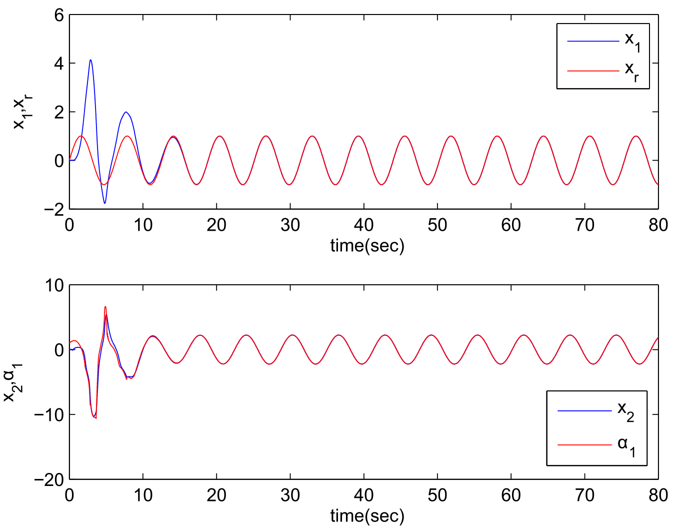

Fig. 1. Curves of x_1, x_r and x_2, α_1.

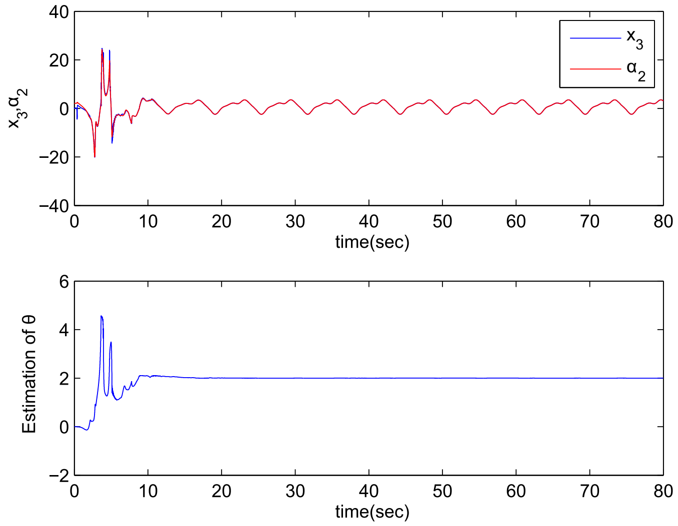

Fig. 2. Curves of x_3, α_2 and $\hat{\theta}$.

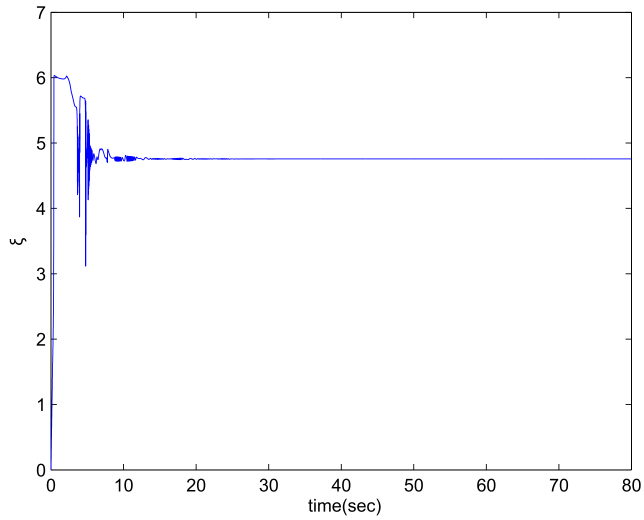

Fig. 3. Curves of ξ.

Assumptions 1 and 2 (PE condition) can converge to the actual value of the estimated parameters, with unknown control direction. Although simulation example (29) illustrates the identification problem for strict feedback nonlinear system with unknown control direction, the conservatism of the PE condition in Assumption 2 is not discussed. Especially, when the PE condition in Assumption 2 does not hold, the convergence of the estimation of unknown parameters is not considered. Therefore, we give the following

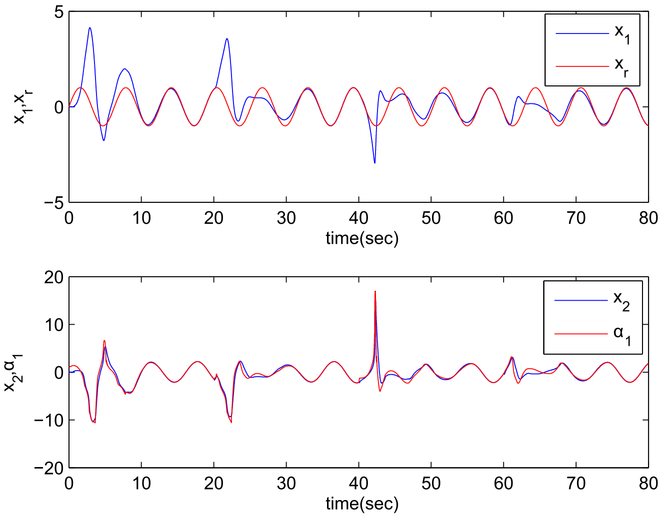

Fig. 4. Curves of x_1, x_r and x_2, α_1 with switches of control direction.

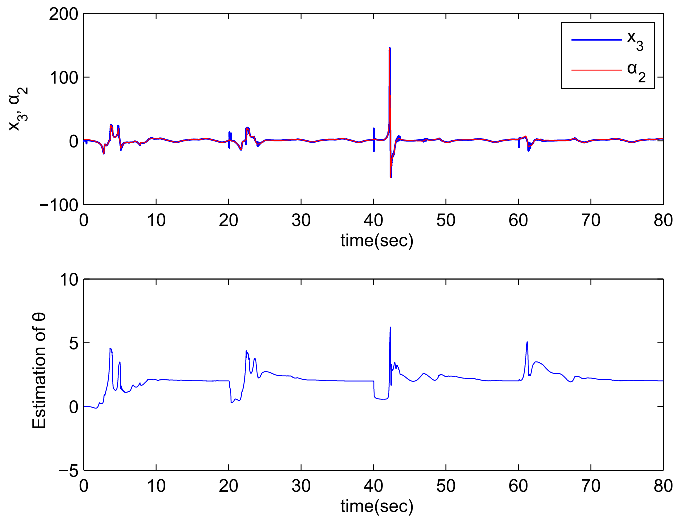

Fig. 5. Curves of x_3, α_2 and $\hat{\theta}$ with switches of control direction.

example:

$$
\begin{aligned}
\dot{x}_1 &= x_2 + \theta_1 x_1 + \theta_2 x_1^2,\\
\dot{x}_2 &= x_3 + \theta_3 x_1 \sin x_2,\\
\dot{x}_3 &= bu + \theta_4 x_2 x_3,
\end{aligned}
\tag{30}
$$

where θ_1, θ_2, θ_3 and θ_4 are unknown parameters. b denotes the unknown control gain.

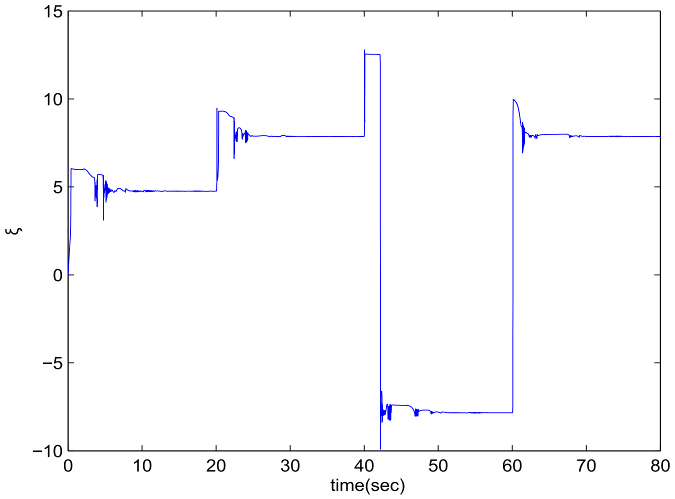

Fig. 6. Curves of ξ with switches of control direction.

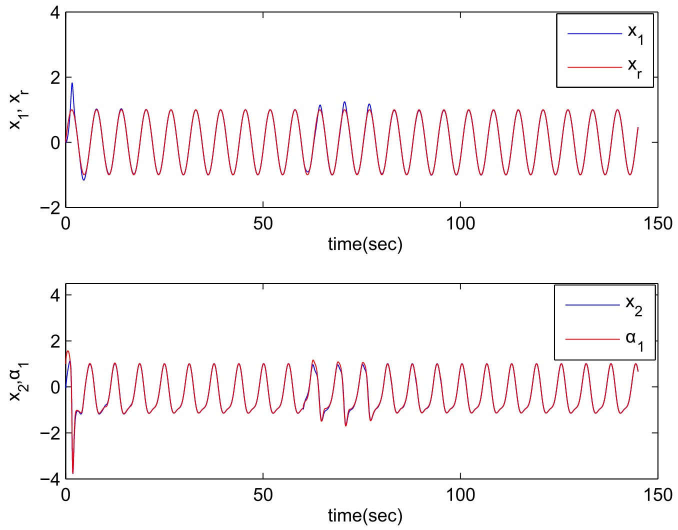

Fig. 7. Curves of x_1, x_r and x_2, α_1.

From (1), $\theta = [\theta_1, \theta_2, \theta_3, \theta_4]^{\mathrm{T}}$, $f_1(x_1) = [x_1, x_1^2, 0, 0]^{\mathrm{T}}$, $f_2(\bar{x}_2) = [0, 0, x_1 \sin x_2, 0]^{\mathrm{T}}$ and $f_3(x) = [0, 0, 0, x_2 x_3]^{\mathrm{T}}$.

Considering Assumption 2 (PE condition), yields

$$\int\limits_{t-T}^{t} f_1(x_r) f_1^{\mathrm{T}}(x_r)\,\mathrm{d}\tau = \begin{bmatrix} \int_{t-T}^{t} \phi(x_r)\phi^{\mathrm{T}}(x_r)\,\mathrm{d}\tau & 0 \\ 0 & 0 \end{bmatrix},$$

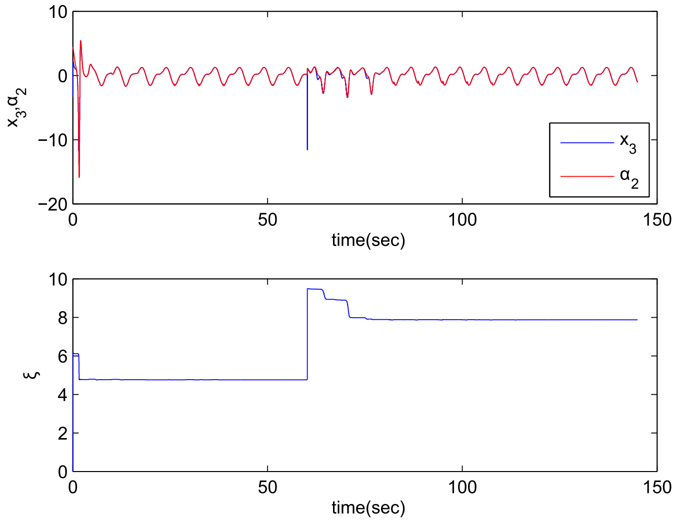

Fig. 8. Curves of x_3, α_2 and ξ.

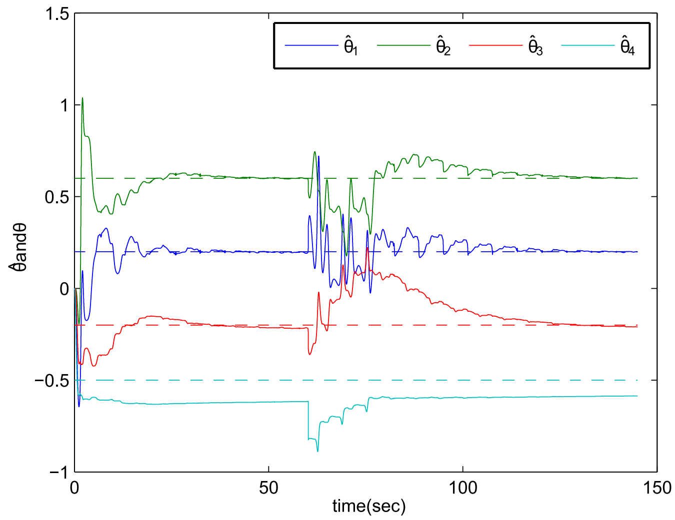

Fig. 9. Curves of $\hat{\theta}$ (solid line) and θ (dashed line).

where $\phi(x_1) = [x_1, x_1^2]^{\mathrm{T}}$. Obviously, PE condition does not hold for plant system (30). Therefore, the identification of unknown parameters vector θ is not guaranteed. However, we can easily conclude that the estimations of unknown parameters θ_1 and θ_2 corresponding to $\phi(x_1)$ converge to its actual values in plant system with $\phi(x_1)$ satisfying Assumption 2. Actually, we can construct the Lyapunov function in (18) and (19) by using $\phi(x_r)$ and $\tilde{\Theta} = [\tilde{\theta}_1, \tilde{\theta}_2]^{\mathrm{T}}$ instead of $f_1(x_r)$ and $\tilde{\theta}$. Under PE condition on $\phi(x_1)$, following Theorem 2, yields $\dot{V} \leqslant -\sum_{j=1}^{n} k_j z_j^2 - (\mu/(2T))\tilde{\Theta}^{\mathrm{T}}\tilde{\Theta} + g_0(bN(\xi)+1)\dot{\xi}$. Thus, $\tilde{\theta}_1$ and $\tilde{\theta}_2$ converge to zero.

To illustrate this property, we assume that $\theta_1 = 0.2$, $\theta_2 = 0.6$, $\theta_3 = -0.2$, $\theta_4 = -0.5$ and reference signal is $x_r = \sin t$. With the reference signal, it is easily seen that there exist constants $T = 2\pi$ and $\mu = (3/4)\pi$ such that $\int_{t-T}^{t} \phi(x_r)\phi^{\mathrm{T}}(x_r)\,\mathrm{d}\tau \geqslant \mu I$. The control direction is $b = -1$ for all $t \in [0s, 60s)$ and we give a switch at $t = 60s$, then $b = 1$ for all $t \in [60s, 145s)$. Following the controller design procedure in (2)–(5), adaptive laws (7) and (8), choose the control coefficients: $k_1 = 2$, $k_2 = 2$, $k_3 = 2$, $\Gamma = \mathrm{diag}(1,1,1)$ and the simulation results are shown in Figs. 7–9, respectively. Figure 7 shows the curves of x_1 and x_r and curves of x_2, α_1. The variation of ξ and curves of x_3 and α_2 is presented in Fig. 8. The estimation of unknown parameters: $\hat{\theta}_1$, $\hat{\theta}_2$, $\hat{\theta}_3$, $\hat{\theta}_4$ are shown with solid line in Fig. 9, while the actual value of θ is presented with dashed line. From Fig. 9, the estimation: $\hat{\theta}_1$, $\hat{\theta}_2$ can converge to the actual value of θ_1, θ_2 with variation of control direction. However, $\hat{\theta}_4$ does not converge to θ_4. Also, $\hat{\theta}_3$ can achieve the actual value θ_3, which can not be concluded from Theorem 2.

5 Conclusions

In this paper, based on Nussbaum function technique and backstepping design procedure, an adaptive controller for strict feedback system with unknown control direction has been proposed. It has been proved that the tracking error is asymptotically convergent to zero. Based on the conclusion above, the parameters identification problem has been studied. A new simple PE condition has been obtained with a novel Lyapunov function. Under the PE condition, the parameters estimations have been proved, by using the property of Nussbaum function and Barbalat's lemma, to be asymptotically convergent to the actual values of parameters. Finally, simulation examples have been given to show the effectiveness of controller and the conservatism of PE condition.

Acknowledgment. The authors thank the associate editor and the anonymous reviewers for their expertise comments to improve the quality of our work.

References

1. W.S. Chen, C.Y. Wen, S.Y. Hua, et al., Distributed cooperative adaptive identification and control for a group of continous-time systems with a cooperative PE condition via consensus, *IEEE Trans. Autom. Control*, **59**(1):91–106, 2014.

2. Z. Ding, Adaptive control of nonlinear systems with unknown virtual control coefficients, *Int. J. Adapt. Control Signal Process.*, **14**(5):505–517, 2000.

3. M. Fu, B. Barmish, Adaptive stabilization of linear systems via switching control, *IEEE Trans. Autom. Control*, **31**(12):1097–1103, 1986.

4. S.S. Ge, J. Wang, Robust adaptive tracking for time-varying uncertain nonlinear systems with unknown control coefficients, *IEEE Trans. Autom. Control*, **48**(8):1463–1469, 2003.

5. B.L. He, J.M. Li, Systematic method of parameter convergence in strict-feedback nonlinear systems, *J. Syst. Sci. Math. Sci.*, **31**(5):501–511, 2011.
6. P.A. Ioannou, J. Sun, *Robust Adaptive Control*, Prentice Hall, Upper Saddle River, NJ, 1996.
7. I. Kanellakopoulos, P.V. Kokotovic, A.S. Morse, Systematic design of adaptive controllers for feedback linearizable systems, *IEEE Trans. Autom. Control*, **148**(11):1241–1253, 1991.
8. M. Kristic, I. Kauellakopoulos, P.V. Kokotovic, Adaptive nonlinear control without over parameterization, *Syst. Control Lett.*, **19**(3):177–185, 1992.
9. M. Krstic, I. Kauellakopoulos, P.V. Kokotovic, *Nonlinear and Adaptive Control Design*, John Wiley & Sons, New York, 1995.
10. A. Loría, R. Kelly, A.R. Teel, et al., Uniform parametric convergence in the adaptive control of mechanical systems. Discussion and Commentary, *Eur. J. Control*, **11**(2):87–111, 2005.
11. A. Loría, E. Panteley, D. Popovic, et al., δ-persistency of excitation: A necessary and sufficient condition for uniform attractivity, *Proceedings of the 41st IEEE Conference on Decision and Control, Las Vegas, NV, December 10–13, 2002*, IEEE, 2002, pp. 3506–3511.
12. R. Lozano, B. Brogliato, Adaptive control of a simple nonlinear system without a priori information on the plant parameters, *IEEE Trans. Autom. Control*, **37**(1):30–37, 1992.
13. Y.P. Luo, Y.C. Hung, Control synchronization and parameter identification of two different chaotic systems, *Nonlinear Dyn.*, **73**(3):1507–1513, 2013.
14. F. Mazenc, M. Malisoff, M. De Queiroz, Uniform global asymptotic stability of adaptive cascaded nonlinear systems with unknown high-frequency gains, *Nonlinear Anal., Theory Methods Appl.*, **74**(4):1132–1145, 2011.
15. B. Mårtensson, The order of any stabilizing regulator is sufficient a priori information for adaptive stabilization, *Syst. Control Lett.*, **6**(2):87–91, 1985.
16. B. Mårtensson, Remarks on adaptive stabilization of first-order adaptive systems, *Syst. Control Lett.*, **14**(1):1–7, 1990.
17. J. Mei, M.H. Jiang, B. Wang, et al., Finite-time parameter identification and adaptive synchronization between two chaotic neural networks, *J. Franklin Inst.*, **350**(6):1617–1633, 2013.
18. D. Mudgett, A.S. Morse, Adaptive stabilization of linear systems with unknown high frequency gains, *IEEE Trans. Autom. Control*, **30**(6):549–554, 1985.
19. J. Na, X.M. Ren, Y.Q. Xia, Adaptive parameter identification of linear SISO systems with unknown time-delay, *Syst. Control Lett.*, **66**(1):43–50, 2014.
20. R.D. Nussbaum, Some remarks on a conjecture in parameter adaptive control, *Syst. Control Lett.*, **3**(5):243–246, 1963.
21. J.J.E. Slotine, W. Li, Adaptive robot control: A new perspective, in *Proceedings of the 26th IEEE Conference On Decision And Control, Los Angeles, CA, December 9–11, 1987*, IEEE, 1987, pp. 192–198.
22. D.A. Suarez, R. Lozano, Adaptive control of nonminimum phase systems subject to unknown bounded disturbances, *IEEE Trans. Automa. Control*, **41**(12):1830–1836, 1996.

23. X.M. Wu, Z.Y. Sun, F. Liang, et al., Online estimation of unknown delays and parameters in uncertain time delayed dynamical complex networks via adaptive observer, *Nonlinear Dyn.*, **73**(3):1753–1768, 2013.

24. X.D. Ye, J.P. Jiang, Adaptive nonlinear design without a priori knowledge of control directions, *IEEE Trans. Autom. Control*, **43**(11):1617–1621, 1998.

25. Q.J. Zhang, J. Luo, L. Wan, Parameter identification and synchronization of uncertain general complex networks via adaptive-impulsive control, *Nonlinear Dyn.*, **71**(1–2):353–359, 2013.

26. Y. Zhang, C. Wen, Y.C. Soh, Adaptive backstepping control design for systems with unknown high-frequency gain, *IEEE Trans. Autom. Control*, **45**(12):2350–2354, 2000.